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Categorical Spectral Analysis of Periodicity in Human and Viral Genomes

by

Elizabeth Howe

THESIS

Submitted in partial satisfaction of the requirements for the degree of

MASTER OF SCIENCE

in

Biological and Medical Informatics

in the

GRADUATE DIVISION

of the

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Dedication and Acknowledgments

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Categorical Spectral Analysis of Periodicity in Human and Viral Genomes

Elizabeth Howe

Abstract

Periodicity in nucleotide sequences arises from regular repeating patterns that may reflect important structure and function. Although a three-base periodicity in coding regions has been known for some time and has been used as the basis for powerful gene prediction algorithms, its origins are still not fully understood. Here we show that, contrary to common belief, amino acid bias and codon usage bias are insufficient to create base-three periodicity. This paper rigorously applies the method of spectral envelope to systematically characterize the contributions of codon bias, amino acid bias, and protein structural motifs to the three-base periodicity of coding sequences. Additionally, we show how the method of spectral envelope can be used to trace the evolution of viral genomes and monitor global sequence changes without having to align to previously known genomes. This method also detects reassortment events, such as those that led to the 2009 pandemic H1N1 virus.
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Abstract

Periodicity in nucleotide sequences arises from regular repeating patterns that may reflect important structure and function. Although a three-base periodicity in coding regions has been known for some time and has been used as the basis for powerful gene prediction algorithms, its origins are still not fully understood. Here we show that, contrary to common belief, amino acid bias and codon usage bias are insufficient to create base-three periodicity. This paper rigorously applies the method of spectral envelope to systematically characterize the contributions of codon bias, amino acid bias, and protein structural motifs to the three-base periodicity of coding sequences. Additionally, we show how the method of spectral envelope can be used to trace the evolution of viral genomes and monitor global sequence changes without having to align to previously known genomes. This method also detects reassortment events, such as those that led to the 2009 pandemic H1N1 virus.

1 Introduction

Recent advancements in molecular biology have made a vast amount of functional and structural information in genomic nucleotide sequences available at multiple levels. Analyzing the sequences that become available is important because it helps us identify subtle features specific to coding sequences, regulatory sites, introns, and intergenic regions; sequence analysis also facilitates our understanding of the principles that have guided DNA sequence evolution. A staple in traditional sequence analysis is local alignment techniques. However, a major problem with these techniques is that they represent DNA molecules as two-dimensional units, despite their three-dimensional structure and well-known long-range interactions.

For example, several recent studies highlight the limitations of local alignment techniques. The conformational states of DNA under negative superhelical stress critically depend on long-distance coupling of base-pairs [1]. Furthermore, traditional motif analyses have failed to explain how long non-coding RNAs find their target genomic locations and direct chromatin remodeling. In addition, motif analyses can account for only half of nucleosome-free regions in yeast [2]. Even though nucleosomes have high preferences for specific DNA sequences in vitro, probabilistic nucleosome-DNA interaction model of this study has limited predictive abilities, and might benefit from incorporating relations between DNA duplex destabilization and nucleosome organization. The duplex destabilization energy is associated with long genomic distance because it comes from both long-range torsional and short-range stacking energies of base pairs. In a similar vein, it should be known that aligning nucleosomal DNA can produce spurious
periodicity of dinucleotides. Particular dinucleotides have been hypothesized to be present at helical
turns of DNA (about 10 bp periods) of nucleosomes and to aid in nucleosome formation. However, this
periodicity is likely to be a byproduct of the alignment and likely to be missing in individual nucleosomes;
for example, if we randomly generate 3000 sequences and align them by shifting them to the left or right
by at most 5 bp, we can create spurious periodic AA counts. More rigorous approaches are needed to
study this phenomenon. In summary, these limitations suggest that new alignment-free perspectives are
necessary to extract information from genomic sequences.

One compelling idea is to analyze sequences from a dual picture of frequency space and study hidden
periodic features without having to perform local alignments. This idea of searching for patterns in
frequency space has revealed two main periodicities: an approximate 10.5-base periodicity, which relates
to DNA helical repeats to facilitate nucleosome formation, and the three-nucleotide periodicity, which we
will focus in this paper. The three-nucleotide periodicity describes the property that many DNA regions
have a prominent power spectrum peak at frequency 1/3 arising from the fact that codons are three
nucleotides long. The three-nucleotide periodic phenomenon is commonly associated with protein-coding
regions and has intrigued many biologists for several decades [3–8].

