Lawrence Berkeley National Laboratory

Recent Work

Title

BINDING ENERGIES OF LIGHT HYPERFRAGMENTS

Permalink

<https://escholarship.org/uc/item/889303zr>

Authors

Dietrich, K.
Mang, H.J.

Publication Date

1963-07-01

University of California

Ernest O. Lawrence Radiation Laboratory

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545*

BINDING ENERGIES OF LIGHT HYPERFRAGMENTS

Berkeley, California

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

BINDING ENERGIES OF LIGHT HYPERFRAGMENTSK. DIETRICH[†], H. J. MANG*Lawrence Radiation Laboratory, University of California, Berkeley^{††}*
and

R. FOLK

Lehigh University, Bethlehem, Pennsylvania^{††}

Received 1 July 1963

Abstract: The binding energies of ${}_A\text{H}^3$, ${}_A\text{He}^4$ and ${}_A\text{H}^4$, and ${}_A\text{He}^6$ are calculated using an independent pair approximation assuming the interactions are described by spin-dependent nucleon-nucleon and A -nucleon potentials with square-well shapes outside of hard cores. The calculated binding energies are presented as functions of the depths of the A -nucleon potentials for two possible ranges of these potentials. The experimental binding energies then determine the actual potential depths. The case that the singlet potential is stronger than the triplet potential is favoured for both ranges treated. This result leads to a prediction of zero spin for ${}_A\text{He}^4$ and ${}_A\text{H}^4$ and no bound excited states for these four-particle hyperfragments. The two-particle A -nucleon system also turns out to be unbound.

1. Introduction

Several authors ^{1, 2, 12)} have discussed the determination of a two-particle potential which describes the A -nucleon interaction ^{††} at low energies from the binding energies of the light hyperfragments ⁴⁾ (see table 1). We present another such analysis using a method that allows us to handle hard-core potentials and to include all two-body correlations. A potential shape and range are assumed and the potential depth is adjusted to match the experimental binding energies. Since there are very few experimental data ⁵⁾ for the two-body problem, we are forced to examine these more complicated systems of the A particle combined with two or more nucleons.

The three-nucleon problem is an example where the results of such an analysis are sensitive to the chosen form of the potential. So far, results of analyses using simple central potentials can be made to agree ⁶⁻⁸⁾ with both two- and three-nucleon low energy data only if the potential has a hard core. The potentials which are able to explain all two-nucleon data ⁹⁾ do have hard cores. However, these are not simple central potentials, and therefore the central potentials used here are effective low ener-

[†] On leave from the University of Frankfurt/M, Germany.

^{††} Work started while the authors were at the Institute for Theoretical Physics, Heidelberg, Germany.

^{†††} An extensive survey of the problems on hypernuclei and the hyperon-nucleon interaction is given by R. H. Dalitz ³⁾.

gy potentials. It seems clear [†] that any analysis that attempts to determine an effective low energy potential from binding energy data should assume a potential shape with a hard core.

There are only limited two-body data for the A -N interaction, and we do not therefore have the same direct argument in favour of a hard core in this case. Starting with an assumption of global symmetry at the elementary particle level, several authors have argued ¹¹⁾ that the A -N potential has a hard core if the N-N potential has one.

TABLE 1
The experimental binding energies

Species	B_A (MeV) ^{a)}	B_A (MeV) ^{b)}	B_A (MeV) ^{c)}
${}_A\text{H}^3$	0.21 ± 0.20		0.21 ± 0.20
${}_A\text{H}^4$	2.11 ± 0.10	2.35 ± 0.30	2.26 ± 0.20
${}_A\text{He}^4$	2.40 ± 0.11	2.25 ± 0.27	
${}_A\text{H}^5$	3.10 ± 0.07	3.25 ± 0.18	3.10 ± 0.07

^{a)} Crayton *et al.* ⁴⁾.

^{b)} Bhowmik *et al.* ⁴⁾.

^{c)} Values used to obtain results given in tables 5 and 6.

Thus Lichtenberg ²⁾ contends that any hyperfragment calculation should use only hard-core potentials. He carries out a variational calculation for ${}_A\text{H}^3$ using exponential potentials with hard cores. For computational convenience he chooses the range of the A -N potential much larger than predicted by meson field theory and also chooses a trial wave function which does not allow for asymmetries of the A and nucleon motions. Thus, we expect a more complicated variational calculation ¹²⁾ to show appreciable changes in the potential depth determined.

In their extensive variational calculations of ${}_A\text{H}^3$, ${}_A\text{H}^4$ and ${}_A\text{He}^4$, and ${}_A\text{He}^5$, Dalitz and Downs ¹⁾ use central potentials without hard cores. Their trial wave function for the three-body case ${}_A\text{H}^3$ is sufficiently flexible to reproduce the short range two-body correlations as well as the proper asymptotic behaviour. For the four- and five-body hyperfragments, however, they are forced to simplify the trial wave function to a product of a core function for the nucleons times a single particle function for the A particle. This is certainly not valid if the potential has a hard core.

Any adequate variational calculation with hard-core potentials of the four- and five-body problems is so laborious that the present authors decided to use instead a method first introduced by Brenig ¹³⁾ and applied very successfully by Mang and Wild ⁸⁾ to the three- and four-nucleon problems. This approximation scheme treats the interaction of each pair of particles in some average background field due to the other par-

[†] Recently ¹¹⁾, velocity dependent nucleon-nucleon potentials without hard cores have been proposed.

ticles, which assumes that three-and-more body correlations are not important. This is, of course, not the same as another assumption that is made, that three-body forces [†] are not important.

In this paper, it is assumed that the two-body interactions are adequately described by central potentials with a square-well shape outside of a hard core and a mean spin dependence. Only the light hyperfragments ${}_A\text{H}^3$, ${}_A\text{H}^4$ and ${}_A\text{He}^4$, and ${}_A\text{He}^5$ are treated; the heavier hyperfragments are more difficult to analyse because the additional nucleons are not in the S shell.

In the next section, expressions are obtained for the binding energies of these light hyperfragments in terms of one- and two-particle wave functions. With the assumption that three-and-more particle correlations are not important, the Schroedinger equation for the system is reduced to a set of coupled equations for these one and two particle wave functions. These equations are then solved in sect. 3 by a self-consistent perturbation technique and the wave functions are used to evaluate the energy. In sect. 4 we discuss the results and examine the accuracy of this method.

2. The One- and Two-Particle Equations

In this section the ground state energy of a hyperfragment is expressed in terms of the one and two body operators of the Hamiltonian acting on one and two body wave functions rather than on the total wave function and a set of coupled equations for these wave functions is established. Before we start this, let us state the problem precisely by giving the Hamiltonian for the system.

2.1. ASSUMPTION OF SYMMETRY AND THE SPIN-AVERAGED HAMILTONIAN

If all two-particle potentials were identical, spherically symmetric, attractive potentials, the lowest energy state would be the state of greatest space symmetry allowed by the Pauli principle since then the particles are more likely to be closer together in order to minimize the potential energy. For the light hyperfragments, even though the singlet and triplet N-N and Λ -N forces are not the same (see subsect. 3.4), we assume that the ground state is symmetric in the space interchange of the nucleons. A discussion of this approximation has been given ¹⁵⁾ for the three and four nucleon systems. If this assumption were relaxed, the numerical work involved would be increased a great deal, but it would still be a workable problem for present computers.

The ground state of the three-body hyperfragment has ¹⁾ a total isobaric spin zero, which means for our assumption of space symmetry that the neutron and proton spins are coupled to give a spin 1 state. This spin is in turn coupled with the spin $\frac{1}{2}$ state of the lambda particle so that the total spin I is $\frac{1}{2}$ or $\frac{3}{2}$ depending on the relative strengths of the singlet and triplet Λ -nucleon potentials. A similar argument leads to $I = 0$ or 1

[†] There is still disagreement about the relative strengths of the Λ -N interaction and the interaction of a Λ particle with a pair of nucleons as determined by field theory. The effect of possible three-body forces on the binding energies of hypernuclei is discussed by R. Dalitz and by A. R. Bodner and S. Sampathar ¹⁴⁾.

for the ground state of the four-body hyperfragment, with the spins of the two identical nucleons coupled to the antisymmetric spin zero state. Finally, for ${}_A\text{He}^5$ the spins of the two pairs of identical nucleons are each coupled to give the spin zero state, and thus $I = \frac{1}{2}$ for this case.

The A -nucleon potential is assumed to be of the form

$$V_{NA} = (V_s P_s + V_t P_t) v_{NA}(r_{NA}),$$

where V_s and V_t are constants measuring the strengths of the singlet and triplet interaction and P_s and P_t are the usual spin projection operators. A similar expression is assumed for the nucleon-nucleon interaction. As discussed above, the spin states of the hyperfragments are determined if we assume that either $V_s > V_t$ or $V_t < V_s$. This analysis is carried out for both these assumptions in the hope that the results will determine which is correct.

With the spin state determined or assumed as discussed above, the spin projection operations of the two-body potentials can be evaluated. In this way the problem reduces to a Schrodinger equation with a spin-averaged A -nucleon potential which is given in table 2 and with a spin averaged nucleon-nucleon potential which is just the triplet potential for ${}_A\text{H}^3$ and is one-half the sum of the triplet and singlet potentials for the other hyperfragments considered here. The nucleons can now be treated as distinguishable; i.e., neutron or proton with spin up or down.

