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**TEACHING MATHEMATICS IN EARLY GRADES:
BELIEFS AND PRACTICES RELATED TO STUDENTS' ASSETS**

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Teaching Mathematics Instruction in Early Grades:

Beliefs and Practices Related to Students' Assets

By Brittany Caldwell

Abstract

This dissertation focused on mathematics instruction in early grades (Pre-K - 3rd grade) by exploring teachers' beliefs about mathematics teaching and the teaching practices of one teacher. I specifically examined teachers' beliefs and one teacher's practices that reflect asset-based views of students and support mathematics learning with understanding. This work contributes to early mathematics education research by documenting teachers' beliefs about mathematics, documenting one accomplished teacher's practices, and comparing that teacher's practices when teaching in-person and online.

In my analysis of teachers' beliefs (Chapter 2), *Early Grades Teachers' Beliefs About Mathematics, Language, and Emergent Bilinguals*, I explored the research question: what are early grades teachers' professed beliefs about mathematics, language, student thinking, students' out-of-school experiences, and students' home and everyday language practices, in particular for EBs? I documented teachers' professed beliefs related to mathematics and EBs through one survey and one interview. I was particularly interested in characterizing teachers' beliefs about mathematics (Schoen & LaVenia, 2019) and their beliefs about language (Fernandes, 2020). Through analysis of teachers' beliefs, I found

that the 20 teachers in this study held varying degrees of asset-based views of EBs. All the teachers responded to the survey with at least 74% of their non-neutral responses in ways that reflect an asset-based view. I identified and sorted teachers' total percentage of asset-based responses on the survey across four categories which include 1) some asset-based views, 2) many asset-based views, 3) most asset-based views, and 4) all asset-based views. In addition to what I found in the survey responses, the interviews clarified and provided more detailed descriptions of their beliefs. From the interviews, I found that teachers held beliefs about students' assets and teaching mathematics with EBs related to students' everyday and home language, students' backgrounds and experiences, mathematics vocabulary, and supporting EBs. Teachers described their views on using students' assets in two ways: allowing students' assets in the classroom or drawing on students' assets for mathematics learning.

In the analysis of teaching practices during in-person instruction (Chapter 3), *An Account of an Accomplished Teacher's In-Person Mathematics Instruction in a First Grade Classroom: Drawing on Students' Assets*, I explored the nature of in-person mathematics instruction for five weeks in Ms. C's first-grade class. The research questions that guided this analysis include: what was the nature of mathematics instruction in a first-grade classroom with an accomplished teacher? and, how did an accomplished teacher draw on students' assets (student thinking and experiences)? This analysis provided evidence that Ms. C 1) created opportunities for students to develop conceptual understanding, 2) used teacher moves and was highly responsive to students and their thinking, 3) established norms around participating in the class, making mistakes, and efficiency, and 4) drew on students' experiences outside of the classroom.

During that period of in-person instruction, there was evidence that Ms. C's teaching practices aligned with her professed beliefs documented through the survey and interview. In particular, her teaching practices reflected the low "transmissionist", low "facts first", and low "fixed instructional plan" beliefs documented in previous research (from the belief constructs, Schoen & LaVenia, 2019). The central features of Ms. C's in-person teaching practices also aligned with research-based recommendations for effective mathematics teaching (e.g., Hiebert & Carpenter, 1992; Aguirre et al., 2012; Moschkovich, 2013; Turner et al., 2016; Wager, 2013). The vignettes in Chapter 3 provide detailed examples of her teaching practices and illustrate how Ms. C drew on students' assets to create opportunities for mathematics learning with understanding.

In the analysis comparing in-person and online teaching practices (Chapter 4), *A Comparison of Mathematics Instruction In-Person and Online with First-Grade Students*, I described how Ms. C adapted and facilitated mathematics instruction online with first-grade students, including one EB. I explored the research questions: 1) What were the differences between classroom routines and mathematics activities in person compared to online during COVID-19 in an early grades classroom? 2) Did Ms. C enact math instruction online that aligns with her professed beliefs? If so, how? 3) What was the nature of online math instruction for the EB in Ms. C's classroom? My observations support the claim that most of the central features of Ms. C's mathematics instruction documented during in person teaching (i.e., teaching for conceptual understanding, using teacher talk moves, establishing norms, and using students' experiences) were similar, even when the lessons looked different. For example, I observed Ms. C consistently eliciting student thinking and strategies while problem-solving both in-person and online. She also used a variety of teacher moves, such as revoicing and questioning, to guide students to uncover patterns and identify information that they noticed. In terms of the professed beliefs documented in Chapter 2, two of the belief constructs observed to align with her teaching practices in person (Schoen & LaVenia, 2019), Ms. C's low "transmissionist" and low "facts first" beliefs, were also reflected in the observations of her teaching practices online. However, institutional constraints impacted her teaching practices in ways that resulted in less alignment with her low "fixed instructional plan" belief, documented in Chapter 2 and observed during in-person teaching in Chapter 3. There were structural and policy differences between the two settings. Despite those differences, this analysis showed that many of the central features of Ms. C’s mathematics teaching practices persisted even when she transitioned to online teaching during a pandemic with her first-grade students.

Acknowledgments

This dissertation was made possible by the wonderful support and collaboration with Ms. C. Ms. C invited me into her classroom during in-person instruction and then again invited me back to observe her online teaching during a global pandemic. Ms. C had never taught online prior to the shift to remote instruction during COVID-19. Her willingness to have me observe made this dissertation possible.

I express profound gratitude for the years of support and guidance from my advisor, Judit Moschkovich. She pushed me to be a better scholar. Her guidance taught me to think critically about mathematics teaching, think about equity, and think about supporting emergent bilingual students in early grades. She also surrounded me with a network of scholars, creating many opportunities for me. I will forever be grateful for her hard work and dedication.

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Chapter 1: Introduction, Conceptual Framing, and Methods

This dissertation focused on mathematics instruction in early grades (Pre-K - 3rd grade) by exploring teachers' beliefs about mathematics teaching and the teaching practices of one teacher. I specifically examined teachers' beliefs and one teacher's practices that reflect asset-based views of students and support mathematics learning with understanding. This work contributes to early mathematics education research by documenting teachers' beliefs about mathematics, documenting one accomplished teacher's practices, and comparing that teacher's practices when teaching in-person and online.

Teachers' beliefs are important because they have been documented as impacting teaching practices (Schoenfeld, 2002; Raymond, 1997) and equity in early math classrooms (e.g., Lee & Ginsburg, 2007; Raymond, 1997; Roesken-Winter, 2013; Stipek, Givvin, Salmon, & MacGyvers, 2001; Törner, Rolka, Roeskin, & Sriraman, 2010). Research has highlighted the importance of drawing on students' experiences and backgrounds to provide access to content (e.g., Aguirre et al., 2012; Turner et al., 2012), 2) and leveraging students' home and everyday language to support student learning (e.g., Brenner, 1998; de Araujo et al., 2018; Turner & Celedón-Pattichis, 2011; Moschkovich, 2013; Moschkovich, 2015b). Teaching practices that reflect these recommendations often align with asset-based beliefs. Specifically, teachers with asset-based beliefs view children as coming to the classroom with valuable experiences and resources for learning mathematics rather than needing to catch up or replace experiences from home. Having asset-based views are important as they often align with teaching practices that create opportunities for children to draw upon their resources to make meaning for mathematics.

However, beliefs and practices do not always align. Therefore, it was important to document an accomplished teacher's practices and examine whether and how those practices reflected her professed beliefs. The descriptive study of in-person teaching is also important for practice as these examples can serve as cases for professional development. Comparing her instruction in-person and online was relevant to current debates on how the pandemic impacted teaching practices.

Students, particularly children in elementary school, perform mathematics tasks significantly better when taught using student-centered approaches (Polly, Margerison, & Piel, 2014). Documenting one teacher's student-centered practices provides a detailed account of teaching practices and evidence of how they align with her professed beliefs. The vignettes of Ms. C's teaching practices can also be used to ground conversations during professional development. In the vignettes, I illustrate how Ms. C. elicited and drew on student thinking and solutions to focus discussions. This practice of eliciting and using student thinking has been documented as an effective method for supporting mathematics learning in early grades (e.g., Fennema et al., 1996).

Even before the COVID-19 pandemic, elementary school teachers were largely unprepared and had few experiences teaching online (Leary et al., 2020; McAllister & Graham, 2016). Once teachers started teaching online, many challenges impacted their instruction (DeCoito & Estaiteyeh, 2022b). Research has found that teachers prioritized covering content rather than using effective teaching such as student-centered approaches (DeCoito & Estaiteyeh, 2022a). Although research recommends student-centered approaches (Polly, Margerison, & Piel, 2014), the sudden shift online may have left many children without many opportunities to access this kind of teaching. Building upon and incorporating students' thinking into early grades mathematics instruction can support mathematics learning with understanding (Carpenter, Hiebert & Moser, 1983; Fennema et al., 1996). In a study exploring the problem-solving strategies used by kindergarteners, findings show that young children successfully solved a broad range of whole number word problems (addition, subtraction, multiplication, and division) and often used direct modeling (Carpenter et al., 1993). When teachers know more about children’s mathematical thinking (e.g., how children use direct modeling), they are better equipped to support young children’s mathematics learning (Fennema et al., 1996; Phillip et al., 2007).

More research is needed to document early grades teaching centered on student thinking and conceptual understanding. Similarly, there is much we do not know about online teaching with young learners in elementary school during the pandemic. This dissertation explored the professed beliefs of accomplished early grades teachers to document experienced teachers' beliefs about mathematics, language, and EBs. I also use classroom vignettes to illustrate the in-person teaching practices of one accomplished, first-grade teacher, Ms. C., and then compare her teaching in-person to online during the pandemic. I more thoroughly describe the papers in this dissertation at the end of this introduction chapter.

Review of the Literature

In the following sections, I summarize relevant research on classroom mathematics instruction, provide an overview of recommendations for effective mathematics teaching, and describe tensions documented in teaching mathematics. To frame the data on teachers' beliefs, I also summarize literature on early grades teachers' beliefs by discussing work related to Cognitively Guided Instruction (CGI) and typical beliefs held by early grades teachers about mathematics.

Effective Mathematics Teaching in Early Grades

Mathematics is an essential part of children's everyday lives (Bishop, 1982; Ginsburg, 2006), and young children's involvement with mathematics predicts later academic achievement (Duncan et al., 2007). Many researchers have highlighted that a variety of mathematics content, such as counting, operations, shape, spatial relations, patterning, and working with data, is important to learn in early grades (e.g., Bishop, 1982; Clements & Sarama, 2007; Perry & Dockett, 2002). Children also participate in mathematical practices outside of the classroom, such as counting, locating, measuring, designing, playing, and explaining (Bishop, 1982). Beyond providing children with opportunities to engage with a variety of mathematics topics in early grades (Clements & Sarama, 2007; Perry & Dockett, 2002), researchers recommend that young children have access to *rigorous* and *hybrid* mathematics content. *Rigorous* mathematics content is cognitively demanding and focused on conceptual understanding; *hybrid* mathematics refers to content that bridges students' informal strategies (Carpenter et al., 1993; Carpenter, Fennema & Franke, 1996), their home and community funds of knowledge (Civil, 2002; Civil, 2007; Gonzalez, Andrade, Civil, & Moll, 2001), and mathematics children do in the classroom. Drawing on *rigorous* and *hybrid* mathematics content expands ideas of "what" mathematics is essential for young children to learn in early grades. Access to *rigorous* mathematics while including *hybrid* mathematics (i.e., children's informal strategies and children's mathematical and language practices from the home and community) can create opportunities to learn mathematics content taught in the classroom while also shifting the content itself. This positions children as "ready for school" when they enter early grades as the mathematics content inside schools reflect this dual focus (home and school, every day and academic).

To support mathematics learning in early grades, research-based recommendations include that mathematics instruction be equitable (Gutiérrez, 2009; Moschkovich, 2013), focused on understanding (Hiebert, 1990; Hiebert & Carpenter, 1992; Kilpatrick & Swafford & Findell, 2002), centered on student thinking (Carpenter et al. 1993; Fennema et al., 1996), include multiple activities (Fuson, 1988; Fuson, 1991; Fuson, 2009; Wager, 2013), and leverage students' Funds of Knowledge (Civil, 2007; González, Andrade, Civil, & Moll, 2001; Turner et al., 2016). I overview the relevant research that grounds these features of effective mathematics instruction.

Research on equity in mathematics classrooms recommends that all children be afforded opportunities to engage with high-quality mathematics instruction. Related to these opportunities is holding an asset-based view of students regardless of their backgrounds and prior experiences. Children draw on many "repertories of practice" as they learn new things (Gutiérrez & Rogoff, 2003, p. 22). Children's "repertoires of practice" shape the way they interact in schools, but also change through these interactions. An emphasis in schools should be on "helping students develop dexterity in using both familiar and new approaches" (Gutiérrez and Rogoff, 2003, p. 23). Making space in the classroom for students' "repertories of practice" leverages students' resources and creates meaningful opportunities for children to learn mathematics with understanding. For this study, I particularly looked for the ways teachers drew on children's assets related to 1) students' experiences and backgrounds (e.g. Aguirre et al., 2012; Turner et al., 2012) and 2) students' home and everyday language (e.g., Brenner, 1998; de Araju et al., 2018; Turner & Celedón-Pattichis, 2011; Moschkovich, 2013; Moschkovich, 2015b).

Aligned with an asset-based view of students, Gutiérrez (2009) highlights four dimensions of equity: access, achievement, identity, and power. Equitable mathematics does more than show students how to engage with school mathematics by pushing against traditional conceptions of what mathematics is in school (Gutiérrez, 2009). Equitable mathematics, according to Gutiérrez (2009), helps students "play the game" and "change the game" (p. 5). Students can do both when mathematics instruction provides access to mathematics concepts and practices while simultaneously leveraging students' experiences to form the context of school mathematics.

To illustrate, Lipka et al., (2005) found that flexibility with participation structures in the classroom by allowing children to use joking or humor as well as non-verbal communication (i.e., drawing on home literacy practices) and using meaningful context that was relevant to children's lives such as fish basket weaving (i.e., drawing on home mathematical practices) supported improved academic performance in mathematics and fostered positive relationships between the teacher and the students. In this example, the literacy and mathematics practices that children were familiar with became the focus. This changed the classroom's mathematics content from decontextualized and irrelevant to meaningful and motivating content for students, while also providing access to mathematics concepts and practices promoted in school.

Drawing on the equity framework developed by Gutiérrez (2009), Moschkovich (2013) offers a set of principles and guidelines for teaching mathematics equitably to emergent bilinguals (EBs)[1]. Central to these principles is affording EBs opportunities to engage with mathematics. To define equitable teaching practices for EBs, Moschkovich identifies two aspects: 1) supporting mathematical reasoning, conceptual understanding, and discourse and 2) broadening participation for EBs. Moschkovich (2013) asserts, "to support mathematical reasoning, conceptual understanding, and discourse, classroom practices need to provide all students with opportunities to participate in mathematical activities that use multiple resources to do and learn mathematics" (p. 46). Aligned with these two frameworks, equitable teaching practices

[1] I use the term EBs to discuss linguistically diverse children to challenge most of the literature looking at this population and labeling this group as English Learners (ELs). The term EBs includes all children, birth to age 8, who are learning English in addition to another language. The term dual language learner is often used to identify children birth to age 5, therefore I use the term EL to include both children who are learning their primary language and secondary language simultaneously and children who start to learn English upon entering school. The word "learner" imbues a deficit frame that positions the individual as lacking something. Additionally, using "English" learner can privilege English over other languages and ways of using language that children bring with them as they learn. I use EBs as a more inclusive alternative to the previous terms ELs or Limited English Proficient, while still recognizing that there may be faults with this wording and the meaning it imbues.

increase student participation in mathematical practices. Simultaneously, including new student voices and ideas can shift the explored content and ideas.

Below I summarize research on four features of effective mathematics teaching. These features include 1) creating opportunities to develop conceptual understanding, 2) focusing on student thinking during mathematics, 3) including multiple activities, and 4) leveraging students' "funds of knowledge."

1. Creating Opportunities to Develop Conceptual Understanding

Research tells us that mathematics lessons must focus on conceptual understanding and move beyond only teaching procedural fluency (Hiebert, 1990; Hiebert & Carpenter, 1992). Procedural fluency is using appropriate algorithms for mathematics exercises and includes knowing when and how to use the procedure and being able to use the skill "flexibly, accurately, and efficiently" (Kilpatrick & Swafford & Findell, 2002). Conceptual understanding, however, includes students constructing meaning for the procedure they are using by creating connections among representations, applying a procedure to other problems, and explaining why and how they solved a problem a particular way (Hiebert & Carpenter, 1992). Students who develop conceptual understanding, before procedural fluency, tend to remember the procedures better and can use procedures to apply to other mathematics problems (Hiebert, 1990; Hiebert & Carpenter, 1992). Procedural fluency is an important strand of mathematical proficiency; however, without conceptual understanding, children will rely on memorized facts and skills, which are highly prone to errors.

2. Focusing on Student Thinking During Mathematics

Research from the last several decades recommends that early grades mathematics instruction focus on student thinking. Building upon and incorporating students' thinking into early grades mathematics instruction can support mathematics learning with understanding (Carpenter, Hiebert & Moser, 1983; Fennema et al., 1996). Young children can successfully solve a broad range of whole number word problems (addition, subtraction, multiplication, and division) and often use direct modeling (Carpenter et al., 1993). Direct modeling is a strategy for problem-solving that involves concrete objects (e.g., blocks, Unifix cubes) or written inscriptions to represent the action in a word problem (Carpenter et al., 1993). Similarly, research exploring children's learning of number and counting (Fuson 1988; Fuson, 1991) and strategies with whole number addition problems (Secada, Fuson, & Hall, 1983) has revealed a great deal about children's early mathematics thinking and learning. For example, given time and opportunities to solve various addition and subtraction word problems, children start to employ more efficient strategies (counting-on) (Secada, Fuson & Hall, 1983). Children can be successful with addition, subtraction, multiplication, and division word problems when given opportunities to solve them while getting the support they need (e.g., manipulatives, time, interactions with a peer or teacher) (Carpenter et al., 1993; Secada, Fuson, & Hall, 1983).

When teachers know more about children's mathematical thinking, they are better equipped to support young children's mathematics learning (Fennema et al., 1996; Phillip et al., 2007). Teachers need to know how students think about early mathematics to use their thinking to support learning. Teachers can better understand children's mathematical thinking through professional development designed to give teachers opportunities to think specifically about children's thinking related to mathematics. In a long-term (4-year) professional development study focused on children's mathematical thinking about numbers and operations, teachers participated in intensive workshops overviewing ways to think about and solve word problems (Fennema et al., 1996). This professional development program supported teachers in employing more student-centered approaches to mathematics instruction (Fennema et al., 1996). Further, findings revealed that children in classrooms with teachers in the program showed higher achievement gains in mathematics than in classrooms with teachers not in the program (Fennema et al., 1996).

3. Including Multiple Activities

Children can learn mathematics through multiple experiences inside and outside of the classroom. In the classroom, mathematics instruction may seem only to include a scheduled "math time" specific to school, yet mathematics learning also happens through other means. In a study exploring mathematics learning in preschool classrooms, Wager (2013) identified three spaces where children engage with and learn mathematics, including 1) instructional time, 2) engagement with math games, manipulatives, or math objects, and 3) free play. During instructional time teachers give "teacher-initiated explicit instruction of mathematics" (Wager, 2013, p. 167). Teachers may practice "seeding various interest areas with materials to encourage mathematical thinking" (Wager, 2013, p.168). During free play, teachers can engage in "observing and responding (mathematizing) children's mathematics that occurred in play" (Wager, 2013, p.168).

Children must experience mathematics in different and multiple activities across these spaces because these can influence learners' beliefs about mathematics and their dispositions towards mathematics. If mathematics instruction is heavily structured and scripted, children may only experience mathematics as correct answers, memorization, and speed (Parks & Bridges-Rhoads, 2012). Having different activities can also help children see mathematics in everyday life, not just during mathematics time in school. An essential part of learning mathematics is developing a "productive disposition" towards mathematics (Gresalfi, 2009; Kilpatrick, Swafford, and Findell, 2001). Productive disposition includes the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick, Swafford, & Findell, 2001, p. 116). Children must see themselves as mathematics learners. Experiencing mathematics through different and multiple activities can give more opportunities for children to grapple with mathematics and develop a productive disposition.

4. Leveraging Students' Funds of Knowledge

Teachers should draw on the mathematical activities children engage with in their homes and communities to support mathematics learning for all students (Civil, 2007; González, Andrade, Civil, & Moll, 2001). This includes the language and literacy practices children use at home (Brenner, 1998; Razfar, 2013; Moschkovich & Nelson-Barber, 2009; Turner & Celedón-Pattichis, 2011) as well as the mathematical activities (Civil, 2007; González, Andrade, Civil, & Moll, 2001). As Moschkovich and Nelson-Barber (2009) note, "teachers need to know something about students' home, community, social, and cultural values and practices and how these may influence classroom interactions" (p. 113). Identifying what students bring with them to the classroom as resources, instead of deficits, is vital for supporting children who come from cultures or communities different from their own (Moschkovich & Nelson-Barber, 2009).

Culturally Sustaining Pedagogy (CSP) builds upon the framework of Culturally Relevant Pedagogy (Ladson-Billings, 1995) and asserts that teaching practices and pedagogy should not only reframe deficit thinking towards an asset-based approach but also work to *sustain* the cultural and linguistic competencies that individuals bring to school (Paris, 2012). According to this view, rather than working to replace one's mathematical ideas with more technical or academic versions, mathematical ideas become a part of the individual's resources to understand concepts better. By mathematizing family practices and drawing on the mathematical activities children use at home, leveraging students' everyday mathematical and language practices into mathematics instruction aligns with CSP. As teachers draw on children's mathematical practices from their homes and communities, these practices become a part of school mathematics, which changes how mathematics looks in the classroom. Bringing in the mathematics of the children and their communities is one concrete way to shift "what counts" as mathematics in the classroom by acknowledging aspects of identity and power in the mathematics classroom (Gutiérrez, 2009). Aligned with CSP, the home and community practices are preserved and sustained when teachers focus on the mathematical activities that children engage with outside of school and embed these activities into instruction.

Tensions in Teaching Mathematics

Teachers face multiple tensions that impact teaching practices during mathematics instruction. For example, scripted curriculum materials and enforced pacing guides have been shown to impact teacher autonomy and alignment between teachers' practices and their beliefs about mathematics instruction (Parks & Bridges-Rhodes, 2012; Pease-Alvarez & Samway, 2008). In this section, I describe previous research on contextual constraints that have been shown to impact teaching practices.

Students' backgrounds can impact teaching practices in preschools. Lee & Ginsburg (2007) found that the socioeconomic status (SES) of families impacted teachers' views and teaching practices in different preschools. Specifically, teachers serving low-SES families tended to have deficit views of students and therefore focused on academics in preschool with attempts to help children "catch up" (Lee & Ginsburg, 2007). Whereas preschool teachers serving middle-SES families tended to hold more asset-based views of their students and their teaching practices were often more playful and less focused on academics (Lee & Ginsburg, 2007).

Other contextual factors that can impact teaching practices, even practices that do not align with teachers' beliefs, include curriculum materials (Parks & Bridges-Rhodes, 2011) and policy mandates that impact curriculum implementation (Pease-Alvarez & Samway, 2008). Parks & Bridges-Rhodes (2012) found that when a district adopted a heavily scripted literacy program, a teacher with more student-centered beliefs changed her teaching practices even during mathematics instruction to align with a more procedurally-focused, teacher-centered instruction. Similarly, Pease-Alvarez and Samway (2008) explored the beliefs and practices of a group of teachers when a district-mandated reading program was implemented and enforced. They found that teachers' practices and agency changed. The teachers focused less on the needs of their students and more on the instruction that was dictated by the teaching manual. All the teachers in this study reported they felt a loss of teacher agency and autonomy over their teaching (Pease-Alvarez & Samway, 2008).

Teachers' beliefs can impact their teaching practices. In the following section, I describe research that has characterized typical elementary school teachers' beliefs about mathematics teaching and learning.

Teachers' Beliefs about Teaching Mathematics

Teachers' beliefs about teaching reflect a variety of views of learning and teaching, along a continuum of traditional views to contemporary views (Oaks & Lipton, 1999; Staub & Stern, 2002). In mathematics education researchers have referred to contemporary views as "reform" views or beliefs (Philipp, 2007). Teachers who describe traditional beliefs discuss learning and teaching mathematics as creating teacher-centered, direct-transmission classrooms with an emphasis on procedures, getting the right answers, and disregarding mistakes as simply wrong (Oaks & Lipton, 1999). Teachers who hold contemporary or reform views report beliefs that teaching should be student-centered and students must construct their own meaning for mathematics problems rather than receiving a standard procedure to solve them (Oaks & Lipton, 1999). Teachers with underlying contemporary/reform views believe that students learn by building upon pre-existing ideas and that learning happens when students are actively engaged with mathematics (e.g., using manipulatives, writing, talking, exploring) (Bransford, Brown & Cocking, 2000; Donnovan & Bransford, 2005).

CGI is one teaching framework centered on building upon student thinking about mathematics (Carpenter et al., 1983; Carpenter, Fennema & Franke, 1996; Fennema et al., 1996). This approach has greatly re-shaped teachers' beliefs and early instruction to center on student thinking and strategies children employ for whole number word problems. CGI is important because of the implications it has for students' mathematics outcomes as students in classes more grounded in CGI revealed higher achievement gains than control groups (Fennema et al., 1996).

Teachers who have been trained in CGI methods and use them are often more equipped to support mathematics learning with understanding for their students as CGI has been found to support teachers shifting from direct-transmission models to more cognitive-constructivist models of teaching (Fennema at al., 1996). Beyond this, teachers who have had professional development in CGI and use this approach in their classrooms tend to emphasize understanding of concepts rather than learning specific skill sets (Fennema et al., 1996). Teachers employing the CGI framework typically start with a word problem related to whole numbers (e.g., joining, separating, multiplication, measurement division) and let children solve the problem on their own. Strategies for solving the problem are then elicited by the teacher and often shared with the whole group. This approach focuses on the children's strategies for solving the word problems.

Fennema et al., (1996) explored how the CGI program supported early grades teachers in forming beliefs grounded in CGI. These beliefs related to students' problem-solving, drawing on student solutions and strategies, and the role of the teacher. The four-year teacher development program focused on supporting teachers in learning about children's mathematical thinking. This involved intensive workshops focused on children's mathematical thinking as well as support from a CGI staff member assigned to each school. The CGI staff member was involved with teachers on a regular basis as a coach and guide during meetings and discussions of children's mathematics learning. Results revealed that the CGI program helped 18 out of the 21 teachers form beliefs and instructional practices more aligned with CGI. For example, seven of the 21 teachers discussed beliefs that children can solve problems and that instruction should be based on student strategies and student thinking. These 18 teachers shifted from traditional to reform views, specifically from direct-transmission views to more cognitive-constructivist views of teaching and their observed mathematics teaching shifted from teaching a specific procedural skill set to supporting student conceptual understanding.

Complicating the traditional and contemporary/reform continuum, recent research has identified more specific beliefs related to mathematics teaching which include “transmissionist” beliefs, “facts first” beliefs, and “fixed instructional plan” beliefs (Schoen & LaVenia, 2019). Beyond categorizing teachers’ general beliefs about teaching as oriented to teacher-centered versus student-centered instruction, “transmissionist” beliefs, “facts first” beliefs, and “fixed instructional plan” beliefs have been used to characterize typical beliefs that elementary teachers hold specifically related to mathematics teaching (e.g., Philipp et al., 2007; Schoen & LaVenia, 2019). Schoen & LaVenia (2019) developed the beliefs constructs tool which included these three belief scales to characterize the ways over 200 teachers discussed beliefs related to mathematics instruction after they participated in professional development focused on CGI. The first scale is related to “transmissionist” views, which reflects the extent to which teachers believe they should guide students toward a single standard algorithm. The CGI approach supported teachers in learning to elicit and draw on student thinking to guide students away from viewing mathematics problems as having only a single standard solution, and support learning with conceptual understanding (Fennema et al., 1996). Schoen & LaVenia (2019) also identified beliefs related to how teachers viewed the relationship between facts, skills, and problem-solving which they included in the “facts first” scale. In this scale, teachers with low “facts first” beliefs viewed problem-solving as a place where students would develop skills and learn facts and reported that facts and skills were not a pre-requisite to problem-solving. Finally, the teachers also reported a “fixed instructional plan” belief, describing how one must adhere to the scope and sequencing of curriculum materials. Having a low “fixed instructional plan” belief meant that teachers reported that they adjusted the curriculum based on their assessments of student understanding and to support student learning with understanding. This is more in alignment with CGI, due to the focus on eliciting and using student thinking during instruction.

Teaching is complex and can be impacted by many factors including institutional constraints, tensions, and beliefs. This dissertation focuses on teaching practices from an accomplished teacher to characterize her teaching practices, compare her teaching practices to her professed beliefs, and identify some factors that impacted her teaching.

Theoretical Framing for the Dissertation

A situated, sociocultural perspective (Lave & Wenger, 1991; Vygotsky, 1978) frames this dissertation study. In this perspective, learning and teaching are social practices. A sociocultural perspective acknowledges the role of cultural tools that teachers take up and use to support learning (e.g., curriculum materials, manipulatives, participation structures, teacher moves). I used two frameworks for the two sets of analyses in this dissertation. For the analyses of beliefs in Chapters 2 and 4, I used assumptions about teachers’ professed beliefs

(Phillip, 2007), the lens of language orientations (Fernandes, 2020), and belief constructs from CGI research (Schoen & LaVenia, 2019). For the analyses of teaching practices in Chapters 3 and 4, I used assumptions and a focus on teacher moves and norms from enacted curriculum research (Remillard and Heck, 2014), the four research-based recommendations for effective mathematics teaching (summarized above), and the lens of Multiple Mathematical Knowledge Bases (MMKB) (Aguirre et al., 2012; Turner et al., 2012). In the following sections, I overview how I used these frameworks.

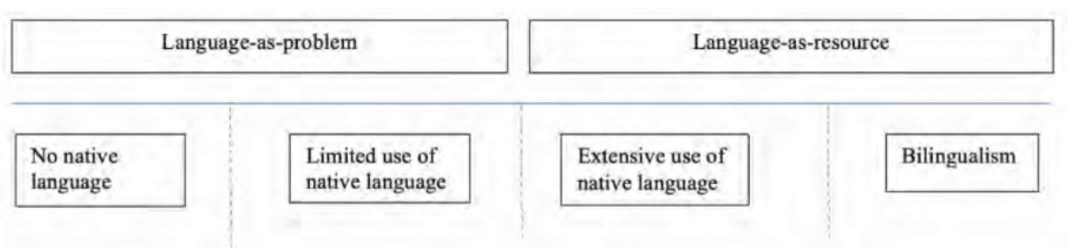
Professed Beliefs

I explored teachers professed beliefs related to mathematics, language, and EBs. I defined beliefs using the work from Phillip (2007) as "psychologically held understandings, premises, or propositions about the world that are thought to be true [...] as lenses that affect one's view of some aspect of the world or as dispositions toward action" (p. 259). I looked specifically at professed beliefs (Speer, 2005; Gonzalez Thompson, 1984) about mathematics, language, mathematics learning, and mathematics teaching. Professed beliefs are explicitly stated by teachers, rather than attributed beliefs inferred by researchers (Speer, 2005). I acknowledge that beliefs can impact teaching practice (Schoenfeld, 2002; Raymond, 1997), shaping the ways teachers facilitate mathematics instruction in early grades.

To analyze beliefs, I used the language orientations framework (Fernandes, 2020) and belief constructs (Schoen & LaVenia, 2019). The language orientations framework includes four orientations that range from viewing language as a problem to viewing language as a resource (Figure 1) which was originally created using responses from pre-service teachers. In my study, I used this framework to look at in-service teachers. On the farthest side of the continuum where language is viewed as a problem, Fernandes (2020) highlighted pre-service teachers' orientations that reflected "no native language." In this group, the teachers felt there was no room for native language in a mathematics classroom. Next, pre-service teachers with a "limited use of native language" believed that it was ok for students to use their native language in some situations, but the goal was to replace their language with English in the mathematics classroom. Moving towards views of language as a resource, Fernandes (2020) found that some pre-service teachers held beliefs that reflected "extensive use of native language" where teachers supported any language use in the classroom and the goal was to learn mathematics regardless of language. The final group had beliefs that reflected and promoted "bilingualism" where the pre-service teacher promoted native language use because they believed that native languages support mathematics learning. While Fernandes (2020) only included native language in these constructs, I expanded this work to also include everyday ways of communicating. This is because many researchers (e.g., Au & Kawakami, 1985; Barwell, 2005; Brenner, 1998; de Araujo, Roberts, Willey, and Zahner, 2018; Gutiérrez, Baquedano-López & Tejeda, 1999; Lipka et al., 2005; Razfar, 2013; Turner & Celedón-Pattichis, 2011) have found that inviting all of students linguistic resources into the classroom, including native language and language practices (e.g., talk story, joking, storytelling, dialects), supports engagement and learning.

Figure 1

Language Orientations Construct (Fernandes, 2020)



I also used Schoen & LaVenia's (2019) "belief constructs" framework to characterize the professed beliefs Ms. C reported related to her mathematics instruction. The three scales that make up this framework include 1) "transmissionist", 2) "facts first", and 3) "fixed instructional plan." The "transmissionist" scale represents the extent to which teachers believe they should guide students towards a single standard solution. For example, a teacher with low "transmissionist" beliefs often described a down-up approach where they use the thinking and strategies of students to inform teaching. Low transmissionist beliefs align with research on student thinking that has shown that students perform better when they experience this type of teaching (Fennema et al., 1996; Polly, Margerison, & Piel, 2014). The "facts first" scale represents teachers' beliefs about the role of learning facts in problem-solving. Teachers with low "facts first" report or hold the belief that learning and teaching problem-solving creates opportunities for students to develop meaning for facts as students progress through the problem-solving process. A teacher holding or reporting high "facts first" beliefs would instead believe that students must learn basic facts before they can solve problems. Finally, the "fixed instructional plan" scale represents the extent to which teachers agree that they must adhere to the scope and sequencing of topics and pace them according to the curriculum. A low "fixed instructional plan" reflects a belief that teaching is more effective when teachers make adaptations to prescribed scope and sequencing based on student assessments and needs. In my description of Ms. C's beliefs, I use these three scales to characterize how she discussed and reported her mathematics teaching during her interviews and on her survey.