For example, it has led to Shepherd’s proposal that the ancestral forms of present-day genes might
have consisted of repeating RNY (purine-any-pyrimidine) triplets [3]. Specifically, for three species of
viruses, and somewhat for prokaryotes and eukaryotes, properties in phase and amplitude of the periodic
variations belonging to the relative positions of purines and pyrimidines suggested that vestiges of an
earlier coding system are still visible. The phase information of the correlations was the same in all the
genomes tested and indicates that all primitive codons were of the form RNY [3]. Such a system is called
a comma-less code, and would have benefited primitive translation mechanisms since translatable codons
RNY could only belong to one of three possible reading frames. As a support, it was further shown that
the presence of RNY periodicity can be used to identify the correct reading frame [4]. One may question
whether enrichment of RNY codons could be the result of natural selection instead of a primitive code,
in that any remnants of a repeating RNY gene structure would have been erased by mutation. However,
when frameshift mutations that disrupt the patterned RNY bias are removed, the periodic correlations
are increased in amplitude [4]. Shepherd’s work may indicate how rhythmic correlations can offer novel
perspectives in how life first functioned. Thus, patterns recurring at a period of three bases may suggest
principles behind gene evolution from small building blocks.
Furthermore, powerful modern algorithms utilize the base-three periodicity to predict protein-coding regions in unannotated genomes [9–12], since noncoding DNA regions lack the prominent power spectrum peak at frequency $1/3$. These modern algorithms can be broken down into four main steps: converting DNA sequences into numerical sequences such that biological information is retained; processing these numerical sequences by choosing window shape and length if necessary; extracting the period-3 property in order to differentiate protein-coding regions from noncoding regions (for example, short-time Fourier transform or wavelet analysis); and finally a classification step [11]. The Voss representation technique is one of the most popular ways to convert a DNA sequence into a set of numerical sequences [11]. In this technique, the DNA sequence is mapped to four binary indicator sequences, one for each of the four nucleotides. For each binary indicator sequence, the presence of a certain nucleotide means we need a 1 while the absence of that nucleotide means we need a 0. However, as mentioned in the Methods and Materials section, this brings a bias of preassigning numerical values to nucleotides which we will avoid. It is interesting to note that some algorithms have found subregions with prominent period-3 spectrum that were considered to be noncoding regions; these were hypothesized to be unannotated coding regions or pseudogenes [11].

Carefully understanding the source and pattern of the 3-base periodicity in genomic sequences thus represents an important and interesting problem in biology. As described below, however, several explanations for the periodicity are now available, providing sometimes conflicting views and creating confusion in the field. This paper presents a rigorous mathematical analysis to clarify our understanding and demonstrates the utility of analyzing genomes in the frequency space.

Some competing explanations for the three-base periodicity and our extensions are:

- **(G-non G-N) repeat which may be important for ribosomal RNA guiding [5].** The motif (G-non G-N) is found to belong to 130 diverse species of animals, plants, bacteria, viruses, organelles, plasmids, and transposons. A local disruption of this periodical (G-non G-N) pattern is strongly correlated with instances of ribosome slippage. Interestingly, this (G-non G-N) pattern is found to disappear in the area of ribosome slippage sites. The (G-non G-N) pattern reemerges immediately downstream of the slippage site, but now in a new frame, reflecting the new translation reading frame. This may indicate that the (G-non G-N) repeat in mRNA is needed to monitor the correct reading frame during translation.

  **Note:** In this paper, however, we show that the most predominant triplet in proteins that show
statistically significant periodicity is in fact NWS (any-[A/T]-[G/C]).

- **Periodic G or C base at third codon position** [13]. Lio et al. searched for subcodes while viewing the DNA sequences as being composed of “strong” (S=G or C) and “weak” (W=A or T) base pairs. The three-nucleotide periodicity of S was found only in protein-coding sequences and primarily in structural genes playing important roles in cell metabolism. This periodicity was specifically prevalent in sequences of prokaryotes living in extreme environments. The authors suggested that the conservation of the periodicity is due to increased stability and translation accuracy, since the G and C pairing in the third codon position can help messenger RNA bind more effectively to ribosomes.

  **Note:** In line with this observation, we also find that NWS repeats are prevalent in human protein domains that show statistically significant three-base periodicity.

- **Amino acid usage bias: the preponderance of only a few amino acids in a given protein** [7]. Tsonis et al. randomly selected 100 proteins from the Protein Information Resource (PIR) database. Their analysis showed that in 80% of these proteins, only four amino acids, which share the same first base in their codons, make up 32% of the protein length. As for the rest of the analyzed proteins, three or fewer amino acids, possibly having different first bases in codons, make up nearly 40% of the protein length. In comparison, introns do not exhibit this same structure, and the authors concluded that this is why non-coding sequences do not possess the period-3 property.

