UNIVERSITY OF CALIFORNIA

Los Angeles

Essays of Game Theory and Its Applications in Political Economy

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Economics

by

Xinyu Fan

2018

© Copyright by

Xinyu Fan

2018

ABSTRACT OF THE DISSERTATION

Essays of Game Theory and
Its Applications in Political Economy

by

Xinyu Fan

Doctor of Philosophy in Economics

University of California, Los Angeles, 2018

Professor Simon Adrian Board, Chair

The dissertation consists of three essays on game theory with applications in political economy. It sheds lights on how strategic interactions among individuals in the society influence the formation of organizational structures and cultural practices. In many cases, the implementation of desirable institutions cannot be relied on the good hearts of the individuals in power. Game theoretical analysis thus serves as a useful benchmark, to deepen our understanding of *whether* benevolence can be achieved out of the most extreme cases of self-interest, and more importantly, *how*.

The first chapter studies the inevitable path towards centralization after power struggles. Power struggles are modeled as iterative coalition formation in which players use their power

to form alliances, eliminate others, and split resources, where formal commitment is impossible. Players can strategically give away power, i.e., burn power to invite new alliances or buy off key members to avoid expulsion. The paper shows that the stability of a power structure relies on the existence of a vested interested group that is capable of adjusting to rule within a smaller circle, but chooses not to do so because the weak outsiders cede power and rents to the strong insiders in order to deter regime changes. This ensures the survival of the weak, at the cost of increased inequality. We show that perturbations on equilibrium structures follow the Iron Law of Oligarchy: regardless of the immediate directions of power shifts, power often ends up more concentrated to a few elite members. The model helps to explain the reproduction of a ruling minority over and over again after regime changes.

The second chapter (joint work with Feng Yang) discusses how a mid-tier officer strategically promotes his subordinates to build up reputation when the big boss is watching him. We show that promotion can be a signaling tool for the superior officer, where he can strategically postpone promoting the subordinate to shift blame and enhance his own reputation. Furthermore, with top-down personnel control, the promoter has extra incentives to shirk, knowing that information manipulation is always an option in the future.

The third chapter (joint work with Lingwei Wu) explores the economic origins of gender-biased social norms, in the context of foot-binding, a painful custom that persisted in historical China that reshaped the feet of girls during early childhood. We present a unified theory to explain the key stylized facts about foot-binding, and investigates its historical dynamics driven by a gender-asymmetric mobility system in historical China (the Civil Examination System). The exam system marked the transition from heredity aristocracy to meritocracy. We show that foot-binding was gradually adopted by women as a social ladder in response to such changes of men's upward social mobility.

The dissertation of Xinyu Fan is approved.

Hugo Andres Hopenhayn

Moritz Meyer-ter-Vehn

Marek G Pycia

Tomasz Marek Sadzik

Daniel Simon Treisman

Simon Adrian Board, Committee Chair

University of California, Los Angeles

2018

DEDICATIONS

To my parents, Ling Lin and Yi Fan,

to my grandmother Bingxing Zhang, my grandfather Guangyi Lin.

And to my grandmother Yuanying Lu, my grandfather Jiuhong Fan in heaven,

I miss you.

Contents

1 Elite Persistence, Power Struggles and Coalition Dynamics **1**

1.1 Introduction 1

1.1.1 Literature 9

1.2 Model 11

1.2.1 Set-up 11

1.2.2 Existence and uniqueness of equilibrium 15

1.3 Elite Persistence in Small-sized Coalitions 17

1.3.1 Three-Agent Coalitions 17

1.3.2 Characterization 21

1.3.3 Elite Persistence 24

1.4 Elite Persistence in Large-sized Coalitions 26

1.4.1 Equilibrium Characterization 26

1.4.2 Elite Persistence 28

1.5 Applications 32

1.5.1 Expropriation vs. Development 32

1.5.2 Domestic vs. International Conflicts 35

1.5.2.1 Foreign Threat 35

1.5.2.2 Foreign Ally 37

1.6 Discussions 40

1.7 Concluding Remarks 42
1.8 Appendix 45
1.9 References 53

2 Strategic Promotions in Hierarchies **57**

2.1 Introduction 57
2.2 Benchmark Model 62
2.2.1 Environment 62
2.2.2 Analysis 67
2.3 Active Ruler and Bureaucrat 71
2.3.1 Environment 71
2.3.2 Analysis 73
2.3.3 Transparency 75
2.4 Empirical Evidence 76
2.4.1 Data and Measures 78
2.4.2 Empirical Model 79
2.4.3 Results 82
2.4.4 Robustness Checks 83
2.5 Conclusion 84
2.6 Appendix 86
2.6.1 Survey Time 86
2.6.2 Proofs 87
2.6.3 Pseudo Promotions and Bureaucratic Redundancy 94
2.7 References 97

3 The Economic Motives of Foot-binding **101**

3.1 Introduction 101
3.2 Background 107

3.2.1 Foot-binding and The Stylized Facts . . . 107
3.2.2 Social Stratification and The Civil Examination . . . 109
3.3 The Theory . . . 113
3.3.1 Marriage Market with Inherited Status . . . 119
3.3.2 Restricted Examination System . . . 120
3.3.3 Meritocratic Examination System . . . 122
3.3.4 The Decline of Foot-binding . . . 127
3.3.5 Summary . . . 128
3.4 Empirical Analysis . . . 130
3.4.1 Data: Cross-sectional Variation . . . 130
3.4.2 Empirical Strategy . . . 134
3.4.3 Empirical Results . . . 136
3.4.4 A Spatial Discontinuity Case . . . 138
3.5 Extensions . . . 138
3.5.1 Hierarchical Structure of the Exam System . . . 139
3.5.2 Continuous Choice of Foot-binding . . . 140
3.5.3 Multiple Competition Tools: Dowry and Foot-binding . . . 142
3.6 Conclusion and Discussion . . . 144
3.7 Maps, Figures and Tables . . . 146
3.8 Appendix . . . 160
3.8.1 Foot-binding in Historical Accounts . . . 160
3.8.2 Flipped Rankings under High Meritocracy . . . 163
3.8.3 Proofs of Propositions and Corollaries . . . 164
3.8.4 Appendix Tables and Figures . . . 176
3.9 References . . . 189

List of Figures

1.1 Timing of The Game 15

2.1 Set-up of The Game 64
2.2 Trust in the Central Government, WVS 2012 80
2.3 Survey Time in Chinese Provinces 86

3.1 Regional Variation of Foot-binding Prevalence 146
3.2 The Civil Exam System and the Social Ladder 148
3.3 Men's Mobility Trend: Evidence from CBDB 149
3.4 Matching Illustration 150
3.5 Foot-binding Prevalence and Exam Meritocracy α 151
3.6 Foot-binding Prevalence and Disutility 152
3.7 Sample Counties by Prevalence of Foot-binding 153
3.8 The Case of Jiangsu Province 154
3.9 The Distribution of Ballads on Foot-binding 182
3.10 The Distribution of Bound Feet Beauty Contest (Sai Zu Hui) 183
3.11 Northern Song Entry Exams and Location of Archaeological Lotus Shoes . . 184
3.12 The Proportion of Degree Holders from Commoners' Families: Jinshi 185
3.13 The Proportion of Degree Holders from Commoners' Families: Juren 186
3.14 Regional Distribution of Rice/Wheat Suitability in China 187
3.15 Regional Distribution of Cotton and Flax Suitability in China 188

List of Tables

2.1 Type Formation for Ruler (Promoter) and Bureaucrat (Promotee) 73

2.2 Diff-in-Diffs Estimation: Promotion and Promoter's Reputation 83

3.1 Number of days of man-labor required per crop hectare for major crops . . . 147

3.2 Out-of-Archive and In-Archive Counties 155

3.3 Ambiguous-Info. Counties and Clear-Info. Counties 156

3.4 Descriptive Statistics 157

3.5 Crop Suitability, Exam Quotas and Foot-binding Prevalence 158

3.6 Exam Quotas and Foot-binding Prevalence: Prefecture Fixed Effects 159

3.7 Historical Sources of Foot-binding: A Comparison 177

3.8 The Income of Gentry Class by Occupation 178

3.9 Proportion of Gentry and Literati by Province 179

3.10 Correlation Table of Variables 180

3.11 Crop Suitability, Exam Quotas and Foot-binding Prevalence: Control For Sex Ratio 181

ACKNOWLEDGMENT

First and foremost, I would like to thank my advisor Simon Adrian Board. It has been a great honor and privilege to be Simon's student. As a researcher in training, I enjoy the freedom of trying out different routes; And as a Ph.D student, I would also need guidance to proper paths. Simon makes sure of the perfect balance of the two. I often come up with wild ideas during the studies. And Simon is always there for me, however weird the idea might be, and teaches me to discover the economic insights underneath. So thank you, Simon, for being the best advisor I could have ever hoped for.

I am also deeply indebted to Marek G Pycia, Tomasz Marek Sadzik, and Moritz Meyer-ter-Vehn. They have enlightened me academically on research, encouraged me when things are not going as expected, and have supported me continuously throughout the job market season. I would never forget the conversation in Tomasz's office days before my first pro-seminar, or the Skype calls with Marek from Switzerland, or the taxi ride with Moritz in Philadelphia. Furthermore, I would like to express my sincere gratitude to Hugo Andres Hopenhayn and Joe Ostroy, for supporting me during the Ph.D studies, and also for writing the letters for me when I applied to the program in 2012. Thanks for making this wonderful journey possible. I also greatly appreciate the inspiring conversations with Ichiro Obara, Jay Lu, and the participants of the theory proseminars.

As a theorist interested in political economy, I benefit greatly from the professors in political science. In particular, I would like to thank Daniel Simon Treisman and Michael Chwe for opening a whole new field for me. I would also like to thank James Tong for the nice chats and the get-together dinners. My interests in political economy was seeded in my college years, for which I thank Jacques Crémer, Clement Wong, and in particular, my undergraduate advisor, Chenggang Xu.

This dissertation would have not been possible without my co-authors, Feng Yang (Chapter 2) and Lingwei Wu (Chapter 3). We jointly show how theoretical models could speak

to the facts of the real world. Having the privilege working with them enlightens me of the kind of theorist I would strive to be. Their work attitude has been a motivation beyond this dissertation. So thank you, Feng and Lingwei.

My journey at UCLA has been special because of the friends and colleagues within and outside of the department. I thank Wenshu Guo, Zhuoran Lu, Omer Ali, Nick Doran, Jing Hang, Byeong-Hyeon Jeong, Ting Ji, Daehyun Kim, Ioannis Kospenaris, Fanghua Li, Xinyu Li, Chen Liu, Yingju Ma, Liyan Shi, Yangbo Song, Jesper Sørensen, Yuan Tian, Rongzhang Wang, Meng Xu, Menghan Xu, Yizhi Xu, Tianyu Yin, Wenbo Zhou, and in particular, Edward McDevitt, Chiara Paz, and Juliana Smith, for making this department a big happy family. I also thank Wayne Yuan Gao, Lei Bao, Chencen Cai, Longyan Chen, Xiao Chen, Fen Guo, Hui Guo, Jialing Hu, Hao Ji, Datong Jia, Jia Li, Shuoran Li, Siyun Lin, Yuqi Liang, Chunhui Liu, Zhenwei Liu, Cheng Mao, Danqing Mei, Xin Nie, Ning Ruan, Tashi Sun, Evelyn Wang, Zeyuan Yu, Hongwei Zhang, Meimei Zhang, Jun Zhao, and Yifei Zhao for their support along the way. Last but not least, I thank Jasper and Vivian for being my enduring source of inspiration.

VITA

2009-2012	Bachelor of Economics and Finance,
	University of Hong Kong
2012-2014	Master of Arts in Economics
	University of California, Los Angeles
2013-2017	Teaching Assistant, Associate, Fellow
	University of California, Los Angeles
2016	UCLA Departmental Excellence in Teaching Prize
2017	UCLA Departmental Outstanding Teaching Assistant Award
2017-2018	UCLA Dissertation Year Fellowship

Chapter 1

Elite Persistence, Power Struggles and Coalition Dynamics

> "*Even when the discontent of the masses culminates in a successful attempt to deprive the bourgeoisie of power, this is ... effected only in appearance; always and necessarily there springs from the masses a new organized minority which raises itself to the rank of a governing class.*"
>
> Robert Michels, *Political Parties: A sociological study of the oligarchical tendencies of modern democracy,* 1915, p.390.

1.1 Introduction

Why do social and political hierarchies often prove extremely difficult to eradicate? In many cases, revolutions that aim to install social equality quickly reproduce the same type of hierarchies that the revolutionaries sought to destroy. In George Orwell's terms, it turns out "some animals are more equal than others". The Bolshevik Revolution, rather than establishing the rule of the proletariat, quickly degenerated into an oligarchy led by Lenin, and later a dictatorship of Stalin for 30 years (Pipes 2011). Likewise, the French revolutionaries, after overthrowing their king, soon produced an emperor (De Tocqueville 1856). Meanwhile,

elites tend to persist after grasping power, as in the al Assads in Syria, or the Kims in North Korea. The durability of unequal power distribution has often been attributed to culture - the "serf mentality" of Russians (Havel 1985) or Confucian values in Asia (Zhang et.al 2005) - or to the rapacious energy of dominant individuals (Hobbes 1651). The break-down of hierarchies, as suggested in the seminal paper of Acemoglu, Egorov, and Sonin (2008, hereafter AES), can take place when the alliance of the weak collectively expel the strong in power struggles and ensure no further power struggles within such alliance. That is, to sustain a regime when no agreements are credible, power itself should become the *de facto* commitment device. However, in AES, players have fixed power levels, thus are unable to respond to elimination threats by adjusting to a lower-ranked position, like Nikolai Bukharin of the Soviet Union after 1929 (Gregory 2013); or by pledging allegiance to others, like the Sénat Conservateur to Napoleon Bonaparte after the coup of 18 Brumaire (Doyle 1989). Furthermore, it remains a puzzle why centralization of power often restores soon after the old regime falls, or in Michels' (1915) words, "always and necessarily there springs from the masses a new organized minority which raises itself to the rank of a governing class".

The main contribution of this paper is to connect power adjustment and the inevitable trend to centralization in power struggles. The paper explains how the ability to *adjust* power serves as a commitment device in power struggles, which leads to the convergence of power to an elite group, and how such elite group persists during perturbations. Power struggles are modeled as an iterative coalition formation process where a group of players use their power to form alliances, eliminate others, and split fixed resources according to relative power if they survive the power struggles. Power struggles follow the law of the jungle: players cannot commit to protect oneself from others, because there is no one to enforce the agreements. However, players have the option to adjust power: in particular, they can strategically give away power, i.e. burn power, before participating in power struggles to eliminate others and split resources. By doing so they can credibly invite new alliances or buy off key members in a coalition, in order to avoid expulsion. In equilibrium, the weak

outsiders cede power (and consequently rents) to a few strong insiders who are capable of ruling in a smaller circle, until the insiders are sufficiently compensated to stay in the status quo. This ensures the survival of the weak, at the cost of increased inequality. Thus the elite class emerges. We characterize the equilibrium power structures, and show regardless of the immediate directions of power perturbations, power often ends up more concentrated to a few elite members. Thus the elites persist during ups and downs. This is sometimes known as the Iron Law of Oligarchy.

The emergence of the elites results as follows. In power struggles without commitment, any alliance that intends to eliminate others has to ensure the current alliance survives future power struggles from within. Therefore, a stable regime change often involves power restructuring within the regime changers prior to a revolution or a coup. In particular, a strong player sometimes has to give away power to credibly assure his allies that he is not a future threat, in the effort of forming a smaller circle to split the rents. This, on the other hand, gives the excluded players an opportunity to buy off the strong by ceding power preemptively: the strong players receive enough benefits from the power ceding thus are reluctant for further regime changes. That is, the flexibility of power adjustment serves as a commitment device of threatening to form a smaller ruling circle. In response, the players outside the ruling circle (outsiders) have to give away their power until some of the players (elites) in the smaller ruling circle prefers the status quo. As a result, power becomes more concentrated, and a hierarchy is established in equilibrium between the outsiders and the elites. After the elites establish their prestigious positions, any perturbations that make them stronger naturally centralize the power structure. Meanwhile, any perturbations that weaken their positions re-establish the incentives to eliminate the outsiders, which forces the outsiders further cede power to restore the prestige of the elites. As a result, the hierarchy persists, regardless of the initial direction of the power shifts.

To be specific, the model consists of a coalition of agents, who split a fixed rent of one

according to their respective, endowed levels of power. For example, in a 4-agent coalition with power structure $(2, 4, 5, 10)$, the first agent (with power level of 2) claims rents of $\frac{2}{2+4+5+10} = \frac{2}{21}$, while the rest of the agents claim $\frac{4}{21}$, $\frac{5}{21}$ and $\frac{10}{21}$, respectively. If agents are unsatisfied with current rent distribution, they can engage in power struggles in order to attempt to change the structure. Power struggles are modeled as an iterative process of two meetings. The first meeting is called a *power burning* meeting, where each agent sequentially proposes a power adjustment scheme to a sub-coalition of agents (an alliance) to prepare for the coming struggles. We assume agents only adjust power downwards (eg. disarm an army, resign from office, migrate to a foreign country, etc.), and refer to this process as *power burning*. A burning scheme is enacted only with unanimous consent from the included agents. Otherwise no power is burnt and it turns to the next agent to propose. The burning meeting continues until no agent nor sub-coalition wishes to further burn power.

The second meeting is an *elimination meeting*. Each agent sequentially proposes a new, and potentially smaller ruling coalition. The proposal passes if the coalition commands strict majority power and only with the unanimous consent of all agents within the coalition. Otherwise the proposal is rejected and it turns to the next agent to propose. If all proposals are rejected, the game ends and the rents are split between agents based on the current power structure. If a new coalition is formed, all excluded agents are eliminated (eg. retired, exiled, or executed). The power struggle cycles continue in the new coalition: first the burning meetings and then the elimination meeting. An equilibrium is reached when no further burning or eliminations take place. Agents collect payoffs in equilibrium according to their status: A surviving agent splits the rents proportional to his power; an eliminated agent receives a large and negative payoff.

We show that a Sub-game Perfect Nash Equilibrium (SPE) always exists for the power struggle game, and the equilibrium structure is unique given initial power structure and the sequence of moves. We then study the formation of equilibrium structure to show how hierarchies resurge in equilibrium, and how the elites maintain their prestige during power

perturbations.

We illustrate the main economic forces using the following example. In the power struggle game, agents seek to form smaller alliances for higher rents while watching out for future eliminations. Recall the example of $(2, 4, 5, 10)$, which resembles a hierarchy with an (initial) elite agent holding power of 10, two medium agents holding power of 4 and 5, and a weak agent holding power of 2. Based on the rules of power struggles, the alliance of the non-elites, $(2, 4, 5)$ is strong enough to overthrow the elite $(2 + 4 + 5 > 10)$ in revolution and split the rent within the alliance.

Power struggles do not stop after the old hierarchy falls. Suppose the old elite is eliminated, in the new regime of $(2, 4, 5)$, a new alliance between agents with power 4 and 5 is powerful enough to eliminate the weak agent. However, the coalition $(4, 5)$ is not stable after the elimination of the weak, because the stronger agent with power 5 can now expel the agent with power 4 and become a dictator. Furthermore, there is no way for the stronger agent to credibly commit not to do so. Anticipating further power struggles, $(4, 5)$ is not a profitable deviation for the agent with power 4. The current structure $(2, 4, 5)$ is thus stable. This is the idea of self-enforcing coalitions in AES.

However, when agents are allowed to burn power, agents behave differently. First, during the fall of the old hierarchy, the agent with power 10, facing elimination, can cede power completely to survive, $(2, 4, 5, 10) \to (2, 4, 5, 0)$. Next, in the new regime, $(2, 4, 5, 0)$, the agent with power 5 could credibly commit to the alliance by burning power down to 4, thus inducing a post-burning alliance of $(4, 4)$. Such an alliance is not only rent-improving for both participants, but also safe from future power struggles because no individual is powerful enough to expel the other one. We call such alliances *stable improving* coalitions.

Anticipating the formation of the stable improving alliance, the excluded agent has to cede power for survival. He continues to do so until the insider becomes indifferent between the status quo and the alternative. In the example, the agent with power 2 burns power to

1, $(2,4,5,0) \to (1,4,5,0)$, so that strong insider (with power 5) is rent-indifferent between the current regime $\left(\frac{5}{1+4+5}\right)$ and the stable improving alternative $\left(\frac{4}{4+4}\right)$. The new power structure $(1,4,5,0)$ is the equilibrium structure. Lastly, the agent who possesses power 1 in equilibrium still enjoys greater rents than in the original power structure $\left(\frac{1}{10} > \frac{2}{21}\right)$, that is why the agent signs up bringing down the old hierarchy in the first place: it is not because that he anticipates peace ever after, but that he expects more rents even after future struggles settle.

We characterize the equilibrium structure for small-sized coalitions, and characterize the boundaries for large-sized equilibrium structures. When coalition sizes are small, a power structure reaches equilibrium in power struggles only when there does not exist a strictly stable improving alternative. Furthermore, the shape of the regime necessarily converges to one of the three forms: a dictatorship where all power is concentrated in one individual; an oligarchy where half of the power is concentrated in one individual; or a polyarchy where all individuals has identical power levels. We then show that any power perturbations on the equilibrium structure, regardless of direction, leads to power centralization to the already powerful. This resembles the Matthew effect (Merton 1968) where the rich become richer, and the poor become poorer. In the literature of organizational structures, it is also referred as the Iron Law of Oligarchy (Michels 1915)[1]. When the coalition sizes get larger, more flexible power structures can be supported in equilibrium. In particular, it is possible that a strictly stable improving alternative exists, but fails to be implemented because the transitional path can be blocked by early-moving powerful agents that guard the regime. We then categorize the shapes that equilibrium structures can take, and provide conditions that the Iron Law of Oligarchy holds in larger coalitions.

To see this in the model, recall the equilibrium structure in the previous example $(1,4,5)$

[1]In reality, Magaloni and Kricheli (2010) record the increasingly rampant one-party rules in modern times. Svolik (2012) and Kendall-Taylor and Frantz (2016) highlight the persistence of successors' power after the deaths of old rulers.

(here we ignore the agent with zero power as he does not participate in the rent distributions). Suppose a negative perturbation applies to the elite, $(1, 4, 5) \to (1, 4, 5 - \epsilon)$. Now the elite has incentives to ally with the agent with power 4 and eliminate the weak because their alliance $(4, 4)$ is now strictly stable improving ($\frac{1}{2} > \frac{5-\epsilon}{1+4+5-\epsilon}$). To block this alliance, the weak again cedes power until the new structure becomes $(1 - \epsilon, 4, 5 - \epsilon)$, where the elite restores his rent after the negative shocks, thus is reluctant to launch any regime changes. Similarly, suppose a positive perturbation hits the weak, $(1, 4, 5) \to (1 + \epsilon, 4, 5)$. The alliance $(4, 4)$ again becomes strictly stable improving, which forces the weak to cede back the increased power. The stable structure recovers to $(1, 4, 5)$, and the weak remains weak.

On the other hand, suppose a positive perturbation hits the elite, $(1, 4, 5 + \epsilon)$, or a negative perturbation hits the weak $(1 - \epsilon, 4, 5)$, then the elite has unchecked power to eliminate the rest of the coalition ($5+\epsilon > 1+4$) and becomes a dictator. To avoid elimination, the two other agents cede power completely to the elite, by burning down to 0, and the equilibrium structure becomes $(0, 0, 5 + \epsilon)$ and $(0, 0, 5)$, respectively. Therefore, a perturbation, regardless of direction, has a bias towards centralization, and a small perturbation may collapse the power structure to a dictatorship.

Even though the model is abstract, the insights apply to many real life scenarios. For example, the founding of the Song Dynasty in imperial China captures all of the previous forces. In 960 AD, at the Bridge of Chen, a military general named Zhao Kuangyin of Zhou Dynasty was persuaded by his lieutenants to take the throne and became the new emperor. His lieutenants put a yellow dragon robe, which represented imperial power, on General Zhao. The incident turned out a peaceful coup as the sitting emperor, 9-year-old Chai Zongxun, agreed to cede power. Chai stepped down and lived a peaceful life thereafter (Paludan 1998). Known as the Coup of the Dragon Robe, this encapsulates the change $(2, 4, 5, 10) \to (2, 4, 5, 0)$. However, the power struggles did not stop when General Zhao became Emperor Zhao. Two years later, Emperor Zhao encouraged some of his old

lieutenants to retire in a grand banquet. The lieutenants agreed with "the tears of joy and regret (of not volunteering to do so earlier)", ceding their military command back to the regime (Tao Li 2004, Vol.1). The regime became more centralized, which captures the changes $(2, 4, 5) \rightarrow (1, 4, 5)$. After grasping the military control, Emperor Zhao further took advantage of a small mistake in local jurisdiction to centralize both administrative and jurisdiction control, and soon seized all the power to himself. This last episode captures $(1 - \epsilon, 4, 5) \rightarrow (0, 0, 5)$ (Tao Li 2004, Vol.14).

Having the model at our service, we apply our framework to consider two real-life scenarios and provide testable predictions for power struggles and their consequences. The first scenario focuses on the tradeoff between economic development and extraction. While an intuitive tradeoff exists between efficiency and equality, power struggles introduce additional concerns of survival for the rulers. We show that, when the initiating power struggle is a policy choice for the leadership, a ruler with weak control over the society may support *more* economic development. That is, a weak ruler can be forced to focus on economic decentralization instead of extraction for fear of losing power in the extractions, which in turn leads to a weaker ruler and the cycle continues. Correspondingly, we predict a far-sighted ruler should adopt a "sow and reap" strategy when he is strong: aim for economic development for some time, then initiate power struggles before the potential opponents gets too powerful, and resume economic development afterwards. One possible pattern following such strategy is the cycling between decentralization (reform) and centralization (extraction). Meanwhile, it predicts an authoritarian ruler, after consolidating power, should aim for economic development to enlarge the pie. The early years of Bashar al Assad regime in Syria, and the recent developments under the Kim Jong-un regime in North Korea are consistent with the prediction.

The second scenario is the domestic conflict resolution in presence of international influence. Intuitively, imminent foreign threats (Nazis on the Soviet Union, Mongolian riders

at the gate, etc.) soften domestic conflicts because a united domestic power stands more chances against the threat. However, when external forces are no longer a threat, domestic coalitions resume their power struggles. As a result, conflicts can be inefficiently prolonged if the involving parties concern the purges should the conflict be resolved. We suggest the power struggles in post-war military leaderships could be available for empirical tests. On the other hand, an agent facing expulsion threats can make himself indispensable in the power balance, by inviting a strong foreign force to settle the disputes. As an illustration, we point to the regime changes in Caribbean colonies where the increasingly powerful citizenry induced the elite to cede power back to the British Crown (Ashdown 1979, Carvalho and Dippel 2016).

We conclude the paper by discussing the important assumptions made in the model, as well as the future directions of research. In particular, we highlight the impact of elimination costs (eg. casualty of war) in power struggles. Power struggles are cost-free as a simplifying assumption in the model. However, when elimination costs are positive, centralized regimes are more likely to be formed. We also expect a non-monotonic relationship between the costs of conflicts and the frequency of conflicts: conflicts may first be more frequent, and then less so as elimination costs increase. Implications on international relations are also discussed.

1.1.1 Literature

Our discussion of power struggles is related to a number of literature. We echo the models of power struggles (Acemoglu Egorov Sonin 2006, 2008, 2012, Jandoc and Juarez 2013, Guimaraes and Sheedy 2017), in that participants concern not only the immediate impact of power structure changes, but also the aftermath, i.e. the struggles after struggles[2]. Building

[2]The far-sight in elimination for resources are also seen in the game of truels, or three-person duels (Kilgour 1977, Amengual and Toral 2006). Three players form a circle, where each of them points his gun to the "downstream" player, and has his head aimed by the "upstream" one. A balance of terror can be maintained where no player pulls the trigger, which shares the insight of far-sighted power struggles.

on AES, this is the first paper that allows agents to adjust their power to impose elimination threats or to respond to such threats. Furthermore, we identify the novel role of power burning as a commitment device. While power, as a commitment device, guarantees stability of coalitions in AES, power burning as an additional commitment device in this paper further shapes the structure of a stable coalition, and its tendency towards power centralization.

Our model is also among the first papers to combine the roles of power in rent distribution (Baron and Ferejohn 1989) and the roles of power in endogenously determining coalition composition (Pycia 2012, Ray and Vohra 2015). The seminal work by Baron and Ferejohn (1989) argues the resource distribution in legislature depends on exogenous participation constraints of involving parties. And one has to accommodate the necessary alliances to rule. Bueno de Mesquita et al. (2003) extend the insight to political survival and argue that the ruler has to accommodate the interests of his smallest winning coalition. However, it remains unclear how the participation constraints and the sizes of winning coalitions are determined. Our paper provides a micro-foundation of how participation constraints and winning coalitions are endogenously formed in power struggles, thus linking the distribution role of power to the coalition formation role.

In the literature of elite persistence in organizations, especially non-democracies, Acemoglu and Robinson (2006, 2008) and Robinson (2012) emphasize the office-holding elites implement policies and investments that favor themselves in the future. The insights are also confirmed in empirical studies of democracies, in the context of United States Congress (Dal Bó, Dal Bó, and Snyder 2007). Their models focus on the competition between an existing elite and, essentially, a representative citizen. This paper adds to the discussion by showing how the elites emerge and persist from a generic coalitional structure. To the best of the author's knowledge, this is also the first paper to show that the persistence of elites can be established and sustained through power struggles and far-sighted coalition formations in a zero-sum game, without opportunities for elites to accumulate power through investments (eg. Robinson 2008).

Our model aims to provide a framework of regime changes without explicit assumptions on institutional details. In the classical literature about regime changes, a menu of established regime choices are usually prerequisites: Acemgolu and Robinson (2005, 2013) on democracy vs. dictatorship; Fearon (2011) on ballot vs. bullet, etc. Discussions based on established regimes provide clear-cut comparisons, but implicitly assume away the origin of institutions, i.e. where does the menu come from. On the other hand, there are growing attempts to discuss regime changes abstract from institutional details (eg. Aghion and Bolton 2003, Aghion, Alesina and Trebbi 2004, Gehlbach, Sonin and Svolik 2016). Following the latter approach, this paper contains no presumed forms of institutions, thus adding to the literature by showing how familiar institutional designs may emerge as equilibrium outcomes.

1.2 Model

This section details the set-up of the model and discusses the existence and uniqueness of the equilibrium after power struggles. Our model of power struggles provides opportunities for players to adjust their power, form alliances, and eliminate other players. In brief, players first participate in *power burning meetings* where they sequentially propose downward power adjustment schemes until no coalition wishes to burn power further. Players then participate in *elimination meetings* where they sequentially form alliances to eliminate unallied members in order to gain higher relative power. Equilibrium is reached when no players wish to further adjust power or to eliminate others.

1.2.1 Set-up

Players, power, coalitions. We denote the set of finite *agents* by N_o. Agents are individuals or representatives for groups. A *coalition*, $N = \{1, 2, ..., n\}$, is a group of agents, $N \subset N_o$. Each agent in the coalition is endowed with some power $0 \leq \gamma_i < \infty$. γ_i is discrete

and exists on small grids $\mathbb{G} = \{0, \epsilon, 2\epsilon, ...\}$[3]. Agents are labeled from weak to strong in the *power vector*, $\gamma_N = (\gamma_1, \gamma_2, ..., \gamma_n)$, $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$. The power of a coalition N is simply the sum of its members' powers, $\sum_{i \in N} \gamma_i$. Within coalition N, a sub-coalition $C \subset N$, where $\gamma_{i \in C} = \gamma_{i \in N}$ for all $i \in C$, is *winning* if $\sum_{i \in C} \gamma_i > \sum_{j \in N \setminus C} \gamma_j$. That is, a sub-coalition is winning if it commands strict majority power. Furthermore, we denote $N_+ \subset N$ as the sub-coalition in which players have strictly positive power levels.

Players move sequentially in the game. An *agenda* $s \in S$ specifies the sequence of moves and is fixed in a game [4]. We refer to a particular agenda as *top-down* when the (initially) strongest agent always moves first, followed by the second strongest, all the way down the hierarchy to the least powerful agent. An agenda is imposed on individuals, but not on power structures.

Strategies. Agents take actions in two kinds of meetings, iteratively. The game starts with stage-j coalition N_j, where in the first stage $N_1 = N$, and proceeds in the following steps:

1. [**Burning meeting**]: The burning meeting takes place in rounds. The history is visible to all agents. In Round 1, each agent, i, sequentially makes a burning proposal, $0 \leq \rho_i^{j,1}\left(\gamma_{B_{i,1}}^{j,0} | h_{1,s_i}\right) \leq \gamma_{B_{i,1}}^{j,0}$, to agents in sub-coalition $B_{i,1} \subset N$. h_{1,s_i} denotes the history up to Round 1 according to agenda s before i burns. Then agents in $B_{i,1}$ cast their votes sequentially according to s. Each agent votes yes or no. The proposal is passed if and only if all agents included in the proposed coalition vote yes. If passed, the proposal is implemented

[3]We assume power is discrete to cut the game into a finite horizon for the existence of equilibrium. In real life, power is often discrete, such as the size of an army or the population of one's followers. We also assume the sizes of grids to be uniform. It is useful to think of grids to be very small such that the power adjustment can almost be continuous, and $\frac{1}{\epsilon} \in \mathbb{N}$, so that all integers are available in the grids.

[4]We assume a fixed agenda because turbulent power struggles often take place in a short period of time, where the sequence of moves of are hard to change due to geographic limits, etc. More discussions follow in Section 1.6.

and power is burnt accordingly; otherwise no power adjustments take place and we proceed to the next agent. A round finishes after all agents have made their proposals, and then it proceeds to the next round. In Round 2, each agent continues to make burning proposals $0 \leq \rho_i^{j,2}\left(\gamma_{B_{i,2}}^{j,1}|h_{2,s_i}\right) \leq \gamma_{B_{i,2}}^{j,1}$, following s. The process continues until Round $t+1$, where no burning takes place: that is, for all $i \in N_j$, either $\rho_i^{j,t+1}\left(\gamma_{B_{i,t}}^{j,t}|h_{t,s_i}\right) = \gamma_{B_{i,t}}^{j,t}$, $t \geq 1$, or no proposal is passed. A burning meeting lasts for at least two rounds, but ends in finite rounds because of the discreteness of power. When the burning meeting ends, the game proceeds to Step 2.

2. [**Elimination meeting**]: The elimination meeting also takes place in rounds. Following the same agenda s, skipping any eliminated agents or agents with zero power, an agent, i, makes a proposal of a winning sub-coalition $P_i^j \subset N_j$ as the new ruling coalition. Agents then cast their votes sequentially according to s. Each agent in P_i^j votes yes or no. The proposal is passed if and only if all agents in P_i^j vote yes. The game proceeds to Step 3 below if the proposal is accepted; otherwise it turns to the next agent s_{i+1} according to s to propose. If no individual proposal is accepted in a round, the status quo proposal $P^j = N_j$ is accepted automatically. As a result, an elimination meeting always ends in one round.

3. Agents not in the proposal are eliminated. If the size of winning coalition shrinks, i.e. $|P_i^j| < |N_j|$, we go to stage $j+1$, where $N_{j+1} = P_i^j$, and start from Step 1 again. If $|P_i^j| = |N_j|$, we go to Step 4.

4. Payoffs are assigned.

Payoffs. Staying in the coalition generates office-holding rents. The total rents to share are fixed and are normalized to 1. Agents receive rents proportional to their relative power if they survive in power struggles. For an agent who survives in coalition N with power structure γ, he receives payoffs $U_i(N,\gamma) = \frac{\gamma_i}{\sum_{i\in N}\gamma_i}$. An eliminated agent receives $U_i = -n$, which captures the large costs of losing offices.

During the elimination meetings, if multiple non-proposing agents have the same power

levels, we assume they have equal probabilities to be included in an elimination proposal. Given the large and negative payoff of being eliminated, an agent does not gamble for probabilistic inclusion in the stable coalition[5]:

Assumption 1. *Non-proposing agents with the same power levels have equal probabilities to be included in an elimination proposal.*

We also apply the following tie-breaking assumptions when an agent receives the same rents or has the same power levels with another agent in elimination meetings:

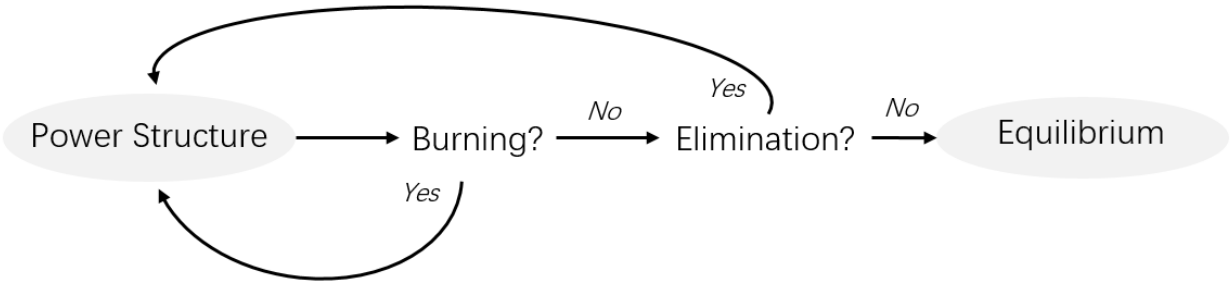
Assumption 2. *Agents always prefer a coalition where they burn less power when receiving the same rents.*

Assumption 3. *Agents always prefer a coalition formed with fewer meetings when the rents received and the power burnt are the same.*

Agents stop attending meetings after they burn down to zero power. Assumption 2 captures the costly nature of burning power, for instance, the dismissal costs of a disarmament. Assumption 3 captures the meeting costs. These costs are usually small in scale relative to the rents, so we only consider them when breaking the ties. The lexicographic tie-breaking assumptions help to identify a unique stable structure in equilibrium.

[5]For example, suppose there is a coalition with power structure $(3, 3, 3)$ and the agenda is top-down. Any two agents can form a winning coalition and eliminate the remaining agent. When agent 1 chooses his alliance, he is indifferent with choosing agent 2 and 3, thus assigning probability of $\frac{1}{2}$ to include either one. Both agents 2 and 3 face negative expected utility $\frac{1}{2}\left(\frac{1}{2} - n\right) < 0$. Under top-down agenda, the late-mover of the two, i.e. agent 3, has a better response by burning down to 0 and guarantees payoff of 0.

Figure 1.1: Timing of The Game



1.2.2 Existence and uniqueness of equilibrium

The solution concept is sub-perfect equilibrium (SPE). A coalition N^* with its power structure γ^* is in equilibrium if it survives the iterations of power burning and eliminations: $(N^*, \gamma^*) \in \phi(N, \gamma|s)$, where ϕ is the correspondence generated by the power struggle game, N is the initial coalition, γ is the initial power structure, and s is the agenda. Power struggles do not last forever because the power levels are discrete and thus the room for adjustment is limited. Therefore, power structures stabilize in finite numbers of meetings. Furthermore, with the tie-breaking assumptions, we are able to select the unique equilibrium that aims to minimize power burning and the number of meetings.

Theorem 1.1. *There exists a SPE of the power struggle game. For a given agenda, s, and under Assumptions 1-3, the game induces a unique equilibrium power structure for initial coalition N with power structure γ, that is, $(N^*, \gamma^*) = \phi(N, \gamma|s)$.*

Proof. See Appendix. □

Theorem 1.1 shows that a unique outcome is reached after power struggles if the initial power structure and the agenda are fixed. Therefore, the direction of coalition dynamics is clear. Denote the initial power structure of a coalition as the *status quo*. By Theorem 1.1,

if multiple equilibrium structures are reached starting from the same status quo, it is either due to the agenda differences, or the violations of at least one of the three assumptions. On the other hand, multiple status quos may converge to the same equilibrium structure. In the next section we discuss the impact of different agendas on equilibrium structures and the convergence of status quos, in detail.

Given the significant costs of being eliminated, the losing agents in power struggles always cede power completely to the rest of the coalition when they are about to be expelled, and forego the rights of claiming rents[6]. Therefore, we expect to see no elimination taking place on equilibrium path.

Corollary 1.1. *No elimination takes place on equilibrium path. In any equilibrium* $(N^*, \gamma^*) = \phi(N, \gamma|s)$, $N^* = N$.

Proof. See Appendix. □

This is in contrast with AES where agents not in the winning coalition are eliminated because they are unable to adjust their power to survive. In our model, agents are allowed to give up their power to create new balances. Furthermore, agents who do not have elimination concerns may also give up power to trigger favorable regime changes, thus enriching the potential stable structures generated under power struggles. Surprisingly, such richness sharpens the predictions we can make towards equilibrium structures, and we predict that the dynamics of equilibrium structures in response to external shocks often have a tendency to centralize. This will be the focus of our discussion in the next two sections.

Hereafter we suppress notations when there is no confusion about the coalition size or the agenda referred to, and denote the equilibrium structure as $\gamma^* = \phi(\gamma)$. Also, we refer to

[6]A good way of burning power for a politics-involving oligarch is perhaps selling your assets and emigrating to a foreign country, as Roman Abramovich, the current owner of Chelsea Football Club in Britain, did to his Russian conglomerate.

"power struggles taking place" as the case where some power is burnt during the formation of equilibrium, that is, $\gamma \neq \phi(\gamma)$.

1.3 Elite Persistence in Small-sized Coalitions

1.3.1 Three-Agent Coalitions

As motivations for the general characterization, we first analyze the equilibrium structure of a 3-agent coalition. 3-agent coalition is the minimal organization size that allows both competition for higher rents, and alliances for survival.

Consider a generic 3-agent coalition wherein agents have different power levels. We identify the agents as the weak (w), the medium (m) and the elite (e) by their power levels. They form a coalition with initial power structure[7] $\gamma = (\gamma_w = a, \gamma_m = b, \gamma_e = c)$, where $0 < a < b < c$. In the absence of power struggles, the *elite* receives rent of $\frac{c}{a+b+c}$; the *medium* receives $\frac{b}{a+b+c}$; and the *weak* receives $\frac{a}{a+b+c}$.

During power struggles, less powerful agents tend to form alliances to defeat the strong, while watching out for future power struggles within the alliance. Such alliance may not always be feasible: when $a + b < c$, the collective force of the weak and the medium fails to compete with the elite. Facing the threat of elimination, they have to cede power to the elite completely, by burning down to 0, $(a, b, c) \to (0, 0, c)$. As a result, the equilibrium structure becomes $(\gamma_w = 0, \gamma_m = 0, \gamma_e = c)$. An extreme hierarchy is formed, which we naturally refer to as a dictatorship.

When $a + b > c$, no single agent is powerful enough to dictate, but any two-agent alliance is powerful enough to eliminate the third. A myopic weak agent thinks that an alliance with the medium agent could collectively expel the elite, and then receive higher payoffs in the new regime $(\gamma_w = a, \gamma_m = b)$, because $\frac{a}{a+b} > \frac{a}{a+b+c}$ and $\frac{b}{a+b} > \frac{b}{a+b+c}$. However, a far-sighted weak

[7] In the following part of the paper, we use "power structure" and "regime" interchangeably.

agent understands that, without credible commitment, the medium agent becomes dominant after ousting the elite, and will proceed to eliminate the weak. As a result, the weak agent expects zero payoff. In anticipation, the alliance of (a, b) is not rent-improving. Similarly, no other two-agent alliance is stable at current power levels, which makes the original structure (a, b, c) stable in power struggles. This is the same as the idea of self-enforcing coalitions in AES.

To solve the commitment problem, suppose agents can strategically burn power to prepare for the coming struggles. Each agent understands a two-agent alliance is stable only if they are of equal power: each agent receives rent of $\frac{1}{2}$, which is rent-improving for both participants. Meanwhile, the post-burning structure has to be powerful enough to oust their opponents. For example, when $a < \frac{c}{2}$, an alliance between the weak and the medium, $\left(\gamma_w^{'}, \gamma_m^{'}\right) = (a, a)$ fails to eliminate the elite. However, an alliance where the elite burns down to match power with the medium, $\left(\gamma_m^{'}, \gamma_e^{'}\right) = (b, b)$, is powerful enough to triumph in the power struggles [8].

Anticipating the formation of (b, b) and his consequent elimination, the weak agent adjusts power to block the alliance. He does so by burning power until the elite becomes rent-indifferent in the current coalition as in the alternative $(\frac{1}{2})$. To be precise, the weak burns to $c - b$[9], which adjusts the regime to $\gamma^{'} = \left(\gamma_w^{'} = c - b, \gamma_m^{'} = b, \gamma_e^{'} = c\right)$. The structure is in equilibrium since no agent wishes to further burn power or eliminate other agents. In this case, the flexibility in power adjustment provides credible threats for the elite to burn power, which further leads to the weak agent burning power.

For any agenda, we can follow the practice above to look for rent-improving alliances that are powerful enough to win the current power struggles, and are stable in future ones. If such

[8]There are other options of stable and rent-improving alliances, such as $\left(\gamma_m^{'}, \gamma_e^{'}\right) = (b - \epsilon, b - \epsilon)$ for small ϵ. However, the tie-breaking assumption in Section 1.2.1 selects the outcome with the least power burnt, i.e. (b, b).

[9]Recall that we are in the case where $a + b > c$, thus $c - b \geq 0$.

an alliance exists, excluded agents have to cede power for survival. For 3-agent coalitions, we completely characterize the equilibrium structures as follows:

Proposition 1.1. *Starting from a generic power structure* $\gamma = (\gamma_w = a, \gamma_m = b, \gamma_e = c)$, $0 < a < b < c$, *the equilibrium structure converges to one of the three forms:*

1. *a dictatorship:* $\gamma^* = (0, 0, c)$ *when* $a + b < c$;
2. *a duopoly:* $\gamma^* = (a, a, 0)$ *when* $a + b \geq c$, $a > \frac{c}{2}$, $s = (m, w, e)$ *or* $s = (w, m, e)$;
3. *an oligarchy:* $\gamma^* = (c - b, b, c)$ *otherwise.*

Proof. See Appendix. □

As the proposition shows, when the initial structure features an unbalanced power, the society necessarily falls into a dictatorship, without external forces to enforce rules or commitments. The other social classes fail to collectively compete with the elite, and must rally behind one agent for survival. This resembles the *established autocrats* in Svolik (2012), when the autocrats have no comparable competitors. The complete power ceding to the ruling elite is also consistent with Acemoglu and Robinson's (2017) recent study, which shows regimes turn despotic when the power of the state is significantly stronger than the power of society.

On the other hand, when the initial power distribution is relatively equal, the equilibrium structure often ends up an oligarchy. In an oligarchy, the rents received by the weak is irrelevant of the initial power he possesses. Instead, it is ironed down by the power difference between the medium and the elite agents. Power ironing suggests that a variety of initial structures ends up with the same equilibrium, which may explain why only a few common forms of organizational structures are observed across geographic locations, cultures, and initial power endowments. This is in contrast with a classic paradigm of regime changes where the small collectively overthrow the strong and rule without concern of future power changes. During the formation of an oligarchy, the alliance between the less powerful (the weak and the medium) fails to compete with the alliance between the powerful (the medium

and the elite). This is not because of the lack of ability to overthrow the elite, but because of the survival concerns in the future. In this sense, power burning widens participation at the cost of power centralization.

The balance between the elite and the alliance of the rest of the coalition in oligarchies resembles the *contested autocracies* in Svolik (2012) where the autocrat's power is delicately balanced with other players. A natural question is whether and how the balance of power may break down and degenerate to unchecked power, i.e. the transition from contested to established autocracies. We address this question in Section 1.3.2 below.

