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Subjective mortality risk and bequests

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Abstract

This paper investigates the ability of subjective expectations about life expectancy to predict wealth holding patterns in later life. Based on panel data from the Asset and Health Dynamics among the Oldest Old, we estimate a structural life-cycle model with bequests. Each individual's subjective survival rates in the future are estimated with data on his belief of survival probabilities to a target age. This estimation is build upon a Bayesian updating method developed in Gan et al. (2005). We find that life-cycle model using subjective survival rates performs better than using life-table survival rates in predicting wealth holdings. This result suggests that subjective survival expectations play an important role in deciding consumption and savings. In addition, the estimation results show that most bequests are involuntary or accidental.

JEL classification:

D91; C81

Keywords

Subjective mortality risk; Bequest; Life-cycle model; Median regression

1. Introduction

The main goal in this paper is to investigate the empirical relevance of subjective survival rates as determinants of consumption, saving and bequests by the older population. Several previous studies have used life tables (Skinner, 1985; Hurd, 1989; Palumbo, 1999; De Nardi et al., 2010). Yet it is unlikely that each individual has the same beliefs as those summarized by a life table. In this paper, we estimate a life cycle model of Yaari (1965) by using both life

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tables and individual subjective survival curves. The out-of-sample predictions on assets suggest that individuals' behavior is more consistent with their individual subjective beliefs than life tables.

The model investigated here includes a component of bequest motives as in Yaari (1965) and Hurd (1989). A significant portion of household wealth is passed from one generation to another by bequests. According to Kotlikoff and Summers (1981), 80% of household wealth was inherited. Gale and Scholz (1994) estimate that total bequests were $105 billion in the US in 1986. Hurd and Smith (2002) find that the elderly anticipate leaving roughly 40% of their wealth in bequests. Although bequest plays an important role in household wealth accumulation, there is no consensus in the literature on the significance of bequest motives. Some people (Hamermesh and Menchik, 1987; Kotlikoff and Summers, 1981; Kopczuk and Lupton, 2007; De Nardi, 2004; Ameriks et al., 2011) argue that the bequest motive is important while others (Hurd, 1989; De Nardi et al., 2010; Lockwood, 2012) claim that it is economically trivial, and most bequests are accidental or involuntary. The second goal of this paper is to identify bequest motives by comparing the wealth path among those individuals who have children and those individuals who do not have children. We find that bequest motives are very small, indicating most bequests are involuntary or accidental.

This paper applies individual subjective survival rates to estimate a structural life-cycle model of saving and consumption that includes a bequest motive. A large panel data set, the Asset and Health Dynamics among Oldest Old (AHEAD) collected data on people who were born between 1890 and 1923 and their spouses (regardless of age) including information on individuals' expectations of a wide range of future events.[1] Respondents in the survey are asked about their subjective chances of living to a certain age. Earlier work, such as Hamermesh (1985), Hurd et al. (1998), Hurd and McGarry (1995, 2002) and Gan et al. (2005) (GHM hereafter) have studied the relationship between subjective probabilities and actual survival rates.[2] These papers have found that, on average, individual subjective survival probabilities are consistent with life tables, varying appropriately with known risk factors and having predictive power for actual mortality beyond that contained in a life table. Therefore, there is important information content in these responses on subjective survival probabilities. A remaining question is whether individuals behave as they respond to the survey questions.

Individual subjective survival rates are obtained by respondents self-report about their belief of survival probabilities to target ages. However, the subjective survival probabilities have serious focal response problems: many individuals tend to give responses of 0.0 and 1.0. These focal responses cannot be directly used in analyzing life-cycle models where survival probabilities are required. To eliminate focal biases, GHM suggest a Bayesian updating method. For each individual in the AHEAD data set, GHM estimate an "optimism" index. Compared to the life table survival probability, an individual may overestimate or underestimate his/her survival probability. The estimated "optimism" indices show

[1]See Soldo et al. (1997).

[2]Hamermesh (1985) was the first to investigate how people's subjective survival probabilities are related to life tables and what the implications of the subjective probabilities are.

significant individual heterogeneity, and can be applied to derive individuals' subjective survival probabilities without focal biases. The individualized survival curves developed in GHM are used to estimate a life cycle model with bequests in this paper.

Understanding people's bequest motives is very important for public policies. Kotlikoff (1988) asserts that inherited wealth plays an important and perhaps dominant role in US wealth accumulation. Bequests may hold a key answer to the social security problem that baby boomers may face: they may eventually receive significant estates from their parents such that their dependence on social security may be reduced.

Predicting whether a large portion of wealth will be passed from one generation to the next generation requires knowledge of the motives for bequests.[3] As pointed out in the literature (Hamermesh and Menchik, 1987; Kotlikoff, 1988; Hurd, 1989), a large amount of bequeathed wealth does not necessarily imply a substantial motive for bequests. Without a well-functioning annuity market, people will have to save against mortality risk, and the resulting bequests could be involuntary.[4] If most bequests are in fact involuntary or accidental, the value of the bequeathed wealth may decrease in the future as the annuity market further develops.[5] In addition, it is also possible that people may change their perceptions of stock market risks after the recent financial crisis. In that case, more people may move into annuities, and the total amount of bequeathed wealth will decrease.

The rest of the paper is organized as follows. In Section 2, we introduce a life-cycle model with bequests. Our emphasis is on how to estimate such a model. Section 3 presents the estimation results. In particular, Section 3.1 introduces the data that will be used in the paper. Three key variables are used in the empirical variables: wealth, income and subjective survival probabilities. In Section 3.2, we present parameter estimates based on various estimation methods. Section 3.3 calculates the bequest incentives based on estimates from Section 3.2. In Section 3.4, we conduct out-of-sample predictions and simulate the consumption and wealth trajectories under various sets of parameter estimates. Finally, we summarize the results of this paper in Section 4.

2. The model

Our starting point is the standard life-cycle model with bequests as in Yaari (1965) and Hurd (1989). Let the utility function of a retired individual be:

[3]Various incentives for bequest are offered in the literature. Some argue that bequests serve as incentives to younger generations to provide appropriate care for older generations (Cox, 1987; Bernheim et al., 1985). Others argue that bequests are mainly motivated by altruism.

[4]One reason of the little presence of private annuity market in the US, as argued in earlier papers by Friedman and Warshawsky (1988, 1990), and later by Mitchell et al. (1999), is because the present value of annuity payout is significantly lower than that of annuity premium, although Mitchell et al. (1999) show a large reduction in the difference between the payout and the premium in 1990s.

[5]Poterba (1997) documents that variable annuity premium payments increased by a factor of five during the period 1988–1993.

$$\sum_{t=0}^{N} \beta^t U(c_t) s_t + \sum_{t=0}^{N} \beta^t B(w_{t+1}) m_{t+1}$$

(1)

where s_t is the subjective probability that the individual will be alive at time t. m_{t+1} is the subjective mortality rate at time $t + 1$: $m_{t+1} = s_t - s_{t+1}$. The subjective maximal number of periods an individual can survive is N. The time discount factor is denoted as β. Consumption at time t is denoted as c_t, and wealth at the beginning of time t is denoted as w_t. The first term in (1) is the present value of utility from consumption conditional on survival; and the second term in (1) is the present value of the utility from leaving a bequest of w_{t+1} conditional dying at t+1. The utility from a bequest, $B(w_{t+1})$, is increasing in w_{t+1}.

This model only applies to singles. The corresponding model for couples is much more complicated because it has to account for bequeathing by a couple to the next generation, and also for providing to a surviving spouse.[6]

As in Hurd (1989), we further assume a borrowing constraint such that bequeathable wealth cannot become negative. The constraint imposed on borrowing indicates that future Social Security benefits cannot be used as collateral for a consumption loan. This constraint arises from the fact that all heads of households in the sample are older than 70 years old in 1993 when the survey started, and in the US, Social Security benefits cannot be used as collateral. Such a constraint imposes an important boundary condition in our analysis:

$$w_t = (1 + r) w_{t+1} + A_{t-1} - c_{t-1} \geq 0$$

(2)

where w_t is the wealth accumulated at the end of time $t - 1$ and A_{t-1} is annuity income at time $t - 1$.