Teaching Practices

I also explored the teaching practices of one accomplished teacher. Drawing on a sociocultural perspective, I assume that teaching is enacted. Remillard and Heck (2014) describe teacher enactment of their practices as the "enacted curriculum." Enacted curriculum acknowledges the various systems that impact teaching and learning in the classroom and bring the relationships between these systems to the forefront. Remillard and Heck (2014) define enacted curriculum as “the interactions between teachers and students around the tasks of each lesson and accumulated lessons in a unit of instruction, is analogous to the performance of play, complete with the idiosyncrasies and unpredictable elements of live performance” (p. 713). The enacted curriculum is part of the operational curriculum (teachers’ plans and actions to carry out the instruction) rather than the official curriculum (Remillard & Heck, 2014).

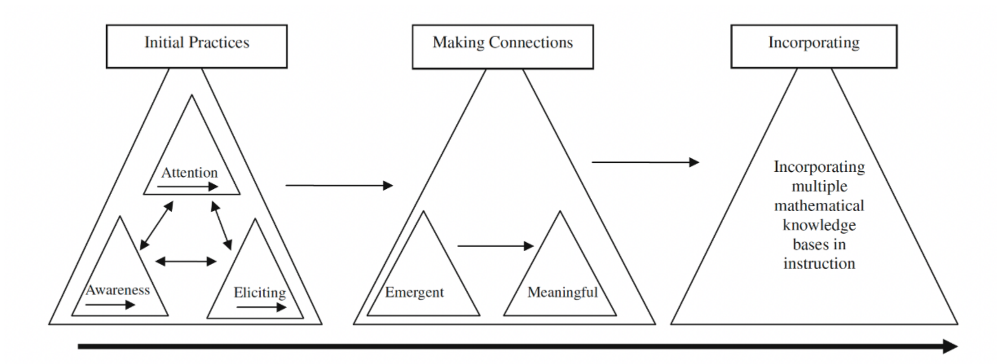
There are four dimensions of enacted curriculum: 1) mathematics, 2) instructional interactions and norms, 3) teacher’s pedagogical moves, and 4) the use of resources and tools (Remillard & Heck, 2014). The mathematics includes both the content and the mathematical practices during instruction (Remillard & Heck, 2014). The second dimension, the instructional interactions and norms, consists of the interactions between the teacher, the students, the task, and the norms in the classroom (Remillard & Heck, 2014). Teachers play an important role in creating a classroom culture conducive to learning mathematics with understanding and broadening participation in mathematics. Classroom interactions and norms include creating mathematics discourse that enables discussion and deliberation of mathematical ideas, developing social and socio-mathematical norms that support learning opportunities, and building positive relationships (Franke, Kazemi, and Battey (2007). The interactions and norms in a classroom are framed by a teacher's pedagogical moves, the third dimension of enacted curriculum.

The teacher's pedagogical moves are the actions of a teacher that shape what mathematics is covered and how (Remillard & Heck, 2014). The pedagogical moves that are the focus of this study include the moves that create opportunities for students to participate in mathematics instruction. This is also related to the fourth dimension of enacted curriculum, the use of resources and tools, including the physical, technological, linguistic, and other supports employed during instruction (Remillard & Heck, 2014). Inviting students to use their linguistic resources (familiar language practices, participation structures, and home languages) creates opportunities for students to draw on more resources for learning mathematics in the classroom. Many things impact the ways teachers enact curriculum, including beliefs about mathematics teaching and learning, accessibility of resources, and contextual opportunities and constraints (Remillard & Heck, 2014).

To analyze teaching practices (observations) I used the Multiple Mathematical Knowledge Bases framework developed by Turner et al. (2012) and expanded upon by Aguirre et al. (2012). Children's MMKB include "multiple understandings and experiences that have the potential to shape and support students' mathematics learning" (Turner et al., 2016, p. 49). These understandings and experiences include student thinking and strategies as well as students' experiences outside of the classroom (Turner et al., 2012). The MMKB framework identifies three different ways that teachers draw on children's MMKB which reflect increasingly more integration into instruction (Figure 2) (Aguirre et al., 2012; Turner et al., 2012).

Figure 2

Multiple Mathematical Knowledge Bases (MMKB) Learning Trajectory



The first phase reflects *initial practices*. Initial practices include attention, awareness, and eliciting children's MMKB. Attention includes what teachers attend to and what they notice, whereas awareness is about how teachers interpret children's MMKB. Eliciting involves both the questioning strategies teacher use and the ways teachers interact with children and families. Specific to practices reflective of this phase, teachers may draw on students' mathematics thinking or students' experiences rather than both (Turner et al., 2012). Deficit views can inhibit movement along the learning trajectory beyond initial practices (Turner et al., 2012). As teachers move beyond initial practices, teachers *make connections* between children's MMKB and instruction. These start as emergent connections and shift towards meaningful connections that reflect mathematically rich problem-solving experiences that draw on children's mathematical thinking and mathematical experiences from outside of school. Aguirre et al. (2012) complicate this phase by introducing an intermediate category, *transitional connections,* that teachers make that are more than just brief attempts to connect to MMKB but are not fully meaningful. The last phase, and the goal of the learning trajectory, is *incorporating.* Teachers who incorporate children's MMKB show orientations towards attending to MMKB, awareness of children's resources, and effective practices for eliciting and connecting with children's MMKB. I used this framework to characterize the teaching practices by paying particular attention to if and how Ms. C, the focal teacher, draws on students' thinking and experiences during instruction.

Turner et al. (2016) identified specific ways teachers draw on MMKB and characterized the connections that teachers made to children's MMKB. The categories identified in this work align with the teachers' learning trajectory as they provide concrete ways teachers enact the various levels by outlining how children's MMKB can be used in school mathematics in ways that are 1) based on assumptions, 2) reflect layering or mathematizing, or 3) uncovering mathematical activities (Aguirre et al., 2012; Turner et al., 2012; Wager, 2012). Teachers may draw on children's MMKB by making assumptions about familiar or relevant contexts. The questions that reflect assumptions look similar to those found in a textbook (If I have 5 blocks and give you 3, how many do I have?) but reference items assumed to be familiar to all children, like jellybeans, pennies, or balls. Teachers may also draw on knowledge of familiar objects or activities, which also looks textbook-like but references things that the teacher knows to be a student's favorite or of high interest to the child. Drawing on assumptions or familiar objects reflects a shallow interpretation of leveraging students' experiences in mathematics instruction. Thus, as teachers move towards more meaningful connections to children's MMKB they participate in mathematizing family practices. Contexts that the students participate in like the nail salon, eating dinner, shopping, or doing laundry can be used and the focus becomes the mathematics used in these activities. Turner et al. (2016) identified that mathematics is *layered* onto the activity in this category. Last, teachers can identify mathematical activities that children themselves are engaging in. When teachers draw on the mathematics that students use at home, such as keeping track of allowance, these connections serve as the clearest link to how children might use mathematics outside of school.

Methodology

This dissertation study brings together three analyses (Chapters 2, 3, and 4) focused on early grades mathematics instruction. Across these three chapters (originally 3 papers[2]), I focused on beliefs and teaching practices. These three chapters use different data sources and analyses: one is an analysis of teachers' beliefs using a survey and interviews (Chapter 2); two are analyses of teaching practices using observations (Chapters 3 and 4). In Chapter 2, I examined the professed beliefs about mathematics and language of a group of teachers (n=20). In Chapters 3 and 4, I documented and analyzed the teaching practices of one teacher, Ms. C. I drew on my analysis of beliefs from Chapter 2 to explore whether and how Ms. C's teaching practices aligned with her professed beliefs when teaching in-person right before March 2020 and online during the pandemic. In this section, I summarize the data collection, research questions, and analysis for each chapter.

To account for context, it is important to "study mathematical thinking and learning in the settings in which it naturally and regularly occurs without intervention" (Moschkovich, 2019, p. 65). In this approach, the researcher seeks to uncover the complicated nature of phenomena and gives voice to the participants (Borko, Liston & Whitcomb, 2007). The phenomena I explored were beliefs and mathematics teaching practices. Starting with the data collection phase and then into the analysis phase, "patterns are developed

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[2] I planned to write 3 separate papers for this dissertation. However, through data collection and analysis I found that the three proposed papers were more connected than I had originally envisioned. Therefore, I wrote the three analysis chapters in a paper format while also referencing the analysis across the papers.

inductively from the data and deductively from the conceptual framework" (Borko, Liston & Whitcomb, 2007, p. 5). I drew from research on teachers' beliefs and early grades mathematics teaching to deductively identify patterns. Specifically, these themes and patterns related to the frameworks I used: for beliefs, language orientations (Fernandes, 2020)) and belief constructs (Schoen & LaVenia, 2019); for teaching practices, recommendations for effective mathematics teaching (summarized in an earlier section) and MMKB (Aguirre at al., 2012; Turner et al., 2012). Additionally, themes and codes were also inductively uncovered from the teachers' responses during interviews, to the survey, and for the observations of teaching. The codes and themes I used to characterize the data drew on descriptive and in-vivo coding strategies (Saldaña, 2013). Descriptive codes include a description and the function of the code (Saldaña, 2013). In-vivo codes draw on the participants' words verbatim to reflect what participants say (Saldaña, 2013). I intentionally did this to highlight both what the teachers said and highlight the features of what they said and did.

In the next section, I provide an overview of the participants, data collection, and data analysis for the study of beliefs (chapter 2) and the analyses of teaching practices (Chapters 3 and 4).

Teachers' Beliefs: Participants, Data Collection, and Analysis

I originally planned to use this data on teachers' beliefs as part of the selection process for subsequent studies of teaching practices in more than one classroom. Due to the pandemic, I was unable to follow that initial plan and only gained access to one classroom. Therefore, I focused the analyses of teaching practices on one, accomplished first-grade teacher, Ms. C. in two settings, in-person and then later online during the pandemic.

Participants and Data Collection

The participants for the survey/interview chapter of this dissertation [3] included a group of 20 early grades teachers (Pre-K through 3rd grade) across California. I used purposeful selection to recruit teachers as I intentionally recruited teachers that had been teaching a minimum of 5 years and were teaching in preschool through 3rd grades (at the time of the survey). I used this eligibility criteria because I was interested in the beliefs of accomplished teachers. All 20 teachers responded to a survey online through google forms. The survey asked teachers about background information and then included a set of statements related to mathematics, language, and EBs that the teachers responded to using a five-point Likert scale ranging from strongly disagree to strongly agree. Originally, I had planned on recruiting two or three participants from this group of teachers to conduct the teaching practices studies (as I describe in the next section). Due to school closures and COVID-19 my plans shifted, and I did not observe in multiple classrooms from this participant group.

[3] Originally called Paper 1, now it is Chapter 2.

Instead, I used existing classroom observations from one teacher in this group, Ms. C, and then I observed her online instruction after school closures (I discuss these teaching practices studies in the following section).

Of these 20 participants, four participants were preschool teachers, three were kindergarten teachers, five were first grade teachers, one was the distance/online teacher for third grade, one teacher had a kindergarten-first grade combination class, and the remaining six did not identify their current grade as this was a question I added in after some of the teachers had already filled out the survey. Given the eligibility criteria, all teachers had been teaching over five years with the range between five and 36 years.

Five of the 20 teachers also participated in a semi-structured interview to clarify survey responses and elaborate on teaching strategies and approaches to supporting EBs. The teachers had all been teaching for at least five years in early grades, ranging from 5 years to 36 years of teaching. Half of the participants reported being monolingual, and the other half reported being multilingual. Most of the teachers (80%) reported having EBs in their class (when they took the survey), and one teacher reported that all her students were EBs. I did not collect any more background information on the teachers beyond what I reported here.

Data Analysis

I used qualitative methods and descriptive statistics of survey responses and interpretive analysis of the interviews to analyze this data. Starting with the larger participant population (n=20), I wrote descriptive summaries about demographic information, student and teaching information, and participants' responses to the survey. To analyze the survey responses, I first coded the responses for whether they aligned with an asset-based view. I drew on the literature about beliefs (summarized above and in chapter 2) that aligned with asset-based views of students to identify whether agreement or disagreement with each statement on the survey reflected an asset-based view. In particular, I drew on the recommendations from research related to teachers' beliefs about 1) students' experiences and backgrounds (e.g. Aguirre et al., 2012; Turner et al., 2012), 2) students' home and everyday language (e.g., Brenner, 1998; de Araju et al., 2018; Turner & Celedón-Pattichis, 2011; Moschkovich, 2013; Moschkovich, 2015), and 3) teaching mathematics vocabulary (Moschkovich, 2013, Moschkovich, 2015).

I coded each participants' response as aligning or not aligning to an asset-based view[4]. I then calculated the percentage of asset-based responses for each participant. For this calculation, I only looked at non-neutral responses

[4] Having binary categories (asset-based views versus non-asset-based views) does not consider the complexities of teachers' beliefs. However, I used these categories to characterize or sort teachers' responses to the statements on the survey. I then looked for more detail and complicated these categories by looking at interview responses.

(responses that fell on either side of neutral) and divided the number of asset-aligning responses by the total number of non-neutral. For example, one participant had 21 responses that aligned with asset-based views, one response that aligned with a deficit-based view, and one response that was neutral. To calculate the percentage of asset-based responses for this individual I divided their asset-based responses (21) by the total of non-neutral responses (22) which gave me a percentage of 95%. I made the decision to take out neutral responses from this calculation because neutrality does not fit with either an asset-based or deficit-based view on this scale. From the survey, I then identified specific survey responses that revealed beliefs about students' backgrounds and experiences, students' language, supporting EBs, and vocabulary. To characterize these participants' professed beliefs, I analyzed beliefs related explicitly to mathematics learning for EBs.

Drawing on the findings from the survey responses, I then transcribed the interviews with particular attention to the responses associated with 1) students' background and experience, 2) students' language, 3) mathematics vocabulary, and 4) support for EBs. I narrowed my focus to look at the responses related to my frameworks. This included descriptions of students' experiences and interests (drawing on the MMKB framework) and related to students' language (drawing on the language orientations framework). I also looked at how teachers talked about support more generally for EBs to see if there were commonalities across participants. After transcribing and identifying what each teacher said in their interview related to these categories, I looked across participants for common themes and variations in responses.

I also used Schoen & LaVenia's (2019) "belief constructs" framework to characterize the professed beliefs Ms. C reported related to her mathematics instruction. I compared the characteristics of the scales ("transmissionist", "facts first", and "fixed instructional plan") to the responses that Ms. C shared during her interview and on the survey. Once I compared Ms. C's responses to the scales, I identified Ms. C's beliefs related to these scales. Ms. C's responses related to low "transmissionist", low "facts first", and low "fixed instructional plan" beliefs. I only did this analysis for Ms. C as she was the focal teacher for the teaching practices analyses.

The analysis in Chapter 2 provides an overview of the beliefs of the larger participant group. I also focus on Ms. C's professed beliefs that she reported in her interviews and on a survey. I describe the beliefs Ms. C held about EBs. I then describe how Ms. C's beliefs relate to the "belief constructs" framework (Schoen & LaVenia, 2019). Specifically, I describe Ms. C's low "transmissionist", low "facts first", and low "fixed instructional plan" beliefs. These beliefs align with beliefs that other elementary teachers hold about mathematics instruction that are grounded in the Cognitively Guided Instruction

(CGI) approach (Schoen & LaVenia, 2019) which focuses on student thinking and solutions as the starting point for instruction (e.g., Fennema et al., 1996).

Teaching Practices: Participant, Data Collection and Analysis

Participant: Ms. C

The two analyses of teaching practices (Chapters 3 and 4) focus on the teaching practices of one teacher. I used several criteria to establish Ms. C as an accomplished teacher: her extensive time in the classroom (13th year in 2019-2020, 14th year in 2020-2021 as a first-grade teacher), background in professional development (as both a participant and developer), and pedagogical perspective. Ms. C participated in professional development at a Cognitively Guided Instruction (CGI) Summer Conference for one year, attended monthly CGI math circle meetings, attended 10 years of the California Mathematics Council Conferences[5], and went to many school/district-sponsored professional development meetings. She also served on the leadership team for the local Math Project through an established research University where she taught and helped plan the summer institutes. She was also the after-school mathematics club teacher.

[5] This is an annual mathematics conference aimed at supporting teachers as they explore new ideas and mathematics activities.

Ms. C had a particular pedagogical perspective that drew on some aspects of CGI and on other aspects, such as including playful activities and bringing in her students' interests and experiences outside of the classroom. CGI is one teaching framework centered on student thinking about mathematics (Carpenter et al., 1983; Carpenter, Fennema & Franke, 1996; Fennema et al., 1996) that has been shown to be effective at supporting student learning outcomes (Fennema et al., 1996).

The School

Ms. C worked at an elementary school in California. The school was located in the central coast in a suburban area. Data collection started during the 2019-2020 school year and continued through the 2020-2021 school year. During the 2019-2020 school year, the school had 548 students from transitional kindergarten to fifth grade. The school served 113 students that qualified for free and reduced lunch (21%) and 49 designated English Learners (9%) (Ed-Data, 2020). There were an additional 43 students (8%) at the school that were designated Fluent English Proficient. Most of the students that were designated English Learners (n = 26) spoke Spanish as their first language. During the 2019-2020 school year Ms. C taught 23 children in a kindergarten/first-grade combination class with 10 first-graders and 13 kindergarteners. While Ms. C. described the majority of her students as white and middle-class from English-speaking families, she also told me that two were low-income and three were

English learners from Spanish-speaking Latinx families. She also said that a handful of students were of Latinx backgrounds who were quite proficient in English.

During the 2020-2021 school year, Ms. C's school enrolled 468 students. In the school, 122 students (26.1%) qualified for free and reduced lunch, 31 students (6.6%) were designated English Learners, and 24 students (5.1%) were designated Fluent English Proficient (Ed-Data, 2021). Most (n = 25) of the designated English Learners spoke Spanish as their primary language. During 2020-2021, the elementary school Ms. C worked at started completely remote due to COVID-19. Ms. C's first-grade class had 20 children, of which six were low-income, four were designated English Learners, one was Latinx, and one was special needs. Ms. C taught mathematics with small groups of five students at a time for 30 minutes, two times a week, in an online format using Zoom. The observations took place with the same group of five students. I used purposeful selection when I chose the group to observe, as this group included an EB.

The Classroom

An analysis of the central features of Ms. C's practices are presented in Chapter 3. Here, I only provide a summary description of the classroom context. One of the findings is that in many ways Ms. C's mathematics instruction drew on elements of CGI. For example, she consistently drew on students' thinking and elicited students' solutions when they were problem-solving. She also used students' contributions as the focus of many whole group discussions. Ms. C drew on student thinking and student solutions throughout her lessons, yet the structure of her lessons looked different from a typical CGI lesson. Instead of having the students start off by solving a problem on their own and then working collaboratively as a group, Ms. C started with the whole group collaboration and then had students work independently. Even during whole group discussions, Ms. C did not focus on a single, standard approach to solving problems. Rather, she invited students to share how they solved problems in the whole group activity. This likely worked because of the classroom norms and classroom culture that Ms. C established around participation and contributing solutions even when students were unsure of answers. In Ms. C's classroom, mistakes were sources of conversations, not something to be avoided. Beyond this, students were given structures which framed appropriate behavior for contributing during a whole-class discussion but also had the freedom to get up and take care of personal needs. The focus on learning, rather than strict behavior management, created an environment in Ms. C's classroom where the students openly shared their thinking and solutions during whole-group activities.

In addition to drawing on student thinking and solutions, Ms. C also included playful mathematics activities frequently. Playful activities are important for learning in early grades (Wager & Parks, 2014; Perry & Dockett, 2002; Wager & Parks, 2016; Vygotsky, 1978), although they are not explicitly a part of the CGI approach. Bringing in playful activities is an aspect of Ms. C pedagogical perspective that slightly differed from "typical" CGI teaching. Playful mathematics was a large part of Ms. C's mathematics instruction. At least once a week, students were invited to play mathematics games which sometimes included whole-group activities and other times were small-group board games. Even during free time, which I often observed after the scheduled mathematics time, many of the students would continue to play the mathematics games that Ms. C introduced or played with objects (e.g., blocks, ramps, whiteboards, board games) where they still participated in mathematical practices (building, designing, explaining, counting). In Ms. C's classroom, mathematics was in many ways framed as being playful, which also connected mathematics to the experiences her students had outside the classroom.

Ms. C drew on the experiences her students had outside of the classroom to contextualize mathematics problems. For example, Ms. C often used the names of her students in word problems and brought in activities she knew they did outside of the classroom (e.g., soccer during PE, afterschool clubs). Although the playful activities (e.g., board games) may not have directly mirrored the activities her students did outside of school, bringing in play was another way Ms. C connected to students' interests and experiences. Brining in students' interests and experiences is not a part of the CGI approach, which highlights another way that Ms. C's pedagogical perspective differed.

Data Collection and Analysis

I collected all three types of data from Ms. C (observations, interviews, and the survey). She took the survey, talked with me during two interviews (one before in-person instruction in 2020 and one before online instruction in 2021), and welcomed me into her classroom (in-person and online) for observations. The observational data was my primary data source when looking at the central features of Ms. C's teaching practices. The analyses of teaching practices were qualitative in nature. I used interpretive methodologies for data analysis. The analysis started during data collection and continued through the writing of the papers. Every day after I observed, I wrote analytic memos. I made spreadsheets where I summarized information and developed initial codes from the observations related to the content, emergent themes, questions, and structure. I then employed case study analysis using analytic induction. I wrote memos and detailed descriptive summaries across all the data sources for the teaching practices study in Ms. C's classroom.

The questions that guided my memos and initial analysis included: How was mathematics structured during the week, day, and hour? How did the teacher position children during whole-class activities focused on mathematics, or how did the teacher talk about students' roles during mathematics instruction?

Did the teacher provide or talk about opportunities for children to discuss mathematical ideas? Did the teacher talk about or encourage children to take time to grapple with concepts related to mathematics? What norms did the teacher establish around mathematics? Who held the authority in the classroom around mathematics? Did the teacher talk about or actively elicit and explore children's ideas and strategies? What types of materials did the teacher use (or discuss using) to support mathematics learning? I highlighted features and themes from the summaries that became the basis for data reduction and further analysis. After completing my initial analysis of the observations, I used Ms. C's interview and survey responses to clarify my findings.

In Chapters 3 and 4, I examine Ms. C's teaching practices. Chapter 3 describes the central features of her teaching practices using vignettes to illustrate how Ms. C enacted curriculum in person in a first-grade classroom prior to COVID-19. In Chapter 3, I also use classroom observation data to describe the features of Ms. C's instruction and how Ms. C's teaching practices aligned with her professed beliefs. In Chapter 4, I drew on observational data to compare Ms. C's in-person and online teaching practices and the alignment between her professed beliefs and her teaching practices.

Previews of the Three Analyses Chapters: Research Questions, Findings, and Contributions

This section provides an overview of the three analysis chapters. I review the research questions for each analysis and provide a short summary of the main findings. I end by suggesting possible contributions, which will be discussed in more detail in Chapter 5 (Discussion and Implications).

Preview of Chapter 2: Teachers' Beliefs

In Chapter 2, *Early Grades Teachers' Beliefs about Mathematics, Language, and Emergent Bilinguals*, I documented the professed beliefs of 20 early grades teachers. I drew on a conceptual framework of professed beliefs (Phillip, 2007; Speer, 2005) and used the lens' of language orientations (Fernandes, 2020) and belief constructs (Schoen & LaVenia, 2019) to analyze beliefs. The research question I explored was: what are early grades teachers' professed beliefs about mathematics, language, student thinking, and students' early, out-of-school experiences with mathematics, particularly for EBs?

The teachers in this study displayed varying degrees of asset-based responses (74%-100%) to the survey and discussed beliefs related to 1) students' backgrounds and experiences, 2) students everyday and home languages, 3) mathematics vocabulary, and 4) supporting EBs. Teachers described their beliefs about students' assets (experiences and home/everyday language) in ways that aligned with either allowing students' assets in the classroom or drawing on students' assets to support mathematics learning. This study corroborates previous research that also found teachers' beliefs as situated on a continuum (e.g., Fernandes, 2020; Oaks & Lipton, 1999; Staub & Stern, 2002) and extends previous work (e.g., Fernandes, 2020) by focusing on teachers' beliefs about language beyond native language to include everyday language. For the purposes of this dissertation, this study of beliefs also set up the closer ethnographic teaching practices studies of Ms. C and the comparison of Ms. C's teaching practices and her professed beliefs.

This work is particularly important as there is little literature focused on teachers' beliefs about supporting EBs' mathematics (Fernandes, 2020). The analysis in Chapter 2 adds to the research literature about teachers' beliefs about mathematics instruction for EBs. While there is a great deal of literature on teachers' beliefs, and more specifically about teachers' beliefs related to mathematics teaching (e.g., Ambrose, 2004; Lee & Ginsburg, 2007; Raymond, 1997; Roesken-Winter, 2013; Staub & Stern, 2002; Stipek, Givvin, Salmon, & MacGyvers, 2001; Philipp, 2007) there are only a few studies that explore the beliefs of early grades teachers related specifically to language and mathematics instruction with EBs (e.g., Fernandes, 2020).

Preview of Chapter 3: Teaching Practices In Person

In Chapter 3, *An Account of an Accomplished Teacher's In-Person Instruction in a First-Grade Classroom: Drawing on Students' Assets,* I characterized the nature of mathematics instruction by exploring the research questions: 1) what was the nature of mathematics instruction in a first-grade classroom with an accomplished teacher? and 2) how did an accomplished teacher draw on students' assets (student thinking and experiences)?

Ms. C's teaching practices aligned with recommendations from research on early grades mathematics teaching (e.g., Aguirre et al., 2012; Carpenter et al., 1993; Carpenter, Fennema & Franke, 1996; Fennema et al., 1996; Perry & Dockett, 2002; Turner et al., 2012; Wager, 2013) including the following: Ms. C 1) created opportunities for students to develop conceptual understanding, 2) used teacher moves and was highly responsive to student contributions, 3) established participation norms and socio-mathematical norms related to mistakes and efficiency, and 4) drew on students' experiences. The central features of Ms. Cs teaching practices in person aligned with her professed beliefs related to low "transmissionist," low "facts first," and low "fixed instructional plan."

The analysis in this chapter makes several contributions. First, it provides a detailed picture of an accomplished teacher's practices. These vignettes and the accompanying analyses can be used to ground discussions in teacher education and professional development. Second, I provide a critique of representing teacher learning, change, or practices using a linear trajectory or progression. The analysis of in-person teaching practices highlights the complexities of teaching and complicates the representation of teaching in models used in research. Any model of teacher change, teacher learning, or teaching practices needs to include the complexity of teaching.

Preview of Chapter 4: Comparing Teaching Practices In Person and Online

In Chapter 4, I compare Ms. C's instruction online and in-person. I found that the features of her instruction were similar even though her teaching in many ways looked different. Beyond this, I found that Ms. C's teaching practices online aligned with her beliefs except for her low "fixed instructional plan" belief. I discuss potential reasons for this difference and its implications for practice in Chapter 5 (Discussion and Implications).

In Chapter 4[6], *Teaching Early Grades Mathematics Online During a Global Pandemic,* I compared Ms. C's teaching practices and alignment with her professed beliefs both in-person and online. The research questions I explored included: 1) What were the differences between classroom routines and mathematics activities in person compared to online during COVID-19 in an early grades classroom? 2) Did Ms. C enact math instruction online that aligned with her professed beliefs? If so, how? And 3) What was the nature of online math instruction for EBs in Ms. C's classroom?

A comparison of online and in-person instruction showed that the central features of Ms. C's enacted curriculum (i.e., teaching for conceptual

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[6] Chapter 4 was originally the third paper.

understanding, using teacher moves, establishing norms, and using students' experiences outside the classroom) persisted online, even though some features looked different. Although the supplemental materials that Ms. C brought into her in-person instruction were absent online, the smaller, still frequent strategies that Ms. C used during mathematics instruction still positioned the students, in particular, the EB I call Samuel[7], as competent and created opportunities for frequent and varied participation. Ms. C's online teaching continued to reflect an asset-based approach to teaching mathematics with children in early grades.

Despite the persistence of the central features of Ms. C's teaching practices both in-person and online, there were some tensions that I observed during Ms. C's online mathematics instruction. Ms. C's in-person teaching practices aligned with her professed beliefs. However, Ms. C's teaching practices online did not align with her low "fixed instructional plan" belief. I observed constraints online that Ms. C faced that I did not observe during her in-person teaching. The main constraint that I observed included unclear guidance and uncertainty from the administration as teachers navigated future plans for transitioning from online back to in-person teaching. Related to this, the administration asked teachers to cover the same content as the other teachers in their grade. This constraint resulted in limiting Ms. C's autonomy to supplement

[7] All names are pseudonyms.

and adjust the content that she covered online. During her interview prior to COVID-19, Ms. C discussed her low “fixed instructional plan” belief and her teaching practices in-person aligned with this belief. This was evident in the ways she supplemented and adjusted the curriculum materials to meet the needs of her students while teaching in person. However, Ms. C’s teaching practices online looked different as she taught with more fidelity to the scope and sequencing of the curriculum materials. The way her teaching practices did not align with her low “fixed instructional plan” belief demonstrates how the constraints she faced online impacted her teaching. This tension between Ms. C’s belief and her teaching practices was due to institutional constraints including a lack of administrative guidance and overt control from the administration regarding pacing and covering content.

This study adds to existing literature that has looked at the institutional constraints that can impact teaching (e.g., Parks & Bridges-Rhodes, 2012; Pease-Alvarez & Samway, 2008) and contributes to the more recent focus of research on the impacts of COVID-19 and the shift to online schooling (e.g., DeCoito & Estaitayeh, 2022a; DeCoito & Estaitayeh, 2022b). The implications for professional development and contributions to research on mathematics instruction in early grades include providing professional development for teachers that is specific to drawing on students’ experiences outside of the classroom and informing stakeholders of the effectiveness of drawing on student thinking and experiences to support mathematics learning with understanding, discussed in detail in Chapter 5.

My Positionality and the Impact of COVID-19

It is essential to be as transparent as possible when doing research, including identifying my position as a researcher. I used to be a preschool teacher and acted as an informal math coach for the other preschool teachers in my school, so I did have some insights and casual connections to teachers in this study. My position as a doctoral candidate and a researcher during the study put me in a position of power when I worked with the teachers in this study. Specifically for my relationship with Ms. C, I was introduced to Ms. C through a mutual colleague years before I started collecting data for my dissertation. She was also a participant in a smaller research project I had done during the second year of my doctoral program. Over the years of getting to know Ms. C, I started developing a friendly relationship where we would talk about things not related to teaching and math. I got to know which baseball team she liked and the hobbies she enjoyed. While I did not spend time with Ms. C outside of school, she and I had developed a more casual relationship over the years, which helped facilitate some of the conversations we had, especially as she openly shared her feelings and frustrations once schools transitioned online due to COVID-19.

Beyond my positionality, this dissertation was significantly shaped by the COVID-19 pandemic. My original plan for this dissertation was to explore how two to four experienced (five or more years of teaching), early grades teachers created opportunities in their classroom for EBs to learn mathematics with understanding. The original research questions included:

1. What are teachers' beliefs about mathematics, students' thinking, and students' Funds of Knowledge?
2. What is the nature of mathematics instruction in early grades classrooms?
3. Do teachers draw on students' Funds of Knowledge (e.g., student thinking, informal strategies, and home mathematics and/or language practices), and if so, how?
4. Do teachers particularly draw on the Funds of Knowledge of EBs, and if so, how?

To explore these questions, I planned to observe in person and interview the teachers across two units of instruction (about six weeks for each teacher). Before, during, and after observations, I planned to interview each teacher. The goal was to observe teachers from the same grade as they taught similar content. I had to revise my original plans when schools suddenly shifted to remote instruction.

The participants in my study changed, my plan for data collection shifted, and the focus of my dissertation changed when schools suddenly moved online. I had planned to recruit the larger participant group from the beliefs study and then purposefully select teachers from this group for the in-depth teaching practices studies. This purposeful selection would have been based on 1) teaching experience, 2) beliefs about mathematics, language, and EBs, and 3) EBs in the classroom. Due to schools closing, this plan changed. I was not able to observe in-person instruction as in-person instruction was temporarily stopped and even once it resumed, there were strict limitations that only allowed faculty and staff on most school grounds. During this time even parents had restricted access on campus. Therefore, I pivoted from original plan of comparing different teachers and classrooms towards a comparison of teaching modes (in-person and online) of one accomplished teacher, Ms. C. My original observations of Ms. C's teaching in-person took place during a feasibility study.

Due to the changes in data availability, I had to move away from my original plan to focus on EBs. I did not take notes explicitly on EBs during the feasibility study during Ms. C's in-person instruction. Rather, I took notes more generally on all the students. Even when I did focus on EBs during my observations of Ms. C's online instruction, Ms. C only had one EB that participated in online mathematics instruction. Since I did not collect explicit data on EBs during in-person instruction and only observed one EB during online instruction, I had to move away from the focus on EBs throughout the dissertation. The focus instead shifted towards a comparison study of in-person and online mathematics instruction.

The pandemic impacted more than just where and how I collected data. My conversations with teachers often included stories about moving to online instruction, even before I asked questions about this. Teachers often responded to questions with phrases like, "because I have to do this online" or "if I were in my classroom, I would have….”. COVID-19 impacted what teaching looked like. It affected what research about teaching looked like. It impacted our lives, work, and health (physical and mental). The effects of the pandemic were present across all parts of this dissertation, from the study's design to the findings. Lastly, I originally planned to write three separate papers. However, that did not work as well as expected because the analyses were more connected than I had originally envisioned. I also reference and build upon the findings across the papers (e.g., I first analyzed for professed beliefs and then compare teaching practices and beliefs to explore if and how they aligned). The current version of the dissertation has three chapters: Chapter 2: Early Grades Teachers' Beliefs about Mathematics, Language, and Emergent Bilinguals, Chapter 3: An Account of an Accomplished Teachers In-Person Instruction in a First Grade Classroom: Drawing on Students' Assets, and Chapter 4: A Comparison of Mathematics Instruction In-Person and Online with First-Grade Students.

