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Three-dimensional characteristics of the mean jet flows induced by an unconfined pitching cantilever plate in a quiescent fluid

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Abstract- Jet flows induced by pitching cantilever plates provide a power-efficient solution for fluid acceleration and cooling enhancement. In such applications, the time-averaged (mean) properties of the induced jet flows are of great importance. We report a combined experimental and numerical study on the three-dimensional characteristics of the mean jet downstream of a harmonically pitching cantilever plate in a quiescent fluid and correlate them with the transient 3D vortex structures emanated from the trailing and side edges. Our Particle Image Velocimetry (PIV) and 3D numerical simulations reveal that the mean induced jet has two distinct regions – a shrinking region immediately downstream of the trailing edge followed by an abrupt expansion region – separated by the so-called necking point. We investigate the transient 3D wake vortex evolution downstream of the plate to help elucidate the physics underlying the geometry of the mean jet. Our observations suggest that the breakdown of the shed vortex structure and the particular reorientation of the consequent sub-structures are the primary factors dictating the shape of the jet. Investigations of the effect of several geometric and vibrational factors on the mean jet characteristics suggest that the cantilever width and the amplitude of the oscillations predominantly control the mean jet boundary. The results presented in this study improve our understanding of the complicated three-dimensional geometry of the induced mean jet in oscillating cantilevers and facilitate the optimized design of the devices that operate based on this principle, such as piezoelectric fans.

Keywords: 3D flows, 3D jet, electronics cooling, fluid acceleration, fluid jet, horseshoe vortex, IPMCs, piezoelectric fans, resonating cantilevers, vibrating thin plates, vorticity.

1. Introduction

The characteristic of flows generated by vibrating thin plates has been studied extensively in the past, due in part to its importance in understanding the natural behavior of aquatic and flying species. They also have enabled innovative employments of biomimetic structures in such applications as piezoelectric fans for electronics cooling [1], [2], micro air vehicles (MAVs) [3], swimming robots [4], fluid acceleration using the ionic polymer metal composites (IPMCs) [5]–[7], and energy harvesting [8]. In applications that require net flow generation, such as fluid acceleration using piezoelectric fans or IPMCs, understanding the complex three-dimensional geometry of induced time-averaged jets and underlying transient wake vortex structures is critical for systematic designs of such devices.

Many past studies focused primarily on the 2D thrust and drag signature of wakes and their interactions with oscillating objects, with limited attention to the three-dimensionality of induced jets downstream. In their studies [9], [10] on the two-dimensional vortex patterns generated by a pitching airfoil in a non-zero free stream, Bohl and Koochesfahani observed that changes in the patterns of vortices being shed from airfoil trailing edges correlate with the transition from the drag-producing to thrust-producing downstream flows. Complementary to these previous studies, Schnipper et al. [11] visualized different vortical patterns under wide ranges of vibration frequencies and amplitudes and constructed a corresponding phase diagram. Heathcote et al. [12] used flexible flapping plates in a quiescent fluid to analyze the effects of the vibration amplitude and beam stiffness on thrust generation. Eastman et al. [13] studied thrust force generation by vibrating cantilever plates and proposed a correlation between the thrust and the vibration amplitude and frequency. Shrestha et al. [14] studied lateral vortex shedding in a pitching plate with a fixed geometry using the particle image velocimetry (PIV) and proposed a 2D vortex regime map to describe the observed vortex patterns. These patterns were correlated with aerodynamic drag forces on the plate. Dewey at al. [15] experimentally studied the power efficiency of thrust generation using flexible and rigid pitching plates.

Other studies focused on 2D velocity profiles downstream of pitching plates. Kim et al. [16], [17] and Choi et al. [18] studied the mechanisms of vortex generation and propagation in the 2D wake of a pitching plate in a quiescent fluid. They visualized induced flow fields that result from the interaction of the shed vortices. Dehdari Ebrahimi et al. [19], [20] used PIV and 2D numerical simulation to study the vortex evolution at the trailing edge of pitching cantilever plates along their mid-span planes over wide ranges of frequencies, amplitudes, and plate lengths. They observed three distinct vortex regimes in the wake as they increased the Reynolds number and demonstrated that these vortex regimes influence flow generation ability of pitching plates. Stafford and Jeffers [21] used PIV to investigate 2D flow fields and pressure contours in the downstream flows induced by a piezoelectric fan.