It is typical in this literature to assume a constant risk aversion utility function $U(c_t) = c_t^{1-\gamma} / (1 - \gamma)$. Income from annuities such as Social Security is assumed to be constant. The marginal utility of a bequest, denoted as α, is dependent on how many children the person has:

[6]Estimating the couple's bequest motive is our next research objective.

$$B_w \equiv \alpha \equiv \frac{\partial B}{\partial w} = 1_{\text{children}}(\alpha_0 + \alpha_1 * \text{No. of children}),$$

(3)

where 1_{children} is an indicator function. The assumption that the bequest motive exists only if the person has any children is important to identify the model. Otherwise, the identification may only come from the functional form assumptions.[7] Besides the identification advantage, this simple specification is chosen for two reasons. First, despite the potential presence of bequest motive, there is little empirical evidence of a more complex motive of altruism. Second, this specification implies that bequests are luxury good: as wealth increases, the marginal utility from consumption decreases relative to the marginal utility from bequests α. Given a reasonable large value of α, wealthy people have strong motive to save and less wealthy people can still leave bequests when they die prematurely.

The maximal age that a person may live, denoted as N, is obtained when the person's subjective survival rate $s_t < 0.0001$. Different agents have different maximum ages N since their subjective survival rates are different. Given the interest rate r, income A, and the parameter values of β, γ, and α, the paths of wealth are contingent on the initial wealth w_0. The analysis of the solution of the discrete model is similar to that of the continuous model in Hurd (1989). Here we only state how to estimate the model.

Estimating the model requires at least two waves of wealth data for each individual. We use wealth data in wave 2 and wave 3 to estimate the model. The wave 4 wealth data is used for out-of-sample prediction.[8] The wealth level in wave 2 serves as the initial wealth w_0. We use backward induction to find the trajectories of the wealth and consumption. For a given set of parameter values β, γ, and α, we can obtain the trajectories of wealth $\{w_t^b, t = 1, \ldots, N+1\}$, where the superscript b indicates the value is calculated from backward induction. We then compare w_3^b at the trajectory with the observed wave 3 wealth w_3. We use the subscript 3 because in our data set the interval between the two waves of wealth is 3 years. The parameter set that minimizes the difference between w_3^b and w_3 is our estimates.

There are three types of consumptions paths corresponding to low, medium, and high wealth. We discuss these three different cases in the discrete model:

(1) In the first case, the bequest is strictly positive even if the individual survives to the greatest age possible: i.e., $w_{N+1} > 0$. Then the consumption trajectory satisfies:

[7] Kopczuk and Lupton (2007) examine the possible unobserved heterogeneity in bequest motives.

[8] There is good evidence that wave 1 wealth data in AHEAD underestimate financial asset ownership and hence the value of financial assets, so we do not use wave 1 (Rohwedder et al., 2004).

$$c_t^{-\gamma} s_t = \alpha \sum_{i=t}^{N} \beta^{i-t} (1+r)^{i-t} m_{i+1}.$$

(4a)

The consumption trajectory that satisfies (4a), $\{c_t^*\}$, and actually initial wealth, w_0, generate the wealth path

$$w_{t+1} = (1+r)^{t+1} w_0 + \sum_{i=0}^{t} (1+r)^{t-i} (A_i - c_i^*) > 0.$$

(4b)

Eq. (4a) shows that if the wealth level at $N + 1$ is strictly positive, the consumption trajectory depends on the subjective survival rate but is independent of initial wealth w_0. This occurs because the marginal utility from consumption (left-hand-side) at time t equals the present value of the marginal utility from bequests, which is assumed to be independent of wealth level. The wealth trajectory, w_t^b, can be calculated from Eq. (4b), which shows that wealth trajectories vary according to the initial wealth w_0. Fig. 1 shows typical consumption and wealth trajectories. Wealth monotonically increases and consumption monotonically decreases with age, but other patterns are possible. The only requirement for this case is that wealth is strictly positive at any time in this person's life span.

The minimal level of initial wealth that corresponds to the consumption path (4a) is w_0^*, given by:

$$w_0^* = \sum_{i=0}^{N} (1+r)^{-i-1} (c_i^* - A) > 0.$$

Any initial wealth larger than $w_0 > w_0^*$ will produce a consumption path $\{c^*\}$ as in (4a), and will lead to $w_{N+1} > 0$. Note that both N and w_0^* vary as individual subjective survival rate varies.

(2) In the second case, although the bequest is zero at the time of death, ($w_{N+1} = 0$), the borrowing constraint is not binding; that is, the wealth level is strictly positive for any $t < N + 1$. The consumption path satisfies:

$$c_t^{-\gamma} s_t = \beta(1+r)c_{t+1}^{-\gamma} s_{t+1} + \alpha m_{t+1}, \quad \text{for } t = 0, 1, \ldots, N-1$$

(5a)

$$w_{N+1} = (1+r)^{N+1} w_0 + \sum_{i=0}^{N} (1+r)^{N-i}(A_i - c_i) = 0$$

(5b)

$$w_t > 0, \qquad \text{for } t = 1, 2, \ldots, N.$$

(5c)

Eq. (5b) states that the consumption trajectory should lead to zero wealth level at time $N+1$: the person will leave no bequest should he or she live to the greatest age possible. Fig. 2 illustrates one case where wealth reaches zero exactly at the maximum possible age. Consumption in Fig. 2 first increases and then decreases as mortality risk becomes large. However, it is possible that consumption monotonically decreases if the time discount factor is small.

There will be a range of initial wealth and associated consumption paths that satisfy (5a)–(5c). The intuition for this result will be discussed when we provide the estimation algorithm (Step 2 in the algorithm. See Appendix A). Let w_0^* be the largest of these values so that any value of larger than w_0^* leads to $w_{N+1} > 0$ and the consumption path will be independent of w_0. Let $\hat{w}_0$ be the smallest of those values so that any smaller value of initial wealth causes the wealth to reach 0 before $N+1$. Let $\{\hat{c}\}$ and $\{\hat{w}\}$ be the individual's consumption and wealth trajectories associated with $\hat{w}_0$, and $\{c^*\}$ and $\{w^*\}$ be the individual's consumption and wealth trajectories associated with w_0^*. Therefore, in the case of medium wealth, the consumption trajectory must lie between $\{\hat{c}\}$ and $\{c^*\}$, and the wealth trajectory must lie between $\{\hat{w}\}$ and $\{w^*\}$.

(3) Lastly, we consider the case that the borrowing constraint is binding. Let T be the time when bequeathable wealth is exhausted. The consumption path is found from the solutions to four equations, (6a)–(6d):

$$c_t = A, \qquad \text{for } t = T, \ldots, N,$$

(6a)

$$c_t^{-\gamma} s_t = \beta(1 + r) c_{t+1}^{-\gamma} s_{t+1} + \alpha m_{t+1}, \quad \text{for } t = 0, 1, \ldots, T - 2,$$

(6b)

$$w_T = (1 + r)^T w_0 + \sum_{i=0}^{T-1} (1 + r)^{T-1-i} (A_i - c_i) = 0$$

(6c)

$$w_t > 0, \qquad \text{for } t = 1, 2, \ldots, T - 1 .$$

(6d)

In this case consumption and wealth will eventually decline. Fig. 3 illustrates possible consumption and wealth trajectories in this case.

Each individual in our sample has a different subjective survival curve. Therefore, every individual's critical value of wealth is different. We search to find out his/her critical wealth value, and then calculate his/her consumption and wealth trajectories. Our objective is to find a set of parameter values that minimize the difference between the predicted second wave wealth, w_3^b, and the observed second wave wealth, w_3. We consider two different objective functions: mean square loss function and the absolute value loss function.

$$\min_{\alpha,\beta,\gamma} \sum_i \left(w_{3i} - w_{3i}^b\right)^2$$

(7a)

$$\min_{\alpha,\beta,\gamma} \left\{ \sum_i \left| w_{3i} - w_{3i}^b \right| \right\}.$$

(7b)

The mean square loss function in (7a) is the one used in Hurd (1989). The absolute value loss function in (7b) corresponds to median regression. The advantage for median regression over the mean regression is that median regression is robust to outliers. A more general way to deal with outliers is to use quantile regression. Estimates from different quantiles can be used to evaluate individual heterogeneities at different wealth levels. The results of quantile regression are presented in Appendix B.

We apply the Quasi-Newton method to mean square loss objective function (7a) and Nelder–Mead Simplex method to absolute value loss objective function (7b). For any given set of parameters, β, γ, and α, we need to find the predicted wave 3 wealth for each individual. The detailed algorithm to find w_3^b is given in Appendix A.

We briefly discuss how to estimate the covariance matrix. Let the parameter set be denoted as $\delta = (\gamma, \beta, \alpha)^T$, and let the covariance matrix be Ω. It is straightforward to obtain the covariance matrix for estimates based (7a). The covariance matrix from median regression in (7b) is given by:

$$\Omega = \frac{1}{4 f_u^2(0)} \left(E\left[\left(\frac{\partial w_3^b}{\partial \delta} \right) \left(\frac{\partial w_3^b}{\partial \delta} \right)^T \right] \right)^{-1}$$

(8)

where $f_u(0)$ is the density of the error term u evaluated at 0. The error term u is defined as $u = w_3 - w_3^b$. Empirically, we first conduct a non-parametric kernel regression, and then evaluate the obtained density function at 0 to get $f_u(0)$. The expectation part can be calculated by sample average. Since no explicit solutions exist for the derivative $\partial w_3^b / \partial \delta$, numerical derivatives are used in the calculation.

