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A Trapped-Ion Mode

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February 24, 1977

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OVERLAP OF BOUNCE RESONANCES
AND THE MOTION OF IONS IN A TRAPPED-ION MODE

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February 24, 1977

ABSTRACT

Motion near a separatrix (the boundary between closed and open orbits) is studied both theoretically and numerically for parameters appropriate to the dissipative trapped-ion mode. We observe disappearance of an invariant in a stochastic layer surrounding the separatrix, as a result of overlapping bounce resonances. The fraction of ions lying within the stochastic layer is large even for a mode of relatively small amplitude.

Dissipative trapped-particle instabilities¹ are thought to cause anomalous transport in tokamaks. The transport rates are determined by the amplitude of the fluctuating electric field of the nonlinearly saturated instability. It is therefore important to consider nonlinear processes which might lead to saturation at relatively low levels.

In this Letter we report studies of a nonlinear process not previously considered, as far as we know, for

the trapped-ion mode. Some stabilizing effects of this process have been studied by us already, but estimates of saturation levels are reserved for publication elsewhere, because details of trapped-particle instability theory are required.

The equations of motion which we study describe other physical situations of current interest. The equations were first used² to study the one-dimensional motion of a particle in two electrostatic waves with different amplitudes and phase velocities. This configuration is of interest³ in the theory of radio-frequency heating. The equations were used⁴ for a tokamak to estimate the fraction of a magnetic island (due to a tearing mode) in which field lines would be braided. In the general theory of stochasticity,⁵ study of motion near a separatrix is recognized as being of fundamental importance.

We study the guiding center motion of an ion in a magnetic field whose amplitude varies sinusoidally, the usual model⁶ for a tokamak of large aspect ratio and circular cross section. The inclusion of guiding center drifts off a field line would not change the physics of the process being studied, so we ignore those drifts. Our calculations are thus in the spirit of local theory, in which radial excursions are assumed sufficiently small. We ignore collisions in order to isolate the effects, including

pitch-angle scattering, due to the electric field of the mode.

A trapped-ion mode causes perturbations in the motion of the ion. We assume a single mode is present, with toroidal and poloidal mode numbers l and m , respectively. As the ion moves along a field line, it feels the potential

$$\Phi(\Theta, t) = -\Phi_0 \cos[(m-lq)\Theta - \omega t + \eta], \quad (1)$$

where Θ is the poloidal angle, q the safety factor on the magnetic surface on which the ion moves, ω the mode frequency, and η a phase angle related to the ion's initial toroidal angle.

The equations of motion are derivable from the Hamiltonian

$$H(\Theta, p, t) = H_0(\Theta, p) + e\Phi(\Theta, t) \quad (2)$$

$$H_0 = (p/qR_0)^2/2M - \mu\Delta B \cos \Theta,$$

where p is the momentum conjugate to Θ , R_0 the major radius of the tokamak, e and M the ion charge and mass, ΔB the modulation amplitude of the magnetic field, and μ the magnetic moment. Since μ is conserved during the motion, it plays the role of a parameter here.

A more convenient form of Hamiltonian (2) for analytic work uses action and angle variables. We express the unperturbed Hamiltonian H_0 in terms of the longitudinal action

$$J(H_0) = (2\pi)^{-1} \oint p \, d\Theta,$$

which, except for constant factors, is $\oint v_{||} \, ds$, in standard

notation. The explicit expressions^{2,7} for J and the canonically conjugate angle variable ϕ involve elliptic integrals and K , defined by $2K^2 \equiv (1 + H_0/\mu\Delta B)$. The variables J and ϕ are defined in such a way that an ion which changes from trapped to circulating does not change its J or ϕ discontinuously. We denote by ω_b the frequency $\partial H_0/\partial J$ of the unperturbed motion. For a trapped ion ($K < 1$), ω_b is the frequency of bouncing between magnetic mirrors. For a circulating ion ($K > 1$), is half the frequency of transiting from one minimum of the magnetic field to the next. The bounce frequency of a deeply trapped ion ($K=0$) is $\omega_b(0) = (\mu\Delta B/M)^{1/2}/qR_0$.

In terms of the action-angle variables, Hamiltonian (2) appears

$$H(\phi, J, t) = H_0(J) + e\dot{\Phi}(\phi, J, t) \quad (3)$$

$$\dot{\Phi} = -\dot{\Phi}_0 \sum_n V_n(J) \cos(n\phi - \omega t + \eta) \quad .$$

The n th bounce resonance occurs when the n th term is slowly varying in time. From H_0 we have $\dot{\phi} = \omega_b(J)$, and thus the condition for bounce (or transit) resonance is

$$\omega_b(J) = \omega/n \quad . \quad (4)$$

Since $\omega_b(J) \rightarrow 0$ as $K \rightarrow 1$ for both trapped and circulating ions, there is a double infinity of values of J near the separatrix ($K=1$) which satisfies (4). The separation between resonances n and $n+1$ is

$$\delta_n \equiv \omega_b(J_n) - \omega_b(J_{n+1}) = \omega/n(n+1) \quad ,$$

where J_n is the value of J satisfying (4).