Chapter 2: Early Grades Teachers' Beliefs about Mathematics, Language, and Emergent Bilinguals[8]

Many factors impact how teachers orchestrate their mathematics instruction. One factor that has been the focus of many studies in mathematics education is teachers' beliefs (e.g., Lee & Ginsburg, 2007; Madni et al., 2005; Raymond, 1997; Roesken-Winter, 2013; Schmeisser et al., 2013; Staub & Stern, 2002; Stipek, Givvin, Salmon, & MacGyvers, 2001; Törner, Rolka, Roeskin, & Sriraman, 2010; Vacc & Bright, 2012). Beliefs can impact teaching practice (Schoenfeld, 2002; Raymond, 1997), shaping the ways teachers facilitate mathematics instruction in early grades. Following this, teachers' instruction can impact students' dispositions towards mathematics (Gresalfi, 2009). A teacher who believes that mathematics is about correct answers and working quickly to solve problems may pass these same beliefs to students. In contrast, a teacher who believes mathematics is about problem-solving and learning through mistakes would likely create more opportunities for students to develop productive dispositions which includes the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in

[8] Originally Paper 1

diligence and one’s own efficacy” (Kilpatrick, Swafford, & Findell, 2001, p. 116).

Schoen & LaVenia (2019) characterized the ways over 200 teachers discussed beliefs related to mathematics instruction after they participated in professional development focused on CGI. Schoen & LaVenia (2019) developed the “belief constructs” tool to identify and characterize key beliefs of elementary teachers related to teaching mathematics with understanding. The tool included three scales 1) “transmissionist”, 2) “facts first”, and 3) “fixed instructional plan.” Typically, teachers with beliefs that reflect low “transmissionist”, low “facts first”, and low “fixed instructional plan” hold views that are grounded in the CGI approach for teaching mathematics that centers on uncovering and drawing on student thinking and solutions for problem-solving.

Teachers’ beliefs are shaped by the setting and context where they teach. Lee and Ginsburg (2007) found that teachers in middle-SES preschools were more likely to support activities relevant to the students’ interests and were more focused on the social aspect rather than the academic aspect, of preschool. The teachers from middle-SES preschools viewed their students as coming from homes with educational resources that prepared them for school. In comparison, teachers serving low-SES preschools were more likely to highlight the importance of academics and direct instruction in preschool. Aligned with deficit views, the teachers from low-SES preschools positioned their students as coming from disadvantaged homes and needing to catch up. This distinction is crucial for EBs in poor schools as they may not have opportunities to draw on their full set of resources to learn mathematics if their teachers hold deficit views of them. Teachers' deficit views of students can limit students' access to quality mathematics instruction and opportunities to learn mathematics with understanding (Lee & Ginsburg, 2007; Turner et al., 2012). Instead, teachers need to hold asset-based views of their students, including EBs, to fully support their mathematics learning. Research has highlighted the importance of 1) drawing on EBs' experiences and backgrounds to provide access to content (e.g. Aguirre et al., 2012; Turner et al., 2012), 2) leveraging students' home and everyday language to support student learning (e.g., Brenner, 1998; de Araju et al., 2018; Turner & Celedón-Pattichis, 2011; Moschkovich, 2013; Moschkovich, 2015a), and 3) teaching vocabulary as connected to concepts and not isolated (Moschkovich, 2013, Moschkovich, 2015a). When teachers hold beliefs that align with these recommendations, they have asset-based views of their students and may become more likely to provide opportunities for mathematics and language learning in the classroom.

Teachers' beliefs about language and language learning can impact EBs' opportunities to participate in mathematics instruction. There are beliefs that may be unproductive for supporting the needs of EBs in the mathematics classroom. For example, a belief that mathematics should be taught by an

English as a Second Language/ English Language Development (ESL/ELD) teacher may reflect a view that children need to learn English before they can participate in mathematics. This could also lead to placing a student in an ESL/ELD class with a teacher unprepared to teach mathematics. There is specific content and pedagogical content knowledge related to teaching mathematics and these may not be present from teachers with little training in mathematics and in a classroom where the main and often the only focus is on teaching English.

Fernandes (2020) identified a continuum related to teachers' beliefs about students' home languages. Using the Mathematics Education of English Learners Scale (MEELS) (Fernandes & McLeman, 2012), Fernandes (2020) found that pre-service teachers' beliefs about the use of home languages in the classroom fell into one of four categories that reflect beliefs about native language use in mathematics classrooms: no native language, limited use of native language, extensive use of native language, and bilingualism. In his study, two out of 31 participants were characterized as having beliefs related to no native language use in the classroom whereas nearly half (n=15) were characterized as limited native language use (Fernandes, 2020). These two categories were highlighted to show an expansion of previous work (Ruiz, 1984) highlighting teachers' beliefs that focus on a deficit view (i.e., language as a problem) of students' native language. In contrast, participants who identified language as a resource shared beliefs related to extensive use of native language in the classroom (n=10) and bilingual use in the classroom (n=3) (Fernandes, 2020). The teachers' beliefs that were uncovered in this study were complex and included mixed views of students' native language use in the mathematics classroom. Fernandes (2020) highlights a need for more research to look at teachers' beliefs about language and mathematics instruction with EBs.

Given the need identified by Fernandes (2020), in this chapter, I analyzed early grades teachers' beliefs about mathematics, language, and students' assets. I explored the research question: what are early grades teachers' professed beliefs about mathematics, language, student thinking, students' out-of-school experiences, and students' home and everyday language practices, in particular for EBs?

Framework

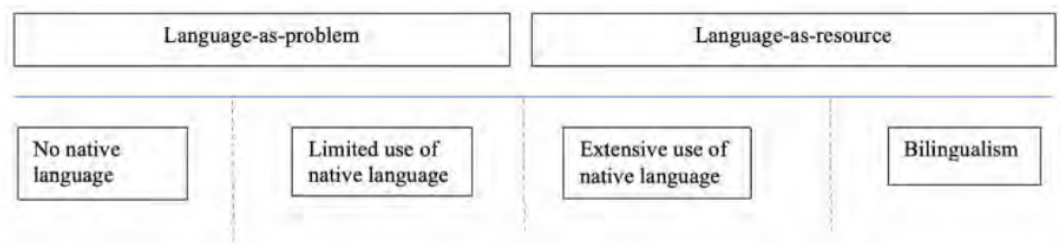
In this study I looked at beliefs through the lens of Multiple Mathematical Knowledge Bases (MMKB) (Aguirre et al., 2012; Turner et al., 2012), the language orientations construct (Fernandes, 2020), and the belief constructs (Schoen & LaVenia, 2019). The following section overviews how I defined beliefs and how I used the frameworks to inform the design of my study.

I drew on Philipp (2007) to define beliefs as "psychologically held understandings, premises, or propositions about the world that are thought to be true [...] beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action" (p. 259). I looked specifically at professed beliefs that were explicitly stated by teachers, rather than inferred attributed beliefs (Speer, 2005; Gonzalez Thompson, 1984). I identified professed beliefs by exploring what teachers said about their mathematics instruction, student learning, and language.

The primary purpose of this study was to look at teachers' beliefs about language and mathematics for EBs. Therefore, I used the belief constructs (Schoen & LaVenia, 2019) and language orientations construct (Fernandes, 2020) with the associated MEELS instrument (Fernandes & McLeman, 2012) to design the data collection protocols and to look at the data. For the design of the study, I adapted MEELS to develop a survey and to inform the interview questions that I used for data collection. I used MEELS to include items related to native language use in the mathematics classroom, fairness of supporting EBS, and teaching strategies for EBs. The language orientations framework (Fernandes, 2020) was the primary lens I used to look at teachers' responses. This framework includes four orientations that range from viewing language as a problem to viewing language as a resource (Figure 1). On the farthest side of the continuum where language is viewed as a problem, Fernandes (2020) highlighted pre-service teachers' orientations that reflected "no native language." In this group, the teachers felt there was no room for native language in a mathematics classroom. Next, pre-service teachers with a "limited use of native language" believed that it was acceptable for students to use their native language in some situations, but the goal was to replace their language with English in the mathematics classroom. Moving towards views of language as a resource, Fernandes (2020) found that some pre-service teachers held beliefs that reflected "extensive use of native language" where teachers supported any language use in the classroom and the goal was to learn mathematics regardless of language. The final group had beliefs that reflected and promoted "bilingualism" where the pre-service teacher promoted native language use because they believed that native languages support mathematics learning. While Fernandes (2020) only included native language in these constructs, I expanded this work to also include everyday ways of communicating. This is because many researchers (e.g., Au & Kawakami, 1985; Barwell, 2005; Brenner, 1998; de Araujo, Roberts, Willey, and Zahner, 2018; Gutiérrez, Baquedano-López & Tejeda, 1999; Lipka et al., 2005; Razfar, 2013; Turner & Celedón-Pattichis, 2011) have found that inviting all of students' linguistic resources into the classroom, including native language and language practices (e.g., talk story, joking, storytelling, dialects), supports engagement and learning.

Figure 1

Language Orientations Construct (Fernandes, 2020)



In addition to the language orientations framework (Fernandes, 2020), I used Schoen & LaVenia's (2019) "belief constructs" framework to characterize the professed beliefs Ms. C reported related to her mathematics instruction. I only used this framework to look at Ms. C's beliefs as she was the focal case for the teaching practices analyses (Chapters 3 & 4). The three scales that make up the belief constructs include 1) "transmissionist", 2) "facts first", and 3) "fixed instructional plan." The "transmissionist" scale represents the extent to which teachers believe they should guide students towards a single standard solution. For example, a teacher with low "transmissionist" beliefs often described a down-up approach where they use the thinking and strategies of students to inform teaching. Low transmissionist beliefs align with research on student thinking that has shown that students perform better when they experience this type of teaching (Fennema et al., 1996; Polly, Margerison, & Piel, 2014). The "facts first" scale represents teachers' beliefs about the role of learning facts in problem-solving. Teachers with low "facts first" report or hold the belief that learning and teaching problem-solving creates opportunities for students to develop meaning for facts as students progress through the problem-solving process. A teacher holding or reporting high “facts first” beliefs would instead believe that students must learn basic facts before they can solve problems. Finally, the “fixed instructional plan” scale represents the extent to which teachers agree that they must adhere to the scope and sequencing of topics and pace them according to the curriculum. A low “fixed instructional plan” reflects a belief that teaching is more effective when teachers make adaptations to prescribed scope and sequencing based on student assessments and needs. In my description of Ms. C’s beliefs, I use these three scales to characterize how she discussed and reported her mathematics teaching during her interviews and on her survey.

In addition to teachers’ beliefs about language and mathematics, I also included opportunities for teachers to share their thoughts about students’ experiences and interests related to mathematics outside of the classroom. MMKB includes students’ “multiple understandings and experiences that have the potential to shape and support students’ mathematics learning” (Turner et al., 2016, p. 49). One key feature of MMKB is students’ interests and experiences outside of the classroom. To elicit teachers’ reactions about students’ experiences, I intentionally asked questions and included statements about this in my data collection protocols.

Methodology

This is a qualitative study where I used the frameworks of MMKB (Aguirre et al., 2012; Turner, et al., 2012) and language orientations (Fernandes, 2020) to design the study and analyze the data. Data for this study came from survey and interview responses from a group of experienced early grades teachers. My analysis of these responses drew on descriptive statistics and interpretive analysis. I focused on the words the teachers said and their responses to statements and questions. One guiding assumption I have about the teachers in this study is that these teachers are professionals who navigate many things that can impact their instruction that may or may not align with their beliefs about mathematics instruction with EBs. I intentionally did not count or run statistical analyses on the data because of the small sample size (n=20) and because I purposefully selected these teachers in a fashion that made them not representative of the larger teaching force (e.g., in terms of years of teaching, experiences with professional development). In this section, I provide an overview of the participants, the data that was collected, and my methods for analyzing the data.

Participants

I recruited participants using convenience sampling from sources including mutual colleagues, individuals I knew, and a well-established social media platform. During recruitment, I identified specific criteria for eligibility.

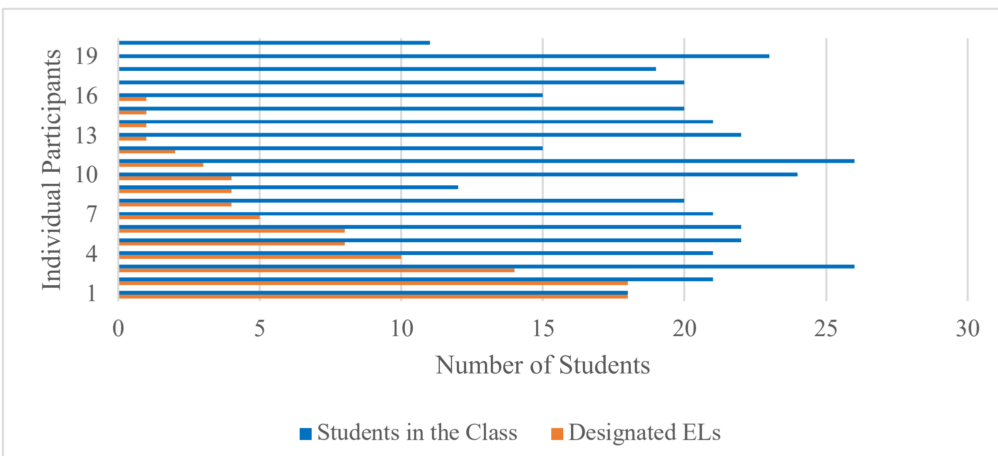
This included teaching for at least 5 years in an early grades classroom (Pre-K through 3rd grade) and current placement in an early grades classroom. I also told the teachers that the study was about mathematics and language. In mathematics education research, there are a plethora of studies done with pre-service teachers (e.g., Ambrose, 2004; Parks & Wager, 2015; Turner et al., 2016). Therefore, I intentionally selected experienced teachers (five or more years of teaching) to explore their beliefs and practices. Recruitment started at the beginning of the 2020-2021 school year and it took about 6 months to have 20 participants agree to participate. Due to the difficulty with recruitment, I stopped recruiting once I had 20 participants.

All twenty teachers responded to a survey online through google forms. Of these 20 participants, four participants were preschool teachers, three were kindergarten teachers, five were first grade teachers, one was the distance/online teacher for third grade, one teacher had a kindergarten-first grade combination class, and the remaining six did not identify their current grade as this was a question I added after some of the teachers had already filled out the survey. Given the eligibility criteria, all teachers had been teaching for over five years with a range between 5 and 36 years. While most of the teachers reported being monolingual, six teachers reported speaking and understanding Spanish and English, three reported speaking and understanding French and English, one reported familiarity with Mandarin, and one reported “other”. Half of the participants reported being monolingual and the other half reported being multilingual.

Related to the students' demographics, the teachers reported on their students' EL[9] designation and family background to the best of their knowledge. While four teachers reported having no designated ELs in their class, others taught classes of only ELs (Figure 3). Of the 20 teachers, 16 of them had at least one EL in their class during the 2020-2021 school year. Figure 2 below shows the total enrolled children (range of 11-26 students) compared with the designated ELs in each classroom.

Figure 2

Number of Students Compared to Designated ELs in Each Class



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[9] In the survey Emergent Bilinguals (EBs) were referred to as "ELs" to remain consistent with terms that the teachers were familiar with. I mirror this language when I share the responses to align more closely with the teachers' responses. However, I use the term EBs throughout the rest of the paper to highlight an alternative term that frames young multilingual learners using an asset-based view and shifts away from the English-dominant way of labeling.

Data Collection

There were two sources of data for this study that came from survey and interview responses. For the survey, I recruited 20 teachers as participants. The items on the survey included demographic information (e.g., years of teaching experience, professional development, student information) (see Appendix B) and statements about mathematics, student learning, teaching, the role of language in learning math, and emergent bilinguals (see Appendix C). Teachers responded using a 5-point Likert scale ranging from strongly agree to strongly disagree. Some survey items were adapted from MEELS (Fernandes & McLeman, 2012). MEELS was used by Fernandes (2020) as a data collection tool as he developed the language orientations framework. Fernandes (2020) used MEELS to identify pre-service teachers' beliefs about mathematics and language instruction with EBs (Fernandes, 2020).

From the participant group of 20, only five agreed to be interviewed after taking the survey. Therefore, I interviewed these five teachers using a semi-structured interview. The five participants self-selected to participate in the interview and were not selected to be representative of the larger sample. Questions on the interview asked teachers to reflect on their mathematics teaching, students learning, views of the role of language in learning mathematics, and supporting emergent bilinguals (see Appendix A).

Data Analysis

To analyze the survey, I first coded the responses as aligning with an asset-based view. I drew on the literature outlined in the introduction chapter and review of the literature in this chapter about effective teaching and about beliefs that align with asset-based views of students. I used previous research to identify if agreement or disagreement with each statement reflected an asset-based view. In particular, I drew on the recommendations from research related to 1) students' experiences and backgrounds (e.g., Aguirre et al., 2012; Turner et al., 2012), 2) students' home and everyday language (e.g., Brenner, 1998; de Araujo et al., 2018; Turner & Celedón-Pattichis, 2011; Moschkovich, 2013; Moschkovich, 2015a), and 3) teaching mathematics vocabulary (Moschkovich, 2013, Moschkovich, 2015a). My codes for asset-based views for each statement is outlined in the table below (Table 1).

Table 1

My Interpretations of Responses that Align with Asset-Based Views

Statements where Agreement Reflects an Asset-Based View	**Statements where Disagreement Reflects an Asset-Based View**
Teachers and schools need to learn about the mathematical practices from students' families and communities.	Students should learn math vocabulary before they learn math concepts.
Students learning English should use their primary language and everyday ways of talking to engage with mathematics content.	Language demands for English Learners (ELs) in math only occur in word problems.
It is fair for ELs to get accommodations on math tests (e.g., extra time, use of dictionary).	Math is not language intensive.

ELs can be effectively taught math in English before they are fluent in English.	The math work of ELs and non-ELs should be graded the same way.
When teaching ELs, I should use a variety of math vocabulary.	I should teach math to ELs and non-ELs in the same way.
When teaching ELs, I should provide different opportunities (e.g. small groups, one-on-one with the teacher) for them to explain their thinking in English.	ELs should be taught math by ESL/ELD (English as a Second Language / English Language Development) teachers.
I should adjust the language in math problems to ensure ELs understand.	Accommodating the needs of ELs in the math classroom can slow down the learning of other students.
When there are ELs in my classroom, my lesson plans should address both the math content and the English needed.	ELs will not learn English quickly if I allow them to speak their native language in my math class
It is helpful to restate the math explanations given by ELs during class discussions.	When teaching ELs, I should focus more on basic computations than on problem solving activities.
When interacting with ELs, I should focus on the math in their explanations instead of their language.	When ELs switch between languages to explain their mathematical thinking, it shows a lack of mathematical understanding.
The different ways that ELs learned math (e.g., multiplication, addition, solving problems) in their homes is a valuable resource in the math class.	When ELs switch between languages to explain their mathematical thinking, it shows a lack of language fluency.
An ELs background and experiences are valuable resources to help all students learn math.	

For some of the statements it was easier to determine which response (agreement or disagreement) reflected an asset-based view. For example, the statement, “ELs will not learn English quickly if I allow them to speak their native language in my math class," does not reflect an asset-based view, and there is a great deal of literature that has shown that ELs should have access to their native language in the classroom (e.g., Barwell, 2005; Brenner, 1998; Turner & Celedón-Pattichis, 2011). Therefore, it was evident to me that disagreeing or strongly disagreeing with this statement aligned with an asset-based view. I found it more challenging to decide if agreement or disagreement with certain statements reflected an asset-based view. For the statement, "I should teach math to ELs and non-Els in the same way," I found more challenging to decide if agreement or disagreement reflected an asset-based view. ELs should have access to the same mathematics content as their monolingual counterparts. However, ELs also need specific support to engage with that content, which may look different. Therefore, I determined that disagreement aligned with more of an asset-based view for this statement. When I came to a statement that was not fully obvious to me, I referred to the literature from the introduction chapter and the introduction to this paper and made an informed decision. The way I determined how a statement did or did not reflect an asset-based view came from my interpretation of the literature on this topic. I do not think these categories are hard and fast. Additionally, I acknowledge that as teachers took this survey they interpreted the statements, and their interpretations were impacted by their experiences and ideas related to effective teaching. Thus, I did not solely rely on the survey for this study but also used the interviews to complicate the survey results and give a more detailed picture of these teachers' beliefs.

After I determined if agreement or disagreement revealed an asset-based view for each statement, I coded each participants response as aligning or not aligning to an asset-based view. I then calculated the percentage of asset-based responses for each participant. For this calculation, I only looked at non-neutral responses (responses that fell on either side of neutral) and divided the number of asset-aligning responses by the total number of non-neutral. For example, one participant had 21 responses that aligned with asset-based views, one response that aligned with a deficit-based view, and one response that was neutral. To calculate the percentage of asset-based responses for this individual I divided their asset-based responses (21) by the total of non-neutral responses (22) which gave me a percentage of 95%. I made the decision to take out neutral responses from this calculation because neutrality does not fit with either an asset-based or deficit-based view on this scale. If a person identified many neutral responses and the rest of their responses aligned with an asset-based view, it did not seem accurate for their percentage of asset-based beliefs to be lower if the only other responses were neutral.

To further examine the results of the survey, I used interview data from participants who agreed to be interviewed. To analyze participants' answers to the interview questions I narrowed my focus to look at the responses related to my frameworks. This included descriptions of students' experiences and interests (drawing on the MMKB framework) and related to students' language (drawing on the language orientations framework). For Ms. C I also analyzed her interview and survey responses using the belief constructs (Schoen and LaVenia, 2019) as she was the focal teacher for the analyses of teaching practices in Chapters 3 and 4. Beyond this, I also looked at how teachers talked about support more generally for EBs to see if there were commonalities across participants. I went through each interview from beginning to end and transcribed each section that related to beliefs about students' experiences, students' language, supporting EBs, and mathematics vocabulary. I then summarized teachers' responses and quotes using descriptive codes. From these summaries, I looked across the participants to identify themes and look at the details of how these five teachers talked about their beliefs related to mathematics, students' experiences, students' language, and supporting EBs. Since I only interviewed five of the 20 teachers, these descriptions are not reflective of my entire participant group and cannot be generalized to other groups. However, these descriptions offer detailed ways in which these five teachers described students' assets related to mathematics and can provide insights into the beliefs that impact instruction for EBs

Teachers' Beliefs

In the following section, I provide an overview of the findings from the survey and the interviews. The survey responses revealed that the teachers in this study held varying degrees of asset-based views of students classified as English Learners. All the teachers responded with at least 74% of their non-neutral responses in ways that reflect an asset-based view. I first overview how I sorted and characterized the participants based on percentages of asset-based survey responses. Beyond what I found in the survey responses, the interviews with five of the 20 teachers clarified and provided more detailed descriptions of their beliefs particularly related to supporting EBs' mathematics learning. From the interviews, I found that teachers held beliefs about students' assets and teaching mathematics with EBs related to students' everyday and home language, students' backgrounds and experiences, mathematics vocabulary, and supporting EBs.

Having an Asset-Based View (Survey Responses)

Teachers in this study held varying degrees of asset-based views of their students. Across the responses, there was consensus (everyone responded similarly) for two statements in the survey. Everyone in this group either disagreed or strongly disagreed with the statement, "When ELs switch between languages to explain their mathematical thinking, it shows a lack of mathematical understanding." Similarly, all the participants either agreed or strongly agreed with the statement, "When there are ELs in my classroom, my lesson plans should address both the math content and the English needed." This shows that across this sample, 100% of the teachers held beliefs that switching between languages does not show a lack of mathematical understanding and that mathematics teaching should include a focus on both the math and English needed. These two beliefs reflect asset-based views of students because they acknowledge that students may use multiple languages regardless of their mathematical understanding, and that instruction should include a focus on both math and language.

Beyond these two statements, I identified and sorted teachers' total percentage of asset-based view responses across four categories that are similar to but vary from those outlined by Fernandes' (2020) language orientations framework. Since Fernandes (2020) categorized his participants' responses related to their beliefs about students' native language, my categories had to be expanded to include students' assets more generally, including mathematical practices from their homes and communities, everyday language, and teaching practices that align with drawing on students' assets in mathematics instruction. My four categories include 1) some asset-based views, 2) many asset-based views, 3) mostly asset-based views, and 4) all asset-based views. Three of the participants (15%) responded in ways that 70-76% of their responses reflected asset-based views (some asset-based views group). Two of the participants (10%) responded with 80-89% of their responses reflecting asset-based views (most asset-based views group). Most of the participants feel into the third category, the many asset-based views group, with 12 of the 20 teachers (60%) who responded with 90-95% answers that aligned with an asset-based view. Three participants (15%) responded with 100% of their non-neutral answers in ways that reflected asset-based views (all asset-based views). In the following section I discuss responses to the survey and identify specifics about each group.

Some Asset-Based Views Group (15%)

In this group, there was consensus around some statements that reflect an asset-based view, which I include in the following table (Table 2). Additionally, this group shared some consensus around the other statements that did not reflect asset-based views. Two of the three teachers in this group agreed that they should grade the work of ELs and non-ELs the same ways. Two of the three also expressed beliefs that math should be taught to ELs by an ESL/ELD teacher. Finally, two of the three expressed beliefs that when a student switches between languages it reflects a lack of language fluency.

Table 2

Consensus Across Members of the "Some Asset-Based View Group"

Statement	**Responses**
It is fair for ELs to get accommodations on math tests (e.g., extra time, use of dictionary, etc.).	Agree/Strongly Agree
Accommodating the needs of ELs in the math classroom can slow down the learning of other students.	Disagree/Strongly Disagree
When there are ELs in my classroom, my lesson plans should address both the math content and the English needed.	Agree/Strongly Agree

When teaching ELs, I should provide different opportunities (e.g., small groups, one-on-one with the teacher, etc.) for them to explain their thinking in English.	Agree/Strongly Agree
It is helpful to restate the math explanations given by ELs during class discussions.	Agree/Strongly Agree
When ELs switch between languages to explain their mathematical thinking, it shows a lack of mathematical understanding.	Disagree/Strongly Disagree
An ELs background and experiences are valuable resources to help all students learn math.	Agree/Strongly Agree

Many Asset-Based Views Group (10%)

Two of the teachers answered the survey with 84-89% of their responses reflecting asset-based views of students. Since there were only two in this category, I provide an overview of the responses for each person. The first teacher answered 17 of the statements in ways that reflected asset-based views and two in ways that did not. Beyond this, they selected neutral responses for four of the statements. This teacher strongly disagreed that it is helpful to restate the math explanations given by ELs during class discussions. This teacher agreed that language demands only occur in word problems. These two responses do not align with an asset-based view while the 17 other responses from this individual did align with an asset-based view. The other teacher from this category responded to 16 statements in ways that reflected an asset-based view, three statements that did not reflect an asset-based view, and four statements with neutral responses. This teacher disagreed that teachers and schools need to learn about the math practices from students' homes and communities. They agreed that math work should be graded the same way and that switching between languages demonstrates a lack of language fluency. These three beliefs do not reflect an asset-based view, although this teachers' responses on 16 other items did reflect an asset-based view.

Mostly Asset-Based Views Group (60%)

This was the largest category with 12 (60%) of the teachers fitting into this group. This group included teachers that responded with 90-95% of their non-neutral scores reflecting asset-based views of their students. The teachers in this group had consensus around beliefs including: teaching should include attention to math and language, native language use in the classroom is acceptable, teaching should not focus on basic computations more than problem-solving, the different ways ELs learned math in their home is valuable, and when ELs code-switch it does not show a lack of mathematical understanding. In addition to the responses that reflected asset-based views of their students, each of these teachers held some beliefs that did not reflect an asset-based views. Three of the 12 participants in this group responded that the math work of ELs and monolingual students should be graded the same. Two identified that math should be taught by an ESL/ELD teacher. Two disagreed that it's helpful to restate math explanations from ELs. Three identified that switching between languages reflects a lack of language fluency. And two responded that they should focus on the language in ELs' explanations instead of the math.

All Asset-Based Views Group (15%)

This group included three teachers. These teachers responded to the statements with either neutral or responses that reflected asset-based views for every single statement. One of the three teachers responded to 24 out of the 24 questions all with responses that reflected an asset-based view. One of the teachers responded to 19 of the statements in ways that align with asset-based views, and responses to four of the statements with a neutral answer. The last person in this group responded to 15 of the questions in ways that reflected asset-based views and responded to five in ways that reflected neutrality.

Students' Assets in the Mathematics Classroom (Survey and Interview Responses)

In the following section, I discuss the ways teachers talked about their beliefs related to students' backgrounds and experiences, students' everyday and home language, mathematics vocabulary, and supporting EBs (Table 3). The first two categories (students' backgrounds/experiences and students' everyday/home language) came directly from the frameworks I used for this study (language orientations and MMKB). The other two categories emerged directly form the data. In this section I draw on some survey data, but primarily discuss the ways teachers talked about these beliefs during their interviews. I only interviewed a subset of the teachers from the larger participant group so these descriptions may not reflect the beliefs from the larger group of teachers in this study.

Table 3

Teachers Professed Beliefs During Interviews

	Students' Backgrounds	Students' Language	Mathematics Vocabulary	Supports for EBs
Ms. C	Games, afterschool clubs, student names	Demonstration and using everyday language to support learning	Embedded and using everyday language to support "math words"	Access to the same content at peers
Ms. G	Fractions in cooking, pets, student names. "Materials reflect experiences my kids have"	Demonstration and using everyday language to support learning	Embedded and using everyday language to support "academic language"	Visuals and discussions. Opportunities to make sense.
Mr. J	Technology	Demonstration	Embedded	Visuals, time with student teachers, and purposeful seating.
Ms. L	Sports, video games, home dept, building, bowling	Demonstration	Embedded	Visuals and concrete materials
Ms. R	Scavenger hunts on zoom, money	Demonstration	Embedded	Visuals and concrete materials

When I took a more in-depth look at these five teachers' interview responses, I found that their responses mostly aligned with the categories I used to describe the survey responses (e.g., Ms. C was in the group with All Asset-

Based Views). However, the responses from one participant, Ms. G, did not fit into these original categories. Ms. G was a part of the group that responded to 75% of the survey items in ways that reflected an asset-based view (Some Asset-Based Views Group). However, when I interviewed her, she explicitly talked about drawing on students' assets to support mathematics and language learning. In the survey Ms. G disagreed with the two statements that teachers need to learn about the mathematical practices from students' homes and communities and that the ways student learned math in their homes was a valuable resource for learning mathematics in the classroom. However, in her interview she talked about having her students interview their parents to "see how they use math in their life." From the interview, she found out that one of the parents was a construction worker who frequently built things. Ms. G then talked about how she used this information and an important practice (measure twice and cut once) during her unit on measurement. Ms. G even mentioned, "I want the materials that my kids have to reflect the experiences my kids have as well as a window to the world." So, despite her response on the survey that seemed to reflect a deficit view, her responses during her interview showcased how Ms. G elicited information from her students and their families and intentionally used those practices during her mathematics instruction. Similarly, Ms. G disagreed that she should rewrite story problems to help ELs understand when she took the survey, yet her interview responses highlight that she did adjust word problems. In her interview, she said that she frequently "rewrites story problems…[and] support[s] with pictures" so that ELs could relate and understand the question. The case of Ms. G illustrates the complexities of identifying asset-based views and beliefs. Beyond this, the responses from Ms. G reveal the limited, and potentially misleading, interpretations of survey data without any other source to confirm or complicate findings.

Beliefs about Students' Backgrounds and Experiences

There was strong consensus among the participants (19/20 agreed/strongly agreed) that students' backgrounds, experiences, and the ways they learned mathematics in their homes were valuable for mathematics learning. All five participants who were interviewed identified some connections they make to students' backgrounds and experience by using this information to contextualize mathematics problems and make them more relatable. These connections are aligned with the "initial practices" from the MMKB framework because these teachers are acknowledging that mathematics should be relatable, yet these connections are somewhat surface-level as they do not necessarily support mathematics learning.

In line with the category from MMKB of "making meaningful connections," two of the participants discussed ways they uncovered the mathematical practices that their students' saw and did in their homes and talked about using these practices in instruction. Ms. G stated repeatedly, "I want the materials that my kids have to reflect the experiences my kids have as well as a window to the world." She went on to explain that she had her students interview their parents about the mathematics they do at home. From this interview, Ms. G learned that "a lot of people use fractions in cooking. Parents talked about how they use math in their work but I don't know how much they can relate to accounting. I think there was a builder or two in there so that is like a gold mine right there. So, we talked about measure twice and cut once." Ms. G talked about using these connections in her mathematics instruction to the students' experiences in their homes, in particular the mathematical practices that came from a community of builders.

In a similar fashion, Ms. L talked about "changing every lesson there is" by highlighting critical features and routines while making the context relevant and accessible. She drew on a student's experiences and mathematical practices outside of school when she connected to a sport they played, such as bowling. She identified that one of her students used math as they kept score. While she didn't explicitly discuss how she brought this into her instruction, she talked about bringing other experiences from students' homes into her instruction, such as student's trips to Home Depot and building with family members. Connections like these, that reflect specific experiences from students, bridge the mathematical practices from students' homes with their experiences learning mathematics in the classroom and can broaden access to mathematics content (Turner et al., 2016).

Beliefs about Students' Home and Everyday Language

There was consensus among the participants (16/20) that students could use their "primary language" and "everyday ways of talking" during math. Beyond this there was complete consensus (20/20) that switching between languages did not show lack of mathematical understanding. These responses reflect that many of the participants in the sample held beliefs that EBs' primary language and everyday ways of talking about math should be allowed, if not encouraged in a mathematics classroom. This is an important finding because it aligns with recommendations from the research that shows when ELs can use their home and everyday language to participate in mathematics, they have more opportunities to learn mathematics with understanding (e.g., Brenner, 1998; Hicks, 1995; Michaels, 1981; Turner & Celedón-Pattichis, 2011)

From the interviews, the ways teachers talked about students' home and everyday language focused mainly on students' demonstration of understanding. All the participants who were interviewed (n=5) shared that students could demonstrate their understanding of mathematics using home and everyday language. Mr. J said, "any way they can demonstrate it they are accepted." Similarly, Ms. R shared, "they have to explain it, they can show me if they have to." These statements and the responses on the survey revealed that these teachers held beliefs that students can demonstrate their understanding of mathematics with the language resources they have (e.g., everyday language, home language, gestures, drawings).