TABLE 2
The spin-averaged A -nucleon potentials

$V_s > V_t$	${}_A\text{H}^3$	$I = \frac{1}{2}$	$V_{NA}^3 = \frac{1}{4}V_t + \frac{3}{4}V_s$
	${}_A\text{H}^4, {}_A\text{He}^4$	0	$V_{NA}^4 = \frac{1}{2}V_t + \frac{1}{2}V_s$
	${}_A\text{He}^5$	$\frac{1}{2}$	$V_{NA}^5 = \frac{3}{4}V_t + \frac{1}{4}V_s$
$V_t > V_s$	${}_A\text{H}^3$	$\frac{3}{2}$	$V_{NA}^3 = V_t$
	${}_A\text{H}^4, {}_A\text{He}^4$	1	$V_{NA}^4 = \frac{5}{6}V_t + \frac{1}{6}V_s$
	${}_A\text{He}^5$	$\frac{1}{2}$	$V_{NA}^5 = \frac{3}{4}V_t + \frac{1}{4}V_s$

In order to formulate the problem so that it describes a hyperfragment with 2,3 or 4 nucleons, we would need a cumbersome notation. Instead let us consider the four-particle hyperfragments, ${}_A\text{H}^4$ and ${}_A\text{He}^4$, as an example, and in the cases where the change to the ${}_A\text{H}^3$ and ${}_A\text{He}^5$ problems is not obvious, the corresponding relations are given in the appendix. Coulomb forces are ignored.

Thus, we consider

$$\begin{aligned}
 E\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) &= H\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \\
 &= \left[\sum_{i=1}^3 T_N(i) + T_A(4) + \sum_{i>j=1}^3 v_{NN}(r_{ij}) + \sum_{i=1}^3 v_{NA}(r_{i4}) + \frac{1}{2}M\omega_s^2 \left(\frac{\sum_{i=1}^4 m_i \mathbf{r}_i}{M} \right)^2 \right] \\
 &\quad \times \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4), \quad (1)
 \end{aligned}$$

where

$$\begin{aligned}
 T_N(i) &= -\frac{\hbar^2}{2m_N} \nabla_{\mathbf{r}_i}^2, \quad T_A(4) = -\frac{\hbar^2}{2m_A} \nabla_{\mathbf{r}_4}^2, \\
 r_{ij} &= |\mathbf{r}_i - \mathbf{r}_j|,
 \end{aligned}$$

and where v_{NN} and v_{NA} are the spin-averages of the assumed nucleon-nucleon and A -nucleon potentials, respectively. Clearly, we have labelled the lambda particle as particle 4, the mass of particle i as m_i , and the total mass of the hyperfragment as M . We emphasize again that Ψ is the ground state.

The last term in the Hamiltonian is an artificial centre-of-mass potential^{8, 16)} which has no effect on the internal motion of the hyperfragment, but it enables us to use shell model wave functions as a first approximation to the asymptotic behaviour of Ψ . We must remember, however, to subtract the ground state energy of this centre-of-mass oscillator $\frac{3}{2} \hbar\omega_s$ from the total energy of the system when we calculate the internal energy.

The Hamiltonian for the internal motion alone is

$$H_{\text{int}} = H - H_s,$$

where H_s is the Hamiltonian for the centre-of-mass motion,

$$H_s = -\frac{\hbar^2}{2M} \left[\sum_{i=1}^4 \nabla_{\mathbf{r}_i} \right]^2 + \frac{1}{2}M\omega_s^2 \left(\frac{\sum_{i=1}^4 m_i \mathbf{r}_i}{M} \right)^2.$$

Thus, for the internal motion we have

$$\begin{aligned}
 E_{\text{int}} \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) &= \left[\sum_{i=1}^3 T_N(i) + T_A(4) + \sum_{i>j=1}^3 v_{NN}(r_{ij}) + \sum_{i=1}^3 v_{NA}(r_{i4}) \right. \\
 &\quad \left. + \frac{\hbar^2}{2M} \left(\sum_{i=1}^4 \nabla_{\mathbf{r}_i} \right)^2 \right] \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4). \quad (2)
 \end{aligned}$$

It is convenient to regroup the terms of eq. (1). Using the identity

$$\left(\sum_{i=1}^4 m_i \mathbf{r}_i \right)^2 = M \sum_{i=1}^4 m_i r_i^2 - \sum_{i>j=1}^4 m_i m_j r_{ij}^2,$$

eq. (1) can be written

$$E\Psi(r_1, r_2, r_3, r_4) = \left[\sum_{i=1}^3 (T_N(i) + \frac{1}{2}m_N\omega_s^2 r_i^2) + T_A(4) + \frac{1}{2}m_A\omega_s^2 r_4^2 \right. \\ \left. + \sum_{i>j=1}^3 u_{NN}(r_{ij}) + \sum_{i=1}^3 u_{NA}(r_{i4}) \right] \Psi(r_1, r_2, r_3, r_4), \quad (1')$$

where

$$u_{NN}(r) = v_{NN}(r) - \frac{1}{2} \frac{m_N^2 \omega_s^2}{M} r^2, \\ u_{NA}(r) = v_{NA}(r) - \frac{1}{2} \frac{m_N m_A \omega_s^2}{M} r^2.$$

2.2. THE REDUCTION TO THE ONE- AND TWO-PARTICLE EQUATIONS

Two complete sets of single particle wave functions are needed for the application of the method developed by Mang and Wild⁸⁾ to our problem. We label these functions $\phi_k^N(r)$ and $\phi_k^A(r)$ where k stands for the set of three eigenvalues needed to characterize each function of the complete set. These functions will be chosen to be eigenfunctions of oscillator Hamiltonians. The motivation for this choice will become clearer later on.

The reduction to the one- and two-particle equations is accomplished by projecting eqs. (1') and (2) onto the space spanned by products of the functions ϕ_0^N and ϕ_0^A , where $k=0$ will mean $(n, l, m) = (0, 0, 0)$ for our later choice of oscillator functions. The one- and two-body wave functions are defined as

$$\phi_N(r_1) = \int \phi_0^N(r_2) \phi_0^N(r_3) \phi_0^A(r_4) \Psi(r_1, r_2, r_3, r_4) d^3 r_2 d^3 r_3 d^3 r_4, \quad (3)$$

$$\phi_A(r_4) = \int \phi_0^N(r_1) \phi_0^N(r_2) \phi_0^N(r_3) \Psi(r_1, r_2, r_3, r_4) d^3 r_1 d^3 r_2 d^3 r_3, \quad (4)$$

$$\psi_{NN}(r_1, r_2) = \int \phi_0^N(r_3) \phi_0^A(r_4) \Psi(r_1, r_2, r_3, r_4) d^3 r_3 d^3 r_4, \quad (5)$$

$$\psi_{NA}(r_1, r_4) = \int \phi_0^N(r_2) \phi_0^N(r_3) \Psi(r_1, r_2, r_3, r_4) d^3 r_2 d^3 r_3. \quad (6)$$

Note that ϕ_N and ϕ_A are radially symmetric functions. The three-body projections ψ_{NNN} and ψ_{NNA} are similarly defined.

To start the actual reduction, we take the scalar products of $\phi_0^N(r_1) \phi_0^N(r_2) \phi_0^N(r_3) \phi_0^A(r_4)$ with eqs. (1') and (2) and obtain

$$\begin{aligned}
 E = & 3 \int \phi_0^N(r) [T_N(r) + \frac{1}{2} m_N \omega_s^2 r^2] \phi_N(r) d^3 r + \int \phi_0^A(r) [T_A(r) + \frac{1}{2} m_A \omega_s^2 r^2] \phi_A(r) d^3 r \\
 & + 3 \int \phi_0^N(r_1) \phi_0^N(r_2) u_{NN}(r_{12}) \psi_{NN}(r_1, r_2) d^3 r_1 d^3 r_2 \\
 & + 3 \int \phi_0^N(r_1) \phi_0^A(r_4) u_{NA}(r_{14}) \psi_{NA}(r_1, r_4) d^3 r_1 d^3 r_4, \quad (7)
 \end{aligned}$$

$$\begin{aligned}
 E_{\text{int}} = & 3 \int \phi_0^N(r) \left[T_N(r) + \frac{\hbar^2}{2M} \nabla_r^2 \right] \phi_N(r) d^3 r + \int \phi_0^A(r) \left[T_A(r) + \frac{\hbar^2}{2M} \nabla_r^2 \right] \phi_A(r) d^3 r \\
 & + 3 \int \phi_0^N(r_1) \phi_0^N(r_2) \left[v_{NN}(r_{12}) + \frac{\hbar^2}{M} \nabla_{r_1} \cdot \nabla_{r_2} \right] \psi_{NN}(r_1, r_2) d^3 r_1 d^3 r_2 \\
 & + 3 \int \phi_0^N(r_1) \phi_0^A(r_4) \left[v_{NA}(r_{14}) + \frac{\hbar^2}{M} \nabla_{r_1} \cdot \nabla_{r_4} \right] \psi_{NA}(r_1, r_4) d^3 r_1 d^3 r_4, \quad (8)
 \end{aligned}$$

where Ψ has been normalized so that

$$\int \phi_0^N(r_1) \phi_0^N(r_2) \phi_0^N(r_3) \phi_0^A(r_4) \Psi(r_1, r_2, r_3, r_4) d^3 r_1 d^3 r_2 d^3 r_3 d^3 r_4 = 1. \quad (9)$$

Thus if ϕ_N , ϕ_A , ψ_{NN} and ψ_{NA} were known, E and E_{int} could be evaluated.