These past 2D studies, however, did not provide a complete picture of the evolution of jet-like flows downstream of oscillating plates. Past 3D studies demonstrated that vortices shed from the trailing and lateral edges of a pitching plate form horseshoe structures that interact with each other as they propagate downstream. Buchholz et al. [22]–[26] observed that each horseshoe is significantly deformed under the influence of two subsequent horseshoes: an opposite-sign interaction with the horseshoe shed immediately after and a like-sign interaction with the horseshoe that follows with 2π phase difference. Green et al. [28] and King and Green [29] performed 3D PIV measurements combined with a Lagrangian Coherent Structure (LCS) analysis on trapezoidal pitching plates in a free stream and provided 3D graphical representations of the vortex structures in the wake, confirming the aforementioned interactions. Taira and Colonius [31] used numerical simulations to investigate transient vortex structures shed from a pitching plate in a free stream starting from rest. By studying the three-dimensional horseshoe vortex structures emanating from a pitching plate in a quiescent fluid, Agarwal et.al [32] showed that the complex interactions reported in the previous studies are highly influenced by free stream flows and that they are largely absent in quiescent fluids. Oh et al. [33], [34] used the moving mesh method to perform 3D numerical simulation and analyze vortex structures and flow fields in the wake of pitching plates in a quiescent fluid confined by two end walls. They reported unusual 3D shapes of mean jet flows downstream of these plates.

Although these past studies provided useful pictures of transient 3D vortex structures, detailed quantitative investigation of different factors affecting the shapes and other characteristics of downstream jets and their underlying mechanisms have been largely lacking. In the present work, we systematically investigate the 3D characteristics of induced jets in the wake of a pitching cantilever plate in a quiescent fluid and correlate these characteristics with transient vortex structures and their temporal evolution. We perform PIV experiments together with 3D numerical simulation using the immersion boundary method to visualize and extract wake structures downstream of plates under different vibrational (frequency and amplitude) and geometric (width and length) parameters. Our study also helps elucidate spatial evolution of the time-averaged jet shape.

The rest of the paper is organized as follows: In Section 2 we describe the parameters and experimental setup used in the present study. Section 3 provides a description of the geometry and other details of our numerical simulation. Section 4 contains the results and discussions regarding the fluid jet and wake vortex structures observed in the experiments and numerical studies. Summary and conclusions are presented in Section 5.

2. Experiment Parameters and Setup

The vibrational characteristics of a pitching plate are described by its frequency, f, and amplitude, A. We limit ourselves to small amplitude vibrations and represent oscillation of the trailing edge position y as

$$y = A\cos(\overbrace{2\pi f t}^{\phi}) \tag{1}$$

where φ is the oscillation phase. The characteristic trailing edge (or "tip") velocity is defined as $u_{\text{tip}} = fA$. The main length scale relevant to 3D vortex shedding from the trailing edge is the width of the plate, w. The thickness of the plate, t, only has secondary effects on the vortex generation. As we shall discuss later in Sec.4, the length of the plate, l, also has relatively small influence on the characteristics of downstream jet flows for the plates considered in the present study (Table 1). The aspect ratio of the plate is defined as $AR = w/L$. The normalized amplitude, α, which represents the aspect ratio of the oscillation envelope, is defined as

$$\alpha = \frac{A}{w} \tag{2}$$

and the oscillatory Reynolds number, Re, is defined as

$$Re = \frac{u_{\text{tip}}\, w}{\nu} = \frac{fAw}{\nu} \tag{3}$$

We used a commercial piezoelectric actuator (Steminc Inc., SMPF61W20F50) to oscillate cantilever plates about their resonance frequencies. The actuator is 20 mm wide and 23 mm long (Fig. 1a). Flexible thin plates made of mylar with different dimensions were prepared in-house and attached to the actuator using cyanoacrylate glue. Table I lists the geometric parameters of the blades (width, w, and length, l, thickness, t) along with their vibrational characteristics (frequency, f, and amplitudes, A) used in the present study. Different resonance frequencies for the same plate (for example, Cases 7 and 8 in Table I) are obtained by changing the plates' mass distribution using a method reported in a previous study [2].

Small-amplitude sinusoidal voltage waves from a function generator (Model 33220A, Agilent) were amplified using a high-voltage amplifier (Model PZD700A, TREK) before being fed to the actuator (Fig.1b). The desired vibration amplitudes wer obtained by tuning the input voltage and measured optically with an uncertainty of ± 0.1 mm less than, 10% of the minimum amplitude used in this study.

The 1 mm-thick illumination sheet for our PIV experiments (Fig.1b) was generated using a 500 mW continuous wave laser (Hercules, LASERGLOW Technologies) with a wavelength of 532 nm. Successive frames were recorded using a high-speed camera (Phantom VEO-640L) capable of acquiring 16-bit, 4-megapixel images. The frame rate in each experiment was set to capture at least 70 frames per each full period of plate oscillation, equivalent to framerates > 6300 fps. A shutter speed of 80 μs was used to reduce the effects of motion blur.

