Algebraic reasoning in 3- to 5-year-olds

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Abstract

The current study asks when children begin to understand that when an object is added to a set, the numerosity of the set has increased regardless of set size. This knowledge can be expressed algebraically as ‘x + 1 > x’. In Experiment 1, 3- to 5-year-old children were asked to reason about transformations (i.e., addition, subtraction, rearrangement) performed on a visible set of objects. We found that 5-year-olds were able to reason about how each transformation affected numerosity, and 4-year-olds showed limited understanding. In Experiment 2, children were asked to reason about transformations performed on a hidden set of objects. Similar results were found. Together, we showed that the ability to reason about number algebraically develops gradually between the ages of 3 and 5. Implications for number word acquisition were discussed.

Keywords: algebraic reasoning, preschoolers, number concept, numerical transformations

Introduction

In the last few decades, research on the development of numerical cognition has focused mostly on the representation and reasoning about particular numbers such as ‘two’, ‘five’, and ‘fifty’. For example, researchers have examined how the Approximate Number System (ANS) represents numbers (Dehaene, 1997; Xu & Spelke, 2000), the role of counting in children’s concept of number (Fuson, 1988; Gelman & Gallistel, 1978; Wynn, 1990, 1992), and how Parallel Individuation, an object tracking system, may be involved in number word learning (Carey, 2009; Le Corre & Carey, 2007). However, our knowledge of number is not restricted to particular numbers; it also involves knowledge about number that applies to *all possible numbers*. For example, for any given set of objects with numerosity x, x changes whenever an object is added to or subtracted from the set; this knowledge applies regardless of what numerosity x refers to. Thus, whether the set contains 1, 5, or 100 objects, when an object is added to the set, the numerosity has increased. We can represent this with algebraic expressions ‘x + 1 > x’. The present studies investigate the developmental trajectory of children’s understanding of algebraic number.

Previous research has shown that sometime between the preschool and kindergarten years, children begin to show signs of knowledge that adding one element to a set increases its numerosity. In one study, 5-year-olds were shown a box of objects labelled with a number word. The experimenter then performed a transformation that changed the numerosity (e.g., taking away one object; a numerical transformation) or a transformation that did not change the numerosity (e.g., shaking the box; a non-numerical transformation), and asked whether there was still the same number of objects in the box (Lipton & Spelke, 2006). Crucially, the researchers used numbers that were beyond children’s counting range (e.g., 127) to ensure that children could not recruit particular facts about number words to succeed on the task. They found that children correctly responded with the original number word after non-numerical transformations, but a different number word after numerical transformations. This finding shows that 5-year-olds understand the circumstances under which a change in number word is licensed. On the assumption that children understand how number words represent numerosities, this suggests that they understand when the numerosity has changed.

Some studies suggest that younger children have similar knowledge; however, the evidence is weaker. Sarnecka and Gelman (2004) presented older 2- and 3-year-olds with one set of objects placed in an opaque box and labeled it with a large (e.g., five, six) number word that was outside the counting range of children. Then, the experimenter performed a numerical (adding one object, subtracting one object) or a non-numerical (shaking) transformation. Children were asked whether the original or a new, alternative number word applied, e.g., “Now how many moons – is it five or six?” They found that children correctly chose the alternative number word after the numerical transformation and the original number word after the non-numerical transformation.

