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Essays on Industrial Policy and Innovation in an Open Economy

By

SEUNGJIN BAEK  
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**Abstract**

Just like people, industries have a lifecycle. My dissertation explores how trade and industrial policy affect innovation and welfare across the industry lifecycle—often called the “product cycle.” Recently, new frontier technologies such as artificial intelligence, robotics, and green energy technologies have been rapidly emerging in what Klaus Schwab has labelled the ‘Fourth Industrial Revolution’ (Schwab, 2017). This dissertation suggests that by taking into account the industry lifecycle, policy can have a more significant impact on the welfare of not only the implementing country but also its counterparts, compared to a time when new industries have become well-established and mature.

In the first chapter, I develop a simple open economy model that incorporates productivity dynamics suggested by the industry lifecycle. In this lifecycle, productivity is low and does not grow significantly in the early stage, then after a radical innovation, it grows very fast for a while before tapering off. The model suggests important policy implications. First, considering industry lifecycle when designing industrial policy is important since the growth potential and degree of externality vary depending on the stage of the targeted industry’s lifecycle. Second, policymakers need to take into account the difference in timing when policy costs and benefits occur. The model shows that industrial policy reduces instantaneous utility in the short run due to distortions created by the policy, but it can increase overall welfare by accelerating innovation in the targeted industry in the long run. Third, home industrial policy can increase foreign welfare through the terms-of-trade effect, meaning the foreign country can benefit from the lower home product price due to home innovation.

In the second chapter, I present a general framework for analyzing the welfare effects of industrial policy when a country is hastening to catch up to the technological frontier, versus racing to create new technologies. The model in this chapter, which incorporates industry lifecycle theory into an open economy macroeconomic model by Corsetti et al. (2007), provides distinct welfare implications in two scenarios: *catch-up* and *frontier technology races*. In the former scenario, the targeted industry is nascent with high growth potential at home, but mature abroad. In contrast,

in the latter scenario, both the home and foreign industries have high growth potential and are in competition with each other. For the home country, a production subsidy accelerates innovation in the targeted industry and thus can enhance welfare in both scenarios, despite a trade-off between short-term losses and long-term gains. For the foreign country, in the catch-up scenario, a home production subsidy unambiguously increases foreign welfare. Conversely, in the scenario of frontier technology races, it may induce a beggar-thy-neighbor effect by delaying innovation abroad. In such circumstances, the foreign country responds by implementing aggressive countervailing policies to mitigate the negative spillover effects. If both countries instead cooperatively support the industry, the welfare outcome is a Pareto improvement compared to the Nash equilibrium.

In the third chapter, I explore the reasons why many countries support industries essential for transitioning to a green economy, despite the cost of converting to green energy and the opportunities for free-riding on other countries' carbon abatement. By incorporating the negative externalities from greenhouse gas emissions into the open-economy macroeconomic model developed in Chapter 2, I analyze the welfare effects of industrial policies that subsidize production of capital goods (like solar panels or wind turbines) used to produce green energy. The model predicts that a production subsidy for the green capital goods industry is desirable for the home country, as it accelerates innovation in the industry and consequently green energy adoption. This acceleration at home delays innovation abroad, generating a beggar-thy-neighbor effect, despite the environmental benefits from home innovation. Thus, in a Nash equilibrium, both nations competitively raise production subsidies, improving welfare in both countries by reducing distortions created by the subsidy and greenhouse gas emissions. A cooperative equilibrium still yields a Pareto improvement, given the incomplete resolution of the free-riding problem in the Nash equilibrium. To quantitatively analyze the welfare and environmental effects of policies implemented by the US and the EU, I estimate the innovation timing elasticity, showing for the first time that the pace of innovation increases with the number of firms operating in an industry. The estimate is sufficiently high to shift the optimal national policy from free-riding to subsidizing green capital goods production in the quantitative analysis.

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## CHAPTER 1

# Industrial Policy in the context of Industry Lifecycle: Simple Model

### 1.1. Introduction

China's economy has grown rapidly during the last three decades, with annual real GDP growth exceeding 9 percent from 1991 to 2021 (IMF 2022). The Chinese government has conducted a range of assertive industrial policies for manufacturing sectors during this period, which many studies such as Rodrik (2006), Gabriele (2010), and Felipe et al. (2013) argue has played significant role in attaining such rapid growth.

The government continues trying to upgrade China's industrial structure further using tools of industrial policy. For example, Made in China 2025 is a national plan to develop China's manufacturing industry, implementing a transition from labor-intensive workshops into a technology-intensive powerhouse. In response to such efforts by China to become a global leader in high tech industries, many politicians and scholars in advanced economies have begun calling for implementation of new industrial policy to foster high tech industries or preserve aging manufacturing sectors.

In this context, a central question emerges: How do these industrial policies affect the welfare of both the country conducting the policies and its counterpart countries? In theory, when there is an externality in an industry, the optimal industrial policy in the implementing country can be designed in a way that the social marginal cost equals the social marginal benefit. Thus, it is necessary to understand what externality may exist in the industry that the government wants to support in order to study the welfare effect of the industrial policy. However, an externality in an industry need not remain the same while the industry develops. If we can find a generalized pattern of the development path of industries, an externality in an industry is likely to change along with the stages of the generalized industry lifecycle. Studies such as Klepper (1996) and Akcigit and

Kerr (2018) argue that if an industry already becomes mature, thus it is in the late stage of what they call the industry lifecycle, then a small number of large-sized firms are likely to operate in the industry. Such firms have a higher incentive to focus on R&D for process innovation, which is principally designed to lower a firm's average cost of production. Since process R&D is related to minor innovation and the marginal effectiveness of process R&D spending usually decreases over time, the social benefit from supporting the industry may be limited in this case.

Welfare effects of industrial policy for counterpart countries are also important factors to consider. If an industrial policy decreases the welfare of its counterpart countries, those countries will respond by taking measures to offset the effects of the policy. However, there are not many papers that study the welfare effect of industrial policy for counterpart countries.

In this context, this paper attempts to figure out how externalities arising from innovation in an industry change along with the industry lifecycle and incorporate the productivity dynamics derived from it into a general equilibrium model. With the model, this paper studies the effects of an industrial policy on the welfare of the implementing country and its counterpart countries. The findings support industrial policies targeted towards industries in the early stages of the lifecycle being engaged in product innovation.

There are some novelties in this paper. First, it builds a growth model that incorporates productivity dynamics, taking into account the industry lifecycle. The growth models in existing literature usually assume a fixed growth rate. However, such an assumption is not realistic considering that the growth rate of an industry changes over time. The second contribution is that this paper explicitly studies the welfare effect of the counterpart country in order to find conditions under which an industrial policy can be welfare-improving for both the country conducting the policy and its counterpart countries.

## **1.2. Literature Review**

This literature review pertains to both Chapter 1 and Chapter 2.

### **1.2.1. Industrial Policy and Welfare.**

**Long-run Growth.** This paper builds on and contributes to the literature that studies how industrial policy affects productivity and welfare of the home country under open economy in

dynamic view, which includes Redding (1999) and Melitz (2005). The literature that emphasizes innovation as the engine of growth (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) is also closely related in that the main source of welfare improvement in the model is innovation and the consequential increase in productivity. A novel feature of the model in this paper is that the type of innovation and the effect of innovation on aggregate productivity in an industry change depending on where the industry places along its lifecycle while other literature assumes that the type of innovation or the parameter determining the degree of innovation in an industry are time invariant. The model in this paper also supplements Melitz (2005) by suggesting a plausible shape of learning curve. He suggests whether industrial policy to protect an infant industry increases home welfare or not depends on the shape of learning curve in the industry and the degree of substitutability between home and foreign goods, but does not present how the learning curve usually looks.

This paper is related to the literature such as Rodrik (2006), Aghion et al. (2015), Atkeson and Burstein (2019), Choi and Levchenko (2021), and Lane (2022) which studies the role of industrial policy in economic growth. Especially, since empirical results of Rodrik (2006) and Aghion et al. (2015) suggest that policies that support younger and high growth potential sectors tend to increase the targeted industry's productivity more, this paper complements their results. Rodrik (2013) also provides empirical evidence that younger manufacturing sectors exhibit more rapid labor productivity growth. However, based on industry lifecycle theory and the empirical finding from Korea's Heavy and Chemical industry drive in Section 2.5, this paper is different in that actual productivity increase in the targeted industry is realized with a lag after policy intervention.

From the feature of the lagged realization of productivity increase based on industry lifecycle, this paper contributes to literature which suggests empirical evidence on short-run and long-run welfare trade off from industrial policy (Kim et al., 2021; Choi and Levchenko, 2021). This paper provides a theoretical foundation on the trade off between short-run distortion and long-run gain from growth and emphasizes the importance of welfare analysis of industrial policy in dynamic view.

**Optimal Industrial Policy.** Another related literature is the one that studies the optimal mix of industrial and trade policy. Bartelme et al. (2021) and Lashkaripour and Lugovskyy (2023)

suggest optimal industrial and trade policy to maximize the welfare by exploiting the differences in scale economies across industries and improving terms of trade in static view. Although the static scale economies utilized in Lashkaripour and Lugovskyy (2023) also operate in the short run in my model<sup>1</sup>, my focus lies on a novel form of externality within a dynamic framework. This externality operates through a mechanism where increased production and innovation activities among firms in the targeted industry lead to heightened learning-by-doing and knowledge diffusion within the sector. Consequently, it fosters radical innovation, accelerating the industry’s transition to a high-productivity stage. Thus, importantly, this externality is related with “how quickly” innovation can occur in the targeted industry due to industrial policy, rather than “how much higher” productivity can increase due to industrial policy, which is the focus of the existing literature.

Bai et al. (2023) is notable for providing optimal innovation and trade policy in a dynamic setting and elucidating the home country’s incentive to influence foreign innovation to maximize its welfare. However, my paper distinguishes itself in two significant ways. First, the degree of productivity increase due to innovation varies depending on the industry lifecycle, leading to distinct policy implications in two scenarios: catching-up and frontier technology races. Second, this paper also examines the welfare of the foreign country, thus delving into the strategic policy decisions of both the home and foreign countries.

**1.2.2. Industry Lifecycle and Innovation.** This paper is related to the literature which studies the stylized pattern of industry dynamics (Vernon, 1966; Abernathy and Utterback, 1978; Gort and Klepper, 1982; Jovanovic and MacDonald, 1994; Klepper, 1996; Antràs, 2005; Eriksson et al., 2021). This paper contributes to the literature by incorporating the theory on how innovation in an industry changes along its lifecycle into a canonical open macroeconomic model to analyze its implications on the aggregate economy, while most of the literature focuses on regularities along the lifecycle with partial equilibrium analysis.

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<sup>1</sup>There can be complementarity or trade-off between the new externality presented in this paper (dynamic scale economy) and the static scale economies discussed in Lashkaripour and Lugovskyy (2023). For instance, suppose a high growth potential industry has a lower elasticity of substitution across varieties within the industry (micro elasticity of substitution) compared to other industries. In this case, if the government supports the industry to accelerate innovation (dynamic scale economy), more labor will be allocated to the industry from other industries in response to the policy. From a static perspective, based on Lashkaripour and Lugovskyy (2023), it implies a reallocation of resources from industries with low scale economies to those with high scale economies, illustrating complementarity between static and dynamic scale economies. In contrast, if the micro elasticity of substitution of the targeted industry is higher than others, there exists a trade-off between static and dynamic scale economies.

In this aspect, the literature which develops a growth or trade model that incorporates multiple types of innovations with a consideration of industry dynamics (Krugman, 1979; Klette and Kortum, 2004; Atkeson and Burstein, 2010; Akcigit and Kerr, 2018; Hsieh et al., 2021) is also closely related to this paper. This paper contributes to the literature in two aspects. First, this paper derives policy implications based on welfare analysis. The implications are novel in that it provides a criteria regarding the right “timing” of industrial policy. Second, it studies how industrial policy affects the foreign innovation and welfare as well as those for the home country. While Krugman (1979) studies the effects of innovation in North (developed country) on the welfare for South (developing country), his framework is appropriate for analyzing only an asymmetric case where a developing country tries to catch up a developed country by imitation of technology. This paper also analyzes the case where two symmetric countries compete to take the leadership in an industry using a frontier technology.

**1.2.3. Growth and International Welfare Spillover.** My work contributes to literature which studies international spillovers from productivity increases in the home country (Ghironi and Melitz, 2005; Matsuyama, 2007; Corsetti et al., 2007). This paper makes two contributions to the existing literature. First, it provides a theoretical framework to analyze the effects of industrial policy targeting a specific industry by extending the model of Corsetti et al. (2007) to a multi-industry model. Second, it suggests a novel channel through which a productivity improvement in the home country affects foreign welfare. In my model, if there is a productivity increase in an industry in the home country, it causes a decrease in the mass of firms in the same industry in the foreign country. Due to shrinking of firm activity in the foreign industry, innovation will be delayed in the industry and net foreign welfare accordingly decreases in this dynamic setting. This channel provides the possibility of “beggar-thy-neighbor” while the other effects from the existing literature (e.g. effects based on terms of trade and love for variety) still apply in the model.

### 1.3. The Model

**1.3.1. Closed Economy Model.** I start with a simple closed economy model. It is assumed that there are two sectors. In sector 1, there is a continuum of industries which are in the early stage of the industry lifecycle. Sector 2 consists of an industry where innovation hardly happens.

An example of sector 2 could be agriculture. I will explain the meaning of being in the ‘early’ stage of the industry lifecycle in detail in the productivity dynamics part.

1.3.1.1. **Demand.** The utility function of the representative consumer at time  $t$  has the following form.

$$(1.1) \quad u(c_{1t}, c_{2t}) = c_{1t}^\alpha c_{2t}^{1-\alpha}$$

where  $c_{1t} \equiv n \times \exp\left(\int_0^n \ln c_{1i,t}^{\frac{1}{n}} di\right)$  is the consumption of aggregate sector 1 product<sup>2</sup> and  $c_{2t}$  is the consumption of sector 2 product. Here,  $c_{1i,t}$  represents the consumption for sector 1-industry  $i$  product, and  $n$  represents the mass of industry in sector 1.

The welfare of the economy is represented by the following equation:

$$(1.2) \quad W = \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t})$$

where  $\beta$  is the discount rate.

1.3.1.2. **Supply.** There is only one factor of production, labor, which is mobile across industries. The total number of labor is fixed at  $\bar{L}$ . Firms in the same industry have the same linear constant-returns-to-scale production function.

$$(1.3) \quad q(L_{1i,t}) = a_{1i,t} L_{1i,t} \text{ for industry } i \in [0, n] \text{ in sector 1}$$

$$(1.4) \quad q(L_{2i,t}) = \bar{a} L_{2t} \text{ for sector 2}$$

where  $L_{1i,t}$  is the number of units of labor used for industry  $i$  in sector 1 at time  $t$  and  $L_{2t}$  is the number of units of labor used for sector 2 at time  $t$ .  $a_{1i,t}$  represents the a productivity level of industry  $i$  in sector 1 at time  $t$ . I assume that the productivity level for sector 2 is fixed over time at  $\bar{a}$ .

Labor and product markets are assumed to be perfectly competitive. We can reconstruct the supply side for sector 1 in a simpler way. Let  $L_{1t}$  denote the number of labor units used in sector 1. The utility function is Cobb-Douglas, and the expenditure share for each industry in sector 1 is

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<sup>2</sup>Since  $\exp\left(\int_0^n \ln c_{1i,t}^{\frac{1}{n}} di\right)$  is the geometric average of consumptions from all industries in sector 1,  $n \times \exp\left(\int_0^n \ln c_{1i,t}^{\frac{1}{n}} di\right)$  can be interpreted as the aggregate consumption from sector 1.

the same, which is  $1/n$ . Thus, the expenditure for the product of industry  $i$  in sector 1 should be the same for all  $i$ . Let  $E$  denote the expenditure for sector 1-industry  $i$  product. Expenditure is the same as revenue in equilibrium. Also, since goods markets are perfectly competitive, revenue should be the same as cost. Thus, the following condition holds.

$$(1.5) \quad E = p_{1i,t}q_{1i,t} = w_t L_{1i,t} \text{ for all } i \in [0, n]$$

From (1.5), it follows that the number of workers used in every industry in sector 1 should be the same in equilibrium. In other words, the following conditions must hold in equilibrium.

$$(1.6) \quad L_{1i,t} = \frac{L_{1t}}{n} \text{ for all } i \in [0, n]$$

By using condition (1.6), we can derive the following equilibrium conditions.

$$(1.7) \quad c_{1i,t} = a_{1i,t} L_{1i,t} = a_{1i,t} \frac{L_{1t}}{n}$$

By using condition (1.7), we can rewrite the aggregate consumption in sector 1 as follows:

$$(1.8) \quad c_{1t} = n \times \exp\left(\int_0^n \ln c_{1i,t}^{\frac{1}{n}} di\right) = \exp\left(\int_0^n \ln a_{1i,t}^{\frac{1}{n}} di\right) L_{1t}$$

Since  $\exp\left(\int_0^n \ln a_{1i,t}^{\frac{1}{n}} di\right)$  is the geometric average of productivities of all industries in sector 1, we can derive the aggregate production function for sector 1.

$$(1.9) \quad q(L_{1t}) = a_{1t} L_{1t}, \text{ where } a_{1t} \equiv \exp\left(\int_0^n \ln a_{1i,t}^{\frac{1}{n}} di\right)$$

Here,  $a_{1t}$  can be interpreted as average productivity of sector 1. From now on, I will use this sector level aggregate production function (1.9) instead of industry level production function (1.3) for sector 1.

**1.3.1.3. Productivity Dynamics.** A novel feature of this model is that it does not assume a constant growth rate for the aggregate productivity of an industry. As we have seen in the literature regarding industry lifecycle, it is reasonable to assume that the aggregate productivity of an industry changes as follows: Aggregate productivity grows slowly at the inception of the industry.

However, its growth rate increases rapidly after some point. Then, the growth rate declines, and the industry's aggregate productivity hardly grows at all in the late stage of the industry lifecycle.

To capture these productivity dynamics, I refer to Jovanovic and MacDonald (1994) regarding how innovations happen along the industry lifecycle. Let  $\overline{a_{high}}$  and  $\overline{a_{low}}$  denote the high and low productivity of industry  $i$  in sector 1. It is assumed that all firms have low productivity,  $\overline{a_{low}}$ , in the early stage of the industry lifecycle. The productivity does not increase until a refinement occurs. After the refinement occurs at time  $k$ , all firms in the industry acquires high productivity,  $\overline{a_{high}}$ , with probability  $r$ .

To build intuition on welfare effects of an industrial policy, suppose we start from some initial condition where all industries in sector 1 are in an early stage. The condition has the following meanings: refinement hasn't arrived in sector 1 yet, and thus the average productivity in sector 1 stays at the lower bound,  $\overline{a_{low}}$ .

Now, suppose that refinement in sector 1 happens at  $t = k$ . Under these assumptions, the average productivity of sector 1 will grow over time as follows. Please refer to Appendix A.1 for how to derive (1.10).

$$(1.10) \quad \text{Sector 1: } a_{1t} = (\overline{a_{high}})^{1-(1-r)^{t-k+1}} (\overline{a_{low}})^{(1-r)^{t-k+1}}, \text{ when } t \geq k$$

Jovanovic and MacDonald (1994) argues that the technology is refined from outside the industry. Thus, the refinement is assumed to occur exogenously in the paper. However, in most cases, a huge innovation from outside the industry cannot be applied immediately. Knowledge stock is required to be accumulated for the application of the outside innovation to production technologies in the industry. I assume that the knowledge stock in an industry in the early stage of the industry lifecycle depends on the cumulative production in the industry. This assumption is based on the learning-by-doing effect, as in Melitz (2005). Formally, the applicable refinement is assumed to occur when the cumulative production exceeds a certain level as follows.

ASSUMPTION 1. *Refinement occurs in the early stage of industry lifecycle if  $Q_{1t} \geq \bar{Q}$  where  $Q_{1t} \equiv \sum_{j=0}^t q_{1,j}$  which is the cumulative production of sector 1 until time  $t$ .*



In the spirit of Melitz (2005), the level of productivity in this model depends on the cumulative level of production in the sector,  $Q$ . However, here, it is related with the probability of a boost in productivity, which comes through the emergence of a refinement or dominant design that enables standardization in production as in Jovanovic and MacDonald (1994), reflected in a higher  $a$ . Even though the timing of refinement is stochastic in Jovanovic and MacDonald (1994), I assume that the timing of refinement is deterministic process, as seen in Assumption 1, for simplicity. It is important to note that a refinement does not imply any boost in productivity. It simply shifts the sector to a state where there is a positive probability of a boost in productivity. This probabilistic process is given by Equation (1.10) above. So in my framework, the emergence of a refinement makes it easier for firms to make their production process more efficient, but the existence of a refinement does not in itself make the process more efficient. We could think of a refinement as an industry-specific thing, like the invention of the personal computer, which made it much easier for firms to find ways to improve productive efficiency than the earlier mainframe. One could also think of a refinement as applying more broadly to a sector. An example could be an assembly line. This refinement in the layout of production, as it diffused over time, made it possible for a wide variety of industries within the manufacturing sector to make their individual production processes more efficient. For the services sector, the emergence of the internet has had a similar effect.

In this model, if an industry produces more in the early stage of the industry lifecycle, the cumulative production of the industry increases faster and thus the refinement happens earlier. This is the key mechanism through which an industrial policy can improve welfare of the implementing country in this model.

1.3.1.4. *Static Equilibrium.* Since labor is nonspecific between the all industries, the labor market equilibrium is as follows:

$$(1.11) \quad \text{Sector 1: } w_t = p_{1t}a_{1t}$$

$$(1.12) \quad \text{Sector 2: } w_t = p_{2t}\bar{a}$$

where  $w_t$  is the wage, and  $p_{it}$  is the price of the aggregate sector  $i$  product at time  $t$ .

The income of the representative consumer is  $w_t\bar{L}$ . By combining the labor market equilibrium condition, we can get  $w_t\bar{L} = p_{1t}a_{1t}\bar{L} = p_{2t}\bar{a}\bar{L}$ . Since the utility function is Cobb-Douglas function,

the representative consumer will spend fraction  $\alpha$  of the consumer's income for buying aggregate sector 1 product,  $c_{1t}$ , and spend fraction  $1 - \alpha$  of their income for buying sector 2 product,  $c_{2t}$ . Consequently, the equilibrium conditions for goods markets are derived as follows.

$$(1.13) \quad \text{Sector 1: } L_{1t} = \alpha \bar{L}$$

$$(1.14) \quad \text{Sector 2: } L_{2t} = (1 - \alpha) \bar{L}$$

The equilibrium for goods markets also satisfies the resource constraint:

$$(1.15) \quad L_{1t} + L_{2t} = \bar{L}$$

The static equilibrium is the prices and wage  $\{p_{1t}, p_{2t}, w_t\}$  and labor allocation  $\{L_{1t}, L_{2t}\}$  such that equilibrium conditions from (1.11) to (1.15) are satisfied.

1.3.1.5. **Industrial Policy.** According to Pack and Saggi (2006), industrial policy is defined as “any type of selective intervention or government policy that attempts to alter the structure of production toward sectors that are expected to offer better prospects for economic growth than would occur in the absence of such intervention.” In this paper, I assume that government wants to promote a targeted industry and use industrial policies to achieve its goal. For simplicity, I only analyze the effect of production subsidy among various industrial policies.

Assume that government gives a production subsidy to firms in sector 1, and the production subsidy rate is  $s$ . I additionally assume the government corrects a lump-sum tax from the consumers to fund the production subsidy. Since every market is perfectly competitive, the labor market equilibrium condition for sector 1, (1.11), will change by the introduction of the policy as follows:

$$(1.16) \quad w_t = (1 + s)p_{1t}a_{1t}$$

By using condition (1.16), goods market equilibrium conditions are changed as follows:

$$(1.17) \quad \text{Sector 1: } L_{1t} = \frac{1 + s}{1 + \alpha s} \alpha \bar{L}$$

$$(1.18) \quad \text{Sector 2: } L_{2t} = \frac{1}{1 + \alpha s} (1 - \alpha) \bar{L}$$

1.3.1.6. **Welfare Effects.** The production subsidy has two effects in terms of welfare. The first effect is the static resource reallocation effect. As evident in conditions (1.13) and (1.17), the production subsidy for sector 1 leads to increased labor usage in that sector within the economy. Since all markets are perfectly competitive, such reallocation of resources introduces distortions in the static aspect. This effect is further illustrated by comparing instantaneous utility with and without the production subsidy.

$$(1.19) \quad \text{Instantaneous utility without the production subsidy: } u_t = \alpha^\alpha (1 - \alpha)^{1-\alpha} \bar{L}$$

$$(1.20) \quad \text{Instantaneous utility with the production subsidy: } u_t = \frac{(1+s)^\alpha}{1+\alpha s} \alpha^\alpha (1 - \alpha)^{1-\alpha} \bar{L}$$

It is evident that the instantaneous utility without the production subsidy, as shown in (1.19), is strictly greater than that with the production subsidy, as depicted in (1.20).

The second effect is the early innovation effect. It's worth noting that we begin from the initial condition where refinement has not yet occurred in sector 1. In this scenario, the production subsidy accelerates the arrival of refinement in sector 1. Consequently, the economy experiences earlier productivity improvements in the dynamic aspect.

Overall, if the second effect outweighs the first, the industrial policy improves the welfare of the economy.

1.3.1.7. **Illustrative Example.** I construct an illustrative example of the model to examine the welfare effects of a production subsidy in the context of industry lifecycle. For the parameters of the closed economy model, I set  $\alpha = 0.5$  and  $\beta = 0.99$ . The total labor supply is given by  $\bar{L} = 1$ .

In terms of productivity dynamics, I assume that all industries in sector 1 are in the early stage of industry lifecycle. This implies that the aggregate productivity in all industries in sector 1 are currently low,  $\overline{a_{low}}$ . I set  $\overline{a_{low}} = 5$  and  $\overline{a_{high}} = 10$ . Additionally, the productivity level of sector 2 is assumed to be fixed at  $\bar{a} = 10$ .

As explained in the model, the timing of refinement depends on the cumulative production of the sector.

$$(1.21) \quad \text{The refinement arrives at time } t \text{ if } Q_{1,t-1} \geq 50, \text{ where } Q_{1,t-1} \equiv \sum_{j=0}^{t-1} q_{1,j}$$

Let  $k$  denote the time when refinement arrives in sector 1. The probability that an industry in sector 1 can achieve high productivity after the refinement, denoted by  $r$ , is set to 0.05. Consequently, the law of motion for the average productivity of sector 1 is as follows:

$$(1.22) \quad \begin{aligned} t < k: a_{1t} &= \overline{a_{low}} = 5 \\ t \geq k: a_{1t} &= 10^{1-(0.95)^{t-k+1}} 5^{(0.95)^{t-k+1}} \end{aligned}$$

**Equilibrium without Policy Intervention.** If there is no policy intervention, firms in sector 1 and 2 hire 0.5 units of labor, respectively, for all time period. The refinement occurs at  $t = 21$ .

**Equilibrium with Policy Intervention.** Now, I assume that the government provides a production subsidy to all firms in sector 1 until the refinement occurs. The production subsidy rate is assumed to be 20%. In this scenario, before the refinement occurs, sector 1 utilizes 0.545 units of labor, which is greater than in the case without the production subsidy. Conversely, sector 2 utilizes 0.455 units of labor.

Due to the production subsidy, firms in sector 1 produce more, resulting in the refinement occurring earlier than in the case without policy intervention: The refinement occurs at  $t = 20$ . After the refinement, the government ceases to provide the subsidy, and thus the equilibrium dynamics are the same as those after the refinement without policy intervention.

**Welfare Comparison.** When there is no policy intervention, the cumulative welfare is 452.6. With the production subsidy, the cumulative welfare increases to 453.4. Thus, the industrial policy is welfare-improving.

To derive implications from the results, I examine the dynamics of instantaneous utility for both cases. Without government intervention and before refinement, the instantaneous utility is 3.54. It decreases to 3.52 when the government provides a production subsidy to firms in sector 1. This decline in instantaneous utility reflects the resource reallocation effect resulting from the distortion caused by the production subsidy.

After the refinement occurs in sector 1, the productivity of industries in the sector increases over time. Thus, earlier refinement should be beneficial for the economy. By implementing the industrial policy, the refinement arrives one period earlier, from  $t = 21$  to  $t = 20$ . This reflects the early innovation effect.

In summary, the industrial policy initially makes the economy worse off, but the long-run gains from faster growth in the targeted industry offset the earlier loss. In this example, the long-run gain exceeds the initial loss, making the industrial policy eventually welfare-increasing.

In this model, when either (i) the industry has high productivity growth potential (high  $\overline{a_{high}}$ ) or high  $r$ , (ii) learning-by-doing is strong, or (iii) the discount factor is small (high  $\beta$ ), the industrial policy can be welfare-improving for the economy.

**1.3.2. Two Country Model.** One of the purposes of this paper is to examine the welfare effects of an industrial policy on a country's trading partners. To analyze this, an open economy model is necessary. Additionally, in the model, if the country implementing an industrial policy is small, it has no effect on the welfare of the rest of the world. Thus, I extend the closed economy model to a model involving two large countries.

1.3.2.1. *Demand.* The preference is identical for both the domestic and foreign representative consumers. The utility function of the representative consumer at time  $t$  has the following form.

$$(1.23) \quad u(c_{1t}, c_{2t}) = c_{1t}^\alpha c_{2t}^{1-\alpha}$$

where  $c_{it}$  is the consumption of the aggregate sector  $i$  product at time  $t$ . Domestic and foreign products within each sector are imperfect substitutes. Domestic and foreign goods are aggregated in the following way.

$$(1.24) \quad \text{Sector 1: } c_{1t} = \left( c_{1h,t}^\epsilon + c_{1f,t}^\epsilon \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$(1.25) \quad \text{Sector 2: } c_{2t} = \left( c_{2h,t}^\epsilon + c_{2f,t}^\epsilon \right)^{\frac{\epsilon}{\epsilon-1}}$$

where  $c_{1h,t} \equiv n \times \exp \left( \int_0^n \ln c_{1ih,t}^{\frac{1}{n}} di \right)$  represents the aggregate domestic sector 1 goods at time  $t$ , and  $c_{1f,t} \equiv n \times \exp \left( \int_0^n \ln c_{1if,t}^{\frac{1}{n}} di \right)$  represents the aggregate foreign sector 1 goods at time  $t$ . Here,  $c_{1ih,t}$  and  $c_{1if,t}$  represent the consumption for sector 1-industry  $i$  product for the home and foreign country respectively, and  $n$  represents the mass of industries in sector 1 which are the same for home and foreign country. The value of  $\epsilon$  measures the degree of substitutability between domestic and foreign goods within an industry. The lower is  $\epsilon$ , the more differentiated are the domestic and

foreign product. The welfare of each country is represented by the following equation:

$$(1.26) \quad W = \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t})$$

where  $\beta$  is the discount rate, which is same for home and foreign country.

1.3.2.2. **Supply.** There is only one factor of production: labor, which is mobile across industries but not across countries. The total number of labor is fixed in both the home and foreign countries, at  $\bar{L}_h$  and  $\bar{L}_f$  respectively. All firms in the same industry within the same country have the same linear constant-returns-to-scale production function. Applying the same logic as in the closed economy model, I will use a sector-level aggregate production function for sector 1 instead of an industry-level production function for this open economy model.

$$(1.27) \quad \text{Sector 1 in the home country: } f(L_{1h,t}) = a_{1h,t}L_{1h,t}$$

$$(1.28) \quad \text{Sector 2 in the home country: } f(L_{2h,t}) = \bar{a}L_{2h,t}$$

$$(1.29) \quad \text{Sector 1 in the foreign country: } f(L_{1f,t}) = a_{1f,t}L_{1f,t}$$

$$(1.30) \quad \text{Sector 2 in the foreign country: } f(L_{2f,t}) = \bar{a}L_{2f,t}$$

where  $L_{ih,t}$  and  $L_{if,t}$  are the number of units of labor used in sector  $i$  at time  $t$  in the home and foreign country, and  $a_{1h,t} \equiv \exp\left(\int_0^n \ln a_{1hi,t}^{\frac{1}{n}} di\right)$  and  $a_{1f,t} \equiv \exp\left(\int_0^n \ln a_{1fi,t}^{\frac{1}{n}} di\right)$  are the average productivity level in sector 1 at time  $t$  in the home and foreign countries, respectively. Here,  $a_{1ih,t}$  and  $a_{1if,t}$  represent the productivity of industry  $i$  in sector 1 in the home and foreign countries, respectively.

1.3.2.3. **Productivity Dynamics.** Productivity dynamics are the same for the open economy model as for the closed economy model. Let  $\overline{a_{high}}$  and  $\overline{a_{low}}$  denote the high and low productivity levels of all industries in sector 1 in the home and foreign countries, respectively. Suppose we start from an initial condition where all industries in sector 1 in the home country are in an early stage, but sector 1 in the foreign country is mature. In detail, refinement hasn't yet occurred in sector 1 in the home country, so the productivity of all industries in the sector remains at the lower bound,  $\overline{a_{low}}$ . However, refinement has already occurred in sector 1 in the foreign country, and the productivity of all industries in the sector has reached the upper bound.

The productivity of industries in sector 1 in the home country does not increase until refinement occurs. After refinement occurs at time  $k$ , an industry acquires high productivity,  $\overline{a_{high}}$ , with probability  $r$ . Under these assumptions, the average productivity of sector 1 in the home and foreign countries will change over time as follows.

$$(1.31) \quad \text{Sector 1 in the home country: } a_{1t} = (\overline{a_{high}})^{1-(1-r)^{t-k+1}} (\overline{a_{low}})^{(1-r)^{t-k+1}}, \text{ when } t \geq k$$

$$(1.32) \quad \text{Sector 1 in the foreign country: } a_{1t} = \overline{a_{high}} \text{ for all } t$$

I further assume that the timing of refinement depends on the cumulative production in the sector. Formally, refinement occurs when the cumulative production exceeds a certain level, as follows.

ASSUMPTION 2. *Refinement occurs in the early stage of industry lifecycle if  $Q_{1h,t} \geq \bar{Q}$ , where  $Q_{1h,t} \equiv \sum_{j=0}^t q_{1h,j}$  represents the cumulative production of sector 1 in the home country until time  $t$ .*

As in the closed economy model, if an industry produces more in the early stage of the industry lifecycle, the cumulative production of the industry increases faster, leading to earlier refinement. This earlier productivity increase in the implementing country can improve the welfare of both the country and its trading partner in this model.

1.3.2.4. **Static Equilibrium.** Labor and product markets are perfectly competitive. Since labor is non-specific across all industries in each country, the labor market equilibriums are as follows:

$$(1.33) \quad \text{Sector 1 in the home country: } w_{h,t} = p_{1h,t} a_{1h,t}$$

$$(1.34) \quad \text{Sector 2 in the home country: } w_{h,t} = p_{2h,t} a_{2h,t}$$

$$(1.35) \quad \text{Sector 1 in the foreign country: } w_{f,t} = p_{1f,t} a_{1f,t}$$

$$(1.36) \quad \text{Sector 2 in the foreign country: } w_{f,t} = p_{2f,t} a_{2f,t}$$

where  $w_{h,t}$  and  $w_{f,t}$  are the wages in the home and foreign countries, respectively, and  $p_{ih,t}$  and  $p_{if,t}$  are the prices of the aggregate sector  $i$  product at time  $t$  in the home and foreign countries,

respectively. Let  $x_t$  denote  $p_{1h,t}/p_{1f,t}$ , which is the relative price between domestic and foreign goods in sector 1, and  $y_t$  denote  $p_{2h,t}/p_{2f,t}$ , which is the relative price between domestic and foreign goods in sector 2.  $x_t$  and  $y_t$  can be rewritten in the following way using the labor market equilibrium conditions in the home and foreign countries.

$$(1.37) \quad x_t = \frac{w_{h,t} a_{1f,t}}{w_{f,t} a_{1h,t}}$$

$$(1.38) \quad y_t = \frac{w_{h,t} a_{2f,t}}{w_{f,t} a_{2h,t}}$$

Then, we can get the relative wage between domestic and foreign country using the above equations.

$$(1.39) \quad \frac{w_{h,t}}{w_{f,t}} = x_t \frac{a_{1h,t}}{a_{1f,t}} = y_t \frac{a_{2h,t}}{a_{2f,t}}$$

The income of the domestic representative consumer is  $w_{h,t}\bar{L}_h$ . By combining the labor market equilibrium conditions, we can obtain  $w_{h,t}\bar{L}_h = p_{1h,t}a_{1h,t}\bar{L}_h = p_{2h,t}a_{2h,t}\bar{L}_h$ . Similarly, the income of the foreign representative consumer is  $w_{f,t}\bar{L}_f = p_{1f,t}a_{1f,t}\bar{L}_f = p_{2f,t}a_{2f,t}\bar{L}_f$ .

Since the utility function is a Cobb-Douglas function, the representative consumers in the home and foreign countries will spend a fraction  $\alpha$  of their income on purchasing aggregate sector 1 products,  $c_{1t}$ , and spend a fraction  $1 - \alpha$  of their income on purchasing sector 2 products,  $c_{2t}$ . Additionally, the domestic and foreign products from each sector are aggregated using the constant elasticity of substitution functional form. Thus, out of the income allocated for buying products from sector 1, the consumer will allocate a fraction  $\frac{1}{1+x_t^{\epsilon-1}}$  for buying domestic goods from sector 1 and a fraction  $\frac{x_t^{\epsilon-1}}{1+x_t^{\epsilon-1}}$  for buying foreign goods from sector 1. Similarly, out of the income allocated for buying products from sector 2, the consumer will allocate a fraction  $\frac{1}{1+y_t^{\epsilon-1}}$  for buying domestic goods from sector 2 and a fraction  $\frac{y_t^{\epsilon-1}}{1+y_t^{\epsilon-1}}$  for buying foreign goods from sector 2. Consequently, the equilibrium conditions for the goods market are derived as follows.

$$(1.40) \quad \text{Domestic goods from sector 1: } \frac{\alpha}{1+x_t^{\epsilon-1}} \left( a_{1h,t}\bar{L}_h + \frac{1}{x_t} a_{1f,t}\bar{L}_f \right) = a_{1h,t}L_{1h,t}$$

$$(1.41) \quad \text{Foreign goods from sector 1: } \frac{\alpha x_t^{\epsilon-1}}{1+x_t^{\epsilon-1}} (x_t a_{1h,t}\bar{L}_h + a_{1f,t}\bar{L}_f) = a_{1f,t}L_{1f,t}$$



$$(1.42) \quad \text{Domestic goods from sector 2: } \frac{1 - \alpha}{1 + y_t^{\epsilon-1}} \left( \bar{a}\bar{L}_h + \frac{1}{y_t} \bar{a}\bar{L}_f \right) = \bar{a}L_{2h,t}$$

$$(1.43) \quad \text{Domestic goods from sector 1: } \frac{(1 - \alpha)y_t}{1 + y_t^{\epsilon-1}} (y_t \bar{a}\bar{L}_h + \bar{a}\bar{L}_f) = \bar{a}L_{2f,t}$$

There are two resource constraints in the model.

$$(1.44) \quad \text{Resource constraint in the home country: } L_{1h,t} + L_{2h,t} = \bar{L}_h$$

$$(1.45) \quad \text{Resource constraint in the foreign country: } L_{1f,t} + L_{2f,t} = \bar{L}_f$$

The static equilibrium consists of the prices and wages,  $\{p_{1h,t}, p_{2h,t}, p_{1f,t}, p_{2f,t}, w_{h,t}, w_{f,t}\}$ , and the labor allocation,  $\{L_{1h,t}, L_{2h,t}, L_{1f,t}, L_{2f,t}\}$ , such that the labor market equilibrium conditions from (1.33) to (1.36), and the goods market equilibrium from (1.40) to (1.43), and resource constraints, (1.44) and (1.45), are satisfied.

1.3.2.5. **Industrial Policy.** As in the closed economy model, I assume that the government provides a production subsidy to firms in sector 1, with the production subsidy rate denoted as  $s$ . Additionally, I assume that the government imposes a lump-sum tax on consumers to fund the production subsidy. Since every market is perfectly competitive, only the labor market equilibrium equation (1.33) will change with the introduction of the policy, as follows.

$$(1.46) \quad w_{h,t} = (1 + s)p_{1h,t}a_{1h,t}$$

1.3.2.6. **Welfare Effects.** In the home country, there are resource reallocation effects and early innovation effects, as in the closed economy model. However, in the two-country model, there is an additional effect: the terms of trade effect. If we assume that the home country has a comparative advantage in sector 2, then it is a net importer of products from sector 1. In this case, an increase in productivity in sector 1 in the home country improves the terms of trade for the home country.

For the foreign country, there is a benefit from the home country's productivity improvement. The increase in productivity in the home country means that consumers in the foreign country can also purchase products from sector 1 at a cheaper price. However, under the assumption that the foreign country has a comparative advantage in sector 1, the terms of trade for the foreign country will deteriorate.

In summary, for the home country, the early innovation and terms of trade effects increase, but the resource allocation effect decreases its welfare. For the foreign country, the early innovation effect of the home country increases, and the terms of trade effect decreases its welfare. Thus, we can see that the early innovation effect plays a crucial role in improving the welfare of both the home and foreign countries through industrial policy.

**1.3.2.7. Illustrative Example.** For parameters of the two-country model, I set  $\alpha = 0.5$  and  $\beta = 0.99$ . I refer to the Armington elasticity of the machinery and electronics industry estimated using TSLS in Feenstra et al. (2018), which are 2.01 and 2.40, respectively, and set  $\epsilon = 2$ . Total labor supply is given by  $L_h = 1$  and  $L_f = 1$ . Thus, the size of the home and foreign country is assumed to be the same.

Regarding productivity dynamics, I assume that all industries in sector 1 in the home country are in the early stage of the industry lifecycle. It means the average productivity in sector 1 is low,  $\overline{a_{low}}$ . It is assumed that sector 1 in the foreign country is mature. Thus,  $a_{1f,t} = \overline{a_{high}}$  for all  $t$ . I set  $\overline{a_{low}} = 5$  and  $\overline{a_{high}} = 10$ . Again, the productivity level of sector 2 for the home and foreign country is fixed at  $\bar{a} = 10$ .

The timing when refinement happens depends on the cumulative production of the sector.

$$(1.47) \quad \text{The refinement arrives at time } t \text{ if } Q_{1h,t-1} \geq 50, \text{ where } Q_{1h,t-1} \equiv \sum_{j=0}^{t-1} q_{1h,j}$$

Let  $k$  denote the time when refinement in sector 1 in the home country arrives. The probability that an industry can achieve high productivity after the refinement,  $r$ , is set to 0.05. Consequently, the law of motion for the productivity of sector 1 is as follows:

$$(1.48) \quad \begin{aligned} \text{Sector 1 in the home country: } & t < k, a_{1h,t} = \overline{a_{low}} = 5 \\ & t \geq k, a_{1h,t} = 10^{1-(0.95)^{t-k+1}} 5(0.95)^{t-k+1} \end{aligned}$$

$$(1.49) \quad \text{Sector 1 in the foreign country: for all } t, a_{2h,t} = \overline{a_{high}} = 10$$

**Equilibrium without Policy Intervention.** In the home country, before refinement arrives in sector 1, industries in sector 1 hire 0.407 units of labor and those in sector 2 hire 0.593 units of labor. Thus, the home country produces 2.04 units of aggregate domestic sector 1 product and 5.93

units of domestic sector 2 product. The foreign country uses 0.578 workers in sector 1 and 0.422 workers in sector 2. Accordingly, the foreign country manufactures 5.78 units of aggregate foreign sector 1 product and 4.22 units of foreign sector 2 product. This static equilibrium before the arrival of the refinement coincides with the result of the classical Ricardian model. The home country has a comparative advantage in sector 2, and the foreign country has a comparative advantage in sector 1. Thus, the home country uses more resources for producing products in sector 2 and becomes a net exporter of sector 2 products. In contrast, the foreign country uses more resources for producing products in sector 1 and becomes a net exporter of sector 1 products.

The refinement happens at  $t = 25$ . After the arrival of the refinement, the home country gradually uses more labor to produce goods in sector 1, reflecting increased productivity in sector 1. Since the comparative advantage becomes the same for both countries in the long run, the home and foreign countries will use the same amount of labor for sector 1 and 2 in the long run.

**Equilibrium with Policy Intervention.** Now, I assume that the government in the home country provides a production subsidy to all industries in sector 1 until the refinement happens. The production subsidy rate is assumed to be 20%. In this case, before the refinement happens, the home country naturally uses more labor to produce products in sector 1 than in the case without a production subsidy. 0.458 units of labor are used in sector 1, and 0.542 units of labor are used in sector 2. The home country manufactures 2.29 units of aggregate domestic sector 1 product and 5.42 units of domestic sector 2 products. In the foreign country, 0.537 workers are hired in sector 1, and 0.463 workers are hired in sector 2. Thus, 5.37 units of aggregate foreign sector 1 product are manufactured, and 4.63 units of foreign sector 2 product are manufactured.

Because of the production subsidy, industries in sector 1 produce more, and thus refinement occurs earlier than in the case without policy intervention. The refinement happens at  $t = 22$ . After refinement, the government stops providing the subsidy, and thus the equilibrium dynamics are the same as those after refinement without policy intervention.

**Welfare Comparison.** When there is no policy intervention, the welfare of the home and foreign countries is 927.73 and 977.08, respectively. In the case with a production subsidy, the welfare of both the home and foreign countries increases to 930.80 and 977.37, respectively. This indicates that the industrial policy is welfare-improving for both the home and foreign countries.

TABLE 1.1. Welfare Comparison

	w/o subsidy	with subsidy	welfare
Home	927.73	930.80	Improved
Foreign	977.08	977.37	Improved

Notes: The numbers in the cells represent the welfare level in each case.

To draw implications from the results, I examine the instantaneous utility before refinement occurs. When the government does not implement any policy and refinement has not yet occurred, the utility of the home country is 7.867. It decreases to 7.804 when the government provides a production subsidy to industries in sector 1. This decline in instantaneous utility in the home country results from the distortion caused by the production subsidy. It is surprising that instantaneous utility also decreases in the foreign country upon implementation of the home industrial policy. In the case without the policy, the instantaneous utility in the foreign country is 9.331. However, it falls to 9.278 when the home country implements the industrial policy. The home country produces more products in sector 1 with the production subsidy, leading to a decrease in the relative price of products from sector 1. Since the foreign country is a net exporter of goods in sector 1, this means that the terms of trade worsen for the foreign country. This effect causes the instantaneous utility of the foreign country to decrease. In summary, the industrial policy results in a welfare decrease for both the home and foreign countries before the refinement in sector 1 in the home country.

TABLE 1.2. Temporarily Loss from Distortion before the Arrival of Refinement

Stage	Home country			Foreign country		
	$L_{1h,t}$	$L_{2h,t}$	$u_{h,t}$	$L_{1f,t}$	$L_{2f,t}$	$u_{f,t}$
w/o subsidy	0.407	0.593	7.867	0.578	0.422	9.331
with subsidy	0.458	0.542	7.804	0.537	0.463	9.278

After the refinement arrives in sector 1 in the home country, the productivity of the industries in the sector increases over time. This means consumers in both the home and foreign countries can consume more products from sector 1 at a cheaper price. Thus, if the refinement occurs earlier, it is beneficial for both the home and foreign countries. Because of the industrial policy, the refinement happens faster than in the case without the policy. This effect benefits both the home and foreign countries.

TABLE 1.3. Gain from Early Arrival of Refinement in Sector 1 in Home Country

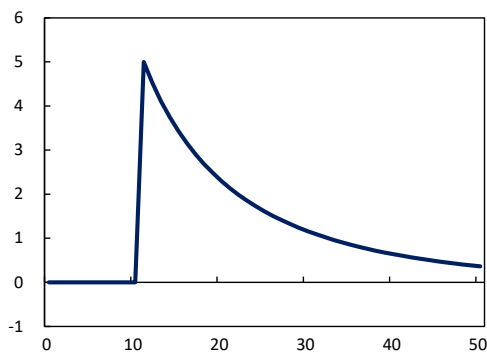
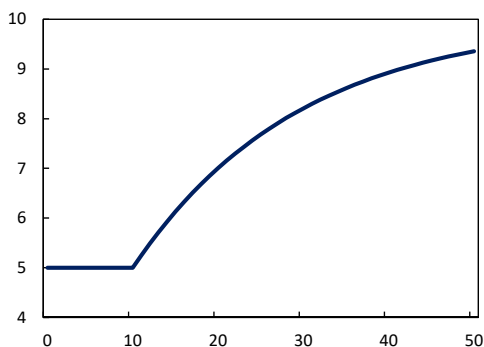
	w/o subsidy	with subsidy
Arrival time	$t = 25$	$t = 22$

In summary, the industrial policy initially makes both the home and foreign countries worse off temporarily, but the long-run gains from faster growth in the targeted industry offset the earlier losses of the two countries. In this numerical example, the long-run gain is larger than the initial loss, and thus the industrial policy eventually increases welfare for both the home and foreign countries. In this model, as in the closed economy model, when: (i) the industry has a high productivity growth potential (high  $\overline{a_{high}}$  or high  $r$ ), (ii) learning-by-doing is strong, or (iii) the discount factor is small (high  $\beta$ ), the industrial policy can improve welfare for both the home and foreign countries. However, compared to the closed economy model, the terms of trade effect plays an important role, especially for the foreign country.

#### 1.4. Empirical Examination about Productivity Dynamics

The results from the model are based on assumptions regarding how productivity changes over time. Therefore, I examine time series data on productivity for certain industries. Given that many countries aim to promote high-tech industries through industrial policies, I focus on high-tech industries such as Semiconductor and Other Electronic Components, Computer and Peripheral Equipment, Communication Equipment, Electronic Instruments, and Industrial Machinery. I use Multifactor Productivity data from the U.S. Bureau of Labor Statistics, spanning from 1987 to 2019.

Figure 1.1 illustrates the productivity dynamics of sector 1 in the home country without policy intervention, as depicted in the numerical example above. The left panel illustrates the dynamics of productivity levels, while the right panel depicts the dynamics of productivity growth rates for the industry. As shown in Figure 1.1, in the model, productivity remains low and does not grow significantly in the early stages of the lifecycle. However, after refinement occurs, productivity begins to increase. The growth rate of productivity is initially very high following the refinement's arrival and gradually declines over time.



(a) Aggregate Productivity Level in the Model

(b) Aggregate Productivity Growth in the Model

FIGURE 1.1. Average Productivity Dynamics of Sector 1 in the Home Country in the Model

Figure 1.2 and 1.3 display the dynamics of productivity levels and productivity growth rates for the five high-tech industries. The solid lines represent the actual data for each year, while the dotted lines represent the five-year moving average of the actual data. The observed patterns closely resemble those suggested by the model for all five industries.

The analysis of productivity in high-tech industries indicates that the productivity dynamics proposed by the model closely align with the observed data. Furthermore, in line with industry lifecycle theory, there is likely a positive externality linked to productivity in the early stages of industry development. This suggests that the implications derived from the model in this paper are likely to hold validity.

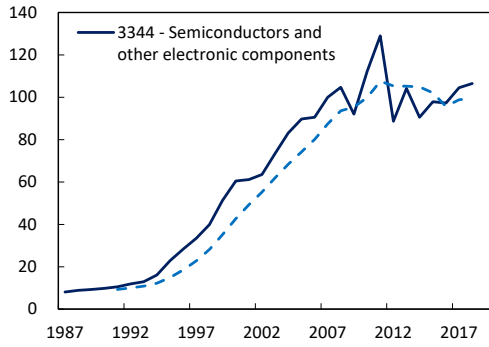
### 1.5. Conclusion and Discussions

In this chapter, I develop a simple open economy model that incorporates productivity dynamics suggested by the industry lifecycle. In this lifecycle, productivity is low and does not grow significantly in the early stage, then after a radical innovation, it grows very fast for a while before tapering off. This model is novel in two main aspects. First, the parameters related to innovation endogenously change based on the stage of the industry lifecycle. This allows the model to provide realistic policy implications. For example, if the growth rate or probability of innovation is constant over time, industrial policy has permanently positive or negative welfare effects, which is not

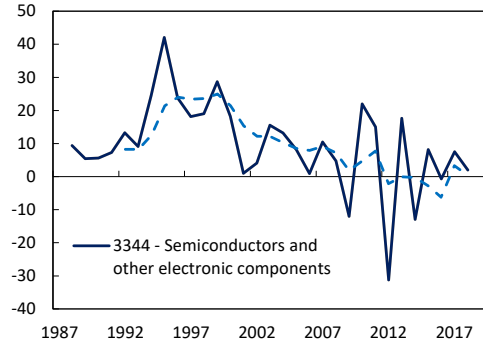
realistic. Second, it focuses on the welfare of other countries, a subject that much of the existing literature overlooks.