3. Data and estimation results

3.1. Data

Our data set consists of the second, third and fourth waves of the AHEAD sample. We do not employ wave 1 data because there is good evidence that the first wave of AHEAD underreported asset holdings (Rohwedder et al., 2004). To select our sample, we use the following sample selection criteria: (1) Because the model in this paper applies only to singles, our sample only includes people who are alive and who are singles in both wave 2 and wave 3. (2) Total wealth or non-housing wealth is non-negative in wave 2 and wave 3. (3) Responses to the survival probability question in wave 2 are valid. When total wealth is used as one of the selection criteria, the number of valid observations is 1903. When we consider non-housing wealth, the number of observations decreases to 1752. Among these valid observations in wave 2 and wave 3, only 1460 of them are still valid in wave 4.

Three key variables are used in this paper: household wealth, income, and individual subjective survival curves. We now discuss these three variables in detail.

(1) The Wealth and Income Data—The AHEAD survey is a panel survey of older Americans. The wave 1 survey of AHEAD was conducted in 1993. The initial sample of AHEAD includes a sample of people who were 70 years old or more in 1993 (and their spouses regardless of age). The wave 2 survey was conducted in 1995, and wave 3 and wave 4 were conducted in 1998 and 2000, respectively.

The AHEAD data set provides more than 10 categories of wealth data. In household surveys such as AHEAD a relatively large portion of people do not provide valid responses to all wealth questions (Juster and Smith, 1997; Chand and Gan, 2003). AHEAD uses a sequence of questions to bracket a wealth item. Although this technique is very successful in reducing non-response rates, it requires serious effort to impute the wealth values. Chand and Gan (2002) discuss various imputation methods. The imputed wealth and income data used in this paper are obtained from Adams et al. (2003).

In Table 1, we list summary statistics of the total wealth and the wealth net of housing wealth. For each wave of wealth, we list the mean, median, variance, minimum and maximum values. From Table 1, mean wealth decreases slightly between wave 2 and wave 3, 4.5% for total wealth and 2.5% for non-housing wealth. The median wealth decreases more than mean wealth. Between wave 2 and wave 3, median wealth decreased by 14% and 15% for total wealth and non-housing wealth, respectively. Because of sample attrition and missing reports, the sample size decreases more than 20% between wave 3 and wave 4. In addition, because of large fluctuation of financial market between these two waves, a significant number of households experienced wealth losses. The minimum non-housing wealth in wave 4 is −$157,895. The mean non-housing wealth decreases around 30% but median wealth decreases only 6.2% from wave 3 to wave 4.

As Table 1 indicates, median wealth is less than half of mean wealth, reflecting the positive skewness that exists in the asset distribution. More specifically, the median is respectively 35%, 32% and 48% of mean total wealth in waves 2, 3 and 4 and 20%, 14%, and 19% of the mean non-housing wealth in waves 2, 3, and 4.

In Table 2, we list age, the number of children and income. The average age of respondents in the second wave is 79 years of old. Although heads of households in our sample have to be at least 72 years in wave 2, their spouses who may be younger are also included in the sample. The number of people in our sample who are younger than 72 years old is 46 (2.63% of the sample). Among all the people in our sample, 80.2% have children. The average number of children in our sample is 2.55. One household has 16 children. Second wave income is used as a measure of people’s annuity income. The mean income level is $18,107 with a large standard deviation of $22,873.

(2) Individual Subjective Survival Probability—In this paper, for each individual, we construct two survival curves: the life-table survival curve and the subjective survival curve. The life-table survival curve is directly obtained from the life table.The subjective survival curve is obtained from GHM. Here we briefly describe the subjective survival curve.

One innovation in two surveys (Health and Retirement Study and AHEAD) is that they include questions about the respondent’s subjective probabilities about events in the future. In particular, each respondent is asked about his/her perceived probability of surviving to a target age that is between 10 and 15 years in the future. Hurd and McGarry (1995, 2002) found that the average subjective survival probabilities are very close to the life table predictions and varied with known risk factors and determinants of mortality. The subjective survival probabilities also predict actual survival correctly. Although on average these subjective probabilities are consistent with life tables, they can vary substantially. People from some groups are optimistic while those from some other groups are pessimistic. For example, Hurd and McGarry (1995) found that nonwhite individuals reported significantly higher subjective survival probabilities than white individuals after controlling a variety of variables.

AHEAD suffers a serious problem called focal bias at the individual level (Hurd and McGarry, 2002; Gan et al., 2005). In all age groups, a substantial fraction of respondents give focal responses of 0.0 and 1.0. These responses cannot represent the respondents’ true probabilities and are used directly in life-cycle model estimation because the distribution of true probabilities should be continuous and cannot be either zero or one.

GHM develop a Bayesian updating method to recover each individual’s “true” subjective probability. Given the same age and sex, different people may have very different subjective survival probabilities. Some of the differences may be due to the health and wealth situations of individuals; some others may simply reflect personality. GHM assume that the econometrician does not know an individual’s true belief regarding his or her survival probability but knows the distribution of those beliefs — the Bayesian prior distribution. The individual reports a survival probability based on his or her true beliefs. The difference between the true and reported beliefs represents measurement error.

GHM obtain the posterior distribution from the prior distribution based on the self-reported survival probabilities. GHM then apply the posterior distribution of survival probabilities to observed mortality among the panel to estimate parameter values that best characterize each individual's belief as to survival probabilities in each year.

For each individual in their data set (AHEAD), GHM estimate an "optimism" index. Compared to the life table survival probability, an individual may overestimate or underestimate his/her survival probability. The estimated "optimism" indices in GHM show that significant individual heterogeneity exists in the AHEAD population. These indices then can be used to derive individuals' subjective survival rates in each year without focal biases.

GHM consider four different optimism indices: age scaling and hazard scaling indices in constrained and unconstrained forms. Age scaling reflects the fact that individuals may think of themselves as aging more or less rapidly than the average person of their age and gender. Hazard scaling refers to the fact that individuals may think of themselves as facing an annual mortality risk that bears a fixed relationship to the average for persons of their age and gender. Both indices can be constrained so that the average belief coincides with the predictions of life tables.

In this paper, we use the "unconstrained hazard-scaling" index because GHM find it had the best predictive power of actual survival experience among all four indices. In particular, let the current age of individual i be a. His subjective survival probability to age $a + t$ is given by:

$$s_{ia}(t) = \exp\left(-\int_0^t \lambda_{ia}(a + r)dr\right),$$

where $\lambda_{ia}(a + t)$ is the hazard function at age $a + t$. Further, let the individual's life table hazard be $\lambda_{i0}(a + t)$. The "unconstrained hazard-scaling" in GHM assumes that: $\lambda_{ia}(a + t) = \psi_i \lambda_{i0}(a + t)$ where ψ_i is the individual's optimism index. If $\psi_i > 1$, this individual is said to be "pessimistic"; if $\psi_i < 1$, then this person is "optimistic". Table 2 has the summary statistics of the optimism index estimated from responses in wave 2.

The mean and median of ψ_i are .659 and .663, respectively. People in this sample are on average more optimistic about their survival probabilities than the life table implies. A more optimistic person may save more than a life-table person would do. If we use an observed sequence of wealth to estimate our model, the estimates based on subjective survival curves should indicate a lower time discount factor and/or lower bequest motive than the estimates based on life tables.

In a simple life cycle model, GHM show that ignoring individual heterogeneities may result in bias estimates. In this paper, we apply both the subjective survival probability developed in GHM and the life table survival probability.

3.2. Estimation results

Our main results exclude housing wealth. In principle, at the extreme of very high transaction costs, it is difficult to change the consumption level of housing.[9] Therefore, holding of housing wealth would simply reflect initial conditions and differences between the rate of housing appreciation and the general inflation rate. Excluding housing wealth from bequeathable wealth would give a better idea of the change in desired wealth holdings than would be found from including housing wealth.[10]

In Table 3, we report the estimates of our model using non-housing wealth and assuming a fixed interest rate $r = 0.04$. We will test the robustness of our estimates later by using different interest rates. In Panel (A) of Table 3, we apply median regression to estimate the model using both subjective and life-table survival curves. Although the marginal utility of bequests is estimated to be almost zero in both cases, other parameter estimates vary significantly. Using life-table survival curve yields a higher time discount rate than using subjective survival curves. This is expected because people subjectively overestimate their survival probabilities relative to the life table. They behave accordingly by saving more to prepare for a longer lifespan, rather than valuing future consumption more than current-period consumption as implied by the estimates based life-table survival curves.