However, in another study with a slightly different paradigm, the experimenter used two sets of objects rather than one, and labelled one of the two sets with a number word (e.g., “This tray has [five] sheep”; Condry & Spelke, 2008). Children between the ages of 3 and 3 ½ were tested, and number words that were outside of their known number word range (e.g., five, seven) were used. A numerical or non-numerical transformation was performed on the labelled set, and the experimenter asked children to point to the tray that was labelled by either the original number word (e.g., five) or a different number word (e.g., six). Using the two-set task, 3-year-olds were equally likely to choose the labelled and the unlabelled tray regardless of the type of transformation, suggesting that they may not understand how transformation affects number word meanings. Some have proposed that the success on the one-set task could be explained by domain-general pragmatic reasoning, rather than domain-specific numerical reasoning (Brooks, Audet, & Barner, 2013). Despite the factors that may explain these conflicting findings, most previous studies on children's understanding of number require them to interpret number words in the tasks. Children's apparent difficulty in mastering the logic of number words may reflect a lack of knowledge of how number words represent number, and might not constitute evidence for a lack of numerical knowledge per se. Moreover, studies on children's verbal numerical comparison have found that 4-year-olds may not understand that "ten" is more than "six" (Le Corre & Carey, 2007; Schaeffer, Eggleston, & Scott, 1974). Thus, even if children apply a different, larger number word after the addition of one object, this does not necessarily show that they understand the numerosity has increased as a result of addition.

The current experiments investigate the developmental trajectory of children's understanding of algebraic number – specifically, the knowledge that when an object is added to a set, it necessarily follows that the numerosity has increased. In two experiments, children's reasoning about changes in numerosities of visible sets (Experiment 1) and hidden sets (Experiment 2) was investigated. Given that in previous studies, children demonstrate conflicting behaviour in tasks involving number words, the current experiments did *not* require children to interpret or use number words.

Experiment 1 – Visible Sets

To investigate when children begin to understand that adding one element to a set of objects increases its numerosity regardless of set size, children were asked to reason about numerical and non-numerical transformations of visible sets of objects that differed in set size – small and large. Small number trials always began with a set of 2 objects, and large number trials always began with a set of 15 objects. For both small and large sets, children observed four kinds of transformation: addition of an object ('Plus 1'), subtraction of an object ('Minus 1'), addition and subtraction of the same object ('Minus A Plus A'), and moving one object ('Move 1'). After the transformation, children were asked whether there were more objects in the set.

Given that children cannot rely on the ANS for reasoning about large sets because a numerical ratio of 15:14 (in the case of subtraction) or 15:16 (in the case of addition) is beyond the range of ratio discrimination that preschoolers can perform (see Halberda & Feigenson, 2008), their performance on large number trials is critical for determining whether children are capable of reasoning about algebraic number. However, if they fail at reasoning about large sets, it could be due to task demands. To ensure that task demands are not an issue, small sets were included. Given that children can recruit Parallel Individuation to reason about small sets, if they succeed with small sets but fail with large sets, this provides evidence that they do not reason about number algebraically. However, if they fail with both small and large sets, this suggests that their failure may be due to a difficulty in understanding the task. Importantly, children were prevented from counting.

Method

Participants Forty-eight children between the ages of 3 and 5 participated. There were 19 3-year-olds (range = 3;1 to 3;11; mean = 3;6), 14 4-year-olds (range = 4;0 to 4;11; mean = 4;3), and 15 5-year-olds (range = 5;0 to 5;11; mean = 5;6). They were recruited at daycare centres in Kitchener-Waterloo and nearby areas.

Design and Procedure At the beginning of the session, children were introduced to two puppets – Winnie the Pooh and Giraffe. Children first completed two familiarization trials, followed by 16 experimental trials. The purpose of the familiarization trials was to test whether children can reason about changes made to a single object. During familiarization, Winnie the Pooh took out a ball that could be made bigger and smaller (Hoberman's Sphere), and asked Giraffe to make it bigger. On one trial, Giraffe made it bigger; on another trial, Giraffe made it smaller. The direction of the change was counterbalanced across participants. After each trial, children were asked, "Does Winnie the Pooh have a bigger ball now?" A 'yes' or 'no' response was recorded. If children answered incorrectly, feedback was given. Almost all children succeeded on the two familiarization trials with no feedback. One child failed one of the trials but succeeded on a second attempt, and was included in the analyses.

The experimental trials began with a dialogue between Winnie the Pooh and Giraffe.

Winnie the Pooh: Giraffe, I have some toys to show you. Do you want to see?
Giraffe: Yeah, sure!
Winnie the Pooh: Look, here are my [blocks] and I want more [blocks]. Can you help me?
Giraffe: Yeah, let me help you. (Giraffe performs the transformation).

