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PHOTOELECTRONS IN THE UPPER ATMOSPHERE: A FORMULATION INCORPORATING EFFECTS OF TRANSPORT

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Abstract—An efficient scheme is described for computation of photoelectron energy spectra. The method, similar to one used in radiative transfer theory, incorporates effects due to spatial and angular redistribution of electrons. These effects play a dominant role at heights above 400 km.

1. INTRODUCTION

Absorption of solar radiation at wavelengths below 1000 Å plays a major role in both the chemistry and thermodynamics of the atmosphere above an altitude of about 120 km. A significant fraction of the absorbed energy appears initially in photoelectrons. The ultimate fate of this energy depends on a complex sequence of collision processes and may involve significant spatial transport of the fast electrons.

Photoelectrons are released with an average initial energy of about 10 eV. Energy is lost to neutral species, through inelastic collisions which may contribute in part to the excitation of day airglow (Dalgarno *et al.*, 1969). Collisions with ambient electrons represent a selective heat source which maintains an electron temperature exceeding the corresponding temperatures for ions and neutrals. Considerable attention has been focused in recent years on attempts to model the spatial and energy distribution of photoelectrons.

Initial efforts were directed primarily towards lower altitudes where the photoelectron energy distribution is set mainly by local production and loss (Hanson and Johnson, 1961; Hanson, 1963; Dalgarno, McElroy and Moffett, 1963). Hanson (1963) first drew attention to the potential importance of transport, and recent literature (Nisbet, 1968; Nagy and Banks, 1970; Banks and Nagy, 1970; Cicerone and Bowhill, 1971; Cicerone *et al.*, 1973; Mantas, 1975; Mantas and Bowhill, 1975; Swartz, 1976; Oran and Strickland, 1976; Lejeune and Wormser, 1976) has emphasized attempts to devise a unified theory for both local and non-local regimes. A variety of approaches have been devised to treat the complexities introduced by transport, including models based on Monte Carlo techniques, analogies with molecular diffusion, and theories for

radiative transfer. The most complete models for transport are perhaps those described by Mantas (1975), Oran and Strickland (1976) and Lejeune and Wormser (1976). Victor *et al.* (1976) and Oran and Strickland (1976) give up to date descriptions of the atomic processes which affect the electron energy distribution, incorporating very recent laboratory and theoretical information on relevant cross sections. Their models make use of extreme ultraviolet solar radiative fluxes as measured by Hinteregger (1977) on the Atmospheric Explorer Satellites (Dalgarno *et al.*, 1973), and were used to analyze direct measurements of the photoelectron spectrum obtained by Doering *et al.* (1975) and Doering *et al.* (1976) with instrumentation on the same satellites. Peterson *et al.* (1977) have recently reported measurements of the photoelectron spectrum which arises due to electrons released from the magnetic conjugate ionosphere.

We shall now develop a model for photoelectrons which allows for a relatively complete description of the relevant transport and collisional physics. The model can be applied to study processes at either low or high altitude, and may be used to investigate interhemispheric transport. It will be applied elsewhere to an analysis of data from Atmospheric Explorer.

2. THE TRANSPORT EQUATION

The photoelectron spectrum may be characterized by a function $\phi(\mathbf{r}; E, \hat{n}, t)$ which defines the number of electrons moving with energy E , in direction \hat{n} , at time t , and position \mathbf{r} . The function ϕ is analogous to the specific intensity which appears in the theory of radiative transfer, and has units of electrons $\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{eV}^{-1}$. In general ϕ is a function of 7 variables. We shall restrict attention here to steady state solutions. We shall further

assume that photoelectrons are confined to gyrate around a well defined magnetic field line. The radius of gyration is taken to be small compared to scales for spatial inhomogeneity in the background atmosphere. The direction of motion along field lines may be defined by a single quantity, the pitch angle α . The photoelectron spectrum at an arbitrary position along a given field line may be specified then in terms of three parameters: x , the displacement from some reference position; α , the pitch angle; and E , the energy.

