A Four-site Higgsless Model with Wavefunction Mixing

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Motivated by models of holographic technicolor, we discuss a four-site deconstructed Higgsless model with nontrivial wavefunction mixing. We compute the spectrum of the model, the electroweak triple gauge boson vertices, and, for brane-localized fermions, the electroweak parameters to $\mathcal{O}(M_W^2/M_\rho^2)$. We discuss the conditions under which αS vanishes (even for brane-localized fermions) and the (distinct but overlapping) conditions under which the phenomenologically interesting decay $a_1 \to W\gamma$ is non-zero and suppressed by only one power of (M_W/M_ρ).

I. INTRODUCTION

Higgsless models of electroweak symmetry breaking [1] may be viewed as "dual" to more conventional technicolor models [2, 3] and, as such, provide a basis for constructing low-energy effective theories to investigate the phenomenology of a strongly interacting symmetry breaking sector [4, 5]. One approach to constructing such an effective theory, the three-site model [6], includes only the lightest of the extra vector mesons typically present in such theories – the meson analogous to the ρ in QCD. An alternative approach is given by "holographic technicolor" [7], which potentially provides a description of the first two extra vector mesons – including, in addition to the ρ, the analog of the a_1 meson in QCD.

In this note we consider consider a four-site "Higgsless" model [8] illustrated, using "moose notation" [9], in fig. 1. We show how, once an L_{10}-like "wavefunction" mixing term for the two strongly-coupled $SU(2)$ groups in the center of the moose is included, we can reproduce the features of the holographic model – including the vanishing of the parameter αS for brane-localized fermions and the existence (whether or not $\alpha S = 0$) of the potentially interesting decay $a_1 \to W\gamma$.

II. THE MODEL

The Lagrangian for the model consists of several parts. First, the usual nonlinear sigma model link terms

$$\mathcal{L}_\pi = \frac{f_1^2}{4}\left[\mathrm{Tr} D^\mu \Sigma_1 D_\mu \Sigma_1^\dagger + \mathrm{Tr} D^\mu \Sigma_3 D_\mu \Sigma_3^\dagger\right] + \frac{f_2^2}{4}\mathrm{Tr} D^\mu \Sigma_2 D_\mu \Sigma_2^\dagger \ . \quad (1)$$

Next, the gauge-boson kinetic energies

$$\mathcal{L}_{gauge} = -\frac{1}{4}\left(\vec{W}_{0\mu\nu}^2 + \vec{W}_{1\mu\nu}^2 + \vec{W}_{2\mu\nu}^2 + \vec{W}_{3\mu\nu}^2\right) \ , \quad (2)$$

where we denote the weakly-coupled $SU(2) \times U(1)$ fields by $\vec{W}_0$ and $\vec{W}_3 \equiv B$ (by convention, $i = 3$ vanishes for the charged sector), and the strongly coupled $SU(2)$ fields by $\vec{W}_{1,2}$. And finally, there is an L_{10}-like mixing between the

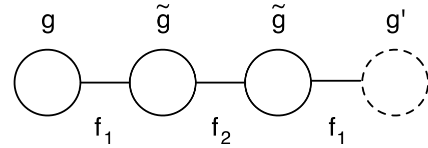

FIG. 1: The "moose" diagram [9] for the $SU(2)^3 \times U(1)$ model considered in this note. The solid circles represent $SU(2)$ groups; the dashed circle, a $U(1)$ group; the "links", $SU(2) \times SU(2)/SU(2)$ non-linear sigma models. In order to be phenomenologically realistic [10], we work in the limit $g, g' \ll \tilde{g}$; in this limit the model also has an approximate parity symmetry. We consider brane-localized fermions, which couple only the the $SU(2) \times U(1)$ at the ends of the moose, and add an L_{10}-like "wavefunction mixing" term to mix the two strongly-coupled $SU(2)$ groups in the middle two sites.

middle two sites

$$\mathcal{L}_\varepsilon = -\frac{\varepsilon}{2}\mathrm{Tr}\left[\vec{W}_{1\mu\nu}\Sigma_2 \vec{W}_2^{\mu\nu}\Sigma_2^\dagger\right] \ , \quad (3)$$

where in this calculation we treat ε as an $\mathcal{O}(1)$ parameter. This model has a "parity" (more precisely, a G-parity) symmetry in the $g = g' = 0$ limit, under which $\vec{W}_1^\mu \to \vec{W}_2^\mu$, $\Sigma_1 \to \Sigma_3^\dagger$, and $\Sigma_2 \to \Sigma_2^\dagger$. In the limit $f_2 \to \infty$,[1] this model reduces to the three-site model considered in [6].

