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Author
Goebel, Charles.

Publication Date
1956-01-11

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Contract No. W-7405-eng-48

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Charles Goebel

January 11, 1956

UCRL-3260

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Charles Goebel

Radiation Laboratory, University of California
Berkeley, California

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ABSTRACT

Selection rules governing the annihilation of a nucleon-antinucleon pair into pions, photons, and heavy mesons are derived from the conservation of angular momentum, parity, charge parity and isotopic spin. Special attention is given to the probability of annihilation purely into neutral pions.

SELECTION RULES FOR NN̄ ANNIHILATION[*]

Charles Goebel

Radiation Laboratory, University of California
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We consider annihilation of nucleonium, i.e. a nucleon-antinucleon pair, from a state of definite angular momentum, parity, isotopic spin, and charge parity (if neutral) into systems of pions, photons, and heavy mesons.

States of Nucleonium

A state of a weakly interacting nucleon-antinucleon pair can be described by the quantum numbers total angular momentum j, spin s, orbital angular momentum ℓ, and by a specification of the isotopic states, i.e., $p\bar{n}$, $n\bar{p}$, $p\bar{p}$, or $n\bar{n}$. From these we can express the parity $\mathcal{P} = -(-)^{\ell}$, the isotopic spin $i = 1$ or 0, the charge parity (in the case of neutral nucleonium, i.e., $i_z = 0$) $C = (-)^{s+\ell}$, and the "CT" parity[1] $CT = (-)^{s+\ell+i}$. If the pair is interacting strongly, then s, ℓ, and the "isotopic specification" are no longer good quantum numbers, but $\mathcal{P}$, i, and CT

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[*] This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] See Pais and Jost, Phys. Rev. **87**, 871 (1952). C is the eigenvalue of a rotation of $180°$ in i-spin space around an axis transverse to the 3 or "charge" axis. Only systems with $i_z = 0$ have a T parity, and then $T = (-)^i$. CT is the product of C and T, and is useful in that if the T-rotation is taken to be around the 2 axis all three pions have odd CT parity.

(and C if $i_z = 0$) are good quantum numbers if we neglect electromagnetic interaction. For instance if we call a nucleonium state $^3_3P_2$ we really mean the state having $j = 2$, $i = 1$, $\mathcal{P} = +1$, $CT = -1$ (and $C = +1$ if $i_z = 0$).

Decay into Pions

The pion has CT parity -1, so that an N pion state has $CT = (-)^N$. The pion has an intrinsic spatial parity -1, so that an N pion state has $\mathcal{P} = (-)^N \mathcal{P}_{orbital}$. For two pions, j equals $\mathcal{l}$ because the pion is spinless, therefore $\mathcal{P} = (-)^j$; for three pions having $j = 0$, $\mathcal{P}_{orbital}$ equals $+1$ and therefore $\mathcal{P} = -1$.

Thus a nucleonium state having $CT = \{+ , -\}$ can decay into an $\{$ even, odd $\}$ number of pions; with the qualifications that two pions are impossible if $(-)^j \mathcal{P} = -1$ (which is always the case for spin singlet nucleonium), and three pions are impossible if the state is $(0 +)$. Selection rules based on CT are not absolute in the presence of electromagnetic interaction, but decays violating them will be slower than the corresponding radiative decays.[1]

Table I lists the least number of pions into which each nucleonium state can decay; a state that decays into some (even or odd) number of pions can decay into any larger (even or odd) number of pions also. For states of higher odd or even $\mathcal{l}$ the table would be precisely the same as given for P or D state respectively, except that the forbidding of three mesons from $^3_3P_0$ is not repeated, since the corresponding higher $\mathcal{l}$ states do not have $j = 0$. If for a given $\mathcal{l}$ one weights the four spin states equally, and either weights the four isotopic spin states equally ("mixed" nucleonium, e.g. p incident on D) or gives equal weight to the two $i_z = 0$

TABLE I: Characteristics of the Allowed Final States from Nucleonium Annihilation