The change in photoelectron intensity, $d\phi$, along a differential path length, ds , is given by

$$d\phi(E, x, \alpha) = -\kappa(E, x)\phi(E, x, \alpha) ds + \frac{1}{2}\chi(E, x) \int_0^\pi P(E, \alpha, \alpha')\phi(E, x, \alpha') \sin \alpha' d\alpha' ds + \tilde{S}(E, x, \alpha) ds, \quad (1)$$

where the individual terms on the right hand side account for collisional loss of electrons from the initial beam (E, x, α), scattering of electrons from other pitch angles (E, α'), and production of electrons (E, α) either by photoionization or by inelastic scattering from (E', α'). The quantity $\kappa(E, x)$ denotes the inverse of the collisional mean free path; $\chi(E, x)$ is the inverse of the mean free path limited to elastic scattering; $P(E, \alpha, \alpha')$ defines the probability of an elastic scatter from pitch angle α into α' . The change of ϕ along the trajectory s may be written in the form

$$\frac{d\phi}{ds} = \mu \frac{\partial \phi}{\partial x} + \frac{d\mu}{ds} \frac{\partial \phi}{\partial \mu}, \quad (2)$$

where

$$\mu = \cos \alpha, \quad (3)$$

or in the form

$$\frac{d\phi}{ds} = \mu \frac{\partial \phi}{\partial x} - \frac{(1-\mu^2)}{2} \frac{d \log B}{dx} \frac{\partial \phi}{\partial \mu}, \quad (4)$$

where B is the strength of the magnetic field. The second term on the right hand side of (4) allows for the change in pitch angle along a trajectory as constrained to satisfy the adiabatic invariance of the magnetic moment.

The transfer equation may be recast now in the form

$$\mu \frac{\partial \phi}{\partial \tau}(E, \tau, \mu) - \frac{(1-\mu^2)}{2} \frac{d \log B}{d\tau} \frac{\partial \phi}{\partial \mu}(E, \tau, \mu) = \phi(E, \tau, \mu) - \frac{\tilde{\omega}(E, \tau)}{2} \int_{-1}^1 P(E, \mu, \mu')\phi(E, \tau, \mu') d\mu' - S(E, \tau, \mu), \quad (5)$$

where τ is a dimensionless path length analogous to the optical depth parameter in theories of radiative transfer,

$$d\tau(E, x) = -\kappa(E, x) dx, \quad (6)$$

$\tilde{\omega}(E, \tau)$ is a scattering efficiency similar to the single scattering albedo in transfer theory,

$$\tilde{\omega}(E, x) = \frac{\chi(E, x)}{\kappa(E, x)} \quad (7)$$

and S is given by

$$S(E, x, \mu) = \frac{\tilde{S}(E, x, \mu)}{\kappa(E, x)}. \quad (8)$$

The dimensionless path length τ is uniquely related to the displacement x along field lines for any given value of E , and may be simply located in terms of a latitude-altitude coordinate system. The intensity ϕ at energy E depends implicitly on the intensity at higher energies E' through the source term S in (5).

3. METHOD OF SOLUTION

It is convenient to introduce further quantities j , h , P^+ , P^- , S^+ and S^- defined as follows:

$$j(\tau, \mu) = \frac{1}{2}\{\phi(\tau, +\mu) + \phi(\tau, -\mu)\} \quad (9)$$

$$h(\tau, \mu) = \frac{1}{2}\{\phi(\tau, +\mu) - \phi(\tau, -\mu)\} \quad (10)$$

$$P^\pm(\tau, \mu') = \frac{1}{2}\{P(+\mu, \mu') \pm P(-\mu, \mu')\} \quad (11)$$

$$= \frac{1}{2}\{P(\mu, +\mu') \pm P(\mu, -\mu')\} \quad (12)$$

$$S^\pm(\tau, \mu) = \frac{1}{2}\{S(\tau, +\mu) \pm S(\tau, -\mu)\} \quad (13)$$

with $0 < \mu \leq 1$ in (9), (10), (11) and (13), and with $0 < \mu' \leq 1$ in (12). Equation (12) reflects the assumption of reciprocity, i.e. scattering from $\mu \rightarrow \mu'$ is equivalent to scattering from $\mu' \rightarrow \mu$. With these definitions we have

$$\frac{1}{2} \int_{-1}^1 P(\mu, \mu')\phi(\tau, \mu') d\mu' = \int_0^1 P^+(\mu, \mu')j(\tau, \mu') d\mu' + \int_0^1 P^-(\mu, \mu')h(\tau, \mu') d\mu' \quad (14)$$

and

$$\mu \frac{\partial j}{\partial \tau} - \frac{(1-\mu^2)}{2} \frac{d \log \beta}{d\tau} \frac{\partial j}{\partial \mu} = h(\tau, \mu) - \tilde{\omega}(\tau) \int_0^1 P^-(\mu, \mu')h(\tau, \mu') d\mu' - S^-(\tau, \mu), \quad (15)$$