Then the experimenter asked, "Does Winnie the Pooh have more [blocks] now?" Children gave a 'yes' or 'no' response. Stimuli included eight different kinds of objects – pom poms, bows, buttons, red Lego blocks, yellow Lego blocks, bells, rocks, and hearts. Real objects were used. All objects were presented on a coloured sheet of letter-sized paper (see Figure 1).

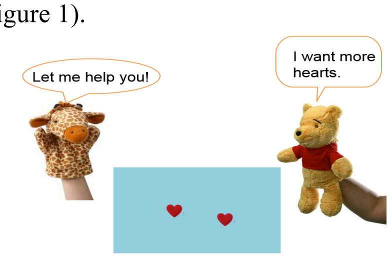

Figure 1: An illustration of the experimental set-up.

Children received both small and large number trials, which were conducted in blocks. The order of the blocks was counterbalanced between children. For each number block, the experimenter transformed the set in one of four ways: (a) by moving one object ('Move 1'); (b) by adding one object ('Plus 1'); (c) by removing one object ('Minus 1'); and (d) by moving one object outside of the sheet and then putting it back into the set of objects ('-A+A). There were two trials for each transformation, making a total of 16 trials for each child. There were two possible item orders, and the order of trial type was randomized such that no two consecutive trials were of the same trial type.

Results

Overall proportion correct Participants received a score of 1 for each trial that they answered correctly. The maximum score is 8 for both the small and large number trials.

Preliminary analyses found no order or gender effects (p's $> .49$), so these variables were not included in subsequent analyses. First, to examine the performance on small and large number trials, a repeated measures ANOVA using proportion correct as the dependent variable, with Set Size (Small and Large) as a within-subjects factor, and Block Order (Small first vs. Large first) and Age Group (3-year-olds vs. 4-year-olds vs. 5-year-olds) as between-subjects factors was conducted. There was a main effect of Age Group, $F(2,42) = 8.33$, $p < .001$. Tukey HSD revealed that 3-year-olds (57.6%) did not differ significantly from 4-year-olds (58.9%), but both groups performed significantly worse than the 5-year-olds (85.8%). There was a significant main effect of Set Size, $F(1,42) = 36.02$, $p < .001$. Children were better at reasoning about small sets (78.0%) than large sets (57.0%). No interactions were found. There was also a main effect of Block Order, $F(1,42) = 8.25$, $p = .027$, indicating that children performed better overall when they were tested on small number trials first (small block first 74.6%; large block first: 60.4%).

To examine children's ability to reason about algebraic number, their performance on large number trials was tested against chance (50%). Separate analyses were conducted for each of the three age groups. One-sample t-tests showed that only 5-year-olds (79.2%) performed significantly above chance on large number trials, $t(15) = 3.95$, $p <.001$. Both the 3- and the 4-year-olds were not different from chance, $M = 46.1\%$, $t(18) = -.71$, $p = .49$, $M = 45.5\%$, $t(13) = -.66$, $p =.52$, respectively.

However, one may argue that 3-year-olds simply do not understand the task (e.g., difficulty of reasoning about the puppet's intentions). To address this, one-sample t-tests were performed for small number trials. Results showed that children from all age groups demonstrated above chance performance: 3-year-olds: $M = 69.1\%$, $t(18) = 3.27$, $p = .004$; 4-year-olds: $M = 72.3\%$, $t(13) = 2.96$, $p = .011$; 5-year-olds: $M = 92.5\%$, $t(14)=9.38$, $p < .001$.