PIV experiments were conducted in a large sealed transparent chamber made of acrylic plates (30 cm × 20 cm × 8 cm) to reduce interference from the side walls [35]. Seeding particles were generated by evaporating a solution of water and glycerin (30% glycerin in volume). The particles were then allowed to settle for about a minute before conducting any experiment to minimize initial disturbance from particle injection or previous experiments. The chamber was mounted on a linear translation stage with a resolution of 0.01mm in order to move the plate relative the illumination sheet (Fig.1b).

An open-source MATLAB software package, PIVlab [36], is used to analyze the captured video images. 64×64-pixel windows with 50% overlap are cross-correlated in two successive frames using the Advanced Discrete Fourier Transform technique to obtain the direction and magnitude of particles' displacements in sub-areas. The calculated velocity fields are next post-processed by manually filtering the outlier data and replacing them with interpolated equivalents in areas where improper lighting condition leads to inaccurate displacement vectors. An algorithm based on penalized least squares method [37] is employed to reduce the noise in the flow fields.

The PIV setup and image processing procedure are validated using several benchmarks [38]. In general, the bias error from window deformation and random errors from the cross-correlation algorithms are the main sources of uncertainty in the image processing steps. But for three-dimensional flows, the out-of-plane velocities are generally a larger source of error. These out-of-plane flows cause seeding particles to leave the illumination sheet in-between successive images, leading to random error. Assuming a very conservative estimate for the out-of-plane velocity of 1 m/s (of the same order as the maximum expected streamwise velocities) and a minimum framerate of 6300 fps, particles are projected to travel at most 0.16 mm or 16% of the illumination sheet thickness between successive frames. Other studies values as high as 50% [28]. An overall uncertainty of 0.14 pixel is estimated for the displacement vectors obtained in out PIV experiments. Considering other uncertainties, such as pixel to distance conversion and camera framerate accuracy, we estimate a total uncertainty in velocity measurements of 0.016 m/s.

To construct 3D vortex structures from the results of our PIV experiments, we first extract 2D PIV data from 21 planes uniformly spaced 1 mm apart along the span of the plate (normal to the z axis, Fig.1c). The identified vortex cores in each plane are then interpolated to form 3D vortex structures. A similar technique was reported in [28] and [29]. Further discussion of this technique is provided in Sec.4.

Fig. 1. (a) Schematic of the piezoelectric actuator and the attached plate. (b) Schematic of the PIV experiment setup. (c) Visualization planes for the 3D PIV data extraction used in the present study.

Table I. Geometric and vibrational characteristics of the plates used in the present study and the corresponding dimensionless parameters. The two last columns indicate whether PIV or numerical simulation results are available for each case. The vibration amplitude A is varied among Cases 1, 2, 11 and 12 for otherwise identical plates; the plate width w for Cases 1, 3 and 4; the plate length l for Cases 1, 5 and 6; and the vibration frequency f for Cases 1, 7, and 8.

Case No.	l (mm)	w (mm)	t (mm)	f (Hz)	A (mm)	α	Re	AR	PIV	Sim.
1	20	20	0.2	70	2.5	0.123	230	1	•	•
2	20	20	0.2	70	1.4	0.07	131	1	•	•
3	20	15	0.2	70	2.5	0.164	172	0.75	•	•
4	20	30	0.2	70	2.5	0.082	344	1.5	•	•
5	15	20	0.2	70	2.5	0.123	230	1.33	•	•
6	30	20	0.2	70	2.5	0.123	230	0.67	•	•
7	20	20	0.2	40	2.5	0.123	131	1	•	•
8	20	20	0.2	100	2.5	0.123	328	1	•	•
9	15	20	0.2	70	1.8	0.092	172	1.33	×	•
10	30	20	0.2	70	3.7	0.185	345	0.67	×	•
11	20	20	0.2	70	0.6	0.03	56	1	×	•
12	20	20	0.2	70	0.2	0.01	19	1	×	•

3. Numerical Simulations

Three-dimensional numerical simulations are performed using a commercial computational fluid dynamics package (Ansys CFX). We use the immersed boundary method to circumvent large computational overheads associated with re-meshing and distorted nodes in the moving mesh methods. The numerical simulation domain is $3w \times 2w \times w$ (Fig.2a). We confirm that the domain is large enough to avoid complicating effects at the boundaries (HOW DID YOU VERIFY THIS?). The open boundary condition is specified on the outer walls of the computation domain to allow free fluid motion in directions normal to the domain boundaries. The relative pressure on these boundaries is specified as zero.

