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Application of a new hybrid non-linear Muskingum model to flood routing

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This paper introduces a hybrid non-linear Muskingum model for flood routing. The proposed hybrid model has more degrees of freedom for fitting observed data than other non-linear Muskingum models. The main goal of this work is to develop a comprehensive model for outflow routing. The proposed hybrid model's predictive skill is evaluated with experimental, real and multimodal hydrograph-routing problems. The results confirm the predictive skill of the hybrid model based on the minimisation of the sum of the square deviation (SSD) between observed and routed outflows, the sum of the absolute value of the deviations (SAD) between the observed outflow and the computed outflow, and the deviations between the peak of the routed and actual outflows (DPO). Results from this study show the hybrid model improved the SSD by 79, 15 and 5%, SAD by 50, 2 and 5%, and the DPO by 77, 4 and 34% compared with the best alternative Muskingum model in solving the experimental, real and multimodal example problems, respectively.

Notation

I	inflow to the river reach
i	time steps
K	travel time coefficient
O	outflow from the river reach
O_i	observed outflows in step i
O_i^{P}	predictive outflows in period i
S	reach storage
S_i^{C}	storage corrector
S_i^{P}	storage predictor
$\mathrm{Time}_{O_{\mathrm{P}}}$	time of peak of the observed outflows
$\mathrm{Time}_{O_{\mathrm{P}}^{\mathrm{C}}}$	time of peak of the calculated outflows
wi_{-1}	weighting coefficient of time steps for inflow values
wi_0	weighting coefficient of time steps for inflow values
wo_{-1}	weighting coefficients for outflow values
wo_0	weighting coefficients for outflow values
ws_{-1}	weighting coefficient
ws_0	weighting coefficient
ws_1	weighting coefficient
X	weighting factor describing the importance of inflow and outflow
α_1	exponent parameter
α_2	exponent parameter
β	exponent parameter

1. Introduction

One of the non-structural methods to manage a river's flooding is flood forecasting, which relies on flood routing for the prediction of hydrograph propagation along river reaches (Tewolde and Smithers, 2006). Hydraulic and hydrologic methods are available for flood routing (Karahan *et al.*, 2013). The hydraulic approach is based on the numerical solution of either the advective-diffusion equations or the one- or two-dimensional Saint-Venant equations of gradually varied unsteady flow in open channels (Cunge, 1980). These approaches require measurements of flow depth and discharge using elaborate stream gauging. Therefore, they cannot be applied in places that lack topographical and channel–geometric data. Hydraulic methods are also relatively cumbersome computationally, but yield accurate flood-routing predictions. The hydrologic routing method, on the other hand, relies on the principle of continuity between discharge and water storage in river reaches. The hydrologic routing method is relatively simple and has good predictive accuracy (Chow *et al.*, 1988). The Muskingum model is the best-known hydrologic routing method. It includes several versions. It requires model calibration involving observed hydrographs (Das, 2004). Versions of the non-linear Muskingum model reviewed herein are the first (NL1), second (NL2), third (NL3) and fourth (NL4) formulations, which were introduced, respectively, by Chow (1959), Gill (1978), Easa (2014) and Bozorg-Haddad *et al.* (2015a). These four non-linear Muskingum model formulations differ in their number of parameters, which for the models NL1, NL2, NL3 and NL4 are, respectively, three, three, four and seven. Bozorg-Haddad *et al.* (2015a) reported that the NL4 model has better predictive accuracy than other non-linear Muskingum formulations based on the sum of the square deviations (SSD) between the observed outflow and the computed outflow, the sum of the absolute value of the deviations (SAD) between the observed outflow and the computed outflow, and the deviations between the peak of the routed and actual flows (DPO). This is so because the NL4 non-linear Muskingum model has more degrees of freedom than the other three models (NL1, NL2 and NL3). Hamedi *et al.* (2016) attempted to improve the estimation of the predicted outflow hydrograph with various non-linear Muskingum models. Muskingum generalised non-linear models reported by Hamedi *et al.* (2016) exhibited excellent flood-hydrography prediction capacity based on the SSD, SAD and DPO fitting criteria.

Tung (1985) introduced a non-linear Muskingum simulation model. Hamedi *et al.* (2014) and Hamedi *et al.* (2016) implemented Tung's (1985) simulation method and improved its boundary conditions. Bozorg-Haddad *et al.* (2015b) reduced the predictive error of reach storage with a non-linear Muskingum model.

In practical applications the calibration step is of utmost importance when applying non-linear Muskingum models (Chow *et al.*, 1988). Various methods for estimating the non-linear Muskingum parameters have been reported over the last few decades. Non-linear optimisation methods are classified into three groups. The first group consists of mathematical search methods such as the segmental least-squares method (S-LSQ), non-linear least-squares method (N-LSQ), Lagrange multiplier (LM), Broyden–Fletcher–Goldfarb–Shannon (BFGS), Nelder Mead simplex method (NMS) and the generalised reduced gradient method (GRG). These methods are beset by convergence to local optima when the initial estimate of the global solution is poor. The second group of non-linear Muskingum parameter estimation methods comprises phenomenon-mimicking algorithms, such as the pattern search algorithm (PS), the genetic algorithm (GA), harmony search (HS), particle swarm optimisation (PSO), parameter-setting-free harmony search (PSF-HS), immune clonal selection (ICS) algorithm, differential evolution (DE), coco search (CS), shuffled frog-leaping algorithm (SFLA), cuckoo search (CS) algorithm and honey-bee mating optimisation (HBMO). The convergence of the latter algorithms is slow due to their random search nature (Barati, 2012). The third group of parameter estimation methods consists of hybrid algorithms that combine phenomenon-mimicking algorithms and mathematical search methods, such as the combination of the genetic algorithm (GA) with NMS (GA-NMS), combining GA with GRG (GA-GRG) and HS combination with BFGS (HS-BFGS). These algorithms offer the advantages of both types of optimisation methods (i.e. phenomenon-mimicking algorithms and mathematical techniques), while offsetting their disadvantages.