Remarks on agenda. When agents have fixed power levels, the outcomes of power struggles are independent of agenda (See AES). However, as Proposition 1.1 suggests, when burning power is possible, the formation of equilibrium does depend on agenda[10]: the sequence of moves determines which alternate alliance can be formed first. As Olson (2000, p.xii) writes, the opportunities to consolidate authority "...occurs at times that Polish reformer Leszek Balcerowicz refers to as *moments of extraordinary politics*." Furthermore, the same initiate power structure could lead to drastically different equilibrium structures. For instance, starting from the same initial power structure (a, b, c) when $a > \frac{c}{2}$, two alliances are stable and rent-improving: $\left(\gamma_w^{'}, \gamma_m^{'}\right) = (a, a)$ and $\left(\gamma_m^{'}, \gamma_e^{'}\right) = (b, b)$. When the elite moves last, i.e. $s = (m, w, e)$ or (w, m, e), the weak and the medium agent preempt to form the alliance of (a, a), which forces the elite agent to respond by ceding power. The equilibrium structure becomes $(a, a, 0)$. However, in all other agendas, for example, $s = (w, e, m)$, after the medium agent burns power to a, the elite can react by mimicking the first-mover and also burn down to a. This leads to a new structure, (a, a, a). The tie-breaking assumption suggests that the last-mover should yield, thus blocking the alliance of $\left(\gamma_w^{'}, \gamma_m^{'}\right) = (a, a)$ in

[10]This is consistent with the literature of agenda setting. McCombs and Shaw (1972), followed by Scheufele (1999), Scheufele and Tewksbury (2007), and recently Helfer (2016) discuss the role of media, in determining whether the relevant parties are aware of the potential changes of other members, or the potential policies that are put into motion. In the context of Kazakhstan and Kyrgyzstan, Schatz (2009) shows the authoritarian elites make effective use of agenda setting tools to strengthen the rule.

the first place. This illustrates the survival concerns of the elite. When the initial power structure is highly skewed, the elite is always free of survival concerns. However, when the initial power structure is relatively equal, the elite could continue to expropriate the weak and strengthen the rule with advantageous timing, but may lose power otherwise. We show that the probability of successful expropriations has important implication in how the elite determines the economic development policies in Section 1.5.1.

The first-mover advantages in mobilizing allies during regime changes are consistent with reality. In the attempted coup d'état against Recep Erdoğan of Turkey in July 2016, the failed efforts of the rebellions to block social media and the swift reaction of Erdoğan[11] jointly determined the failure of the coup. As a comparison, the 1969 Libyan coup succeeded because the coup soldiers cut off the connection between King Idris and his potential supporters, which made King Idris the last mover in power struggles.

1.3.2 Characterization

The solution of 3-person coalitions sheds lights on the characterization of more general coalition structures. In general, agents seek alliances to eliminate others and split resources within a smaller circle. To prevent future struggles, the power within the alliance should be restructured before elimination. Similar with Proposition 1.1, we follow an induction approach. Suppose we have solved all $(n-1)$-agent equilibrium structures under all agendas $(n \geq 4)$, we now turn to solve the equilibrium for a n-agent coalition under a specific agenda s, by examining whether a sub-coalition wishes to eliminate the rest of the agents. As in 3-agent coalitions, a sub-group of agents in a n-agent coalition choose to form a smaller

[11]One hour after the coup started, rebellion soldiers occupied Taksim Square in central Istanbul and then inside the buildings of the state broadcaster, the Turkish Radio and Television Corporation (TRT), in Ankara, trying to control the media. They also tried to block social media such as Twitter, Facebook and YouTube. At the time, Erdoğan was on holiday outside Turkey. Two hours after the coup, Erdoğan did a Facetime interview with CNN Türk, in which he called upon his supporters to fight against the coup soldiers. Erdoğan returned to Istanbul five hours after the coup, and later wiped out the coup soldiers with the support of Turkish military.

coalition among themselves if they are able to adjust into a power structure which is strong enough to win in the current power struggles, is stable in all future struggles, and raises rents for participating agents. Formally, we define power structures satisfying such properties to be *stable improving*:

Definition 1.1. $C \subset N$ with γ_C is *stable improving* given N, γ, s, if there exists $\gamma_C^{'} \leq \gamma_C$, s.t. $\sum \gamma_{i\in C}^{'} > \sum \gamma_{j\in N\backslash C}$, and $U_i\left(C, \gamma_C^*\right) \geq U_i\left(N, \gamma\right)$ for all $i \in C$, where $\gamma_C^* = \phi\left(\gamma_C^{'}|s\right)$.

In particular, (C, γ_C) is *strictly stable improving*, if $U_i\left(C, \gamma_C^*\right) > U_i\left(N, \gamma\right)$ for all $i \in C$.

Notice that in the definition of stable improving coalitions, the adjustable structure $\gamma_C^{'}$ might not coincide with equilibrium structure γ_C^*. That is, agents may aim for temporarily unstable deviation, if the stable deviation after the deviation is desirable. This echoes the idea of far-sighted stability in Chwe (1994). It is crucial when in some power structures, $\gamma_C^{'}$, is winning while γ_C^* is not $(\sum \gamma_{i\in C}^{'} > \sum \gamma_{i\in N\backslash C} \geq \sum \gamma_{i\in C}^*)$, that agents in C are willing to deviate to $\gamma_C^{'}$ to eliminate agents in $N\backslash C$ and further adjust to γ_C^*.[12] Therefore, the non-existence of strictly stable improving coalitions is stricter than rent-improving for deviating agents because it requires the deviation to be in equilibrium as well. But it is weaker than coalition-proof equilibrium (Bernheim et. al 1987) because it allows for deviations after deviation.

For small-sized coalitions, we show that there is a one-to-one relationship between strictly stable improving coalitions and equilibrium power structures.

Theorem 1.2. *For $|N| \leq 7$, a coalition is in equilibrium if and only if there does not exist a sub-coalition C that is strictly stable improving. Moreover, equilibrium power structures always take one of the following three forms:*

[12] An example is $\gamma = (3, 4, 6, 6, 6)$, when the agenda is bottom-up, i.e. from the weakest agent to the strongest. A stable improving sub-coalition C has power structure $\gamma^{'} = \left(\gamma_1^{'} = 3, \gamma_2^{'} = 4, \gamma_3^{'} = 6\right)$, and $\gamma_C^* = (\gamma_1^* = 2, \gamma_2^* = 4, \gamma_3^* = 6)$. Anticipating this, agent 4 and 5 have to burn down to avoid elimination, and the equilibrium is $(2, 4, 6, 0, 0, 0)$. In equilibrium, the first agent burns power $(3 \to 2)$, but enjoys higher rents $\left(\frac{2}{12} > \frac{3}{25}\right)$.

1. *a dictatorship:* $\gamma_i^* > 0$, *and* $\gamma_j^* = 0$ *for all* $j \neq i$;
2. *an oligarchy:* $\gamma_i^* = \sum_{j \neq i} \gamma_j$, *for some* $i \in N$;
3. *a polyarchy:* $N_+^* \in \left\{2^k\right\}$, $k \in \mathbb{N}_+$, *and* $\gamma_i^* = \gamma_j^*$, *for all* $i, j \in N_+^*$

Proof. See Appendix. □

There are two takeaways from Theorem 1.2. First, in small-sized coalitions, stability is guaranteed by that all sub-coalitions with regime-changing capabilities have at least one agent prefer to stay in the status quo. The key reason behind the result is the degree of freedom in terms of power restructuring is limited when coalition sizes are small. As shown later, the theorem fails to hold for larger coalitions because a relatively equal structure, with large numbers of players, enables a powerful non-elite group to check and balance potential deviations. Second, for small-sized coalitions, equilibrium structures converge to familiar institutional designs. In most cases either there is an unequivocal dictator, or there is a strongman centered around by several less powerful agents who check and balance the strong. This shows that the daily observations of dictatorships and authoritarian regimes are necessary equilibrium outcomes, instead of random dice rolls. There are, occasionally, polyarchies formed in equilibrium. In fact, it is easy to show that the three forms of institution designs of any sizes are all equilibrium structures.

Corollary 1.2. *Dictatorships, oligarchies and polyarchies of all sizes are equilibrium power structures.*

But as we show in the next sections, the reverse is not true: equilibrium structures can take more forms than the above three intuitive designs. Moreover, polyarchies are fragile and have tendencies to fall into oligarchies or dictatorships in face of exogenous shocks, while power in oligarchies also tends to centralize upon perturbation. Therefore, equilibrium outcomes for small-sized coalitions after power struggles not only converge to certain shapes, but also converges to a certain direction: centralization.

1.3.3 Elite Persistence

In this section we analyze how small-sized equilibrium structures respond to power perturbations. Power perturbations are small positive or negative shocks on individual agents. We show that, regardless of the equilibrium structure and the direction of the perturbation, power in the post-perturbation structure always becomes weakly more concentrated to the already powerful.

Using 3-agent coalition as an example, when the equilibrium structure is a dictatorship and the dictator is perturbed, either $(0, 0, c + \epsilon)$ or $(0, 0, c - \epsilon)$ remains an equilibrium and remains a dictatorship. When the equilibrium is a polyarchy, any perturbation makes the slightly stronger group advantageous in power struggles. As a result, the balance between almost equal players are highly fragile. For instance, a perturbation on a duopoly makes the slightly stronger agent dictate, i.e. $(a + \epsilon, a, 0) \to (a + \epsilon, 0, 0)$ or $(a - \epsilon, a, 0) \to (0, a, 0)$.

When the equilibrium structure is an oligarchy, power is balanced between the elite circle and the non-elites. We separate the perturbations into the ones that increases equality within the coalition, and the ones that reduce equality. For perturbations that increase equality, suppose a small positive shock $\epsilon > 0$ hits the weak, creating new power structure $(c - b + \epsilon, b, c)$, or a small negative shock hits the elite, creating new power structure $(c - b, b, c - \epsilon)$. In both cases, the elite agent receives lower rents than before. Thus the stable and rent-improving alliance $\left(\gamma_m^{'}, \gamma_e^{'}\right) = (b, b)$ becomes more attractive to the elite. Applying Proposition 1.1, the post perturbation equilibrium structures are $(c - b + \epsilon, b, c) \to (c - b, b, c)$ and $(c - b, b, c - \epsilon) \to (c - b - \epsilon, b, c - \epsilon)$, respectively. In both cases, the elite agent restores rents of $\frac{1}{2}$. Next, for perturbations that reduces equality, such as $(c - b - \epsilon, b, c)$ or $(c - b, b, c + \epsilon)$, by Proposition 1.1 the structures directly collapse to dictatorships: $(c - b - \epsilon, b, c) \to (0, 0, c)$ and $(c - b, b, c + \epsilon) \to (0, 0, c + \epsilon)$. That is, small changes in favor of equality are usually ineffective in oligarchies, as long as the benefits of the vested interest group are satisfied. However, small changes in favor of inequality might

lead to drastic descent into dictatorship because an unchecked power is created. In either case, the prestige of the elite persists, if not strengthens. It is easy to show that the result generalizes to dictatorships, oligarchies and polyarchies of all sizes. Denote the set of all dictatorships, oligarchies and polyarchies as $\mathbb{D}$, we have the following theorem.

Theorem 1.3. *For equilibrium structure $\gamma_N^* \in \mathbb{D}$ of any size, power is weakly more centralized after perturbation on any individual agents.*

That is, there exists some agent $i \in N$, where $U_i(\gamma_N^*) \geq U_j(\gamma_N^*)$ for all $j \neq i$, and there exists a threshold, $\underline{\eta} > 0$, such that for any perturbation $\eta \in \mathbb{G}$, $\eta \in \left(-\underline{\eta}, 0\right) \cup \left(0, \underline{\eta}\right)$, on any $j \in N$, after the perturbation, $U_i\left(\phi\left[\left(\gamma_j^* + \eta, \gamma_{-j}^*\right)\right]\right) \geq U_i(\gamma^*)$, and $U_i\left(\phi\left[\left(\gamma_j^* + \eta, \gamma_{-j}^*\right)\right]\right) \geq U_j\left(\phi\left[\left(\gamma_j^* + \eta, \gamma_{-j}^*\right)\right]\right)$.

Proof. See Appendix. □

Theorem 1.3 shows that a most powerful agent before perturbation remains so after perturbation, regardless the direction of the power shifts. Furthermore, the agent has weakly higher relative power, thus weakly more rents. Combining Theorem 1.2 and 1.3, the elite persistence for small-sized coalitions are inevitable after power perturbations.

Corollary 1.3. *For any equilibrium structure with sizes $|N| \leq 7$, power is power is weakly more centralized after perturbation on any individual agents.*

In reality, the instances where a weakened elite regains power are not uncommon. In authoritarian regimes, leadership is often transited from experienced old rulers to less powerful successors; for example, from Kim Jung-il to Kim Jung-un of North Korea, and from Hafez to Bashar al-Assad in Syria. Both sons were less experienced in politics and were not raised as successors to the throne for the majority of their youth. However, both managed to seize power while the old lieutenants ceded theirs. Similar persistence of elite families

and identities are documented in other countries[13]. In the next section we discuss, in more detail, why and how the persistence can be sustained in a general coalition with larger sizes.

1.4 Elite Persistence in Large-sized Coalitions

In Section 1.3 we characterized small-sized ($|N| \leq 7$) equilibrium structures and showed that power concentrates to the elite regardless of the direction of perturbations. Therefore, the position of the elite persists after perturbations. In this section, we characterize the boundaries of the equilibrium structures for larger-sized coalitions ($|N| \geq 8$). With the boundaries we categorize the shapes that equilibrium structures can take. Among the more flexible equilibrium structures, we provide a tight condition about when the elite persistence in small-sized coalitions extends to a general environment.

1.4.1 Equilibrium Characterization

For larger-sized coalitions, while a power structure remains stable when any sub-coalition fails to form a strictly stable improving alternative, there are other ways to sustain stability. For instance, when there exists a stable improving alternative, but such smaller coalition is blocked by the mimicking behavior of a prior-moving agent according to the agenda. Denote $N_{\succ_s i}$ as the set of agents who move before agent i in agenda s. The existence of stable improving sub-coalitions means some agents are capable and willing to form their own alliances and expel the others. However, their restructuring attempt may trigger respective burning from other agents in the coalition, which then leaves the original sub-coalition undesirable. In fact, the equilibrium structure can be bounded by the non-existence of strictly stable improving sub-coalitions and the non-existence of an early-moving blocking agent in such sub-coalitions.

[13]See Stone (1990) and Paige (1997) for Central American countries; Dalton (1968) for Liberia; Woodward (1955) and Wiener (1982) for US South after the civil war.

Theorem 1.4. *For $|N| \geq 8$, a coalition is in equilibrium if there does not exist a sub-coalition C that is strictly stable improving, and only if there does not exist a sub-coalition C that is strictly stable improving and $\gamma'_{i\in C} > \max\{\gamma_j \mid j \in N_{\succ_s i} \backslash C\}$ for all $i \in C$.*

Proof. See Appendix. □

A summary of the proof follows. If a coalition (N, γ) starts in equilibrium with a stable improving sub-coalition, C, then agents in such sub-coalition should burn down to γ'_C to raise their rents. They are able to do so because there is no powerful early-mover who can block the burning process by mimicking ($\gamma'_{i\in C} > \max\{\gamma_j \mid j \in N_{\succ_s i} \backslash C\}$). Therefore, the original power structure cannot be in equilibrium. On the other hand, if a power structure does not have a stable improving sub-coalition, then for every adjustable winning sub-coalition, C, such that $\gamma'_C \leq \gamma_C$ and $\sum \gamma'_{i\in C} > \sum \gamma_{i\in N\backslash C}$, there is some agent who enjoys no more rents in the deviation than in the current power structure. Consequently, the deviation proposal fails to pass and the original structure is in equilibrium. However, the theorem does not eliminate the possibility of an equilibrium structure where a rent-improving regime change is blocked by a powerful agent who has first-mover advantage, and enjoys more rents in the status quo.

Theorem 1.4 establishes the boundaries of any equilibrium structure after power struggles. More importantly, it categorizes the two ways to support an equilibrium structure in general. The first way is the non-existence of strictly stable improving coalitions, as in the small-sized coalitions: every coalition with regime-changing capabilities has an agent prefers the status quo. It frees the agents from the concern of "my friend today becomes my foe tomorrow". The concept resembles that of AES but carries two major differences: 1. AES does not allow deviations after deviations because power levels are fixed; 2. The algorithm in AES is independent from agenda, while in our algorithm different agendas leads to different equilibrium structures. The second way to support an equilibrium structure is to have one (or more) powerful early-moving agent, or a *Steward*, who prefers staying in the current

power structure and is able to block a strictly stable improving coalition by threatening to mimic their burning choices.

1.4.2 Elite Persistence

In Section 1.3, the reason for the elite persistence in a small-sized coalition is that the regime-changing ability of the elite circle remains intact during equality-enhancing perturbations. Moreover, power struggles triggered by perturbations only end when the outsiders cede just enough power to accommodate the insiders. In this section, we show that such intuitions carry to a general environment where power burning actually takes place during the formation of equilibrium.

When an agent burns power for survival, there are two cases. In the first case, he faces inevitable elimination thus has to burn all of his power and give away the rights to claim rents. For example, in the power structure $(1, 2, 6, 8)$, the agent with power 1 can avoid elimination only by burning power to 0. In the second case, he burns power partially to block a stable improving sub-coalition. As in Section 1.3, the agent with power 2 in $(2, 4, 5)$ burns power partially to block the stable improving coalition of $(4, 4)$, which creates the equilibrium structure $(1, 4, 5)$. For the block to succeed, some agents in the original stable improving coalition should be indifferent between the rents received in the deviation and the post-burning structure. In general, for a coalition N, with initial structure γ and equilibrium structure γ^*, recall N_+ as the coalition of agents with strictly positive power. We have the following lemma.

Lemma 1.1. *When power is partially burnt during power struggles, some agent in the equilibrium structure receives the same rents in the current structure as in a stable improving alternative. That is, if* $0 < \gamma_i^* < \gamma_i$, *for some* $i \in N$, *then there exists* $C \subsetneqq N_+$ *with* $\gamma_C^{'}$, *where* $\gamma_C^{'} \leq \gamma_C^*$ *and* $\sum_i \gamma_{i\in C}^{'} > \sum_i \gamma_{i\in N\setminus C}^*$, *such that for some* $j \in C$, $U_j\left(N, \gamma^*\right) = U_j\left(C, \phi\left(\gamma_C^{'}\right)\right)$

Proof. See Appendix. □

Lemma 1.1 captures a key economic force within the model. Agents, facing the threat of elimination, give away power to the extent that the deviating agents are indifferent between the rents from current structure versus from the stable improving sub-coalition. Denote the set of agent(s) whose deviation constraint binds as G, $G = \{x \in C | U_x(N, \gamma^*) = U_x\left(C, \phi\left(\gamma_C^{'}\right)\right)\}$. We call the agents in G "Guardians" because they are the elites who guard the stable structure from deviations. Denote the complement set of C in N as $A = N \backslash C$. We call agents in A "Accommodators" because they give away power to accommodate the guardians. Applying this terminology to the example in Section 1.1, where $\gamma^* = (\gamma_w^* = 1, \gamma_m^* = 4, \gamma_e^* = 5)$. The stable improving coalition with power structure $\gamma_C^{'} = \left(\gamma_m^{'} = 4, \gamma_e^{'} = 4\right)$ is blocked by the accommodator, w, who partially burns $\gamma_w^* = 1 < \gamma_w = 2$, to accommodate the guardian e, who enjoys the same rents in the equilibrium $\left(\frac{5}{1+4+5}\right)$ as in the deviation $\left(\frac{4}{4+4}\right)$. In Michels' (1915) words, the guardians exist because " (the) society cannot exist without a dominant or political class, and that the ruling class, while its elements are subject to frequent partial renewal, nevertheless constitutes the only factor of sufficiently durable efficacy in the history of human development."

As with small-sized coalitions, we are interested in whether shocks that weaken the strong (the guardians) and shocks that empowers the weak (the accommodators) actually lead to increase of equality in equilibrium. In both cases, we focus on equilibrium structures where partial power is burnt during the formation of equilibrium; that is, the set of guardians is non-empty.

Suppose a small negative shock hits one of the guardians. There are two possible outcomes. First, if there are multiple guardians and one of them is weakened, then the rest of the guardians now enjoy even higher rents than in the stable improving alternative. Consequently, the deviation remains unattractive to them and no power struggles take place. Second, suppose there is only one guardian who is hit by a small, negative shock. The weakened guardian receives fewer rents, thus allowing for a *strictly* stable improvement. As a result, the accommodators respond by burning their own power to appease the guardian.

Formally, we have the following theorem.

Theorem 1.5. *If* $0 < \gamma_i^* < \gamma_i$ *for some* $i \in N$*, and* G *is a singleton, then there exists a threshold,* $\underline{\eta} > 0$*, such that for any negative perturbation* $\eta \in \mathbb{G}$*,* $0 < \eta < \underline{\eta}$*, on* $x \in G$ *,* $U_x(\gamma^*) = U_x\left(\phi\left[\left(\gamma_x^* - \eta, \gamma_{-x}^*\right)\right]\right)$.

Proof. See Appendix. □

Theorem 1.5 illustrates the persistence of the elite after they are weakened. The elite recovers from negative shocks, in terms of the rents received, because the stable improving coalition remains intact after the shock. Thus, the "outside option" of the elite remains unchanged, and the accommodators bear the burden of the negative shock upon the power of the elite. In Michels' (1915) words, "The government, or, ... the state, cannot be anything other than the organization of a minority. It is the aim of this minority to impose upon the rest of society a legal order which is the outcome of the exigencies of dominion and of the exploitation of the mass." Such idea, known as "The Iron Law of Oligarchy", is well documented in the studies of power structures. In fact, Kendall-Taylor and Frantz (2016) studied all 79 dictatorships between 1946 and 2012, and concluded, "...a leader's death in office almost never leads to significant near-term liberalization. Likewise, only rarely does it spell the end of the regime or precipitate instability in the form of coups or protests. On the contrary, authoritarian regimes have proven to be remarkably resilient when a leader dies." In the meantime, the trend of organizations towards oligarchy continued to spread throughout the first decade of the twenty-first century. According to Magaloni and Kricheli (2010), one-party regimes have now become the most common type of authoritarian rule, constituting 57% of the authoritarian regimes during 1950–2006 and 33% of the total number of regimes in the world.

Meanwhile, if a positive shock hits the guardian, he always receives at least as many rents as before, because the guardian always has the option to burn up the positive shock and returns to the previous equilibrium structure. Therefore, an immediate corollary follows

from Theorem 1.5 is that, regardless of the directions of perturbations, the power of the strong always gets weakly more centralized.

Corollary 1.4. *If $0 < \gamma_i^* < \gamma_i$ for some $i \in N$, and G is a singleton, then there exists $\underline{\eta} > 0$, such that $U_x(\gamma^*) \leq U_x\left(\phi\left[\left(\gamma_x^* + \eta, \gamma_{-x}^*\right)\right]\right)$, for any $\eta \in \mathbb{G}$, $\eta \in \left(-\underline{\eta}, \underline{\eta}\right)$ applied on $x \in G$.*

Next, we consider the other side of the same coin: the shock that increases the power of an accommodator. When an accommodator is empowered, the guardian is necessarily weakened, which again triggers a strictly improving sub-coalition.

Theorem 1.6. *If $0 < \gamma_i^* < \gamma_i$ for some $i \in N$, then there exists $\overline{\eta} > 0$ such that $U_x(\gamma^*) = U_x\left(\phi\left[\left(\gamma_j^* + \eta, \gamma_{-j}^*\right)\right]\right)$, for any $\eta \in \mathbb{G}$, $0 < \eta < \overline{\eta}$ and $\gamma_j^* < \gamma_j$, $j \in A$.*

Proof. See Appendix. □

Theorem 1.6 illustrates that the policy of empowering the weak to increase social equality might not work, because the stable improving coalition involving the elites always needs to be accommodated. To an extreme it can be totally ineffective (See Section 1.3). This theorem provides a unified explanation of the consolidation of autocratic regimes facing opportunities in favor of the non-elites. Such opportunities may be trade, as in Guatemala and Mexico in the 19th century; or the abolition of slavery as in West Africa (Robinson 2012); or the abolition of Mita in Peru (Dell 2008). Therefore, one should not take the rise of the non-elites for granted, when a favorable junction arrives. In fact, as long as the elite circle remains intact, empowerment of the weak seldom works.

Additional remarks. Sometimes social empowerment does induce dramatic changes of the political landscapes. Such empowerment usually involves significant increase of power, or changing both power *and* agenda at the same time. In the Arab Spring, the Tunisian government under Ben Ali, focused on foreign direct investment, open trades, and open media, which not only empowered the masses but also changed domestic political agendas. The success of the revolution reveals the possible coordination between social empowerment and agenda-settings.

1.5 Applications

In this section, we explore the implications of adding elements of real life to the framework. In Section 1.5.1, we introduce the possibility of production, and discuss the boundary of power struggles under the tradeoff between expropriation and economic development. In Section 1.5.2, we consider the impact of a foreign force on domestic power structures. Throughout Section 1.5, we assume that initiating power struggles is a *choice* for some particular agent in the coalition[14], and we examine how that choice interacts in the presence of other forces. We also aim to generate testable predictions, in order to demonstrate that the framework goes beyond a technical practice.

1.5.1 Expropriation vs. Development

Power struggles aim to change how the pie is split, whereas production aims to enlarge the pie. If the choices are mutually exclusive, a natural trade-off exists. From a social welfare perspective, production is always preferred to power struggles. Intuitively, a ruler prioritizes economic development only after he is free from survival concerns (Gehlbach and Keefer 2011)[15]. In the section, we show that this intuition may fail. A ruler free from survival concerns may focus on expropriations, while a ruler with survival concerns may focus on economic development.

Players. Consider a society consisting of three representative agents: the elite agent (e) who rules; the magistrate (m) who helps to rule; and the worker (w) who produces. Denote

[14]In reality, initiation of power struggles can indeed be a policy choice. The national development of China was guided by the principle of "taking class struggles as central tasks" since 1949, especially during the Cultural Revolution, before the ruling party decided to "concentrate on modernization" in 1977. Vietnam, on the other hand, turned to "Renovation" in 1986 with the goal of creating a "socialist-oriented market economy", after 15 years of leftist policies.

[15]For more recent events, North Korea announced its plan to boost economic development, after Kim Jong-Un consolidated power.

the initial power structure $\gamma_0 = (\gamma_w^0 = k + h, \gamma_m^0 = 1 - k, \gamma_e^0 = 1)$, where $k > 0$, $h \geq 0$. By Proposition 1.1, the structure is in equilibrium when $h = 0$, thus h measures how decentralized power is from the equilibrium. We also assume that $k + h < 1 - k$, such that the magistrate holds more power than the worker.

Strategies. The game begins with the initial power structure γ_0 and agenda s. The elite chooses whether to initiate power struggles. If power struggles are initiated, the game is played as in Section 1.2, and agents receive rents according to the equilibrium structure. If the ruler chooses not to initiate power struggles, then the worker produces. Production increases the worker's power by $\beta > 0$. And production plus the rents are distributed according to $\gamma_0^{'} = (k + h + \beta, 1 - k, 1)$.

Payoffs. There is an existing pie of size π to share. Production enlarges the pie by $\alpha\gamma_w^0$, where $\alpha > 0$ is a measure of productivity.

An intuitive approach suggests a direct comparison between the increased rents from expropriation (power struggles) and the increased size of the pie, due to production. However, as Proposition 1.1 shows, the elite, in a 3-agent coalition, may lose power in the struggles, which makes expropriation risky. We therefore distinguish elites by their likelihood of survival in power struggles, and provide their optimal decisions in the following proposition:

Proposition 1.2. *Rulers with survival concerns prioritize economic growth over expropriation more often than those without survival concerns. That is:*

When $s \neq (m, w, e)$ and $s \neq (w, m, e)$, the ruler focuses on production if and only if

$$\frac{1}{2}\pi\beta + \left(\frac{1}{2}\pi - \alpha\right) \leq k < \frac{1 - h}{2}$$

.

When $s = (m, w, e)$ or $s = (w, m, e)$, the ruler focuses on production if and only if

$$\min\left\{\frac{1}{2}-h,\ \frac{1}{2}\pi\beta+\left(\frac{1}{2}\pi-\alpha\right)\right\}\leq k<\frac{1-h}{2}$$

.

Proof. Without concerns of losing power, the ruler conducts a direct comparison between enlarging the pie and getting a larger share of the existing pie: $\frac{1}{2+h+\beta}\left[\alpha\left(k+h\right)+\pi\right]>\frac{1}{2}\pi$, which gives the left hand side of the first inequality, $k>\frac{1}{2}\pi\beta+\left(\frac{1}{2}\pi-\alpha\right)h$. The right hand side of the first inequality is given by the assumption that $k+h<1-k$.

When the ruler has survival concerns, he decentralizes whenever the worker is strong enough to overthrow him in agenda $s=(m,w,e)$ or $s=(w,m,e)$, therefore he decentralizes whenever $k+h>\frac{1}{2}$. When $k+h\leq\frac{1}{2}$, the elite ruler has no survival concerns and behaves the same as in the first case. Combining the two we get the expression on the left hand side of the second inequality. □

It is intuitive that power struggles are more likely to be reduced by economic development: when productivity (α) is higher, when the existing pie is smaller, and when the empowerment from production is insignificant (β is low). As a result, high value-added agents are less likely to be expropriated[16], whereas a regime abundant in natural resources often suffers from power struggles.

The key implication of the proposition is that elite rulers, facing survival concerns, favor economic development in a larger set of initial power structures. This is because the elite ruler is forced to prioritize economic development when expropriation would trigger a revolution to overthrow the ruler.

Prediction: Timing of economic reform. The analysis above generates testable predictions of when a regime turns from "extractive" to "inclusive", per Acemoglu and

[16]For the opposite direction, a Chinese proverb best describes the ruler's strategy: after the cunning hare is killed, the hound is boiled.

Robinson (2013) terminology. When the current power structure is highly skewed, elite rulers behave in the same manner under all agendas: promoting economic growth when productivity is high, and extracting rents from others when existing resources are abundant. When the current structure is less skewed, elite rulers, under the agendas where they have survival concerns, tend to prioritize economic development more often than others. On the other hand, we predict a strong ruler follows a "sow and reap" strategy in a dynamic environment. He should aim for economic development for a number of periods, then initiate power struggles before the potential opponents become too powerful, and, finally, resume economic development thereafter. This strategy could explain the variations, and sometimes the cyclicality, of development policies seen in a regime across time or across different regimes. Furthermore, it sheds lights on logic behind periodic campaigns initiated by the ruling elites, aiming to restore social order.

1.5.2 Domestic vs. International Conflicts

In the previous discussions, power struggles occur domestically; i.e., the grand coalition is fixed. In this section, we consider how domestic power struggles are affected by the presence of external power. It is convenient to think of power struggles as conflicts such as wars or negotiations over resources. Intuitively, imminent foreign threats soften domestic conflicts, because a united domestic power has a better chance against the threats. On the other hand, an agent to be expelled can make himself indispensable in the power balance by inviting a strong foreign force to settle the disputes. We thus argue that the both external threats and international cooperation may help alleviate domestic power struggles.

1.5.2.1 Foreign Threat

Players. Considers a domestic country with three warlords, w, m and e. The current power structure is $\gamma_0 = (\gamma_w = k + h, \gamma_m = 1 - k, \gamma_e = 1)$. The agenda is top-down. A foreign

country with power level g, $2 < g < 2 + h$, attacks the domestic country with probability $0 \leq \mu \leq 1$. The war is won by domestic forces if and only if they are collectively more powerful than the enemy $\gamma_p + \gamma_m + \gamma_e \geq g$.

Strategies. The game proceeds as follows: 1. Domestic agents conduct power struggles as in Section 1.2; 2. The foreign country attacks; 3. The resource is distributed to the winner(s).

Payoffs. Winners of the war share the resources according to their power levels, and the loser gets nothing.

It is easy to see that a foreign threat softens domestic power struggles because $g > 2 > 2 - 2k$, thus warlords m and e are no longer interested in seeking the power burning alliance $(1 - k, 1 - k)$ because such an alliance fails to defeat the invader. Consequently, the domestic agents set aside power struggles and join hands when foreign forces are likely to invade.

Proposition 1.3. *Power struggles are softened when external threat is high. In particular, power struggles are ceased when $\mu > \underline{\mu}$, where $\underline{\mu} = \frac{h}{2+h}$.*

Proof. The cut-off is determined by $\left(1 - \underline{\mu}\right) \frac{1}{2} = \frac{1}{2+h}$, which gives $\underline{\mu} = \frac{h}{2+h}$. □

There are two implications of this proposition. First, the cut-off is independent of the level of external threat g, because should the invasion takes place, the outcome is always binary. Therefore, when the threat is credible ($2 < g < 2 + h$), domestic conflict resolution depends on the probability of invasion (μ), instead of th strength of the invader[17]. Second, the cut-off increases in h, because when the invasion does not take place, a resource-rich warlord further incentivizes more extraction.

Prediction: Post-war struggles. We predict that, in general, power struggles are more likely to take place shortly after the end of conflicts and less likely shortly before the start

[17]We leave an alternative model, with probabilistic victory, for future research.

of conflicts. On the other hand, if a rent-seeking agent anticipates the purge after conflict resolution, it is then in his interest to prolong the conflict, even if it is socially inefficient. Below we provide historical evidence of this prediction.

Georgy Zhukov was one of the most important military commander of the Soviet Union in World War II. He led and won multiple battles, including the Battle of Berlin, which end the War in Europe. During the war, Zhukov held several prestigious positions in the army and in the political system because of his outstanding military talents. He served as Chief of the General Staff and then Deputy Commander-in-Chief. However, after the War, Zhukov was viewed by Stalin as a potential threat to his leadership because of his popularity in the army. Stalin replaced Zhukov with Vasily Sokolovsky as Commander-in-Chief of the Soviet Ground Forces. Zhukov was assigned to command the Odessa Military District, away from Moscow and lacking in strategic significance and troops. Zhukov's military career during the 1940s illustrates how power struggles respond to external threats. Power struggles can be softened when the threat is imminent, and will resume after the threat is exterminated.

Soviet Union is not a lone example. In China, during World War II, Kuomintan and the Communist Party of China, who were previously in civil war, united forces against Japan. The two parties resumed the civil war after defeating the Japanese. Moreover, in the 7th century, the Spanish kingdoms ceased the conflict to fight against the Moors.

1.5.2.2 Foreign Ally

Players. Consider a similar domestic country with three political parties, representing the workers, the middle class, and the elite with initial domestic structure $\gamma_0 = (\gamma_w = k+h, \gamma_m = 1-k, \gamma_e = 1)$. The agenda is (m, w, e). The elite has connections with a foreign ally (a), who holds power $1 < d < 2$. For simplicity, the foreign ally is assumed to move after domestic players, if it is invited to participate in domestic affairs. Thus, the revised agenda is (m, w, e, a).

Strategies. The game proceeds as follows: 1. The elite decides whether to invite a foreign

ally; 2. Power struggles take place as in Section 1.2, either in a 3-agent or a 4-agent coalition; 3. Payoffs are assigned.

Payoffs. Agents split rents of 1 according to their power levels after the power struggles.

By Proposition 1.1, when $k+h > \frac{1}{2}$, the worker and the middle class form an alliance to overthrow the elite in a 3-agent coalition. Anticipating the elimination, the elite has incentive to invite the foreign ally, which transforms the power structure to $(d+k-2, 1-k, 1, d)$. This is the equilibrium of the 4-agent coalition after power struggles. The elite receives $\frac{1}{2d} > 0$, which is preferred to ceding power completely, as they would in the 3-agent coalition.

In general we have the following result for any coalition with domestic conflict, $\phi(\gamma) \neq \gamma$.

Proposition 1.4. *Any domestic conflict can be stopped by a foreign power. That is, for any coalition N with domestic conflicts such that $\phi(\gamma|s) \neq \gamma$, there exists an external agent with power level γ_f such that $\phi((\gamma, \gamma_f)|s) = (\gamma, \gamma_f)$.*

Proof. Take $\gamma_f = \sum \gamma_{i \in N}$, by Theorem 1.2, $\phi((\gamma, \gamma_f)) = (\gamma, \gamma_f)$ □

By the construction of the proof, an agent can make himself indispensable in the power balance by inviting a more powerful outsider to participate local business. Consequently, he may maintain his local prestige and avoid the potential purges otherwise. Notice that despite the disappearance of conflict, this might not always be in the interest for the elite to do so. The proposition also illustrates how domestic power struggles make it easier for foreign countries to penetrate and guarantee a large slice, if not the largest slice, of pie.

Prediction: International interventions. We predict that the local elites may cede power to a foreign superior power when their domestic rule is threatened. In addition, we predict that the prestige of the local elites may persist among local communities after inviting a foreign power to join domestic affairs. Below we provide evidences from the 19th century Caribbean islands and modern-day Sri Lanka.

In the 19th century, there were fourteen Caribbean, British sugar colonies[18]. Here the sugar cane plantation owners were the dominant social class, and slaves were the major source of the labor supply. In 1833, the British parliament passed *An Act for the Abolition of Slavery*, which emancipated the slaves. Consequently, the poor were significantly empowered and began to play a larger role in local governance. According to Carvalho and Dippel (2016), "Starting with Montserrat in 1861 and ending with Grenada in 1876, all but one of the powerful assemblies voluntarily dissolved themselves, and invited the Crown to write a new constitution for them with a legislature appointed by the governor". Ashdown (1979) argues that "the colonies gave up their elected assemblies voluntarily, for in most cases the white, privileged classes preferred direct imperial government to the government of the colored classes who were slowly obtaining greater representation in the legislative councils."

Conversely, if the current domestic conflict cannot be resolved by domestic powers alone, a local force may invite external allies to eliminate the threats for them. The civil war in Sri Lanka had lasted for more than 25 years, and was ended with the external help from China, who "provided lethal military equipment and post-war diplomatic support" [19]). In exchange for their aid, Sri Lanka has signed a $1.1billion deal with China for the control and development of the southern deep-sea port of Hambantota, which has potential military uses[20]. For more examples of the power balancing role of external intervention, see Levine and Modica (2016).

---

[18]Namely, Antigua, Barbados, Jamaica, Montserrat, Nevis, St. Kitts, the Virgin Islands, Dominica, Tobago, St. Vincent, Grenada, Trinidad, St. Lucia, and Guyana. For a detailed discussion of the historical background, see Carvalho and Dippel (2016).

[19]See: https://www.brookings.edu/blog/order-from-chaos/2015/05/01/india-v-china-in-sri-lanka-lessons-for-rising-powers/

[20]See: http://www.bbc.com/news/world-asia-40761732

1.6 Discussions

Commitments. It is important to point out that, although commitment is not possible during power adjustment and power struggles, we do assume the implementation of a burning outcome when a proposal reaches unanimous agreement. Alternatively, if we assume agents can only make individual burning choices, Theorem 1.2 fails to hold and the set of stable structures enlarges. The intuition is that a stable improving coalition may not be implemented if other undesirable deviations are triggered during the transition path. We provide a numerical example in Appendix B. Such possibility suggests that regime changes can be blocked by deviations on the transition path, even with a desirable destination.

Elimination costs. Another assumption we make in the model is costless eliminations. It is an approximation of the case where rents received by staying in the coalition significantly outweigh the potential costs of elimination. Adding a positive elimination cost does not change the resurgence of hierarchies nor the persistence of elites in perturbations. However, drastic regime changes from oligarchies to dictatorships take place less frequently with positive elimination costs. To see this, suppose every elimination costs $\kappa > 0$ to each agent involved, the equilibrium structure of a 3-agent coalition $(2, 4, 5)$ becomes $\left(1 + \frac{20\kappa}{1-2\kappa}, 4, 5\right)$ because the stable improving alternative of $(4, 4)$ only guarantees rent of $\frac{1}{2} - \kappa$ instead of $\frac{1}{2}$. As a result, if the elite agent is hit by a small positive perturbation, $\left(1 + \frac{20\kappa}{1-2\kappa}, 4, 5 + \epsilon\right)$, the perturbed structure remains in equilibrium since $1 + \frac{20\kappa}{1-2\kappa} + 4 > 5 + \epsilon$. That is, positive elimination costs offer equilibrium structures some slackness in how they can perturbed[21]. Meanwhile, such slackness breaks down when perturbations accumulate to a tipping point that an imbalanced power is formed, eg. $5 + \sum_t \epsilon_t > 1 + \frac{20\kappa}{1-2\kappa} + 4$, we thus expect to see

[21]Of course, costs of elimination may depend on the power difference of involving agents, and may be asymmetric even for agents with same power, due to, for instance, geographical reasons. Caselli et. al (2015) showed that conflicts are more likely when the resources of interest located near the border of two countries because it is less costly for the attacking side to occupy the resources.

the sudden change of the regime - in this very example, the sudden fall to dictatorship after several periods of oligarchies.

Apart from a uniform elimination cost, it is also reasonable to relate elimination costs to power comparison. Intuitively, conflict is more costly for parties with almost equal strength, and is less so if one is stronger than the other by far. Following the 3-agent example with initial power structure $(2, 4, 5)$, suppose the elimination technology has cost $c(\gamma_A, \gamma_B) = \frac{\gamma_A}{\gamma_B}$, for both coalitions A and B, where $\gamma_B \geq \gamma_A$, then it is easy to show that $(4, 5)$ now becomes stable, and the equilibrium structure is $(0, 4, 5)$. As the example shows, elimination costs can be a commitment device in itself. As elimination costs increase, we may first see more conflicts, as more oligarchic structures can be supported, and then fewer conflicts, as it is too costly to change the regime.

The non-monotonic relationship sheds lights on the national security strategies in international relations: for a small country to survive in power struggles, it only pays off to develop weapons of mass destruction, instead of marginal improvement in the military. Linking further to Section 1.5.1, if a small country gets access to a high elimination cost technology to ensure his safety, the country can then turn to economic development, knowing that it can keep all the gains. The situation in North Korea, is arguably heading towards this direction.

Boundaries of burning. A key difference between our model and AES is that we allow for fully flexible power adjustment while AES assumes fixed power levels[22]. The real world lies somewhere between the two. One way to reconcile the two models is to introduce a lower bound of power burning: individuals can adjust their power only above a lower bound.

[22]It is true that sometimes power struggles involve executing your enemies such as in the Great Purge in the Soviet Union. But in many other cases the struggles end with the relevant parties ceding power but keeping their lives. Even in Soviet Union, the power transition since Nikita Khrushchev, despite the struggles, had been largely without blood shed. The same applies to the transition after Cultural Revolution in China, the transition from Chiang Ching-kuo to Lee Teng-hui in Taiwan, the transition from Thein Sein to Aung San Suu Kyi in Myanmar, and the power consolidation of Bashar al Alssad in Syria.

The lower bound may be determined by blood lines, religious beliefs, ethnicity (Montalvo and Reynal-Querol 2005), institutional legacies (Evans 1995), etc. For example, if one has a royal name or is of royal blood, it is often not enough for him or her to cede power and claim loyalty to the current ruler. The leader sometimes ends up eliminating the bloodline to tie up the loose ends. This practice endures from the sibling killing tradition in Ottoman Empire to the assassination of Kim Jung-Nam in February, 2017, and the recent arrest of Prince Alwaleed bin Talal and Prince Mutaib bin Abdullah of Saudi Arabia.

1.7 Concluding Remarks

The goal of this paper is to study the persistence of elites in coalition dynamics. It does so by constructing a model of power struggles, where players use their power to eliminate others and split resources in a coalition. The modeling innovation is to allow players to cede power when they are about to be expelled. A power structure is in equilibrium when the opponents of the current structure fail to adjust to a stable, rent-improving alternative. We characterize stable structures in general and show that power often ends up more concentrated to a few elite members when equilibrium structures are perturbed. We use the model to explain why social and political hierarchies often prove extremely difficult to eradicate and why, in many cases, revolutions that aim to install social equality quickly reproduce the same type of hierarchies that the revolutionaries sought to destroy.

As final remarks, we discuss several other directions we would like to explore in the future and their respective intuitions. First, the current analysis focuses on non-targeted transfers (burning)[23]. A direct generalization would be to allow targeted transfers, where a member

[23]Burning $\Delta = \gamma_i - \rho_i(\gamma_i)$ pieces of power for agent i is equivalent to cede power $\frac{\Delta}{n-1}$ uniformly to every other agent in the coalition. Burning power is a special case of non-targeted power transfer. We adopt power burning as the only way of power transfer to emphasize the difficulty of such process. Unlike physical assets, power is often hard to transfer from one to another, for instance, for an army to switch allegiance to another commander. Other examples of burning power include resigning or exiting from office (Steve Bannon), migrating to a foreign country (Leon Trotsky), abandoning access to weapons (chemical, nuclear

can transfer power precisely to another individual. We conjecture that the qualitative results of the model will not change, because allowing targeted transfers provides a cheaper way to accommodate pivotal players. For example, if free transfers are allowed and we start from a 3-agent coalition $(3, 4, 5)$ under a top-down agenda, the cheapest way for agent 1 to accommodate agent 3 is to directly transfer 1 piece of power; that is, to turn the power structure to $(2, 4, 6)$. It differs from the original equilibrium structure $(1, 4, 5)$, but they share the feature of being oligarchies.

Second, the coalition in the current framework is fully connected: every agent can form alliances with everyone else. It would be interesting to investigate how particular network structure affects coalition formation outcomes for the following reasons: First, restrictions on coalition structures are pervasive in reality. Members in a large society may not know to each other or the impact of power burning is only restricted within a sub-group. In the meantime, power struggles, or conflicts of wars can be means to access to a larger coalition, as described in Dziubinski et. al (2017). Second, particular network structures speak to different types of collective action problems. For instance, if a sub-coalition has a "line" structure (eg. a command chain), the sequence of moves is in the coalition. Meanwhile, in a "star" structure (eg. a radio station with its audience), only the first mover is clear, but not the rest of the sequence.

Third, the agents in the current model are purely rent-maximizing. One further practice is to add in commitment types, or the idealists, that aim to prevent conflicts. The simple intuition applies that if the powerful happens to be a commitment type, such as George Washington, then conflict resolution can be straightforward. A more interesting question to ask is perhaps how few commitment types are needed to ensure a peaceful environment. Along the same line, is long-lasting peace possible, if people believe in the existence of some commitment types in the coalition.

---

and other weapons of mass destruction), etc.

Lastly, power metric is one-dimensional in the model. The same discussion can be extended to multi-dimensional, such as to allow an agent to possess some economic power, as well as some political power. Economic power can be invested in building political power and vice versa. The dynamics of the two-way transition are expected to further shed lights on the transition from one institution to another.

1.8 Appendix

Appendix A: Omitted proofs

Proof of Theorem 1.1.

Proof. **Existence.** First, observe that the game has finite horizon. The burning meetings last for finite rounds because power is discrete, so there are at most $\frac{\prod_{i=1}^{n}\gamma_i}{\epsilon^n}$ rounds of burning meetings. The elimination meetings last for finite rounds because there are finite proposals and finite voters, and if no proposal is accepted the status quo is enforced. By Zermelo's Theorem, a finite horizon game of perfect information admits a SPE.

Uniqueness of equilibrium structure. Given agenda s, suppose there exists another equilibrium $\left(N^{'},\gamma^{'}\right) \neq (N^*,\gamma^*)$. There are two cases. First, $N^* \neq N^{'}$. Without loss of generality assume that $N^* \subsetneqq N^{'}$. Then there exists some agent i such that $i \in N^{'}$ but $i \notin N^*$. Therefore in the first equilibrium $U_i = -n$, which cannot happen in equilibrium because agent i can simply burn power down to zero in the previous burning meeting such that $U_i = 0$, which is an improvement. Therefore (N^*,γ^*) cannot be in equilibrium. By the same logic we know that $N^* = N$, because otherwise any agent in $N\backslash N^*$ has a better response by burning down to zero.

Consequently, elimination does not take place on equilibrium path. Now suppose $N^* = N^{'}$ but $\gamma^* \neq \gamma^{'}$. Denote the first time (round) that an agent burns power differently on equilibrium path as t, the member as j, strategy as $s^*_{j|h_t}$, $s^{'}_{j|h_t}$. Therefore the member is indifferent between two burning choices, $U_j\left(s^*_{j|h_t}\right) = U_j\left(s^{'}_{j|h_t}\right)$, which implies that $\frac{\gamma^*_j}{\sum\gamma^*_i} = \frac{\gamma^{'}_j}{\sum\gamma^{'}_i}$. Furthermore, the tie-breaking rule suggests that $\gamma^*_j = \gamma^{'}_j$. So j cannot burn power differently. Neither can any other members, $\gamma^* = \gamma^{'}$. Contradiction.