Initial Nucleonium State			Pions[a]	K Mesons and Pions					
				Scheme A: the θ is (0+)				Scheme B: the θ is (1-)[d]	
				$K\bar{K}$[b]	$K\bar{K}\pi$[c]			$\theta\bar{\theta}$	$\tau\bar{\theta}$ / $\theta\bar{\tau}$
	C	CT			like K mesons	unlike K mesons			
$^3_3S_1$	−	+	$2p$	like	p_0p	$\binom{sp}{ps}$		p	p
$^3_1S_1$	−	−	$3p^2$	like	p_1p	$\binom{s_1p}{p_1s}$		p	p
$^1_3S_0$	+	−	$5s^4$	unlike	s_0s			p	p
$^1_1S_0$	+	+	$4p^3$	unlike	s_1s			p	p
$^3_3P_{0,1,2}$	+	−	$5sp^3;3sp;3pd$	like;unlike;like	$-;\binom{s_0p}{p_1s};\binom{p_1d}{d_0p}$	$ss;pp;\binom{sd}{pp}{ds}$		s;d;s	−;s;d
$^3_1P_{0,1,2}$	+	+	$2s;4sp^2;2d$	like;unlike;like	$-;s_1p;d_1p$	$s_1s;p_1p;\binom{s_1d}{p_1p}{d_1s}$		s;d;s	−;s;d
$^1_3P_1$	−	+	$4sp^2$	unlike	$\binom{s_1p}{p_0s}$	pp		s	s
$^1_1P_1$	−	−	$3sp$	unlike	p_1s	p_1p		s	s
$^3_3D_{1,2,3}$	−	+	$2p;4\binom{p^3}{spd};2f$	like;unlike;like	$p_0p;\binom{s_0d}{p_0p}{d_0s};d_1d$	$\binom{sp}{ps};\binom{pd}{dp};\binom{pd}{dp}$		p;p;p	p;p;f
$^3_1D_{1,2,3}$	−	−	$3p^2;3\binom{p^2}{sd};3d^2$	like;unlike;like	$p_1p;p_1p;f_1f$	$\binom{s_1p}{p_1s};\binom{p_1d}{d_1p};\binom{p_1d}{d_1p}$		p;p;p	p;p;f
$^1_3D_2$	+	−	$3\binom{p^2}{sd}$	unlike	p_1p	$\dfrac{pd}{dp}$		p	p
$^1_1D_2$	+	+	$4\binom{p^3}{spd}$	unlike	d_1d	$\dfrac{p_1d}{d_1p}$		p	p

[a] The prefixed number is the least possible number of pions; the lowest - ℓ configuration is given for this least number state.

[b] "Like" signifies that the pair $K\bar{K}$ must be either $\theta\bar{\theta}$ or $\tau\bar{\tau}$; "unlike" that the pair is $\theta\bar{\tau}$ or $\tau\bar{\theta}$.

[c] The lowest - ℓ configuration is given, the first letter being the orbital state of the $K\bar{K}$ pair, with its isotopic spin as a subscript; the second letter is the orbital state of the pion around the K meson pair. Where the isotopic spin of the $K\bar{K}$ pair can be either 0 or 1 it is not stated.

[d] The lowest - ℓ orbital state is given.

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states("neutral" nucleonium, e.g. $\bar{p}$ incident on H) one finds the ratios

of the number of states with least pion numbers of 2, 3, 4, or 5 given in

Table II.

TABLE II

Relative Numbers of Nucleonium States Decaying into each Pion Multiplicity.

$N_{min} \backslash \ell$	"Mixed" nucleonium				"Neutral" nucleonium		
	0	1	even > 0	odd > 1	0	1	even or odd > 1
2	4.5	1	3	1	3	2	2
3	1.5	4.5	3	5	3	3 2/3	4
4	0.5	2	2	2	1	2	2
5	1.5	0.5	0	0	1	1/3	0

If the annihilation takes place in a small volume, then low orbital

angular momenta in the final state will be favored. The lowest - ℓ

configurations are listed in Table I for the least pion number states;

the lowest - ℓ configuration for a state of 2n additional pions simply

has an additional factor of s^{2n}.

It is of some interest to estimate the proportion of decays leading

to pure π^0 states, since these will yield the entire nucleonium rest

mass as visible energy in a Cerenkov counter. Since $C = +1$ for a system

of neutral pions, and (for a neutral system) we can write $CT = (-)^i C = (-)^N$,

a system of N neutral pions has $i = \{0, 1\}$ if N is $\{$even, odd$\}$.

If we neglect the space state of the N-pion final state, and thus assume that all isotopic spin states are equally populated, we can deduce the probability P_N that an N pion state is a pure π^o state, given that $i_z = 0$ and $i = \left\{ 0, 1 \right\}$ if N is $\left\{ \text{even, odd} \right\}$.