$$\begin{aligned} & \mu \frac{\partial h}{\partial \tau} \frac{(1-\mu^2)}{2} \frac{d \log B}{d \tau} \frac{\partial h}{\partial \mu} \\ & = j(\tau, \mu) - \bar{\omega}(\tau) \int_0^1 P^+(\mu, \mu') j(\tau, \mu') d\mu' \\ & \quad - S^+(\tau, \mu), \quad (16) \end{aligned}$$

where (15) and (16) may be obtained by summing and differencing (5) for positive and negative values of μ . The intensity ϕ is retrieved by

$$\phi(\tau, \pm \mu) = j(\tau, \mu) \pm h(\tau, \mu), \quad (17)$$

where $0 < \mu \leq 1$. Other quantities of physical significance include the mean intensity, J , given by

$$J(\tau) = \int_0^1 j(\tau, \mu') d\mu' = \frac{1}{2} \int_{-1}^1 \phi(\tau, \mu') d\mu' \quad (18)$$

and the flux, πF , defined by

$$\begin{aligned} \pi F(\tau) &= 4\pi \int_0^1 h(\tau, \mu') \mu' d\mu \\ &= 2\pi \int_{-1}^1 \phi(\tau, \mu') \mu' d\mu'. \quad (19) \end{aligned}$$

For the limiting case of transport in a constant strength magnetic field, equations (15) and (16) reduce to a form identical to the equation of transfer for a plane parallel inhomogeneous atmosphere. With the further assumption that the functions P and S be even in μ , (15) and (16) may be combined to give

$$\begin{aligned} & \mu^2 \frac{d^2 j}{d\tau^2}(\tau, \mu) \\ & = j(\tau, \mu) - \bar{\omega}(\tau) \int_0^1 P^+(\mu, \mu') j(\tau, \mu') d\mu' \\ & \quad - S^+(\tau, \mu). \quad (20) \end{aligned}$$

The integral in (20) may be replaced by a suitable quadrature formula. The second order integrodifferential equation reduces then to a set of coupled ordinary second order differential equations and may be solved by straightforward numerical techniques, as described for example by Auer (1967) and Prather (1974).

For the more general problem in which P may contain odd terms in μ , equations (15) and (16) can no longer be combined, and we must solve directly for both j and h . Replacing integrals by quadrature formulae as before, we have

$$\begin{aligned} & \mu_k \frac{d j_k}{d \tau} \frac{(1-\mu_k^2)}{2} \frac{d \log B}{d \tau} \left(\frac{\partial j}{\partial \mu} \right)_k \\ & = h_k - \bar{\omega} \sum_{l=1}^m g_l P_{kl}^- h_l - S_k^- \quad (21) \end{aligned}$$

and

$$\begin{aligned} & \mu_k \frac{d h_k}{d \tau} \frac{(1-\mu_k^2)}{2} \frac{d \log B}{d \tau} \left(\frac{\partial h}{\partial \mu} \right)_k \\ & = j_k - \bar{\omega} \sum_{l=1}^m g_l P_{kl}^+ j_l - S_k^+, \quad (22) \end{aligned}$$

where subscripts k and l indicate quantities to be evaluated at either μ_k or μ_l , the pivotal points of the quadrature formula, and g_l denotes corresponding weights.

We selected a modified Gaussian quadrature formula (Sykes, 1951) which allows an exact evaluation of polynomial integrands of order $2m-1$, with m quadrature points distributed over the range $0 \leq \mu \leq 1$:

$$\int_0^1 F(\mu) d\mu = \sum_{k=1}^m g_k F(\mu_k). \quad (23)$$

Derivatives of j and h with respect to μ , evaluated at the quadrature points, may be expressed (Chandrasekhar, 1960, p. 365) as linear combinations of the functions j and h at the quadrature points as follows:

$$\left(\frac{\partial F}{\partial \mu} \right)_k = \sum_{l=1}^m D_{kl} F_l. \quad (24)$$

Thus

$$\begin{aligned} & \mu_k \frac{d j_k}{d \tau} \frac{d \log B}{d \tau} \sum_{l=1}^m D_{kl}' j_l \\ & = h_k - \bar{\omega} \sum_{l=1}^m g_l P_{kl}^- h_l - S_k^- \quad (25) \end{aligned}$$

and

$$\begin{aligned} & \mu_k \frac{d h_k}{d \tau} \frac{d \log B}{d \tau} \sum_{l=1}^m D_{kl}' h_l \\ & = j_k - \bar{\omega} \sum_{l=1}^m g_l P_{kl}^+ j_l - S_k^+ \quad (26) \end{aligned}$$

with $k = 1, \dots, m$. The coefficients D_{kl}' are related to D_{kl} according to

