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Nationwide threshold of representation

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Abstract

How large must parties be to achieve minimal representation in a national assembly? The degree of institutional constraints is reflected indirectly by the number of seat-winning parties (n) and more directly by the threshold of representation (T), defined as the vote level at which parties have a 50–50 chance to win their first seat. The existing theoretical threshold formulas use district-level reasoning and therefore overestimate the nationwide threshold. This study extends the theory to the nationwide level. In addition to district magnitude (M), the number of electoral districts and hence assembly size (S) emerge as important variables. When all seats are allocated in M-seat districts, $T=75\%/[(M+1)(S/M)^{0.5}]$ and $n=(MS)^{0.25}$. T and n are connected by $T=75\%/[n^2+(S/n^2)]$. These theoretical expectation values are tested with 46 durable electoral systems.

Keywords: Effective thresholds; Number of parties; District and national levels; Small party representation

1. The problem

As shown by Duverger (1954), electoral systems affect party systems. In Sartori's (1976) terminology, electoral systems can be "feeble" or "strong", meaning that they can be permissive or inhospitable to small parties. How large should parties be, to be entitled to representation in a national assembly? Should 1% of the nationwide votes suffice, or 3 or 5%? And if a certain cutoff level is felt desirable, then how can it be approached through institutional design?

One obvious means is to stipulate a nationwide legal threshold, but this has been used relatively rarely. Most often the combined effect of electoral rules and other factors brings about a zone of nationwide vote shares where parties sometimes succeed and sometimes fail in gaining representation. Within this zone, an average thres-

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hold of representation can be defined where parties have a 50–50 chance of winning their first seat. It is an implicit threshold determined largely by electoral rules, but the form of its dependence on these rules has remained fuzzy. It will be shown, indeed, that the relationships previously proposed can be misleading, if used for institutional design in newly democratizing countries.

The objective of this study is to establish and test a logically derived relationship between relatively simple electoral rules and the nationwide threshold of representation. The normative issue—which barrier height is optimal under a given country's circumstances—is not discussed. But if a certain level is deemed desirable, then the present results will help to determine whether the proposed institutions are conducive to the desired outcome.

In what ways does the nationwide threshold of representation matter? An important aspect of a democratic political system is how open it is to the entry of small new parties. Extreme openness may splinter the party landscape, while extreme closure may produce staleness. A high barrier may also affect the perception of fairness and hence legitimacy of the regime. The implicit threshold introduced by electoral rules has appreciable impact on some countries having few parties, while others have many. This is why extensive theoretical work on such thresholds has been carried out, starting with Rokkan (1968). However, this previous work was limited to the district level, where calculations were more manageable. The present study extends the theory to the national level, which is of more interest regarding nationwide openness.

A related issue is how many parties receive at least minimal representation (one seat). A low barrier does not necessarily cause many small parties to form so as to take advantage of it, but on average the number of seat-winning parties can be expected to decrease with increasing threshold. The question is, how many seat-winning parties can be rationally expected for a given threshold? Although coalition forming and other power politics depend mainly on parties that are much above minimal representation, a large number of small parties in the assembly still makes the process of reaching majority more cumbersome.[1]

In sum, regarding the nationwide threshold of representation we have three interconnected issues:

1. How high is the nationwide threshold of votes at which a party can be expected to achieve minimal representation, and how does it depend on electoral rules?
2. How many parties does the given electoral system accommodate at the national assembly level?

[1] The number of parties in a legislature can be expressed in several ways. For purposes of power relations and coalition formation the number of "relevant" (Sartori, 1976) or "effective" (Laakso and Taagepera, 1979) parties is more suitable. However, the total number of seat-winning parties also matters for stability of democracies in the long run. It may affect the perception of fairness of the regime. This was the driving force behind the change of electoral rules in New Zealand; in 1996 the effective threshold was lowered, and more parties achieved representation.

3. What is the average relationship between the height of the threshold and the number of parties that succeed in surpassing it?

A theoretical answer to these questions would help us design electoral rules that yield a number of assembly parties roughly in a desired ballpark. Thus the question has both theoretical interest and also potential applicability. The specific objective of this article is to bridge the gap between the district-level equations pioneered by Rokkan (1968) and the nationwide realities. The resulting equations are tested for 46 fairly stable electoral systems.

2. Previous work

A political system's openness to small parties can be empirically measured in two aforementioned ways: the nationwide threshold of representation (T) and the number of seat-winning parties (n).

For the number of seat-winning parties the operational procedure is straightforward (at least in principle): count the parties (and independents) that win at least one seat in the national assembly. For the nationwide threshold of representation proceed as follows (Taagepera, 1989). Consider many elections where the same electoral rule is used. Find the nationwide vote share T that satisfies the following condition. The number of cases where parties won a seat with less than T equals the number of cases where parties failed to win a seat with more than T. This empirically determined value is distinct from the legal threshold (T_L) used in some countries.

A lower threshold should correspond to more parties represented, and this has indeed been observed. On the average, Taagepera (1989) found that

$$n=(60\%/T)^{0.5}, \tag{1}$$

T being in per cent. But what are the theoretical reasons behind such an empirical relationship? And which features of electoral rules and other factors determine the values of T and n in the first place?

For the number of seat-winning parties Taagepera and Shugart (1993, p. 459) have suggested a dependence on district magnitude (M) and the number of assembly seats (S). For simple electoral rules, where M seats are allocated in each district, the average theoretical expectation, broadly confirmed by data, is

$$n=(MS)^{0.25}. \tag{2}$$

Regarding thresholds, investigation began much earlier, but no explanation for the empirically observed nationwide threshold values was reached. Reasonably enough, the inquiry began at the district level, and theoretical expressions for thresholds were worked out for various specific allocation formulas (Rokkan, 1968; Rae et al., 1971; Rae, 1971; Grofman, 1975; Lijphart and Gibberd, 1977; Laakso, 1979a,b; Lijphart, 1986). Approximations that applied to most allocation formulas followed (Taagepera and Shugart, 1989; Lijphart, 1994). The most recent formulation (Taagepera, 1998a)[2] for the average threshold of achieving representation in a district is

$$T' = 75\%/(M+1). \quad \text{[district level]} \tag{3}$$

Primes are used here to designate quantities at district level, as distinct from nationwide. And this is precisely the point at which previous studies (including mine) went off the track by confusing the district and nationwide levels. Effective thresholds calculated at *district* level (on the basis of district magnitude) were counted as equivalent in their effect to legal thresholds stipulated *nationwide*. This is most visible in Lijphart (1994),[3] but indirectly my work (Taagepera and Shugart, 1989, in particular) commits the same error—and over 10 years no one has pointed it out. What may look obvious in retrospect remained uncorrected.