So far we have shown that an equilibrium structure exists and is unique for given N, γ, s. There might exist multiple SPE's, because in the voting stage, if a previous member included

in the proposal votes no, and the next member is indifferent between yes and no because the proposal is rejected either way. But the constitution of the equilibrium structure remains the same. □

Proof of Corollary 1.1.

Proof. See the first part of the "uniqueness" proof of Theorem 1.1. □

Proof of Proposition 1.1.

Proof. When the elite agent is very powerful, it naturally dictates. When the elite agent is not powerful enough, the key is whether the weak and the medium agent could form a powerful alliance against him.

When $a \leq \frac{c}{2}$, an alliance between the weak and the medium is not powerful enough. So the unique stable and rent-improving coalition remains $\left(\gamma_m^{'} = b, \gamma_e^{'} = b\right)$. The consequent equilibrium is $(c-b, b, c)$.

Now we consider the cases under $a > \frac{c}{2}$. When $s = (m, w, e)$ or (w, m, e), the weak and the medium can form the sub-coalition $\left(\gamma_w^{'} = a, \gamma_m^{'} = a\right)$ before the elite moves. Facing (a, a, c), the elite, as the last-mover, has no choice but to burn down to zero because gambling for probabilistic inclusion is never optimal. Therefore the equilibrium structure is $(a, a, 0)$. In any other agenda where the elite is not the last mover, the elite can always mimic the medium agent's burning strategy and secure rent of $\frac{1}{2}$. As a result, the weak agent should always accommodate the elite by burning down to make up the power difference between the elite and the medium agent. Therefore the equilibrium structure is $(c-b, b, c)$. □

Proof of Theorem 1.2.

Proof. We start to follow an induction approach to characterize the equilibrium structure for coalitions with size $|N| \leq 7$. First of all, the theorem holds for any 3-agent coalition, as suggested in Proposition 1.1. And it is easy to check that all the three forms are indeed

equilibrium structures. Denote the set of structures that fall in either one of the three forms as Γ.

For coalitions of all sizes, when there exists some i such that $\gamma_i > \sum_{j \neq i} \gamma_j$, then the equilibrium structure naturally converges to a dictatorship. In the following discussion we thus focus on the cases where $\gamma_i \leq \sum_{j \neq i} \gamma_j$. Without loss of generality we reorder the agents from weakest to strongest, as in the original power structures.

When $|N| = 4$, suppose there is an equilibrium structure, $\gamma^{'} \notin \Gamma$. The most powerful agent, which we denote γ_4, without loss of generality, enjoys strictly rents strictly less than $\frac{1}{2}$, because otherwise the structure shall end up in a dictatorship. Consequently, the two most powerful players, which we denote γ_3 and γ_4, have incentives to form a rent-improving alliance $\left(\gamma_3^{'} = \gamma_3, \gamma_4^{'} = \gamma_3\right)$. Therefore, the original structure cannot be in equilibrium.

When $|N| = 5$, similarly, suppose there exists an equilibrium structure $\gamma^{'} \notin \Gamma$, then the most powerful agent has rents $U_5 < \frac{1}{2}$. If $2\gamma_4 > \gamma_1 + \gamma_2 + \gamma_3$, then γ_5 can threaten to burn down to match power with γ_4, thus resulting an oligarchy. If $2\gamma_4 \leq \gamma_1 + \gamma_2 + \gamma_3$, it has to be true that $2\gamma_5 \geq \gamma_1 + \gamma_2$. When $2\gamma_5 > \gamma_1 + \gamma_2$, there exists an oligarchy with $\left(\gamma_5^{'} = \gamma_5, \gamma_4^{'}, \gamma_3^{'}\right)$ where $\gamma_4^{'} + \gamma_3^{'} = \gamma_5$ that is strictly rent-improving, therefore the original structure is not in equilibrium. When $2\gamma_5 = \gamma_1 + \gamma_2$, the original structure is equal-powered, thus it is easy to show that a polyarchy of 4 agents is a profitable deviation.

When $|N| = 6$, suppose $U_6 < \frac{1}{4}$, then a profitable deviation is again a 4-agent polyarchy. Therefore, $U_6 \geq \frac{1}{4}$. Furthermore, $U_5 < \frac{1}{4}$, otherwise an oligarchy is formed. Consequently, $U_4 < \frac{1}{4}$, and $U_1 + U_2 + U_3 \leq \frac{1}{2}$. It cannot be the case that $U_1 + U_2 + U_3 = \frac{1}{2}$, in which case it is an equal-powered coalition, which contradicts with $U_6 \geq \frac{1}{4}$. Therefore, $U_1 + U_2 + U_3 < \frac{1}{2}$, and $2U_6 \geq \frac{1}{2}$. It is a profitable deviation to a 3-agent oligarchy centered around agent 6.

When $|N| = 7$, similarly we know that $U_7 \geq \frac{1}{4}$. Furthermore, $U_6 < \frac{1}{4}$, $U_5 < \frac{1}{4}$. Suppose $U_1 + U_2 + U_3 + U_4 \geq \frac{1}{2}$, then $U_5 < \frac{1}{6}$. Consequently, $U_4 < \frac{1}{6}$, then it is profitable to deviate to a 4-agent oligarchy centered around agent 7. Suppose $U_1 + U_2 + U_3 + U_4 < \frac{1}{2}$, then it is immediate that a 3-agent oligarchy centered around agent 7 is profitable.

When $|N| = 8$, the theorem no longer holds because a relatively equal structure now enables a powerful non-elite group to check and balance potential deviations. As a counter-example, consider the following coalition structure: $(5.2, 5.1, 5, 5, 5, 5, 5, 5)$. □

Proof of Theorem 1.3.

Proof. For dictatorships, pick the dictator as agent i.

For oligarchies, pick the agent enjoys half of the rent in the original equilibrium as agent i, and the rest follows from the analysis for 3-agent coalitions.

For polyarchies of size 2^k, all agents are equally powerful in the original equilibrium. For $k = 1$, any perturbation collapses the regime into a dictatorship. For $k = 2$, positive perturbation turns the regime into an oligarchy centered around the empowered agent (which we pick as agent i); negative perturbation turns the regime into a polyarchy of size 2, with two earliest-moving non-weakened agents. In this case we pick either one of them as agent i.

For $k \geq 3$, if the perturbation is positive, pick the empowered agent as agent i, who now becomes the uniquely most powerful agent. If the perturbation is negative, pick the earliest-moving non-weakened agent as agent i, because the post-perturbation regime is a polyarchy of size 2^{k-1} in which i is included. □

Proof of Theorem 1.4.

Proof. For a general coalition N,

"If" condition.

Suppose there does not exist a stable improving sub-coalition, that is, for any winning sub-coalition $C \subset N$ such that $\gamma_C^{'} > \gamma_{N \setminus C}$, where $\gamma_i^{'} \leq \gamma_i$, and $\gamma_C^* = \phi\left(\gamma_C^{'}\right)$, there exists some agent $i \in C$ that $U_i\left(C, \gamma^*\right) \leq U_i\left(N, \gamma\right)$. If $U_i\left(C, \gamma^*\right) < U_i\left(N, \gamma\right)$, then agent i rejects the proposal when C is proposed in the elimination meeting. If $U_i\left(C, \gamma^*\right) = U_i\left(N, \gamma\right)$, should a equilibrium with inner circle C is formed, the agents have to participate in at least 3 rounds in burning meeting, which is dominated by maintaining the status quo by the tie-breaking

assumptions. Therefore, agent i rejects the proposal in the elimination meeting as well. Anticipating the rejection, other members in C have no incentives to adjust power in the burning meeting, thus the status quo coalition remains unchanged. So the original structure is in equilibrium.

"Only if" condition.

Suppose a coalition N with γ is in equilibrium, and there is a sub-coalition C with adjusted structure $\gamma_C^{'}$ that is stable improving. Pick any agent in C who makes burning proposals in the burning meeting: if he proposes $\gamma_C^{'}$, it is a better response for all involving agents in C to agree on the proposal and secures a rent-improving payoff. He is able to do so because $\gamma_{i\in C}^{'} > \max\{\gamma_j \mid j \in N_{\succ_s i} \backslash C\}$ therefore no prior-moving agent can mimic the move. Thus the original structure is not equilibrium, contradiction. □

Proof of Lemma 1.1.

Proof. Since some power is burnt, then for any adjustable sub-coalition $C \subset N$ such that $\gamma_C^{'} > \gamma_{N\backslash C}^{*}$ and $\gamma_C^{'} \leq \gamma_C^{*}$, there exists some agent $j \in C$ that $U_j(N, \gamma^*) \geq U_j\left(C, \phi\left(\gamma_C^{'}\right)\right)$.

If $U_j(N, \gamma^*) = U_j\left(C, \phi\left(\gamma_C^{'}\right)\right)$ for some j, the statement is proved. If $U_j(N, \gamma^*) > U_j\left(C, \phi\left(\gamma_C^{'}\right)\right)$ for all j and all such sub-coalition C, pick agent i as the last agent who burns partial power, $0 < \gamma_i^* < \gamma_i$, and slightly increase his power in his last burning choice. Then the inequalities for all j's still hold (because i is the last agent burning partial power. A complete burner does not change the result, otherwise they do not burn completely) and agent i enjoys more rent. Thus it is a better response for i. Agent i can continue to do so until for some $j^{'}$, $U_{j'}(N, \gamma^*) = U_{j'}\left(C, \phi\left(\gamma_C^{'}\right)\right)$. □

Proof of Theorem 1.5.

Proof. By Lemma 1.1, $U_x(N, \gamma^*) = U_x\left(C\phi\left(\gamma_C^{'}\right)\right)$. Notice that $\gamma_{x\in C}^{'} < \gamma_{x\in N}^{*}$, otherwise $\frac{\gamma_x^*}{\sum \gamma_{i\in N}^*} = \frac{\gamma_x^{'}}{\sum \gamma_{i\in C}^{'}}$, $C = N_+$thus is not a proper sub-coalition. Consequently, $\gamma_C^{'}$ is adjustable after the small shocks $0 < \eta < \underline{\eta_0} = \gamma_{x\in N}^{*} - \gamma_{x\in C}^{'}$.

There are two cases to consider.

First, if $\left(C, \gamma_C^{'}\right)$ is the only deviation coalition whose deviation constraint binds. That is, for any winning sub-coalition $\widetilde{C} \neq C$ such that $\sum \gamma_{i\in\widetilde{C}}^{'} > \sum \gamma_{i\in N\backslash\widetilde{C}}^{*}$, $\gamma_{\widetilde{C}}^{'} \leq \gamma_{\widetilde{C}}^{*}$, we have $U_j\left(N, \gamma^*\right) > U_j\left(\widetilde{C}, \phi\left(\gamma_{\widetilde{C}}^{'}\right)\right)$ for some $j \in \widetilde{C}$. Then for small enough grid ν, after a small negative shock hits x, we still have we have $U_j\left(N, \gamma^{*,\eta}\right) > U_j\left(\widetilde{C}, \phi\left(\gamma_{\widetilde{C}}^{'}\right)\right)$, where $\gamma^{*,\eta} = \left(\gamma_x^* - \eta, \gamma_{-x}^*\right)$. This means that $\left(C, \gamma_C^{'\eta}\right)$ is the only stable improving sub-coalition. Thus agents in $N\backslash C$ has to burn additional power to accommodate x, until $U_x\left(N, \gamma^*\right) = U_x\left(C, \phi\left(\gamma_C^{'}\right)\right)$. In particular, denote the set of $\widetilde{C}$ as $\widetilde{\mathcal{C}}$, denote $\eta_{\widetilde{C}}$ as the largest shock that makes $U_{j\in N}\left(\gamma^{*,\eta}\right) \geq U_{j\in\widetilde{C}}\left(\phi\left(\gamma_{\widetilde{C}}^{'}\right)\right)$ for some $j \in \widetilde{C}$, we can calculate $\underline{\eta_1} = \min\left\{\eta_{\widetilde{C}}\right\}_{\widetilde{C}\in\widetilde{\mathcal{C}}}$.

Second, if there are other deviation coalitions whose deviation constraint binds. There are two scenarios: 1. the binding agent is not x. Then after the shock the deviation constraint is slack. We go back to the first case; 2. the binding agent is x, then we calculate the upper bound of the shock size by $\underline{\eta_2} = \min\left\{\gamma_{x\in N}^* - \gamma_{x\in\widehat{C}}^{'}\right\}_{\widehat{C}\in\widehat{\mathcal{C}}}$, where $\widehat{\mathcal{C}}$ is the set of deviation coalitions with binding agent x.

Lastly we denote $\underline{\eta} = \min\left\{\underline{\eta_0}, \underline{\eta_1}, \underline{\eta_2}\right\}$. □

Proof of Theorem 1.6.

Proof. We once again try to show that $\left(C, \gamma_C^{'\eta}\right)$ remains the unique stable improving sub-coalition after the shock. The proof is similar with that of Theorem 1.5. A small positive shock from the agents in A makes $\left(C, \gamma_C^{'}\right)$ feasible but keeping incentive constraints of other winning coalitions slack. Denote the threshold derived from such process as $\overline{\eta_1}$. The next step is to check whether an empowered agent j brings additional stable improving coalitions to consider. Pick $\overline{\eta_2} < \gamma_j - \gamma_j^*$, suppose there is such a new stable improving coalition $\widehat{C}$, where $j \in \widehat{C}$, $\widehat{\gamma}_j < \gamma_j$, then we know that for agent j, γ_j^* is a better response than $\widehat{\gamma}_j$. Therefore $\left(C, \gamma_C^{'\eta}\right)$ remains the unique stable improving coalition to consider. Lastly we denote $\overline{\eta} = \min\left\{\overline{\eta_1}, \overline{\eta_2}\right\}$. □

Appendix B. Additional Examples

Suppose that in the burning meeting an agent can only adjust his own power downwards, i.e. $B_i \equiv \{i\}$, the following example shows that there exists a equilibrium which includes a stable improving sub-coalition.

Consider a committee with 4 members. Examples can be the Board of Directors in a corporation or standing members in national security council. The committee decides a policy which generates rent of 1 to share among the members. The initial formal power structure is $\gamma = (\gamma_x = 4, \gamma_y = 4, \gamma_z = 4, \gamma_w = 6)$, where we refer to x, y, z, w as four committee members, with w being the leader of the committee, or the "first among the equals", an expression used from Joseph Stalin to Saddam Hussein. The agenda is top-down, that is, w moves first, followed by z, y and x. As in the main text, members try to form stable improving alliances to increase their relative power. A feasible option can be among y, z and w, with underlying new power balance $\gamma_{C'} = \left(\gamma_y^{'} = 3, \gamma_z^{'} = 3, \gamma_w^{'} = 6\right)$, which itself is a equilibrium . However, to achieve this, both y and z has to adjust power. Suppose y adjusts first in the transition to $\gamma_z^{'} = 3$. When it is z's turn to adjust, he faces power structure $\gamma^{'} = (4, 4, 3, 6)$, which he now has an opportunity to seek for a new alliance with w *only*, which generates $\gamma_{C"} = \left(\gamma_y^{"} = 4, \gamma_w^{"} = 4\right)$ and an eventual equilibrium of $(0, 4, 2, 6)$. Agent y enjoys more rents in inner circle $C^{'}$ than in $C"$ since $\frac{4}{12} > \frac{3}{12}$. Therefore the transition cannot continue. Furthermore, anticipating y's incentive to switch alliances (and leaves z with rents $\frac{2}{12}$), z no longer wishes to initiate power struggles that centers around $C^{'}$. Thus the original power structure stays stable.

This is a key difference of the sequential coalition formation game comparing with standard cooperative coalition formation. In cooperative game literature, it is assumed that upon

a rent-improving allocation the involving agents can commit to form a coalition to collect the rents. It is, however, not the case in this model, because during the formation of such coalition, some agents might have received higher rents in the transition when other agents burn power, and consequently choose to divert to another deviation which may be not desirable from the perspective of the initiator. Such motive provides an additional economic force to support a wider range of institutional arrangement. In the literature, this is closest to Chwe (1994) in the discussion of the credibility of selecting a deviation strategy.

1.9 References

Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin. *Coalition Formation in Political Games.* Technical Report, National Bureau of Economic Research, 2006.

Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin. Coalition Formation in Nondemocracies. *The Review of Economic Studies*, 75(4):987-1009, 2008.

Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin. Dynamics and Stability of Constitutions, Coalitions, and Clubs. *The American Economic Review*, 102(4):1446-1476, 2012.

Acemoglu, Daron, and James A Robinson. *Economic Origins of Dictatorship and Democracy.* Cambridge University Press, 2005.

Acemoglu, Daron, and James A Robinson. De Facto Political Power and Institutional Persistence. *The American Economic Review*, 96(2):325-330, 2006.

Acemoglu, Daron, and James A Robinson. Persistence of Power, Elites, and Institutions. *The American Economic Review*, 98(1):267-293, 2008.

Acemoglu, Daron, and James A Robinson. *Why Nations Fail: The Origins of Power, Prosperity, and Poverty.* Crown Business, 2013.

Acemoglu, Daron, and James A Robinson. *The Emergence of Weak, Despotic and Inclusive States.* 2017.

Aghion, Philippe, Alberto Alesina, and Francesco Trebbi. Endogenous Political Institutions. *The Quarterly Journal of Economics*, 119(2):565-611, 2004.

Aghion, Philippe and Patrick Bolton. Incomplete Social Contracts. *Journal of the European Economic Association*, 1(1):38-67, 2003.

Ashdown, Peter. *Caribbean History in Maps.* [Port of Spain], Trinidad: Longman Caribbean, 1979.

Bernheim, B Douglas, Bezalel Peleg, and Michael D Whinston. Coalition-proof Nash Equilibria i. Concepts. *Journal of Economic Theory*, 42(1):1-12, 1987.

Bueno De Mesquita, Bruce, et al. *The logic of Political Survival*. MIT press, 2005.

Carvalho, Jean-Paul, and Christian Dippel. *Elite Identity and Political Accountability: A Tale of Ten Islands.* Technical report, National Bureau of Economic Research, 2016.

Caselli, Francesco, Massimo Morelli, and Dominic Rohner. The Geography of Interstate Resource Wars. *The Quarterly Journal of Economics*, 130(1):267-315, 2015. 34

Chwe, Michael Suk-Young . Farsighted Coalitional Stability. *Journal of Economic Theory*, 63(2):299-325, 1994.

Dal Bo, Ernesto, Pedro Dal Bo, and Jason Snyder. Political Dynasties. *The Review of Economic Studies*, 76(1):115-142, 2009.

Dalton, George. History, Politics, and Economic Development in Liberia. *The Journal of Economic History*, 25(4):569-591, 1965.

De Tocqueville, Alexis. *Ancien regime and the French revolution.* Penguin UK, 2008.

Dell, Melissa. *The Mining Mita: Explaining Institutional Persistence.* Manuscript, Department of Economics, MIT, 2008.

Doyle, William. *The Oxford History of The French Revolution.* Oxford Paperbacks, 1989.

Dziubinski, Marcin, Sanjeev Goyal, and David EN Minarsch. *The Strategy of Conquest.* 2017.

Evans, Peter B. *Embedded Autonomy: States and Industrial Transformation.* Princeton University Press, 2012.

Fearon, James D. Self-enforcing Democracy. *The Quarterly Journal of Economics*, 126(4):1661-1708, 2011.

Gehlbach, Scott, and Philip Keefer. Investment Without Democracy: Ruling-party Institutionalization and Credible Commitment in Autocracies. *Journal of Comparative Economics*, 39(2):123- 139, 2011.

Gehlbach, Scott, Konstantin Sonin, and Milan W Svolik. Formal Models of Nondemocratic Politics. *Annual Review of Political Science*, 19:565-584, 2016.

Gregory, Paul R. *Politics, Murder, and Love in Stalin's Kremlin: The Story of Nikolai Buk-*

harin and Anna Larina. Hoover Press, 2013.

Guimaraes, Bernardo, and Kevin D Sheedy. Guarding The Guardians. *The Economic Journal*, 2017.

Havel, Vaclav, and Paul Wilson. The Power of The Powerless. *International Journal of Politics*, 15(3/4):23-96, 1985.

Helfer, Luzia. Media Effects on Politicians: An individual-level Political Agenda-setting Experiment. *The International Journal of Press/Politics*, 21(2):233-252, 2016.

Hobbes, Thomas. *Leviathan*. A&C Black, 2006.

Jandoc, Karl, and Ruben Juarez. Self-enforcing Coalitions with Power Accumulation. *International Journal of Game Theory*, 1-29, 2013.

Kendall-Taylor, Andrea, and Erica Frantz. When Dictators Die. *Journal of Democracy*, 27(4):159-171, 2016.

Kilgour, DM. Equilibrium Points of Infinite Sequential Truels. *International Journal of Game Theory*, 6(3):167-180, 1977.

Levine, David K, Salvatore Modica, et al. *An Evolutionary Model of Intervention and Peace*. Technical report, UCLA Department of Economics, 2016.

Magaloni, Beatriz, and Ruth Kricheli. Political Order and One-party Rule. *Annual Review of Political Science*, 13:123-143, 2010.

McCombs, Maxwell E, and Donald L Shaw. The Agenda-setting Function of Mass Media. *Public Opinion Quarterly*, 36(2):176-187, 1972.

Merton, Robert K, et al. The Matthew Effect in Science. *Science*, 159(3810):56-63, 1968.

Michels, Robert. *Political Parties: A Sociological Study of The Oligarchical Tendencies of Modern Democracy*. Hearst's International Library Company, 1915.

Olson, Mancur. *Power and Prosperity: Outgrowing Communist and Capitalist Dictatorships*. Basic books, 2000.

Paige, Jerey M. *Coffee and Power. Revolution and The Rise of Democracy in Central America,* 1997.

Paludan, Ann. *Chronicle of The Chinese Emperors: The Reign-by-reign Record of The Rulers of Imperial China*, 1998.

Pipes, Richard. *Russia under The Bolshevik Regime*. Vintage, 2011.

Pycia, Marek. Stability and Preference Alignment in Matching and Coalition Formation. *Econometrica*, 80(1):323-362, 2012.

Ray, Debraj, and Rajiv Vohra. The Farsighted Stable Set. *Econometrica*, 83(3):977-1011, 2015.

Robinson, James A. Elites and Institutional Persistence. *The role of elites in economic development*, pages 29-52, 2012.

Schatz, Edward. The Soft Authoritarian Tool Kit: Agenda-setting Power in Kazakhstan and Kyrgyzstan. *Comparative Politics*, 41(2):203-222, 2009.

Scheufele, Dietram A. Framing as A Theory of Media Effects. *Journal of Communication*, 49(1):103-122, 1999. 36

Scheufele, Dietram A, and David Tewksbury. Framing, Agenda Setting, and Priming: The Evolution of Three Media Effects Models. *Journal of Communication*, 57(1):9-20, 2007.

Stone, Samuel Z. *The Heritage of The Conquistadors: Ruling classes in Central America from the conquest to the Sandinistas*. Univ of Nebraska Pr, 1990.

Svolik, Milan W. *The Politics of Authoritarian Rule*. Cambridge University Press, 2012.

Tao, Li. *Xu zizhi tongjian changbian. Collected Data for A Continuation of The Comprehensive Mirror for Aid in Government*, 2004.

Wiener, Jonathan M. *Social Origins of The New South: Alabama, 1860-1885*. 1982.

Woodward, Comer Vann. *The Strange Career of Jim Crow*. Oxford University Press, USA, 1955.

Zhang, Yan Bing, Mei-Chen Lin, Akihiko Nonaka, and Khisu Beom. Harmony, Hierarchy and Conservatism: A Cross-cultural Comparison of Confucian Values in China, Korea, Japan, and Taiwan. *Communication Research Reports*, 22(2):107-115, 2005.

Chapter 2

Strategic Promotions in Hierarchies

2.1 Introduction

The determinants of promotions are one of the central topics in the study of bureaucracies. While most studies focus on the bilateral interactions between the promoter and the promotee, the incentives facing the promoter are often more complex and are understudied. A promoter, when deciding whether to promote a subordinate, often needs to consider how such actions send signals to other parties of concern. In a bureaucratic hierarchy, a fundamental feature embedded is the top-down accountability: mid-tier officers care about their reputation in the eyes of the superior, which may distort the incentives of his own actions. In the meantime, officers sitting on the top of the hierarchy also strive to improve their public support in many political contexts. Unsurprisingly, they do so in democracies because public support channels into electoral votes. Even in authoritarian regimes, rulers care about their popularity because they live under the shadow of mass political unrest and electoral defeat (Geddes, Wright and Frantz 2014)[1]. Given performance, politicians may strategically claim

[1]Geddes, Wright, and Frantz's (2014) data set shows that uprisings, rebellions and election defeats jointly account for 51% of the 223 post-World War II dictatorial regime collapses.

credit for themselves and shift blame to others in order to gain public support[2].

Consequently, we ask the question that how a mid-tier official should promote his subordinates to build up reputation when the big boss is watching him, and whether such maneuver can be effective, when the citizens are sophisticated to interpret promotion decisions as informative signals of promoters[3]. Intuitively, a promoter with reputation concerns tries to act like a "good" officer, by adopting the same promotion strategy that a good officer would choose. An immediate consequence is promotional inefficiencies, where deserved promotions are delayed, in order for the promoter to take the credit. Furthermore, having such personnel control tool, the promoter has extra incentives to shirk, in face of competent subordinate, knowing that information manipulation is always an option in the future. Our analysis thus demonstrates that the strategic promotion decreases government responsiveness.

We build a model to study an organizational structure in which observable performance reflects joint types of the superior and the subordinate, in which the superior cares about his reputation among the audience outside the organization and determines the subordinate's career prospects. This structure resembles the relationship between a popularity-minded authoritarian ruler and his appointed local agent, or between an elected politician and his appointed bureaucrat. Citizens outside the government form the audience[4]. The superior's reputation is the public's perception of his ability to deliver good performance. Citizens

[2]Of the two strategies, shifting blame is more frequently deployed due to "negativity bias" (e.g., Nielsen and Moynihan 2017)

[3]In reality, the efforts to shift blame may not be obviously effective. For instance, in an authoritarian regime with multiple levels of government, the central government may shift blame to local agents in the presence of undesirable governance outcomes (Cai 2008; Li 2006). Although the top-down blame-shifting may sometimes work, over time citizens may grow more sophisticated and stop accepting such naïve propaganda: Being aware that the central government decides local officials' career prospects, they do not expect the latter to defend themselves when the central government shifts blames; this in turn hinders the credibility of the propaganda. As an example, Chinese local officials will risk their career prospects if they openly defend themselves in the presence of central government's blame-shifting (Li 2016, p.115).

[4]Alternatively, one could think of the organization as a multi-level government, where the mid-tier official makes the promotion decision of a lower-ranking official, while caring about his or her reputation in the eyes of a high-ranking official. In this case, the high-ranking official is the audience.

measure ability based on desirable attributes, such as competence and non-corruptness, among others, while the superior has the ability to promote the subordinate. Our model highlights a novel role of promotion and lack thereof: promotion as a signal of the promoter's type.

Specifically, we assume there are a central ruler, a local bureaucrat, and a representative citizen. Our model assumes asymmetric information between the state (ruler and bureaucrat) and the citizen. The citizen observes only the revealed governance outcomes, but not knowing the exact types of the principal (central ruler) and its agents (local bureaucrat). Information is complete within the state. Since local governance depends on joint types of central and local governments, it is difficult for the citizen to identify what role each government has played when local governance is mediocre; in other words, the citizen knows that either the central or the local government is bad, but does not know which one it is[5]. In particular, when the citizen cannot distinguish between a good principal and bad agent versus a bad principal and good agent, the principal may manipulate promotion decisions to signal his type to the citizen. To do this, bad principals mimic what good principals do, i.e. they do not promote the agent. Consequently, good agents are denied deserved promotions. In the benchmark model, we assume that types are fixed, and that the ruler only uses promotion as a signaling tool to build his own reputation. Next, we allow types to be "chosen" based on the individual's talent, which adds a moral hazard problem before the signaling problem: a competent promoter has fewer incentives to respond timely to citizens' demands because of the ability to manipulate reputation in the future.

The model generates the following testable predictions: a strong ruler always promotes bureaucrats to important positions in an efficient way, namely, promoting the strong only.

[5]Existing studies have recognized the difficulty of citizens in inferring types of multiple governments based on jointly determined governance outcomes. For instance, Birney (2014) argues that Chinese citizens have difficulty in inferring the corruptness of local officials when observing non-implementation of the village election law. This is because the non-implementation can be due to either corrupt local officials or mandates from irresponsive high-ranking officials.

A weak ruler is likely to prevent deserved promotions to mimic a strong ruler when reputation concerns are significant and when the efficiency benefits from promotion are small. Consequently, when local government performance sends a mixed signal (e.g. when citizens cannot distinguish a team of good ruler and bad bureaucrat from a team of bad ruler and good bureaucrat) promotion is a "bad" signal for ruler's type because a strong ruler would never promote a weak bureaucrat. Therefore, citizens feel less positively toward the ruler once promotion is observed.

We provide supportive evidence for the reputation decreasing effect of promotion, using a novel research design of subnational leader turnovers in China. China fits the domain of our model because it has a centralized personnel control system managed by the ruling party and an economically decentralized system with local performance determined by both central and local governments (e.g., Birney 2014; Xu 2011). Moreover, the Chinese central government cares heavily about its own reputation, as poor reputation may lead to increased demands for regime-level political changes (Li 2004, 2011). To our knowledge, we are the first to investigate the reputation effect of promotions in a centralized personnel control system, such as the Chinese government. At the general level, the theory also speaks to governments with appointed officials in both democracies and non-democracies.

Our analysis makes several contributions to the literature. First, our study speaks to the burgeoning literature on information management in dictatorships. Although politicians in democratic governments also manage information to enhance their image, citizens in authoritarian regimes face low transparency and media freedom, which makes them more susceptible to governmental information management. An authoritarian ruler may improve his reputation and gain public support through actual performance improvement, information manipulation, or both (Chen and Xu 2017; Guriev and Treisman 2015). While some resilient authoritarian rulers are dedicated to the challenge of sustaining good economic performance, they also spend substantial financial resources on information manipulation to convince the ruled of their various desirable attributes (e.g., Gilley 2008; Shambaugh

2007). Researchers have identified various information management tools, such as signaling their strength through propaganda (Huang 2015), concealing the information about economic (under)performance (Hollyer, Rosendorff, and Vreeland 2015), or censoring the media (Lorentzen 2014). We contribute to this strand of the literature by studying a specific information manipulation technique: strategically promoting or demoting local agents.

Second, our work contributes to the literature on strategic promotions. A classical paradigm focuses on the tradeoff between promoting for competence versus loyalty (Egorov and Sonin 2011). Another objective of strategic promotion is to prevent poaching. Waldman (1984) argues that in corporate environments, firms are more reluctant to promote workers if they anticipate poaching from competitors because promotions send a positive signal of the worker's quality. Our paper is closer to Waldman (1984) since we both consider the signaling role of promotions; instead of promotion as a signal for the promotee, this paper considers promotion as a signal for promoter.

Third, our study speaks to two strands of literature on political institutions that consider strategic interactions among players. The first is on the important role that various authoritarian institutions, such as regime-sanctioned parties and legislatures, play in authoritarian survival (e.g., Gandhi and Przeworski 2007; Geddes 2003; Magaloni 2008; Svolik 2012). We underscore how an often-overlooked institutional design can strengthen dictatorial rule in a specific way. The central government - but less so the local government - must maintain its good reputation to hold off demands for political change and to sustain authoritarian rule (Li 2004). Along these lines, we propose that a centralized personnel control system can sustain dictatorial rule by providing a chance for the central government to maintain good reputation through manipulating promotion. The second is on the various tools to increase government responsiveness and accountability, which among others may include increased media coverage (Snyder and Strömberg 2010), enhanced transparency (Bordignon and Minelli 2001), and the separation of power (Persson, Roland, and Tabellini 1997). This paper argues that a unilateral signaling tool available to the principal dampens his responsiveness

and accountability because he can send signals to shift blame after shirking. Blame shifting is more effective when the principal knows the agents well because he can determine when to send the signal. Consequently, intra-governmental transparency (or effective monitoring) may reduce responsiveness of the superior.

2.2 Benchmark Model

2.2.1 Environment

Consider a game of incomplete information with three players: a central ruler (r) of competence type θ^r, a local bureaucrat (b) of competence type θ^b, and a representative citizen[6]. Competence type can be high or low, $\theta^i \in \{h, l\}$, $h > l$, and is fixed in this benchmark model[7]. Competence types are known within the regime, but remain unknown to the citizen. The ruler and the bureaucrat deliver types as performance. Then the ruler determines whether to promote the bureaucrat or not. Higher types are preferred for promotion. The payoff from the promotion is discussed in more detail below.

The citizen cares about his ruler's competence, of which he holds some prior belief. His job is to infer the ruler's type from two signals: performance and promotion decision. The ruler then receives a payoff which is proportional to his expected type.

Formally, the timing of the game is as follows:

1. Nature determines the central ruler's type $\theta^r \in \{h, l\}$ and the local bureaucrat's type $\theta^b \in \{h, l\}$, from the same pool that $Pr(\theta^i = h) = \mu,\ i = r, b$. Both the ruler and the bureaucrat observe their own and each other's type. The citizen observes neither, but holds correct prior belief that the ruler is of high type with probability μ.

[6]We use *leader*, *superior*, *central government* and *principal* interchangeably to refer to the central ruler. Meanwhile, *subordinate*, *local official*, and *agent* all refer to the local bureaucrat.

[7]We relax this assumption in the extended model.

2. Local performance is delivered (non-strategically) as the sum of types: $X = \theta^r + \theta^b$.

3. The citizen observes performance X.

4. The ruler decides whether to promote the bureaucrat. The ruler chooses promotion probability $p\left(\theta^r, \theta^b\right) \in [0, 1]$, which is jointly determined by the types of ruler and bureaucrat.

5. The citizen observes the promotion outcome $P\left(\theta^r, \theta^b\right) \in \{0, 1\}$, and update his belief of the ruler.

6. Payoffs are assigned. For the ruler, the payoff is jointly determined by the gains from promotion and the gains from reputation. The details of payoff are discussed below.

Next, we explain the key elements in the model.

Types. We interpret types as the attributes that the citizen prefers the ruling authority to have. Standard interpretation in the literature includes competence, non-corruptness, and vision[8]. We assume type information is common knowledge to the ruler and the bureaucrat, but not to the citizen. This is because frequent interactions between the ruler and the bureaucrat through meetings, external events, etc. usually generate rich information about types; the citizen, as an outsider, does not have these interactions and thus often has limited information[9].

Local Performance. We assume that local governance performance is the joint product of the central ruler and local bureaucrat's types. This assumption reflects the fact that both central policies and local execution affect governance outcomes. However, one should be cautious to interpret types as efforts, because types are fixed in the benchmark model. In the extended model, we relax the assumption and allow players to choose types, which makes

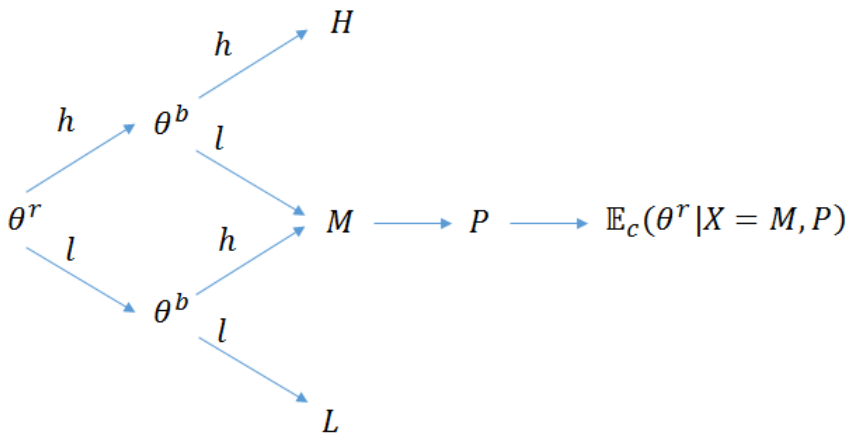
[8]We suppress potentially multi-dimensional competence to a one-dimensional parameter θ for the convenience of discussions. Assuming θ to be a vector of multiple attributes does not change the results qualitatively.

[9]Sometimes such interactions within the bureaucracy also fail to perfectly reveal the type information. The extended model also analyzes such situations.

the effort interpretation appropriate.

We assume that performance is the sum of types. Given the binary types, local performance takes three values $\{2l, h+l, 2h\}$, denoted as $\{L, M, H\}$ respectively. When local performance is Low, the citizen immediately infers that $\theta^r = \theta^b = l$. Similarly, the citizen infers that $\theta^r = \theta^b = h$ after High performance. The signal is jammed only when performance is Mediocre, because the citizen does not know who contributes h and who contributes l. Thus $Pr\left(\theta^r = h | X = M\right) = \frac{1}{2}$. The citizen needs the additional signal from promotion to make better inferences. The inference process and timing are shown in Figure 2.1.

Figure 2.1: Set-up of The Game



Promotion. Promotion has two roles in the model: it facilitates the ruler's job and provides information about the ruler's type. By promoting a local bureaucrat to a more important office, the central ruler gains additional helping hands. It is straightforward to see that high-type bureaucrats are more preferred in this regard because low-type bureaucrats may not fit in the higher office, thus doing more harm than good. Also, the ruler receives praises for having a keen eye for talent[10]. However, promotion also creates a local power

[10]The praises can be regarded as another form of reputation (on the ruler's ability to discover talents). We interpret reputation as the perceived quality of individual attributes (competence, non-corruptness, etc.). Therefore, we aggregate the praises into the net gains of promotion, $g\left(\theta^b\right)$, introduced next.

vacuum because the central ruler is responsible for finding a proper replacement. In this case, promoting a good bureaucrat makes it harder to find an equally competent successor. For the ruler, the (net) gains from promotion need to balance both considerations. In the benchmark model and extended models, we assume that the first consideration dominates[11]. That is, if we denote the net gains from promotion for the ruler as $g\left(\theta^b\right)$, we assume $g^{'} > 0, g\left(h\right) > 0, g\left(h\right) > 0 > g\left(l\right)$. In particular, the second part of the inequality suggests that an ignorant bureaucrat placed at high office can be detrimental. We assume that the ruler chooses a promotion probability $p\left(\theta^r, \theta^b\right) \in [0, 1]$, to accommodate both pure and mixed strategy equilibrium. The citizen does not observe p, but instead the realization of the promotion decision, $P \in \{0, 1\}$, which is binary and $P = 1$ with probability p, $P = 0$ with probability $(1 - p)$. Consequently, the citizen updates belief about the ruler's reputation, i.e. the expected type of the ruler, as $\mathbb{E}_c\left(\theta^r | X, P\right)$. Some justifications about modeling reputation are seen below.

Reputation. We assume a non-strategic citizen who infers the type of the central ruler. The role of a non-strategic citizen in evaluating the ruler is common in the literature, both in theoretical works (Gehlbach and Simpser 2015; Little 2015) and empirical investigations (e.g., Li 2004). Unlike the existing literature, we do not focus on the survival of the ruler (Edmond 2013). Instead, we interpret the ruler's preference toward being perceived as a higher type as his reputation concern.

Payoff. In the benchmark model, the ruler is the only active player. We assume that the ruler's payoff is the weighted average of gains from local performance, promotion and reputation. In particular:

$$\max_{p(h,l)} \widetilde{U}_r\left(\theta^r\right) = \left(1 - \alpha - \beta\right) X\left(\theta^r, \theta^b\right) + \alpha p\left(\theta^r, \theta^b\right) g\left(\theta^b\right) + \beta \mathbb{E}_c\left(\theta^r | X, P\right)$$

[11]We consider an alternative possibility that the local power vacuum is a bigger concern for the central ruler in the Appendix.

The first term corresponds to the gains from local performance. The proportion is fixed because both the ruler and the bureaucrat simply deliver their types. The second term corresponds to the ruler's expected efficiency gains from promotion if the ruler chooses a promotion probability $p\left(\theta^r, \theta^b\right)$ to promote a bureaucrat of type θ^b. The third term corresponds to ruler's reputation gains, which is proportional to the ruler's perceived type in the eyes of the citizen, after observing performance X and promotion outcome P. α and β are the weights assigned to each term. We normalize the total weights to one.

As discussed above, when the signal is clear, i.e. $X = H$ or $X = L$, the citizen perfectly learns the ruler's type and there is no incentive for the ruler to manipulate promotion. The ruler bases the decision purely on promotion gains, $g\left(\theta^b\right)$. Therefore, promotion is efficient and based purely on merits, which means $p\left(\theta^r, \theta^b\right) = 1$ if and only if $\theta^b = h$; $p\left(\theta^r, \theta^b\right) = 0$ if and only if $\theta^b = l$. In other words, bureaucrats get the promotions they deserve.

The interesting case is when the signal is jammed, i.e. $X = M$. From now on we only consider this case. There are two scenarios. First, a high-type ruler could be paired with a low-type bureaucrat, ignoring the performance term in the payoff function[12]. Then, the strong ruler faces the following (revised) maximization problem:

$$\max_{p(h,l)} \widetilde{U}_r(h) = \alpha p(h,l)\, g(l) + \beta \mathbb{E}_c\left(\theta^r | X = M, P\right)$$

Second, a low type ruler is paired with a high type bureaucrat. Similarly, the weak ruler faces the following problem:

$$\max_{p(l,h)} \widetilde{U}_r(l) = \alpha p(l,h)\, g(h) + \beta \mathbb{E}_c\left(\theta^r | X = M, P\right)$$

[12]This is without loss of generality because they just deliver their types. In other words, there are no choices to make.

2.2.2 Analysis

Our solution concept is Perfect Bayesian Equilibrium. To analyze the ruler's promotion decision, we first look at the citizen's beliefs after mediocre performance, $X = M$. Regardless of μ, $Pr(\theta^r = h|X = M) = \frac{1}{2}$. Next, the ruler makes his promotion decision ($p(h,l)$ for strong ruler, $p(l,h)$ for weak ruler) and collects promotion gains. The citizen makes a conjecture about the strategy of the ruler, $\widehat{p}(h,l)$ and $\widehat{p}(l,h)$, then uses the conjecture to update his beliefs about the ruler after seeing the promotion outcome. In equilibrium, the strong (weak) ruler should maximize his payoff given the strategy that a weak (strong) leader would have chosen and given the citizen's conjectures. In the meantime, the citizen's conjectures should be correct, i.e. coinciding with the ruler's strategy.

Formally, given the citizen's conjecture, denote the belief of a strong ruler after seeing promotion $P = 1$ as q_P, that after non-promotion $P = 0$ as q_{NP},

$$q_P := Pr(\theta^r = h|X = M, P = 1) = \frac{\widehat{p}(h,l)\frac{1}{2}}{\widehat{p}(h,l)\frac{1}{2} + \widehat{p}(l,h)\frac{1}{2}} = \frac{\widehat{p}(h,l)}{\widehat{p}(h,l) + \widehat{p}(l,h)}$$

$$q_{NP} := Pr(\theta^r = h|X = M, P = 0) = \frac{1 - \widehat{p}(h,l)}{2 - \widehat{p}(h,l) - \widehat{p}(l,h)}$$

Consequently, $\mathbb{E}_c(\theta^r|X = M, P = 1) = q_P h + (1 - q_P) l$, $\mathbb{E}_c(\theta^r|X = M, P = 0) = q_{NP} h + (1 - q_{NP}) l$

In equilibrium, we require:

$$\begin{aligned}
\widehat{p}(h,l) &= \arg\max_{p(h,l)} \widetilde{U}_r(h) \\
&= \arg\max_{p(h,l)} \alpha p(h,l) g(l) + \\
&\beta\left(p(h,l)\left[q_P h + (1 - q_P) l\right] + \left[1 - p(h,l)\right]\left[q_{NP} h + (1 - q_{NP}) l\right]\right)
\end{aligned}$$

and

$$\begin{aligned}\widehat{p}(l,h) &= \arg\max_{p(l,h)} \widetilde{U}_r(l) \\ &= \arg\max_{p(l,h)} \alpha p(l,h)\, g(h) + \\ &\beta\left(p(l,h)\left[q_P h + (1-q_P)\, l\right] + \left[1-p(l,h)\right]\left[q_{NP} h + (1-q_{NP})\, l\right]\right)\end{aligned}$$

We also make off-equilibrium path belief assumptions to be consistent with the perfect information case. In particular, if $\widehat{p}(h,l) = \widehat{p}(l,h) = 1$, citizen assigns any deviation to non-promotion probability one of being strong; if $\widehat{p}(h,l) = \widehat{p}(l,h) = 0$, citizen assigns any deviation to promotion probability one of being weak.

Intuitively, in face of mediocre performance, promotion harms the strong ruler ($g(l) < 0$) and benefits the weak ruler ($g(h) > 0$). Therefore, without reputation concerns, the weak ruler promotes, while the strong one does not. However, since citizens interpret non-promotion as a signal of strength, the weak ruler may have incentives to mimic the strong one by not promoting a well-deserved bureaucrat. Anticipating this, the citizen should downgrade his belief of a strong ruler upon observing a promotion. The adjustment process ends when the weak ruler is indifferent between promotion and non-promotion (i.e., does not want to mimic the strong ruler any further), and the citizen's conjecture is correct. On the other hand, a strong ruler has no incentive to feign weakness by promoting undeserving bureaucrats, because it is not beneficial in terms of promotion or reputation gains. We summarize the above intuition in the following proposition.

Proposition 2.1. *In equilibrium, the strong ruler never promotes. The weak ruler promotes in the following manner:*

he never promotes when $\frac{\alpha g(h)}{\beta(h-l)} \leq \frac{1}{2}$*;*

he promotes with probability $2 - \frac{\beta(h-l)}{\alpha g(h)}$ *when* $\frac{1}{2} < \frac{\alpha g(h)}{\beta(h-l)} < 1$*;*

he always promotes when $\frac{\alpha g(h)}{\beta(h-l)} \geq 1$.

The reputation of the ruler always (weakly) decreases after promotion.

Proof. See Appendix. □

Therefore, in equilibrium, there may be insufficient promotions due to the ruler's reputation concerns. This is consistent with the "bad reputation" literature (e.g. Ely and Välimäki 2003) in which players manipulate decisions when they are concerned about their reputation. It also provides a new driving force to explain insufficient promotions, in additional to the incentivization or networks arguments (see Ting 2003). Proposition 2.1 generates several interesting comparative statics. Recall that manipulation is more severe when the weak ruler is more likely to decline promotions. This occurs when the ruler places more weights on reputation than promotion gains ($\frac{\beta}{\alpha}$ is larger). It makes intuitive sense: when reputation concerns are intrinsically more important, the incentive to manipulate promotion is higher, resulting in increased manipulations. If we interpret β as a measure of the extent to which politicians care about their reputation among citizens, then Proposition 2.1 suggests that manipulation is more severe in democracies due to electoral pressure[13]. Manipulation is also more severe when the performance of the weak officer is worse (l is lower). That is, when the outside option of revealing true type is less attractive, the ruler tends to hide weakness more.

The above two comparative statics are straightforward. The comparative statics of the value of a high type, h, are less so. On one hand, higher h triggers larger promotion gains $g(h)$, thus incentivizing promotion; on the other hand, high h indicates higher reputation gains, which discourage promotion. If $g(h)$ is concave, which suggests diminishing marginal returns to officer quality, then higher h induces less promotion because reputation gains dominate promotion gains. If $g(h)$ is convex, which suggests that high-quality officers can

[13]This is because authoritarian rulers usually have alternative strategies to stay in office, such as using repression.

exponentially improve governance, then the promotion probability is U-shaped in h: it decreases first because reputation gains dominate, and increases later because the increasing return from governance becomes significant.

Thus far we have only discussed the role of promotion (and lack thereof) as a signaling tool. In theory, promotion and demotion are two sides of the same coin. One could expect a bad ruler to mimic the demotion pattern of a good one, i.e., if demotions are allowed, the incompetent ruler may demote his competent agents out of reputation concerns. Therefore, we do not discuss demotion separately. Instead, we proceed to discussing an alternative to demotions, namely pseudo promotions, which can be even more prevalent in the political context.