$$D_{kl}' = \frac{(1-\mu_k^2)}{2} D_{kl}. \quad (27)$$

The dependent variables j_k and h_k in (25) and (26) are functions of τ and E . The spatial (τ) dependence of j_k and h_k may be evaluated using a finite difference representation for (25) and (26) over an appropriate τ grid. If we ignore, for the moment, complications introduced by the angular derivatives in (15) and (16), we may note that spatial derivatives of j or h may be evaluated directly in terms of

local values for h or j . Thus, j and h need only be evaluated at alternate points along the grid τ_j :

$$\left(\frac{dj_k}{d\tau}\right)_i = \frac{j_k(\tau_{i+1}) - j_k(\tau_{i-1})}{\tau_{i+1} - \tau_{i-1}} = \sum_i A_{ki}(\tau_i) h_i(\tau_i) \quad (28)$$

and

$$\left(\frac{dh_k}{d\tau}\right)_{i+1} = \frac{h_k(\tau_{i+2}) - h_k(\tau_i)}{\tau_{i+2} - \tau_i} = \sum_i B_{ki}(\tau_{i+1}) j_i(\tau_{i+1}), \quad (29)$$

where i is an even integer. The matrices A_{ki} and B_{ki} are defined by the form of the right hand sides of (25) and (26). Angular derivatives of j and h are evaluated as averages of values at adjacent points.

Values of j at the boundaries, τ_0 , may be related to h at adjacent interior points using a Taylor series expansion:

$$\begin{aligned} h(\tau_0 \pm \Delta, \tau) &= h(\tau_0, \mu) \pm \Delta \left[\frac{\partial h}{\partial \tau} + \frac{d\mu}{d\tau} \frac{\partial h}{\partial \mu} \right]_{\tau_0} \\ &= h(\tau_0, \mu) \pm \frac{\Delta}{\mu} \left[j(\tau_0, \mu) \right. \\ &\quad \left. - \tilde{\omega} \int_0^1 P^+(\mu, \mu') j(\tau_0, \mu') d\mu' - S^+(\tau_0, \mu) \right]. \quad (30) \end{aligned}$$

Sufficiently deep in the atmosphere, the electron intensity may be regarded as isotropic, such that

$$h(\tau_0, \mu) = 0. \quad (31)$$

This condition, applied at both ends of a field line, provides the necessary boundary conditions. If the incident intensity, ϕ^* , is specified at some high altitude, then we can introduce an alternate boundary condition using (30), with

$$h(\tau_0, \mu) = j(\tau_0, \mu) - \phi^*. \quad (32)$$

Solution of the transfer equation now requires inversion of a matrix equation. The matrix has dimensions $nm \times nm$, where n gives the number of spatial grid points and m is the number of angular quadrature points. However, the matrix is block tri-diagonal with block sizes $m \times m$, and the system of equations may be solved directly using an efficient scheme described by Auer (1967). The method has the advantage that computational time increases only linearly as a function of the number of spatial grid points.

4. ENERGY DEPENDENCE

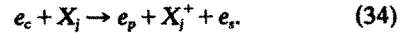
The interdependence of photoelectron intensity at a specific energy, and electron intensities at other energies, arises through the source function \tilde{S} in

equation (1). An electron may lose energy due to an inelastic collision with an atmospheric species j . Suppose that the process α has cross section $Q_{j\alpha}$ and involves a discrete energy loss $\Delta E_{j\alpha}$. In this case the photoelectron intensity at energy $E + \Delta E_{j\alpha}$ produces electrons of energy E . The rate at which electrons are inserted at energy E is then given by

$$\begin{aligned} \tilde{S}_{j\alpha}(E, \tau, \mu) &= \frac{1}{2} n_j(\tau) Q_{j\alpha}(E + \Delta E_{j\alpha}) \\ &\quad \times \int_{-1}^1 \phi(E + \Delta E_{j\alpha}, \tau, \mu') R(\mu, \mu') d\mu'. \quad (33) \end{aligned}$$

where $n_j(\tau)$ denotes the number density of species j at τ and $R(\mu, \mu')$ defines the probability of scattering from μ' to μ which may be related to the differential scattering cross section.