In unitary gauge (with $\Sigma_1 = \Sigma_2 = \Sigma_3 \equiv \mathcal{I}$), the $\mathcal{L}_\varepsilon$ term above corresponds to wavefunction-mixing of the fields $\vec{W}_i$,

$$\mathcal{L} = -\frac{1}{4}\vec{W}_{i\mu\nu}\tilde{Z}_{ij}\vec{W}_j^{\mu\nu} - \frac{1}{2}\vec{W}_{i\mu}M_{ij}^2\vec{W}_j^\mu \ , \quad (4)$$

with

$$\tilde{Z} = \begin{pmatrix} 1 & & & \\ & 1 & \varepsilon & \\ & \varepsilon & 1 & \\ & & & 1 \end{pmatrix} \ . \quad (5)$$

To avoid ghosts, we require $\tilde{Z}$ to be positive-definite, and hence $|\varepsilon| < 1$.

[1] For fixed values of $2/f_1^2 + 1/f_2^2$, see eqn. (23).

III. MASSES AND MIXING ANGLES

The eigenstates corresponding to the quadratic part of Lagrangian in eqn. (4) satisfy the generalized eigenvalue equation

$$M^2 \vec{v}_n = m_n^2 \tilde{Z} \vec{v}_n \ , \tag{6}$$

where $\vec{v}_n$ is a vector in site-space with components v_n^i. The superscript i labels the sites, running from 0 to 2 for charged-bosons ($n = W^\pm, \rho^\pm, a_1^\pm$), and 0 to 3 for neutral ones ($n = Z^0, \rho^0, a_1^0, \gamma$). If we choose eigenvectors normalized by $\vec{v}_n^T \tilde{Z} \vec{v}_m = \delta_{nm}$, the gauge-eigenstate ($W^i_\mu$) and mass-eigenstate ($W'_{n\mu}$) fields are related by

$$W^i_\mu = \sum_n v_n^i W'_{n\mu} \ . \tag{7}$$

A. The $g = g' = 0$ Limit

Consider first the $g = g' = 0$ limit, in which we can determine the leading contributions to the heavy gauge-boson masses. Due to the parity symmetry in this limit, we expect the eigenvectors to be proportional to $\vec{W}_1^\mu \pm \vec{W}_2^\mu$. Applying the normalization condition $\vec{v}_n^T \tilde{Z} \vec{v}_m = \delta_{nm}$, we find a parity-even eigenvector (the "ρ")

$$\vec{\rho}^\mu = \frac{1}{\sqrt{2(1+\varepsilon)}} \left(\vec{W}_1^\mu + \vec{W}_2^\mu \right) \ , \tag{8}$$

with mass

$$m_\rho^2 = \frac{\tilde{g}^2}{4} \frac{f_1^2}{1+\varepsilon} \ , \tag{9}$$

and a parity-odd eigenvector (the "a_1")

$$\vec{a}_1^\mu = \frac{1}{\sqrt{2(1-\varepsilon)}} \left(\vec{W}_1^\mu - \vec{W}_2^\mu \right) \ , \tag{10}$$

with mass

$$m_{a_1}^2 = \frac{\tilde{g}^2}{4} \frac{f_1^2 + 2f_2^2}{1-\varepsilon} \ . \tag{11}$$

We note that the ρ and a_1 are degenerate for

$$\varepsilon = -\frac{f_2^2}{f_1^2 + f_2^2} \ , \tag{12}$$

a value satisfying the constraint $|\varepsilon| < 1$. As ε becomes more negative, the a_1 becomes lighter than the ρ.

B. The Photon

Examining the eigenvalue eqn. (6) we see that the wavefunction factor $\tilde{Z}$ affects the normalization of a massless eigenvector, but not the orientation. We see, therefore, that the photon must be of the form

$$A_\mu = \frac{e}{g} W^3_{0\mu} + \frac{e}{\tilde{g}} W^3_{1\mu} + \frac{e}{\tilde{g}} W^3_{2\mu} + \frac{e}{g'} B_\mu \ , \tag{13}$$

or

$$(v_\gamma)^T = \left(\frac{e}{g} \ , \ \ \frac{e}{\tilde{g}} \ , \ \ \frac{e}{\tilde{g}} \ , \ \ \frac{e}{g'} \right) \ . \tag{14}$$

The electric charge e is, then, determined from the normalization condition to be

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} + \frac{2(1+\varepsilon)}{\tilde{g}^2} \ . \tag{15}$$

Examining the photon-couplings, we see that the unbroken gauge-generator has the expected form $Q = T^3 + T^3_1 + T^3_2 + Y$.