Two of the teachers also discussed the role of students' home and everyday language as a resource for learning. Ms. G's response illustrates this belief very clearly as she described how she treated students' home and everyday language as both a resource for demonstrating what students know but also as a resource for learning language and content:

> It seems like the latest and greatest thinking is that you're accepting kids' natural language and however they try to describe things but you're also providing structured activities in which to move that forward. So, you are revoicing what you hear kids saying to support taking on that academic language. (Ms. G's interview transcript)

Beliefs about students' home and everyday languages have the potential to impact how teachers position students and their contributions in the classroom. The beliefs that the teachers in this study discussed related to language are important as they align with as asset-based view of their students. Beyond this, these teachers' responses highlighted two features of beliefs about students' home language and everyday language. First, that demonstration of understanding should go beyond "academic vocabulary" (Ms. G) and second that students' home and everyday language should be used to support learning of mathematics (Ms. C) and content language (Ms. G).

Beliefs about Mathematics Vocabulary

A theme that emerged directly from the data related to teachers' beliefs about vocabulary. Participants were specifically asked to respond to two survey questions about mathematics vocabulary which included: 1) "When teaching ELs, I should use a variety of math vocabulary," and 2) "Students should learn math vocabulary before the learn math concepts." More than half of the participants identified that they should teach a variety of math vocabulary (n=11), and more than half (n=11) either disagreed or strongly disagreed that they should teach math vocabulary before teaching math concepts.

Instead, during the interview teachers talked about embedding vocabulary into their mathematics lessons and teaching it alongside mathematics content. Mr. J described, "terms are referred to over and over. They are hearing them over and over every day." In an example of this, Mr. J referred to a science lesson, "Now we are going to move onto dirt. What's your hypothesis? That is just a fancy word for guess." Mr. J shared that he used the words he wanted his students to learn by integrating them into the lessons and what he said. In a similar way, Ms. L shared, "it gets embedded in what I am doing all the time. In small groups, I get everyone talking about, 'what does that mean?'" This practice of keeping vocabulary embedded within activities is important as it maintains the interconnectedness of mathematical concepts and vocabulary and it can create more opportunities for EBs to make meaning for mathematics vocabulary (Moschkovich, 2013; Moschkovich, 2015).

Related to students' everyday and home languages, Ms. C and Ms. G also talked about the role of students as they learned mathematics vocabulary. In addition to embedding mathematics vocabulary into lessons, they described how students' everyday and home language should be guided with "structured activities in which to move that forward" (Ms. G's interview transcript). Ms. G talked about supporting a progression towards "academic language" by drawing on students' ways of communicating. Ms. C also talked about students using their everyday and home language to make meaning for "math words." Both Ms. C and Ms. G talked about revoicing students' contributions to highlight connections to vocabulary.

Beliefs About Supporting EBs

Another category that emerged from the interview data related to teachers' beliefs about supporting EBs. Four of the five teachers who were interviewed talked about using visuals to support EBs during mathematics instruction. Two of the teachers also discussed bringing in concrete objects to support mathematics learning. Beyond this, one teacher talked about intentionally grouping the EBs in their classroom to support engagement. For example, Mr. J said, "sitting them next to a student who talks but will not overshadow them.” In this statement, he acknowledged that EBs should both have models of English speakers but also have opportunities to participate. Another teacher, Ms. G explicitly shared a strategy, the three-reads strategy, that she used with EBs when they were working on word problems. She said, “the three reads. Our first read is making sense of the situation and talking about it.” Ms. G then went onto explain that the other two reads included identifying what was being asked and then figuring out what they needed to do to solve the problem. The three-reads approach is an effective approach to supporting EBs engagement with word problems. One teacher, Ms. C, had a slightly different response. Instead of offering specific strategies she used for supporting EBs, she said she did the same things with all her students. Ms. C responded to the question about how she supported EBs in her classroom with:

> “I feel like they are all learning English. I don’t really do anything different for my ELs versus my other students. Because they are so young, they all need. So, when we use sentence frames, we all use sentence frames. We are all equal here.”

Unlike the other teachers who very clearly identified the differentiated support they provide for EBs in their class, Ms. C said that she provided the same support to all her students.

Description of Ms. C’s Beliefs using Survey and Interview Data

In the following section, I overview Ms. C's professed beliefs that she reported in her interviews and on her survey. I focus on Ms. C in this section as she was the focal teacher for the analyses of teaching practices (Chapters 3 and 4). First, I describe the beliefs Ms. C held about mathematics, language, and EBs. I then describe how Ms. C's beliefs relate to the "belief constructs" framework (Schoen & LaVenia, 2019). Ms. C held low "transmissionist", low "facts first", and low "fixed instructional plan" beliefs. These beliefs align with beliefs that other elementary teachers hold about mathematics instruction that are grounded in the CGI approach (Schoen & LaVenia, 2019). In chapters 3 and 4, I describe alignment between Ms. C's teaching practices and her professed beliefs.

Ms. C's Beliefs about Students' Assets

Ms. C responded to the survey with 100% of her non-neutral responses reflecting asset-based views of her students. This characterization comes from both the language orientations framework (Fernandes, 2020) and the MMKB framework (Aguirre et al., 2012; Turner et al., 2012). These responses aligned with the group of "all asset based-views." Looking more closely at Ms. C's interview responses, her descriptions of her beliefs highlight different ways she drew on students' assets. In this section, I summarize the beliefs that Ms. C held related to students' backgrounds, students' language, mathematics vocabulary, and supporting EBs (see Table 4).

Table 4

Ms. C's Beliefs about Supporting EBs

Students' Backgrounds (MMKB)	Students Language (Language Orientations)	Mathematics Vocabulary (From Interview and Survey Data)	Supports for EBs (From Interview Data)
Games, afterschool clubs, student names	Demonstration and using everyday language to support learning	Embedded and using everyday language to support "math words"	Access to the same content as peers, opportunities to talk

Related to students' backgrounds, Ms. C talked about the importance of bringing in students' backgrounds and experiences. She discussed bringing in students' experiences by contextualizing mathematics problems with familiar activities. Another way Ms. C reported that she drew on students' experiences was by bringing in playful activities during mathematics instruction, typically, through games. As evidenced through her discussion of student experiences, Ms. C reported a belief that student experiences were allowed and useful during mathematics, but not the focus. This aligns with the analysis category from MMKB "allowing students assets" as the focus she reported was on bringing in these experiences, but not necessarily using them to support mathematics learning. Research has found that when teachers fully incorporate students' experiences from outside the classroom they focus on the mathematical practices that students participate in and bring these practices into the classroom (Turner et al., 2012; Turner et al., 2016). This was not how Ms. C talked about (or was observed bringing in student experiences, see Chapter 3 for that analysis), rather

her reported use of student experiences more closely reflected surface-level applications.

Related to students’ language, Ms. C's survey responses reflected a belief related to drawing on students’ assets to support mathematics learning. Ms. C identified in her survey that teachers should 1) provide EBs opportunities to participate and talk, and 2) focus on math, rather than correct language, from EBs' math explanations. Ms. C talked in her interview about both allowing her students to use “whatever way they could” to demonstrate that they understood a concept, but she also talked about the importance of students using their everyday language to make meaning of “math words”. Ms. C's interview and survey responses reflected a belief related to students’ language that aligned with a “limited use of language” (Fernandes, 2020). Like the limited use of language orientation, Ms. C reported that she invited her students to use any language in the classroom, although she talked about wanting her students to learn English and mathematics vocabulary. In the interview, she talked about allowing and encouraging students to use their everyday and home language, but maintaining these practices was not evident in the way she discussed students’ everyday and native language.

In the interview, Ms. C professed a belief that mathematics vocabulary should be embedded into mathematics instruction. During the interview, Ms. C made connections between students’ everyday language and vocabulary and she talked about students using their everyday and home language to make meaning for “math words.” Ms. C also discussed (and demonstrated in her teaching, see Chapter 3) revoicing students’ contributions to highlight connections to vocabulary. Revoicing is one effective approach to supporting students’ mathematics learning (de Araujo et al., 2018; Moschkovich, 2015) and it provides opportunities for students to hear and use more precise mathematical language and can support participation in mathematical discussions (Moschkovich, 2015b).

Related to supporting her EB students, in the interview Ms. C reported a belief that she should treat all her students the same as they were all learning English. Ms. C said, “I feel like they are all learning English. I don’t really do anything different for my ELs versus my other students. Because they are so young, they all need. So, when we use sentence frames, we all use sentence frames. We are all equal here.” In this statement, Ms. C was saying that she doesn’t believe in treating her EBs differently as they also received good instruction. However, recommendations for equitable mathematics instruction with EBs include considering the histories and backgrounds of EBs (Moschkovich, 2013), and it would likely be difficult for Ms. C, a monolingual, presumably white, middle-class woman, to have a deep understanding of her students' histories and background without getting to know them individually and specifically eliciting this information from her EB students[10].

Ms. C's Beliefs about Mathematics: Relationship to Belief Constructs

In addition to looking at Ms. C's beliefs related to language and mathematics with EBs, I also characterized Ms. C's professed beliefs about mathematics, students' thinking, and teaching by situating her beliefs using the belief constructs framework (Schoen & LaVennia, 2019). Based on Ms. C's interview and survey responses, I found that Ms. C held low "transmissionist," low "facts first," and low "fixed instructional plan" beliefs. I summarize these professed beliefs in the following section.

Related to the "transmissionist" scale, during the interview Ms. C talked about letting her students solve problems and share their solutions to support their learning. This reflects low "transmissionist" beliefs. Beyond this, Ms. C also held low "facts first" beliefs. Ms. C said that she often started her lessons with a discussion, "pair, share," or a "number talk." Finally, related to "fixed instructional plan" beliefs, Ms. C talked about using the curriculum as a guide and discussed supplemental activities she brought into the classroom, like games. In her interview, Ms. C shared that she would use the homework page,

[10] While I cannot make claims about Ms. C's interpretation of "I don't do anything different" my conjecture is that she was well-intentioned in this statement.

which offered more practice, only when she noticed that her students needed extra practice. She said she often skipped the homework pages in the student handbook as they were repetitive. She told me during an initial interview before any observations, “if they know it, they don’t need to do it.” In her initial interview, Ms. C talked about using the students’ demonstration of a concept to guide her planning instead of the scope and sequencing of the curriculum materials. She said, “I go off of the previous days lesson. Sometimes if I know I did one lesson, and it was hard I will do a repeat or reteach it. I can pull a small group to the back too.” Ms. C talked about the curriculum as a guide to the concepts she needed to teach, but she used her students to guide how and when she would revisit or move ahead in the materials.

Ms. C’s low “transmissionist,” “facts first,” and “fixed instructional plan” beliefs align with the typical, CGI-aligned beliefs elementary teachers hold about mathematics instruction. In Chapter 3, I discuss Ms. C’s teaching practices and consider how they align with her professed beliefs that I described in this chapter. In chapter 4, I show how Ms. C’s teaching practices related to following the scope and sequencing of the curriculum materials looked different when teaching online highlighting how her teaching online conflicted with her belief related to a “fixed instructional plan.”

Teachers’ Descriptions of Students’ Assets: Allowing vs. Drawing on Assets for Mathematics

In this study of teachers' beliefs, I found that teachers held varying degrees of asset-based views of their EBs and described beliefs specific to students' backgrounds and experiences, students' everyday and home languages, mathematics vocabulary, and supporting EBs. In this section, I synthesize these findings and highlight how these teachers described their beliefs related to using students' assets in mathematics instruction in two ways: 1) allowing students' assets in the classroom and 2) drawing on students' assets for mathematics learning.

I developed these two categories in part from the MMKB framework. The MMKB framework (Aguirre et al., 2012; Turner et al., 2012; Turner et al., 2016) identified how pre-service teachers talked about using students' thinking and experiences in their lesson plans. My first category, allowing students assets in the classroom, is in part related to the "initial practices" category of the MMKB framework. The teachers in my study talked about using students' experiences as context for mathematics problems and that students could use home and everyday language to demonstrate that they understood mathematics. These beliefs acknowledge that students' experiences and language are valuable and important, but not necessarily a central part of learning mathematics. My second category, drawing on students' assets for mathematics learning, in many ways reflects the "meaningful connections" and "incorporating" categories from the MMKB framework. This category more closely aligns with beliefs that math and language practices should be used to support mathematics learning that reflects what students do in their homes and communities.

Allowing Students' Assets in the Classroom

Consistently across the five teachers who were interviewed, descriptions about students' assets revealed that they all allowed students' assets in the classroom. For example, they all discussed ways that they would allow students to demonstrate their understanding or share an answer using any way they could. All the teachers also talked about using students' backgrounds and experiences as context for mathematics problems. Two of the five teachers did not provide explicit details of using the mathematical practices from the students' lives or drawing on students' linguistic resources to support learning. One teacher talked about using students' names and another teacher said that their students probably used technology in their homes. Previous work has characterized ways various types of everyday mathematical practices can be used in mathematics instruction including using context that is 1) based on assumptions, 2) reflects layering or mathematizing, or 3) uncovering mathematical activities (Aguirre et al., 2012; Turner et al., 2012; Wager; 2012). Some of the ways these teachers talked about contextualizing problems reflected surface-level uses of information, some of which were likely based on assumptions about students rather than informed by information that was gathered from them. While these teachers talked about students' using their assets in the classroom, there were no clear connections to content or learning for three of the five teachers. These responses in some ways reflect asset-based views of their students and can potentially act as an entry point for teachers to more fully incorporate students' assets into mathematics instruction. For example, acknowledging that eliciting and attending to students' experiences outside of the classroom is a part of leveraging students' experiences in meaningful ways. Eliciting and attending to may act as an entry point for teachers to start to make these meaningful connections (Turner et al., 2012; Turner et al., 2016).

Drawing on Students' Assets for Mathematics Learning

Three of the five teachers (Ms. G, Ms. L, and Ms. C) discussed drawing on students' assets for learning mathematics and mathematics vocabulary beyond just allowing students' assets in the classroom. Related to students' background and experiences, Ms. G and Ms. L made clear connections to the student's home life as they talked specifically about the mathematical practices from students' homes (measurement in building and keeping track of score in bowling) and highlighted ways they used these practices in instruction (a unit on measurement and during discussions). When teachers identify the mathematics that students use at home, such as keeping track of allowance or keeping score in bowling, and use this information during instruction they connect to ways students use mathematics outside of school (Turner et al., 2016). Ms. G and Ms. L made connections to mathematical practices that their students saw or used at home and talked about how they brought these practices into their instruction. This aligns with the third category of using MMKB, uncovering mathematical activities. For EBs, this approach to teaching mathematics can broaden access to mathematical content by making it familiar and positioning student' activities from their homes as a valuable resource for learning.

Related to students' home and everyday language, Ms. G and Ms. C talked about the ways they encouraged students to learn both mathematics content and vocabulary by drawing on students' linguistic resources. Both teachers talked about the practice of revoicing to support students as they learned "math words" (Ms. C) and "academic language" (Ms. G). Revoicing is one effective approach to supporting students' mathematics learning (de Araujo et al., 2018; Moschkovich, 2015b) as it provides opportunities for students to hear and use more precise mathematical language and can support participation in mathematical discussions (Moschkovich, 2015b) . When EBs have access to all their linguistic resources in the classroom they have more opportunities to make meaning for mathematics content and language. In one study comparing two groups, the students from classrooms where home languages were accepted and encouraged outperformed students in classrooms where home languages were not accessible for classroom learning (Turner & Celedón-Pattichis, 2011). The importance of EBs having access to all their linguistic resources is frequently documented in the literature for student success (e.g., Brenner, 1998;

de Araujo et al., 2018; Turner & Celedón-Pattichis, 2011; Moschkovich, 2013; Moschkovich, 2015a).

Conclusion

This study explored early grades teachers' professed beliefs about mathematics, student thinking, and students' early, out of school experiences with mathematics, particularly for EBs. The teachers in this study displayed varying degrees of asset-based responses (74%-100%) to the survey and discussed beliefs related to 1) students' backgrounds and experiences, 2) students everyday and home languages, 3) mathematics vocabulary, and 4) supporting EBs. During the interviews, teachers described their beliefs about students' assets (experiences and home/everyday language) in ways that aligned with either allowing students' assets in the classroom or drawing on students' assets to support mathematics learning.

This study corroborates and extends previous research (e.g., Fernandes, 2020; Turner et al., 2016). The research on teachers' use of students' MMKB (Aguirre et al., 2012; Turner et al., 2012; Turner et al., 2016) has highlighted characteristics of drawing on MMKB (students' thinking and mathematical experiences outside of school). The existing research on MMKB primarily focuses on how MMKB is integrated into instruction (Aguirre et al., 2012; Turner et al., 2012; Turner et al., 2016) and ways that curriculum creates "spaces" to draw on students MMKB (Land et al., 2018). This study adds to that research by providing examples of how teachers talk about using students' linguistic resources in addition to their experiences outside of school.

The findings specific to teachers' descriptions of allowing or drawing on students' home and everyday languages in the classroom (to demonstrate understanding or as a resource to make meaning) corroborates other findings related to teachers' beliefs. I compared the findings from my survey to those reported by Fernandes (2020). Like Fernandes' (2020) findings, most of his participants fit within the two middle categories, which reflected a combination of seeing students' language as a problem and seeing it as a resource. In my sample, 85% of the participants fit within these "blended" categories which is similar to his finding of 80% of his participants holding views with blended beliefs.

Differing from his study, I did not have any teachers that responded in ways that reflected a "no native language" orientation. Fernandes (2020) found that 7% of his participants fell into this group that held beliefs that native language should not be used in the classroom for mathematics learning. In my sample and in the sample from Fernandes (2020) 15% of the teachers demonstrated views that align with a bilingual orientation. As discussed in the "drawing on students' assets for mathematics learning" category in this analysis, the teachers in this group discussed using the experiences and language of students to support mathematics content and language. The teachers who talked about students using their home language to make meaning for mathematics and for “math talk” or “academic language” also expressed beliefs that all linguistic resources should be used to learn. My findings corroborate the work by Fernandes (2020) in that the teachers' beliefs discussed in this chapter also fall within a continuum in the ways they view students’ language in a mathematics classroom.

Fernandes (2020) conducted his study with pre-service teachers. My study extends this work as it was done with veteran teachers. This analysis also extends the previous work as it looks at teachers’ beliefs about assets more generally. This includes teachers’ beliefs about native language in addition to other assets that can be used to support mathematics learning, such as everyday language and experiences outside the classroom. Beyond this, the survey and interview responses from Ms. G differed which suggests that interviews can be used to clarify and confirm survey responses.

This analysis is limited by the small sample size (n=20), and these findings cannot be generalized to the larger teacher population. For this dissertation, the analysis of the survey and interview data in this chapter sets up and frames a closer focal study of one teacher, Ms. C, in the following two chapters. In Chapters 3 and 4, I use the analysis of Ms. C’s beliefs from this chapter to explore if and how her beliefs align with her teaching practices.

Chapter 3: An Account of an Accomplished Teachers' In-Person Mathematics Instruction in a First-Grade Classroom: Drawing on Students' Assets[11]

Children's early mathematics education is vitally important for success in schooling. The National Council of Teachers of Mathematics (NCTM) and the National Association for the Education of Young Children (NAEYC) gave a collaborative position on early mathematics education that stresses the importance of high-quality, accessible, and challenging early mathematics education for all children to create a strong foundation for future learning (NAEYC & NCTM, 2002; NAEYC & NCTM, 2010). This position is supported by longitudinal data showing that preschool mathematics achievement is a strong predictor of later academic success in both mathematics and literacy (Duncan et al., 2007). At the same time, when children don't have opportunities to participate in mathematics early on, they often do not catch up (Schoenfeld & Stipek, 2011).

There is also a strong need to support and engage all students in early grades mathematics regardless of their language classification or family background. Early childhood educators (ECEs), defined here as those who work with children ages 0-8, are more frequently being asked to support learning of

[11] Originally this was Paper #2

mathematics with understanding for all their students. Teachers, in particular ECEs, are underprepared and unsupported (de Araujo et al., 2018) to teach math in a way that draws on a variety of students' assets (student thinking, linguistic resources, and everyday experiences) especially when students come from homes and communities that differ from their own (Turner et al., 2012; Turner et al., 2016). To address this challenge and contribute to the growing body of literature on teaching mathematics by drawing on students' assets in early grades classrooms, I explore the mathematics instruction of one accomplished teacher in a first-grade classroom. This study of Ms. C's teaching practices provides a detailed example of an accomplished teacher's instruction that draws on students' assets. This chapter contributes to the research literature by describing the complexities of teaching mathematics and complicating the trajectory of teaching models. The analysis also contributes to practice; these vignettes can be used to ground discussions of teaching in real classroom cases, which is a long-term practice in mathematics teacher education and professional development (e.g., Ambrose, 2004; Ambrose, Clements & Philipp, 2004; Philipp et al., 2007). The research questions for this study were: 1) what was the nature of mathematics instruction in a first-grade classroom with an accomplished teacher? and 2) how did an accomplished teacher draw on students' assets (student thinking and experiences)?

Review of the Literature

This study is informed by research that shows that effective mathematics instruction for a diverse group of learners includes instruction that is equitable (Gutiérrez, 2009; Moschkovich, 2013), focuses on understanding (Hiebert, 1990; Hiebert & Carpenter, 1992; Kilpatrick, Swafford & Findell, 2002), centers on student thinking (Carpenter et al. 1993; Fennema et al., 1996), includes various playful, hands-on, and meaningful activities (Fuson, 1988; Fuson, 1991; Fuson, 2009; Wager, 2013), and leverages students' Funds of Knowledge (Civil, 2002; González, Andrade, Civil, & Moll, 2001; Turner et al., 2016).

Previous work has explored the ways pre-service teachers draw on students' Multiple Mathematical Knowledge Bases (MMKB), both students' thinking and mathematical experiences outside of school, in ways that create opportunities for instruction to reflect the recommendations above (Aguirre et al., 2012; Turner et al., 2012; Turner et al., 2012). When teachers draw on children's MMKB they can provide access to the mathematics content while also positioning the children as mathematical thinkers and learners (Turner et al, 2016). Teachers may attempt to connect mathematics to something familiar, yet these connections may or may not actually be relevant for a specific child. For example, when teachers make assumptions about familiar objects or even if they draw on the interests of their students, the mathematics may resemble word problems typically seen in textbooks (Turner, et al., 2016). In this way, drawing on children's MMKB may be interpreted and employed in shallow or superficial ways. Mathematics instruction may reflect assumptions about the mathematical practices in children's homes rather than the actual practices in which children participate. Similarly, misinterpretations of leveraging MMKB may result in teachers feeling pressured to get to know each student's mathematical practices (through interviews or home visits).

Teachers need to know their students and take the time to uncover the mathematics they are already doing, but this does not necessitate teachers doing home visits and interviews with every student, every year. Instead, as teachers start to draw on their students' MMKB, they may come to recognize the importance of connecting mathematics to real-life content and connect to MMKB in more meaningful ways. Turner et al., (2016) argue that "posing these kinds of problems [that draw on MMKB], even if they do not always mirror the specific ways that children and families engage in mathematics reasoning outside of school, opens a space for children to talk about their out-of-school mathematics practices" (p. 68). This creates an entry point for children. Using these familiar contexts may require less effort on the part of the child and they can focus more on mathematics (Turner et al., 2016). When teachers mobilize children's MMKB in their instruction, they support mathematics learning for all of their students by drawing on the assets students bring with them into the classroom. As teachers draw on children's mathematical practices from their homes and communities, these practices then become a part of school mathematics, and this changes how mathematics looks in the classroom. Bringing in the mathematics of the children and their communities is one concrete way to shift "what counts" as mathematics in the classroom by acknowledging aspects of identity and power (Gutiérrez, 2009). Early grade teaching that aligns with this research positions all students as thinkers and doers of mathematics and frames the ideas and practices children bring to the classroom as resources for mathematics learning, rather than something to overcome.

Framework

A situated, sociocultural perspective (Lave & Wenger, 1991; Vygotsky, 1978) frames this study; in this perspective, learning and teaching are social practices. This perspective highlights the impact of the social context on how teachers enact mathematical practices, like teaching measuring or counting (Parks & Bridges-Rhodes, 2012). A sociocultural perspective also acknowledges the role of cultural tools that teachers take up and use to support learning (curriculum materials, manipulatives, teacher moves, student thinking).

The sociocultural perspective frames my work through the assumptions I have and the data I collected. I looked at interactions in the classroom. This includes the interactions the teacher had with her students, the interactions the teacher had with the curriculum, and the ways students interacted with the teacher and mathematics. A key feature of the sociocultural perspective that frames this study, is the view that curriculum is enacted, in contrast to the notion of curriculum as only predefined and fixed. Remillard and Heck (2014) define enacted curriculum as "the interactions between teachers and students around the tasks of each lesson and accumulated lessons in a unit of instruction, is analogous to the performance of play, complete with the idiosyncrasies and unpredictable elements of live performance" (p. 713). There are four dimensions of enacted curriculum: 1) mathematics, 2) instructional interactions and norms, 3) teacher's pedagogical moves, and 4) the use of resources and tools (Remillard & Heck, 2014). The enacted curriculum is part of the operational curriculum (teachers' plans and actions to carry out instruction), rather than the official curriculum (Remillard & Heck, 2014).

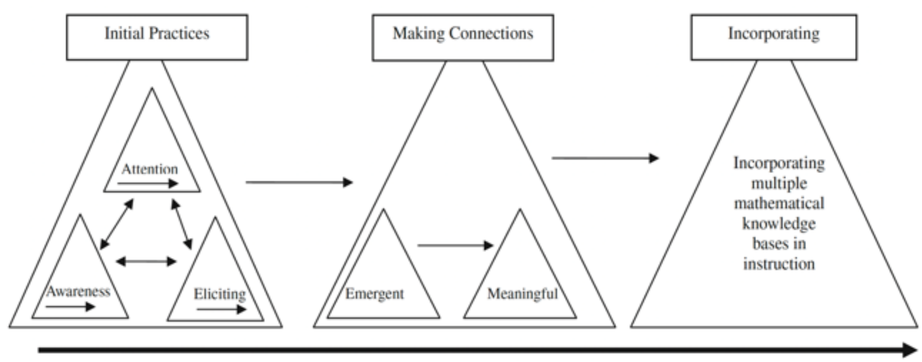
Enacted curriculum highlights the various systems that impact teaching and learning in the classroom while bringing the relationships between these systems to the forefront. To frame enacted curriculum, I used MMKB as a lens to look at Ms. C's mathematics instruction. MMKB includes students' "multiple understandings and experiences that have the potential to shape and support students' mathematics learning" (Turner et al., 2016, p. 49). The MMKB framework brings together work on student thinking and student experiences outside of school.

Drawing on children's mathematical thinking (the first part of the MMKB construct) to support learning largely comes out of the work around

Cognitively Guided Instruction (CGI) (Carpenter et al., 1983; Carpenter et al., 1996; Fennema et al., 1996). The second part of the MMKB construct draws on the literature on children's mathematical Funds of Knowledge (Civil, 2007; González, Andrade, Civil, & Moll, 2001) which asserts that mathematics instruction that reflects students' social worlds is more accessible to a diverse group of students. The framework includes three phases that reflect different ways that teachers draw on students' MMKB (Figure 1). These phases include 1) initial practices, 2) making connections, and 3) incorporating.

Figure 1

MMKB Learning Trajectory (Aguirre et al., 2012; Turner et al., 2012)



As teachers start to employ initial practices (Turner et al., 2012), such as eliciting children's mathematical thinking and their mathematical experiences outside of the classroom, their mathematics instruction will start to reflect an attempt to connect to children's practices. These attempts to connect to children's interests are important and reveal a shift towards asset-based views of children (Turner, et al., 2016). As teachers develop an understanding of children's MMKB, they often start to make emergent and then meaningful connections between students' MMKB and instruction (Turner et al., 2012). Turner et al. (2016) and Wager (2011) similarly found that teachers can meaningfully draw on mathematical activities that children engage in at home and in their communities. Engaging students in activities that resemble the embedded activities of children's homes takes time to get to know the students and the ways in which they use mathematics in their everyday lives. By mathematizing family practices and drawing on the mathematical activities children use at home, teachers are fully incorporating MMKB into mathematics instruction.

This model has been used to describe teacher change. For this study, I used MMKB to design my study by paying attention to how Ms. C drew on student thinking and experiences. I also used the MMKB framework when I looked at the data, which I describe more fully in the data analysis section of this paper. I did not use this framework to look at teacher change, which is how it was used previously (Turner et al., 2012; Turner et al., 2016), but instead used it as a lens to look at the teaching practices of an accomplished teacher.

Methodology

For this study, I employed an interpretive, ethnographic methodology in a naturalistic setting. In this approach, researchers seek to uncover the complicated nature of phenomena and give voice to the participants (Borko, Liston & Whitcomb, 2007). Starting with the data collection phase and then into the analysis phase, "patterns are developed inductively from the data and deductively from the conceptual framework" (Borko, Liston & Whitcomb, 2007, p. 5). Therefore, for analyzing the data I used both inductive and deductive approaches. Deductively, I drew from the MMKB framework (Aguirre et al., 2012; Turner et al., 2012; Turner et al., 2012) to look for teaching practices that leveraged students' thinking (e.g., Carpenter et al., 1993; Fennema et al., 1996) and students' "Funds of Knowledge" (Civil, 2002; González, Andrade, Civil, & Moll, 2001; Turner et al., 2016). The way I looked at students' "Funds of Knowledge" was similar to how this construct was conceptualized as a part of the MMKB framework. Turner et al. (2012) and Aguirre et al. (2012) looked at both teachers' use of student thinking and students' experiences and interests that could support engagement with mathematics as a part of their MMKB framework. Similarly, to identify students' "Funds of Knowledge" I did not look inside the homes and communities of the students, rather I looked at the ways a first-grade teacher elicited information from her students and included connections to the experiences students had outside of the classroom related to mathematics. I also inductively uncovered teaching practices that were related to but did not fit within the MMKB framework (i.e., establishing classroom norms and creating opportunities to develop conceptual understanding). I used the lens of MMKB to look at my data, to inform how I did my observations, to guide the questions I asked, but I also stepped back from this lens to document the interactions in the classroom that did not neatly fit within the MMKB framework.

Participants and Setting

This study focused on the mathematics instruction of one accomplished early grades teacher, Ms. C, in a first-grade classroom. Ms. C's time in the classroom, background in professional development (as both a participant and developer) and pedagogical perspective are the criteria I used to establish Ms. C as an accomplished teacher. As I more thoroughly discussed in the introduction chapter, Ms. C's pedagogical perspective drew on a CGI approach and other aspects such as including playful activities and bringing in her students' interests and experiences outside of the classroom.

In many ways Ms. C's mathematics instruction drew on elements of CGI which was evident in her frequent use of student thinking and solutions throughout activities. However, the structure of her lessons looked different from a typical CGI lesson. Bringing in playful activities is another aspect of Ms. C pedagogical approach that slightly differed from "typical" CGI teaching. Mathematics was framed as playful in Ms. C's classroom, which also connected mathematics to the experiences her students had outside the classroom. Ms. C drew on the experiences her students had outside of the classroom to contextualize mathematics problems.

During the 2019-2020 school year, Ms. C had been teaching in first grade for 13 years, and that was the first year she also had a combination class with kindergarteners. Ms. C identified that she was monolingual and taught in English. I did not collect other demographic information from Ms. C. Ms. C was very experienced with mathematics instruction. She attended professional development related to mathematics for over 10 years (California Math Council Conference, Cognitively Guided Instruction, and the local Mathematics Project) and attended one conference specific to supporting English Learners in math. Beyond this, Ms. C held an active position of leadership in a mathematics professional development program (the Mathematics Project held at a local university). In this role, she attended monthly meetings where she was a co-leader with a group of experienced teachers and university faculty to develop and facilitate professional development sessions every summer.

In 2019-2020, Ms. C taught 23 children in a kindergarten/first-grade combination class with 10 first-graders and 13 kindergarteners. In the K/1 class, Ms. C reported having two students from low-income families, three EBs, one student with extreme anxiety, and students with Latinx backgrounds (although she did not specify the number or other specific information). The focus of this study was mathematics instruction with the 10 first-grade students after the kindergarten students left for the day. Ms. C had scheduled mathematics time for the first-grade students every day in the afternoon. Although math time was planned from 1:30-2:00 pm, lessons often extended well beyond the 30-minute limit.