The result was an overestimate of nationwide effective threshold implied by district magnitude, as compared to nationwide legal threshold. This overestimate becomes obvious if applied to single-seat districts, where Lijphart (1994, pp. 17 and 28) proposed an effective threshold of about 35%, while Taagepera and Shugart (1989, p. 117) went as high as 50%. This is indeed the typical observed range for winning the seat when only one seat is at stake, be it an individual district in assembly elections or a presidential election—see examples in Table 1. Regarding the entire nationwide assembly, however, small parties are observed to have an even chance

Table 1
Sample average thresholds of representation in individual one-seat districts with plurality rule

Elections	Number of cases	Threshold (%)
Parliament, UK 1983	650	39.0
Congress, US 1970	435	49.6
Presidency, US 1824–1975	38	47.2

Sources: Taagepera (1998a) and calculations based on Diamond (1975). The number of seat-winning parties is one by definition.

[2] This formula has been mentioned previously by Lijphart (1994, p. 183) as a personal communication by Taagepera.

[3] Tables in Lijphart (1994, pp. 22, 31, 34–35 and 44) list in the same "effective threshold" column nationwide legal thresholds and district-level values calculated from district magnitudes (using a formula akin to Eq. (3)), whichever is higher. By this criterion (Lijphart, 1994, p. 22) Finland's magnitude-imposed effective threshold (5.4%) suggests that small Finnish parties would love to switch to nationwide proportional representation (PR) with a 4% legal threshold. In fact, the smallest of them would hate it, because under the present rules they tend to obtain a seat when they reach a mere 1.3% of the *nationwide* vote (because of sufficient concentration in one single district). On the other hand, those parties certain of surpassing the 4% hurdle would love it indeed, because they would avoid the less than PR that befalls most Finnish parties below the break-even point of 17% votes.

of winning at least one parliamentary seat with 7% of nationwide votes in pre-1996 New Zealand and around 1% in larger countries.[4]

These observations boil down to two general statements:

1. The nationwide effective threshold is lower than the district-level effective threshold of the same formal height based on district magnitude; and, indirectly,
2. A district-level legal threshold offers a lower barrier than a nationwide legal threshold of the same formal height.[5]

Once it is pointed out, the difference between the district and nationwide levels may look so obvious as not worth being presented as some new discovery. However, no one pointed out the frequent confusion, prior to Taagepera (1998a). Meanwhile, as components of "effective magnitude" (Taagepera and Shugart, 1989) or "effective threshold" (Lijphart, 1994), the theoretically calculated district-level thresholds, intermixed with nationwide legal thresholds, have been used to establish correlations with such presumed outputs of electoral systems as the effective number of parties and disproportionality between vote and seat shares. The results have been encouraging (see, for example, Lijphart, 1994, pp. 107–117). However, the actual degree of correlation may have been underestimated because of the incongruous values of thresholds used.

The present study offers a new theoretical formula for nationwide threshold of representation, one that goes a long way to approach reality. The key lies in taking into account not only the magnitude but also the number of electoral districts and hence the total number of seats in the assembly. In the process, the observed relationship between the number of seat-winning parties and representation threshold (Eq. (1)) also receives a theoretical basis (and is modified).

Fig. 1 illustrates the improvement made in the case of single-seat districts. The nationwide threshold of representation (T) is plotted against the number of electoral districts (E) in the country. The previous theoretical estimates of 35 to 50% are seen to fit the average single district but obviously offer no guidance whatsoever at the nationwide level. In contrast, the formula to be derived in this study comes close to the basic trend at both levels.[6]

[4] Once the issue is couched in this form, one may object: "Surely Lijphart (1994) and Taagepera and Shugart (1989) weren't saying that a party has to win 35% of the nationwide vote before it wins a single seat, nationwide." No, they didn't—they just didn't call attention to the distinction, when dealing with single-seat districts. And when dealing with multiseat districts (where the distinction isn't obvious), they did fall into the trap of comparing *nationwide* legal thresholds with *district-level* effective thresholds. See specific example in [3].

[5] A longer discussion of these proposals is available from the author.

[6] Data for Fig. 1 come from Tables 1 and 2, the latter to be presented later. The curve shown in Fig. 1 results from Eq. (8) (developed later on in this study) when M=1.