With such advantages, the model suggests important policy implications. First, considering industry lifecycle when designing industrial policy is important since the growth potential and degree of externality vary depending on the stage of the targeted industry's lifecycle. Second, policymakers need to take into account the difference in timing when policy costs and benefits occur. The model shows that industrial policy reduces instantaneous utility in the short run due to distortions created by the policy, but it can increase overall welfare by accelerating innovation in the targeted industry in the long run. Third, home industrial policy can increase foreign welfare through the terms-of-trade effect, meaning the foreign country can benefit from the lower home product price due to home innovation.

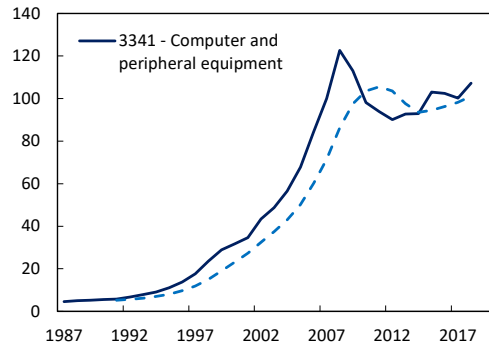
Despite the novelties, there is ample room for improvement in this simple model. First, monopolistic competition can be introduced for the market structure. This can be beneficial because (i) the model can allow for a microfoundation for investment in innovation, such as R&D investment by firms, for which some degree of monopolistic power is required, and (ii) by introducing a free entry condition, the model can capture changes in the entry and exit of firms, which are important for determining the stage of the lifecycle based on industry lifecycle theory (e.g., Jovanovic and MacDonald, 1994; Klepper, 1996). Second, it is interesting to compare various policy tools. In particular, since R&D subsidy targets boosting innovation activity, it is interesting to compare the welfare effects of R&D subsidy with production subsidy and to figure out conditions under which each type of tool is preferred. In the next chapter, I develop a general model framework reflecting these points.



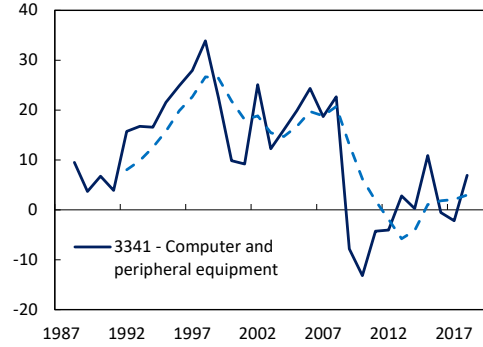
(a) Semiconductor Sector Productivity Level



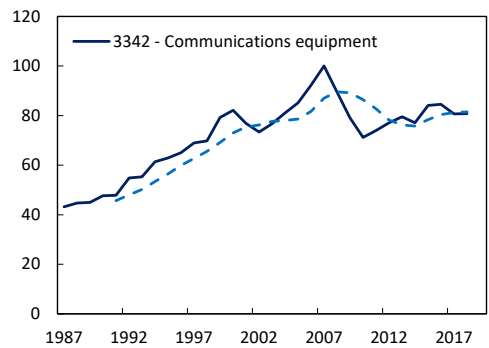
(b) Semiconductor Sector Productivity Growth



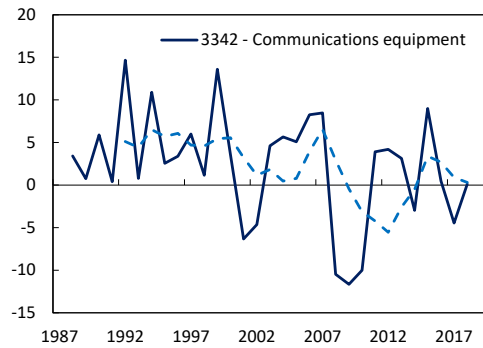
(c) Computer Sector Productivity Level



(d) Computer Sector Productivity Growth



(e) Communication Equipment Sector Productivity Level

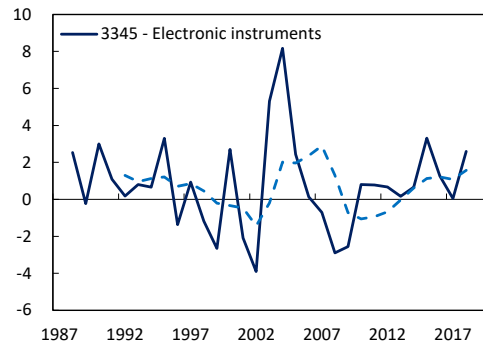
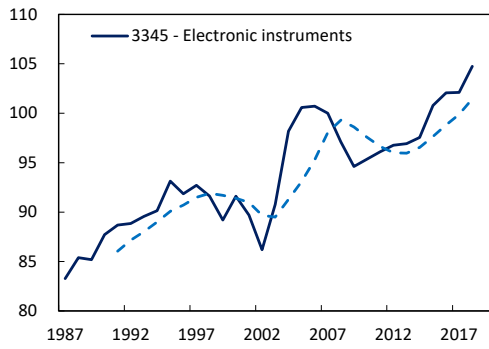


(f) Communication Equipment Sector Productivity Growth

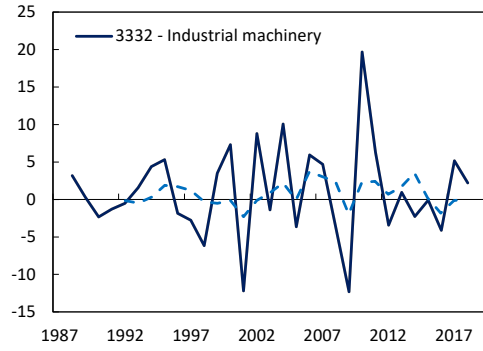
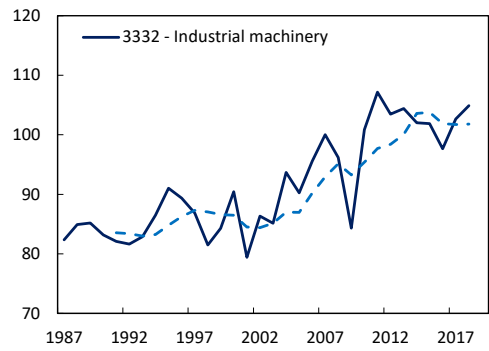
FIGURE 1.2. Productivity Dynamics of High Tech Industries

Notes: The dotted line represents the five-year moving average of the actual data.





(a) Electrical Instrument Sector Productivity Level (b) Electrical Instrument Sector Productivity Growth



(c) Industrial Machinery Sector Productivity Level (d) Industrial Machinery Sector Productivity Growth

FIGURE 1.3. Productivity Dynamics of High Tech Industries

Notes: The dotted line represents the five-year moving average of the actual data.

## Industrial Policy in the context of Industry Lifecycle: Catch-Up versus Frontier Technology Races

### 2.1. Introduction

Based on the discussion points in the previous chapter, this chapter presents a general framework for analyzing the welfare effects of industrial policy for both the domestic economy and its counterpart countries. This chapter demonstrates that the welfare effects depend on where the targeted industry happens to be in its lifecycle in the home country and abroad. Especially, the welfare analysis demonstrates that the welfare effects are very different for the foreign country in two cases: *catch-up* vs *frontier technology races*. Subsidies that aid catch-up have positive spillovers for trading partners, while frontier races targeting firms trying to advance a technological frontier are more likely to have a beggar-thy-neighbor effect.

This chapter suggests a theoretical link between the growth-boosting effect of industrial policy and industry lifecycle theory. Regarding how industrial policy affects economic growth, Rodrik (2006) presents some stylized facts. First, he presents empirical evidence supporting the idea that growth accelerations are associated with the increase in the share of manufacturing industries in an economy. He emphasizes that industrial policies played an important role in successfully achieving such structural changes in some countries such as China and India. Second, he argues that the level of productivity with which a good is produced does converge to that of advanced countries. Moreover, the paper argues that the lower the unit value of goods a country initially produces, the greater is the growth the country will experience. These stylized facts suggest that industrial policy is more likely to succeed in boosting economic growth and eventually improving a country's welfare when it targets innovative, young industries.

That implication can be theoretically supported by industry lifecycle theory such as Abernathy and Utterback (1978) and Jovanovic and MacDonald (1994). The theory predicts that productivity

in an industry remains low in the early stage of the industry lifecycle but then increases rapidly following a radical innovation. In this context, if industrial policy encourages major innovations in a young industry to occur earlier, the industry can accelerate the transition to the high-productivity stage, thereby boosting aggregate economic growth. On the other hand, theory suggests that productivity organically tapers off in an industry in the late stage of its lifecycle. Thus, if industrial policy supports a mature industry, such a policy would not effectively stimulate economic growth. My model is built on this theoretical background.

Another important aspect of the research question concerns how growth-stimulating industrial policy affects the welfare of a foreign country. Existing literature studying the international spillover from home productivity improvement (Ghironi and Melitz, 2005; Matsuyama, 2007; Corsetti et al., 2007) suggests that the welfare of the foreign country is primarily affected by changes in the terms of trade. Those studies mostly consider two conflicting effects for the foreign welfare. First is a “direct effect”, whereby domestic productivity improvement lowers the prices of home products, benefiting foreign consumers. Second is a “indirect effect” in which the domestic wage increases relative to the foreign wage due to increased labor demand, causing the foreign countries’ terms of trade to deteriorate. While my model encompasses these mechanisms for the international spillover of domestic productivity increase, it introduces a novel channel through which a change in the mass of firms in an industry influences the timing of innovation in that industry. Based on this channel, the model suggests the possibility of beggar-thy-neighbor effects stemming from the home country’s growth-stimulating industrial policy.

I also analyze how the foreign country responds to the domestic industrial policy. Policymakers should consider this seriously because if an industrial policy diminishes the welfare of its counterpart countries, those countries may respond by implementing measures to offset the effects of the policy. I introduce a game situation where the home and foreign country compete to foster the same industry. The Nash equilibrium in this game demonstrates the consequent policy competition in which both countries respond to each other’s policy decisions by more aggressively supporting the targeted industry. The analysis suggests if two countries cooperatively support the industry, the welfare outcome is a Pareto improvement compared to the Nash equilibrium.

The chapter is structured as follow. Section 2.2 presents model setup and Section 3.3 analyzes the welfare effect of industrial policy under the influence of the industry lifecycle. Section 2.4 derives theoretical implications through model simulation. Section 2.5 examines Korea's industrial policy in 1970s to check whether the outcomes are consistent with the central mechanism of my model. Section 2.6 compares the welfare effects of a production and R&D subsidy. Section 3.5 concludes.

## 2.2. The Model

I nest the model of industry lifecycle in the model of open economy spillovers by Corsetti et al. (2007) to analyze welfare effects of an industrial policy by taking the industry lifecycle into account.

In the model, the world economy consists of two countries, home and foreign. In each country, there are households, firms, and a government. The size of households is  $L$  in the home country and  $L^*$  in the foreign country.

Since industrial policy supports targeted industries by design, I extend the model in Corsetti et al. (2007) to a model with two industries. Industry 1 in the home country is a young industry with high growth potential. In contrast, industry 2 in the home country is a mature industry where innovation has tapered off. An example of an industry like industry 2 in the model could be agriculture.

In Case 1, both foreign industry 1 and 2 are matured. The home and foreign country can be thought of as a developing and developed country respectively in this case. Thus, home industry 1 is trying to catch up to the technological frontier in industry 1 in this case. I will relax this assumption later and consider another case where the developmental states of the home and foreign country are similar and those countries are competing with each other.

Throughout the paper, I set the home and foreign country wage as numeraire in the home and foreign country respectively for convenience.

**2.2.1. Firms.** There is a continuum of firms with mass  $n_{i,t}$  in industry  $i$  in the home country at time  $t$ . Similarly,  $n_{i,t}^*$  denotes the mass of firms in industry  $i$  in the foreign country at time  $t$ . The mass of firms in each industry in each country is endogenously determined within the model.

There is only one factor of production, labor, which is mobile across industries. The productivity is assumed to be the same for all firms in each industry in each country for simplicity. However, there is a difference in productivity between industries and between countries. Under these assumptions, the firm producing variety  $h$  in industry  $i$  in the home country has the following linear constant returns to scale production function.

$$(2.1) \quad y_{i,t}(h) = a_{i,t}l_{i,t}(h)$$

where  $l_{i,t}(h)$  is labor used in the production for variety  $h$  in industry  $i$ .

A firm also needs to hire  $\frac{1}{v_{i,t}}$  units of labor each period regardless of its amount of production. Firms allocate this fixed cost to different activities, depending on their industry's stage along the lifecycle. As suggested in Abernathy and Utterback (1978) and Klepper (1996), firms in the early stage of the lifecycle need to invest in R&D for product innovation. At this stage, a "dominant design," defined as a product design widely accepted by consumers, has not yet emerged. Consequently, numerous firms producing different varieties enter the industry and focus on product R&D to establish a dominant design. Once a dominant design is established, the industry transitions to a high-productivity stage where the production process becomes highly standardized, and manufacturing productivity rapidly increases. Thus, the fixed cost at this stage is primarily used for process R&D or for the construction and maintenance of large facilities. For simplicity, it is assumed that this fixed cost remains constant over time and is the same for all firms across all industries in each country. Given wage  $w_h$ , the fixed cost  $q_{i,t}(h)$  is then,

$$(2.2) \quad q_{i,t}(h) = \frac{w_h}{v_{i,t}} = \frac{1}{v}$$

All firms in each country sell their product to home and foreign consumers. When a firm sells its product abroad, it entails iceberg cost,  $\tau$ . Therefore, the resource constraint for home variety  $h$  in industry  $i$  at time  $t$  is as follows.

$$(2.3) \quad y_{i,t}(h) \geq Lc_{i,t}(h) + (1 + \tau)L^*c_{i,t}^*(h)$$

where  $c_{i,t}(h)$  and  $c_{i,t}^*(h)$  represent the consumption of the home variety  $h$  in industry  $i$  at time  $t$  by the home and foreign representative consumers, respectively.  $L$  and  $L^*$  denote the number of people in the home and foreign countries, respectively.

Let  $p_{i,t}(h)$  denote the price for home variety  $h$  in industry  $i$  at time  $t$  in home market which is expressed in terms of home wages, and  $p_{i,t}^*(h)$  denote the price for home variety  $h$  in industry  $i$  at time  $t$  in the foreign market which is expressed in terms of foreign wages.  $\epsilon_t$  is the exchange rate which is defined as the relative price of foreign labor in terms of home labor units<sup>1</sup>. The operating profit in domestic labor units, which is revenue minus variable cost, of a home firm producing variety  $h$  in industry  $i$  at time  $t$  is:

$$(2.4) \quad \Pi_{i,t}(h) = p_{i,t}(h)c_{i,t}(h)L + \epsilon_t p_{i,t}^*(h)c_{i,t}^*(h)L^* - l_{i,t}(h)$$

Similar expressions hold for the firms in foreign country.

**2.2.1.1. Innovation and Productivity.** A novel feature of the model in this paper is that the productivity of firms in the industry with high-growth potential endogenously increases. As we have seen in the literature regarding industry lifecycle, it is reasonable to assume that the productivity of firms in an industry changes depending on where the industry is in its lifecycle as follows: Productivity grows slowly for some time after the inception of the industry. However, it increases rapidly after major or radical innovations occur. Then, the growth rate gradually declines and stabilizes at a low level. To capture these productivity dynamics, I refer to Jovanovic and MacDonald (1994) regarding how innovations happen along the industry lifecycle.

Let  $\overline{a_{i,low}}$  and  $\overline{a_{i,high}}$  denote low and high productivity in industry  $i$ . To build intuition for analyzing welfare effects of the growth-boosting industrial policy in the context of industry lifecycle, I define two stages along the cycle, “the early stage” and “high-productivity stage”, as follows.

**DEFINITION 1.** *Industry  $i$  is in “the early stage” of an industry lifecycle, which means*

- i) *The productivity of firms in the industry stays at the lower bound,  $\overline{a_{i,low}}$ , in this stage.*
- ii) *Firms in the industry at this stage have growth potential, as their productivity can increase to  $\overline{a_{i,high}}$  following the occurrence of a refinement.*

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<sup>1</sup>For example, if the foreign wage is twice as expensive as the home wage when compared in a common currency, the exchange rate,  $\epsilon_t$ , is 2. In a model where the home wage is the only numeraire and the world uses one common currency (meaning the exchange rate is not introduced in the model),  $\epsilon_t$  is equivalent to the foreign wage.

DEFINITION 2. *When a refinement occurs in industry  $i$ , the industry enters “the high-productivity stage” of the lifecycle, which means*

- i) *Upon the occurrence of a refinement in the industry, all firms immediately achieve high productivity,  $\overline{a_{i,high}}$ .<sup>2</sup>*
- ii) *Productivity of firms in the industry does not grow any more within the lifecycle.*

Based on Jovanovic and MacDonald (1994) and Abernathy and Utterback (1978), refinement is defined as a major technological innovation which creates a dominant design in the industry. After the emergence of the dominant design, firms which successfully create or follow the dominant design have a broad consumer base but the other firms which fail to do it exit. The surviving firms in the industry need to produce a large quantity of their goods and start to focus their R&D on improving manufacturing productivity. Accordingly, these process R&D efforts lead a significant increase in productivity within the industry.

How does the refinement occur? Jovanovic and MacDonald (1994) argue that the technology is refined from outside the industry. Thus, the refinement is assumed to occur exogenously in their paper. However, in most cases, a huge innovation from outside the industry cannot be applied immediately within the industry. Thus, one might reasonably assume that the timing of refinement in industry  $i$  depends on accumulated knowledge stock in the industry,  $Q_{i,t}$ . Formally, the relevant refinement is assumed to occur as follows.

ASSUMPTION 3. *Refinement occurs at time  $t_i^r$  when the accumulated knowledge stock in industry  $i$ ,  $Q_{i,t}$ , exceeds a threshold,  $\bar{Q}_i$ , for the first time:  $Q_{i,t_i^r} \geq \bar{Q}_i$  and  $Q_{i,t_i^r-j} < \bar{Q}_i$  for  $0 < j < t_i^r$*

Additionally, I assume that knowledge stock in an industry is accumulated by firms' activities in the industry and thus it is increasing with the accumulated number of operating firms in the industry as follows.

ASSUMPTION 4.  $Q_{i,t} = Q_i \left( \int_0^t n_{i,j} dj \right)$  where  $Q_i(\cdot)$  is an increasing function

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<sup>2</sup>As in Jovanovic and MacDonald (1994), it is more realistic to assume that firms get high productivity with a probability after the refinement. However, since results of the model do not change qualitatively with the assumption in Definition 2, I use the simplified assumption for convenience.

Assumption 4 is critical to later results and is based on three motivations. First, knowledge stock in an industry is accumulated as R&D efforts of firms in the industry are accumulated. This is in line with Grossman and Helpman (1991) in that the commercial exploitation of technology requires a substantial investment from firms. As mentioned in Section 2.2.1, each firm devotes its fixed cost to product R&D before the emergence of a dominant design. Since every firm spends the same fixed cost in the model, the total accumulated R&D efforts of firms is proportional to the accumulated number of operating firms ( $\int_0^t \int_0^{n_{i,j}} q_{i,j}(h) dh dj = \frac{1}{v} \int_0^t n_{i,j} dj$ ). In the early stage of an industry lifecycle, firms continuously experiment to design a new consumer product. Even though one firm cannot directly see another firm's R&D, it can still learn from consumers' response to the firm's product. Second, the timing of refinement also depends on the cumulative production in the industry, which is related with the learning-by-doing effect in Melitz (2005). In the model, the productivity of all firms in each industry is assumed to be the same and thus the amount of production by each firm in an industry is also the same in equilibrium ( $y_{i,t}(h) = y_i$ ). Accordingly, the cumulative production in the industry is proportional to the accumulated number of operating firms ( $\int_0^t \int_0^{n_{i,j}} y_{i,j}(h) dh dj = \int_0^t \int_0^{n_{i,j}} y_i dh dj = y_i \int_0^t n_{i,j} dj$ ). Therefore, here, cumulative industry production is positively correlated with the probability of the emergence of a refinement or dominant design that enables standardization in production as in Jovanovic and MacDonald (1994), reflected in a higher industry productivity parameter,  $a_{i,t}$ . Lastly, having more firms operate generates more competition. Aghion et al. (2015) argues that more competition in an industry causes firms to increase their efforts to innovate, boosting productivity growth.

Based on Assumption 3 and 4, the timing of refinement in industry  $i$ ,  $t_r$ , is determined as follows

$$(2.5) \quad t_i^r = \frac{Q_i^{-1}(\bar{Q}_i)}{n_{i,s1}}$$

where  $n_{i,s1}$  is the mass of firms in industry  $i$  in a period in the early stage.



**2.2.2. Households.** The utility function of the representative consumer in the home country at time  $t$  has the following form.

$$(2.6) \quad U_t = \frac{C_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} - l_t$$

where  $C_t$  is a comprehensive consumption index at time  $t$ ,  $\psi$  is the intertemporal elasticity of substitution, and  $l_t$  is labor supply by the representative consumer at time  $t$ . The comprehensive consumption index aggregates industry-level consumption indexes,  $C_{1,t}$  and  $C_{2,t}$ , in the following way:

$$(2.7) \quad C_t = \left[ C_{1,t}^{1-\frac{1}{\sigma}} + C_{2,t}^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where  $\sigma$  is the elasticity of substitution across industries. The industry  $i$  consumption index,  $C_{i,t}$ , is a composite of varieties produced in industry  $i$  in each country,  $C_{hi,t}$  and  $C_{fi,t}$ , as follows.

$$(2.8) \quad C_{i,t} = \left[ C_{hi,t}^{1-\frac{1}{\eta}} + C_{fi,t}^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where  $\eta$  is the elasticity of substitution between the origin-specific industry-level consumption aggregates, which is referred to “macro” elasticity of substitution in Feenstra et al. (2018). Lastly, the home and foreign industry-level consumption aggregates,  $C_{hi,t}$  and  $C_{fi,t}$ , are assumed to be

$$(2.9) \quad C_{hi,t} = \left[ \int_0^{n_{i,t}} c_{i,t}(h)^{1-\frac{1}{\gamma}} dh \right]^{\frac{\gamma}{\gamma-1}}, \quad C_{fi,t} = \left[ \int_0^{n_{i,t}^*} c_{i,t}(f)^{1-\frac{1}{\gamma}} dh \right]^{\frac{\gamma}{\gamma-1}}$$

where  $\gamma$  is the elasticity of substitution across varieties of an industry in each country, which I call as “micro” elasticity of substitution as in Feenstra et al. (2018).<sup>3</sup> It is worth to mention that the micro elasticity of substitution is the same regardless of industries and countries in the model. As in Lashkaripour and Lugovskyy (2023), if the micro elasticity of substitution differs across industries, then each industry has different static scale economy due to the difference. Thus, when a resource allocation changes in an economy due to industrial policy, the difference in static scale economies

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<sup>3</sup>The model in Corsetti et al. (2007) assumes that micro elasticity of substitution in an industry is the same as macro elasticity of substitution. In that case, when  $\gamma$  is large, which is not unusual, industrial structure changes too radically by productivity shocks. For this reason, I set an assumption that micro elasticity of substitution can be different from macro elasticity of substitution. The more generalized model has an advantage that industrial structure changes gradually by external shocks.

across industries generate an additional welfare effect. Since this paper focuses on welfare effects caused by hastening the occurrence of innovation (a dynamic scale economy), I abstract from policy incentives based on differences in the micro elasticity of substitution (a static scale economy) by making  $\gamma$  the same across industries. I also assume  $\gamma > \eta$  which means that varieties produced in the same country are more substitutable than varieties produced in different countries (i.e., consumers view Ford and Chevrolet as closer substitutes than Ford and Fiat.)

Households own all firms in their own country. Each household receives an equal share of profits of all firms in their country:

$$(2.10) \quad \Pi_t \equiv \int_0^{n_{1,t}} \Pi_{1,t}(h)dh + \int_0^{n_{2,t}} \Pi_{2,t}(h)dh$$

The representative consumer maximizes its utility (3.1) in time  $t$  subject to the following budget constraint<sup>4</sup>:

$$(2.11) \quad P_t C_t = l_t + \Pi_t - \frac{T_t}{L}$$

where  $T_t$  is a lump-sum tax which I will explain in detail in the next section.

By solving the representative consumer's utility maximization problem, the representative consumer's demand for  $C_{i,t}$ ,  $C_{hi,t}$ ,  $C_{fi,t}$ ,  $c_{i,t}(h)$ ,  $c_{i,t}(f)$ , and labor supply,  $l$ , satisfies the first-order conditions:

$$(2.12) \quad C_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\sigma} C_t$$

$$(2.13) \quad C_{hi,t} = \left( \frac{P_{hi,t}}{P_{i,t}} \right)^{-\eta} C_{i,t}, \quad C_{fi,t} = \left( \frac{P_{fi,t}}{P_{i,t}} \right)^{-\eta} C_{i,t}$$

$$(2.14) \quad c_{i,t}(h) = \left( \frac{p_{i,t}(h)}{P_{hi,t}} \right)^{-\gamma} C_{hi,t}, \quad c_{i,t}(f) = \left( \frac{p_{i,t}(f)}{P_{fi,t}} \right)^{-\gamma} C_{fi,t}$$

$$(2.15) \quad P_t C_t^{\frac{1}{\psi}} = w_t = 1$$

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<sup>4</sup>Since consumers do not save in the model, they maximize their current utility in each period by only considering their current budget constraint.

where  $P_t$ ,  $P_{i,t}$ , and  $P_{j,t}$  are the utility-based consumer price index (CPI), the industry  $i$  composite price index, and the country  $j$  specific industry  $i$  composite price index, which are defined as the minimum expenditure required to purchase one unit of the comprehensive consumption index,  $C_t$ , the industry  $i$  composite consumption index,  $C_{i,t}$ , and country  $j$  specific industry  $i$  composite aggregate,  $C_{j,t}$  respectively.  $P_t$ ,  $P_{i,t}$ , and  $P_{j,t}$  can be expressed as:

$$(2.16) \quad P_t = \left[ P_{1,t}^{1-\sigma} + P_{2,t}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

$$(2.17) \quad P_{i,t} = \left[ P_{hi,t}^{1-\eta} + P_{fi,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

$$(2.18) \quad P_{hi,t} = \left[ \int_0^{n_{i,t}} p_{i,t}(h)^{1-\gamma} dh \right]^{\frac{1}{1-\gamma}}, \quad P_{fi,t} = \left[ \int_0^{n_{i,t}^*} p_{i,t}(f)^{1-\gamma} df \right]^{\frac{1}{1-\gamma}}$$

Households provide labor in a competitive market both for fixed-cost related and production activities and thus the resource constraint for labor is:

$$(2.19) \quad Ll_t \geq \int_0^{n_{1,t}} \frac{y_{1,t}(h)}{a_{1,t}} dh + \int_0^{n_{2,t}} \frac{y_{2,t}(h)}{a_{2,t}} dh + \int_0^{n_{1,t}} q_{1,t}(h) dh + \int_0^{n_{2,t}} q_{2,t}(h) dh$$

Again, similar expressions hold in the foreign country.

**2.2.3. Government and Industrial Policy.** In this paper, as the definition of industrial policy in Pack and Saggi (2006)<sup>5</sup>, I assume *ex ante* that the government wants to promote industries with high growth potential and uses industrial policy to achieve its goal. In this section, I first analyze the impact of a production subsidy as an example of one type of industrial policy. I will also examine the effect of an R&D subsidy in a later section.

Assume that the government provides a production subsidy to firms in industry 1 until refinement occurs in the industry. The production subsidy rate for industry  $i$  at time  $t$  is  $s_{i,t}$ . I assume the government levies a lump sum tax from the consumers to fund the production subsidy. The

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<sup>5</sup>According to Pack and Saggi (2006), industrial policy is defined as “any type of selective intervention or government policy that attempts to alter the structure of production toward sectors that are expected to offer better prospects for economic growth than would occur in the absence of such intervention.”

amount of the total subsidy at time  $t$ ,  $S_t$ , is:

$$(2.20) \quad S_t = \int_0^{n_{1,t}} s_{1,t} p_{1,t}(h) y_{1,t}(h) dh + \int_0^{n_{2,t}} s_{2,t} p_{2,t}(h) y_{2,t}(h) dh = s_{1,t} n_{1,t} p_{1,t} y_{1,t} + s_{2,t} n_{2,t} p_{2,t} y_{2,t}$$

Thus government budget constraint is as follows:

$$(2.21) \quad s n_{1,t} p_{1,t} y_{1,t} = T_t$$

**2.2.4. Static Equilibrium.** I will describe the equilibrium without any production subsidy first and then explain how equilibrium conditions change by introducing the production subsidy.

**2.2.4.1. Equilibrium without Production Subsidy.**

**Prices.** Firms face monopolistic competition where a firm sets its price with constant markups over marginal costs as follows:

$$(2.22) \quad p_{i,t}(h) = \frac{\gamma}{\gamma-1} \frac{1}{a_{i,t}} \equiv p_{i,t}, \quad \epsilon_t p_{i,t}^*(h) = (1+\tau) \frac{\gamma}{\gamma-1} \frac{1}{a_{i,t}} = (1+\tau) p_{i,t}$$

$$(2.23) \quad p_{i,t}^*(f) = \frac{\gamma}{\gamma-1} \frac{1}{a_{i,t}^*} \equiv p_{i,t}^*, \quad \frac{p_{i,t}(f)}{\epsilon_t} = (1+\tau) \frac{\gamma}{\gamma-1} \frac{1}{a_{i,t}^*} = (1+\tau) p_{i,t}^*$$

where  $\tau$  represents the trade cost.

Using (2.22) and (2.23), the country  $j$  specific industry  $i$  composite price index in the home and foreign country,  $P_{j,i,t}$  and  $P_{j,i,t}^*$ , can be expressed respectively:

$$(2.24) \quad P_{hi,t} = p_{i,t} n_{i,t}^{\frac{1}{1-\gamma}}, \quad P_{fi,t} = (1+\tau) \epsilon_t p_{i,t}^* n_{i,t}^* \frac{1}{1-\gamma}$$

$$(2.25) \quad P_{hi,t}^* = (1+\tau) \frac{p_{i,t}}{\epsilon_t} n_{i,t}^{\frac{1}{1-\gamma}}, \quad P_{fi,t}^* = p_{i,t}^* n_{i,t}^* \frac{1}{1-\gamma}$$

and industry  $i$  composite price index in the home and foreign country,  $P_{i,t}$  and  $P_{i,t}^*$ , are:

$$(2.26) \quad P_{i,t} = \left( p_{i,t}^{1-\eta} n_{i,t}^{\frac{1-\eta}{1-\gamma}} + \phi (\epsilon_t p_{i,t}^*)^{1-\eta} n_{i,t}^* \frac{1-\eta}{1-\gamma} \right)^{\frac{1}{1-\eta}}$$

$$(2.27) \quad P_{i,t}^* = \left( p_{i,t}^*{}^{1-\eta} n_{i,t}^* \frac{1-\eta}{1-\gamma} + \phi \left( \frac{p_{i,t}}{\epsilon_t} \right)^{1-\eta} n_{i,t}^{\frac{1-\eta}{1-\gamma}} \right)^{\frac{1}{1-\eta}}$$

where  $\phi \equiv (1 + \tau)^{1-\eta}$ .

Based on (2.16), (2.22), (2.23), (2.26) and (2.27), we can see that the utility based CPI in home and foreign country,  $P_t$  and  $P_t^*$ , are determined by five endogenous variables,  $n_{1,t}$ ,  $n_{2,t}$ ,  $n_{1,t}^*$ ,  $n_{2,t}^*$ , and  $\epsilon_t$ .

**Zero-Profit Conditions.** Free entry implies that profit is zero in equilibrium. Thus, a firm's operating profit in each industry should equal the fixed cost in both the home and foreign country, as follows:

$$\begin{aligned}
(2.28) \quad \Pi_{i,t}(h) &= \frac{p_{i,t}(h)y_{i,t}(h)}{\gamma} \\
&= \frac{p_{i,t}(h)}{\gamma} [c_{i,t}(h)L + (1 + \tau)c_{i,t}^*(h)L^*] \\
&= \frac{1}{\gamma} \left( \frac{p_{i,t}(h)}{P_{hi,t}} \right)^{1-\gamma} \left[ \left( \frac{P_{hi,t}}{P_{i,t}} \right)^{1-\eta} \left( \frac{P_{i,t}}{P_t} \right)^{1-\sigma} P_t^{1-\psi} L + \phi \epsilon_t^\eta \left( \frac{P_{hi,t}}{P_{i,t}^*} \right)^{1-\eta} \left( \frac{P_{i,t}^*}{P_t^*} \right)^{1-\sigma} P_t^{*1-\psi} L^* \right] \\
&= \frac{1}{v}
\end{aligned}$$

$$\begin{aligned}
(2.29) \quad \Pi_{i,t}^*(f) &= \frac{p_{i,t}^*(f)y_{i,t}^*(f)}{\gamma} \\
&= \frac{p_{i,t}^*(f)}{\gamma} [c_{i,t}^*(f)L^* + (1 + \tau)c_{i,t}(f)L] \\
&= \frac{1}{\gamma} \left( \frac{p_{i,t}^*(f)}{P_{fi,t}^*} \right)^{1-\gamma} \left[ \left( \frac{P_{fi,t}^*}{P_{i,t}^*} \right)^{1-\eta} \left( \frac{P_{i,t}^*}{P_t^*} \right)^{1-\sigma} P_t^{*1-\psi} L^* + \phi \epsilon_t^{-\eta} \left( \frac{P_{fi,t}^*}{P_{i,t}} \right)^{1-\eta} \left( \frac{P_{i,t}}{P_t} \right)^{1-\sigma} P_t^{1-\psi} L \right] \\
&= \frac{1}{v^*}
\end{aligned}$$

**Balance of Payment Equilibrium Condition.** I assume balanced trade in the model where the value of a country's imports is the same as the value of its exports. Thus, the following equation holds in the equilibrium.

$$(2.30) \quad (1 + \tau)L^* [p_{1,t}(h)c_{1,t}^*(h)n_{1,t} + p_{2,t}(h)c_{2,t}^*(h)n_{2,t}] = \epsilon_t(1 + \tau)L [p_{1,t}^*(f)c_{1,t}(f)n_{1,t}^* + p_{2,t}^*(f)c_{2,t}(f)n_{2,t}^*]$$

Firm Size and Labor. From (2.22), (2.23), (2.28), and (2.29), the size of a firm from industry  $i$  in the home or foreign country, respectively, is determined as follows:

$$(2.31) \quad y_{i,t}(h) = \frac{(\gamma - 1)a_{i,t}}{v}, \quad y_{i,t}^*(f) = \frac{(\gamma - 1)a_{i,t}^*}{v^*}$$

Based on (2.31), the amount of labor hired for production by a firm in industry  $j$  in the home and foreign country is:

$$(2.32) \quad l_{i,t}(h) = \frac{\gamma - 1}{v}, \quad l_{i,t}^*(f) = \frac{\gamma - 1}{v^*}$$

Since every firm also hires  $\frac{1}{v}$  and  $\frac{1}{v^*}$  unit of labor for fixed cost activities at home and abroad respectively, the aggregate labor demand in each economy ( $L_t$  and  $L_t^*$ ) is given by<sup>6</sup>

$$(2.33) \quad L_t = \frac{\gamma(n_{1,t} + n_{2,t})}{v}, \quad L_t^* = \frac{\gamma(n_{1,t}^* + n_{2,t}^*)}{v^*}$$

From the aggregate labor demand (2.33) and the representative consumer's budget constraint ( $P_t^{1-\psi} = l_t$ ,  $P_t^{*1-\psi} = l_t^*$ ), labor supply from the representative consumer in the home and foreign country is determined as follow

$$(2.34) \quad l_t = \frac{\gamma(n_{1,t} + n_{2,t})}{Lv} = P_t^{1-\psi}, \quad l_t^* = \frac{\gamma(n_{1,t}^* + n_{2,t}^*)}{L^*v^*} = P_t^{*1-\psi}$$

**Definition of Equilibrium.** The system of five equations, which are two zero-profit conditions (one for each industry) in the home country from (2.28), two zero-profit conditions in the foreign country from (2.29), and the balance of payment equilibrium (C.30), determines the five endogenous variables,  $n_{1,t}$ ,  $n_{2,t}$ ,  $n_{1,t}^*$ ,  $n_{2,t}^*$ , and  $\epsilon_t$  as functions of exogenous variables,  $v$ ,  $v^*$ ,  $L$ ,  $L^*$ ,  $a_{1,t}$ ,  $a_{2,t}$ ,  $a_{1,t}^*$  and  $a_{2,t}^*$ .

This equilibrium will be used in the welfare analysis for the period when the home government stops giving subsidies following the emergence of the refinement.

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<sup>6</sup>Total labor demand from a firm is  $\frac{\gamma}{v}$ , which the sum of the labor for production ( $\frac{\gamma-1}{v}$ ) and fixed cost activities ( $\frac{1}{v}$ ), in both industries in the home country. Similarly, total labor demand from a firm in both industries is  $\frac{\gamma}{v^*}$  abroad.

2.2.4.2. **Equilibrium with Production Subsidy.** When the home government gives a subsidy to firms in industry 1, the representative firm in the home industry 1 changes its price to:

$$(2.35) \quad p_{1,t}(h) = \frac{\gamma}{\gamma-1} \frac{1}{(1+s)a_{1,t}} = p_{1,t}, \quad \epsilon p_{1,t}^*(h) = (1+\tau) \frac{\gamma}{\gamma-1} \frac{1}{(1+s)a_{1,t}} = (1+\tau)p_{1,t}$$

Also, the profit function of firms in industry 1 changes from (2.28) as follows:

$$(2.36) \quad \begin{aligned} \Pi_{1,t}(h) &= \frac{(1+s)p_{1,t}(h)y_{1,t}(h)}{\gamma} \\ &= \frac{(1+s)p_{1,t}(h)}{\gamma} [c_{1,t}(h)L + (1+\tau)c_{1,t}^*(h)L^*] \\ &= \frac{1+s}{\gamma} \left( \frac{p_{i,t}(h)}{P_{hi,t}} \right)^{1-\gamma} \left[ \left( \frac{P_{hi,t}}{P_{i,t}} \right)^{1-\eta} \left( \frac{P_{i,t}}{P_t} \right)^{1-\sigma} P_t^{1-\psi} L + \phi \epsilon_t^\eta \left( \frac{P_{hi,t}}{P_{i,t}^*} \right)^{1-\eta} \left( \frac{P_{i,t}^*}{P_t^*} \right)^{1-\sigma} P_t^{*1-\psi} L^* \right] \\ &= \frac{1}{v} \end{aligned}$$

The equations expressing the relationship between the size of a firm and the amount of labor it hires for production are still given by (2.31) and (2.32).

The expression for the labor supply is also same as that in the equilibrium without a production subsidy, but the representative consumer's budget constraint in the home country changes to  $P_t^{1-\psi} + \frac{T_t}{L} = l_t$ :

$$(2.37) \quad l_t = \frac{\gamma(n_{1,t} + n_{2,t})}{Lv} = P_t^{1-\psi} + \frac{T_t}{L}$$

Now, the system of equilibrium equations which determines the five endogenous variables,  $n_{1,t}$ ,  $n_{2,t}$ ,  $n_{1,t}^*$ ,  $n_{2,t}^*$ , and  $\epsilon_t$ , as functions of exogenous variables,  $v$ ,  $v^*$ ,  $L$ ,  $L^*$ ,  $a_{1,t}$ ,  $a_{2,t}$ ,  $a_{1,t}^*$  and  $a_{2,t}^*$ , consists of (2.36), the zero profit condition for industry 2 in the home country (2.28), two zero profit conditions for each industry in the foreign country (2.29) and balance of payments equation (C.30).

This equilibrium will be employed in the welfare analysis for the period during which the home government provides subsidies to firms in Industry 1, prior to the occurrence of refinement.

### 2.3. Welfare Analysis

In the model, the central mechanisms through which industrial policy affects the welfare of the home and foreign country is a change in the timing of innovation in the two countries. For this reason, a dynamic model is necessary to analyze the welfare effects of industrial policy. I assume that the model is in continuous time and the welfare of the representative consumer is defined as following:

$$(2.38) \quad \ln W = \int_0^{\infty} e^{-\rho t} \ln U_t dt$$

where  $\rho$  denotes discount rate.

Since the equations in the system of equilibrium are non-linear, it is hard to provide a general closed form solution. Thus, as in Corsetti et al. (2007), I set a symmetric initial condition where  $v = v^* = L = L^* = a_{1,t} = a_{2,t} = a_{1,t}^* = a_{2,t}^* = 1$  and  $s = s^* = 0$ . With this initial condition, there is a symmetric equilibrium such that  $\epsilon_t = 1$ ,  $n_{1,t} = n_{2,t} = n_{1,t}^* = n_{2,t}^*$ ,  $l_t = l_t^* = P_t^{1-\psi} = P_t^{*1-\psi} = \gamma(n_{1,t} + n_{2,t}) = \gamma(n_{1,t}^* + n_{2,t}^*)$ .

I impose restrictions on  $\psi$  such that  $\psi < 1$ . This restriction is necessary to reflect a stylized fact documented in industry lifecycle theory by Jovanovic and MacDonald (1994) and Klepper (1996) that the mass of firms decreases after significant productivity improvements.<sup>7</sup> Additionally, this condition, in conjunction with  $\gamma > 1$ , satisfies  $\gamma > \psi$ . This implies that labor supply is not excessively elastic in comparison to the elasticity of demand for goods, and it ensures that entry of new firms exerts downward pressure on profits.

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<sup>7</sup>In the model, when  $a_{1,t}$  increases, there are three effects: income, relative wage, and substitution effect which I will define in detail in the next section. The direction of substitution effect is unambiguous for each industry. The productivity increase in industry 1 makes the industry 1 composite price index decrease relative to the industry 2 composite price index. This leads the demand for varieties and the mass of firms in industry 1 to increase, and affects industry 2 in the opposite way. However, the direction of the income and relative wage effect depends on  $\psi$ . The productivity increase leads to an increase in the real wage which creates the income effect. In addition, it causes firms in the home country to demand more labor, which makes the home wage to increase relative to the foreign wage. The direction of these two effect is the same for both industries. If  $\psi > 1$ , an increase in the real wage leads to an increase in the total expenditure of the representative consumer ( $P_t C_t = P_t^{1-\psi}$ ) and it leads to an increase in the mass of firms in both industries. In contrast, If  $\psi < 1$ , the income effect makes the mass of firms in both industries decrease. Overall, in order that the mass of firms decreases in industry 1 after a productivity increase,  $\psi$  must be smaller than 1. Thus, considering all three effects, when  $\psi > 1$ , the productivity increase in the home industry 1 causes the mass of firms in in the industry to increase always. Since this is contrast to the stylized fact, I exclude the case of  $\psi \geq 1$ .



**2.3.1. Case 1: Catch-Up.** I assume that industry 1 in the home country is young and has high growth potential. Productivity of firms in industry 1 in the home country is initially low, so  $a_{1,t} = \overline{a_{1,low}} = 1$ . After a refinement occurs at time  $t_r(s)$ , the productivity of firms in the home industry 1 jumps to  $a_{1,t} = \overline{a_{1,high}} = k > 1$ . It is important to note that the timing of refinement,  $t_r(s)$ , depends on the production subsidy because the mass of operating firms in industry 1 varies by the subsidy, which changes the speed of accumulation of the knowledge stock in the industry. In contrast to the home industry 1, I start with an assumption that the home industry 2 and both industry 1 and 2 in the foreign country are mature, which means that innovations rarely occur in those industries and thus productivity of those industries does not change over time:  $a_{2,t} = a_{1,t}^* = a_{2,t}^* = 1$  for all  $t$ . This case is appropriate for analyzing the situation where the home and foreign country are a developing and developed country respectively, and the home industry 1 is trying to **catch up** to the technological frontier in the foreign industry 1. In Case 2, I will assume instead that the foreign industry 1 is also still young, creating a **frontier technology race**.

Under the above mentioned assumptions, there are two stages over time in Case 1. The stage 1 and 2 are defined:

DEFINITION 3.

- *Stage 1* ( $0 < t < t_r(s)$ ) is a time period when a refinement has not yet occurred in the home industry 1 ( $a_{1,t} = 1$ )
- *Stage 2* ( $t \geq t_r(s)$ ) is a time period after an occurrence of a refinement in the home industry 1 ( $a_{1,t} = k > 1$ )

Based on Definition 3, the welfare of the home and foreign representative consumer can be rewritten as follows.

$$(2.39) \quad \ln W = \int_0^{t_r(s)} e^{-\rho t} \ln U_1 dt + \int_{t_r(s)}^{\infty} e^{-\rho t} \ln U_2 dt$$

$$(2.40) \quad \ln W^* = \int_0^{t_r(s)} e^{-\rho t} \ln U_1^* dt + \int_{t_r(s)}^{\infty} e^{-\rho t} \ln U_2^* dt$$

where  $U_1$  and  $U_2$  ( $U_1^*$  and  $U_2^*$ ) denote the home (foreign) representative consumer's utility in stage 1 and 2 respectively. In what follows, I take a first-order approximation of this model in the

neighborhood of the initial symmetric equilibrium mentioned above and analyze the local effects of industrial policy:

$$(2.41) \quad \frac{d \ln W}{ds} = \underbrace{\int_0^{t_r(0)} e^{-\rho t} \frac{d \ln U_1}{ds} dt}_{\text{Short-run resource reallocation effect}} \quad \underbrace{-e^{-\rho t_r(0)} (\ln U_2 - \ln U_1) \frac{dt_r(s)}{ds}}_{\text{Long-run gain from speeding up innovation}}$$

$$(2.42) \quad \frac{d \ln W^*}{ds} = \underbrace{\int_0^{t_r(0)} e^{-\rho t} \frac{d \ln U_1^*}{ds} dt}_{\text{Short-run resource reallocation effect}} \quad \underbrace{-e^{-\rho t_r(0)} (\ln U_2^* - \ln U_1^*) \frac{dt_r(s)}{ds}}_{\text{Long-run gain from speeding up innovation}}$$

As it can be seen from (C.40) and (3.31), the production subsidy has two effects in terms of welfare at home and abroad: a short-run resource reallocation effect ( $\int_0^{t_r(s)} e^{-\rho t} \frac{d \ln U_1}{ds} dt$  and  $\int_0^{t_r(s)} e^{-\rho t} \frac{d \ln U_1^*}{ds} dt$  respectively) and a long-run gain from speeding up innovation ( $-e^{-\rho t_r(0)} (\ln U_2 - \ln U_1) \frac{dt_r(s)}{ds}$  and  $-e^{-\rho t_r(0)} (\ln U_2^* - \ln U_1^*) \frac{dt_r(s)}{ds}$  respectively). I will look into each effect in detail.

**2.3.1.1. Short-run Resource Reallocation Effect.** When the home government subsidizes firms in industry 1, the policy affects resource allocation in both the home and foreign countries. Since a refinement hasn't occurred yet, there is no growth effect in this stage. In the welfare aspect, the short-run resource reallocation effect for the home and foreign country is reflected in the first term in equation (C.40) and (3.31) ( $\int_0^{t_r(s)} e^{-\rho t} \frac{d \ln U_1}{ds} dt$  and  $\int_0^{t_r(s)} e^{-\rho t} \frac{d \ln U_1^*}{ds} dt$  respectively). I analyze how the production subsidy affects the mass of firms, prices and the relative wage and how such changes eventually affect the instantaneous welfare in Stage 1 at home and abroad. The detailed method and results are presented in Appendix C.4.

**Resource Reallocation.** Change in the mass of firms in the home and foreign industry 1 in Stage 1 plays a central role in determining overall welfare effects of the production subsidy in the model. Thus, I first explain how the mass of firms in the home and foreign industry 1 changes by the subsidy. Regarding this, there are three effects in the model: an income effect, a relative wage effect and a substitution effect. The sign and size of the three effects for each industry are specified as follows:

- The income effect, which corresponds to the first term in (C.73), (B.26), (C.74) and (B.28) for each industry, is  $\frac{(\gamma-1)\psi}{4(\gamma-\psi)}$  for every industry.

- The relative wage effect, which corresponds to the second term in (C.73), (B.26), (C.74) and (B.28) for each industry, is  $\frac{(\gamma-1)\psi}{4(\gamma-\psi)} \left(1 + \frac{2\gamma\phi(1-\psi)}{\Delta}\right)$  for the both home industries and  $-\frac{(\gamma-1)\psi}{4(\gamma-\psi)} \left(1 + \frac{2\gamma\phi(1-\psi)}{\Delta}\right)$  for the both foreign industries.
- The substitution effect, which corresponds to the third term in (C.73), (B.26), (C.74) and (B.28) for each industry, is  $\frac{\gamma-1}{\gamma-\sigma} \left(\sigma + \frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)$  and  $-\frac{\gamma-1}{\gamma-\sigma} \left(\sigma + \frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)$  for the home industry 1 and 2,  $-\frac{\gamma-1}{\gamma-\sigma} \left(\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)$  and  $\frac{\gamma-1}{\gamma-\sigma} \left(\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)$  for the foreign industry 1 and 2.

The following lemma shows that the income effect is derived from the world market clearing condition (2.43) where the left hand side is the world consumption value and the right hand side is the world production value in the home currency.

$$(2.43) \quad P_t^{1-\psi} + \epsilon_t P_t^{*1-\psi} = p_{1,t} y_{1,t} n_{1,t} + p_{2,t} y_{2,t} n_{2,t} + \epsilon_t (p_{1,t}^* y_{1,t}^* n_{1,t}^* + p_{2,t}^* y_{2,t}^* n_{2,t}^*)$$

LEMMA 1. *In the first-order approximation of the world market clearing condition (2.43) with respect to  $s$ , the relative wage effect cancels out between countries, and the substitution effect cancels out between sectors. Consequently, the income effect solely solves the first-order approximation equation.*

I add an intuitive explanation for this effect. When the home government provides a 1 percent production subsidy to firms in industry 1 ( $s = 0.01$ ), firms in the home industry 1 reduce their prices by 1 percent, as seen in (2.35). The 1 percent drop in  $p_{1,t}$  directly decreases the left hand side in (2.43) by  $\frac{1}{4}(\psi - 1)$  percent. This is because i) a 1 percent decrease in  $p_{1,t}$  leads to a  $\frac{1}{2}$  percent decrease in the home utility based CPI,  $P_t$ , and ii) a  $\frac{1}{2}$  percent decrease in  $P_t$  causes the home demand,  $P_t^{1-\psi}$ , to change by  $\frac{1}{2}(\psi - 1)$  percent, and iii) the home demand accounts for  $\frac{1}{2}$  of the world demand. On the other hand, the 1 percent drop in  $p_1$  directly causes the world revenue, the right hand side in (2.43), to decrease by  $\frac{1}{4}$  percent. This is because i) the revenue of the home industry 1,  $p_{1,t} y_{1,t} n_{1,t}$ , decreases by 1 percent, and ii) the revenue of the home industry 1 accounts for  $\frac{1}{4}$  of the world revenue. Overall, the world experiences an excess demand of  $\frac{\psi}{4}$  percent. To eliminate this excess demand, the mass of firms in all industries in both the home and foreign country needs to change uniformly by  $\frac{(\gamma-1)\psi}{4(\gamma-\psi)}$ . This is because a 1 percent increase in the mass of firms in all industries in both the home and foreign country causes the world demand to change

by  $\frac{1-\psi}{1-\gamma}$  percent and the world revenue to change by 1 percent, thereby reducing the world excess demand by  $\frac{\gamma-\psi}{\gamma-1}$ .

The relative wage effect is attributable to the change in the relative wage,  $\epsilon$ . The following lemma demonstrates that the relative wage effect is derived from the balance of payment equilibrium condition (C.30).