In addition to the discrete excitation of an atmospheric species j , we must also allow for ionizing collisions which produce a continuum of electron energies,



In this case the higher energy scattered electron is arbitrarily designated by the subscript p (primary). Energies of the incident (c), primary (p), and secondary (s) electrons are constrained to satisfy the relation

$$E_p + E_s + \Delta E_{j\alpha} = E_c, \quad (35)$$

where $\Delta E_{j\alpha}$ denotes the ionization potential for the relevant reaction (j, α). The source of primary electrons at E due to (j, α) is given by

$$\begin{aligned} \tilde{S}_{j\alpha}(E, \tau, \mu) &= \frac{1}{2} n_j(\tau) \int_{E+\Delta E_{j\alpha}}^{2E+\Delta E_{j\alpha}} \frac{dQ_{j\alpha}(E', E)}{dE} \\ &\quad \times \int_{-1}^1 \phi(E', \tau, \mu') R(\mu, \mu') d\mu' dE', \quad (36) \end{aligned}$$

where the differential cross section is normalized such that the total ionization cross section for electrons incident at E_c is given by

$$Q_{j\alpha}(E_c) = \int_{E_c - \Delta E_{j\alpha}}^{E_c - \Delta E_{j\alpha}} \frac{dQ_{j\alpha}(E_c, E_p)}{dE_p} dE_p. \quad (37)$$

For the electron ionization reaction (34) the total number of electrons is not conserved and, in effect, we produce a new (secondary) electron for each reaction. The secondary electrons are included in the source term \tilde{S} in a manner parallel to that for primary electrons. The integration in (36) for this case extends over all incident energies greater than $2E + \Delta E_{j\alpha}$. The total source of electrons due to the inelastic collisions of higher energy electrons with atmospheric species is calculated by summing the

individual source terms $\tilde{S}_{j\alpha}$ over all species and processes.

Electrons may arrive at energy E also as a consequence of collisions with ambient electrons. Energy degradation here is caused by a continuous succession of infinitesimal scatters with thermal electrons in the surrounding plasma. If we restrict ourselves to a set of discrete energy levels, $E_1 < E_2 < \dots$, then the rate at which electrons cascade through successive levels varies inversely with their width. The source term due to this process for electrons in the energy interval E_i is given by

$$\tilde{S}(E_i, \tau, \mu) = (E_{i+1} - E_i)^{-1} v^{-1} \frac{dE}{dt}(E_{i+1}, \tau) \times \frac{1}{2} \int R(\mu, \mu') \phi(E_{i+1}, \tau, \mu') d\mu', \quad (38)$$

where v denotes electron speed at E_{i+1} , and $dE/dt(E_{i+1}, \tau)$ is the energy loss rate at E_{i+1} , a function of the plasma temperature and density.

The source terms from inelastic collisions with neutral species and continuous loss to the ambient plasma are derived ultimately from photoproduction at higher energies. Direct photoproduction at E may be incorporated in \tilde{S} in a fairly straightforward fashion by adding a primary photoelectron source calculated from measured values for the incident solar flux (Hinteregger, 1977) and absorption/ionization cross sections for major atmospheric gases as summarized for example by Victor *et al.* (1976).

The inverse mean free path κ is defined in terms of the totality of collision processes which may affect an electron at energy E , and is given by

$$\kappa(E, \tau) = \sum_{j,\alpha} n_j Q_{j\alpha} + \sum_j n_j Q_j^{el} + (v \Delta E)^{-1} \frac{dE}{dt}, \quad (39)$$

where Q_j^{el} denotes the cross section for elastic scattering of electrons with energy E by atmospheric species j and the summation over α includes all possible inelastic collisions, excitation plus ionization. The third term on the right hand of (39) arises from collisions with ambient electrons and depends on the energy interval width ΔE as described above. The inverse scattering mean free path χ is given by

$$\chi(E, \tau) = \sum_j n_j Q_j^{el}. \quad (40)$$

In calculating the source terms and mean free paths, the collisional processes were treated in a manner similar to Victor *et al.* (1976) who computed photoelectron intensities in the limit of negligible transport. This limit, often called the local

approximation, is valid at lower altitudes where the mean free path of the photoelectron is small. As may be seen from equation (5), when transport terms are dropped and the intensity is taken as isotropic,

$$\phi^{local}(E, \tau) = \frac{S(E, \tau)}{1 - \bar{\omega}(E, \tau)} = \frac{\tilde{S}(E, \tau)}{\kappa(E, \tau) - \chi(E, \tau)}. \quad (41)$$

Thus, the local approximation is determined only by the source term \tilde{S} and by inelastic opacity sources.

Up to this point we considered the energy space as a continuum of states and introduced the concept of energy intervals only in connection with the treatment of continuous loss. We propose to divide the energy spectrum into a number of energy intervals, each characterized by a mean energy E_i . Redistribution of inelastically scattered electrons assumes that electrons are uniformly distributed in the various energy bins. However, cross sections for all electrons in a given interval are assigned values appropriate to the mean energy E_i .