Equations for ϕ_N and ψ_{NN} follow by taking the scalar products of $\phi_0^N(r_2) \phi_0^N(r_3) \phi_0^A(r_4)$ and then $\phi_0^N(r_3) \phi_0^A(r_4)$ with eq. (1'). These are

$$\begin{aligned}
 E \phi_N(r_1) = & [T_N(1) + \frac{1}{2} m_N \omega_s^2 r_1^2] \phi_N(r_1) \\
 & + 2 \int \phi_0^N(r_2) [T_N(2) + \frac{1}{2} m_N \omega_s^2 r_2^2 + u_{NN}(r_{12})] \psi_{NN}(r_1, r_2) d^3 r_2 \\
 & + \int \phi_0^A(r_4) [T_A(4) + \frac{1}{2} m_A \omega_s^2 r_4^2 + u_{NA}(r_{14})] \psi_{NA}(r_1, r_4) d^3 r_4 \\
 & + \int \phi_0^N(r_2) \phi_0^N(r_3) u_{NN}(r_{23}) \psi_{NNN}(r_1, r_2, r_3) d^3 r_2 d^3 r_3 \\
 & + 2 \int \phi_0^N(r_2) \phi_0^A(r_4) u_{NA}(r_{24}) \psi_{NNA}(r_1, r_2, r_4) d^3 r_2 d^3 r_4, \quad (10)
 \end{aligned}$$

$$\begin{aligned}
 E \psi_{NN}(r_1, r_2) = & [T_N(1) + T_N(2) + \frac{1}{2} m_N \omega_s^2 (r_1^2 + r_2^2) + u_{NN}(r_{12})] \psi_{NN}(r_1, r_2) \\
 & + \int \phi_0^N(r_3) [T_N(3) + \frac{1}{2} m_N \omega_s^2 r_3^2 + u_{NN}(r_{13}) + u_{NN}(r_{23})] \psi_{NNN}(r_1, r_2, r_3) d^3 r_3 \\
 & + \int \phi_0^A(r_4) [T_A(4) + \frac{1}{2} m_A \omega_s^2 r_4^2 + u_{NA}(r_{14}) + u_{NA}(r_{24})] \psi_{NNA}(r_1, r_2, r_4) d^3 r_4 \\
 & + \int \phi_0^N(r_3) \phi_0^A(r_4) u_{NA}(r_{34}) \Psi(r_1, r_2, r_3, r_4) d^3 r_3 d^3 r_4. \quad (11)
 \end{aligned}$$

By the same technique, analogous equations for ϕ_A , ψ_{NA} , ψ_{NNN} and ψ_{NNA} can be obtained.

Up to this state, no approximations have been made in our attempt to find the ground state energy of eq. (1'). We now introduce some simplifications for which the justification is discussed in sect. 4.

Since ψ_{NNN} , ψ_{NNA} and Ψ appear in eqs. (10) and (11), this reduction of eq. (1') requires the complete ground state wave function in order to solve for the one and two particle functions. However, if the three- and four-particle correlations are not important we may replace the three and four body functions by products of one- and two-particle functions, the two-particle functions being inserted where two-particle potentials appear. These approximations ("independent pair model") have been discussed by several authors^{17, 13, 8)}.

Using these approximations in eqs. (10) and (11) and then replacing ψ_{NN} and ψ_{NA} by product approximations where this is allowed by the above discussion, the one- and two-body equations reduce to (a bar is put above the functions to denote that the equations are not exact)

$$\varepsilon_N \bar{\phi}_N(r_1) = [T_N(1) + \frac{1}{2}m_N \omega_s^2 r_1^2 + \frac{2}{3}U_{NN}(r_1) + \frac{1}{3}U_{NA}(r_1)] \bar{\phi}_N(r_1), \quad (12)$$

$$\varepsilon_A \bar{\phi}_A(r_4) = [T_A(4) + \frac{1}{2}m_A \omega_s^2 r_4^2 + U_{AA}(r_4)] \bar{\phi}_A(r_4), \quad (13)$$

$$\begin{aligned} \varepsilon_{NN} \bar{\psi}_{NN}(r_1, r_2) = [T_N(1) + T_N(2) + \frac{1}{2}m_N \omega_s^2 (r_1^2 + r_2^2) + u_{NN}(r_{12}) + \frac{1}{3}U_{NA}(r_1) \\ + \frac{1}{3}U_{NA}(r_2) + \frac{1}{3}U_{NN}(r_1) + \frac{1}{3}U_{NN}(r_2)] \bar{\psi}_{NN}(r_1, r_2), \end{aligned} \quad (14)$$

$$\begin{aligned} \varepsilon_{NA} \bar{\psi}_{NA}(r_1, r_4) = [T_N(1) + T_A(4) + \frac{1}{2}\omega_s^2 (m_N r_1^2 + m_A r_4^2) + u_{NA}(r_{14}) \\ + \frac{2}{3}U_{NN}(r_1) + \frac{2}{3}U_{AA}(r_4)] \bar{\psi}_{NA}(r_1, r_4), \end{aligned} \quad (15)$$

where

$$U_{NN}(r_1) \bar{\phi}_N(r_1) = 3 \int \phi_0^N(r_2) u_{NN}(r_{12}) \bar{\psi}_{NN}(r_1, r_2) d^3 r_2, \quad (16)$$

$$U_{NA}(r_1) \bar{\phi}_N(r_1) = 3 \int \phi_0^A(r_4) u_{NA}(r_{14}) \bar{\psi}_{NA}(r_1, r_4) d^3 r_4, \quad (17)$$

$$U_{AA}(r_4) \bar{\phi}_A(r_4) = 3 \int \phi_0^N(r_1) u_{NA}(r_{14}) \bar{\psi}_{NA}(r_1, r_4) d^3 r_1. \quad (18)$$

It is our next task to solve this set of coupled equations for $\bar{\phi}_N$, $\bar{\phi}_A$, $\bar{\psi}_{NN}$, and $\bar{\psi}_{NA}$ (this is done in the next section) and then to use these approximate one- and two-body wave functions to evaluate the energy in eqs. (7) and (8). As argued in ref. ⁸⁾, the functions ϕ_0^N and ϕ_0^A can be chosen so that the average or long range behaviour of the one or two particle functions is reflected in their projections on ϕ_0^N and ϕ_0^A . In order that the approximate one and two particle solutions of eqs. (12) to (15) show this same

“average” behaviour, we choose their normalizations to be the same as for the exact solutions; i.e.,

$$\int \phi_0^N(r) \bar{\phi}_N(r) d^3r = 1, \quad (19)$$

$$\int \phi_0^A(r) \bar{\phi}_A(r) d^3r = 1, \quad (20)$$

$$\int \phi_0^N(r_1) \phi_0^N(r_2) \bar{\psi}_{NN}(r_1, r_2) d^3r_1 d^3r_2 = 1, \quad (21)$$

$$\int \phi_0^N(r_1) \phi_0^A(r_4) \bar{\psi}_{NA}(r_1, r_4) d^3r_1 d^3r_4 = 1. \quad (22)$$

Before trying to solve for these approximate wave functions, let us show that the energy expression of eq. (7) is given approximately by the eigenvalues of eqs. (12) to (15). If we take the scalar products of $\phi_0^N(r_1)$ with eq. (12), $\phi_0^A(r_4)$ with eq. (13), $\phi_0^N(r_1)\phi_0^N(r_2)$ with eq. (14) and $\phi_0^N(r_1)\phi_0^A(r_4)$ with eq. (15), and use the normalizations of eqs. (19) to (22), we get the eigenvalues ε_N , ε_A , ε_{NN} and ε_{NA} in terms of integrals involving the kinetic and potential energies. If it is assumed that the approximate one and two particle wave functions are related in the same way as the exact functions, i.e.,

$$\bar{\phi}_N(r_1) \approx \int \phi_0^N(r_2) \bar{\psi}_{NN}(r_1, r_2) d^3r_2, \quad (23)$$

$$\bar{\phi}_N(r_1) \approx \int \phi_0^A(r_4) \bar{\psi}_{NA}(r_1, r_4) d^3r_4, \quad (24)$$

$$\bar{\phi}_A(r_4) \approx \int \phi_0^N(r_1) \bar{\psi}_{NA}(r_1, r_4) d^3r_1, \quad (25)$$

then the eigenvalues are given in terms of the same four integrals that appear in eq. (7) except that the approximate one and two particle wave functions replace the exact ones. Since there are four integrals and four eigenvalues, the integrals can be expressed in terms of the eigenvalues. Substituting these results into eq. (7), we get

$$E \approx 3\varepsilon_{NN} + 3\varepsilon_{NA} - 6\varepsilon_N - 2\varepsilon_A. \quad (26)$$

This equation along with eqs. (7) and (8) provide three expressions for the binding energy of the hyperfragment. The errors introduced by the approximations used to find the solution enter into these three energy expressions in different ways, and thus the agreement obtained will provide a test of the size of these errors.

3. The Perturbation Solution

Eqs. (14)-(17) are not easy to solve because of the non-local potential terms. In this section these terms are approximated by local potentials, chosen so as to represent

the non-local terms as well as possible. The integration in the definition of the non-local potentials smooths out the strong, short range behaviour of the two-body potentials u_{NN} and u_{NA} , and thus we can choose smooth local potentials for our approximation.

3.1. THE LOCAL POTENTIALS

Three oscillator potentials are chosen so that the differences

$$[U_{NN}(r) - \frac{1}{2}m_N\omega_1^2 r^2]\bar{\phi}_N(r) \equiv P_1(r)\bar{\phi}_N(r),$$

$$[U_{NA}(r) - \frac{1}{2}m_N\omega_2^2 r^2]\bar{\phi}_N(r) \equiv P_2(r)\bar{\phi}_N(r),$$

$$[U_{AA}(r) - \frac{1}{2}m_A\omega_3^2 r^2]\bar{\phi}_A(r) \equiv P_3(r)\bar{\phi}_A(r),$$

are small as defined below. Here "small" does not mean that the magnitude is small since constant potentials could be added to each of the oscillator potentials without changing anything except to add constants to the eigenvalues of eqs. (14)-(17). This does not change the wave functions or the matrix elements in the perturbation expansion below and therefore we do not include these constants⁸).