LEMMA 2. *In the first-order approximation of the balance of payment equilibrium condition (C.30) with respect to  $s$ , the income effect cancels out between countries, and the substitution effect cancels out between sectors. Consequently, the relative wage effect solely solves the first-order approximation equation.*

Due to the production subsidy, more firms enter industry 1 in the home country and hire additional labor. This results in an increase in the relative wage of domestic labor. In other words, it leads to a decrease in  $\epsilon_t$ , as indicated in (B.29). The relative wage effect impacts both the domestic and foreign industries in the opposite direction but with equal magnitude. In addition to the income and substitution effects,  $\epsilon_t$  decreases until the balance of payment equilibrium (C.30) is restored.

The substitution effect arises as firms enter and exit each industry to satisfy the zero profit condition again, as stated in (2.44) and (2.45).

$$(2.44) \quad \frac{1+s}{\gamma} \left( \frac{p_{1,t}(h)}{P_{h1,t}} \right)^{1-\gamma} \left[ \left( \frac{P_{h1,t}}{P_{1,t}} \right)^{1-\eta} \left( \frac{P_{1,t}}{P_t} \right)^{1-\sigma} P_t^{1-\psi} + \phi \epsilon_t^\eta \left( \frac{P_{h1,t}}{P_{1,t}^*} \right)^{1-\eta} \left( \frac{P_{1,t}^*}{P_t^*} \right)^{1-\sigma} P_t^{*1-\psi} \right]$$

$$= \frac{1}{\gamma} \left( \frac{p_{2,t}(h)}{P_{h2,t}} \right)^{1-\gamma} \left[ \left( \frac{P_{h2,t}}{P_{2,t}} \right)^{1-\eta} \left( \frac{P_{2,t}}{P_t} \right)^{1-\sigma} P_t^{1-\psi} + \phi \epsilon_t^\eta \left( \frac{P_{h2,t}}{P_{2,t}^*} \right)^{1-\eta} \left( \frac{P_{2,t}^*}{P_t^*} \right)^{1-\sigma} P_t^{*1-\psi} \right]$$

$$(2.45) \quad \frac{1}{\gamma} \left( \frac{p_{1,t}^*(f)}{P_{f1,t}^*} \right)^{1-\gamma} \left[ \left( \frac{P_{f1,t}^*}{P_{1,t}^*} \right)^{1-\eta} \left( \frac{P_{1,t}^*}{P_t^*} \right)^{1-\sigma} P_t^{*1-\psi} + \phi \epsilon_t^{-\eta} \left( \frac{P_{f1,t}^*}{P_{1,t}} \right)^{1-\eta} \left( \frac{P_{1,t}}{P_t} \right)^{1-\sigma} P_t^{1-\psi} \right]$$

$$= \frac{1}{\gamma} \left( \frac{p_{2,t}^*(f)}{P_{f2,t}^*} \right)^{1-\gamma} \left[ \left( \frac{P_{f2,t}^*}{P_{2,t}^*} \right)^{1-\eta} \left( \frac{P_{2,t}^*}{P_t^*} \right)^{1-\sigma} P_t^{*1-\psi} + \phi \epsilon_t^{-\eta} \left( \frac{P_{f2,t}^*}{P_{2,t}} \right)^{1-\eta} \left( \frac{P_{2,t}}{P_t} \right)^{1-\sigma} P_t^{1-\psi} \right]$$

LEMMA 3. *In the first-order approximation of the zero profit conditions (2.44) and (2.45) with respect to  $s$ , the income and relative wage effects cancel out between sectors. Consequently, the substitution effect solely solves the first-order approximation equation.*

Since the production subsidy makes home varieties in industry 1 relatively cheaper, the home and foreign consumers substitute industry 1-foreign varieties with industry 1-home varieties. This leads to an increase in the mass of firms in home industry 1 and a decrease in that in foreign industry 1. In contrast, foreign varieties become relatively cheaper in industry 2 because of the increase in relative wage of home labor. It leads to a decrease in the mass of firms in home industry 2 and an increase in that in foreign industry 2. This substitution continues until the profit of firms in industry 1 and 2 becomes the same in both home and foreign country.

Overall, by combining the three effects, the mass of firms in each industry changes as follows.

PROPOSITION 1. *A production subsidy causes the mass of home industry 1,  $n_{1,t}$ , to unambiguously increase and that of foreign industry 1,  $n_{1,t}^*$ , to unambiguously decrease. Changes in  $n_{2,t}$  and  $n_{2,t}^*$  are ambiguous.*

PROOF. See Appendix C.4 □

The direction of the substitution effect is opposite from the two other effects for  $n_{2,t}$  and the direction of relative wage effect is opposite from the two other effects for  $n_{2,t}^*$ . Thus, the overall effect of a production subsidy on  $n_{2,t}$  and  $n_{2,t}^*$  depends on the relative sizes of three effects. It is important to note that any increase in  $n_{1,t}$  caused by the production subsidy leads to faster accumulation of knowledge stock and more learning by doing in home industry 1, which eventually makes a refinement occur faster in home industry 1.

**Welfare Effect.** In Stage 1, the production subsidy's welfare effect on the home country is summarized in the next proposition.

PROPOSITION 2. *The welfare effect of production subsidy for the home country in Stage 1 is ambiguous and it depends on the values of parameters as seen in (B.33). For example, if  $\gamma \rightarrow \infty$ , the production subsidy decreases home welfare in the neighborhood of the initial equilibrium. In contrast, if  $\phi = 0$  or  $\gamma = \eta$ , the welfare effect of production subsidy is positive.*

PROOF. See Appendix C.4

□

Since firms have a monopolistic power, the production subsidy makes firms' prices in the targeted industries closer to their marginal costs, which generates an efficiency gain in the home country. In addition, home consumers experience a relative wage gain from the policy. However, at the same time, they have to pay tax which the government charges to finance the subsidy. Overall, in Stage 1, the welfare effect of the production subsidy for the home country depends on the degree of those gains and losses. For example, when markets are highly competitive ( $\gamma \rightarrow \infty$ ), the welfare gain from making the prices closer to the marginal costs vanishes. However, the relative wage, which is mostly affected by the macro elasticity substitution ( $\eta$ ) does not decrease a lot. Thus, the welfare of the home country decreases by  $\frac{\psi\phi}{2\eta-1+2\psi\phi-\phi}$ . Also, it is worth to note that the welfare effect is always positive under autarky ( $\phi = 0$ ) unlike that in the open economy since home consumers do not pay the portion of the tax which subsidizes the consumption of foreign consumers under autarky.

The following proposition summarizes the production subsidy's welfare effect on the foreign country in Stage 1.

**PROPOSITION 3.** *A production subsidy unambiguously increases the foreign country's welfare in Stage 1 in the open economy ( $\phi > 0$ ).*

PROOF. See Appendix C.4

□

For foreign country's welfare, the home country's production subsidy decreases foreign country's utility based CPI as shown in (C.78). However, it also decreases the foreign wage relative to the home wage, which deteriorates foreign country's term of trade. Overall, if trade is not entirely restricted ( $\phi > 0$ ), the former effect is greater than the latter one. Thus, in an open economy, home's subsidy increases the welfare of the foreign country.

**2.3.1.2. Long-run Gain from Speeding up Innovation.** After a refinement occurs, the productivity of firms in home industry 1 jumps to  $k$  from 1. The refinement occurs earlier in the home industry 1 by the production subsidy provided to the industry since the subsidy causes industry-level R&D efforts and learning-by-doing effects to be accumulated faster, and accelerates competition in the industry.

PROPOSITION 4. *A production subsidy causes the home industry 1 to move from the early stage (Stage 1) to the high-productivity stage (Stage 2) faster.*

PROOF.

$$(2.46) \quad \frac{dt(s)}{ds} = -\frac{Q_1^{-1}(\bar{Q}_1)}{n_{1,s1}(s)^2} \frac{dn_{1,s1}(s)}{ds} = -t(s) \frac{d \ln n_{1,s1}(s)}{ds} < 0 \quad \left( \because \frac{d \ln n_{1,s1}(s)}{ds} > 0 \text{ as shown in (C.73)} \right)$$

where  $n_{1,s1}(s)$  means the mass of the home industry 1 in Stage 1. □

The following corollaries explains the direction and size of the early innovation effect.

COROLLARY 1. *The early innovation in the home industry 1 by the home production subsidy increases the welfare of the home and foreign country.*

COROLLARY 2. *The size of gain from early innovation in the home industry 1 depends on (i) how much the instantaneous utility of the home and foreign representative consumer increases after home innovation ( $\ln U_2 - \ln U_1$  and  $\ln U_2^* - \ln U_1^*$  respectively), (ii) how much sooner the refinement occurs in the home industry 1 because of the home subsidy ( $\frac{dt_r(s)}{ds}$ ) and (iii) how long it takes to occur the refinement without the home subsidy in the home industry 1 ( $t_r(0)$ ).*

To build intuition for the condition (i) in Corollary 2, even though productivity radically increases after innovation, I look into the local effects of a productivity increase by taking a first-order approximation of the model in the neighborhood of the initial symmetric equilibrium. Table B.2 in Appendix B.2 shows the results.

Innovation in the home industry 1 increases both home and foreign country's welfare to the first order as seen in (B.42) and (B.44). Since  $a_{1,t}$  jumps a lot in Stage 2 by nature, it is necessary to see whether the results in Table B.2 are maintained even when  $a_{1,t}$  radically goes up. I present, through model simulations under a reasonable parameterization, how mass of firms, relative wage, utility based CPI and instantaneous utility at home and abroad change while  $a_{1,t}$  increases in Section 2.4. The result shows that the more  $a_{1,t}$  rises, the more the both the home and foreign utility in Stage 2 increase relative to Stage 1.

Condition (ii) means that the earlier the production subsidy makes a refinement arrive in the targeted industry, the more successful the policy will be. Equation (2.5) ( $t_r(s) = \frac{Q_1^{-1}(\bar{Q}_1)}{n_{1,s1}(s)}$ ) tells that

how much earlier an industrial policy make a refinement occur depends on how much the policy increases the mass of firms in the targeted industry in the early stage.

The intuition from condition (iii) is that the timing that the targeted industry transits to the high growth stage without policy supports,  $t_r(0)$ , should not be too far away from now. In other words, the home targeted industry should not be too far from the frontier in order for the policy to be successful. If it takes too long time for the targeted industry to achieve high productivity, the cost by the production subsidy becomes larger and the benefit from innovation is discounted a lot.

**2.3.1.3. Overall Welfare Effect.** Table 2.1 summarizes the overall welfare effect of production subsidy in Case 1. This result has two main implications for the home country. First, it suggests that if industrial policy supports a young industry with high growth potential, it can increase productivity growth and welfare in the long-run. This is in line with the argument of Aghion et al. (2015). Second, despite the long-run benefit, it provides an explanation on why industrial policy is not desirable under certain circumstances even for promoting catch-up in developing countries. The policy is likely to cause welfare loss while subsidising targeted industry, and it takes time before productivity actually increases. If the short-run welfare loss dominates the long-run gain from growth, the policy will eventually decrease the home welfare. This supports the importance of dynamic analysis for evaluating industrial policy.

In contrast to the home country, the industrial policy unambiguously increases the welfare of the foreign country. This suggests that developing countries' industrial policy fostering catching up in industries which are already matured in developed countries is beneficial to developed countries.

TABLE 2.1. Summary of Welfare Effect in Case 1 (Catch-up)

Effect	Home	Foreign
Short-run resource reallocation effect	(+)/(-)	(+)
Long-run gain from speeding up innovation	(+)	(+)
Overall effect	(+)/(-)	(+)

**2.3.2. Case 2: Frontier Technology Races.** Now, I assume that the foreign industry 1 is also young and has high growth potential like the home industry 1. Since the industry is operating at the technological frontier in both countries, this sets up a race to take leadership in the industry



through innovation. The effects of the subsidy are distinct from the catch-up situation in Case 1. What follows explains how.

Under the symmetric initial condition, without the home country's industrial policy, a refinement occurs at the same time in the home and foreign industry 1 ( $t_r(0) = t_r^*(0)$ ). However, if the home government gives a production subsidy to firms in industry 1, resources will be re-allocated in the home and foreign country in response to the policy. Before a refinement occurs in one of the home and foreign country, equilibrium is the same as that in Stage 1 (short-run) in Case 1, implying that the short-run *resource reallocation effect* in Case 2 is also the same as that in Case 1.

As it can be seen in (C.73) and (C.74) in Appendix C.4, the production subsidy causes the mass of firms in the home industry 1,  $n_{1,t}$ , to increase and the mass of firms in foreign industry 1,  $n_{1,t}^*$ , to decrease in Stage 1. This leads the innovation to occur earlier in the home industry 1 and to be delayed in the foreign industry 1 (i.e. if  $s > 0$ ,  $t_r(s) < t_r^*(s)$ ). The following proposition formally shows the home country's production subsidy causes the innovation in the home industry 1 to occur earlier but that in the foreign country to be delayed in Case 2.

**PROPOSITION 5.** *Production subsidy causes the home industry 1 to move from the early stage to the high-productivity stage but it delays the transition of the foreign industry 1.*

**PROOF.**

$$(2.47) \quad \frac{dt(s)}{ds} = -\frac{Q_1^{-1}(\bar{Q}_1)}{n_{1,s1}(s)^2} \frac{dn_{1,s1}(s)}{ds} = -t(s) \frac{d \ln n_{1,s1}(s)}{ds} < 0$$

$$(2.48) \quad \frac{dt_r^*(s)}{ds} = -\frac{dt_r(s)}{ds} \left( \frac{n_{1,s1}^*(s)}{n_{1,s2}^*} - 1 \right) - t_r(0) \frac{n_{1,s1}^*(s)}{n_{1,s2}^*} \frac{d \ln n_{1,s1}^*(s)}{ds} > 0$$

where  $n_{1,sj}(s)$  and  $n_{1,sj}^*(s)$  are the mass of the home and foreign industry 1 in Stage  $j$ . □

Based on Proposition 5, three stages can be defined over time as follows.

**DEFINITION 4.**

- *Stage 1* ( $0 < t < t_r(s)$ ): *A refinement has not occurred yet in both the home and foreign industry 1* ( $a_{1,t} = 1$ ,  $a_{1,t}^* = 1$ )

- Stage 2 ( $t_r(s) \leq t < t_r^*(s)$ ): A refinement occurred in the home industry 1 but has not occurred yet in the foreign industry 1 ( $a_{1,t} = k$ ,  $a_{1,t}^* = 1$ )
- Stage 3 ( $t_r^*(s) \leq t$ ): A refinement occurred in both the home and foreign industry 1 ( $a_{1,t} = k$ ,  $a_{1,t}^* = k$ )

Given the above definition, the welfare of the home and foreign representative consumer can be re-expressed as:

$$(2.49) \quad \ln W = \int_0^{t_r(s)} e^{-\rho t} \ln U_1 dt + \int_{t_r(s)}^{t_r^*(s)} e^{-\rho t} \ln U_2 dt + \int_{t_r^*(s)}^{\infty} e^{-\rho t} \ln U_3 dt$$

$$(2.50) \quad \ln W^* = \int_0^{t_r(s)} e^{-\rho t} \ln U_1^* dt + \int_{t_r(s)}^{t_r^*(s)} e^{-\rho t} \ln U_2^* dt + \int_{t_r^*(s)}^{\infty} e^{-\rho t} \ln U_3^* dt$$

where  $U_j$  ( $U_j^*$ ) denotes the home (foreign) representative consumer's utility in stage  $j$ . The local effect of production subsidy on the home and foreign welfare in the neighborhood of the initial equilibrium is respectively:

$$(2.51) \quad \frac{d \ln W}{ds} = \underbrace{\int_0^{t_r(0)} e^{-\rho t} \frac{d \ln U_1}{ds} dt}_{\text{Short-run resource reallocation effect}} \quad \underbrace{-e^{-\rho t_r(0)} (\ln U_2 - \ln U_1) \frac{dt_r(s)}{ds}}_{\text{Gain from early home innovation}} \quad \underbrace{-e^{-\rho t_r^*(0)} (\ln U_3 - \ln U_2) \frac{dt_r^*(s)}{ds}}_{\text{Loss from delayed foreign innovation}}$$

$$(2.52) \quad \frac{d \ln W^*}{ds} = \underbrace{\int_0^{t_r(0)} e^{-\rho t} \frac{d \ln U_1^*}{ds} dt}_{\text{Short-run resource reallocation effect}} \quad \underbrace{-e^{-\rho t_r(0)} (\ln U_2^* - \ln U_1^*) \frac{dt_r(s)}{ds}}_{\text{Gain from early home innovation}} \quad \underbrace{-e^{-\rho t_r^*(0)} (\ln U_3^* - \ln U_2^*) \frac{dt_r^*(s)}{ds}}_{\text{Loss from delayed foreign innovation}}$$

**2.3.2.1. Loss from Delaying Foreign Innovation.** In the equation (2.51) and (2.52), the analysis for the first two terms, the short-run resource allocation and gain for early home innovation effect, are the same as Case 1. Thus, in this section, I only add explanation on welfare analysis for the third term ( $-e^{-\rho t_r^*(0)} (\ln U_3 - \ln U_2) \frac{dt_r^*(s)}{ds}$  and  $-e^{-\rho t_r^*(0)} (\ln U_3^* - \ln U_2^*) \frac{dt_r^*(s)}{ds}$ , respectively) from delayed innovation in the foreign country.

**COROLLARY 3.** *The delayed innovation in the foreign industry 1 by the home production subsidy decreases the welfare of the home and foreign country.*

COROLLARY 4. *The size of loss from delayed foreign innovation depends on (i) how much the instantaneous utility of the home and foreign representative consumer increase after foreign innovation ( $\ln U_3 - \ln U_2$  and  $\ln U_3^* - \ln U_2^*$  respectively), (ii) the length of the delay before the refinement occurs in the foreign industry 1 attributable to the home subsidy ( $\frac{dt_r^*(s)}{ds}$ ) and (iii) how long the refinement takes to occur the refinement initially without the home subsidy in the foreign industry 1 ( $t_r^*(0)$ ).*

2.3.2.2. **Overall Welfare Effect.** Table C.1 summarizes the effects of the home production subsidy on the home and foreign welfare. Because of the loss from delayed foreign innovation, the production subsidy is less likely to increase home welfare compared with Case 1. However, based on the result from in Figure 2.2a, since the degree of positive spillover from the counterpart country's productivity increase is relatively small, this additional negative effect does not change much policy implications for the home country.

In contrast, the delayed innovation in the foreign country can affect quite negatively foreign welfare. It is interesting in Case 2 that the negative effect from delayed innovation can dominate the other two positive effects and consequently the home production subsidy has a beggar-thy-neighbor effect. The higher the growth potential of the targeted industry, the more a home production subsidy increases the home welfare but decreases the foreign welfare. In such circumstances, the foreign government will respond to the home policy by conducting countervailing policy to offset the beggar-thy-neighbor effect, which means the home and foreign country are in a game situation.

In the next section, I study under what conditions home production subsidy increases or decreases the home and foreign welfare through model simulation for Case 1 and 2.

TABLE 2.2. Summary of Welfare Effect in Case 2

Effect	Home	Foreign
Short-run resource reallocation effect	(+)/(-)	(+)
Gain from speeding up home innovation	(+)	(+)
Loss from delaying foreign innovation	(-)	(-)
Overall effect	(+)/(-)	(+)/(-)

## 2.4. Model Simulation

In this section, I study how production subsidy affects the welfare<sup>8</sup> at home and abroad with a reasonable parameterizations.

**2.4.1. Catch-Up Revisited.** As shown in the previous section, since production subsidy in Case 1 unambiguously beneficial for the foreign country in the model, I will focus on under what conditions production subsidy increases home welfare in this section.

**Calibration.** I set parameter values which are taken from standard values in the literature. The elasticity of substitution between domestic varieties within an industry is set at  $\gamma = 4$  and the elasticity of substitution between the home- and foreign-specific industry-level aggregate is set at  $\eta = 2.5$  based on Feenstra et al. (2018). The elasticity of substitution between industries is set at  $\sigma = 1.36$  following Redding et al. (2021). The trade cost is set at  $\tau = 0.25$  following Obstfeld and Rogoff (2001). Since intertemporal elasticity of substitution is usually set between 1/2 and 1, it is set at  $\psi = 0.75$ . The discount rate is set at  $\rho = 0.042$ .

I use an initial condition,  $v = v^* = 1$  and  $L = L^* = 10$ . Initially, I assume that productivity of each industry in the home and foreign country is set at  $a_{1,t} = 6.67$  and  $a_{2,t} = a_{1,t}^* = a_{2,t}^* = 10$ , which reflects a situation where the home industry 1 is currently less productive than the foreign industry 1 but is trying to catch up. I assume that productivity increases 50 percent<sup>9</sup> after the refinement ( $a_{1,t} = 10$  if  $t \geq t_r$ ).

I assume a function for knowledge stock reflecting decreasing marginal accumulation from firms' activity as follows

$$(2.53) \quad Q_{1,t} = \sqrt{\int_0^t n_{1,j} dj}$$

The threshold for the level of knowledge stock which realizes refinement in the home industry 1 is set at  $\bar{Q}_1 = 1.8$ . This makes the refinement occur around  $t = 10$  without a policy support in the model. I will use these values of parameters as a benchmark.

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<sup>8</sup>To define the welfare in equation (3.26), the utility in (3.1) should be positive. However, under the assumption of  $\psi < 1$ , the utility is negative in the initial equilibrium. To address this issue, I employ a different utility function,  $U_t' = -\frac{1}{v_t}$ , for model simulation. Since this new utility function is a monotonic increasing transformation of the original utility function, it does not affect the analysis presented in the previous sections.

<sup>9</sup>The 50 percent growth matches the long-run treatment effect of Korean government's HCI drive on the productivity of those industries

**Short-run Resource Reallocation Effect.** As Table 2.1 shows, the short-run welfare effect of production subsidy is important regarding whether the subsidy is beneficial for the home country or not. Thus, I conduct an experiment to discern how home utility in Stage 1 changes as the production subsidy rate to industry 1 varies. The simulation results using the benchmark parameters show that the production subsidy increases home utility in Stage 1 (pre-refinement) until  $s = 0.08$ . However, as seen from Figure 2.1a, the increase in the home utility, which is 0.1% at  $s = 0.08$ , is negligible in size. Also, when the production subsidy rate exceeds 0.08 ( $s > 0.08$ ), home utility starts to fall increasingly rapidly. If  $s > 0.16$ , the home instantaneous utility becomes even less than that without the production subsidy. In contrast, the foreign utility in Stage 1 is increasing further while  $s$  increases.

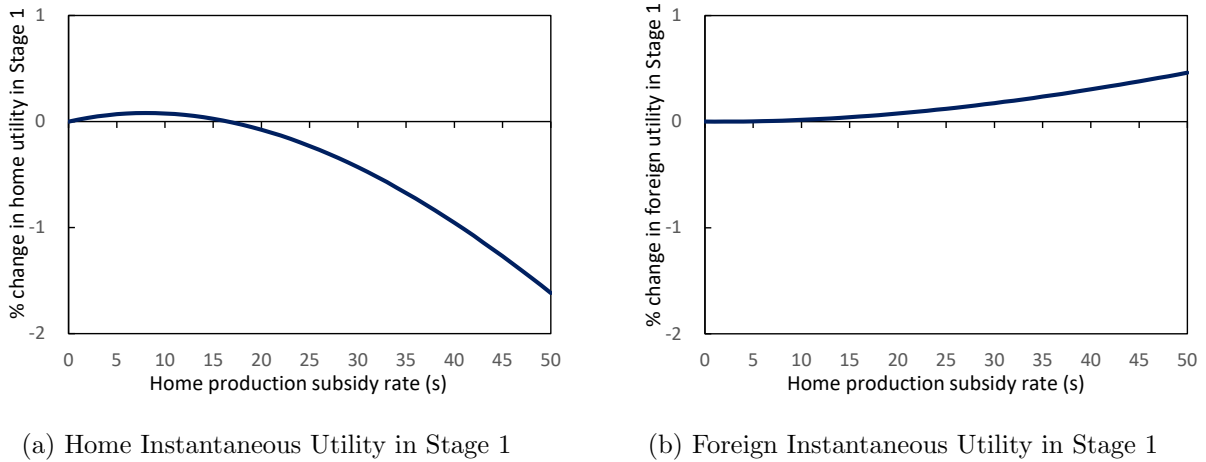


FIGURE 2.1. Instantaneous Utility in Stage 1 (Pre-refinement) while Changing Production Subsidy Rate  $s$

Notes: Figure 2.1a and 2.1b show how the home and foreign instantaneous utility in Stage 1 change while  $s$  increases.

**Long-run Gain from Speeding up Innovation.** As seen in Corollary 1, both the home and foreign country unambiguously profit from innovation in the home industry 1 being hastened by a production subsidy. Here, I focus on which factors affect the size of long-run gain from earlier home innovation.

Condition (i) in Corollary 2 means that, for both the home and foreign country, the more utility increases in Stage 2 compared with Stage 1, the more a home production subsidy increases

both countries' welfare. As Figure 2.2a shows, the degree to which utility increases in Stage 2 is determined by the degree of productivity improvement in the targeted industry after innovation. I note two implications from the result in Figure 2.2a. First, if the growth potential of the targeted industry is high enough, the size of the utility increase in Stage 2 (post-refinement) is much larger than the size of utility losses from the tax in Stage 1 as seen in Figure 2.1a and 2.2a.<sup>10</sup> Second, the direct utility gains in the home country from home innovation is much larger than its spillover effect in the foreign country since a productivity increase in the home country causes the home (foreign) country's terms of trade improvement (deterioration) as seen in Figure 2.2b.

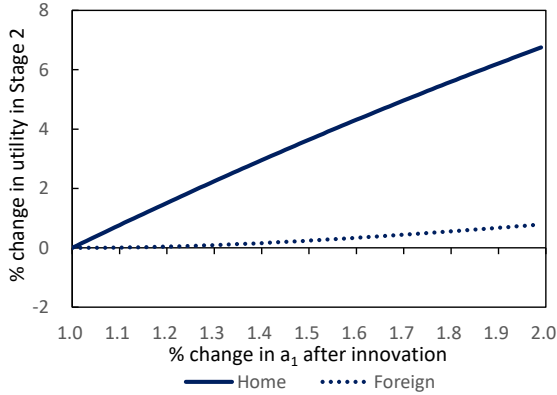
Regarding Condition (ii) in Corollary 2, a higher subsidy induces earlier innovation as seen in Figure 2.2c and 2.2d. It is worth noting that this clearly presents the trade off between short-run loss and long-run gain by the policy. As Figure 2.1a and 2.2d show, a higher subsidy rate leads to larger short-term losses due to increased taxation in Stage 1 but a larger long-run gain from earlier innovation in targeted industry.

**Overall Welfare Effect.** Figure 2.3 shows how the overall home and foreign welfare changes as the production subsidy rate  $s$  increases. Using the benchmark parameters, the optimal production subsidy rate for the home country is 0.37. Under the optimal subsidy rate, the home and foreign welfare increase 10.1% and 2.1%, respectively. In this case, a home production subsidy increases both home and foreign welfare and thus the foreign country does not need to respond to the home policy.

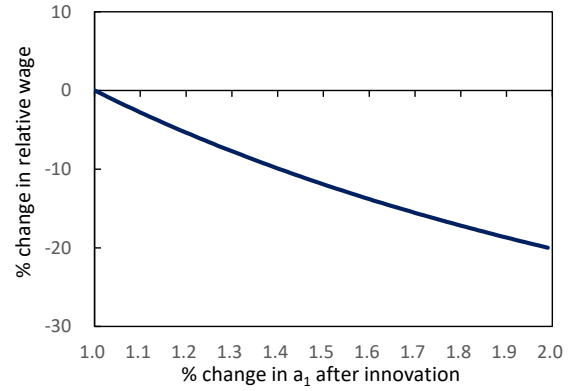
The simulation results for Case 1 suggest that if industrial policy does not have a sufficiently large growth-boosting effect in the targeted industry, the policy can only increase home welfare modestly at best. This is why the timing of industrial policy is important for success of the policy. If industrial policy supports an already-matured industry, it is hard to expect a growth-boosting effect in the industry. This is consistent with theoretical and empirical analysis of Bartelme et al. (2021) and Lashkaripour and Lugovsky (2023). When considering additional policy costs such as resource misallocation "within" a targeted industry as argued in Kim et al. (2021), the following policy implication becomes more apparent: policy makers need to pay great attention to how industrial policy can boost growth of targeted industry from a dynamic standpoint as well as how

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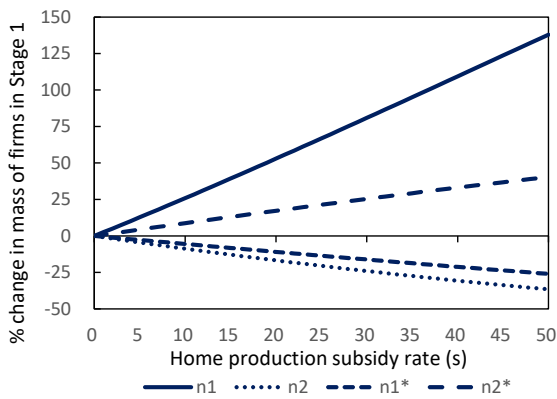
<sup>10</sup>For example, the home utility in Stage 1 decreases by 1.6% with  $s = 0.5$  which is a high subsidy rate. In contrast, the home utility in Stage 2 increases by 3.7% if productivity in the targeted industry rises 50% after innovation.



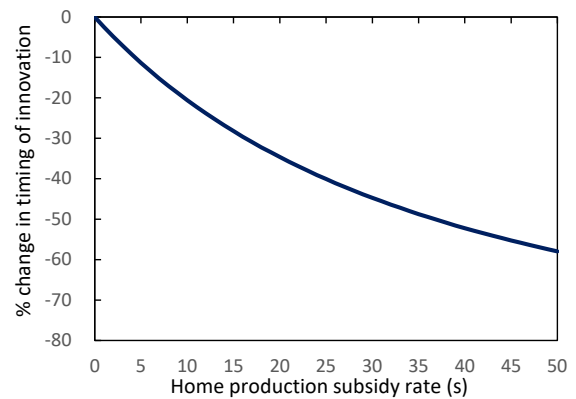
(a) Home and Foreign Utility in Stage 2



(b) Relative Wage in Stage 2



(c) Mass of Firms in Stage 1



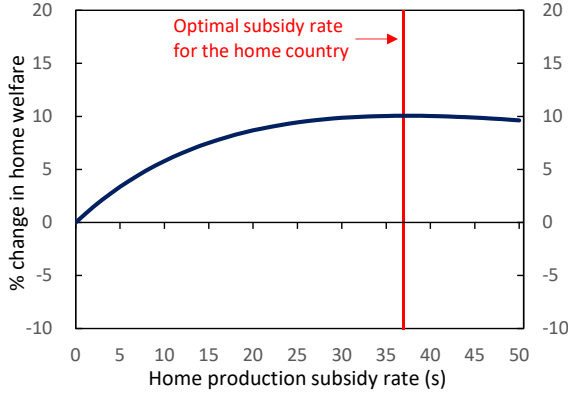
(d) Timing of Innovation in Home Industry 1

FIGURE 2.2. Factors Affecting Long-run Gain in Stage 2 (Post-refinement)

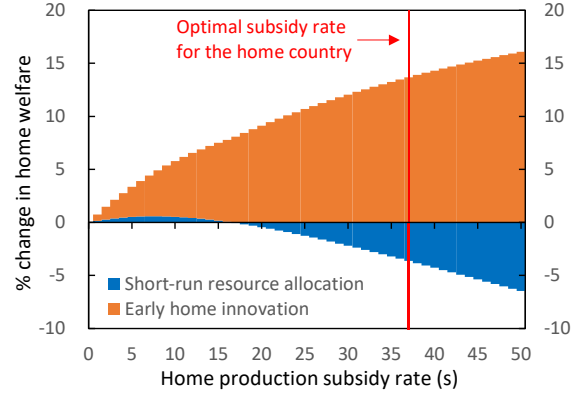
Notes: Figure 2.2a and 2.2b show how the home and foreign utility, and the relative wage in Stage 2 change while the degree of productivity improvement in Stage 2 in the home industry 1, which is percentage change in  $a_{1,t}$  in Stage 2, varies. Figure 2.2c and 2.2d present how the mass of firms in Stage 1 and timing of innovation in the home industry 1 change while production subsidy rate  $s$  increases.

the policy affects economic efficiency in a static setting. In addition, where the targeted industry is in its lifecycle is an important criterion for determining whether the policy will produce growth-boosting effect or not.

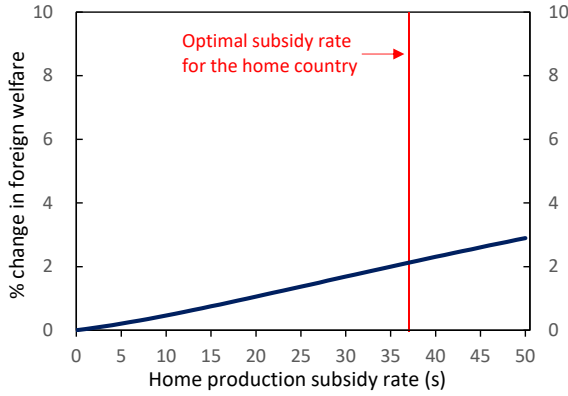
**Sensitivity Analysis.** I also conduct sensitivity analyses to study under what conditions industrial policy can increase home welfare in Case 1, especially focusing on the role of the micro and macro elasticity of substitution ( $\gamma$  and  $\eta$  respectively in the model).



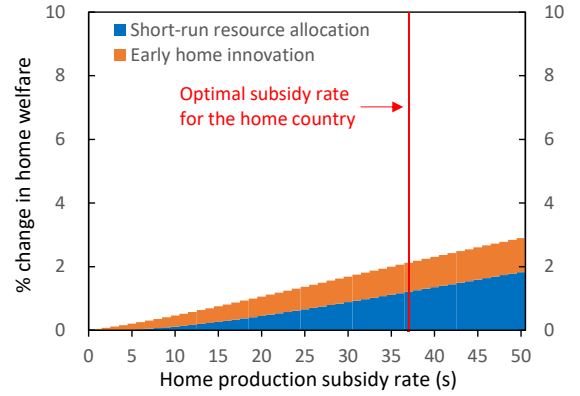
(a) Home Welfare



(b) Decomposition of Home Welfare Change



(c) Foreign Welfare



(d) Decomposition of Foreign Welfare Change

FIGURE 2.3. Overall Welfare Change by Home Production Subsidy in Catch-up

Notes: Figure 2.3a and 2.3c show how the overall home and foreign welfare change while home production subsidy rate  $s$  increases in Case 1. Figure 2.3b and 2.3d present the decomposition of the home and foreign welfare change into 'short-run resource allocation effect' and 'early home innovation effect'. The red line in the figures represents home country's optimal subsidy rate.

Given a micro elasticity of substitution ( $\gamma$ ), a higher macro elasticity of substitution ( $\eta$ ) magnifies the positive effects on home welfare in two ways. First, with a higher  $\eta$ , a production subsidy causes the mass of firms in the targeted industry to increase more because of higher substitution between the home and foreign products, which leads a refinement to occur earlier in the industry. Second, under a higher  $\eta$ , the relative home wage increases more in both Stage 1 and 2 as seen in the comparative statics results (B.29) and (B.40), and thus the home country benefits more from



improvement in the terms of trade. It is worth mentioning that this result contrasts with the implication in Melitz (2005) that the welfare benefit from a production subsidy to an infant industry *decreases* with the level of product differentiation between home and foreign goods. The difference is mainly caused by the above-mentioned effects from change in relative wage or terms of trade do not occur in the small open economy setting of Melitz (2005). This implies that considering changes in relative wage or terms of trade in a large-country open economy model instead of a small open economy model provides quite different welfare implications.

On the other hand, given a macro elasticity of substitution ( $\eta$ ), a higher micro elasticity of substitution ( $\gamma$ ) dampens home welfare gains from a domestic subsidy in two ways. First, it makes the benefit from correcting monopolistic market power in Stage 1 smaller since the inefficiency caused by monopolistic market power is vanishing with higher  $\gamma$ .<sup>11</sup> Second, a production subsidy is not capable of significantly increasing the mass of firms in the targeted industry subject to high  $\gamma$ . This is because the profit margin of firms in an industry with  $\gamma$  is already low, and thus, a production subsidy does not substantially encourage the entrance of new firms.

Figure 2.4 shows well the relationships explained above. Based on the results in Feenstra et al. (2018) that the range of estimates of macro elasticity of substitution falls mostly between 1 and 4, Figure 2.4 indicates that a production subsidy increases home welfare under most plausible parameterizations in the case where the targeted industry's productivity rises 50% after innovation. Even though a production subsidy is likely to be beneficial to the home country in this case, it is worth noting that there is a chance, which is not negligible, that the policy decreases home welfare. If productivity increases only 25% after the refinement, the probability that a production subsidy worsens home welfare increases, as seen in Figure 2.4. Policy makers have to be cautious of providing a production subsidy—especially when the substitutability between home and foreign goods is low and thus domestic firms have difficulties expanding their market share in the export market even with policy support.

**2.4.2. Frontier Technology Races Revisited.** For the benchmark case for frontier technology races (Case 2), I use the same values of parameters as the benchmark for catch-up (Case 1):

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<sup>11</sup>If the production subsidy rate exceeds  $\frac{1}{\gamma-1}$ , the subsidy is already causing distortion even in a partial equilibrium sense. In this case, higher  $\gamma$  brings about more distortion in Stage 1.

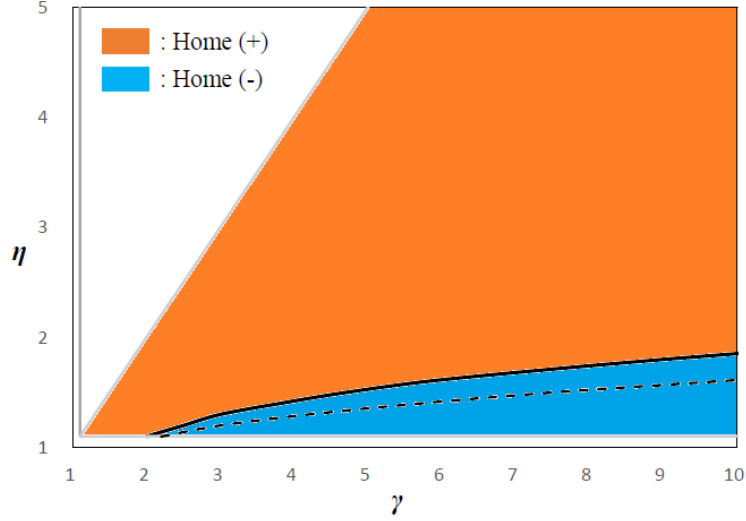


FIGURE 2.4. Sensitive Analysis in Catch-up (Effect of  $s = 0.1$  on the Home Welfare,  $k = 1.5$  vs  $k = 1.25$ )

Notes: Figure 2.4 shows whether a 10 percent production subsidy rate ( $s = 0.1$ ) increases home welfare under  $k = 1.5$  and  $k = 1.25$  respectively while micro and macro elasticity of substitution ( $\gamma$  and  $\eta$  respectively) change. In the figure, the dotted line represents the boundary line in the case of  $k = 1.5$ .

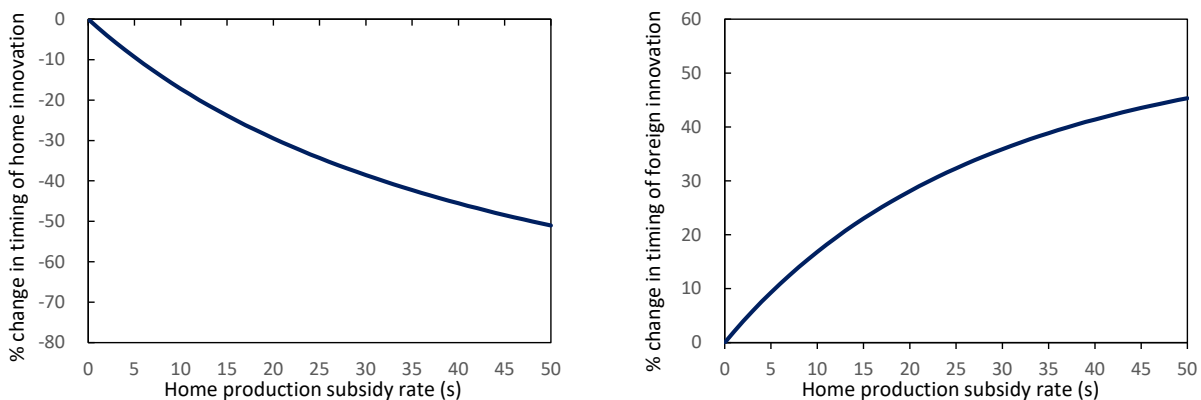
$\gamma = 4$ ,  $\eta = 2.5$ ,  $\sigma = 1.36$ ,  $\psi = 0.75$ ,  $\tau = 0.25$  and  $\rho = 0.042$ . It is again assumed that  $v = v^* = 1$  and  $L = L^* = 10$ .

The initial productivity of each industry in the home and foreign country is set at  $a_{1,t} = a_{1,t}^* = 6.67$  and  $a_{2,t} = a_{2,t}^* = 10$ . As in Case 1, I assume that productivity increases by 50 percent after the refinement in the home and foreign industry 1 ( $a_{1,t} = 10$  if  $t \geq t_r$ ,  $a_{1,t}^* = 10$  if  $t \geq t_r^*$ ). Based on the assumption of the symmetric initial condition, the knowledge stock accumulation function is the same in the home and foreign industry 1:

$$(2.54) \quad Q_{1,t} = \sqrt{\int_0^t n_{1,j} dj}, \quad Q_{1,t}^* = \sqrt{\int_0^t n_{1,j}^* dj}$$

The threshold for the level of knowledge stock required for realization of a refinement are  $\bar{Q}_1 = \bar{Q}_1^* = 2.1$  which makes the refinement occur around  $t = 10$  without any home or foreign policy intervention.

As shown in Figure 2.5a and 2.5b, a higher production subsidy rate causes a refinement to occur earlier in the home industry but later in the foreign industry 1. The delayed foreign innovation negatively affects the home and foreign welfare.



(a) Timing of Innovation in Home industry 1

(b) Timing of Innovation in Foreign Industry 1

FIGURE 2.5. Timing of Home and Foreign Innovation if Home only Subsidizes a Frontier Industry

Notes: Figure 2.5a and 2.5b show how the timing of refinement in the home and foreign industry 1 change while  $s$  increases.

Figure 2.6 shows the home and foreign welfare change in Case 2 while production subsidy rate  $s$  increases. In the benchmark case, the home country's optimal subsidy rate is 0.30 and home welfare increases by 7.1% under the optimal subsidy rate which is not much different from Case 1. This implies that the negative spillover from delayed foreign innovation does not affect home welfare a lot. However, under the home country's optimal production subsidy rate, foreign welfare decreases by 2.0%. It is also worth noting that if  $0 < s < 0.68$ , the subsidy decreases foreign welfare. This implies that home industrial policy is highly likely to adversely affect foreign welfare in the case of frontier technology races.

**Policy Reaction.** The above result naturally provides an implication that if a country supports a young industry with high-growth potential in which home and foreign firms are competing to take initiative, the foreign country has to conduct countervailing policy in order for its industry not to drop out of the race. Considering the importance of accumulation of data for recent frontier

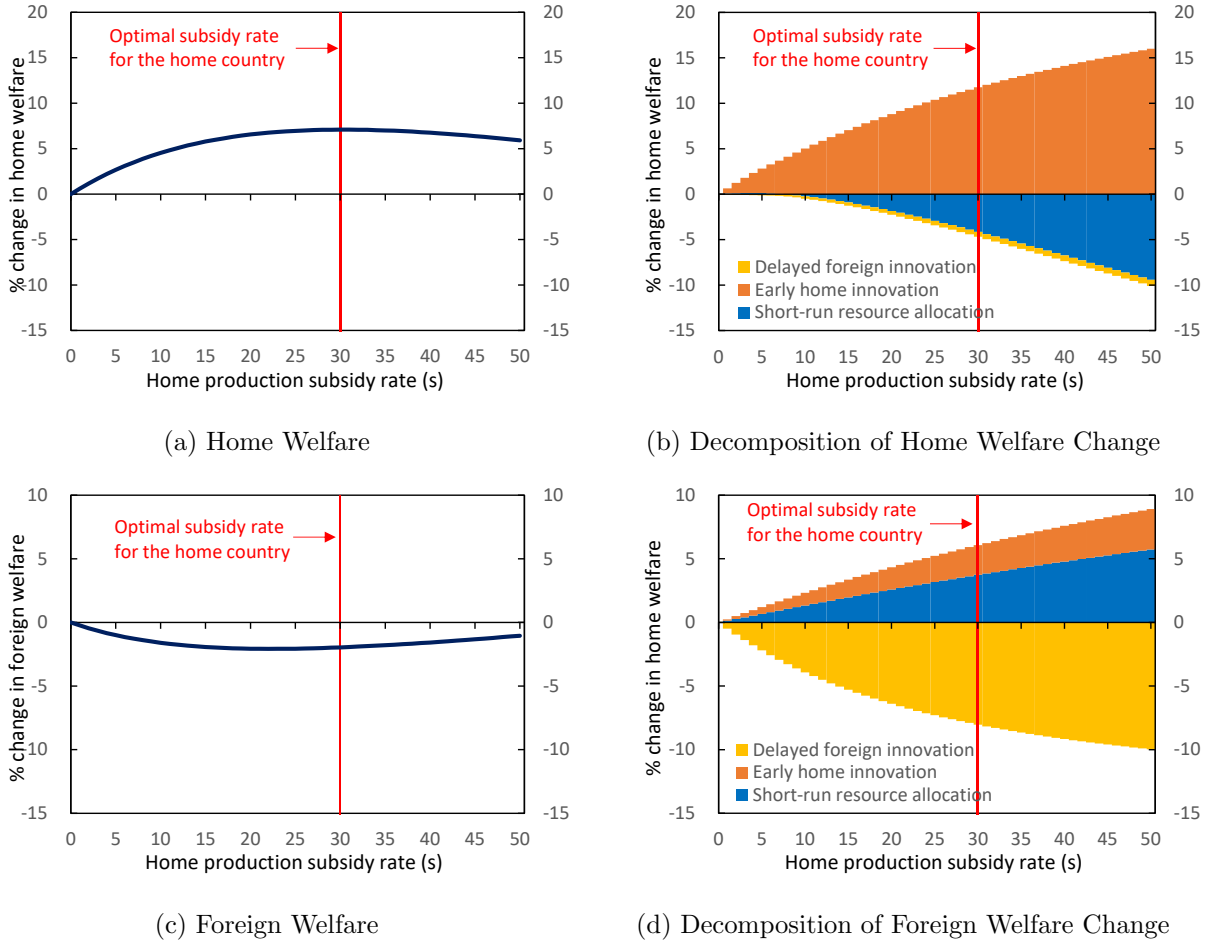


FIGURE 2.6. Overall Welfare Change from Unilateral Home Production Subsidy in Frontier Technology Races

Notes: Figure 2.6a and 2.6c show how the overall home and foreign welfare change while the home production subsidy rate  $s$  increases in Case 2. Figure 2.6b and 2.6d present the decomposition of the home and foreign welfare change into ‘short-run resource allocation effect’, ‘early home innovation effect’, and ‘delayed foreign innovation effect’. The red line in the figures represents home country’s optimal subsidy rate.

technologies such as Artificial Intelligence and machine learning, such policy reaction becomes even more necessary.

In this context, I analyze the equilibrium in a game situation where both the home and foreign government set a production subsidy rate to foster domestic industry 1. In Figure 2.7, I present the home and foreign country’s reaction curves showing each country’s optimal production subsidy rate given any subsidy rate chosen by the other country. The Nash equilibrium in this situation

is established at the point where the two curves intersect. The equilibrium shows well the policy competition between two countries in which each country responds to the other country's production subsidy rate by more aggressively setting its own subsidy rate. It is interesting that both home and foreign welfare increase in the Nash equilibrium compared with the benchmark case where the home country sets its optimal production subsidy rate but the foreign country does not conduct any policy. This is because the short-run benefit from the counterpart country's aggressive production subsidy outweighs the negative spillover effects on innovation for both countries.

I also show how the equilibrium changes if the two countries cooperatively conduct industrial policy. Surprisingly, they set a higher production subsidy rate in the cooperative equilibrium than in the Nash equilibrium. The reason for this is that each country internalizes positive spillovers to the counterpart country when deciding its production subsidy rate. As seen in Table 2.3, the welfare outcome in the cooperative equilibrium is a Pareto improvement compared with the Nash equilibrium.

The above results imply that a policy competition to promote specific economic sectors can be beneficial worldwide if the sector is young in both countries and has high growth-potential. Interestingly, this implication is opposite to a common criticism of industrial policy that it is not efficient for every country to foster the same industry.

TABLE 2.3. Home and foreign welfare in each equilibrium in Figure 2.7

	Benchmark (No foreign reaction)		Nash		Cooperative	
	Home	Foreign	Home	Foreign	Home	Foreign
Short-run resource reallocation	-4.2%	+3.7%	+0.8%	+0.8%	-1.8%	-1.8%
Timing of home innovation	+11.8%	+2.3%	+9.3%	+9.3%	+12.8%	+12.8%
Timing of foreign innovation	-0.5%	-8.0%				
Overall	+7.1%	-2.0%	+10.1%	+10.1%	+11.0%	+11.0%

Notes: This table presents the home and foreign welfare change in each equilibrium compared with the initial equilibrium.

**Sensitivity Analysis.** As in Case 1, where the home country subsidizes an industry to catch up with frontier technology, I conduct a sensitivity analysis for Case 2, wherein the home country subsidizes efforts to expedite the discovery of new technology. The micro and macro elasticities of substitution play a pivotal role in determining the welfare effects of industrial policy on the

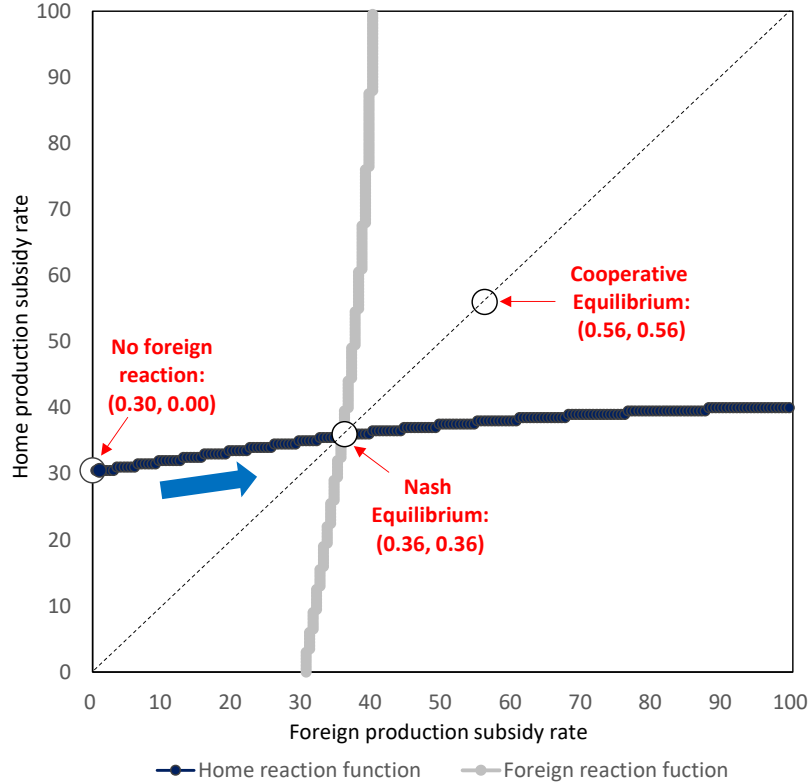


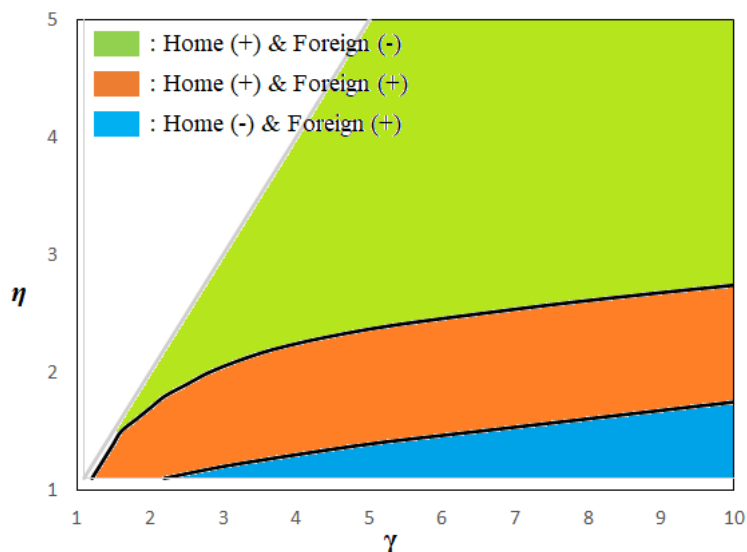
FIGURE 2.7. Equilibrium under Policy Competition

Notes: Figure 2.7 presents the home and foreign country's reaction functions, which show each country's optimal production subsidy rate given any subsidy rate chosen by the other country.

foreign country. A higher  $\eta$  reduces foreign welfare, while a higher  $\gamma$  enhances it. Given a specific  $\gamma$ , a high  $\eta$  facilitates substitution between home and foreign firms, accelerating innovation at home but impeding it abroad. Moreover, a higher  $\eta$  leads to a larger increase in the home wage compared to the foreign wage, resulting in more favorable terms of trade for the home country and a correspondingly less favorable situation for the foreign country. On the other hand, given a specific  $\eta$ , higher  $\gamma$  implies a more positive effect on reducing the price for the foreign country due to both the home subsidy and increased home productivity. However, it also indicates a greater subsidy expenditure for the home country.