C. The W-boson

Next, we consider a perturbative evaluation of the electroweak boson eigenvectors and eigenvalues, computed in powers of $x = g/\tilde{g}$. We start with the W-boson; the charged-boson mass matrix is given by

$$M_W^2 = \frac{\tilde{g}^2}{4} \begin{pmatrix} x^2 f_1^2 & -x f_1^2 & 0 \\ -x f_1^2 & f_1^2 + f_2^2 & -f_2^2 \\ 0 & -f_2^2 & f_1^2 + f_2^2 \end{pmatrix} \ . \tag{16}$$

To $\mathcal{O}(x^2)$ we find

$$\begin{aligned} v_W^0 &= \left[1 - \frac{f_1^4 + 2(1+\varepsilon) f_1^2 f_2^2 + 2(1+\varepsilon) f_2^4}{2(f_1^2 + 2f_2^2)^2} \, x^2 \ , \right] \\ v_W^1 &= x \, \frac{f_1^2 + f_2^2}{f_1^1 + 2f_2^2} W_1 \ , \\ v_W^2 &= x \, \frac{f_2^2}{f_1^2 + 2f_2^2} W_2 \ , \end{aligned} \tag{17}$$

where we have computed, but do not display, the corrections of $\mathcal{O}(x^3)$ to the last two components. For the corresponding eigenvalue we find

$$m_W^2 = \frac{g^2}{4} \frac{f_1^2 f_2^2}{f_1^2 + 2f_2^2} \left[1 - \frac{f_1^4 + 2(1+\varepsilon) f_1^2 f_2^2 + 2(1+\varepsilon) f_2^4}{(f_1^2 + 2f_2^2)^2} \, x^2 \right] \ . \tag{18}$$

D. The Z-boson

The neutral gauge-boson mass matrix is

$$M_Z^2 = \begin{pmatrix} x^2 f_1^2 & -x f_1^2 & 0 & 0 \\ -x f_1^2 & f_1^2 + f_2^2 & -f_2^2 & 0 \\ 0 & -f_2^2 & f_1^2 + f_2^2 & -x \tan\theta f_1^2 \\ 0 & 0 & -x \tan\theta f_1^2 & x^2 \tan^2\theta f_1^2 \end{pmatrix} \ . \tag{19}$$

where we have defined the angle θ by $g'/g \equiv \tan\theta$. Note that θ is the *leading order* weak mixing angle; we will later define a weak mixing angle θ_Z that is better suited to comparison with experiment. We have computed the Z-boson eigenvector to $\mathcal{O}(x^3)$ – as the result is complicated, and the algebra unilluminating, we do not reproduce it here. For the Z-boson mass, we find

$$m_Z^2 = \frac{g^2}{4\cos^2\theta}\frac{f_1^2 f_2^2}{f_1^2+2f_2^2}\left[1 - \frac{(3-\varepsilon)f_1^4 + 4(1+\varepsilon)(f_1^2 f_2^2 + f_2^4) + (1+\varepsilon)(f_1^2+2f_2^2)^2\cos 4\theta}{4(f_1^2+2f_2^2)^2}\, x^2 \sec^2\theta\right] . \qquad (20)$$

IV. THE ELECTROWEAK PARAMETERS

From eqn. (7), we can compute the couplings of the mass-eigenstate electroweak gauge-bosons to fermions. For brane-localized fermion couplings of the form

$$\mathcal{L}_f = g_0 \vec{J}_L^\mu \cdot \vec{W}_\mu^0 + g' J_Y^\mu B_\mu , \qquad (21)$$

we find the mass-eigenstate W-boson couplings $g_W^f = g_0 v_W^0$ and the Z-boson couplings

$$g_Z^f = g v_Z^0 I_3 + g' v_Z^3 Y = g I_3 (v_Z^0 - \tan\theta v_Z^3) + g' v_Z^3 \mathcal{Q} . \qquad (22)$$

We may then compute the on-shell precision electroweak parameters at tree-level to $\mathcal{O}(x^2)$, using the definitions and procedures outlined in [10, 11]. The values of electric charge, eqn. (15), and m_Z^2, eqn. (20), are given above, and we find the Fermi constant

$$\sqrt{2} G_F = \frac{1}{v^2} = \frac{2}{f_1^2} + \frac{1}{f_2^2} , \qquad (23)$$

where $v \approx 246$ GeV.

