Title
Optimal Power Allocation in Distributed Sensing (SEN 3)

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Optimal Power Allocation in Distributed Sensing

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Problem Description: Minimize BLUE MSE subject to a total network power constraint

Introduction to Parameter Estimation

In this work, we consider the problem of optimal power allocation for parameter estimation and detection in a distributed sensor network setting. For the simple star topology, an analysis of the effect of the measurement noise variance on the optimal power allocation policy is presented. Relaying nodes are introduced to form more complicated branch, tree and linear topologies (depicted in Figure 1). Analytical solutions for these cases for both amplify-and-forward (AF) and estimate-and-forward (EF) transmission protocols are intractable, and thus asymptotically optimal (for increasing measurement noise variance) solutions are derived.

- Simple signal model for the star topology
  The received signal at the fusion center from the $i^{th}$ sensing node is:
  $y_i = \sqrt{P_i h_i (\theta + z_i)} + n_i$
  - $\theta$ is the deterministic scalar parameter to be estimated
  - $z_i$ and $n_i$ are zero-mean and unknown PDF noise terms, independent
  - $h_i$ is non-random channel attenuation factor known at fusion center (FC)
  - $P_i$ is power gain factor, $0 \leq P_i \leq 0$, $\forall i$
- The best linear unbiased estimator (BLUE) is optimal
  Since the measurement and channel noise terms are only defined using second-order statistics, the best linear unbiased estimate is the optimal linear estimator.
- Constrained optimization problem is considered
  The generic optimization problem, for any topology, is
  minimize MSE subject to $\sum_{i=1}^{N} P_i = P_T$

Proposed Solution: Optimal solution evolves from sensor selection to power equalization

Analysis of the Star Topology

- The MSE for the star topology, and optimal solution
  \[ \text{MSE} = \left( \sum_{i=1}^{N} \frac{1}{\sigma^2_i + 1/r_i} \right)^{-1} \]
  \[ P_i^* = \frac{1}{r_i \sigma^2_i} \left( \sqrt{r_i} \nu^* - 1 \right)^+ \]
  where $r_i = |h_i|^2/\sigma^2_i$ is the channel SNR for the $i^{th}$ node.
  The solution is obtained using Lagrangian optimization and KKT conditions, and $\nu^*$ is the optimal Lagrange multiplier for the equality constraint

- Optimal power allocation strategy evolves from a waterfilling solution to power equalization as measurement noise increases
  The evolution of the optimal solution is shown as a function of the measurement noise variance in Figure 2. The no measurement noise case is an example of extreme waterfilling; only the sensor with the strongest SNR is active, and sensor selection is optimal for $\sigma^2 = 0$.

Asymptotically optimal solutions

- Solutions for branch, tree and linear networks are intractable
  - Use solution techniques and results from star topology to develop the asymptotically optimal (for increasing measurement noise variance) solutions to these more complex topologies.
- For low measurement noise in tree topologies, branch selection is optimal; and in the case of linear network, sensors further away from the fusion center remain inactive.
- As the measurement noise increases, all sensors become active. Power equalization is optimal for the leaves of a branch topology; for a linear network, weighted power equalization is optimal.

Extending optimizations to complex topologies

- Introduce relay transmission protocols
  Consider the simple two-hop linear network shown in Figure 1.
  - Using amplify-and-forward (AF), the FC receives:
    \[ y_{FC,AF} = \sqrt{P_0 h_0 (\sqrt{P_1 h_1 (\theta + z_1)} + n_1)} + n_0 \]
  - For the estimate-and-forward (EF) protocol, the signal model is:
    \[ y_{FC,EF} = \sqrt{P_0 h_0 (\theta + w)} + n_0 \]
  where $\hat{\theta}_R = \theta + w$ is the BLUE estimate formed at the relay.

Asymptotic solution techniques

- For low measurement noise in tree topologies, branch selection is optimal; and in the case of linear network, sensors further away from the fusion center remain inactive.
- As the measurement noise increases, all sensors become active. Power equalization is optimal for the leaves of a branch topology; for a linear network, weighted power equalization is optimal.

Topology comparisons

- Figure 4 illustrates that more branching with shorter hops is preferred to linear topologies which use longer direct hops.