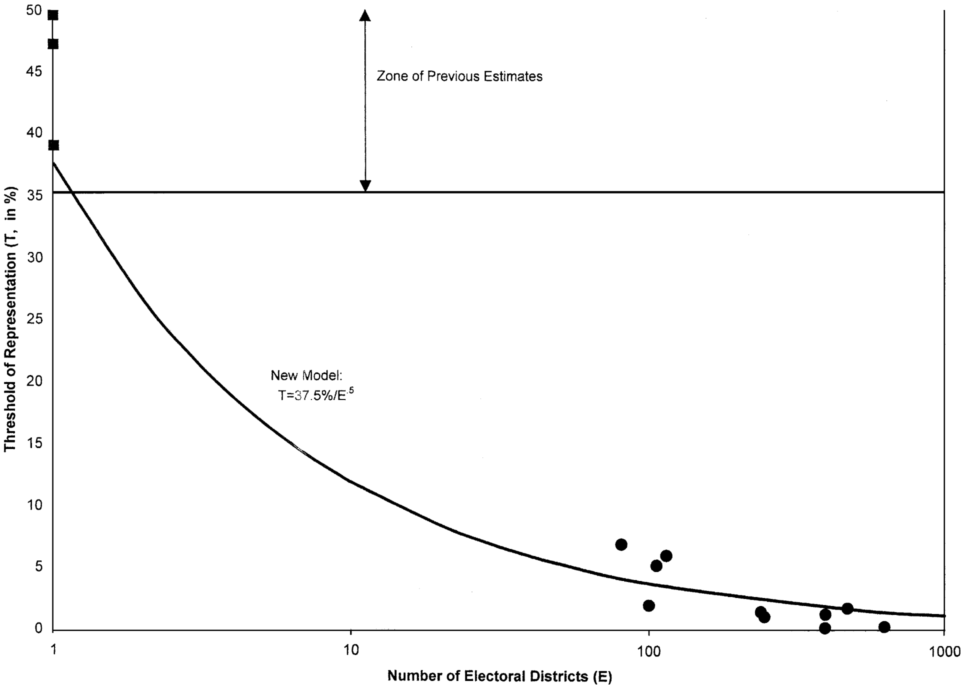

Fig. 1. Nationwide threshold of representation for single-member districts: previous estimates, new equation, and empirical data. Data from Tables 1 and 2.

3. The rational model

The threshold of representation (T) and the number of seat-winning parties (n) both depend on the institutional constraints imposed by the electoral rules but also on the learned responses by political actors and various country-specific factors. The theoretical considerations presented here deal only with the institutional constraints. As such, the results should be expected to fit the worldwide median outcomes for n and T, but not all individual countries.

3.1. The boundary values method of logical model building

The model building approach applied here is reasoning by boundary values, as used previously by Taagepera and Shugart (1993). This "ignorance-based quantitative model" is explained in detail by Taagepera (1999). The boundary values approach is used frequently in some physical sciences (e.g. electrodynamics). Consider the extreme values a variable could possibly take and draw conclusions from these constraints.

As a specific example, in a single 25-seat district the number of seat-winning parties can in principle range from 1 to 25. These are the logical boundary values. It is unlikely that a single party would capture all 25 seats, or that 25 different parties would win one seat each. The median value of actual observations is likely to be far away from the conceptually possible extremes. For reasons elaborated at length by Taagepera (1999), the geometric mean here is the best guess, in the absence of any other information. For M=25, this means about five parties winning seats, and five seats for the average party. This is the "expectation value" in the sense of actual values being distributed around it in a balanced way.

It will be seen that by judicious use of this boundary values approach we can determine expectation values for n and T at a given magnitude and number of districts. Can we expect real countries to fit such an outrageously mechanical model? There is little to lose in trying, given that no other model to predict n and T has been proposed. Once the institutional baseline is established we can check whether the median country fits and measure the degree to which various countries deviate from the baseline. Whatever the outcome, our knowledge will be enhanced, compared to a capitulating statement that "Anything can happen."

3.2. The number of seat-winning parties

This subsection retraces the reasoning in Taagepera and Shugart (1993), where the boundary values method is applied in two stages. Assume that seat allocation uses some PR or semi-PR rule (rather than multi-seat plurality). In line with the example in the previous subsection, the expectation value in one single district is

$$n' = M^{0.5}. \quad \text{[district level]} \tag{4}$$

Now consider the simplest possible nationwide situation: a country divided into E electoral districts, all with magnitudes fairly close to the average magnitude (M). Thus the assembly has S=EM seats. Apply again the boundary values approach. If, instead of the actual districts, nationwide seat allocation were used (meaning a single district of magnitude S), $S^{0.5}$ parties would be expected to win seats according to Eq. (4) above. If, on the other hand, the votes distribution in all actual districts is identical, then the same $M^{0.5}$ parties would win seats in each district and nationwide, too. Therefore, the logically possible range for n is $M^{0.5}<n<S^{0.5}$. In the absence of any other information about the vote distribution among the districts, the geometric mean of the boundary values is again the most plausible value:

$$n = (MS)^{0.25} = n'E^{0.25} = (S^2/E)^{0.25} = (M^2E)^{0.25}, \quad \text{[nationwide]} \tag{5}$$

Eq. (5) adds several alternative expressions to Eq. (2), the result obtained by Taagepera and Shugart (1993). This rational model would be validated if the actual data points are distributed roughly equally above and below the expectation value.

3.3. Nationwide threshold of representation

Now we come to the core of this study. Assume again E districts of magnitude M. First consider the effect of a legal threshold applied in each district separately

(T_L'). Make it higher than the threshold inherent in district magnitude (Eq. (3)): $T_L' > T' = 75\%/(M+1)$. This means that surpassing T_L' guarantees a seat.

What would be the corresponding nationwide threshold? Again, locate the logical boundaries. Consider a party whose nationwide votes share falls just short of T_L'. If the votes were evenly distributed among the districts, this party would have a lesser share than T_L' everywhere and would narrowly fail to win in each district. This is the unluckiest case possible. Now consider a party whose nationwide vote share is barely above T_L'/E. If all these votes were concentrated into one single district, this party would narrowly win a seat. This is the luckiest possible case. Thus the logical boundaries are

$$T_L'/E < T < T_L. \tag{6}$$

Taking again the geometrical mean of the possible extremes (for reasons explained in Taagepera, 1999) yields

$$T = T_L'/E^{0.5}. \tag{7}$$

Now choose T_L' to be exactly equal to the district-level effective threshold imposed by magnitude: $T_L' = T' = 75\%/(M+1)$. Under such conditions the district-level legal threshold and district magnitude have essentially the same effect. Hence, when M determines the district-level threshold, Eq. (7) should still apply:

$$T = T'/E^{0.5} = 75\%/[(M+1)E^{0.5}] = 75\%/[(M+1)(S/M)^{0.5}], \quad \text{[nationwide]} \tag{8}$$

the latter form resulting from $S=EM$. The first variant of this equation looks like the district-level one (Eq. (3)), except for insertion of $E^{0.5}=(S/M)^{0.5}$.

This is the central theoretical result in this study. The assumptions undergirding Eq. (8) are the following.