  **Note:** Table 1 shows the bias in the genome-wide amino acid usage in human. We will demonstrate in this paper that even if all 64 codons are equally represented, simply removing stop codons from a fixed reading frame in random sequences can create fictitious periodicity. We also use Markov Chain Monte Carlo (MCMC) minimization to show that amino acid bias by itself is not sufficient to create periodicity.

- **Species-specific synonymous codon usage bias** [15] Eskesen et al. simulated random coding sequences by sampling triplet nucleotides from the codon usage frequencies estimated from *D. melanogaster, H. sapiens,* and *C. elegans,* and found that the periodicity observed in these simulations was highly similar to the periodicity in the actual exons. The authors also simulated coding sequences by sampling non-stop codons at equal frequency, and found only minor periodicities in A and T nucleotides. Lastly, the authors simulated sequences by sampling all codons at equal
frequency, including the stop codons, and found that the DNA periodicity completely disappeared. Based on these results, the authors suggested that the period-3 property is due to the codon usage bias specific to a given organism. The species-specific codon usage bias may potentially be due to different tRNA abundance [16]. Codons that have more cognate tRNA are more likely to be translated with greater accuracy because they shorten the length of ribosome pausing during the elongation step and thus reduce the chance of incorrect amino acid incorporation. However, Ohno observed that the number of tRNAs can quickly change via gene duplication and that the codon bias may actually be an artifact of primitive heptameric repeats [6].

Note: As the above discussion shows, codon usage bias is only one component of the periodicity, and we will quantify the contribution of codon usage bias.

• **Position of amino acids and the synonymous codons within coding sequences** [14]. Trotta found in all organisms he examined (*M. tuberculosis*, *E. coli*, *B. subtilis* and *S. cerevisiae*) that the periodicity in protein-coding sequences is not caused only by codon usage frequency. Instead, the true periodicity was found to be higher than the periodicity predicted only by codon usage frequency. Trotta suggested two further causes of periodicity to explain this difference: amino acid position bias and the location of the synonymous codons. It is worth noting that an earlier study by Yin *et al.* [26] determined that the 3-base periodicity is not affected by the ordering of the amino acids. However, the study reached this conclusion by only examining a single gene (*GroE*), and failed to actually quantify the extent to which amino acid position contributes to periodicity.

Note: Here in our paper we confirm that, although it plays a small role, amino acid positioning is indeed a component of the 3-base periodicity. Future work should elucidate the exact role of the positioning of synonymous codons within coding sequences.

• **Differential multinomial probabilities of nucleotides across the three codon positions** [17]. Gutierrez *et al.* analyzed the Fourier spectra of the four numerical series obtained by assigning a particular nucleotide to 1 and all others to 0; they found that the height of the spectrum at period 3 is proportional to the variance of each nucleotide across codon positions. It suggests that an asymmetric distribution of nucleotides across the three codon positions provides a mathematical explanation of the periodicity.

Note: By decomposing the multinomial probabilities into a sum product of conditional and marginal
probabilities, we will show that there exist infinitely many combinations of codon and amino acid frequencies that lead to uniform multinomial probabilities across codon positions. We will thus show that codon and amino acid biases are not sufficient to create the three-base periodicity.

Ultimately, in this paper, we offer a complete quantitative analysis of sequence characteristics that contribute to the period-3 property. Although a spectral analysis was performed for a few sequences in [7], the Fourier spectrum of categorical data crucially depends on how the data are represented as real numbers. The first mathematically rigorous method for studying periodicity in categorical data was developed by Stoffer et al. [8] and previously applied to DNA sequence analysis [8,18]. By taking advantage of multi-core processing in modern computers, we have used this method called spectral envelope, explained in the Methods section, to compute the spectrum of all coding DNA sequences and select regulatory regions in the human genome. CpG islands are shown to have different classes of spectra, depending on their evolutionary origin.

We further develop a projection method for visualizing global sequence evolution and show how our approach can be used to trace the spectral evolution of H1N1 influenza A virus. In addition to gradual shifts in genomic sequence content, we demonstrate how to classify abrupt jumps corresponding to reassortments, such as those that led to the 2009 pandemic H1N1 virus. This new approach is able to summarize genome-wide sequence evolution in a concise manner in a way that does not require sequence alignments.

2 MATERIALS AND METHODS

2.1 Spectral envelope calculation

Even though Fourier analysis and signal processing are deep-rooted techniques in the physical sciences, they are relatively new techniques in the field of molecular biology. A sequence of DNA can be thought of as a categorical time series where the categories are the members of the nucleotide alphabet. In order to use spectral analysis to study the periodic patterns of a DNA sequence, the categorical time series needs to be converted to a numerical series; this is called a categorical scaling that assigns a number to each category. The choice of scaling is important because different scalings will affect the resulting spectrum and may cause us to focus on different periodic aspects of the sequence. The spectral envelope method is
a rigorous method that bypasses this bias of preassigning numerical values to nucleotides (i.e., subjective scaling). That is, the spectral envelope method may help us determine which nucleotide alphabets make up the signals, since ideally we should be considering all different possible scalings that expose various characteristics of the DNA.