For the width of resonance n we ignore all other resonances and calculate from (3) the width of the "secondary" separatrix of resonance n . This separatrix divides the ϕ -plane into regions in which the phase $n\phi - \omega t$ is either bounded or unbounded in time. The width (in frequency) of resonance n is

$$\Delta_n = 4 \left| e \Phi_0 V_n(J_n) \partial \omega_b / \partial J_n \right|^{\frac{1}{2}}.$$

When resonances overlap, an invariant⁸ ceases to exist, which allows motion to be stochastic.⁹ A rough guide for the onset of stochastic motion is that the resonance width exceed the separation between resonances:⁵

$$\frac{1}{2}(\Delta_n + \Delta_{n+1}) \approx \Delta_n > \delta_n. \quad (5)$$

We choose the typical¹⁰ value $m - lq = \frac{1}{2}$ and look up¹¹ the explicit formulas for $V_n(J)$. Condition (5) then predicts overlap of all resonances with n greater than a critical n depending on Φ_0 . After some manipulation, we find from (5) how close an ion must be to the primary separatrix for its motion to be stochastic:

$$|E - \mu B_M| < (128/\pi^2) e \Phi_0 \omega / \omega_b(0). \quad (6)$$

In (6), E is the ion's energy and B_M is the maximum field on the magnetic surface on which the motion occurs.

The theory based on (5) actually predicts that the part of the stochastic layer lying outside the separatrix should be roughly half the width given by (6) for the inside part.

We find, however, that (6) describes our numerical results for both parts of the layer. The agreement is best for ions near the separatrix, where approximations used in the theory

are valid.

Refs. 3 and 4 gave a method for calculating the width of the stochastic layer for other values of $m-lq$. An expression analogous to (6) given in those works is valid in the limit of large $\omega/\omega_b(0)$. In deriving (6) we have used the opposite limit, which is appropriate to motion in a trapped-ion mode.

For our numerical calculations we choose

$$e\Phi_0 = 0.05 (\Delta B/B_0) T_i, \quad (7)$$

$$m-lq = \frac{1}{2}, \quad \omega = \frac{1}{2} \omega_{bi},$$

where $\omega_{bi} = (T_i \Delta B/B_0 M)^{1/2} / qR_0$, with T_i the ion temperature and B_0 the field at the magnetic axis. We have chosen Φ_0 comparable to the mode amplitudes at which other¹² nonlinear processes become important. We integrate the equations of motion derivable from (2) and convert the resulting $\Theta(t)$ and $p(t)$ to $\varphi(t)$ and $\bar{I}(t)$ using the explicit expressions mentioned above.

Stochastic effects can be strong for low-energy ions even for the relatively weak mode amplitude (7). The strength of the perturbation in (2) is measured by the parameter $e\Phi_0/\mu\Delta B$, which, for given Φ_0 , increases as μ decreases. Note also that $\omega/\omega_b(0) = (T_i/\mu B_0)^{1/2} \omega/\omega_{bi}$ increases as μ decreases.

In Fig. 1 we show the increase in width of the stochastic layer as μ is decreased. Using the trajectory

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information $\Phi(t)$ and $J(t)$, we plot a point whenever ωt is a multiple of 2π . When the points for a given trajectory lie on a curve, an invariant exists, while if they fill an area, an invariant does not exist. This surface of section method was used in earlier work⁸ on the overlap of cyclotron resonances, where more discussion appears.

The analytic form of the invariant is not simply J when a trapped-ion mode is present, as can be seen from Fig. 1. Near the large islands, which represent the resonances $\omega = \omega_b$, even the topology of the invariant curves changes, from open curves to closed ones. But the invariant ceases to exist, only where multiple resonances overlap. The bounce resonances discussed above cause disappearance of the invariant in the vicinity of the separatrix.

In Fig. 2 we show the region in velocity space in which stochastic effects are important. The horizontal axis is the parameter $H_0/\mu\Delta B$ which measures the depth of trapping in the magnetic well. The heavy lines surrounding the separatrix ($H_0 = \mu\Delta B$) give the observed width of the stochastic layer for a range of μ values. The widths were measured from plots such as Fig. 1. The dashed lines show the location of the ions satisfying $\omega = \omega_b$ and thus the location of the large islands in Fig. 1. The width of these islands has not been indicated on Fig. 2. The light lines are contour levels showing the number of ions in each small

area on Fig. 2; a Maxwellian distribution and $B_0/\Delta B = 4$ were assumed. We see that a large fraction of the ions lies within the stochastic region.

The single-mode assumption used in (1) is not crucial for the effects discussed above. If many modes were present, many more resonances (4) would appear, increasing the tendency for the motion to be stochastic. Plots like Fig. 1 show no qualitative changes when we multiply (1) by $g(\Theta) = \frac{1}{2}(1 + \cos \Theta)$ to model the ballooning of the mode on the outside of the torus.

We believe the overlap of bounce resonances is the physical mechanism behind some of the results observed in Ref. 13, where no explanation was given. Detrapping of particles due to ion sound turbulence¹⁴ also relies on the presence of electric field energy at multiples of ω_b . In contrast to Ref. 14 we have shown here that stochastic effects can occur in the presence of even a single mode. The strong perturbations of ion trajectories seen in Fig. 1 and the importance of resonant ions in trapped-ion mode theory suggest to us that the nonlinear effects shown here might profoundly influence saturation of the mode.

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FIGURE CAPTIONS

Fig. 1. Trajectories of ions in the presence of a trapped-ion mode with parameters given by (7). X 's indicate initial conditions for the trajectories, which are represented by plotting a point each wave period. A hand-drawn curve connecting the points shows existence of an invariant. The line at $J = 8/\pi$ is the separatrix. The parameter $\eta = 0$.

(a) $\mu B_0 = T_i$, (b) $\mu B_0 = \frac{1}{2} T_i$.

Fig. 2. Extent in velocity space of the stochastic region (between the heavy lines) for a trapped-ion mode given by (7).

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Fig. 1

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Fig. 2

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