Pseudo promotion refers to the tactic of replacing an unsatisfactory local official by transferring him to an idle but privileged position at the same or sometimes higher level. For instance, an authoritarian ruler often deems demotion undesirable because it significantly disincentivizes agents from performing well (Cai 2014). Some autocracies, like China, have a long tradition of avoiding demotion of local officials unless there is a catastrophic event (Landry 2008; Mei and Pearson 2014). Pseudo promotion is so commonly deployed in China that a Chinese phrase was even invented to describe the very situation: *Mingsheng anjiang* ("promote to demote"). Moreover, there is evidence that the public is familiar with the tactic and understands its implications[14]. Pseudo promotion thus gives the ruler an opportunity to select a new local official. In the Appendix, we show that pseudo promotions increase the promoter's reputation. In pseudo promotions, a low-type ruler mimics a high-type one, leading to over-promotion and bureaucratic redundancy.

[14]Yizi Chen's (2013) memoir suggests that students and young intellectuals proposed moving old leaders to a consultative organization to accelerate China's market reform.

2.3 Active Ruler and Bureaucrat

2.3.1 Environment

In this section, we discuss an extended model, where we allow both the ruler and the bureaucrat to actively choose efforts in delivering local performance. The ruler and the bureaucrat enter the game with some talents given by Nature. Talents can be h (high/strong) or l (low/weak). We model effort choice as a privilege of strong officers (both central and local): Strong officers can choose to exert high effort (work) or low effort (shirk), but weak officers can only choose low effort. Choosing high effort comes with a cost $c > 0$ while choosing low effort is costless. We assume the ruler chooses effort first, the bureaucrat second[15]. Once effort is chosen, it becomes the officer's types and is fixed thereafter. Then we play the game as in the benchmark model.

Formally, the revised timing is as follows:

1. Nature draws the central ruler's talent $\tau^r \in \{h, l\}$ and the local bureaucrat's talent $\tau^b \in \{h, l\}$ from the same pool. Talent is common knowledge for the ruler and the bureaucrat but not for the citizen. The citizen holds prior belief μ that an officer is of high talent. The two draws of talent are independent.

2. The central ruler chooses effort, which then becomes his type $\theta^r \in \{\tau^r, l\}$. This is observed by the local bureaucrat but not the citizen.

3. The local bureaucrat chooses effort, which then becomes his type $\theta^b \in \left\{\tau^b, l\right\}$. This is observed by ruler but not the citizen.

4. Local performance is delivered as the sum of types: $X = \theta^r + \theta^b$.

5. The citizen observes performance X.

[15]Apart from effort choice, another key assumption is that the choices are sequential with the ruler choosing before the bureaucrat. This makes sense because local bureaucrats often need to follow central policies or guidelines from above.

6. The ruler decides whether to promote the bureaucrat. The promotion probability $p\left(\theta^r, \theta^b\right) \in [0,1]$ is jointly determined by the types of ruler and bureaucrat[16].

7. The citizen observes the promotion outcome $P\left(\theta^r, \theta^b\right) \in \{0,1\}$, and updates his beliefs about the ruler.

8. Payoffs are assigned.

Payoffs. With the new structure, we specifies ruler's and bureaucrat's payoffs as follows:

$$U_r\left(\tau^r, \theta^r\right) = (1-\alpha-\beta)\, X\left(\theta^r, \theta^b\right) + \alpha p\left(\theta^r, \theta^b\right) g\left(\theta^b\right) + \beta \mathbb{E}_c\left(\theta^r | X, P\right) - c\mathbb{I}\left(\theta^r = h\right) \mathbb{I}\left(\tau^r = h\right)$$

$$U_b\left(\tau^b, \theta^b\right) = (1-\gamma)\, X\left(\theta^r, \theta^b\right) + \gamma P\left(\theta^r, \theta^b\right) w - c\mathbb{I}\left(\theta^b = h\right) \mathbb{I}\left(\tau^b = h\right)$$

The ruler's payoff differs from that in the simple model only by the cost of effort. We model the bureaucrat's payoff as a weighted-average (with weight γ) of local performance and wage increase (w) upon promotion. We leave out reputation concern for the bureaucrat to emphasize that reputation building is a more important task for the central ruler[17].

There are two additional assumptions we make in the extended model. The first is that moral hazard only applies to high talent individuals. The assumption highlights the fact that moral hazard is less of an issue for less capable individuals because their efforts contribute little to the eventual outcome. It also simplifies the analysis. The rules of type formation are illustrated in Table 2.1.

[16]To be precise, the promotion probability is jointly determined by the types of the ruler and bureaucrat, both of which are functions of each individual's talent and consequent effort choices.

[17]Alternatively, one may assume that reputation concern exists for bureaucrat, but with a small weight that can be essentially ignored.

Table 2.1: Type Formation for Ruler (Promoter) and Bureaucrat (Promotee)

Talent (for both r and b)	Effort (Yes or No)	Types formed
High	Yes	High
High	No	Low
Low	Yes	Low
Low	No	Low

The second assumption is that the ruler chooses efforts before the bureaucrat. This reflects the fact that central policies are usually established before local execution, and the fact that local bureaucrats can often infer the ruler's efforts from policy details. The assumption is especially reasonable in authoritarian regimes, where local bureaucrats often have to govern under the restrictions of central instructions.

2.3.2 Analysis

We use backward induction. When the types are formed, ruler and bureaucrat play the same game as in the simple model. In the type-forming stage, the bureaucrat trades off the benefit of shirking with the decrease in performance and promotion opportunities. The ruler trades off the benefit of shirking with the decrease in performance and reputation loss, but may recover some of his reputation by manipulating promotions. In particular, we are interested in finding equilibria where the ruler does shirk because of the manipulation option later.

We first set up a benchmark with only the moral hazard problem of effort choice where no strategic promotion takes place. The intuition of the strategic interaction is as follows: when reputation can no longer be manipulated (types are revealed after chosen), the ruler's signaling tool to hide weakness is no longer effective. Thus, when cost of exerting effort is low, a talented ruler should always work hard. For the bureaucrat who has no access to promotion decisions, his only consideration is the cost of effort. We therefore have the following proposition.

Proposition 2.2. *In the non-signaling benchmark, when the cost of effort is not too high,*

high talent individuals always work. That is, there exists $\widetilde{c} > 0$ *such that* $\theta^i = h$ *whenever* $\tau^i = h$*, and* $c \leq \widetilde{c}$*,* $i \in \{r, b\}$*.*

Proof. See Appendix. □

Now consider what happens when types are not revealed after they are chosen, and the ruler can manipulate promotion to fake reputation. The existence of signal jamming explained in the simple model provides an opportunity for the weak ruler to earn a better reputation than he otherwise deserves. However, it also provides a talented ruler with incentives to shirk and choose to behave like a weak ruler and to exploit the reputation gains from manipulating promotion. In this case, the ruler's signaling tool has negative externality on the moral hazard problem and induces shirking from the ruler. The following proposition constructs precisely such an equilibrium: the strong ruler chooses to shirk in the hope that he can manipulate promotion to maintain a decent reputation later.

Proposition 2.3. *When cost of effort is not too large or too small, there exists a top-shirking equilibrium where a talented ruler shirks when he observes a talented local bureaucrat, and cheats the citizen by not promoting the good-performing bureaucrat. Formally, When* $c \in [\underline{c}, \overline{c}]$*, where* $\overline{c} \leq \widetilde{c}$*, there exists an equilibrium in which:*

$\theta^b = h$ *if and only if* $\tau^b = h$

$\theta^r = h$ *if and only if* $\theta^b = l$ *and* $\tau^b = h$

$p(h, l) = 0$

$p(l, h) = 2 - \mu - (1 - \mu) \frac{\beta(h-l)}{\alpha g(h)}$

Proof. See Appendix. □

Proposition 2.3 speaks to the negative externality of promotion as a signaling tool for the central ruler. When the bureaucrat chooses his level of efforts, he plays a simple career concern game similar to Holmstrom (1999) in which the incentive of choosing costly effort

is two-fold: the immediate benefit in higher output (better local performance) and the long-term higher promotion probabilities. When the ruler has the promotion tool, on the one hand, he can incentivize higher effort from the bureaucrat while doing less himself and manipulating promotions to maintain his reputation. By selecting low efforts, the ruler effectively pushes the bureaucrat to exert effort while saving some costs of effort himself. The downside of his reputation risk is then mitigated by not promoting the high-performing bureaucrat.

There are several interesting implications of this "active shirking of the ruler". First, the phenomenon takes place only when both the ruler and the bureaucrat are of high types: if the ruler is of low type, then he does not have a choice; and if the ruler is of high type but the bureaucrat is of low type, shirking from the top destroys his reputation completely by fully revealing the types. Second, the cost of effort must be mild: if the cost is too high, shirking is always the best choice; if the cost is too low, working and grabbing full reputation gains is always the best choice.

2.3.3 Transparency

Suppose now that the type information can also be private. In particular, we focus on the case where there is asymmetric information between the ruler and bureaucrat: the bureaucrat can perfectly observe the type of the ruler, but not vice versa[18]. Formally, the ruler has probability $\nu \in [0, 1]$ of knowing the bureaucrat's type before choosing his effort level. With probability $(1 - \nu)$, bureaucrat's type is private information.

With such asymmetry of information, we revisit Proposition 2.3. In Proposition 2.3, the incentive for the ruler to shirk comes from the ability to manipulate promotion when the bureaucrat is high-type and exerts effort. In all other cases, the ruler either cannot shirk or

[18]One justification of such asymmetry is that it is easier for the bureaucrat to identify the type of the ruler through meetings (where the ruler talks most of the time) or policy documents. But it is hard for the ruler to validate the types of the bureaucrat before he observes the local performances.

chooses not to shirk. If the ruler does not know the bureaucrat's type, he has fewer incentives to shirk. That is, if the information regarding the bureaucrat's type is less transparent, the ruler's manipulation tool is less effective, decreasing the ruler's incentive to shirk. In this sense, intra-governmental transparency (or effective monitoring) reduces the responsiveness of the superior, which can be bad for social welfare.

To put it formally we have the following corollary.

Corollary 2.1. *When the ruler does not know the bureaucrat's talent for sure, the more uncertain the ruler is, the less likely that he shirks. In particular, the ruler never shirks when the bureaucrat completely hides his talent, $\nu = 0$, when the prior belief is not too high, $\mu < \overline{\mu}$.*

Proof. See Appendix. □

Corollary 2.1 generates another prediction: intra-government opacity, namely a ruler with limited access to detailed information about his agents, could reduce shirking. In particular, the moral hazard problem may be completely mitigated when the ruler has no information about the bureaucrat. On the other hand, institutions that enable a ruler to collect detailed information about his agents, such as the Organization Department within the Communist Party of China (CPC), will increase intra-governmental transparency and make shirking more likely.

2.4 Empirical Evidence

Building on the existing literature (Li 2016) that rulers care about citizens' perceptions about them, our theoretical model shows that with sophisticated citizens who are capable of updating promotion decisions as signals, the promoter would manipulate promotion signals for reputation building, which further leads to promotional inefficiency and less responsiveness. In the empirical part, our main goal is to solidify the assumption that citizens indeed

perceive promotions as useful signals and use them to update their trust on the government. In particular, we provide supportive evidence to the model by showing that citizens do downgrade their perception after observing promotions. Combined with these empirical findings, our result justifies that reputation concern could exist in promotion decisions of a central ruler.

In this section, we provide some empirical evidence for the theory. To restate, our theory has two major empirical implications: a ruler's reputation declines after promoting a bureaucrat with mediocre observable performance, and the ruler reduces effort when the bureaucrat is talented and the effort cost is mild. While lacking proper data to examine the latter proposition (since both effort and talent are difficult to measure), we focus on testing the former one, using a novel research design in China.

As a preview of the results, using the 2012 World Value Survey (WVS) administrated in China during a narrow three-month window in coincidence with promotion of two top provincial leaders, we find that citizens in the promotion provinces (which happened to have mediocre governance) were more likely to reduce their trust in the central government after observing the promotion than those in non-promotion provinces did.

As described below, the limited data may render the evidence more suggestive than conclusive. However, the evidence lends reasonably strong support to our theory. First, the empirical pattern, novel to studies on bureaucratic control and public opinions in China, fits well with our theory. Second, the empirical finding remains robust after considering two influential theses regarding determinants of promotion of Chinese local officials, namely competence and networks. Third, we do not find similar effects of promotion on trust in other political institutions as we have found on respondents' trust in the central government. The last two increase our confidence that the empirical pattern is unlikely driven completely by channels other than what our theory has suggested.

2.4.1 Data and Measures

Linking model setups to the empirics, the promoter is the Chinese central government and the subordinates are top provincial leaders, either party secretaries or governors. To test the predictions, we searched for feasible data among often used and publicly available social surveys[19]. Our target survey should meet two criteria: (1) it has appropriate measures of the central government's reputation, and (2) there are promotions of provincial leaders during the survey period.

The WVS 2012 data in China met both requirements. The survey interviewed 2274 individuals in China between November 16, 2012, and January 21, 2013[20]. During the survey period, promotions of provincial leaders occurred in Shanxi and Guangxi on December 18, 2012, but not in any other provinces[21]. More specifically, the Governor of Shanxi, Wang Jun, was promoted to the Party Secretary of Inner Mongolia, while the Party Secretary of Guangxi, Guo Shengkun, was promoted to the Party Secretary of the Ministry of Public Security (and was appointed as the Minister of Public Security on December 28, 2012)[22]. 62

[19]We searched among recent waves of the Asian Barometer Survey (ABS), the Chinese General Social Survey (CGSS) and the World Value Survey (WVS).

[20]In the original survey, 26 were interviewed before or on November 15, 2012. There were turnovers of provincial leaders in several provinces on November 15, immediately after the CPC's 18th National Congress. But only two out of the 26 respondents were from provinces with turnovers (on November 15), so the sample is not big enough to employ a DiD estimation. Thus, we focused on the estimating the effect of promotions on December 18 and dropped the 26 observations.

[21]A couple of provinces, such as Inner Mongolia, Jilin, Fujian and Sichuan, had promotions of provincial leaders on November 15, 2012 but no turnovers between November 16, 2012 and January 21, 2013, thus coded as provinces with no promotion. Their difference from other provinces experiencing no turnovers on either November 15 or December 18 will be captured by province fixed effects and will not affect our estimation.

[22]The promotion of Guo is less clear to the public as that of Wang because the former was not recognized until December 24, 2012 when he participated in a meeting under his new title. We respond to this problem in two ways: first, as all interviews conducted in Guangxi Province were in January 2013 (Appendix Figure A), re-coding the treatment date for Guangxi Province to December 24, 2012 does not change the result; second, as shown below, we report an additional set of results with Guangxi excluded from the sample, and the result does not change.

interviews were conducted on or before December 18, 2012, and around 211 after the date in these two provinces with promotions; there were 1244 and 757 interviews conducted before and after that date, respectively, in the other provinces. (Figure 2.7.1 in the Appendix shows the timing of the surveys by province.) It is also worth noting that the regional GDP growth rates of Shanxi and Guangxi ranked as 20th and 18th out of the 31 provincial units in 2012, which fits our definition of mediocre performance[23].

As the promotion announcement date was not known to the public *a priori*, the fact that there were interviews conducted both before and after such date provides an opportunity to employ a difference-in-differences (DiD) strategy to estimate the reputation dampening effect of promotions.

2.4.2 Empirical Model

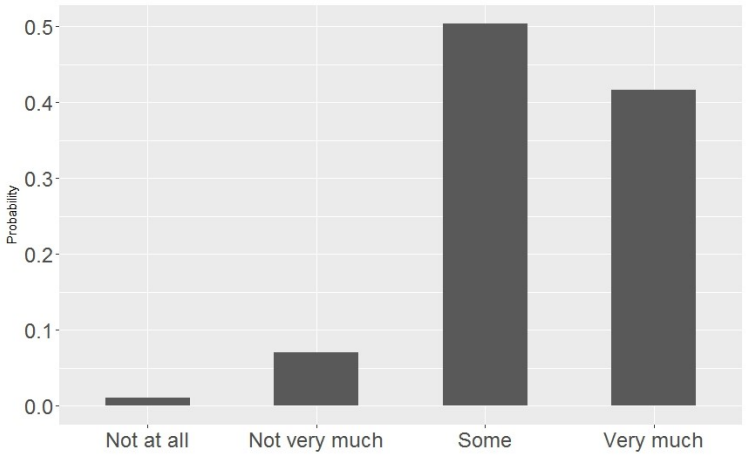
We employ a DiD with province and week fixed effects:

$$y_{i,p,t} = \gamma_p + \lambda_t + \beta \left(Promotion_p \times Post_t \right) + X_{i,p,t}\delta + \epsilon_{i,p,t}$$

The dependent variable ($y_{i,p,t}$) is the individual i's trust in the central government from province p and interviewed in week t. The values range from 1 ("Not at all") to 4 ("Very much"). Figure 2.2 shows the distribution of the dependent variable. As discussed in previous observational and experimental studies, although political fear in China might affect respondents' answers to sensitive questions, like those concerning political trust, its magnitude is found too small to undermine the "validity" of the trust measurement (Shi 2001, p. 407; Tang 2016, p. 150).

[23]It is widely accepted that GDP growth is one of the most important indicators of performance of Chinese local officials in the reform era (e.g., Li and Zhou 2005). The growth data is from the National Bureau of Statistics of China, http://data.stats.gov.cn/easyquery.htm?cn=E0103 (May 2017).

Figure 2.2: Trust in the Central Government, WVS 2012



γ and λ denote the province and week fixed effects, respectively, which capture time-invariant province effects and common time trends[24]. *Promotion* takes the value of 1 if an individual is from a province where promotion happened during the time of the survey and is 0 otherwise; *Post* takes the value of 1 if an individual takes the survey after the promotion date and is set to 0 otherwise. The Promotion and Post variables per se are dropped because of their collinearity with the week and province dummies.

β, the coefficient of interest, is the estimator for the DiD effect of the treatment on the treated. It captures the idea that promotion decision of the central government leads citizens to downgrade its reputation and is expected to be negative. By conducting such test, we assume that citizens in each province update their beliefs about the central government's type (i.e., θ^r) based only on promotion outcomes of their provincial officials, but not on similar information from other provinces[25]. One may concern that citizens may acquire additional information from neighboring provinces to update their beliefs about the central government.

[24]Excluding week fixed effect and including only the treatment time dummy yield qualitatively the same findings.

[25]This is consistent with our model setup that there is only a central government and a local one. Consequently, citizens only infer the central government's type from local performance.

However, the Chinese central government often has different policies for different provinces, causing its perceived type to vary across regions. Accordingly, the information from other provinces becomes less helpful for citizens to infer the central government's type in their home provinces.

$X_{i,p,t}$ is a set of individual-level controls, including age, gender, education, income, party membership, residence in an urban area, daily news consumption, and general social trust, which have generally been controlled for in previous studies of political trust in China (e.g., Li 2004, 2011). We also report estimation results after controlling for six variables measuring trust in domestic political institutions: trust in the civil service, trust in the army, trust in the police, trust in the courts, trust in political parties and trust in the legislature[26].

Additionally, we consider two influential narratives about the determinants of officials' promotion in China. One narrative argues that an official's promotion chances are mainly determined by his competence (Li and Zhou 2005) while another argues that the chance is largely determined by networks with superior officials (Shih, Adolph, and Liu 2012). We are less concerned with the former because if true, there is no obvious reason to expect trust in the central government to decline, as stated in Proposition 2.1. But the network narrative indeed poses a challenge: If the narrative is valid and citizens indeed update their beliefs about the central government's quality after observing promotions, intuitively, promoting well-connected officials will decrease the central government's reputation, which generates an observationally equivalent prediction. Specifically, immediately after Wang Jun's promotion, Li Xiaopeng, the son of the former Premier Li Peng, was appointed as the acting Governor of Shanxi. We are concerned that citizens may view the promotion of Wang Jun as making room for the promotion of Li Xiaopeng, and downgrade the central government's reputation because of the perceived role of connections in Li's promotion. To mitigate these concerns,

[26]We prefer the estimation results without controlling for those trust variables because they may be affected by the treatment; thus, including them may introduce post-treatment bias.

we control for the respondents' perceived importance of networks in determining success to ensure that our findings are not completely driven by the network narrative[27].

Finally, the usual parallel trend assumption for DiD estimation applies here. Specifically, we assume that treated provinces would have followed similar trends in central government trust to the control provinces in the absence of the promotion. This assumption is plausible in our case where the promotion announcement date is unexpected and the time window under consideration - around two months - is narrow. We provide additional evidence for this assumption in the robustness check section.

2.4.3 Results

Table 2.2 summarizes the major findings. Because treatment is assigned at the provincial level, standard errors are clustered by province to avoid potential correlation of errors within provinces[28]. We report three sets of results estimated using different samples: (1) the full sample, (2) a subsample after excluding one of the two treated provinces, Guangxi, and (3) a sub-sample keeping only the four weeks during which the treated provinces have observations[29]. The DiD estimates are found statistically significant across all the models. In our preferred estimation, model 2 of Table 2.2, promotion decreases individual's trust in the central government by 0.266 points on a one to four scale. Given that the mean trust in

[27]Interestingly, we find that the promotions significantly increase respondents' perception of the importance of networks, which supports the network narrative. But because the reputation-decreasing effect of promotion still exists after controlling for the perceived importance of network, our findings are not driven by the network narrative.

[28]Due to the small number of province clusters (i.e., 24 or fewer), we implement standard error clustering by using the partial function in ivreg2 of Stata (to partial out week and province dummies first).

[29]We run the regression with Guangxi excluded because all corresponding respondents were interviewed after the treatment date (see Appendix Figure 2.3), and we want to ensure that the finding is not completely driven by Guangxi observations. Similarly, by using the four-week subsample, we demonstrate that observations interviewed during weeks when no interviews are taken in treated provinces are not driving the main finding.

central government is around 3.325, among individuals with the mean level of political trust, promotions decrease trust in central government by around 8 percent.

Table 2.2: Diff-in-Diffs Estimation: Promotion and Promoter's Reputation

	Full sample			Excluding Guangxi			Four weeks only		
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Promotion × Post (β)	-0.267***	-0.266***	-0.267***	-0.268***	-0.265***	-0.268***	-0.322***	-0.431***	-0.256***
	(0.062)	(0.071)	(0.037)	(0.063)	(0.078)	(0.037)	(0.067)	(0.099)	(0.062)
Dependent variable mean	3.324	3.325	3.312	3.334	3.336	3.323	3.297	3.305	3.291
Individual-level controls		yes	yes		yes	yes		yes	yes
Trust in domestic inst.			yes			yes			yes
Week fixed effects	yes	yes	yes	yes	yes	yes	yes	yes	yes
Province fixed effects	yes	yes	yes	yes	yes	yes	yes	yes	yes
Observations	2,072	1,639	1,460	1,910	1,510	1,358	1,230	965	849
R-squared	0.002	0.034	0.630	0.002	0.037	0.644	0.003	0.060	0.622

Note: Dependent variable is trust in the central government; individual-level controls include age, gender, education, income, party membership, residence in an urban area, daily news consumption, general social trust, and perceived importance of networks; trust in domestic institutions include six variables that measure trust in the civil service, the army, the police, courts, political parties and the legislatures, respectively; robust standard errors in parentheses are clustered at the province level; R-squared is computed after the week dummies and province dummies are partialled out, thus undereportting the explanatory power; *** p<0.01, ** p<0.05, * p<0.1.

2.4.4 Robustness Checks

We conducted two robustness checks (results reported in the Appendix). First, following Bechtel and Hainmueller (2011), we probe the plausibility of the parallel trend assumption with a falsification test that estimates a DiD regression for the pre-treatment period. If the falsification DiD estimate is not significant, the parallel-trend assumption holds in the pre-treatment period; thus, it will be more plausible to assume that the treated and control units approximately follow parallel trends for the post-treatment period, especially given the short time window under investigation. Specifically, we propose a hypothetical treatment date exactly in the middle of the pre-treatment period and estimate the DiD regression

using the pre-treatment sub-sample. The insignificant falsification DiD estimate supports the parallel trends assumption.

Second, we show that the effect of promotion on trust in the central government does not apply to other political institutions or actors, such as the civil service, the army, the police, courts, political parties, the legislature, the APEC and the UN. The null effect on other political trust variables increases our confidence that the main finding is not driven by some unobserved mechanisms that change general political trust, where the trust in central government is one of the components, and that it is likely through our proposed mechanism.

2.5 Conclusion

We have shown that promotion can be used as an information management tool by popularity-seeking rulers to manipulate reputation. We demonstrated the intuition with our formal model and our empirical results supported that sophisticated citizens understand rulers' strategies, and adjust their evaluations accordingly.

Our theory also predicts that strategic promotion may result in declining government responsiveness and may provide room for the ruler to implement undesirable social policies. Surprisingly, intra-governmental transparency (effective monitoring) within the hierarchy has a negative impact on the ruler's responsiveness. Due to space and data limits, we do not test these predictions in this paper and will leave them for future research.

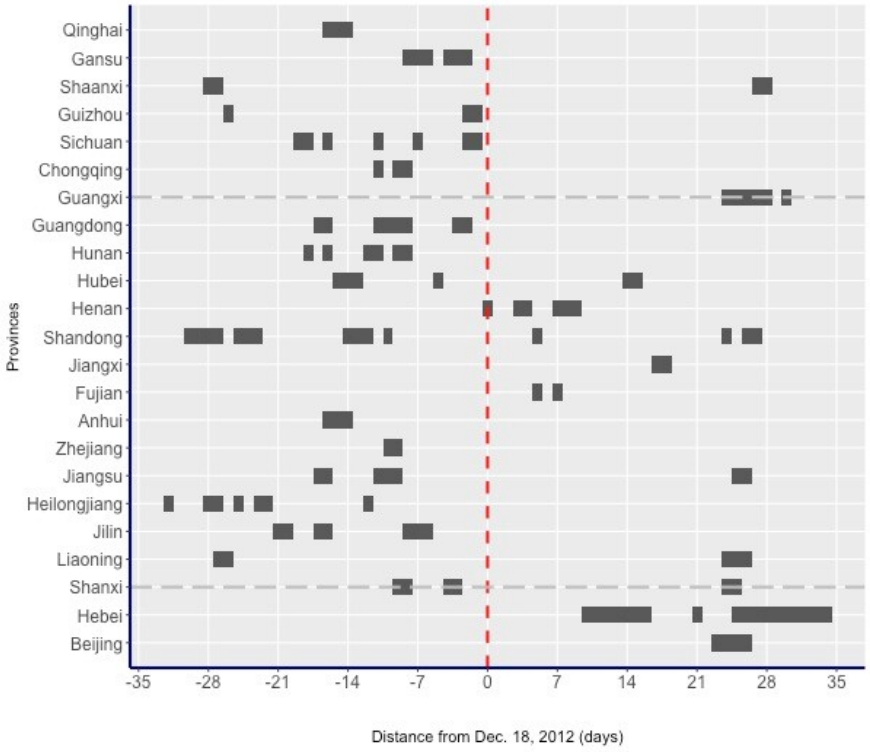
We note that our formalization focuses on promotion decisions as the only signaling tool that the ruler possesses. In the real life, there are, of course, other channels for the ruler to signal strength and to hide weakness, such as employing the strength-signaling propaganda (Huang 2015), strategically censoring the media (Lorentzen 2014), or flexing muscles in diplomatic disputes. In this paper, we highlight the existence of one of these channels, and point to the decrease of government responsiveness as a problem common to all of these channels.

Beyond the authoritarian context, our theory could be applied to all organizations that share such structure. For instance, the theory may advance our understanding of the relationship between an elected politician and his appointed bureaucrats in democracies and how the former shifts blame to the latter by delaying deserved promotions, or in a corporation where mid-tier officers delay promotions to claim credit in front of his boss.

2.6 Appendix

2.6.1 Survey Time

Figure 2.3: Survey Time in Chinese Provinces



This figure presents the dates (centered on December 18, 2012) when the World Value Survey (WVS) was administrated in each province between late 2012 and early 2013. Negative values indicate interviews conducted before December 18 while the positive ones indicate interviews after such date.

2.6.2 Proofs

Proof of Proposition 2.1.

Proof. Rearranging the terms to get:

$$\begin{aligned}\max_{p(h,l)} \widetilde{U}_r(h) &= \left(\alpha g(l) + \beta\left[q_P h + (1-q_P) l - q_{NP} h - (1-q_{NP}) l\right]\right) p(h,l) + constant \\ &= \left(\alpha g(l) + \beta (q_P - q_{NP})(h-l)\right) p(h,l) + constant\end{aligned}$$

and

$$\max_{p(l,h)} \widetilde{U}_r(l) = \left(\alpha g(h) + \beta (q_P - q_{NP})(h-l)\right) p(l,h) + constant$$

Now suppose $q_P \geq q_{NP}$. Then $p(l,h) = 1$, the weak always promotes. Then to maintain $q_P \geq q_{NP}$ in equilibrium, it has to be that $p(l,h) = p(h,l) = 1$. However, it is easy to see that the strong ruler has profitable deviation to $p(h,l) = 0$ because by our assumption of off-equilibrium path belief, now $q_{NP} = 1$. Contradiction.

Therefore $q_P < q_{NP}$. Then from the strong ruler's problem, $p(h,l) = 0$. For the weak ruler, there are three cases:

1. $\alpha g(h) + \beta (q_P - q_{NP})(h-l) > 0$. Then $p(l,h) = 1$. In equilibrium the citizen has the correct conjecture, therefore $q_P = 0$, $q_{NP} = 1$. Such equilibrium exists if and only if $\frac{\alpha g(h)}{\beta(h-l)} \geq 1$.

2. $\alpha g(h) + \beta (q_P - q_{NP})(h-l) = 0$. In equilibrium, $q_P = \frac{\widehat{p}(h,l)}{\widehat{p}(h,l)+\widehat{p}(l,h)} = 0$, $q_{NP} = \frac{1-\widehat{p}(h,l)}{2-\widehat{p}(h,l)-\widehat{p}(l,h)} = \frac{1}{2-p(l,h)}$. To plug black, $p(l,h) = 2 - \frac{\beta(h-l)}{\alpha g(h)}$. This equilibrium exists if and only if $\frac{1}{2} < \frac{\alpha g(h)}{\beta(h-l)} < 1$

3. $\alpha g(h) + \beta (q_P - q_{NP})(h-l) < 0$. In this case we have a pooling equilibrium where $p(l,h) = p(h,l) = 0$. Consequently and based on our off-equilibrium path assumption,

$q_P = 0$, $q_{NP} = \frac{1}{2}$. Such equilibrium exists if and only if $\frac{\alpha g(h)}{\beta(h-l)} \leq \frac{1}{2}$

Lastly, $q_P < q_{NP}$ means that the reputation of the ruler always decreases after promotion.

□

Proof of Proposition 2.2.

Proof. To achieve the result we require the following.

1. When $\tau^r = l, \tau^b = l$, no choice to make.

2. When $\tau^r = h, \tau^b = l$, $(1-\alpha-\beta)(h+l) + \beta l - c \geq 2(1-\alpha-\beta)l + \beta l$

3. When $\tau^r = l, \tau^b = h$, $(1-\gamma)(h+l) + \gamma w - c \geq 2(1-\gamma)l$

4. When $\tau^r = h, \tau^b = h$, since ruler moves first. We use backward induction. Given $\tau^r = h$, we require bureaucrat to choose high effort, that is: $2(1-\gamma)h + \gamma w - c \geq (1-\gamma)(h+l)$. Then it is bureaucrat's dominant strategy to choose to work. Next we turn to the ruler. For the ruler to work, we require: $(1-\alpha-\beta)(h+h) + \alpha g(h) + \beta h - c \geq (1-\alpha-\beta)(l+h) + \alpha g(h) + \beta l$.

To combine the inequalities above, we have:

$c \leq \widetilde{c} = \min\{(1-\alpha-\beta)(h-l), (1-\gamma)(h-l) + \gamma w\}$. □

Proof of Proposition 2.3.

Proof. There are four cases to consider.

1. $\tau^r = l, \tau^b = l$

This is the simplest case. There is no effort choice for ruler or bureaucrat. Type is perfectly learned. No promotion is made. Denote the payoff function as $U_i(\tau^i, \theta^i)$, $i \in \{r, b\}$, the payoffs are:

$$U_r(l,l) = 2(1-\alpha-\beta)l + \beta l = (2-2\alpha-\beta)l$$

$$U_b(l,l) = 2(1-\gamma)l$$

2. $\tau^r = h, \tau^b = l$

Then the ruler needs to decide whether to exert effort or not. Again denote citizen's conjecture as q_P and q_{NP}.

We look for the case where the ruler now chooses $\theta^r = h$. If ruler does exert effort, he gets:

$$U_r(h,l) = (1-\alpha-\beta)(h+l) + \beta(q_{NP}h + (1-q_{NP})l) - c$$

We require:

$$(1-\alpha-\beta)(h+l) + \beta(q_{NP}h + (1-q_{NP})l) - c \geq (2-2\alpha-\beta)l$$

That is,

$$c \leq (1-\alpha-\beta+\beta q_{NP})(h-l)$$

3. $\tau^r = l, \tau^b = h$

Here we require the talented bureaucrat exert effort. That is,

$$U_b(h,h) = (1-\gamma)(h+l) + \gamma\widehat{p}(l,h)w - c \geq 2(1-\gamma)l$$

That is,

$$c \leq (1-\gamma)(h-l) + \gamma\widehat{p}(l,h)w$$

And in this case the ruler's payoff is:

$$(1-\alpha-\beta)(h+l) + \beta(q_{NP}h + (1-q_{NP})l)$$

because in equilibrium the weak ruler is indifferent between mimicking the strong ruler

or not.

4. $\tau^r = h, \tau^b = h$

Here we require the talented ruler shirks and talented bureaucrat exert effort. We divide the analysis into two parts.

First, we show the conditions under which $\theta^b = h$ after $\theta^r = l$. We require the same condition as in 2.2.3:

$$c \leq (1-\gamma)(h-l) + \gamma \widehat{p}(l,h) w$$

Next, we calculate ruler's payoff if $\theta^b = h$ and $\theta^r = h$. It is straightforward to see that if the ruler exerts efforts, the bureaucrat will exert efforts because $(1-\gamma)(h+h) + \gamma - c > (1-\gamma)(h-l) + \gamma \widehat{p}(l,h) w - c$. Then, we can compare ruler's payoff between working and shirking, and we require that ruler prefers to shirk:

$$U_r(l,h) = (1-\alpha-\beta)(h+l) + \beta(q_{NP}h + (1-q_{NP})l) \geq (1-\alpha-\beta)(h+h) + \beta h - c = U_r(h,h)$$

That is,

$$c \geq (1-\alpha-\beta q_{NP})(h-l)$$

So now let's recap: when the citizen sees a mediocre performance $X = M$, he knows there are the following possibilities: 1. With probability $\mu(1-\mu)$, a high talent ruler who works and a low talent bureaucrat. In this case we shall see no promotion; 2. With probability $\mu(1-\mu)$, a low talent ruler and a high talent bureaucrat who works. In this case ruler promotes with probability $p(l,h)$; 3. With probability μ^2, a high talent ruler who shirks and a high talent bureaucrat who works. In this case ruler also promotes with probability $p(l,h)$.

We want to construct an equilibrium where all the following conditions hold at the same time.

$$c \leq (1 - \alpha - \beta + \beta q_{NP})(h - l)$$

$$c \leq (1 - \gamma)(h - l) + \gamma \widehat{p}(l, h) w$$

$$c \geq (1 - \alpha - \beta q_{NP})(h - l)$$

$$\widehat{p}(h, l) = \arg\max_{p(h,l)} \alpha p(h, l) g(l) + \beta (p(h, l)[q_P h + (1 - q_P) l] + [1 - p(h, l)][q_{NP} h + (1 - q_{NP}) l])$$

$$\widehat{p}(l, h) = \arg\max_{p(l,h)} \alpha p(l, h) g(h) + \beta (p(l, h)[q_P h + (1 - q_P) l] + [1 - p(l, h)][q_{NP} h + (1 - q_{NP}) l])$$

$$q_P := \frac{\widehat{p}(h, l) \mu (1 - \mu)}{\widehat{p}(h, l) \mu (1 - \mu) + \widehat{p}(l, h)[\mu (1 - \mu) + \mu^2]} = \frac{\widehat{p}(h, l)(1 - \mu)}{\widehat{p}(h, l)(1 - \mu) + \widehat{p}(l, h)}$$

$$q_{NP} := \frac{[1 - \widehat{p}(h, l)] \mu (1 - \mu)}{[1 - \widehat{p}(h, l)] \mu (1 - \mu) + [1 - \widehat{p}(l, h)][\mu (1 - \mu) + \mu^2]} = \frac{[1 - \widehat{p}(h, l)](1 - \mu)}{[1 - \widehat{p}(h, l)](1 - \mu) + [1 - \widehat{p}(l, h)]}$$

$$p(l, h) = \widehat{p}(l, h) \in (0, 1)$$

$$p(h,l) = \widehat{p}(h,l) = 0$$

Similar with the simple model, the solution of the equilibrium boils down to solving

$$\alpha g(h) + \beta (q_P - q_{NP})(h-l) = 0$$

That is,

$$p(l,h) = 2 - \mu - (1-\mu)\frac{\beta(h-l)}{\alpha g(h)}$$

In the meantime, from

$$c \leq (1 - \alpha - \beta + \beta q_{NP})(h-l)$$

$$c \leq (1-\gamma)(h-l) + \gamma \widehat{p}(l,h) w$$

we know that denote $\overline{c} = \min\{(1 - \alpha - \beta + \beta q_{NP})(h-l), (1-\gamma)(h-l) + \gamma \widehat{p}(l,h) w\}$, $\overline{c} \leq \widetilde{c} = \min\{(1-\alpha-\beta)(h-l), (1-\gamma)(h-l) + \gamma w\}$. □

Proof of Corollary 2.1.

Proof. We only need to look at the case where information is hidden and the ruler has high talents. There are two cases.

If the bureaucrat's talent is seen, then everything is the same as in the original equilibrium.

If the bureaucrat's talent is hidden, the condition for the ruler to shirk is:

$$\mu U_r(l,h) + (1-\mu) U_r(l,l) \geq \mu U_r(h,h) + (1-\mu) U_r(h,l)$$

Comparing with the shirking condition in original equilibrium:

$$U_r(l,h) \geq U_r(h,h)$$

Since $U_r(l,l) \leq U_r(h,l)$ (Case 2 in the proof of Proposition 2.3), the condition is harder to satisfy than in the original equilibrium. In particular, when $U_r(l,l) < U_r(h,l)$ and $\mu = 0$, the inequality is reversed: the ruler never shirks. By continuity there exists $\overline{\mu}$ such that the ruler never shirks whenever $\mu \leq \overline{\mu}$. □

2.6.3 Pseudo Promotions and Bureaucratic Redundancy

Here we consider an alternate set-up. In the simple model we assume $g(h) > 0 > g(l)$, which corresponds to promotions where the role at a higher office is more important, or fits only for strong individuals. But sometimes the higher office may not necessarily be more powerful, nor difficult to harness. We argue that it is possible for some promotions that $g(l) > 0 > g(h)$. we refer the promotions satisfying this condition "pseudo promotions", while we refer the promotions in the simple model as "real promotions". Below we explain where the name comes from.

Pseudo promotion is created for the purpose of allocating those promoted officers without offering them real power. The reason behind such design is for replacement of local bureaucrat: if the local office is vital and a weak bureaucrat is sitting on it, the ruler has incentive to replace him, even with reasonable costs. However, demotion is often unusual and too costly in such centralized regime. Consequently, the ruler can pseudo promote the bureaucrat to some idle position with fancy titles, thus leaving the local office an opportunity to select a new one.[30] It is straightforward that in this case only pseudo promoting weak bureaucrats achieves the purpose ($g(l) > g(h)$). Pseudo promoting strong bureaucrat only brings additional local uncertainty because the local successor may not be high type ($0 > g(h)$). Using the same solution techniques as in the simple model, we have the following corollary.

Corollary 2.2. *For pseudo promotions, in equilibrium, the strong ruler always promotes after mediocre local performance. And the reputation of the ruler always weakly increases after promotion.*

[30] A similar motive applies to the teacher selection in public schools of the United States. Principals of schools send out the bad teachers, i.e. the lemons, in the hope for better replacement, because sacking teachers are very costly. See *The Economist.* 2014. "Dance of the lemons", March 5th, http://www.economist.com/news/united-states/21599005-reformers-want-make-it-easier-sack-bad-teachers-dance-lemons (August 15 2017).

Proof. Rearranging the terms to get:

$$\max_{p(h,l)} \widetilde{U}_r(h) = \left(\alpha g(l) + \beta\left[q_P h + (1-q_P) l - q_{NP} h - (1-q_{NP}) l\right]\right) p(h,l) + constant$$

$$= \left(\alpha g(l) + \beta (q_P - q_{NP})(h-l)\right) p(h,l) + constant$$

and

$$\max_{p(l,h)} \widetilde{U}_r(l) = \left(\alpha g(h) + \beta (q_P - q_{NP})(h-l)\right) p(l,h) + constant$$

Now suppose $q_P \leq q_{NP}$. Then $p(l,h) = 0$, the weak never promotes. Then to maintain $q_P \leq q_{NP}$ in equilibrium, it has to be that $p(l,h) = p(h,l) = 0$. However, it is easy to see that the strong ruler has profitable deviation to $p(h,l) = 1$ because by our assumption of off-equilibrium path belief, now $q_P = 1$. Contradiction.

Therefore $q_P > q_{NP}$, which means that the reputation of the ruler always increases after pseudo promotion. □

The over-promotion under pseudo promotions sheds new light on explaining redundancy: it is a price to pay for the flexibility of personnel control. Given that demotions are hard, the superior officer sets aside idle positions at insignificant departments to accommodate the replaced local officials. Once the local officials are pseudo promoted, the superior officer is able to find potentially capable newcomers to fill in the position. Such strategic redundancy provision predicts that redundancy is not necessary an indicator of system failure. Instead, it could be a promising sign of effective replacement. For instance, we predict that redundancy could be positively correlated with local economic performance, especially when the local performance is vital to the regime. Our predictions is in stark contrast with conventional views that staff redundancy is an outcome of patronage (Ang 2016) or a tool used by local government to bargain over fiscal resources with upper-level government (Guo 2008).

The strategic roles of bureaucratic redundancy is understudied in existing literature, except for Ting (2003), who discusses redundancy as a strategic design for the principal to overcome conflict of interests with the agents. This paper adds to the literature by emphasizing redundancy as a byproduct of effective signaling in strategic promotions. That is, redundancy itself may be a good signal of competence.

2.7 References

Ang, Yuen Y. 2016. "Co-optation & Clientelism: Nested Distributive Politics in China's Single-Party Dictatorship." *Studies in Comparative International Development* 51:3 (September): 235-256.

Bechtel, Michael M. and Jens Hainmueller. 2011. "How Lasting Is Voter Gratitude? An Analysis of the Short-and Long-Term Electoral Returns to Beneficial Policy." *American Journal of Political Science* 55(4):852-868.

Birney, Mayling. 2014. "Decentralization and Veiled Corruption under China's 'Rule of Mandates'." *World Development* 53:55-67.

Bordignon, Massimo and Enrico Minelli. 2001. "Rules Transparency and Political Accountability." *Journal of Public Economics*, 80(1):73-98.

Cai, Yongshun. 2008. "Power Structure and Regime Resilience: Contentious Politics in China." *British Journal of Political Science* 38(3):411-432.

Cai, Yongshun. 2014. *State and Agents in China: Disciplining Government Officials*. Stanford University Press.

Chen, Jidong and Yiqing Xu. 2017. "Information Manipulation and Reform in Authoritarian Regimes." *Political Science Research and Methods* 5(1):163–178.

Chen, Yizi. 2013. *The Memoir of Chen Yizi*. Hong Kong: New Century Press.

Edmond, Chris. 2013. "Information Manipulation, Coordination, and Regime Change." *The Review of Economic Studies* 80(4):1422–1458.

Egorov, Georgy and Konstantin Sonin. 2011. "Dictators and Their Viziers: Endogenizing The Loyalty-Competence Trade-Off." *Journal of the European Economic Association* 9(5):903-930.

Ely, Jeffrey C. and Juuso Välimäki. 2003. "Bad Reputation." *The Quarterly Journal of Economics* 118:3 (August): 785-814.

Gandhi, Jennifer and Adam Przeworski. 2007. "Authoritarian Institutions and the Survival of Autocrats." *Comparative Political Studies* 40:11 (November): 1279-1301.

Geddes, Barbara. 2003. *Paradigms and Sand Castles: Research Design in Comparative Politics*. University of Michigan Press.

Geddes, Barbara, Joseph Wright, and Erica Frantz. 2014. "Autocratic Breakdown and Regime Transitions: A New Data Set." *Perspectives on Politics* 12(2):313-331.

Gehlbach, Scott and Alberto Simpser. 2015. "Electoral Manipulation as Bureaucratic Control." *American Journal of Political Science* 59(1):212-224.

Gilley, Bruce. 2008. "Legitimacy and Institutional Change: The Case of China." *Comparative Political Studies* 41(3):259-284.

Guo, Gang. 2008. "Vertical Imbalance and Local Fiscal Discipline in China." *Journal of East Asian Studies* 8:1 (April): 61-88.

Guriev, Sergei and Daniel Treisman. 2015. "*How Modern Dictators Survive: An Informational Theory of the New Authoritarianism.*" NBER Working Paper No. 21136.

Hollyer, James R., B. Peter Rosendorff, and James Raymond Vreeland. 2015. "Transparency, Protest, and Autocratic Instability." *American Political Science Review* 109(4):764-784.

Holmstrom, Bengt. 1999. "Managerial Incentive Problems: A Dynamic Perspective." *The Review of Economic Studie*s 66(1):169-182.

Huang, Haifeng. 2015. "Propaganda as Signaling." *Comparative Politics* 47(4):419-444.

Landry, Pierre F. 2008. *Decentralized Authoritarianism in China: The Communist Party's Control of Local Elites in the Post-Mao Era*. Cambridge University Press.

Li, Hongbin and Li-An Zhou. 2005. "Political Turnover and Economic Performance: The Incentive Role of Personnel Control in China." *Journal of Public Economics* 89(9/10):1743-1762.

Li, Cheng. 2006. "Think National, Blame Local: Central-Provincial Dynamics in the Hu Era." *China Leadership Monitor* 17(1)1-24.

Li, Lianjiang. 2004. "Political Trust in Rural China." *Modern China* 30(2):228-258.

Li, Lianjiang. 2011. "Distrust in Government Leaders, Demand for Leadership Change, and Preference for Popular Elections in Rural China." *Political Behavior* 33(2):291-311.

Li, Lianjiang. 2016. "Reassessing Trust in the Central Government: Evidence from Five National Surveys." *The China Quarterly* 225:100-121.

Little, Andrew T. 2015. "Fraud and Monitoring in Non-competitive Elections." *Political Science Research and Methods* 3(1):21-41.

Lorentzen, Peter. 2014. "China's Strategic Censorship." *American Journal of Political Science* 58(2):402-414.

Magaloni, Beatriz. 2008. "Credible Power-Sharing and the Longevity of Authoritarian Rule." *Comparative Political Studies* 41(4/5):715-741.

Mei, Ciqi, and Margaret M. Pearson. 2014. "Killing a Chicken to Scare the Monkeys? Deterrence Failure and Local Defiance in China." *The China Journal* 72:75-97.

Nielsen, Poul A., and Donald P. Moynihan. 2017. "How Do Politicians Attribute Bureaucratic Responsibility for Performance? Negativity Bias and Interest Group Advocacy." *Journal of Public Administration Research and Theory* 27(2):269-283.

Persson, Torsten, Gérard Roland, and Guido Tabellini. 1997. "Separation of Powers and Political Accountability." *The Quarterly Journal of Economics* 112(4):1163-1202.

Shambaugh, David. 2007. "China's Propaganda System: Institutions, Processes and Efficacy." *The China Journal* 57:25-58.

Shi, Tianjian. 2001. "Cultural Values and Political Trust: A Comparison of the People's Republic of China and Taiwan." *Comparative Politics* 33(4):401-419.

Shih, Victor, Christopher Adolph, and Mingxing Liu. 2012. "Getting Ahead in the Communist Party: Explaining the Advancement of Central Committee Members in China." *American Political Science Review* 106(1):166-187.

Snyder Jr., James M. and David Strömberg. 2010. "Press Coverage and Political Accountability." *Journal of Political Economy* 118(2):355-408.