The only non-zero precision electroweak parameter parameter is αS [12], for which we find

$$\frac{\alpha S}{4 s^2} = \frac{\varepsilon f_1^4 + 2(1+\varepsilon) f_1^2 f_2^2 + 2 f_2^4 (1+\varepsilon)}{(f_1^2+2f_2^2)^2}\, x^2 , \qquad (24)$$

As expected [5, 7], we can choose ε so that αS vanishes for any given value of f_1/f_2

$$\varepsilon \to -\frac{2(f_2^4 + f_1^2 f_2^2)}{f_1^4 + 2 f_1^2 f_2^2 + 2 f_2^4} , \qquad (25)$$

while satisfying $|\varepsilon| < 1$.

Note, however, that the value of the low-energy parameter $|\varepsilon|$ that makes αS vanish is of order one, larger than would be expected by naive dimensional analysis [13]. This result is consistent with investigations of continuum 5d effective theories [14, 15], and with investigations of plausible conformal technicolor "high-energy completions" of this model using Bethe-Salpeter methods [16, 17], both of which suggest that $\alpha S > 0$ and that it may not be possible to achieve very small values of αS. We note also that the result is consistent with the expectation of [18, 19], since the value of ε required for αS to vanish results in axial-vector mesons which are lighter than the vector mesons.[2]

V. TRIPLE BOSON VERTICES

A. Electroweak Vertices

Consider the electroweak vertices γWW and ZWW. To leading order, in the absence of CP-violation, the triple gauge boson vertices may be written [23]

$$\begin{aligned}
\mathcal{L}_{TGV} &= -ie\frac{c_Z}{s_Z}\left[1+\Delta\kappa_Z\right] W_\mu^+ W_\nu^- Z^{\mu\nu} \\
&- ie\left[1+\Delta\kappa_\gamma\right] W_\mu^+ W_\nu^- A^{\mu\nu} \\
&- ie\frac{c_Z}{s_Z}\left[1+\Delta g_1^Z\right](W^{+\mu\nu}W_\mu^- - W^{-\mu\nu}W_\mu^+) Z_\nu \\
&- ie(W^{+\mu\nu}W_\mu^- - W^{-\mu\nu}W_\mu^+) A_\nu , \qquad (26)
\end{aligned}$$

where the two-index tensors denote the Lorentz field-strength tensor of the corresponding field. In the standard model, $\Delta\kappa_Z = \Delta\kappa_\gamma = \Delta g_1^Z \equiv 0$. Note that the expressions for κ_Z and g_1^Z involve $c_Z \equiv \cos\theta_Z$ and $s_Z \equiv \sin\theta_Z$, as defined by

$$c_Z^2 s_Z^2 = \frac{e^2}{4\sqrt{2} G_F M_Z^2}, \qquad (27)$$

rather than the leading order mixing angle θ.

Let us begin with the coupling of the photon of the form $(W^{+\mu\nu}W_\mu^- - W^{-\mu\nu}W_\mu^+)A_\nu$. In terms of the wavefunctions $v_{\gamma,W}$, this coupling is proportional to

$$g_\gamma = \sum_{i,j} g_i v_\gamma^i v_W^i \tilde{Z}_{ij} v_W^j . \qquad (28)$$

From eqn. (14), we have $g_i v_\gamma^i \equiv e$ and therefore, by applying the normalization condition $\vec{v}_W^T \tilde{Z} \vec{v}_W = 1$, we

[2] An alternative approach, Degenerate BESS [20, 21], produces degenerate vector and axial mesons and $\alpha S = 0$ using a different theory without unitarity delay [10] – see "case I" described in [22].

obtain $g_\gamma \equiv e$ independent of any choice of the four-site parameters — as required by gauge-invariance and consistent with the form of eqn. (26).

Next, we evaluate $\Delta\kappa_\gamma$, with

$$e\,[1+\Delta\kappa_\gamma] = \sum_{i,j} g_i (v_W^i)^2 \tilde{Z}_{ij} v_\gamma^j = e \sum_{i,j} \frac{g_i}{g_j} (v_W^i)^2 \tilde{Z}_{ij} \ , \tag{29}$$

for which we calculate

$$\Delta\kappa_\gamma = \frac{\varepsilon\, f_1^4}{(f_1^2+2f_2^2)^2}\, x^2 = \frac{\varepsilon\, v^4}{f_2^4}\, x^2 \ . \tag{30}$$

Note that this vanishes in the absence of wavefunction mixing ($\varepsilon \to 0$), and also in the "three-site" limit ($v/f_2 \to 0$), as consistent with [6].