1. At district level, Eq. (3) is a fair approximation: $T'=75\%/(M+1)$.
2. There are E districts of fairly equal magnitude M.
3. Given that the nationwide threshold T imposed by M must logically be lower than T' but higher than T'/E, the best approximation for T is T' divided by the square root of E (Eq. (6) to first part of Eq. (8)).

3.4. The number of seat-winning parties vs. threshold of representation

Eqs. (5) and (8) connect an output variable (n or T) to institutional input variables (M, S, E). But we can also derive expressions that predict a specific average relationship between the two output variables, T and n, once either magnitude or assembly size is given. Indeed, given that $E^{0.5}=n^2/M=S/n^2$ (according to Eq. (5)), we also have the following alternative forms:

$$T = 75\%/[n^2(1+1/M)] \quad \text{[nationwide]} \tag{9}$$

and

$$T = 75\%/[n^2 + (S/n^2)]. \quad \text{[nationwide]} \tag{10}$$

These equations supply a theoretical explanation for the empirical relationship $n=(60\%/T)^{0.5}$ observed earlier (Eq. (1)). Indeed, Eq. (9) reduces to Eq. (1) when $M=4$, which is close to the median district magnitude observed.

According to Eq. (10), as threshold increases, the number of seat-winning parties decreases, as one would expect, but the assembly size also matters. This effect is small, though. For $M=1$, Eq. (9) yields $T=75\%/(2n^2)$, while for $M=S$ it can be shown that $T=75\%(n^2+1)$ results.[7] The allowed zone delimited by these boundaries is quite narrow.[8] The actual variations in the T–n relationship can be expected to be much wider than those caused by assembly size, because Eq. (9) represents only the expected average, and various factors extraneous to electoral rules enter.

The contrast between district-level and nationwide thresholds is the most marked for systems with numerous single-seat districts, as illustrated in Fig. 1. In the case of multi-seat districts the contrast is smaller (because the number of districts is much lower) but still appreciable. For Finland ($E=15$ and $M=14$, when omitting the single-seat Aland district), district-level $T'=75\%/14=5.4\%$, while at the national level $T=75\%/[(14+1)3.9]=1.3\%$ (cf. [3]).

When a country uses a single nationwide district the difference between district and nationwide thresholds vanishes, according to the equations above, as well it should, unless a legal threshold overrides the magnitude. Thus in the case of Denmark the nationwide legal threshold of 2% overrides the threshold based on nationwide PR allocation of 175 seats, which is 75%/176=0.4%.

The reader may react skeptically to the theoretical equations presented, because (1) it cannot be so utterly mechanical, devoid of specifically political considerations; and (2) it better not be, because otherwise it would take the fun out of the study of politics. But this would be an excessive interpretation of the present model. First, the model based on institutional constraints deals only in long-term averages: it does not predict the number of parties that will win seats in tomorrow's elections. Second, the model does not pretend to explain everything even about the average threshold of representation in a given country. To do so would be unreasonable in view of the obvious effects of culture, history, and sociopolitical and geographic heterogeneity. But it would be equally unreasonable to deny that electoral rules impose some constraints.[9]

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[7] Proof: When $M=S$ Eq. (8) yields $T=75\%/(M+1)$, while Eq. (5) yields $n^2=M$. Hence $T=75\%(n^2+1)$.

[8] This theoretically allowed zone is shown in Fig. 3, to be introduced later in connection with empirical testing.

[9] Are these equations truly theoretical, or do empirical considerations slip in? No fitting of empirical data is involved—we deal with algebraic quantities based on conceptual extreme values. The only place where this may not seem to be the case is Eq. (3) with its numerical value "75%". Where does this number come from? For a specified allocation rule, the exclusion and inclusion thresholds are extreme possibilities that can be calculated theoretically (at district level). The average threshold T' in Eq. (3) represents a judicious median for these theoretical exclusion and inclusion thresholds for many usual allocation rules, and judgment does enter. This is why Taagepera and Shugart (1989), Lijphart (1994) and Taagepera (1998a) reached slightly different expressions, depending on how they weighted the various

How much of the total variation is explained by the institutional constraints? This is to be found out from empirical testing, to be presented next. It will be seen that appreciable playroom is left to other factors, while still showing that M and E have considerable predictive power regarding the average outcomes of elections.

4. Empirical testing

4.1. The data

As data base, Mackie and Rose (1991) was chosen. The test involves measured n and T for all those elections where basically the same electoral rules were maintained for at least four elections.[10] Three groups of electoral systems are distinguished:

1. seats fully allocated in districts;
2. seats allocated nationwide, subject to legal threshold; and
3. more complex allocation rules.

4.1.1. Seats fully allocated in districts

For this group the average district magnitude (M) and the number of districts (E) are reasonably well defined. Hence Eqs. (5) and (8) can be used to calculate the number of seat-winning parties and the threshold of representation, respectively. The values of n and T calculated from Eqs. (5) and (8) will be designated as baseline values (n_B and T_B). They will be compared to values actually measured (n_A and T_A).

Table 2 lists all countries and periods in Mackie and Rose (1991) that qualify. The 14 cases with $M>1$ and the 16 cases with $M=1$ are kept separate. M and E are shown, and the electoral systems are listed in the increasing order of E. Assembly size is $S=EM$. Further shown are the baseline number of seat-winning parties n_B calculated from Eq. (5) and the actual number n_A. Similarly, the baseline threshold T_B calculated from Eq. (8) is shown along with the actual threshold of representation (T_A).

theoretical extremes. However, these possibly empirically motivated shifts are small (see graph in Taagepera, 1998a, p. 396). We are essentially dealing with theory-based averages even there.

[10] The procedure is direct for n_A. Count the seat-winning parties for each election, and take the average over elections that used the same electoral rules. Difficulties arise when many seats in the data source are listed under "Other". The square root of the number of such seats was added to n_A. The resulting range of error is largest for Japan and Ireland. T_A is determined as follows (Taagepera, 1989). For a period during which the same electoral rule was used, find the vote share T_A such that the number of cases where any party won a seat with less than T_A equals the number of opposing cases where any party failed to win a seat with more than T_A. At this votes level a party has a 50% probability of winning a seat. To measure T_A, cases must exist where parties narrowly win or miss their first seat, and the "Other" category presents further difficulties.