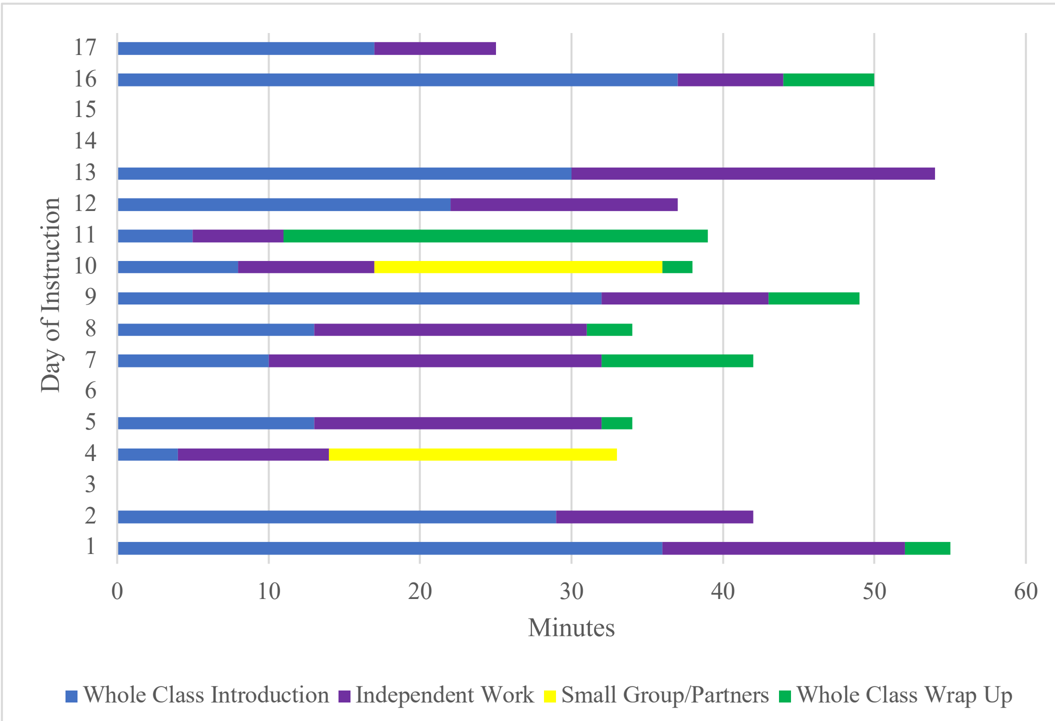
The mathematics unit I observed related to place value, number comparisons, addition, and subtraction. Ms. C used the school's adopted curriculum, Engage NY. From Engage NY, the unit I observed was called Module 4 from the first-grade mathematics curriculum. Ms. C identified that the timing and length of the unit made it an excellent unit for me to observe. In Ms. C's class, this unit lasted for six weeks. In total, mathematics lessons for the unit occurred over 17 days of instruction[12]. I observed 13 of the 17 days of instruction. During the observations, I took detailed notes, including quotes during many parts of the lessons. The daily structure of the lessons was similar each day as Ms. C always introduced each lesson with a whole-class activity that typically included a whole-class discussion with students and her using manipulatives (Figure 2). Students often used unifix cubes, mini whiteboards, base-ten blocks, or small objects for counting collections during the introductory whole-class activities. Following the whole-class introduction, students typically did independent work. However, on two of the 13 days, the students participated in a small group or partner activity. On most days, (8 out of 13) Ms. C

[12] The unit was cut short due to school closures for COVID-19.

concluded her lessons with a whole-class wrap-up where she asked how the activity went or revisited topics that she had observed students struggling with.

Figure 2

Structure of Mathematics Time (Minutes Per Activity)



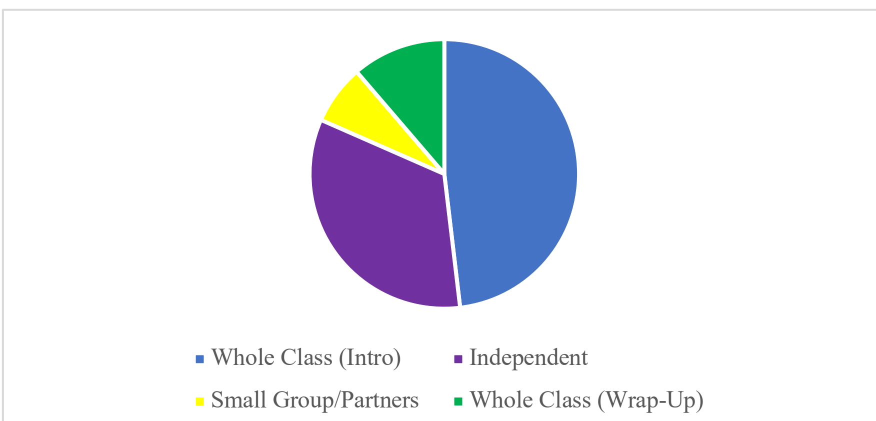
On average, the mathematics lessons lasted 41 minutes, ranging from 25 to 55 minutes. Most of the mathematics lessons were whole-class activities (59%), with the whole-class introduction taking up the greatest time overall (48%) (Figure 3). Students had opportunities to explore activities related to comparing numbers, solving word problems, and decomposing numbers in various settings (whole-class, independent time, and small group). On days 1-4, activities focused on numbers, counting, and grouping quantities of tens and ones. For example, on the first day of the unit, the students started the lesson by counting and organizing groups of objects and then did a gallery walk to look at everyone's work. After a discussion of grouping using "friendly numbers" and how to represent quantities in an efficient way, Ms. C had the students work individually on a worksheet where they circled groups of 10 objects. Ms. C then wrapped up the lesson with another whole-class discussion about grouping and efficiency.

Days 5-9 included activities related to number comparisons with greater than and less than. On the ninth day of the unit, Ms. C worked with the whole class when they discussed greater than, less than, one more, one less, ten more, and ten less using a 100s chart. In the discussion, the students talked about patterns they noticed and how they could solve comparison questions. Ms. C then had the students work on a worksheet individually where they were prompted to compare numbers using the greater than and less than symbols. The students then returned to the carpet for a whole class wrap-up of the lesson where Ms. C asked the students to tell her about successes and challenges with the lesson.

The last seven days of the unit (10-17) included a variety of activities centered on addition and subtraction. For example, on the 16th day Ms. C started the whole-class discussion with a word problem related to addition. She had the students use unifix cubes and whiteboards to solve a problem and then the students and Ms. C discussed their strategies. After this Ms. C had the students work through a small set of word problems individually before they returned to the carpet for another whole-class discussion. During the wrap-up activity, the students returned to their whiteboards and unifix cubes to discuss addition problems with the "result unknown" and talked about strategies for solving these problems using the unifix cubes. Before ending the lesson, Ms. C prompted the students to share their reasoning when they discussed solutions. In later sections of this chapter, I provide more detailed vignettes of selected activities from Ms. C's classroom to illustrate the features of Ms. C's mathematics instruction.

Figure 3

Distributions of Activities (Across the Unit)



Data Collection

The sources of data for this study include Ms. C's interview responses and my in-person, classroom observations across one mathematics unit. Prior to observations, I interviewed Ms. C and asked her to complete a background information form. The interview was semi-structured and asked questions about mathematics, student thinking, and teaching (Appendix A). The goal of the interview was to get a sense of her teaching practices and hear about her instructional decision-making. I also asked her about hypothetical situations to learn about how she approached interactions with students. For example, one question asked, "if a student were to say I am not good at math, how would you respond?" The background information form included questions related to Ms. C's education, teaching history, professional development experiences, and student information (Appendix B).

After the interview, I collected observational data in Ms. C's classroom during mathematics lessons with her first-grade students. Observations took place during scheduled mathematics time across one mathematics unit. This time was designated for mathematics instruction, although Ms. C told me that she often incorporated mathematics into other activities throughout the day (e.g., counting as children lined up, doing the calendar, making graphs and charts). I created and used an observation form to focus my observation notes (Appendix D). In the form, I noted the date, topic, materials used, and the number of students in the class each day. I also listed guiding questions to keep in mind as I took observation notes. Some of the guiding questions included: What is the structure of the lesson? Does the teacher draw on students thinking? How? When? Does the teacher connect to out-of-school math? Is the focus on procedures or conceptual understanding? These questions were informed by the MMKB framework.

Data Analysis

I used a qualitative approach for analysis. Each day I took observational notes, wrote memos, and summarized the lesson in a spreadsheet with information related to the content, teaching, and emergent themes. Some themes were related to the four features of effective mathematics teaching and to the MMKB framework (using teacher moves to elicit student thinking and drawing on students' experiences), while other themes emerged from the data (creating opportunities for conceptual understanding and establishing norms around mathematics and participation). This set of data (notes, memos, and spreadsheet) served as my preliminary data set for the 13 days I observed.

Analysis occurred in four phases for this study: 1) observing, writing memos, and identifying initial codes, 2) reducing the data and writing descriptive narratives of selected lessons, 3) identifying codes for close analysis, and 4) comparing the descriptive narratives and selected lessons for themes and making claims about findings. During phase one I developed the cases where I took observational notes, wrote memos, and wrote initial codes in a spreadsheet. After I collected all the observational data, I started phase two where I transcribed using my observation notes and reduced the data. I reduced the data by focusing only on the themes that related to MMKB as well as codes that were frequent, even if they did not fit with MMKB (e.g., establishing norms). I picked themes that aligned with MMKB and with research on effective mathematics instruction because I wanted to focus on those features of her instruction. After I identified the themes, I wrote detailed summaries of specific lessons, interactions, and activities that illustrated those themes. I intentionally selected lessons and vignettes that related to the themes I pursued to include in my summaries. Then during phase four, I identified all the themes that I planned to pursue (creating opportunities for conceptual understanding, using teacher moves and being responsive, establishing classroom norms around participation and mathematics, and drawing on students' experiences), and then looked across those four themes to make claims about Ms. C's practice. I used the interview responses from Ms. C to clarify and confirm the claims I made about her instruction.

The Nature of Mathematics Instruction

In the following section I highlight four features of Ms. C's instruction that align with recommendations from the research about effective mathematics instruction in early grades (e.g., Aguirre et al., 2012; Carpenter et al., 1993; Carpenter, Fennema & Franke, 1996; Fennema et al., 1996; Perry & Dockett, 2002; Turner et al., 2012; Wager, 2013). These features include: 1) creating opportunities for students to develop conceptual understanding, 2) using teacher moves and being highly responsive to student contributions, 3) establishing participation norms and socio-mathematical norms related to mistakes and efficiency, and 4) drawing on students' experiences outside the classroom. The vignettes of Ms. C's instruction provide detailed examples of an accomplished teacher's practice. These vignettes can be used to ground discussions of teaching in real classroom cases.

The features of instruction illustrated in this section show that the MMKB framework can be used to look at teaching practice (in addition to how it has been previously used to look at teacher change, see Turner et al., 2016). Two of the features (feature two: using teacher moves and being highly responsive, and feature four: drawing on students' interests and experiences) relate specifically to the MMKB framework. The other two features (creating opportunities for conceptual understanding and establishing classroom norms), although related to MMKB, are slightly different from the components outlined as a part of MMKB and emerged from the data on Ms. C's instruction.

Creating Opportunities for Conceptual Understanding

Ms. C created opportunities for the students in her classroom to develop conceptual understanding. One way she did this was by creating many activities where students connected multiple representations (e.g., the 100s chart activity, unifix cubes with word problems, using drawings, counting collections). Creating opportunities to connect multiple representations is a documented strategy for supporting conceptual understanding (Jansen, Gallivan & Miller, 2018; Powell & Nurnberger-Haag, 2015). Research tells us that it is important that mathematics lessons focus on conceptual understanding (Hiebert, 1990; Hiebert & Carpenter, 1992) because without conceptual understanding children will rely on memorized facts and skills which are highly prone to errors. Beyond this, to support students with learning mathematics in early grades, equitable instruction creates opportunities for developing conceptual understanding (Moschkovich, 2013). Moschkovich (2013) asserts, “to support mathematical reasoning, conceptual understanding, and discourse, classroom practices need to provide all students with opportunities to participate in mathematical activities that use multiple resources to do and learn mathematics” (p. 46). The classroom practices that Ms. C used in her instruction created these opportunities for developing conceptual understanding for all the students in her class.

Throughout the unit, Ms. C created opportunities for the students to develop conceptual understanding by incorporating physical and drawn representations for support during her instruction. Every day she included some type of manipulative (e.g., counting collections, unifix cubes, base ten blocks). She also provided many opportunities for students to make connections across multiple representations (e.g., physical objects and the corresponding numeral, using manipulatives to solve word problems). She often used and had the students use manipulatives and drawings to illustrate ideas in activities. The following example showcases Ms. C’s use of physical objects during a counting activity where students were asked to organize various items in a way that could easily be counted by others.

Vignette 1: Connecting Representations with Counting Collections

During the very first activity of the unit, Ms. C had the students play with, count, and organize a group of objects (e.g., paintbrushes, jacks, marbles). As a part of this activity, she asked the students to sort their objects in an organized and clear way so that others would be able to identify the quantity without having to count out each object.

Ms. C: If Ms. Brittany [me, the researcher] were to come over, is there a way you can show her other than just writing it? [One child lined the objects up in groups of 5] How can you show Ms. Brittany how you can show her? How did you get that number? Did you just pick a number?

Student: I counted by 2

Ms. C: Can you show me that number? [Ms. C sits down with a child and asks them how they got a specific quantity]

Student: I have 80

Ms. C: How are you going to show your friends? [approaching another student] Is there a way you can organize your gems so we can see that there are 47.

After circulating among the children, Ms. C directed everyone's attention to one student's strategy and asked them to look at the objects and try the strategy. Ms. C then had the students do a gallery walk (walk around to other students' desks to see the organization of the objects) to identify whether the organization was easy, medium, or hard to count. After the children walked around, they returned to the carpet and shared their ideas about the organization of the objects. Ms. C identified one student's work with organizing the jacks.

Ms. C: Let's go over the jacks and see [Figure 4].

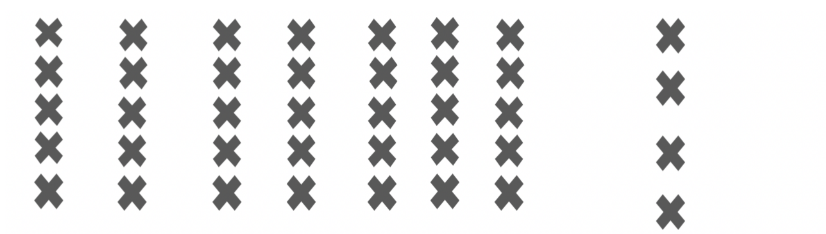
Student: There is one row with one less. [Ms. C prompted them to count]

All Students: 5, 10, 15, 20, 25, 30, 35, 36, 37, 38, 39

Ms. C: When we see a break, we know that we have to count differently. Some of us thought this was hard and some saw it was easy.

Figure 4

How a Student Organized the Jacks



After this discussion, Ms. C brought the class back to the carpet where they discussed the activity. In this follow up she introduced the term "efficient" which she brought up routinely throughout the rest of the unit. In this lesson, Ms. C connected the term efficient with the practice of clearly organizing objects (physical or drawn) to represent a quantity in a way that was clear to an observer. Drawing on the student's strategy (Figure 4), the jacks were organized so that observers could quickly count by five. The remaining four jacks were spaced out to visually make them recognizable as individual objects to count (Figure 4). Rather than telling the students the most efficient way to sort their objects or even how the organization was related to the quantity, she let the children play with the objects and sort them and then discussed ways to display their objects to others. She gave the students time to discuss their strategies for organizing and to look at the organization other students. This created opportunities for students to make connections between the drawing, the objects, and the quantity. In this way, she elicited student thinking and built on a student's strategy for organizing. This vignette illustrates one way that Ms. C created opportunities for her students to develop conceptual understanding through connecting multiple representations.

Vignette 2: Connecting Representations using a 100s Chart

Another example of Ms. C focusing on conceptual understanding was when children were using a 100s chart to make meaning for place value and structure of the number system. Instead of focusing on teaching a specific procedure or showing one correct way of figuring out the problem (n+10, n-10, n+1, n-1) Ms. C used a commonly used representation of the number system, a 100s chart (Figure 5). After the students tried out various numbers (e.g., find 65, what's 10 more, and what's 10 less), the students started to notice a "trick" that any number for n+10 would come directly below the given number on the 100s chart.

Ms. C:	What number is 10 more than 46? If you don't know you can count. Student 9 can you count 10 more.
Student 9:	[silently counts] 56.

Ms. C: Go ahead and color 56. Now put your finger on 46. You are going to do 10 less.

Student 2: Oh, I get it!

Ms. C: Some of you saw the trick and some of you knew the trick.

Student 2: When you go here if it's 10 more you go down one.

Ms. C: Why do you go down one?

Student 2: Because when you go down one you know

Ms. C: So, what happens when you go down one?

Student 2: Well, it's 10.

Figure 5

A Blank Numbers Chart Given to Students

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

In this vignette, Ms. C used guiding questions like "why did you do down one?" and "what happens?" to focus students' attention on the relationship between the numbers and their placement on the 100s chart, thus focusing on the base-ten structure of the chart. These focused questions can guide students as they make meaning of representations and learn about place value. Ms. C could have also asked questions that were more focused on tens and ones to guide the students' attention more intentionally, although this was not evident in this interaction. This vignette once again illustrates a feature of instruction typical in this classroom: connecting representations.

One tension of teaching this vignette illustrates is providing guidance for students while also allowing children to discover things on their own. In this vignette, Ms. C was leading students to connect the representations rather than the students discovering this connection on their own. There are many factors that can impact a teacher's decisions to let children discover or providing guidance during mathematics instruction. These can include limited time for scheduled mathematics instruction, teachers' beliefs about how children learn, teachers' beliefs about effective mathematics teaching, and other constraints that impact teaching and teacher autonomy. Focusing students' attention through questioning is important, as Ms. C did, yet illustrates a tension between letting children really explore and create meaning on their own versus guiding or even telling students what to look for and how to look for it.

In these vignettes, connecting representations through the counting collections and the 100s chart, Ms. C created opportunities for her students to make meaning for representations of numbers. She asked them to use different representations of numbers and connected their work to the concept of base ten or the base-ten structure. She used strategies such as asking guiding questions (e.g., What did you notice? Why did you go down one? What happens?) and letting the children do the mathematical thinking without providing correct answers or teaching a standard algorithm. These teacher moves, and others that Ms. C used, have been documented as important for supporting conceptual understanding (de Araujo et al., 2018; O'Connor & Michaels, 2017). This vignette is only one example that illustrates how this teacher used teacher moves to create opportunities for students to develop their conceptual understanding of number.

Using Teacher Moves

Ms. C used several teaching moves documented in previous research to support conceptual understanding during her instruction including: 1) being highly responsive to students' contributions, 2) eliciting student thinking, and 3) asking for explanations and justifications. Throughout the unit Ms. C displayed responsiveness to the children and their contributions and used many teacher moves to support engagement with activities. A teacher's pedagogical moves are the actions that shape what mathematics is covered and how (Remillard & Heck, 2014). Ms. C's pedagogical moves and interactions were reflective of and responsive to the students themselves (e.g., using predictable participation and grouping structures, familiar language, and familiar contexts in activities).

Ms. C supported students' participation in discussions and engagement with mathematical ideas by drawing on students' thinking, contributions, and experiences. Ms. C's pedagogical choices positioned students' thinking, their contributions, their demonstration of understanding, and familiar contexts as central to instruction. For example, Ms. C employed various teacher moves like eliciting student thinking during discussions, using open-ended questions (what do you notice?), using guiding questions, modeling, positioning contributions as valuable, asking for explanations, and being highly responsive to students and their contributions during the activity with the 100s chart. Teachers moves such as these are important for creating opportunities for students to participate in mathematical discussions (Kazemi & Hints, 2014; Chapin, O'Connor & Anderson, 2009).

Vignette 3 (Part 1): Eliciting Student Thinking while Comparing Numbers

On the ninth day of instruction, Ms. C started a lesson on greater than and less than, number relations, and ten more/ten less and one more/one less. She handed each child a laminated 100s chart (Figure 6) and sat with them in a circle during the whole group discussion. The 100s Chart Lesson is a strong example of Ms. C's responsiveness and teacher moves. This lesson was aimed at supporting students' understanding of place value and working on activities comparing numbers, using a mathematical tool (the 100s chart), and making sense of representations. At the beginning of the lesson Ms. C explicitly guided the students by directing them to color in specific numbers and finding 10 more, 10 less, one more, and one less. She used the numbers that the students picked as the starting number which built on the students' contributions.

Ms. C invited the students to guide the lesson through her open-ended question: "what do you notice?" Ms. C took the time to explore the ideas that students brought up about the 100s chart (Figure 6). This vignette illustrates how Ms. C elicited student thinking and created opportunities for her students to compare numbers.

Ms. C: What do you notice that's neat or cool?

Student 1: It goes all the way down that all of those numbers end with a number one.

Ms. C: You notice that the numbers in this row all end with one. Anyone else?

Student 2: Well, it goes 2,3,4,5,6,7,8,9

Ms. C: Great. So, pick a number.

Student 3: Two

Ms. C: Let's all count the numbers down [the page]

All Students: 2,12,22,32,42,52,62,72,82,92

Student 4: The numbers starting the number repeats.

Student 3: [raises hand]

Ms. C: I already called on you, can I call on someone else who hasn't talked?

Student 5: 12,23,34,45,56,67, 78, 89 [Figure 6]

Ms. C: Let's count this way [diagonally].

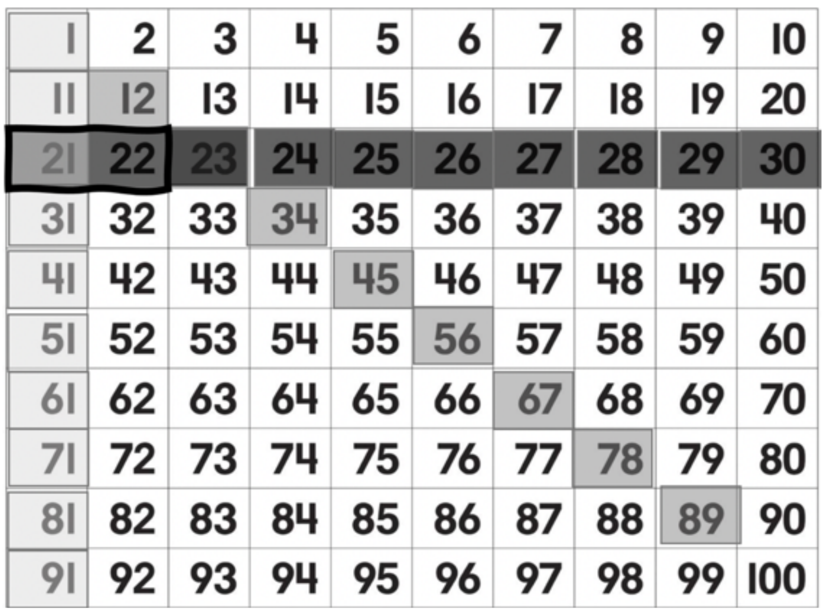
All Students: 12,23,34,45,56,67, 78, 89

Student 6: 21 and 22, the number is almost the same

Ms. C: He notices that the number in the tens place stays the same and the number in the ones place changes.

Figure 6

Patterns on the 100s Chart That Students Noticed



As shown in the transcript above, one student noticed a pattern when counting diagonally (12,23,34,45….). Another student noticed a pattern that the number in the 10s place remained the same when counting across (21,22,23….). During this interaction, the student said, "21 and 22, the number is almost the same." Ms. C revoiced and elaborated on this student's contribution by stating, "He notices that the number in the tens place stays the same and the number in the ones place changes." By giving space to the students' ideas, Ms. C not only positioned their contributions as important but also gave students time to make meaning of the representation (the 100s chart). Revoicing is one effective approach to supporting students' mathematics learning (de Araujo et al., 2018; Moschkovich, 2015; O'Connor & Michaels, 2017).

Vignette 3 (Part 2): Open Exploration of the 100s Chart

One tension evident in this vignette is the role of explicit guidance during this activity. When introducing the activity Ms. C was more explicit than towards the end. Towards the end of the lesson, Ms. C did not direct the children as explicitly as she did in the beginning. Instead, she asked them another open-ended questions, "so, what are we supposed to do?"

Ms. C:	Let's find the number 64 and color it in. So, what are we supposed to do?
All Students:	One more [color in 65]
Ms. C:	And then what are we going to do after we do one more?
All Students:	One less [color in 63]
Ms. C:	Now what's next?
All Students:	10 more
Ms. C:	If you don't remember how to find 10 more than count 10 more.
Student 3:	74 [color in 74]
Ms. C:	Ok, back to 64. And now what are we going to do?
All Students:	10 less [color in 54]

Ms. C then pointed to their colored 100s charts and asked them to tell her what they noticed on their papers.

Ms. C: Ok look at your paper. What do you notice something that's on your paper? [Figure 7]

Student 4: They are all going like that [motions up and down]

Ms. C: What is that?

All Students: A cross.

Student 4: If you go across the numbers are the same and if you go down the second numbers are the same.

Ms. C: So, if you go across the number in the tens place is the same and if you go up and down the number in the ones place is the same.

Figure 7

The Cross-Like Figure When Coloring in 10 More, 10 Less, 1 More and 1 Less

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

In this vignette, Ms. C reminded the students that they could use different strategies for finding these numbers (counting or using the “trick”). Some children noticed a visual pattern that when they found all the numbers (10 more, 10 less, one more, one less) related to a given number that a cross-like figure appeared (See Figure 7). By using this “trick” some students quickly found all the numbers to color. Directing the students to this figure Ms. C asked, “what do you notice something that’s on your paper?” The students motioned up and down as some mentioned, “they are all going like that”. Another student pointed out a pattern related to the highlighted numbers in the “cross.” The student noticed, “if you go across the numbers are the same and if you go down the second numbers are the same.” Ms. C revoiced this comment with more precise language, “So, if you go across the number in the tens place is the same and if you go up and down the number in the ones place is the same.”

Although Ms. C was trying to support understanding of place value, she did not explicitly state the ones and tens place of a given number nor did she focus on a procedural method of determining place value. Instead, she used the contributions and patterns that the students noticed as examples of the tens and ones place. This vignette illustrates that Ms. C’s teaching focused on making meaning rather than telling the students how to do things step-by-step. Vignette 3 is only one example, yet Ms. C often used activities in her class (e.g., the sparkle game, worksheets, whiteboard practice with number bonds, using unifix cubes) to embed mathematical concepts within activities without explicitly telling the students what concepts they were working on or defining the concepts procedurally.

Vignette 4: Eliciting Student Strategies and Being Responsive to Students

Another example of Ms. C's responsiveness and use of teacher moves came from a lesson on addition word problems. The following excerpt from the Word Problems Lesson illustrates the way that Ms. C elicited student thinking and used student strategies to support problem-solving. On the 16^{th} day of the unit, Ms. C led a lesson about addition and word problems. The lesson included a whole group activity that lasted 32 minutes, a short independent work time for seven minutes, and a wrap-up of the lesson for six minutes. Ms. C started the lesson with a whole-class activity that included whiteboards and unifix cubes for everyone. During this whole-class activity, Ms. C included word problems that used the names of students and familiar activities and objects like afterschool activities and sports equipment. This vignette illustrates how Ms. C elicited feedback from the students and collaborated with her students on a word problem by eliciting student strategies and asking students to explain their strategies.

Ms. C: Was it easier to build out cubes and write on the white boards or do the tape diagrams on the paper? [Ms. C elicits feedback from each child – most say the cubes and the white boards]. We will take a break from our math [notebook] and just use cubes and whiteboards. Will[13] went to Kids in Nature [a program at their school] and knitted four scarves. Miranda went to the girls and boys club [another afterschool program] and painted 3 pictures. Sonia went to the library and read 2 books. Sam went to the CK and went on the monkey bars one time. Show me all the different combinations. How many different activities did our friends do after school?

After this question, the students showed a thumbs up that they had answers and some of the students verbally answered 10 once they were called on by Ms. C.

Ms. C: How did you know this is 10?

Student 1: I know that 4 plus 3 is 7 and then I put these 3 together and saw that it was 10 all together

Student 2: 4 plus 3 is 7 and 2 more is 9 and 1 more is 10.

Student 3: So, I saw 4 and 1 and 3 and 2. I did groups of 5 and it was easy to see 10.

This vignette illustrates one of the teacher moves that Ms. C often used during her mathematics instruction – eliciting student thinking and solution strategies. Instead of stopping at the answer (10) she asked the students *how* they knew the answer was 10. Multiple students shared their strategies which included:

Student 1: 4+3=7 and 7 +3 =10

Student 2: 4+3=7 and 7 +2=9 and 9+1=10

Student 3: 4+1=5 and 3+2=5 so 5+5=10

[13] All names are pseudonyms.

Vignettes 3 and 4 exemplify the teaching moves Ms. C used regularly during her instruction. During activities, Ms. C supported meaning-making by using teacher moves such as revoicing, questioning, eliciting student thinking and solutions, and asking for students' reasoning and often continued conversations with the students until they demonstrated some understanding of the ideas. These teacher moves have been documented as crucial for giving students opportunities to engage in productive mathematical discussions (Kazemi & Hints, 2014; Chapin, O'Connor & Anderson, 2009). Ms. C's responsiveness and teacher moves structured discussions as a place for learning and a place that created opportunities for students to problem-solve.

Establishing Norms

Ms. C created norms around participation and mathematics that made a space where children could take risks and be brave to contribute and make meaning for mathematics. Ms. C established two socio-mathematical norms (Franke, Kazemi & Battey, 2007; Yachel & Cobb, 1996) in her classroom including 1) positioning mistakes as useful for learning, and 2) promoting efficiency. Additionally, she gave children time to grapple with mathematics content where the focus was on supporting understanding (rather than memorizing), and she elicited student thinking and used it for instruction. Ms. C also put in place many norms for participating during mathematics instruction. For example, Ms. C often reminded the students how they should use manipulatives and participate in discussions. One day when the students were using unifix cubes Ms. C covered the expectations and norms for these manipulatives. She asked the students to remember to have a "calm body" and when using the cubes not to stack them in tall towers or use them for building. She also reminded the students that when they knew an answer, instead of shouting out "we show a thumbs up." The norms that were established (both participation norms and socio-mathematical) created an environment that made space for students to persevere in problem-solving, use manipulatives and other tools for making meaning of mathematical concepts, and participate in mathematical discussions.

I observed students participating and often making mistakes, and Ms. C modeled that mistakes were acceptable and important for learning. Ms. C encouraged students to be brave and take mathematical risks. This was demonstrated by students asking questions and contributing to conversations even when they did not have an answer. Further, by not focusing on the product, but rather the process of solving a problem, Ms. C's classroom got messy and loud. There were often lots of marks on the board where Ms. C would draw and then redraw and cross out items. Similarly, there was often a lot of noise in the classroom. This was noise from the children discussing and laughing. Noise also came from movement of items, like pencils and desks, as well as from children physically moving their bodies from place to place. This classroom was not a place where Ms. C exhibited excess control over the students' bodies, instead the classroom norms encouraged student participation and autonomy. This autonomy paired with clear expectations created a classroom culture where I observed that the students listened to Ms. C, participated in mathematics, and took care of their personal needs (e.g., getting up to go to the restroom, getting items from their backpacks, standing up to get some wiggles out) in a way that was not disruptive.

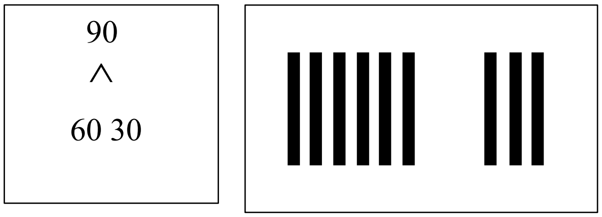
Vignette 6: Establishing a Norm that Mistakes are for Learning

During the unit, Ms. C established norms around mistakes that 1) mathematics is about the process, not the product, 2) mathematics is to be practiced in low-stakes activities, and 3) learning happens through mistakes. In addition to frequently saying "mistakes are for learning," Ms. C used mathematics instruction to discuss students thinking when they made mistakes.

During an activity where students practiced representing addition problems with number bonds, equations, and drawings, one student struggled to draw the picture for the equation 60+30=90. Number bonds are one way to represent addition problems by highlighting how a quantity can be decomposed into smaller quantities (Figure 8). As shown in Figure 8, Ms. C started by putting the number bond up on the board:

Figure 8

A Number Bond (Left) and Picture Representation (Right) of 60+30=90



Ms. C then asked one student, Laura, to write a picture that represented the number bond. Laura drew the correct number of ten sticks to represent the number bond (Figure 8). Then Ms. C wanted the students to practice representing the number bond in an equation that asked for the number of “tens” rather than a traditional numeral (___tens +____tens =___tens). To do this, the students needed to identify that there were six tens (60) plus three tens (30) to make a total of nine tens (90). Ms. C asked the students what number came first. A student wrote on the big white board 60 tens +30 tens = 90 tens. While this equation was correct, it was not a match to the problem they were working on (60+30=90 or 6 tens + 3 tens = 9 tens). After the student wrote 60 tens +30 tens = 90 tens Ms. C asked the students, “is this true or false?” and then asked them to explain.

Student 1: Because 60 tens is bigger than 100

Student 2: False. This number needs to say 6 tens instead of 60 tens and 3 tens instead of 30 tens.

After these explanations, all but one student, Laura, agreed that the statement was false. Ms. C then called Laura up to the board and asked her to draw out 60 tens +30 tens= 90 tens and asked her to look at her original drawing to see if they matched. Without drawing out 90 tens, Laura stated that the pictures were not the same and changed her answer.

In this activity, Ms. C used a mistake as an opportunity for students to explain their reasoning and compare picture representations of equations. Ms. C frequently emphasized that mathematics was not about producing an answer but rather the work or the process of solving the problem. Later in the unit, Ms. C reminded the students that, “what’s important to me today is not the answer.” Ms. C asked the students to reflect and talk about their thinking rather than emphasizing wrong answers.

Another way that Ms. C reinforced the norm that mistakes are for learning was by modeling mistakes herself to the whole group and highlighting her mistakes. For example, during an activity with unifix cubes, Ms. C incorrectly counted the cubes and then explicitly pointed out her mistake to the students. On another day, when the students gave Ms. C feedback about a lesson, one student identified having difficulty with orienting the greater than and less than signs. Ms. C responded by saying, “it’s good to make mistakes because that’s how our brains grow.” In addition to the constant modeling of mistakes, focusing on working through mistakes, and explicitly saying that mistakes support learning, Ms. C had posters in her classroom that said, “This is a Mistakes Making Place,” “Mistakes are OK,” and “Mistakes are for Learning.”

The constant reminders about the importance of mistakes and ways that mistakes were treated during instruction moved the focus away from avoiding mistakes towards embracing mistakes. Children were freed up to try things without the worry of always getting it right. The focus was not on the answer but rather the process.

Focusing on Efficiency

At the very beginning of the unit, Ms. C introduced the idea of efficiency. During the first lesson, students worked on counting collections and discussed ways to count the objects (Vignette 1). Ms. C asked the children to discuss the most efficient way to count and display their collections. After her introduction to the word efficiency on Day 1, she revisited this word and discussed its meanings on days 2, 4, 6, and 7[14]. The following vignette follows the moments when Ms. C brought up efficiency during whole group discussions.

