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Publication Date
1964-06-01

UCRL-11499

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Berkeley, California

Proceedings of the International Conference on High Energy Physics - Dubna, USSR. 1964.

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AEC Contract No. W-7405-eng-48

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June 15, 1964

Paper to be presented at the 1964 International Conference on High Energy Physics, to be held at Dubna, U. S. S. R., August 5-15, 1964.

NON-REGGE TERMS IN THE VECTOR CHANNEL OF THE VECTOR-SPINOR THEORY*

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Following the results of the previous talk, it is of interest to inquire whether there is a Kronecker-delta term in the vector channel of the vector-spinor theory. If it were absent from the vector channel, as it was from the spinor channel, one would have a theory with elementary particles but with no Kronecker-delta terms in <u>any</u> channel. We would then <u>not</u> be able to define a bootstrap theory as one with no Kronecker-delta terms in any channel. Our conclusion will be, however, that the vector channel, unlike the spinor channel, is not exceptional. There is a Kronecker-delta term in this channel.

We first observe that the diagram, Fig. 1, is gauge invariant by itself and has an asymptotic behavior which is not in conflict with

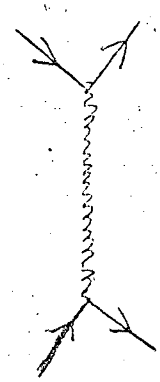

Fig. 1. A diagram with a vector-meson intermediate state.

* This work done under the auspices of the U. S. Atomic Energy Commission.

the unitarity equation. The reasoning by which we proved the absence of a Kronecker-delta term in the spinor channel cannot therefore be applied here, since such reasoning depended on the correlation between Fig. 1 and other diagrams. Of course, this does not prove that the Kronecker-delta term really does not cancel, and one has to examine the problem further in order to establish the existence of such a term.

Gell-Mann et al. have given a general criterion for the cancellation of the Kronecker-delta term. Suppose we represent the potential in the appropriate channel as follows:

	Sense	Nonsense	
Sense	$-a\,\delta_{j,1}$	$\frac{b}{\sqrt{j-1}}$	+ Terms regular at $j = 1$.
Nonsense	$\frac{b}{\sqrt{j-1}}$	$\frac{c}{j-1}$	

In the previous talk we have shown that the potential is of this form. The nonsense-nonsense and sense-nonsense elements become infinite at $j = 1$. The sense-sense amplitude does not become infinite, as it does not involve the value $\ell = -1$, but it will contain a Kronecker-delta singularity. Figure 1 will give such a singularity. The criterion for the disappearance of the Kronecker-delta term in the scattering amplitude is then

$$ac = b^2 . \qquad (1)$$

If there is more than one nonsense state, a and b will be square matrices and b a rectangular matrix. The appropriate generalization of (1) is then

$$a = b^T c^{-1} b . \qquad (2)$$

Gell-Mann et al. did not show conclusively that (1) or (2) is a sufficient condition for the cancellation of the Kronecker-delta term, as there were some uncertainties about subtractions. However, these equations are necessary conditions.

When we attempt to apply these criteria to the vector-meson channel, we are faced with the circumstance that there are no two-particle nonsense states in this channel. In other words, there is no pair of particles with the quantum numbers of the vector meson for which $\sigma_1 + \sigma_2 - 1 = 1$. If therefore we limit ourselves to two-particle intermediate states, we shall certainly not cancel the Kronecker-delta term, since there is no diagram to cancel Fig. 1. The possibility has been raised that three-particle intermediate states might effect a cancellation. The three-meson state has $\sigma_1 + \sigma_2 + \sigma_3 - 2 = 1$ so, by analogy with the criteria $\sigma_1 = n$ for a one-particle state, $\sigma_1 + \sigma_2 - 1 = n$ for a two-particle state, we may expect that the three-photon state gives a singularity at $j = 1$.

Let us therefore investigate whether the criterion of Gell-Mann et al. can possibly be satisfied with three-meson intermediate states. We shall examine a spinor antispinor sense state and a three-meson nonsense state. The sense-sense amplitude is given by Fig. 1 in lowest order perturbation theory. The sense-nonsense and nonsense-nonsense amplitudes are given by Figs. 2(a) and 2(b). We notice that

Fig. 2. Coupling to the three-vector state.

$$\text{Sense-Sense} = O(g^2)$$
$$\text{Sense-Nonsense} = O(g^3)$$
$$\text{Nonsense-Nonsense} = O(g^6) \; .$$

Thus Eq. (2) cannot possibly be satisfied, since the two sides are of different orders in the coupling constant. It follows that the Kronecker-delta term cannot cancel.

The possibility has also been raised that the disconnected diagram, Fig. 3, may be relevant for the nonsense-nonsense amplitude.

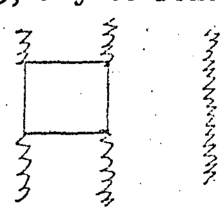

Fig. 3. A disconnected diagram for the three-vector amplitude.

Such a diagram at any rate involves the correct power of the coupling constant to effect a cancellation. However, if we define z as the cosine of the angle of scattering of the disconnected particle, the scattering amplitude will contain a factor $\delta(z - 1)$. The partial-wave amplitude will therefore be independent of the angular momentum j. Since the quantity c in (1) or (2) is the residue of a pole in j, the amplitude represented by Fig. 3 will not contribute to c, and the lowest-order perturbation is given by Fig. 2(b).

There is one further point which must be cleared up before our results are established. The criteria, (1) or (2) have been derived on the assumption that there are a finite number of nonsense states. More precisely, they have been derived on the assumption that a finite number of trajectories begin to move from the value of j in question as the coupling is turned on. We must therefore investigate whether a three-particle intermediate state may not cause an infinite number of trajectories to move from a particular value of j as the coupling is turned on. We have investigated this question and have found that a finite number—more precisely, two—trajectories begin to move from $j = \sigma_1 + \sigma_2 + \sigma_3 - 2$ as the coupling is turned on. No trajectories move from any higher value of j. The criterion of Gell-Mann et al. can therefore be applied, and our reasoning is correct. As a matter of fact, the trajectories which move from $j = \sigma_1 + \sigma_2 + \sigma_3 - 2$ belong to the 2-1 representation of the permutation group π_3. If therefore, the three particles are all identical, there is no trajectory at all which moves from the value $j = \sigma_1 + \sigma_2 - 1$. Thus, in a theory with only one vector meson, there is no three-particle trajectory which moves from $j = 1$. In a theory with more than one elementary vector meson there may be such trajectories, but the criterion of Gell-Mann et al. shows that the Kronecker-delta term is not cancelled.