Table 2
Baseline and actual numbers of seat-winning parties and thresholds of representation – electoral systems with seats fully allocated in districts

Country, period, and number of elections	Average district magnitude (M)	Number of districts (E)	Seat-winning parties (n)		Threshold (T, in %)	
			Base[a]	Actual	Base[b]	Actual
M>1						
Luxembourg[c] 1922–51, 7	13.4	2	4.3	4.7	3.7	3.4
Luxembourg 1919–89, 11	13.7	4	5.2	5.5	2.6	2.2
Malta 1921–45, 6	4	6.2	3.2	3.0	6.0	8.8
Malta 1947–87, 11	5	10.0	4.0	3.2	4.0	3.9
Finland 1907–87, 30	14	15	7.2	6.8	1.3	1.3
Norway 1953–85, 9	7.8	19.5	5.9	6.3	1.9	2.6
Norway 1921–49, 8	7.5	20	5.8	6.5	2.0	1.3
Portugal 1975–87, 7	11.3	22	7.3	6.9	1.3	1.4
Switzerland 1919–87, 19	7.8	25	6.3	10.5	1.7	0.7
Sweden 1952–68, 6	8.9	26	6.7	5.7	1.5	1.5
Sweden 1908–48, 14	8.2	28	6.6	5.2	1.5	0.8
Ireland 1922–89, 24	3.5	42.7	4.8	8.2	2.6	1.0
Spain 1977–86, 4	6.7	52	7.0	12.8	1.4	0.4
Japan 1928–86, 22	4.0	120	6.6	10.0	1.4	0.7
M=1						
Australia 1901–17, 7	1	75	2.9	3.4	4.3	ND
New Zealand 1890–1987, 32	1	80.8	3.0	3.5	4.2	6.9
Netherlands 1888–1913, 8	1?	100	3.2?	6.5	3.8?	2.0
Australia 1919–87, 28	1	106.0	3.2	3.7	3.6	5.2
Norway 1882–1903, 8	1	114.4	3.3	2.9	3.5	6?
Denmark 1901–18, 7	1	117.6	3.3	4.7	3.5	ND
Norway 1906–18, 5	1	123.6	3.3	5.7	3.4	ND
Sweden 1887–1905, 8	1+	226.4	3.9	2.8	2.5	ND
US 1828–82, 28	1	239.9	3.9	3.0	2.4	1.5
Canada 1878–1988, 31	1	246.9	4.0	4.4	2.4	1.1
Germany 1871–1912, 13	1	395.8	4.5	13.6	1.9	0.2
US 1884–1936, 27	1	396.2	4.5	3.3	1.9	1.3
US 1938–88, 26	1	435.2	4.6	2.5	1.8	ND
France 1958–81, 7	1	470.1	4.7	6.7	1.7	1.8
Italy 1895–1913, 6	1?	508	4.8?	5.1	1.7	ND
UK 1922–1987, 19	1	628.2	5.0	6.4	1.5	0.3

Periods with essentially constant electoral rules and average M and E were determined mainly based on Nohlen (1978) and Lijphart (1994). $S=EM$. Actual n and T were determined on the basis of Mackie and Rose (1991). The question marks reflect unclarities regarding district magnitude. Notation "1+" for M indicates the presence of a few multi-seat plurality districts. The 57 US House elections are divided into three separate time periods despite the rules remaining the same, because E increased appreciably and other changes in political conditions are possible over almost two centuries.

ND=not determinable because of dearth of parties narrowly winning or missing a seat.

[a] n_B calculated from Eq. (5).

[b] T_B calculated from Eq. (8).

[c] Elections covering alternating halves of the country.

4.1.2. Nationwide allocation subject to legal threshold

A different situation arises when the seats are effectively allocated by PR in a single nationwide district, subject to a legal threshold sufficiently high to block some small parties that otherwise would obtain representation. Here the legal threshold T_L imposes itself as baseline T_B. Since the legal threshold overrides the effect of M, n_B cannot be calculated from Eq. (5). However, n_B can be calculated from Eq. (10) and the nationwide legal threshold. This theoretical n_B can be compared to the measured value, n_A. The results for electoral systems of this type in Mackie and Rose (1991) are shown in Table 3, listed by increasing legal threshold.[11]

4.1.3. Complex allocation rules

In other electoral systems, seat allocation takes place partly at district and partly at national (or other supra-district) level. The rules can be complex, and hence the effect of the number and magnitude of districts is overridden by other features. Hence Eqs. (5) and (8) cannot be applied.

A remarkable aspect of Eq. (10), however, is that, although it results from two equations both of which involve the number and magnitude of districts (Eqs. (5) and (8)), E and M cancel out in a way that leaves assembly size as the only parameter. This means that Eq. (10) can be applied formally to complex electoral systems where the effect of district magnitude is overridden by other features. Such extension puts the theory to further test. Will complex electoral systems still follow the pattern of

Table 3
Legal and actual thresholds of representation, and baseline and actual numbers of seat-winning parties—electoral systems with nationwide seat allocation subject to nationwide legal threshold

Country, period, and number of elections	Assembly size (S)	Threshold (T, in %)		Seat-winning parties (n)	
		Legal	Actual	Base[a]	Actual
Netherlands 1956–81, 10	150	0.67	0.67	10.5	10.6
Netherlands 1918–52, 9	100	0.75	0.7	9.9	10.4
Israel 1949–88, 12	120	1	1.0	8.6	12.4
Denmark 1964–88, 12	175	2	2.0	6.0	8.5
Sweden 1970–88, 7	349.3	4	4.0	4.2	5.3
Germany 1961–87, 8	496.9	5	5.0	3.7	3.2

Periods with essentially constant electoral rules were determined mainly based on Nohlen (1978) and Lijphart (1994). Actual n and T were determined on the basis of Mackie and Rose (1991).
[a] n_B calculated from legal T through Eq. (10).