Use of discrete energy bins introduces some modification to the source terms as given in equations (33) and (36). In order to obtain the contribution due to an inelastic collision process (j, α), we sum equation (33) over all higher energy bins which overlap with bin E downgraded in energy $\Delta E_{j\alpha}$. Each term in the sum is weighted by the fractional overlap in energy space, and is divided by the width of the lower bin of energy E . The integral over higher energies in the source term due to electron ionization is replaced by a sum, where each contributing bin is weighted according to both energy width and relevant differential cross section. The cross section is normalized (equation 37) also by a summation over the range of primary energy bins. Complications arise if the energy loss associated with an inelastic process is smaller than the energy bin size. We must be careful in this case to introduce an effectively elastic process to allow for an energy loss process in which a fraction of the electrons remain within the original energy bin. The primary production of photoelectrons (eV^{-1}) from photoionization is readily expressed as a uniform source over a given energy bin by summing the total number of electrons produced within the energy interval and dividing by the bin width.

Although use of discrete energy intervals ignores the continuity of the $\phi(E)$ vs E relationship, there are also distinct advantages: small scale structure in the primary photoionization is automatically averaged; the mean free path for continuous loss processes is easily determined (equation 39); and the

different energy bins may be easily separated for computational convenience. Furthermore, with the exception of the ionization-produced secondaries, use of energy bins guarantees conservation of photoelectrons cascading through energy bins as they lose energy to the surrounding media.

5. RESULTS AND DISCUSSION

We described a technique for solving the problem of photoelectron spatial transport and spectral degradation. The method is both efficient and accurate. The spatial dimension is divided into a set of discrete ordinates. The photoelectron intensity for a specific energy at each point in space and at a set of angle points was calculated directed. The selected range of the energy spectrum was covered completely by a set of energy bins. The photoelectron intensity in the highest energy bin was calculated first by assuming that the only source term was derived from photoionization. Thereafter, the intensity at each successively lower energy bin was computed, including source terms from higher energy intensities in addition to photo-ionization. The entire process is direct and non-iterative, and calculates the photoelectron intensities along a magnetic field line for the chosen spectrum of energies. The accuracy of the solution depends both on the spatial step size and on the energy bin width.

Energy bins were spaced uniformly between 1 and 100 eV. Angular redistribution functions were taken as isotropic, as was the primary photoionization source. Figure 1 shows a comparison of present results with calculations by Victor *et al.* (1976) at an altitude of 256 km. The agreement is satisfactory despite the relatively coarse energy resolution (2 eV) employed here. The agreement at low altitudes provides an important check on the methodology employed to solve the general transport problem, in that both calculations employ similar data on relevant cross sections, and the influence of transport should be unimportant at this altitude.

Figures 2-4 summarize results obtained at a succession of higher altitudes. We show a comparison of results from the general theory with those for the local approximation, with two choices, 1 eV and 5 eV, for the energy resolution element. The local approximation agrees with the general formulation at 248 km. It deviates somewhat at the lowest energies at 427 km where the mean free path is relatively large. The general theory allows for migration of electrons from lower altitudes, where the

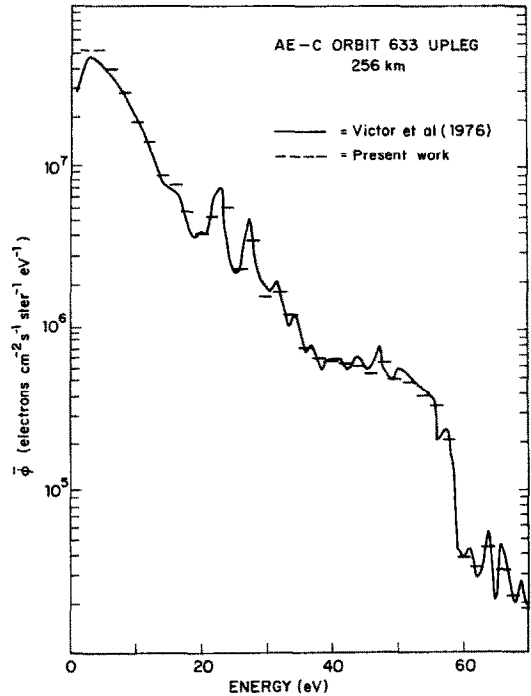


FIG. 1. ANGULAR MEAN PHOTOELECTRON INTENSITY $\bar{\phi}$ AS A FUNCTION OF ENERGY.