N	2	3	4	5	6	7	8	9	10
P_N	0.3333	0.2000	0.0667	0.0286	0.0095	0.0037	0.0012	0.0005$^-$	0.0001$^+$

This table is derived in the Appendix. Then, still neglecting the effects of space states, we can write the probability P_o that neutral nucleonium, decaying into pions, decays into neutral pions only:

$$\text{even } \ell \, , \quad > 2: \qquad P_o = \sum \frac{1}{8} P_N (\alpha_N^{(3)} + \alpha_N^{(4)})$$

$$\text{odd } \ell \, , \quad > 1: \qquad P_o = \sum \frac{1}{8} P_N (2\alpha_N^{(2)} + 3\alpha_N^{(3)} + \alpha_N^{(4)})$$

$$\text{average } \ell, \qquad : \qquad P_o = \sum \frac{1}{8} P_N (\alpha_N^{(2)} + 2\alpha_N^{(3)} + \alpha_N^{(4)})$$

where the $\alpha_N^{(m)}$ are the branching ratios for decay into N pions of a state that decays into at least m pions (N - m even). The estimation of the α_N by the Lepore-Neuman statistical theory[2] indicates them to be

[2] J. V. Lepore, M. Neuman, and R. Stuart, Phys. Rev. 94, 788 (1954).

See the forthcoming paper by J. V. Lepore and D. H. Holland.

rapidly decreasing functions of N; i.e., $\alpha_{m+2}^{(m)} \ll \alpha_m^{(m)}$, so that $\alpha_N^{(m)} \approx \delta_{Nm}$. Further, since the phase space for a final state of two heavy mesons (see next section) is nearly the same as for two pions, the at-least-four-pion decay states would not decay into pions on this theory. Taking the estimations[2] $\alpha_2^{(2)} = 3/7$, $\alpha_3^{(3)} = \frac{1}{2}$, and all others zero, we find 0.033 for the probability that neutral nucleonium decays (directly) into π^o's only.

Decay into Heavy Mesons

We assume Gell-Mann's scheme, assigning $i = \frac{1}{2}$ to both the $\theta(\longrightarrow 2\pi$'s) and the $\tau(\longrightarrow 3\pi$'s), which we call collectively K mesons. The spin and parity of the two are known to the following extent. The θ must have $\mathcal{P} = (-)^j$ in order to decay into two pions, and further, j must be even for θ^o to decay into two neutral pions.[3] The τ seems to be $(0 -)$ on the basis of Dalitz's analysis of the observed correlation in the final state. We shall accept the τ as $(0, -)$ and consider the possibilities that the θ is either $(0, +)$ or $(1, -)$, calling the first assumption $\left[\theta \text{ is } (0 +) \right]$ scheme A, the second $\left[\theta \text{ is } (1 -) \right]$ scheme B.

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[3] J. Osher (unpublished) has evidence from an experiment similar to/that of Collins $\left[\text{Proceedings of the Rochester Conference, 1955, Interscience, New York, p.139} \right]$ that the $\theta^o \longrightarrow \pi^o \pi^o$ decay does occur, with lifetime and abundance of the same order as for $\theta^o \longrightarrow \pi^+ \pi^-$. Furthermore, from the theoretical point of view, if the θ has odd spin its two-pion final state would necessarily have $i = 1$, so that the different lifetimes of θ^+ and θ^o would be inexplicable under charge independence.

In either case, a state of two K mesons has $i = 0$ or 1. In Scheme A, a pair $\theta\bar{\theta}$ or $\tau\bar{\tau}$ (which we call <u>like</u> pairs) has $\mathcal{P} = (-)^\ell = (-)^j$, and $CT = (-)^{\ell+i}$ (and $C = (-)^\ell$ if $i_z = 0$). The pairs $\theta\bar{\tau}$ or $\tau\bar{\theta}$ (<u>unlike</u> pairs) are presumably interconvertible, as there is no selection-rule bar between them, and thus we can form linear combinations having $\mathcal{P} = -(-)^\ell$, and $CT = \pm 1$. In Scheme B, a pair $\theta\bar{\theta}$ has spin states $s_\theta = 0, 1,$ or 2, so that $CT \left[=(-)^{\ell+s_\theta+i} = (-)^{s_\theta+i}\mathcal{P} \right]$ can be ± 1 for a given i, j and $\mathcal{P}$. The unlike pairs in Scheme B have $\mathcal{P} = (-)^\ell$. Thus for a given $j \neq 0$, $\mathcal{P}$ can be either ± 1 (CT likewise); but if $j = 0$, then $\ell = s = 1$, therefore $\mathcal{P} = -1$.