We use tetrahedral meshes of spatially varying sizes along the trailing edge: 0.005*w* in 7 spheres of radius 0.125*w* centered along the trailing edge; 0.01*w* in 3 intermediate spheres of radius 0.25*w*; and 0.025*w* in the rest of the domain (Fig.2b). Mesh independence was assessed by comparing the peak velocities calculated from the time-averaged velocity profiles of a representative case after 10 full cycles for different mesh sizes. Less than 3% variation was obtained for mesh sizes half (??) the chosen set of values.

The cantilever plate is approximated as a rigid body rotating about a pivot point at its base. This approximation is justified by the predominance of the lowest-order vibration mode. Fig.2c compares an actual plate deflection obtained from one of our experiments with that from the rigid body approximation at an exaggerated amplitude. Our approximation is deemed reasonable because, in pitching slender bodies, it is the motion of the trailing edge and its vicinity that primarily determines the wake properties and thrust or drag signature and not intermediate motions along the plate [39]. This approximation was also shown to successfully predict the characteristics of a two-dimensional jet from a cantilevered plate [19].

The first-order upwind scheme is used for advection terms and the first-order backward Euler scheme is used for time marching. The time step in the computations is automatically adjusted such that the Courant number remains smaller than one in all computational cells.

Fig. 2. (a) Numerical simulation domain. (b) Mesh used in the present numerical simulation. (c) Comparison between the actual plate deflection and the rigid body approximation (images are not to scale).

4. Results

4.1 ~~Time-averaged Three-dimensional jet (NEED more specific subtitle)~~

Fig.3a shows a typical time-averaged velocity profile of a pitching cantilever plate over 10 cycles of oscillations obtained from numerical simulations (upper half) and PIV experiments (lower half) for Case 1 (Table 1). The white solid lines in the lower image correspond to the projected streamlines from the experimental results. We define the jet boundary by identifying and interpolating the loci of flow entrainments (white circles along the streamlines on the *x-z* plane, lower left Fig.3a) where the direction of the time-averaged velocity changes from +*x* to –*x*. The negative time-average velocities outside the jet boundary are rather unique to jets produced by pitching plates. The boundary of other jets is typically defined using a threshold value of streamwise velocity, e.g. 1% of the momentum-averaged velocity.

The height of plate in a) and b) MUST match! Part a (x-z plane) should also have "non-contributing" width labeled.

Fig. 3. (a) Time-averaged velocity profile of a pitching cantilever plate in the *x-z* and *x-y* planes (Case 1). The thick white arrows indicate the tip-to-tip displacement of the trailing edge (= 2*A*). (b) Time-averaged jet boundary projected on the *x-z* plane.

Figure 3 suggests that the induced mean jet forms two distinct regions downstream. Close to the trailing edge, the mean jet boundary in the *x-z* plane shrinks linearly in the spanwise direction towards the midspan line as the flow proceeds in the +*x* direction, while staying nearly independent of *x* in the *x-y* plane. The average velocity in this region is nearly uniform, with small fluctuations around the average value, u_{ave} (Fig. 4a). The magnitude of this average velocity is approximately a linear function of the tip velocity with a slope of ~5.6, as shown in the inset of Fig. 4a. Further downstream, starting at approximately x/A ~ 1.7, the jet boundary stops shrinking in the *x-z* plane and forms two parallel lines. This coincides with an abrupt expansion of the jet boundary on the *x-y* plane, accompanied by a significant change in the direction of the velocity vectors. We refer hereafter to the borderline between the two regions as the "necking point", defined approximately by the intersection of two tangent lines to the jet profile as shown in Fig. 3b. The streamwise location of the necking point is referred to as the necking length and serves as a quantitative measure of the streamwise extent of the jet.

The streamlines form two spiral curves on the top and bottom of the oscillation envelope on the *x*-*y* plane (Fig.3a, lower right subplot). These spirals indicate spanwise flow toward the midspan of the plate. These two spiral streamlines confine the flow to a high velocity region between them and form the boundary of the jet on the *x*-*y* plane upstream of the necking point. As we shall show later, these spirals and the associated change in direction of the flow are related to the transient tilting of vortex structures shed from the trailing edge. Downstream of the necking point, the streamwise velocity component decreases significantly (Fig. 4a) due to the mixing facilitated by interactions of the vortices.

We also note from Fig. 3 (on the *x*-*z* plane), not all parts of the trailing edge, along its width, contribute to net positive flow generation. The two regions near the corners in fact experience weakly negative streamwise velocities. The width of these regions is defined as the non-contributing width. We calculate the non-contributing widths using our numerical results because plate motion and light reflection make PIV results unreliable right in the vicinity of the trailing edge.