Figure 2.8 illustrates how the relative sizes of these elasticities influence which country benefits from the home country's subsidy. Considering estimates of the micro and macro elasticity of substitution in Feenstra et al. (2018), where the former mostly lies between 1 and 10 and the

latter mostly lies between 1 and 4, the sensitivity analysis shows that the home production subsidy worsens foreign welfare under wide range of parameters.



Effect of  $s = 0.1$  on the home and foreign welfare,  $k = 1.5$

FIGURE 2.8. Sensitive Analysis in Frontier Technology Races

Notes: Figure 2.8 shows whether 10 percent of production subsidy rate ( $s = 0.1$ ) increases the home and foreign welfare while micro and macro elasticity of substitution ( $\gamma$  and  $\eta$  respectively) change.

## 2.5. Empirical Examination of Catch-Up

I revisit the Heavy and Chemical Drive (HCI drive), conducted by the Korean government during the period 1973-1979, through the lens of this model. I examine the policy in a simple event study framework to check whether outcomes are consistent with the central mechanism in my model.

**HCI Drive.** The Korean government launched its HCI drive in 1973, which aimed to support 6 industries: steel, non-ferrous metal, petrochemical, machinery, shipbuilding, and electronics. Supports included tax cuts, foreign credit allocation and providing new industrial complexes for those industries.<sup>12</sup> The table in Appendix B.3 provides a detailed description of the targeted industries. The HCI drive unexpectedly ended after the assassination of President Park in October

<sup>12</sup>See Kim et al. (2021), Choi and Levchenko (2021) and Lane (2022) for more details.

1979. Among various policy supports, a tax cut is closely related to this paper. Baek and Kim (2023) estimates the wage subsidy rate if tax supports were implemented in the form of wage subsidies, based on the effective marginal corporate tax rate provided by Yoo (1991). Using the method proposed by Baek and Kim (2023), production is estimated to be about 11 percent.<sup>13</sup>

There are several reasons why the HCI drive is a good example to use for the empirical examination to check whether the policy outcomes are consistent with the central mechanism in the catch up case in the model. First, heavy and chemical industries were literally infant industries even compared with other industries in Korea at the beginning of the drive. During the 1960s, Korean economy had grown, led by labor-intensive light industries such as textiles, wearing apparel, and leather products. With the comparative advantage in light industries at that time (Lane (2022)), the HCI drive was criticized by many economists and businesses even when President Park was alive. In addition, since firms in Korea did not have any experience manufacturing HCI products, some even argued the government ought to foster light industries further based on the advantage of low labor costs. Second, different policy implications can be drawn depending on the time horizon. The HCI drive is often considered to be a successful industrial policy in the long-run aspect (Kim and Leipziger (1997), Choi and Levchenko (2021), and Lane (2022)). Table 2.4 presents how HCI's value added, labor, and capital stock share in total manufacturing change during 1970-1990. As shown in the table, those shares significantly increased. However, as observed by Kim et al. (2021), the policy also causes inefficiency such as resource misallocation within the targeted industries, which was a reason why the policy was withdrawn right after President Park's assassination. In this context, it is plausible that there might be conflicting effects from the policy in the short-run and the long-run. Lastly, the timing of the policy can be used to identify its impact (Choi and Levchenko (2021), and Lane (2022)): i) the policy was partly initiated by external political event that President Nixon announced the end of direct U.S. military support for Asia-Pacific allies in 1969 and ii) it unexpectedly ended right after the assassination of President Park.

**2.5.1. Data.** I compare outcomes in the targeted industries with those in the non-targeted industries to see how resource allocation, output and productivity changed during and after the HCI

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<sup>13</sup>In this model, profit is proportional to output, and thus corporate tax can be transformed into a production subsidy.



TABLE 2.4. HCI's Value added, Labor, Capital Stock Share in Total Manufacturing

Year	Value Added	Labor Input	Capital Stock
1970	39.7%	28.3%	44.1%
1990	57.3%	43.5%	60.1%

Source: Pyo et al. (1993)

drive. For this comparison between the targeted and the non-targeted industries in Korea, I use 1) annual data for the number of plants and workers, and valued added for 28 Korean manufacturing industries between 1967 and 1986 from Statistics Korea's annual Mining and Manufacturing Survey and 2) annual data on total factor productivity for 28 Korean manufacturing industries between 1970 and 1986 from Pyo et al. (1993).

I also analyze how the wage and real exchange rate of South Korea changed relative to other countries during and after the HCI drive. For the comparison between South Korea and other countries, I utilize annual wage data from the Organization for Economic Cooperation and Development (OECD) between 1970 and 1986, and annual real exchange rate data from the Bank for International Settlements (BIS) between 1967 and 1986. Additionally, as controls, I incorporate working population data from the OECD and monetary policy rate data from the International Monetary Fund (IMF).

**2.5.2. Empirical Specifications.** As in Kim et al. (2021) and Lane (2022), I use a difference-in-difference estimation to explore the effect of the HCI drive on output, input, and productivity in the targeted industries compared with the non-targeted industries. The following equation is the difference-in-difference specification I use:

$$(2.55) \quad \log Y_{it} = \alpha + \sum_{j=\{1967-1971\} \cup \{1973-1990\}} \beta_j [D_i \times Year_t^j] + \delta_i + \delta_t + \epsilon_{it}$$

where  $Y_{it}$  is outcome variable for industry  $i$  in year  $t$ , and  $D_i$  is dummy variable which is equal to one if the industry were treated and otherwise zero. 9 industries are subject to the targeted industries out of 28 industries (See Appendix B.3 for details).  $Year_t^j$  is a year dummy variable.  $\delta_i$  and  $\delta_t$  are industry and time fixed effects respectively. Since I don't include year 1972 in the set of year dummy variables,  $\beta_j$  means the differential evolution of targeted and non-targeted industries relative to 1972. Standard errors are clustered by industry.

I also use another difference-in-difference estimation to see how the HCI drive affected wage and real exchange rate of Korea relative to other countries.

$$(2.56) \quad \log Y_{ct} = \alpha + \sum_{j=\{1967-1971\} \cup \{1973-1990\}} \beta_h [D_c \times Year_t^j] + \gamma_c + \delta_t + \log X_{ct} + \epsilon_{ct}$$

where  $Y_{ct}$  is outcome variable for country  $c$  in year  $t$ , and  $D_c$  is dummy variable which is equal to one if the country is Korea and otherwise zero.  $Year_t^j$  is a year dummy variable.  $\delta_c$  and  $\delta_t$  are country and time fixed effects respectively.  $X_{ct}$  represents controls, which include the working-age population for the wage equation and the monetary policy rate for the real exchange rate equation. Similarly,  $\beta_h$  means the differential evolution of Korea and other countries relative to 1972. Standard errors are clustered by country.

**Findings.** Figure 2.9 plots  $\beta_j$  for four outcome variables from the equation (2.55), which are labor input, number of plants, value added, and TFP, and  $\beta_h$  for two outcome variables from the equation (2.56), which are wage and real exchange rate, with 95% confidence interval.

The results from the difference-in-difference estimations are consistent with the model's central mechanisms. First, the pattern of productivity growth in the targeted industries matches the productivity evolution described in Section 3.2.3.1. Total factor productivity of the targeted heavy and chemical industries increased significantly relative to that of non-treated manufacturing industries with persistence after 9 years from the implementation of the HCI drive—later than the end of the policy. In addition, after 9 years, the total factor productivity kept increasing even though labor input and the number of plants didn't increase much. These imply that, as industry lifecycle theory anticipates, 1) even though the HCI were young and had high growth-potential, some amount of time was required for a realization of productivity improvement and 2) the productivity growth in the targeted industries was maintained at such higher rate for a while after it outpaced that in other matured industries. From there results, it can be reasonably inferred that the targeted industries moved from the early stage to the high-productivity stage around 1982.

Second, the estimation results on labor input, number of plants and value added also align with the model's predictions on resource allocation. Figures 2.9a, 2.9b and 2.9c demonstrate that labor input, number of plants, and value added of the targeted HCI increased significantly relative to the non-targeted manufacturing industries almost immediately after the policy implementation. Similar

to the comparative statics results in (C.73), even though productivity of targeted industry did not significantly increase immediately after the policy support, value added significantly increased due to increased resource usage, such as greater labor input and a higher number of firms entering the targeted industry. After the end of the HCI drive, while labor input and the number of plants in the targeted industries moved similarly to those in the non-targeted industries, value-added in targeted industries continued to outpace that in the non-targeted industries due to improvements in productivity. This suggests that the targeted industry transitioned to a high-growth stage, leading to a significant increase in productivity compared to the non-targeted industries.

Third, the movement of Korea's wage and real exchange rate compared with other countries is consistent with the model's mechanisms. Figures 2.9e and 2.9f show that 1) Korea's real exchange rate depreciated and 2) Korea's wage increased relative to other countries right after the policy implementation and after 1982 when the targeted industries were likely to transition to a high-growth stage. Since the period of the HCI drive and after 1982 correspond to Stage 1 and 2 in the model, respectively, the results match the comparative statics results in (B.29), (B.32), (B.40), and (B.43). The positive spillovers of the home country's industrial policy for the foreign country primarily materialize through a depreciation of the home country's real exchange rate in the model, as observed in Figure 2.9c. As the model predicts, foreign countries were likely to benefit from lower prices of Korean products in both the short and long run. In the short run, industrial policy increases the home wage, but price decreases due to subsidies dominate the effect of the wage increase. Similarly, in the long run, improvements in productivity increase the home wage, but price decreases due to productivity increases dominate the effect of wage increase.

## 2.6. Extention: Production Subsidy vs R&D Subsidy

In this section, I study how a home R&D subsidy affects innovation and welfare both at home and abroad, compared to a home production subsidy. For this policy comparison, I focus solely on Case 2, where both the home and foreign industry 1 are in their nascent stages and have high growth potential—the frontier technology races scenario—but only the home country provides the subsidy.

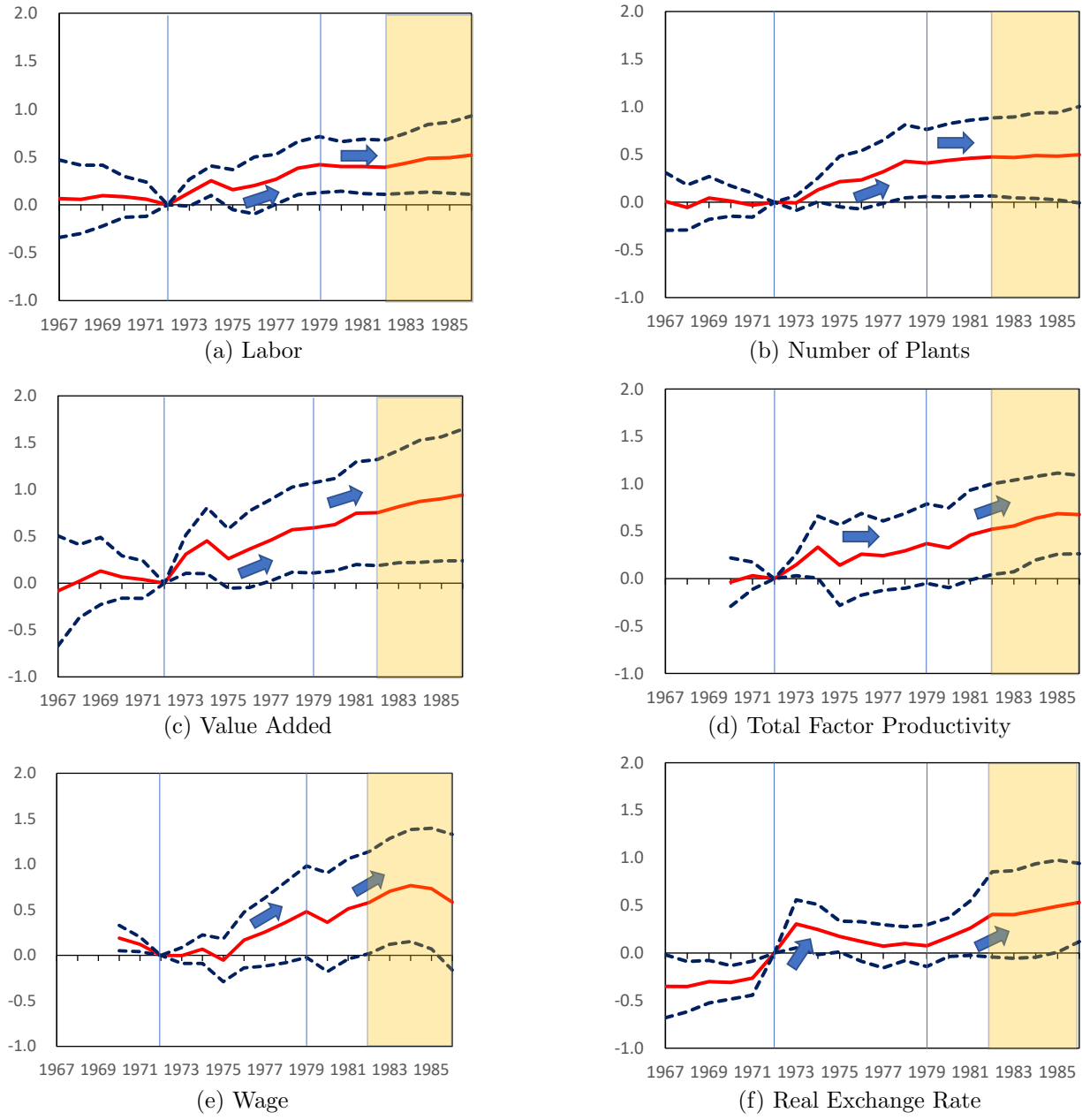


FIGURE 2.9. Impact of HCI Drive, [1973-1979]

Notes: Figure 2.9a, 2.9b, 2.9c and 2.9d plot the estimated coefficients  $\beta_j$  along with a 95 percent confidence interval from equation (2.55). Figure 2.9e and 2.9f plot  $\beta_h$  along with a 95 percent confidence interval from equation (2.56). The vertical lines indicate the start and end year of the HCI drive. The shaded area represents the period after the evolution of targeted industries' productivity becomes significantly different relative to non-targeted industries.

Since the fixed cost in equation (3.12) is assumed to be devoted to R&D cost in the early stage of the industry lifecycle, an R&D subsidy works by decreasing the fixed cost in the model. Thus, the zero profit condition for a firm in the home industry 1 changes to equation (2.57).

$$\begin{aligned}
(2.57) \quad \Pi_{1,t}(h) &= \frac{p_{1,t}(h)y_{1,t}(h)}{\gamma} \\
&= \frac{p_{1,t}(h)}{\gamma} [c_{1,t}(h)L + (1 + \tau)c_{1,t}^*(h)L^*] \\
&= \frac{1}{\gamma} \left( \frac{p_{i,t}(h)}{P_{hi,t}} \right)^{1-\gamma} \left[ \left( \frac{P_{hi,t}}{P_{i,t}} \right)^{1-\eta} \left( \frac{P_{i,t}}{P_t} \right)^{1-\sigma} P_t^{1-\psi} L + \phi \left( \frac{P_{hi,t}}{\epsilon_t P_{i,t}^*} \right)^{1-\eta} \left( \frac{P_{i,t}}{P_t^*} \right)^{1-\sigma} P_t^{*1-\psi} L^* \right] \\
&= (1 - s_d) \frac{1}{v}
\end{aligned}$$

where  $s_d$  is the R&D subsidy rate. In contrast to a production subsidy, an R&D subsidy does not affect the price of the home varieties in industry 1 and thus the pricing equation (2.22) remains unchanged.

I again assume that the home government provides an R&D subsidy to firms in industry 1 until a refinement occurs in the industry. Under this assumption, a R&D subsidy has different effects only in Stage 1 (short-run) compared with a production subsidy. In more detail, it differently affects the home and foreign welfare by changing i) the instantaneous utility in Stage 1 ( $U_1$  and  $U_1^*$ ) and ii) the timing of refinement in the two countries ( $t_r$  and  $t_r^*$ ) in a different way. Thus, I focus on how the short-run reallocation effect is different under a R&D subsidy. The comparative statics results for R&D subsidy are reported in Table B.3 in Appendix B.4.

The results in Table B.3 show that a R&D subsidy has qualitatively similar effects with a production subsidy. First, it unambiguously increases the mass of firms in the home industry 1 and decreases that in the foreign industry 1, which causes innovation to occur earlier in the home industry 1 and to be delayed in the foreign industry 1. Second, R&D subsidy can either increase or decrease the home instantaneous utility in Stage 1 while it unambiguously increases the foreign instantaneous utility in Stage 1.

Even though the direction of the aforementioned two effects are the same for both policies, the magnitude of those effects is different depending on which policy is implemented. For a fair comparison, it is necessary to compare effects of two policies given the same amount of required

tax. The following equations present the change in tax by introducing a production and a R&D subsidy respectively in the neighborhood of the initial equilibrium.

$$(2.58) \quad \frac{dT}{ds} = \frac{\gamma}{v} n_{1,t}, \quad \frac{dT}{ds_d} = \frac{1}{v} n_{1,t}$$

From the above equations, to the first order near the initial equilibrium, the relation between a production and a R&D subsidy rate which cause the same amount of tax is derived as follows.

$$(2.59) \quad \gamma ds = ds_d$$

Intuitively, this relationship is satisfied because the government subsidizes total revenue, which is equivalent to  $\frac{\gamma}{v}$  in the model, with a production subsidy while it only subsidizes the fixed cost ( $\frac{1}{v}$ ), which is equivalent to the operating profit, with a R&D subsidy. Equation (2.59) implies police-makers can set a R&D subsidy rate at  $\gamma$  times higher than a production subsidy rate with the same amount of a consequent tax.

Bringing the previous results from equation (2.59), Table C.2 and Table B.3 together allows me to compare the effects of two policies analytically. The following propositions show how the two policies differently affect the home and foreign economies.

**PROPOSITION 6.** *To the first order in the neighborhood of the initial equilibrium, if a production and a R&D subsidy impose the same amount of tax, i) the home utility in Stage 1 under the production subsidy is greater than that under the R&D subsidy, and ii) the R&D subsidy increases the mass of firms in the home industry 1 more than the production subsidy, and consequently iii) the R&D subsidy causes a refinement to occur more earlier in the home industry 1 than the production subsidy.*

$$(2.60) \quad dU_1^p - dU_1^d |_{T_p=T_d} = \frac{1}{2} P_t^{1-\psi} (\gamma - 1) ds > 0$$

$$(2.61) \quad d \ln n_{1,t}^p - d \ln n_{1,t}^d |_{T_p=T_d} = -(\gamma - 1) ds < 0$$

$$(2.62) \quad dt_r^p - dt_r^d |_{T_p=T_d} = t_r(0)(\gamma - 1) ds > 0$$

PROOF. See Appendix B.5

□

PROPOSITION 7. *To the first order in the neighborhood of the initial equilibrium, if a production and a R&D subsidy impose the same amount of tax, i) the policy effect on the foreign utility in Stage 1 and the mass of firms in the foreign industry 1 is the same under both policies, and ii) the R&D subsidy causes the refinement to be more delayed in the foreign industry 1 than the production subsidy.*

$$(2.63) \quad dU_1^{*p} - dU_1^{*d} |_{T_p=T_d} = 0$$

$$(2.64) \quad d \ln n_{1,t}^{*p} - d \ln n_{1,t}^{*d} |_{T_p=T_d} = 0$$

$$(2.65) \quad dt_r^{*p} - dt_r^{*d} |_{T_p=T_d} = - \left( \frac{n_1^*(1)}{n_1^*(2)} - 1 \right) t_r(0)(\gamma - 1) ds < 0$$

PROOF. See Appendix B.5

□

where a variable with superscript p (or d) means that corresponding to a production subsidy (or R&D subsidy).

For the home country, Proposition 6 shows a R&D subsidy is more effective for increasing the mass of firms in the targeted industry and consequently hastening innovation more than a production subsidy. This mainly arises from two reasons. First, as mentioned above, the a R&D subsidy rate is set at  $\gamma$  times higher than a production subsidy with the same amount of a consequent tax. Second, decrease in price by firms in the targeted industry in response to a production subsidy prevents new entry to the industry in some degree.

On the other hand, a R&D subsidy causes more distortion in Stage 1 than a production subsidy with the same amount of subsidy because it changes more the resource allocation in Stage 1. In short, a R&D subsidy hastens the long-run innovation more at the cost of greater distortion in the short-run. The following equation summarizes the difference in welfare effect for the home country

between two policies.

$$(2.66) \quad d \ln W^p - d \ln W^d \Big|_{T_p=T_d} = (\gamma - 1) \left[ \underbrace{\frac{1}{2} \frac{1}{\rho} (1 - \psi) \left( 1 - e^{-\rho t_r(0)} \right)}_{\text{Short-run resource reallocation : Production } > \text{ R\&D}} \underbrace{- e^{-\rho t_r(0)} (\ln U_2 - \ln U_1) t_r(0)}_{\text{Early home innovation : Production } < \text{ R\&D}} \right. \\ \left. + \underbrace{e^{-\rho t_r(0)} (\ln U_3 - \ln U_2) \left( \frac{n_1^*(1)}{n_1^*(2)} - 1 \right) t_r(0)}_{\text{Delayed foreign innovation : Production } > \text{ R\&D}} \right] ds$$

Since each policy has its advantages and disadvantages compared with the other, which policy is better for the home welfare depends on the values of parameters. If the targeted industry's growth potential is very high, the positive welfare effect from the early home innovation will dominate the other effects for the home country. Thus, the higher the growth-potential of the targeted industry is, the better R&D subsidy is for the home welfare.

Regarding the foreign country, Proposition 7 shows, to the first order, there is no difference in the effects on the foreign utility and the mass of firms in the foreign industry 1 between two policies in Stage 1. However, a R&D subsidy causes more faster transition from Stage 1 to Stage 2, which makes innovation be more delayed in the foreign industry 1. The following equation shows how differently the two policies affect the foreign welfare.

$$(2.67) \quad d \ln W^{*p} - d \ln W^{*d} \Big|_{T_p=T_d} = e^{-\rho t_r(0)} t_r(0) (\gamma - 1) \left[ \underbrace{-(\ln U_2^* - \ln U_1^*)}_{\text{Early home innovation : Production } < \text{ R\&D}} + \underbrace{(\ln U_3^* - \ln U_2^*) \left( \frac{n_1^*(1)}{n_1^*(2)} - 1 \right)}_{\text{Delayed foreign innovation : Production } > \text{ R\&D}} \right] ds$$

From equation (2.67), an implication can be drawn that a production subsidy is likely to be better for the foreign welfare than a R&D subsidy because i) the direct innovation effect  $(\ln U_3^* - \ln U_2^*)$  is much greater than the spillover effect  $(\ln U_2^* - \ln U_1^*)$ , and ii) the mass of firms in the foreign industry 1 drops a lot in transition from Stage 1  $(n_1^*(1))$  to Stage 2  $(n_1^*(2))$  due to the productivity jump in the home industry 1. Overall, contrary to the home welfare, the higher the growth potential



of the targeted industry is, the better a production subsidy is for the foreign welfare. Table 2.5 summarizes the welfare effect comparison between the two policies.

TABLE 2.5. Summary of welfare effect comparison between production and R&D subsidy

Effect	Home		Foreign	
	Production	R&D	Production	R&D
Short-run resource reallocation effect		$\gamma$		$=$
Gain from speeding up home innovation		$\lambda$		$\lambda$
Loss from delaying foreign innovation		$\gamma$		$\gamma$

**2.6.1. Model Simulation.** The analysis in the previous section implies that the growth potential of the targeted industry plays an important role in determining the relative performance of two policies. In this context, I carry out numerical exercises for two scenarios. Three conditions are different for each scenario: productivity growth by innovation, required time for innovation without policy, and discount rate. Table 2.6 presents the difference in those conditions in detail. I use the same values for all the other parameters as those in the benchmark case in Section 2.4.2.

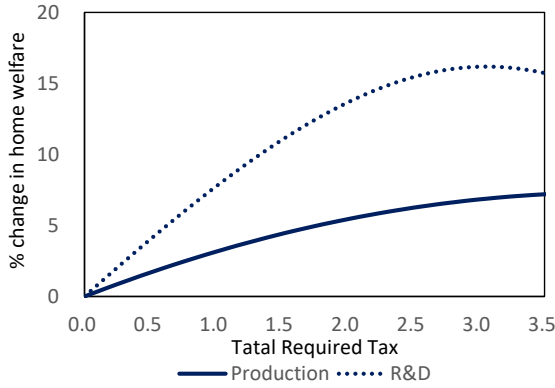
TABLE 2.6. Conditions in two scenarios

Condition	Scenario 1	Scenario 2
Productivity growth by innovation ( $\frac{a_{1,t}(2)}{a_{1,t}(1)}$ )	1.5	1.05
Required time for innovation without policy ( $t_r(0)$ )	10	20
Discount rate ( $\rho$ )	0.042	0.111

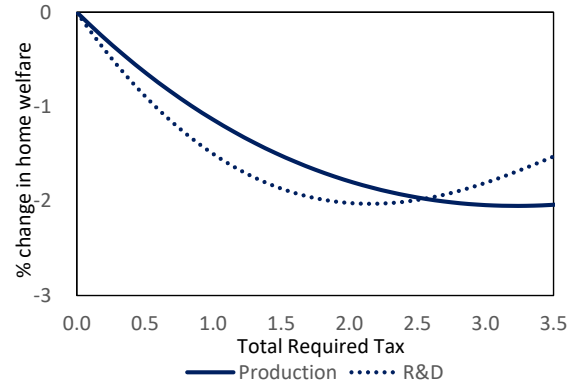
In the first scenario, relative to the second scenario, the growth potential of the targeted industry is higher and less time is required for innovation without any policy support, and discount rate is smaller.

Figure 2.10 shows how each policy affects the home and foreign welfare under each scenario. For an appropriate comparison, the welfare is calculated by controlling the total amount of subsidy the government has to provide until emergence of innovation to be the same under each policy. The results show a conflict of interest between the home and foreign country regarding choice of policy instrument. If the targeted industry has very high growth-potential (Scenario 1), a R&D subsidy performs better for the home welfare than a production subsidy by hastening innovation more, which worsens the foreign welfare more. In contrast, the growth-potential of the targeted

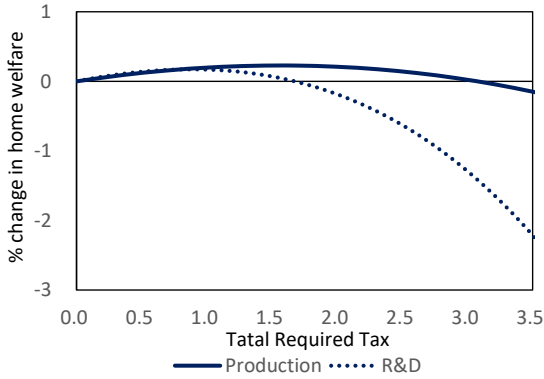
industry is not that high (Scenario 2), a production subsidy is superior to a R&D subsidy for the home country while a R&D subsidy performs slightly better for the foreign country.



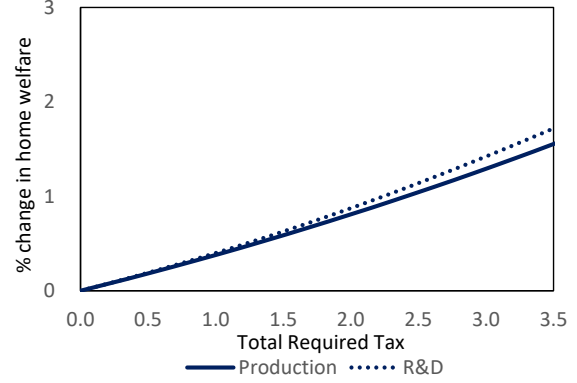
(a) Home Welfare in Scenario 1



(b) Foreign Welfare in Scenario 1



(c) Home Welfare in Scenario 2



(d) Foreign Welfare in Scenario 2

FIGURE 2.10. Comparison of Welfare Effect: Production vs R&D Subsidy

Notes: Figure 2.10a and 2.10b (Figure 2.10c and 2.10d) show the change in the home and foreign welfare in Scenario 1 (Scenario 2) while the overall tax amount which the home government has to spend in Stage 1 changes under production and R&D subsidy respectively.

## 2.7. Conclusion

In this paper, I study how the timing of industrial policy affects its welfare effects on the home and foreign country with a consideration of industry dynamics based on industry lifecycle theory. To answer this question, I propose an open economy macroeconomic model incorporating industry lifecycle theory. In this model, the home industrial policy affects the home and foreign welfare by hastening or delaying the timing of innovation in the targeted industry.

I derive several policy implications from the model. First, if industrial policy supports a young and high growth-potential industry, it hastens innovation in the industry and thus has a positive long-run growth effect. Even with the positive growth effect, overall welfare effect can be positive or negative since the policy can cause short-run welfare loss before realization of the innovation. This suggests the importance of welfare analysis of industrial policy in a dynamic view.

Second, home industrial policy can increase or decrease the foreign welfare, depending on whether the targeted industry is already mature in the foreign country. If the industry is already mature abroad, the home policy unambiguously increases the foreign welfare. This result is consistent with the positive international spillovers from home productivity increases suggested by Corsetti et al. (2007) with the standard Dixit-Stiglitz consumption index. On the other hand, if the foreign industry is also young and has high growth-potential, the home policy can worsen the foreign welfare by delaying innovation in the foreign industry. This is a novel channel through which home industrial policy causes “beggar-thy-neighbor” consequences.

Third, home industrial policy can trigger a policy competition between the home and foreign country. In the case where home industrial policy has a beggar-thy-neighbor effect, the model predicts that the foreign country responds by implementing more aggressive policy to offset the negative spillovers from the home policy and take a leadership in the high growth-potential industry. This gives an explanation why advanced countries are actively seeking to support high-tech industries in response to recent Chinese industrial policy, while they did not for China’s catching-up policy of manufacturing industries such as steel and shipbuilding. The model suggests that if the home and foreign country cooperatively support the high growth-potential industry in such a game situation, the welfare outcome is a Pareto improvement compared with the Nash equilibrium.

I also compare welfare effects of a production and R&D subsidy on home and foreign country. The results show that a R&D subsidy is more effective for hastening innovation in the targeted home industry than a production subsidy at the cost of more short-run distortion. Thus, the higher the growth potential of the targeted industry is, the better a R&D subsidy performs than a production subsidy by more accelerating R&D activities and competition in the industry. This mechanism works in the opposite way for the foreign country. Since the home R&D subsidy delays

more innovation in the foreign industry which also has high growth-potential, the foreign country benefits much later from the rapid productivity growth after innovation.

The implications derived in this paper can be useful for policy decision making since it provides a criteria for deciding “a right timing” to policy makers. For example, based on Vernon (1966) or Klepper (1996), if policy makers observe that firms start to set up their production facilities in other countries with lower wage rate, or the market share of a few large firms increases, or firms focus on process R&D in the industry that they are considering to support, it indicates the industry is maturing and thus it might be not good time to support the industry in aspect of long-run growth. However, I have to admit that the measurements of industry lifecycle suggested by the existing literature might not perfectly work for establishing industrial policy. For future research, providing a measurement method of industry lifecycle in the context of policy can complement this paper.

# Transition to a Green Economy: Policy Competition and Cooperation

## 3.1. Introduction

Reducing carbon dioxide emissions is an urgent necessity, given that carbon dioxide is the main contributor to greenhouse gas emissions and a key factor in global warming and climate change (Ozturk and Acaravci (2010)). Consequently, many countries are promoting green energy-related industries such as solar panels, wind turbines, and electric vehicle batteries. For instance, the United States (US) passed the Inflation Reduction Act (IRA), which provides subsidies to US businesses, households, and sub-national governments for investments leading to reduced greenhouse gas emissions. In response to the IRA, the European Union (EU) unveiled its Green Deal Industrial Plan (GDIP), followed by a newly announced subsidy scheme for solar panels, batteries, wind turbines, electrolyzers, and heat pumps.

Given that environmental issues inherently involve free-riding problems and the costs associated with transitioning to production systems using green energy sources have been considered high, the recent competitive implementation of policies supporting a transition to a green economy is noteworthy. Many studies, such as Cline (1992), Nordhaus and Yang (1996), Carraro and Egenhofer (2007), Yang (2008), Nordhaus (2010), Weitzman (2014), and Farrokhi and Lashkaripour (2021) explain that an individual country entirely bears the costs of abating greenhouse gas emissions but the avoidance of climate damage is a worldwide public good, which leads to strong incentives for free-riding. In addition, Shwom et al. (2010) and Nordhaus (2010) show a common belief that environmental policies will be costly and thus have adverse effects on the economy and employment. In this context, this paper aims to examine why many countries are competitively promoting the green industry and whether one country's policy triggers reactions from others. To answer

these questions, this paper investigates the welfare effects of recent industrial policies on both the implementing country and its counterparts.

For this analysis, I incorporate industry lifecycle theory into an open economy macroeconomic model from Corsetti et al. (2007), as in Baek (2023), for two reasons. First, industries that are essential for transitioning to a green economy can reasonably be considered to be in the early stages of their lifecycle and to have high-growth potential. This is because renewable energy usage remains low in most countries, despite an evident trend toward increasing green energy consumption.<sup>1</sup> This view is also supported by governments who state their expectations for the high growth potential and growing market size of those industries as one of the major reasons to support them, in addition to environmental urgency. Second, based on industry lifecycle theories such as Abernathy and Utterback (1978) and Klepper (1996), it is important to note that industries in their early stages, although possessing high growth potential, do not experience rapid growth immediately. Rather, their productivity or quality improves gradually at first and then escalates dramatically following radical innovations within the industry. This suggests that the benefits of industrial policies aimed at supporting nascent industries may take time to materialize. Consequently, it is essential to consider the welfare effects that occur during the ‘transition’ from one steady state to another, as induced by a policy, when evaluating the overall welfare effects of industrial policy. It is worth mentioning that Gillingham and Stock (2018) is closely aligned with the two points mentioned above. The paper suggests that while boosting demand for the technology in the early stages through subsidies may incur high initial costs, it can yield dynamic benefits by accelerating the realization of economies of scale or learning-by-doing, as demonstrated by the case study of solar photovoltaic panels.<sup>2</sup>

Taking into account environmental problems and industry lifecycle factors, the model in this paper exhibits distinct features compared to other canonical models in an open economy setup such as those proposed by Lashkaripour and Lugovsky (2023), Bartelme et al. (2021), Farrokhi

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<sup>1</sup>The renewable energy usage rate was 12% in 2020 in the US and 18% in 2020 in the EU according to the Energy Information Administration (EIA) and Eurostat.

<sup>2</sup>Gillingham and Stock (2018) suggests that (i) the demand for solar photovoltaic panels significantly increased due to policies that provided aggressive financial support for installing rooftop photovoltaic arrays, such as the German Energiewende and the California Solar Initiative, starting around 2002, and (ii) that this growth in sales likely led to a substantial decrease in panel prices after about 2007 through economies of scale and learning-by-doing.

and Lashkaripour (2021) and Bai et al. (2023). First, the model in this paper incorporates two externalities. One is the negative externality caused by greenhouse gas emissions from production using conventional energy. The other is knowledge spillover or industry-level learning-by-doing in the targeted industry, which can accelerate the transition from the early stage to the high-growth stage. These two externalities are similar to the ‘double externality’ concept in Nordhaus (2021). I will show that the knowledge spillover and industry-level learning-by-doing act as catalysts helping the world overcome the free-riding problem associated with the reduction of greenhouse gas emissions. Further, the objective of industrial policy in this paper differs from those in other studies. In this paper, governments aim to accelerate innovation in the targeted industry to leverage the knowledge spillover or industry-level learning-by-doing and hasten the abatement of damaging emissions. Because the innovation leads to productivity growth, this goal is ultimately linked to economic growth. This stands in contrast to the objective of reallocating resources to maximize gains from economies of scale in a static context, as pursued in industrial policies in other papers. Third, given this different policy objective, the welfare effects of industrial subsidies outlined in other canonical models still apply in this model. However, they are considered either as costs or as supplementary benefits incurred to accelerate innovation in the targeted industry.

With these novel features, this paper contributes to the literature in three ways. First, the model formally explains why both the home and counterpart countries are competitively supporting the targeted industry under these circumstances. Even though each country can benefit from the other’s innovation through (i) the reduction of greenhouse gas emissions and (ii) terms of trade gains from accessing cheaper products from the other country, the benefits of accelerating domestic innovation are greater due to the large economies of scale achieved by accelerated productivity growth and a consequent expansion in domestic green energy adoption. Therefore, supporting the green capital industry is desirable for the home country. In contrast, the home production subsidy delays foreign innovation and has a high probability of incurring a beggar-thy-neighbor effect on the foreign country. Thus, both countries are naturally engaged in a competitive game where each country sets its production subsidy to maximize its own welfare, given the other country’s subsidy level. In the Nash equilibrium, both countries end up setting production subsidies higher than the optimal rate they would choose if the other country did not react. Yet, ironically, each country’s

welfare level increases in the Nash equilibrium compared to the level under the optimal subsidy given no countering subsidy abroad, due to efforts to take the lead in green technology. This result contrasts with the conventional prisoner’s dilemma that characterizes many environmental conservation issues.

Second, this paper estimates an innovation timing elasticity, which denotes the responsiveness of the timing of innovation to an increase in the number of firms operating in the industry. This elasticity provides a novel interpretation regarding the benefits of industrial policy. Even though this elasticity is closely related to knowledge spillover, learning-by-doing, or economies of scale within an industry, it captures such concepts in a distinct way from existing literature. Many studies, such as Caballero and Lyons (1992), Irwin and Klenow (1994), Bartelme et al. (2021), and Lashkaripour and Lugovsky (2023), estimate a parameter that determines the degree of within-industry spillover. The parameter in those prior studies is constant, regardless of the industry’s stage in its lifecycle. Moreover, the related effects are either realized simultaneously or just one period after resources have been allocated to the industry. In contrast, the elasticity estimated in this paper is specific to the early stage of an industry’s lifecycle. The impact of within-industry spillover is not immediate; rather, it serves to shorten the time required for radical innovation in the industry. In this way, even though the policy’s role is limited only to accelerating innovation, it can be significant when the targeted industry is in its early stage of the lifecycle.

Third, this paper provides a quantitative analysis of recent industrial policies conducted by the US and the EU. The results are consistent with the model’s predictions that the welfare effects of one-sided industrial policies benefit the country implementing the policy but worsen the welfare of the counterpart. Accordingly, both countries set higher production subsidies in the Nash equilibrium. In this equilibrium, the welfare of both countries increases, and global greenhouse emissions decrease further compared to the case where only one country supports the green industry. In addition, the analysis shows that there is still a possibility for Pareto improvement and further reduction in greenhouse gas emissions through cooperative policy.

Related Literature. This paper contributes to the literature on industrial policy, innovation, and growth (Redding, 1999; Melitz, 2005; Rodrik, 2006; Aghion et al., 2015; Atkeson and Burstein, 2019; Choi and Levchenko, 2021; Lane, 2022; Bai et al., 2023). Among these, Bai et al. (2023)



is the closest to this paper in two aspects: it characterizes optimal innovation and trade policy in a ‘dynamic setting’ and focuses on the environment when ‘new technology’ emerges. Bartelme et al. (2021) and Lashkaripour and Lugovskyy (2023) are also closely related to this paper in that the welfare effects based on economies of scale, which arise during the reallocation of resources to the targeted industry, operate similarly in the model presented in this paper. However, this paper differs from the existing literature in that it incorporates the additional negative externality arising from greenhouse gas emissions, which is the focus of the environmental literature (Cline, 1992; Nordhaus and Yang, 1996; Stern, 2007; Carraro and Egenhofer, 2007; Garnaut, 2008; Yang, 2008; Nordhaus, 2010; Acemoglu et al., 2012; Acemoglu et al., 2016; Farrokhi and Lashkaripour (2021)), and analyzes the interaction between the innovation-related externality and the environmental externality. While Acemoglu et al. (2012) and Acemoglu et al. (2016) analyze the optimal policy path in a dynamic setup considering both innovation and the environment, these papers employ a closed-economy model and do not examine the international spillover of the home policy and the subsequent strategic policy responses of counterpart countries, which constitute the main focus of this paper. Farrokhi and Lashkaripour (2021) is notable for providing an optimal policy with a multi-country, multi-industry model in a game theory framework and comparing global welfare and environmental outcomes under two proposals: carbon border taxes and a climate club proposed in Nordhaus (2015). However, the paper does not consider dynamic technological innovation related to green transition.

This paper is also related to the literature on the estimation of knowledge spillovers and learning-by-doing within industries (Caballero and Lyons, 1992; Irwin and Klenow, 1994; Lieberman, 1987; Lindström, 2000; Thornton and Thompson, 2001; Lieberman, 1987; Bartelme et al., 2021). Bartelme et al., 2021 is most relevant in this respect as it estimates an elasticity related to external economies of scale (known as a scale elasticity) and quantitatively analyzes welfare gains from optimal industrial policy based on the estimates. This scale elasticity is concerned with an immediate change from one steady state to another in a static setting. In contrast, the innovation timing elasticity estimated in this paper pertains to the ‘transition path’ to reach a designated new steady state (high-growth stage). Thus, this study focuses on ‘how much faster’ the transition is

completed, offering a novel rationale for industrial policy, as opposed to focusing on ‘how much higher’ the level of innovation is.

This paper contributes to the literature that studies the stylized patterns of industry dynamics (Vernon, 1966; Abernathy and Utterback, 1978; Gort and Klepper, 1982; Jovanovic and MacDonald, 1994; Klepper, 1996; Antràs, 2005; Eriksson et al., 2021). Baek (2023) provides a general framework which incorporates the theory on how innovation in an industry changes along its life cycle into a canonical open macroeconomic model. This paper is an application of Baek (2023) to the green industrial policy, also extending it by providing a model-based empirical specification to estimate the sensitivity of the timing of innovation to industry lifecycle dynamics.

Lastly, this paper is related to the literature that studies the international transmission of a home country’s productivity increase to a foreign country (Ghironi and Melitz, 2005; Matsuyama, 2007; Corsetti et al., 2007). The mechanisms in Ghironi and Melitz (2005) and Corsetti et al. (2007) operate similarly in this model, making home innovation beneficial to the foreign country from a static perspective.

The paper is structured as follows. Section 2 reviews the related literature. Section 3 presents the model setup, and Section 4 analyzes the welfare effects of industrial policy. Section 5 presents the estimation of innovation timing elasticity and provides a quantitative analysis of recent industrial policies by the US and the EU. Section 6 concludes.

### 3.2. The Model

In the model, the world economy consists of two countries, home and foreign. Each country comprises households, final goods firms, green capital goods firms, and a government. The size of the households is denoted by  $L$  in the home country and  $L^*$  in the foreign country.

The green capital goods market is crucial in the model because innovation takes place in this market, and the government aims to accelerate this innovation within its jurisdiction. I nest the model of the industry lifecycle within the model of open economy spillovers by Corsetti et al. (2007) to analyze the welfare effects of an industrial policy by taking the industry lifecycle into account.

Throughout the paper, I set the home country wage as the numeraire for convenience.

**3.2.1. Households.** The utility function of the representative consumer in the home country at time  $t$  has the following form which is separable in consumption and pollution as in Keeler et al. (1971), Cabo et al. (2015), and Hassler et al. (2016).

$$(3.1) \quad U_t = C_t e^{-l_t - \kappa \left( \int_{I_t}^1 Y_{i,t}^e di + \int_{I_t^*}^1 Y_{i,t}^{e*} di \right)}$$

In equation (3.1),  $C_t$  represents the aggregate consumption of the final good at time  $t$ , and  $l_t$  is the labor supply by the representative consumer at time  $t$ . The exponent  $-\kappa \left( \int_{I_t}^1 Y_{i,t}^e di + \int_{I_t^*}^1 Y_{i,t}^{e*} di \right)$  captures the negative externalities arising from the production activities of manufacturing sectors that utilize conventional energy, such as fossil fuels, in both domestic and foreign countries.<sup>3</sup> In this term,  $\kappa$  represents the degree of negative externality from the production using conventional energy while  $Y_{i,t}^e$  and  $Y_{i,t}^{e*}$  denote production from the manufacturing sector  $i$  using conventional energy in the home and foreign countries, respectively, and  $I_t$  and  $I_t^*$  represent the cutoff index<sup>4</sup> for manufacturing sectors using green energy in the home and foreign countries, respectively. Details on production and the share of final goods producers who adopt green capital will be provided in a later section.

The aggregate final good comprises agricultural and manufacturing final goods. The agricultural good, denoted by  $C_t^A$ , is a final good and identical across countries of origin. I include the agricultural good in this benchmark model to facilitate tractability for analytical results, but I remove it in numerical simulations later. In contrast, manufactured final goods, denoted by  $C_t^M$ , consist of final goods from a continuum of sectors. The home good differs from the foreign good even though products in a specific sector from the same origin are identical. Consumers allocate their consumption between these two types of final goods with consumption shares of  $\iota$  and  $1 - \iota$ , as shown in the following equation.

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<sup>3</sup>Since environmental issues caused by greenhouse gases are global, I assume that the degree of negative impact from production using conventional energy is the same, whether the production takes place in the home or foreign countries.

<sup>4</sup>These cutoff indices also represent the share of manufacturing sectors that use green energy in the home and foreign countries.

$$(3.2) \quad C_t = (C_t^A)^\iota (C_t^M)^{1-\iota}$$

The representative consumer allocates a share of expenditure  $\beta$  to domestic composite manufacturing final goods and  $1 - \beta$  to foreign composite manufacturing final goods, as represented by:

$$(3.3) \quad C_t^M = (C_{h,t}^M)^\beta (C_{f,t}^M)^{1-\beta}$$

where  $C_{h,t}^M$  and  $C_{f,t}^M$  denote the domestic and foreign composite manufacturing final goods, respectively.

The domestic composite manufacturing final goods are aggregated through a Cobb-Douglas function, using a continuum of final goods in sector  $i$  that range from 0 to 1 and have a mass of one, as follows:

$$(3.4) \quad C_{h,t}^M = e^{\int_0^1 \ln C_{hi,t}^M di}$$

where  $C_{hi,t}^M$  is the consumption of the domestic final goods from sector  $i$ .

Households supply labor in a competitive market, serving both fixed-cost-related activities and production activities. I will elaborate on this in the next section. The labor supply is determined endogenously within the model.

Households own green capital goods firms located in the home country, which operate under monopolistic competition, in their own country. Each household receives an equal share of the profits from all firms in the green capital goods sector in their country:<sup>5</sup>

$$(3.5) \quad \Pi_t \equiv \int_0^{n_t} \Pi_t^g(\omega) d\omega$$

where  $n_t$  denotes the number of firms in the domestic green capital goods market at time  $t$ , and  $\Pi_t^g(\omega)$  represents the profit of a domestic green capital goods firm producing variety  $\omega$ .

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<sup>5</sup>Profits from all firms will be zero in equilibrium because firms' productivities are identical in the green capital goods markets, and a zero-profit condition will be imposed.

The representative consumer maximizes its utility (3.1) in time  $t$  subject to the following budget constraint<sup>6</sup>:

$$(3.6) \quad P_t C_t = w_t l_t + \Pi_t - \frac{T_t}{L}$$

where  $T_t$  is a lump-sum tax used to provide subsidies to the green capital goods industry. I will explain the subsidy in detail in a later section.

Similar expressions hold in the foreign country.

**3.2.2. Final Goods Production.** Final goods markets are perfectly competitive. The agricultural final good is produced with one unit of labor.

$$(3.7) \quad Y_t^A = L_t^A$$

For manufacturing final goods, firms in all sectors use labor and energy for production. Firms in each sector can choose to use either green or conventional energy. If a firm decides to use green energy, it must employ green capital, denoted by  $Z^g$ , to generate the energy itself. When a firm opts to use conventional energy, denoted by  $E$ , it can purchase this energy from the market at a price of  $\psi$ . It is assumed that productivity from using green capital varies across sectors.<sup>7</sup> Specifically, productivity from green capital in sector  $i$ , denoted by  $\lambda_i$ , decreases as  $i$  increases.<sup>8</sup> The production function for final goods in sector  $i$  varies depending on the type of capital used, as follows:

$$(3.8) \quad \begin{aligned} Y_{i,t}^g &= A_{i,t} \left( L_{i,t}^g \right)^\alpha \left( \lambda_i Z_{i,t}^g \right)^{1-\alpha} \\ Y_{i,t}^e &= A_{i,t} \left( L_{i,t}^e \right)^\alpha \left( E_{i,t} \right)^{1-\alpha} \end{aligned}$$

where  $A_{i,t}$  represents the productivity of sector  $i$  at time  $t$ ,  $L_{i,t}^k$  indicates the amount of labor employed in sector  $i$  at time  $t$ , depending on its energy type.  $E_{i,t}$  and  $Z_{i,t}^g$  are the amounts of conventional energy and green capital used in sector  $i$  at time  $t$ , respectively. For simplicity, I

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<sup>6</sup>Since consumers do not save in the model, they maximize their current utility in each period by only considering their current budget constraint.

<sup>7</sup>For example, the iron and steel manufacturing industry is one of the most energy- and carbon-intensive industries. Thus, it is more difficult for such industries to use renewable energy for their production compared to other industries.

<sup>8</sup>This implies the production cost of using green capital increases as  $i$  increases.

assume that capital fully depreciates in each period. Firms in each sector choose whether they will use conventional or green energy depending on their costs relative to productivity. The index for the marginal adopting sector,  $I_t$ , serves as the green energy adoption ratio in the manufacturing sectors. The green energy adoption ratio is determined so that the marginal adopting sector ( $i = I_t$ ) has a productivity for green capital,  $\lambda_{I_t}$ , satisfying the following equation:

$$(3.9) \quad \frac{P_t^g}{\lambda_{I_t}} = \psi_t$$

where  $P_t^g$  represents the price of aggregated green capital goods in the home country at time  $t$  and  $\psi_t$  represents the price of conventional energy. It is worth mentioning that this mechanism is similar to that in Helpman and Trajtenberg (1998), which studies the welfare effects of the diffusion of a new general purpose technology. In this regard, the increase in green energy adoption can be interpreted as the diffusion of a general purpose technology.

**3.2.3. Green Capital Goods Production.** The green capital goods sector is based on the model presented in Corsetti et al. (2007). The green capital market operates under monopolistic competition, where varieties are aggregated using the CES function.

$$(3.10) \quad Z_t^g = \left( \int_0^{n_t} z_t^g(\omega)^{\frac{\gamma-1}{\gamma}} d\omega + \int_0^{n_t^*} z_t^g(\omega_f)^{\frac{\gamma-1}{\gamma}} d\omega_f \right)^{\frac{\gamma}{\gamma-1}}$$

where  $\gamma$  represents the elasticity of substitution across varieties in the green capital goods market, and  $n_t$  and  $n_t^*$ <sup>9</sup> indicate the mass of firms in the home and foreign green capital goods market at time  $t$ , respectively. Here,  $\omega$  and  $\omega_f$  denote the home and foreign varieties, respectively.