Adding and subtracting these oscillator potentials where appropriate, eqs. (14)-(17) can be written

$$\varepsilon_N \bar{\phi}_N(r) = [T_N(r) + \frac{1}{2}m_N(\omega_s^2 + \frac{2}{3}\omega_1^2 + \frac{1}{3}\omega_2^2)r^2 + \frac{2}{3}P_1(r) + \frac{1}{3}P_2(r)]\bar{\phi}_N(r), \quad (27)$$

$$\varepsilon_A \bar{\phi}_A(r) = [T_A(r) + \frac{1}{2}m_A(\omega_s^2 + \omega_3^2)r^2 + P_3(r)]\bar{\phi}_A(r), \quad (28)$$

$$\begin{aligned} \varepsilon_{NN} \bar{\psi}_{NN}(r_1, r_2) = [T_N(1) + T_N(2) + \frac{1}{2}m_N(\omega_s^2 + \frac{1}{3}\omega_1^2 + \frac{1}{3}\omega_2^2)(r_1^2 + r_2^2) + u_{NN}(r_{12}) \\ + \frac{1}{3}P_1(r_1) + \frac{1}{3}P_1(r_2) + \frac{1}{3}P_2(r_1) + \frac{1}{3}P_2(r_2)]\bar{\psi}_{NN}(r_1, r_2), \end{aligned} \quad (29)$$

$$\begin{aligned} \varepsilon_{NA} \bar{\psi}_{NA}(r_1, r_4) = [T_N(1) + T_A(4) + \frac{1}{2}m_N(\omega_s^2 + \frac{2}{3}\omega_1^2)r_1^2 + \frac{1}{2}m_A(\omega_s^2 + \frac{2}{3}\omega_3^2)r_4^2 \\ + u_{NA}(r_{14}) + \frac{2}{3}P_1(r_1) + \frac{2}{3}P_3(r_4)]\bar{\psi}_{NA}(r_1, r_4). \end{aligned} \quad (30)$$

The P -terms are to be treated as perturbations.

3.2. THE SELF-CONSISTENCY CONDITIONS

Since the two-particle potentials u_{NN} and u_{NA} are included in the zero order problem, the short range correlations are included in the zero order two-particle wave functions. The quantities ω_1 , ω_2 and ω_3 are to be adjusted to give the correct "average" behaviour of the wave functions.

The first step in fixing the average behaviour of the wave functions is the choice of normalization. We can obtain the desired normalization of the one-particle wave functions as given by eqs. (19) and (20) by choosing the complete sets ϕ_k^N and ϕ_k^A to be normalized eigenfunctions of the zero order one-particle Hamiltonians; i.e., of eqs. (27) and (28) with the P -terms missing. Then in the usual perturbation treatment, ϕ_0^N and ϕ_0^A do not appear in the higher order corrections, and thus the normalization relations (19) and (20) are satisfied. We normalize the zero order two-particle wave

functions so that eqs. (21) and (22) are satisfied by these zero order solutions. In this case, however, the higher order perturbation corrections will change the normalizations of the two-particle functions slightly.

Before the next step can be carried out, a definite criterion for the correct average behaviour must be chosen. The eqs. (23)-(25) relate the average behaviour of the two-particle functions to that of the one-particle functions. Let us see what are the consequences of requiring that the projections of the zero order two-particle functions give approximately the one-particle functions. To do this we take the scalar product of ϕ_0^N with the zero order form of eq. (29), and if the projected equation is to be the same as eq. (27) for the one-particle function $\bar{\phi}_N$, then we must require that

$$\int \phi_0^N(r_2) [u_{NN}(r_{12}) - \frac{1}{6} m_N \omega_1^2 (r_1^2 + r_2^2)] \bar{\psi}_{NN}^0(r_1, r_2) d^3 r_2 \\ = (\frac{2}{3} P_1(r_1) + \frac{1}{3} P_2(r_1)) \bar{\phi}_N(r_1) + c \bar{\phi}_N(r_1), \quad (31)$$

where $\bar{\psi}_{NN}^0$ is the solution to the zero order form of eq. (29) and c is a constant. Repeating this procedure with the scalar product of ϕ_0^A with eq. (30) and ϕ_0^N with eq. (30) and comparing these projected equations with eqs. (27) and (28) respectively, we get two relations similar to eq. (31). Since there are only three parameters ω_1 , ω_2 and ω_3 to adjust, we cannot hope to satisfy these three relations for all values of the position variable r . We cannot even require that the two sides of eq. (31) when treated as perturbations on the zero order equations for $\bar{\phi}_N$ and the projection of $\bar{\psi}_{NN}^0$, respectively, lead to the same first order corrections to the wave functions. However, one parameter can be adjusted so that the component of the first order correction which is proportional to ϕ_{100}^N be the same for both perturbation terms, or that

$$2 \int \phi_{100}^N(r_1) \phi_0^N(r_2) u_{NN}(r_{12}) \bar{\psi}_{NN}^0(r_1, r_2) d^3 r_1 d^3 r_2 \\ + \int \phi_{100}^N(r_1) \phi_0^A(r_4) u_{NA}(r_{14}) \bar{\psi}_{NA}^0(r_1, r_4) d^3 r_1 d^3 r_4 \\ - \frac{1}{6} m_N (2\omega_1^2 + \omega_2^2) \int \phi_{100}^N(r) r^2 \phi_0^N(r) d^3 r \\ = \int \phi_{100}^N(r_1) \phi_0^N(r_2) [u_{NN}(r_{12}) - \frac{1}{6} m_N \omega_1^2 (r_1^2 + r_2^2)] \bar{\psi}_{NN}^0(r_1, r_2) d^3 r_1 d^3 r_2. \quad (32)$$

A similar treatment of the two distinct projections of eq. (30) leads to two similar conditions. We have singled out the perturbation term proportional to ϕ_{100}^N because this term is expected to be the largest since it is the first spherically symmetric excited state of the zero order Hamiltonian.

The parameters ω_1 , ω_2 and ω_3 can be adjusted so that eq. (32) and the two similar relations mentioned above are satisfied. If the terms

$$K_{ij}^{NN} \equiv \int \phi_{i00}^N(r_1) \phi_{j00}^N(r_2) \bar{\psi}_{NN}^0(r_1, r_2) d^3 r_1 d^3 r_2, \quad (33)$$

$$K_{ij}^{NA} \equiv \int \phi_{i00}^N(r_1) \phi_{j00}^A(r_4) \psi_{NA}(r_1, r_4) d^3 r_1 d^3 r_4, \quad (34)$$

with $(i, j) = (1, 0), (2, 0), (0, 1), (0, 2),$ and $(1, 1)$, are negligible[†] compared to unity, these three conditions reduce to

$$\frac{1}{2} m_N \omega_1^2 \int \phi_{100}^N(r) r^2 \phi_{000}^N(r) d^3 r = 3 \int \phi_{100}^N(r_1) \phi_{000}^N(r_2) u_{NN}(r_{12}) \bar{\psi}_{NN}^0(r_1, r_2) d^3 r_1 d^3 r_2, \quad (35)$$

$$\frac{1}{2} m_N \omega_2^2 \int \phi_{100}^N(r) r^2 \phi_{000}^N(r) d^3 r = 3 \int \phi_{100}^N(r_1) \phi_{000}^A(r_4) u_{NA}(r_{14}) \bar{\psi}_{NA}(r_1, r_4) d^3 r_1 d^3 r_4, \quad (36)$$

$$\frac{1}{2} m_A \omega_3^2 \int \phi_{100}^A(r) r^2 \phi_{000}^A(r) d^3 r = 3 \int \phi_{000}^N(r_1) \phi_{100}^A(r_4) u_{NA}(r_{14}) \bar{\psi}_{NA}(r_1, r_4) d^3 r_1 d^3 r_4, \quad (37)$$

which are the actual conditions used to determine the parameters. This choice of parameters should lead to one- and two-particle wave functions whose average behaviour is approximately correct as can be tested by inserting them into eqs. (23)-(25). In subsect. 4.1 the deviations from these relations are discussed and a numerical measure of this deviation is given in table 4.

It is important to note that the conditions (35), (36) and (37) are the same conditions that make the first order corrections for $\bar{\phi}_N$ and $\bar{\phi}_A$ orthogonal to ϕ_{100}^N and ϕ_{100}^A , respectively. For oscillator wave functions,

$$\int \phi_{j00}(r) [T(r) + \frac{1}{2} m \omega^2 r^2] \phi_{000}(r) d^3 r = 0 \quad \text{for } j \geq 2,$$

and thus this means that the self-consistency conditions (35) to (37) force the first order corrections to the single-particle energy terms in eq. (7) to be zero. Since the self-consistent conditions (35) to (37) eliminate ϕ_{100}^N and ϕ_{100}^A from the first order corrections to the single particle functions, we expect these conditions to make ϕ_{000}^N and ϕ_{000}^A good approximations to $\bar{\phi}_N$ and $\bar{\phi}_A$. As given by eqs. (44) and (45) and table 4, the other first order corrections are found to be small.

3.3. THE SOLUTION OF THE ZERO ORDER EQUATIONS

The solutions of the zero order parts of eqs. (27) and (28) are the familiar oscillator wave functions with the energy eigenvalues

$$\varepsilon_N^{(n, l, m)} = \left(\frac{4n + 2l + 3}{2} \right) \hbar \sqrt{\omega_s^2 + \frac{2}{3} \omega_1^2 + \frac{1}{3} \omega_2^2}, \quad (38)$$

$$\varepsilon_A^{(n, l, m)} = \left(\frac{4n + 2l + 3}{2} \right) \hbar \sqrt{\omega_s^2 + \omega_3^2}. \quad (39)$$

[†] These terms are given in table 4.

More details of the solutions are given in ref. ⁸). The zero order form of eq. (29) separates in the centre-of-mass and relative coordinates of the two particles, which is the reason for the choice of oscillator potentials as the local potentials. The centre-of-mass motion is also described by an oscillator wave function. The relative motion of the two particles is more complicated, but for the choice of a square well outside of a hard core for the shape of v_{NN} and v_{NA} , Mang and Wild ⁸) have given the solution in terms of common functions.