[11] These countries may be divided into formal districts, but nationwide compensatory seats impose nationwide PR, subject to legal threshold (and various other stipulations in some countries). Inclusion of T_A, measured as before, in Table 3 may look superfluous, but it isn't so. Many legal thresholds (e.g. in Denmark and Germany) have escape clauses for parties with sufficient regional vote concentrations. Therefore, the equality of legal and actual thresholds cannot be taken for granted, even though no deviations occur in Table 3. Regarding applicability of Eq. (10), see [12].

Eq. (10)?[12] For this part of the test, Table 4 lists the actual values (n_A and T_A) for complex systems available in Mackie and Rose (1991). Countries are listed by increasing assembly size.

We now have two ways to evaluate the agreement of actual values with the theoretical model for nationwide threshold of representation:

- T_A vs. $(M+1)E^{0.5}$, using data in Table 2 (and, marginally, Table 1)—Fig. 2; and
- n_A vs. T_A, using data in all four tables—Fig. 3.

As a preparatory step, the relationship between the number of seat-winning-parties and *MS* (Eq. (2)) as proposed by Taagepera and Shugart (1993) is checked. In Table 2 the values of n_B and n_A are shown separately for systems with $M>1$ and $M=1$. The actual number of seat-winning parties exceeds the expected number for 19 cases and falls short for 11. The median difference n_A-n_B is +0.35 parties for $M>1$ countries and +0.75 parties for $M=1$ countries, the overall median being 0.4 parties. Given that n_A-n_B ranges from −2.1 to +9.1 parties for individual countries, its overall

Table 4
Actual number of seat-winning parties and threshold of representation—electoral systems with multi-tier or otherwise complex allocation of seats

Country, period, and number of elections	Assembly size (S)	Actual seat-winning parties (n_A)	Threshold (T_A, in %)
Iceland 1934–87, 18	55.7	4.7	3.3
Denmark 1920–50, 13	146.7	6.5	0.8
Austria 1923–70, 11	165	3.7	3.0
Austria 1971–86, 5	183	3.2	3.3?
Belgium 1919–87, 22	204.9	7.7	0.8
Greece 1974–89, 5	300	5.2	1.1
Germany 1949–57, 3[a]	462	7.2	1.8
Italy 1919–21, 2	522	10.5	0.4
Germany 1920–33, 8	541.4	12.9	0.2
Italy 1946–87, 11	611.5	11.4	0.4

Periods with essentially constant electoral rules were determined mainly based on Nohlen (1978) and Lijphart (1994). Actual n and T were determined on the basis of Mackie and Rose (1991).
[a] Gradual shift from *Land* level to nationwide PR and 5% threshold.

[12] Why would one expect Eq. (10) still to work, even when its theoretical underpinnings (number and magnitude of districts) no longer are in evidence? Recurrent attempts have been made to approximate the major effects of complex electoral rules with simple rules, using the notion of “effective magnitude” (Taagepera and Shugart, 1989) or “effective threshold” (Lijphart, 1994). The underlying assumption is that, in its effects on party representation, any complex set of electoral rules broadly corresponds to a simple one, reduced to a number of equal-magnitude districts, even though the equivalent E and M may be hard to pin down. If so, then the premises of Eq. (10) are not “false premises” in the case of complex electoral rules but hidden premises. If such an extension should be unwarranted, then it would become manifest in the form of a wide disagreement with measured values.

median is relatively close to zero. The model based on M and S alone looks like a fair first approximation.

4.2. Nationwide threshold of representation

The actual threshold falls short of the expected in 16 cases and exceeds it in eight cases in Table 2. The median difference $T_A - T_B$ is -0.3% for $M>1$ countries and -0.75% for $M=1$ countries, the overall median being -0.35%. Given that $T_A - T_B$ ranges from -1.8% to $+2.8\%$ for individual countries, the median difference is relatively close to zero. But let us look at the detailed picture.

Fig. 2 shows T plotted against $(M+1)E^{0.5}$, so that Eq. (8) corresponds to a straight line. Both axes are on logarithmic scale.[13] There are two conceptual limits. For a single one-seat district ($M=E=1$) we have $(M+1)E^{0.5}=2$; this is the lowest possible value of $(M+1)E^{0.5}$. Also T cannot exceed 50%. This is illustrated by the single-district values in Table 1, which are also shown in Fig. 2. Among the $M>1$ systems, those with rather high E (Spain, Ireland, Switzerland and Japan) deviate the most.

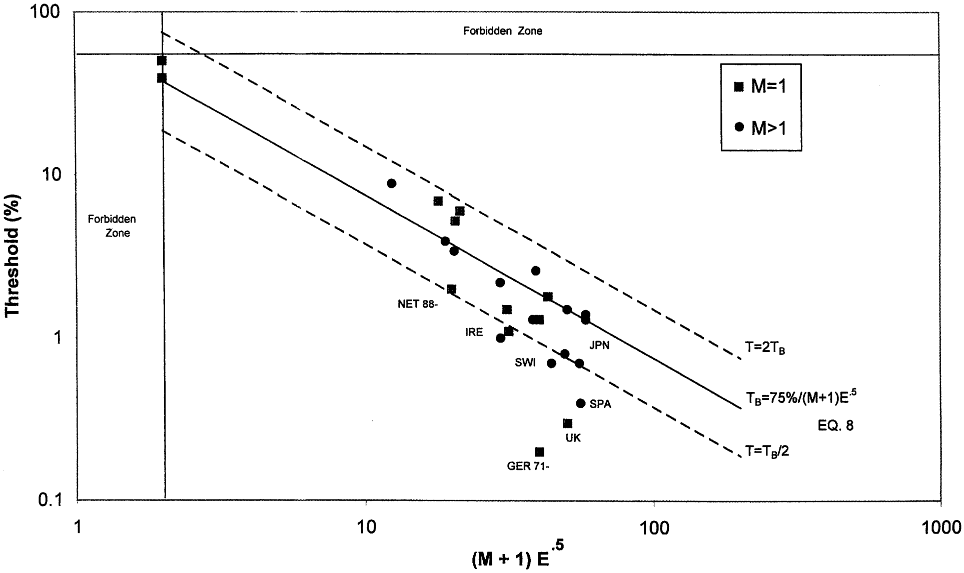

Fig. 2. Nationwide threshold of representation in terms of the number and average magnitude of electoral districts: Eq. (8) and data mainly from Table 2 but also Table 1.