Gill (1978) applied the LSQ to estimate the parameters of the NL2 model. Gavilan and Houck (1985) estimated the NL1 and NL2 models' parameters with the standard search method (SS). Tung (1985) employed the PS method based on the parameters of the model NL2. Yoon and Padmanabhan (1993) reported several methods to estimate the non-linear Muskingum parameters. Mohan (1997) employed the GA to estimate the parameters of the NL2 model. Kim *et al.* (2001) applied the HS algorithm to estimate the NL2 model parameters. The results of the HS algorithm estimates were better than those calculated with the GA based on fitting criteria such as the SSD, SAD, the DPO and deviations between the peak of routed and actual flows times (DPOT). Das (2004) employed the LM to estimate the NL1 and NL2 model parameters. Cheng *et al.* (2005) applied an adaptive network-based fuzzy inference system (ANFIS) to forecast long-term discharge of monthly river flow. The comparison of ANFIS and artificial neural networks indicated the superiority of the ANFIS model. Geem (2006) applied the BFGS method to estimate the NL2 parameters based on the minimisation of the SSD. Chu and Chang (2009) implemented the PSO algorithm for estimating the NL2 model parameters. Luo and Xie (2010) applied the immune clonal selection algorithm (ICSA) to estimate the NL2 model parameters. Barati (2011) applied the NMS method to calibrate the NL2 model. Geem (2011) found the PSF-HS method superior to the HS methods in calibrating the NL2 model. Barati (2012) reported GA-NMS for estimating the NL3model parameters. Xu *et al.* (2012) reported the DE method to estimate the parameters of the NL2 model. Orouji *et al.* (2013) calibrated the NL2 model with SA and SFLA. Karahan *et al.* (2013) improved the BFGS method to estimate the parameters of the NL2 model with the HS-BFGS hybrid method. Barati (2013) calibrated the NL1 and NL2 models with the GRG method. Easa (2014) calibrated the NL3 model with the GA-GRG method. Easa (2014) stated that the use of the combined GA-GRG method produces the best calibration of the NL3 model (see, also, Easa (2014)). Bozorg-Haddad *et al.* (2015a) combined SFLA with NMS to estimate the parameters of the model NL3. They concluded that the SFLA-NMS was an accurate estimator of non-linear Muskingum parameters. Fotovatikhah *et al.* (2018) presented a comprehensive survey about application of conventional artificial intelligence and computational intelligence methods in flood management systems. The results showed the efficiency of these methods for flood prediction.

Mosavi *et al.* (2018) focused on state-of-the-art of machine learning models in flood prediction to give insight into the most suitable models. Yaseen *et al.* (2019) applied an enhanced version of an extreme learning machine (EELM) model in river flow forecasting. Then the comparison of EELM to classical ELM and the support vector machine (SVR) indicated the superiority of EELM.

In recent years, several authors have proposed new relations and parameters for the non-linear Muskingum models with the aim of achieving improved model calibrations and predictions (Easa, 2013; Hamedi *et al.*, 2014). These works are among a range of publications focusing on various aspects of water resources management (Ahmadi *et al.*, 2015; Akbari-Alashti *et al.*, 2014; Beygi *et al.*, 2014; Bozorg-Haddad *et al.*, 2009, 2013, 2015c, 2015d; Fallah-Mehdipour *et al.*, 2013a, 2013b, 2014; Farhangi *et al.*, 2012; Jahandideh-Tehrani *et al.*, 2015; Orouji *et al.*, 2014a, 2014b; Sabbaghpour *et al.*, 2012; Soltanjalili *et al.*, 2011). This study presents a new hybrid non-linear Muskingum model for improved flood routing prediction. The predictive skill of the proposed flood-routing model is evaluated with three case studies.

2. Methods

The hybrid model and the method for estimating its parameters are introduced in this section.

2.1 The hybrid non-linear Muskingum model

The Muskingum flood-routing model is based on the principle of continuity between discharge and reach storage. Two linear equations between continuity and storage define the classic linear Muskingum model

1. $$\frac{\mathrm{d}S}{\mathrm{d}t} \approx \frac{\Delta S}{\Delta t} = I - O$$

2. $$S = K[XI + (1 - X)O]$$

where S is the reach storage; I is the inflow to the river reach; O is the outflow from the river reach; t denotes time; X is a weighting factor describing the importance of inflow and outflow; K is a travel time coefficient that ranges between zero and 0·5 in river reaches. Yoon and Padmanabhan (1993) showed that a non-linear Muskingum model is more appropriate for flood routing when reach storage streamflow is not linear. Equation 3 was proposed by Hamedi *et al.* (2016) for the non-linear (hybrid) Muskingum model herein named NL42

3. $$S_i = K\left[X_1\left(C_1 I_i^{\alpha_1}\right) + X_2\left(C_1 I_{i+1}^{\alpha_1}\right) + (1 - X_1 - X_2)\left(C_2 O_i^{\alpha_2}\right)\right]^{\beta}$$

where i denotes the time steps and there are N time steps ($i = 0$, 1, 2, …, N); C_1 and C_2 are coefficients of inflows and outflows, respectively; X_1 and X_2 are weighting factors measuring the degree of inflows importance in the time intervals i and $i+1$, respectively; and α_1, α_2 and β are exponent parameters that account for the non-linearity of the flood wave. The NL42 model improves the relation between storage and stream flow relative to other non-linear Muskingum models. The hybrid non-linear Muskingum model supplements the NL42 model of Equation 3 with several other models as follows.

