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COMMENT ON "A SIMPLE MODEL FOR THE 1/f-TYPE POWER SPECTRUM" BY M. AGU

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Comment on "A Simple Model for the 1/f-Type Power Spectrum" by M. Agu

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ABSTRACT

A Gaussian distribution of S, where $1/D_oS$ is a relaxation time, does not yield the 1/f spectrum.

The conclusion of M. Agu[1] that a Gaussian distribution of S, where $\tau=1/D_oS$ is a relaxation time, yields a 1/f power spectrum is incorrect. In 1950, A. van der Ziel[2] showed that a distribution of relaxation times, τ, proportional to $1/\tau$ gave the required 1/f spectrum.

Briefly, a variable, A_s, which obeys a Langevin equation of the form

$$\frac{d}{dt}A_s+D_oSA_s=0, \qquad (1)$$

has an autocorrelation function

$$\langle A_s(t)A_s(t+t')\rangle=\langle(\Delta A)^2\rangle\exp(-|t'|D_oS), \qquad (2)$$

and a power spectrum

$$<|A_s(\omega)|^2> = \frac{<(\Delta A)^2> D_o S/\pi}{\omega^2 + D_o^2 S^2}, \tag{3}$$

where $<(\Delta A)^2>$ is independent of S. For an ensemble of many such systems with different values of S occurring with probability P(S), the ensemble spectrum

$$<|A(\omega)|>^2 = \frac{<(\Delta A)^2> D_o}{\pi} \int \frac{P(S) S dS}{\omega^2 + D_o^2 S^2}. \tag{4}$$

The inclusion of negative values of S is physically unreasonable in a Langevin equation. For $P(S)=N\exp(-S^2/\sigma)$ for $S \geq 0$,

$$<|A(\omega)|^2> = \frac{N<(\Delta A)^2>}{2D_o \pi} e^x \int_x^\infty \exp(-t)\, dt/t \tag{5}$$

where $x = \omega_o^2 / D_o^2 \sigma$. In the limit of $\omega \to 0$

$$<|A(\omega)|^2> \to \frac{N<(\Delta A)^2>}{2D_o \pi} \ln\left(\frac{D_o^2 \sigma}{\omega^2}\right), \tag{6}$$

which is not the 1/f spectrum.

If, instead of an ensemble of systems of different S, one considers a single system with an S that fluctuates in time, one must solve the more complicated equation,

$$\frac{d}{dt} A(t) + D_o S(t) A(t) = 0. \tag{7}$$

Solutions to Eq. (7) are readily found only in two limits. If the correlation time for fluctuations in S, τ_s, is much greater than the average relaxation time, $1/D_o\langle S\rangle$, S will be constant for the decay of any given fluctuation in A(t) and Eq. (4) applies. If, on the other hand, $\tau_s \ll 1/D_o\langle S\rangle$, the fluctuations in S are averaged over a single decay in A(t) and

$$\langle|A(\omega)|^2\rangle = \frac{\langle(\Delta A)^2\rangle D_o\langle S\rangle/\pi}{\omega^2+D_o^2\langle S\rangle^2} . \qquad (8)$$

The "explanation" of the 1/f spectrum as the proper distribution of time constants has been known for some time. Although mathematically correct, it has not been of great help in developing physical theories for the occurrence of 1/f noise. It is as difficult to explain the proper distribution of time constants as the 1/f spectrum itself.

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REFERENCES

1. M. Agu, Phys. Lett. 51A (1975) 77.
2. A. Van der Ziel, Physica 16, (1950) 359.

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