[13] The logarithmic scale is used in Figs. 2 and 3 for three interconnected reasons. (1) It avoids a crowding of points at the lower values and thus brings the general trend better into evidence. (2) Equal relative (per cent) deviations of actual values from baseline correspond visually to equal distances on the graph, thus giving a more adequate idea of the relative deviation. (3) The baseline relationship corresponds to a straight line (or nearly so, in Fig. 3) on log–log, making it easier to evaluate the deviations from baseline.

Among the M=1 systems, Imperial Germany and the UK fall much below the baseline.

In 19 cases out of 24 the institutional theory predicts the actual threshold of representation within a factor of two.[14] However, with few exceptions at low combinations of M and E (New Zealand, Australia, Norway 1882–1903, and Malta 1921–45) the theoretical baseline tends to represent the upper limit on the actual T rather than the average. In the case of Imperial Germany the actual T_A is lower than T_B by a factor of almost 10.[15] Either the impact of institutional constraints must be revised, or political factors tend to work mainly in the direction of lowering the actual threshold. The latter may be the case, for the following reasons.

Small parties can desist from running candidates in the hopeless districts, concentrating their resources in the potentially winnable. As a result, their nationwide vote share may decrease over time. If they still win a seat, it is now won with a reduced percentage of the nationwide vote, so that the measured T_A decreases. This effect is strongest when the number of districts is high, as it is in most M=1 systems.[16] The small parties may find it harder to drop some districts completely when the number of districts is low (either because M is large or the total number of assembly seats is rather low, as in pre-1996 New Zealand). In such a case the pattern described by institutional constraints is preserved.

4.3. Number of seat-winning parties vs. threshold of representation, nationwide

The lower the nationwide threshold, the more parties may succeed in winning at least one seat. The rational model expresses it through Eq. (10). Sometimes the relationship is driven by the number and magnitude of districts (Table 2). Sometimes a nationwide legal threshold predominates and imposes a value of n (Table 3). Sometimes the determining factors are more complex, but the actual n and T can still be measured (Table 4). It makes sense to plot all these actual values of n versus T to see whether the data in the three tables produce similar patterns.

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[14] It may be asked why an expression such as “within a factor of 2” is used rather than apply the statistical measures of goodness of fit more widespread in political science. The answer is that the latter sometimes do not tell the entire story or tell different stories depending on how they are introduced. The central question is: If one uses the model presented to estimate the threshold of representation in an unknown country, how close can one expect to be? “Within a factor of 2” answers that question in a way that is easy to envisage.

[15] Visibly, a much better statistical fit in Fig. 2 could be obtained with a parabolic curve, but it would lack theoretical justification. Moreover, it would depend heavily on a few outliers (Imperial Germany, the UK, Spain).

[16] Single-seat districts have a high exclusion threshold (worst case possibility), and this helps to explain why they often lead to two-party hegemony. However, in the presence of multiple competitors they also have a fairly low inclusion threshold (best case opportunity), and this may enable independents and local parties to win seats. In the presence of three nationwide parties, a local independent in a UK district faces an inclusion threshold of only 25% of district votes—which translates into a mere 0.04% of the nationwide vote (cf. Grofman, 1999; Taagepera, 1998b). The same extends to small regional parties, which may explain the low actual nationwide thresholds in Imperial Germany and the UK.

Fig. 3 shows *n* plotted against *T*, both on logarithmic scales. The data from different tables are plotted using different symbols. The extreme curves corresponding to Eq. (9) are shown—those with M=1 and M=S. Except for the UK, the data points are within a factor of two of the center of the zone delineated by these extremes. Agreement is excellent for M>1 systems: 10 out of 14 points are within the theoretically predicted zone. Complex systems also fit well: 4 points out of 10 are in the zone, and the rest are balanced on both sides. For M=1 systems the points are widely but evenly scattered on both sides of the theoretical zone. The legal threshold systems is the only category where the number of seat-winning parties at a given threshold tends to be high.

Imperial Germany, the extreme outlier in Fig. 2, falls in line in Fig. 3. While its observed *n* and *T* deviate from expectations based on the number of districts, they do so in a self-consistent way.[17] The same is true of several other outliers of Fig. 2 (Spain, Switzerland, Ireland, Japan, and The Netherlands 1888–1913). In conjunction with the fair agreement for complex systems, the hunch is confirmed that the height of the hurdle and the number of parties surpassing it are uniquely connected.