The photoelectron spectrum of Victor *et al.* (1976, solid line) is compared with the present work at a 2 eV resolution. The theoretical models are calculated at 256 km and are based on the AE-C orbit 633 upleg.

primary source is largest, and gives intensities which exceed those from the local approximation. This effect is most obvious in Fig. 4 at an altitude of 689 km where intensities obtained in the transport model are larger by almost an order of magnitude, than those found with the local assumption. Figures 2-4 also illustrate the dependence of results on the computational choice of energy resolution.

This paper was intended to demonstrate an efficient scheme for computation of the electron energy spectrum. We employed a number of approximations. Most serious was the assumption that the redistribution functions should be isotropic. A more strongly forward peaked scattering function, appropriate for example for photoelectron-thermal electron collisions, would enhance the effects of transport. Anisotropy may be readily incorporated in the present scheme as described earlier. A detailed comparison of model with observation is postponed to a subsequent report in which the assumptions of isotropy will be relaxed.

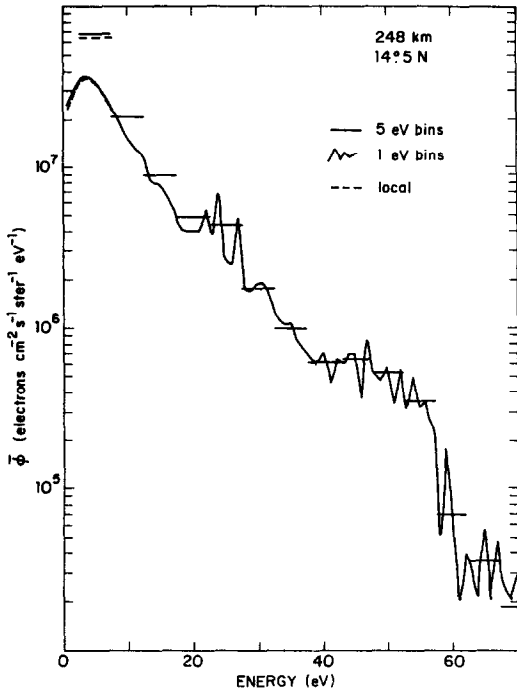


FIG. 2. ANGULAR MEAN PHOTOELECTRON INTENSITY $\bar{\phi}$ AT 298 km AND 14.5°N AS A FUNCTION OF ENERGY FOR 1 AND 5 eV ENERGY BINS.

The altitude–latitude points employed here and in Figs. 3 and 4 lie along a single geomagnetic field which intersects AE–E orbit 394. The complete solution (solid line) is compared with the local solution (dashed line). All scattering processes — elastic and inelastic — are assumed isotropic.

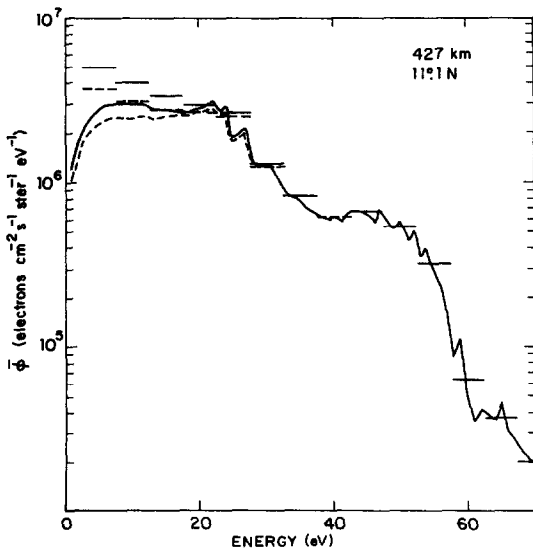


FIG. 3. ANGULAR MEAN PHOTOELECTRON INTENSITY $\bar{\phi}$ AT 427 km AND 11.1°N. SEE FIG. 2.

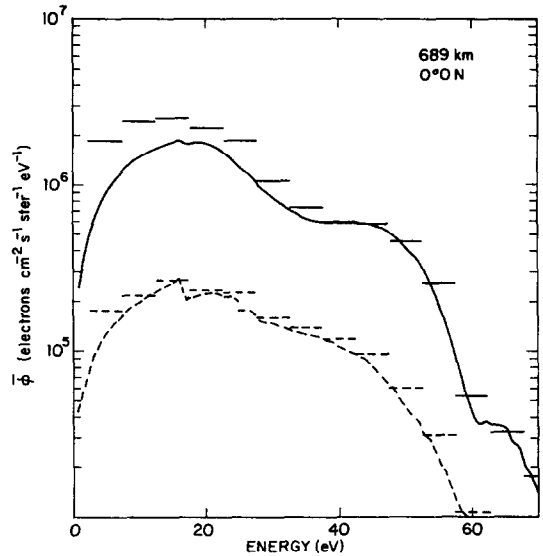


FIG. 4. ANGULAR MEAN PHOTOELECTRON INTENSITY $\bar{\phi}$ AT 689 km AND 0.0°N. SEE FIG. 2.