Svolik, Milan W. 2012. *The Politics of Authoritarian Rule*. Cambridge University Press.

Tang, Wenfang. 2016. *Populist Authoritarianism: Chinese Political Culture and Regime Sustainability*. Oxford University Press.

Ting, Michael M. 2003. "A Strategic Theory of Bureaucratic Redundancy." *American Journal of Political Science* 47(2):274-292.

Waldman, M. 1984. "Job Assignments, Signaling, and Efficiency." *The RAND Journal of Economics* 15(2):255-267.

Xu, Chenggang. 2011. "The Fundamental Institutions of China's Reforms and Development." *Journal of Economic Literature* 49(4):1076-1151.

Chapter 3

The Economic Motives of Foot-binding

"*Foot-binding as a ladder of success for women thus mirrored the fate of the civil service exam, a similar vehicle for men...Foot-binding was useful for social climbing, not mountain climbing.*"

- Dorothy Ko, *Cinderella's Sisters: A Revisionist History of Footbinding*, *Chapter 6,* 2005

3.1 Introduction

Gender-biased social norms reflect the evolution of gender inequality. Carrying significant disutility for women, gender-biased social norms have existed widely across the globe and in different historical periods, with profound impacts on women's economic, social, physical, and psychological well-being. For instance, as a cruel procedure that violates a woman's rights to health and security, female genital mutilation (FGM) has a long history and still persists in Africa, the Middle East and Asia (WHO, 2012). In other historical contexts, we also observe corsets in Victorian Europe and the U.S. and foot-binding in historical China. These practices share striking similarities: all involve devastating body modifications, all are practiced at young ages for girls, all have significant marriage market implications, and all carry significant individual and social costs. Moreover, if we extend our horizon beyond cruel

body modifications, we observe other gender-biased norms that harm women's welfare, for instance high dowries in post-modernization India, which often result in dowry death, female infanticide, and related violence (e.g. Bloch and Rao 2002, Anderson 2003, Bhalotra et al. 2016). Given that some such norms were successful eliminated (corsets and foot-binding) while others persist (FGM and high dowries in India), one may wonder about the origins of gender-biased social norms, as well as the institutional and economic factors that determine their spread, persistence, and disappearance. This paper sheds light on these questions by examining foot-binding, a representative gender-biased social norm in historical China that had been successfully eradicated in the early 20^{th} century.

As a painful practice in historical China, foot-binding targeted girls whose feet were reshaped systematically during their early childhood. Originating from a female dancer in imperial palace during the Five Dynasties (907-960), foot-binding persisted for nearly a millennium in historical China. Initially appreciated as a major icon of feminine beauty, from the early 20^{th} century onwards, it increasingly was viewed as a brutal cultural practice reflecting the oppression of women by a masculine-dominated society. Considerable efforts were made by the government and social activists to eradicate the practice (e.g. Yang, 2012), and scholars from multiple disciplines have provided perspectives to explain its popularity. Many explanations focus on a specific aspect of foot-binding, but none can explain the timing of its emergence and decline, as well as regional and class differences in its prevalence, within a consistent framework.

This paper presents a unified economic theory of foot-binding that can explain variation in its practice over time, among classes, and across regions. Foot-binding is modeled as a premarital investment made by girls' parents for marriage market competition, where there is a key trade-off between higher expected marrying-up benefits with foot-binding and disutility from the pain of foot-binding and the labor opportunity cost in non-sedentary work (e.g., farmland work). A key feature of the theory is that it explicitly models how changes in the gender-specific social mobility system affect matching in the marriage market and thus the

marrying-up benefits of foot-binding for upper class and lower class women.

To explain the dynamics of foot-binding in response to changes in marriage market competition, we consider a gender-asymmetric shock to the social mobility system in historical China – the Civil Examination System (in Chinese, the *Keju*, 607-1905). The exam system triggered a transition from heredity aristocracy to meritocracy. By taking and passing the exams at different levels, talented males could climb the social ladder while those who failed the exams would move downwards. We model how the change in the exam system generated a more heterogeneous distribution in the quality of men than of women, and triggered intensive marriage market competition among women which increased their premarital investments. Foot-binding, which embodies both Confucianism moral codes and men's aesthetic appreciation of women, was adopted to distinguish themselves in the marriage market and served as a social ladder for them to climb up.

In addition to the marriage market value of foot-binding, we also consider the labor opportunity cost of foot-binding by examining different types of women's labor in diverging agricultural regimes. Given that foot-binding deforms women's feet, it sharply limits physical mobility thus precludes them from engaging in intensive non-sedentary activities (e.g. rice farming), while having much less of an effect on sedentary activities such as household handicraft production. Therefore, among lower class women who played an active income-earning role, foot-binding prevalence exhibited regional variations driven by different agricultural regimes. In particular, foot-binding of lower class women was highly prevalent in regions where women specialized in sedentary labor (e.g. handicrafts in Northern China), and less popular in regions requiring labor-intensive farmland work (e.g. rice cultivation in the Pearl River Delta).

To test the model predictions, we draw upon a wide range of quantitative and qualitative sources, including cross-sectional data from Republican archives on county-level foot-binding prevalence, as well as historical, anthropological and archaeological evidence. In particular, we use cross-sectional variation in the exam quota of entry-level degree holders as a proxy

for the proportion of men succeeding in the exams (Kun et al., 1899) to capture the marriage market benefits from foot-binding. Empirically, we find that more exam quotas predict a higher incidence of foot-binding at the county level, with a one standard deviation increase in exam quota leading to a 6 percentage point increase in the probability of foot-binding. As additional evidence, we use archaeological findings to show that foot-binding shoes of upper class women are more likely to have occurred in regions with more exam quotas. In addition, local folk ballads reveal a systematic association between foot-binding and marrying up benefits.

To empirically test the impact of labor opportunity cost on foot-binding, we combine the cross-sectional data on county-level foot-binding prevalence with the agricultural suitability index developed by GAEZ (Global Agro-Ecological Zones, FAO) to test whether labor-intensive crop cultivation can be protective for women in terms of foot-binding. We find that a greater suitability for rice (a major labor-intensive crop) relative to wheat predicts less foot-binding prevalence, and this finding is robust across multiple model specifications, sub-samples, and inclusion of a rich set of county-level socioeconomic, demographic and geographic variables. Specifically, a one standard deviation increase in relative rice suitability lowers the probability of foot-binding by 9 percentage points. As an additional check to rule out potential confounding factors, we use a qualitative example of spatial discontinuity in Jiangsu Province to support our causal interpretation of this relationship.

Lastly, we present three extensions of the model. The first adopts a three-layer hierarchy of exam results rather than a binary structure, reflecting high, medium and low rankings in the entry-level exam. Second, we take foot-binding as a continuous practice to explain variation in the size of bound feet. (i.e. large versus small). Finally, we introduce multiple tools for marriage market competition, allowing foot-binding and dowry payments to coexist. Results from our extensions are qualitatively similar to our baseline model, yielding richer implications for explaining differences in foot-binding practice.

Our study makes contributions to several different literatures. First, it increases under-

standing of the origins of gender inequality, by showing that a gender-asymmetric mobility system can generate gender-biased cultural customs that carry high disutility for women. Previous scholarly work has explained the origins of gender norms from agriculture activities (e.g. Boserup, 1970; Qian, 2008; Alesina, Giuliano and Nunn, 2013; Xue 2017), marriage market competition (e.g. Grossjean and Khattar, 2016), and education and technology (e.g. Goldin 1990, 2006; Goldin and Katz 2002). Our paper provides a case similar to the association between the caste system and high dowries in post-modernization India (Anderson, 2003), and therefore serves an additional case to explain the historical roots of gender norms (see Giuliano, 2017 for a summary in this field).

Second, our research contributes to the literature on the economics of marriage and premarital investments (e.g. Becker, 1973, 1974; Peters and Siow 2002; Iyigun and Walsh, 2007; Chiappori et al. 2009; Hoppe et al. 2009; Bhaskar and Hopkins 2016). We add to this literature by showing that non-monetary utility transfers (foot-binding) can be one type of premarital investment. The perspective is similar to that of Mariani (2010) who takes women's chastity as a special type of premarital investment. By zooming in foot-binding as a case study, we examine the impact of a specific shock in mobility system on the evolution of foot-binding, providing both theoretical and empirical analysis.

Third, our work contributes to the growing literature in cultural economics, including Carvalho's (2013) explanation of the New Veiling movement among Muslim women; Chesnokova and Vaithianathan (2010), Bellemare et. al. (2015) and Poyker (2016) on the motives and persistence of FGM; and studies by Rao (1993), Edlund (2000) and Anderson (2003, 2007) on dowries in India. In this regard, our paper facilitates understanding of how destructive gender-biased cultural practices can arise, considering similar cultural practices such as corsets in the Victorian period in the US and Europe[1]. While foot-binding and

[1]The Victorian period also was a peak period for gender-asymmetric social mobility (e.g. Galor and Moav, 2006; Lindert, 2000).

corsets have disappeared, understanding the evolution of foot-binding is helpful to policy makers seeking to eradicate harmful customs such as FGM and high dowries.

Last but not least, this paper contributes to the literature on foot-binding by providing a unified economic theory of the practice, and a quantitative assessment using a new historical data set. It builds upon previous studies of foot-binding have come from different disciplines such as history, anthropology (e.g. Blake, 1994; Gates, 2001; Bossen et al., 2011; Brown et al., 2012; Yang, 2012; Bossen and Gates, 2017), sociology (e.g. Mackie, 1996), and literature (e.g. Wang, 2002; Ko, 2005), all of which provide useful insights and evidence[2]. In the economic literature, the earliest analysis of foot-binding is from Veblen (1899), who considered foot-binding to be a parallel phenomenon to Victorian corsets in Europe and America, where in both cases the investment was a symbol of social status. Another discussion of foot-binding is by Cheung (1972), who argues that it reduces the cost of enforcing property rights over wives. However, both of these studies are limited in that neither systematically reviewed the stylized facts of foot-binding and cannot explain foot-binding's temporal and regional variations.

The rest of this paper is organized as follows. Section 3.2 provides a brief review of foot-binding history and documents key stylized facts, as well as a historical description of the Civil Examination System. Section 3.3 presents the model, and Section 3.4 discusses the data and empirical results. Three extensions of our theory are discussed in Section 3.5, and

[2]Previous explanations of foot-binding emphasized its aesthetic, erotic, social and cultural motives. Perhaps the most widely acknowledged interpretation for foot-binding is the aesthetic and erotic motive (e.g. Yao, 1936; Levy, 1960; Ko, 2005; Gates, 2008). A second explanation of foot-binding is the biological motive, that it was restrictive in order to guarantee women's purity and enhance women's loyalty to their husbands (Daly, 1978; Mackie, 1996). The prevalence of foot-binding also has been viewed as a consequence of the revival of Confucianism to control women (Chen, 1928), or as a way to consolidates the patriarchal kinship system (Greenhalgh, 1977). In addition, the ethnicity origin of foot-binding argues that the Han Chinese women wanted to distinguish themselves from invading northern barbarians with such dressing code (Ebrey, 1990). Recent anthropological work has revealed an association with women's household handicraft (Ebery, 1990; Mann, 1997; Gates, 2001; Bossen et al.,2011; Bossen and Gates, 2017). In particular, Bossen and Gates (2017) argue that foot-binding was adopted to boost the economic contribution of girls in household production, especially for handicrafts which call for sedentary labor.

Section 3.6 concludes.

3.2 Background

3.2.1 Foot-binding and The Stylized Facts

The custom of foot-binding targeted girls whose feet were reshaped systematically during their early childhood, usually from age 5 to 12. The process is often initiated and practiced by mothers or grandmothers and can last for years, during which the bones and muscles were gradually modified towards a certain shape. A pair of bound feet carries lifetime painfulness, and is often accompanied by other types of health issues (e.g. infections). There were even cases when the girl became handicapped and unable to walk without the assistance of others for the rest of her life. However, given the significant disutility and welfare loss induced by foot-binding, the appreciation of foot-binding by men and the society as a whole stayed mysterious[3].

In terms of the earliest evidence of foot-binding, the greatest consensus is that it emerged during the Five Dynasties (907-960) (e.g., Ko, 2005). *Yao Niang*, who was a dancer of the palace, wrapped her feet tightly to support them and enhance their daintiness. In its early phase, foot-binding was often practiced by dancing girls and prostitutes for the purpose of entertaining men. Starting in the Song dynasty (960-1279), foot-binding gradually gained popularity among upper class women[4]. Later, the practice diffused from the upper class to

[3]The obsession for foot-binding could be depicted from numerous articles, poems, and prose from the Six Dynasties (420-589) to the late Qing (1636-1911). Bound feet were often the smaller the better, and the highest class of bound feet was labeled as the "*Three-Inch Golden Lotus*". A throughout checklist for beautiful bound feet includes the following seven items: thin, tiny, sharp, arched, scented, soft and straight (Yao, 1936). Readers could also refer to Zhang (2015) for a summary of poems and prose on the appreciation over foot-binding.

[4]Archaeological evidence show that bureaucrats' wives and daughters have all practiced foot-binding (Ebrey,1990).

the lower class during the Ming and Qing dynasties. While the Qing government tried to prohibit foot-binding in 1636, 1638 and 1664, these prohibitions were largely ignored. In the late Qing, with a surging influence of foreign influence and domestic reforms, intellectuals and social activists raised awareness about the detrimental effects of foot-binding. In the 1930s, the Republican government officially prohibited this practice by implementing nation-wide anti-foot-binding campaigns. By the middle of the 20^{th} century, foot-binding finally had faded into the past.

To establish the stylized facts of foot-binding, we investigate a wide range of historical sources and present a detailed summary in Appendix 3.9.1. Briefly, four stylized facts merit emphasis: (1) Time variation: while scattered instances of foot-binding can be found in earlier historical periods, it did not gain popularity until the post-Song period; (2) Class variation: foot-binding was first adopted by the upper class and later by the lower class, and its prevalence within the upper class was greater that within the lower class; (3) Regional variation: among lower class women, the prevalence of foot-binding varies by region. As illustrated by Figure 3.1, foot-binding was most prevalent in Northern China, while it was much less prevalent in Southern China, especially in the Pearl River Delta[5]; (4) Size variation: the higher the social status, the more tightly bound the feet[6]. In addition, within the upper class, the size of feet grew smaller over time. In Song dynasty, the shape of bounded feet were thin and straight, which was referred to as *knife-slim feet*. However, in the Qing dynasty, the shape of bound feet became highly curved, which were often called the *Three-inch Lotus*[7].

[5]This pattern has been well established by scholars (e.g. Qian, 1969; Hu Pu-An; Ko, 2005; Davin, 1976; Turner, 1997). Another piece of evidence illustrating the intensity of foot-binding in Northern China was the *Bound Feet Beauty Contest* (in Chinese, *Sai Zu Hui*). This unique beauty contest was often organized in either early spring or Autumn (Yao, 1991), when women proud of their lotus feet came out to show off their feet in public. At the same time, crowds of men came to appreciate the lotus feet one by one to decide the winner (see Appendix Figure 3).

[6]The smallest bounded feet were often found in Beijing, Hebei, Shandong, Shanxi, and Shaanxi, and the largest ones were often found in Suzhou, Hangzhou, Jiaxing, and Songjiang (Qian, 1969).

[7]One may also observe variation in terms of ethnicity. While most Han Chinese women intensively

>>Figure 3.1. Regional Variation of Foot-binding Prevalence<<

3.2.2 Social Stratification and The Civil Examination

In this paper, we focus on a gender-biased institutional shock: the Civil/Imperial Examination System (the *Keju* exams)[8], which was a meritocratic elite recruitment system adopted by Chinese emperors triggering changes in social mobility. The system was established during the Sui (581-618) and the Tang (618-907) dynasties, consolidated and expanded during the Song (960-1276) and fully institutionalized during the Ming (1368-1644) and Qing (1644-1911). During the post-Song period, the most important central and local officials and bureaucrats were selected through this system.

Several features of the exam system are worth highlighting. First, only men were eligible to participate in the examination[9]. Second, almost men of all class were eligible to take the exams regardless of family background, social status or age[10]. Third, the number of officials selected and the criteria of the exams changed over time. In particular, during the Sui (581-618) and the Tang (618-907), only a very limited proportion of officials were selected by the exams, while after the Song (960-1279) there was a significant increase in recruitment

practiced foot-binding, the ethnically minority women living in Canton, Fujian and Yunnan do not bind their feet at all. However, following the Han Chinese, some Manchurian women also imitate the foot-binding practice. It was also recorded that some Hui people in Gansu province also did foot-binding. So overall, there's no uniform pattern of foot-binding adoption by ethnic minorities. For the consistency of analysis, in the following we will focus on the Han Chinese women and their foot-binding practice.

[8]For a general description of the examination system, see Magone (2015). On the *Keju* system and changing social mobility as a consequence, see Ho (1962), Hartwell (1982), Elman (2000, 2013), Kung and Jiang (2015) and recently Shiue(2017). For the impact of the abolition of *Keju*, see Bai and Jia (2016). For the cultural persistence of the exams, see Chen, Kung and Ma (2017).

[9]Except for the exams hosted by the Taiping regime during their rebellion (1851-1864) that permitted women to take the exams, which is geographically and temporally very limited.

[10]One exception is men with pariah status, defined as people who are exluded from the four major catagories of Chinese society: the *shi* (gentry scholars), the *nong* (peasant farmers), the *gong* (artisans and craftsmen), and the *shang* (merchants and traders). The proportion of pariah status is very low in the whole population.

size, and the procedures became much more meritocratic[11] (Chafee,1995). Forth, the exam system structure was hierarchical. Specifically, during the post-Song period, there were three levels of exams including: (i) the Licensing exam (*Tongshi*) at the county level, those who passed the exam were entitled as *Literati* (*Shengyuan* or *Xiucai*), and the number of degree holders at this level is subject to quotas assigned by the central government; (ii) the Qualifying exam (*Xiangshi*) at the provincial level, where those who passed were entitled as *Recommended man* (*Juren*); (iii) the Academy exam (*Huishi*), where those earned the title *Presented Scholar* (*Jinshi*)[12]. Finally, with respect to content, the exams mainly focused on canonical texts from the *Five Classics* (*Wujing*) and the *Four Books* (*Sishu*)[13]. With the above features, the exam system served as a recruitment system that was highly inclusive and meritocratic.

In terms of regional variation in elite recruitment, the exam was regulated by a quota system during the Ming and Qing dynasties. Just as the structure of the exams was hierarchical, the quota system was also hierarchical at the level of *Literati*, *Recommended Man* and *Presented Scholar*. The quota allocation at the entry level of the exam was determined by the following factors: population, tax obligations, and historical talent distribution (Chang, 1955, Shang, 2004, Liang and Zhang, 2013). While there were modifications to the quota allocation driven by sociopolitical shocks, the overall quota allocation pattern across regions

[11]The key improvements to the exam system made by the Song include: (1) the local examiners were relocated once a year, in order to prevent collusion between candidates and examiners; (2) during the examination period, examiners had no contact with the candidates (*Suo Yuan*,); (3) the names and birth place information of candidates on the exam papers were obscured, to create an anonymous grading process (*Hu Ming*); (4) the answers of candidates were transcribed, making it impossible to identify specific candidates by their handwriting (*Teng Lu*); (5) if candidates were the sons or close relatives of examiners, an alternate examiner would be assigned (*Bie-Tou Shi*).

[12]The final national level exam is the Palace exam (*Dianshi*), which is hosted in front of the emperor. This level of exam aims at ranking the Presented scholars (Jinshi) into three levels.

[13]However, there still existed variation in the exams in terms of the way these classics were tested and the criteria. During the Sui and the Tang, the gracefulness of the articles written by candidates was the key. After the Song, the major content for the exams had been focused on memorizing the classics of Confucianism. In the Qing dynasty, the exam content and format were highly standardized.

was stable during the Ming and Qing period.

In addition to the exam system as the major elite recruitment channel during the post-Song dynasties, there also existed alternative ways to become government officials or obtain exam degrees, including purchasing exam degrees by making donation to the government (especially in times of disasters or wars), or inheriting their senior relative's position. However, those cases were relatively rare, and very often they were assigned with less important positions in the bureaucratic system. Moreover, the career prospects of these informal degree holders were not as good as those with formal degrees through passing the examination, so they were with lower social status in general[14].

Social Stratification. The exam system generated a social hierarchy and deeply affected social mobility in historical China, serving as a major social ladder for men to climb up. Figure 3.2 provides an illustration of this social ladder based upon Chang (1955), mapping each level of exam degree holders to their corresponding positions in the social hierarchy. Under the exam system, degrees and achievements through the exam played a key role in determining the social status of men.

>>Figure 3.2. The Civil Exam System and the Social Ladder<<

The upper class consisted of degree holders and officials had the greatest prestige, influence, and wealth in the society. About two percent of the population belonged to this stratum (Chang, 1955). In general, government officials and higher level degree holders (i.e. *Presented Scholars* and *Recommended Men*) belonged to the upper gentry class, and the lower level degree holders (mainly consisted of *Literati*, sometimes *Tribute Students*, those who were of the Imperial Academy) were the lower gentry class. The gentry class played a dominant role in local administration and enjoyed many aspects of privileges. They were distinguished from commoners in style of dress and costumes, were protected against insults

[14]For an in-depth discussion comparing these two types of degree holders (those with formal and informal degrees), see Chang (1955).

from commoners and interference from officials, and were exempted from coercive labor service, and also excused from paying the poll tax (Hsu, 1970). The income differences of a commoner's family and a gentry family is also profound. In Appendix Table 3.8, we present the relative income of gentry versus commoners based on Chang (1962). The ratio of average annual income of gentry to the average annual income of a male labor varies from 1.5 to 36[15]. In particular, for three most popular occupations of gentry - gentry services, secretaries to officials, and teaching - the ratios range from 20 to 50, revealing large income disparities across classes. We do similar calculations within gentry class, and show that gentry at different rankings had a clear hierarchical structure as well.

Social Mobility. There has been rich studies of to what extent the exam system has affected social mobility[16]. Most studies reveal a profound association between the exam system and the social mobility, especially by comparing the pre-Song and post-Song dynasties. For post-Song dynasties, scholars used a wide variety of sources to illustrate within-dynasty variation of mobility introduced by the exam system[17]. To get a sense of how the exam system changed social mobility across dynasties, we examine data from the China Biographical Database Project (CBDB)[18] to calculate a surname fractionalization index among Chinese figures and celebrities (mostly officials and scholars) across dynasties. By using the sur-

[15]By examining the aggregate income profile of the 19th century elite, Chang (1962, Table 26) illustrated that the major income sources of the upper class came from officeholding in the imperial bureaucracy, landownership, mercantile activities, gentry services, teaching, services as secretaries to officials, and other gentry services (i.e. practice of traditional medicine and scholarship awardees). In terms of income composition, the upper gentry class enjoyed higher proportions of their income from landownership, officeholding in the imperial bureaucracy, mercantile activities, while the key income resource of lower gentry class were teaching, secretaries services and other gentry services.

[16]For reference in social history, see Ho (1962), Hartwell (1982), Hymes (1986) and Elman (1991, 2000). For recent studies in the field of economic history, see Kung and Jiang (2015) and Shiue (2017).

[17]See appendix 9.2 for further discussion.

[18]The CBDB is an online relational database with biographical information about approximately 360,000 individuals as of April 2015, primarily from the 7th through 19th centuries. It is a relational database of biographical information for China before the early twentieth century. This version: 2015-03-18.

names of these people as a measure of social mobility, we hypothesize that a concentrated distribution of surnames indicates restricted mobility, while a high level of fractionalization of surnames captures greater mobility. We focus on the vast majority of the population that are of the Han ethnicity, which are identified as those having single character surnames (as a proxy), and calculate a surname fractionalization index following Alesina et. al (2003). As illustrated by Figure 3.3, we see a jump in the fractionalization index during the Song[19]. For dynasties of the post-Song period, the surname fractionalization index stays relatively stable.

>>Figure 3.3. Men's Mobility Trend: Evidence from CBDB<<

3.3 The Theory

In this section, we develop a unified theory of foot-binding which models foot-binding as a premarital investment by the bride's family to compete for better marriage opportunities when the quality dispersion of grooms is larger than that of brides. We first present the setup of the model, and then consider how a gender-biased social mobility institution triggered the evolution of foot-binding in both the upper and lower classes.

Preferences: Foot-binding, Beauty and Women's Virtue. Men's preference for foot-binding was a complex combination of appreciation of women's beauty and feminine virtue. With respect to the aesthetic value of foot-binding, there exists a long-standing admiration of women's small feet and elegant gait, traceable in Chinese poems, prose and other forms of literature. Walking slowly and daintily, even timidly, was appreciated from an

[19]Here's an illustrative example: during the Tang, there were 369 chancellors in total, distributed among 98 families; while the Song's 134 chancellors were distributed among 126 families. Among 1935 scholar-officials, those from non-bureaucratic families constituted around 55.12%. The cases with multiple generations all being chancellors during Tang were very common, i.e. "Many families with more than ten chancellors", but very rare during the Song (only three cases).

aesthetic perspective, while walking fast and boldly was considered vulgar[20]. By reshaping the feet with huge pain and physical restriction, foot-binding helps women approach this aesthetic goal. The specific obsession over foot-binding and tiny feet was depicted from the Six Dynasties (420-589) to the late Qing (1636-1911)[21].

In addition to aesthetic value, foot-binding was also considered as carrying a "*vector of status*", as Mann (1997) puts it. It was a symbol of elegance and good breeding (e.g. Mann, 1997, Ko, 2005), and a mark of status and virtue (Bossen and Gates, 2017). Since foot-binding is a painful process for women to undertake, well-shaped and tiny bound feet also help to reveal their endurance, obedience and submissiveness, which conform with the moral codes of women suggested by Confucianism[22]. Taken together, as a package of beauty and women's feminine virtue, foot-binding captured the key elements of men's moral and aesthetic appreciation of women.

Setup. We look at a society with two classes: the upper class and the lower class, denoted by $\tau \in \{u,\ l\}$. The upper class occupies share μ of the population, with the remainder in the lower class $(1-\mu)$. As the upper class occupies a small proportion of the population, we assume $\mu < \frac{1}{2}$. We normalize both the potential bride's and groom's population to 1, and assume the sex ratio to be 1 and identical across classes[23]. We index individual women by i

[20]In addition to its aesthetic values, foot-binding is also considered to enhance women's erotic value due to men's foot fetishism (Howard Levy, 1966).

[21]Bound feet were often the smaller the better, and the highest class of bound feet was labeled as the "*Three-Inch Golden Lotus*". A throughout checklist for beautiful bound feet includes the following seven items: thin, tiny, sharp, arched, scented, soft and straight (Yao, 1936). Readers could also refer to Zhang(2015) for a summary of poems and prose on the appreciation over foot-binding.

[22]However, Neo-Confucianism itself did not propose foot-binding, that none of its classics had advocated foot-binding. Cheng, Yi (1033-1107), one of the leading philosophers of Neo-Confucianism in the Song, was against foot-binding practice and his female family members till the Yuan dynasty(1271-1368) did not conduct foot-binding.

[23]Though the *de facto* sex ratio of male to female could be greater than one (especially considering the existence of historical female infanticide), assumptions about sex ratios do not drive our theoretical results. Our main results go through as long as there is greater quality heterogeneity of males than that of females, after the exam system is established.

and individual men by j, respectively.

Marriage. Traditional Chinese marriages are arranged by brides' and grooms' families, and are facilitated by matchmakers[24]. Thus, we take the decision makers to be the bride and groom's parents. We also assume that matching between brides and grooms in the marriage market is one-to-one[25]. Next, we define the quality attributes of brides and grooms, respectively. In particular, we posit two quality attributes – family socioeconomic status θ and ability γ, where $\theta, \gamma \in \{h, l\}$, and $h > l > 0$. We assume that the ability distribution is independent of class and gender, and the proportion of individuals with high ability h revealed by the exams is p. We further assume that $p < \mu$, reflecting the fact that the examination is highly selective and elite recruitment through the examination system is not perfect[26].

When parents seek marriages for their daughters, the quality of a man q_j is the weighted average of his family status θ_j and ability γ_j, where the relative weight for ability of the groom is α^G, so that $q_j = \alpha^G \gamma_j + (1 - \alpha^G)\theta_j$. The quality of a women q_i, on the other hand, is determined by her family status, ability as well as her labor value affected by foot-binding, thus $q_i = \alpha^B \gamma_j + (1 - \alpha^B)\theta_j + \beta_{\tau_j}(\mathbb{I}_{FB})l$, where α^B is the relative weight on ability

[24]Matchmakers have a long history in historical China (Chen, 1936), the earliest evidence can be dated back to the Zhou dynasty (1046-256 BCE). Matchmakers played a key role in the marriage market matching regardless of social class and geographic regions, and they are often local senior women with good connections with families and established reputation. There are two types of matchmakers: (1) official matchmakers, who help facilitating matching in the marriage market, taking records of local marriage market matching, and dealing with law suits regarding women and marriage; (2) private matchmakers, with a focus on marriage market matching only and helping to proceed marital procedures.

[25]This is motivated by China's historical marriage patterns, which are monogamous with multiple concubines. This paper focuses on the market for wives, not that of concubines. The marriage markets for wives and concubines are separated and very different. To marry a wife, the groom's family needs to go through formal marital procedures (Three Letters and Six Etiquettes, *San Shu Liu Li*). A wife has well established economic and legal status and cannot be easily divorced. For concubines, the marital procedures are often informal, and they are often bought from brothel and other places. Concubines have low economic and legal status compared with wives and can be easily dismissed or traded.

[26]This assumption of talent composition does not affect our baseline analysis, yet only changes the marriage market competition pattern when there's a high degree of meritocracy (when $\alpha > \frac{1}{2}$), which case is presented in Appendix 9.2.

for brides, l is women's labor value, and $\beta_{\tau_j}(\mathbb{I}_{FB})$ captures the return to women's labor as a function of foot-binding as well as the husband's social class τ_j. In particular, we assume that $\beta_u = 0$, or that women who marry upper class men do not play a laborious role in household production. However, for lower class families, since foot-binding impedes labor, we assume that $\beta_w|_{\mathbb{I}_{FB}=0} = \beta_0 > \beta_w|_{\mathbb{I}_{FB}=1} = \beta_1 > 0$, or that women with natural feet have greater labor value than those with bound feet. In the equation for women's quality, we also assume women's meritocracy parameter $\alpha^B = 0$, since the return to women's intellectual ability is almost zero for most of the historical period we consider (women cannot be officials or be involved in commerce). Combining the two sides, for bride i marrying groom j, the matching value of their marriage is $v(q_i, q_j)$. Since α^B is zero in the following analysis, we suppress the notation of α^G to be α, so the matching value function can be simplified to be $v(q_i, q_j) = v\left(\theta_i + \beta_{\tau_j}(\mathbb{I}_{FB})\, l,\ \alpha\gamma_j + (1-\alpha)\theta_j\right)$.

Taking the benefits and costs of foot-binding for both grooms and brides into account, the total marriage benefit for each side in the market is the following:

$$\text{Bride: } V_B(q_i, q_j, \mathbb{I}_{FB,i}) = v\left(\theta_i + \beta_{\tau_j}(\mathbb{I}_{FB,i})\, l,\ \alpha\gamma_j + (1-\alpha)\theta_j\right) - \mathbb{I}_{FB,i}C; \text{ and}$$

$$\text{Groom: } V_G(q_i, q_j, \mathbb{I}_{FB,i}) = v\left(\theta_i + \beta_{\tau_j}(\mathbb{I}_{FB,i})\, l,\ \alpha\gamma_j + (1-\alpha)\theta_j\right) + \mathbb{I}_{FB,i}B.$$

Foot-binding creates disutility for brides (denoted by C) and affects their labor value; on the other hand, it serves as a lump-sum utility transfer to grooms (denoted by B).

Disutility of Foot-binding. The bride's family has the option to bind their daughter's feet for better marriage opportunities[27]. Foot-binding generates a uniform benefit $B > 0$ to the grooms, and induces disutility $C > 0$ to the bride. We assume $B \leq C$ to reflect the extremely painful and unnatural process of foot-binding, and it is socially costly. Often mothers and grandmothers bound the feet of their daughters/granddaughters, so the process

[27]There are, of course, other options to seek for marriage opportunities, such as dowry. Here we assume away dowry to highlight the role of foot-binding and its evolution over the years. An extension of how dowry fits in the model is provided in Section 5.

itself is not economically costly and no medical specialists are needed[28]. Thus, both rich and poor households could afford the cost of the "surgery" of foot-binding[29].

Equilibrium. In our model, an equilibrium specifies a matching of the whole population and foot-binding choices of every woman, where three conditions are satisfied: (1) feasibility: all candidates in the marriage pool are paired; (2) optimal foot-binding: no bride will be better off in expected marital payoffs by altering her foot-binding choice; and (3) no blocking pairs: no one in a pair has an incentive to find a better partner who also prefers the new pair. We only consider pure strategy equilibrium, because foot-binding is a childhood commitment and cannot be altered upon marriage. Formally, we have the following definition and assumptions:

Definition 3.1. An equilibrium in the game is a matching $M := \{(i,j)\}_{i\in I, j\in J}$ and a set of foot-binding choices $\{\mathbb{I}_{FB,i}\}_{i\in I}$ such that:

1. $\forall i \in I,\ \exists j \in J\ s.t.\ (i,j) \in M$; Symmetrically, $\forall j' \in J,\ \exists i' \in I\ s.t.\ (i',j') \in M$.
2. Given $\{\mathbb{I}_{FB,k}\}_{k\neq i}$, $\forall i \in I,\ \mathbb{E}_{j\in J}\left[V_B\left(q_i, q_j, \mathbb{I}_{FB,i}\right)\right] \geq \mathbb{E}_{j'\in J}\left[V_B\left(q_i, q_{j'}, 1-\mathbb{I}_{FB,i}\right)\right]$.
3. Given $\{\mathbb{I}_{FB,i}\}_{i\in I}$, there does not exist $(i,j),(i',j') \in M$, such that $V_B\left(q_{i'}, q_j, \mathbb{I}_{FB,i'}\right) \geq V_B\left(q_{i'}, q_{j'}, \mathbb{I}_{FB,i'}\right)$, $V_G\left(q_{i'}, q_j, \mathbb{I}_{FB,i'}\right) \geq V_G\left(q_i, q_j, \mathbb{I}_{FB,i}\right)$

To make the analysis more tractable, we make the following assumptions on the matching value function v and foot-binding disutility C:

Assumption 1. (Reservation utility). *Getting married always dominates staying*

[28]Essential tools for foot-binding includes long and sliced cloth, scissors, sewing kit and a couple of shoes with decreasing size.

[29]One may also take the girl's family labor into consideration in this framework. In particular, as girls could start work from around age 10 until getting married at age 16, their labor value is considerable. Incorporating this into our model, we consider that foot-binding not only induces disutility to girls, but also brings some utility flow by increasing girl's labor, denoted as T. Here T is the difference in the girl's labor value with and without foot-binding. Bossen and Gates (2017) claimed T to be *positive*, since handicraft work is very boring and it's hard to have children sitting all day working. In that case, the bride's marriage benefit function would be: $V_B\left(q_i, q_j, \mathbb{I}_{FB,i}\right) = v\left(\theta_i + \beta_{\tau_j}\left(\mathbb{I}_{FB,i}\right) l, (1-\alpha)\theta_j + \alpha\gamma_j\right) - \mathbb{I}_{FB,i}(C-T)$. Thus a positive T could offset the disutility of foot-binding to some extent, making the adoption of foot-binding in lower-class families more likely.

single. The marriage benefits to brides and grooms are both larger than their reservation utilities, $V_B > \underline{V_B}$ *and* $V_G > \underline{V_G}$*, where* $\underline{V_B} > 0$ *and* $\underline{V_G} > 0$.

Assumption 2. **(Complementarity).** *v is continuous, convex and differentiable. In particular,* $v_1, v_2, v_{11}, v_{22}, v_{12}, v_{21} > 0$. *That is, v is increasing in both arguments and v exhibits complementarity, so that for* $q_1 \neq q_2$, $v(q_1, q_1) + v(q_2, q_2) > v(q_1, q_2) + v(q_2, q_1)$. *Viewing men and women symmetrically, we also have* $v(q_1, q_2) = v(q_2, q_1)$.

Assumption 3. (Marrying up is beneficial). *For any* $\beta \in \{\beta_0, \beta_1\}$*, linear combinations of h and l are greater than women's quality,* $xh + (1-x)l > l + \beta l$, $\forall x \in (\underline{x}, 1]$, *where* $\underline{x} < \frac{1}{2}$.

That is, judging from the matching value, the gap between h and l is sufficiently large that a lower class woman's quality is always dominated by the gain of marrying into an upper class groom's family.

Timing. The timing of the game is as follows: (1) at $t = 1$, the bride's family decides whether to bind the feet of their daughter (age 5 to 12). (2) at $t = 2$, the groom's quality is *partially* realized and matching in the marriage market takes place. The reason for a partial quality realization is based upon the timing of actual marriages. Namely, the marriage age for men is roughly 16-25 (Guo, 2000), an age range during which a man may have just started his career in academics, business or the bureaucratic system, but not yet achieved a well-established position. For instance, taking the lowest level exam (the Licensing exam) and obtaining the degree usually happens between the ages of 16 and 25 (Chang, 1955). So while men's abilities can be revealed by their performance on the exam at an early age, there still exists uncertainly over men's ability at the time of marriage. Therefore, the quality revealing at time $t = 2$ may be partial. With these assumptions, in the next section we analyze the dynamics of foot-binding in different institutional environments.

3.3.1 Marriage Market with Inherited Status

We start with the baseline case of inherited status, where the relative importance of ability α equals zero and the only determinant of individual quality in the marriage market is family socioeconomic status for both men and women. This case corresponds to the Wei-Jin, Southern and Northern Dynasties (220-589), when commoners had little chance of becoming elite, the elite group was almost closed, and hereditary clans were well-established[30]. In this case, the marriage market competition is illustrated by Figure 3.4(a), where both men and women perfectly inherit their family status, without mobility in either class. The equilibrium in this case is characterized by the following proposition.

>>Figure 3.4. Matching Illustration<<

Proposition 3.1. *In a segregated society with inherited status, there exists an equilibrium where matching is positive assortative in family socioeconomic status and there is no foot-binding.*

Proof. See Appendix.

In this case, the quality of both brides and grooms within each class is homogeneous. The matching pattern is endogamy, with upper class brides matched with upper class grooms, and lower class brides matched with lower class grooms. This matching is stable since no one has an incentive to deviate or stay single. Moreover, there is no incentive for brides to make costly premarital investments, because the complementarity of the marriage output function guarantees that the benefit of a lower class bride to marry up is always less than the loss to an upper class groom of marrying down, which means the lower class brides cannot compensate the upper class groom enough to marry her. This reflects the marriage market outcomes

[30]During this era, the elite recruitment system was mainly the Nine Ranks-Rectifier System (Miyazaki,1977), in which men were ranked by local rectifiers (*Zhongzheng*, in Chinese) for promotion towards officials, mainly based on their family socioeconomic status and moral conduct. Rectifiers belong to one type of government officials, who are in most cases central officials working in local prefectures.

well during this period, wherein powerful clans enjoyed hereditary political privilege and were interconnected through marriages. Thus, marriages were highly assortative in terms of family socioeconomic status, marriage across elites and commoners was very rare, and there was no costly beauty investment (foot-binding) by the bridal families (Zhang, 2003).

3.3.2 Restricted Examination System

In this section, we focus on the Civil Examination System, which is a gender-biased shock in social mobility that significantly transformed the marriage market. Specifically, we model the introduction of the system as a shock to the quality distribution of grooms. In the pre-exam era, the groom's value was fully determined by his family's socioeconomic status, while during the post-exam era, a groom's quality became more heterogeneous as a result of an increased return to ability (that is, $\alpha > 0$). In the post-exam period, grooms could be divided into four quality categories, as illustrated in Figure 3.4(b): (i) *the champions*–the upper class grooms with high ability; (ii) *the fallen elites*–the upper class grooms with low ability; (iii) *the new elites*–the lower class grooms with high ability and (iv) *the new lower class*–the lower class grooms with low ability. Fitting into historical contexts, we consider two cases when the exam system has low meritocracy (i.e., the pre-Song period) and high meritocracy (i.e., the post-Song period).

As reviewed in Section 3.2.2, although the the exam system was a shock to men's mobility, its institutional features significantly changed over time, characterized by the pre-Song and post-Song period. During the pre-Song period, i.e. the Sui (581-618) and Tang (618-907), the size of recruitment via exams was very limited and the degree of meritocracy of the exams was low[31]. The upper class status remained primarily hereditary. We summarize the

[31]At the beginning of the Sui, the emperor decided to recruit three persons from each prefecture each year, judged by the quality of their written articles. By the end of the Sui, the exam system was reshaped to recruit talent based on formal examinations. During the Tang, the exams were better organized with the most popular category of exams being Presented Scholars. During this era, the exam system produced a total of 6617 Presented Scholars. However, less than 10% of officials were from the exam system. Moreover,

equilibrium for this scenario in Proposition 3.2.

Proposition 3.2. (Restricted Meritocracy). *When meritocracy plays a minor role, i.e. there exists a cutoff value* $\alpha_1 < \frac{1}{2}$*, no one practice foot-binding in either class when* $\alpha \leq \alpha_1$*. The matching equilibrium features positive assortative matching in family socioeconomic status.*

Proof. See Appendix.

Proposition 3.2 reveals the case when family status is a dominant factor in men's quality, and the benefit of marrying up is still too small for women of both classes compared to the disutility of foot-binding. The proof of this proposition consists of two parts, where the first part describes the nature of the competition and the second part solves for the cut-off value of α. First, due to complementarity between the groom's and the bride's quality, there will be no cross-class marriage, i.e. a lower class woman marrying a champion or a fallen elite, since given a fixed value of α, the lower class bride's marrying up benefit cannot compensate for the loss to the upper class men from marrying down. Therefore, the only competition is within-class. Further, since α determines the quality gap of heterogeneous types of men within class, a larger α results in higher marrying up gain within class. Thus fixing the value of α, complementarity also means that the marrying up gain for upper class women is always higher than that for lower class women, so that the former always have a greater incentive to compete for better marriages. Therefore, given the fixed cost of foot-binding C, there exists a cut-off value α_1, so that at this point upper class women are indifferent to do foot-binding, where α_1 balances the expected marrying up benefits and the disutility cost, formally $(1-p)[v(h,h) - v(h,(1-\alpha_1)h + \alpha_1 l)] = C$.

due to a highly unequal distribution of educational resources, many successful exam-takers were still from elite families.

3.3.3 Meritocratic Examination System

As described in Section 3.2, the Song dynasty (960-1127) introduced major changes to the exam system, with respect to the scope of recruitment, procedural fairness and content (Chafee, 1995)[32]. This meritocratic exam system was fully institutionalized during the Ming (1368-1644) and the Qing (1644-1911) dynasties. The reformed exam system exhibited a much greater degree of meritocracy, together with a revolution in printing technology that lowered book prices (Ebrey, 1990) and an expansion of private schools (e.g. Chen and Kung, 2016). All of these changes contributed to a significant increase in social mobility during the post-Song period, or a higher α in our model. Consequently, the quality of men became increasingly dispersed, which triggered competition among brides in the marriage market. Potential brides sought ways to differentiate themselves from one another, which led to the adoption of foot-binding.

The next step is to understand how large was the shock of the post-Song exam system to the marriage market. As the exam system is hierarchical, we start from the Licensing exams at the entry level, which is usually taken at a young age. According to Chang (1955), among the Literati (*Xiucai*), 63% were of marriageable ages, i.e. 16-25. So the *Literati* (degree holders at the county level) were the most relevant group of potential grooms[33]. Our question is: among all men of marriageable ages, how large is the proportion of *Literati* degree holders? Based on Chang (1955)'s numbers, we estimate this proportion by making assumptions about population structure. Take Hunan as an example. The county quota for *Literati* degrees in 1812 was 2225 per exam, with exams held twice every three years.

[32]In terms of recruitment scale, during the three centuries of the Song the system produced almost 100,000 *Jinshi*, around 15 times the number recruited during the Tang dynasty. In contrast, during the Tang, the number of selected *Jinshi* each year was around 20-30, and sometimes less than 10. With a high level of meritocracy in elite recruitment, more than 50% of officials during the Song were selected by the exam system.

[33]Some very gifted young men even became degree holders of higher level exams (e.g. the *Recommended men* or *Presented scholar*s) at a young age.

Given that its population in 1842 was 20 million, if we take a lower bound assumption of the sex ratio to be 110 men per 100 women and an upper bound to be 130, and assume that the proportion of males aged 16-25 account for 20% of the total male population, we estimate the number of *newly* produced *Literati* as a share of men of marriageable ages, ranging from 0.098% to 0.106%. While this may not seem very large at first glance, we should remember that this calculation is for the *flow* of marriageable *Literati*, not the *stock* of the *Literati* among the marriageable population. In addition, one could take the entrance exams multiple times and compete in the marriage market for multiple years. Therefore, being "talented" at young ages perceived by the bridal families is in fact more complex. In Table 3.9, we perform this calculation for all provinces in the Qing dynasty[34].

As an illustration of how men's performance in the exams were viewed by bridal families, we can find also evidence in local ballads (Zhang, 2015). For instance, In *Zhangde* county (Henan), a ballad goes: "*If you have lotus feet, you could marry a Literati* (*Xiucai*), *eating bread with meat; If you have large feet, you would marry a blind man, eating bran with chili*". In historical narratives, such phrases were often used to illustrate how mothers persuaded themselves and their daughters to bind their feet. Based upon the collection by Zhang (2015), we present Figure 3.9 as a summary of the regional distribution of foot-binding related ballads.

Following Proposition 3.2, we analyze optimal foot-binding under a meritocratic exam system. Proposition 3.3 formally characterizes the prevalence of foot-binding in the upper class and its diffusion to the lower class. However, since increasing α would naturally flip the ranking of men if $\alpha \geq \frac{1}{2}$, we first consider the foot-binding equilibrium without flipped ranking. To facilitate such an analysis without changing the key economic forces of our model, we introduce an additional assumption:

[34]In addition to the proportion of *Literati* degree holders among all males of marriageable ages, we also calculate the proportion of gentry (defined as men with degrees higher than the *Literati*) among male population. See Table A3 for more details.

Assumption 4. (Bounded Disutility). *Foot-binding disutility has an upper bound, defined by* $\overline{C} = (1-p)[v(l, \frac{h+l}{2})) - v(l+\beta_0 l, l)]$, *so that women from both classes will take up foot-binding when* $\alpha \leq \frac{1}{2}$.

Specifically, Assumption 4 bounds the disutility of foot-binding. The role of this assumption is to ensure both upper and lower class women will take up foot-binding when there is no flipped ranking. Without this assumption (e.g. when $C > \overline{C}$), we will see in Appendix 3.8.2 that lower class women will never take up foot-binding, which does not fit the historical facts.

Proposition 3.3. (Prevalence of Foot-binding). *Foot-binding is first adopted by upper class women and later by lower class women. In particular, there exists a cutoff* α_2, *where* $\alpha_1 < \alpha_2 \leq \frac{1}{2}$, *when lower class women starts foot-binding. The proportion of foot-binding women of upper class is* $r_u = min\{\frac{p}{C}[v(h,h) - v(h,(1-\alpha)h+\alpha l)],\ 1\}$ *when* $\alpha \in \left[\alpha_1, \frac{1}{2}\right]$, *and the proportion of foot-binding women of lower class is* $r_w = min\{\frac{p[v(l,(1-\alpha)l+\alpha h) - v(l+\beta_1 l, l)]}{C + v(l+\beta_0 l, l) - v(l+\beta_1 l, l)},\ 1\}$ *when* $\alpha \in [\alpha_2, \frac{1}{2}]$. *Further,* $0 < r_w \leq r_u \leq 1$.

Proof. See Appendix.