Panel (B) in Table 3 lists the estimates when the mean regression method is used. The marginal utilities of bequest in this panel are much larger than those estimated in Panel (A), which imply strong bequest motives. Another observation in Panel (B) is that the time discount factor is estimated to be significantly larger than 1, indicating that people value future consumption more than current consumption, and that of the time discount factor is higher when the life table survival curve is used.

It is important to note that in a life-cycle model of time-varying survival probabilities, a time discount factor that is larger than 1 does not imply necessarily non-stationary growth in either consumption or wealth. Kocherlakota (1990) shows that it is possible that people still prefer current consumption to future one even with $\beta > 1$, as long as output or income grows at a rate that is sufficiently high. Kocherlakota's discussion is based on an infinitely lived representative agent. In our model, the individual agent has constant income levels. From Eq. (1), even with $\beta > 1$, the rate of consumption growth will turn negative at the time when the hazard rate $-\Delta ln s_t$ is large enough.

The empirical reason to have such an unusual time discount factor is that non-housing wealth during the sample period declined by only 2.5%. Given the constant interest rate of 4%, matching such a small decrease in wealth requires the individual to have an incentive to save. This saving incentive has to come from a large time discount factor. One major drawback, we suspect, is the interest rate we use: the return to capital investment may not

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9Indeed, some researchers found very little housing decumulation except at widowing (Venti and Wise, 2004).

10For completeness, however, we also estimated the model over total wealth, which includes housing asset. The results over total wealth actually are very close to those over non-housing wealth. For example, the estimates over total wealth and subjective survival rates for parameters risk-averse coefficient γ, time discount factor β, and bequest motive parameter α_0, and α_1 are 0.9088 (.1066), 0.9468 (.0641), 4.9759e–7 (.00126), 1.0272e–6 (.00075), respectively (standard errors are in parentheses).

have been 4% during our sample period. However, how to formally incorporate varying interest rate requires a model of portfolio choice, which is beyond the scope of this paper.

In summary, mean regression yields very different parameter estimates from median regression. More specifically, mean regression suggests very large desired bequests while the median regression implies almost zero bequest motives. The difference is undoubtedly due to the large influence of the households at the top of the wealth distribution when the estimation method is mean regression. Increasing wealth between the waves among just a few high-wealth households will require a substantial bequest parameter.

In Table 4, we list results from median regressions with varying interest rates. The risk-averse parameters and the time discount factor are very close to the reference value when interest rate changes from .02 to .06. Within this range of interest rates the marginal utility of bequests is very small.

In the following section, we will try to understand the economic significance of the bequest motive by some simulation exercises.

3.3. Bequest simulations

Among the four parameters we estimate, it is relatively easy to understand the economic significance of the risk-aversion parameter γ and the time discount factor β. To understand the effect of γ and β on bequests, consider a familiar consumption growth equation in the absence of the bequest motive: $\Delta \ln c_t \approx (r + \ln \beta + \Delta \ln s_t)/\gamma$. Given the survival rate s_t and the risk-aversion parameter γ, a larger β will increase algebraically the slope of the consumption path and because of the lifetime budget constraint, initial consumption will have to be reduced. Thus more wealth will be held and so bequests will increase. Although the effect of the time discount factor β on bequests is clear, the effect of the risk-averse parameter on bequests is ambiguous. When the consumption path is decreasing a larger γ will increase algebraically the slope of the consumption path causing more wealth to be held and increasing bequests. When the consumption path is increasing a larger γ will flatten the consumption path causing initial consumption to be higher but later consumption to be lower. Therefore, the total effect on bequests or wealth holdings for γ is ambiguous. It is important to note that a change in bequests because of a change in either γ or β is a change in accidental bequest.

A non-accidental bequest is measured by the marginal utility of bequests. The larger the values of the α, the larger is the bequest motive.

Hurd, 1989 uses two methods to measure the economic significance of marginal utility of bequest, α:

$$\sum (1 + r)^{-t} [\hat{w}_t(\hat{\alpha}) - \hat{w}_t(0)]\, m_t \tag{9a}$$

$$\sum [\hat{w}_t(\hat{\alpha}) - \hat{w}_t(0)]\, s_t$$

(9b)

where $\hat{\alpha} \equiv 1_{\text{children}} (\hat{\alpha}_0 + \hat{\alpha}_1 * \text{No of children})$. In (9a) and (9b), $\hat{w}_t(\hat{\alpha})$ is the optimal wealth trajectory given initial wealth and the estimated values of parameters. The term is defined in a similar way except that the marginal utility of bequests is zero. Eqs. (9a) and (9b) represent two different ways to understand the effect of bequests. In (9a), we calculate the present value of the increase in bequests due to a bequest motive. In (9b), we calculate the population difference in wealth holdings with and without a bequest motive. In Table 5, we calculate the effect of a bequest motive for a particular individual: a male at age 79 whose initial wealth is $35,000 and whose income is $12,000. The individual has two children. The optimism index of this individual is 0.6594.

The results in Table 5 are presented in three different panels, grouped by their estimation methods. In the first three rows, (R1)–(R4), we let the marginal utility of bequests vary. In particular, row (R1) corresponds to a bequest motive estimated from (A1) in Table 3 where subjective mortality risk is used. We let the time discount factor vary in rows (R5)–(R7), and let the risk averse parameter vary in rows (R8)–(R10). The marginal utility of bequest parameter has a significant impact on the level of desired bequests and on the difference in wealth holdings. In rows (R1)–(R4) where the risk aversion parameter (γ) and the time discount factor (β) are estimated using the median regression, the desired bequest rises from almost zero to $125,278 and the difference in wealth holding increases from $1 to $1082,618 when the marginal utility of bequests increases from 2.47E-06 to 1. The effects of varying the marginal utility of bequests on desired bequests and on wealth holdings are very large. When the marginal utility of bequests is 1, the consumption path decreases slowly, from $1211 at age 79 to $1013 at age 109, which implies that the agent saves 90%–95% of annuity income ($12,000). In contrast, when the marginal utility of bequests takes the value from median regression with subjective mortality risk, the consumption path drops quickly, from $21,766 at age 79 to the annuity level of $12,000 at age 86. The large bequest parameters are from the mean regression. While they may describe well the changes in population wealth holdings between waves, they do not describe well the behavior of a typical person as in our example. We take this example as additional evidence that the median regression is more appropriate for describing the behavior of most households.

In rows (R5)–(R7), we allow the time discount factor to vary while keeping the risk aversion parameter constant. The marginal utility of bequests is constant at 0.001. In this case, desired bequests increase from $2.58 to $1408 when the time discount factor increases from 0.7 to 1.2. The result that a larger time discount factor is related to a higher desired bequest is consistent with the prior discussion. Finally, in rows (R8)–(R10), we consider the effect of the risk aversion parameter γ. A larger γ implies a more risk averse agent. When γ increases from 0.5 to 2.0, the desired bequest increases from $5.80 to $518.5.

In summary, simulation results show that a higher marginal bequest motive, larger time discount factor, and larger risk aversion parameter all increase the level of desired bequests. But there are important interaction effects: when the bequest parameter is large, say 0.001, a modest increase in the discount factor or in risk aversion can lead to a large increase in desired bequests and in differences in wealth holdings.

3.4. Consumption/wealth trajectory and out-of-sample predictions

A typical way to evaluate parameter estimates from different methods is to conduct out-of-sample predictions. We used wealth data in wave 2 and wave 3 to obtain parameter estimates. We will now use the estimated parameters to predict the wealth values in wave 4, and compare the predicted wealth to observed wealth in wave 4. Table 6 has the comparison results using various criteria. Each column in Table 6 reports results based on a given set of parameter estimates. The columns numbered A1, A2, B1, or B2 correspond to the estimates listed in Panel A and Panel B in Table 3. These estimates differ in their estimation method and their survival probabilities. The out-of-sample calculation is based on the same survival probability as the parameter estimates are. For example, if the set of parameters is obtained based on subjective survival probability, the out-of-sample calculation is also based on the subjective survival probability.

Parameter estimates in Columns (A1) and (A2) are from median regressions while Columns (B1) and (B2) are from mean regressions. From the first panel in Table 6, (A1) and (A2) have smaller absolute errors and smaller mean square errors than (B1) and (B2), regardless of error types. Furthermore, (A1) and (A2) have a lower sum of absolute errors for low wealth people and a larger sum of absolute errors for high wealth people than (B1) and (B2). This is expected because mean square regressions tend to fit high-wealth observations better because the large wealth values are magnified by the square operation.