Vignette 7: Revisiting Efficiency throughout the Unit

On Day 2 Ms. C did a whole group check-in about the lesson from the day before. Ms. C then did a short review for a child who was absent. During this review, she summarized a discussion the students had around “friendly

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[14] Days 3 and 5 were not observed.

numbers" and identified that counting by 10 was one of the most efficient ways for organizing and grouping a lot of objects to count. Again, on Day 4 Ms. C did a recap at the end of the math lesson discussing efficient ways of grouping objects to count and add together.

On Day 6, Ms. C revisited the term efficient. The students were working on a worksheet where they were asked to make visual representations of numbers, represent visuals and numerals, and compare two numbers/sets using greater than and less than. As she circulated around the classroom, Ms. C found that some students were having difficulty. She announced to the whole class, "on your math page you have 32 and 17. What is the most efficient way to draw out 32?" A student responded, "by 10s" and Ms. C drew on the whiteboard three tall rectangles and two small dots representing three groups of 10 and two ones. In this interaction, Ms. C only used the word efficient, unlike previous instances where she also mentioned faster and easier. She then had the students share and write representations they identified as the most efficient way to represent the number 32.

In another lesson on Day 7 of the unit, Ms. C reminded the students of the term efficient during a whole group interaction. After passing out small whiteboards to each child, she asked a child to pick a number. They picked the number 25. She asked the students to represent 25 in a drawing using their "best" way. One child started drawing 25 individual boxes. Ms. C stopped the student and asked her if she represented the number efficiently. Most of the other students had two tall rectangles and five small circles. The child erased the individual boxes and drew out two rectangles and five boxes. The class continued with a few more examples, including 52, 11, 92, and 100. In this example, the students had opportunities to continue grappling with the idea of efficiency and how efficiency was related to representations of quantities using drawings of base-10 blocks and connections between these pictures and the corresponding number. Beyond this, Ms. C created an opportunity for the students to think about the concept of base ten and practice grouping by ten.

In this example, Ms. C helped her students think about efficiency, but the focus on efficiency was less student-centered than other activities in the class. This again illustrates a tension in teaching that entails making decisions to guide students or letting them discover and make meaning on their own. Instead of asking the student who was drawing individual boxes how their representation was an efficient way to show 25, she stopped her from drawing and asked if it was efficient. The student did not have an opportunity to reason or justify her representation but instead she just erased and drew 25 the same way as the other students. This could have been an even more fruitful opportunity for this student to make meaning of efficiency by having them consider and reflect on different ways to represent 25. It is possible that this child did make this connection, but it was not obvious in this interaction.

In these explicit examples of efficient or efficiency, Ms. C encouraged students to calculate quantities through counting or addition using methods that are “faster” “easier” or “the best way.” The way Ms. C used “efficient” encouraged students to organize objects and drawings to represent a quantity without having to count each individual unit and then to communicate the quantity to others. During the first lesson, Ms. C also brought attention to a student’s comment that it was more efficient to count smaller numbers using smaller units (e.g., count by one, two, or five). Ms. C added to this student’s contribution by highlighting an important feature of efficiency: the total quantity impacts the “most efficient” way to count or calculate. In these examples, efficient was used to describe counting faster or easier and representing quantities in a clear way. In all these instances, Ms. C guided the students to attend to the structure of the number system to represent and count.

There are several tensions in focusing on efficiency. Often students’ strategies are not initially the most efficient way to solve a math problem, thus a focus on efficiency can position students’ strategies as incorrect or ineffective. This tension between students’ strategies and Ms. C’s practice of focusing on efficiency was evident in Ms. C’s classroom. While Ms. C embedded the term efficient within activities and introduced it throughout different lessons, framing efficient as the “best way” may have led students to the belief that there was only one way to represent a solution efficiently. Identifying something as the “best” way can give students an impression of a single, correct way to approach counting, organizing, or representing quantities. I do not think this was her intention, since Ms. C talked with the students about different strategies that could be efficient. However, since she often used the phrase, “the best way,” and equated this with efficiency, students may have interpreted efficiency as referring to a single approach. The introduction and use of the term efficiency highlight how Ms. C embedded mathematics vocabulary within activities and as connected to the content. This approach to teaching vocabulary is important as it promotes a view that mathematics language and mathematics content are interconnected (Moschkovich, 2013; Moschkovich, 2015; Walqui & Bunch, 2019). However, the terms Ms. C used to talk about efficiency could have been more carefully selected, lessening the chance of students developing a view that there was only a single or standard approach to counting, organizing, or representing quantities efficiently.

Throughout the unit, Ms. C drew on the curriculum and altered the mathematics lessons to support engagement with the content. She often adjusted the sequencing from the math workbook, especially if she felt that students needed more time. Ms. C added in various activities beyond the math workbook and spent most of the structured math time having group discussions and group activities with everyone on the carpet. Ms. C structured daily lessons similarly across the unit (i.e., moving from whole-class to independent/small group work, and then whole-class again). The participation norms (like explaining answers, agreeing/disagreeing with solutions/strategies, and ways to have productive conversations and not "be mean" or have comments that were irrelevant or off-topic) maintained a predictable structure that created a culture around mathematics that learning was fun, challenging, and arose from mistakes. Similarly, the socio-mathematical norms around making mistakes and efficiency supported mathematics instruction that was less about figuring out teacher expectations and instead about learning math and making meaning for new ideas.

Drawing on Students' Experiences

As a part of her mathematics instruction, Ms. C used students' experiences and interests outside the classroom to contextualize mathematics problems. She did this by bringing in students' names, familiar activities, and play. In this section, I illustrate how Ms. C contextualized her mathematics lessons using her students' experiences.

During one lesson when students were working through word problems, Ms. C changed the names in the problems to the names of her students and changed the context of the problems to match familiar contexts (e.g., after-school activities or PE). When working on the problem, 12+3=__, she helped the students visualize by contextualizing with PE: "Let's think about Coach Sam. If Coach Sam has 12 soccer balls on the field and finds 3 more how many soccer balls does Coach Sam have?” Ms. C often changed the context of word problems to use the students in her class and the activities they participated in during school (e.g., PE, soccer, and afterschool clubs), objects they were familiar with (e.g., using small toys when doing counting collections), and revisiting activities or games from earlier in the year or previous classes.

Ms. C also drew on familiar contexts and students’ experiences by incorporating playful activities during mathematics instruction. One way she brought in playful activities was through her use of games during her mathematics instruction. Ms. C often commented that games were a useful tool for mathematics instruction with young children as she reflected, “I try to have them play games at least once a week for math.” During the unit, I observed four different games: 1) Sparkle Game, 2) Race to the Finish, 3) Index Card Game, and 4) 10 more/10 less, 1 more/1 less. Ms. C used these games to give the students time to participate in mathematical practices (e.g., counting, adding, subtracting, and comparing numbers). Games can be an interactive way to supplement the mathematics curriculum and give alternative and playful ways to learn mathematical concepts and engage in mathematical practices (e.g., greater than/less than; 10 more 10 less; number structure; addition/subtraction; counting) (Ramani & Siegler, 2008) and can reveal students mathematical thinking (Anderson and Gold, 2006).

One game that Ms. C brought into a lesson was the game Race to the Finish. This game was played as a partner game with a game board and dice. To play the game one player rolled the dice and got a number that represented the moves they could make around the gameboard (e.g., roll a four and move four spaces). Upon reaching the designated spot, the student then counted to the given number using groups of ten (e.g., 35 is one ten, two tens, three tens, 1,2,3,4,5,). The partners competed to get to the end of the game board first. During this game, students practiced decomposing numbers and grouping by tens and ones.

The games that Ms. C brought into mathematics instruction were playful and low-stakes places where the students could practice mathematics (counting and comparing numbers). Board games like these have been found to support children's number development (Ramani & Siegler, 2008) and are one way Ms. C brought play into the classroom. Making mathematics fun, bringing in playful activities, and creating low-stakes spaces (e.g., they were not graded activities and were not under constant supervision of the teacher) invited students into the mathematics activities. These games were not about using precise mathematical language or always getting the right answer but instead offered an inviting space for all students to practice and play with their peers.

Beyond games, Ms. C often had children play with objects as part of her mathematics instruction. When students would work with unifix cubes, she sometimes let them play with the manipulatives before they used them for modeling mathematical problems. During three occasions when I observed children during free play, I noticed that children continued to play while they participated in mathematical practices like designing, building, or counting. Wager (2013) found that children often engage in spontaneous mathematical activities as they engage in free play and may also build upon ideas and activities that have been introduced by the teacher. In Ms. C's classroom, I noticed that the children often spontaneously participated in mathematical activities and played with the mathematics games they played with earlier in the day. Play is an important part of childhood and children learn a lot during play (Wager & Parks, 2014; Perry & Dockett, 2002; Wager & Parks, 2016; Vygotsky, 1978)

Ms. C's Teaching Practices Reflected Her Professed Beliefs

In the following section, I discuss Ms. C's teaching practices and her professed beliefs. I found that Ms. C's teaching practices (features of instruction) aligned with her professed beliefs. In chapter 2, I discussed how Ms. C's beliefs related to belief constructs (Schoen & LaVenia, 2019) about mathematics instruction and aligned with low "transmissionist", low "facts first", and low "fixed instructional plan" views. In this section, I describe how her in-person teaching practices aligned with those beliefs described in Chapter 2.

When looking at Ms. C's teaching practices during in-person classroom observations, I found that she employed a student-centered approach in which she adapted her instruction and she covered the curriculum based on the needs and learning of her students. Ms. C frequently elicited student thinking and strategies and supported her students to see various solutions to problems. Ms. C used students' solutions as the focus of whole-group discussions. For example, as I discussed in Vignette 3, Ms. C elicited student thinking while the students were comparing numbers on a 100s chart to support students to make meaning for place value. Similarly, Ms. C would use student contributions to focus whole-class discussions. These teaching practices align with Ms. C's low "transmissionist" beliefs.

Ms. C's teaching practices also aligned with her low "facts first" beliefs. Rather than focusing on skills, I observed Ms. C often started scheduled mathematics time with open-ended conversations and opportunities for students to think about the problems they were going to solve. During my observations of Ms. C's in-person mathematics instruction, Ms. C typically started her lessons with an open-ended exploration of materials or problems. For example, Vignette 1 illustrates the ways Ms. C used counting collections to give her students opportunities to develop conceptual understanding by connecting multiple representations. Also, in Vignette 2 I showed how students connected multiple representations using a 100s chart as they solved addition problems. Rather than having a procedurally focused approach to teaching, Ms. C created many opportunities for students to develop conceptual understanding by connecting multiple representations. Ms. C often focused on problem-solving, and I did not see her focus on procedures or facts when teaching in person. Ms. C's focus on problem-solving rather than memorizing facts is evidence of the alignment between Ms. C's teaching practices and her professed beliefs related to low "facts first."

Finally, Ms. C's teaching practices also aligned with her low "fixed instructional plan" belief. Through my classroom observations, I saw that Ms. C used the curriculum materials as a guide, rather than a script for her in-person instruction. She attended to the students' needs to determine if and how she progressed through the materials. She often brought in supplemental materials, typically games, to add interactive activities to mathematics. She also did not adhere to the scope and sequencing of the curriculum materials. Ms. C would often skip over the designated homework pages when she saw that her students had mastered a topic. Other times, if students demonstrated that they needed more time or additional practice with topics, Ms. C would use these homework pages during the scheduled mathematics time to revisit topics or give additional practice. In-person, Ms. C's teaching practices aligned with her low "fixed instructional plan" belief.

Drawing on Students' Assets: Challenges and Successes

In the previous sections, I described the central features of Ms. C's teaching practices and how these practices aligned with her professed beliefs. Ms. C 1) created opportunities for students to develop conceptual understanding by connecting representations, 2) used teacher moves and was responsive to students and their thinking, 3) established participation norms and socio-mathematical norms related to making mistakes and efficiency, and 4) drew on students' experiences and incorporated play. In this next section, I describe how these features relate to previous research and how this analysis contributes to research and practice in early mathematics education.

In particular, the case of Ms. C illustrates how one teacher drew on her students' assets (student thinking, solutions, and experiences) to create opportunities for students to learn mathematics with understanding. The vignettes provide detailed examples that can be used to ground discussions of teaching practices for teacher education and professional development. This study also illustrates how the MMKB framework was useful for looking at teaching practices. Beyond this, the analysis of in-person teaching provides the details of Ms. C's in-person instruction which I will draw on to make comparisons to her online instruction in Chapter 4. In the following discussion, I describe how Ms. C drew on her students' assets but also show how her teaching, like any teaching, was not perfect. Teachers face many challenges and tensions that shape their enacted teaching practice. Beyond this, the study of teaching is complex and the models we currently have for studying teaching need to consider this complexity in the ways they represent teacher change, teacher learning, and teaching practice.

Students' Assets: Student Thinking and Experiences

As illustrated in the analysis of the vignettes, Ms. C enacted her mathematics instruction in ways that were focused on student thinking and brought in student experiences. In every lesson, the focus of conversations was on eliciting and using student contributions. We know from years of research on CGI that focusing mathematics instruction on student thinking and solutions is an effective strategy (Carpenter et al., 1983; Carpenter, Fennema & Franke, 1996) that has been shown to improve student achievement (Fennema et al., 1996). Beyond this Ms. C also brought student experiences into her instruction by including students' names, changing the context of word problems to familiar activities, and by using playful activities in mathematics. In comparison, Ms. C frequently drew on student thinking (almost every lesson observed) and only occasionally brought in student experiences from outside the classroom.

This feature is consistent with previous work that explored pre-service teachers' instructional plans related to drawing on students' MMKB (student thinking and experiences) (Turner, et al., 2016). The study of preservice teachers found that most of the sample (84%) drew on student thinking yet only 61% drew on student experiences after an intervention that supported teachers in eliciting and connecting to both (Turner et al., 2016). They found that students of non-White backgrounds were more likely to receive connections to their MMKB in ways that were based on assumptions (Turner et al., 2016). In this sample, Turner et al. (2016) found that only a small group of the pre-service teachers drew on both students thinking and experiences in meaningful ways that reflected the mathematical practices that the students participated in from their homes and communities. Often, when teachers bring in practices from students' homes and communities into the classroom, the explicit connections to mathematics can make these practices look different and what happens when students participate outside of school can get lost or overshadowed by the explicit academic content (Gonzalez et al., 2001). Ms. C drew on her students' assets throughout her instruction, although her practice focused more on drawing on student thinking than student experiences outside the classroom.

A Student-Centered Approach

In addition to the finding that Ms. C brought in student thinking more frequently than student experiences, Ms. C also faced tensions that shaped the way she enacted instruction in her classroom. While I did not explicitly document the specific tensions that Ms. C faced, previous research has described the challenges and tensions that early educators face when teaching mathematics. Some of the tensions that have been documented in research include a tension between teachers' beliefs and their teaching practices (Lee &

Ginsburg, 2007; Parks & Bridges-Rhoades, 2012), policy and teacher autonomy (Parks & Bridges-Rhoades, 2012; Pease-Alvarez & Samway, 2008; Pease-Alvarez, Samway, & Cifka-Herrera, 2010), and teaching procedurally or conceptually (Fennema et al., 1996; Hiebert, 1990; Hiebert & Carpenter, 1992; Kilpatrick, Swafford, and Findell, 2001). In the next chapter, Chapter 4, I discuss the tensions Ms. C faced related to beliefs and practices as well as tensions related to policy and autonomy. Here I discuss how Ms. C overcame the tension between teaching to support understanding and teaching with fidelity to the curriculum materials.

Ms. C was given curriculum materials (Engage NY) that identified when and how she should facilitate her instruction. While this study did not include an analysis of the curriculum materials, I found that Ms. C explicitly said that she did not teach the materials with fidelity and brought in activities that were not included in the materials. For example, Ms. C frequently said to me that she would skip pages that seemed to be repetitive practice when she felt that her students already understood concepts. She also supplemented the curriculum with activities, usually games. Ms. C took cues from her students when they demonstrated that they understood a concept or when they seemed to need more time to grapple with a topic. Previous research has found that mathematics instruction that is scripted and not focused on student thinking can give students messages that mathematics is only about getting the right answer and memorizing one way to do problems (Parks & Bridges-Rhoads, 2012). Rather, a student-centered approach to mathematics instruction is more effective at supporting learning with understanding than a teacher-centered approach (Polly, Margerison, & Piel, 2014).

Ms. C's mathematics instruction illustrates one way to enact a student-centered approach through her intentional eliciting of student thinking, the establishment of classroom norms, and a focus on conceptual understanding. Ms. C used the curriculum materials as a guide but taught in a way that incorporated many supplemental activities into mathematics instruction. This created opportunities for her students to participate in mathematics. While I don't have specific evidence that all of Ms. C's students participated during mathematics instruction, research suggests that broadening participation in mathematics is a key characteristic in equitable mathematics instruction (Moschkovich, 2013).

To create opportunities for participation, Ms. C established norms and structure that made space for them to participate, made activities accessible by drawing on students' names and familiar contexts, and positioned students' contributions as valuable and important for learning. Ms. C's classroom was not a "safe space" in the sense that students were never confronted with feeling uncomfortable. Rather, Ms. C created a "brave space" where students knew that challenges and struggles were frequent, but it was overcoming these difficulties that made learning possible. While the term "brave space" has not explicitly been highlighted in mathematics education literature, other fields such as feminist studies, religious studies, and social epistemology have defined "brave space" as spaces that seek to uncover the "truth" rather than making an individual feel comfortable (Anderson, 2021). This notion privileges an important mathematical practice of perseverance especially when problems are challenging.

Aligned with the practice of persevering (a Common Core State Standard math practice), an important part of learning mathematics is developing a "productive disposition" towards mathematics (Kilpatrick, Swafford, and Findell, 2001). A productive disposition includes the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick, Swafford, & Findell, 2001, p. 116). Exploring students' dispositions in mathematics classrooms, Gresalfi (2009) investigated the ways students' dispositions change over time through engagement with particular classroom practices. Findings from this study reveal that students' dispositions are not fixed and classroom practice can shape dispositions (Gresalfi, 2009). Specifically, individual participation, small group work, and teacher interventions can shape the way students' dispositions develop in the classroom and influence the opportunities to learn (Gresalfi, 2009). For example, being able to collaborate and work with others contributed to opportunities to engage with mathematics. These findings highlight,

> "If mathematics is about getting the right answer, and a student consistently struggles to find the right answer and has limited access to assistance when he or she is confused, then it seems likely that those students will choose not to continue to engage. In contrast, another student who also often experiences confusion but is constantly able to access and secure meaningful assistance may persist" (Gresalfi, 2009, p. 360).

It is important that children see themselves as mathematics learners. Experiencing mathematics through different activities can give more opportunities for students to grapple with mathematics and develop a productive disposition. Ms. C created an environment in her classroom where students could grapple with mathematics concepts and participate in mathematical practices without feeling pressured or embarrassed. By centralizing the process of learning rather than the correctness of the answer, Ms. C made a welcoming and "brave space" for children to learn and do mathematics.

Teaching is Complex

Many learning trajectories that characterize teacher learning or teacher change are often limited in how they represent the complexities of teaching. Knowledge and learning are typically represented through hierarchical progressions in learning trajectories and too often there is a lack of attention to equity with the learning trajectory approach (Lobato & Walters, 2017). Findings from this study can be used to complicate the categories for the MMKB learning trajectory (Figure 1) (Turner et al., 2012).

While this was not a study of teacher change, the vignettes illustrate alternative ways to expand the use of students' MMKB during mathematics instruction. Ms. C drew on student thinking and students' experiences in ways that don't necessarily reflect one single category of the MMKB learning trajectory. In the model, eliciting and attending to students' MMKB are listed as a part of the *initial* practices in the learning trajectory. However, Ms. C's frequent use of eliciting and attending to students' thinking suggests that these practices may be important across all phases outlined in the MMKB model. Ms. C incorporated students' comments and strategies into her instruction and used these contributions to make decisions about what and how to teach. She also brought in familiar activities from children's lives outside of the classroom (e.g., sports, afterschool clubs, playful activities). Ms. C's teaching practices and her expressed reasoning for making decisions about her teaching reveal that Ms. C was *making connections* to students' MMKB in her instruction. This characterizes Ms. C's instruction as aligned with the middle level of the MMKB learning trajectory, not the primary level. Her frequent use of eliciting and attending to students may only seem like a characterization at the initial practices level (the primary level). This complication affirms some overarching issues with learning trajectories especially as they relate to teachers' practice. Instead, it would be useful for visual models of teaching and teacher change to be represented in ways that are not linear. Drawing on students' thinking and their experiences happens dialectically while teaching. A linear trajectory can oversimplify teaching practice and expertise. While the developers of the MMKB model acknowledge that teacher change is not linear (Aguirre at el., 2012; Turner et al., 2012), the model they created can be interpreted as a hierarchical progression. Any model of teacher change, teacher learning, or teacher practice needs to include the complexity of teaching practices.

Conclusion

This study provides an overview of the teaching practices during mathematics instruction of one accomplished, first-grade teacher, Ms. C. Ms. C had a pedagogical perspective that aligned in some ways with a CGI approach but also included a focus on bringing in meaningful experiences from outside the classroom, like games, to supplement the official curriculum. In this first-grade classroom Ms. C 1) created opportunities for students to develop conceptual understanding by connecting representations, 2) used teacher moves and was responsive to students and their thinking, 3) established participation norms and socio-mathematical norms related to making mistakes and efficiency, and 4) drew on students' experiences. The case of Ms. C illustrates in detail how one teacher drew on her students' assets (student thinking, solutions, and experiences) to create opportunities for students to learn mathematics with understanding.

While Ms. C drew on her students' assets throughout her instruction, her practice focused more on drawing on student thinking than student experiences outside the classroom. Drawing on student experiences in authentic ways for mathematics learning in the classroom is challenging and requires a lot of work from teachers and support for this practice. Teacher educators in both teacher education programs and professional development need to intentionally support teachers as they learn to respectfully elicit and make connections to student experiences from their homes and communities. Beyond the data, I can infer that Ms. C's extensive professional development related to CGI and limited professional development related to connecting to students' experiences outside the classroom may have contributed to this feature of her teaching. I discuss this inference more thoroughly in the discussion chapter (Chapter 5).

This study has implications for how teacher learning, teacher change, and teacher practices are represented in theoretical models. I used the MMKB framework to look at teaching practices, and it was an effective way to frame the study and analyze the data. However, the study of teaching is complex, and the models we currently have for studying teaching need to consider this complexity in the ways they represent change, learning or practice. This study also has implications for teacher education and professional development. The vignettes can be used to illustrate how Ms. C drew on students' assets including their mathematical solutions and thinking. The case of Ms. C may also offer an entry point into conversations about how and why teachers may not as frequently bring in students' experiences. Pre-service teachers can be supported as they plan for instruction that draws on student experiences (Turner et al., 2016). Thus, in-service teachers could also be supported through professional development to make connections to student experiences outside the classroom.

Chapter 4: A Comparison of Mathematics Instruction In-Person and Online with First-Grade Students[15]

During the COVID-19 Pandemic instruction transitioned to hybrid (online and in-person) or solely online formats for most of the 2020-2021 school year. This shift created many new challenges related to teaching and learning, especially in early grades. Of these challenges, providing equitable education for students, in particular underserved and underrepresented groups like Emergent Bilingual students (EBs), received much attention. Equitable (Gutiérrez, 2009; Moschkovich, 2013) early grades mathematics instruction must focus on understanding for all students (Hiebert, 1990; Hiebert & Carpenter, 1992; Kilpatrick & Swafford & Findell, 2002). To facilitate this kind of mathematics instruction, teachers need to draw on student thinking (Carpenter, Hiebert & Moser, 1981; Carpenter et al., 1993; Fennema et al., 1996) by including the strategies children employ and encouraging children to have conversations about the mathematics content in the classroom. Further, early grades mathematics instruction should include various activities (Wager, 2013) such as play (e.g., free play, games) (Kamii, Miyakawa, & Kato 2004; Ramani & Siegler, 2008; Wager & Parks, 2014; Wager & Parks, 2016) and manipulatives (Fuson, 2009; Jordan et al., 2008; Resnick & Omanson, 1978), and multiple representations

[15] Originally Paper 3

(Fuson, 1988; Fuson, 1991; Fuson, 2009; Powel & Nurnberger-Haag, 2015). Having play, using manipulatives, and making connections between multiple representations typically involves hands-on activities in a classroom. Crafting these experiences online was another new aspect of teaching during the global pandemic that many teachers did not have to navigate before. Finally, mathematics instruction should leverage students' Funds of Knowledge (Civil, 2002; González, Andrade, Civil, & Moll, 2001; Turner et al., 2016). Mathematics instruction that aligns with these recommendations from research can help to position all students as thinkers and doers of mathematics with understanding.

However, a fast transition to online instruction created many new challenges for teachers, administrators, parents, and students. Questions arose about equity and quality of instruction as it shifted online. A digital divide became more prominent as there was limited access to computers and the internet for some students (Moldavan, Capraro, & Capraro, 2021) when these online tools became necessary for access to instruction. Teachers were faced with supporting their students, often with limited guidance from administrators and uncertainty about the pandemic and schools. All these challenges shifted the focus towards getting students some type of instruction, even if it did not mirror the activities and opportunities for learning that in-person instruction afforded. To explore in-person and online instruction with young children, this study compared the in-person and online mathematics instruction of one accomplished, first-grade teacher, Ms. C. The research questions that guided this work were:

1) What were the differences between classroom routines and mathematics activities in person compared to online during COVID-19 in an early grades classroom?
2) Did Ms. C enact math instruction online that aligned with her professed beliefs? If so, how?
3) What was the nature of online math instruction for the one EB in Ms. C's classroom?

Framework

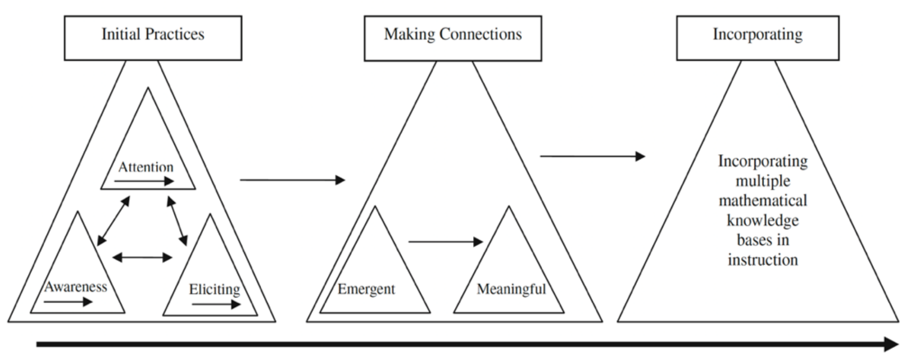
I drew on a sociocultural perspective as a theoretical framework for this study which assumes that learning and teaching happen socially (Lave & Wenger, 1991; Vygotsky, 1978) and that the social context impacts the ways individuals (teachers and students) enact mathematical practices (Parks & Bridges-Rhodes, 2012). Further, this perspective highlights the instrumental role of cultural tools (curriculum materials, manipulatives, teacher moves, norms) for teaching and learning. I used one construct, enacted curriculum, using a lens of Multiple Mathematical Knowledge Bases Framework (MMKB) to explore the enacted curriculum. In this section I discuss how I used MMKB to design and analyze the data related to Ms. C's enacted curriculum.

Using a sociocultural perspective, I make the assumption that curriculum is enacted. Thus, to explore teaching practice, I drew on Remillard and Heck's (2014) concept of enacted curriculum which includes "the interactions between teachers and students around the tasks of each lesson and accumulated lessons in a unit of instruction" (Remillard & Heck, 2014, p. 713). By looking at the four dimensions of enacted curriculum, 1) mathematics, 2) instructional interactions and norms, 3) teacher's pedagogical moves, and 4) the use of resources and tools (Remillard & Heck, 2014), I compared Ms. C's mathematics instruction with first graders in-person and online. I also briefly discuss how Ms. C's beliefs, which I analyzed and discussed in Chapter 2, were enacted differently online. Although beliefs impact practice (Schoenfeld, 2002; Raymond, 1997), there are many other factors that also shape instruction and how teachers enact curriculum. For example, when exploring the connection between a teacher's beliefs, teaching, and curriculum, Park and Bridges-Rhoads (2011) found that when a teacher who expressed the importance of creativity and innovation during mathematics started using an overly scripted literacy curriculum, this led to a shift in teaching practices during mathematics to a narrowed focus on proper grammar and the right answer. Beyond the official curriculum, other things like accessibility of resources and other contextual affordances and constraints can impact the enacted curriculum (Remillard & Heck, 2014).

Similar to my approach to looking at enacted curriculum in Chapter 3, I used the MMKB framework (Aguirre et al., 2012; Turner et al., 2012) as a lens to look at Ms. C's enacted curriculum. MMKB includes students' "multiple understandings and experiences that have the potential to shape and support students' mathematics learning" (Turner et al., 2016, p. 49). The MMKB framework incorporates a focus on student thinking (Carpenter et al., 1983; Carpenter et al., 1996; Fennema et al., 1996). and children's mathematical Funds of Knowledge (Civil, 2007; González, Andrade, Civil, & Moll, 2001) which asserts that mathematics instruction that reflects students' social worlds is more accessible to a diverse group of students. The framework includes three phases that reflect different ways that teachers draw on students' MMKB (Figure 1). These phases include 1) initial practices, 2) making connections, and 3) incorporating.

Figure 1

MMKB Learning Trajectory (Aguirre et al., 2012; Turner et al., 2012)



Initial practices (Turner et al., 2012), include eliciting, awareness, and attention to children's MMKB. Making connections reflects when teachers make emergent and then meaningful connections between students' MMKB and instruction (Turner et al., 2012). Teachers can then move to fully incorporating MMKB into instruction as they meaningfully draw on mathematical activities that children engage in at home and in their communities and engage students in activities that resemble the embedded activities of children's lives outside of the classroom. For both teaching practices analyses (Chapters 3 and 4), I used MMKB to design my study by paying attention to how Ms. C drew on student thinking and experiences. I also used the MMKB framework when I looked at the data. I did not use this framework to look at teacher change, but instead used it as a lens to look at Ms. C's teaching practice.

Methodology

This study is qualitative. For this study I drew on an interpretive, ethnographic approach to document and compare Ms. C's teaching in-person and online. In the following sections I discuss the participants and setting, data collection, and data analysis.

Participants and Setting

Ms. C, a first-grade teacher, was the primary focus of this study. Ms. C had been teaching for 14 years during the 2020-2021 school year. She received professional development at a Cognitively Guided Instruction (CGI) Summer Conference for one year, attended monthly CGI math circle meetings, attended 10 years of the California Math Council Conference, and went to many school/district-sponsored professional development meetings. Ms. C also participated in one summer institute which focused on professional development specifically related to supporting EBs' mathematics learning. In addition to participating in professional development, Ms. C also served on the leadership team for the local Math Project through an established research University where she taught and helped plan the summer institutes. Beyond her professional development, Ms. C was the after-school mathematics club teacher.

Ms. C worked at an elementary school in California with classes ranging from transitional kindergarten to fifth grade. The students Ms. C worked with included two different classes of first-grade students from two different school years. During the 2019-2020 school year, the school enrolled 548 across all grades, of which 113 students qualified for free and reduced lunch (21%), 49 were designated English Learners (9%), and 43 (8%) were designated Fluent English Proficient (Ed-Data, 2021). Most of the students that were designated English Learners (n = 26) spoke Spanish as their first language. During the 2019-2020 school year Ms. C taught 23 children in a kindergarten/first-grade combination class with 10 first graders and 13 kindergarteners. She identified two students as low-income, three students as English Learners, one student with high anxiety, and "some" students with Latinx backgrounds.

During the 2020-2021 school year, Ms. C's school enrolled 468 students. In the school, 122 students (26.1%) qualified for free and reduced lunch, 31 students (6.6%) were designated English Learners, and 24 students (5.1%) were designated Fluent English Proficient (Ed-Data, 2021). Most (n = 25) of the designated English Learners spoke Spanish as their primary language. During

2020-2021, the elementary school Ms. C worked at started completely remote due to COVID-19. Ms. C's first-grade class had 20 children, of which six were low income, four were designated English Learners, one was Latinx, and one was special needs. Ms. C taught mathematics with small groups of five students at a time for 30 minutes, two times a week, in an online format using Zoom. The observations took place with the same group of five students. I used purposeful selection when I chose the group to observe, as this group included an EB.

Data Collection

The primary sources of data for this study included observations (online and in-person), interviews, and survey responses. Data collection happened over two years and across two first-grade classrooms with the same teacher, Ms. C.

The first set of observations occurred during in-person instruction before schools closed due to COVID-19. Observations for this group lasted throughout most of an entire unit on Place Value, Number Comparisons, and Addition and Subtraction (Module 4)[16] for 13 days. The observations during online instruction occurred almost a full year after the initial in-person observations. These observations included all the online structured mathematics time across two mathematics units (Module 3 on Measurement and Module 4 on Place Value,

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[16] The official curriculum materials came from Engage New York, Grade 1, Mathematics Curriculum.

Number Comparisons, and Addition and Subtraction) spanning seven weeks. Both online and in-person observations were cut short due to shifting the format of instruction (going from in-person to online and online to in-person).

The survey was created using an online service (google forms) (Appendix C). The items on the survey included demographic information (e.g., years of teaching experience, professional development, student information) and statements about mathematics, student learning, teaching, the role of language in learning mathematics, and teaching and learning for EBs. Responses were given using a 5-point Likert scale with options to select from strongly agree to strongly disagree. Survey items were adapted from the Mathematics Education of English Learners Scale (MEELS) (Fernandes, 2020; Fernandes & McLeman, 2012). I more thoroughly discussed Ms. C's responses to this survey and her beliefs in Chapter 2.

Ms. C was formally interviewed twice for this study. Once before in-person instruction and once before online instruction about one year apart. The interviews were semi-structured and included questions about teaching, mathematics, and supporting EBs (Appendix A). The interview extended some of the survey items to identify Ms. C's thinking behind her selections and to hear examples of how her beliefs were enacted. For example, I asked Ms. C to tell me about a recent mathematics lesson she taught. I also asked, "how do you support EBs in your class?"