- Apply the NL431 and NL432 models by Hamedi *et al.* (2016) to improve the Muskingum model non-linear boundary conditions in case studies with initial non-uniform flow. In fact, applying these two models causes improvement of the Muskingum model non-linear boundary conditions, not only in cases with uniform initial flow, but also in cases with non-uniform initial flow.
- Implement the NL44 model by Hamedi *et al.* (2016) to reduce the errors in calculating the reach storage.
- Employ the NL45 model to improve the calculation of outflow.

Solving of the hybrid model using the continuity and storage equations of the NL42 model in combination with the models NL431, NL432, NL44 and NL45 (the features of NL431, NL432, NL44 and NL45 are presented in Table 1) is as follows.

- Step 1. Specify the combined model parameters.
- Step 2. Calculate the initial storage: the hybrid model improves the Muskingum model non-linear boundary conditions when the initial flow is non-uniform with the models NL431 (Equation 4) and NL432. The NL431 model sets the initial outflow value equal to a percentage of the initial inflow values $\hat{O}_0 = \lambda I_0$), whereas the NL432

Table 1. Features of NL431, NL432, NL44 and NL45

Model designation	Type Muskingum model	Variant of the model	Model parameters
NL431	NL4	31	(K, X, α_1, α_2, β, C_1, C_2, λ)
NL432	NL4	32	(K, X, α_1, α_2, β, C_1, C_2, λ)
NL44	NL4	4	(K, X, α_1, α_2, β, C_1, C_2, ws_{-1}, ws_0, ws_1 wi_{-1},wi_0, wo_{-1}, wo_0)
NL45	NL4	5	(K, X, α_1, α_2, β, C_1, C_2, ws_{-1}, ws_0, ws_1)

model determines the initial storage value and other parameters values by optimisation.

4. $$S_0 = K[X_1(C_1 I_0^{\alpha_1}) + X_2(C_1 I_1^{\alpha_1}) + (1 - X_1 - X_2)(C_2 \lambda I_0^{\alpha_2})]^{\beta} \quad i = 0$$

5. $$S_0 = \theta$$

in which S_0 denotes the initial reach storage.

- Step 3. Calculate the storage value in the next time step: the NL44 model is employed to reduce the errors in calculating the storage in time step i. Model NL44 saves the calculated storage in time steps $i = 0, 1, 2, \ldots, N$, and the storage in period i is corrected by the storage moving average (SMA). A disadvantage of the non-linear Muskingum model is the inaccurate calculation of storage in some instances. In recent years, several authors have changed the structure of storage modelling to increase the applicability of the non-linear Muskingum model. SMA has been added to the non-linear Muskingum model to reduce the error in storage calculation.

The storages calculated with the NL44 and SMA are called the storage predictor (S_i^{P}) and the storage corrector (S_i^{C}), respectively. S_i^{C} is calculated by the NL44 model based on the predicted values of storage in the periods $i-1$, i and $i+1$, (S_{i-1}^{P}, S_i^{P} and S_{i+1}^{P}), which are calculated with the NL44 model, and S_i^{C} is calculated with a weighted average of S_{i-1}^{P}, S_i^{P} and S_{i+1}^{P}, whereby the weighting coefficients are calculated by optimisation. The calculation of storage in the next time step by the NL44 models is as follows.

Step 3-1. Calculate the rate of change storage predictor ($\Delta S_i^{\mathrm{p}}/\Delta t$) using Equation 6

6. $$\frac{\Delta S_i^{\mathrm{P}}}{\Delta t} = I_i - \frac{1}{C_2}\left\langle \frac{1}{1 - X_1 - X_2}\left\{\left(\frac{S_i^{\mathrm{P}}}{K}\right)^{1/\beta} - [X_1(C_1 I_i)^{\alpha_1} + X_2(C_1 I_{i+1})^{\alpha_1}]\right\}\right\rangle^{1/\alpha_2} \quad i = 0, 1, \ldots, N$$

Step 3-2. Calculate S_i^{P} in next interval using Equation 7

7. $$S_i^{\mathrm{P}} = S_{i-1}^{\mathrm{P}} + \Delta t\left(\frac{\Delta S_{i-1}^{\mathrm{P}}}{\Delta t}\right) \quad i = 1, \ldots, N+1$$

Step 3-3. Repeat steps 3-1 and 3-2. Compare the simulated and predicted storage values in period N and stop the iterative calculations when the values satisfy a convergence criterion.

Step 3-4. Calculate S_i^{C} with Equation 8

8. $$S_i^{\mathrm{C}} = \mathrm{ws}_{-1} S_{i-1}^{\mathrm{P}} + \mathrm{ws}_0 S_i^{\mathrm{P}} + \mathrm{ws}_1 S_{i+1}^{\mathrm{P}} \quad i = 1, \ldots, N$$

where ws_{-1}, ws_0 and ws_1 denote the weighting coefficients applied to S_{i-1}^{P}, S_i^{P} and S_{i+1}^{P}, respectively.