Results in Table 6 can also be used to evaluate the advantage of using subjective survival probabilities instead of life-table survival probabilities. When median regressions are used, parameter estimates based on subjective survival probabilities (A1) produce lower sums of mean square errors and lower sums of absolute errors in out-of-sample prediction of wealth than estimates based on life-table survival curves. In particular, the mean square errors and the absolute errors from subjective survival curves are 42% and 5% less than the corresponding errors from life-table survival curves.

The second and the third panel in Table 6 report comparison results based on predicted mean and predicted median. Although predicted means using both survival curves are lower than the observed mean at wave 4, the mean ($87,033) from subjective survival curves is much closer to the observed mean ($118,112) than the mean ($71,413) from life-table survival curves. Further, we divide the sample into four quartiles according to the wealth level at wave 3, and compare the predicted and observed means in each quartile. In the fourth panel in Table 6, using subjective survival curves produces better predictions than using life-table survival curves in all four quartiles. At the first quartile, the predicted mean using subjective survival curves is $8.6 while the predicted mean using life table is $2385. The observed mean at wave 4 is $—1548. At the second quartile, the predicted mean from subjective survival curves is $7947, which is much closer to the observed mean ($9,091) than the predicted mean from life table ($2385). Similar patterns are observed for the third and fourth quartiles.

When the mean regression method is used, parameter estimates based on subjective survival curves do not have a significant advantage in predicting fourth wave wealth comparing to ones based on life-table survival curves. However, based on either subjective or life-table survival probabilities, the mean regression method produces much larger mean square errors and absolute errors than median regressions. From these results, we conclude that median regression is better than mean regression, and subjective survival probabilities better describe individual saving and bequest decisions than the life-table survival probabilities.

Finally, to better understand how people's consumption and wealth vary, we apply estimates from Table 3 to simulate a hypothetical person's consumption and wealth trajectories in Fig. 3. The hypothetical person we consider is: single male at age 79 with an optimistic index of .6594. He has two children. His initial wealth and income are assumed at the median values in Table 2. In addition, the parameter set for Fig. 3 is obtained from the median regression in Table 3. His consumption level is highest when he starts at age 79, and decreases until he reaches age 85. His wealth decreases and reaches zero at age 85. Above age 85, the person's wealth keeps reaching zero and his consumption equals his annuity income at $12,000. If the person dies before age 85, he leaves some bequest. However, such bequest is accidental since his bequest motive is essentially zero. In all these cases, since the person values future utility lower than current utility, his consumption level peaks at the first year and then decreases until it reaches his annuity income level.

4. Conclusions

Our main goal in this paper is to investigate the importance of an individual subjective survival curve on his/her consumption and saving decisions. A classical life-cycle model with bequests is estimated with the individual-specific subjective survival curves. In almost any life-cycle model, individual mortality risk is an important factor that affects people's decisions. Previous literature assumes that individual mortality risk is the same as life-table mortality risk, ignoring any individual heterogeneity in mortality risk. This assumption may cause biases in parameter estimates. This paper applies the individual subjective survival probability model developed in an earlier paper (GHM). Subjective survival probabilities have significant variations across individuals, and provide explanatory power for actual survival experience beyond life tables. We find that using subjective survival curves produces much better out-of-sample predictions than using life-table survival curves, suggesting that people's consumption and saving decisions are consistent with beliefs about their own mortality risk. This paper applies individual subjective survival curves to a structural life-cycle model. The results of this paper show a strong empirical relevance of subjective survival curves, indicating the importance to take into consideration of this dimension of individual heterogeneity in life cycle models.

The empirical estimates of bequest motives in this model rely on a relative innocuous assumption that only those people who have children have bequest motives. The paper finds that bequest motives are very small, indicating that most bequests are involuntary or accidental.

We do not include health expenditures in the model. Precautionary saving for future health expenditures is another motive for people at advanced ages to save and hold wealth. The unusual time discount rate estimates (as high as 1.076 for mean regression using life table) may be a result of not including health expenditures. However, it is unlikely for this precautionary saving motive to have a large impact on our result that the estimated bequest motive is very small. This is because precautionary savings motive makes individuals' wealth holdings decumulate much slower than those predicted by life-cycle model theoretically (De Nardi et al., 2010), offsetting the effect of bequest motive. We expect that allowing health expenditures would reduce bequest motive.

Another limitation of this study is that we assume that the interest rate is fixed in the model. Certainly the return on capital investment did not maintain constant at the assumed baseline rate of 4% during our sample period. However, it requires a model of portfolio choice to formally incorporate varying interest rate and the data set does not have enough information to estimate such a model.

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Appendix

Appendix A. Algorithm to find the optimal consumption and wealth path

The algorithm to calculate the optimal paths of consumption and wealth in optimal dynamic program problem (1) includes the following three steps.

Step 1: Check the high wealth case, in which a strictly positive bequest is left at the maximum age of life, i.e., $w_{N+1} > 0$.

(1) From Eq. (4a), we calculate the consumption trajectory $\{c_t^b, t = 0, \dots, N\}$.

(2) Substitute the trajectory of consumption $\{c_t^b, t = 0, \dots, N\}$ into Eq. (4b) to get the wealth trajectory $\{w_t^b, t = 1, \dots, N + 1\}$.

(3) If for all $t \in \{1, 2, \dots, N\}$, $w_t^b \geq 0$ and $w_{N+1}^b > 0$, then report w_3^b and go to next observation; else go to *Step 2.*

Step 2: Check the medium wealth case, in which the wealth at the end of maximum age of life is zero, i.e., $w_{N+1} = 0$, and at all other time periods $t \leq N$, $w_t > 0$. We use backward induction to get the consumption and wealth trajectories.

(1) From (5a), c_t ($t = 0, \dots, N - 1$) is a function of c_N by recursive iteration: $c_t = c_t(c_N)$. Substitute the trajectory of consumption $\{c_t(c_N), t = 0, \dots, N - 1\}$ into Eq. (5b) such that wealth level in (5b) now is only a function of c_N. In particular, we have:

$$w_{N+1}(c_N, w_0) = 0. \tag{A.1}$$

Given observed w_0, we can solve (8) to get c_N, denoted as c_N^b. Given c_N^b, we can apply (5a) to iteratively find out $\{c_t^b, t = 0, \dots, N - 1\}$. However, if we do not know w_0, we will have many values of c_N and w_0 such that (8) is satisfied. Among them, the higher bound w_0^* is the maximum of w_0 such that (8) is satisfied and $c_t > 0$ for all $t < N + 1$; the lower bound $\hat{w}_0$ is the smallest w_0 such that (8) is satisfied and $c_t > 0$ for all $t < N + 1$.

(2) If for all $t \in \{0, 1, \dots, N\}$, $c_t^b > 0$, then calculate the wealth trajectory $\{w_t^b, t = 1, \dots, N\}$ from Eq. (2); else go to *Step 3*.

(3) If for all $t \in \{1, 2, \dots, N\}$, $w_t^b > 0$, then report w_3^b and go to the next observation; else go to *Step 3*.

Step 3: Check the low wealth case, in which the wealth reaches zero at a time period $T \le N$. We search all over the possible T from the backward. The method is similar to *Step 2*.

(1) Let $T = N$. From (6b), c_t ($t = 0, \dots, T - 2$) is a function of c_{T-1} by recursive iteration: $c_t = c_t(c_{T-1})$. Substitute the trajectory of consumption $\{c_t(c_{T-1}), t = 0, \dots, T - 2\}$ into Eq. (6c) such that (6c) now is only a function of c_{T-1}. Solve the equation: $w_T = 0$ to get c_{T-1} denoted as c_{T-1}^b. We can get the consumption trajectory $\{c_t^b, t = 0, \dots, N\}$ by applying (6b) with given c_{T-1}^b.

(2) If for all $t \in \{0, 1, \dots, T - 1\}$, $c_t^b > 0$, then calculate the wealth trajectory $\{w_t^b, t = 1, \dots, T - 1\}$ from Eq. (2); else let $T = T - 1$, and repeat (1)–(2).

(3) If for all $t \in \{1, 2, \dots, T - 1\}$, $w_t^b > 0$, then break from the cycle, report w_3^b and go to the next observation; else let $T = T - 1$, and repeat (1)–(3).

Appendix B. Quantile regression and results

The descriptive statistics in Table 1 show that wealth holdings are very positively skewed. This is the main reason for us to carry out median regression. A more general estimation method in dealing with the outliers is quantile regression which allows comparison of bequest behaviors in different quantiles. In this appendix, we present the quantile regression results.