Data Analysis

I analyzed the data for this study to explore the enacted curriculum of one teacher, Ms. C. To analyze the data, I initially went through four phases: 1) data collection and writing memos, 2) data reduction and transcription, 3) writing descriptive narratives, and 4) comparing narratives. Secondary analysis involved revisiting the raw data (interview recordings, survey responses, and observation notes) doing a more in-depth comparison of in-person and online activities

Phase one of data analysis happened as I collected data. I wrote daily memos about the topic covered, the activities used, the interactions, and initial connections to themes (both from the MMKB framework and as they emerged from the data). During phase two I identified the themes for further analysis and wrote vignettes with transcripts related to specific activities. I reduced the data and focused only on the emergent themes that related to MMKB as well as codes that were frequent based on inductive analysis (even if they did not fit with MMKB such as the feature of establishing norms). I did this across observation data for both in-person and online instruction. After I identified the themes that I was going to pursue (i.e., creating opportunities for conceptual understanding, using teacher moves and being responsive, establishing classroom norms around participation and mathematics, and drawing on students' "funds of knowledge") I started phase three where I wrote detailed summaries of specific lessons, interactions, and activities that illustrated the themes. I then wrote narratives about overarching themes and ideas that were presented across the unit (e.g., the introduction of efficiency and how this developed across lessons.). For phase four I compared the narratives from both online and in-person to identify differences and similarities across the two formats to make claims about Ms. C's instruction. As I progressed through phase four, I realized that I needed to revisit the data to take a closer look to compare specific activities, interactions, and the progression of the units to clarify and confirm the findings. I used Ms. C's interviews and survey responses to clarify and confirm the claims I made about her instruction.

In-Person and Online Instruction

In this section I compare Ms. C's online and in-person instruction. This comparison illustrates that the features of Ms. C's enacted curriculum persisted, even though some features looked different, when she transitioned to online teaching with her first-grade students. I first summarize the features of Ms. C's instruction in person which I more thoroughly described in Chapter 3. I then discuss the ways Ms. C's instruction looked similar and other ways that Ms. C's instruction looked different.

Teaching In-Person

During in-person instruction in Ms. C's first grade class prior to schools transitioning to online, Ms. C 1) created opportunities for students to develop conceptual understanding, 2) used teacher moves and was highly responsive to students and their thinking, 3) established norms to related to participating in the class, making mistakes, and efficiency, and 4) drew on students' experiences. The nature of Ms. C's classroom in-person aligned with recommendations from research about supporting mathematics learning with understanding (e.g., Aguirre et al., 2012; Carpenter et al., 1993; Carpenter, Fennema & Franke, 1996; Fennema et al., 1996; Perry & Dockett, 2002; Turner et al., 2012; Wager, 2013). While I provided a complete overview of the nature of mathematics instruction in Ms. C's classroom in-person in Chapter 3, I provide a short summary of the features of Ms. C's instruction below.

To create opportunities for developing conceptual understanding Ms. C facilitated activities that prompted students to connect multiple representations (drawings, manipulatives, equations, and others). This was evident across many activities. For example, Ms. C used counting collections to facilitate a conversation around counting, efficiency, and the organization of quantities to represent numbers. In another activity Ms. C focused on giving students opportunities to make meaning for place value by identifying patterns and making meaning of a 100s chart. Ms. C employed various teacher moves that have been identified in research (e.g., Chapin, O'Connor & Anderson, 2009; de Araujo et al., 2018; Kazemi & Hints, 2014; O'Connor & Michaels, 2017) like eliciting student thinking during discussion, using open ended questions (e.g., what do you notice?), using guiding questions, modeling, positioning contributions as valuable, pressing for explanations, and being highly responsive to students and their contributions during the activity with the 100s chart. Another example of Ms. C's teacher moves and responsiveness was evident when she was teaching a lesson on word problems when she elicited students' thinking and used their contributions to highlight features of solving the problems. Related to norms, Ms. C established norms around mistakes being useful for learning and around efficiency. She also established participation norms in the classroom which created a space where children were invited to take risks and still be respectful of their peers. Finally, Ms. C drew on students' experiences outside the classroom by incorporating playful and familiar activities and connections to the students' lives. One way she consistently did this was by using games to make mathematics playful. Ms. C used games almost every week and identified that learning through games was a large part of her approach to teaching mathematics.

Teaching Online: Similarities to In-Person Instruction

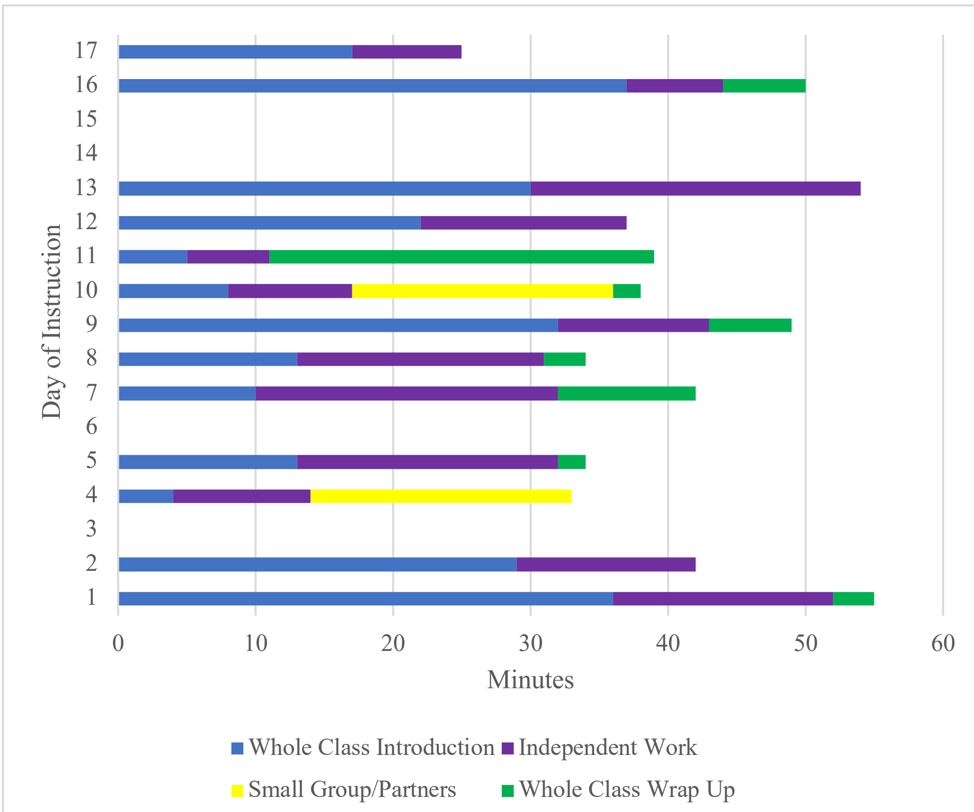
Some similarities emerged through comparison of Ms. C's in-person instruction and her instruction online. Findings highlight that the features of Ms. C's mathematics instruction (i.e., teaching for conceptual understanding, using teacher moves, establishing norms, and using students' experiences outside the classroom) both online and in-person were present, although some differences emerged in the way these looked online. Additionally, Ms. C enacted some of her professed beliefs (about mistakes, games, and participation) in both settings.

Routines of Lessons In-Person and Online

During in-person instruction, Ms. C structured her lessons similarly every day which included: 1) whole-class introduction, then 2) independent or small group work time, and finally 3) whole-class wrap-up (Figure 2). Ms. C always started her lessons with whole-class introduction. Usually, this included a whole class discussion paired with manipulatives (e.g., unifix cubes, base-ten blocks, collections of small toys) and a place to write down and record (e.g., the big white board, individual white boards, a page from the workbook). After the whole group discussion, the students would then go to independent or small group work depending on the activity. Often during this time, the students would work on the workbook page from the official curriculum. Other times they would engage in partner or small group games (e.g., race to 100 game, index card game, 10 more 10 less game). After students completed the independent/small-group time, they typically returned to the carpet for a whole group wrap-up which usually included some sort of check-in when they responded to Ms. C's queries (e.g., How did that go? What was easy? What was hard?). Sometimes Ms. C would share some things she noticed during wrap ups (e.g., student strategies, revisiting concepts). As I illustrated through the vignettes in Chapter 3, Ms. C's lessons typically were student-centered that created many opportunities for her students to develop conceptual understanding. Lessons were planned for 30 minutes each day but often went beyond this. Across my observations, lessons lasted 41 minutes on average.

Figure 2

Structure of In-Person Lessons



Ms. C's online mathematics instruction also had a predictable structure across the lessons. Ms. C and the five students (from the group I observed) would zoom in at the same time every day. She would let students into the meeting once she finished the mathematics lesson with the previous group. Ms.

C always shared her screen containing the content they were covering that day. Most days, this was a workbook page from the student workbook (from the official curriculum materials: Engage NY) (see example workbook page in Figure 3). The workbook pages often included a scenario (e.g., children are talking about their favorite ice cream flavor) and asked mathematical questions related to the scenario. Ms. C started lessons by asking students first to make sense of the workbook page. She would ask questions such as, “what are we being asked to do?”, “what information do we know?” or “what do you notice on this page?” As I will illustrate in the following sections, the students would attend to a variety of information including connections to previous lessons, ways to approach a problem, and what the problems were asking. After making sense of what the questions were asking, Ms. C would then work with the group to help them solve the questions on the workbook page. Typically, Ms. C did not direct students towards a single strategy or answer, but instead asked them to share their approaches to solving the problems. Towards the end of the 30 minutes, Ms. C would often have the students work on problems independently, and then they would all check-in after and share their answers and strategies. On a typical day, the students would progress through multiple workbook pages using this structure. The structured mathematics did not ever go beyond the 30 minutes because Ms. C had other students joining zoom before and after.

Features of Mathematics Instruction were Similar

The following vignette showcases one lesson during Ms. C's online instruction focused on using and understanding graphs and drawing on these representations to solve word problems. In this example, Ms. C created opportunities for her students to develop conceptual understanding, focusing on connecting multiple representations. Similar to her in-person instruction, she also used various teacher moves, including eliciting students thinking and asking questions (open-ended and guiding) to highlight important features of their work.

Vignette 1 (Part 1): Connecting multiple representations using graphs, data, and word problems. Students were asked to use a chart identifying the favorite colors of pretend students from the workbook (Figure 3).

Figure 3

Sample Workbook Page on Representing Data

Name ____________________ Date ______________

A group of people were asked to say their favorite color. Organize the data using tally marks, and answer the questions.

Red	
Green	
Blue	

1. How many people chose red as their favorite color? ________ people like red.
2. How many people chose blue as their favorite color? ________ people like blue.
3. How many people chose green as their favorite color? ________ people like green.
4. Which color received the least amount of votes? __________
5. Write a number sentence that tells the total number of people who were asked their favorite color.

____

EUREKA MATH | Lesson 10: Collect, sort, and organize data; then ask and answer questions about the number of data points. engage^ny 144

Ms. C started this lesson by reminding the students that their work was related to the work they did the previous day when they made a chart representing their favorite games. After this connection, she asked the students, “what do we see on this page?” She prompted the children after they listed things they noticed to “read the chart” before they started on the problems. Ms. C encouraged the children to get in the habit of making sense of representations before they started working with the data through comparisons, addition, and subtraction problems. By doing this she highlighted an important practice by giving the students an opportunity to connect the graphical representation to the data, equations, and comparisons. She also supported students in learning how to approach word problems by reading to understand the problem, identify what is being asked, and using tools like manipulatives or drawings to solve the problem. As the students made sense of the representation, they noticed features such as the labels and descriptions on the chart (red, green, blue) and quantities that were represented. Ms. C then prompted the children to practice representing number with tally marks and reminded the students that they needed to stay organized.

Ms. C: We are going to read the answer and then make the tally marks. Do we remember how tally marks go?

Yu: [Shows tally marks on her paper]

Ms. C: Yes, it’s a tall straight line. Emily, can you wait to go ahead, and then you can go. I want to talk about some strategies to do this. So, I grabbed three colored crayons. It seems a little overwhelming. How can we keep it organized?

Emily: While you count the colors you can cross them off

Ms. C: You can cross them off, yeah. Yeah, Yu?

Yu: [inaudible]

Samuel: We can color them.

Ms. C: Yeah, so Emily said you can cross them off and Samuel said you can color them. So, I am going to draw a line to connect the student to the word. It's up to you friends, we can do the tally mark as we write them, or we can go back and count them. It's up to you. So, you all can go ahead and start to create your tally marks. The only thing I am going to give you is a reminder, what happens when we get to five tally marks?

Dan: We cross one tally mark across the other ones.

Ms. C: Yeah Dan, we cross of the others with the diagonal.

Samuel: I see two green…

Ms. C: [interrupting]I am going to stop you because I don't want the answer yet. Some people move fast, and some go slow. So, I am going to have you hold on to wait for everyone. It's important when we look at graphing and data that we slow down and do our best.

Vignette 1 (Part 2): Discussing the commutative property of addition. After this introduction, the students worked through the word problems for this lesson that asked a variety of questions related to identifying quantities from the graph (how many people chose green as their favorite color?), comparison problems (which color received the least amount of votes?), and problems asking students to represent the data from the graph using a number sentence (write a number sentence to tell us how many people were asked about their favorite color.). As the students worked through representing the data from the graph using an equation Ms. C asked the students about ordering the numbers in a way that was different than the way they were shown on the graph.

Dan: So, we write a number sentence like 6+2+5=blank.

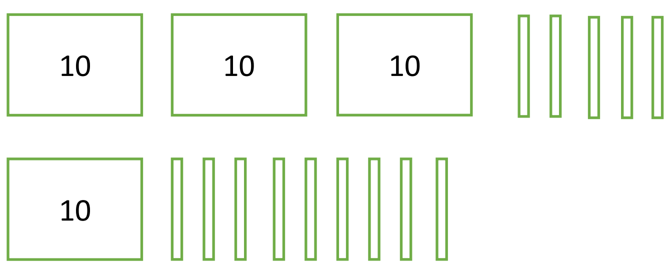
Ms. C: What if someone did 2+5+6=blank. Would this be the same answer?

By asking this question, she prompted the students to think about the commutative property of addition problems - when adding numbers, the sum is the same regardless of the order. As the lesson progressed Ms. C prompted the students to identify features of another graph and using the representation to help solve word problems in the workbook. Through this lesson, Ms. C prompted the children to make sense of graphical representations, use the representations to answer word problems, practice skills like representing numbers using tally marks and organizing data, and think about the properties of addition problems. While the lesson in the workbook asked students to practice word problems, Ms. C elaborated on this when she prompted the students to make connections between the graph and the questions that were posed, had them practice making sense of the representation and had them think about important features of mathematics. This vignette illustrates the way Ms. C used the representations in the workbook (graphs, drawings, and notation of numbers) to create opportunities for the students to develop conceptual understanding. This vignette shows that while this activity looked different from the ways Ms. C often supported conceptual understanding by connecting multiple representations during in-person instruction, she used the available materials coupled with discussion around representations to support conceptual understanding online.

Vignette 2: Establishing norms and eliciting student thinking online. In many ways mirroring the classroom norms that were established in-person, Ms. C established socio-mathematical norms during online instruction about mistakes and efficiency, as well as participation norms about taking risks and sharing thinking about the process, instead of a focus on the answer. In many instances, Ms. C reminded the students that "it's ok to make mistakes" and "mistakes are how our brains grow." She encouraged students to talk through their thinking. In the following vignette, Ms. C used a student's response about the "number of objects" to create opportunities for students to think about features of numbers, place value, and grouping by tens. Students were asked to solve the problem: *Lisa has 3 boxes of 10 crayons as well as 5 extra crayons. Sally has 19 crayons. Sally says she had more crayons, but Lisa disagrees. Who is right?* Ms. C prompted the children to think about ways to solve the problem when she asked them, "what is the best way to find out?" The students identified that they could draw the problem. Ms. C drew on her workbook (that the children could see over Zoom) based on what the children told her (three boxes with the number 10 on it and five lines) (Figure 4). She then asked them how many Lisa had. The students identified that Lisa had 35 based on the drawing. Ms. C then drew one box labeled 10 and an additional nine lines to show what a student in her class identified. The students discussed the answer about who was right. One student observed that they "could" both be right if they were just counting the objects because Lisa had 8 objects and Sally had 10. Ms. C did not disregard this contribution but rather asked the students what the boxes in the drawing represented. The class agreed that the boxes represented a quantity of 10. Another student made a connection between the drawing and using a 10 frame to represent quantities.

Figure 4

Ms. C's Drawing of the Word Problem (Lisa's and Sally's Crayons)



Through this vignette, Ms. C used a drawing to represent the quantities and actions from a word problem. The students had opportunities to practice grouping numbers by 10, an important practice when making meaning of place value, and a strategy to differently represent quantities with manipulatives or drawings. Students then had visual representations to compare the two quantities in their efforts to answer the question. This practice of representing quantities with pictures or manipulatives and doing the actions asked in word problems (adding, subtracting, comparing) is another way that Ms. C created opportunities for students to develop conceptual undersigning. The norms Ms. C reflected in this example included asking students to share their thinking, taking up student contributions (even when they are "wrong"), and representing quantities efficiently. During this lesson and many others, Ms. C often asked students who wanted to share their answers to wait until everyone had time to think. Ms. C usually responded, "hold your answers until we are all ready" or asked questions to push students to think about their responses before sharing. The norms around math and participation made mathematics time online focused on thinking, learning, and discussing, instead of getting the right answers, memorization, and speed.

One dilemma that was evident in this vignette is the tension between students having opportunities to draw or use manipulatives and the limitations of zoom. While Ms. C prompted the students to draw the crayons, she did not ask them to share their drawings. The students may not have drawn the boxes which could be a missed opportunity to represent the problem using drawings. Beyond this, the focus of the discussion was on Ms. C's drawing. Ms. C knew how to represent the problem using the boxes and lines, whereas a student may have done this differently. This vignette illustrates the tension between having the students manipulate or draw representations and the constraints of teaching on zoom.

Vignette 4: Discussing efficiency and engaging in playful mathematics. A key feature of Ms. C's in-person instruction was the use of games during mathematics instruction. Games did not appear as frequently online as they did in person. When I observed in the classroom, I saw four math games, and Ms. C told me about a fifth during in-person instruction compared to one math game used during my observations of the same module during online instruction. However, Ms. C still used games online as a fun and interactive way to draw connections to children's interests and lives outside of school. On the 100th day of school, she had the students play a game, *Race to 100*. She started the lesson by asking about efficient ways to count to 100. The students offered various strategies (count by one, count by 10) and talked about the fastest way to get to 100. Then, the students and Ms. C played the game. She altered the game from partners to teams. Ms. C was on one team and the five students were on the other team. The teams took turns spinning the spinner and progressing towards 100 on the game board. Ms. C used a document camera to show her spinner and the students had spinners that were sent home in packets. When the designated math time ended, Ms. C suggested that the children play the game at home with their families.

During Ms. C's in-person instruction she used games as low-stakes activities that were familiar to the students (playing, working with partners, using a game board, taking turns) to give them opportunities to practice mathematics (counting, adding, subtracting, and comparing numbers). In the same way, Ms. C used the *Race to 100* game as a fun way for students to practice counting and think about efficient ways to get to 100. For example, the students frequently got a "1" on the spinner and could only move one space each time. Ms. C spun higher and progressed towards 100 much faster than her students. She highlighted that her placement on the game board was farther along than the students because she was getting bigger numbers on the spinner. She reminded them that it took fewer turns to get to 100 if the number was larger (e.g., getting a five instead of a one). This gave students a way to think about efficiency and compare ways to progress on a gameboard (e.g., moving one space at a time versus moving five spaces at a time). This vignette shows how Ms. C included playful mathematics into her instruction online. In comparison, she did not use games or playful activities with the same frequency online.

Vignette 5: Using teacher moves like questioning and eliciting student thinking to support an EB during instruction. During Ms. C's online instruction, the small group I observed included one EB student, Samuel. The way Ms. C frequently called upon Samuel, asked for explanations, and focused on his display of mathematical understanding rather than using correct language, reflects some of the ways that Ms. C created opportunities for Samuel to participate and allowed him to use his linguistic resources to engage with mathematics. In the following vignette I highlight how Ms. C used questioning, wait time, and eliciting and attending to student thinking to engage Samuel during mathematics instruction.

During one of the first lessons I observed, Ms. C highlighted a mathematical term, “compare”, after introducing the terms longer than and shorter than. She said:

> “We are going to use another math word. That word is ‘compare’. Instead of using the word ‘look’, we are going to use the word compare. [After the students started doing some comparison of pencils on the page, Ms. C had the students draw the pencils.] Is it going to be longer or shorter? How do I answer the question?”

In response to this question another student answered Ms. C’s question, then Ms. C asked Samuel, “how do we put that in a sentence?” Initially, Samuel did not respond. Ms. C waited for a few moments (using wait time) and then continued the conversation with the students. As the students and Ms. C talked about the workbook problems, Samuel raised his hand and told Ms. C, “Nigel’s is longer than Corey’s.” In this interaction, Ms. C invited Samuel into the conversation and gave him time to respond. Samuel then used the math terms Ms. C introduced (longer than) to make a sentence about the problem they were working on. As the class moved onto the next page, Samuel continued to raise his hand and answer the questions. After a few more questions Ms. C asked Samuel to explain how he knew one object was shorter than the other. Samuel explained that he did it the same way as another student. Ms. C asked him to show her, so Samuel counted grid lines behind the picture to show that one was longer than the other. As she did in this vignette, Ms. C asked Samuel to share his strategies and answers throughout every activity. Ms. C consistently asked Samuel to contribute to the conversation. She asked him to read math problems, use math terms, share his answers, and to explain how he got specific answers.

In another activity, the students were asked to compare and order different animals based on their size (as depicted in the image). Ms. C started the lesson by having Samuel read the first word problem. Instead of asking for the answer, she asked the students, “does anyone want to tell us what they think?” Following this, the students discussed how they could order the 3 animals. The students talked about bears walking on four legs and monkeys being bigger if they were an ape but expressed some confusion because of the way the animals were depicted on the paper and how the question included some given information about the various heights of the animals. Then Samuel contributed, “the bear is shortest than the monkey” Ms. C said, “oh yes because it tells us [in the word problem]”. In this interaction, Samuel did not use the correct form of short to make a statement about the bear and monkey. Rather than highlighting this error, Ms. C expressed agreement with Samuel’s statement and used this to clarify the problem for the other students.

While I did not count the number of contributions from Samuel and the other students, Samuel contributed during every lesson. His contributions often followed being asked to read or share, although sometimes Samuel raised his hand and asked to contribute. During the interviews and check-ins with Ms. C she identified that mathematics instruction should provide EBs opportunities to participate and talk and that she should focus on math, rather than correct language, from EBs math explanations. In her interactions with Samuel, she clearly provided opportunities for him to participate. Beyond this, she explicitly told me that even when Samuel did not use the right words, she felt that he understood the mathematics. The way Ms. C enacted curriculum during mathematics aligned with recommendations from research for broadening participation for EBs by supporting both mathematics and language learning in the classroom (e.g., Fernandes, 2020; Moschkovich, 2013).

These vignettes illustrate some of the ways Ms. C’s instruction was similar in-person and online. Both in-person and online, Ms. C focused on supporting conceptual understanding by connecting multiple representations, she was responsive to students and used teacher moves like eliciting students' thinking, she established norms, and she drew on playful activities to make learning fun and familiar for her students. Even though Ms. C's features of instruction were similar, in many ways her lessons looked different. In the following section, I will provide an overview of some of the ways that Ms. C's instruction was different online.

Teaching Online: Differences from In-Person Instruction

When Ms. C shifted to online teaching, the structure of "mathematics time" was different and the work she did also looked different. Specifically related to structure, the scheduled mathematics time shifted from a predictable progression (whole class>small group or individual work>whole class) to all small group work (groups of five students at a time). The objects that were used also shifted from predominately using hands-on materials (e.g., manipulatives, whiteboards, games) to mainly workbook pages from the student handbook. Finally, when instruction shifted online each child had less time to do math with Ms. C (an average of 30 minutes per day online vs. 40.9 min per day on average during in-person). Online, Ms. C had less autonomy to supplement the official curriculum and pace lessons. School administrators told teachers that there could be changes to students' classroom assignments once they transitioned back to in-person instruction. It was unclear what the district and parents would decide related to structuring this transition. There were conversations about giving parents the option of keeping their children home with remote instruction, moving them to a hybrid structure (some in-person and some online), or having them return entirely in-person. There were other conversations about implementing cohorts with smaller groups of students that would come to school on a rotating schedule. Given the uncertainty of how children would transition to in-person instruction, the teachers paced the lessons the same across the school to ensure that all the students had covered the same content. This meant Ms. C had less autonomy to skip ahead, bring in supplemental materials, or slow down to revisit topics that children were struggling with.

The Structure of Instruction was Different.

The structure of Ms. C's mathematics instruction was different online. Ms. C had a very predictable and consistent structure in person which I described at the beginning of this chapter (Figure 2). For Ms. C's online instruction, the structure was consistent across the lessons (small group) but differed from the structure she used during in-person instruction. Since Ms. C met with her students in groups of five during online instruction, the structure she used during the entire scheduled mathematics time was working with the whole group of five students. Technically Ms. C did not meet with all 20 students in her class at the same time on Zoom. When she met with the groups of five, she progressed through the lessons together with all of them. She did not have some students work independently while she worked with others.

Throughout the lessons, Ms. C had discussions and worked through lessons with everyone in the group.

The Use of Tools Looked Different

Another difference between online and in-person instruction was related to the use of tools. During in-person instruction, Ms. C typically started her lessons by having the students use hands-on manipulatives (unifix cubes, collections of objections, base-ten blocks) and/or tools for writing and drawing (whiteboards, laminated 100s charts). Once the students worked with these materials and discussed them as a group they would then return to their desks and either work on a worksheet from the official curriculum workbook or work with small groups or a partner playing a game. At the end of most of the in-person lessons, Ms. C would have all the students go back to the carpet for whole group check-in when she would either revisit the mathematics or get feedback from the students about how the lesson went.

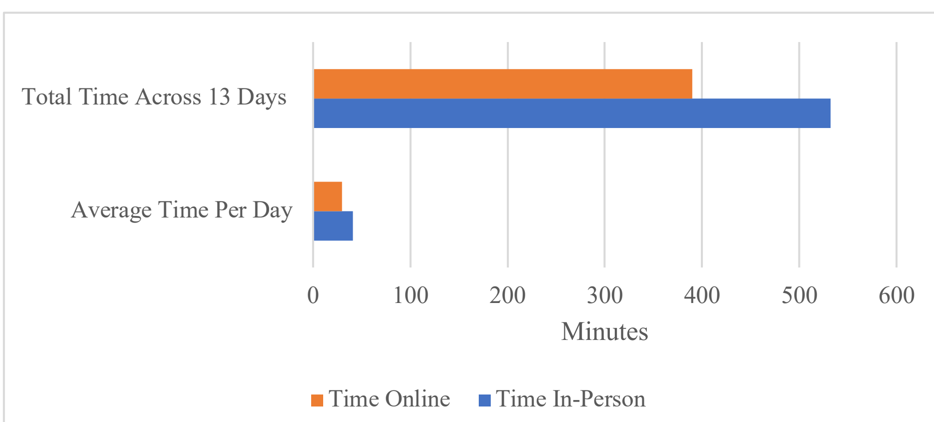
Online, the use of manipulatives and hands-on materials was little to non-existent. While some manipulatives were available for families to pick up (e.g., the printed game board and spinner for the *Race to 100* game), many of the manipulatives that Ms. C frequently used during in-person instruction (e.g., unifix cubes, base-ten blocks, mini whiteboards, counting collections) were not provided by the school. These manipulatives cost a great deal of money. Ms. C purchased and accumulated many of the manipulatives in her classroom over her many years of teaching. It was difficult to afford and distribute these items to her students during COVID-19. Therefore, Ms. C had her students use items from their homes (e.g., pieces of paper, erasers, markers) and things that could be printed (e.g., game boards, spinners) or easily distributed (post-it notes and index cards) for students to have some sort of hands-on experience during mathematics time. During one lesson the students used a piece of paper as a tool for measuring and comparing and during another lesson the students had a spinner that they used to play the *Race to 100* game. Other than that, the students worked together on the workbook pages from the student workbook. Whereas Ms. C often used one workbook page (front and back) per lesson in-person, during online instruction the students progressed through multiple pages in their workbook each day. It is important to note, that given the structural differences of teaching online, the workbook page served as a collaborative written record. This in many ways mirrored the function of a whiteboard during in-person instruction. This written record acted as a shared space for meaning making and the focus of many conversations were on this "object." Since the online conversations centered on the workbook, rather than supplemental activities that drew on hands-on materials, progressing through multiple workbook pages was possible. This difference has implications for curriculum materials and pacing guides, which is discussed later in this paper. Beyond the difference in how students used tools online, the total time spent on mathematics instruction was different online.

Time Designated for Mathematics was Shorter Online

During online instruction, students were given less structured time for mathematics (Figure 5). During online instruction, students participated in mathematics for 30 minutes, two times a week. During in-person instruction, structured math time lasted an average of 40.9 minutes per day and occurred four times a week. While my observations for both in-person and online included a total of 13 days in each format, the entirety of the 13 days during in-person instruction was related to one unit on counting and comparing numbers (Module 4). During the 13 days of observation online, Ms. C completed an entire unit on measurement (Module 3) and got about halfway through the unit on counting and comparing numbers (Module 4).

Figure 5

Comparison of Structured Mathematics Time Online and In-Person



Ms. C's Work was Different

Ms. C told me that her work was different online. Ms. C was selected as the lead mathematics teacher across the first-grade in her school. This meant that she helped facilitate meetings and guide teachers around instruction. This did not give her total control over the mathematics content and pacing, but she did have a leadership role in making these decisions. All the first-grade teachers had frequent meetings about what to cover and the pacing of lessons. This spurred conversations about how quickly to cover content, what content should be covered and how to facilitate the instruction. These meetings created opportunities for the teachers to collaborate around teaching and learning in ways that were not done at this school prior to the move to online instruction.

Beyond the collaboration, the teachers at this school had to cover the same content using the same activities and at the same pace. For Ms. C this meant that she had less autonomy to circle back to topics that students were struggling with or move ahead when students demonstrated understanding and mastery and could not use supplemental activities unless they were agreed upon by the group of first-grade teachers. In part, this was due to the uncertainty of returning to in-person instruction. At this school, teachers were told by the administration that there was a possibility that some students would be placed in different classes with different teachers depending on parents' decisions about returning to in-person instruction. Additionally, the school was navigating decisions around offering different options for attending school (i.e., fully in-person, hybrid in-person and online, or fully online).

Ms. C's autonomy was also impacted because the supplemental materials that Ms. C frequently used during her in-person instruction (e.g., whole class discussions, using manipulatives, and using math games) had to be agreed upon by all the teachers or they would likely not be used. For example, Ms. C and the teachers agreed that on the 100th day of school the students would play the game, *Race to 100*. Comparing the use of games in Ms. C's instruction online and in-person, Ms. C more frequently used games when she taught in-person (five games throughout the module in person compared to one game throughout the module online).

Another, and possibly the most impactful way that Ms. C's work differed, was the reality and changes of everyday life for Ms. C and her students caused by COVID-19. During in-person instruction, Ms. C and I would often check in at the end of the school day and talk about plans for future lessons, how lessons went, students' progress, and sometimes even about events in our lives (e.g., going out of town, getting our hair done, exercising with family members). Online, these check-ins at the end of lessons were more frequently about the stress of uncertainty, the fear of getting sick and dealing with sick students, frustration about the accessibility of the vaccine, and unclear guidance from the school and district. At the beginning of my observations, Ms. C shared that the school was unsure of how and when the students would return to the classroom. Then halfway through my observations (Day 7), Ms. C shared that students would likely go to the same teacher unless there was a special circumstance (e.g., a child remained fully online). In preparation for students returning to in-person, Ms. C shared how her school approached recess. She explained that different classes would only be allowed to play in designated play yard areas. For example, her class could play on the play structure on Mondays and Wednesdays, the blacktop on Tuesdays and Thursdays, and the Field on Fridays. Keeping children and teachers safe became a top priority, so teachers, administrators, and parents were in constant communication and deliberation about how best to do this. I did not have an opportunity to continue my observations of Ms. C's instruction once schools transitioned to in-person. This was due to strict policies that restricted any non-essential personnel from entering the school campus. From conversations with Ms. C, I learned that many precautionary measures were put in place to prioritize students' and teachers' health during this transition. This included mask mandates, social distancing, frequent COVID-19 testing, and reminders to practice hand hygiene.

Ms. C's Online Teaching Practices Did Not Align with Her Belief Related to a Fixed Instructional Plan

I described Ms. C's professed beliefs as they related to the belief constructs (Schoen & LaVenia, 2019) in Chapter 2. To review, Ms. C's beliefs aligned with low “transmissionist”, low “facts first”, and low “fixed instructional plan” views. When looking at Ms. C’s teaching practice in-person, I found that Ms. C’s in-person teaching practice aligned with her professed beliefs. However, Ms. C’s online teaching practices did not align with her low “fixed instructional plan” belief. Through comparison of her teaching practices, I found that in-person Ms. C routinely brought in supplemental activities, skipped lessons, or revisited topics based on the needs of her students. For example, I observed Ms. C supplementing the official curriculum in person at least once a week by including mathematics games. Online I observed Ms. C more closely following the scope and sequencing of the curriculum materials. She had multiple pages of the student workbook that she covered each day and asked the students to complete pages at home if they did not get to cover all of them during the zoom meetings.

During informal conversations at the end of my online observations, Ms. C explained her reasons for following the pacing more closely to the curriculum materials. She explained that due to the uncertainty of returning to in-person schooling, Ms. C was guided by the administration to cover the same content as the other teachers across first-grade. To do this, Ms. C had routine meetings with the teachers where they discussed the lessons and pages they would cover each week. If supplemental materials were brought in, they had to be used across all classrooms. For example, on the 100th day of school, the teachers agreed to use a game (Race to 100) during mathematics time. This was the only supplemental material that I observed during Ms. C's online teaching.