Fig. 4b shows the time-averaged streamwise velocity profiles on the *x*-*z* plane (y = 0) at 5 different x locations. We observe that the velocity profiles before the necking point (x/A = 0.2 and 0.8) exhibit two peaks on either side of the midspan line. In contrast, the velocity profiles after the necking point (x/A = 2.44, 3.25, 4) have only single peaks along the midspan line. These profiles after the necking point exhibit approximately the same shape outboard of $z/w \sim \pm 0.05$, consistent with two nearly parallel lines defining the jet boundary after the necking point.

Fig. 4. (a) Time-averaged streamwise velocity along the midspan line, y = 0 and z = 0 (Case 1). (b) The spanwise profile fo the time-averaged streamwise velocity at 5 locations along the x direction shown in Fig.3b.

The geometry of the jet can be understood by considering the transient three-dimensional wake vortex structures shed from the trailing and side edges of the plate. Fig. 5 shows such structures in a full cycle of oscillation as obtained from our PIV experiments and numerical simulation. The vortex cores are identified using the second invariant of the velocity gradient tensor ∇U, a method known as the Q-criterion method [40]. For consistency, the threshold Q value is set as 10% of the maximum Q value at phase φ = 0. In the numerical simulation results, the color of the vortex structures represents the magnitude of the vorticity in the z-direction, ω_z. We note that, because our PIV results do not resolve the spanwise velocity component, the color only represents the sign of the vortex (clockwise or counter-clockwise). Vortices along the side edges are also not captured in the PIV results.

Fig. 5 shows that vortex formation starts slightly earlier than φ = 0, with counter-clockwise vortices forming in the spanwise direction along the width of the plate at the trailing edge. This was also confirmed in the 2D studies of Kim *et al.* [16] and Dehdari Ebrahimi *et al.* [19]. This spanwise structure is accompanied by two streamwise vortices at the cantilever side edges, which together form a horseshoe structure. The oscillation velocity of the pitching plate linearly increases with the chordwise distance from the leading edge. Therefore, the vortex strength of the legs of the horseshoe increases in the chordwise direction. At the same instance, the previously generated clockwise vortex still exists as a coherent structure. This structure is attached to the cantilever plate at the sharp corners and increasingly separated from the plate towards the midspan. As the cantilever plate continues its downstroke half-cycle towards the neutral position ($\varphi = \pi/2$), the counter-clockwise vortex structure grows and separates from the trailing edge in the midspan while still being attached to the corners. In addition, the clockwise vortex structure from the previous cycle breaks down from the midspan and the corners, forming two coherent sub-structures.

We emphasize that the vertical structure does not disappear at the midspan and corners. Instead, it elongates, forming a hairpin vortex that connects the two sub-structures. This hairpin vortex structure is not shown in Fig. 5 as its strength is below the threshold Q value. The two vortex sub-structures are wrapped around the new spanwise vortex as they are convected downstream. This wrapping effect adds a y-direction component to the vorticity of the sub-structures, inducing a significant flow toward the midspan and a corresponding outward flow in the y-direction. This behavior leads to the abrupt expansion of the jet boundary on the x-y plane (Fig. 3a) discussed earlier. At $\varphi = \pi$, the clockwise vortex sub-structures travel further downstream while losing their core strength owing to diffusion. Due to the low strength of these vortex sub-structures, our PIV visualizations were not able to capture them beyond this instant. At the same instant, the counter-clockwise spanwise structure (shown in red) has gone through the same process and is ready to break. At $\varphi = 3\pi/2$, the counter-clockwise vortex structure breaks and the consequent sub-structures wrap around the new vortex in opposite directions, similar to $\varphi = \pi/2$.

The vortex breakdown and tilting in the wake of pitching plates was also reported by Green *et al.* [28]. However, due to the higher strength of their vortices, vortex breakdown was delayed until after two and a half cycles (as opposed to less than one cycle in the present study). Furthermore, the presence of a free stream in their experiments augmented the vortex transport such that the vortex breakdown and the subsequent jet boundary shrinkage only occurred near the end of their domain.

Fig. 5. Three-dimensional vortex structures obtained from the experimental and numerical simulation results during one full cycle of oscillation (Case 1).