Step 4. Calculate the outflow at the next time step, the hybrid model NL45 reduces the errors incurred in the calculation of the outflow in period i. Equation 9 shows how to calculate the outflow. Notice that the outflow in period i equals the weighted average of the inflows in the intervals i and $i+1$. Equation 9 has three parameters (R_1, R_2 and R_3) that are calculated with optimisation

9. $$\hat{O}_{i+1} = R_1 I_{i+1} + R_2 I_i + R_3 \hat{O}_i$$

This work improves the calculation of outflows with the non-linear Muskingum model NL45. The calculation of outflow in the next time step with the NL45 model is as follows.

Step 4-1. Calculate the predictive outflows in period i (O_i^{P}) with Equation 10

10. $$O_i^{\mathrm{p}} = \frac{1}{C_2}\left\langle \frac{1}{1 - X_1 - X_2}\left\{\left(\frac{S_i^{\mathrm{C}}}{K}\right)^{1/\beta} - [X_1(C_1 I_i)^{\alpha_1} + X_2(C_1 I_{i+1})^{\alpha_1}]\right\}\right\rangle^{1/\alpha_2}$$

Step 4-2. Continue the algorithm and repeat steps 3-4 to 4-1 for as long as the stopping criteria have not been reached.

Step 4-3. Calculate O_i^{C} with Equation 11

11. $$O_i^{\mathrm{C}} = \mathrm{wi}_{-1} I_{i-1} + \mathrm{wi}_0 I_i + \mathrm{wo}_{-1} O_{i-1}^{\mathrm{P}} + \mathrm{wo}_0 O_i^{\mathrm{p}}$$

In which wo_{-1} and wo_0 denote the weighting coefficients for outflow values, and wi_{-1} and wi_0 are weighting coefficient of time steps for inflow values.

Step 4-4. Repeat step 4-3 until step N is reached. The flowchart of the calculation process is presented in Figure 1.

In most of the studies related to simulation non-linear Muskingum models the SSD has been considered as the main objective function to evaluate the performance of the model. Other authors have applied the SAD, DPO and DPOT as supplemental parameters for better evaluation of non-linear Muskingum models.

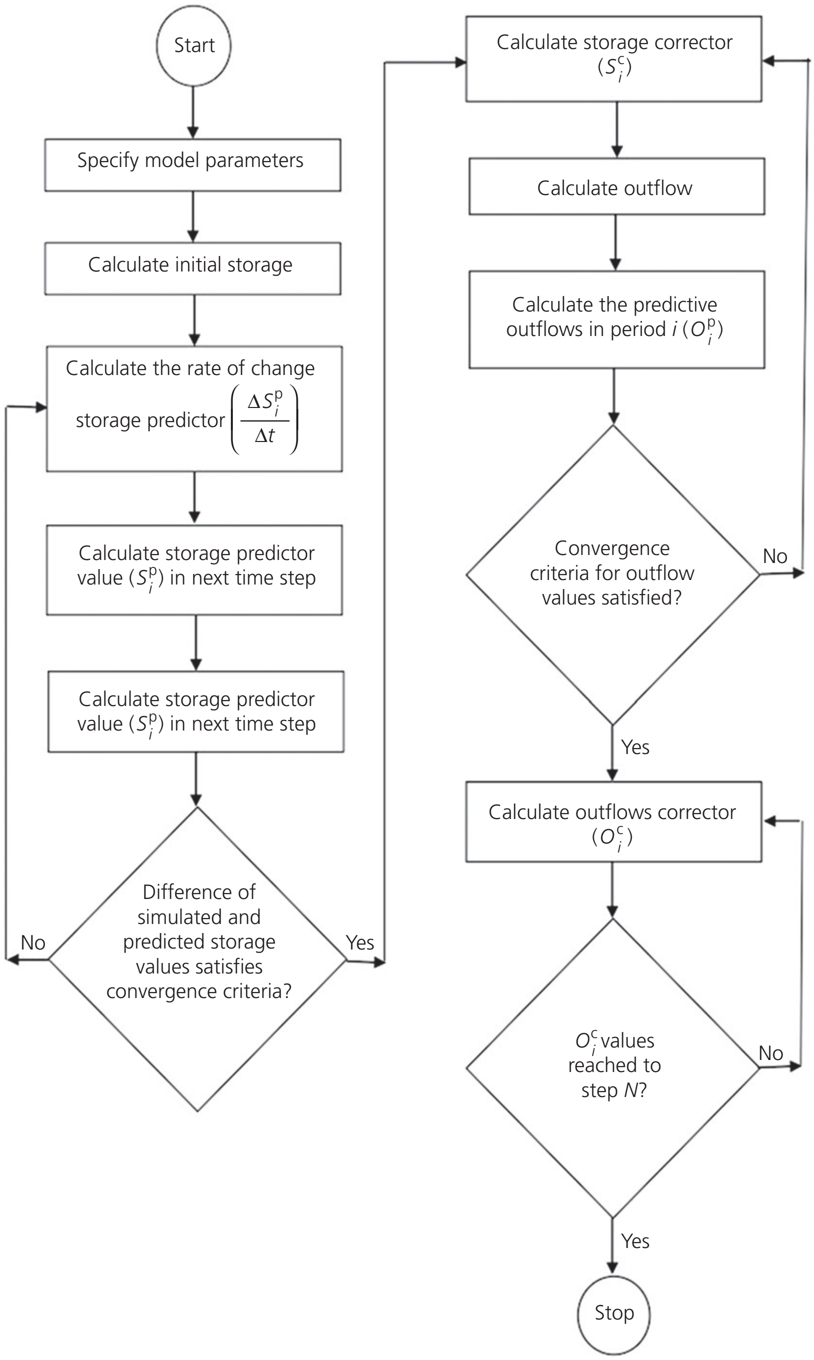

Figure 1. Flowchart of hybrid model calculation process

Considering the cited facts, this study employs the SSD, SAD, DPO and DPOT as the performance criteria for evaluating the predictive skill of the proposed individual non-linear and hybrid non-linear Muskingum models.