In Proposition 3.3, we analyze the case when foot-binding was first adopted by the upper class, or when α is greater than cut-off α_1. Since complementarity implies that the marrying up gain for lower class women is always less than for upper class women, lower class women take up foot-binding at a higher cut-off value α_2. In particular, α_2 is the value of α for which the expected marrying up benefits for lower class women equals the disutility of foot-binding, formally $(1-p)[v(l,(1-\alpha_2)l+\alpha_2 h) - v(l+\beta_0 l, l)] = C$. Intuitively, for any bride of a certain class, her expected payoff when foot-binding, namely marry-up benefits net of foot-binding cost, should equal her expected marriage market payoff without foot-binding. Since $\alpha_1 < \alpha_2$, the upper class women practice foot-binding earlier than the lower class.

For any α in $(\alpha_2, \frac{1}{2}]$, foot-binding was used by women of both classes, and the proportion of foot-binding in the upper class is higher than that of the lower class. Two elements

contribute to this result: (i) the complementarity of brides and grooms, so that lower class women always have fewer marrying-up benefits than upper class women given a fixed α; and (ii) there exists a labor opportunity cost of foot-binding for lower class women, which is not the case for upper class women. In particular, because foot-binding impedes heavy labor and only lower class women play a laborious role in household production, lower class women consider *both* the disutility and the opportunity cost of foot-binding. In equilibrium, the proportions of women foot-binding in the upper and lower classes are pinned down by the net expected benefit of marrying up by foot-binding equaling the disutility of foot-binding.

Historically, Proposition 3.3 captures the situation in the post-Song dynasties, when foot-binding first appeared among upper class women then spread to lower class women. The result in Proposition 3.3 characterizes an asymmetric equilibrium in a symmetric game with many players[35]. This equilibrium can be thought of as the approximate outcome of the play of a specific symmetric mixed-strategy equilibrium, when each woman chooses foot-binding with a probability close to the fraction of foot-binding women within her class in this asymmetric pure-strategy equilibrium. With large numbers, the ex-ante probability and ex-post frequency are approximately the same (Cabral 1988). Based upon the above proposition, we present comparative statics in the following corollary:

Corollary 3.1. *The foot-binding percentage of upper class women r_u and lower class women r_w is non-decreasing with the proportion of high-ability men p, and non-increasing with the disutility of foot-binding C, when $\alpha \in [\alpha_1, \frac{1}{2}]$ for upper class and $\alpha \in [\alpha_2, \frac{1}{2}]$ for lower class. For lower class women, the foot-binding proportion r_w is non-increasing with β_0 when $\alpha_2 \leq \alpha \leq \frac{1}{2}$.*

To test Corollary 3.1's prediction on p for upper class women, we describe archaeological findings of shoes for bound feet. As Ebrey (1990) documented, the evidence shows that

[35]In the sense that if we swap players, their payoffs are not affected.

bureaucrats' wives and daughters had all practiced foot-binding during the Song. In particular, three pairs of shoes for bound feet belonging to upper class women from the Song were found in *Fuzhou* (in Fujian), *Quzhou* (in Zhejiang) and *De'An* (in Jiangxi). Figure 3.11 in appendix illustrates the location of the three sites and the regional distribution of Northern Song degree holders. All three sets of "lotus shoes" were found in regions with a large number of degree holders.

For foot-binding among lower class women, the intuition of Corollary 1 is straightforward – foot-binding is more intense where non-sedentary labor is more valuable (high β_0)[36]. This result can help us to understand the regional variation in foot-binding practice among lower class women. In Section 3.4, we analyze this comparative statics in detail using historical data that captures regional variation in β and p.

Historically, the magnitude of α during the Post-Song period should be less than half for two reasons. First, at the timing of marriage market matching, potential grooms are mostly taking the entry level exams at the county level, which degree can only guarantee a lowest level of gentry class, especially compared to the higher degrees obtained at older ages (e.g. *Recommended Men* or *Presented Scholars*). Therefore, a degree of Literati can hardly dominant the importance of family background at this stage, so that most marriage market matching are still taking place within social class, though the competition within class getting intensified (Chen, 1936). Second, the *de facto* probability of being successful in higher rankings of the exams was different among the upper and lower class men. In a landmark study of this literature, Ho (1962) demonstrates the patterns of social mobility associated with the exam system in the Ming and the Qing, showing that the proportion of degree

[36]For lower class women, the comparative statics of foot-binding proportion r_w in terms of β_1 is less clear. With greater value of sedentary labor (greater β_1), women face lower opportunity cost of foot-binding, yet they also have lower incentive to marry up since now their own labor value is higher. So the direction of the effect of β_1 on r_w depends on the marrying up benefits compared with disutility term C, that when the marrying up benefits is greater than disutility, higher β_1 would lead to more foot-binding; yet when marrying up benefits is less than disutility, higher β_1 would lead to less foot-binding among lower class women.

holders from commoner's family background was below 50% for most of the time during the two dynasties[37]. Recent quantitative findings are largely in line with Ho's perspective (e.g. Kung and Jiang, 2015, Shiue, 2017), that the exam system increased social mobility but with noticeable heterogeneity by social status, that the higher status families had advantages over lower status families in succeeding in exams. For theoretical interest and to obtain a full picture of foot-binding dynamics with high α, we examine the case when $\alpha > \frac{1}{2}$ so that the ranking of groom's quality is flipped in Appendix 3.8.2.

3.3.4 The Decline of Foot-binding

Following the logic of the above model, we characterize the decline of foot-binding as being the consequence of two forces: (1) the decreasing benefit of foot-binding in the marriage market; and (2) the increasing opportunity cost of foot-binding in the labor market.

First, after lasting for more than a thousand years, the gender-biased exam system collapsed in 1905. After the Opium Wars in the mid-19^{th} century, Christian missionaries spread God's message in China. An important part of the missionary work was the establishment of girl's schools, mostly in coastal cities. During the Republican years, girls had greater opportunity to attend school. The increasing equality of educational opportunities promoted women's upward mobility, and women's quality dispersion began to catch up with that of men. In this case, the payoff of foot-binding as a costly beauty investment decreased. In our model, this process can be characterized as α^G and α^B becoming more equal over time.

Another economic force driving Chinese women out of foot-binding was the modern industrialization process. Starting in the late Qing, industrialization imposed transport infrastructure, and more integrated markets, which deeply altered the market structure for

[37]This finding applies to both Presented Scholars (*Jinshi*) and Recommended men (*Juren*). Appendix Figure 3.11 and 3.12 plot out the original findings of Ho (1962, Table 9 and 11). In addition to Ho (1962)'s perspective, other historians argue that Ho overestimated the degree of mobility to some degree.

textile production (Bossen and Gates, 2017)[38]. First, the opportunity cost of foot-binding increased, because women now had to leave their homes to work in distant factories, increasing the value of β_0 [39]. In addition, factory textile products gradually replaced handicrafts decreasing β_1. Overall, the social and economic changes led to a return to the initial situation, in that men's and women's quality distributions became more equal over time. Under these circumstances, foot-binding is no longer a desired tool to compete in the marriage market, as its benefits shrank while its cost increased significantly.

3.3.5 Summary

To summarize the above theoretical results, we present Figure 3.5 which shows how foot-binding prevalence changes with the degree of meritocracy (α) in the exams based upon Propositions 3.1 to 3.3, as well as Proposition 3.7 in Appendix 3.8.2.

>>Figure 3.5: Foot-binding Prevalence and Exam Meritocracy<<

When α is less than α_1, there's no incentive for foot-binding since the benefit fails to cover the fixed cost. When α reaches a cutoff (α_1), upper class women start foot-binding, and the proportion of foot-binding women among the upper-class jumps to a certain level. When α reaches a cutoff ($\alpha_2 \leq \frac{1}{2}$), lower class women start to take up foot-binding, and the proportion of foot-binding of upper class women is higher than that of lower class women. When $\alpha > \frac{1}{2}$ and there is flip in the ranking of new elites and fallen elites, women from

[38]Bossen and Gates (2017) conducted intensive interviews of 1800 rural elderly women in 27 villages (9 provinces), recording their foot-binding history (if any), personal life history, economic activities, and village characteristics. Combining these with data on local geographical, transportation, industrialization and economic development from a wide variety of sources, they provide evidence that the demise of foot-binding is closely related to the development of modern firms and the more integrated market that gradually replaced traditional household handicraft production.

[39]Hill Gates interviewed women in three counties in Fujian province, and found distinguishable patterns of foot-binding varying by time and locality. She writes, "*As machine-woven cloth began to flood the market in the 1930s, even married women sought other sources of income when these were available. Most alternatives required more mobility, demanded more outdoor labor, and allowed more comfort than the pre-industrial labor regime. By the early 1930s, very few Tong'an girls were being bound*".

both classes practice foot-binding, but with the two differences: (1) there is a discontinuity in the proportion of foot-binding upper class women, since now the average quality of men that upper class women are competing for decreases; and (2) there exists a kink in the proportion of foot-binding among lower class women, since now marrying the fallen elites is less attractive for lower class women than before.

One may also wonder what proportion of women practice foot-binding as a function of disutility C, for a fixed value of α. In Figure 3.6, we plot the proportion of foot-binding women in both upper and lower classes as a function of C, given $\alpha \in [\alpha_2,\ \frac{1}{2}]$ when both classes adopt foot-binding.

>>Figure 3.6: Proportion of Women with Foot-binding<<

Figure 3.6 illustrates several interesting points. First, when the disutility of foot-binding C is below the expected marrying up benefits net of opportunity cost for lower class women, i.e. $C \leq p\Delta V_l - \delta$, women of both classes practice foot-binding since the cost is low and is fully offset by the benefits. Here the marrying up benefits ΔV_l is $v\left(l, (1-\alpha)l + \alpha h\right) - v\left(l + \beta_0 l, l\right)$, and labor cost δ is $v(l + \beta_0 l, l) - v(l + \beta_1 l, l)$. Second, when C is in the window of $(p\Delta V_l - \delta,\ p\Delta V_u)$, where ΔV_u is $v(h, h) - v(h, (1-\alpha)h + \alpha l)$, fewer lower class women take up foot-binding as the disutility of foot-binding increases. Moreover, when C is larger than $p\Delta V_u$, fewer upper class women take up foot-binding as disutility increases. Finally, when C is too large (i.e. larger than ΔV_u), neither class will practice foot-binding. However, considering Assumption 4 that imposes an upper bound of C, we have a further restriction that $C \leq \bar{C} = (1-p)[v(l, \frac{h+l}{2}) - v(l+\beta_0 l, l)]$, so that C will sit in the interval of $[p\Delta V_l - \delta, \Delta V_l]$ when α is approaching $\frac{1}{2}$, thus the proportion of foot-binding women of upper class will be in the interval of $(p, 1]$.

3.4 Empirical Analysis

3.4.1 Data: Cross-sectional Variation

The historical facts presented above with regard to changes in social mobility and the marriage market provide us validation for the key theoretical predictions. To further examine our theory, in this section we analyze cross-sectional county data on foot-binding from the Republican archives to empirically assess the comparative statics presented in Corollary 3.1. Specifically, we focus on the regional determinants of foot-binding, using exogeneous variation in the benefits of marrying up (corresponding to p in our theory) and in the opportunity cost of labor (corresponding to β in our theory).

Foot-binding. Data on the prevalence of foot-binding at the county level come from surveys conducted by the Republican government in the early 1930s. In 1928, the Republican government officially prohibited foot-binding nationwide and later conducted a survey to investigate progress on its prohibition (Yang, 2003). This survey was conducted by the Ministry of the Interior, Republic of China, with records kept in the Second Historical Archives of China (SHAC) in Nanjing. Our data covers counties in four provinces, namely Shandong, Chahaer (now part of Hebei province), Hunan and Yunnan.

The main question of interest is the following: "*Before the implementation of the prohibition law, to what extent did women practice foot-binding?*" For instance, in *Gaoyuan* county of Shandong province, the answer is "*seven or eight women out of ten bound their feet*". In *Yongshun* county of Hunan province, the answer is "*women always work along with men, so there was no foot-binding custom*". We code the answers that clearly describe intensive foot-binding practice as "High Prevalence", taking a value of one, and code those that describe foot-binding as being rare or nonexistent as "Low Prevalence", taking a value of zero. With respect to cases with vague information on foot-binding intensity, we code those cases to be "ambiguous". Using the above coding system, we present the regional distribution of

foot-binding prevalence in the counties in the archives in Figure 3.7.

>>Figure 3.7. Foot-binding Regional Prevalence: Sample Counties<<

As shown in Figure 3.7, the distribution of foot-binding prevalence is very similar to that described in the styled facts supported by numerous qualitative sources described in Section 3.2 and Appendix 3.8.1, that Northern China had higher prevalence of foot-binding than Southern China.

Exam Quota. To capture the variation in the size of elite recruitment (p), and therefore the benefits of marrying up related to the exam system, we use one institutional feature of the exam system that created cross-sectional variation during the Ming and Qing: the quota allocation of the exam degree holders. Specifically, the quota system was an institution that regulated and balanced elite recruitment through the exams across regions, and allocation was hierarchical at the level of *Literati*, *Recommended Man* and *Presented Scholar*. In particular, by controlling the fundamental entry-level exam quotas, the imperial government could *de facto* control the size of the local gentry, specifically those with exam degrees above the *Literati* level (Chang, 1955). The variation in exam quota was relatively stable across the Ming and Qing[40], and the main rational for quota allocation at the county level included the following factors: population, tax obligations, and historical talent distribution[41] (Chang, 1955, Shang, 2004, Liang and Zhang, 2013). In our analysis, we use the exam quota at the county level during the late Qing (Kun et al., 1899) as a proxy for the benefits for women to marry up.

Women's Labor. Following our theory, the value of both sedentary and non-sedentary labor determines the opportunity cost of foot-binding for lower class women. Non-sedentary

[40]The Qing government adopted the baseline allocation scheme from the Ming government, yet for some regions, the quota allocation experienced changes in respond to conflicts (e.g. the Taiping Rebellion), exam performances or contribution/donation to the imperial government. But the regional distribution was overall stable during this period.

[41]Bai and Jia (2016) investigates the quota allocation at the prefecture level, using the number of small rivers (given river lengths) and exam performance before the quota system as instrument variables.

labor, namely agricultural work on the farmland, is much more sensitive to foot-binding than sedentary labor, which is mostly household handicrafts that are less affected by foot-binding. Using the regional variation of crops and handicraft sources, we capture the value of non-sedentary labor (β_0) and the value of sedentary labor (β_1) by comparing different agricultural regimes in China.

For cereal crops cultivation, there existed two agricultural regimes in China: rice versus wheat. In particular, in Southern China (Yangtze River and Pearl River Deltas), wetland rice was often the dominant cereal crop; while in Northern China (the Yellow River Basin), wheat, millet, maize and sorghum were the main cereals. As has been documented, cultivating rice requires highly intensive skilled labor (especially with double cropping, e.g. Bray 1994), where women's labor serves as key input and they are often considered to have a comparative advantage in the field because they are more careful and have more dexterous hands, especially valuable in rice cultivation (e.g.transplant rice seedlings). Therefore, in rice regions, women often worked alongside their husbands in the fields and paddies. To illustrate features of crops, we use John Buck's 1937 land utilization survey as a summary in Table 3.1.

>>Table 3.1: Number of Days of Man-labor Required per Crop Hectare for Major Crops<<

Table 3.1 presents number of days of man-labor required per crop hectare for major crops. The average number of labor days required for crops in the Wheat Regime was around 100, while the average number of labor days required in the rice region was almost double[42]. Empirically, we collect agricultural suitability and other county data from multiple sources. For the geographic distribution of crop suitability, we used the FAO's GAEZ (Global Agro-Ecological Zones), including rice, wheat, flax and cotton[43]. Mapping this fact to our model,

[42]It should be noted that in a double cropping system of rice, early and late rice should be counted as a whole.

[43]The suitability index of a certain crop is estimated based on a model, which has been applied considering

the β_0 of rice-growing areas is greater than that of non-rice-growing regions. Figure 3.14 illustrate the regional distribution of suitability index of rice and wheat. In our analysis, we use the relative agricultural suitability index of rice to wheat to proxy non-sedentary labor-intensive agricultural activity.

Next, we consider the value of sedentary labor – women's household handicraft work. The composition of household handicraft products had great variation across regions, depending on the endowment of resources. For instance, handicraft produced in Southwestern China included the spinning and weaving of cotton, hemp, silk, straw hats, mats and shoes, plaited bamboo baskets and pot covers, twisted fiber ropes, woven saddle pads and fish nets, dyed cloth, and so on (Bossen and Gates, 2017). But in general, the dominant handicraft source fibers were cotton and flax, where cotton outputs were in particular with higher economic value than flax outputs (e.g. Xue, 2017). In the empirical analysis, we use cotton and flax suitability as proxy for sedentary labor activity to test our prediction on the opportunity cost of foot-binding (varying β_1). Figure 3.15 illustrates the regional distribution of suitability index of cotton and flax.

Other Variables. In addition to the above variables, we also collect other variables as controls in our analysis. For county population and sex ratio in the early 1930s, we collect such information from Yearbook of Domestic Affairs of Republican China in 1931[44]. To capture the social influence on foot-binding in late-Qing China, we use the records of Christian missionaries in China during the period of 1918-1921 (Stauffer, 1922) as proxies, including the number of churches, vicars, and disciples per 10,000 people. In addition, considering that the newly established firms in the late 19^{th} century and early 20^{th} century could influence foot-binding practice, we use numbers of modern firms in the prefecture level as a proxy (Chang, 1987,1989). Also to reflect the impact of ethnic minority, we use the

the average climate of baseline period 1961-1990 reflecting suitability levels and distributions within grid cells.

[44]The population and sex ratio data for earlier periods on county level are not available.

intensity of the Mongol migration in the 13^{th} century, constructed based on Wu and Cao (1997). Finally, to control for the influence of Republican government on anti-foot-binding, we calculate the great circle distance from each county to Nanjing (which was capital of the Republican government).

3.4.2 Empirical Strategy

In this section, we first discuss the feature of our archival sample, and then present the empirical strategy to identify the impact of marriage market and labor incentives on foot-binding. As discussed in section 3.4.1, our data on foot-binding is constructed from archives conducted by the Republican government covering four provinces, which could induce two potential concerns: first, some counties are missing and not covered by the archive[45]; second, among the counties covered by the archive, some reported clear information on foot-binding, yet some with ambiguous information. For our analysis, sample selection would only be an issue if the selection were based on some unobserved characteristics that are correlated with determinants of foot-binding at county level. As a robustness check for the selection issue of the first type, we conduct a balance check for out-of-archive and in-archive counties by province in Table 3.2. As shown in Table 3.2, the two types of counties do not display significant differences in terms of relative difference suitability of rice to wheat, cotton and flax suitability, and distance to Nanjing for counties of Hunan province. For Shandong, Chahaer and Yunnan, there is no clear pattern in common for selection on the in-archive counties.

>>Table 3.2: Out-of-Archive and In-Archive Counties<<

For the second type of selection issue, we label counties with ambiguous information on

[45]This issue may stem from the wars and disturbances during the early 20^{th} century, that the archives of some counties are missing, or they didn't conduct the survey in the first place. But unfortunately, we cannot distinguish such difference based on the sources we have.

foot-binding as in the "ambiguous category", and conduct t-test for key variables. Table 3.3 shows that none of the variables show significant differences in mean across the two groups of counties.

>>Table 3.3. Ambiguous-Info. Counties and Clear-Info. Counties<<

In the next step, we use those counties with clear information on foot-binding as main analytical sample, and descriptive statistics of key variables on the province and county level are shown in Table 3.4.

>> Table 3.4: Descriptive Statistics<<

As shown in Table 3.4, counties with higher prevalence in foot-binding on average have higher suitability for cotton and flax, more entry-level exam quotas, and less relative suitability of rice to wheat. Given those statistics are informative, we formally analyze the determinants of foot-binding using the following regression specification:

$$Footbinding_c = \alpha + \beta Exam\ Quota_c + \gamma Suitability_c + \mu X_c + \epsilon_c$$

Here foot-binding prevalence in county c is a dummy variable, taking one for high prevalence, and zero for low or no prevalence. Our key explanatory variables are the difference in the suitability of rice compared to wheat, cotton and flax, as well as the size of the county exam quota, where all are standardized to compare the magnitudes of their effects. X_c is a vector of control variables, including the number of modern firms, international influence (proxy by Christianity activity) and geographic characteristics.

Next, we consider identifying the effects of exam quota and crop suitability on foot-binding. For crop suitability, these indexes are exogenous, as they are compound indicators that capture exogenous and predetermined geographical and ecological factors. For exam quota, the major identification challenge here is whether the factors determined quota allocation at county level are orthogonal to determinants of foot-binding. As discussed in Section

3.2.2, the assignment of quota across regions is not random but based on factors including population, tax obligations, historical talent distribution, and political stability[46]. Therefore, it is possible that the quota may be confounded with unobservables that capturing historical talent distribution or political stability, which may affect foot-binding at the same time. To understand the endogeneity issue on exam quota, we use the following strategies: (1) we progressively add county level socioeconomic variables as a richer set of control, including province and prefecture fixed effects in different specifications; (2) we test for selection on unobservables for quota allocation at the county level using Oster (2016)'s approach.

3.4.3 Empirical Results

Table 3.5 shows the baseline results using the above empirical specification, where robust standard errors are reported. As in Column 1, one standard deviation's increase in the relative suitability of rice to wheat leads to 9.4 percentage points lower probability in foot-binding prevalence, and the impact of cotton suitability as a proxy of local handicraft intensity has a slightly larger magnitude. For county exam quota, one standard deviation's increase in the quota leads to 6 percentage point's higher incidence in foot-binding prevalence. Columns 2, 3, 4, 5 progressively add variables capturing Christianity activities, socioeconomic variables (size of modern firms and historical Mongolion migration intensity), geographical distance to Nanjing and province fixed effects. Most are not statistically significantly different from zero. At the same time, the impacts of rice relative suitability, cotton suitability and county exam quota are highly stable. To sum up, this set of results supports the comparative statics of foot-binding prevalence with respect to both the proportion of high ability men and the opportunity cost of foot-binding in labor.

[46]For instance, the exam quotas are more generously given to the remote and border provinces, e.g. Yunnan and Guizhou, for the sake of maintaining social stability of these regions. Investigating the impact of abolition of the exam system on social uprisings at different locality, Bai and Jia (2016) uses the variation in prefecture level exam quotas and finds that a higher density of quota were associated with a higher probability of revolution participation after the abolition.

>> Table 3.5: Crop Suitability, Exam Quotas and Foot-binding Prevalence<<

Next, considering the assignment of county exam quota may be endogeneous to foot-binding by being confounded with unobserved socio-economic characteristics, we use prefecture fixed effect and also use Oster (2016)'s approach to evaluate the degree of selection on unobservables compare to observables to alleviate this concern. The results are presented in Table 3.6.

>> Table 3.6: Crop Suitability, Exam Quotas and Foot-binding: Prefecture Fixed Effects<<

In Column 1 of Table 3.6, we use the full sample and the only explanatory variables are county exam quota and county population, where a one standard deviation's increase in the quota increases foot-binding prevalence by 7 percentage points. In Columns 2, we add prefecture fixed effects, and find the magnitude of the exam quota's effect staying the same, while the R^2 increases a lot compared with Column 1. In Column 3, we further add the suitability variables, yet find no statistical significance of their impacts. This is not surprising, however, since the variation in suitability within prefecture is small given the relatively high homogeneity in terms of geographical conditions. In Column 4, we add other control variables, and find the coefficient of quota still very stable. In lower panel of Table 3.6, we practice Oster (2016)'s test for selection on unobservables. The assumption of the test is we have a proportional selection relationship of selection on observables and unobservables, so selection on observables is informative about selection on unobservables. Let the proportion of selection be δ and label our key coefficient (of the exam quota) be β, and R_{max} be the R^2 from a hypothetical regression that includes both observables and unobservables, the testing statistics are presented for Columns 2, 3 and 4. For Column 2, the δ for $\beta = 0$ and R_{max}=1 is 2.5, which passes the common heuristic value of 1. The bias-adjusted β, given $\delta = 1$ and $R_{max} = 1$ is 0.07, which is similar to our original estimates. For Column 3, we have similar magnitude and direction of selection on observables and unobservables; for Column 4, the testing results are very similar. Overall, the analysis in

Table 3.6 adds towards our analysis in Table 3.5 that county exam quotas play a positive role in determining foot-binding prevalence, and presents the degree to which selection in county exam quota as a concern is limited.

3.4.4 A Spatial Discontinuity Case

We next extend the empirical analysis by using a case of spatial discontinuity. As Figure 3.8 illustrates, Jiangsu Province lies at the border on the Northern and Southern China, along the Huai River. Thus, it is a province with both wheat and rice cultivation. The northern part of Jiangsu mainly cultivates wheat, while rice dominates its southern part. With respect to foot-binding practice, according to the provincial gazetteer of Jiangsu published in the early Republican years, except for urban areas, the Northern Huai River region had the highest prevalence of foot-binding, and the Yangtze River region had very little foot-binding. Women living north of the Huai River worked within the household, and women living south of the river worked in farming. The case of Jiangsu adds a piece of evidence to our understanding of female labor specialization and foot-binding prevalence.

>>Figure 3.8. A Map of Jiangsu Province<<

To sum up, our major finding in Section 3.4 is that greater relative suitability of rice to wheat was associated with lower prevalence of foot-binding. On the other hand, a higher suitability for cotton was correlated with a higher popularity of foot-binding. These findings are robust across model specifications and different sub-samples, as well as to the inclusion of a rich set of county-level socioeconomic and geographic variables.

3.5 Extensions

In this section we provide three extensions of our baseline model. First, we assume a finer hierarchy of *exam* results rather than a binary structure, where individual intellectual

capability can take three values, i.e. $\gamma \in \{High,\ Medium,\ Low\}$. Second, we allow the choice of foot-binding to be continuous rather than binary. Finally, we introduce multiple price tools for premarital investment: dowry and foot-binding. In particular, we examine the interaction of these two competition tools and how this can change the strategy of bridal families in equilibrium.

3.5.1 Hierarchical Structure of the Exam System

Previously, we assume a binary exam structure, i.e. $\gamma \in \{h, l\}$, corresponding to high and low ability captured by the exams. Here in the first extension, we replace the binary structure with multi-level achievement structure, i.e. $\gamma \in \{H,\ M,\ L\}$, representing three levels of ability that reflecting high, medium and low rankings in the Licensing exam. To capture a pyramid structure of talents, we assume the probability of getting a high score H is p (the same as before), that of getting a medium score M is q, and $0 < p < q < 1 - p - q$. Further, we focus on the case when $\alpha < \frac{1}{2}$ to fix the ranking of grooms and present the analysis comparable to Section 3.3.3. With the new assumptions, the quality distribution of grooms are described in Table 3.7, with labels A-F provided for each group of grooms:

>>Table 3.7: Groom's Quality Distribution with Hierarchical Ranking<<

Quality Ranking	l	$(1-\alpha)l + \alpha m$	$(1-\alpha)l + \alpha h$	$(1-\alpha)h + \alpha l$	$(1-\alpha)h + \alpha m$	h
Category	F	E	D	C	B	A
Male Population	$(1-\mu)(1-p-q)$	$(1-\mu)q$	$(1-\mu)p$	$\mu(1-p-q)$	μq	μp

Proposition 3.4. *With hierarchical exam rankings, the quality of grooms is further spread. Women of both classes will practice foot-binding, and it is more intensive among upper class. Moreover, the prevalence of foot-binding is higher than the benchmark case for women of both classes.*

Proof. See Appendix.

When $\alpha < \frac{1}{2}$, upper class women seek for marriages of grooms with ranking A and B within class, and lower class women seek for those with ranking D and E within class. Similar to Proposition 3.3, here the no-cross-class marriage is a result of the complementarity between brides and grooms. To pin down the proportions of foot-binding women in each class, we have reasoning process similar to that of Proposition 3.3. Intuitively, in equilibrium upper class brides bind their feet to the extent that the expected payoff of marrying-up net of foot-binding costs equals to non-foot-binding marriage outcomes. For lower class brides, the cost of foot-binding for lower class brides includes both disutility and the opportunity cost of labor, driving the proportion of foot-binding less than that from upper-class brides together with the complementarity. In the end, upper class women have higher prevalence in foot-binding of lower class women. Comparing the prevalence of Proposition 3.4 and Proposition 3.3, we also have that a higher proportion of foot-binding women in both classes in this extension, which is directly driven by the additional marrying up benefits of the M-type talent men within class. Given that the *de facto* talent measures of grooms could be a richer set (considering the number of their trials in taking the exam, their ranking in the exam), the impact of the exams on marriage market is actually much larger than the impact calculated purely based on the number of degree holders, so our quantification in Section 3.3.3 provides a lower bound on the marriage market effects of the exams.

3.5.2 Continuous Choice of Foot-binding

In this section we analyze adoption of foot-binding when the choice is continuous instead of binary. As before, we assume that foot-binding is costly and there are only two ability levels revealed by the exam (H and L). The key difference is we allow the disutility of foot-binding to be a continuous function of binding intensity, $c(b)$, with the following assumption:

Assumption 5. (Cost of Continuous Foot-binding) (1). $c(b) = 0$ for $b \in [0, \underline{b})$, $0 < c(b) < c\left(\overline{b}\right)$ for $b \in \left[\underline{b}, \overline{b}\right]$, and $c(b) = c\left(\overline{b}\right)$ for $b \in (\overline{b}, \infty)$; (2). $c' > 0$, $c'' < 0$ for

$b \in \left[\underline{b}, \overline{b}\right]$.

The first property says that a small degree of foot-binding, i.e.$b \in [0, \underline{b})$, is not costly until it becomes visible enough. The practice of foot-binding is physically subject to a point when it is physically impossible to further bind the feet, i.e. $b = \overline{b}$. The second property says that the cost of foot-binding increases in binding intensity and is concave. To simplify our analysis here, we assume the benefits of foot-binding $B(b) = c(b)$. Following similar logic, now the counterpart we have of Assumption 4 (bounded disutility) is the following:

Assumption 6. (Bounded Disutility with Continuous Foot-binding). *The cost of foot-binding is positive but bounded from above, i.e.* $0 < c(\overline{b}) \leq \overline{C}_{cont}$*, so that lower class women take up foot-binding when* $\alpha \leq \frac{1}{2}$*. In particular,* $\overline{C}_{cont}$ *is defined by* $\overline{C}_{cont} = v(l, \frac{h+l}{2}) - v(l + \beta_0 l, l)$.

The return to lower class women's labor value with continuous foot-binding now is defined by the following:

$$\beta = \begin{cases} \beta_0 & if \quad b < \underline{b} \\ \beta_1 & if \quad b \geq \underline{b} \end{cases}$$

Therefore, following a logic similar to that in Section 3.3.1 and 3.3.2, we define following proposition for the case of continuous foot-binding:

Proposition 3.5. (Continuous Foot-binding). *When foot-binding is continuous:*

(1) There's no foot-binding in upper class below cutoff $\tilde{\alpha_1}$*, and no foot-binding in lower class below cutoff* $\tilde{\alpha_2}$*, where* $\tilde{\alpha_1} < \tilde{\alpha_2}$.

(2) When $\alpha \in [\tilde{\alpha_1}, \frac{1}{2}]$*, a population of* μp *upper class women take up foot-binding with binding intensity* b_u*; when* $\alpha \in [\tilde{\alpha_2}, \frac{1}{2}]$*,* $(1-\mu)p$ *lower class women take up foot-binding with intensity* b_l*, where* $b_u > b_l$.

(3) When $\alpha \in (\frac{1}{2}, 1)$*, a population of* μp *upper class women choose binding intensity* b_C*,* $(1-\mu)\,p$ *choose binding intensity* b_{NE}*, and* $(\mu - p)$ *choose no foot-binding; for lower*

class women, $p(1-\mu)$ *choose foot-binding with intensity* b_{FE}*. In particular,* $b_C > b_{NE}$ *and* $b_C > b_{FE}$*.*

Proof. See Appendix.

With continuous foot-binding, brides can more precisely differentiate themselves in the marriage market. In the binary model, signal jamming takes place among upper class brides competing for champions and new elites. In the case of continuous foot-binding, upper class brides separate themselves by adopting a higher intensity of foot-binding to compete for champions, and a lower intensity to compete for the new elites. Since the population of champions and new elites combined is smaller than that of upper class brides (i.e. $p < \mu$), Bertrand competition stops when foot-binding intensities completely absorb any marrying-up benefits. This result hinges on the assumption that foot-binding is able to provide large enough benefits to achieve separation, since otherwise brides can pool to maximal binding intensity, and we are back to the binary case. Therefore, the continuous foot-binding case predicts within-class variation in foot-binding intensity. Historically, this echoes with our last stylized fact, that the higher the social status was the bride, the smaller their feet were.

3.5.3 Multiple Competition Tools: Dowry and Foot-binding

In this section, we allow for dowry payments in addition to foot-binding to see if the existence of alternative means of competition in the marriage market alters our earlier results. To accommodate full generality, we assume continuous foot-binding. To highlight the interactions between dowry and foot-binding, we assume away the small labor value differences under different levels of foot-binding ($b \geq \underline{b}$), that is, $\beta = 0$. The marriage values for two sides with both dowry and foot-binding are as follows:

$$\text{Brides' Utility Function: } V_B(q_i, q_j, b_{FB,i}, d) = v(\theta_i, \alpha\gamma_j + (1-\alpha)\theta_j) - c(b_{FB,i}) - u(d_i)$$

$$\text{Grooms' Utility Function: } V_G(q_i, q_j, b_{FB,i}, d) = v(\theta_i, \alpha\gamma_j + (1-\alpha)\theta_j) + b(b_{FB,i}) + u(d_i)$$

Here we do not include the budget constraint of the bride's family to highlight the substitution between dowry and foot-binding when there are interior solutions. One can easily add in budget constraints, and the qualitative results of the analysis remain unchanged.

Now the bride's family needs to decide both foot-binding intensity $b_{FB,i}$ and the size of the dowry payment d_i. Since dowry is usually offered as a direct monetary transfer, we assume $u^{'} > 0$, $u^{''} < 0$ and $u^{'}(0) = 1$, that is, a monetary transfer has diminishing marginal returns. Adopt Assumption 5 and 6 in the continuous foot-binding, we have the following proposition:

Proposition 3.6. (Foot-binding and Dowry). *When both dowry and foot-binding are available, we have the following cases:*

(1) There's no foot-binding nor dowry payment in either class when $\alpha = 0$. After a cutoff of $\tilde{\alpha_1}$ for upper class and $\tilde{\alpha_2}$ for lower class, bride's family use dowry $d^*_{0,u}$ and $d^*_{0,l}$ respectively, where $\tilde{\alpha_1} < \tilde{\alpha_2}$; for given $\alpha \in [\tilde{\alpha_1}, \tilde{\alpha_2}]$, $d^*_{0,u} > d^*_{0,l}$.

(2) When $\alpha \in [\tilde{\alpha_1}, \frac{1}{2}]$, a population of μp upper class women choose binding intensity b^*_u and dowry size d^*_u; when $\alpha \in [\tilde{\alpha_2}, \frac{1}{2}]$, $(1-\mu)p$ lower class women choose binding intensity b^*_l and dowry size d^*_l;

(4) When $\alpha \in (\frac{1}{2}, 1)$, a population of μp upper class women choose binding intensity b^*_C, dowry size d^*_C; $p(1-\mu)$ choose binding intensity b^*_{NE}, dowry size d^*_{NE}; and $(\mu - p)$ choose no foot-binding and pay no dowry. For lower class women, $p(1-\mu)$ choose binding intensity b^*_{FE}, dowry size d^*_{FE}. In particular, $b^*_C > b^*_{NE}$, $b^*_C > b^*_{FE}$, $d^*_C > d^*_{NE}$, and $d^*_C > d^*_{FE}$.

Proof. See Appendix.

The above proposition carries several new insights. First, as foot-binding investment requires a fixed cost, it will be adopted later than dowry payments. In this sense, dowry payments are more flexible than foot-binding. Second, when both tools play a role in marriage competition, both foot-binding intensity and dowry payments will increase with the degree of competition. In fact, this is exactly what we have observed since the Song dyn-

asty, that more and more historical accounts include parent's complains about high dowries required to marry their daughters for both class (e.g. Guo, 2000). Third, being qualitatively consistent with our previous results, upper class women still have greater foot-binding intensity and larger dowries compared to lower class women.

3.6 Conclusion and Discussion

Economic incentives can deeply influence cultural practices. This paper explains the economic motives of foot-binding in historical China by developing a model of the marriage market that also considers labor opportunity cost and treats foot-binding as a premarital investment. We show that foot-binding is not only a cultural practice that carries aesthetic, erotic and social motives, but also has a deep economic rationale. The key trade-off considered in the foot-binding decision is the following: (1) foot-binding can improve marriage market outcomes for women by making themselves more attractive; and (2) foot-binding has a significant disutility and can have a large labor opportunity cost depending on specific agricultural regimes and household handicraft patterns. We analyze several cases which provide insights into the evolution of foot-binding. In a stratified society where marriage is completely assortative in terms of family status and there is no incentive for foot-binding. The introduction of a gender-biased examination system significantly increased men's mobility and quality dispersion. Historically, this led to the emergence of foot-binding among upper class women. When meritocracy increases further, as happened historically in China, marrying-up benefits continue to increase, and foot-binding diffuses from upper class women to lower class women, exactly the sequence observed in China.

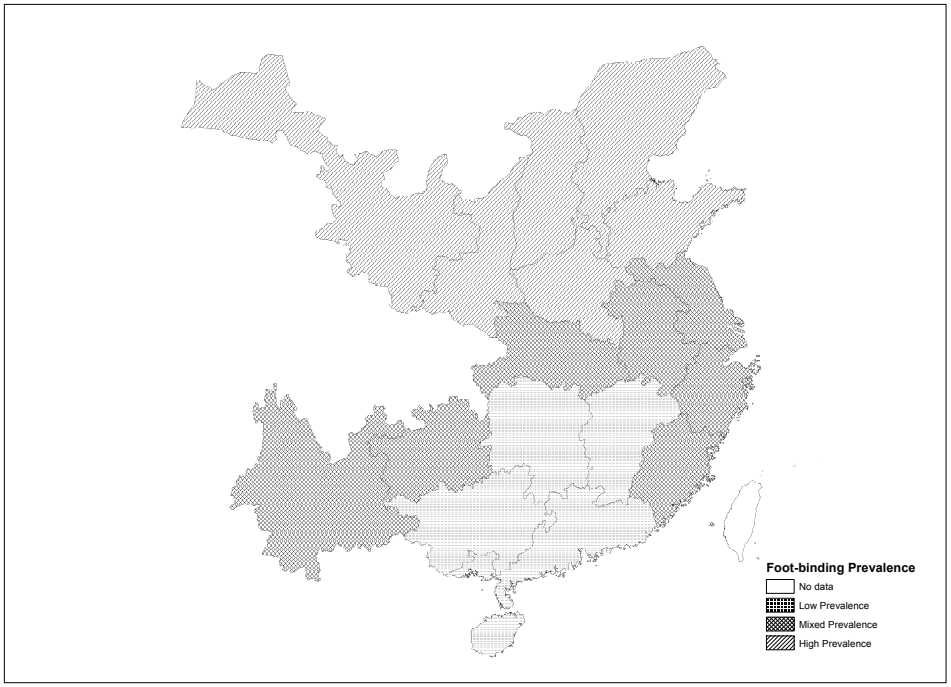
For lower class women, the opportunity cost of foot-binding also plays a significant role in decision-making. In the model, we consider two agricultural regimes. Where women's farmland work dominants (e.g. rice-growing regions), foot-binding is a much less attractive investment for women. In areas with less agricultural labor demand in farming (e.g. the

wheat-growing regions) or where women can specialize in household handicraft production which can be done with sedentary labor, foot-binding is more popular. Thus, different agricultural regimes and gender labor specialization translate into differential adoption of foot-binding among women. Based on the predictions of our model, our empirical analysis using county-level from the Republican archives shows that higher suitability of rice relative to wheat and higher suitability for household handicraft are associated with less/more foot-binding among lower class women respectively, and the county exam quota predicts a higher incidence of foot-binding. The results are robust to using different samples and control variables. The empirical findings directly test and support our theoretical predictions, that the prevalence of foot-binding among lower class women increases with the marriage market prospects and decreases with the labor opportunity cost of foot-binding.

Overall, our paper sheds light upon cultural practices from an economic perspective, showing that an equal mobility institution without gender bias, as well as women's active participation in economic activities can be protective against cultural customs that carry high disutility for women. Future work may focus on the dynamic interactions between foot-binding, dowries and sex ratios, and further examine similar cultural practices in our context, i.e. the corsets. We believe the analysis of foot-binding introduced here will yield broader insights for future studies in women's development.

3.7 Maps, Figures and Tables

Figure 3.1: Regional Variation of Foot-binding Prevalence



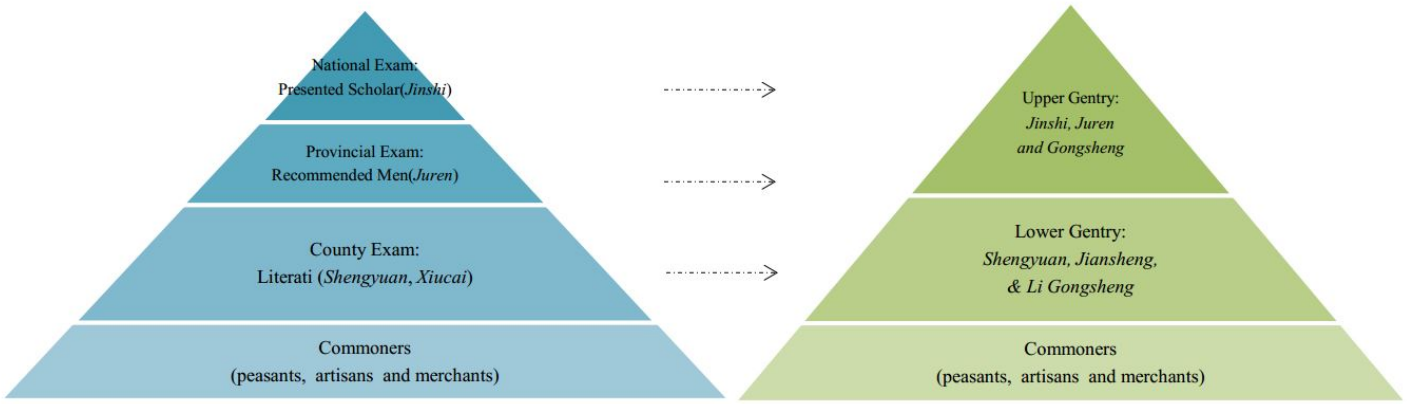
Note: The foot-binding prevalence variable on province level is based on historical qualitative evidence, including Hu(1936); Qian(1969); Xu(1984); Ko(2005); Davin(1976); Lu(1987); Yao(1936); Turner(1997)

Table 3.1: Number of days of man-labor required per crop hectare for major crops

Major Crops in Rice Region						
	Rice	Rice, early	Rice, late	Rice, glutinous		
Average	186.4	120.7	104.9	201.6		
Major Crops in Wheat Region						
	Wheat	Spring Wheat	Maize	Kaoliang	Millet	Millet, proso
Average	95.4	114.6	108.7	101.9	103.9	127.8

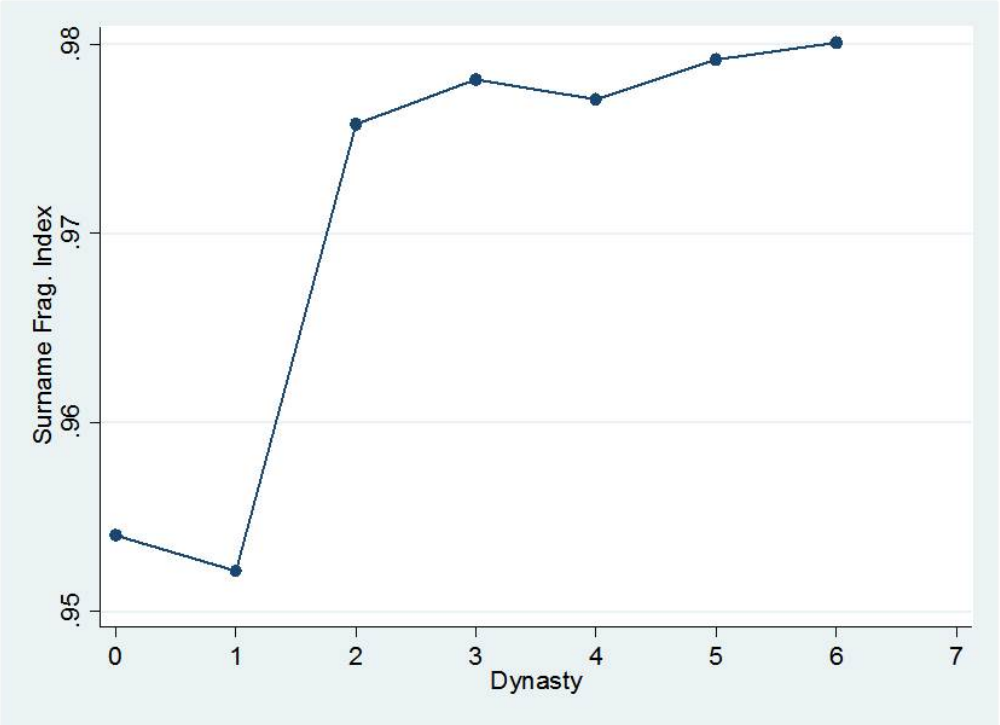
Source: Buck, J. L. (1937). Land Utilization in China: A Study of 16,786 Farms in 168 Localities, and 38,256 Farm Familites in Twenty-two Provinces in China, 1929-1933–Statistics (Vol. 2). Commercial Press, Limited, Agents in the United States, The University of Chicago Press.

Figure 3.2: The Civil Exam System and the Social Ladder



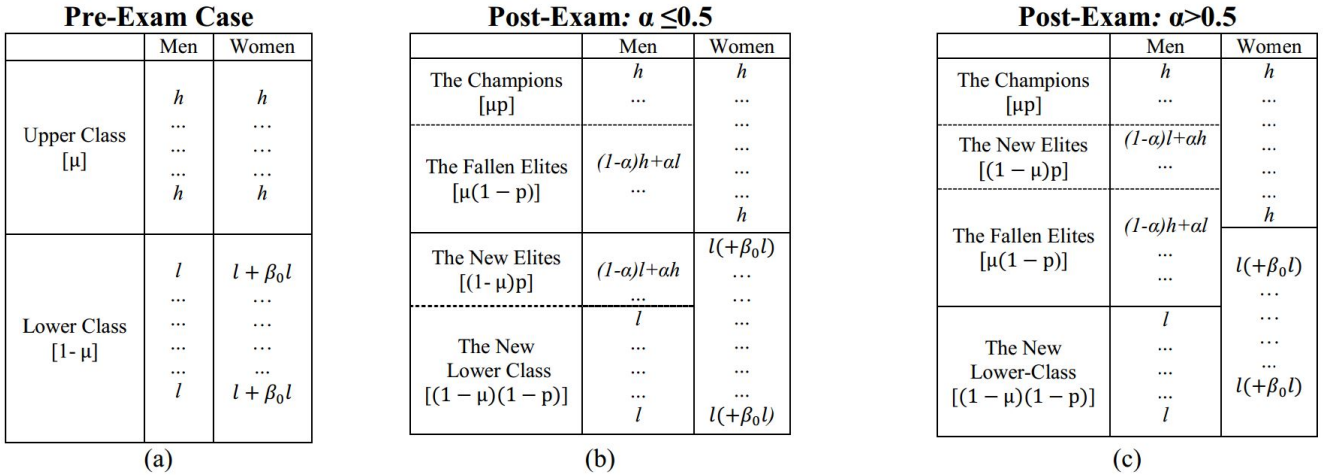
Note: Illustration based on Chang(1955). Description of degree holders of the exam system is presented in Section 3.2, including Presented Scholars (*Jinshi*), Recommended Men (*Juren*), and Literati (*Shengyuan*). Tribute Students (*Gongsheng*) refer to those who are accepted into the Imperial Academy (*Guozijian*), and *Jiansheng* refers to those who purchased the Literati degree without taking actual exams at the entry level.