Firms in the green capital goods market only use labor for production, and the production function is as follows:

$$(3.11) \quad y_t^g(\omega) = a_t^g(\omega)l_t^g(\omega)$$

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<sup>9</sup>As in Corsetti et al. (2007), I assume that all foreign firms serve the home market, while all domestic firms serve foreign markets.

where  $a_t^g(\omega)$  is the productivity of the firm producing variety  $\omega$  at time  $t$  and  $l_t^g(\omega)$  is the labor employed by the firm. I assume that  $a_t^g(\omega)$  is the same for all firms in each country but can be different across countries for simplicity.

A firm also needs to hire  $\frac{1}{v_t}$  units of labor each period, regardless of its amount of production. Firms allocate this fixed cost to different activities depending on their industry's stage along the lifecycle. As suggested in Abernathy and Utterback (1978) and Klepper (1996), firms in the early stage of the lifecycle need to invest in R&D for innovation. A 'dominant design,' defined as a product design widely accepted by consumers, has not yet emerged at this stage. Firms forgo alternative opportunities and operate in the early industry with low profit for some time to secure future higher profits by inventing a dominant design. In this context, without a certain amount of R&D investment, they are unlikely to innovate, and there is no reason to operate in the industry.<sup>10</sup>

For simplicity, it is assumed that this fixed cost remains constant over time and is the same for all firms in each country. Given wage  $w_t$ , the fixed cost  $q_t^g(\omega)$  is then,

$$(3.12) \quad q_t^g(\omega) = \frac{w_t}{v_t} = \frac{1}{v}$$

The operating profit, which is revenue minus cost, of the firm producing variety  $\omega$  at time  $t$  is

$$(3.13) \quad \Pi_t^g(\omega) = p_t^g(\omega)z_t^g(\omega) + e_t p_t^{g*}(\omega)z_t^{g*}(\omega) - l_t^g(\omega)$$

where  $p_t^g(\omega)$  denotes the price for home variety  $\omega$  at time  $t$  in the home market which is denominated in the home currency, and  $p_t^{g*}(\omega)$  denotes the price for home variety  $\omega$  at time  $t$  in the foreign market which is denominated in the foreign currency.  $e_t \equiv \frac{w_t^*}{w_t}$  represents the exchange rate defined as the relative value of the foreign wage in terms of the home wage.

The overall model structure is illustrated in Figure 3.1.

**3.2.3.1. Innovation and Productivity.** Given that the ratio of global renewable energy usage to total primary energy consumption was approximately 15 percent in 2020 according to the EIA, and considering that many countries and international organizations, such as the US and the EU, are targeting net-zero greenhouse gas emissions—defined as balancing the amount of emitted

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<sup>10</sup>Empirical regularity, as suggested by Akcigit and Kerr (2018), supports this assumption. It indicates a substantial decline in innovation-intensity, defined as the number of patents per employee, with increasing firm size among innovative firms.

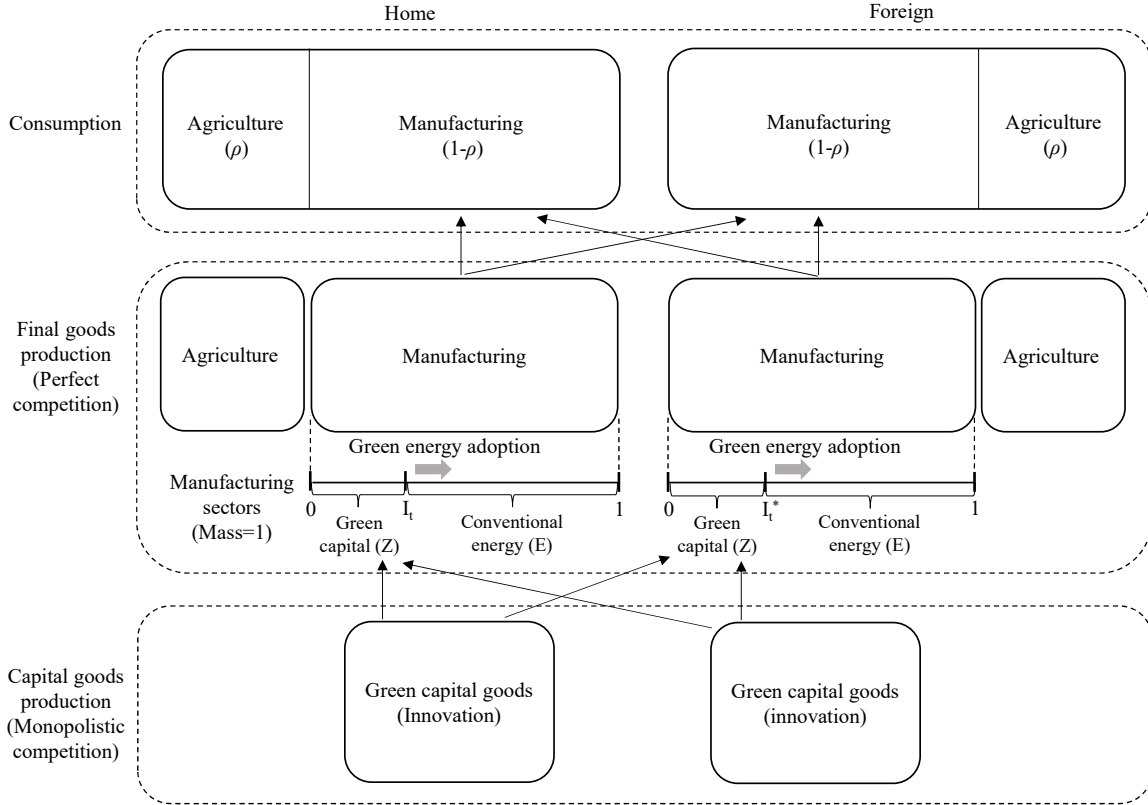


FIGURE 3.1. Model Structure

greenhouse gases with the amount removed from the atmosphere—it is reasonable to expect that the green capital industry is still in the early stages of its lifecycle.

As in Baek (2023), I assume that the productivity of the green capital industry endogenously changes as predicted by the industry lifecycle theory such as Jovanovic and MacDonald (1994) and Abernathy and Utterback (1978): Productivity grows slowly for some time after the industry's inception; then it increases rapidly following radical innovations; and finally, it gradually declines and stabilizes at a low level. This pattern of productivity change is assumed as follows.

*ASSUMPTION 5. The productivity of a firm producing variety  $\omega$  remains constant at the current level  $a_t^g(\omega)$  until its accumulated knowledge stock, denoted by  $k_t(\omega)$ , exceeds a certain threshold,  $\bar{K}$  at time  $t^r(\omega)$ . After reaching  $t^r(\omega)$ , the firm's productivity jumps to  $\bar{a}_H$  with probability  $p^r$ . For simplicity, I assume that  $p^r = 1$ .*



In this context, a key objective of industrial policy is to expedite the accumulation of knowledge in the targeted industry, thereby hastening its transition to the high-growth stage. The accumulation of a firm’s knowledge stock, denoted by  $k_t(\omega)$ , is assumed to depend not only on the firm’s own R&D efforts but also on industry-wide R&D activities (or learning-by-doing). There are three reasons why I assume that knowledge diffusion primarily occurs domestically. First, numerous countries endeavor to restrict technology diffusion in high-growth potential industries. For instance, in recent years, the United States has implemented several restrictions on technology sales to Chinese firms, invoking national security concerns. Central industries affected include telecommunications (Huawei) and semiconductor manufacturing. Second, according to industry lifecycle theories as presented in Vernon (1966) and Klepper (1996), firms are small during the initial stages of the industry lifecycle, and the production of new products largely takes place in the innovator’s home country. Consequently, major channels for international knowledge diffusion, such as trade (Eaton and Kortum, 2006; Buera and Oberfield, 2020; Cai et al., 2022)<sup>11</sup>, multinational production firms (Lind and Ramondo, 2023), FDI (Javorcik, 2004; Fons-Rosen et al., 2017), or migration (Bahar and Rapoport, 2018) are less likely to be in effect. Third, in a scenario where the degree of domestic knowledge diffusion is stronger than international knowledge diffusion (Branstetter, 2001), results of this paper do not qualitatively change. Accordingly, the following assumption is made.

ASSUMPTION 6.  $k_t(\omega) = k\left(\int_0^t q_j^g(\omega) dj, \int_0^t Q_j^g dj\right)$  where  $\int_0^t q_j^g(\omega) dj$  represents the cumulative R&D expenditure by firm  $\omega$ , and  $\int_0^t Q_j^g dj$  represents the cumulative R&D expenditure by all firms in the green capital goods industry.  $k(\cdot)$  is an increasing function with respect to both arguments.

Under the assumption that the fixed costs are allocated to R&D investment in the early stage, the cumulative R&D expenditure for both an individual firm and the green capital goods industry can be expressed as follows:

$$(3.14) \quad \int_0^t q_j^g(\omega) dj = \int_0^t \frac{1}{v} dj, \quad \int_0^t Q_j^g dj = \int_0^t \frac{1}{v} n_j dj$$

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<sup>11</sup>According to the 2012 United States benchmark input-output table (BEA), approximately 78% of domestic usage of products associated with energy-related capital goods originates from domestic sources.

Furthermore, following Irwin and Klenow (1994), I assume that a firm's knowledge stock function takes the following form:

$$(3.15) \quad k_t(\omega) = \left( \int_0^t q_j^g(\omega) dj \right)^{\zeta_1} \left( \int_0^t Q_j^g dj \right)^{\zeta_2}$$

Utilizing Assumption 5<sup>12</sup> and equations (3.14) and (3.15), the timing of innovation is given by:

$$(3.16) \quad t^r = t^r(\omega) = v \bar{K}^{\frac{1}{\zeta_1 + \zeta_2}} n_{i,t}^{-\frac{\zeta_2}{\zeta_1 + \zeta_2}}$$

I define the *innovation timing elasticity* as the elasticity of the timing of innovation with respect to the industry's total cumulative R&D expenditure (or production), denoted by  $\epsilon_{t^r}$ . The elasticity is given as follows:

$$(3.17) \quad \epsilon_{t^r} \equiv \frac{d \ln t^r}{d \ln \int_0^t Q_j^g dj} = \frac{d \ln t^r}{d \ln n_t} = -\frac{\zeta_2}{\zeta_1 + \zeta_2}$$

It is worth mentioning that the innovation timing elasticity is determined by comparing the relative contributions of the industry's total cumulative R&D expenditure (or production) to that of each firm's own cumulative R&D expenditure (or production) in constructing the firm's knowledge stock. This elasticity is the most important parameter for successful industrial policy in the model. I will estimate this elasticity for quantitative analysis in a later section.

**3.2.4. Industrial Policy.** As mentioned earlier, this paper focuses on the role of industrial policy in fostering growth in targeted industries. Both domestic and foreign governments offer production subsidies to firms in the green capital industry at rates denoted by  $s$  and  $s^*$ , respectively. These subsidies aim to facilitate either a significant decrease in the price of green capital goods or a notable improvement in their quality through innovation. Such innovation leads to a reduction in the cost of utilizing green energy, encouraging more final goods sectors to adopt production processes that leverage green energy. The subsidy continues either until an innovation occurs in the targeted industry or until the other country discontinues its provision. In this paper, I examine the impact of production subsidies as a representative example of industrial policy.<sup>13</sup>

<sup>12</sup>Based on Assumption 5, the industry's timing of innovation aligns with that of an individual firm ( $t^r = t^r(\omega)$ ).

<sup>13</sup>Refer to Baek (2023) for analysis with an R&D subsidy using a similar model to this paper.

In addition, I assume the government levies a lump-sum tax from consumers to fund the production subsidy. The total subsidy amount at time  $t$ , denoted as  $S_t$ , is given by

$$(3.18) \quad S_t = \int_0^{n_t} s p_t^g(\omega) y_t^g(\omega) d\omega = s n_t p_t^g(\omega) y_t^g(\omega)$$

Thus, the government's budget constraint is as follows:

$$(3.19) \quad s n_t p_t^g(\omega) y_t^g(\omega) = T_t$$

**3.2.5. Equilibrium.** For convenience, I establish the following assumptions. These will be employed in Section 3.3 to derive closed-form analytical results from the model. However, all these assumptions will be relaxed in Section 3.4 where I move to numerical simulations.

ASSUMPTION 7. (*symmetry*)  $v = v^* = L = L^* = A_{i,t} = A_{i,t}^* = 1$  and  $\psi_t = \psi_t^* = \psi$ .

ASSUMPTION 8. *Trading of final goods does not incur any trade costs, while green capital goods are subject to them.*

ASSUMPTION 9.  $\iota > (1 - \iota)(1 - \alpha)I_t$  and  $\iota > (1 - \iota)(1 - \alpha)I_t^*$

ASSUMPTION 10.  $\beta = \frac{1}{1+\phi}$

ASSUMPTION 11. (*Innovation*) *Before innovation occurs in a country's green capital market,  $a_t^g = a_t^{g*} = 1$  and  $I_t = I_t^* = \underline{I}$ . Once innovation takes place,  $a_t^g$  and  $a_t^{g*}$  increase to  $\bar{a}_H$  and  $I_t$  and  $I_t^*$  increase to  $\bar{I}$ .*

Assumption 9 implies that the size of the agricultural final goods market is larger than that of the green capital goods market in each country. This assumption, combined with Assumption 8, ensures that the relative wage between the home and foreign countries remains constant at 1 ( $w_t^* = 1, e_t = 1$ ).<sup>14</sup>

Assumption 10 implies that the degree of home bias for the final goods is the same as that of the capital goods. This assumption simplifies the analytic solutions in the welfare analysis.

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<sup>14</sup>When  $\rho > (1 - \rho)(1 - \alpha)I_t$  and  $\rho > (1 - \rho)(1 - \alpha)I_t^*$  are met, assume that no firm operates in the green capital market in a given country due to a lack of competitiveness, leading the final goods firms in that country to import all their green capital from the other country. Even under this scenario, with  $e_t = 1$ , a positive amount of agricultural final goods are produced in both countries given the larger size of the agricultural final goods market compared to the green capital market in both nations. Therefore,  $e_t$  remains at 1.

Assumption 11 provides a simplified representation of the innovation dynamics discussed in Section 3.2.3.1. While  $I_t$  and  $I_t^*$  could be endogenously determined by the productivity structure characterized by  $\lambda_i$ , even in the short run, employing Assumption 11 is reasonable because when a technology is in the early stage, its substitutability with the existing dominant technology is low. This can also be observed in the estimation of  $\sigma$  in Section 3.4.3.  $\sigma$  is much higher in the early stage than in the later stage.<sup>15</sup> Additionally, Assumption 11 significantly simplifies the welfare analysis.

Similar to Corsetti et al. (2007), the system of equations for equilibrium can be simplified to a system of two zero-profit conditions, (3.20) and (3.21), involving two endogenous variables,  $n_t$  and  $n_t^*$ . Solving for these variables enables us to determine all other variables. See Appendix C.1 for the formal derivation.

$$(3.20) \quad \Pi_t^g(\omega) = \frac{(1+s)(1-\iota)(1-\alpha)}{\gamma} \left[ \left( \frac{p_t^g(\omega)}{P_t^g} \right)^{1-\gamma} I_t + \phi \left( \frac{p_t^g(\omega)}{P_t^{g*}} \right)^{1-\gamma} I_t^* \right] = 1$$

$$(3.21) \quad \Pi_t^{g*}(\omega_f) = \frac{(1+s^*)(1-\iota)(1-\alpha)}{\gamma} \left[ \left( \frac{p_t^{g*}(\omega_f)}{P_t^{g*}} \right)^{1-\gamma} I_t^* + \phi \left( \frac{p_t^{g*}(\omega_f)}{P_t^g} \right)^{1-\gamma} I_t \right] = 1$$

where  $\phi \equiv (1+\tau)^{1-\gamma}$  and  $\tau$  represents the trade costs, and the prices are as follows.

$$(3.22) \quad p_t^g(\omega) = \frac{1}{1+s} \frac{\gamma}{\gamma-1} \frac{1}{a_t^g}$$

$$(3.23) \quad p_t^{g*}(\omega_f) = \frac{1}{1+s^*} \frac{\gamma}{\gamma-1} \frac{1}{a_t^{g*}}$$

$$(3.24) \quad P_t^g = \left( n_t (p_t^g(\omega))^{1-\gamma} + \phi n_t^* (p_t^{g*}(\omega_f))^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

$$(3.25) \quad P_t^{g*} = \left( n_t^* (p_t^{g*}(\omega_f))^{1-\gamma} + \phi n_t (p_t^g(\omega))^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

The equilibrium is described in detail in Appendix C.1.

### 3.3. Welfare Analysis

In this section, using the equilibrium obtained in Section 3.2.5, I analyze the welfare effects of home and foreign production subsidies on their own and the counterpart country's welfare.

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<sup>15</sup>Assumption 11 is equivalent to  $\sigma = \infty$  in the short-run (or in the early stage of the product's lifecycle).

The welfare functions for the home and foreign country are defined as follows:

$$(3.26) \quad \ln W = \int_0^{\bar{T}} e^{-\rho t} \ln U_t dt$$

$$(3.27) \quad \ln W^* = \int_0^{\bar{T}} e^{-\rho t} \ln U_t^* dt$$

where  $\rho$  denotes the discount rate. I set the discount factor to zero ( $\rho = 0$ ) in this section for two reasons. First, much of the existing literature (Cline, 1992; Stern, 2007; Garnaut, 2008) sets the discount factor at almost zero for environmental issues, as global environmental changes inevitably affect the welfare of future generations. This calibration is based on the idea that all generations should be treated equally. Second, this assumption makes welfare analysis provide simpler and more intuitive results. With a positive discount factor, results in a game where both the home and foreign set subsidy rate to maximize their own welfare become complicated since costs and benefits of the policy exist in both the short run and long run. Under the assumption of  $\rho = 0$ , I define  $\bar{T}$  as the last period<sup>16</sup> for the welfare analysis to prevent the welfare measure from approaching infinity. In Section 3.4, I will allow the model to have a small discount factor and infinity time horizon as in Stern (2007) and Garnaut (2008). It is also assumed that  $s = s^* = 0$  in the initial conditions.

In what follows, I first examine the case where only the home country provides a production subsidy to the green capital industry and assess its welfare effects. Then, I analyze how the welfare effects change when both the home and foreign countries provide production subsidies to maximize their own welfare.

**3.3.1. Industrial Policy in the Absence of a Foreign Response.** In this section, only the home country offers a production subsidy to the green capital industry ( $s \geq 0$ ,  $s^* = 0$ ). Given the results in the comparative statics in Table C.2, the following proposition is derived.

PROPOSITION 8. *In the initial conditions of  $s = s^* = 0$ , the home production subsidy accelerates home innovation ( $\frac{dt^r(s,s^*)}{ds} < 0$ ) and delays foreign innovation ( $\frac{dt^{r^*}(s,s^*)}{ds} > 0$ ).*

PROOF. Based on equation (C.73) and (C.74), and  $\epsilon_{tr} < 0$ , we can prove the following:  
 $\frac{dt^r(s,s^*)}{ds} = \frac{dt^r(s,s^*)}{d \ln n_t} \frac{d \ln n_t}{ds} = \epsilon_{tr} \frac{d \ln n_t}{ds} < 0$  and  $\frac{dt^{r^*}(s,s^*)}{ds} = \frac{dt^{r^*}(s,s^*)}{d \ln n_t^*} \frac{d \ln n_t^*}{ds} = \epsilon_{tr} \frac{d \ln n_t^*}{ds} > 0$  □

<sup>16</sup>The time  $\bar{T}$  can be interpreted as marking the end of one cycle of dominant technology. With an infinite time horizon, an industry is likely to experience several cycles of changing dominant technologies.

This conflict regarding innovation timing serves as the central mechanism triggering policy competition between the home and foreign countries in this model, a point that will be explained in this section. Based on Proposition 8, I introduce three stages as follows:

DEFINITION 5.

- *Early State* ( $0 < t < t^r(s, s^*)$ ): Neither the domestic nor the foreign green capital industries have experienced innovation.
- *Leading Stage* ( $t^r(s, s^*) \leq t < t^{r^*}(s, s^*)$ ): Innovation has occurred in the domestic green capital industry but not yet in the foreign industry.
- *Mature Stage* ( $t^{r^*}(s, s^*) \leq t$ ): Both the domestic and foreign green capital industries have innovated.

Following this definition, the welfare functions of the home and foreign countries can be expressed as:

$$(3.28) \quad \ln W = \int_0^{t^r(s, s^*)} \ln U_E dt + \int_{t^r(s, s^*)}^{t^{r^*}(s, s^*)} \ln U_L dt + \int_{t^{r^*}(s, s^*)}^T \ln U_M dt$$

$$(3.29) \quad \ln W^* = \int_0^{t^r(s, s^*)} \ln U_E^* dt + \int_{t^r(s, s^*)}^{t^{r^*}(s, s^*)} \ln U_L^* dt + \int_{t^{r^*}(s, s^*)}^T \ln U_M^* dt$$

where  $U_S$  and  $U_S^*$  denote the utility level of the home and foreign countries in stage  $S$ , respectively. Thus, the welfare effect of the home production subsidy can be decomposed into three parts as follows:

$$(3.30) \quad \frac{d \ln W}{ds} = \underbrace{\int_0^{t^r(s, s^*)} \frac{d \ln U_E}{ds} dt}_{\text{Short-run resource reallocation effect}} \underbrace{-(\ln U_L - \ln U_E) \frac{dt^r(s, s^*)}{ds}}_{\text{Earlier domestic innovation effect}} \underbrace{-(\ln U_M - \ln U_L) \frac{dt^{r^*}(s, s^*)}{ds}}_{\text{Delayed foreign innovation effect}}$$

$$(3.31) \quad \frac{d \ln W^*}{ds} = \underbrace{\int_0^{t^r(s, s^*)} \frac{d \ln U_E^*}{ds} dt}_{\text{Short-run resource reallocation effect}} \underbrace{-(\ln U_L^* - \ln U_E^*) \frac{dt^r(s, s^*)}{ds}}_{\text{Earlier domestic innovation effect}} \underbrace{-(\ln U_M^* - \ln U_L^*) \frac{dt^{r^*}(s, s^*)}{ds}}_{\text{Delayed foreign innovation effect}}$$

I will analyze each effect in detail in this section.

**Short-run Resource Reallocation Effect.** In the initial condition, the welfare effects of the home subsidy for both the home and foreign countries are derived as follows. See Appendix C.3.1

for the derivation and determination of the sign of each term.

$$(3.32) \quad \int_0^{t^r(0,0)} \frac{d \ln U_E}{ds} dt = t^r(0,0) \left[ \underbrace{-\frac{d \ln P_E}{ds}}_{\text{CPI: (+)}} \underbrace{-(1-\iota)(1-\alpha)I_E}_{\text{Tax: (-)}} \right] = t^r(0,0) \frac{(1-\iota)(1-\alpha)I_E}{\gamma-1} > 0$$

$$(3.33) \quad \int_0^{t^r(0,0)} \frac{d \ln U_E^*}{ds} dt = t^r(0,0) \underbrace{\left( -\frac{d \ln P_E^*}{ds} \right)}_{\text{CPI: net zero effect}} = 0$$

Equation (3.32) and (3.33) show that the home production subsidy causes home welfare to increase but has a net zero effect on foreign welfare in the early stage. For the home country, the second term in equation (3.32),  $-(1-\iota)(1-\alpha)I_E$ , indicates the loss from collecting tax. To finance the tax, the representative household must supply more labor, resulting in a loss of utility. Despite this cost, the economy initially benefits more from the decrease in the utility-based CPI, which occurs for two reasons: first, the subsidy directly reduces the price of green capital, which in turn lowers the price of final goods produced using green capital; second, it increases the mass of the green capital industry, further reducing the utility-based CPI. This latter mechanism aligns with the channel described in Lashkaripour and Lugovskyy (2023), where an economy benefits from a production subsidy by exploiting economies of scale through an increasing number of firms in the targeted industry.<sup>17</sup>

In contrast, the foreign utility-based CPI does not change due to the tension between terms of trade gain and loss from economies of scale. The foreign country benefits as foreign consumers and final goods firms can purchase cheaper final goods and green capital from the home country. However, as seen in equation (C.74), the home subsidy leads to a decrease in the number of foreign green capital goods firms, resulting in a loss from economies of scale. Overall, the net effect is zero.

**Earlier Home Innovation Effect.** As seen in Proposition 8, the home production subsidy increases the mass of firms in the home green capital industry, thereby hastening innovation in the industry. The welfare effect caused by earlier home innovation is derived as follows. See Appendix

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<sup>17</sup>This is well represented by the term,  $\frac{(1-\iota)(1-\alpha)I_E}{\gamma-1}$ , in the home short-run resource reallocation effect. The component  $(1-\iota)(1-\alpha)I_E$  represents the size of the targeted industry, while  $\frac{1}{\gamma-1}$  is the scale elasticity, as cited in Lashkaripour and Lugovskyy (2023). Consequently, this term indicates that the positive effect in the early stage amplifies as the product of scale elasticity and sector size increases.

C.3.2 for the derivation and determination of the sign of each term.

$$(3.34) \quad -(\ln U_L - \ln U_E) \frac{dt^r(s, s^*)}{ds} = t^r(0, 0) \left[ \underbrace{-\ln P_L + \ln P_E}_{\text{CPI: (+)}} + \underbrace{\kappa(\bar{I} - \underline{I})Y_i^e}_{\text{Less emission:(+)}} \right] \underbrace{\left( -\epsilon_{tr} \frac{d \ln n_E}{ds} \right)}_{\text{Earlier home innovation: (+)}} > 0$$

$$(3.35) \quad -(\ln U_L^* - \ln U_E^*) \frac{dt^r(s, s^*)}{ds} = t^r(0, 0) \left[ \underbrace{-\ln P_L^* + \ln P_E^*}_{\text{CPI: (+)}} + \underbrace{\kappa(\bar{I} - \underline{I})Y_i^e}_{\text{Less emission:(+)}} \right] \underbrace{\left( -\epsilon_{tr} \frac{d \ln n_E}{ds} \right)}_{\text{Earlier home innovation: (+)}} > 0$$

For the home country, gains follow innovation due to a reduction in greenhouse gas emissions. This is attributed to an increased use of green capital in the home country ( $\underline{I} \rightarrow \bar{I}$ ), leading to the production of fewer final goods utilizing conventional energy sources. Moreover, the home country experiences a decline in the utility-based CPI, a change driven unambiguously by two primary factors: first, enhanced productivity in green capital production directly reduces the price of home green capital, thereby decreasing the prices of final goods made with green capital; and second, a surge in the number of domestic green capital goods firms—resulting from increased competitiveness against foreign counterparts and an expanding green capital market—enhances economies of scale, further driving down the prices of goods produced using green capital.

The foreign country shares the emission reduction gains observed in the home country, even though its own greenhouse gas emissions remain unchanged. It also benefits from improved terms of trade. A decline in competitiveness leads to a reduction in the number of foreign green capital firms, resulting in losses from economies of scale. However, the advantage of accessing more affordable final goods and green capital from the home country outweighs the negative effects of reduced economies of scale. This outcome regarding international welfare spillover from home productivity improvement aligns with the findings presented in Ghironi and Melitz (2005) and Corsetti et al. (2007). Taking into account these effects, the foreign country also benefits from earlier home innovation.

**Delayed Foreign Innovation Effect.** The home production subsidy causes the number of operating firms in the foreign green capital industry to decrease, and this delays innovation in the



foreign green capital industry. The welfare effect related to this is represented in the equations below. See Appendix C.3.3 for their derivation and determination of the sign of each term.

$$(3.36) \quad -(\ln U_M - \ln U_L) \frac{dt^{r*}(s, s^*)}{ds} = t^r(0, 0) \left[ \underbrace{-\ln P_M + \ln P_L}_{\text{CPI: (-)}} + \underbrace{\kappa(\bar{I} - \underline{I})Y_i^e}_{\text{Less emission: (+)}} \right] \underbrace{\left( -\epsilon_{tr} \frac{d \ln n_E^*}{ds} \right)}_{\text{Delayed foreign innovation: (-)}}$$

$$(3.37) \quad -(\ln U_M^* - \ln U_L^*) \frac{dt_g^r(s, s^*)}{ds} = t^r(0, 0) \left[ \underbrace{-\ln P_M^* + \ln P_L^*}_{\text{CPI: (+)}} + \underbrace{\kappa(\bar{I} - \underline{I})Y_i^e}_{\text{Less emission: (+)}} \right] \underbrace{\left( -\epsilon_{tr} \frac{d \ln n_E^*}{ds} \right)}_{\text{Delayed foreign innovation: (-)}} < 0$$

Foreign innovation has two conflicting effects on home welfare. First, the loss of comparative advantage decreases home welfare. When a foreign country succeeds in innovation and catches up to home productivity, the home country loses its comparative advantages in the green capital goods market. Since the home green capital firms already possess high productivity, the negative effects from a loss in global market share and a consequent decrease in economies of scale dominate the terms of trade gains stemming from the foreign innovation. This is notable as it contrasts with the impacts of earlier home innovation on foreign welfare. Furthermore, this effect aligns with the findings in Bai et al. (2023). The model in Bai et al. (2023) suggests imposing higher tariffs on the sectors where the home country has comparative advantages. This aims to discourage innovation efforts in foreign sectors and delay their catch-up. Second, the home country benefits from the reduction of greenhouse gas emissions by the foreign manufacturing sectors. Thus, the overall effect of foreign innovation depends on the relative magnitudes of these two conflicting effects. Finally, the welfare effects from delayed foreign innovation are opposite because the net impacts of the previously explained two effects are delayed.

Foreign innovation unambiguously boosts foreign welfare, as the foreign country benefits not only from reduced greenhouse gas emissions but also from a decline in the utility-based CPI due to enhanced competitiveness of its green capital goods firms. However, since these advantages materialize later, the effect of delayed foreign innovation diminishes foreign welfare.

3.3.1.1. **Overall Welfare Effect.** By summing the above explained three effects, the overall welfare effect of home production subsidy for the home and foreign country is determined. The following proposition shows that in the initial condition, the home production subsidy unambiguously increases home welfare.

PROPOSITION 9. (*The necessity of industrial policy*) In the initial condition of  $s = s^* = 0$ ,  $\frac{d \ln W}{ds} > 0$ .

PROOF. See Appendix C.2.1 □

Intuitively, this result occurs because 1) the gain from its own innovation is greater than the welfare effect of foreign innovation ( $\ln U_L - \ln U_E > \ln U_M - \ln U_L$ ) and 2) the degree to which the home production subsidy accelerates home innovation is greater than that of delaying foreign innovation ( $-\frac{d \ln t_r}{ds} > \frac{d \ln t_r^*}{ds}$ ).

In contrast, a home production subsidy has the potential to decrease foreign welfare, as demonstrated in the following proposition.

PROPOSITION 10. (*Beggar-thy-neighbor*) When  $s = s^* = 0$ , if the following condition is satisfied,  $\frac{d \ln W^*}{ds} < 0$ .

$$(3.38) \quad \underbrace{(-\ln P_M^* + \ln P_L^*) \epsilon_{tr} \frac{d \ln n_E^*}{ds}}_{\substack{\text{The absolute value of loss in CPI} \\ \text{in the mature stage}}} > \underbrace{(-\ln P_L^* + \ln P_E^*) \left( -\epsilon_{tr} \frac{d \ln n_E}{ds} \right)}_{\substack{\text{Gain in CPI} \\ \text{in the leading stage}}} + \underbrace{-\epsilon_{tr} (\kappa(\bar{I} - \underline{I}) Y_i^e)}_{\substack{\text{Net gain} \\ \text{from less emission}}}$$

PROOF. See Appendix C.2.2 □

As Proposition 10 demonstrates, in the initial condition, the direction of the welfare effect of a home production subsidy on the foreign country is not definite. The left-hand side of the above inequality (3.38) represents the absolute value of the foreign country's loss experienced through the utility-based CPI by its delayed innovation.

Meanwhile, the terms on the right-hand side illustrate the gains that the foreign country derives from earlier home innovation. As elucidated above, this gain encompasses reductions in the utility-based CPI. Furthermore, even though the home and foreign countries experience an equal degree of gain and loss in terms of greenhouse gas emissions ( $\kappa(\bar{I} - \underline{I}) Y_i^e$ ) from earlier home innovation and

delayed foreign innovation, the degree to which the home production subsidy accelerates home innovation is greater than that by which it delays foreign innovation ( $-\frac{d \ln t_r}{ds} > \frac{d \ln t_r^*}{ds}$ ). Consequently, there exists a net gain from an environmental aspect.

Accordingly, the overall welfare effect of a home production subsidy on foreign welfare is contingent upon the balance of these losses and gains.

TABLE 3.1. Summary of Welfare Effects of One-Sided Home Production Subsidy

Effect	Home			Foreign		
	(+)	(-)	Sum	(+)	(-)	Sum
Short-run resource allocation	Scale $\uparrow$	Tax, ToT $\downarrow$	(+)	ToT $\uparrow$	Scale $\downarrow$	0
Earlier home innovation	Productivity $\uparrow$ , Scale $\uparrow$ , CO <sub>2</sub> $\downarrow$	ToT $\downarrow$	(+)	ToT $\uparrow$ , CO <sub>2</sub> $\downarrow$	Scale $\downarrow$	(+)
Delayed foreign innovation	Scale $\uparrow$	ToT $\downarrow$ , CO <sub>2</sub> $\uparrow$	(+)/(-)	ToT $\uparrow$	Productivity $\downarrow$ , Scale $\downarrow$ , CO <sub>2</sub> $\uparrow$	(-)
Overall effect	Productivity $\uparrow$ , Scale $\uparrow$ , CO <sub>2</sub> $\downarrow$	Tax, ToT $\downarrow$	(+)	ToT $\uparrow$ , CO <sub>2</sub> $\downarrow$	Productivity $\downarrow$ , Scale $\downarrow$	(+)/(-)

Table 3.1 summarizes the welfare effects of a home production subsidy with no foreign reaction. It is worth discussing why the benefit derived from its own innovation is substantially larger than the welfare gain from the other country’s innovation in this model. First, green capital goods function as a ‘general purpose technology’. After innovation, the adoption ratio of green capital goods in the home country rises from  $\underline{I}$  to  $\bar{I}$ . Since energy use is essential for production, an increase in productivity of this general purpose technology, coupled with its expanded usage, significantly impacts the overall manufacturing sectors through the input-output linkage. This mechanism is akin to that described in Helpman and Trajtenberg (1998). Second, ‘home bias’ plays an important role. As the domestic market size for green capital goods expands ( $(1-\iota)(1-\alpha)\underline{I} \rightarrow (1-\iota)(1-\alpha)\bar{I}$ ) due to innovation, domestic firms in the targeted industry benefit exclusively from the growth in the home market, owing to home bias. This dynamic is evident as an increase in  $I_t$  results in a surge in the number of domestic green capital goods firms while concurrently reducing the number of foreign green capital goods. Consequently, home bias enhances the gains from economies of scale.

It is also important to mention the importance of taking into account the industrial lifecycle and the innovation timing elasticity when designing industrial policies. The model suggests that the success of the industrial policy hinges crucially on the speed at which the policy fosters innovation

in the country, especially when the targeted industry is in its nascent stage. This can be formally demonstrated in the model by the fact that the net growth effect, which is the sum of earlier home innovation and delayed foreign innovation effects, is proportional to the innovation timing elasticity. This also indicates that the greater the innovation timing elasticity, the more aggressively the home country should offer production subsidies, and the more vigorously the foreign country should adopt counterbalancing policies or even more actively support its industry.

In this context, I will introduce a game situation in the next section where both the home and foreign countries provide a production subsidy to maximize their own welfare.

### 3.3.2. Policy Competition and Cooperation.

**Policy Competition.** When each country provides a production subsidy to its green capital goods firms, it affects the welfare of the other country by affecting resource allocation and thus innovation timing in the industry. Thus, I will find a Nash equilibrium in this game situation where each country maximizes its welfare by choosing a production subsidy rate, given the other country's production subsidy rate. I will again focus on the symmetric condition,  $s = s^* \geq 0$ . The following proposition and corollary show the Nash equilibrium of the game.

PROPOSITION 11. (*Policy competition*) Under the condition of symmetric production subsidy ( $s = s^*$ ), there exists a unique positive value  $\bar{s}$  satisfying  $\frac{d \ln W}{ds} = 0$ . The value of  $\bar{s}$  is given by

$$(3.39) \quad \bar{s} = \frac{\frac{1}{t^r(0,0)} \frac{d \ln W}{ds} \Big|_{s=s^*=0}}{(1-\iota)(1-\alpha) \underline{I} \frac{d \ln n_E}{ds} \Big|_{s=s^*=0}}$$

where  $\frac{d \ln W}{ds} \Big|_{s=s^*=0}$  represents the welfare effect under the initial condition  $s = s^* = 0$ .

PROOF. See Appendix C.2.3 □

COROLLARY 5. When  $s = s^* = \bar{s}$ ,  $\frac{d \ln W}{ds} = \frac{d \ln W^*}{ds^*} = 0$ , which means that these subsidy rates satisfy Nash equilibrium.

Proposition 11 indicates that, up to  $\bar{s}$ , there is an incentive for each country to increase its production subsidy more than the other country does. This result is interesting in the context that, even though there is an incentive for free-riding in the game—a situation that arises because each country not only benefits equally from reducing greenhouse gas emissions regardless of where

the reduction occurs, but also experiences gains from terms of trade based on the other country's innovation without bearing any cost—every country ends up striving to reduce greenhouse gas emissions. This is because the benefits of earlier innovation, securing a larger share of the global market, and exploiting economies of scale<sup>18</sup> derived from an increase in the mass of operating firms outweigh the benefits of free-riding.

**Policy Cooperation.** I analyze how welfare effects change when the home and foreign countries cooperatively adjust their production subsidies to the same degree. The following proposition and corollaries demonstrate that the cooperative equilibrium is Pareto-superior to the non-cooperative equilibrium.

PROPOSITION 12. (*Policy cooperation*) *When the home and foreign country cooperatively change subsidy rates ( $s_c = s = s^*$  and  $ds_c = ds = ds^*$ ),  $\frac{d \ln W}{ds_c} = \frac{d \ln W^*}{ds_c^*} > 0$  at  $s = s^* = \bar{s}$ .*

PROOF. See Appendix C.2.4 □

COROLLARY 6. *The optimal cooperative subsidy rate is greater than the optimal independent subsidy rate ( $\bar{s} < \bar{s}_c$ ).*

COROLLARY 7. *The welfare under the cooperative equilibrium is Pareto improved compared to the welfare under the Nash equilibrium.*

When each country sets its production subsidy independently, the marginal cost of the policy incrementally increases as the subsidy rate increases. This is because, when a country aims to make its firms more competitive through higher subsidies, despite these firms having the same productivity as those in the foreign country, additional distortion is created. This distortion is eliminated when the two countries coordinate their production subsidies. Proposition 12 implies that, when the two countries set their production subsidies cooperatively, innovation occurs marginally later compared to a scenario where independent policies are set at the same subsidy rate level. However, both countries can benefit more by eliminating both the additional distortion and the loss caused by delaying the other country's innovation.

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<sup>18</sup>These benefits also indicate the high risk of falling behind in the essential industry.

Moreover, since the benefit of the cooperative policy outweighs its cost for each country at the subsidy rate in the Nash equilibrium,  $\bar{s}$ , they are incentivized to extend their support for the green energy industry. Consequently, innovation occurs more swiftly in both countries compared to the Nash equilibrium.

### 3.4. Quantitative Analysis

In this section, I quantitatively analyze how recent industrial policies supporting the green capital industry affect both home and foreign countries. I use the US and the EU as representatives of home and foreign countries, respectively, given that these regions are actively supporting industries related to the transition to a green economy.

The innovation timing elasticity is the most important parameter in determining the welfare effects of industrial policy in the model. Thus, I first estimate the parameter based on the model. Then, I use this estimate to quantify welfare gains from unilateral versus cooperative action. Finally, I rerun these counterfactuals with several adjustments: I make the relative wage fully endogenous by removing the agricultural good that anchors it, introduce the production of capital goods used for conventional energy, and generalize consumption preferences from Cobb-Douglas to generalized CES.

#### 3.4.1. Estimation of Innovation Timing Elasticity.

3.4.1.1. *Data.* For the estimation of innovation timing elasticity, I require industry-country data for each year on the following variables: the share of the industry's export value relative to the total export value in that industry, the number of operating firms in the industry, and the wage level in the industry. To obtain this data, I use two data sources: UN Comtrade Database and the UNIDO Industrial Statistics Database, which I explain in detail.

**UN Comtrade.** I obtain data on the share of export value for an industry in a country relative to the total export value for that industry from the UN Comtrade Database. To maintain consistency in the share over time, I initially fix the set of countries under consideration to prevent fluctuations in the share due to changes in the countries included in the data. Based on value-added data from the UNIDO Industrial Statistics Database, I select 67 countries that account for 98.7%

of the total value added in the dataset. The list of selected countries is provided in Appendix C.6. Accordingly, I only use data on transactions occurring between these countries from 1962 to 2020.

Since both the importing and exporting countries report transactions, theoretically, there are two data entries for a single transaction. Following the approach of Feenstra et al. (2005), I prioritize using the importing partners' reported data whenever possible. If this data is unavailable, I use the export data reported by the exporting country if available.

For industry classification, I employ the SITC classification, as it allows for analysis over the longest time period. To merge this data with the UNIDO Industrial Statistics Database—which utilizes the 2-digit ISIC Rev 3 classification for industries—I match the 4-digit SITC industry codes to the 2-digit ISIC codes using a matching program provided by Liao et al. (2020). Subsequently, using the 2-digit ISIC classification, I calculate the share of export value for an industry in a country relative to the total export value for that industry for all the years under consideration.

**UNIDO Industrial Statistics Database.** The UNIDO Industrial Statistics Database provides data from 1963 to 2020 for 174 countries, including annual output, value-added, gross fixed capital formation, employment, wages, and the number of establishments across 23 manufacturing industries. The dataset adheres to the 2-digit ISIC Rev 3 system for industry classification. For the estimation, I utilize the data on the number of establishments and wages. Given that the raw data are reported in U.S. dollars, there is no need for currency conversion, facilitating direct usage of the data.

*Step 1: Estimate Industry-Country Innovation Indicator.* I estimate the innovation timing elasticity in three steps. In the first step, I estimate the innovation indicator, which captures both the quality and productivity levels, for an industry in a specific country within a given year. I refer to Khandelwal et al. (2013) for this estimation, who utilize trade data to estimate the quality of products in various industries. Their approach hinges on the intuition that, conditional on price, a product with a higher quantity is perceived to be of higher quality.

I adopt their intuition with some variations. Firstly, I aim to capture not only quality but also productivity improvements. Therefore, rather than controlling for price, I control for production cost, using wages as a proxy. Secondly, I rely on the export share in the global export market, data derived from the UN Comtrade Dataset, instead of quantity. This is because the latter is

often missing or inconsistently reported, making it difficult to construct reliable and sufficiently long continuous time series. Thirdly, I control for the number of operating firms to better capture an (average) firm's innovation capacity. Thus, the underlying principle of the methodology in this paper is that a higher share in the export market, conditional on production cost and the number of establishments, indicates higher quality or productivity of an average firm.

Building on the intuition described above, I derive the equation for estimating the innovation indicator. Similar to Khandelwal et al. (2013), assume that products from industry  $k$  are globally aggregated in the export market in the following way.

$$(3.40) \quad Q_{k,t} = \left[ \sum_{i=1}^N (\xi_{ik,t} Q_{ik,t})^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$$

where  $Q_{k,t}$  denotes the composite product of industry  $k$  in the global export market at time  $t$ .  $Q_{ik,t}$  denotes the aggregated products produced in industry  $k$  in country  $i$  at time  $t$ , and  $\xi_{ik,t}$  represents the average quality of products produced in industry  $k$  in country  $i$  at time  $t$ .  $N$  stands for the number of countries in the global export market. Then, the innovation indicator for industry  $k$  in country  $i$  can be derived from the share of exports from industry  $k$  in country  $i$  out of the total global export value in the industry,  $s_{ik,t}$ , as follows:

$$(3.41) \quad \begin{aligned} s_{ik,t} &\equiv \frac{P_{ik,t} Q_{ik,t}}{P_{k,t} Q_{k,t}} = \xi_{ik,t}^{\gamma-1} \left( \frac{P_{ik,t}}{P_{k,t}} \right)^{1-\gamma} = \xi_{ik,t}^{\gamma-1} \left( \frac{\frac{\gamma}{\gamma-1} n_{ik,t}^{\frac{1}{1-\gamma}} w_{ik,t} a_{ik,t}}{P_{k,t}} \right)^{1-\gamma} \\ &= \left( \frac{\gamma}{\gamma-1} \right)^{1-\gamma} \left( \underbrace{\xi_{ik,t} a_{ik,t}}_{\text{Innovation indicator}} \right)^{\gamma-1} n_{ik,t} w_{ik,t}^{1-\gamma} P_{k,t}^{\gamma-1} \end{aligned}$$

where  $P_{k,t}$  and  $P_{ik,t}$  represent the price for the industry  $k$ -composite product in the global export market and the price for the aggregated products produced in industry  $k$  in country  $i$  at time  $t$ , respectively. Additionally,  $n_{ik,t}$ ,  $w_{ik,t}$ , and  $a_{ik,t}$  denote the number of firms, the wage level, and the productivity level in industry  $k$  in country  $i$  at time  $t$ . It is worth mentioning that identifying the innovation indicator is identical to identifying productivity in the model in Section 3.2. Despite



the model not accounting for quality differences, as demonstrated in equation (3.41), the change in the innovation indicator has the same welfare effect as a change in productivity.

Taking the log of equation (3.41) yields the following expression:

$$(3.42) \quad \ln s_{ik,t} + (\gamma - 1) \ln w_{ik,t} - \ln n_{ik,t} = (1 - \gamma) \ln \frac{\gamma}{\gamma - 1} + (\gamma - 1) \ln \xi_{ik,t} a_{ik,t} + (\gamma - 1) \ln P_{k,t}$$

In equation 3.42,  $(\gamma - 1) \ln P_{k,t}$  is replaced with the industry-year fixed effect as it is common to all countries in industry  $k$  at time  $t$ . Additional country-year fixed effects are incorporated to control for other nationwide shocks. Consequently, the innovation indicator can be estimated as the residual from the following OLS regression, assuming a specific value for  $\gamma$ :

$$(3.43) \quad \ln s_{ik,t} + (\gamma - 1) \ln w_{ik,t} - \ln n_{ik,t} = \delta_{k,t} + \delta_{i,t} + e_{ik,t}$$

where  $\delta_{k,t}$  represents industry-year fixed effects, and  $\delta_{i,t}$  represents country-year fixed effects.

Based on Lashkaripour and Lugovsky (2023), I set  $\gamma = 4.71$ . This value represents the average of the estimates for the manufacturing sector's elasticity of substitution across countries in Lashkaripour and Lugovsky (2023). After the OLS regression, the logarithm of the innovation indicator is derived as follows:

$$(3.44) \quad \ln \widehat{\xi_{ik,t} a_{ik,t}} = \frac{\hat{e}_{ik,t}}{\gamma - 1}$$

In Appendix C.7, I provide Figure C.1, which shows the innovation indicators for industries in the United States, for reference.

**Step 2: Identify Industry-Country Innovation Timing.** With the log difference of the estimated industry-country innovation indicators, denoted as  $d \ln \widehat{\xi_{ik,t} a_{ik,t}}$ <sup>19</sup>, I calculate two separate metrics. The first metric,  $LI_{ik,t}$ , represents the average growth rate of the indicator over the last 10 years, spanning from  $t - 10$  to  $t - 1$ , and the second metric,  $FI_{ik,t}$ , illustrates the average growth rate over the subsequent 10 years, from  $t$  to  $t + 9$ . The formulas are as follows:

$$(3.45) \quad LI_{ik,t} = \frac{\sum_{t-10}^{t-1} d \ln \widehat{\xi_{ik,s} a_{ik,s}}}{10}, \quad FI_{ik,t} = \frac{\sum_t^{t+9} d \ln \widehat{\xi_{ik,s} a_{ik,s}}}{10}$$

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<sup>19</sup>This indicates the growth rate of innovation indicators.

Then, the innovation timing is identified as the year when the difference in the average growth rates between the last and the following 10 years, denoted as  $FI_{ik,t} - LI_{ik,t}$ , is largest for each industry-country time series. Given that the average length of the time series for the industry-country innovation indicator is about 27 years, it is reasonable to assume that there is only one lifecycle within each series.

To ensure robust identification of innovation timing, I have implemented several measures. First, I exclude industry-country observations where the  $FI_{ik,t}$  value is negative at the identified innovation timing, as this cannot be considered indicative of innovation. Second, I omit the identified innovation timings for industry-country pairs if their  $FI_{ik,t} - LI_{ik,t}$  value at the identified innovation timing<sup>20</sup> falls within the lowest 10 percent of all such values across industry-country pairs. This decision is based on the fact that the cutoff for the lowest 10 percent of the differences in the average growth rate between the last and the following 10 years is about 1.7 percent. Therefore, if the increase in the average growth rate during the 10 years after the identified innovation timing is smaller than this cutoff, such cases are not considered indicative of innovation. For the robustness check, I modify this cutoff and examine how the elasticity of innovation timing changes. As indicated in Table 3.2, the elasticity does not vary significantly with different cutoff values.

**Step 3: Estimate the Innovation Timing Elasticity.** From the above steps, I obtain the estimated industry-country innovation timing, denoted as  $t_{ik}^r$ . This is used to estimate the innovation timing elasticity. Building on equation (3.16) the following equation is derived:

$$(3.46) \quad \ln t_{ik}^r = \ln v_k + \frac{1}{\zeta_1 + \zeta_2} \ln \bar{K}_{ik} + \frac{\zeta_2}{\zeta_1 + \zeta_2} \ln n_{ik}$$

To apply the above equation in regression analysis, I need data reflecting the knowledge stock threshold for innovation for each industry-country pair, denoted as  $\bar{K}_{ik}$ . I assume that  $\bar{K}_{ik}$  is determined as follows:

$$(3.47) \quad \ln \bar{K}_{ik} = \theta_k + \theta_i + \zeta_3 \ln \xi_{ik,t_0} a_{ik,t_0}$$

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<sup>20</sup>As explained, this value is the largest for each industry-country time series.

where  $\lambda_{ik,t_0} a_{ik,t_0}$  represents the initial level of the innovation indicator, and  $\theta_k$  and  $\theta_i$  represent industry and country fixed effects, respectively.

Applying equation (3.47) to equation (3.46), equation (3.46) can be reformulated as:

$$(3.48) \quad \begin{aligned} \ln t_{ik}^r &= \ln v_k + \frac{\theta_k}{\zeta_1 + \zeta_2} + \frac{\theta_i}{\zeta_1 + \zeta_2} + \frac{\zeta_3}{\zeta_1 + \zeta_2} \ln \xi_{ik,t_0} a_{ik,t_0} + \frac{\zeta_2}{\zeta_1 + \zeta_2} \ln n_{ik} \\ \Rightarrow \ln t_{ik}^r &= \delta_k + \delta_i + \beta_1 \ln \widehat{\xi_{ik,t_0} a_{ik,t_0}} + \beta_2 \ln n_{ik} + e_{ik} \end{aligned}$$

where  $\delta_k$  represents industry fixed effects, and  $\delta_i$  represents country fixed effects.

To proceed to the final step of the estimation, I once again compile data for  $n_{ik}$  from the UNIDO Industrial Statistics Database. Given my focus on knowledge spillover within industries, I choose to use ‘establishment share’—defined as the ratio of the number of establishments in industry  $k$  in country  $i$  at time  $t$  to the total number of establishments in country  $i$  at time  $t$ —rather than using the absolute number of establishments. This choice is based on the possibility that a smaller country with a more concentrated industry could potentially experience greater spillover effects despite having fewer establishments compared to a larger country. To reflect the cumulative knowledge stock and avoid pinpointing the innovation time based on shocks specific to a given year, I calculate a 10-year average of this establishment share, encompassing the period from  $t^r - 10$  to  $t^r - 1$ , to use for  $n_{ik}$ .

Additionally, I select industries that correspond to the green capital goods sector in the model. For this purpose, I use energy-relevant trade data provided by the International Trade Administration. First, I collect all ten-digit HS codes under categories including renewables, thermal power, and the battery supply chain. By matching these HS codes to 2-digit ISIC Rev 3 codes, I identify the sectors of Chemicals, Fabricated Metals, Machinery and Equipment, Computers and Electronics, Electrical Machinery, Motor Vehicles, and Other Transport Equipment as the green capital goods sectors.