12. $$\text{Min. SSD} = \sum_{i=1}^{N} \left(O_i - O_i^{\mathrm{C}}\right)^2 \quad i = 0, 1, 2, \ldots, N$$

13. $$\text{SAD} = \sum_{i=1}^{N} \left|O_i - O_i^{\mathrm{C}}\right| \quad i = 0, 1, 2, \ldots, N$$

14. $$\text{DPO} = \left|O_{\mathrm{P}} - O_{\mathrm{P}}^{\mathrm{C}}\right|$$

15. $$\text{DPOT} = \left|\text{Time}_{O_\text{P}} - \text{Time}_{O_\text{P}^\text{C}}\right|$$

in which O_i and O_i^C denote the observed and predicted outflows in step i, respectively; O_P and O_P^C represent the peak outflow observation and peak outflow prediction, respectively; and Time_{O_P} and $\text{Time}_{O_\text{P}^\text{C}}$ denote the times of peak of the observed and calculated outflows, respectively. DPO and DPOT are central parameters in flood-routing and flood-damage calculation.

2.2 Optimisation procedure of the hybrid non-linear Muskingum model

Excel solver has been successfully applied in hydraulic and hydrologic problems. The solutions obtained by solver are also accurate with short computational time. Excel

Table 2. Optimal results obtained with the hybrid non-linear Muskingum model for the first case study

I	t: h	I_i: m³/s	O_i: m³/s	S_i^P: m³/s	$\Delta S_i^\text{P}/\Delta t$: m³/s	S_i^C: m³/s	O_i^P: m³/s	O_i^C: m³/s	$(O_i - \hat{O}_i)^2$: m³/s	$\lvert O_i - \hat{O}_i \rvert$: m³/s
0	0	22	22	118·13	7·50	160·71	22·00	22·00	0·00	0·00
1	6	23	21	163·11	7·25	257·70	18·86	21·01	0·00	0·01
2	12	35	21	206·63	23·55	455·89	23·64	20·98	0·00	0·02
3	18	71	26	347·93	58·55	752·82	25·96	26·05	0·00	0·05
4	24	103	34	699·23	79·04	1125·40	29·78	33·94	0·00	0·06
5	30	111	44	1173·46	69·41	1511·16	39·92	44·03	0·00	0·03
6	36	109	55	1589·93	49·63	1793·86	53·55	54·95	0·00	0·05
7	42	100	66	1887·73	23·86	1936·55	66·52	66·02	0·00	0·02
8	48	86	75	2030·91	−3·77	1936·91	77·89	75·18	0·03	0·18
9	54	71	82	2008·28	−26·13	1811·03	86·22	81·76	0·06	0·24
10	60	59	85	1851·53	−40·40	1591·38	89·19	84·87	0·02	0·13
11	66	47	84	1609·11	−48·58	1328·76	88·07	84·29	0·08	0·29
12	72	39	80	1317·63	−48·36	1052·98	82·26	80·04	0·00	0·04
13	78	32	73	1027·44	−43·67	801·83	73·47	72·96	0·00	0·04
14	84	28	64	765·41	−35·30	587·91	62·54	63·76	0·06	0·24
15	90	24	54	553·62	−27·03	427·12	51·70	53·90	0·01	0·10
16	96	22	44	391·47	−17·78	311·93	41·77	44·38	0·15	0·38
17	102	21	36	284·78	−10·55	237·27	33·31	36·12	0·01	0·12
18	108	20	30	221·45	−6·36	193·05	27·31	29·56	0·20	0·44
19	114	19	25	183·31	−3·76	167·79	23·62	25·04	0·00	0·04
20	120	19	22	160·73	−1·83	110·82	21·20	22·16	0·02	0·16
21	126	18	19	149·77	−14·01	78·59	15·36	19·02	0·00	0·02
—	—	—	—	65·70	—	—	—	—	—	—
Sum	—	—	—	—	—	—	—	—	0·65	2·66

Observed inflow and outflow data from Wilson (1974)

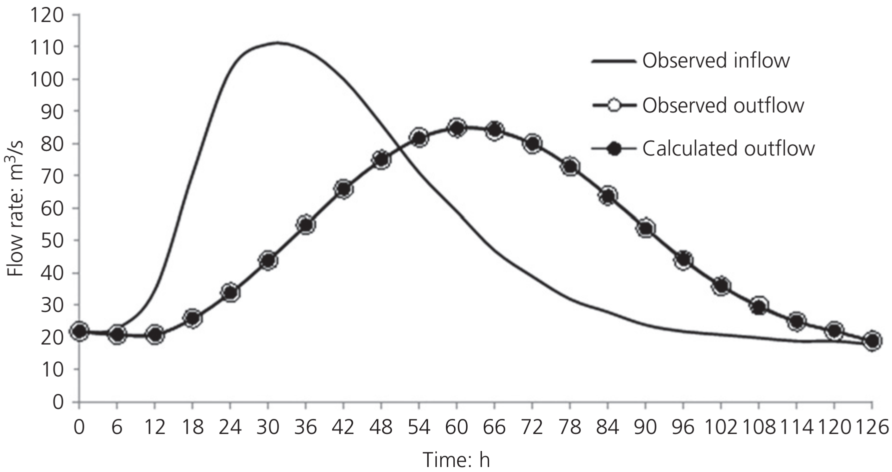

Figure 2. Comparison of the observed and calculated hydrographs obtained with the hybrid model for the first case study (observed inflow and outflow data from Wilson (1974))

solver is applied in this study to estimate the parameters of the hybrid model.