The zero order form of eq. (30) does not separate in the centre-of-mass and relative coordinates, but instead it becomes

$$\begin{aligned} \varepsilon_{NA}^0 \bar{\psi}_{NA}^0(\mathbf{r}, \mathbf{R}) = & \left\{ \left[-\frac{\hbar^2}{2(m_N + m_A)} \nabla_{\mathbf{R}}^2 + \frac{1}{2}(m_N + m_A) \left(\omega_s^2 + \frac{2}{3} \frac{(m_N \omega_1^2 + m_A \omega_s^2)}{(m_N + m_A)} \right) R^2 \right] \right. \\ & + \left[-\frac{\hbar^2(m_N + m_A)}{2m_N m_A} \nabla_{\mathbf{r}}^2 + v_{NA}(r) + \frac{m_N m_A}{(m_N + m_A)} \left(\frac{m_N \omega_s^2}{3m_N + m_A} + \frac{m_A \omega_1^2 + m_N \omega_s^2}{3(m_N + m_A)} \right) r^2 \right] \\ & \left. + \frac{2}{3} \frac{m_N m_A}{(m_N + m_A)} (\omega_1^2 - \omega_s^2) \mathbf{R} \cdot \mathbf{r} \right\} \bar{\psi}_{NA}^0(\mathbf{r}, \mathbf{R}). \end{aligned} \quad (40)$$

In order to solve this equation, the $\mathbf{R} \cdot \mathbf{r}$ term is treated as a perturbation. The zero order form of eq. (40) can then be separated in the centre-of-mass and relative coordinates and can be solved in the same way as $\bar{\psi}_{NN}^0$ is obtained ⁸). Since the average of $\mathbf{R} \cdot \mathbf{r}$ over all directions is zero, the first order perturbation corrections to the energy ε_{NA}^0 are zero and the first order corrections to the wave function are not spherically symmetric but vary as $\mathbf{R} \cdot \mathbf{r}/Rr$. Since the remainder of the integrand of the energy expression (7) which contain $\bar{\psi}_{NA}$ is "nearly" spherically symmetric (see appendix 1), this first order correction to the wave function will not contribute much to the energy. This is presented as the justification for ignoring the $\mathbf{R} \cdot \mathbf{r}$ term in our calculation, which allows us to use the two-body functions given in ref. ⁸).

3.4. THE TWO-PARTICLE POTENTIALS

As mentioned above the two-body potentials v_{NN} and v_{NA} are chosen to be square-well potentials outside of a hard core; i.e.,

$$v(r) = \begin{cases} -\infty & \text{for } r < c \\ -V_0 & \text{for } c < r < c + b \\ 0 & \text{for } r > c + b. \end{cases}$$

Because we are only treating a symmetric ground state, there is no distinction between Wigner or Majorana forces. The two-body potentials that appear in the above equations are the spin-averaged potentials as expressed in table 2. For convenience, c and b are chosen to be the same for the singlet and triplet potentials. For the singlet and triplet N-N potentials, we assume

$$c = 0.4 \text{ fm},$$

and for the singlet and triplet N- Λ forces, we assume

$$c = 0.2 \text{ fm.}$$

With this choice for the hard core, the other parameters of the N-N potentials are chosen ^{7, 18)} to fit the scattering lengths and effective ranges of the N-N triplet and singlet interaction. It is assumed that the singlet np, nn and pp interaction are the same. We use the same parameters as used by Mang and Wild ⁸⁾; and for the ${}^4\text{He}$ and ${}^4\text{H}$ and the ${}^4\text{He}^5$ calculations, this means we use the same spin averaged N-N potential as they used in their calculations,

$$V_{\text{NN}} = 49 \cdot 60 \text{ MeV}$$

with

$$b_{\text{NN}} = 1.5 \text{ fm.}$$

For the ${}^4\text{H}^3$ calculation, the nucleons are in a relative triplet state and so we must use only the triplet potential, which has

$$V_{\text{NN}} = 77 \cdot 81 \text{ MeV}$$

with

$$b_{\text{NN}} = 1.29 \text{ fm.}$$

It is the goal of this paper to determine some information about the N- Λ potential, but since there are only three experimental results (see table 1), we cannot hope to find the ranges and depths of the singlet and triplet interactions. Instead let us set the intrinsic range ^{7, 18)} of the N- Λ interaction equal to the Compton wavelength of two π mesons,

$$1.4843 \text{ fm,}$$

on the assumption that the low energy interaction ³⁾ is due to the exchange of two π mesons in the meson theory picture. This corresponds to

$$b = 1.0843 \text{ fm.}$$

With the ranges assumed, we then calculate the binding energies of the hyperfragments as functions of the triplet and singlet N- Λ potential depths. We repeat this procedure assuming the N- Λ interaction is due to the exchange of a K meson, or for

$$b = 0.4411 \text{ fm.}$$

The results of the calculations actually give the binding energies in terms of the depths of the spin averaged N- Λ potentials. The formulae in table 2 are then solved for the singlet and triplet potential depths. Since there are only two depth parameters, it may not be possible to match all three binding energies. It is hoped that this will be possible for one of the two columns of table 2, either $V_t > V_s$ or for $V_s > V_t$, but not for both. This would allow us to solve for the triplet and singlet depths uniquely.

3.5. INTEGRALS

The integrals in the energy eqs. (7) and (8) using the zero order wave functions and in the self-consistency conditions (35), (36) and (37) which contain only the one-particle functions are elementary integrals. Since the zero order two-particle wave functions are given as the product of a function of the centre of mass times a function of the relative coordinates, the integrations involving the two-particle functions are carried out in these coordinates. The integrations over the centre-of-mass coordinates are carried out in the appendix, which reduces the integrals to the same integrals over the relative coordinates as are worked out in ref. ⁸).

4. Results and Discussion

The results of the calculations are given in figs. 1 to 6. These figures show the binding energies of the three, four and five body hyperfragments as functions of the depths of the spin-averaged A - N potentials as defined in table 2 for the two ranges of this potential that are discussed in subsect. 3.4. As marked on these figures, the energies were obtained from the three energy expressions, eqs. (7), (8) and (26). The centre-of-mass energy $\frac{3}{2} \hbar \omega_s$ has been subtracted from the total energy as given by eqs. (7) and (26).

In all three cases, the ground state energy of the nucleons alone has been subtracted from the ground state energy of the hyperfragment in order to get the binding energies given in figs. 1 to 6. The binding energies of the three and four nucleon systems with the same N - N potentials that are used here are given by Mang and Wild ⁸):

$$\text{binding energy}^\dagger \text{ of } H^3 = 6.58 \text{ MeV,}$$

$$\text{binding energy of } He^4 = 27.60 \text{ MeV.}$$

The experimental value of the binding energy of the deuteron can be used for the two-nucleon ground state energy because the triplet N - N potential was chosen to yield this value.

4.1. ACCURACY

The agreement between the energies calculated using the total Hamiltonian eq. (7) and the energies calculated using the Hamiltonian for the internal motion only, eq. (8), is a measure of the errors introduced by the approximations made. This check tells us mainly how accurately the centre-of-mass motion is described. In addition, the calculation was carried out for different values of ω_s with the potential depths held fixed. Since ω_s is the oscillator parameter for the centre-of-mass potential, the total energy of eq. (7) should change so that $\Delta E / \hbar \Delta \omega_s = \frac{3}{2}$. The values obtained for this ratio are given in table 3.

Another estimate of the possible errors resulting from the approximations used can be made by using the relations (23) to (25) between the single- and two-particle wave

[†] The value reported by Mang and Wild is incorrect. The result given here is the corrected value.

Fig. 1. The dependence of the binding energy of ${}^4\text{H}^3$ on the well depth of the spin-averaged potential for $b_{NA} = 1.0843$ fm.

Fig. 2. The dependence of the binding energy of ${}^4\text{H}^3$ on the well depth of the spin-averaged potential for $b_{NA} = 0.4411$ fm.

Fig. 3. The dependence of the binding energy of the four-body hyperfragment on the well depth of the spin-averaged potential for $b_{NA} = 1.0843$ fm.

Fig. 4. The dependence of the binding energy of the four-body hyperfragment on the well depth of the spin-averaged potential for $b_{NA} = 0.4411$ fm.

Fig. 5. The dependence of the binding energy of ${}^4\text{He}^6$ on the well depth of the spin-averaged potential for $b_{NA} = 1.0843$ fm.

Fig. 6. The dependence of the binding energy of ${}^4\text{He}^6$ on the well depth of the spin-averaged potential for $b_{NA} = 0.4411$ fm.