Proportion of 'yes' responses on large number trials To further explore the development of children's algebraic reasoning, we analyzed whether children's responses differed on the four different transformation types for the large number trials. Friedman's ANOVA with the four transformation types as within-subject variables was computed separately for each age group. The dependent variable was proportion of 'yes' responses. If children can reason about number algebraically, they should understand that 'Plus 1' is the only transformation that increases numerosity, and respond 'yes' only in the case of 'Plus 1'. Thus, for this analysis, we asked whether children were able to differentiate 'Plus 1' from the other transformation types that do not increase the numerosity of a set. A Bonferroni correction (adjusted alpha = .017 for each age group) was applied. Figure 2 displays the proportion of 'yes' responses for each transformation type for large number trials.

For 3-year-olds, responses did not vary across the four transformation types ($\chi2$ (3) = 7.34, $p = .062$). For 4-year-olds, responses varied across the four transformation types ($\chi2$ (3) = 24.3, $p < .001$). Wilcoxon signed rank tests revealed that 4-year-olds were sensitive to the difference between 'Plus 1' and 'Minus 1' ($p = .009$), but not between 'Plus 1' and '-A+A' and 'Move 1' (p's $> .038$). For 5-year-olds, responses varied across the four transformation types ($\chi2$ (3) = 30.2 $p < .001$). Wilcoxon signed rank tests revealed that 5-year-olds were able to differentiate between all the transformation types (all p's $< .003$).

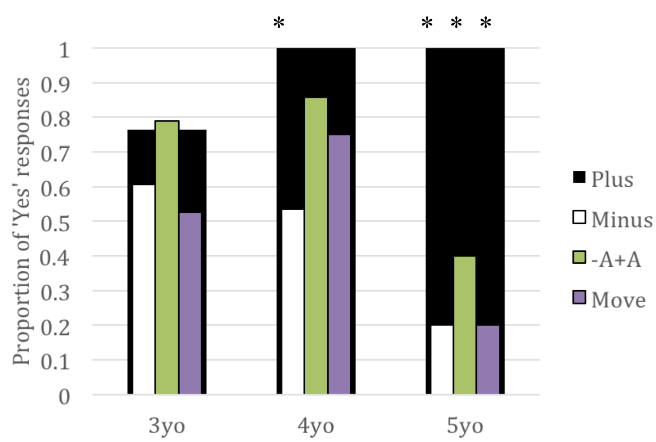

Figure 2: Proportion of 'yes' responses for each transformation type for large number trials by age group. To facilitate comparison, responses on 'Minus 1', '-A+A', and 'Move' trials were plotted *on top of* responses on the 'Plus 1' trial.

Discussion

Using the visible-set experiment, we found that 3-year-olds can reason about the effects of transformations on small sets, but not on large sets, suggesting that they lack the ability to reason about algebraic number. By age 4, children are capable of differentiating between addition and subtraction, but they fail to recognize that taking away an individual and returning it back to the set does not result in a change of numerosity of the set. Five-year-olds are able to recognize that addition increases the numerosity of the set, but other transformations do not. Results from this experiment show that the ability to reason about algebraic number begins to emerge by age 4, and is fully in place by age 5.

The current results raise interesting questions about why 4-year-olds understand the effects of addition and subtraction, but fail to understand that the addition of one element and subtraction of one element does not increase the numerosity. There are two possible explanations. First, children may succeed on the 'Plus 1' and 'Minus 1' trials, but fail on the '-A+A' trial because the latter type of transformation involves two steps – i.e., the subtraction *and* addition of an individual, whereas the former transformations only involve one step. Thus, reasoning about the effect of '-A+A' may be computationally more complex than 'Plus 1' and 'Minus 1'. Another possibility is that 4-year-olds may be responding based on a 'last-action' heuristic. For example, whenever the last action is adding an object, children respond that there are 'more', and whenever the last action is taking away an object, they respond that it is 'not more'. However, this possibility does not explain children's failure to differentiate between 'Plus 1' and 'Move'. Nevertheless, to examine these two possibilities, we added another type of transformation in Experiment 2. Specifically, we included a transformation that involved the removal of 3 individuals and addition of 1 individual ('-3+1'). If children's difficulty with the '-A+A' trial is due to the fact that this transformation involves two steps, then they should also fail on '-3+1'. If they are responding based on a 'last-action' heuristic, then they should respond that there are more objects after the '-3+1' trial.