Parameter estimation in Excel is applied with two search methods, namely, the GRG and evolutionary solver (EV). Over many years GRG has been proven as one of the most reliable approaches for solving complex non-linear problems. The second search method is EV, which is a hybrid of genetic and evolutionary algorithms. This paper overcomes the shortcomings of the GRG and EV methods by resorting to a hybrid approach (EV–GRG), which in the first stage estimates the initial parameters with EV. The solution vector obtained by EV is used as the initial solution vector in the GRG approach. In the next stage the final solutions are estimated with the GRG approach (Barati, 2013).

Table 3. Comparison of the objective functions and parameter values obtained by several models corresponding to the first case study

Model	NL4	NL42	NL431	NL432	NL44	NL45	Hybrid
K	0·48	0·79	0·55	0·49	0·61	1·33	1·17
X_1	0·08	0·02	0·08	0·81	0·08	0·39	0·03
X_2	—	0·01	—	—	—	—	0·88
$(\lambda)\theta$	—	—	0·96	24	—	—	118
σ	—	—	—	—	—	—	—
ws_{-1}	—	—	—	—	−0·19	—	0·57
ws_0	—	—	—	—	1·28	—	−0·11
ws_1	—	—	—	—	−0·09	—	0·54
wi_{-1}	—	—	—	—	—	0·25	0·15
wi_0	—	—	—	—	—	−0·01	0·00
wo_0	—	—	—	—	—	0·60	0·32
α_1	0·70	0·80	0·70	0·73	0·89	0·86	0·60
α_2	0·43	0·37	0·43	0·44	0·59	0·77	0·43
β	3·82	4·37	3·80	3·73	2·86	2·18	3·76
C_1	0·62	1·00	0·60	0·58	0·55	0·81	1·23
C_2	0·74	1·00	0·71	0·72	0·57	0·98	0·90
SSD	5·44	4·81	5·17	3·19	4·27	4·93	0·65
SAD	6·69	6·52	6·96	5·35	6·81	6·36	2·66
DPO	0·05	0·03	0·05	0·07	0·08	0·04	0·13
DPOT	0	0	0	0	0	0	0

Table 4. Comparison of the computed outflows obtained with the several models for the first case study

				$\hat{O}_i$: m³/s						
i	t: h	I_i: m³/s	O_i: m³/s	NL4	NL42	NL431	NL432	NL44	NL45	Hybrid
0	0	22	22	22·00	22·00	21·02	22·00	22·00	22·00	22·00
1	6	23	21	22·00	22·00	21·55	20·65	21·94	21·99	21·01
2	12	35	21	22·38	22·18	22·18	21·78	21·79	22·22	20·98
3	18	71	26	26·21	25·73	26·15	26·05	25·51	25·63	26·05
4	24	103	34	34·02	33·94	34·03	34·09	34·29	33·93	33·94
5	30	111	44	43·69	43·97	43·68	43·71	43·75	44·06	44·03
6	36	109	55	55·34	55·34	55·33	55·32	55·17	55·03	54·95
7	42	100	66	65·98	65·85	65·98	65·96	65·86	65·98	66·02
8	48	86	75	75·02	74·98	75·02	75·01	75·06	75·15	75·18
9	54	71	82	81·78	81·84	81·79	81·79	81·91	81·77	81·76
10	60	59	85	85·05	85·03	85·06	85·07	85·09	84·96	84·87
11	66	47	84	84·07	84·21	84·07	84·08	84·07	84·17	84·29
12	72	39	80	80·17	80·12	80·16	80·15	79·99	80·00	80·04
13	78	32	73	72·81	72·85	72·80	72·79	72·72	72·90	72·96
14	84	28	64	63·90	63·83	63·88	63·88	63·82	63·87	63·76
15	90	24	54	53·95	54·03	53·94	53·96	54·09	54·07	53·90
16	96	22	44	44·50	44·51	44·50	44·51	44·59	44·47	44·38
17	102	21	36	35·96	35·98	35·97	35·98	36·00	35·96	36·12
18	108	20	30	29·43	29·45	29·44	29·44	29·42	29·37	29·56
19	114	19	25	24·93	24·89	24·93	24·92	24·81	24·83	25·04
20	120	19	22	21·88	21·79	21·89	21·86	21·63	21·82	22·16
21	126	18	19	20·24	20·20	20·25	20·23	20·16	20·23	19·02

2.3 Applications and results of the hybrid non-linear Muskingum model

Three experimental, real and multimodal hydrograph-routing problems were chosen as case studies to evaluate the performance proposed hybrid non-linear Muskingum model. The non-linear relation between S and $(XI+(1-X)O)$ in the experimental, real and multimodal problems was a reason for choosing them as case studies. These three problems have been solved in previous studies by other authors, thus providing comparison values for this study's results.