Fig. 6 illustrates the temporal evolution of the y component of vorticity on the x-z plane over a half pitching cycle. Considering symmetry, we only show half of the period of oscillations, $1/f$, in this figure. The projections of the vortex sub-structures described above are shown as high vorticity regions in this plane. These high vorticity regions are concentrated along the jet boundary and are the primary reasons for the spanwise shrinking of the jet shape. We note that, unlike our numerical simulation results, the instantaneous PIV results are asymmetric about the midspan line. This asymmetry may be due to imperfections in the plate (such as uneven mass distribution due to manufacturing or assembly flaws) or uncontrolled secondary flows in the chamber. However, these disturbances do not significantly influence the mean flow behavior, shown previously in Fig. 3.

Fig. 6. Temporal evolution of the y-component vorticity in x-z plane over a half pitching cycle.

To further elucidate the three-dimensionality of vortex shedding from the trailing edge and clarify the presence of a non-contributing width, we compare the instantaneous spanwise vorticity contours in 2D PIV results along the span of the plate. Fig.7 demonstrates these contours and the velocity vectors at $\varphi = 3\pi/2$ on three different planes at indicated spanwise locations. Near the midspan ($z/w = 0$), the vortices have higher strength and are more separated from the trailing edge. These vortices exhibit features resembling the propagating vortices observed in previous work [19]. Towards the corners of the thin plate, the vortex is inclined more toward the streamwise direction and remains attached to the trailing edge, characteristic of the non-propagating regime reported in [19]. An appreciable decrease in the strength of the spanwise vortex component and the velocity magnitude is also evident. Similarly, Green *et al.* [28] observed a change from 2S thrust producing vortex alignment near the midspan to 2S drag producing alignment close to the corners for their trapezoidal pitching plates in moving water. We can attribute the existence of non-contributing parts close to the spanwise extremes of the pitching plate to the presence of such non-propagating vortices at the plate corners.

Fig. 7. Spanwise vorticity contours at three different spanwise locations at $\varphi = 3\pi/2$.

4.2 XXXX

In the following sub-sections, we investigate the effect of several vibrational (amplitude and frequency) and geometric (width and length of the cantilever) factors on the three-dimensional shape of the jet and the vortex behavior at the trailing edge.

4.2.1 Effects of vibration amplitude

The two cases illustrated in Fig. 8, corresponding to Cases 1 and 2 in Table I, differ only in their vibration amplitudes. The extent of the jet boundary in the streamwise direction, as qualified by the necking length in Fig.8, is highly dependent on the amplitude. The necking length increases from 3.2 mm for the lower amplitude (corresponding to $x/A{\sim}2.3$) to 4.2 for the larger amplitude (corresponding to $x/A{\sim}1.7$), that is, a 30% increase for a 75% increase in the amplitude. The non-contributing width also depends strongly on the amplitude, changing from 1.18 mm (z/w= 0.44) to 2.66 mm (z/w=0.36). A more detailed discussion of the amplitude dependence of the necking length and the non-contributing width is presented in Section YYY.

Indicate the value of A inside each plot

Fig. 8. Normalized jet boundary projected on the x-z plane for different amplitudes: (a) Case 1 (A =2.5 mm) and (b) Case 2 (A = 1.4 mm).

Fig. 9 shows the vortex core structures emanated from the trailing and side edges of the plates for the two cases presented in Fig. 8. In these cases, the maximum and minimum height of the vortex structures at the midspan approximately coincide with $y = \pm A$. Furthermore, the vortex structures in both cases arch over the width of the plate and connect to the sharp corners regardless of their vibration amplitude. As a result, the curvature (bending) of the vortex structure is more pronounced at the larger amplitude. The normalized amplitude, α, serves as a quantitative determinant of the curvature of the vortex structures.

In addition, the strength of the vortex structures increases with vibration amplitude, as indicated by the color of the structures, enabling them to travel further downstream before they dissipate by diffusion (??). The combination of these two effects, i.e. increased curvature of the structures and higher vortex strength, causes the flow to stretch more in both the x and y directions at higher amplitudes.

Space the two rows further apart. Indicate the amplitude for each row (on the right??)

Fig. 9. Three-dimensional vortex structures obtained from numerical simulation during one full cycle of oscillation at different amplitudes (A = 1.4 and 2.5 mm, corresponding to Cases 2 and 1) in the respective rows.

4.2.2 Effects of plate width

Fig. 10 shows the jet boundary for the plates that only differ in their widths. The spanwise extent of the mean jet conforms linearly to the width of the plate, with an exception of areas close to the trailing edge, where the effect of the non-contributing width of the plate is more pronounced. The non-contributing width is a strong function of the amplitude but has only a slight dependence on the width, increasing only 27% when the width is increased from 12.7 mm (is it 12.7 mm or 15 mm??? Not consistent with Table I) to 30 mm. Fig. 10 also suggests that increasing the plate width elongates the mean jet in the streamwise direction, as indicated by the locations of the necking points. The necking length of the largest width (w = 30 mm) is ~76% larger than that of the narrowest plate (w = 12.7 mm).