Figure 3.3: Men's Mobility Trend: Evidence from CBDB



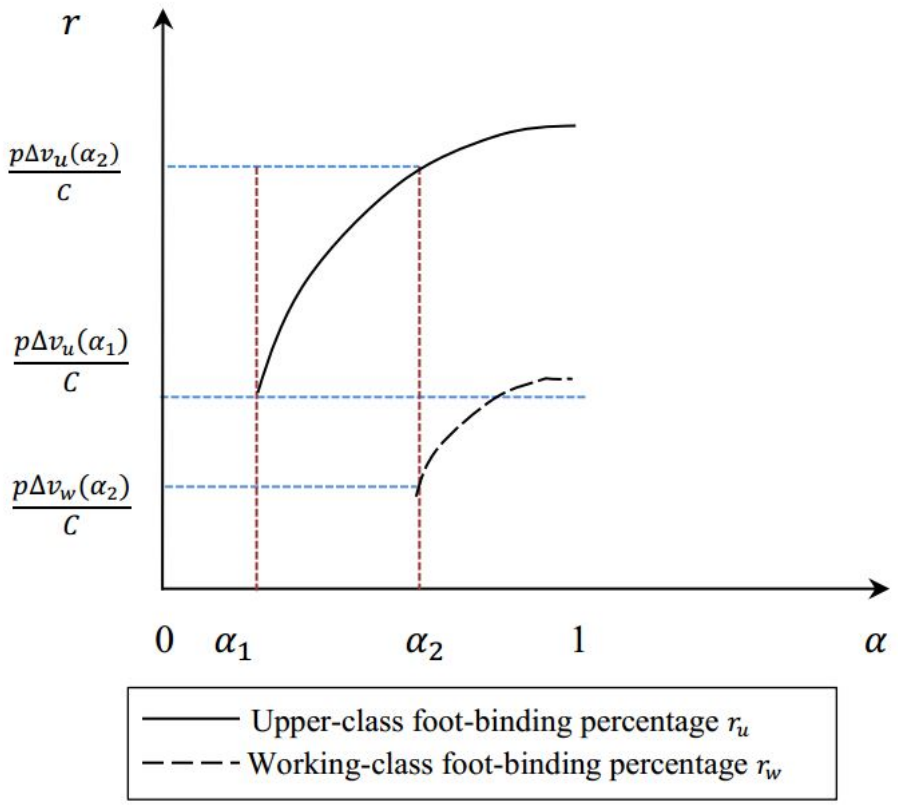
Note: In the above graph, 0 =Qin-Han Dynasty (BC221-220), 1 =Sui-Tang Dynasty (581-907), 2 =Song Dynasty (960-1127), 3 =Yuan Dynasty (1271-1368), 4 =Ming Dynasty (1368-1644), 5 =Qing Dynasty (1636-1911) and 6 =Post-Qing Dynasty (1911-1949). Data source: CBDB (version: 2015-03-18). We calculate the surname fractionalization index with a sample of male celebrities with single character surnames (a proxy of Han ethnicity). The surname fractionalization index is constructed following Alesina et. al (2003): $Frac_d = 1 - \sum_{i=1}^{N} s_i^2$, where s_i is the share of surname group i in dynasty d.

Figure 3.4: Matching Illustration



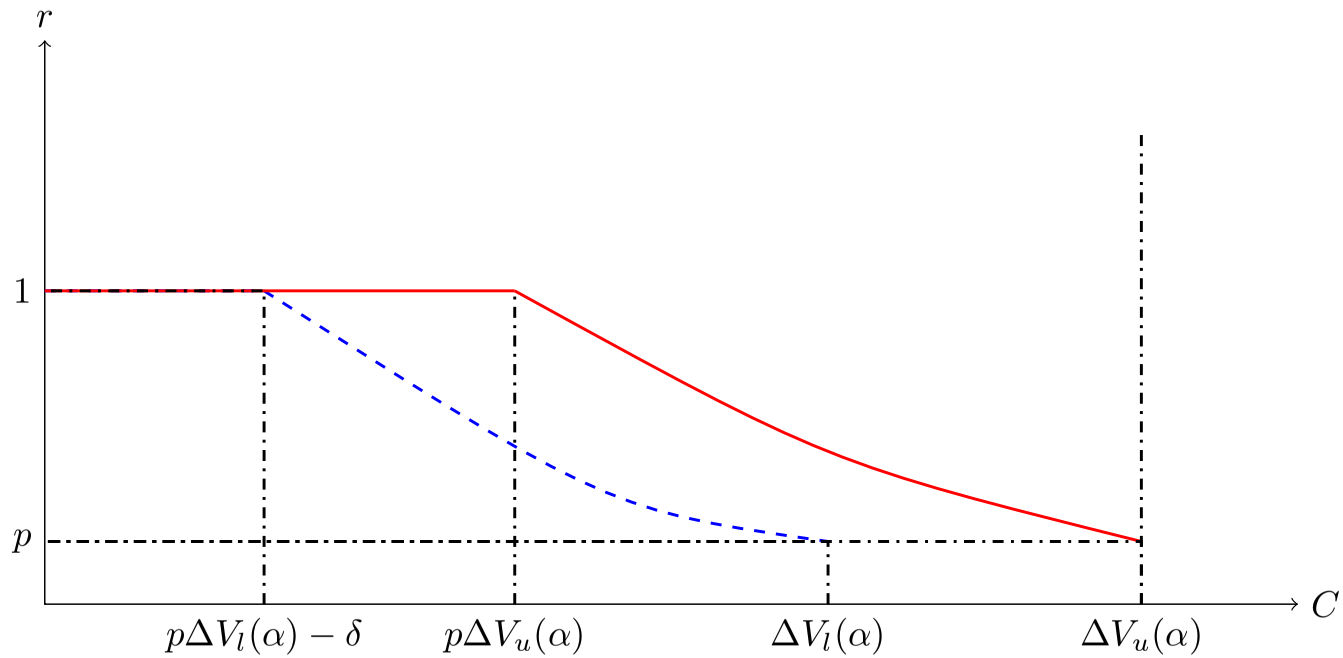
Note: We use brackets for lower class women's quality since their quality depends on their husbands' wealth status. In the case of marrying a wealthy enough husband (with quality measure higher than h), the wife do not have to work in the household. Panel (a) is corresponding to Proposition 3.1 (the pre-Exam case), Panel (b) refers to Proposition 3.2 (Restricted Meritocracy) and 3.3 (Prevalence of Foot-binding), and Panel (c) characterizes
Proposition 3.7 (Flipping Ranking).

Figure 3.5: Foot-binding Prevalence and Exam Meritocracy α



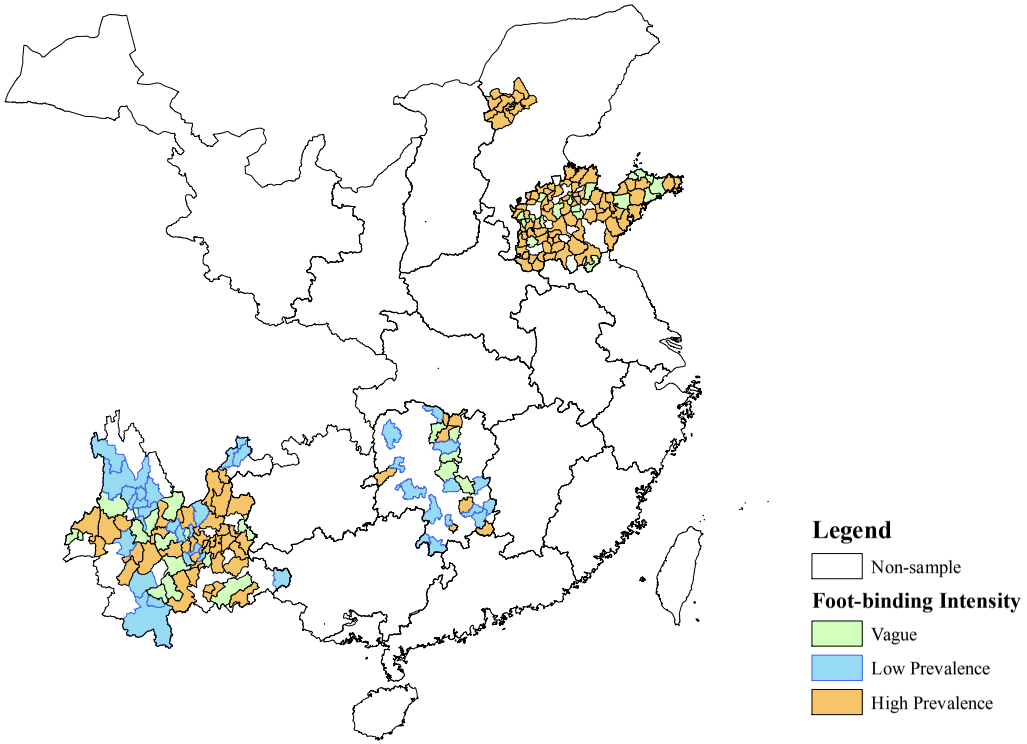
Note: This figure is drawn based on Proposition 3.1, 3.2, 3.3 and 3.7 (in appendix). The horizontal axis is α, or the relative weight of talent for men determined by the mobility institutions (i.e. the exam system). The vertical axis is the proportion of foot-binding within class. Here $\Delta V_u = v(h,h) - v(h,(1-\alpha)h + \alpha l)$, and $\Delta V_l = v(l,(1-\alpha)l + \alpha h) - v(l+\beta_0 l, l)$. Proposition 1 characterizes the case when $\alpha = 0$, Proposition 3.2 refers to the case when $\alpha \in (0, \alpha_1]$, Proposition 3.3 captures the case when $\alpha \in (\alpha_1, \frac{1}{2}]$ for upper class women and $\alpha \in (\alpha_2, \frac{1}{2}]$ for lower class women, and Proposition 3.7 describes the case when $\alpha \in (\frac{1}{2}, 1)$. Here the solid line in red denotes upper class women, while the dashed line in blue denotes lower class women.

Figure 3.6: Foot-binding Prevalence and Disutility



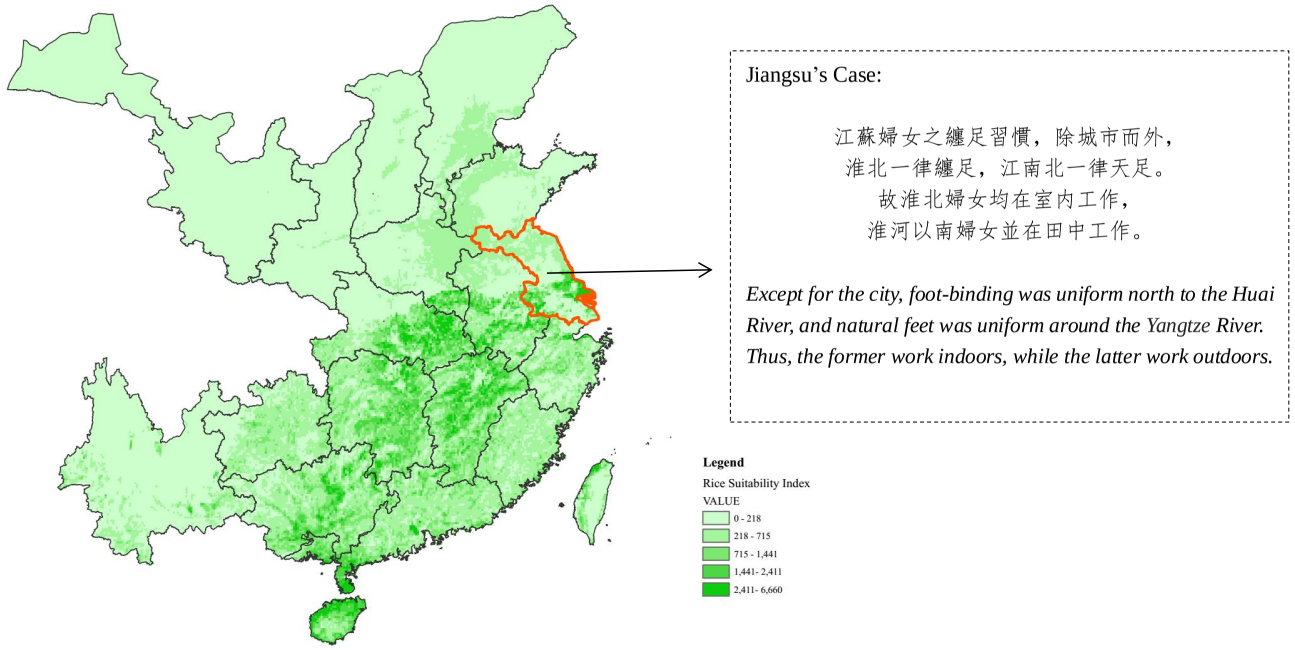
Note: This figure of foot-binding prevalence is drawn given the α discussed in Proposition 3.3. The red solid line denotes upper class women, while the blue dashed line denotes lower class women. Here $\Delta V_u(\alpha) = v(h,h) - v(h,(1-\alpha)h + \alpha l)$, which is the marrying up gain for upper class women. The marrying up gain for lower class women is $\Delta V_l(\alpha) = v(l,(1-\alpha)h + \alpha l) - v(l + \beta_0 l, l)$. The labor opportunity cost of foot-binding δ is $[v(l + \beta 0 l, l) - v(l + \beta 1 l, l)]$.

Figure 3.7: Sample Counties by Prevalence of Foot-binding



Note: This figure of foot-binding prevalence is drawn given the α discussed in Proposition 3.3. The red solid line denotes upper class women, while the blue dashed line denotes lower class women. Here $\Delta V_u(\alpha) = v(h,h) - v(h,(1-\alpha)h + \alpha l)$, which is the marrying up gain for upper class women. The marrying up gain for lower class women is $\Delta V_l(\alpha) = v(l,(1-\alpha)h + \alpha l) - v(l + \beta_0 l, l)$. The labor opportunity cost of foot-binding δ is $[v(l + \beta 0 l, l) - v(l + \beta 1 l, l)]$.

Figure 3.8: The Case of Jiangsu Province



Note: The data in this map comes from geographic distribution of crop suitability from the FAOs GAEZ (Global Agro-Ecological Zones). The two rivers denotes Huai River and Yangtzi River from north to south respectively. The source of this qualitative case is provincial gazetteer of Jiangsu province published in the Republican years.

Table 3.2: Out-of-Archive and In-Archive Counties

	Out-of-Archive Counties		**In-Archive Counties**		**T-test**
	N1	**Mean1**	**N2**	**Mean2**	**p-value**
Shandong					
Relative Suitability	17	-0.86	90	-0.979	0.62
Cotton Suitability	17	1.056	90	0.938	0.15
Flax Suitability	17	0.933	90	0.968	0.85
County Exam Quota	17	13.471	87	15.356	0.03
Log Dist. To Nanjing	17	6.464	90	6.482	0.58
Hunan					
Relative Suitability	45	2.412	26	2.601	0.43
Cotton Suitability	45	-0.447	26	-0.415	0.32
Flax Suitability	45	-0.333	26	-0.417	0.2
County Exam Quota	42	14.476	25	15.16	0.56
Log Dist. To Nanjing	45	6.875	26	6.848	0.4
Chahaer					
Relative Suitability	142	-0.583	10	-0.697	0.4
Cotton Suitability	142	0.429	10	-1.74	0
Flax Suitability	142	0.38	10	0.451	0.66
County Exam Quota	126	16.571	10	15.8	0.56
Log Dist. To Nanjing	142	6.824	10	7.029	0
Yunnan					
Relative Suitability	29	0.957	80	-0.143	0
Cotton Suitability	29	-0.856	80	-1.127	0.07
Flax Suitability	29	-1.241	80	-1.245	0.98
County Exam Quota	7	11.143	69	13.217	0.4
Log Dist. To Nanjing	29	7.64	80	7.581	0

Note: This table provides a balance check for out-of-archive and in-archive counties by the four provinces covered by the Republican archives. The key variables for comparison include standardized relative suitability (Rice-Wheat), cotton and flax suitability, county exam quota. Column 5 provides p-value for the t-test between two groups. All variables are standardized except for log distance to Nanjing (capital of the Republican government).

Table 3.3: Ambiguous-Info. Counties and Clear-Info. Counties

	Ambiguous-Info. Counties		**Clear-Info. Counties**		**T-test**
	N1	**Mean1**	**N2**	**Mean2**	**p-value**
Relative Suitability (Rice-Wheat)	38	0.048	168	-0.242	0.31
Cotton Suitability	38	-0.014	168	-0.199	0.36
Flax Suitability	38	-0.083	168	-0.093	0.96
County Exam Quota	35	-0.309	156	-0.071	0.22
Log Dist. To Nanjing	38	6.975	168	6.983	0.93

Note: This table provides a balance check for ambiguous-info. counties and clear-info. counties, among all counties covered by the Republican archives. Column 5 provides p-value for the t-test between two groups. Key variables include standardized relative suitability index (Rice-Wheat) and county exam quota, cotton and flax suitability index. All variables are standardized except for log distance to Nanjing.

Table 3.4: Descriptive Statistics

	Low Prevalance		High Prevalance		T-test	
Crop Suitability	**N1**	**Mean**	**N2**	**Mean**	**P-value**	**Sources**
Agricultural suitability variables						
Relative Suitability (Rice-Wheat)	38	1.134	130	-0.645	0	A
Cotton Suitability	38	-0.921	130	0.013	0	A
Flax Suitability	38	-0.969	130	0.163	0	A
Socio-economic variables						
County Exam Quota	33	-0.437	123	0.027	0.02	B
County Population (1000 ppl.)	38	-0.188	130	0.001	0.28	C
County Sex Ratio	38	-0.094	130	0.107	0.31	C
Number of Christian Churches	36	-0.27	127	0.043	0.09	D
Number of Vicars	36	-0.242	127	-0.036	0.14	D
Number of Disciples (per 10,000 people)	36	-0.159	127	0.029	0.34	D
Number of Modern Firms	38	-0.254	130	-0.179	0.46	E
Mongol Historical Migration Intensity	38	-0.777	130	-0.146	0	F
Geographical Characteristics						
Log Distance to Nanjing (km)	38	7.327	130	6.882	0	G

Note: This table provides descriptive statistics of key variables for counties covered by the Republican archives. Column 5 provides p-value for the t-test between two groups. Key variables include standardized relative suitability index (Rice-Wheat) and county exam quota, cotton and flax suitability index. All variables are standardized except for log distance to Nanjing.

Table 3.5: Crop Suitability, Exam Quotas and Foot-binding Prevalence

	Sample= All Counties				
	Dependent Variable: Foot-binding in Rural Area (1=High Prevalence)				
	(1)	(2)	(3)	(4)	(5)
Relative Suitability (Rice-Wheat)	-0.094***	-0.095***	-0.095***	-0.089***	-0.088**
	(0.024)	(0.025)	(0.025)	(0.025)	(0.039)
Cotton Suitability	0.109***	0.105***	0.105**	0.129**	0.177**
	(0.027)	(0.028)	(0.045)	(0.051)	(0.078)
Flax Suitability	0.053	0.055	0.055	0.066	0.014
	(0.038)	(0.040)	(0.051)	(0.051)	(0.054)
County Exam Quota	0.060*	0.063*	0.063*	0.061†	0.058†
	(0.034)	(0.036)	(0.038)	(0.038)	(0.038)
County Population	-0.077***	-0.060*	-0.060*	−0.057†	-0.022
	(0.023)	(0.033)	(0.034)	(0.035)	(0.042)
Number of Christian Churches		0.026*	0.026*	0.017	0.022†
		(0.014)	(0.014)	(0.014)	(0.014)
Number of Vicars		-0.047	-0.047	-0.045	-0.060
		(0.045)	(0.046)	(0.049)	(0.046)
Number of Disciples		-0.010	-0.010	-0.010	-0.010
		(0.036)	(0.036)	(0.036)	(0.034)
Log Distance to Nanjing (km)			0.001	0.102	-0.049
			(0.166)	(0.191)	(0.336)
Number of Modern Firms				0.002	0.011
				(0.028)	(0.026)
Mongol Historical Migration Intensity				0.058*	-0.008
				(0.032)	(0.036)
Constant	0.782***	0.777***	0.771	0.088	1.057
	(0.028)	(0.028)	(1.147)	(1.316)	(2.238)
Prov.FE	N	N	N	N	Y
Observations	153	153	153	153	153
R-squared	0.348	0.351	0.351	0.362	0.396

Note: *** p<0.01, ** p<0.05, * p<0.1, † p<0.15. Parentheses include robust standard errors. Dependent variable is a dummy variable indicating high prevalence of foot-binding, and key explainatory variables include relative suitability index and county exam quota, where both are standardized. A set of control variables includes standardized county population, cotton and flax suitability index, number of modern firms, Mongol historical migration intensity, Christianity activity variables, as well as log distance to Nanjing.

Table 3.6: Exam Quotas and Foot-binding Prevalence: Prefecture Fixed Effects

	Dependent Variable: Foot-binding Prevalence (1=High 0=Low)			
	Sample= All Counties			
	(1)	(2)	(3)	(4)
County Exam Quota	0.070*	0.070**	0.084**	0.087**
	(0.041)	(0.033)	(0.033)	(0.034)
County Population	0.009	-0.044	-0.050*	-0.037
	(0.049)	(0.029)	(0.028)	(0.040)
Relative Suitability (Rice-Wheat)			0.069	0.062
			(0.045)	(0.047)
Cotton Suitability			0.014	0.024
			(0.123)	(0.131)
Flax Suitability			0.066	0.068
			(0.040)	(0.044)
Constant	0.795***	1.158***	1.221***	-1.435
	(0.032)	(0.075)	(0.147)	(2.249)
Prefecture FE.	No	Yes	Yes	Yes
Other Control Variables	No	No	No	Yes
Observations	153	153	153	153
R-squared	0.035	0.720	0.730	0.732
Test Unobservable Selection		Column 1&2	Column 1&3	Column 1&4
δ for $\beta = 0$ given $R_{max} = 1$		2.528	3.312	2.861
Bias-Adjusted β, given $\delta = 1$ and $R_{max} = 1$		0.07	0.102	0.117

Note: *** p<0.01, ** p<0.05, * p<0.1. Robust standard errors in parentheses. Dependent variable is a dummy variable indicating high prevalence of foot-binding within county. Both county exam quota and county population are standardized. Prefecture fixed effects are controlled in 2 to 4 columns. The test on selection on unobservables is based on Oster (2016). A set of control variables includes standardized county population, cotton and flax suitability index, number of modern firms, Mongol historical migration intensity, Christianity activity variables, as well as log distance to Nanjing.

3.8 Appendix

3.8.1 Foot-binding in Historical Accounts

As we have discussed above, foot-binding was an unpresentable and cryptic topic to discuss in public domain, and even taboo in official genres. Therefore, there is a lack of systematic record of foot-binding by the imperial government during the Song, Ming and Qing. In this section, we will summarize and compare different sources recorded and compiled by a variety of authorship on foot-binding, trying to piece out the picture of foot-binding and cross-check the evidence one from another.

Overall, sources on foot-binding have significant variation, ranging from semi-official records, privately compiled history, personal history, literature, and folklore, to archaeological evidence. Appendix Table 3.7 provides a summary, presenting the types and contents, authorship/compilers, time range, accessibility and potential bias. It could be clearly seen that those sources are very different by nature, thus relying merely on one type of the sources may result in bias in understanding foot-binding. As listed as 1 in Appendix Table 3.7, the first type of sources on foot-binding we examine is official records by the Republican government. In the early 20th century, the Republican government prohibited foot-binding and surveyed to investigate the process of prohibition from 1931 to 1934 (Yang, 2003).In this process, the local county government conducted survey on previous foot-binding prevalence and the progress of anti-foot-binding campaign, then sent reports to the Republican government, which were later compiled by the Ministry of The Interior. This is the primary source used by this paper for empirical analysis.

The second type of the sources on foot-binding is from gazetteers (on province, prefecture, and county level), which is semi-official compared with governmental records. The authors of gazetteers are mostly local gentries, scholars, and officials, and the compiling were largely based on local files, records, and personal observations. Foot-binding in gazetteers was barely

mentioned from the Song to mid-Qing dynasty, and mostly mentioned during the late Qing and Republican Years. For gazetteers published during the late Qing, most of the text on foot-binding appears in the chapter of "local customs". During the Republican years, the gazetteers almost uniformly mentioned foot-binding as a bad custom along with description on anti-footbinding campaigns. Taken together, only around 230 localities (mostly counties) have foot-binding texts in gazetteers, and most of the content is anti-foot-binding, not about the pre-existing culture of foot-binding. Therefore, while gazetteers could be a good source for supplementary purpose, they will be most useful for understanding the anti-footbinding process.

The third type of our sources on foot-binding are jottings (in Chinese, Biji), literature and privately compiled history. The authors of such type are mostly scholars, historians, philologists, and sometimes even amateurs of foot-binding, e.g., Qian, Yong (1759-1844), Xu, Ke(1869-1928), Hu, Pu-an (1878-1947), Yao, Ling-hsi(1899-1963). Their accounts cover the widest aspect of foot-binding history, and are irreplaceable in the value of first-hand observation on foot-binding. Their accounts of foot-binding are often a key source for contemporary scholars, including Dorothy Ko(2005) and Susan Mann(1997).

The fourth type of the sources is anthropology, personal history, and interviews (e.g. see a rich source summarized by Turner, 1997, and Gates, 2015; Bossen and Gates, 2017 for contemporary interviews with foot-binding women). The authors of this type of sources include foreign visitors/scholars, Christian missionaries, travelers, and foot-binding women themselves. The key merit of this type of source is its first-hand observation and experiences. For instance, Bossen and Gates (2017) show their interviews with 1800 rural senior women in 27 villages (9 provinces), consulting their foot-binding history (if any), personal life history, economic activities, village characteristics, and so on. This source is again irreplaceable in helping us to understand foot-binding on the grassroots.

The fifth source we look at is the folkloristic evidence and oral literature, i.e., ballads (e.g. see Zhang, 2015). The authors of the ballads are the folk. Compiled by domestic or foreign

observers, the folk ballads described foot-binding in various ways, often bringing remarkable insight into this phenomenon. However, the drawback of this source is also clear--we do not know its time span and what kind of ballads survived and why. Thus the folk ballads will be taken as complementary sources when we examine our theory and empirical predictions.

The final source we discuss in this paper is the archaeological evidence (bound feet shoes/lotus shoes) (see Ebrey 1992 for example). The clear advantage of this source is that it is the only source that can help us to date back to the early dynasties (e.g., the Song, 960-1279). By examining the owner of the shoes, her family status, as well as the location of shoes, this type of source will be important for us to understand foot-binding at its earlier phase.

3.8.2 Flipped Rankings under High Meritocracy

Section 3.3 of our paper has analyzed cases when $\alpha < \frac{1}{2}$. To get a full picture of foot-binding dynamics with high α, we further examine the case when $\alpha > \frac{1}{2}$ so that the ranking of groom's quality is flipped. Using the group labels in Figure 3.4(b), now the marriage market ranking from the highest to lowest is: *the champions, the new elites, the fallen elites*, and *the new lower class*. An illustration of the equilibrium matching for this case can be found in Figure 3.4(c). Proposition 3.7 characterizes the foot-binding practice when the exam system is highly meritocratic and the society has high mobility.

Proposition 3.7. *When men's competence dominates family status* ($\frac{1}{2} < \alpha < 1$)*, foot-binding is adopted by women of both classes, and is more prevalent in the upper class. The proportion of foot-binding women in the upper class is* $r_u = min\{\frac{p}{C}\{\mu[v(h,h) - v(h,(1-\alpha)h + \alpha l)] + (1-\mu)[v(h,(1-\alpha)l + \alpha h) - v(h,(1-\alpha)h + \alpha l)]\},\ 1\}$ *and in the lower class is* $r_w = min\{\frac{p[v(l,(1-\alpha)h+\alpha l) - v(l+\beta_1 l, l)]}{C + v(l+\beta_0 l, l) - v(l+\beta_1 l, l)},\ 1\}$*. Further,* $0 < r_w \leq r_u \leq 1$.

In this case, upper class brides compete for two types of men - the champions and new elites. The remaining upper class brides then compete with lower class brides for the fallen elites. In equilibrium, the expected net payoff from foot-binding in the marriage market is equal to the disutility of foot-binding, which condition pins down the proportion of foot-binding women in the upper and lower classes. With similar logic to that of Proposition 3.3, the proportion of upper class women who bind their feet is greater than that of lower class women. It should be noted that, when the upper class women compete for two types of men with only one tool for competition (the binary foot-binding option), the marriage benefits between marrying champions or new elites cannot be further absorbed by the competition. In the extensions of our model in Section 3.5, we will discuss foot-binding as a continuous competition tool, and also consider the availability of multiple competition tools (i.e. dowry payments) for further investigation.

Corollary 3.2. *The foot-binding percentage of upper class women and of lower class women is non-decreasing in the proportion of high ability men p, and non-increasing with the disutility of foot-binding C when* $\frac{1}{2} < \alpha < 1$. *For lower class women, the foot-binding percentage is non-increasing in* β_0.

Corollary 3.2 establishes comparative static predictions for the case of $\frac{1}{2} < \alpha < 1$. It delivers similar predictions to Corollary 3.1. Thus, the key comparative statics are robust whether α is larger than one half or not.

3.8.3 Proofs of Propositions and Corollaries

Proof of Proposition 3.1.

Proof. In the pre-exam period, we start from the matching where upper class brides are matched with upper class grooms, lower class brides are matched with lower class groom, and there is no foot-binding. We check such matching is in equilibrium.

The only players who have incentive to seek for better marriage opportunities are the lower class brides, because the upper class brides are already paired with the best grooms possible. If one of them chooses to foot-binding, then the condition that an upper class groom is willing to switch for her is:

$$v(l,h) + B > v(h,h) \Rightarrow B > v(h,h) - v(l,h)$$

That is, the foot-binding benefits outweigh the costs of marry-down. And the condition that the lower class bride is willing to choose foot-binding is:

$$v(l,h) - C > v(l+\beta_0 l, l) \Rightarrow C < v(l,h) - v(l+\beta_0 l, l) < v(l,h) - v(l,l)$$

However, since $B \leq C$, if the two conditions hold, we have $v(h,h) - v(l,h) < v(l,h) - v(l,l)$, which contradicts with v being complementary. Thus lower class bride has no incentive to choose foot-binding. The matching is in equilibrium.

However, there might be multiple equilibria in the game. To require the equilibrium to be unique, we need an additional assumptions on the value function that $v(x,x) \geq v(x+a, x-a)$ for all $0 < a \leq x$.

Among all assortative matching outcome the non-foot-binding matching is obviously unique because without competition there is no need to bind feet. If there exists another non-assortative matching, then there exists at least a pair (h,l) and a pair (l,h). To start with, the lower class bride in (l,h) must have bound her feet, because otherwise the groom should choose an upper class bride.

If the h bride deviates to pair up with the h groom, then the payoff difference is $v(h,h) - v(h+\beta l, l)$. By Assumption 3, when $x = 1$, $h - l > \beta l$, $v(h,h) - v(h+\beta l, l) \geq v(h+\beta l, h-\beta l) - v(h+\beta l, l) > v(h+\beta l, l) - v(h+\beta l, l) = 0$ therefore $v(h,h) - v(h+\beta l, l) > 0$, regardless of foot-binding choices. So the h bride has incentive to deviate.

Next we check whether the h groom prefers the h bride to the original l bride. There are two cases: (1) the h bride has bound feet. Then the h groom prefers the h bride immediately, and the original matching is not stable; (2) the h bride has natural feet. Then the h groom compares a non-foot-binding upper class bride and a foot-binding lower class bride: if the groom prefers the foot-binding lower class bride, it means $v(h,h) \leq v(l,h) + B$, in which case for the lower class bride $C \geq v(h,h) - v(l,h) > v(l,h) - v(l,l)$. Therefore the lower class bride prefers not to bind the feet and marry a lower class groom in the first place. Again, the original matching is not in equilibrium. Thus uniqueness is proven under the additional assumption.

Proof of Proposition 3.2.

Proof. The proof is constructed as follows. First, we discuss any possible competition between or within class. Second, we determine the conditions of no foot-binding in upper class and lower class respectively. Lastly we compare the conditions to prove the proposition.

First, due to complementarity (Assumption 2), the lower class bride never seeks for cross-

class marriage. Specifically, for a lower class bride matched with, for example, a champion through foot-binding, the marrying-up benefit of the bride's side should be able to compensate the marrying-down loss of the champion, i.e.: $v(l,h) - v(l+\beta_0 l, l) > v(h,h) - v(l,h)$. However, due to complementarity, this will not be feasible. Thus cross-class marriage is not feasible, and there only exists competition within class.

Second, we examine within class competition among women, that upper brides are matched with champions and fallen elites, lower class brides are matched with new elites and new lower class.

Lemma 1. *There exists α_1 such that no foot-binding is adopted in intra-class marriage in upper class when $\alpha < \alpha_1$.*

For the upper bride to stay in class without foot-binding, we require the cost of foot-binding is too large compared with the benefit of marrying up, that is, $v(h,h) - C \leq pv(h,h) + (1-p)\, v(h,(1-\alpha)h + \alpha l)$, then $C \geq (1-p)\left[v(h,h) - v(h,(1-\alpha)h+\alpha l)\right]$. Denote the cutoff value of α as α_1.

Lemma 2. *There exists α_2 such that no foot-binding is adopted in intra-class marriage in lower class when $\alpha_1 < \alpha_2 \leq \frac{1}{2}$.*

For the lower class bride to stay in class without foot-binding, we require when everyone else choose no foot-binding, a lower class bride has no incentive to do so either:

$v(l,(1-\alpha)l + \alpha h) - C \leq pv(l,(1-\alpha)l+\alpha h) + (1-p)\, v(l+\beta_0 l, l)$,

then $C \geq (1-p)\left[v(l,(1-\alpha)l+\alpha h) - v(l+\beta_0 l, l)\right]$. Denote the cutoff value as α_2.

Since v is convex, we have $v(h,h) - v(h,(1-\alpha)h+\alpha l) > v(l,(1-\alpha)l+\alpha h) - v(l,l) > v(l,(1-\alpha)l+\alpha h) - v(l+\beta_0 l, l)$, where the first equality is guaranteed by mean value theorem. Therefore $\alpha_1 < \alpha_2 < \frac{1}{2}$. Here $\alpha_2 < \frac{1}{2}$ is directly guaranteed by Assumption 4, where $C < \bar{C} = (1-p)\left[v\left(l, \frac{l+h}{2}\right) - v(l+\beta_0 l, l)\right]$. □

Proof of Proposition 3.3

Proof. Follow the proof of Proposition 3.2, we know that when $\alpha \leq \alpha_1$ there is no foot-

binding in upper class, and when $\alpha_1 < \alpha \leq \alpha_2$, there is foot-binding in upper class but not in the lower class. When $\alpha_2 \leq \alpha \leq \frac{1}{2}$ there are foot-binding in both class. Here it remains to show the foot-binding percentage in upper class and lower class, denoted as r_u and r_l respectively.

For upper class, when $\alpha \in [\alpha_1, \frac{1}{2}]$, in equilibrium the percentage of foot-binding women should make the marriage benefit with and without foot-binding indifferent, if the solution is interior. That is:

$$\frac{p}{r_u} v\left(h,h\right) + \left(1 - \frac{p}{r_u}\right) v\left(h,(1-\alpha)h + \alpha l\right) - C = v\left(h,(1-\alpha)h + \alpha l\right)$$

Thus $r_u = \frac{p}{C}\left[v\left(h,h\right) - v\left(h,(1-\alpha)h + \alpha l\right)\right]$. Of course there is a natural upper bound of r_u, where everyone binds their feet. So, $r_u = min\{\frac{p}{C}\left[v\left(h,h\right) - v\left(h,(1-\alpha)h + \alpha l\right)\right],\ 1\}$.

For lower class, when $\alpha \in [\alpha_2, \frac{1}{2}]$, in equilibrium the percentage of foot-binding women should make the marriage benefit with and without foot-binding indifferent. That is:

$$\frac{p}{r_l} v\left(l,(1-\alpha)l + \alpha h\right) + \left(1 - \frac{p}{r_l}\right) v\left(l + \beta_1 l, l\right) - C = v\left(l + \beta_0 l, l\right), \text{ thus}$$
$$r_l = \frac{p[v(l,(1-\alpha)l+\alpha h) - v(l+\beta_1 l, l)]}{C + v(l+\beta_0 l, l) - v(l+\beta_1 l, l)}.$$

Again, combining the boundary case, we have:

$$r_l = min\{\frac{p[v(l,(1-\alpha)l+\alpha h) - v(l+\beta_1 l, l)]}{C + v(l+\beta_0 l, l) - v(l+\beta_1 l, l)},\ 1\}$$

Lastly, by complementarity of v, we have foot-binding is less prevalent in the lower class, i.e. $r_l \leq r_u$.

Proof of Proposition 3.4.

Proof. The proof is constructed as follows. First, we examine the possibility of inter-class marriages when $\alpha < \frac{1}{2}$. With complementarity (Assumption 2), the lower class bride never seeks for cross-class marriage. In particular, a lower class bride cannot compensate the marrying-down loss of ranking A and B grooms (in Table 3.4) through foot-binding. Thus cross-class marriage is not feasible, and there only exists competition within class.

Second, we examine within class competition among women, that upper brides are matched with ranking A and B grooms, lower class brides are matched with D and E grooms.

Lemma 3. *There exists $\alpha_1^{Ext.}$ such that no foot-binding is adopted among upper class women when $\alpha < \alpha_1^{Ext.}$.*

For the upper bride to stay in class without foot-binding, we require the cost of foot-binding is too large compared with the benefit of marrying up, that is, $v(h,h) - C \leq pv(h,h) + qv(h,(1-\alpha)h+\alpha m) + (1-p-q)\,v(h,(1-\alpha)h+\alpha l)$, then

$C \geq (1-p)\left[v(h,h) - v(h,(1-\alpha)h+\alpha l)\right] - q[v(h,(1-\alpha)h+\alpha m) - v(h,(1-\alpha)h+\alpha l)]$.

Denote the cutoff value of α as $\alpha_1^{Ext.}$, thus $\alpha_1 < \alpha_1^{Ext.}$.

Lemma 4. *There exists $\alpha_2^{Ext.}$ such that no foot-binding is adopted among lower class women when $\alpha_1^{Ext.} < \alpha_2^{Ext.} \leq \frac{1}{2}$.*

For the lower class bride to stay in class without foot-binding, we require when everyone else choose no foot-binding, a lower class bride has no incentive to do so either:

$$v(l,(1-\alpha)l+\alpha h) - C \leq$$
$$pv(l,(1-\alpha)l+\alpha h) + qv(l,(1-\alpha)l+\alpha m) + (1-p-q)\,v(l+\beta_0 l, l),$$

$$\text{then } C \geq (1-p)\left[v(l,(1-\alpha)l+\alpha h) - v(l+\beta_0 l, l)\right] - q[v(l,(1-\alpha)l+\alpha m) - v(l+\beta_0 l, l)].$$

Denote the cutoff value as $\alpha_2^{Ext.}$.

Since v is convex, $\alpha_1^{Ext.} < \alpha_2^{Ext.}$. Moreover, to make sure that lower class women adopt foot-binding when α is less than half for the multiple talent case, the upper bound of disutility is $C^{Ext.} = (1-p)\left[v\left(l,\frac{l+h}{2}\right) - v(l+\beta_0 l, l)\right] - q[v\left(l,\frac{l+m}{2}\right) - v(l+\beta_0 l, l)]$, which is smaller than $\bar{C}$ given in Assumption 4. Thus, we have $\alpha_1^{Ext.} < \alpha_2^{Ext.} < \frac{1}{2}$.

Upper class brides now compete on grooms with ranking A and B. The lower class brides compete on grooms with ranking D and E within class. Suppose equilibrium foot-binding percentages in both class are r_u^{Ext} and r_l^{Ext}, respectively, then apply the indifference condition and assume interior solution:

$\frac{p}{r_u}v\left(h,h\right)+\frac{q}{r_u}v\left(h,(1-\alpha)l+\alpha m\right)+\left(1-\frac{p+q}{r_u}\right)v\left(h,(1-\alpha)h+\alpha l\right)-C=v\left(h,(1-\alpha)h+\alpha l\right)$,

so that the proportion of foot-binding women in upper class is:

$$r_u^{Ext}=\frac{p[v(h,h)-v(h,(1-\alpha)h+\alpha l)]+q[v(h,(1-\alpha)l+\alpha m)-v(h,(1-\alpha)h+\alpha l)]}{C}.$$

Add the boundary, we have:

$$r_u^{Ext}=min\{\frac{p[v(h,h)-v(h,(1-\alpha)h+\alpha l)]+q[v(h,(1-\alpha)l+\alpha m)-v(h,(1-\alpha)h+\alpha l)]}{C},\ 1\}.$$

For lower class women, in equilibrium the percentage of foot-binding women should make the marriage benefit with and without foot-binding indifferent. That is:

$\frac{p}{r_l}v\left(l,(1-\alpha)l+\alpha h\right)+\frac{q}{r_l}v\left(l,(1-\alpha)l+\alpha m\right)+\left(1-\frac{p+q}{r_l}\right)v\left(l+\beta_1 l,l\right)-C=v\left(l+\beta_0 l,l\right)$,

so that $r_l=\frac{p[v(l,(1-\alpha)l+\alpha h)-v(l+\beta_1 l,l)]+q[v(l,(1-\alpha)l+\alpha m)-v(l+\beta_1 l,l)]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)}$.

Add the boundary, we have:

$$r_l^{Ext}=min\{\frac{p[v(l,(1-\alpha)l+\alpha h)-v(l+\beta_1 l,l)]+q[v(l,(1-\alpha)l+\alpha m)-v(l+\beta_1 l,l)]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)},\ 1\}.$$

By complementarity of v it is easy to check that $r_l^{Ext}\leq r_u^{Ext}$ in this case. Moreover, comparing the proportion of foot-binding women in either class in this case with our baseline model, it's easy to check: $r_u<r_u^{Ext}$ and $r_l<r_l^{Ext}$.

Proof of Proposition 3.5.

Proof. (1), (2) and (3): Following the logic of the proof of Proposition 1, when $\alpha=0$ there exists no incentive to practice foot-binding. In addition, following our complementarity assumption, any inter-class marriage is not feasible, so that the competition is merely within-class.

As there exists a fixed-cost for foot-binding, namely $c\left(\underline{b}\right)$, upper class women will only start foot-binding when $\alpha\in[\tilde{\alpha_1},\frac{1}{2}]$, where $\tilde{\alpha_1}$ is defined by $c\left(\underline{b}\right)=v(h,h)-v(h,(1-\tilde{\alpha_1})h+\tilde{\alpha_1}l)$, when μ of upper class women take up foot-binding, and their foot-binding intensity choice b_u perfectly absorbs the marry-up benefit: $c(b_u)=v(h,h)-v(h,(1-\alpha)h+\alpha l)$. Similarly, when $\alpha\in[\tilde{\alpha_2},\frac{1}{2}]$, where $\tilde{\alpha_2}$ is defined by $c\left(\underline{b}\right)=v(l,(1-\tilde{\alpha_2})l+\tilde{\alpha_2}h)-v(l+\beta_0 l,l)$, μ

of lower class women take up foot-binding, and their foot-binding intensity choice b_l perfectly absorbs the marry-up benefit: $c(b_l) = v(l, (1-\alpha)l + \alpha h) - v(l + \beta_0 l, l)$.

(4) When $\alpha \in (\frac{1}{2}, 1)$, that we have a flipping ranking of new elites and fallen elites, the upper class women are now competing for both the champions and the new elites, while the lower class women is competing for fallen elites left. Now the Bertrand competition leads to the unique outcome where all rents from marrying up are dissipated. That is,

$$c\left(b_C\right) = v\left(h, h\right) - v\left(h, (1-\alpha)h + \alpha l\right)$$

$$c\left(b_{NE}\right) = v\left(h, (1-\alpha)l + \alpha h\right) - v\left(h, (1-\alpha)h + \alpha l\right)$$

It is immediate that both binding intensities increase with α. In this case, upper class brides are indifferent between $b = 0, b = b_C$ or $b = b_{NE}$. Therefore the share of brides who adopt the corresponding intensities should coincide with the share of champions and new elites. The same logic applies to lower class brides, where

$$c\left(b_{FE}\right) = v\left(l, (1-\alpha)h + \alpha l\right) - v\left(l + \beta_0 l, l\right)$$

The share of lower class bride's population who bind their feet should also coincide with the rest of the fallen elites, which has size $\mu + (1-\mu)\, p - \mu = (1-\mu)\, p$. Due to complementarity, we have $b_C > b_{NE}$, and $b_C > b_{FE}$. When α is slightly larger than $\frac{1}{2}$, $b_{FE} > b_{NE}$, and when α is close to 1, $b_{FE} < b_{NE}$. □

Proof of Proposition 3.6.

Proof. (1) Similar logic of the proof of Proposition 3.1.

(2) As the foot-binding investment calls for a fix amount of cost $c(\underline{b})$, it will not be used until the marry-up benefits within class can compensate the minimum level of cost. Following similar analysis of Proposition 6, upper class women start foot-binding when $\alpha \in [\tilde{\alpha_1}, \frac{1}{2}]$,

where $\tilde{\alpha_1}$ is defined by $c\left(\underline{b}\right) = v(h,h) - v(h,(1-\tilde{\alpha_1})h + \tilde{\alpha_1}l)$, and lower class women take up foot-binding when $\alpha \in [\tilde{\alpha_2}, \frac{1}{2}]$, where $\tilde{\alpha_2}$ is defined by $c\left(\underline{b}\right) = v(l,(1-\tilde{\alpha_2})l+\tilde{\alpha_2}h) - v(l+\beta_0 l, l)$.

For upper class, when $\alpha \in (0, \tilde{\alpha_1})$, bride's family only uses dowry payments. The dowry payments will perfectly absorb the marrying up benefit, so now the dowry by the upper class brides is defined by: $u(d^*_{0,u}) = v(h,h) - v(h,(1-\alpha)h + \alpha l)$. Similarly, when $\alpha \in (0, \tilde{\alpha_2})$, the lower class brides' family only use dowry payments. The dowry payment level in lower class is defined by $u(d^*_{0,l}) = v(l,(1-\alpha)l + \alpha h) - v(l+\beta_0 l, l)$. By complementarity of v it is easy to check that $d^*_{0,u} > d^*_{0,l}$ given a certain level of $\alpha \in (0, \tilde{\alpha_1})$.

(3): When $\alpha \in [\tilde{\alpha_1}, \frac{1}{2}]$, both foot-binding and dowry payments will be used by upper class women; and when $\alpha \in [\tilde{\alpha_2}, \frac{1}{2}]$, both price tools will be used by lower class women. We can construct proof in following steps:

Step 1. We solve the continuous "package benefit" choice problem where

$$\text{Bride: } V_B\left(q_i, q_j, h_i\right) = v\left(\theta_i,\ \alpha\gamma_i + (1-\alpha)\theta_i\right) - h_i$$

$$\text{Groom: } V_G\left(q_i, q_j, h_i\right) = v\left(\theta_i,\ \alpha\gamma_j + (1-\alpha)\theta_j\right) + h_i$$

where bride's family chooses an effective marriage package benefit h_i. Denote the solution of the above problem h_i^* .

Step 2. We solve the substitution problem between foot-binding and dowry choice.

$$\min_{b_i, d_i} c\left(b_i\right) + u\left(d_i\right)$$

$$s.t.\ c\left(b_i\right) + u\left(d_i\right) \geq h_i^*$$

Solving Step 1 is exactly the same as in Section 3.5.2. So it only remains to show how the optimal b_i and d_i is solved. Given h_i^*, this minimization problem is solved when the following two equations hold:

$$c^{'}(b_i) = u^{'}(d_i)$$

$$c(b_i) + u(d_i) = h_i^*$$

With both $c(\cdot)$ and $u(\cdot)$ concave, it is immediate that the optimal b_i^* and d_i^* increase with h_i^*.