2.4 Case study 1: experimental problem

The example presented in this section was introduced by Wilson (1974) and expanded by Yoon and Padmanabhan (1993). This case study has a 6 h period ($\Delta t = 6$) and the number of steps $N = 21$. The flow regime is steady at the beginning of the flood. Thus, the hybrid NL432 model is intended to improve the handling of the boundary conditions. Table 2 lists the calculated and observed outflows, plus the SSD, SAD and DPO, which are equal to 0·65, 2·66 and 0·13, respectively. Figure 2 depicts the observed inflow and outflow hydrographs and the calculated hydrograph. It is seen in Figure 1 that the hydrograph routing achieved with the hybrid model almost matches the observed outflow hydrograph. Table 2 lists the model parameters, coefficients and performance criteria calculated with the non-linear Muskingum models and the hybrid non-linear Muskingum model. According to Table 3 the lowest (best) SSD and SAD values correspond to the hybrid model. The SSD and SAD values for the hybrid model are, respectively, 79 and 50% lower (improved) than those for the NL43 model. Table 4 presents the outflows estimated with the various non-linear Muskingum

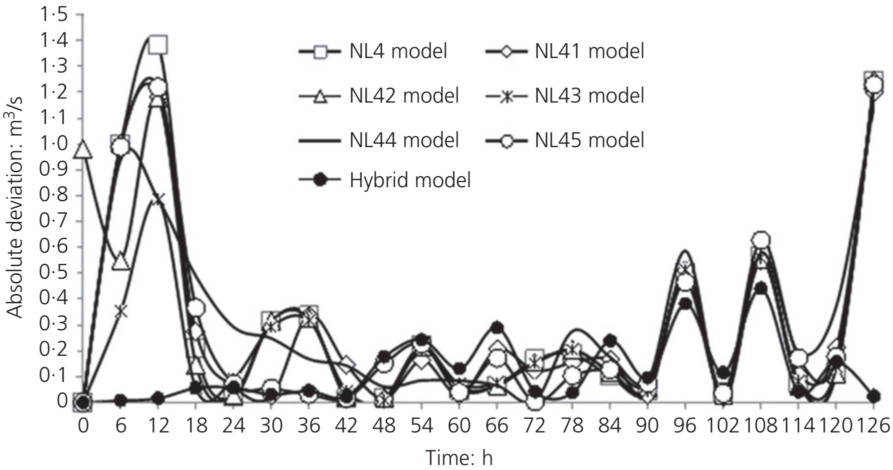

Figure 3. Comparison of the SAD values calculated with the several models for the first case study

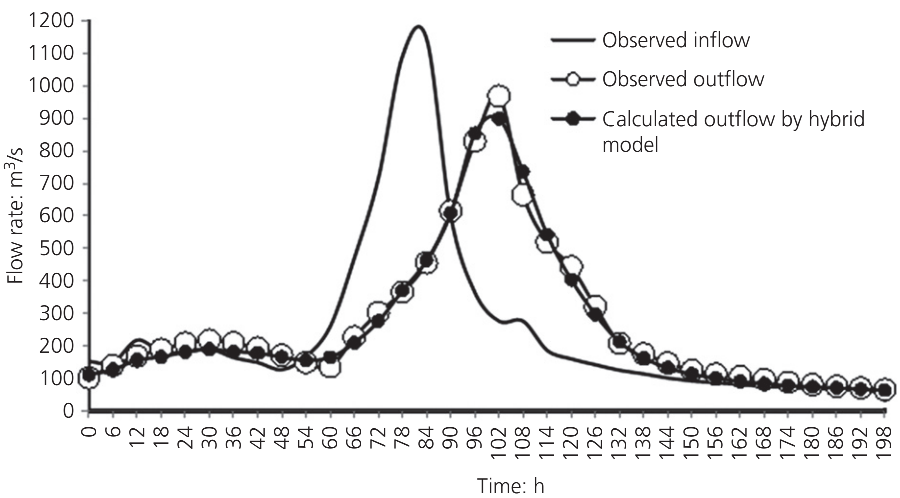

Figure 4. Comparison of the observed and calculated hydrographs obtained with the hybrid model for the second case study

models and the proposed hybrid non-linear Muskingum model. The results shown in Table 3 establish the superior predictive accuracy of the latter model.

Figure 3 depicts a comparison of the SAD values calculated with the several models for the first case study. Figure 3 demonstrates that the proposed hybrid model achieved a better (lower) absolute deviation (AD) than the other routing models.

2.5 Case study 2: real flood-routing problem

The second study refers to a flood that occurred in 1960 in the river Wye, England. The 69·75 km stretch of the River Wye from Erwood to Belmont has no tributaries and very small lateral inflow (O'Donnell *et al.*, 1988). This case study has a 6 h time step ($\Delta t = 6$) and $N = 33$. Figure 4 depicts a comparison of the observed and calculated hydrographs obtained with the hybrid model for the second case study. The hybrid model's predictions match the observed outflow very closely.