Indicate the value of w in each subplot!

Fig. 10. Normalized jet boundary projected on the x-z plane for different widths corresponding to (a) Case 3 (w = 12.7 mm), (b) Case 1 (w = 20 mm), and (c) Case 4 (w = 30 mm).

Fig. 11 shows the downstream wake vortex structures for plates of different widths. From the frontal views of the vortex structures at $\varphi = 0$ and $\varphi = \pi$, we note that the maximum and minimum height of the vortex structures in the midspan are almost independent of the width. That is, the vortex structures have nearly the same normal distance from the trailing edge at these phases. ~~For the narrowest width (w = 12.7 mm), the vortex structure attaches to the corners of the cantilever with a sharp slope, causing more curvature of the structures at lower width, which in turn makes the two wrapped sub-structures in $\varphi = \pi/2$ and $\varphi = 3\pi/2$ closer to each other with a relatively small angle. For the intermediate width (w = 20 mm), the vortex core at the midspan is connected to the trailing edge corners at a milder slope, and thus, the consecutive vortex sub-structures are further apart and form a wider angle. For the widest plate (w = 30 mm), the vortices form a nearly flat structure around the midplane and the flow may be approximated as two-dimensional. The separated vortex structures has weaker coherence for plates with smaller widths and more likely to break from the mid-span.~~

(I HAVE NO IDEA WHAT YOU ARE DESCRIBING. WHAT IS SHARP SLOPE? WHAT IS LOWER WIDTH? Etc etc POOR/INACCURATE WORD CHOICES AND CONVOLUTED LONG SENTENCEs MAKE THEM IMPOSSIBLE TO UNDERSTAND)

Space the rows further apart. Indicate the width for each row (on the right??)

Fig. 11. Three-dimensional vortex structures obtained from numerical simulation during one full cycle of oscillation for different plate widths in the respective rows: w = 12.7 mm (Case 3), 20 mm (Case 1), and 30 mm (Case 4).

Effect of length

Our experimental and numerical simulation results show that, for the plates considered in the present study, the plate length has relatively small effects on the geometry of the jet. Fig. 12 shows the jet boundaries downstream of the pitching plates of different lengths, showing no considerable change. Only 3% increase in the necking length and 18% increase in the non-contributing width are observed when the plate length is increased from 15 mm to 30 mm.

Indicate the value of l inside each subplot!

Fig. 12. Normalized jet boundary projected on the *x-z* plane for different lengths: l = 15 (Case 5), 20 (Case 1), 30 mm (Case 6).

Fig. 13 shows the wake vortex structures for different plate lengths examined. The curvature of the vortex structure is nearly identical in all three cases, indicating that the amplitude and the width are the primary length scales affecting the wake. But the coherence of the vortex structures is weaker at longer plate lengths and the structures generated by the longer plates are more likely to break earlier.

Space the rows further apart. Indicate the length for each row (on the right??)

Fig. 13. Three-dimensional vortex structures obtained from numerical simulations during a full cycle of oscillation for different lengths in the respective rows: l = 15 (Case 5), 20 (Case 1), 30 mm (Case 6).

4.2.3 Effect of vibration frequency

Fig. 14 shows the jet boundary for three different frequencies of the pitching motion. Similar to the the plate length, changing the frequency does not strongly affect the boundary of the jet. However, the velocity magnitude of the jet is almost linearly proportional to the frequency as previously shown in Fig. 4. The slight up-down asymmetry observed in the PIV results at the lowest frequency is believed to be an experimental artifact caused by the misalignment of the plate with respect to the illumination sheet.

Indicate the value of *f* inside each subplot!

Fig. 14. Normalized jet boundary projected on the *x-z* plane for different frequencies: f = 40 (Case 7), 70 (Case 1), and 100 Hz (Case 8).

4.2.4 Necking length and non-contributing width

The normalized necking length for all the cases examined in this study are presented in Fig. 15a. The necking length varies approximately with α as

$$\frac{x_{NP}}{A} = n\,\alpha^{m} \tag{4}$$

The power law curve fits to our experimental and numerical simulation results are shown with the solid and dashed lines, respectively. The exponent, m, is equal to -0.55, suggesting that the necking length is influenced to similar degrees by the plate width and vibration amplitude ($x_{NP} \sim A^{0.45}w^{0.55}$). The coefficient n is slightly (~17%) larger for the experimental data. Fig.15b shows that the normalized non-contributing width follows a similar power law trend ($w^* = 0.74\,A^{0.85}w^{0.15}$). The correlation suggests that the non-contributing width is predominantly governed by the amplitude of the pitching motion and is only slightly affected by the plate width.