Experiment 2 – Hidden Sets

Experiment 1 suggests that the capacity to reason about numerical transformations without representing any particular numerosities may begin to emerge around age 4, and is fully in place at age 5. However, the visible-set experiment could underestimate children's ability to reason about algebraic number because some of the transformations required children to resolve a conflict between perception and their conceptual understanding of numerical transformations. For example, in the case of moving one object, children may experience conflict between their perceptual reasoning (i.e., the set of objects takes up more space) and conceptual reasoning (i.e., the numerosity has not changed even though objects are moved). And this may explain why 3- and 4-year-olds responded that there were more elements around 50-70% of the time after one object was moved. Nevertheless, children's performance on the '-A+A' trial in Experiment 1 suggests that the reasoning conflict explanation does not fully explain the results. On the '-A+A' trial, the pre- and post-transformation sets looked perceptually the same, yet 3- and 4-year-olds said there were more elements post-transformation 80% of the time. Moreover, 3-year-olds performed poorly on the 'Minus 1' trials, despite the fact that perceptual and conceptual reasoning coincide. This makes it unlikely that such a conflict can account for all of the results in the previous experiment. Nevertheless, to ensure that the perceptual aspect of sets does not interfere with children's numerical judgments, in Experiment 2, objects were presented in an opaque box. Then a numerical or a non-numerical transformation was performed, and the experimenter asked if there were more objects in the box. This also provides another test for children's reasoning about algebraic number.

Method

Participants Fifty-two children between the ages of 3 and 5 participated. There were 21 3-year-olds (range = 3;0 to 3;11; mean = 3;7), 17 4-year-olds (range = 4;1 to 4;11; mean = 4;5), and 14 5-year-olds (range = 5;0 to 6;1; mean = 5;8). They were recruited at daycare centres in Kitchener-Waterloo and nearby areas. An additional three children were excluded from the analyses for failing twice on a familiarization trial (n = 2) and object trials during the test phase (n = 1; see below).

Design and Procedure The design of the hidden-set experiment was similar to the visible-set experiment except that the sets children had to reason about could not be seen. This experiment began with the same familiarization phase using Hoberman's Sphere, followed by experimental trials. Twelve kinds of objects were used in the study: buttons, rocks, Lego blocks, bows, sticks, pom poms, flowers, leaves, stars, beads, shells, bells.

After familiarization, children were told that they were going to play a game with a box and some objects. The experimenter first showed that the box was empty, then she transferred objects from an opaque cup into the box, and said, "I'm going to put [bells] into the box". The experimenter closed the box, and asked, "Are there more [bells] in the box now?" The experimenter then performed a transformation, and asked again, "Are there more [bells] in the box now?" The purpose of repeating the same test question twice is that in piloting, we found that some children had difficulty parsing the event into appropriate time points for comparison. For example, it appeared that they sometimes compared the initial state of the box (i.e., an empty state) to the post-transformation state. To scaffold children into comparing the post-transformation state to the state of the box that was immediately before transformation, we asked the test question directly before and after transformation. Responses to the first question were not analyzed.

Five kinds of transformation were performed: adding one object ('Plus 1'), removing one object ('Minus 1'), taking away one object and putting it back ('-A+A'), taking away three objects and putting another one back ('-3+1'), and knocking on the box ('Knock'). For 'Plus 1', the experimenter added an identical object to the box. For 'Minus 1', the experimenter removed an object and hid it under the table. For '–A+A', the experimenter took an object out of the box, put it in front of the child, and put it back into the box. For '-3+1', the experimenter took out 3 objects, put them in front of the child, quickly showed him/her that the box was not empty, and added an identical object.

The purpose of the '-3+1' trial was to examine if children's difficulty with reasoning about the effects of transformation is specific to a change of one individual object. Children's performance on '-3+1' trial also addresses the question of whether their difficulty with the '-A+A' trial in the visible-set experiment can be explained by the 'two-step' processing account or the 'last-action' heuristic account.