Similarly we may compute Δg_1^Z and $\Delta\kappa_Z$, and we find

$$\begin{aligned}\Delta g_1^Z &= \Delta\kappa_Z + \frac{\varepsilon f_1^4 \tan^2\theta_Z\, x^2}{(f_1^2+2f_2^2)^2} \ , \qquad (31)\\ &= -\frac{(\varepsilon s_Z^2 f_1^4 + (1+\varepsilon) f_1^2 f_2^2 + (1+\varepsilon) f_2^4)}{(f_1^2+2f_2^2)^2 \cos(2\theta_Z)} \frac{x^2}{c_Z^2} \ ,\end{aligned}$$

where the difference between θ and θ_Z is irrelevant to this order. Note that $\Delta g_1^Z - \Delta\kappa_Z$ vanishes when $\varepsilon \to 0$, and also, as expected [6], in the "three-site" limit $f_2 \to \infty$.

B. ρ, $a_1 \to W + \gamma$

Finally, we consider the $(\rho, a_1) - W - \gamma$ couplings that motivated this study. Electromagnetic gauge-invariance implies that the coupling of the form $(\rho^{+\mu\nu} W^-_\mu - W^{-\mu\nu}\rho^+_\mu) A_\nu$ must vanish. Analogous to eqn. (28) we find that the $\rho - W - \gamma$ and $a_1 - W - \gamma$ couplings of this form are proportional to $\vec{v}_W^T \tilde{Z} \vec{v}_{\rho,a_1} \equiv 0$, and therefore vanish identically.

There is no reason, however, that terms proportional to $(\rho^+_\mu,\, a^+_{1\mu})\, W^-_\nu A^{\mu\nu}$ must vanish [5, 7]. In this case, we find

$$e\,\kappa_{\gamma W\rho} = \sum_{i,j} g_i v_W^i v_\rho^i \tilde{Z}_{ij} v_\gamma^j = e \sum_{i,j} \frac{g_i}{g_j} v_W^i v_\rho^i \tilde{Z}_{ij} \ , \tag{32}$$

and similarly for the a_1. Computing these couplings to $\mathcal{O}(x^3)$, we find

$$\kappa_{\gamma W\rho} = -\frac{\varepsilon(1+\varepsilon)^{3/2} f_1^4}{2\sqrt{2}(f_1^2+2f_2^2)(\varepsilon f_1^2 + (1+\varepsilon) f_2^2)}\, x^3 \tag{33}$$

$$\kappa_{\gamma W a_1} = \frac{\sqrt{2}\,\varepsilon\, v^2}{\sqrt{1-\varepsilon\, f_2^2}}\, x \ . \tag{34}$$

Note that both couplings vanishes in the $\varepsilon \to 0$ and $f_2 \to \infty$ limits. Furthermore, while the $\rho - W - \gamma$ coupling is typically small ($\mathcal{O}(x^3)$), we find the $a_1 - W - \gamma$ coupling is only suppressed by x, consistent with [5, 7]. If the value of ε corresponds (25) to $\alpha S = 0$, then $\kappa_{\gamma W a_1}$ is

$$\kappa_{\gamma W a_1} = -\frac{2\sqrt{2} v^2 (f_1^2+f_2^2)\ x}{(f_1^2+2f_2^2)\sqrt{f_1^2+2f_1^2 f_2^2+2f_2^2}} . \tag{35}$$

As mentioned earlier, for this value of ε, the a_1 state is lighter than the ρ.

VI. SUMMARY

We have introduced a deconstructed Higgsless model with four sites and non-trivial wavefunction mixing, and have shown that it exhibits key features of holographic technicolor [5, 7]. The electroweak parameter αS vanishes for a value of the wavefunction mixing at which the a_1 is lighter than the ρ – even if all fermions are brane-localized. Furthermore, the model includes the decay $a_1 \to W\gamma$, suppressed by only one power of (M_W/M_ρ), in contrast with an $(M_W/M_\rho)^3$ suppression of the decay $\rho \to W\gamma$. These decays are of potential phenomenological interest at LHC.

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