Using this dataset and the estimation equation (3.48), I obtain the estimation results, which are presented in Table 3.2.

In the baseline case represented in column (1) in Table 3.2, as mentioned earlier, I use  $\gamma = 4.71$  and set the cutoff value for the differences in the average growth rate between the following and

TABLE 3.2. Estimation Results for Innovation Timing Elasticity

	log of innovation time					
	(1) $\gamma = 4.71$	(2) $\gamma = 2.5$	(3) $\gamma = 7.5$	(4) $\gamma = 10$	(5) $\gamma = 4.71$	(6) $\gamma = 4.71$
Cutoff for innovation	lowest 10%	lowest 10%	lowest 10%	lowest 10%	lowest 5%	lowest 25%
log of initial innovation indicator	-0.410 (0.292)	-0.363** (0.146)	-0.356 (0.356)	-0.168 (0.405)	-0.588** (0.279)	-0.223 (0.375)
log of establishment share	-0.303*** (0.085)	-0.334*** (0.082)	-0.225** (0.092)	-0.235** (0.093)	-0.309*** (0.083)	-0.258** (0.106)
Observations	215	211	203	206	226	173
$R^2$	0.461	0.493	0.468	0.444	0.446	0.512

Notes: Standard errors in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . The estimation includes industry fixed effects and country fixed effects. For the results presented in columns (1), (2), (3), and (4), the CES parameter  $\gamma$  varies while the cutoff for innovation is set at the lowest 10% of  $FI_{ik,t} - LI_{ik,t}$  which is calculated based on Equation (3.45). For the results in columns (5) and (6), the innovation timing data consist of values above the lowest 5% and 25% of  $FI_{ik,t} - LI_{ik,t}$ , respectively, while the value of the CES parameter  $\gamma$  is fixed at 4.71.

the last 10 years, which is considered indicative of innovation, at the lowest 10%. For a robustness check, I alter these values and examine how the estimate of innovation timing elasticity varies. As shown in Table 3.2, the estimates are significant across all specifications and do not change significantly depending on the values of  $\gamma$  and the cutoff for innovation, ranging from -0.23 to -0.33.

Based on equation (3.16), the innovation timing elasticity reflects the relative contribution of industry knowledge spillover from the industry's overall R&D efforts to the construction of a firm's knowledge stock; however, it does not provide information about the return to scale of R&D expenditure. According to the estimation result in baseline case, it can be interpreted that about 30% of a firm's knowledge stock is constructed from industry knowledge spillovers, making the firm's own R&D expenditure about 2.3 times more important than industry knowledge spillover. This implies that while knowledge spillover plays an important role in the accumulation of a firm's knowledge stock, the firm's own R&D effort is essential for innovation.

The results for the initial innovation indicator,  $\hat{\beta}_1$ , indicate that this estimate is not statistically significant in many specifications, including the baseline specification. This suggests that while

industries with higher initial levels of productivity or quality appear to innovate faster, the time required for industry-wide radical innovation is not significantly influenced by these initial levels of productivity or quality.

**3.4.2. Model Generalization.** In this section, I generalize the baseline model in Section 3.2 by aligning it more closely with other canonical models, such as Lashkaripour and Lugovskyy (2023) and Bartelme et al. (2021), and by relaxing all the simplifying assumptions made in Section 3.2.5.

For this purpose, first, I introduce another capital goods sector that produces capital goods for conventional energy sources as in Hötte (2020). In the baseline model, the economy experiences only benefits from increasing economies of scale in the green capital sector as it transitions to a high-growth stage. By introducing a conventional capital goods sector, the gains from innovation are reduced compared to the baseline model, as the economy also experiences losses from decreasing economies of scale in the conventional capital sector. It is noteworthy that the elasticity of substitution across varieties in both the green and conventional capital goods sectors is set to the same value, a detail I will explain in the calibration section. Consequently, there is no incentive to reallocate resources for maximizing economies of scale, as described in the mechanisms of Lashkaripour and Lugovskyy (2023) and Bartelme et al. (2021). In this context, the policy environment is more conservative in the static aspect compared to those in the referenced papers.

Second, I ensure that the green energy usage ratios,  $I_t$  and  $I_t^*$ , are endogenously determined at all stages. To account for this, I introduce the following productivity structure for green energy adoption:

$$(3.49) \quad \lambda_i = \bar{\lambda}(1 - i)^\sigma$$

where  $\bar{\lambda}$  is a constant, and  $\sigma$  is the parameter governing the degree of increasing difficulty in green energy adoption as the industry index  $i$  increases.

Third, I relax the assumption of constant relative wages by eliminating agricultural final goods. The relative wage is now determined by the ensuing balance-of-payment equilibrium condition.

Fourth, I endogenously determine the consumption shares of domestic and foreign manufacturing goods by introducing the following CES structure.

$$(3.50) \quad C_t^M = \left( C_{h,t}^M \frac{\eta-1}{\eta} + C_{f,t}^M \frac{\eta-1}{\eta} \right)^{\frac{\eta}{\eta-1}}$$

Lastly, I allow a positive discount rate and an infinity time horizon.

Detailed equations for the generalized model are provided in Appendix C.8.

**3.4.3. Calibration.** Using the estimates from the previous section, I set the innovation timing elasticity,  $\epsilon_{tr}$ , to -0.30. Regarding the degree of productivity increase after innovation, I use a 30% productivity growth rate post-innovation in the green capital industry ( $a^g = 1.3$ ). With this productivity improvement after innovation, the model matches the projection in Nijssse et al. (2023) of a relative price decrease in green capital goods compared to conventional energy in the long run, corresponding to the stage where both the US and the EU succeed in innovation in the green capital goods sector. I set the required time for a transitional innovation in the initial equilibrium, where the US and EU do not provide any subsidies, at 12 years for the US and 8 years for the EU, respectively. This is based on a similar identification method used in Section 3.4.1, utilizing the projected renewable energy shares in the US and EU from Nijssse et al. (2023). For  $\sigma$ , the parameter determining the difficulty of green energy adoption, it is reasonable to assume that green energy adoption does not significantly change due solely to production subsidies before innovation, but becomes more flexible after innovation. To reflect this, I estimate  $\sigma$  in the early stage using historical prices of green and conventional energy provided by International Energy Agency (IEA) and  $\sigma$  in the leading and mature stage with the projection data for the green and conventional energy prices provided by Nijssse et al. (2023). The estimate of  $\sigma$  is 4.45 in the early stage and 1.14 in the leading and mature stage, respectively. The extent to which negative externality from carbon emission reduces welfare in the initial equilibrium is set at -2% based on the estimation using the social cost of carbon provided in Nordhaus (2017).

Apart from the aforementioned parameters, I choose parameter values based on standard values found in the literature. I set discount rate,  $\rho$ , at 0.1% according to Stern (2007). Based on Lashkaripour and Lugovskyy (2023), the elasticity of substitution between varieties in the green and conventional capital sectors is set to  $\gamma = 6.40$ . This value represents the average elasticity of

substitution across varieties for the seven sectors: Chemicals, Fabricated Metals, Machinery and Equipment, Computers and Electronics, Electrical Machinery, Motor Vehicles, and Other Transport Equipment. For the elasticity of substitution across countries in manufacturing final goods sectors, I set  $\eta = 4.71$ , also based on Lashkaripour and Lugovskyy (2023). This value is the average elasticity of substitution across countries for manufacturing sectors. Based on energy expenditure share data from OECD (2022), I set the energy cost share at  $1 - \alpha = 0.12$ . Data from the Energy Information Administration (EIA) and Eurostat indicate that, in 2018, the renewable energy share of total energy production was 0.12 in the United States and 0.21 in EU countries.

**Industrial Linkage Structure.** For counterfactual analysis, information on the industrial linkage structure between the US and the EU is necessary. This includes several key shares: the share of home-produced final goods in total final good consumption; the share of domestic sales in total sales of home-produced final goods; the share of home-produced capital goods in total capital formation; the share of sales to the domestic market in total sales of home-produced capital goods<sup>21</sup>; and the share of export value for each type of capital goods and final goods in total exports.

To get data on these shares, I use OECD Inter-Country Input-Output (ICIO) Tables for the year 2018. Since I focus on the welfare effects between the US and the EU, I only use the input-output linkage between the US and the EU. To obtain shares related to final goods, I use data on final consumption expenditure of households, non-profit institutions serving households, and general government from domestically produced products and products imported from the counterpart country. To obtain shares related to capital goods, I use gross fixed capital formation data from domestically produced products and products imported from the counterpart country.

Information on parameters and industrial linkage structure data used for the quantitative analysis is summarized in Table 3.3.

**3.4.4. Counterfactual Analysis.** For the counterfactual analysis, I employ the method presented by Dekle et al. (2008). As in Bartelme et al. (2021), it is assumed in the equilibrium observed in the ICIO dataset for the year 2018 that both the US and the EU do not provide production

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<sup>21</sup>I assume that for both green and conventional capital goods, the share of home-produced capital goods in total capital formation and the share of sales to the domestic market in total home-produced capital goods are the same.

TABLE 3.3. Parameters and Economics Linkage Structure

Parameters	Description	Value		Source
$\epsilon_{tr}$	Innovation timing elasticity	-0.30		Estimation
$a_g$	Innovation effect	1.3		Estimation (Nijse et al., 2023)
$\sigma$	Green energy adoption friction	4.45 / 1.14		Estimation (IEA, Nijse et al., 2023)
$N$	Environmental externality	-2%		Estimation (Nordhaus, 2017)
$\rho$	Time discount rate	0.1%		Stern (2007)
$\gamma$	CES parameter for capital goods	6.40		Lashkaripour and Lugovskyy (2023)
$\eta$	CES parameter for final goods	4.71		Lashkaripour and Lugovskyy (2023)
$1 - \alpha$	Energy cost share	0.12		OECD (2022)
Initial conditions	Description	US	EU	Source
$I_0$ ( $I_{f0}$ )	Green energy usage ratio	0.12	0.21	EIA, Eurostat
$t^r$ ( $t_f^r$ )	Required time for innovation	12	8	Estimation (Nijse et al., 2023)
$s_{hh}^{fc}$ ( $s_{ff}^{fc}$ )	Final goods domestic demand	0.93	0.97	OECD ICIO
$s_{hh}^{fp}$ ( $s_{ff}^{fp}$ )	Final goods domestic sales	0.96	0.94	OECD ICIO
$s_{hh}^{zc}$ ( $s_{ff}^{zc}$ )	Capital goods domestic demand	0.92	0.97	OECD ICIO
$s_{hh}^{zp}$ ( $s_{ff}^{zp}$ )	Capital goods domestic sales	0.97	0.93	OECD ICIO
$s_h^{fe}$ ( $s_f^{fe}$ )	Final goods export	0.37	0.66	OECD ICIO
$s_h^{ge}$ ( $s_f^{ge}$ )	Green capital export	0.02	0.05	OECD ICIO
$s_h^{ee}$ ( $s_f^{ee}$ )	Conventional capital export	0.11	0.29	OECD ICIO

Notes: Innovation effect refers to productivity growth in the green capital goods sector after innovation. For the parameter governing the difficulty of green energy adoption ( $\sigma$ ), the first value is applied in the early stage and the second value is applied in the leading and mature stages.

subsidies. The detailed system of equations that determines counterfactual changes is provided in Appendix C.9.

Figure 3.2 and Table 3.4 show several equilibria under various scenarios and the welfare changes in each scenario. The results offer a compelling explanation for why the US and the EU are actively and competitively supporting essential industries for a transition to a green economy. If each country conducts industrial policy without any reaction from the other, the optimal subsidy rates would be 0.23 and 0.18 for the US and the EU, respectively. However, when each country attempts to set its subsidy rate to maximize its own welfare, while taking into account the other country's subsidy, there is a Nash equilibrium. In this equilibrium, both the US and the EU set a higher subsidy rate, which is 0.3 for the US and 0.25 for the EU, than in the scenario where each implements a production subsidy alone. This outcome arises because the welfare gain from



a country's own innovation significantly outweighs the welfare gain derived from free-riding on the other country's innovation. This can be demonstrated by examining the welfare effects in scenarios with optimal subsidies and no reaction from the other country. In those scenarios, the welfare gain from advancing its own innovation stands at 3.44% and 1.51% for the US and the EU, respectively, while the welfare gains from the other country's earlier innovation are only 0.34% and 0.83%.

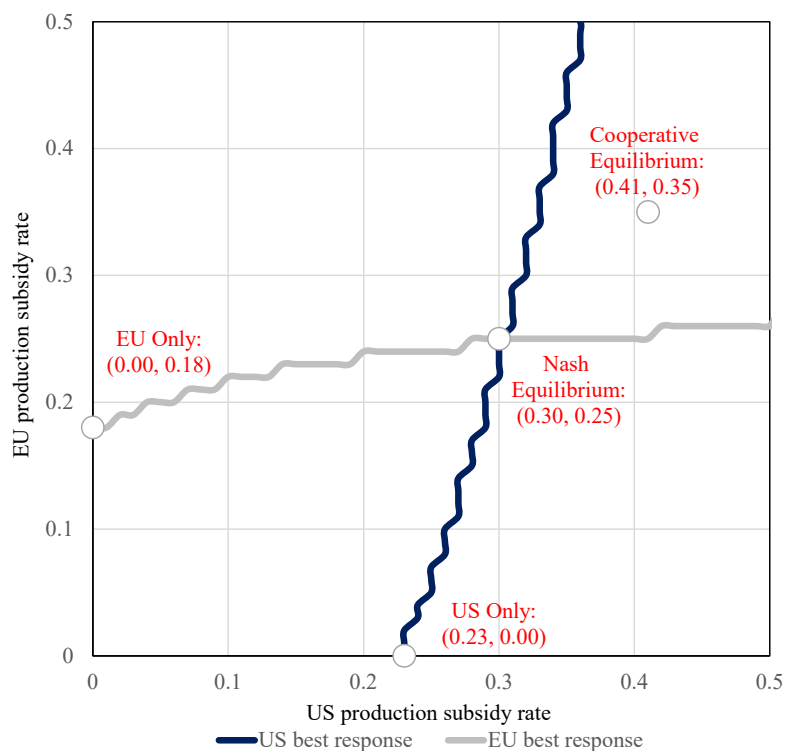


FIGURE 3.2. Equilibrium in Various Scenarios ( $\epsilon_{tr} = -0.3$ )

Notes: Figure 3.2 presents the US and the EU's best functions under the generalized model, which show each country's optimal production subsidy rate given any subsidy rate chosen by the other country.

The results also indicate that if each country implements an optimal subsidy and the other country does not respond to the policy, the policy can have a beggar-thy-neighbor effect. The EU's optimal subsidy decreases the US's welfare by 0.62%. This is because the loss from delayed innovation by the EU's subsidy is large for the US, amounting to -1.26% for the US. Interestingly, the EU is much less damaged from delayed innovation by the US's subsidy (-0.69%) than the US, which allows the EU to have a positive welfare change by the US's subsidy even though it is small. This

TABLE 3.4. Welfare of the US and the EU in Each Equilibrium

Stage	US only		EU only		Nash		Cooperative	
	US	EU	US	EU	US	EU	US	EU
Early	-0.13%	+0.73%	+0.30%	+0.22%	+0.39%	+1.30%	-0.33%	+0.91%
Leading	-0.18%	-0.69%	+0.34%	+1.51%	+0.27%	+1.25%	+0.42%	+1.83%
Mature	+3.44%	+0.83%	-1.26%	-0.31%	+3.37%	+0.82%	+4.04%	+0.98%
Overall	+3.12%	+0.87%	-0.62%	+1.42%	+4.05%	+3.40%	+4.13%	+3.76%

Notes: This table illustrates the variations in the welfare of the US and the EU at each scenario compared to the initial equilibrium under the baseline model.

result suggests that the loss from delayed innovation is larger for the country without a comparative advantage. Industrial linkage data in Table 3.3 indicate that the EU has a comparative advantage in both the capital goods and manufacturing goods sectors. This interpretation is supported by the observation that the innovation timing in the green capital goods sector in the US is delayed by about 1 year (from year 12 to year 13) due to the EU’s policy, while the timing in the EU is delayed by about 0.5 year (from year 8 to year 8.5).

Additionally, it is notable that the cooperative equilibrium leads to a Pareto improvement, increasing the welfare gain from 4.05% to 4.13% for the US and from 3.40% to 3.76% for the EU. This finding aligns with results observed in games concerning the environmental issues in the existing literature. In many cases, environmental issues are likely to give rise to a free-riding problem, preventing any entity from taking appropriate measures to resolve the environmental problems even though they could improve their utility by addressing the issue cooperatively. Thus, a cooperative policy is desirable even though such cooperation is hard to maintain (Nordhaus and Yang 1996; Carraro and Egenhofer 2007; Yang 2008; Farrokhi and Lashkaripour, 2021). Similarly, since the free-riding problem is not entirely resolved in the Nash equilibrium, there exists a potential for Pareto improvement through cooperative policy. However, it is also important to note that the scope for improvement through cooperative policy in this case is limited, as both countries have already established high production subsidies as a result of their competition.

Lastly, this analysis sheds light on the environmental effects of industrial policy, in addition to the welfare effects. The numbers in Table 3.5 represent the reduction in greenhouse gas emissions as a proportion of total emissions over 13 years (from 2018 to 2030) in the initial equilibrium.<sup>22</sup> Since

<sup>22</sup>This can also be interpreted as a yearly decrease in global greenhouse gas emissions by that amount over 13 years.

it is desirable for each country to support the green capital industry, greenhouse gas emissions decrease in any equilibrium. However, this reduction is not substantial when only one country supports the green industry; although such a policy accelerates a country’s own innovation, it also delays that of the other country. The former leads to a decrease in emissions, while the latter causes an increase, yielding a net effect that is not significantly large. As shown in Table 3.5, greenhouse gas emissions decrease much more significantly in both the Nash and cooperative equilibria.

TABLE 3.5. Change in Greenhouse Gas Emission in Each Equilibrium

	US only	EU only	Nash	Cooperative
Greenhouse gas emission	-7.76%	-1.34%	-11.77%	-14.39%

Notes: This table illustrates the changes in the global greenhouse gas emission at each equilibrium compared to the initial equilibrium under the baseline model.

3.4.4.1. *Shift from Free-riding to Active Subsidizing.* The most significant difference between the results of the quantitative analysis in this paper and those found in the existing literature is that each country actively provides support to the green capital goods sector, even without any arrangement for cooperation. This international policy dynamic leads to a faster decrease in carbon emissions, contrasting with a situation where each country opts for free-riding on another country’s efforts to resolve environmental problems. This difference is primarily due to the existence of knowledge spillover and the potential for faster innovation. Consequently, the extent of each country’s efforts depends on the magnitude of this externality, which can be characterized by the value of the innovation timing elasticity. In this context, I will examine the role of the innovation timing elasticity in-depth.

For this purpose, I first investigate how each country’s optimal policy changes when there is no externality to hasten innovation timing in the green capital goods sector, meaning  $\epsilon_{tr} = 0$ . Figure 3.3 presents the results for equilibria under various scenarios with the condition of  $\epsilon_{tr} = 0$ .

The best response function of the US and the EU is clearly different from that in Figure 3.2. First, each country chooses a much smaller subsidy rate across all scenarios. Second, each country does not respond significantly to the other country’s increase in production subsidy. This is evidenced by the observation that the subsidy rate only increases from 0.09 to 0.10 for the US and from 0.09 to 0.11 for the EU in the unique Nash equilibrium. In contrast, in Figure 3.2, within

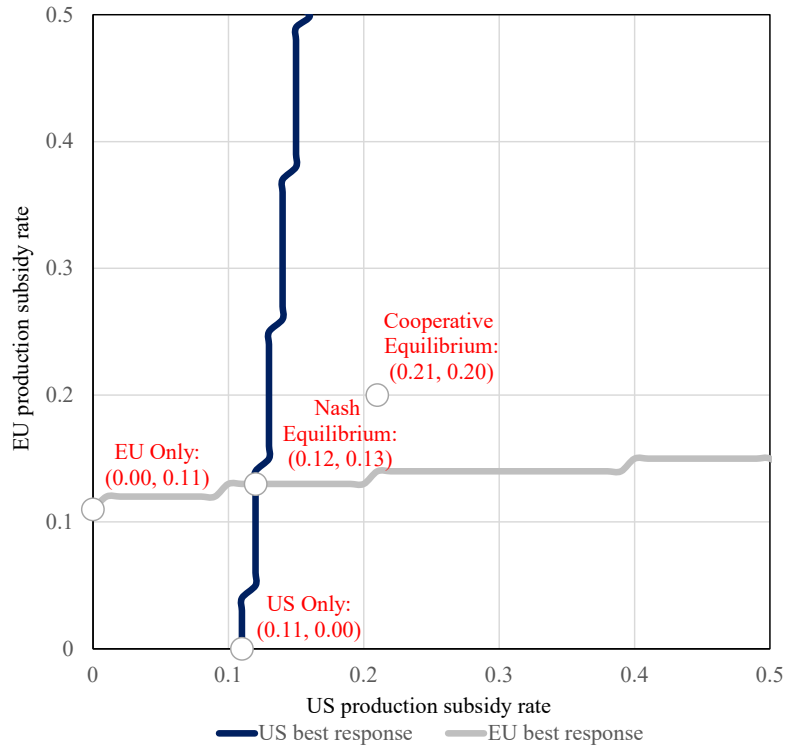


FIGURE 3.3. Equilibrium under Various Scenarios ( $\epsilon_{tr} = 0$ )

Notes: Figure 3.3 displays the best-response functions for the US and the EU under the generalized model with  $\epsilon_{tr} = 0$ . These functions indicate each country's optimal production subsidy rate, given any subsidy rate selected by the other country.

the range of multiple Nash equilibria, if one country increases its subsidy rate, the other country responds by increasing its subsidy rate by the same amount.

To derive intuition, I also construct a simple competitive game where the US and the EU choose between two options: free-riding ( $s = 0$ ) versus active subsidizing ( $s = 0.41$ ,  $s^* = 0.35$ )<sup>23</sup>. Table 3.6 shows the payoff matrices of this game under various values of  $\epsilon_{tr}$ .

In the case of  $\epsilon_{tr} = 0$ , since there is no externality related to innovation, free-riding becomes the dominant strategy for both the US and the EU. Thus, choosing free-riding by both countries is the unique Nash equilibrium. This result aligns with the documented difficulties in overcoming free-riding problems in the setting where only environmental externality exists (Nordhaus and Yang 1996; Carraro and Egenhofer 2007; Yang 2008). In the case of  $\epsilon_{tr} = -0.15$ , due to the externality

<sup>23</sup>These are the subsidy rates chosen by the US and EU in the cooperative equilibrium with  $\epsilon_{tr} = -0.3$ .

TABLE 3.6. Competitive Game between the US and the EU under Various  $\epsilon_{tr}$

		(A) $\epsilon_{tr} = 0$	
		EU	
		$s^* = 0.00$	$s^* = 0.35$
US	$s = 0.00$	(0%, 0%)	(0.61%, -2.01%)
	$s = 0.41$	(-3.89%, 1.22%)	(-0.39%, 1.09%)

		(B) $\epsilon_{tr} = -0.15$	
		EU	
		$s^* = 0.00$	$s^* = 0.35$
US	$s = 0.00$	(0%, 0%)	(-2.09%, -1.80%)
	$s = 0.41$	(-2.03%, 0.49%)	(2.01%, 2.49%)

		(C) $\epsilon_{tr} = -0.3$	
		EU	
		$s^* = 0.00$	$s^* = 0.35$
US	$s = 0.00$	(0%, 0%)	(-1.85%, -0.49%)
	$s = 0.41$	(0.76%, -0.60%)	(4.13%, 3.76%)

Notes: Subtables (a), (b), and (c) represent the payoff matrices of the US and the EU under the conditions of  $\epsilon_{tr} = 0$ ,  $\epsilon_{tr} = -0.15$ , and  $\epsilon_{tr} = -0.3$ , respectively. In each cell, the first value is the payoff of the US, and the second value is the payoff of the EU. The bordered cells represent the Nash equilibrium in each game.

from knowledge spillover, free-riding is no longer a dominant strategy, and there are two Nash equilibria: either both countries free-ride or both actively subsidize. In the case of  $\epsilon_{tr} = -0.3$ , the timing elasticity is high enough such that active subsidizing becomes the dominant strategy for the US, and choosing active subsidizing by both countries is the unique Nash equilibrium. These results demonstrate that the estimated innovation timing elasticity is sufficiently high to shift national optimal policy from free-riding to active subsidizing. Additionally, considering the externality related to growth potential, which varies depending on the lifecycle stage of an industry, is important for designing industrial policy.

### 3.5. Conclusion

This paper aims to explain the reasons behind an interesting phenomenon: many countries are competitively supporting industries essential for a transition to a green economy, despite the policy costs and the possibility of free-riding. This phenomenon indicates that each country has strong

organic incentives to implement industrial policy—incentives that outweigh the costs stemming from the policy and benefits of free-riding. As many governments emphasize, these incentives stem from the high-growth potential and growing market size of the targeted industries. Industries may possess such high-growth potential exclusively in the early stages of their lifecycle. For this reason, industry lifecycle theory needs to be incorporated into the general equilibrium models of industrial policy focused on the green energy transition.

The policy aimed at fostering a transition to a green economy is inevitably linked to another source of externality: greenhouse gas emissions. Accordingly, the model in this study incorporates two sources of externality: knowledge spillovers within the industry and environmental externalities. Among these two types of externalities, the former plays a crucial role in triggering policy competition between countries. The central mechanism behind this competition is that a domestic production subsidy accelerates the timing of innovation in the home country while delaying it in the foreign country. This dynamic aligns with Bai et al. (2023), where a country aims to enhance its own welfare by influencing the innovation efforts of other countries. Furthermore, this dynamic highlights the importance of analyzing the welfare effects of domestic policies on other countries, a topic that has received less emphasis in most of the existing literature but is a focal point in this paper. This emphasis is particularly persuasive, given that it is unrealistic to assume that foreign countries will remain passive in response to attempts by the home country to stifle R&D efforts in promising sectors.

This paper also sheds light on the importance of welfare analysis during the transition from one steady state to another. In the model, even though the outcome in the steady state is the same regardless of policy intervention, the welfare effect from policy is substantial. This is somewhat surprising, given that the innovation timing elasticity is not large—estimated to be around  $-0.30$ —and firms play a significantly larger role in accumulating knowledge stock. This occurs because if innovation takes place earlier in an industry in one country, it negatively impacts the same industry in the counterpart country. Thus, even though the foreign industry eventually innovates, the transition period—when the foreign country has yet to innovate—becomes longer, and the benefits of taking the lead first become larger. In this regard, the innovation timing elasticity can provide important information for designing industrial policy.

The model in this paper can be used to study more general issues. One example is the welfare effects arising from the evolution of new technologies, such as Artificial Intelligence. The mechanism for endogenous technology adoption, similar to Helpman and Trajtenberg (1998), can capture the welfare effects of emerging new technologies within the context of an open economy. I hope this model can be applied to analyze many other interesting topics.

## APPENDIX A

### Appendix for Chapter 1

#### A.1. Deriving dynamics of average productivity of sector 1, $a_{1t}$

As defined in  $a_{1t} \equiv \exp\left(\int_0^n \ln a_{1i,t}^{\frac{1}{n}} di\right)$ ,  $a_{1t}$  represents the geometric average productivity of all industries in sector 1. After the refinement occurs in sector 1,  $r$  percentage of low productivity industries are transformed into high productivity industries in each period. The table below shows the change in the proportion of high productivity industries and low productivity industries in sector 1 over time subsequent to the refinement occurring at  $t = k$ .

TABLE A.1. Proportion of High and Low Productivity Industries after a Refinement

Time	High Productivity	Low Productivity
$k$	$r$	$1 - r$
$k + 1$	$r + r(1 - r)$	$(1 - r)^2$
$k + 2$	$r + r(1 - r) + r(1 - r)^2$	$(1 - r)^3$
$\vdots$	$\vdots$	$\vdots$
$t > k + 2$	$r \sum_{j=1}^{t-k+1} (1 - r)^{j-1} = 1 - (1 - r)^{t-k+1}$	$(1 - r)^{t-k+1}$

By utilizing the time path of the proportion, we can derive the geometric average productivity of all industries in sector 1 as follows.

$$(A.1) \quad a_{1t} = (\overline{a_{high}})^{1-(1-r)^{t-k+1}} (\overline{a_{low}})^{(1-r)^{t-k+1}}, \text{ when } t \geq k$$



## APPENDIX B

### Appendices for Chapter 2

#### B.1. Comparative Statics for Change in $s$

An equilibrium with a production subsidy at each period is given by  $\{n_{1,t}, n_{2,t}, n_{1,t}^*, n_{2,t}^*, \epsilon_t\}$  that satisfies the following five equations: the zero profit condition for industry 1 in the home country from (2.36), the zero profit condition for industry 2 in the home country from (2.28), two zero profit conditions for each industry in the foreign country from (2.29) and balance of payment equilibrium from (C.30).

As in Corsetti et al. (2007), I set a symmetric initial condition where  $v = v^* = L = L^* = a_{1,t} = a_{2,t} = a_{1,t}^* = a_{2,t}^* = 1$  and  $s = 0$ . With this initial condition, there is a symmetric equilibrium such that  $\epsilon_t = 1$ ,  $n_{1,t} = n_{2,t} = n_{1,t}^* = n_{2,t}^*$ ,  $l_t = l_t^* = P_t^{1-\psi} = P_t^{*1-\psi} = \gamma(n_{1,t} + n_{2,t}) = \gamma(n_{1,t}^* + n_{2,t}^*)$ . I take a first-order approximation of this model in the neighborhood of this initial symmetric equilibrium and analyze the local effects of industrial policy.

For computational convenience, I extend the system of equilibrium into 24 equations with 24 endogenous variables. These variables include the mass of firms in each industry,  $\{n_{1,t}, n_{2,t}, n_{1,t}^*, n_{2,t}^*\}$ , the exchange rate ( $\epsilon_t$ ), the composite bundle of home varieties and the composite bundle of foreign varieties for the home representative consumer in each industry,  $\{C_{h1,t}, C_{h2,t}, C_{f1,t}, C_{f2,t}\}$ , the composite bundle of home varieties and the composite bundle of foreign varieties for the foreign representative consumer in each industry,  $\{C_{h1,t}^*, C_{h2,t}^*, C_{f1,t}^*, C_{f2,t}^*\}$ , the price index of the composite bundle of home varieties for the home representative consumer in each industry,  $\{P_{h1,t}, P_{h2,t}\}$ , the price index of the composite bundle of foreign varieties for the foreign representative consumer in each industry,  $\{P_{f1,t}^*, P_{f2,t}^*\}$ , the composite prices index in industry,  $\{P_{1,t}, P_{2,t}, P_{1,t}^*, P_{2,t}^*\}$ , utility based CPI,  $\{P_t, P_t^*\}$ , and the real exchange rate,  $RER_t$ . I take a first-order approximation of the system with respect to  $s$  and obtain the following equations.

$$(B.1) \quad (2.36): \gamma(1 + \phi) + \gamma(1 + \phi) \frac{d \ln P_{h1,t}}{ds} + \frac{d \ln C_{h1,t}}{ds} + \phi \frac{d \ln C_{h1,t}^*}{ds} = 0$$

$$(B.2) \quad (2.28): \gamma(1 + \phi) \frac{d \ln P_{h2,t}}{ds} + \frac{d \ln C_{h2,t}}{ds} + \phi \frac{d \ln C_{h2,t}^*}{ds} = 0$$

$$(B.3) \quad (2.29): \gamma(1 + \phi) \frac{d \ln P_{f1,t}^*}{ds} + \frac{d \ln C_{f1,t}^*}{ds} + \phi \frac{d \ln C_{f1,t}}{ds} = 0$$

$$(B.4) \quad (2.29): \gamma(1 + \phi) \frac{d \ln P_{f2,t}^*}{ds} + \frac{d \ln C_{f2,t}^*}{ds} + \phi \frac{d \ln C_{f2,t}}{ds} = 0$$

$$(B.5) \quad \frac{d \ln P_{h1,t}}{ds} + \frac{d \ln C_{h1,t}^*}{ds} + \frac{d \ln P_{h2,t}}{ds} + \frac{d \ln C_{h2,t}^*}{ds} = 2 \frac{d \ln \epsilon_t}{ds} + \frac{d \ln P_{f1,t}^*}{ds} + \frac{d \ln C_{f1,t}}{ds} + \frac{d \ln P_{f2,t}^*}{ds} + \frac{d \ln C_{f2,t}}{ds}$$

$$(B.6) \quad (2.12), (2.13), (2.15): \frac{d \ln C_{h1,t}}{ds} = -\eta \frac{d \ln P_{h1,t}}{ds} + (\eta - \sigma) \frac{d \ln P_{1,t}}{ds} + (\sigma - \psi) \frac{d \ln P_t}{ds}$$

$$(B.7) \quad (2.12), (2.13), (2.15): \frac{d \ln C_{h2,t}}{ds} = -\eta \frac{d \ln P_{h2,t}}{ds} + (\eta - \sigma) \frac{d \ln P_{2,t}}{ds} + (\sigma - \psi) \frac{d \ln P_t}{ds}$$

$$(B.8) \quad (2.12), (2.13), (2.15): \frac{d \ln C_{f1,t}}{ds} = -\eta \frac{d \ln \epsilon_t}{ds} - \eta \frac{d \ln P_{f1,t}^*}{ds} + (\eta - \sigma) \frac{d \ln P_{1,t}}{ds} + (\sigma - \psi) \frac{d \ln P_t}{ds}$$

$$(B.9) \quad (2.12), (2.13), (2.15): \frac{d \ln C_{f2,t}}{ds} = -\eta \frac{d \ln \epsilon_t}{ds} - \eta \frac{d \ln P_{h2,t}^*}{ds} + (\eta - \sigma) \frac{d \ln P_{2,t}}{ds} + (\sigma - \psi) \frac{d \ln P_t}{ds}$$

$$(B.10) \quad (2.12), (2.13), (2.15): \frac{d \ln C_{h1,t}^*}{ds} = \eta \frac{d \ln \epsilon_t}{ds} - \eta \frac{d \ln P_{h1,t}}{ds} + (\eta - \sigma) \frac{d \ln P_{1,t}^*}{ds} + (\sigma - \psi) \frac{d \ln P_t^*}{ds}$$

$$(B.11) \quad (2.12), (2.13), (2.15): \frac{d \ln C_{h2,t}^*}{ds} = \eta \frac{d \ln \epsilon_t}{ds} - \eta \frac{d \ln P_{h2,t}}{ds} + (\eta - \sigma) \frac{d \ln P_{2,t}}{ds} + (\sigma - \psi) \frac{d \ln P_t}{ds}$$

$$(B.12) \quad (2.12), (2.13), (2.15): \frac{d \ln C_{f1,t}^*}{ds} = -\eta \frac{d \ln P_{f1,t}^*}{ds} + (\eta - \sigma) \frac{d \ln P_{1,t}^*}{ds} + (\sigma - \psi) \frac{d \ln P_t^*}{ds}$$

$$(B.13) \quad (2.12), (2.13), (2.15): \frac{d \ln C_{f2,t}^*}{ds} = -\eta \frac{d \ln P_{h2,t}^*}{ds} + (\eta - \sigma) \frac{d \ln P_{2,t}^*}{ds} + (\sigma - \psi) \frac{d \ln P_t^*}{ds}$$

$$(B.14) \quad (2.18), (2.35): \frac{d \ln P_{h1,t}}{ds} = \frac{1}{1 - \gamma} \frac{d \ln n_{1,t}}{ds} - 1$$

$$(B.15) \quad (2.18): \frac{d \ln P_{h2,t}}{ds} = \frac{1}{1 - \gamma} \frac{d \ln n_{2,t}}{ds}$$

$$(B.16) \quad (2.18): \frac{d \ln P_{f1,t}^*}{ds} = \frac{1}{1 - \gamma} \frac{d \ln n_{1,t}^*}{ds}$$

$$(B.17) \quad (2.18): \frac{d \ln P_{f2,t}^*}{ds} = \frac{1}{1 - \gamma} \frac{d \ln n_{2,t}^*}{ds}$$

$$(B.18) \quad (2.17): (1 + \phi) \frac{d \ln P_{1,t}}{ds} = \frac{d \ln P_{h1,t}}{ds} + \phi \left( \frac{d \ln \epsilon_t}{ds} + \frac{d \ln P_{f1,t}^*}{ds} \right)$$

$$(B.19) \quad (2.17): (1 + \phi) \frac{d \ln P_{2,t}}{ds} = \frac{d \ln P_{h2,t}}{ds} + \phi \left( \frac{d \ln \epsilon_t}{ds} + \frac{d \ln P_{f2,t}^*}{ds} \right)$$

$$(B.20) \quad (2.17): (1 + \phi) \frac{d \ln P_{1,t}^*}{ds} = \frac{d \ln P_{f1,t}^*}{ds} + \phi \left( -\frac{d \ln \epsilon_t}{ds} + \frac{d \ln P_{h1,t}}{ds} \right)$$

$$(B.21) \quad (2.17): (1 + \phi) \frac{d \ln P_{2,t}^*}{ds} = \frac{d \ln P_{f2,t}^*}{ds} + \phi \left( -\frac{d \ln \epsilon_t}{ds} + \frac{d \ln P_{h2,t}}{ds} \right)$$

$$(B.22) \quad 2 \frac{d \ln P_t}{ds} = \frac{d \ln P_{1,t}}{ds} + \frac{d \ln P_{2,t}}{ds}$$

$$(B.23) \quad 2 \frac{d \ln P_t^*}{ds} = \frac{d \ln P_{1,t}^*}{ds} + \frac{d \ln P_{2,t}^*}{ds}$$

$$(B.24) \quad \frac{d RER_t}{ds} = \frac{d \ln \epsilon_t}{ds} + \frac{d \ln P_t^*}{ds} - \frac{d \ln P_{1,t}}{ds}$$

I obtain the results presented in Table C.2 by solving the aforementioned system of 24 equations. Additionally, I follow the same procedure to obtain the results shown in Table B.2 and Table B.3.

TABLE B.1. Comparative Statics: Production subsidy

$$(B.25) \quad \frac{d \ln n_{1,t}}{ds} = \frac{1}{2} \left[ \frac{(\gamma-1)\psi}{2(\gamma-\psi)} + \frac{(\gamma-1)\psi}{2(\gamma-\psi)} \left( 1 + \frac{2\gamma\phi(1-\psi)}{\Delta} \right) + \frac{\gamma-1}{\gamma-\sigma} \left( \sigma + \frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] > 0$$

$$(B.26) \quad \frac{d \ln n_{2,t}}{ds} = \frac{1}{2} \left[ \frac{(\gamma-1)\psi}{2(\gamma-\psi)} + \frac{(\gamma-1)\psi}{2(\gamma-\psi)} \left( 1 + \frac{2\gamma\phi(1-\psi)}{\Delta} \right) - \frac{\gamma-1}{\gamma-\sigma} \left( \sigma + \frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right]$$

$$(B.27) \quad \frac{d \ln n_{1,t}^*}{ds} = \frac{1}{2} \left[ \frac{(\gamma-1)\psi}{2(\gamma-\psi)} - \frac{(\gamma-1)\psi}{2(\gamma-\psi)} \left( 1 + \frac{2\gamma\phi(1-\psi)}{\Delta} \right) - \frac{\gamma-1}{\gamma-\sigma} \left( \frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] < 0$$

$$(B.28) \quad \frac{d \ln n_{2,t}^*}{ds} = \frac{1}{2} \left[ \frac{(\gamma-1)\psi}{2(\gamma-\psi)} - \frac{(\gamma-1)\psi}{2(\gamma-\psi)} \left( 1 + \frac{2\gamma\phi(1-\psi)}{\Delta} \right) + \frac{\gamma-1}{\gamma-\sigma} \left( \frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right]$$

$$(B.29) \quad \frac{d \ln \epsilon_t}{ds} = -\frac{1}{2} \frac{[(\eta-\psi)(1-\phi) + (\eta-1)(1+\phi)]\gamma}{\Delta} < 0$$

$$(B.30) \quad \frac{d \ln P_t}{ds} = \frac{1}{2} \frac{1}{1-\psi} \left( \frac{d \ln n_{1,t}}{ds} + \frac{d \ln n_{2,t}}{ds} - 1 \right) = -\frac{1}{2} \left[ \frac{\gamma}{\gamma-\psi} \left( 1 - \frac{\psi(\gamma-1)\phi}{\Delta} \right) \right] < 0$$

$$(B.31) \quad \frac{d \ln P_t^*}{ds} = \frac{1}{2} \frac{1}{1-\psi} \left( \frac{d \ln n_{1,t}^*}{ds} + \frac{d \ln n_{2,t}^*}{ds} \right) = -\frac{1}{2} \frac{\gamma}{\gamma-\psi} \left[ \frac{\psi(\gamma-1)\phi}{\Delta} \right] \leq 0$$

$$(B.32) \quad \frac{d RER_t}{ds} = \frac{1}{2} \frac{\psi(1-\phi)\gamma}{\Delta} > 0$$

$$(B.33) \quad \frac{d U_1}{ds} = -P_t^{1-\psi} \left( \frac{d \ln P_t}{ds} + \frac{1}{2} \right) = \frac{1}{2} P_t^{1-\psi} \left[ \frac{\gamma}{\gamma-\psi} \left( 1 - \frac{\psi(\gamma-1)\phi}{\Delta} \right) - 1 \right]$$

$$(B.34) \quad \frac{d U_1^*}{ds} = -P_t^{*1-\psi} \frac{d \ln P_t^*}{ds} \geq 0$$

$$(B.35) \quad \Delta \equiv (2\eta-1)[\gamma-\psi(1-\phi)] + 2\psi\phi(\gamma-\eta) - \gamma\phi > 0$$

## B.2. Comparative Statics for Change in $a_{1,t}$

TABLE B.2. Comparative Statics: change in  $a_{1,t}$

$$(B.36) \quad \frac{d \ln n_{1,t}}{d \ln a_{1,t}} = \frac{1}{2} \left[ -\frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} - \frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} \left( 1 - \frac{2\psi(\gamma-1)\phi}{\Delta} \right) + \frac{\gamma-1}{\gamma-\sigma} \left( \sigma - 1 + \frac{2\phi(\gamma-1)(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right]$$

$$(B.37) \quad \frac{d \ln n_{2,t}}{d \ln a_{1,t}} = \frac{1}{2} \left[ -\frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} - \frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} \left( 1 - \frac{2\psi(\gamma-1)\phi}{\Delta} \right) - \frac{\gamma-1}{\gamma-\sigma} \left( \sigma - 1 + \frac{2\phi(\gamma-1)(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] < 0$$

$$(B.38) \quad \frac{d \ln n_{1,t}^*}{d \ln a_{1,t}} = \frac{1}{2} \left[ -\frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} + \frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} \left( 1 - \frac{2\psi\phi}{\Delta} \right) - \frac{\gamma-1}{\gamma-\sigma} \left( \frac{2\phi(\gamma-1)(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] < 0$$

$$(B.39) \quad \frac{d \ln n_{2,t}^*}{d \ln a_{1,t}} = \frac{1}{2} \left[ -\frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} + \frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} \left( 1 - \frac{2\psi(\gamma-1)\phi}{\Delta} \right) + \frac{\gamma-1}{\gamma-\sigma} \left( \frac{2\phi(\gamma-1)(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right]$$

$$(B.40) \quad \frac{d \ln \epsilon_t}{d \ln a_{1,t}} = -\frac{1}{2} \frac{[(\gamma-\psi)(1-\phi) + (\gamma-1)(1+\phi)](\gamma-1)}{\Delta} \in \left( -\frac{1}{2}, 0 \right)$$

$$(B.41) \quad \frac{d \ln P_t}{d \ln a_{1,t}} = \frac{1}{2} \frac{1}{1-\psi} \left( \frac{d \ln n_{1,t}}{ds} + \frac{d \ln n_{2,t}}{ds} \right) = -\frac{1}{2} \frac{\gamma-1}{\gamma-\psi} \left[ 1 - \frac{\psi(\gamma-1)\phi}{\Delta} \right] < 0$$

$$(B.42) \quad \frac{d \ln P_t^*}{d \ln a_{1,t}} = \frac{1}{2} \frac{1}{1-\psi} \left( \frac{d \ln n_{1,t}^*}{d \ln a_{1,t}} + \frac{d \ln n_{2,t}^*}{d \ln a_{1,t}} \right) = -\frac{1}{2} \frac{\gamma-1}{\gamma-\psi} \left[ \frac{\psi(\gamma-1)\phi}{\Delta} \right] \leq 0$$

$$(B.43) \quad \frac{d RER_t}{d \ln a_{1,t}} = \frac{1}{2} \frac{\psi(1-\phi)(\gamma-1)}{\Delta} > 0$$

$$(B.44) \quad \frac{d U_1}{d \ln a_{1,t}} = -P_t^{1-\psi} \frac{d \ln P_t}{d \ln a_{1,t}} > 0$$

$$(B.45) \quad \frac{d U_1^*}{d \ln a_{1,t}} = -P_t^{*1-\psi} \frac{d \ln P_t^*}{d \ln a_{1,t}} \geq 0$$

$$(B.46) \quad \Delta \equiv (2\eta-1)[\gamma-\psi(1-\phi)] + 2\psi\phi(\gamma-\eta) - \gamma\phi > 0$$

As in Section 2.3.1.1, I add an intuitive explanation on how resource allocation is affected by an increase in  $a_{1,t}$ . When productivity increases in the home industry 1, as in the short-run (Stage 1), there are three effects: an income effect, a relative wage effect and a substitution effect. Again, Equation (2.43) helps to understand an income effect. 1 percent increase in productivity of firms in the home industry 1 leads to change the world demand by  $\frac{1}{4}(\psi - 1)$ . However, since the home government does not give subsidy anymore, there is no direct change in the right hand side. Thus, with the assumption  $\psi < 1$ , the excess supply exists by  $\frac{1}{4}(1 - \psi)$ . To clear the excess supply, the mass of firms in all industries in both the home and foreign country needs to uniformly decrease by  $\frac{(\gamma-1)(1-\psi)}{4(\gamma-\psi)}$ .

The relative wage effect causes the mass of firms in the home industries ( $n_{1,t}$  and  $n_{2,t}$ ) to decrease and that in the foreign industries ( $n_{1,t}^*$  and  $n_{2,t}^*$ ) to increase. After innovation, the increase in productivity in the home industry 1 leads home firms to demand more labor and thus the relative wage of the home labor increases. This is represented by a decrease in  $\epsilon_t$  in (B.40). With  $\psi < 1$ , when real wage increases, the mass of firms decreases.

The substitution effect is qualitatively the same as that in Stage 1. The increase in productivity in the home industry 1 makes the industry 1 home varieties relatively cheaper than the industry 1 foreign varieties. Thus, the home and foreign consumers substitutes the industry 1 foreign varieties with the industry 1 home varieties, which increases the mass of firm in the home industry 1 and decreases the mass of firms in the foreign industry 1. In contrast, the decrease in  $\epsilon_t$  leads the home and foreign consumers to substitute the industry 2 home varieties with the industry 2 foreign varieties. Because of this substitution, the mass of firms in the home industry 2 decreases and that in the foreign industry 2 increases.

By taking the three effects into account together, the mass of firms in the home industry 2,  $n_{2,t}$ , and that in the foreign industry 1,  $n_{1,t}^*$ , unambiguously decreases. Changes in  $n_{1,t}$  and  $n_{2,t}^*$  are ambiguous since the direction of the substitution effect is opposite from the other two effects for  $n_{1,t}$  and the direction of the income effect is opposite from the other two effects for  $n_{2,t}^*$ .

### B.3. List of Treated and Untreated Industries during HCI Drive

Industry Name	HCI
Manufacture of food products	N
Manufacture of beverages	N
Manufacture of tobacco products	N
Manufacture of textiles, except apparel	N
Manufacture of wearing apparel	N
Manufacture of leather and fur articles	N
Manufacture of footwear	N
Manufacture of wood and of products of wood and cork; except furniture	N
Manufacture of furniture	N
Manufacture of pulp, paper and paper products	N
Printing and service activities related to printing	N
Manufacture of chemical products	Y
Manufacture of other chemical products	N
Manufacture of refined petroleum products	Y
Manufacture of coke and briquettes	Y
Manufacture of Rubber	N
Manufacture of Plastic	N
Manufacture of ceramic products	N
Manufacture of glass and glass products	N
Other non-metallic mineral products	N
Manufacture of basic iron and steel	Y
Manufacture of basic precious and other non-ferrous metals	Y
Manufacture of fabricated metal products, except machinery and equipment	Y
Manufacture of machinery and equipment n.e.c.	Y
Manufacture of electronic and electrical equipment	Y
Manufacture of transport equipment	Y
Manufacture of medical, precision and optical instruments	N
Other manufacturing	N

#### B.4. Comparative Statistics: R&D subsidy

TABLE B.3. Comparative Statistics: R&D subsidy

$$(B.47) \quad \frac{d \ln n_{1,t}}{ds_d} = \frac{1}{2} \left[ \frac{\gamma-1}{2(\gamma-\psi)} + \frac{\gamma-1}{2(\gamma-\psi)} \left( 1 + \frac{2\psi\phi(1-\psi)}{\Delta} \right) + \frac{\gamma-1}{\gamma-\sigma} \left( 1 + \frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] > 0$$

$$(B.48) \quad \frac{d \ln n_{2,t}}{ds_d} = \frac{1}{2} \left[ \frac{\gamma-1}{2(\gamma-\psi)} + \frac{\gamma-1}{2(\gamma-\psi)} \left( 1 + \frac{2\psi\phi(1-\psi)}{\Delta} \right) - \frac{\gamma-1}{\gamma-\sigma} \left( 1 + \frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right]$$

$$(B.49) \quad \frac{d \ln n_{1,t}^*}{ds_d} = \frac{1}{2} \left[ \frac{\gamma-1}{2(\gamma-\psi)} - \frac{\gamma-1}{2(\gamma-\psi)} \left( 1 + \frac{2\psi\phi(1-\psi)}{\Delta} \right) - \frac{\gamma-1}{\gamma-\sigma} \left( \frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] < 0$$

$$(B.50) \quad \frac{d \ln n_{2,t}^*}{ds_d} = \frac{1}{2} \left[ \frac{\gamma-1}{2(\gamma-\psi)} - \frac{\gamma-1}{2(\gamma-\psi)} \left( 1 + \frac{2\psi\phi(1-\psi)}{\Delta} \right) + \frac{\gamma-1}{\gamma-\sigma} \left( \frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right]$$

$$(B.51) \quad \frac{d \ln \epsilon_t}{ds_d} = -\frac{1}{2} \frac{(\eta-\psi)(1-\phi) + (\eta-1)(1+\phi)}{\Delta} < 0$$

$$(B.52) \quad \frac{d \ln P_t}{ds_d} = \frac{1}{2} \frac{1}{1-\psi} \left( \frac{d \ln n_{1,t}}{ds_d} + \frac{d \ln n_{2,t}}{ds_d} - 1 \right) = -\frac{1}{2} \left[ \frac{1}{\gamma-\psi} \left( 1 - \frac{\psi(\gamma-1)\phi}{\Delta} \right) \right] < 0$$

$$(B.53) \quad \frac{d \ln P_t^*}{ds_d} = \frac{1}{2} \frac{1}{1-\psi} \left( \frac{d \ln n_{1,t}^*}{ds_d} + \frac{d \ln n_{2,t}^*}{ds_d} \right) = -\frac{1}{2} \frac{1}{\gamma-\psi} \left[ \frac{\psi(\gamma-1)\phi}{\Delta} \right] \leq 0$$

$$(B.54) \quad \frac{dU_1}{ds_d} = -P_t^{1-\psi} \left( \frac{d \ln P_t}{ds_d} + \frac{1}{2} \right) = \frac{1}{2} P_t^{1-\psi} \left[ \frac{1}{\gamma-\psi} \left( 1 - \frac{\psi(\gamma-1)\phi}{\Delta} \right) - 1 \right]$$

$$(B.55) \quad \frac{dU_1^*}{ds_d} = -P_t^{*1-\psi} \frac{d \ln P_t^*}{ds_d} \geq 0$$

$$(B.56) \quad \Delta \equiv (2\eta-1)[\gamma-\psi(1-\phi)] + 2\psi\phi(\gamma-\eta) - \gamma\phi > 0$$



### B.5. Proof of Proposition 6 and 7

I derive the difference in policy effects on the variables of interest by utilizing the previous results from equation (2.59), Table C.2 and Table B.3 together. For a suitable comparison, I calculate the difference under the condition that the total amount of subsidy, which the home government must provide until the occurrence of the refinement, is the same for both policies.