There were three trials for each transformation type. The type of transformation was pseudo-randomized such that no two consecutive trials were of the same transformation type. Children received one of two randomized orders of trials.

In addition to trials that required children to judge whether there were more elements, children were given three object trials as control trials. These trials were designed to ensure that children understood the task instructions. They involved the addition of a pen, the removal of a pen, and knocking on the box. The experimenter asked, "Is there a red pen in the box now?" after each trial. For addition, the experimenter first put a blue pen, and then a red pen into the box; for the removal of a pen, the experimenter put both the blue and the red pens into the box, and then removed the red pen; for knocking, the experimenter put the blue pen in the box, and knocked on the box. Children were allowed to make one error across the three trials. Altogether, only four children made an error; they were included in the analyses. One child made two errors and was excluded.

Results and Discussion

Preliminary analyses found no order or gender effects (p's > .31), so these variables were not included in subsequent analyses.

Proportion of 'yes' responses Similar to the visible-set experiment, we analyzed whether children's responses differed on the five different transformation types using Friedman's ANOVA, followed by Wilcoxon signed rank tests. The dependent variable was proportion of 'yes' responses. A Bonferroni correction (adjusted alpha = .0125 for each age group) was applied. Figure 3 displays the proportion of 'yes' responses for each transformation type.

For 3-year-olds, responses did not vary across the five transformation types ($\chi2$ (4) = 8.87, p = .064). An inspection of Figure 3 suggests an overall 'yes' bias for 3-year-olds. For 4-year-olds, responses varied across the five transformation types ($\chi2$ (4) = 23.43, p < .001). Wilcoxon signed rank tests revealed that 4-year-olds were able to differentiate 'Plus 1' from 'Minus 1' and from '-3+1' (all p's < .007). The results also revealed that they were marginally able to differentiate between addition and 'Knock' (p = .018). However, they did not recognize that '-A+A' does not increase the numerosity of the set. For 5-year-olds, responses varied across the five transformation types (χ2 (4) = 31.35, p < .001). 5-year-olds can distinguish 'Plus 1' from most transformation types (all p's < .009), except for '-A+A' (p = .18).

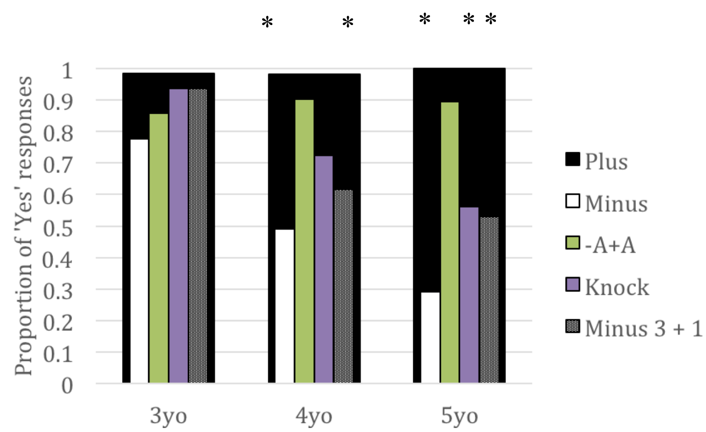

Figure 3: Proportion of 'yes' responses for each transformation type by age group in Experiment 2.

Consistent with the previous experiment, the current experiment showed that 3-year-olds fail to reason about algebraic number, and this ability emerges at age 4. It is important to note that 4-year-olds in this experiment distinguish between addition and '-3+1' but continue to have difficulty with reasoning about '-A+A'. This suggests that children's difficulty with reasoning about the effect of '-A+A' in the previous experiment cannot be explained by the fact that '-A+A' involves two steps, because '-3+1' also involves two steps. Given that the last action of '-A+A' and '-3+1' both involve the addition of an individual, these results also suggest that children are likely not relying on a 'last-action' heuristic.