TABLE 3

The dependence of the total energy on the strength of the centre-of-mass potential

	$V_{NA}(\text{MeV})$	$b_{NA}(\text{fm})$	$\hbar\omega_b(\text{MeV})$	$E(\text{MeV})$ from eq. (7)	$\frac{\Delta E}{\hbar\Delta\omega_b}$
${}^A\text{H}^3$	55.56	1.0843	13.489 14.717	19.080 20.973	1.54
${}^A\text{H}^4, {}^A\text{He}^4$	50	1.0843	14.717 18.024	14.014 19.049	1.52
	364	0.4411	14.717 18.024	14.662 19.638	1.505
${}^A\text{He}^5$	44	1.0843	19.745 20.813	-1.193 0.438	1.53
	44	1.0843	19.745 27.924	-1.193 11.195	1.51
	46	1.0843	19.745 27.924	-2.681 9.682	1.51
	337	0.4411	19.745 20.813	-1.229 0.375	1.502

TABLE 4

Comparison ^{a)} of the average behaviour of the one- and two-particle functions

	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
K_{i0}^{NN}	1	0.000683	0.0118	-0.00247	-0.00194
K_{i0}^{NA}	1	-0.0298	0.0182	0.000546	0.0288
K_{0i}^{NA}	1	0.0365	0.0135	0.00210	0.0130
a_{NN}^i		0	0.0414	-0.00795	-0.00648
a_{NA}^{i0}		0	0.0648	0.00313	-0.0526
a_{NA}^{0i}		0	0.0445	-0.0140	-0.0386
$\frac{2}{3}a_{NN}^i + \frac{1}{3}a_{NA}^{i0} - K_{i0}^{NA}$		-0.000683	0.0374	-0.00179	-0.0199
$\frac{2}{3}a_{NN}^i + \frac{1}{3}a_{NA}^{i0} - K_{i0}^{NA}$		0.0298	0.0310	-0.00481	-0.0506
$a_{NA}^{0i} - K_{0i}^{NA}$		-0.0365	0.0310	-0.0161	-0.0516

^{a)} Example given for the four-body hyperfragment with $V_{NA}^4 = 50$ MeV, $b_{NA} = 1.0843$ fm, and $\hbar\omega_b = 18.02$ MeV.

functions. An example of this check is given in table 4 for the single-particle wave functions correct to first order and the zero order two-particle functions. The integrals $K_{i,j}^{NN}$ and $K_{i,j}^{NA}$ are defined in eqs. (33) and (34), while

$$a_{NN}^j = \frac{1}{\varepsilon_N^0 - \varepsilon_N^j} \left[3 \int \phi_{j00}^N(r_1) \phi_{000}^N(r_2) u_{NN}(r) \bar{\psi}_{NN}^0(r_1, r_2) d^3 r_1 d^3 r_2 \right. \\ \left. - \frac{1}{2} m_N \omega_1^2 \int \phi_{j00}^N(r) r^2 \phi_{000}^N(r) d^3 r \right], \quad (41)$$

$$a_{NA}^{j0} = \frac{1}{\varepsilon_N^0 - \varepsilon_N^j} \left[3 \int \phi_{j00}^N(r_1) \phi_{000}^A(r_4) u_{NA}(r_{14}) \bar{\psi}_{NA}^0(r_1, r_4) d^3 r_1 d^3 r_4 \right. \\ \left. - \frac{1}{2} m_N \omega_2^2 \int \phi_{j00}^N(r) r^2 \phi_{000}^N(r) d^3 r \right], \quad (42)$$

$$\dot{a}_{NA}^{0j} = \frac{1}{\varepsilon_A^0 - \varepsilon_A^j} \left[3 \int \phi_{000}^N(r_1) \phi_{j00}^A(r_4) u_{NA}(r_{14}) \bar{\psi}_{NA}^0(r_1, r_4) d^3 r_1 d^3 r_4 \right. \\ \left. - \frac{1}{2} m_A \omega_3^2 \int \phi_{j00}^A(r) r^2 \phi_{000}^A(r) d^3 r \right], \quad (43)$$

where $\varepsilon_N^j = \varepsilon_N^{j00}$ and $\varepsilon_A^j = \varepsilon_A^{j00}$ of eqs. (38) and (39). Using these symbols, the first-order single-particle wave functions are

$$\bar{\phi}_N^0(r) + \bar{\phi}_N^{(1)}(r) = \phi_{000}^N(r) + \sum_{j=2}^{\infty} (\frac{2}{3} a_{NN}^j + \frac{1}{3} a_{NA}^{j0}) \phi_{j00}^N(r), \quad (44)$$

$$\bar{\phi}_A^0(r) + \bar{\phi}_A^{(1)}(r) = \phi_{000}^A(r) + \sum_{j=2}^{\infty} a_{NA}^{0j} \phi_{j00}^A(r). \quad (45)$$

These functions and $\bar{\psi}_{NN}^0$ and $\bar{\psi}_{NA}^0$ are used in eqs. (23) to (25). The terms $\frac{2}{3} a_{NN}^j + \frac{1}{3} a_{NA}^{j0} - K_{j0}^{NN}$, $\frac{2}{3} a_{NN}^j + \frac{1}{3} a_{NA}^{j0} - K_{j0}^{NA}$, and $a_{NA}^{0j} - K_{NA}^{0j}$ that are given in table 4 are the differences between the coefficients in the expansions of the expressions obtained on the two sides of eqs. (23) to (25) in terms of ϕ_{j00}^N , ϕ_{j00}^N and ϕ_{j00}^A , respectively. These coefficients are a few percent of unity. This means that the average behaviour of the approximate wave functions probably differs from that of the exact one and two particle functions by the same few percent.

From the above discussion, we can only give a rough estimate of the accuracy of the curves given in figs. 1 to 6. The energy eigenvalue calculated from an approximate eigenfunction is usually more accurate than the function used. Thus, we do not expect the fractional errors in the calculated values of the kinetic and potential energies to be as large as the coefficients that are given in table 4 for the errors in the wave functions. The error following from the few percent discrepancy in the average behaviour of the wave functions shows up in the difference between the energies found by eqs. (7) and (8) and the energy found by eq. (26) since the relations (23) to (25) were used to obtain eq. (26) from eq. (7). We use the difference in these energies as the estimated error of these calculations. The agreement between the energies obtained

from eqs. (7) and (8) and the results reported in table 3 show that the error in the energy of the centre-of-mass motion is negligible except for the case reported by fig. 1.

4.2. THE Λ -NUCLEON POTENTIAL

Using the energy curves in figs. 3 to 6 that were obtained from eq. (8) and are labeled H_{int} and the experimental binding energies given in table 1, the values of the well depth of the singlet and triplet Λ -N potential were calculated from the formulae of

TABLE 5
 V_s and V_t for different spins of the four-body hyperfragment

	$b_{NA} = 0.4411 \text{ fm}$	Spin-averaged potential (MeV)	V_s (MeV)	V_t (MeV)	S_s	S_t
$V_s > V_t$	${}^{\Lambda}\text{He}^4, {}^{\Lambda}\text{H}^4(I=0)$	$V_{NA}^4 = 375.8 \pm 2$	455	297	0.938	0.61
	${}^{\Lambda}\text{He}^5 (I = \frac{1}{2})$	$V_{NA}^5 = 336.2 \pm 3$	± 12	± 8		
$V_t > V_s$	${}^{\Lambda}\text{He}^4, {}^{\Lambda}\text{H}^4(I=1)$	$V_{NA}^4 = 375.8 \pm 2$	-20	455		0.938
	${}^{\Lambda}\text{He}^5 (I = \frac{1}{2})$	$V_{NA}^5 = 336.2 \pm 3$	± 40	± 12		
<hr/>						
	$b_{NA} = 1.0843 \text{ fm}$					
$V_s > V_t$	${}^{\Lambda}\text{He}^4, {}^{\Lambda}\text{H}^4(I=0)$	$V_{NA}^4 = 51.8 \pm 1$	67.7	35.9	0.845	0.45
	${}^{\Lambda}\text{He}^5 (I = \frac{1}{2})$	$V_{NA}^5 = 43.9 \pm 1$	± 5	± 3		
$V_t > V_s$	${}^{\Lambda}\text{He}^4, {}^{\Lambda}\text{H}^4(I=1)$	$V_{NA}^4 = 51.8 \pm 1$	-28	67.7		0.845
	${}^{\Lambda}\text{He}^5 (I = \frac{1}{2})$	$V_{NA}^5 = 43.9 \pm 1$	± 20	± 5		

TABLE 6
Mean well depth V_{NA}^3

	$b_{NA} = 0.4411 \text{ fm}$	V_{NA}^3 from V_{AN}^4 and V_{NA}^5	V_{NA}^3
$V_s > V_t$	${}^{\Lambda}\text{H}^3$	$V_{NA}^3 = 415.4 \pm 7 \text{ MeV}$	$414.9 \pm 3 \text{ MeV}$
$V_t > V_s$	${}^{\Lambda}\text{H}^3$	$V_{NA}^3 = 455.0 \pm 8$	414.9 ± 3
<hr/>			
	$b_{NA} = 1.0843 \text{ fm}$		
$V_s > V_t$	${}^{\Lambda}\text{H}^3$	$V_{NA}^3 = 59.8 \pm 3$	60.3 ± 1
$V_t > V_s$	${}^{\Lambda}\text{H}^3$	$V_{NA}^3 = 67.7 \pm 5$	60.3 ± 1

table 2 and are given in table 5. The possible deviations reported for these values reflects both the experimental and calculation uncertainties. Note that there are two possible results depending on the spin of the four-body hyperfragment. Each of

these results leads to a predicted value of the spin-averaged Λ -N potential for the three-body hyperfragment, where we make use of the relations in table 2. These predicted values are compared in table 6 with the values obtained from the H_{int} curves of figs. 1 and 2 and the experimental binding energy of ${}_{\Lambda}\text{H}^3$. A brief account of these results has been given elsewhere ¹⁹). The small differences between the values reported there and here are due to the small changes in the accepted values for the binding energies.

One concludes that the case $V_s > V_t$ is slightly favoured for both the force ranges treated. If the values of V_s and V_t for this case are accepted, we can determine if an excited state of the $({}_{\Lambda}\text{He}^4, {}_{\Lambda}\text{H}^4)$ doublet exists for $I = 1$. For this $I = 1$ state, we would have

$$V_{\text{NA}}^4 = \frac{5}{6}V_t + \frac{1}{6}V_s = \begin{cases} 323 \text{ MeV for } b = 0.4411 \text{ fm} \\ 41.2 \text{ MeV for } b = 1.0843 \text{ fm,} \end{cases}$$

which are considerably less than the well depths given in figs. 3 and 4 as needed for binding the $({}_{\Lambda}\text{He}^4, {}_{\Lambda}\text{H}^4)$ doublet. As discussed by Dalitz ³), this probable result that the $({}_{\Lambda}\text{He}^4, {}_{\Lambda}\text{H}^4)$ doublet has zero angular momentum in its ground state and has no bound excited states allows one to conclude from the observed creation of these hyperfragments by the interaction of K mesons with the He^4 nuclei that the relative K- Λ parity is negative.