Table 5 lists the parameters, coefficients and performance criteria achieved with the various non-linear Muskingum models and the hybrid non-linear Muskingum model. It is seen in Table 6 that the proposed hybrid non-linear Muskingum

Table 5. Comparison of the performance criteria and parameter values obtained with several routing models for the second case study

Model	NL4	NL42	NL431	NL432	NL44	NL45	Hybrid
K	0·60	1·56	0·60	0·61	0·66	0·91	1·94
X_1	0·609	0·66	0·64	0·61	0·64	0·44	0·55
X_2	—	–0·09	—	—	—	—	0·49
$(\lambda)\theta$	—	—	0·66	1053	—	—	0·73
σ	—	—	—	—	—	—	—
ws_{-1}	—	—	—	—	0·12	—	0·12
ws_0	—	—	—	—	0·78	—	0·60
ws_1	—	—	—	—	0·10	—	0·29
wi_{-1}	—	—	—	—	—	–0·04	0·04
wi_0	—	—	—	—	—	0·05	–0·14
wo_0	—	—	—	—	—	0·63	0·73
α_1	1·05	0·99	1·04	1·05	1·04	1·13	1·03
α_2	1·16	1·13	1·17	1·16	1·17	1·11	1·09
β	1·40	1·37	1·40	1·40	1·38	1·31	1·28
C_1	1·00	1·04	1·00	1·02	1·12	0·98	1·13
C_2	1·00	0·62	1·03	1·00	1·08	1·05	0·91
SSD	30 894	28 853	28 136	30 804	29 402	23 741	19 953
SAD	732	705	668	723	701	683	621
DPO	73	73	76	73	72	78	69
DPOT	1	1	1	1	1	0	0

Table 6. Comparison of the objective functions and parameter values obtained by several models for the third case study

Model	NL4	NL42	NL431	NL432	NL44	NL45	Hybrid
K	0·007	0·47	0·08	0·08	2·70	10·00	0·58
X_1	5×10^{-6}	0·06	5×10^{-7}	5×10^{-7}	0·07	0·93	0·08
X_2	—	–0·03	—	—	—	—	0·93
$(\lambda)\theta$	—	—	0·71	69	—	—	0·78
σ	—	—	—	—	—	—	—
ws_{-1}	—	—	—	—	–0·36	—	1·03
ws_0	—	—	—	—	1·28	—	–0·25
ws_1	—	—	—	—	0·08	—	0·22
wi_{-1}	—	—	—	—	—	0·82	0·62
wi_0	—	—	—	—	—	0·13	0·02
wo_0	—	—	—	—	—	0·03	0·21
α_1	3·12	1·80	3·12	1·12	1·00	1·00	1·63
α_2	1·42	1·18	1·42	1·42	1·00	1·00	1·06
β	1·00	1·004	1·00	1·00	1·10	1·05	1·02
C_1	1·00	0·08	1·00	1·00	1·00	8·44	0·18
C_2	1·00	0·87	1·00	1·00	1·00	0·91	1·05
SSD	69 861	52 469	67 253	69 538	50 871	33 312	28 855
SAD	934	890	940	988	930	789	721
DPO	51	10	30	30	21	21	9
DPOT	0	0	0	0	0	0	0

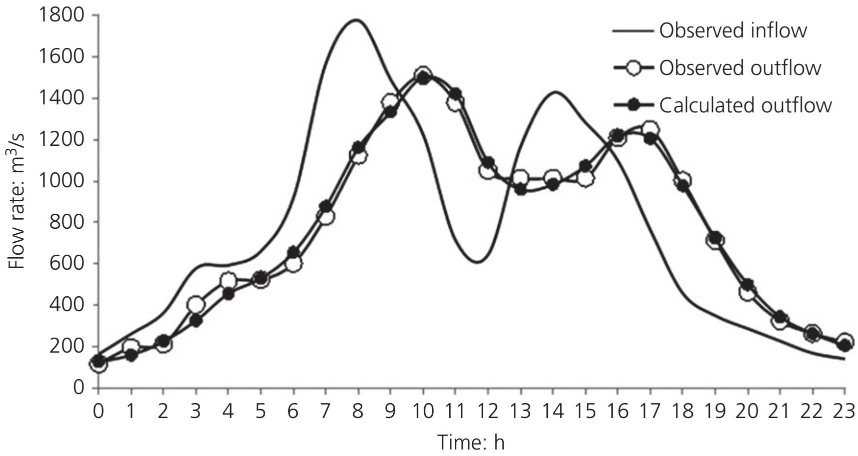

Figure 5. Comparison of the observed and calculated hydrographs obtained with the hybrid model for the third case study

model outperformed the other routing models based on the values of the SSD, SAD and DPO.

2.6 Case study 3: multimodal problem

This flood hydrograph was reported by Viessman and Lewis (2003). This case study has a 1 h period time step ($\Delta t = 1$) and the number of steps is $N = 23$. The hydrograph outflow exhibits peaks at 10 and 17 h. Figure 5 portrays a comparison between the observed outflow and routed hydrographs with the various non-linear models implemented herein. It is seen in Figure 5 that the predictive skill of the hybrid model is of high quality.

Table 6 lists the values of the SSD, SAD and DPO obtained with the various routing models. Table 6 shows that the hybrid non-linear Muskingum models' values of SSD, SAD and DPO were better (smaller magnitude) than those achieved with the other routing models.

3 Conclusions

This study introduced a hybrid non-linear Muskingum model for flood routing that improves the calculation of outflow hydrographs relative to other Muskingum routing models. The proposed hybrid non-linear Muskingum model improves some of the functions available in the NL42, NL431, NL432, NL44 and NL45 Muskingum models. The prediction of outflow hydrographs corresponding to three case studies established the overall superior performance of the proposed hybrid non-linear Muskingum model compared with other non-linear routing models judged by performance criteria embodied by the SSD, SAD, DPO and DPOT.

The predictive skill of the hybrid model has established its capability in flow routing. It is recommended herein that this new hybrid model should be applied instead of other non-linear Muskingum models in flow routing.

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