(B.57)

$$\begin{aligned}
dU_1^p - dU_1^d &= \frac{1}{2}P_t^{1-\psi} \left[ \frac{\gamma}{\gamma-\psi} \left( 1 - \frac{\psi(\gamma-1)\phi}{\Delta} \right) - 1 \right] ds - \frac{1}{2}P_t^{1-\psi} \left[ \frac{1}{\gamma-\psi} \left( 1 - \frac{\psi(\gamma-1)\phi}{\Delta} \right) - 1 \right] ds_d \\
&= \frac{1}{2}P_t^{1-\psi} \left[ \frac{\gamma}{\gamma-\psi} \left( 1 - \frac{\psi(\gamma-1)\phi}{\Delta} \right) - 1 \right] ds - \frac{1}{2}P_t^{1-\psi} \left[ \frac{1}{\gamma-\psi} \left( 1 - \frac{\psi(\gamma-1)\phi}{\Delta} \right) - 1 \right] \gamma ds \\
&= \frac{1}{2}P_t^{1-\psi} (\gamma-1) ds > 0
\end{aligned}$$

(B.58)

$$\begin{aligned}
d \ln n_{1,t}^p - d \ln n_{1,t}^d &= \frac{1}{2} \left[ \frac{(\gamma-1)\psi}{2(\gamma-\psi)} + \frac{(\gamma-1)\psi}{2(\gamma-\psi)} \left( 1 + \frac{2\gamma\phi(1-\psi)}{\Delta} \right) + \frac{\gamma-1}{\gamma-\sigma} \left( \sigma + \frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] ds \\
&- \frac{1}{2} \left[ \frac{\gamma-1}{2(\gamma-\psi)} + \frac{\gamma-1}{2(\gamma-\psi)} \left( 1 + \frac{2\psi\phi(1-\psi)}{\Delta} \right) + \frac{\gamma-1}{\gamma-\sigma} \left( 1 + \frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] ds_d \\
&= \frac{1}{2} \left[ \frac{(\gamma-1)\psi}{2(\gamma-\psi)} + \frac{(\gamma-1)\psi}{2(\gamma-\psi)} \left( 1 + \frac{2\gamma\phi(1-\psi)}{\Delta} \right) + \frac{\gamma-1}{\gamma-\sigma} \left( \sigma + \frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] ds \\
&- \frac{1}{2} \left[ \frac{\gamma-1}{2(\gamma-\psi)} + \frac{\gamma-1}{2(\gamma-\psi)} \left( 1 + \frac{2\psi\phi(1-\psi)}{\Delta} \right) + \frac{\gamma-1}{\gamma-\sigma} \left( 1 + \frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] \gamma ds \\
&= -(\gamma-1) ds < 0
\end{aligned}$$

$$(B.59) \quad dt_r^p - dt_r^d = -t_r(0) \left( d \ln n_{1,t}^p - d \ln n_{1,t}^d \right) = t_r(0)(\gamma-1) ds > 0$$

(B.60)

$$\begin{aligned}
dU_1^{*p} - dU_1^{*d} &= \frac{1}{2}P_t^{*1-\psi} \left[ \frac{\gamma}{\gamma-\psi} \left( \frac{\psi(\gamma-1)\phi}{\Delta} \right) \right] ds - \frac{1}{2}P_t^{*1-\psi} \left[ \frac{1}{\gamma-\psi} \left( \frac{\psi(\gamma-1)\phi}{\Delta} \right) \right] ds_d \\
&= \frac{1}{2}P_t^{*1-\psi} \left[ \frac{\gamma}{\gamma-\psi} \left( \frac{\psi(\gamma-1)\phi}{\Delta} \right) \right] ds - \frac{1}{2}P_t^{*1-\psi} \left[ \frac{1}{\gamma-\psi} \left( \frac{\psi(\gamma-1)\phi}{\Delta} \right) \right] \gamma ds = 0
\end{aligned}$$

(B.61)

$$\begin{aligned}
& d \ln n_{1,t}^{*p} - d \ln n_{1,t}^{*d} \\
&= \frac{1}{2} \left[ \frac{(\gamma-1)\psi}{2(\gamma-\psi)} - \frac{(\gamma-1)\psi}{2(\gamma-\psi)} \left( 1 + \frac{2\gamma\phi(1-\psi)}{\Delta} \right) - \frac{\gamma-1}{\gamma-\sigma} \left( \frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] ds \\
&- \frac{1}{2} \left[ \frac{\gamma-1}{2(\gamma-\psi)} - \frac{\gamma-1}{2(\gamma-\psi)} \left( 1 + \frac{2\psi\phi(1-\psi)}{\Delta} \right) - \frac{\gamma-1}{\gamma-\sigma} \left( \frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] ds_d \\
&= \frac{1}{2} \left[ \frac{(\gamma-1)\psi}{2(\gamma-\psi)} - \frac{(\gamma-1)\psi}{2(\gamma-\psi)} \left( 1 + \frac{2\gamma\phi(1-\psi)}{\Delta} \right) - \frac{\gamma-1}{\gamma-\sigma} \left( \frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] ds \\
&- \frac{1}{2} \left[ \frac{\gamma-1}{2(\gamma-\psi)} - \frac{\gamma-1}{2(\gamma-\psi)} \left( 1 + \frac{2\psi\phi(1-\psi)}{\Delta} \right) - \frac{\gamma-1}{\gamma-\sigma} \left( \frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2 + 4\phi(\gamma-\eta)} \right) \right] \gamma ds = 0
\end{aligned}$$

(B.62)

$$\begin{aligned}
dt_r^{*p} - dt_r^{*d} &= - \left( \frac{n_{1,t}^*(1)}{n_{1,t}^*(2)} - 1 \right) (dt_r^p - dt_r^d) - t_r(0) \frac{n_{1,t}^*(1)}{n_{1,t}^*(2)} (d \ln n_{1,t}^{*p} - d \ln n_{1,t}^{*d}) \\
&= - \left( \frac{n_{1,t}^*(1)}{n_{1,t}^*(2)} - 1 \right) t_r(0) (\gamma-1) ds < 0
\end{aligned}$$

## APPENDIX C

### Appendices for Chapter 3

#### C.1. Equilibrium

Prices. Prices for green capital goods are set by imposing a constant markup over marginal costs as follows:

$$(C.1) \quad p_t^g(\omega) = \frac{1}{1+s} \frac{\gamma}{\gamma-1} \frac{1}{a_t^g}, \quad e_t p_t^{g*}(\omega) = p_t^{g*}(\omega) = (1+\tau) \frac{1}{1+s} \frac{\gamma}{\gamma-1} \frac{1}{a_t^g} = (1+\tau) p_t^g(\omega)$$

$$(C.2) \quad p_t^{g*}(\omega_f) = \frac{1}{1+s^*} \frac{\gamma}{\gamma-1} \frac{1}{a_t^{g*}}, \quad \frac{p_t^g(\omega_f)}{e_t} = p_t^g(\omega_f) = (1+\tau) \frac{1}{1+s^*} \frac{\gamma}{\gamma-1} \frac{1}{a_t^{g*}} = (1+\tau) p_t^{g*}(\omega_f)$$

where  $\tau$  represents trade cost, and prices with an asterisk are denominated in foreign currency.

The price of aggregated green capital goods for each country is as follows:

$$(C.3) \quad P_t^g = \left( n_t (p_t^g(\omega))^{1-\gamma} + \phi n_t^* (e_t p_t^{g*}(\omega_f))^{1-\gamma} \right)^{\frac{1}{1-\gamma}} = \left( n_t (p_t^g(\omega))^{1-\gamma} + \phi n_t^* (p_t^{g*}(\omega_f))^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

(C.4)

$$P_t^{g*} = \left( n_t^* (p_t^{g*}(\omega_f))^{1-\gamma} + \phi n_t \left( \frac{p_t^g(\omega)}{e_t} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} = \left( n_t^* (p_t^{g*}(\omega_f))^{1-\gamma} + \phi n_t (p_t^g(\omega))^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

where  $\phi \equiv (1+\tau)^{1-\gamma}$

Prices for agricultural final goods are 1 in both the home and foreign countries ( $P_t^A = 1, P_t^{A*} = 1$ ). By solving the manufacturing final goods firms' cost minimization problem, their prices are determined for the home and foreign countries by

$$(C.5) \quad \begin{aligned} P_{hi,t}^M &= \frac{1}{A_{i,t}} \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \lambda_i^{\alpha-1} P_t^{g1-\alpha} = \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \lambda_i^{\alpha-1} P_t^{g1-\alpha}, \quad 0 \leq i < I_t \\ P_{hi,t}^M &= \frac{1}{A_{i,t}} \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \psi_t^{1-\alpha} = \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \psi_t^{1-\alpha}, \quad I_t \leq i \end{aligned}$$

$$(C.6) \quad \begin{aligned} P_{fi,t}^{M*} &= \frac{1}{A_{i,t}^*} \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \lambda_i^{\alpha-1} P_t^{g*1-\alpha} = \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \lambda_i^{\alpha-1} P_t^{g*1-\alpha}, \quad 0 \leq i < I_t^* \\ P_{fi,t}^{M*} &= \frac{1}{A_{i,t}^*} \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \psi_t^{*1-\alpha} = \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \psi_t^{*1-\alpha}, \quad I_t^* \leq i \end{aligned}$$

The home and foreign aggregated manufacturing final goods price are

$$(C.7) \quad P_{h,t}^M = e^{\int_0^1 \ln P_{hi,t}^M di}$$

$$(C.8) \quad P_{f,t}^{M*} = e^{\int_0^1 \ln P_{fi,t}^{M*} di}$$

The composite manufacturing final goods price in the home and foreign country are

$$(C.9) \quad P_t^M = \beta^{-\beta} (1-\beta)^{-\beta+1} (P_{h,t}^M)^\beta (e_t P_{f,t}^{M*})^{1-\beta} = \beta^{-\beta} (1-\beta)^{-\beta+1} (P_{h,t}^M)^\beta (P_{f,t}^{M*})^{1-\beta}$$

$$(C.10) \quad P_t^{M*} = \beta^{-\beta} (1-\beta)^{-\beta+1} (P_{f,t}^{M*})^\beta \left( \frac{P_{h,t}^M}{e_t} \right)^{1-\beta} = \beta^{-\beta} (1-\beta)^{-\beta+1} (P_{f,t}^{M*})^\beta (P_{h,t}^M)^{1-\beta}$$

The utility-based CPI is given by

$$(C.11) \quad P_t = \iota^{-\iota} (1-\iota)^{-\iota+1} (P_t^M)^{1-\iota}$$

$$(C.12) \quad P_t^* = \iota^{-\iota} (1-\iota)^{-\iota+1} (P_t^{M*})^{1-\iota}$$

Final goods Markets. By solving the representative consumer's utility maximization problem, we derive the following final goods demands:

$$(C.13)$$

$$\text{Aggregated final goods: } C_t = P_t^{-1}, \quad C_t^* = P_t^{*-1}$$

$$(C.14)$$

$$\text{Agricultural final goods: } C_t^A = \iota, \quad C_t^{A*} = \iota$$

$$(C.15)$$

$$\text{Home manufacturing sector } i \text{ goods: } C_{hi,t}^M = \frac{(1-\iota)\beta}{P_{hi,t}^M}, \quad C_{hi,t}^{M*} = \frac{(1-\iota)(1-\beta)e_t}{P_{hi,t}^M} = \frac{(1-\iota)(1-\beta)}{P_{hi,t}^M}$$

$$(C.16)$$

$$\text{Foreign manufacturing sector } i \text{ goods: } C_{fi,t}^{M*} = \frac{(1-\iota)\beta}{P_{fi,t}^{M*}}, \quad C_{fi,t}^M = \frac{(1-\iota)(1-\beta)}{e_t P_{fi,t}^{M*}} = \frac{(1-\iota)(1-\beta)}{P_{fi,t}^{M*}}$$

Accordingly, the equilibrium conditions for the final goods markets are as follows:

$$(C.17) \quad \text{Agricultural final goods: } Y_t^A + Y_t^{A*} = C_t^A + C_t^{A*}$$

$$(C.18) \quad \text{Home manufacturing sector } i \text{ goods: } Y_{i,t}^k = C_{hi,t}^M + C_{hi,t}^{M*}, \quad k \in \{g, e\}$$

$$(C.19) \quad \text{Foreign manufacturing sector } i \text{ goods: } Y_{i,t}^{k*} = C_{fi,t}^{M*} + C_{fi,t}^M, \quad k \in \{g, e\}$$

Green Capital Goods and Conventional Energy Market. By solving the final goods firm's cost minimization problem with green capital as its energy input, the following demand for green capital goods varieties  $\omega$  and  $\omega_f$  is derived.

$$(C.20)$$

$$z_t^g(\omega) = \left( \frac{p_t^g}{P_t^g} \right)^{-\gamma} \int_0^{I_t} Z_{i,t}^g di, \quad z_t^{g*}(\omega) = \left( \frac{(1+\tau)p_t^g}{e_t P_t^{g*}} \right)^{-\gamma} \int_0^{I_t^*} Z_{i,t}^{g*} di = \left( \frac{(1+\tau)p_t^g}{P_t^{g*}} \right)^{-\gamma} \int_0^{I_t^*} Z_{i,t}^{g*} di$$

$$(C.21)$$

$$z_t^{g*}(\omega_f) = \left( \frac{p_t^{g*}}{P_t^{g*}} \right)^{-\gamma} \int_0^{I_t^*} Z_{i,t}^{g*} di, \quad z_t^g(\omega_f) = \left( \frac{e_t(1+\tau)p_t^{g*}}{P_t^g} \right)^{-\gamma} \int_0^{I_t} Z_{i,t}^g di = \left( \frac{(1+\tau)p_t^{g*}}{P_t^g} \right)^{-\gamma} \int_0^{I_t} Z_{i,t}^g di$$

The demand for aggregated green capital from sector  $i$  in the home and foreign countries, denoted by  $Z_{i,t}^g$  and  $Z_{i,t}^{g*}$ , is given as follows:

$$(C.22) \quad Z_{i,t}^g = \frac{(1-\alpha)(1-\iota)\beta}{P_t^g} + \frac{(1-\alpha)(1-\iota)(1-\beta)e_t}{P_t^g} = \frac{(1-\alpha)(1-\iota)}{P_t^g}$$

$$(C.23) \quad Z_{i,t}^{g*} = \frac{(1-\alpha)(1-\iota)\beta}{P_t^{g*}} + \frac{(1-\alpha)(1-\iota)(1-\beta)}{e_t P_t^{g*}} = \frac{(1-\alpha)(1-\iota)}{P_t^{g*}}$$

Thus, the equilibrium conditions for the green capital goods markets are:

$$(C.24) \quad y_t^g(\omega) = z_t^g(\omega) + (1+\tau)z_t^{g*}(\omega)$$

$$(C.25) \quad y_t^{g*}(\omega_f) = z_t^{g*}(\omega_f) + (1+\tau)z_t^g(\omega_f)$$

For the conventional energy market, where it is assumed the supply is perfectly elastic at prices  $\psi_t$  and  $\psi_t^*$  in the home and foreign countries, equilibrium is determined by the demand side. The demand for conventional energy from sector  $i$  that decides to use conventional energy is as follows:

$$(C.26) \quad E_{i,t} = \frac{(1-\alpha)(1-\iota)}{\psi_t}$$

$$(C.27) \quad E_{i,t}^* = \frac{(1-\alpha)(1-\iota)}{\psi_t^*}$$

Zero-Profit Conditions. Free entry into the green capital market implies that a firm's profit will be zero in equilibrium. Thus, a firm's operating profit in each industry should equal the fixed cost in both the home and foreign countries, as follows:

$$(C.28) \quad \begin{aligned} \Pi_t^g(\omega) &= \frac{(1+s)p_t^g(\omega)y_t^g(\omega)}{\gamma} \\ &= \frac{(1+s)p_t^g(\omega)}{\gamma} [z_t^g(\omega) + (1+\tau)z_t^{g*}(\omega)] \\ &= \frac{(1+s)(1-\iota)(1-\alpha)}{\gamma} \left[ \left( \frac{p_t^g(\omega)}{P_t^g} \right)^{1-\gamma} I_t + \phi \left( \frac{p_t^g(\omega)}{P_t^{g*}} \right)^{1-\gamma} I_t^* \right] = 1 \end{aligned}$$

$$(C.29) \quad \begin{aligned} \Pi_t^{g*}(\omega_f) &= \frac{(1+s^*)p_t^{g*}(\omega_f)y_t^{g*}(\omega_f)}{\gamma} \\ &= \frac{(1+s^*)p_t^{g*}(\omega_f)}{\gamma} [z_t^{g*}(\omega_f) + (1+\tau)z_t^g(\omega_f)] \\ &= \frac{(1+s^*)(1-\iota)(1-\alpha)}{\gamma} \left[ \left( \frac{p_t^{g*}(\omega_f)}{P_t^{g*}} \right)^{1-\gamma} I_t^* + \phi \left( \frac{p_t^{g*}(\omega_f)}{P_t^g} \right)^{1-\gamma} I_t \right] = 1 \end{aligned}$$

Balance of Payment Equilibrium Condition. I assume balanced trade in the model, whereby the value of a country's imports equals the value of its exports. Thus, the following equation holds at equilibrium.

$$(C.30) \quad \begin{aligned} M_t^{A*} + (1-\iota)(1-\beta) + (1+\tau)p_t^g z_t^{g*}(\omega)n_t &= M_t^A + (1-\iota)(1-\beta) + (1+\tau)p_t^{g*} z_t^g(\omega_f)n_t^* \\ \Rightarrow M_t^{A*} + \frac{\phi n_t p_t^g(\omega)^{1-\gamma}}{(P_t^{g*})^{1-\gamma}}(1-\iota)(1-\alpha)I_t^* &= M_t^A + \frac{\phi n_t p_t^{g*}(\omega_f)^{1-\gamma}}{(P_t^g)^{1-\gamma}}(1-\iota)(1-\alpha)I_t \end{aligned}$$

where  $M_t^A (= \max[0, C_t^A - Y_t^A])$  and  $M_t^{A*} (= \max[0, C_t^{A*} - Y_t^{A*}])$  represent the quantity of agricultural final goods imported in the home and foreign countries, respectively.

Green Energy Adoption. The green energy adoption ratios in the home and foreign countries, denoted by  $I_t$  and  $I_t^*$ , are determined as follows:  $I_t$  and  $I_t^*$  satisfy the following equations.

$$(C.31) \quad \frac{P_t^g}{\lambda_{I_t}} \leq \psi_t \text{ for } i \leq I_t, \quad \psi_t < \frac{P_t^g}{\lambda_{I_t}} \text{ for } I_t < i$$

$$(C.32) \quad \frac{P_t^{g*}}{\lambda_{I_t}^*} \leq \psi_t^* \text{ for } i \leq I_t^*, \quad \psi_t^* < \frac{P_t^{g*}}{\lambda_{I_t}^*} \text{ for } I_t^* < i$$

Resource Constraints. For the home and foreign countries, the resource constraints are given as follows, respectively:

$$(C.33) \quad \begin{aligned} l_t &\geq Y_{i,t}^A + \int_0^{I_t} \alpha P_{hi,t}^M Y_{i,t}^g di + \int_{I_t}^1 \alpha P_{hi,t}^M Y_{i,t}^e di + \int_0^{n_t} \frac{y_t^g(\omega)}{a_t^g} d\omega + \int_0^{n_t} q_t^g(\omega) d\omega \\ &\Rightarrow l_t \geq Y_{i,t}^A + (1 - \iota)\alpha + \gamma n_t \end{aligned}$$

$$(C.34) \quad \begin{aligned} l_t^* &\geq Y_{i,t}^{A*} + \int_0^{I_t^*} \alpha P_{fi,t}^{M*} Y_{i,t}^{g*} di + \int_{I_t^*}^1 \alpha P_{fi,t}^{M*} Y_{i,t}^{e*} di + \int_0^{n_t^*} \frac{y_t^{g*}(\omega_f)}{a_t^{g*}} d\omega_f + \int_0^{n_t^*} q_t^{g*}(\omega_f) d\omega_f \\ &\Rightarrow l_t^* \geq Y_{i,t}^{A*} + (1 - \iota)\alpha + \gamma n_t^* \end{aligned}$$

Tax. From Section 3.2.4, the tax used to finance the production subsidy is determined in the home and foreign countries, respectively, as follows:

$$(C.35) \quad T_t = s n_t p_t^g(\omega) y_t^g(\omega)$$

$$(C.36) \quad T_t^* = s^* n_t^* p_t^{g*}(\omega_f) y_t^{g*}(\omega_f)$$

Definition of Equilibrium. A general equilibrium with the home and foreign production subsidies,  $\{s, s^*\}$ , consists of the home and foreign outputs,  $\{Y_t^A, Y_{i,t}^k, y_t^g(\omega)\}$  and  $\{Y_t^{A*}, Y_{i,t}^{k*}, y_t^{g*}(\omega_f)\}$ , the home and foreign labor supply,  $\{l_t, l_t^*\}$ , and the home and foreign final goods demands,  $\{C_t, C_t^A, C_{hi,t}^M, C_{fi,t}^M\}$  and  $\{C_t^*, C_t^{A*}, C_{hi,t}^{M*}, C_{fi,t}^{M*}\}$ , and the home and foreign green capital goods demands,  $\{z_t^g(\omega), z_t^g(\omega_f), Z_{i,t}^g\}$  and  $\{z_t^{g*}(\omega_f), z_t^{g*}(\omega), Z_{i,t}^{g*}\}$ , and the home and foreign conventional energy demands,  $E_{i,t}$  and  $E_{i,t}^*$ , and the mass of the home and foreign capital goods firms,  $n_t$  and  $n_t^*$ , and the home and foreign green capital usage ratio,  $I_t$  and  $I_t^*$ , and the home and foreign prices,  $\{P_t, P_t^M, P_{h,t}^M, P_{hi,t}^k, P_t^g, p_t^g(\omega)\}$  and  $\{P_t^*, P_t^{M*}, P_{f,t}^{M*}, P_{fi,t}^{k*}, P_t^{g*}, p_t^{g*}(\omega_f)\}$ , and the home and foreign tax,  $T_t$  and  $T_t^*$ , such that equation (C.1)-(C.36) hold.<sup>1</sup>

<sup>1</sup>There are 42 endogenous variables and 42 equations since the following equations each contribute two: (C.13), (C.14), (C.15), (C.16), (C.20), (C.21).

Similar to the approach taken in Corsetti et al. (2007), the equilibrium system of equations defined above can be simplified to two key equations: (C.28) and (C.29). These equations involve two endogenous variables,  $n_t$  and  $n_t^*$ . Solving for these variables allows us to determine all other variables using the equations (C.1)-(C.34). By solving equations (C.28) and (C.29), we derive the following solution.

$$(C.37) \quad n_t = \frac{(1+s)(1-\rho)(1-\alpha)}{\gamma} \frac{I_t + \phi^2 I_t^* - S^{-\gamma} A^{1-\gamma} \phi (I_t + I_t^*)}{1 + \phi^2 - (S^\gamma A^{\gamma-1} + S^{-\gamma} A^{1-\gamma}) \phi}$$

$$(C.38) \quad n_t^* = \frac{(1+s^*)(1-\rho)(1-\alpha)}{\gamma} \frac{I_t^* + \phi^2 I_t - S^\gamma A^{\gamma-1} \phi (I_t + I_t^*)}{1 + \phi^2 - (S^\gamma A^{\gamma-1} + S^{-\gamma} A^{1-\gamma}) \phi}$$

where  $S \equiv \frac{1+s}{1+s^*}$  and  $A \equiv \frac{a_t^g}{a_t^{g^*}}$ .

The solution of the equilibrium is summarized in Table C.1.



TABLE C.1. Summary of Equilibrium

Home	Foreign
$n_t = \frac{(1+s)(1-\iota)(1-\alpha)}{\gamma} \frac{I_t + \phi^2 I_t^* - S^{-\gamma} A^{1-\gamma} \phi (I_t + I_t^*)}{1 + \phi^2 - (S^{-\gamma} A^{\gamma-1} + S^{-\gamma} A^{1-\gamma}) \phi}$	$n_t^* = \frac{(1+s^*)(1-\iota)(1-\alpha)}{\gamma} \frac{I_t^* + \phi^2 I_t - S^{-\gamma} A^{\gamma-1} \phi (I_t + I_t^*)}{1 + \phi^2 - (S^{-\gamma} A^{\gamma-1} + S^{-\gamma} A^{1-\gamma}) \phi}$
$p_t^g(\omega) = \frac{1}{1+s} \frac{\gamma}{\gamma-1} \frac{1}{a_t^g}$	$p_t^{g*}(\omega_f) = \frac{1}{1+s^*} \frac{\gamma}{\gamma-1} \frac{1}{a_t^{g*}}$
$P_t^g = \left( n_t (p_t^g(\omega))^{1-\gamma} + \phi n_t^* (p_t^{g*}(\omega_f))^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$	$P_t^{g*} = \left( n_t^* (p_t^{g*}(\omega_f))^{1-\gamma} + \phi n_t (p_t^g(\omega))^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$
$0 \leq i \leq I_t, P_{hi,t}^M = \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \lambda_i^{\alpha-1} P_t^{g1-\alpha}$	$0 \leq i \leq I_t^*, P_{fi,t}^{M*} = \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \lambda_i^{\alpha-1} P_t^{g*1-\alpha}$
$I_t < i, P_{hi,t}^M = \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \psi_t^{1-\alpha}$	$I_t^* < i, P_{fi,t}^{M*} = \alpha^{-\alpha} (1-\alpha)^{-\alpha+1} \psi_t^{*1-\alpha}$
$P_{h,t}^M = e^{\int_0^1 \ln P_{hi,t}^M di}$	$P_{f,t}^{M*} = e^{\int_0^1 \ln P_{fi,t}^{M*} di}$
$P_t^M = \beta^{-\beta} (1-\beta)^{-\beta+1} \left( P_{h,t}^M \right)^\beta \left( P_{f,t}^{M*} \right)^{1-\beta}$	$P_t^{M*} = \beta^{-\beta} (1-\beta)^{-\beta+1} \left( P_{f,t}^{M*} \right)^\beta \left( P_{h,t}^M \right)^{1-\beta}$
$P_t = \iota^{-\iota} (1-\iota)^{-\iota+1} \left( P_t^M \right)^{1-\iota}$	$P_t^* = \iota^{-\iota} (1-\iota)^{-\iota+1} \left( P_t^{M*} \right)^{1-\iota}$
$E_{i,t} = \frac{(1-\alpha)(1-\iota)}{\psi}$	$E_{i,t}^* = \frac{(1-\alpha)(1-\iota)}{\psi}$
$Z_{i,t}^g = \frac{(1-\alpha)(1-\iota)}{P_t^g}$	$Z_{i,t}^{g*} = \frac{(1-\alpha)(1-\iota)}{P_t^{g*}}$
$z_t^g(\omega) = \left( \frac{p_t^g}{P_t^g} \right)^{-\gamma} \int_0^{I_t} Z_{i,t}^g di$	$z_t^{g*}(\omega) = \left( \frac{(1+\tau)p_t^g}{P_t^{g*}} \right)^{-\gamma} \int_0^{I_t^*} Z_{i,t}^{g*} di$
$z_t^g(\omega_f) = \left( \frac{(1+\tau)p_t^{g*}}{P_t^g} \right)^{-\gamma} \int_0^{I_t} Z_{i,t}^g di$	$z_t^{g*}(\omega_f) = \left( \frac{p_t^{g*}}{P_t^{g*}} \right)^{-\gamma} \int_0^{I_t^*} Z_{i,t}^{g*} di$
$C_t^A = \iota$	$C_t^{A*} = \iota$
$C_{hi,t}^M = \frac{(1-\iota)\beta}{P_{hi,t}^M}, C_{hi,t}^{M*} = \frac{(1-\iota)(1-\beta)}{P_{hi,t}^M}$	$C_{fi,t}^{M*} = \frac{(1-\iota)\beta}{P_{fi,t}^{M*}}, C_{fi,t}^M = \frac{(1-\iota)(1-\beta)}{P_{fi,t}^{M*}}$
$C_t = P_t^{-1}$	$C_t = P_t^{*-1}$
$y_t^g(\omega) = z_t^g(\omega) + (1+\tau)z_t^{g*}(\omega)$	$y_t^{g*}(\omega_f) = z_t^{g*}(\omega_f) + (1+\tau)z_t^g(\omega)$
$Y_{i,t}^k = C_{hi,t}^M + C_{hi,t}^{M*}, k \in \{g, e\}$	$Y_{i,t}^{k*} = C_{fi,t}^M + C_{fi,t}^{M*}, k \in \{g, e\}$
$Y_t^A = l_t - (1-\iota)\alpha - \gamma n_t$	$Y_t^{A*} = l_t^* - (1-\iota)\alpha - \gamma n_t^*$
$T_t = \frac{s\gamma n_t}{1+s}$	$T_t^* = \frac{s^*\gamma n_t^*}{1+s^*}$
$l_t = 1 + T_t$	$l_t^* = 1 + T_t^*$
$0 \leq t \leq t^r, I_t = \underline{I}$	$0 \leq t \leq t^{r*}, I_t^* = \underline{I}$
$t^r < t, I_t = \bar{I}$	$t^{r*} < t, I_t^* = \bar{I}$
$S \equiv \frac{1+s}{1+s^*}, A \equiv \frac{a_t^g}{a_t^{g*}}$	

## C.2. Proofs

**C.2.1. Proposition 9.** Based on the results of comparative statics in Table C.2,  $Y_{i,t}^e$  is fixed in each stage in each country. Thus, the total global greenhouse gas emission in each stage is given by  $\int_{I_t}^1 Y_{i,t}^e di + \int_{I_t^*}^1 Y_{i,t}^{e*} di = (1 - I_t)Y_i^e + (1 - I_t^*)Y_i^e$ , where  $Y_i^e$  denotes the fixed amount of production using conventional energy.

When  $s = s^* = 0$ , based on equations (3.32), (3.34), and (3.36), the welfare effect of a home production subsidy on the home country can be expressed as follows:

$$\begin{aligned}
 \text{(C.39)} \quad \frac{d \ln W}{ds} &= t^r(0, 0) \left[ -\frac{d \ln P_E}{ds} - (1 - \iota)(1 - \alpha)I_E + (-\ln P_L + \ln P_E + \kappa(\bar{I} - \underline{I})Y_i^e) \left( -\epsilon_{tr} \frac{d \ln n_E}{ds} \right) \right. \\
 &\quad \left. + (-\ln P_L + \ln P_E + \kappa(\bar{I} - \underline{I})Y_i^e) \left( -\epsilon_{tr} \frac{d \ln n_E^*}{ds} \right) \right] \\
 &= t^r(0, 0) \left[ \frac{(1 - \iota)(1 - \alpha)I_E}{(\gamma - 1)} + (-\ln P_L + \ln P_E + \kappa(\bar{I} - \underline{I})Y_i^e) \left( -\epsilon_{tr} \frac{d \ln n_E}{ds} \right) \right. \\
 &\quad \left. + (-\ln P_M + \ln P_L + \kappa(\bar{I} - \underline{I})Y_i^e) \left( -\epsilon_{tr} \frac{d \ln n_E^*}{ds} \right) \right]
 \end{aligned}$$

Based on the comparative statics in Table C.2, the welfare can be reexpressed as follows:

$$\begin{aligned}
 \text{(C.40)} \quad \frac{d \ln W}{ds} &= t^r(0, 0) \left[ \frac{(1 - \iota)(1 - \alpha)I_E}{(\gamma - 1)} + (-\ln P_L + \ln P_E) \left( -\epsilon_{tr} \frac{d \ln n_E}{ds} \right) \right. \\
 &\quad \left. + (-\ln P_M + \ln P_L) \left( -\epsilon_{tr} \frac{d \ln n_E^*}{ds} \right) + \kappa(\bar{I} - \underline{I})Y_i^e \right] > 0
 \end{aligned}$$

Based on the results presented in Appendix C.3.2 and C.3.3, we have  $-\ln P_L + \ln P_E > 0$  and  $-\ln P_M + \ln P_L < 0$ . Therefore, we can conclude that the welfare effect of a home production subsidy on the home country near the initial equilibrium is positive. ■

**C.2.2. Proposition 10.** When  $s = s^* = 0$ , based on equations (3.33), (3.35), and (3.37), the welfare effect of a home production subsidy on the foreign country can be expressed as follows:

$$\begin{aligned}
 \text{(C.41)} \quad \frac{d \ln W^*}{ds} &= t^r(0, 0) \left[ (-\ln P_L^* + \ln P_E^* + \kappa(\bar{I} - \underline{I})Y_i^e) \left( -\epsilon_{tr} \frac{d \ln n_E}{ds} \right) \right. \\
 &\quad \left. + (-\ln P_M^* + \ln P_L^* + \kappa(\bar{I} - \underline{I})Y_i^e) \left( -\epsilon_{tr} \frac{d \ln n_E^*}{ds} \right) \right]
 \end{aligned}$$

Additionally, using the results in Table C.2, we can find the condition under which a home production subsidy worsens foreign welfare as follows:

$$\begin{aligned}
& \frac{d \ln W^*}{ds} < 0 \Leftrightarrow \\
\text{(C.42)} \quad & -\epsilon_{tr} (\kappa(\bar{I} - \underline{I})Y_i^e) - (-\ln P_L^* + \ln P_E^*)\epsilon_{tr} \frac{d \ln n_E}{ds} - (-\ln P_M^* + \ln P_L^*)\epsilon_{tr} \frac{d \ln n_E^*}{ds} < 0 \Leftrightarrow \\
& (-\ln P_M^* + \ln P_L^*)\epsilon_{tr} \frac{d \ln n_E^*}{ds} > -(-\ln P_L^* + \ln P_E^*)\epsilon_{tr} \frac{d \ln n_E}{ds} - \epsilon_{tr} (\kappa(\bar{I} - \underline{I})Y_i^e)
\end{aligned}$$

■

**C.2.3. Proposition 11.** Based on equations (3.32), (3.34), (3.36), and the results in Table C.2, the welfare effect of increasing the subsidy rate under the symmetric condition ( $s = s^*$ ) on the conducting country can be rewritten as follows.

$$\begin{aligned}
\text{(C.43)} \quad & \frac{d \ln W}{ds} = t^r(s, s^*) \left( -\frac{d \ln P_E}{ds} - (1 - \iota)(1 - \alpha)I_E - \frac{s}{1 + s} \frac{2\gamma\phi(1 - \iota)(1 - \alpha)I_E}{(1 - \phi)^2} \right) \\
& - t^r(s, s^*)\epsilon_{tr} (-\ln P_L + \ln P_E + \kappa(\bar{I} - \underline{I})Y_i^e) \frac{d \ln n_E}{ds} \\
& - t^r(s, s^*)\epsilon_{tr} (-\ln P_M + \ln P_L + \kappa(\bar{I} - \underline{I})Y_i^e) \frac{d \ln n_E^*}{ds} \\
& = t^r(s, s^*) \left[ \left( \frac{1}{1 + s} \frac{\gamma(1 - \iota)(1 - \alpha)I_E}{(\gamma - 1)} - (1 - \iota)(1 - \alpha)I_E - \frac{s}{1 + s} \frac{2\gamma\phi(1 - \iota)(1 - \alpha)I_E}{(1 - \phi)^2} \right) \right. \\
& - \frac{1}{1 + s} \epsilon_{tr} (-\ln P_L + \ln P_E + \kappa(\bar{I} - \underline{I})Y_i^e) \left( 1 + \frac{2\gamma\phi}{(1 - \phi)^2} \right) \\
& \left. + \frac{1}{1 + s} \epsilon_{tr} (-\ln P_M + \ln P_L + \kappa(\bar{I} - \underline{I})Y_i^e) \frac{2\gamma\phi}{(1 - \phi)^2} \right] \\
& = t^r(s, s^*) \left[ \frac{1}{1 + s} \frac{1}{t^r(0, 0)} \frac{d \ln W}{ds} \Big|_{s=s^*=0} - \frac{s}{1 + s} (1 - \iota)(1 - \alpha)I_E - \frac{s}{1 + s} \frac{2\gamma\phi(1 - \iota)(1 - \alpha)I_E}{(1 - \phi)^2} \right] \\
& = t^r(s, s^*) \left[ \underbrace{\frac{1}{1 + s} \frac{1}{t^r(0, 0)} \frac{d \ln W}{ds} \Big|_{s=s^*=0} - \frac{s}{1 + s} \left( 1 + \frac{2\gamma\phi}{(1 - \phi)^2} \right) (1 - \iota)(1 - \alpha)I_E}_{\textcircled{A}} \right]
\end{aligned}$$

To see how this welfare effect changes while increasing  $s$ , I first take the derivative of  $\textcircled{A}$  with respect to  $s$  as follows.

(C.44)

$$\frac{d\textcircled{A}}{ds} = t^r(s, s^*) \left[ -\frac{1}{(1+s)^2} \frac{1}{t^r(0,0)} \frac{d \ln W}{ds} \Big|_{s=s^*=0} - \frac{1}{(1+s)^2} \left( 1 + \frac{2\gamma\phi}{(1-\phi)^2} \right) (1-\iota)(1-\alpha)I_E \right] < 0$$

Since Proposition 9 shows  $\frac{d \ln W}{ds} \Big|_{s=s^*=0} > 0$ , it can be proven that  $\textcircled{A} \Big|_{s=s^*=0} > 0$ . Thus, it can be proven that there is a unique  $s$  which satisfies  $\textcircled{A} = 0$ .

Then, I take the derivative of  $\frac{d \ln W}{ds}$  with respect to  $s$  as follows.

$$(C.45) \quad \frac{d \frac{d \ln W}{ds}}{ds} = \frac{dt^r(s, s^*)}{ds} \textcircled{A} + t^r(s, s^*) \frac{d\textcircled{A}}{ds}$$

Since  $t^r(s, s^*) > 0$  and  $\frac{dt^r(s, s^*)}{ds} < 0$ ,  $\frac{d \frac{d \ln W}{ds}}{ds} < 0$  as long as  $\textcircled{A} > 0$ . Additionally, when  $\textcircled{A} = 0$ ,  $\frac{d \frac{d \ln W}{ds}}{ds} = t_0$ , and when  $\textcircled{A} < 0$ ,  $\frac{d \frac{d \ln W}{ds}}{ds} < 0$ . Accordingly, it follows that there is a unique  $s$  which satisfies  $\frac{d \ln W}{ds} = 0$ .

From equation (C.43),  $\bar{s}$  is solved as follows.

$$(C.46) \quad \begin{aligned} & t^r(s, s^*) \left[ \frac{1}{1+s} \frac{1}{t^r(0,0)} \frac{d \ln W}{ds} \Big|_{s=s^*=0} - \frac{s}{1+s} \left( 1 + \frac{2\gamma\phi}{(1-\phi)^2} \right) (1-\iota)(1-\alpha)I_E \right] = 0 \\ & \Rightarrow s \left( 1 + \frac{2\gamma\phi}{(1-\phi)^2} \right) (1-\iota)(1-\alpha)I_E = \frac{1}{t^r(0,0)} \frac{d \ln W}{ds} \Big|_{s=s^*=0} \\ & \Rightarrow \bar{s} = \frac{\frac{1}{t^r(0,0)} \frac{d \ln W}{ds} \Big|_{s=s^*=0}}{\left( 1 + \frac{2\gamma\phi}{(1-\phi)^2} \right) (1-\iota)(1-\alpha)I_E} = \frac{\frac{1}{t^r(0,0)} \frac{d \ln W}{ds} \Big|_{s=s^*=0}}{(1-\iota)(1-\alpha)I_E \frac{d \ln n_E}{ds} \Big|_{s=s^*=0}} \end{aligned}$$

■

**C.2.4. Proposition 12.** Based on the comparative statics in Table C.3, the welfare effect of a cooperative production subsidy change at  $\bar{s}$  is as follows.

$$(C.47) \quad \begin{aligned} \frac{d \ln W}{ds^c} \Big|_{s=s^*=\bar{s}} &= t^r(\bar{s}, \bar{s}) \left( -\frac{d \ln P_E}{ds^c} - \frac{d l_E}{ds^c} \right) - t^r(\bar{s}, \bar{s}) \epsilon_{t^r} (-\ln P_M + \ln P_E + 2\kappa(\bar{I} - \underline{I})Y_i^e) \frac{d \ln n_E}{ds^c} \\ &= t^r(\bar{s}, \bar{s}) \left[ \left( \frac{1}{1+\bar{s}} \frac{\gamma(1-\iota)(1-\alpha)I_E}{\gamma-1} - (1-\iota)(1-\alpha)I_E \right) \right. \\ & \quad \left. - \epsilon_{t^r} \frac{1}{1+\bar{s}} \left( \frac{1}{\gamma-1} (\ln \bar{I} - \ln \underline{I}) + \ln \bar{a}_H + 2\kappa(\bar{I} - \underline{I})Y_i^e \right) \right] \end{aligned}$$

Using equilibrium  $\bar{s}$  at equation (C.46), the above equation can be expressed as follows.

$$\begin{aligned}
\left. \frac{d \ln W}{ds^c} \right|_{s=s^*=\bar{s}} &= t^r(\bar{s}, \bar{s}) \frac{1}{1+\bar{s}} \left[ \left( \frac{\gamma(1-\iota)(1-\alpha)I_E}{\gamma-1} - (1+\bar{s})(1-\iota)(1-\alpha)I_E \right) \right. \\
&\quad \left. - \epsilon_{tr} \left( \frac{1}{\gamma-1} (\ln \bar{I} - \ln \underline{I}) + \ln \bar{a}_H + 2\kappa(\bar{I} - \underline{I})Y_i^e \right) \right] \\
&= t^r(\bar{s}, \bar{s}) \frac{1}{1+\bar{s}} \left[ \left( \frac{(1-\iota)(1-\alpha)I_E}{\gamma-1} - \bar{s}(1-\iota)(1-\alpha)I_E \right) \right. \\
&\quad \left. - \epsilon_{tr} (-\ln P_L + \ln P_E + \kappa(\bar{I} - \underline{I})Y_i^e) - \epsilon_{tr} (-\ln P_M + \ln P_L + \kappa(\bar{I} - \underline{I})Y_i^e) \right] \\
&= t^r(\bar{s}, \bar{s}) \frac{1}{1+\bar{s}} \left[ \left( \frac{(1-\iota)(1-\alpha)I_E}{\gamma-1} - \frac{\frac{\gamma(1-\iota)(1-\alpha)I_E}{(\gamma-1)} - (1-\iota)(1-\alpha)I_E}{1 + \frac{2\gamma\phi}{(1-\phi)^2}} \right) \right. \\
(C.48) \quad &\quad \left. - \epsilon_{tr} (-\ln P_M + \ln P_L + \kappa(\bar{I} - \underline{I})Y_i^e) \left( 1 + \frac{\frac{2\gamma\phi}{(1-\phi)^2}}{1 + \frac{2\gamma\phi}{(1-\phi)^2}} \right) \right] \\
&= t^r(\bar{s}, \bar{s}) \frac{1}{1+\bar{s}} \left[ \left( \frac{1}{\gamma-1} - \frac{\frac{\gamma}{\gamma-1} - 1}{1 + \frac{2\gamma\phi}{(1-\phi)^2}} \right) (1-\iota)(1-\alpha)I_E \right. \\
&\quad \left. - \epsilon_{tr} (-\ln P_M + \ln P_L + \kappa(\bar{I} - \underline{I})Y_i^e) \left( 1 + \frac{\frac{2\gamma\phi}{(1-\phi)^2}}{1 + \frac{2\gamma\phi}{(1-\phi)^2}} \right) \right] \\
&= t^r(\bar{s}, \bar{s}) \frac{1}{1+\bar{s}} \left[ \left( \frac{\frac{2\gamma\phi}{(1-\phi)^2}}{(\gamma-1) \left( 1 + \frac{2\gamma\phi}{(1-\phi)^2} \right)} \right) (1-\iota)(1-\alpha)I_E \right. \\
&\quad \left. - \epsilon_{tr} (-\ln P_M + \ln P_L + \kappa(\bar{I} - \underline{I})Y_i^e) \left( 1 + \frac{\frac{2\gamma\phi}{(1-\phi)^2}}{1 + \frac{2\gamma\phi}{(1-\phi)^2}} \right) \right] > 0
\end{aligned}$$

Since both the first term and the second term in the equation above are positive, we can prove that  $\frac{d \ln W}{ds^c} = \frac{d \ln W^*}{ds_c^*} > 0$  at  $s = s^* = \bar{s}$ . ■

### C.3. Welfare Effect Decomposition

**C.3.1. Short-run Resource Reallocation Effect.** By taking the derivative of the short-run resource reallocation effect with respect to  $s$  for both the home and foreign countries in the initial

equilibrium, we obtain the following two equations.

$$\begin{aligned}
\int_0^{t^r(0,0)} e^{-\rho t} \frac{d \ln U_E}{ds} dt &= \frac{1}{\rho} (1 - e^{-\rho t_g^r(0,0)}) \frac{d \ln U_E}{ds} \\
&= \frac{1}{\rho} (1 - e^{-\rho t_g^r(0,0)}) \left[ -\frac{d \ln P_E}{ds} - \frac{dl_E}{ds} - \kappa \int_{\underline{I}}^1 \left( \frac{dY_{i,E}^e}{ds} + \frac{dY_{i,E}^{e*}}{ds} \right) di \right] \\
&= \frac{1}{\rho} (1 - e^{-\rho t_g^r(0,0)}) \left[ -\frac{d \ln P_E}{ds} - (1 - \iota)(1 - \alpha) I_E \right] \\
&= \frac{1}{\rho} (1 - e^{-\rho t_g^r(0,0)}) \frac{(1 - \iota)(1 - \alpha) I_E}{(\gamma - 1)} > 0
\end{aligned} \tag{C.49}$$

$$\begin{aligned}
\int_0^{t^r(0,0)} e^{-\rho t} \frac{d \ln U_E^*}{ds} dt &= \frac{1}{\rho} (1 - e^{-\rho t_g^r(0,0)}) \frac{d \ln U_E^*}{ds} \\
&= \frac{1}{\rho} (1 - e^{-\rho t_g^r(0,0)}) \left[ -\frac{d \ln P_E^*}{ds} - \frac{dl_E^*}{ds} - \kappa \int_{\underline{I}}^1 \left( \frac{dY_{i,E}^e}{ds} + \frac{dY_{i,E}^{e*}}{ds} \right) di \right] \\
&= -\frac{1}{\rho} (1 - e^{-\rho t_g^r(0,0)}) \frac{d \ln P_E^*}{ds} = 0
\end{aligned} \tag{C.50}$$

Based on the results from the comparative statics in Table C.2, we can determine that  $\frac{d \ln U_E}{ds} > 0$  and  $\frac{d \ln U_E^*}{ds} = 0$ .

**C.3.2. Earlier Home Innovation Effect.** First, the earlier home innovation effect for the home and foreign country can be expressed as follows:

$$\begin{aligned}
&-e^{-\rho t_g^r(0,0)} (\ln U_L - \ln U_E) \frac{dt^r(0,0)}{ds} = -e^{-\rho t_g^r(0,0)} t^r(0,0) \epsilon_{tr} (\ln U_L - \ln U_E) \frac{d \ln n_E}{ds} \\
&= -e^{-\rho t_g^r(0,0)} t^r(0,0) \epsilon_{tr} \left[ -\ln P_L + \ln P_E - \kappa(2 - \bar{I} - \underline{I}) Y_i^e + \kappa(2 - 2\underline{I}) Y_i^e \right] \frac{d \ln n_E}{ds} \\
&= -e^{-\rho t_g^r(0,0)} t^r(0,0) \epsilon_{tr} \left[ -\ln P_L + \ln P_E + \kappa(\bar{I} - \underline{I}) Y_i^e \right] \frac{d \ln n_E}{ds}
\end{aligned} \tag{C.51}$$

$$\begin{aligned}
&-e^{-\rho t_g^r(0,0)} (\ln U_L^* - \ln U_E^*) \frac{dt^r(s, s^*)}{ds} = -e^{-\rho t_g^r(0,0)} t^r(0,0) \epsilon_{tr} (\ln U_L^* - \ln U_E^*) \frac{d \ln n_E}{ds} \\
&= -e^{-\rho t_g^r(0,0)} t^r(0,0) \epsilon_{tr} \left[ -\ln P_L^* + \ln P_E^* - \kappa(1 - I_L) Y_{i,L}^e + \kappa(1 - I_E) Y_{i,E}^e \right] \frac{d \ln n_E}{ds} \\
&= -e^{-\rho t_g^r(0,0)} t^r(0,0) \epsilon_{tr} \left[ -\ln P_L^* + \ln P_E^* + \kappa(\bar{I} - \underline{I}) Y_i^e \right] \frac{d \ln n_E}{ds}
\end{aligned} \tag{C.52}$$

To determine the sign of the welfare effect, we must inspect the sign of  $-\ln P_L + \ln P_E$  and  $-\ln P_L^* + \ln P_E^*$ . Using the solutions in Table C.1, we can derive the following closed-form solutions

for the price of aggregated green capital goods in the early and leading stages for each country:

$$(C.53) \quad \ln P_E^g = \ln P_E^{g*} = \frac{1}{1-\gamma} \left( \ln \frac{(1-\iota)(1-\alpha)}{\gamma} + \ln(1+\phi)\underline{I} \right)$$

$$(C.54) \quad \ln P_L^g = \frac{1}{1-\gamma} \left( \ln \frac{(1-\iota)(1-\alpha)}{\gamma} + \ln \frac{(\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2)\bar{I}}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right)$$

$$(C.55) \quad \ln P_L^{g*} = \frac{1}{1-\gamma} \left( \ln \frac{(1-\iota)(1-\alpha)}{\gamma} + \ln \frac{(1 - \bar{a}_H^{\gamma-1}\phi + \bar{a}_H^{\gamma-1}\phi^3 - \phi^2)\underline{I}}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right)$$

where  $\bar{a}_H$  represents the productivity level after innovation.

By applying the above closed-form solutions for the price of aggregated green capital goods,  $-\ln P_L + \ln P_E$  can be expressed as follows:

$$(C.56) \quad \begin{aligned} -\ln P_L + \ln P_E &= \frac{(1-\iota)(1-\alpha)\underline{I}}{1+\phi} \frac{1}{\gamma-1} \left( \ln \frac{(\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2)\bar{I}}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1+\phi)\underline{I} \right) \\ &+ \frac{(1-\iota)(1-\alpha)(\bar{I}-\underline{I})}{1+\phi} \frac{1}{\gamma-1} \left( \ln \frac{(\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2)\bar{I}}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1+\phi)\underline{I} \right) \\ &+ \frac{(1-\iota)(1-\alpha)(\bar{I}-\underline{I})}{1+\phi} \frac{1}{\gamma-1} (\ln \bar{\lambda} - \ln \lambda_{\underline{I}}) \\ &+ \frac{(1-\iota)(1-\alpha)\phi\underline{I}}{1+\phi} \frac{1}{\gamma-1} \left( \ln \frac{(1 - \bar{a}_H^{\gamma-1}\phi + \bar{a}_H^{\gamma-1}\phi^3 - \phi^2)\underline{I}}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1+\phi)\underline{I} \right) \\ &= \frac{(1-\iota)(1-\alpha)\underline{I}}{1+\phi} \frac{1}{\gamma-1} \left( \ln \frac{(\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2)}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1+\phi) \right) \\ &+ \frac{(1-\iota)(1-\alpha)(\bar{I}-\underline{I})}{1+\phi} \frac{1}{\gamma-1} \left( \ln \frac{(\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2)}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1+\phi) \right) \\ &+ \frac{(1-\iota)(1-\alpha)(\bar{I}-\underline{I})}{1+\phi} \frac{1}{\gamma-1} (\ln \bar{\lambda} - \ln \lambda_{\underline{I}}) + \frac{(1-\iota)(1-\alpha)\bar{I}}{1+\phi} \frac{1}{\gamma-1} (\ln \bar{I} - \ln \underline{I}) \\ &+ \frac{(1-\iota)(1-\alpha)\phi\underline{I}}{1+\phi} \frac{1}{\gamma-1} \left( \ln \frac{(1 - \bar{a}_H^{\gamma-1}\phi + \bar{a}_H^{\gamma-1}\phi^3 - \phi^2)}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1+\phi) \right) > 0 \end{aligned}$$

where  $\bar{\lambda} \equiv \frac{\int_{\underline{I}}^{\bar{I}} \lambda_i di}{\bar{I}-\underline{I}}$  represents the average productivity of using green capital for the manufacturing sectors located between  $\underline{I}$  and  $\bar{I}$ .