The column labelled s in table 5 contains the well-depth parameters for the singlet and triplet potentials. In all cases, $s < 1$, which means that the Λ -nucleon system is unbound. The well-depth parameter for the spin-averaged Λ -N potential given in table 6 for the three-body hyperfragment is 0.857 for an intrinsic range of 0.84 fm and 0.754 for an intrinsic range of 1.48 fm. These values of s are for our choice of 0.2 fm for the hard core radius of the Λ -N potential. They are between the values $s = 0.69$ and 0.71 for no hard core as given by Dalitz and Downs ¹) and $s = 0.744$ for $c = 0.2$ fm, $s = 0.851$ for $c = 0.4$ fm and $s = 0.955$ for $c = 0.6$ fm as given by Downs *et al.* ¹²) who used an intrinsic range of 1.5 fm.

The authors thank W. Wild for helpful discussions. They are also indebted to the Fabrik K. Zuse in Bad Hersfeld, to the Technische Hochschule in Karlsruhe, and to the Reactor Station in Karlsruhe for the use of their Zuse Z 22 computers. One of the authors (K.D.) wants to express his thanks to the Bundesministerium für Atomkernenergie and to the Fulbright Commission for financing a stay at the Lawrence Radiation Laboratory. Another of the authors (R.F.) is grateful to the National Science Foundation for a Post-Doctoral Fellowship during which part of this work was done.

Appendix

A.1. INTEGRALS

In this section of the appendix we evaluate many of the integrals that appear in the text. First let us treat the normalization integral for $\bar{\psi}_{\text{NA}}^0$,

$$K_{00}^{\text{NA}} = \int \phi_{000}^{\text{N}}(r_1) \phi_{000}^{\text{A}}(r_4) \bar{\psi}_{\text{NA}}^0(r_1, r_4) d^3r_1 d^3r_4. \quad (\text{A.1})$$

The single-particle functions are the ground state oscillator functions ⁸⁾

$$\phi_{000}^N(r) = \left(\frac{\alpha_N^2}{\pi}\right)^{\frac{3}{2}} e^{-\frac{1}{2}\alpha_N^2 r^2}, \quad \phi_{000}^A(r) = \left(\frac{\alpha_A^2}{\pi}\right)^{\frac{3}{2}} e^{-\frac{1}{2}\alpha_A^2 r^2},$$

where

$$\alpha_N^2 = \frac{m_N}{\hbar} \sqrt{\omega_s^2 + \frac{2}{3}\omega_1^2 + \frac{1}{3}\omega_2^2}, \quad \alpha_A^2 = \frac{m_A}{\hbar} \sqrt{\omega_s^2 + \omega_3^2}.$$

The two-particle function is

$$\bar{\psi}_{NA}^0(\mathbf{r}_1, \mathbf{r}_4) = \left(\frac{\beta_A^2}{\pi}\right)^{\frac{3}{2}} e^{-\frac{1}{2}\beta_A^2 R^2} \psi_{NA}(r) d^3 r d^3 R,$$

where $\mathbf{R}$ and $\mathbf{r}$ are the centre-of-mass and relative coordinates for this two-particle system, and

$$\beta_A^2 = \frac{m_N + m_A}{\hbar} \sqrt{\omega_s^2 + \frac{2(m_N \omega_1^2 + m_A \omega_3^2)}{3(m_N + m_A)}},$$

and $\psi_{NA}(r)$ is the solution ⁸⁾ of the relative motion part of eq. (40) with the $\mathbf{R} \cdot \mathbf{r}$ term missing.

Changing to the centre-of-mass and relative coordinates, the normalization integral (A.1) becomes

$$\left(\frac{\alpha_N^2 \alpha_A^2 \beta_A^2}{\pi^3}\right)^{\frac{3}{2}} \int e^{-\frac{1}{2}(\alpha_N^2 + \beta_A^2)R^2} e^{-\frac{1}{2}\alpha_A^2 r^2} e^{-\lambda \mathbf{R} \cdot \mathbf{r}} \psi_{NA}(r) d^3 r d^3 R,$$

where

$$\alpha_R^2 = \alpha_N^2 + \alpha_A^2, \quad \alpha_r^2 = \frac{m_A^2 \alpha_N^2 + m_N^2 \alpha_A^2}{(m_N + m_A)^2},$$

$$\lambda = \frac{m_A \alpha_N^2 - m_N \alpha_A^2}{m_N + m_A}.$$

The integral over $\mathbf{R}$ can be carried out simply. Using $\mathbf{r}$ as a reference axis to express $\mathbf{R}$ in spherical coordinates, the integrations first over the angular variables and then over the magnitude R are elementary integrals. The result is

$$K_{00}^{NA} = \left(\frac{2\alpha_N \alpha_A \beta_A}{\sqrt{\pi}(\alpha_N^2 + \alpha_A^2 + \beta_A^2)}\right)^{\frac{3}{2}} \int e^{-\frac{1}{2}\delta^2 r^2} \psi_{NA}(r) d^3 r, \quad (\text{A.2})$$

where

$$\delta^2 = \frac{(m_N + m_A)^2 \alpha_N^2 \alpha_A^2 + \beta_A^2 (m_A^2 \alpha_N^2 + m_N^2 \alpha_A^2)}{(m_N + m_A)^2 (\alpha_N^2 + \alpha_A^2 + \beta_A^2)}.$$

With the substitution of m_N for m_A ; α_N for α_A ; and

$$\beta_N^2 = \frac{2m_N}{\hbar} \sqrt{\omega_s^2 + \frac{1}{3}(\omega_1^2 + \omega_2^2)}$$

for β_A^2 ; and $\psi_{NN}(r)$ for $\psi_{NA}(r)$, we obtain the simpler integral K_{00}^{NN} (see eq. (33)) from the expression (A.2); i.e.,

$$K_{00}^{NN} = \left(\frac{2\alpha_N^2 \beta_N}{\sqrt{\pi(2\alpha_N^2 + \beta_N^2)}} \right)^{\frac{3}{2}} \int e^{-\frac{1}{2}\alpha_N^2 r^2} \psi_{NN}(r) d^3 r. \quad (A.3)$$

The function $\psi_{NN}(r)$ is the ground state solution⁸⁾ for the relative motion part of the zero order form of eq. (29). The integrals over the relative coordinates in eqs. (A.2) and (A.3) are essentially the same as those evaluated in ref.⁸⁾

We next reduce the integral K_{10}^{NA} (see eq. (34)) to an integral over the relative coordinates. Using the oscillator wave function solutions⁸⁾ of the zero order forms of eqs. (27) and (28) for the projecting one-particle functions, this integral becomes

$$\begin{aligned} K_{10}^{NA} &= \int \phi_{100}^N(r_1) \phi_{000}^A(r_4) \bar{\psi}_{NA}^0(r_1, r_4) d^3 r_1 d^3 r_4 \\ &= \sqrt{\frac{3}{2}} K_{00}^{NA} - \sqrt{\frac{2}{3}} \alpha_N^2 \left(\frac{\alpha_N^2 \alpha_A^2 \beta_A^2}{\pi^3} \right)^{\frac{3}{2}} \int e^{-\frac{1}{2}(\alpha_N^2 r_1^2 + \alpha_A^2 r_4^2 + \beta_A^2 R^2)} r_1^2 \psi_{NA}(r) d^3 r_1 d^3 r_4 \\ &= \sqrt{\frac{3}{2}} K_{00}^{NA} - 2\sqrt{\frac{2}{3}} \alpha_N^2 \left(\frac{\alpha_N^2 \alpha_A^2 \beta_A^2}{\pi^3} \right)^{\frac{3}{2}} \frac{\partial}{\partial \alpha_N^2} \int e^{-\frac{1}{2}(\alpha_N^2 r_1^2 + \alpha_A^2 r_4^2 + \beta_A^2 R^2)} \psi_{NA}(r) d^3 r_1 d^3 r_4 \\ &= \sqrt{\frac{3}{2}} K_{00}^{NA} - \sqrt{\frac{2}{3}} \alpha_N^2 \left(\frac{2\alpha_N \alpha_A \beta_A}{\sqrt{\pi(\alpha_N^2 + \alpha_A^2 + \beta_A^2)}} \right)^{\frac{3}{2}} \left[\frac{3}{(\alpha_N^2 + \alpha_A^2 + \beta_A^2)} \int e^{-\frac{1}{2}\delta^2 r^2} \psi_{NA}(r) d^3 r \right. \\ &\quad \left. + \frac{\partial \delta^2}{\partial \alpha_N^2} \int e^{-\frac{1}{2}\delta^2 r^2} \psi_{NA}(r) r^2 d^3 r \right], \quad (A.4) \end{aligned}$$

where

$$\frac{\partial \delta^2}{\partial \alpha_N^2} = \frac{\delta^2}{\alpha_N^2(\alpha_N^2 + \beta_A^2)} \left[\alpha_A^2 + \beta_A^2 - \frac{m_N^2 \alpha_A^2 \beta_A^2}{(m_N + m_A)^2 \delta^2} \right].$$

A similar expression for K_{10}^{NA} is obtained from eq. (A.4) by interchanging m_N and m_A and α_N and α_A .

The simpler integral $K_{10}^{NN} = K_{01}^{NN}$ is obtained from eq. (A.4) by substituting α_N , β_N , m_N , and $\psi_{NN}(r)$ for α_A , β_A , m_A and $\psi_{NA}(r)$, respectively.