The quantile regression for our model is to minimize the following absolute value loss function:

$$\min_{\alpha, \beta, \gamma} \left\{ \sum_{i: w_{3i} \geq w_{3i}^b} \theta \left| w_{3i} - w_{3i}^b \right| + \sum_{i: w_{3i} < w_{3i}^b} (1-\theta) \left| w_{3i} - w_{3i}^b \right| \right\},$$

(B.1)

where $0 < \theta < 1$. When $\theta = 1/2$, (B.1) corresponds to median regression (7b). When $\theta = 1/4$, (B.1) corresponds to quartile regression. Quantile regression offers two advantages over the mean regression in Eq. (7a): quantile regression is robust to outliers and can be carried at different quantiles.

The covariance matrix, Ω, from quantile regressions in (B. 1) is given by:

$$\Omega = \frac{\theta(1-\theta)}{f_{u_\theta}^2(0)} \left(E\left[\left(\frac{\partial w_3^b}{\partial \delta} \right) \left(\frac{\partial w_3^b}{\partial \delta} \right)^T \right] \right)^{-1},$$

(B.2)

where δ is the parameter set of $(\gamma, \beta, \alpha, \theta)^T$; $f_{u\theta}(0)$ is the density of the error term evaluated at 0. The error term u_θ is defined as $u_\theta = w_3 - w_3^b$. As in median regression, to estimate the covariance matrix here, we first conduct a non-parametric kernel regression, and then evaluate the obtained density function at 0 to get $f_{u\theta}(0)$. $\partial w_3^b / \partial \delta$ is calculated by numerical derivatives.

Table 7 presents the regression results with a fixed interest rate 0.04. The results show significant heterogeneity in parameters across individuals at different wealth levels. In general, the risk-averse parameter γ decreases with wealth levels, while the estimated time discount factor and bequest motive increase with wealth levels.

To better understand how people's consumption and wealth vary, we apply estimates from Table 7 to simulate a hypothetical person's consumption and wealth trajectories. The hypothetical person we consider is: single male at age 79 with an optimistic index of .6594. He has two children. There are five different figures denoted as Fig. 4a to Fig. 4f. Each figure assumes the person's initial wealth is at one of the 10th, 25th, 50th, 75th, and 90th percentiles. If a person's initial wealth is at an nth (n = 10, 25, 50, 75, 90) percentile, then his income is also assumed at the nth percentile. In addition, the corresponding graph is generated by a parameter set that is obtained from the *n*th quantile regression in Table 7.

Given the difference in their income and wealth levels, and given the difference in their parameter estimates, it is not surprising to have very different trajectories. For the person at 10th percentile wealth level (Fig. 4b), he consumes his wealth ($120) right away (at age 79) because he values his current utility higher than his future utility. After the first year, his consumption equals his annuity income at $5736. His wealth becomes zero at 80 and stays at zero to his life span. His bequest is zero. For the person at 25th percentile (Fig. 4c), he consumes his initial wealth ($4500) in first two years. After age 81, his consumption equals his annuity income at $8000. For the person with median wealth (Fig. 4d), it takes a longer time for this person to consume all his wealth. His wealth decreases and reaches zero at age 85. His consumption level is highest at age 79, and decreases until he reaches age 85. From age 85, the person's wealth reaches zero and his consumption equals his annuity income at $12,000. If the person dies before age 85, he leaves some bequest. However, such bequest is accidental since his bequest motive is essentially zero. In all these cases, since the person values future utility lower than current utility, his consumption level peaks at the first year and then decreases until it reaches his annuity income level.

At the 75th and 90th percentiles, the time discount factor is larger than one, indicating that the person prefers utilities from future periods to utilities from the current period. In this situation, the consumption level may not peak at the first period. In particular, at the 75th percentile (Fig. 4e), the consumption level first increases and then decreases, and it reaches the peak at roughly age 89. The wealth stays roughly the same before age 81, and decreases since then until age 97. After age 81, because his bequest motive is essentially zero, he starts to dissave and his consumption level keeps going up until his mortality risk is large enough that his consumption starts to fall. Finally, the simulated consumption path based on estimates at 90th percentile regression yields miniscule consumption until age 84. His consumption increases since age 84, reaches maximum at age 99, and decreases after age 99. His wealth level peaks at age 93.

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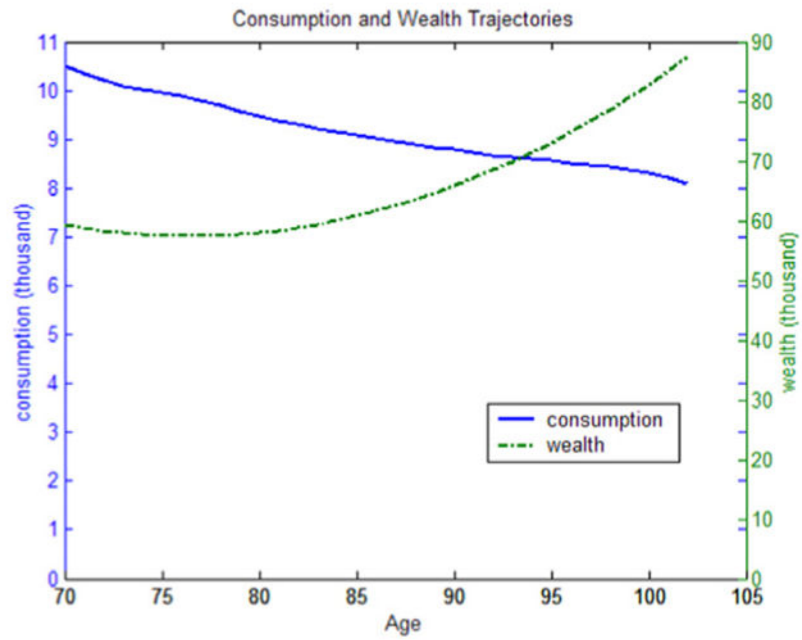

Fig. 1.

Illustration of the positive bequest case.

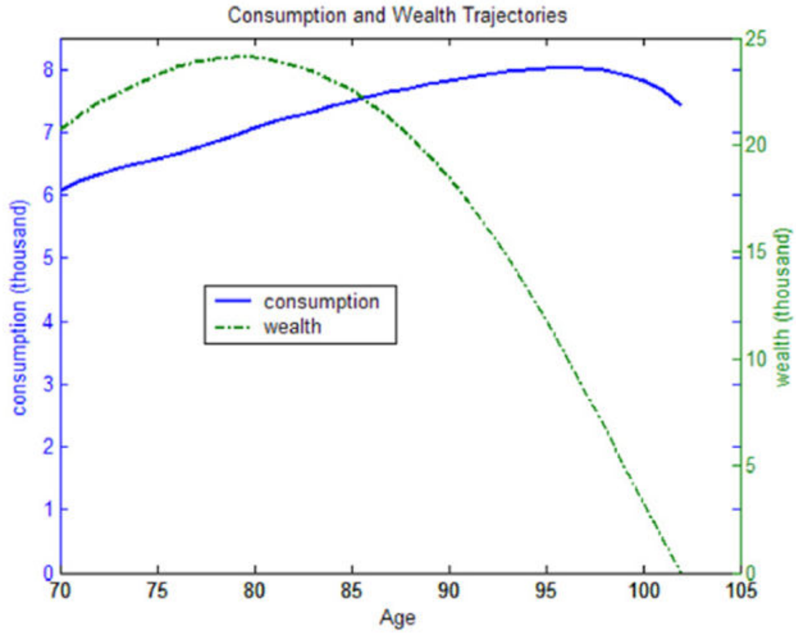

Fig. 2.
Illustration of the zero bequest case (borrowing constraint not binding).

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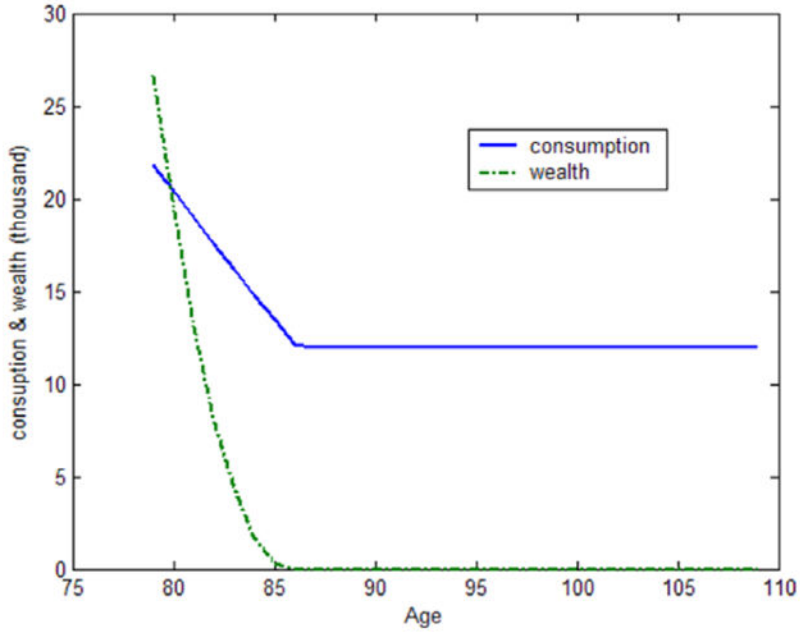

Fig. 3.