Thus in Scheme A, a state having $\mathcal{P} = \left\{ +, - \right\}(-)^j$ can decay into a $\left\{ \text{like, unlike} \right\}$ pair. In Scheme B, any state can decay into a pair $\theta\bar{\theta}$; any state except $(0+)$ can decay into an unlike pair $\theta\bar{\tau}$ or $\tau\bar{\theta}$. A pair $\tau\bar{\tau}$ is the same in both schemes.

Any state can decay into a pair of K mesons plus one or more pions. The lowest - ℓ configuration, along with the isotopic spin of the $K\bar{K}$ pair, is given in Table I.

The proportion of neutral nucleonium decaying into heavy mesons that go ultimately into neutral pions can be easily estimated. In Scheme A, the θ^o would decay into charged or neutral pions in a ratio between $2:1$ and $1:2$, depending on the ratio of $i = 0$ to $i = 2$ in the final state; if charge independence is invoked, the hundred-times-shorter lifetime of θ^o compared to θ^+ implies that θ^o decays almost exclusively through $i = 0$.[4]

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[4] Cf., S. Okubo and R. Marshak, Bull. Am. Phys. Soc. <u>30</u>, No. 8, Q9 (1955).

Therefore the probability that a neutral $\theta\bar{\theta}$ pair will decay into four neutral pions is 1/18. We now have to estimate the proportion of like $K\bar{K}$ that are $\theta\bar{\theta}$ pairs. If at high energies "θ"'s and "τ"'s [i.e. (0 +) and (0 -) K mesons] are produced in equal numbers, the observed ratio of θ^+, τ^+, and $K^+_{\mu2}$ decays, roughly 10:1:20, implies that a "θ+" has a chance of $\approx 2/3$ of decaying into two pions. But then the hundred-times-shorter lifetime of the θ° insures that a "θ$^\circ$" nearly always decays into two pions.[5] Thus in Scheme A, a neutral $K\bar{K}$ pair equally likely to be like or unlike has a chance of 1/72 of decaying into neutral pions. Thus on the Lepore-Neuman statistical theory (see the end of preceding section) the probability for neutral nucleonium to decay indirectly through a K-meson pair into π°'s only is 0.007. In Scheme B, pure π°'s could be produced only from $\tau^\circ \longrightarrow 3\pi^\circ$, which is negligible if τ°'s are as rare as $\tau^\pm$'s.

Radiative Decay

For completeness we shall sketch here the possible radiative decays, even though decays involving photons will be slower than purely mesonic decays by factors of the order of 100 for each photon emitted.

Since interaction with the electromagnetic field need not conserve isotopic spin or CT parity, charged nucleonium can decay into any system of mesons and photons, conserving, of course, charge and strangeness; in addition it must respect the "no 0-0 transition" rule, that a photon plus a $j = 0$

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[5] That is, unless the decay into fermions ($\mu^+\mu^-$ or $\nu\nu$) is also speeded up a hundredfold in the i = 0 state relative to i = 1.

system cannot have $j = 0$. For neutral nucleonium, however, charge parity C is a good quantum number. A photon has $C = -1$, thus a final state consisting of mesons plus N photons has $C = (-)^N C_{\text{meson state}}$.

Thus a neutral nucleonium state having $C = -1$, i.e. spin-space states triplet-even or singlet-odd, can decay into $\pi^0 \gamma$. Any neutral state can decay into $\pi \pi \gamma$, the pions being charged if $C = 1$, and being charged and neutral in the ratio 2:1 if $C = -1$ and $i = 0$.

The decay of neutral nucleonium into a pure photon state is equivalent to positronium annihilation:[6] an N photon state must have $(-)^N = C$, and $(1\pm)$ or (odd $-$) states cannot decay into two photons.

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[6]
C. N. Yang, Phys. Rev. **77**, 242 (1950).