The integrals K_{ij}^{NA} (see eq. (34)) for other values of i and j can be reduced by the same technique as is used above to obtain eq. (A.4). In this reduction, the higher derivatives of δ^2 are needed. The second derivative turns out to be

$$\frac{\partial^2(\delta^2)}{\partial(\alpha_N^2)^2} = -\frac{2}{(\alpha_N^2 + \alpha_A^2 + \beta_A^2)} \frac{\partial \delta^2}{\partial \alpha_N^2},$$

and thus all the higher derivatives can be expressed simply in terms of the first derivative.

While the integrals on the right hand sides of eqs. (35), (36) and (37) are more complicated than the above K -integrals because they contain the two-particle potentials, the integrations over the centre of mass coordinates are identical. Thus, for example, the integration over the centre-of-mass coordinates in the integral

$$\int \phi_{100}^N(r_1) \phi_{000}^A(r_4) u_{NA}(r_{14}) \bar{\psi}_{NA}^0(r_1, r_4) d^3 r_1 d^3 r_4$$

yields the expression (A.4) except that u_{NA} must be included in the integrands of the final integrations over the relative coordinates.

An added complication occurs in the expression for E_{int} , eq. (8), where we need to evaluate

$$\int \phi_{000}^N(r_1) \phi_{000}^A(r_4) (\nabla_{r_1} \cdot \nabla_{r_4}) \bar{\psi}_{NA}^0(r_1, r_4) d^3 r_1 d^3 r_2.$$

Integrating by parts transfers the derivative operators to the single particle functions. Carrying out these differentiations changes the integral to

$$\alpha_N^2 \alpha_A^2 \int \phi_{000}^N(r_1) \phi_{000}^A(r_4) (r_1 \cdot r_4) \bar{\psi}_{NA}^0(r_1, r_4) d^3 r_1 d^3 r_4.$$

Changing to the centre-of-mass and relative coordinates leads to the sum of three integrals since

$$r_1 \cdot r_4 = R^2 - \frac{m_N m_A}{(m_N + m_A)^2} r^2 + \left(\frac{m_A - m_N}{m_N + m_A} \right) \mathbf{R} \cdot \mathbf{r}.$$

The integration over the centre-of-mass coordinates for the second term is the same as for the K -integrals. For the first and third terms, the centre-of-mass integrals can be evaluated by the same technique of integrating first over the angular coordinates of $\mathbf{R}$ and then over the magnitude. In these cases we get

$$\begin{aligned} & \int \phi_{000}^N(r_1) \phi_{000}^A(r_4) R^2 \bar{\psi}_{NA}^0(r_1, r_4) d^3 r_1 d^3 r_4 \\ &= \frac{3}{\alpha_N^2 + \alpha_A^2 + \beta_A^2} + \left(\frac{2\alpha_N \alpha_A \beta_A}{\sqrt{\pi}} \right)^{\frac{1}{2}} \frac{(m_A \alpha_N^2 - m_N \alpha_A^2)^2}{(m_N + m_A)^2 (\alpha_N^2 + \alpha_A^2 + \beta_A^2)^{\frac{1}{2}}} \int e^{-\frac{1}{2} \delta^2 r^2} r^2 \psi_{NA}(r) d^3 r, \\ & \int \phi_{000}^N(r_1) \phi_{000}^A(r_4) \mathbf{R} \cdot \mathbf{r} \bar{\psi}_{NA}^0(r_1, r_4) d^3 r_1 d^3 r_4 \\ &= \left(\frac{2\alpha_N \alpha_A \beta_A}{\sqrt{\pi}} \right)^{\frac{1}{2}} \frac{(m_N \alpha_A^2 - m_A \alpha_N^2)}{(m_N + m_A) (\alpha_N^2 + \alpha_A^2 + \beta_A^2)^{\frac{1}{2}}} \int e^{-\frac{1}{2} \delta^2 r^2} r^2 \psi_{NA}(r) d^3 r. \end{aligned}$$

All the integrals discussed in this appendix have been reduced to integrations over the relative coordinates only. These can be evaluated by the techniques given in ref. ⁸).

A.2. THE THREE- AND FIVE-BODY HYPERFRAGMENT EQUATIONS

The corresponding equations for the three- and five-body hyperfragments can be obtained from the equations given for the four-body hyperfragment by appropriate changes in the coefficients. We list here only those changes which are not obvious giving in each case two new coefficients for each old one. The first is for the three-body system and the second for the five-body case.

In the energy expressions (7) and (8) the factors 3 are replaced by 2 and 4. In eq. (10) the coefficients 2 are replaced by 1 and 3 while the term

$$\int \phi_0^N(r_2) \phi_0^N(r_3) u_{NN}(r_{23}) \psi_{NNN}(r_1, r_2, r_3) d^3 r_2 d^3 r_3$$

does not occur in the three-body case and occurs with a factor 2 in the five-body case. Similarly, in the right hand side of eq. (11), the second and last terms do not occur in the corresponding equation for the three-body hyperfragment while these same terms occur with factors of 2 in the five-body equations. In addition, the five-body equation includes the term

$$\int \phi_0^N(r_3) \phi_0^N(r_4) u_{NN}(r_{34}) \psi_{NNNN}(r_1, r_2, r_3, r_4) d^3 r_3 d^3 r_4.$$

The factors 3 in the non-local potential expressions (16) to (18) are changed to 2 and 4. In eq. (12), the potentials

$$\frac{2}{3} U_{NN} + \frac{1}{3} U_{NA}$$

are replaced by

$$\frac{1}{2} U_{NN} + \frac{1}{2} U_{NA} \quad \text{and} \quad \frac{3}{4} U_{NN} + \frac{1}{4} U_{NA}.$$

The potentials

$$\frac{1}{3} U_{NA}(r_1) + \frac{1}{3} U_{NA}(r_2) + \frac{1}{3} U_{NN}(r_1) + \frac{1}{3} U_{NN}(r_2)$$

in eq. (14) are changed to

$$\frac{1}{2} U_{NA}(r_1) + \frac{1}{2} U_{NA}(r_2)$$

and to

$$\frac{1}{4} U_{NA}(r_1) + \frac{1}{4} U_{NA}(r_2) + \frac{1}{2} U_{NN}(r_1) + \frac{1}{2} U_{NN}(r_2),$$

while the potentials

$$\frac{2}{3} U_{NN}(r_1) + \frac{2}{3} U_{NA}(r_4)$$

in eq. (15) become

$$\frac{1}{2} U_{NN}(r_1) + \frac{1}{2} U_{NA}(r_4) \quad \text{and} \quad \frac{3}{4} U_{NN}(r_1) + \frac{3}{4} U_{NA}(r_4).$$

Finally, the approximate energy expression (26) is

$$E \approx \varepsilon_{NN} + 2\varepsilon_{NA} - 2\varepsilon_N - \varepsilon_A$$

for the three-body hyperfragment and

$$E \approx 6\epsilon_{NN} + 4\epsilon_{NA} - 12\epsilon_N - 3\epsilon_A$$

for the five-body system.

References

- 1) R. Dalitz and B. Downs, *Phys. Rev.* **114** (1959) 593, **111** (1958) 967, **110** (1958) 958
- 2) D. B. Lichtenberg, *Nuovo Cim.* **8** (1958) 463
- 3) R. H. Dalitz, in *Proc. Rutherford Jubilee Int. Conf.* (Heywood and Co., London, 1961) p. 103
- 4) N. Crayton *et al.*, *Phys. Rev.*, to be published;
B. Bhowmik, D. P. Goyal and N. K. Yamdagmi, *Nuclear Physics* **40** (1963) 457
- 5) G. Alexander and W. Lasker, *Lawrence Radiation Laboratory, Physics Note* 301;
O. Dahl, N. Horwitz, D. H. Miller, J. J. Murray and P. G. White, *Phys. Rev. Lett.* **6** (1961) 142
- 6) R. G. Sachs, *Nuclear theory* (Addison-Wesley Publ. Co., Cambridge, Mass., 1953) pp. 188-190
- 7) T. Ohmura (Kikuta), M. Morita and M. Yamada, *Progr. Theor. Phys.* **15** (1956) 222, **17** (1957) 326;
T. Ohmura, *Progr. Theor. Phys.* **22** (1959) 34;
J. M. Blatt and G. Derrick, *Nuclear Physics* **8** (1958) 602
- 8) H. J. Mang and W. Wild, *Z. Phys.* **154** (1959) 182
- 9) J. L. Gammel and R. M. Thaler, *Phys. Rev.* **107** (1957) 291, 1337;
P. S. Signell and R. E. Marshak, *Phys. Rev.* **109** (1958) 1229, **106** (1957) 832
- 10) M. Razavy, G. Field and J. S. Levinger, *Phys. Rev.* **125** (1962) 269
- 11) D. B. Lichtenberg and M. Ross, *Phys. Rev.* **104** (1957) 1714
- 12) B. W. Downs, D. R. Smith and T. N. Truong, *Phys. Rev.* **129** (1963) 2730
- 13) J. Brenig, *Nuclear Physics* **4** (1957) 363
- 14) R. Dalitz, *Ninth Int. Ann. Conf. on High Energy Physics* (Academy of Sciences, USSR, Moscow, 1960), vol. 1, p. 587;
A. R. Bodmer and S. Sampanthar, *Nuclear Physics* **31** (1962) 251
- 15) J. M. Blatt and V. F. Weisskopf, *Theoretical nuclear physics* (Wiley, New York, 1952) p. 193
- 16) H. Lipkin, *Phys. Rev.* **109** (1958) 2071
- 17) L. C. Gomes, J. D. Walecka and V. F. Weisskopf, *Ann. of Phys.* **3** (1958) 241
- 18) J. M. Blatt and J. D. Jackson, *Phys. Rev.* **76** (1949) 18;
L. Hulthén and M. Sugawaro, *Handbuch der Physik*, (Springer, Berlin, 1957) Vol. 39
- 19) K. Dietrich, R. Folk and H. J. Mang, in *Proc. Rutherford Jubilee Int. Conf.* (Heywood and Co., London, 1961) p. 165