Consumption and wealth trajectories at median wealth level. A hypothetical person: male, age 79, 2 kids, optimism index .6594, initial wealth \$35,000, income \$12,000; risk averse γ = 0.9855, time discount β = 0.9420, bequest motive: α_0 = 3.8067e–7, α_1 = 1.0431e–6; desired bequest is \$0.05, and difference in wealth holdings is \$1.17.

	Figure 4-1	Figure 4-2	Figure 4-3	Figure 4-4	Figure 4-5
Description	10th percentile	25th percentile	Median	75th percentile	90th percentile
Risk averse: γ	1.07	1.122	0.9855	0.7703	0.2606
Time discount: β	0.6788	0.8794	0.9420	1.0690	1.2218
Bequest motive:					
α	5.076e-9	1.1209e-8	3.8067e-7	1.2613e-6	0.0012
θ	1.6126e-9	3.6706e-8	1.0431e-6	2.3645e-6	2.3609e-4
Initial wealth	$120	$4,500	$35,000	$125,000	$316,000
Income	$5,736	$8,000	$12,000	$20,000	$35,000
Desired bequest	0	$0.0	$0.05	$5.5151	$1,780.8
Difference in wealth holdings	0	$0.0	$1.17	$95.78	$21,692

Fig. 4a.

Consumption and wealth trajectories (for a hypothetical person: male, age 77, 2 kids, optimal index = .6594).

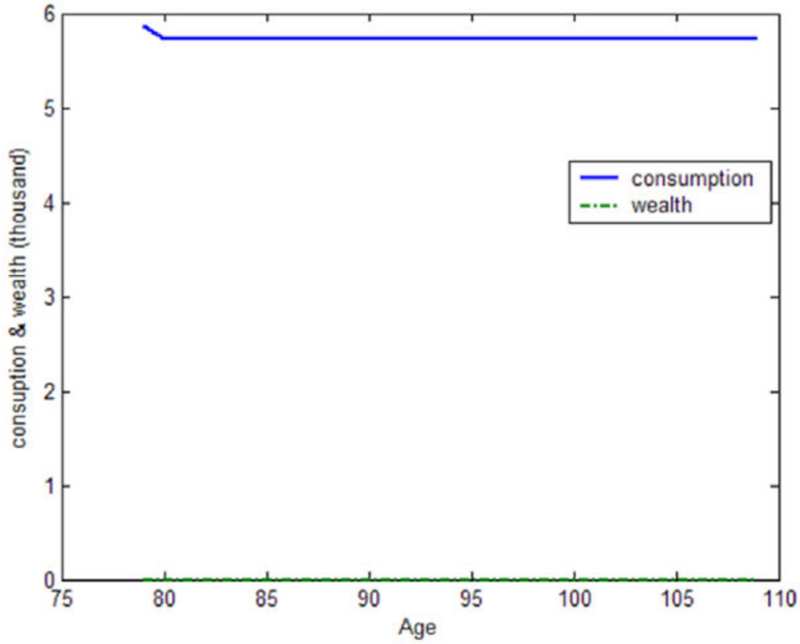

Fig. 4b.
Consumption and wealth trajectories at 10th percentile of wealth.

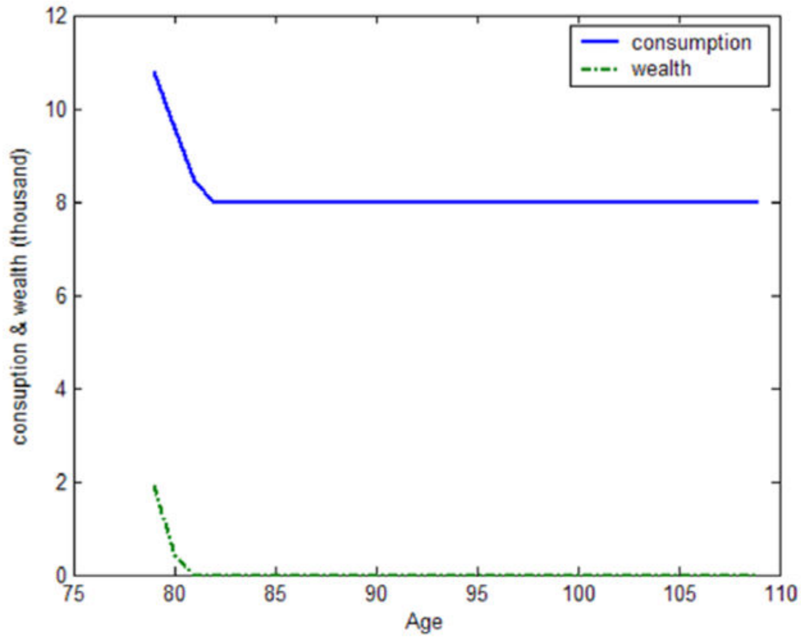

Fig. 4c.
Consumption and wealth trajectories at 25th wealth percentile.

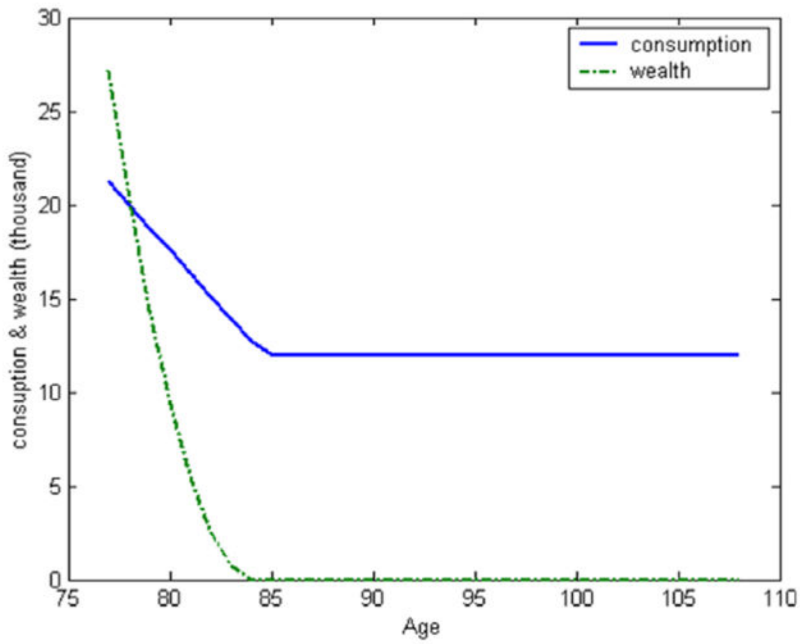

Fig. 4d.
Consumption and wealth trajectories at median wealth level.

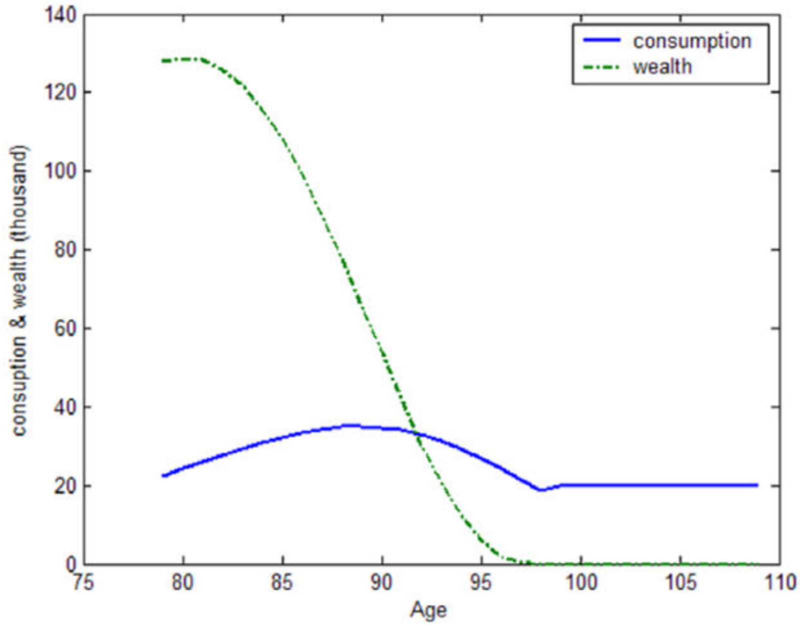

Fig. 4e.
Consumption and wealth trajectories at 75th wealth percentile.