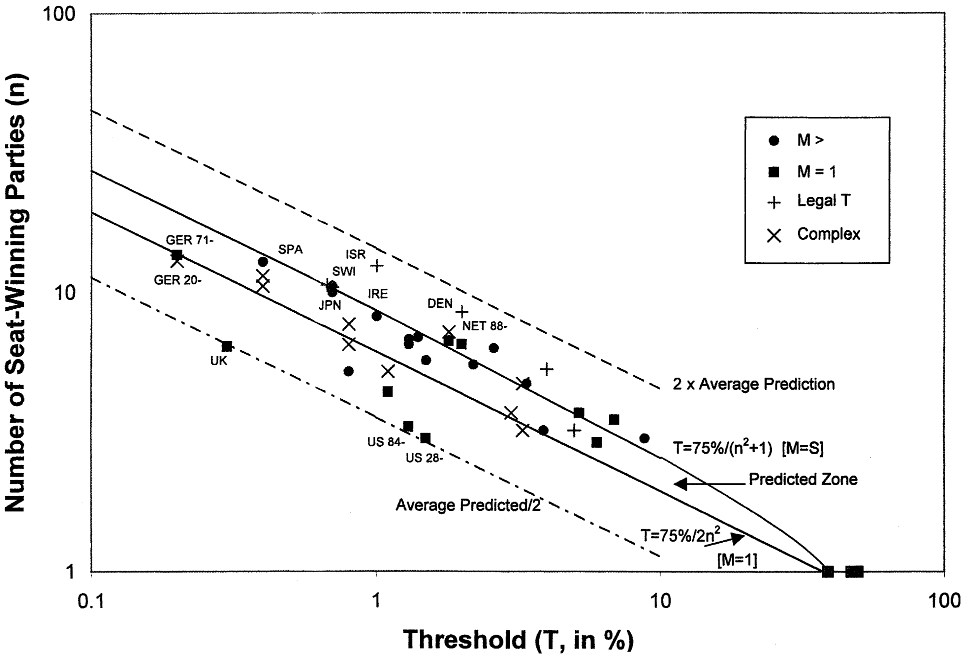

Fig. 3. Nationwide number of seat-winning parties vs. threshold of representation: Eq. (10) and data from Tables 1–4.

[17] Incidentally, both *n* and *T* of Weimar Germany are strikingly close to those of Imperial Germany, despite a major change in electoral rules. It may be random coincidence, or it may suggest that the cultural features that lowered the threshold of representation in Imperial Germany survived it.

The present study has established it in an indirect way, grounding both n and T separately in the number and magnitude of districts, but a more direct and general proof might be possible.

Once the mechanical constraint of the electoral rules is accounted for, the more directly political features are highlighted. One may wonder why all larger Anglo-Saxon countries (the US, the UK, Canada) in Fig. 3 have relatively few seat-winning parties at the given observed threshold, even though they vary in their degree of geographic homogeneity and have quite different party structures and nomination procedures. Among the legal threshold systems, why do Israel, Denmark and Sweden have more parties than expected, while Germany and The Netherlands don't? These are some of the new questions generated by the present analysis.

5. Conclusion

In addition to coalition formation and other power politics (which depend mainly on parties way above the nationwide threshold of representation), the issues of fairness and openness also enter political discourse. This is the reason why many countries, over time, have shifted to more proportional representation. However, the indices of overall deviation from PR do not tell the entire story about the degree of openness to new parties, because these indices depend heavily on the larger parties. The nationwide threshold of representation is the most direct indicator of openness.

So, what has been achieved in this study? Above all, a theoretical formula has been established to estimate the average nationwide vote share needed to win the first seat in the assembly, at a given district magnitude. Scholars ranging from Rokkan (1968) to Lijphart (1994) have thought it worthwhile to strive in this direction, because implicit thresholds are a measure of the openness of the system to smaller parties. However, the earlier formulas were restricted to district level. The transition to the more interesting nationwide level became possible with the realization that not only the (average) magnitude of the districts mattered but also the number of districts. In hindsight, this may look obvious, but it did not enter previous quantitative work.

What can one do with such an equation (Eq. (8))? It offers an avenue toward more informed design of electoral rules. It has been known (and used in institutional engineering) that district magnitude matters in determining how large parties must be to gain representation. But the same average district magnitude leads to easier access when the assembly is larger (meaning more districts). Taking the number of districts into account, one would get closer to the desired outcome (whatever it is). In particular, single-member districts do *not* necessarily produce higher nationwide thresholds than PR. The predicted baseline values in Table 4 cover about the same range for M=1 and M>1. Indeed, the highest actual threshold occurs for M>1 (Malta 1921–45), and the lowest for M=1 (UK 1922–87).

How good is the proposed equation for purposes of institutional design? It is approximate, because many other factors enter, but it is an improvement on what has been available up to now. Agreement with the basic trend, as the number and average magnitude of districts increase, is in evidence in Fig. 2—as well as the extent to which some actual countries diverge.

The formula for nationwide threshold of representation is the main result of this study. But there is a complementary aspect: linking the typical minimum size of the parties that gain representation (the nationwide threshold) to the number of such parties. An earlier formula for the number of parties that can be expected to be present in an assembly has been reconfirmed here. Having now equations both for threshold and the number of parties surpassing this threshold enables us to propose an expected relationship between the two, and Fig. 3 shows the degree of agreement with actual data.

Of what significance is this relationship? It is of course not surprising that more parties surpass the barrier when it is lowered. But how many? For institutional design both the number and minimum size of parties in the assembly are of interest, given that the presence of many marginal-size parties makes coalition formation more difficult. It is therefore of interest to have an estimate of how many parties tend to surpass a given barrier height.

True, an empirical equation for this relationship was already available. So what can we do with the new equations that could not be done before? A new advantage is that the effect of district magnitude is introduced: according to Eq. (9), the same threshold allows for more parties to be represented when the average magnitude is larger. However, as can be seen in Fig. 3, the impact of magnitude on the T–n relationship is so small as to be swamped by the range of random variation.

And so I have to spell out another reason for including this derivative result: having, when faced with an observed empirical relationship (Eq. (1)), the urge to know "why?"—the urge to understand not only *how* things hang together but *why*? Eq. (9) (or, alternatively, Eq. (10)) offers a rational explanation, subject to its underlying assumptions. This means that we understand why threshold and number of seat-winning parties are connected the way they are—and also under what conditions the empirically observed relationship might fail. For instance, if the distribution of district magnitudes is bipolar, then operating with the average magnitude becomes questionable.

The theoretical relations derived from institutional constraints are borne out by data for a variety of electoral rules, mostly within a factor of two. The agreement is especially close when all allocation takes place in districts that are few in number. Some deviation from the equations is to be expected, since they do not take into account the specific seat allocation formulas used and various other aspects of electoral systems, not to mention political responses to the given electoral rules and country-specific factors. Small parties do not necessarily form so as to take advantage of a low threshold. If other factors do cause them to form, they can find ways to counteract the mechanical effect of the electoral rules, but to a limited degree. Many other things matter, along with the electoral system, and the outcome may be partly path dependent.

In view of the sparse input variables (merely the number and magnitude of districts), the surprising aspect is that deviations from the theoretical model are as small as they are. Visibly, some important factors have been pinned down. The strength of other factors can be measured as deviation from baseline expectations.

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