Nevertheless, the current experiment showed that 5-year-olds have trouble reasoning that '-A+A' does not affect the numerosity of a set. This was somewhat inconsistent with findings from the previous experiment, which showed that 5-year-olds understand how adding and removing the same individual does not change the numerosity of visible sets. One possibility is that children were unwilling to answer 'no' to the question, "Are there more [bells] in the box now?" when the numerosity of a set remains the same (as in '-A+A'). For example, a handful of children responded "the same" to the test question about 'more' on the '–A+A' trial, but these same children had no difficulty answering yes/no for addition and subtraction. We speculate that this difficulty may be related to the semantics of adjectives. For example, pairs of adjectives such as tall/short have different entailment patterns than pairs of adjectives such as dead/alive. A person who is not tall does not entail that the person is short, but a person who is not dead does entail that the person is alive. This difference in meaning highlights a distinction that is documented in the semantics literature: tall/short are gradable adjectives and dead/alive are absolute adjectives. It is possible that children are interpreting 'more' as if it is an absolute adjective, when in fact it behaves more similarly to gradable adjectives.

General Discussion

Two experiments explored the development of children's capacity to reason about algebraic number – i.e., that the addition of one object necessarily increases the numerosity of a set regardless of set size. In each of the experiments, children were asked to reason about how different types of transformation changed the numerosity of a set of objects. To assess when children begin to show a capacity to reason about algebraic number, set size (Experiment 1) and the presentation of the set of objects (Experiment 2) was manipulated such that children could not recruit representations of particular numbers to conclude whether the numerosity increased as a result of the transformation. In both experiments, we found that 3-year-olds were not able to reason about algebraic number, and this ability emerges at around age 4, and is fully in place by age 5.

What might account for the developmental differences between ages 3 to 5? One possibility is the experience of schooling. Being in a school environment with structured activities may enhance children's numerical reasoning abilities. However, both 4- and 5-year-olds in our sample received the same amount of schooling, but they demonstrated a different level of understanding of the effects of numerical transformations. Thus, it seems unlikely that schooling alone can explain the developmental differences.

The developmental differences raise important questions regarding the nature of algebraic reasoning. It is possible that children have algebraic representations all along, but they have to learn about particular generalizations. For example, understanding '$x + 1 > x$' does not grant one the knowledge that '$x – 1$' is less than 1. Thus, one could argue that what is developing is not the representation of algebraic number per se, but children's ability to apply numerical principles to algebraic representations. Alternatively, children may have to construct a new type of representations sometime between ages 3 and 5. As the current findings suggest, children are capable of reasoning about small sets (1-3) before reasoning about large sets (14-16) and hidden sets. On the constructivist account, children know that small sets of objects have a particular numerosity, and that there is an order for small sets, i.e., a set of three is more than a set of two, and a set of two is more than a set of one. Moreover, it has also been shown that infants can perform addition over small sets of individuals (Wynn, 1992). Thus, the analogies children observe in the small number range may be powerful enough to allow them to make a crucial induction: any sets of objects will have a numerosity x, which is different from '$x + 1$'.

Lastly, the current study has implications for the literature on number word acquisition. As noted in the Introduction, much previous research on children's understanding of number words is motivated by the assumption that children's understanding of numerical symbols may reveal the nature of their numerical knowledge. The present experiments adopted a different strategy in examining children's numerical knowledge. In particular, children were asked to reason about changes in numerosity, and not how number words change their application in the context of numerical and non-numerical transformations. We found a similar pattern of results – i.e., by 5, children understand how rearranging objects, addition, and subtraction affect the numerosity of sets. This converges with findings from Lipton and Spelke (2006), who showed that 5-year-olds understand that an unknown number word changes when objects are removed from or added to the set, but it remains the same when objects are simply rearranged. This converging result tentatively suggests that 5-year-olds begin to reason about algebraic number at the same time that they understand the meanings of number words outside of their count list.

To sum up, the current experiments are the first to show that children are able to reason about particular numbers much earlier than they can reason about number algebraically.

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