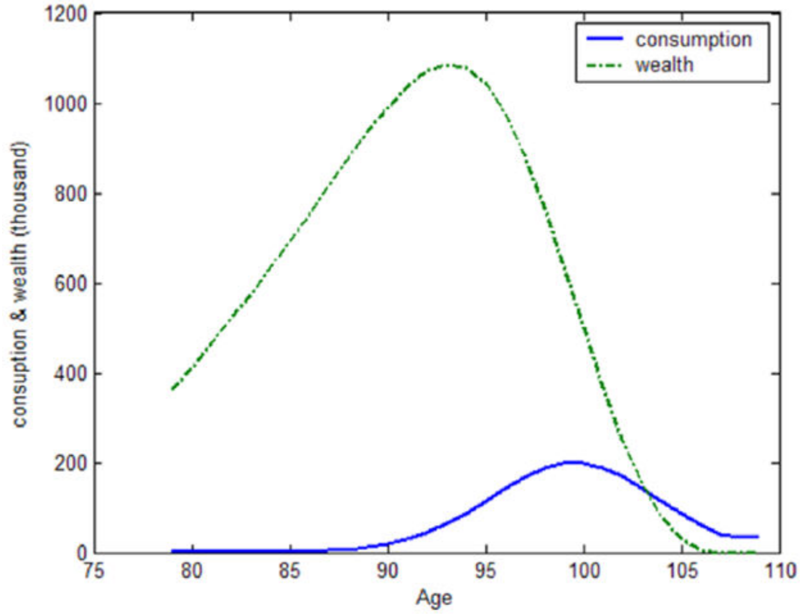

Fig. 4f.
Consumption and wealth trajectories at 90th wealth percentile.

Table 1

Summary statistics of wealth (being alive, single, and nonnegative wealth in the 2nd and 3rd waves; non-missing subjective survival question; in 1995 dollars).

	Wave 2		Wave 3		Wave 4	
	Total wealth	**Non-housing wealth**	**Total wealth**	**Non-housing wealth**	**Total wealth**	**Non-housing wealth**
Mean	221,728	173,042	211,760	168,634	174,428	118,112
Median	78,500	35,000	67,190	23,364	70,746	22,500
Std. Dev	1416,500	1446,572	1299,766	1253,508	404,712	317,598
Min	0	0	0	0	−52,632	−157,895
Max	43,325,000	43,225,000	36,794,393	31,186,916	8368,421	5679,825
No. of Obs.	1903	1752	1903	1752	1460	1460

Table 2

Summary statistics.

	Mean	Std. Dev.	Median	Min	Max
Age of respondents in 1995	79	5.21	78	63	92
Income in wave 2					
Sample of 1903 observations	17,764	22,146	12,000	468	466,000
Sample of 1752 observations	18,107	22,873	12,000	468	466,000
Percentage who have children	80.2%				
Number of children	2.5514	2.3028	2	0	16
Survival probabilities					
optimism index (ψ)	0.6594	0.1176	0.6631	0.4385	1.0906
subjective 3-year survival prob	0.8911	0.0509	0.9026	0.6225	0.9893
life-table 3-year survival prob	0.8347	0.0844	0.8592	0.4175	0.9790
no. of observations in the sample	1752				

Table 3

Estimation results: (marginal utility of bequest = $1_{child}*(\alpha_0 + \alpha_1 *$ No. of kids), interest rate = .04, non-housing wealth).

	Estimation method	Subjective or life table	Risk averse parameter (γ)	Time discount rate (β)	Marginal utility of bequest (α_0)	Marginal utility of bequest (α_1)
A1	Median	Subjective	0.9855 (0.0519)	0.9420 (0.0028)	3.8067e–7 (8.957e–5)	1.0431e–6 (4.6931e–5)
A2	Median	Life table	0.7403 (0.1275)	1.0045 (0.0044)	7.6864e–4 (8.601e–4)	2.1185e–5 (1.7597e–4)
B1	Mean	Subjective	0.7870 (1.544)	1.0546 (0.8767)	1.0008 (0.1525)	1.0022 (0.925)
B2	Mean	Life table	0.7634 (1.295)	1.0763 (0.6890)	0.9986 (0.2316)	0.8941 (0.7546)

Table 4

Robust test with median regression results (varying interest rates, subjective survival rate, non-housing wealth).

Interest rate (r)	Risk averse parameter (γ)	Time discount rate (β)	Marginal utility of bequest (a_0)	Marginal utility of bequest (a_1)
0.02	0.8933 (0.1960)	1.0151 (0.0061)	1.7789e–5 (3.3e–3)	1.8797e–6 (7.9283e–4)
0.03	0.8053 (0.1797)	1.0049 (0.0050)	7.2723e–6 (2.8102e–3)	3.57e–6 (8.4822e–4)
0.04	0.9855 (0.0519)	0.9420 (0.0028)	3.8067e–7 (8.957e–5)	1.0431e–6 (4.6931e–5)
0.05	0.9783 (0.2420)	0.94 (0.0163)	9.7635e–46 (2.6350e–020)	1.3841e–50 (4.8609e–020)
0.06	0.9007 (0.0289)	0.9293 (0.0029)	9.1176e–48 (3.1365e–21)	1.468e–44 (6.1125e–21)

Table 5

Economic significance of marginal utility of bequest (for a hypothetical person: male, age 79, 2 kids, optimism index = 0.6594, initial wealth = $35,000, income = $12,000).

Rows	Risk averse parameter (γ)	Time discount rate (β)	Marginal utility of bequest ($\alpha_0 + 2 * \alpha_1$)	Desired bequest	Difference in wealth holdings
R1	0.9855	0.942	2.4669e–6	$0.05	$1.17
R2	0.9855	0.942	.001	$21.12	$477.22
R3	0.9855	0.942	.1	$32,316	$514,790
R4	0.9855	0.942	1	$125,278	$1082,618
R5	0.9855	0.70	.001	$2.59	$57.26
R6	0.9855	1.00	.001	$80.48	$1434
R7	0.9855	1.20	.001	$1408	$18,238
R8	0.5	0.9420	.001	$5.80	$116.7
R9	1.5	0.9420	.001	$129.5	$2413
R10	2	0.9420	.001	$518.5	$9463

Table 6

Results from out-of-sample predictions.

Models	Med Reg (Subjective) (A1)	Med Reg (Life Table) (A2)	Mean Reg (Subjective) (B1)	Mean Reg (life table) (B2)
Error comparison				
Mean square error	6.5230e8	1.1248e9	2.6798e9	2.7650e9
Absolute error	1.5489e5	1.6440e5	2.6789e5	2.6744e5
Mean comparison				
Predicted mean	87,033	70,719	249,913	247,281
Observed mean		118,112		
Median comparison				
Predicted median	14,795	71,413	96,540	95,1780
Observed median		22,500		
Comparison by quartile[a]				
The first quartile				
Predicted mean	8.6	617.7	33,221	33,791
Observed mean		−1548		
The second quartile				
Predicted mean	7,946.7	2385	74,516	74,004
Observed mean		9091		
The third quartile				
Predicted mean	36,853	23,305	147,202	145,170
Observed mean		53,905		
The fourth quartile				
Predicted mean	3.0189e5	2.5647e5	7.4251e5	7.3481e5
Observed mean		3.5151e5		

[a] The sample is divided into quartiles according to the observed 3rd wave wealth.

Table 7

Quantile regression results: (Marginal utility of bequest = 1_{child} *(α_0 + α_1 * No. of kids), interest rate = .04, non-housing wealth).

Estimation method	**Subjective or life table**	**Risk averse parameter (γ)**	**Time discount rate (β)**	**Marginal utility of bequest (α_0)**	**Marginal utility of bequest (α_1)**
10th percentile	Subjective	1.07 (0.01)	0.6788 (0.2437)	5.076e–9 (3.6102e–4)	1.6126e–9 (1.8606e–4)
25th percentile	Subjective	1.1220 (0.5363)	0.8794 (0.0554)	1.1209e–8 (2.70e–3)	3.6706e–8 (1.30e–3)
75th percentile	Subjective	0.7703 (0.2929)	1.0690 (0.0191)	1.2613e–6 (0.0098)	2.3645e–6 (0.0031)
90th percentile	Subjective	0.2606 (0.1053)	1.2218 (0.0856)	0.0012 (0.0114)	2.3609e–4 (0.04631)