Therefore, when $\alpha \in [\tilde{\alpha_1}, \frac{1}{2}]$, both foot-binding and dowry payments will be used by upper class women. By Bertrand competition, the marry-up benefits will be completely absorbed by these two price tools. There will be μp upper class women choose binding intensity b_u^*, dowry size d_u^*, where b_u^* and d_u^* satisfy the following conditions:

$$c^{'}(b_u^*) = u^{'}(d_u^*)$$

$$c(b_u^*) + u(d_u^*) = v(h,h) - v(h,(1-\alpha)h + \alpha l)$$

When $\alpha \in [\tilde{\alpha_2}, \frac{1}{2}]$, both foot-binding and dowry payments will be used by lower class women, where $(1-\mu)p$ lower class women choose binding intensity b_l^*, dowry size d_l^*, satisfying the following conditions:

$$c^{'}(b_l^*) = u^{'}(d_l^*)$$

$$c(b_l^*) + u(d_l^*) = v(l,(1-\alpha)l + \alpha h) - v(l,l)$$

It is obvious to check that $b_u^* > b_l^*$ and $d_u^* > d_l^*$ when $\alpha \in [\tilde{\alpha_2}, \frac{1}{2}]$.

(4) When $\alpha \in (\frac{1}{2}, 1)$, both foot-binding and dowry payments will be used by both classes, and μp of upper class brides choose binding intensity b_C^*, dowry size d_C^*; $p\,(1-\mu)$ of upper class brides choose binding intensity b_{NE}^*, dowry size d_{NE}^*; and $(\mu - p)$ choose not to bind feet and pay no dowry. For the lower class brides, precisely $p\,(1-\mu)$ of them choose binding intensity b_{FE}^*, dowry size d_{FE}^*. In particular, $b_C^* > b_{NE}^*$, $b_C^* > b_{FE}^*$, $d_C^* > d_{NE}^*$, and $d_C^* > d_{FE}^*$. These values satisfy the following conditions:

$$c^{'}\,(b_C^*) = u^{'}\,(d_C^*), \text{ and } c\,(b_C^*) + u\,(d_C^*) = v(h,h) - v(h,(1-\alpha)h + \alpha l);$$

$$c^{'}\,(b_{NE}^*) = u^{'}\,(d_{NE}^*), \text{ and } c\,(b_{NE}^*) + u\,(d_{NE}^*) = v(h,(1-\alpha)l + \alpha h) - v(h,(1-\alpha)h + \alpha l);$$

$$c^{'}\,(b_{FE}^*) = u^{'}\,(d_{FE}^*), \text{ and } c\,(b_{FE}^*) + u\,(d_{FE}^*) = v(l,(1-\alpha)l + \alpha h) - v(l,l)\ .$$

Proof of Corollary 3.1.

Proof. From Proposition 3, we know that $r_l = min\{\frac{p[v(l,(1-\alpha)l+\alpha h) - v(l+\beta_1 l, l)]}{C + v(l+\beta_0 l, l) - v(l+\beta_1 l, l)},\ 1\}$, so it's immediate that r_l decreases in β_0. Next, since we have $r_l = min\{p + \frac{p[v(l,(1-\alpha)l+\alpha h) - v(l+\beta_0 l, l) - C]}{C + v(l+\beta_0 l, l) - v(l+\beta_1 l, l)},\ 1\}$, thus $\frac{\partial r_l}{\partial \beta_1}$ depends on the numerator. Consider Assumption 4 (Bounded Disutility), we have $\overline{C} = (1-p)[v(l, \frac{h+l}{2})) - v(l+\beta_0 l, l)]$. Therefore, when $C \leq min\{v(l,(1-\alpha)l + \alpha h) - v(l + \beta_0 l, l), \overline{C}\}$, r_l increases with β_1; when $C \in [v(l,(1-\alpha)l + \alpha h) - v(l + \beta_0 l, l), \overline{C}]$, r_l decreases with β_1.

Proof of Proposition 3.7.

Proof. When the α is large enough that generating a flipping ranking between the new elites and fallen elites, that $\frac{1}{2} < \alpha < 1$, the competition will become the following:

First, due to complementarity, a lower class bride never seeks for marriage with champions and new elites. In particular, for a lower class bride matched with a champion through foot-binding, the marrying-up benefit of the bride's side should be able to compensate the marrying-down loss of the champion, i.e.: $v(l,h) - v(l + \beta_0 l, l) > v(h,h) - v(l,h)$. For a lower class bride matched with a new elite through foot-binding, the marrying-up benefits of the bride's side should be able to compensate the marrying-down loss of the new elite, i.e.:

$v(l,(1-\alpha)l+\alpha h)-v(l+\beta_0 l,l)>v(h,(1-\alpha)l+\alpha h)-v(l,(1-\alpha)l+\alpha h)$. However, due to complementarity, both conditions will not be feasible. Thus cross-class marriage is not possible, and there only exists competition for lower class women competing for fallen elites.

Upper class brides now competes on both champions and new elites[47]. The unmarried brides then compete with lower class brides on the fallen elites. Suppose equilibrium foot-binding percentages in both class are r_u and r_l, respectively, then apply the indifference condition and assume interior solution:

$\frac{p}{r_u}v\left(h,h\right)+\frac{(1-\mu)p}{\mu r_u}v\left(h,(1-\alpha)l+\alpha h\right)+\left(1-\frac{p}{\mu r_u}\right)v\left(h,(1-\alpha)h+\alpha l\right)-C=v\left(h,(1-\alpha)h+\alpha l\right)$,

so that the proportion of foot-binding women in upper class is:

$r_u=\frac{p}{C}\{\mu[v(h,h)-v(h,(1-\alpha)h+\alpha l)]+(1-\mu)[v(h,(1-\alpha)l+\alpha h)-v(h,(1-\alpha)h+\alpha l)]\}$.

Add the boundary, we have:

$$r_u=$$

$$min\{\tfrac{p}{C}\{\mu[v(h,h)-v(h,(1-\alpha)h+\alpha l)]+(1-\mu)[v(h,(1-\alpha)l+\alpha h)-v(h,(1-\alpha)h+\alpha l)]\},\ 1\}.$$

For lower class women, in equilibrium the percentage of foot-binding women should make the marriage benefit with and without foot-binding indifferent. That is:

$$\tfrac{p}{r_l}v\left(l,(1-\alpha)h+\alpha l\right)+\left(1-\tfrac{p}{r_l}\right)v\left(l+\beta_1 l,l\right)-C=v\left(l+\beta_0 l,l\right),\text{ so that}$$
$$r_l=\tfrac{p[v(l,(1-\alpha)h+\alpha l)-v(l+\beta_1 l,l)]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)}.$$

Add the boundary, we have:

$$r_l=min\{\tfrac{p[v(l,(1-\alpha)h+\alpha l)-v(l+\beta_1 l,l)]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)},\ 1\}.$$

[47]Here the champions and the new elites act as a whole group for the brides. The reason is that foot-binding in this model is binary thus competition is insufficient. One might expect the upper class brides further compete on the champions using other tools. There are two ways to address the issue: First, if we model foot-binding as a continuous choice, where brides can adopt different sizes of bound-foot, then competition towards the champions drives the brides to bind their feet smaller and smaller. This is also consistent with the historical records. Second, we can further model the dowry process shortly before marriage, where bridal families pay dowry to compete for the champions. Since brides are assumed homogeneous in intrinsic quality, Bertrand competition bids away any extra benefit, and the dowry amount would equalize the marriage value difference between marrying a new elite and marrying a champion. Again such difference is increasing in social mobility parameter α, which suggests that dowry payment increases as Keju system expands. This is also consistent with the stylized facts.

By complementarity of v it is easy to check that $r_l \leq r_u$ in this case. Comparing the proportion of lower class women foot-binding when $\frac{1}{2} < \alpha < 1$ and $\alpha \leq \frac{1}{2}$, we also have for interior solutions, $r_l|_{\frac{1}{2}<\alpha<1} < r_l|_{\alpha\leq\frac{1}{2}}$, since now the fallen elites are less attractive compared to the new elites for lower class women.

Proof of Corollary 3.2.

Proof. When $\frac{1}{2} < \alpha < 1$, recall from Proposition 3.7 that $r_l = min\{\frac{p[v(l,(1-\alpha)h+\alpha l)-v(l+\beta_1 l,l)]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)},\ 1\}$. So it is immediate that r_l decreases in β_0. Next, $r_l = min\{\frac{p[v(l,(1-\alpha)h+\alpha l)-v(l+\beta_1 l,l)]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)},\ 1\} = min\{p + \frac{p[v(l,(1-\alpha)h+\alpha l)-v(l+\beta_0 l,l)-C]}{C+v(l+\beta_0 l,l)-v(l+\beta_1 l,l)},\ 1\}$. Consider Assumption 4 (Bounded Disutility), we have $\overline{C} = (1-p)[v(l, \frac{h+l}{2})) - v(l+\beta_0 l, l)]$. Therefore, when $C \leq min\{v(l,(1-\alpha)h+\alpha l) - v(l+\beta_0 l, l), \overline{C}\}$, r_l increases with β_1; when $C \in [v(l,(1-\alpha)h+\alpha l) - v(l+\beta_0 l, l), \overline{C}]$, r_l decreases with β_1. □

3.8.4 Appendix Tables and Figures

Table 3.7: Historical Sources of Foot-binding: A Comparison

Sources on Foot-binding	**Examples**	**Authors/Compilers**	**Contents**	**Time Range**	**Accessbility**
1. Official records by the Republican Government	Republican government archives(1931-1934)	The Republican government prohibited foot-binding in the 1930s. Original report from local county government. Compiled by the Ministry of Interior, Republic of China.	Describing foot-binding in the history, and anti-footbinding process.	Not restricted, mostly Qing-Republican.	The Second Historical Archives of China, Nanjing
2. Semi-official records: gazetteers (province, prefecture, county)	Historical gazetteers	local gentries, scholars and officials	Foot-binding as a folk practice and anti-footbinding process.	The late Qing-Republican.	Around 230 localities
3. Jottings(Biji), literature, privately compiled history	Qian, Yong(1759-1844), Xu, Ke(1869-1928), Yao,Ling-hsi(1899-1963), Hu,Pu-an(1878-1947)	Scholars, historians, philologists	A wide range of foot-binding history	From the Song to the Republican.	Mostly published during the Qing and Republican.
4.A. Anthropology, personal history, and interviews (historical)	Turner(1997)	Foreign visitors/scholars, missionaries, Travellers, foot-binding women	First-hand sources		See Turner (1997) and Missionary journals.
4.B. Anthropology, personal history, and interviews (contemporary)	Gates(2015), Bossen&Gates(2017)	Personal history described by footbinding women themselves	First-hand sources	Rural elderly women in 27 villages, 9 provinces	e.g. Bossen&Gates(2017).
5. The folkloristic evidence and oral literature, e.g. ballads and bound-feet beauty contest.	Zhang(2015)	The folk. Compiled by domestic or foreign observers.	Describtion in various ways.	Unknown.	Zhang(2015)
6. Archaeological evidence (bound feet shoes/lotus shoes)	Ebrey(1992)	Officials' wives and daughters.	The owener of shoes, her family status, the location of shoes	The Song (960-1279)	Ebrey(1992)

Table 3.8: The Income of Gentry Class by Occupation

			Avg. Annual Income/Avg. Labor Income		
Category	Gentry Occupation	Average Annual Income (in taels)	Col.3/Lower Bound	Col.3/Upper Bound	Col.3/Category 14
(1)	(2)	(3)	(4)	(5)	(6)
1	Rank (Central/local 1st)	180	36	18	12.00
2	Rank (Central/local 2nd)	150	30	15	10.00
3	Rank (Central/local 3rd)	130	26	13	8.67
4	Rank (Central/local 4th)	105	21	10.5	7.00
5	Rank (Central/local 5th)	80	16	8	5.33
6	Rank (Central/local 6th)	60	12	6	4.00
7	Rank (Central/local 7th)	45	9	4.5	3.00
8	Rank (local 8th)	40	8	4	2.67
9	Rank (local 9th or none)	33.11	6.62	3.31	2.21
10	Gentry services	120	24	12	8.00
11	Secretaries to officials	250	50	25	16.67
12	Teaching	100	20	10	6.67
13	Practice of traditional medicine	200	40	20	13.33
14	Scholarship Awardees	15	3	1.5	1.00

Note: Data source: Calculation based on Chang (1962). The lower bound of the male labor annual income is 5 taels and the upper bound is 10 taels.

Table 3.9: Proportion of Gentry and Literati by Province

				%Gentry among Male Pop.		%Quota among Male Pop. (16-25)	
Province	**#Quota**	**#Gentry**	**Pop.**	**SR=110**	**SR=130**	**SR=110**	**SR=130**
Fengtian	131	–	–	–	–	–	–
Zhili	5263	–	36900	–	–	–	–
Jiangsu	2594	75831	29600	0.489%	0.453%	0.084%	0.078%
Anhui	2385	53713	36600	0.280%	0.260%	0.062%	0.058%
Zhejiang	3330	86969	30400	0.546%	0.506%	0.105%	0.097%
Jiangxi	2498	78382	26500	0.565%	0.523%	0.090%	0.083%
Fujian	2196	55152	25800	0.408%	0.378%	0.081%	0.075%
Henan	3017	70874	29100	0.465%	0.431%	0.099%	0.092%
Shandong	3386	64949	36200	0.343%	0.317%	0.089%	0.083%
Shanxi	2842	51366	10300	0.952%	0.882%	0.263%	0.244%
Hubei	2011	52027	28600	0.347%	0.322%	0.067%	0.062%
Hunan	2225	63372	20000	0.605%	0.561%	0.106%	0.098%
Shanxi and Gansu	3450	70269	29800	0.450%	0.417%	0.111%	0.102%
Sichuan	2527	55263	22300	0.473%	0.438%	0.108%	0.100%
Guangdong	2453	77380	21100	0.700%	0.649%	0.111%	0.103%
Guangxi	1885	39638	8100	0.934%	0.866%	0.222%	0.206%
Yunnan	2453	43165	6200	1.329%	1.232%	0.378%	0.350%
Guizhou	1393	25448	4800	1.012%	0.938%	0.277%	0.257%

Note: Data source: Chang (1955). To facilitate our calculation, we take a lower bound assumption of the sex ratio to be 110 men per 100 women and an upper bound to be 130, and assume that the proportion of males aged 16-25 account of 20% of the total male population.

Table 3.10: Correlation Table of Variables

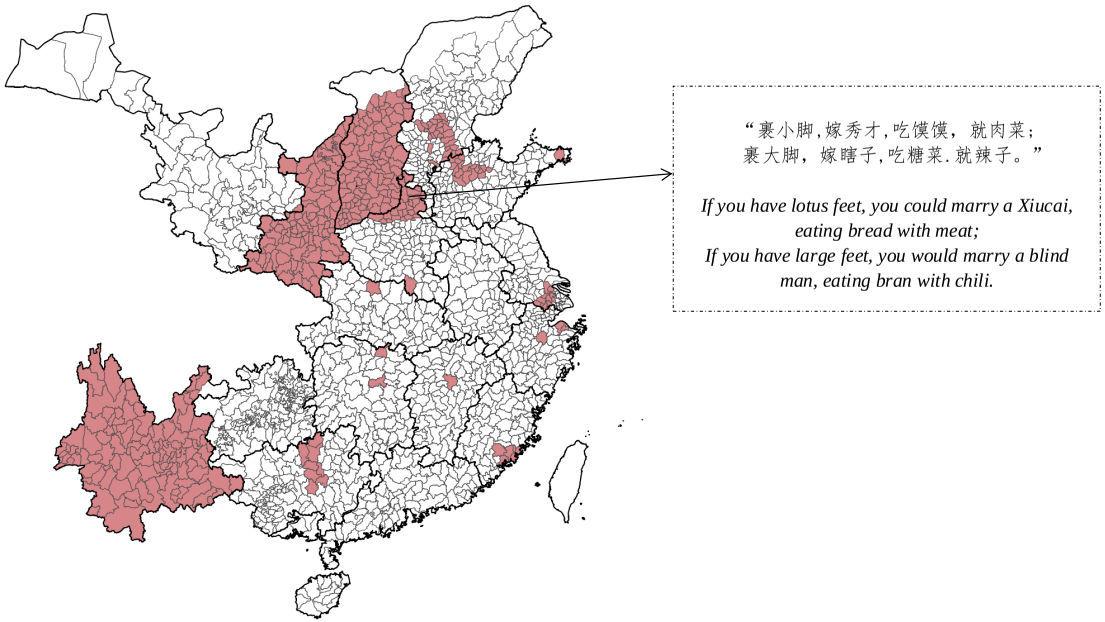
		A	B	C	D	E	F	G	H
A	Relative Suitability (Rice-Wheat)	1							
B	Cotton Suitability	-0.05	1						
C	Flax Suitability	-0.37	0.62	1					
D	Cotton Suitability	-0.15	0.07	0.09	1				
E	County Population	-0.01	0.41	0.49	0.33	1			
F	Log Distance to Nanjing	0.15	-0.82	-0.83	-0.22	-0.56	1		
G	Number of Modern Firms	-0.04	0.08	0.08	0.0906	0.26	-0.17	1	
H	Mongol Historical Migration Intensity	-0.23	0.16	0.32	0.23	0.22	-0.40	0.19	1

Table 3.11: Crop Suitability, Exam Quotas and Foot-binding Prevalence: Control For Sex Ratio

	Sample=Full Archival Sample (Four Provinces)				
	Dependent Variable: Foot-binding in Rural Area (1=High Prevalence)				
	(1)	(2)	(3)	(4)	(5)
Relative Suitability (Rice-Wheat)	-0.100***	-0.100***	-0.100***	-0.093***	-0.088**
	(0.026)	(0.027)	(0.027)	(0.027)	(0.039)
Cotton Suitability	0.114***	0.111***	0.119**	0.143**	0.175**
	(0.029)	(0.031)	(0.052)	(0.058)	(0.078)
Flax Suitability	0.039	0.042	0.049	0.061	0.014
	(0.043)	(0.045)	(0.051)	(0.052)	(0.055)
County Exam Quota	0.059*	0.062*	0.064*	0.063*	0.056†
	(0.035)	(0.036)	(0.038)	(0.038)	(0.038)
County Population	-0.077***	-0.058*	−0.057†	−0.055†	-0.021
	(0.023)	(0.034)	(0.035)	(0.036)	(0.042)
Number of Christian Churches		0.022†	0.023†	0.014	0.027*
		(0.015)	(0.015)	(0.015)	(0.015)
Number of Vicars		-0.048	-0.048	-0.047	-0.060
		(0.046)	(0.046)	(0.049)	(0.046)
Number of Disciples		-0.008	-0.009	-0.009	-0.012
		(0.035)	(0.035)	(0.035)	(0.035)
Log Distance to Nanjing (km)			0.040	0.139	-0.069
			(0.176)	(0.201)	(0.343)
Number of Modern Firms				0.006	0.006
				(0.028)	(0.027)
Mongol Historical Migration Intensity				0.057*	-0.013
				(0.032)	(0.037)
County Sex Ratio	0.029	0.026	0.030	0.028	-0.033
	(0.040)	(0.041)	(0.045)	(0.044)	(0.050)
Constant	0.780***	0.775***	0.501	-0.170	1.192
	(0.029)	(0.029)	(1.215)	(1.387)	(2.281)
Prov. FE.	N	N	N	N	Y
Observations	153	153	153	153	153
R-squared	0.350	0.353	0.353	0.364	0.398

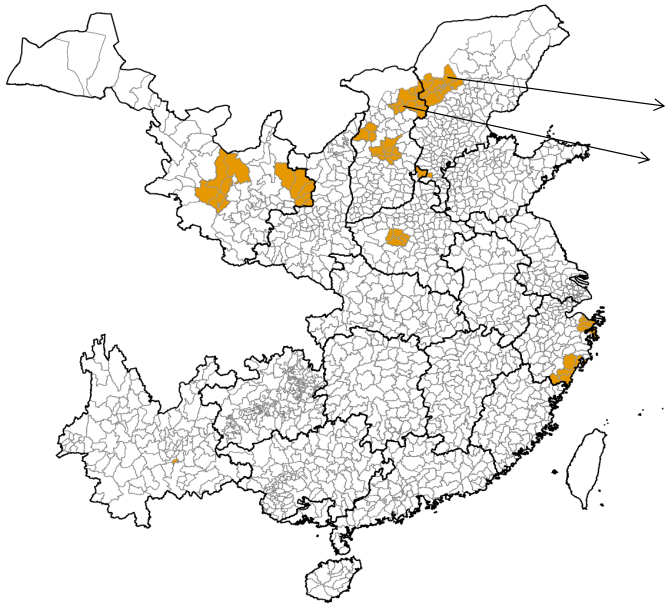
Note: *** p<0.01, ** p<0.05, * p<0.1, † p<0.15. Parentheses include robust standard errors. Dependent variable is a dummy variable indicating high prevalence of foot-binding, and key explainatory variables include relative suitability index and county Shengyuan exam quota, where both are standardized. A set of control variables includes standardized county population, cotton and flax suitability index, number of modern firms, Mongol historical migration intensity, christianity activity variables, as well as log distance to Nanjing. Sex ratio data is from Yearbook of Domestic Affairs(1931).

Figure 3.9: The Distribution of Ballads on Foot-binding



Note: The shaded counties are those with ballads on foot-binding and marriage. Source: Zhang(2015). Base map is CHGIS v4. An example is Zhangde county (Henan: *"If you have lotus feet, you could marry a Literati (Xiucai), eating bread with meat; If you have large feet, you would marry a blind man, eating bran with chili."*.

Figure 3.10: The Distribution of Bound Feet Beauty Contest (Sai Zu Hui)



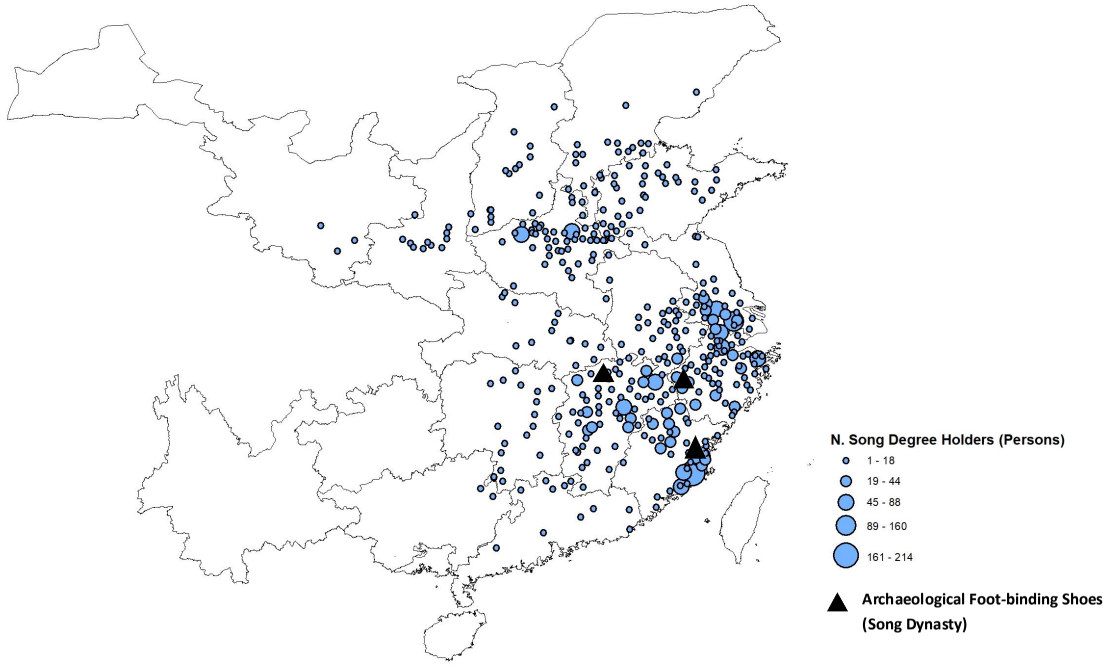


Picture: Contest Scenes 《赛会饬景—双足伸向帘外》
Author: Xing Xing Dian 星兴典
Lines in Picutre:
潘妈一声令下东厢房戈香莲伸向外门蝴蝶履，
赛脚会顿开始西厢屋白金宝晾赛淡绿小弓窝。

With Ms. Pan's ordering, Ge Xianglian of east room lets her feet out,
wearing butterfly shoes;The bound feet beauty contest started,
Bai Jinbao of west room, shows off light green arched shoes.

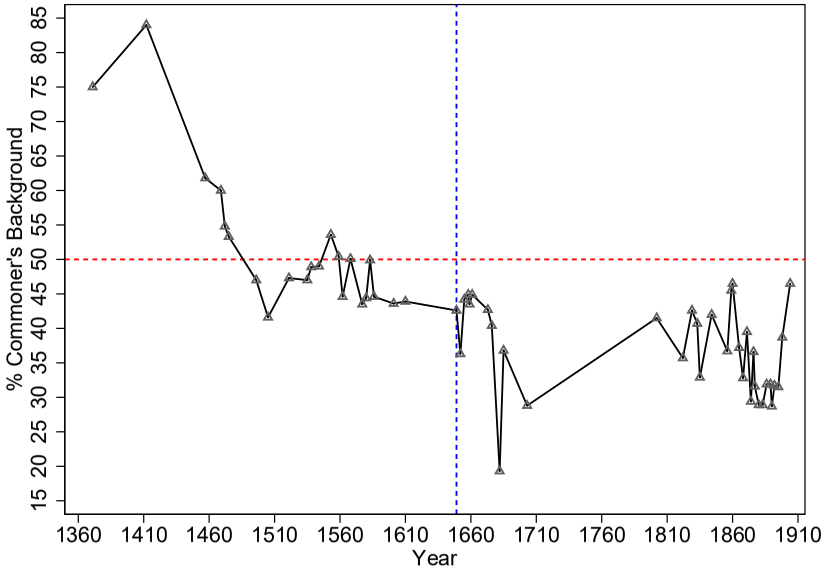
Note: The shaded counties are those with Bound Feet Beauty Contest (in Chinese, *Sai Zu Hui*), as recorded by Zhang(2015),Nagao(1973) and Yao(1936). Base map here is CHGIS v4. The source of picture is from the Foot-binding Forum (organized by Dr. Ke Jisheng), http://www.footbinding.com.tw/.

Figure 3.11: Northern Song Entry Exams and Location of Archaeological Lotus Shoes



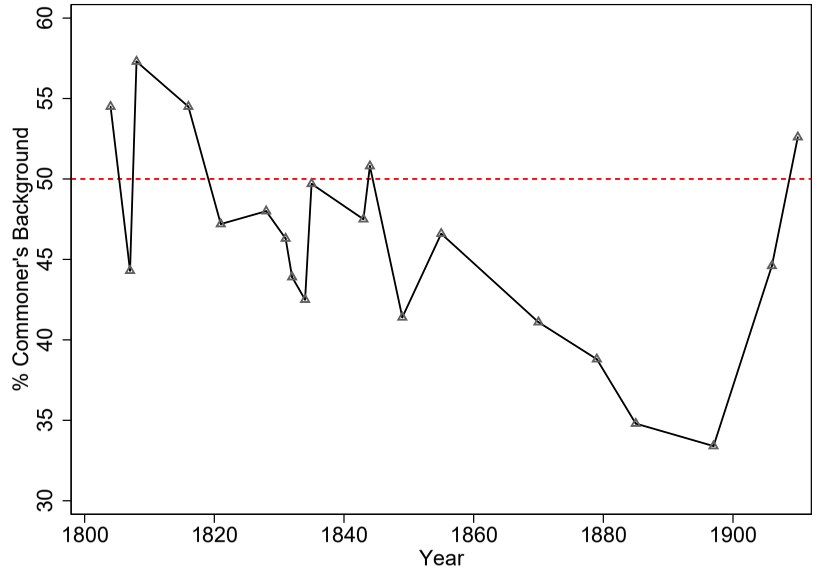
Note: Northern Song Entry Exams(persons) comes from Harvard China Map system, and the original data is from CBDB. The information of archaeological lotus shoes is from Ebrey(1992). Stars denote the locations were lotus shoes were found: Fuzhou (Fujian), Quzhou (Zhejiang) and De'An (Jiangxi).

Figure 3.12: The Proportion of Degree Holders from Commoners' Families: Jinshi



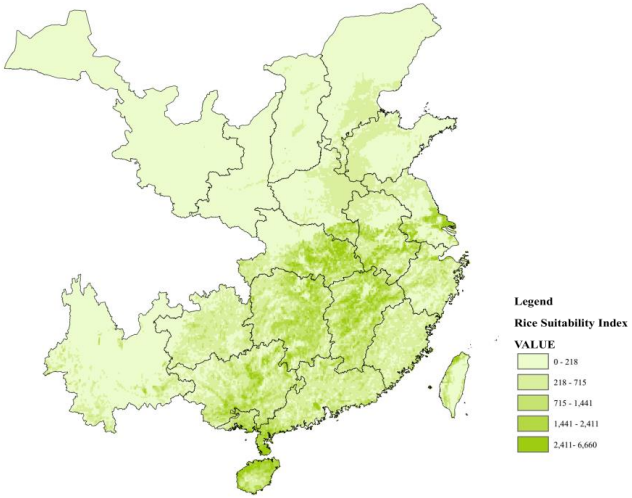
Note: Data source: Ho(1962), Table 9–The Social Backgrounds of *Jinshi* during the Ming and Qing. Here the Y-axis is the proportion of *Jinshi* from type A and B families. Type A families are those without any degree holders among male immediate family members of recent three generations. Type B families are those with Shengyuan degree or Jiansheng (during the Qing) among male immediate family members of recent three generations. Type C families are those with higher degrees than Shengyuan (i.e. Jiansheng, Juren, or Jinshi) among male immediate family members of recent three generations.

Figure 3.13: The Proportion of Degree Holders from Commoners' Families: Juren

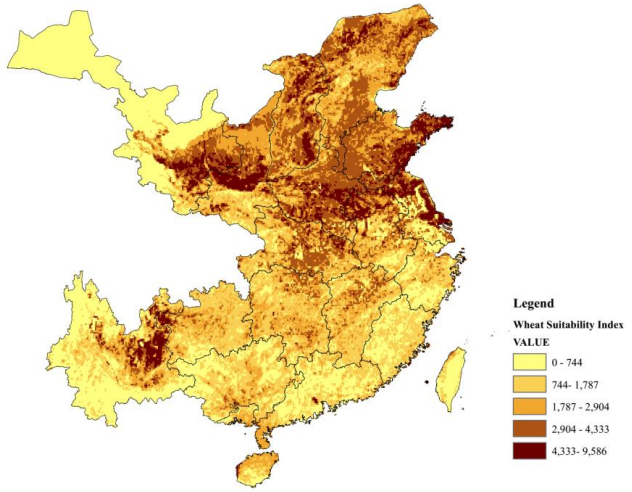


Note: Data source: Ho(1962), Table 11–The Social Backgrounds of *Juren* and *Gongsheng* during the late Qing. Here the Y-axis is the proportion of *Juren* and *Gongsheng* from type A and B families. Type A families are those without any degree holders among male immediate family members of recent three generations. Type B families are those with Shengyuan degree or Jiansheng (during the Qing) among male immediate family members of recent three generations. Type C families are those with higher degrees than Shengyuan (i.e. Jiansheng, Juren, or Jinshi) among male immediate family members of recent three generations.

Figure 3.14: Regional Distribution of Rice/Wheat Suitability in China



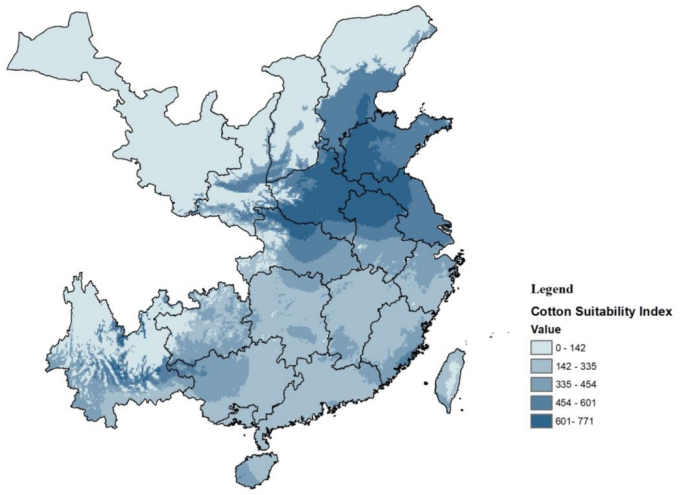
(A) Rice Suitability Index (raw value from GAEZ)



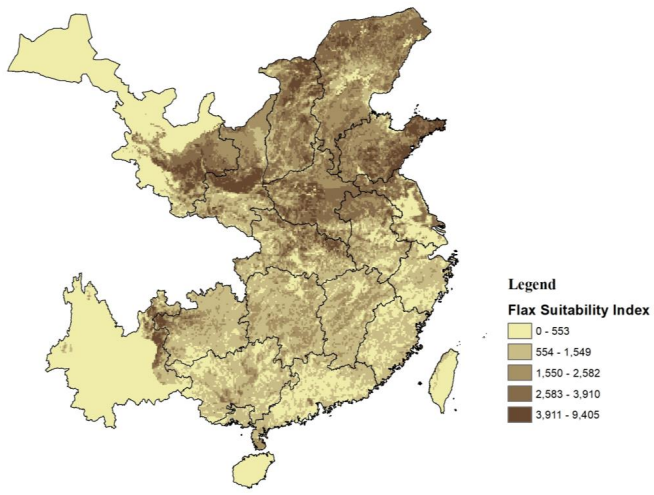
(B) Wheat Suitability Index (raw value from GAEZ)

Note: The data in this map comes from geographic distribution of crop suitability from the FAOs GAEZ (Global Agro-Ecological Zones). The suitability index of a certain crop is estimated based on a model, which has been applied considering the average climate of baseline period 1961-1990 reecting suitability levels and distributions within grid cells. Panel A shows the suitability index for rice, and panel B shows that for wheat. The base map is from CHGIS v4.

Figure 3.15: Regional Distribution of Cotton and Flax Suitability in China



(A) Cotton Suitability Index (raw value from GAEZ)



(B) Flax Suitability Index (raw value from GAEZ)

Note: The data in this map comes from geographic distribution of crop suitability from the FAOs GAEZ (Global Agro-Ecological Zones). The suitability index of a certain crop is estimated based on a model, which has been applied considering the average climate of baseline period 1961-1990 reecting suitability levels and distributions within grid cells. Panel A shows the suitability index for cotton, and panel B shows that for flax. The base map is from CHGIS v4.

3.9 References

Alesina, A., Giuliano, P., & Nunn, N. (2013). On the Origins of Gender Roles: Women and the Plough. *The Quarterly Journal of Economics*, 128(2), 469-530.

Anderson, S. (2003). Why Dowry Payments Declined with Modernization in Europe but are Rising in India. *Journal of Political Economy*, 111(2), 269-310.

———. (2004). Dowry and Property Rights. Department of Economics, University of British Columbia. Mimeo.

———. (2007). Why the Marriage Squeeze Cannot Cause Dowry Inflation. *Journal of Economic Theory*, 137(1), 140-152.

Bai, Y., & Jia, R. (2016). Elite Recruitment and Political Stability: The Impact of the Abolition of China's Civil Service Exam. *Econometrica*, 84(2), 677-733.

Becker, G. S. (1973). A Theory of Marriage: Part I. *The Journal of Political Economy*, 813-846.

———. (1974). A Theory of Marriage: Part II. *The Journal of Political Economy*. S11–S26.

———. (1991). *A Treatise on the Family*. Cambridge, MA: Harvard Univ. Press.

Bellemare, M. F., Novak, L., & Steinmetz, T. L. (2015). All in The Family: Explaining the Persistence of Female Genital Cutting in West Africa. *Journal of Development Economics*, 116, 252-265.

Blake, C. F. (1994). Foot-binding in Neo-Confucian China and the Appropriation of Female Labor. *Signs*, 19(3), 676-712.

Bhalotra, S. R., Chakravarty, A., & Gulesci, S. (2016). The Price of Gold: Dowry and Death in India. *Working Paper*.

Bhaskar, V. and Ed H.. (2016). Marriage as a Rat Race: Noisy Premarital Investments with Assortative Matching. *Journal of Political Economy*. 124:4, 992-1045

Bloch, F., & Rao, V. (2002). Terror as A Bargaining Instrument: A Case Study of Dowry

Violence in Rural India. *The American Economic Review*, 92(4), 1029-1043.

Boserup, E. (2007). *Woman's Role in Economic Development.* Earthscan.

Bossen, L., Xurui, W., Brown, M. J., & Gates, H. (2011). Feet and Fabrication: Footbinding and Early Twentieth-Century Rural Women's Labor in Shaanxi. *Modern China*, 37(4), 347-383.

Bossen, L., & Gates, H. (2017). *Bound Feet, Young Hands: Tracking the Demise of Footbinding in Village China.* Stanford University Press.

Brown, M. J. , Bossen, L., Hill Gates and Damian Satterthwaite-Phillips. (2012). Marriage Mobility and Footbinding in Pre-1949 Rural China: A Reconsideration of Gender, Economics, and Meaning in Social Causation, *The Journal of Asian Studies*, 71(4), 1035-1067

Buck, J. L. (1937). *Land Utilization in China: A Study of 16,786 Farms in 168 Localities, and 38,256 Farm Familites in Twenty-two Provinces in China, 1929-1933--Statistics (Vol. 2).* Commercial Press, Limited, Agents in the United States, The University of Chicago Press.

Cabral, L. M. (1988). Asymmetric Equilibria in Symmetric Games with Many Players. *Economics Letters,* 27(3), 205-208.

Carvalho, J. P. (2013). Veiling. *The Quarterly Journal of Economics*, 128(1), 337-370.

Chaffee, John W. (1995). *The Thorny Gates of Learning in Sung China: A Social History of Examinations.* Albany: State University of New York Press.

Chang, C. (1955). *The Chinese Gentry: Studies on Their Role in Nineteenth Century Chinese Society.* Seattle, WA: University of Washington Press.

———. (1962). *The Income of the Chinese Gentry.* University of Washington Press.

Chang, Y. 1987. "Foreign Industries in Late Chingand the Early Republic of China." *Bulletin of the Institute of Modern History*, Academia Sinica 16:129-249.

———. 1989. "Private Industries in the Late Ching and the Early Republic of China, 1860-1916." *Bulletin of the Institute of Modern History*, Academia Sinica 18:315-561.

Chen, G. (1936). *A History of Chinese Marriage.*[in Chinese, *Zhongguo Hunyin Shi*]. Com-

mercial Press.

Chen, T. and Kung, J. K.S. and Ma, C. (2017). Long Live Keju! The Persistent Effects of China's Imperial Examination System. *Working Paper.*

Chen, T. and Kung, J. K. S.(2016).The Rise of Meritocracy. *Working Paper.*

Cheung, S. N. (1972). The Enforcement of Property Rights in Children, and The Marriage Contract. *The Economic Journal*, 641-657.

CHGIS, Version 4. Cambridge: Harvard Yenching Institute and Fudan Center for Historical Geography, January 2007.

Chiappori, P. A., Iyigun, M., & Weiss, Y. (2009). Investment in Schooling and the Marriage Market. *The American Economic Review*, 99(5), 1689-1713.

Davin, D. (1976). *Woman-work: Women and the Party in Revolutionary China.* Oxford: Clarendon Press.

Daly, M. (1978). Gyn/Ecology. Boston, MA:Beacon Press.

Ebrey, P. (1990). Women, Marriage, and the Family in Chinese History. Pp. 197-223 in *Heritage of China*, edited by P.S. Ropp. Berkeley, CA: University of California Press.

Edlund, L. (2000). The marriage squeeze interpretation of dowry inflation: a comment. *Journal of Political Economy*, 108(6), 1327-1333.

Elman, B. A. (1991). Political, Social, and Cultural Reproduction via Civil Service Examinations in Late Imperial China. *The Journal of Asian Studies*, 50(1), 7-28.

———. (2000). *A Cultural History of Civil Examinations in Late Imperial China.* Berkeley: University of California Press.

———. (2013). *Civil Examinations and Meritocracy in Late Imperial China.* Cambridge, Mass: Harvard University Press.

Gale, D., & Shapley, L. S. (1962). College Admissions and the Stability of Marriage. *The American Mathematical Monthly*, 69(1), 9-15.

Galor, O., & Moav, O. (2006). Das Human-kapital: A Theory of the Demise of the Class Structure. *The Review of Economic Studies*, 73(1), 85-117.

Gao, W. (1997). History of Cotton and Economic Structure in Rural Qing (in Chinese. *Mianye Lishi yu Qingdai Nongcun Jingji Jiegou*). *The Qing History Journal* (*Qingshi Yanjiu*), 4, 002.

Gates, H. (1989). The Commoditization of Chinese Women. *Signs*, 14(4), 799-832.

———. (2001). Footloose in Fujian: Economic Correlates of Footbinding. *Comparative Studies in Society and History*, 43(01), 130-148.

———. (2008). Bound Feet: How Sexy Were They?. *The History of the Family*, 13(1), 58-70.

Giuliano, P. (2017). Gender: An Historical Perspective. *NBER Working Paper 23635.*

Goldin, C. (1990). *Understand the Gender Gap: An Economic History of American Women.* Oxford University Press, Oxford, UK.

———. (2006). The Quiet Revolution That Transformed Women's Employment Education and Family, *American Economic Review Papers and Proceedings*, 96, no. 2, 1–23

Goldin, C., & Katz, L. F. (2002). The Power of the Pill: Oral Contraceptives and Women's Career and Marriage Decisions. *Journal of Political Economy*, 110(4), 730-770.

Grosjean, P. A., & Khattar, R. (2016). It's Raining Men! Hallelujah?. *Working paper.*

Greenhalgh, S. (1977). Bound Feet, Hobbled Lives: Women in Old China. *Frontiers* 2:7-21

Guo, R, Ng, T. and Wong, K. (2015). Sworn Spinsters: An Economic Analysis of Celibacy. *Working paper.*

Guo, S. (2000). *Ethics and Life: The Marriage Relationship of the Qing Dynasty.* [in Chinese, *Lunli yu Shenghuo: Qingdai de Hunyin Guanxi*]. The Commercial Press.

Hartwell, R. (1982). Demographic, Political, and Social Transformations of China,750-1550. *Harvard Journal of Asiatic Studies* 42(2):365-442

Ho, P. (1962). *The Ladder of Success in Imperial China.* New York: Columbia University Press.

Hoppe, H. C., Moldovanu, B., & Sela, A. (2009). The Theory of Assortative Matching Based on Costly Signals. *The Review of Economic Studies*, 76(1), 253-281.

Hsu, I. C. Y. (1970). *The Rise of modern China.* Oxford University Press.

Hu, P. (1936). *The Records of China's Folk Customs* (in Chinese: *Zhonghua Fengsu Zhi*), Vol. 12. Shanghai: Wenyi Press.

Hymes, R. (1986). *Statesmen and Gentlemen: The Elite of Fu-Chou Chiang-Hsi, in Northern and Southern Sung.* Cambridge University Press.

Iyigun, M., & Walsh, P. R. (2007). Building the Family Nest: Premarital Investments, Marriage Markets, and Spousal Allocations. *The Review of Economic Studies*, 74(2), 507-535.

Ko, D. (1997). The Body as Attire: The Shifting Meanings of Footbinding in Seventeenth-century China. *Journal of Women's History*, 8(4), 8-27.

———. (2001). *Every Step a Lotus: Shoes for Bound Feet.* Univ of California Press.

———. (2005). *Cinderella's Sisters: A Revisionist History of Footbinding.* Univ of California Press.

Kun, Gang et al. (Ed.) (1899), *Imperially Established Institutes and Laws of the Great Qing Dynasty* (in Chinese: *Da Qing Hui Dian Shi Li*).

Kung, J. K. S., Jiang, Q. (2015). "The Making of the Gentry Class: Reconsidering the Ladder of Success in Late Imperial China". *Working paper.*

Levy, H. S. (1966). *Chinese Footbinding: The History of a Curious Erotic Custom.* W. Rawls.

Lindert, P. H. (2000). Three centuries of inequality in Britain and America. In A. B. Atkinson & F. Bourguignon, *Handbook of Income Distribution.* Amsterdam: North-Holland

Mackie, G. (1996). Ending Footbinding And Infibulation: A Convention Account. *American Sociological Review,* 999-1017.

Magone, R.(2015). *The Examination System.* Oxford Bibliographies.

Mann, S. (1997). *Precious Records: Women In China's Long Eighteenth Century.* Stanford University Press.

Mariani, F. (2012). The Economic Value of Virtue. *Journal of Economic Growth,* 17(4), 323-356.

Miyazaki, I. (1977). *Studies of the Regulations of the Nine Ranks Bureaucratic System: The Prehistory of the Examination System.* Dobosha.

Nagao, R. (1973). *Enthnography of Chinese Folk Customs.* vol.2.Tokyo: Kokusho Kankokai.

Oster, E. (2016). Unobservable Selection and Coefficient Stability: Theory and evidence. *Journal of Business & Economic Statistics.*1-18.

Pan, G. (1987). *Psychology of Sex.* SDX Joint Publishing Company.

Peters, M., & Siow, A. (2002). Competing Premarital Investments. *Journal of Political Economy*, 110(3), 592-608.

Poyker, M. (2016). Regime Stability and Persistence of Traditional Practices. *Working paper.*

Qian, M. (2004). *State, Imperial Examinations and the Society: An Investigation of the Ming.*[in Chinese*, Guojia, Keju yu Shehui: Yi Mingdai Wei Zhongxin de Kaocha*] Beijing: National Library of China Publishing House.

Qian, N. (2008). Missing Women and the Price of Tea in China: The Effect of Sex-Specific Earnings on Sex Imbalance. *The Quarterly Journal of Economics*, 1251-1285.

Qian, Y. (1969).*Collected words from Luyuan* [in Chinese: *Luyuan conghua*]. Taipei: Guangwen Shuju.

Rao, V. (1993). The Rising Price of Husbands: A Hedonic Analysis of Dowry Increases in Rural India. *Journal of Political Economy*, 101(4), 666-677.

Shang, Yanliu. (2004). *Review on the Civil Exam in Qing China* [in Chinese: *Qingdai Keju Kaoshi Shulu ji Youguan Zhuzuo*]. Tianjin: Baihua Wenyi Press.

Shiue, C. H. (2017). Human capital and fertility in Chinese clans before modern growth. *Journal of Economic Growth*, 22(4), 351-396.

———. (2017). Social mobility in the long-run: An analysis with five linked generations, 1300–1900. Mimeo.

Stauffer, Milton T. (1922). *The Christian Occupation of China: A General Survey of the Numerical Strength and Geographical Distribution of the Christian Forces in China.* China

Continuation Committee, Shanghai.

Turner, C. L. (1997). Locating footbinding: variations across class and space in nineteenth and early twentieth century China. *Journal of Historical Sociology*, 10(4), 444-479.

Veblen, T. (1899). *The Theory of the Leisure Class.* New York: The New American Library.

———. (1894). The Economic Theory of Woman's Dress. pp. 65–77 in L.Ardzrooni (ed.), *Essays in Our Changing Order*, New York: Viking Press.

Wang, P. (2000). *Aching for Beauty: Footbinding in China.* U of Minnesota Press.

World Health Organization (2012), *Female Genital Mutilation*, Fact Sheet #241.

Wu, S. & Cao, S. (1997). *The Migration History of China (Vol. 4).* Fujian People's Press.

Xu, K. (1984). *Qing Petty Matters Anthology* (in Chinese, *Qingbai leichao*). Chung Hwa Book.

Xue, M. M. (2017). High-Value Work and the Rise of Women: The Cotton Revolution and Gender Equality in China. *Working Paper.*

Yang, X. (2012). *The Contested Body: The Anti-Footbinding Movement in modern China* [in Chinese, *Shenti zhi zheng: Jindai Zhongguo fanchanzu de licheng*]. Beijing: Social Sciences Academic Press.

Yao, L. (1936). *A Record of Gathering Radishes* [in Chinese, *Cai Fei Lu*]. 2 vols. Tianjin: Shidai Gongsi Yinxing.

Yao, J. (1991). *Customs of Binding Feet in China* [in Chinese, *Zhongguo Chanzu Fengsu*). Shenyang: Liaoning University Press.

Zhang, B. (2003). *A History of Marriage and Family in Song* [in Chinese: *Songdai Hunyin Jiazu Shilun*]. People's Press.

Zhang, R. (2015). *One Thousand Years of Foot-binding* [in Chinese: *Sancun Jinlian Yiqian Nian*]. Chung Hwa Book.

Zhang, W. and Liang, Z. (2013). Quota System and The Development of Regional Culture. (in Chinese: *Dinge Zhidu yu Quyu Wenhua de Fazhan*). Lijiang Press.