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REFERENCES

- Auer, L. (1967). Improved boundary conditions for the Feautrier method. *Ap. J. Lett.* **150**, L53.
- Banks, P. M. and Nagy, A. F. (1970). Concerning the influence of elastic scattering upon photoelectron transport and escape. *J. geophys. Res.* **75**, 1902.
- Chandrasekhar, S. (1960). *Radiative Transfer*, p. 365. Dover, New York.
- Cicerone, R. J. and Bowhill, S. A. (1971). Photoelectron fluxes in the ionosphere computed by a Monte Carlo method. *J. geophys. Res.* **76**, 8299.
- Cicerone, R. J., Swartz, W. E., Stolarski, R. S., Nagy, A. F. and Nisbet, J. S. (1973). Thermalization and transport of photoelectrons: a comparison of theoretical approaches. *J. geophys. Res.* **78**, 6709.
- Dalgarno, A., McElroy, M. B. and Moffett, R. J. (1963). Electron temperatures in the ionosphere. *Space Sci.* **11**, 463.
- Dalgarno, A., McElroy, M. B. and Stewart, A. I. (1969). Electron impact excitation of the dayglow. *J. atmos. Sci.* **26**, 753.
- Dalgarno, A., Hanson, W. B., Spencer, N. W. and Schmerling, E. R. (1973). The Atmospheric Explorer mission. *Radio Sci.* **8**, 263.
- Doering, J. P., Peterson, W. K., Bostrom, C. O. and Armstrong, J. C. (1975). Measurement of low-energy electrons in the day airglow and dayside auroral zone from Atmospheric Explorer C. *J. geophys. Res.* **80**, 3934.

- Doering, J. P., Peterson, W. K., Bostrom, C. O. and Potemra, T. A. (1976). High resolution daytime photoelectron energy spectra from AE-E. *Geophys. Res. Lett.* **3**, pp. 129.
- Hanson, W. B. and Johnson, F. S. (1961). Electron temperatures in the ionosphere. *Mem. Soc. Sci. Leige*, **5**, p. 390.
- Hanson, W. B. (1963). Electron temperatures in the upper atmosphere. *Space Res.* **3**, 282.
- Hinteregger, H. E. (1977). Solar EUV Flux on AE Computer Files: R74113. EUV.
- Lejeune, G. and Wormser, f. (1976). Diffusion of photoelectrons along a field line inside the plasmasphere. *J. geophys. Res.* **81**, 2900.
- Mantas, G. P. (1975). Theory of photoelectron thermalization and transport in the ionosphere. *Planet. Space Sci.* **23**, 337.
- Mantas, G. P., and Bowhill, S. A. (1975). Calculated photoelectron pitch angle and energy spectra. *Planet. Space Sci.* **23**, 355.
- Nagy, A. F. and Banks, P. M. (1970). Photoelectron fluxes in the ionosphere. *J. geophys. Res.* **75**, 6260.
- Nisbet, J. S. (1968). Photoelectron escape from the ionosphere. *J. atmos. terr. Phys.* **30**, 1257.
- Oran, E. S. and Strickland, D. J. (1976). Calculation of ionospheric photoelectron distribution function. NRL Memo. Rep. 3361.
- Peterson, W. K., Doering, J. P., Potemra, T. A., McEntire, R. W. and Bostrom, C. O. (1977). Conjugate photoelectron fluxes observed on AE-C. *Geophys. Res. Lett.* **4**, 109.
- Prather, M. J. (1974). Solution of the inhomogeneous Rayleigh scattering atmosphere. *Ap. J.* **192**, 787.
- Swartz, W. E. (1976). Thermalization and transport of photoelectrons: a comparison of theoretical Approaches 2. Transport details for isotropic scattering. *J. geophys. Res.* **81**, 183.
- Sykes, J. B. (1951). Approximate integration of the equation of transfer. *Mon. Not. R. astr. Soc.* **111**, 377.
- Victor, G. A., Kirby-Docken, K. and Dalgarno, A. (1976). Calculation of the equilibrium photoelectron flux in the thermosphere. *Planet. Space Sci.* **24**, 679.