Constraints that Impacted Ms. C's Teaching Practice

Although Ms. C's online instruction looked different in many ways, the central features of instruction that I discussed in Chapter 3 persisted online even when the activities looked different. Although the supplemental materials that Ms. C brought into her in-person instruction were absent online, the smaller, still frequent strategies that Ms. C used during mathematics instruction still positioned the students, in particular the EB Samuel, as competent and created opportunities for frequent and varied participation. Ms. C's online teaching continued to reflect an asset-based approach to teaching mathematics with children in early grades. Identifying what students bring with them to the classroom as assets, instead of deficits, supports mathematics learning for children who come from cultures or communities different from the teachers' (Moschkovich & Nelson-Barber, 2009). Despite the persistence of the central features of Ms. C's teaching practice both in-person and online, there were some tensions that I observed during Ms. C's online mathematics instruction.

Through comparison of online and in-person teaching practice, I observed that Ms. C faced constraints online that I did not observe during her in-person teaching. The main constraint that I observed included unclear guidance and uncertainty from the administration as teachers navigated future plans for transitioning from online back to in-person teaching. This constraint resulted in limiting Ms. C's autonomy to supplement and adjust the content that she covered online. Ms. C's teaching practices online looked different as she taught with more fidelity to the scope and sequencing of the curriculum materials. The way her practices did not align with her low "fixed instructional plan" belief demonstrates how the constraints she faced online impacted her teaching. In the following section, I discuss the constraints Ms. C faced and how they created tensions that Ms. C had to navigate during her online teaching.

Tensions Related to Teacher Autonomy

One clear tension that was evident in Ms. C's teaching when she transitioned online related to her autonomy. Limiting teacher autonomy is not new or specifically related to online teaching. Previous research has explored the tensions between teachers' autonomy and institutional constraints even before the pandemic suddenly shifted schooling online. These constraints can include school and district-level policies about using a scripted curriculum (Parks & Bridges-Rhodes, 2011), or enforcement policies about pacing and fidelity to curriculum materials (Pease-Alvarez & Samway, 2008). Unlike this previous research, Ms. C's instruction did not look different online because of a new, heavily scripted curriculum program. Rather, constraints that Ms. C experienced came from administrators dictating how teachers should cover curriculum online. Online I observed that Ms. C did not adjust the scope and sequencing of the curriculum materials in the same ways that she did during her in-person teaching. Further, I observed that online Ms. C did not pace lessons in ways that reflected the needs of her students, she did not bring in supplemental materials, and she did not extend scheduled mathematics time. Instead, Ms. C more closely taught with fidelity to the scope and sequencing of the curriculum materials during her online teaching. Recent research has found that teachers faced challenges during the pandemic related to professional time constraints, personal circumstances, supporting student engagement, navigating technologies, attention to equity, lack of support, and lack of direction from leadership/administration (DeCoito & Estaitayeh, 2022b). In the case of Ms. C, there was both a lack of direction related to the return to in-person schooling as well as an increase of administrative control that dictated that teachers had to teach with more fidelity to the official curriculum. In the next section, I discuss how Ms. C's online teaching practices were less aligned with low "fixed instructional plan" belief because of these institutional constraints.

Online Ms. C Did Not Supplement or Adjust the Curriculum Materials

During online instruction Ms. C did not bring in supplemental activities or adjust the curriculum materials in the same way she did during her in-person teaching. Previous research has found that curriculum materials like pacing guides are often enforced to make sure teachers adhere to curriculum materials with fidelity (Pease-Alvarez & Samway, 2008). Having strict pacing and other prescriptive curriculum materials may not meet the needs of students. In one study, Pease-Alvarez & Samway (2008) explored the beliefs and practices of a group of teachers when a district-mandated reading program was implemented and enforced. They found that teachers' practices and agency changed. Due to the scripted program, the teachers focused less on the needs of their students and more on the instruction that was dictated by the teaching manual. Beyond this, all the teachers reported they felt a loss of teacher agency and autonomy over their teaching (Pease-Alvarez & Samway, 2008).

The supplemental activities that Ms. C facilitated during in-person instruction, even when she extended lessons or was not perfectly aligned with the pacing guide, provided valuable opportunities for students to connect multiple representations, explain their thinking, and use hands-on materials to make meaning for concepts that were covered. For example, Ms. C often included hands-on objects for her students to manipulate during mathematics instruction. Connecting mathematical ideas to objects or other visual representations supports learning conceptually (Carpenter et al., 1993; Hiebert & Carpenter, 1992). Aligned with supporting mathematics learning through engagement with various manipulatives and play, young children need to explore mathematical concepts through various representations. For example, opportunities to explore various representations of number supports the abstraction of the concept of number (Powell & Nurnberger-Haag, 2015). The supplemental activities Ms. C used in-person were often the place of discovery for children as they talked about things they noticed, manipulated objects, and created connections. Given the structural constraints of online instruction (less time and different access to tools like manipulatives), students were given fewer opportunities to delve deeply into mathematics during online instruction.

Online Ms. C Covered More of the Workbook Pages

While the actual time that students had opportunities to participate in scheduled mathematics time was shorter, the pace at which they progressed through content was much faster. During in-person instruction, the students only completed one page from their student workbook each day, and this was only a part of the in-person instruction. Online students covered multiple pages in their student workbook each day. Comparing the scheduled mathematics time in-person and online, Ms. C spent over five weeks on just one module (Module 4) during her in-person instruction, and students had an average of 41 minutes four days a week to do mathematics. Online, the students progressed through the same module (Module 4) in about three weeks and did this during 30-minute sessions twice a week. On a surface level, this may indicate “progress” because Ms. C covered more of the workbook, yet the depth in which they covered materials was not the same. Students completed more pages of the student workbook and Ms. C may have “progressed” more quickly through the units, yet the opportunities for meaning-making and developing connections among representations were not the same online.

No one other than Ms. C, her students, and me were in the zoom meetings that I observed. Ms. C could have resisted teaching with fidelity by changing the lessons or bringing in supplemental activities like she did in-person. Adjusting and supplementing the official curriculum was a teaching practice that I frequently observed during Ms. C's in-person instruction. Creative insubordination (Gutiérrez, 2016) and subtle resistance (Pease-Alvarez & Samway, 2008) have been documented in classrooms when teachers navigate limitations to their autonomy. Pease-Alvarez and Samway (2008) found that teachers participated in subtle acts of resistance to a scripted literacy program by "tweaking" or not using the program behind closed doors. During my online observation I did not see instances of resistance to the constraints that Ms. C faced online. I do not have data about why Ms. C more closely followed the curriculum materials beyond what I discussed here; however, I make some inferences about permeating ideologies related to "preparedness" in the discussion chapter (Chapter 5).

Extending Previous Research

This study adds to existing literature that has looked at the institutional constraints that can impact teaching such as scripted curriculum materials and enforced pacing guides (e.g., Parks & Bridges-Rhodes,2011; Pease-Alvarez &

Samway, 2008). Beyond this, this study also contributes to the more recent focus on the impacts of COVID-19 and the shift to online schooling (e.g., DeCoito & Estaitayeh, 2022a; DeCoito & Estaitayeh, 2022b). Institutional constraints (teaching online, enforced pacing guides) constrained Ms. C's teaching practices in spite of her low "fixed instructional plan" belief.

While Ms. C's teaching practices in each setting looked different in some respects and similar in others, the central features of instruction were similar in both modes (online and in-person). This complicates previous work that found that teachers prioritized covering content and not pedagogical practices when they moved to online teaching (DeCoito & Estaitayeh, 2022a). This study illustrates how Ms. C did both, covered content and prioritized pedagogical practices. Ms. C's practices online did not reflect a teacher-centered or procedural approach to mathematics, rather she still created opportunities for students to develop conceptual understanding by connecting representations. She also continued to employ many teacher moves such as eliciting student thinking and using questions to support student engagement with activities. This study also contributes to the field as it focuses on teaching practices; much of the work during COVID has focused on teacher beliefs, not on practices (DeCoito & Estaitayeh, 2022a; DeCoito & Estaitayeh, 2022b).

Conclusions and Implications

This study compares the in-person and online mathematics instruction of one accomplished, first-grade teacher, Ms. C, with first-grade students. The research questions that guided this work included: 1) What were the differences between classroom routines and mathematics activities in person compared to online during COVID-19 in an early grades classroom? 2) Did Ms. C enact math instruction online that aligns with her professed beliefs? If so, how? And 3) What is the nature of online math instruction for EBs in Ms. C's classroom?

Similarities and differences emerged by comparing Ms. C's in-person and online instruction. The features of Ms. C's mathematics instruction (i.e., teaching for conceptual understanding, using teacher moves, establishing norms, and using students' experiences outside the classroom) both online and in-person were similar, although there were differences in how these features looked. There were also some clear differences between Ms. C's teaching including the structure of "mathematics time" and her work. Specifically related to structure, mathematics time went from a predictable progression (whole class>small group or individual work>whole class) to all small group work (groups of five students at a time). The objects that were used also shifted from predominately hands-on materials (e.g., manipulatives, whiteboards, games) to mainly workbook pages. Finally, when instruction shifted online, each child had less time to do math with Ms. C (an average of 30 minutes per day online vs. 41 min per day on average during in-person). Related to work, Ms. C had less autonomy to supplement the official curriculum and pace lessons. Given the possible structure of how children would return to in-person instruction, the teachers agreed to pace the lessons the same across the school. This meant Ms. C had less autonomy to skip ahead, bring supplemental materials, or slow down to revisit topics that children were struggling with. The similarities across Ms. C's instruction suggest that the central features of Ms. C's teaching persisted online. However, her lessons looked different and there were institutional constraints that impacted how she enacted her beliefs related to a fixed instructional plan.

This work has implications for teacher education and professional development. In particular, teachers need to be viewed as professionals and given the resources they need to support all their students. Policy and teaching materials should address the needs, interests, and understanding of students and move away from prescriptive approaches like scripted lessons and pacing guides meant to enforce teacher implementation with fidelity (Pease-Alvarez, Samway, & Cifka-Herrera, 2010). Policy needs to afford teachers the autonomy to teach their students in ways that support learning. Beyond this, teachers need consistent and ongoing professional development that supports them as they navigate challenges related to their teaching.

Chapter 5: Discussion and Implications

The purpose of this dissertation was to explore the beliefs of early grades educators related to mathematics instruction and the mathematics teaching practices of one teacher. Specifically, I examined how 20 teachers discussed students' assets and how one teacher enacted her mathematics instruction by drawing on her students' assets. Given the timing of this dissertation project, the design and focus of this work drastically changed due to COVID-19. Therefore, the primary impacts on this work stem from the uncertainty of teaching during the pandemic, the lack of preparation for teachers to shift to online teaching, and challenges with supporting opportunities for students to learn mathematics with understanding through online instruction during a pandemic.

When schools shut down in-person operations, teachers were required to shift their instruction completely online with as little as one day's notice. For many teachers, this may have meant that they assumed that they had to reinvent all their teaching practices. Prior to school closures, I walked into classrooms in early grades and saw portraits of students hanging on the walls, desks with crayons and name tags, blocks and games in boxes, and many books. Mathematics teaching practices in many of these classrooms included having children do things with manipulatives, counting and moving objects, writing on whiteboards, and cutting and gluing shapes. Online, using manipulatives, hands-on play and interaction was difficult, and from what I observed in Ms. C's classroom, not as frequent as during in-person lessons. Children had to learn how to work on computers, tablets, or phones, how to mute and unmute, and how to stay focused on a screen with their teacher and classmates in tiny boxes over Zoom.

The pandemic has been documented as having devastating effects related to school access and quality on groups that were already underrepresented and underserved in schools. One longitudinal study of access to technology and live teaching during the pandemic has shown that students from lower socioeconomic backgrounds often had less access to effective internet and technology and fewer interactions with live teachers throughout the school shutdowns (Haderlein et al., 2021). I was originally particularly interested in mathematics instruction for EBs. Due to the changes in the design of the dissertation and the availability of participants, I could not observe teaching practices in classrooms or online with EBs for this dissertation. Instead, I switched to exploring teachers' beliefs about language and mathematics instruction for EBs, and, in the one online classroom I did have access to, I focused on observing that teacher's online instruction with one EB. While the results of the analyses cannot be generalized to supporting mathematics learning for all students or all EBs, the vignettes I analyzed in Chapters 3 and 4 can be used to ground conversations with teachers and support them as they learn to use pedagogical practices for supporting mathematics learning for all students. In this discussion chapter, I first summarize the three analyses and findings from Chapters 2, 3, and 4; I then discuss connections to existing research; lastly, I suggest some implications the dissertation might have for research and practice.

In my analysis of teachers' beliefs (Chapter 2), *Early Grades Teacher's Beliefs About Mathematics, Language, and Emergent Bilinguals*, I explored the research question: what are early grades teachers' professed beliefs about mathematics, language, student thinking, students' out-of-school experiences, and students' home and everyday language practices, in particular for EBs? I documented teachers' professed beliefs related to mathematics and EBs through one survey and one interview. I was particularly interested in characterizing teachers' beliefs about mathematics (Schoen & LaVenia, 2019) and their beliefs about language (Fernandes, 2020). Through analysis of teachers beliefs, I found that the 20 teachers in this study held varying degrees of asset-based views of EBs. All the teachers responded to the survey with at least 74% of their non-neutral responses in ways that reflect an asset-based view. I identified and sorted teachers' total percentage of asset-based responses on the survey across four categories which include 1) some asset-based views, 2) many asset-based views, 3) most asset-based views, and 4) all asset-based views. Three of the participants (15%) responded in ways that 70-76% of their responses reflected asset-based views (Some asset-based views group). Two of the participants (10%) responded with 80-89% of their responses reflecting asset-based views (Most asset-based views group). Most of the participants fell into the third category, the Many asset-based views group, with 12 of the 20 teachers (60%) who responded with 90-95% answers that aligned with an asset-based view. Three participants (15%) responded with 100% of their non-neutral answers in ways that reflected asset-based views (All asset-based views).

In addition to what I found in the survey responses, the interviews clarified and provided more detailed descriptions of their beliefs. From the interviews, I found that teachers held beliefs about students' assets and teaching mathematics with EBs related to students' everyday and home language, students' backgrounds and experiences, mathematics vocabulary, and supporting EBs. Teachers described their views on using students' assets in two ways: allowing students' assets in the classroom and drawing on students' assets for mathematics learning.

In the analysis of teaching practices during in-person instruction (Chapter 3), *An Account of an Accomplished Teacher's In-Person Mathematics Instruction in a First Grade Classroom: Drawing on Students' Assets*, I explored the nature of in-person mathematics instruction during five weeks in Ms. C's first-grade class. The research questions that guided this analysis include: what was the nature of mathematics instruction in a first-grade classroom with an accomplished teacher? and, how did an accomplished teacher draw on students' assets (student thinking and experiences)? This analysis provided evidence that Ms. C 1) created opportunities for students to develop conceptual understanding, 2) used teacher moves and was highly responsive to students and their thinking, 3) established norms around participating in the class, making mistakes, and efficiency, and 4) drew on students' experiences outside of the classroom.

During that period of in-person instruction, there was evidence that Ms. C's teaching practices aligned with her professed beliefs documented through the survey and interview. In particular, her teaching practices reflected the low "transmissionist," low "facts first," and low "fixed instructional plan" beliefs documented in previous research (from the belief constructs, Schoen & LaVenia, 2019). The central features of Ms. C's in-person teaching practices also aligned with research-based recommendations for effective mathematics teaching (e.g., Hiebert & Carpenter, 1992; Aguirre et al., 2012; Moschkovich, 2013; Turner et al., 2016; Wager, 2013). The vignettes in Chapter 3 provide detailed examples of her teaching practices and illustrate how Ms. C drew on students' assets to create opportunities for mathematics learning with understanding.

In the analysis comparing in-person and online teaching practices (Chapter 4), *A Comparison of Mathematics Instruction In-Person and Online with First-Grade Students*, I described how Ms. C adapted and facilitated mathematics instruction online with first-grade students, including one EB. I explored the research questions: 1) What were the differences between classroom routines and mathematics activities in person compared to online during COVID-19 in an early grades classroom? 2) Did Ms. C enact math instruction online that aligns with her professed beliefs? If so, how? 3) What was the nature of online math instruction for the EB in Ms. C's classroom? My observations support the claim that most of the central features of Ms. C's mathematics instruction documented during in person teaching (i.e., teaching for conceptual understanding, using teacher talk moves, establishing norms, and using students' experiences) were similar, even when the lessons looked different. For example, I observed Ms. C consistently eliciting student thinking and strategies while problem-solving both in-person and online. She also used a variety of teacher moves, such as revoicing and questioning, to guide students to uncover patterns and identify information that they noticed. In terms of the professed beliefs documented in Chapter 2, two of the belief constructs observed to align with her teaching practices in person (Schoen & LaVenia, 2019), Ms. C's low "transmissionist" and low "facts first" beliefs, were also reflected in the observations of her teaching practices online. However, institutional constraints impacted her teaching practices in ways that resulted in less alignment with her low "fixed instructional plan" belief, documented in Chapter 2 and observed during in-person teaching in Chapter 3. There were structural and policy differences between the two settings, which I describe in more detail next.

Despite those differences, this analysis shows that many of the central features of Ms. C's mathematics teaching practices persisted even when she transitioned to online teaching during a pandemic with her first-grade students.

Relation to Literature

Beyond the analyses in those three chapters, I have some speculations about teachers' beliefs about students' assets and how these may have been impacted by teachers' experiences with EBs. While I did not find clear patterns related to teachers' asset-based views and their backgrounds or their students, previous research has found that teachers' experiences and the school they taught impacted their beliefs related to students' assets (e.g., Lee & Ginsburg, 2007). Lee and Ginsburg (2007) found that teachers in middle-socioeconomic status preschools were more likely to support activities that were relevant to the students' interests and were more focused on the social aspect, whereas teachers serving low-socioeconomic status preschools were more likely to highlight the importance of academics and direct instruction in preschool. That study showed that teachers' beliefs about students' assets were shaped by students' backgrounds and school characteristics (Lee & Ginsburg, 2007). The analysis of teachers' beliefs (Chapter 2) revealed differences in teachers' beliefs related to students' assets. While I did not collect enough information about the schools or demographic information about students, the findings from Lee and Ginsburg (2007) suggest that working with students from different socioeconomic backgrounds could have impacted the professed beliefs of the teachers in my study. More information about the socioeconomic status of students and schools may have illuminated some patterns around the teachers' beliefs related to asset-based views.

The analysis of teachers' beliefs in Chapter 2 corroborates Fernandes' (2020) study showing that teachers' beliefs about teaching EBs is more complex than viewing language as a problem or as a resource. The analysis of beliefs also extends Fernandes' work (2020) as his study was with pre-service teachers and this study was conducted with veteran teachers. More broadly, this analysis contributes to the research on teachers' beliefs as it focuses on teachers' beliefs about students' assets related to but beyond home language that are important for mathematics learning. The analysis of beliefs can provide insight into teachers' beliefs about a broad range of linguistic practices beyond using home language. For this dissertation, the analysis of beliefs also set up and framed the closer ethnographic look at one teacher's practices, Ms. C, who was the focus of the analysis of teaching practices in Chapters 3 and 4.

Turning to the analyses of teaching practices (Chapters 3 and 4), I found that in both Ms. C's online and in-person instruction she more frequently drew on student thinking than student experiences outside the classroom. This finding is consistent with previous research that found that pre-service teachers more frequently drew on student thinking than student experiences even after an intervention that supported teachers in eliciting and connecting to both (Turner et al., 2016). Turner et al. (2016) found that only a small group of the preservice teachers in their sample drew on both students thinking and experiences in meaningful ways. Even when teachers draw on students' experiences outside the classroom, the connections to mathematics in the classroom can change these practices and make them look different (Gonzalez et al., 2001). The analysis of in-person instruction (Chapter 3) and the comparison of Ms. C's online and in-person (Chapter 4) provides evidence that Ms. C drew on her students' assets throughout her instruction, although her practices focused more on drawing on student thinking than student experiences outside the classroom.

The analysis of in-person teaching practices in Chapter 3 makes two contributions to research. First, I used the MMKB framework for analysis to look at teacher practice and showed how it was an effective tool for exploring teaching practices. MMKB has been used previously to analyze teacher change with pre-service teachers (Aguirre et al., 2012; Turner et al., 2012; Turner et al., 2016) and so this analysis extended that framework by using it as a tool to explore the practices of an accomplished teacher. The analysis of teaching practices in-person also makes a contribution relevant to professional development as it provides a detailed example of teaching practices that draw on students' assets; the vignettes and the analysis can be used to ground discussions of teaching practices during professional development.

The analysis comparing in-person and online teaching practices (Chapter 4) revealed that Ms. C was less able to align instruction with her low “fixed instructional plan” belief documented in Chapter 2; I observed her more closely following the scope and sequencing of the official curriculum materials when she taught online during the pandemic. Parks and Bridges-Rhodes (2012) similarly found that district mandates and school policy, even before the pandemic, had impacts on teachers’ mathematics teaching practices. They found that when a school implemented a heavily scripted literacy curriculum, a teacher with a more “creative” approach to teaching mathematics resorted back to a more scripted approach (Parks & Bridges-Rhodes, 2012). That curriculum was not specifically related to mathematics, yet the heavily scripted literacy curriculum impacted one teacher’s teaching beyond literacy instruction. Even during free play, that teacher used more structured instruction, such as asking students to use complete sentences and count correctly.

This analysis comparing in-person and online instruction also contributes to the literature about online teaching during a pandemic and provides evidence that some teaching practices can persist, even when instruction, constraints, the setting, and the general context of a pandemic make many things look different. Beyond this, the analysis in Chapter 4 provides a case that documents how one teacher maintained many of the research-based teaching practices for effective mathematics teaching during the sudden shift to online instruction during the pandemic. Ms. C's online instruction did not fully switch to the more teacher-centered approach predicted in the literature with the switch to online; instead, she maintained many of the effective teaching practices for supporting learning mathematics with understanding. This result directly contradicts work that found that teachers mainly focused on covering content instead of using effective teaching strategies when they shifted to online teaching during the pandemic (DeCoito & Estaiteyeh, 2022a).

A contribution that both analyses of teaching practices (Chapters 3 and 4) make to theory relates to the way that trajectories of learning to teach are characterized and represented in models (as I discussed in Chapter 3). Knowledge and learning are typically represented through hierarchical progressions and often do not attend to equity (Lobato & Walters, 2017). The analyses of teaching practices (Chapters 3 and 4) illustrate the complexities of teaching by showcasing the various ways that Ms. C drew on students' assets during her mathematics instruction. For example, Ms. C drew on student thinking and students' experiences in ways that don't necessarily reflect one single category of the MMKB learning trajectory. Her frequent use of eliciting and attending to students may only seem like a characterization at the "initial practices" level (the primary level), yet a more wholistic view of her teaching practices reveals how her teaching practices more closely aligned with "making connections." This complicates how the MMKB framework, and any linear model, represents teaching. Drawing on students' thinking and their experiences happens dialectically while teaching, thus models should represent this dialectic relationship rather than a linear one. The developers of the MMKB model acknowledge that teacher change is not linear (Aguirre at el., 2012; Turner et al., 2012), yet the model they created can be interpreted as a hierarchical progression as it is presented in a linear fashion. The two analyses of teaching practices in this dissertation suggest that linear models for teaching practices may not capture the complexity of teaching or fully describe how teachers develop competencies in teaching.

Future Research

The analyses of teaching practices (Chapters 3 and 4) would have been improved with observations of more than one teacher. This was not possible due to the restrictions for in-person observations during the COVID-19 pandemic, and therefore the dissertation focused solely on one teacher's practices. A follow-up study could explore how other early grades teachers enact mathematics curricula with EBs and analyze features of mathematics instruction that might corroborate or extend the features found in this teacher's practices. It would be useful to examine whether and how other accomplished teachers use these or other teaching practices, as well as focus on how they support EBs in their classrooms.

This dissertation focused only on beliefs and teaching practices and did not focus on the learners. Collecting data on student outcomes would extend and clarify the impact of teaching practices and teachers' beliefs on student learning. A follow-up study could explore student learning and identity, particularly for EB students, in early grades classrooms where a teacher enacts any of the teaching practices documented during Ms. C's instruction. Some questions that could guide work looking at students include: Do EBs develop conceptual understanding when they are in a classroom with teaching practices that align with recommendations from research? Are EBs developing productive dispositions towards mathematics? Are EBs participating in activities? What does their participation look like?

Further research could also document in more comprehensive ways how teachers draw on students' assets including student thinking, experiences and interests, and linguistic resources. To frame this dissertation, I combined multiple constructs such as teachers' language orientations (Fernandes, 2020) and belief constructs (Schoen & LaVenia, 2019), with constructs from the MMKB framework (Aguirre et al., 2012; Turner et al., 2012). This shows a need for a more comprehensive way to frame research on teachers' asset-based views and practices. The MMKB framework could be expanded to include linguistic resources, including for example students' home language and their everyday ways of communicating. Additionally, the language orientations construct

(Fernandes, 2020) could be expanded to include a more holistic look at teachers' beliefs about students' linguistic resources, including but not limited to their home languages. While I used multiple constructs to provide a comprehensive account of teachers' beliefs about students' assets, future work could address this issue by creating a more comprehensive framework for documenting how teachers draw on students' assets.

Implications for Practice

This work has implications for practice related to teacher preparation and professional development. The finding that Ms. C more frequently drew on student thinking than student experiences outside the classroom could have been due to the type of professional development that Ms. C participated in. Ms. C had over 10 years of professional development related to student thinking and mathematics, in particular she had many experiences with Cognitively Guided Instruction (CGI) which focuses on student thinking and has shown to be effective (Fennema et al., 1996; Philipp, 2007). However, she only had one summer institute of professional development specific to EBs and mathematics and it was unclear if this included a focus on students' experiences outside of the classroom for mathematics instruction. Ms. C had support to develop pedagogical practices related to drawing on students thinking but was not supported in the same way with her teaching practices related to leveraging her students' experiences outside the classroom. Ms. C may have more frequently leveraged the experiences of her students for mathematics instruction if she had the professional development and support.

The analyses of teaching practices (Chapters 3 and 4) suggest that teachers may need more time to learn multiple ways to use students' experiences during mathematics instruction. Ms. C reported having experiences with professional development related to mathematics (Cognitively Guided Instruction, the Mathematics Project), yet she had less formal support for developing equitable mathematics instruction that drew on students' experiences. Instead, Ms. C had to develop and try practices over her many years of teaching to create opportunities for the EB during her online instruction.

Another implication this dissertation has for practice relates to stakeholders having opportunities to see the effectiveness of drawing on student thinking and experiences to support mathematics learning, rather than an approach to mathematics instruction that reflects a standard, singular approach to learning and teaching. Through the analysis comparing Ms. C's in-person and online instruction, I found that Ms. C faced constraints that led her to enact her curriculum with more fidelity when she moved online during a pandemic compared to her in-person teaching. Adhering to strict pacing in many ways reflects an ideology that all children need to be at the same place or have the same experiences to be prepared to learn mathematics. To promote more productive beliefs and move away from singular notions of "preparedness," policy, standards, and curriculum materials must make space for diverse backgrounds and experiences. Various stakeholders need opportunities to see the effectiveness of drawing on students' experiences outside the classroom to support mathematics learning. When students see their thinking and experiences reflected in instruction, they can more easily access mathematics content (Turner et al, 2012; Turner et al, 2016), and content that reflects students mathematical thinking has been shown to be effective at improving student performance (Fennema et al., 1996).

Appendices

Appendix A: Interview Protocol

INTERVIEW QUESTIONS:

1. Tell me about an effective math lesson you taught in the classroom recently:
    a. How did you plan for the lesson?
    b. Who was in the class?
    c. What activities?
    d. What were you doing?
    e. What were the students doing?
    f. How did you introduce the topic?
    g. How did you ask questions?
    h. How were interactions structured? (group work, individual, whole class)
    i. How did you assess students?
    j. Why did you make these decisions?
2. During a typical week, when does mathematics instruction take place in your classroom?
    a. Is mathematics during a structured math time? Throughout the day?
    b. About how many minutes do you spend on mathematics during a typical day/week?
3. What types of questions do you typically ask your students during mathematics instruction? Can you give some specific examples?
4. During math instruction how to support students with learning math terms?
5. How do you support children learning English during mathematics instruction? Can you give some specific examples?
6. If a student came into your classroom with little to no experience with the English language, how would you engage them in mathematics instruction?
7. Tell me about the ELs in your school.
    a. Were many of them born here?
    b. Are there many immigrants? Refugees?
    c. What countries are they from?
    d. What languages do they speak?
    e. What previous schooling experiences do they have?

8. What do you think is the most important thing for students to learn during mathematics instruction? What do you want your students to know/understand about mathematics when they leave your class?
9. How do you use the school's curriculum when you teach mathematics?
    a. Do you ever change the context of problems? Can you give specific examples?
    b. Do you ever change the math content?
    c. Do you ever change the wording?
    d. Do you ever change the sequencing of lessons?
10. Do you use supplemental materials?
    a. How? In every lesson?
11. What do you think your students struggle with most in math?
    a. What do you think they do well with in math?
12. If a student were to say, "I'm not good at math?" how would you respond?
13. What strategies do you use to determine what children know (are thinking) about mathematics?
14. What are your important goals when you teach math?
15. What mathematics do you think your students engage in outside of the classroom?
    a. How do you know?
16. If you were to give a starting teacher advice about teaching math, what would you tell them?
17. Is there anything else you want to share related to mathematics instruction in your classroom?

Appendix B: Background Information Form

<u>Description:</u> As a part of my dissertation study, I am interested in early grades teachers' beliefs about students' thinking and students' everyday math and language practices, in particular students who are emergent bilinguals. Please complete the questions below:

<u>Demographic Info:</u>

1. By the end of this year I will have been working ________years as a teacher.
2. What was your undergraduate major?
3. What is you highest degree?
    - o Bachelor's Degree
    - o Teaching Credential
    - o Masters and Teaching Credential
    - o Masters without Credential
    - o Doctoral Degree
4. What credentials do you hold?
5. What grade(s) have you taught? (indicate # of years in space)
    - o Pre-K_____
    - o Kindergarten_____
    - o First_____
    - o Second_____
    - o Third_____
    - o Fourth_____
    - o Fifth_____
    - o Sixth_____
    - o Seventh_____
    - o Eighth_____
    - o High School_____

6. List any other experience relevant to teaching math:
7. Have you had any professional development in mathematics education? Include length, frequency, location, organization, etc.
8. Have you had any professional development specifically for teaching English Learners? Include length, frequency, location, organization, etc.
9. Please identify the language communities you belong to:
   Selections for speak, read, write, understand
    a. English
    b. Spanish
    c. French
    d. Mandarin

e. Others (open answer)

Class Information:

1. How many children are in your class this year?
2. Related to children's demographics, can you estimate roughly how many children in your class are low-income? English language learners? Underserved populations (e.g., Latinx, special needs)? Please be as specific as possible.
3. Any other relevant information you would like me to know about the site or about the children you work with?

Future Participation:

1. Would you be interested in participating in a short interview (45-90 minutes) via zoom?
    a. YES/NO
    b. IF YES - Contact info/email:

Appendix C: Survey

NOTE: this survey statements were given to teachers via google forms and responses could be given using a 5-point Likert scale.

Survey Statements:

1=strongly disagree. 2= disagree. 3 =neutral. 4 = agree. 5= strongly agree

1. Teachers and schools need to learn about the mathematical practices from students' families and communities?
2. Students learning English should use their primary language and everyday ways of talking to engage with mathematics content.
3. Students should learn math vocabulary before they learn math concepts.
4. Language demands for ELs in math only occur in word problems.
5. Math is not language intensive.
6. The math work of ELs and non-ELs should be graded the same way.
7. I should teach math to ELs and non-ELs in the same way.
8. It is fair for ELs to get accommodations on math tests (e.g., extra time, use of dictionary, etc.).
9. It is fair to assess ELs' math knowledge using only paper-and-pencil tests.
10. ELs should be taught math by ESL/ELD (English as a Second Language / English Language Development) teachers.
11. Accommodating the needs of ELs in the math classroom can slow down the learning of other students.
12. ELs can be effectively taught math in English before they are fluent in English.
13. When teaching ELs, I should use a variety of math vocabulary.
14. When teaching ELs, I should provide different opportunities (e.g. small groups, one-on-one with the teacher, etc.) for them to explain their thinking in English.
15. I should adjust the language in math problems to ensure ELs understand.
16. When there are ELs in my classroom, my lesson plans should address both the math content and the English needed.
17. ELs will not learn English quickly if I allow them to speak their native language in my math class
18. When teaching ELs, I should focus more on basic computations than on problem solving activities.
19. It is helpful to restate the math explanations given by ELs during class discussions.

20. When interacting with ELs, I should focus on the math in their explanations instead of their language.
21. When ELs switch between languages to explain their mathematical thinking, it shows a lack of mathematical understanding.
22. When ELs switch between languages to explain their mathematical thinking, it shows a lack of language fluency.
23. The different ways that ELs learned math (e.g., multiplication, addition, solving problems) in their homes is a valuable resource in the math class.
24. An ELs background and experiences are valuable resources to help all students learn math.

Appendix D: Observation Protocol

Observation Protocol

Observer: Brittany Caldwell   Date: Time:

Teacher:	
Age/Grade:	
Topic/Content:	
Materials/Tools Used:	

RQ:

1. What are teachers' beliefs about mathematics, students' thinking and students' early, out of school experiences with mathematics?
2. What is the nature of mathematics instruction in early grades classrooms?

QUESTIONS TO FOCUS ON:

1. What was the overall structure of the session?
    a. Structure: Whole group, small group, individual work? Document the overall structure of the sessions: how much time is spent on each of these and the general flow of the session. Who is talking, what's happening, and what are participants doing?
    b. Focus: getting organized, individually working on math, whole group discussion, formative assessment
2. What's the MATH content? Is the focus on concepts or procedures?
    a. P= procedures and skills vs. C= large ideas and concepts
3. Were examples of student work incorporated? Who? When? How?
4. Were there any connections to out of school math? To Students thinking?
5. Scaffolding? How when and what does the teacher scaffold students participation in mathematics instruction? How does the teacher scaffold ELLs participation in mathematics in the classroom?
    a. WHAT? Conceptual understanding or Procedural fuency?
    b. WHEN? Identify the micro (interactional) meso (larger supports throughout a lesson) and macro (across the lessons)
    c. How? Identify concrete examples of these

Time	Notes	Comments/connection to questions

(Add more rows when needed)

<u>Memo:</u>

Wrap-up/concluding thoughts/connection to RQ and research:

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