Given that  $\ln \frac{(\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1} \phi^2)}{1 + \phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1 + \phi) > 0$ ,  $\ln \frac{(1 - \bar{a}_H^{\gamma-1} \phi + \bar{a}_H^{\gamma-1} \phi^3 - \phi^2)}{1 + \phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1 + \phi) < 0$  and  $\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1} \phi^2 > 1 - \bar{a}_H^{\gamma-1} \phi + \bar{a}_H^{\gamma-1} \phi^3 - \phi^2$ , it can be concluded that the sign of  $-\ln P_L + \ln P_E$  is positive.

By using equations (C.53), (C.54) and (C.55),  $-\ln P_L^* + \ln P_E^*$  can be written as follows:

$$\begin{aligned}
(C.57) \quad & -\ln P_L^* + \ln P_E^* = \frac{(1-\iota)(1-\alpha)\underline{I}}{1+\phi} \frac{1}{\gamma-1} \left( \ln \frac{(1-\bar{a}_H^{\gamma-1}\phi + \bar{a}_H^{\gamma-1}\phi^3 - \phi^2)}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1+\phi) \right) \\
& + \frac{(1-\iota)(1-\alpha)\phi\underline{I}}{1+\phi} \frac{1}{\gamma-1} \left( \ln \frac{(\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2)}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1+\phi) \right) \\
& + \frac{(1-\iota)(1-\alpha)\phi(\bar{I}-\underline{I})}{1+\phi} \frac{1}{\gamma-1} \left( \ln \frac{(\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2)}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} - \ln(1+\phi) \right) \\
& + \frac{(1-\iota)(1-\alpha)\phi(\bar{I}-\underline{I})}{1+\phi} \frac{1}{\gamma-1} (\ln \bar{\lambda} - \ln \lambda_{\underline{I}}) + \frac{(1-\iota)(1-\alpha)\phi\bar{I}}{1+\phi} \frac{1}{\gamma-1} (\ln \bar{I} - \ln \underline{I})
\end{aligned}$$

I take the derivative of  $-\ln P_L^* + \ln P_E^*$  with respect to  $\bar{a}_H$ , and its sign is shown to be positive, as follows.

$$\begin{aligned}
(C.58) \quad & \frac{d(-\ln P_L^* + \ln P_E^*)}{d\bar{a}_H} = \frac{(1-\iota)(1-\alpha)\underline{I}}{1+\phi} \left( \frac{-\bar{a}_H^{\gamma-2}\phi}{1-\bar{a}_H^{\gamma-1}\phi} + \frac{\bar{a}_H^{\gamma-2}\phi}{\bar{a}_H^{\gamma-1}-\phi} \right) \\
& + \frac{(1-\iota)(1-\alpha)(1+\phi)\underline{I}}{1+\phi} \left( \frac{(\bar{a}_H^{\gamma-2} - \bar{a}_H^{-\gamma})\phi}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right) \\
& + \frac{(1-\iota)(1-\alpha)\phi(\bar{I}-\underline{I})}{1+\phi} \left( \frac{\bar{a}_H^{\gamma-2}\phi}{\bar{a}_H^{\gamma-1}-\phi} + \frac{(\bar{a}_H^{\gamma-2} - \bar{a}_H^{-\gamma})\phi}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right) \\
& = \frac{(1-\iota)(1-\alpha)(1+\phi)\underline{I}}{1+\phi} \left( \frac{-\bar{a}_H^{\gamma-1}\phi(\bar{a}_H^{\gamma-1}-1)}{\bar{a}_H(1-\bar{a}_H^{\gamma-1}\phi)(\bar{a}_H^{\gamma-1}-\phi)} + \frac{(\bar{a}_H^{\gamma-1} - \bar{a}_H^{1-\gamma})\phi}{\bar{a}_H(1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi)} \right) \\
& + \frac{(1-\iota)(1-\alpha)\phi(\bar{I}-\underline{I})}{1+\phi} \left( \frac{\bar{a}_H^{\gamma-2}\phi}{\bar{a}_H^{\gamma-1}-\phi} + \frac{(\bar{a}_H^{\gamma-2} - \bar{a}_H^{-\gamma})\phi}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right) \\
& = \frac{(1-\iota)(1-\alpha)\phi(1+\phi)\underline{I}}{1+\phi} \left( \frac{\bar{a}_H^{\gamma-1} + 1}{\bar{a}_H(1-\bar{a}_H^{\gamma-1}\phi)(\bar{a}_H^{\gamma-1}-\phi)} \right) \\
& + \frac{(1-\iota)(1-\alpha)\phi(\bar{I}-\underline{I})}{1+\phi} \left( \frac{\bar{a}_H^{\gamma-2}\phi}{\bar{a}_H^{\gamma-1}-\phi} + \frac{(\bar{a}_H^{\gamma-2} - \bar{a}_H^{-\gamma})\phi}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right) > 0
\end{aligned}$$

It is easy to demonstrate that  $-\ln P_L^* + \ln P_E^* = 0$  when  $\bar{a}_H = 1$ . Consequently, we can confirm that  $-\ln P_L^* + \ln P_E^* > 0$ .

Taking into account all the results above, we can demonstrate that the early home innovation effect is positive for both the home and foreign countries.



**C.3.3. Delayed Foreign Innovation Effect.** The delayed foreign innovation effect for the home and foreign country can be expressed as follows:

$$\begin{aligned}
& -e^{-\rho t_g^{r^*}(0,0)}(\ln U_M - \ln U_L) \frac{dt^{r^*}(s, s^*)}{ds} = -e^{-\rho t_g^{r^*}(0,0)} t^r(0,0) \epsilon_{tr} (\ln U_M - \ln U_L) \frac{d \ln n_E^*}{ds} \\
\text{(C.59)} \quad & = -e^{-\rho t_g^{r^*}(0,0)} t^r(0,0) \epsilon_{tr} \left[ -\ln P_M + \ln P_L - \kappa(2 - 2\bar{I})Y_i^e + \kappa(2 - \bar{I} - \underline{I})Y_i^e \right] \frac{d \ln n_E^*}{ds} \\
& = -e^{-\rho t_g^{r^*}(0,0)} t^r(0,0) \epsilon_{tr} \left[ -\ln P_M + \ln P_L + \kappa(\bar{I} - \underline{I})Y_i^e \right] \frac{d \ln n_E^*}{ds}
\end{aligned}$$

$$\begin{aligned}
& -e^{-\rho t_g^{r^*}(0,0)}(\ln U_M^* - \ln U_L^*) \frac{dt^r(s, s^*)}{ds} = -e^{-\rho t_g^{r^*}(0,0)} t^r(0,0) \epsilon_{tr} (\ln U_M^* - \ln U_L^*) \frac{d \ln n_E^*}{ds} \\
\text{(C.60)} \quad & = -e^{-\rho t_g^{r^*}(0,0)} t^r(0,0) \epsilon_{tr} \left[ -\ln P_M^* + \ln P_L^* - \kappa(2 - 2\bar{I})Y_i^e + \kappa(2 - \bar{I} - \underline{I})Y_i^e \right] \frac{d \ln n_E^*}{ds} \\
& = -e^{-\rho t_g^{r^*}(0,0)} t^r(0,0) \epsilon_{tr} \left[ -\ln P_M^* + \ln P_L^* + \kappa(\bar{I} - \underline{I})Y_i^e \right] \frac{d \ln n_E^*}{ds}
\end{aligned}$$

Again, to determine the sign of the delayed foreign innovation effect, we need to inspect the signs of  $-\ln P_M + \ln P_L$  and  $-\ln P_M^* + \ln P_L^*$ . Using the solutions from Table C.1, we can derive the following closed-form solutions for the prices of aggregated green capital goods in the leading and mature stages for each country:

$$\text{(C.61)} \quad \ln P_L^g = \frac{1}{1-\gamma} \left( \ln \frac{(1-\iota)(1-\alpha)}{\gamma} + \ln \frac{(\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1} \phi^2) \bar{I}}{1 + \phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma}) \phi} \right)$$

$$\text{(C.62)} \quad \ln P_L^{g*} = \frac{1}{1-\gamma} \left( \ln \frac{(1-\iota)(1-\alpha)}{\gamma} + \ln \frac{(1 - \bar{a}_H^{\gamma-1} \phi + \bar{a}_H^{\gamma-1} \phi^3 - \phi^2) \underline{I}}{1 + \phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma}) \phi} \right)$$

$$\text{(C.63)} \quad \ln P_M^g = \ln P_M^{g*} = \frac{1}{1-\gamma} \left( \ln \frac{(1-\iota)(1-\alpha)}{\gamma} + \ln(1 + \phi) \bar{a}_H^{\gamma-1} \bar{I} \right)$$

By applying the above closed-form solutions for the price of aggregated green capital goods,  $-\ln P_M + \ln P_L$  can be expressed as follows:

(C.64)

$$\begin{aligned}
-\ln P_M + \ln P_L &= \frac{(1-\iota)(1-\alpha)\bar{I}}{1+\phi} \frac{1}{\gamma-1} \left( \ln(1+\phi)\bar{a}_H^{\gamma-1}\bar{I} - \ln \frac{(\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2)\bar{I}}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right) \\
&+ \frac{(1-\iota)(1-\alpha)\phi\bar{I}}{1+\phi} \frac{1}{\gamma-1} \left( \ln(1+\phi)\bar{a}_H^{\gamma-1}\bar{I} - \ln \frac{(1 - \bar{a}_H^{\gamma-1}\phi + \bar{a}_H^{\gamma-1}\phi^3 - \phi^2)\bar{I}}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right) \\
&+ \frac{(1-\iota)(1-\alpha)\phi(\bar{I} - \underline{I})}{1+\phi} (\ln \bar{\lambda} - \ln \lambda_L)
\end{aligned}$$

I take the derivative of  $-\ln P_M + \ln P_L$  with respect to  $\bar{a}_H$ , and its sign is shown to be negative, as follows.

$$\begin{aligned}
\frac{d(-\ln P_M + \ln P_L)}{d\bar{a}_H} &= (1-\iota)(1-\alpha)\bar{I} \left( \frac{1}{\bar{a}_H} - \frac{(\bar{a}_H^{\gamma-2} - \bar{a}_H^{-\gamma})\phi}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right) \\
&+ \frac{(1-\iota)(1-\alpha)\bar{I}}{1+\phi} \left( -\frac{\bar{a}_H^{\gamma-2}}{\bar{a}_H^{\gamma-1} - \phi} + \frac{\bar{a}_H^{\gamma-2}\phi^2}{1 - \bar{a}_H^{\gamma-1}\phi} \right) \\
&= \frac{(1-\iota)(1-\alpha)\bar{I}}{1+\phi} \left( \frac{(1+\phi)(1+\phi^2 - 2\bar{a}_H^{\gamma-1}\phi)}{\bar{a}_H(1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi)} \right. \\
&\quad \left. - \frac{\bar{a}_H^{\gamma-2}(1 - \bar{a}_H^{\gamma-1}\phi - (\bar{a}_H^{\gamma-1} - \phi)\phi^2)}{(\bar{a}_H^{\gamma-1} - \phi)(1 - \bar{a}_H^{\gamma-1}\phi)} \right) \\
&= \frac{(1-\iota)(1-\alpha)\bar{I}}{1+\phi} \left( \frac{(1+\phi)(1 - \bar{a}_H^{\gamma-1}\phi - (\bar{a}_H^{\gamma-1} - \phi)\phi)}{\bar{a}_H(1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi)} \right. \\
(C.65) \quad &\quad \left. - \frac{\bar{a}_H^{\gamma-1}(1 - \bar{a}_H^{\gamma-1}\phi - (\bar{a}_H^{\gamma-1} - \phi)\phi^2)}{\bar{a}_H(\bar{a}_H^{\gamma-1} - \phi)(1 - \bar{a}_H^{\gamma-1}\phi)} \right) \\
&= \frac{(1-\iota)(1-\alpha)\bar{I}}{1+\phi} \left( \frac{(1+\phi)(1 - \bar{a}_H^{\gamma-1}\phi - (\bar{a}_H^{\gamma-1} - \phi)\phi)}{\bar{a}_H(1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi)} \right. \\
&\quad \left. - \frac{\bar{a}_H^{\gamma-1}(1 - \bar{a}_H^{\gamma-1}\phi - (\bar{a}_H^{\gamma-1} - \phi)\phi)}{\bar{a}_H(\bar{a}_H^{\gamma-1} - \phi)(1 - \bar{a}_H^{\gamma-1}\phi)} - \frac{\bar{a}_H^{\gamma-1}(\phi - \phi^2)}{\bar{a}_H(1 - \bar{a}_H^{\gamma-1}\phi)} \right) \\
&= \frac{(1-\iota)(1-\alpha)\bar{I}}{1+\phi} \left( -\frac{(1 - \bar{a}_H^{\gamma-1}\phi - (\bar{a}_H^{\gamma-1} - \phi)\phi)(1+\phi)\phi^2(\bar{a}_H^{\gamma-1} - 1)^2}{\bar{a}_H(1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi)(\bar{a}_H^{\gamma-1} - \phi)(1 - \bar{a}_H^{\gamma-1}\phi)} \right. \\
&\quad \left. + \frac{\bar{a}_H^{\gamma-1}\phi(1-\phi)^2(1 - \bar{a}_H^{\gamma-1}\phi - (\bar{a}_H^{\gamma-1} - \phi)\phi)}{\bar{a}_H(1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi)(\bar{a}_H^{\gamma-1} - \phi)(1 - \bar{a}_H^{\gamma-1}\phi)} - \frac{\bar{a}_H^{\gamma-1}(\phi - \phi^2)}{\bar{a}_H(1 - \bar{a}_H^{\gamma-1}\phi)} \right) < 0
\end{aligned}$$

It is easy to demonstrate that  $-\ln P_M + \ln P_L = 0$  when  $\bar{a}_H = 1$ . Consequently, we can confirm that  $-\ln P_M + \ln P_L < 0$ .

By using equations (C.54), (C.55) and (C.63),  $-\ln P_L^* + \ln P_E^*$  can be written as follows:

$$(C.66) \quad \begin{aligned} -\ln P_M^* + \ln P_L^* &= \frac{(1-\iota)(1-\alpha)\bar{I}}{1+\phi} \frac{1}{\gamma-1} \left( \ln(1+\phi)\bar{a}_H^{\gamma-1} - \ln \frac{1-\bar{a}_H^{\gamma-1}\phi + \bar{a}_H^{\gamma-1}\phi^3 - \phi^2}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right) \\ &+ \frac{(1-\iota)(1-\alpha)(\bar{I}-\underline{I})}{1+\phi} (\ln \bar{\lambda} - \ln \lambda_{\underline{I}}) + \frac{(1-\iota)(1-\alpha)\bar{I}}{1+\phi} \frac{1}{\gamma-1} (\ln \bar{I} - \ln \underline{I}) \\ &+ \frac{(1-\iota)(1-\alpha)\phi \underline{I}}{1+\phi} \frac{1}{1-\gamma} \left( \ln(1+\phi)\bar{a}_H^{\gamma-1} - \ln \frac{\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} \right) > 0 \end{aligned}$$

Given that  $\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2 > 1 - \bar{a}_H^{\gamma-1}\phi + \bar{a}_H^{\gamma-1}\phi^3 - \phi^2$ ,  $\ln(1+\phi)\bar{a}_H^{\gamma-1} - \ln \frac{1-\bar{a}_H^{\gamma-1}\phi + \bar{a}_H^{\gamma-1}\phi^3 - \phi^2}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} > 0$ , and  $\ln(1+\phi)\bar{a}_H^{\gamma-1} - \ln \frac{\bar{a}_H^{\gamma-1} - \phi + \phi^3 - \bar{a}_H^{\gamma-1}\phi^2}{1+\phi^2 - (\bar{a}_H^{\gamma-1} + \bar{a}_H^{1-\gamma})\phi} < 0$ , it can be concluded that the sign of  $-\ln P_M^* + \ln P_L^*$  is positive.

Overall, the delayed foreign innovation effect is ambiguous for the home country since it benefits from reduced greenhouse gas emissions from the foreign manufacturing sectors, but it suffers losses from the Utility-based CPI. In contrast, the delayed foreign innovation effect reduces foreign welfare.

#### C.4. Comparative Statics for Non-cooperative Change in $s$ under $s = s^*$

As shown in Appendix C.1, an equilibrium with a production subsidy at each period is represented by the set  $\{n_t, n_{1,t}^*\}$ , which satisfies the zero-profit condition for both the domestic and foreign green capital goods firms.

I assume a symmetric condition where  $v = v^* = L = L^* = a_t(\omega) = a_t^*(\omega) = 1$ ,  $I_t = I_t^* = I_E$  and  $s = s^*$ . Given these conditions, there is a symmetric equilibrium such that  $n_t = n_t^*$ ,  $l_t = l_t^* = \gamma n_t = \gamma n_t^* = 1$ . I then take a first-order approximation of this model in the neighborhood of this symmetric equilibrium and analyze the local effects of production subsidy in the early stage.

For computational convenience, I extend the system of equations describing equilibrium to include 6 equations with 6 endogenous variables. These variables include the mass of firms in both the domestic and foreign green capital goods sectors,  $\{n_t, n_t^*\}$ , the price of aggregated green capital goods,  $\{P_t^z, P_t^{z*}\}$ , and the utility-based CPI,  $\{P_t, P_t^*\}$ . I take a first-order approximation of the system with respect to  $s$  and obtain the following equations.

$$(C.67) \quad (C.28): \quad \frac{\gamma(1+\phi)}{1+s} + (\gamma-1) \frac{d \ln P_E^g}{ds} + (\gamma-1)\phi \frac{d \ln P_E^{g*}}{ds} = 0$$

$$(C.68) \quad (C.29): (\gamma - 1) \frac{d \ln P_E^{g*}}{ds} + (\gamma - 1) \phi \frac{d \ln P_E^g}{ds} = 0$$

$$(C.69) \quad (C.3): (1 - \gamma)(1 + \phi) \frac{d \ln P_E^g}{ds} = \frac{d \ln n_E}{ds} - \frac{1 - \gamma}{1 + s} + \phi \frac{d \ln n_E^*}{ds}$$

$$(C.70) \quad (C.4): (1 - \gamma)(1 + \phi) \frac{d \ln P_E^{g*}}{ds} = \frac{d \ln n_E^*}{ds} + \phi \frac{d \ln n_E}{ds} - \frac{(1 - \gamma)\phi}{1 + s}$$

$$(C.71) \quad (C.11): \frac{d \ln P_E}{ds} = (1 - \iota)(1 - \alpha)\beta I_E \frac{d \ln P_E^g}{ds} + (1 - \iota)(1 - \alpha)(1 - \beta) I_E \frac{d \ln P_E^{g*}}{ds}$$

$$(C.72) \quad (C.12): \frac{d \ln P_E^*}{ds} = (1 - \iota)(1 - \alpha)\beta I_E \frac{d \ln P_E^{g*}}{ds} + (1 - \iota)(1 - \alpha)(1 - \beta) I_E \frac{d \ln P_E^g}{ds}$$

By solving the aforementioned six equations, I obtain the solutions presented in Table C.2.

TABLE C.2. Comparative Statics for Non-cooperative Change in  $s$  under  $s = s^*$

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(C.73)	$\frac{d \ln n_E}{ds} = \frac{1}{1 + s} \left[ 1 + \frac{2\gamma\phi}{(1 - \phi)^2} \right] > 0$
(C.74)	$\frac{d \ln n_E^*}{ds} = -\frac{1}{1 + s} \frac{2\gamma\phi}{(1 - \phi)^2} < 0$
(C.75)	$\frac{d \ln P_E^g}{ds} = -\frac{1}{1 + s} \frac{\gamma}{(\gamma - 1)(1 - \phi)} < 0$
(C.76)	$\frac{d \ln P_E^{g*}}{ds} = \frac{1}{1 + s} \frac{\gamma\phi}{(\gamma - 1)(1 - \phi)} > 0$
(C.77)	$\frac{d \ln P_E}{ds} = -\frac{1}{1 + s} \frac{\gamma(1 - \iota)(1 - \alpha)I_E}{(\gamma - 1)} < 0$
(C.78)	$\frac{d \ln P_E^*}{ds} = 0$
(C.79)	$\frac{d \ln Y_{i,E}^g}{ds} = -(1 - \alpha) \frac{d \ln P_E^g}{ds} = \frac{1}{1 + s} \frac{\gamma(1 - \alpha)}{(\gamma - 1)(1 - \phi)} > 0$
(C.80)	$\frac{d \ln Y_{i,E}^e}{ds} = -(1 - \alpha) \frac{d \ln \psi_E}{ds} = 0$
(C.81)	$\frac{d \ln Y_{i,E}^{g*}}{ds} = -(1 - \alpha) \frac{d \ln P_E^{g*}}{ds} = -\frac{1}{1 + s} \frac{\gamma\phi(1 - \alpha)}{(\gamma - 1)(1 - \phi)} < 0$
(C.82)	$\frac{d \ln Y_{i,E}^{e*}}{ds} = -(1 - \alpha) \frac{d \ln \psi_E^*}{ds} = 0$

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### C.5. Comparative Statics for Cooperative Change in $s$ under $s = s^*$

In this Appendix, I analyze the local effects of a production subsidy in the early stage when both the home and foreign countries cooperatively adjust their production subsidies to maximize joint welfare ( $s = s^* = \bar{s}$ ). In this scenario, the system for the first-order approximation is modified as follows.

$$(C.83) \quad (C.28): \frac{\gamma(1+\phi)}{1+s} + (\gamma-1)\frac{d \ln P_E^g}{d\bar{s}} + (\gamma-1)\phi\frac{d \ln P_E^{g*}}{d\bar{s}} = 0$$

$$(C.84) \quad (C.29): \frac{\gamma(1+\phi)}{1+s} + (\gamma-1)\frac{d \ln P_E^{g*}}{d\bar{s}} + (\gamma-1)\phi\frac{d \ln P_E^g}{d\bar{s}} = 0$$

$$(C.85) \quad (C.3): (1-\gamma)(1+\phi)\frac{d \ln P_E^g}{d\bar{s}} = \frac{d \ln n_E}{d\bar{s}} + \phi\frac{d \ln n_E^*}{d\bar{s}} - \frac{(1-\gamma)(1+\phi)}{1+s}$$

$$(C.86) \quad (C.4): (1-\gamma)(1+\phi)\frac{d \ln P_E^{g*}}{d\bar{s}} = \frac{d \ln n_E^*}{d\bar{s}} + \phi\frac{d \ln n_E}{d\bar{s}} - \frac{(1-\gamma)(1+\phi)}{1+s}$$

$$(C.87) \quad (C.11): \frac{d \ln P_E}{d\bar{s}} = (1-\iota)(1-\alpha)\beta I_E \frac{d \ln P_E^g}{d\bar{s}} + (1-\iota)(1-\alpha)(1-\beta)I_E \frac{d \ln P_E^{g*}}{d\bar{s}}$$

$$(C.88) \quad (C.12): \frac{d \ln P_E^*}{d\bar{s}} = (1-\iota)(1-\alpha)\beta I_E \frac{d \ln P_E^{g*}}{d\bar{s}} + (1-\iota)(1-\alpha)(1-\beta)I_E \frac{d \ln P_E^g}{d\bar{s}}$$

By solving the aforementioned six equations, I obtain the solutions presented in Table C.3.

TABLE C.3. Comparative Statics for Cooperative Change in  $s$  under  $s = s^*$

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$$(C.89) \quad \frac{d \ln n_E}{ds^c} = \frac{d \ln n_E^*}{ds^c} = \frac{1}{1+s} > 0$$

$$(C.90) \quad \frac{d \ln P_E^g}{ds} = \frac{d \ln P_E^{g*}}{ds} = -\frac{1}{1+s} \frac{\gamma}{\gamma-1} < 0$$

$$(C.91) \quad \frac{d \ln P_E}{ds} = \frac{d \ln P_E^*}{ds} = -\frac{1}{1+s} \frac{\gamma(1-\iota)(1-\alpha)I_E}{\gamma-1} < 0$$

$$(C.92) \quad \frac{d \ln Y_{i,E}^g}{ds} = \frac{d \ln Y_{i,E}^{g*}}{ds} = \frac{1}{1+s} \frac{\gamma(1-\alpha)}{\gamma-1} > 0$$

$$(C.93) \quad \frac{d \ln Y_{i,E}^c}{ds} = \frac{d \ln Y_{i,E}^{c*}}{ds} = 0$$


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## C.6. Country List Used for Innovation Timing Elasticity Estimation

I provide a list of countries used for estimating innovation timing elasticity in Table C.4. These countries account for 98.7% of the total value added in the UNIDO Industrial Statistics Database for the year 2020.

TABLE C.4. Country List Used for Innovation Timing Elasticity Estimation

Algeria	Argentina	Australia	Austria
Bangladesh	Belarus	Belgium	Brazil
Canada	Chile	China	Hong Kong
Taiwan	Colombia	Croatia	Czechia
Denmark	Egypt	Estonia	Finland
France	Germany	Ghana	Greece
Hungary	Iceland	India	Indonesia
Iran (Islamic Republic of)	Ireland	Israel	Italy
Japan	Kazakhstan	Lithuania	Luxembourg
Malaysia	Mexico	Morocco	Netherlands
New Zealand	Norway	Pakistan	Peru
Philippines	Poland	Portugal	Puerto Rico
Qatar	Republic of Korea	Romania	Russian Federation
Saudi Arabia	Singapore	Slovakia	Slovenia
South Africa	Spain	Sweden	Switzerland
Türkiye	Thailand	Ukraine	United Arab Emirates
United Kingdom	United States of America	Viet Nam	

## C.7. Innovation Indicators

I present innovation indicators for the U.S. manufacturing industries in Figure C.1. Data for other countries are available upon request.

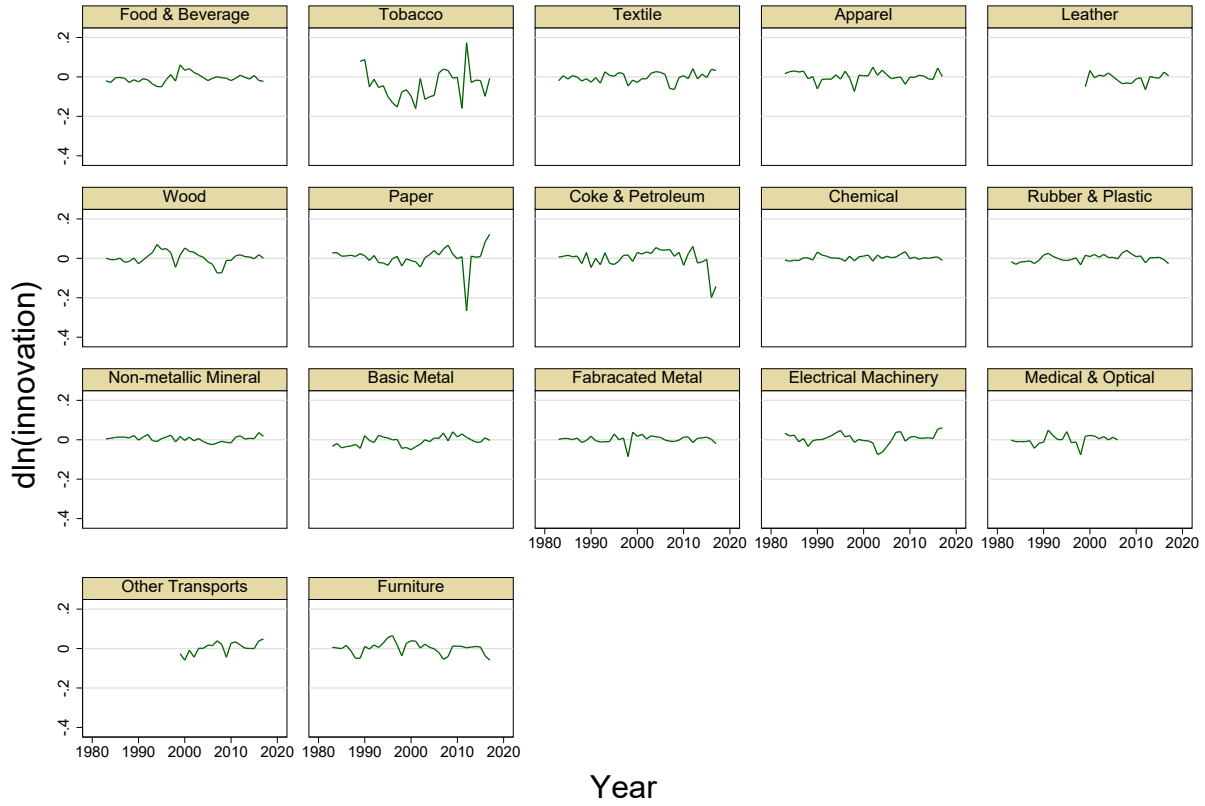


FIGURE C.1. Innovation Indicators for the US Manufacturing Industries

## C.8. Generalized Model

In the generalized model, I make four changes. First, I introduce a conventional capital goods sector in which firms produce capital goods that are used with conventional energy. Similar to the green capital goods sector, the production function of a firm producing a variety  $\omega^e$  in this sector takes the following form.

$$(C.94) \quad y_t^e(\omega^e) = a_t^e(\omega^e)l_t(\omega^e)$$



Conventional capital goods are aggregated using a CES aggregator, similar to the green capital goods sector, as follows.

$$(C.95) \quad Z_t^e = \left( \int_0^{n_t^e} z_t^e(\omega^e)^{\frac{\gamma-1}{\gamma}} d\omega^e + \int_0^{n_t^{e*}} z_t^e(\omega_f^e)^{\frac{\gamma-1}{\gamma}} d\omega_f^e \right)^{\frac{\gamma}{\gamma-1}}$$

$$(C.96) \quad Z_t^{e*} = \left( \int_0^{n_t^{e*}} z_t^{e*}(\omega_f^e)^{\frac{\gamma-1}{\gamma}} d\omega_f^e + \int_0^{n_t^e} z_t^{e*}(\omega^e)^{\frac{\gamma-1}{\gamma}} d\omega^e \right)^{\frac{\gamma}{\gamma-1}}$$

where  $\gamma$  represents the elasticity of substitution between varieties. I set  $\gamma$  to the same value as that in the green capital goods sector.

As in Hötte (2020), it can be assumed that if a final goods firm uses conventional capital, it should also use a natural resource as an additional input. Assuming that  $\psi$  units of labor are required to obtain a unit of natural resource,  $N_{i,t}$ , and given the production function  $Y_{i,t}^e = A_{i,t} L_{i,t}^e \alpha^\xi N_{i,t}^{\alpha(1-\xi)} Z_{i,t}^e 1^{-\alpha}$ , the function can be rewritten as  $Y_{i,t}^e = A_{i,t} L_{i,t}^e \alpha^\xi \left( \frac{L_{i,t}^e}{\psi} \right)^{\alpha(1-\xi)} Z_{i,t}^e 1^{-\alpha} = \frac{A_{i,t}}{\psi^{\alpha(1-\xi)}} L_{i,t}^e \alpha^\xi Z_{i,t}^e 1^{-\alpha}$ . Thus, the production function with conventional capital retains a similar form as that with green capital, as follows.

$$(C.97) \quad Y_{i,t}^e = A_{i,t} (L_{i,t}^e)^\alpha (Z_{i,t}^e)^{1-\alpha}$$

Second, I ensure that the green energy usage ratios,  $I_t$  and  $I_t^*$ , are endogenously determined at all stages. To account for this, I introduce the following productivity structure for green energy adoption:

$$(C.98) \quad \lambda_i = \bar{\lambda}(1-i)^\sigma$$

where  $\bar{\lambda}$  is a constant, and  $\sigma$  is the parameter governing the degree of increasing difficulty in green energy adoption as the industry index  $i$  increases.

Third, I exclude agricultural final goods so that the relative wage varies due to production subsidies or productivity increases. Accordingly, the balance of payment equilibrium condition

changes as follows:

$$\begin{aligned}
& P_{h,t}^M C_{h,t}^{M*} + (1 + \tau) p_t^g z_t^* (\omega^g) n_t^g + (1 + \tau) p_t^e z_t^* (\omega^e) n_t^e + D \\
\text{(C.99)} \quad & = e_t \left[ P_{f,t}^{M*} C_{f,t}^M + (1 + \tau) p_t^{g*} z_t (\omega_f^g) n_t^{g*} + (1 + \tau) p_t^{e*} z_t (\omega_f^e) n_t^{e*} \right]
\end{aligned}$$

where D represents the trade deficit of the home country. As in Bartelme et al. (2021), I assume the trade deficit does not change as we move to the counterfactual equilibrium.

Lastly, manufacturing final goods are aggregated using a CES function instead of a Cobb-Douglas function, as follows:

$$\text{(C.100)} \quad C_t^M = \left( C_{h,t}^M \frac{\eta-1}{\eta} + C_{f,t}^M \frac{\eta-1}{\eta} \right)^{\frac{\eta}{\eta-1}}$$

where  $\eta$  represents the elasticity of substitution between the home and foreign aggregated manufacturing final goods.

### C.9. Exact Hat Algebra for Counterfactual Analysis

The relevant equations for the welfare change caused by production subsidies or productivity changes are as follows: the zero-profit conditions for home and foreign green capital firms (equations (C.28) and (C.29)); the zero-profit conditions for home and foreign conventional capital firms; the aggregated green capital goods prices in the home and foreign countries (equations (C.3) and (C.4)); the aggregated conventional capital goods prices in the home and foreign countries (equations (C.95) and (C.96)); tax in the home and foreign country (equations (C.35) and (C.36)); the utility-based Consumer Price Index (CPI) in the home and foreign countries (equations (C.11) and (C.12)); the equations for green energy usage determination in both the home and foreign countries; and the welfare function for both the home and foreign countries. Accordingly, the following equations are used for counterfactual changes in the early stage.

$$\text{(C.101)} \quad (1 + s)^\gamma \left[ (\hat{P}^g)^{\gamma-1} (\hat{P}^M)^{1-\eta} (\hat{P})^{\eta-1} \hat{I} s_{hh}^{zp} + \hat{e}^\gamma (\hat{P}^g)^{\gamma-1} (\hat{P}^{\hat{M}^*})^{1-\eta} (\hat{P}^*)^{\eta-1} \hat{I}^* (1 - s_{hh}^{zp}) \right] = 1$$

(C.102)

$$(\hat{P}^e)^{\gamma-1} (\hat{P}^M)^{\eta-1} (\hat{P})^{\eta-1} \left( \frac{1 - \hat{I} I_0}{1 - I_0} \right) s_{hh}^{zp} + \hat{e}^\gamma (\hat{P}^e)^{\gamma-1} (\hat{P}^{\hat{M}^*})^{\gamma-1} (\hat{P}^*)^{\gamma-1} \left( \frac{1 - \hat{I}^* I_0^*}{1 - I_0^*} \right) (1 - s_{hh}^{zp}) = 1$$

(C.103)

$$(1 + s^*)^\gamma \left[ (\hat{P}g^*)^{\gamma-1} (\hat{P}\hat{M}^*)^{1-\eta} (\hat{P}^*)^{\eta-1} \hat{I}^* s_{ff}^{zp} + \hat{e}^{-\gamma} (\hat{P}g^*)^{\gamma-1} (\hat{P}\hat{M})^{1-\eta} (\hat{P})^{\eta-1} \hat{I} (1 - s_{ff}^{zp}) \right] = 1$$

(C.104)

$$(\hat{P}e^*)^{\gamma-1} (\hat{P}\hat{M}^*)^{1-\eta} (\hat{P}^*)^{\eta-1} \left( \frac{1 - \hat{I}^* I_0^*}{1 - I_0^*} \right) s_{ff}^{zp} + \hat{e}^{-\gamma} (\hat{P}e^*)^{\gamma-1} (\hat{P}\hat{M})^{1-\eta} (\hat{P})^{\eta-1} \left( \frac{1 - \hat{I} I_0}{1 - I_0} \right) (1 - s_{ff}^{zp}) = 1$$

(C.105)

$$(\hat{P}g)^{1-\gamma} = (1 + s)^{\gamma-1} \hat{n}g s_{hh}^{zc} + (1 + s^*)^{\gamma-1} \hat{e}^{1-\gamma} \hat{n}g^* (1 - s_{hh}^{zc})$$

(C.106)

$$(\hat{P}e)^{1-\gamma} = \hat{n}e s_{hh}^{zc} + \hat{e}^{1-\gamma} \hat{n}e^* (1 - s_{hh}^{zc})$$

(C.107)

$$(\hat{P}g^*)^{1-\gamma} = (1 + s^*)^{\gamma-1} \hat{n}g^* s_{ff}^{zc} + (1 + s)^{\gamma-1} \hat{e}^{\gamma-1} \hat{n}g (1 - s_{ff}^{zc})$$

(C.108)

$$(\hat{P}e^*)^{1-\gamma} = \hat{n}e^* s_{ff}^{zc} + \hat{e}^{\gamma-1} \hat{n}e (1 - s_{ff}^{zc})$$

(C.109)

$$\hat{C} = \frac{(1 - I_0 \hat{I}) \left[ 1 - \left( \frac{1 - I_0 \hat{I}}{1 - I_0} \right)^{\sigma-1} \right]}{(\sigma - 1) I_0 (\hat{I} - 1)} \hat{P}e$$

(C.110)

$$\hat{C}^* = \frac{(1 - I_0^* \hat{I}^*) \left[ 1 - \left( \frac{1 - I_0^* \hat{I}^*}{1 - I_0^*} \right)^{\sigma-1} \right]}{(\sigma - 1) I_0^* (\hat{I}^* - 1)} \hat{P}e^*$$

(C.111)

$$\hat{P}_h^M = (\hat{P}g)^{(1-\alpha)I_0} (\hat{C})^{(1-\alpha)(\hat{I}-1)I_0} (\hat{P}e)^{(1-\alpha)(1-I_0\hat{I})}$$

(C.112)

$$\hat{P}_f^{\hat{M}^*} = (\hat{P}g^*)^{(1-\alpha)I_0^*} (\hat{C}^*)^{(1-\alpha)(\hat{I}^*-1)I_0^*} (\hat{P}e^*)^{(1-\alpha)(1-I_0^*\hat{I}^*)}$$

(C.113)

$$\hat{P}^{1-\eta} = \hat{P}_h^{\hat{M}^{1-\eta}} s_{hh}^{fc} + \hat{e}^{1-\eta} \hat{P}_f^{\hat{M}^*^{1-\eta}} (1 - s_{hh}^{fc})$$

(C.114)

$$\hat{P}^{*1-\eta} = \hat{P}_f^{\hat{M}^*^{1-\eta}} s_{ff}^{fc} + \hat{e}^{\eta-1} \hat{P}_h^{\hat{M}^{1-\eta}} (1 - s_{ff}^{fc})$$

(C.115)

$$\hat{P}g = \left( \frac{1 - \hat{I} I_0}{1 - I_0} \right)^\sigma \hat{P}e$$

(C.116)

$$\hat{P}g^* = \left( \frac{1 - \hat{I}^* I_0^*}{1 - I_0^*} \right)^\sigma \hat{P}e^*$$

$$\begin{aligned} & \hat{e}^\gamma (\hat{P}g)^{\gamma-1} (\hat{P}_f^{\hat{M}^*})^{1-\eta} (\hat{P}^*)^{\eta-1} \hat{I}^* s_h^{ge} + \hat{e}^\gamma (\hat{P}e)^{\gamma-1} (\hat{P}_f^{\hat{M}^*})^{1-\eta} (\hat{P}^*)^{\eta-1} \left( \frac{1 - \hat{I}^* I_0^*}{1 - I_0^*} \right) s_h^{ee} + \\ & + \hat{e}^\gamma (\hat{P}_h^{\hat{M}})^{1-\eta} (\hat{P}^*)^{\eta-1} s_h^{fe} + (1 - s_h^{ge} - s_h^{ee} - s_h^{fe}) \\ & = \hat{e}^{1-\gamma} (\hat{P}g^*)^{\gamma-1} (\hat{P}_h^{\hat{M}})^{1-\eta} (\hat{P})^{\eta-1} \hat{I} s_f^{ge} + \hat{e}^{1-\gamma} (\hat{P}e^*)^{\gamma-1} (\hat{P}_h^{\hat{M}})^{1-\eta} (\hat{P})^{\eta-1} \left( \frac{1 - \hat{I} I_0}{1 - I_0} \right) s_f^{ee} \end{aligned}$$

$$(C.117) \quad +\hat{e}^{1-\eta}(P_f^{\hat{M}^*})^{1-\eta}(\hat{P})^{\eta-1}(1-s_f^{ge}-s_f^{ee})$$

$$(C.118) \quad T_E = \frac{s}{1+s}(1-\alpha)I_0\hat{n}$$

$$(C.119) \quad T_E^* = \frac{s^*}{1+s^*}(1-\alpha)I_0^*\hat{n}^*$$

$$(C.120) \quad \hat{U}_E = \hat{P}^{-1}e^{-T_E}N^{\frac{2-\hat{I}_0-\hat{I}^*I_0^*}{2-\hat{I}_0-\hat{I}_0^*}-1}$$

$$(C.121) \quad \hat{U}_E^* = \hat{P}^{*-1}e^{-T_E^*}N^{\frac{2-\hat{I}_0-\hat{I}^*I_0^*}{2-\hat{I}_0-\hat{I}_0^*}-1}$$

where  $\hat{C}$  and  $\hat{C}^*$  represent the average change in price in sectors that use conventional capital in the initial equilibrium but change their production method because using green capital became cheaper, in the home and foreign country, respectively. In addition,  $s_h^{ge}$ ,  $s_h^{ee}$  and  $s_h^{fe}$  denote the share of export value from the green capital goods, the conventional capital goods, and the manufacturing final goods out of total export value in the home country, respectively. Accordingly,  $1-s_h^{ge}-s_h^{ee}-s_h^{fe}$  represents the share of the trade deficit out of the total export value in the home country.

For the assessment of welfare changes relative to the initial welfare level after innovation in the green capital goods sector of either the home or foreign country, four equations are modified. Specifically, equations (C.101), (C.103), (C.105), and (C.107) are changed as follows:

$$(C.122) \quad a_g^{\gamma-1} \left[ (\hat{P}g)^{\gamma-1}(\hat{P}M)^{1-\eta}(\hat{P})^{\eta-1}\hat{I}s_{hh}^{zp} + \hat{e}^\gamma(\hat{P}g)^{\gamma-1}(P^{\hat{M}^*})^{1-\eta}(\hat{P}^*)^{\eta-1}\hat{I}^*(1-s_{hh}^{zp}) \right] = 1$$

$$(C.123) \quad a_g^{*\gamma-1} \left[ (\hat{P}g^*)^{\gamma-1}(P^{\hat{M}^*})^{1-\eta}(\hat{P}^*)^{\eta-1}\hat{I}^*s_{ff}^{zp} + \hat{e}^{-\gamma}(\hat{P}g^*)^{\gamma-1}(P^{\hat{M}})^{1-\eta}(\hat{P})^{\eta-1}\hat{I}(1-s_{ff}^{zp}) \right] = 1$$

$$(C.124) \quad (\hat{P}g)^{1-\gamma} = a_g^{\gamma-1}\hat{n}\hat{g}^*s_{ff}^{zc} + a_g^{*\gamma-1}\hat{e}^{1-\gamma}\hat{n}\hat{g}(1-s_{ff}^{zc})$$

$$(C.125) \quad (P^{\hat{g}^*})^{1-\gamma} = a_g^{*\gamma-1}\hat{n}\hat{g}^*s_{ff}^{zc} + a_g^{\gamma-1}\hat{e}^{\gamma-1}\hat{n}\hat{g}(1-s_{ff}^{zc})$$

Additionally, since the government no longer provides a production subsidy at these stages, equations (C.118) and (C.119) are not used.

Related to welfare changes after innovation, three cases can be defined. The first case is where the home green capital goods sector has innovated, but the foreign green capital goods sector has not yet ( $\hat{U}_{L1}$  for the home country and  $\hat{U}_{L1}^*$  for the foreign country). The second case is where the foreign green capital goods sector has innovated, but the home green capital goods sector has not yet ( $\hat{U}_{L2}$  for the home country and  $\hat{U}_{L2}^*$  for the foreign country). The third case is where both the

home and foreign green capital goods sector have innovated ( $\hat{U}_M$  for the home country and  $\hat{U}_M^*$  for the foreign country).

Lastly, we can compute the overall welfare effect using the four effects mentioned above for the home country ( $\hat{U}_E, \hat{U}_{L1}, \hat{U}_{L2}, \hat{U}_M$ ) and the foreign country ( $\hat{U}_E^*, \hat{U}_{L1}^*, \hat{U}_{L2}^*, \hat{U}_M^*$ ), respectively. If it is assumed that the US is the home country and the EU is the foreign country, there are 10 cases based on the initial condition in Table 3.3, as follows:

$$\begin{aligned} & \textcircled{1} \quad t_h^r < t_{f0}^r, \quad t_{f0}^r < t_f^r, \quad t_f^r < t_{h0}^r : \\ \text{(C.126)} \quad \hat{W} &= \hat{U}_E^{\frac{1}{\rho}} (1 - e^{-\rho t_h^r}) \hat{U}_{L1}^{\frac{1}{\rho}} (e^{-\rho t_h^r} - e^{-\rho t_{f0}^r}) \left( \frac{\hat{U}_{L1}}{\hat{U}_{L2}} \right)^{\frac{1}{\rho}} (e^{-\rho t_{f0}^r} - e^{-\rho t_f^r}) \left( \frac{\hat{U}_M}{\hat{U}_{L2}} \right)^{\frac{1}{\rho}} (e^{-\rho t_f^r} - e^{-\rho t_{h0}^r}) \end{aligned}$$

$$\begin{aligned} & \textcircled{2} \quad t_h^r < t_{f0}^r, \quad t_{f0}^r < t_{h0}^r, \quad t_{h0}^r < t_f^r : \\ \text{(C.127)} \quad \hat{W} &= \hat{U}_E^{\frac{1}{\rho}} (1 - e^{-\rho t_h^r}) \hat{U}_{L1}^{\frac{1}{\rho}} (e^{-\rho t_h^r} - e^{-\rho t_{f0}^r}) \left( \frac{\hat{U}_{L1}}{\hat{U}_{L2}} \right)^{\frac{1}{\rho}} (e^{-\rho t_{f0}^r} - e^{-\rho t_{h0}^r}) \left( \frac{\hat{U}_{L1}}{\hat{U}_M} \right)^{\frac{1}{\rho}} (e^{-\rho t_{h0}^r} - e^{-\rho t_f^r}) \end{aligned}$$

$$\begin{aligned} & \textcircled{3} \quad t_h^r < t_f^r, \quad t_f^r < t_{f0}^r, \quad t_{f0}^r < t_{h0}^r : \\ \text{(C.128)} \quad \hat{W} &= \hat{U}_E^{\frac{1}{\rho}} (1 - e^{-\rho t_h^r}) \hat{U}_{L1}^{\frac{1}{\rho}} (e^{-\rho t_h^r} - e^{-\rho t_f^r}) \hat{U}_M^{\frac{1}{\rho}} (e^{-\rho t_f^r} - e^{-\rho t_{f0}^r}) \left( \frac{\hat{U}_M}{\hat{U}_{L2}} \right)^{\frac{1}{\rho}} (e^{-\rho t_{f0}^r} - e^{-\rho t_{h0}^r}) \end{aligned}$$

$$\begin{aligned} & \textcircled{4} \quad t_f^r < t_h^r, \quad t_h^r < t_{f0}^r, \quad t_{f0}^r < t_{h0}^r : \\ \text{(C.129)} \quad \hat{W} &= \hat{U}_E^{\frac{1}{\rho}} (1 - e^{-\rho t_f^r}) \hat{U}_{L2}^{\frac{1}{\rho}} (e^{-\rho t_f^r} - e^{-\rho t_h^r}) \hat{U}_M^{\frac{1}{\rho}} (e^{-\rho t_h^r} - e^{-\rho t_{f0}^r}) \left( \frac{\hat{U}_M}{\hat{U}_{L2}} \right)^{\frac{1}{\rho}} (e^{-\rho t_{f0}^r} - e^{-\rho t_{h0}^r}) \end{aligned}$$

$$\begin{aligned} & \textcircled{5} \quad t_{f0}^r < t_f^r, \quad t_f^r < t_h^r, \quad t_h^r < t_{h0}^r : \\ \text{(C.130)} \quad \hat{W} &= \hat{U}_E^{\frac{1}{\rho}} (1 - e^{-\rho t_{f0}^r}) \left( \frac{\hat{U}_E}{\hat{U}_{L2}} \right)^{\frac{1}{\rho}} (e^{-\rho t_{f0}^r} - e^{-\rho t_f^r}) \left( \frac{\hat{U}_M}{\hat{U}_{L2}} \right)^{\frac{1}{\rho}} (e^{-\rho t_h^r} - e^{-\rho t_{h0}^r}) \end{aligned}$$

$$\textcircled{6} \quad t_{f0}^r < t_h^r, \quad t_h^r < t_f^r, \quad t_f^r < t_{h0}^r :$$

$$(C.131) \quad \hat{W} = \hat{U}_E^{\frac{1}{\rho}} (1 - e^{-\rho t_{f0}^r}) \left( \frac{\hat{U}_E}{\hat{U}_{L2}} \right)^{\frac{1}{\rho} (e^{-\rho t_{f0}^r} - e^{-\rho t_h^r})} \left( \frac{\hat{U}_{L1}}{\hat{U}_{L2}} \right)^{\frac{1}{\rho} (e^{-\rho t_h^r} - e^{-\rho t_f^r})} \left( \frac{\hat{U}_M}{\hat{U}_{L2}} \right)^{\frac{1}{\rho} (e^{-\rho t_f^r} - e^{-\rho t_{h0}^r})}$$

$$(C.132)$$

$$\textcircled{7} \quad t_{f0}^r < t_h^r, \quad t_h^r < t_{h0}^r, \quad t_{h0}^r < t_f^r :$$

$$\hat{W} = \hat{U}_E^{\frac{1}{\rho}} (1 - e^{-\rho t_{f0}^r}) \left( \frac{\hat{U}_E}{\hat{U}_{L2}} \right)^{\frac{1}{\rho} (e^{-\rho t_{f0}^r} - e^{-\rho t_h^r})} \left( \frac{\hat{U}_{L1}}{\hat{U}_{L2}} \right)^{\frac{1}{\rho} (e^{-\rho t_h^r} - e^{-\rho t_{h0}^r})} \left( \frac{\hat{U}_{L1}}{\hat{U}_M} \right)^{\frac{1}{\rho} (e^{-\rho t_{h0}^r} - e^{-\rho t_f^r})}$$

$$\textcircled{8} \quad t_f^r < t_{f0}^r, \quad t_{f0}^r < t_h^r, \quad t_h^r < t_{h0}^r :$$

$$(C.133) \quad \hat{W} = \hat{U}_E^{\frac{1}{\rho}} (1 - e^{-\rho t_f^r}) \hat{U}_{L2}^{\frac{1}{\rho} (e^{-\rho t_f^r} - e^{-\rho t_{f0}^r})} \left( \frac{\hat{U}_M}{\hat{U}_{L2}} \right)^{\frac{1}{\rho} (e^{-\rho t_h^r} - e^{-\rho t_{h0}^r})}$$

$$\textcircled{9} \quad t_f^r < t_{f0}^r, \quad t_{f0}^r < t_{h0}^r, \quad t_{h0}^r < t_h^r :$$

$$(C.134) \quad \hat{W} = \hat{U}_E^{\frac{1}{\rho}} (1 - e^{-\rho t_f^r}) \hat{U}_{L2}^{\frac{1}{\rho} (e^{-\rho t_f^r} - e^{-\rho t_{f0}^r})} \left( \frac{\hat{U}_{L2}}{\hat{U}_M} \right)^{\frac{1}{\rho} (e^{-\rho t_{h0}^r} - e^{-\rho t_h^r})}$$

$$(C.135) \quad \textcircled{10} \quad t_f^r = t_{f0}^r, \quad t_h^r = t_{h0}^r : \quad \hat{W} = \hat{U}_E^{\frac{1}{\rho}} (1 - e^{-\rho t_{f0}^r})$$

For the overall foreign welfare effect, the same equations hold with a star for every variable.

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