APPENDIX

Given a system of N pions with total i-spin i_N and $i_z = 0$, we wish to find the probability that all N pions are π^0, assuming that all states of the system with i-spin i_N, $i_z = 0$ are equally probable. The system may be decomposed into a system of one pion of i-spin one plus a system consisting of the remaining N-1. In order that we have $i_N = i$, i_{N-1} must be i, i + 1, or i - 1. Each state of the N - 1 pion system with $i_{N-1} = i$, i + 1, or i - 1 may give rise to a state of the N pion system with $i_N = i$. Corresponding to each such N pion state is a definite probability that all N pions are π^0. The quantities $P^N(i)$ are the averages of these probabilities, taken over all states.

Thus $P^N(i)$ is a sum of terms, each of which is a product of three factors a, b, c, which are identified as follows:

Factor a: the fraction of the states with i_N formed from states with i_{N-1}, i.e., $n^{N-1}(i_{N-1})/n^N(i_N)$, where $n^N(i)$ is the number of states of the N pion system with $i_N = i$, $i_z = 0$.

Factor b: the probability, that $(i_{N-1})_z = (i_1)_z = 0$ for a state of N pions formed from a system of N - 1 pions with i_{N-1} and one pion with i_1. This quantity is symbolized by $\left| \left\langle i_N 0 \mid 0 \, 0 \right\rangle_{1, i_{N-1}} \right|^2$. It can be shown that

$$\left| \left\langle i \, 0 \mid 0 \, 0 \right\rangle_{1, i_N + j} \right|^2 = \frac{i(1 - \delta_{j0}) + \frac{1}{2}(1 + j - \delta_{j0})}{2i + 1 + 2j}$$

where $j = 0, \pm 1$.

Factor c: the probability that if $(i_{N-1})_z = 0$, the $N - 1$ pion system is all π^0. This is just $P^{N-1}(i_{N-1})$.

Multiplying a, b, c, one obtains

$$P^N(i) = \frac{n^{N-1}(i+1)}{n^N(i)} \frac{i+1}{2i+3} P^{N-1}(i+1) + \frac{n^{N-1}(i)}{n^N(i)} \cdot 0 \cdot P^{N-1}(i)$$

$$+ \frac{n^{N-1}(i-1)}{n^N(i)} \frac{i}{2i-1} P^{N-1}(i-1)$$

$$= \frac{1}{n^N(i)} \left[\frac{i+1}{2i+3} n^{N-1}(i+1) P^{N-1}(i+1) \right.$$

$$\left. + \frac{i}{2i-1} n^{N-1}(i-1) P^{N-1}(i-1) \right] .$$

Or, defining

$$f_i^N = n^N(i) P^N(i), \quad \text{one has}$$

$$f_i^N = \left[\frac{i+1}{2i+3} f_{i+1}^{N-1} + \frac{i}{2i-1} f_{i-1}^{N-1} \right] .$$

The f_i^N must satisfy the conditions that $f_0^1 = 1$, $f_i^N = 0$ for $N < 1$ or $i < 0$. Since f_i^N is the probability that a state of $N \pi^0$ has i-spin i, it follows that $\sum_{i=0}^{N} f_i^N = 1$.

The recursion relations for the f's may be solved with the help of a generating function.[7] The solution is

$$f_i^N = \frac{N!(2i+1)}{(N-i)! \, (N+i+1)(N+i-1) \, \ldots \, (N+1-i)} \qquad \text{(for } i \neq 0),$$

$$f_0^N = 1/(N+1) \qquad \text{(for } i = 0).$$

The n_i^N are found from the equations

$$n^N(i) = n^{N-1}(i+1) + n^{N-1}(i) + n^{N-1}(i-1) \qquad \text{(for } i \neq 0),$$

and

$$n^N(0) = n^{N-1}(1) \qquad \text{(for } i = 0),$$

with

$$n^N(N) = 1$$

and

$$n^N(i) = 0 \qquad \text{(if } N < i).$$

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[7] This was pointed out by R. J. Riddell, Jr., who carried out the solution.

TABLE IA: Values of f_i^N

i \ N	1	2	3	4	5	6	7	8	9	10
0		1/3		1/5		1/7		1/9		1/11
1	1		3/5		3/7		1/3		3/11	
2		2/3		4/7		10/21		40/99		---
3			2/5		4/9		14/33		---	
4				8/35		24/77		---		---
5					8/63		---		---	

TABLE IIA: Values of $P^N(i)$

i \ N	2	3	4	5	6	7	8	9	10
0	1/3		1/15		1/105		1/819		1/6633
1		1/5		1/35		1/273		1/2211	