The experimental data seem systematically larger than the numerical simulation results. Why?

Fig. 15. (a) Normalized necking length as a function of the normalized vibration amplitude. The solid and dashed lines correspond to the power law curve fits to the experimental and numerical simulation results, respectively. (b) Normalized non-contributing width obtained from the numerical simulation results as a function of the normalized amplitude. The dashed line represents the power law curve fit.

4.3 Universality of jet shape

Fig. 16 shows the jet boundaries, as projected on the *y-z* plane, at different streamwise locations from the trailing edge. The mean streamwise flow starts as a uniform jet exiting the aperture defined by an oscillation envelope. Before reaching the necking point (Fig. 16 a and b), the jet shrinks linearly in the spanwise (z) direction but remains confined within approximately $\pm A$ in the normal (y) direction.

As the flow passes the necking point (Fig. 16 c – e), the jet boundary expands in the normal (y) direction. The width of the jet boundary (along the z direction), however, remains nearly constant ($0.15w$) after the necking point.

We note that the jet boundaries from all the cases considered in the present study collapse onto nearly universal curves if we normalize the position along the streamwise (x) direction with $(Aw)^{1/2}$, the position along the normal (y) direction by A and the position along the spanwise (z) direction with either w-w^* (before the necking point) or w (after the necking point). Accounting for the non-contributing width (w^*) in the normalization is crucial for obtaining the convergence. After the necking point, the effect of the noncontributing width is diminished.

This result suggests that the boundary of jets generated by pitching cantilever plates (such as piezoelectric fans and IPMCs) can be described using the plate width and vibration amplitude.

Cases and NOT Tests!

Fig. 16. Jet boundaries projected on the *y-z* plane at different streamwise locations (a) $x/(Aw)^{1/2} = 0.07$, (b) $x/(Aw)^{1/2} = 0.29$, (c) $x/(Aw)^{1/2} = 0.86$, (d) $x/(Aw)^{1/2} = 1.14$, and (e) $x/(Aw)^{1/2} = 1.43$.

5. Conclusion

Please rewrite

~~We performed a combined experimental and numerical simulation study on the three-dimensional geometry of the induced mean jet downstream of a pitching cantilever plate and correlated it with the transient vortex behavior in the wake. Our results showed that the time-averaged velocity field forms two distinct regions in the wake identified by their shrinking or expanding nature. The first region extends from the trailing edge to a downstream distance of ~ 0.55$(Aw)^{1/2}$, called the necking point. In this region, the~~

induced fluid jet from the trailing edge shrinks linearly in the spanwise direction towards the midspan line as the flow continues in the streamwise direction. The extent of the jet in the normal direction, however, remains contained within the tip-to-tip displacement of the trailing edge (2*A*) in this region. The velocity profile in this region is approximately uniform and directed in the forward direction with a magnitude of ~5.6 times the tip velocity (*Af*). The second region starts after the necking point. In this region, the jet boundary stops shrinking in the spanwise direction and forms approximately parallel surfaces 0.15*w* apart, and instead, expands abruptly in the normal direction. An important characteristic of the second region is the appreciable decay of the streamwise component of the velocity after the necking point.

To illustrate the physics behind the complicated shape of the mean jet, we investigated the transient vortex formation at the sharp edges of the cantilever. Our 3D visualizations demonstrated that the vorticity propagates into the wake in the form of curved coherent structures emanated from the trailing edge that are initially attached to the cantilever's trailing edge at the corners. These structures lose their coherency due to diffusion and consequently break as they move in the streamwise direction. The resulting sub-structures tilt as they are convected downstream and induce the flow confinement observed in the mean jet.

We systematically studied the effect of several geometric and vibrational parameters on the shape of the induced jet. Our results revealed that the amplitude and the width of the pitching plates are the primary factors affecting the shape of the mean jet. The frequency of the oscillations has minor effects on the mean jet geometry, although linearly affecting the magnitude of the velocity. We observed no significant influence from the length of the cantilevers. We further demonstrated that the jet boundaries for all the tested cases fall into convergent universal curves when the spatial coordinate system is normalized by appropriate parameters.

The present study improves our understanding of the complicated geometry of the mean jet downstream of a pitching plate. Findings presented in this work are of significant importance in applications such as electronics cooling using piezoelectric fans. Placement of the electronic components outside the jet boundary may result in inadequate heat dissipation and overheating of the components.

6. Acknowledgement

7. Declaration of Interest

None.

8. References

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