The spectral envelope technique was first developed in Stoffer et al. [8] in 1993. It is an innovative methodology that is not limited by using simply one predefined scaling; instead, it uses different scalings for different frequencies. The spectral envelope technique finds the scalings that emphasize all the periodic patterns that are encoded in a categorical time series like a DNA sequence.

We will follow the convention used in [8]. Let $\beta : \{A, C, G, T\} \rightarrow \mathbb{R}$ be a real valued function, called the scaling function. Given any DNA sequence $s_1 s_2 \cdots s_n$ of length $n$, where $s_k \in \{A, C, G, T\}$, the scaling function then defines a real-valued series $X_k = \beta(s_k)$. The spectral envelope of $X_k$ is defined as

$$\lambda(\omega) = \max_{\beta \neq 1} \frac{f_\beta(\omega)}{\sigma_\beta^2},$$

where the maximization is taken over $\beta$ not proportional to the identity map that maps all letters to 1, $f_\beta(\omega)$ is the spectral density of $X_k$, and $\sigma_\beta^2$ is the population variance of $X_k$ [8]. The $\beta$ that maximizes the quantity in equation 1 is unique up to scaling and translation. Thus, for DNA sequences, we can use these two degrees of freedom to set $\beta(T) = 0$ and $(\beta(G), \beta(C), \beta(A)) \in S^2_+$, where $S^2_+$ is the upper hemisphere of a unit sphere. The spectral envelope $\lambda(\omega)$ thus gives an upper bound on the spectral energy of any representation of the DNA sequence $s_1 \cdots s_n$ at frequency $\omega$; i.e. it is the maximum possible spectral energy at frequency $\omega$. A spike in the spectral energy may therefore indicate a dominant harmonic. We used a modified version of the R code included in [18] to compute the spectral envelopes presented in this paper; the primary change was using a modified Daniell kernel (0.0625, 0.25, 0.375, 0.25, 0.0625) for smoothing the periodogram. In the following we will use $\lambda$ to denote the sample spectral envelope.

### 2.2 Markov Chain Monte Carlo

Let $a_1 a_2 \cdots a_n$ be a protein sequence, where $a_k$ is an amino acid with $M_k$ codons. Let $s_1 s_2 s_3 \cdots s_{3n}$ be a DNA sequence that codes for this protein. There are $M = \prod_{k=1}^{n} M_k$ distinct DNA sequences that code for the same protein sequence. We would like to find DNA sequences that have the maximum or minimum spectral energy at period 3. Explicitly computing the spectral envelope for all $M$ distinct sequences is
usually not feasible, because this number $M$ is very large even for a short protein sequence. For example, for the protein motif NLNTLTDHNLIDHIAEGTFVQ of length only 22, there are roughly 49 billion distinct synonymous DNA sequences. Thus, we used Markov Chain Monte Carlo (MCMC) methods rather than exhaustive enumeration in order to approximately find the optima of the spectral energy function for each protein motif. In general, MCMC methods consist of algorithms that simulate from a probability density by generating an ergodic Markov chain with this probability density as its equilibrium distribution. The two MCMC methods used in this work are simulated annealing [19] and parallel tempering MCMC [20] (PTMCMC). These two methods are advantageous because these algorithms avoid getting trapped in local minima indefinitely and are easy to implement.

2.2.1 Simulated Annealing

Simulated annealing is guaranteed to find the minimum if the temperature decreases slowly enough, but in practice the technique can only find a decent approximation to a global optimum. In terms of the simulated annealing algorithm, suppose we are looking for the global minimum of a complicated function. We proceed by sampling the large search space for possible solutions according to a candidate distribution. During each sampling we decide whether to update our current solution with this new solution. Initially, when the temperature is large, we accept the proposed solution with relatively high probability; when the temperature decreases however, we are more likely to accept whichever solution is lower.

Specifically, for a given protein motif, the simulated annealing method that was used in this paper is described below:

i) Let $X_0$ be any DNA sequence that codes for the protein motif.

ii) Using the current $X_t$ sequence, generate a candidate sequence $X^*$ by randomly choosing a codon of $X_t$ and replacing it with a synonymous codon.

iii) Calculate the following expression

$$\alpha_{SA} = \min\left[1, e^{-\frac{\lambda(X^*) - \lambda(X_t)}{T_t}}\right]$$

where $\lambda(X^*)$ is the spectral envelope of the candidate sequence $X^*$, $\lambda(X_t)$ is the spectral envelope of the
current sequence $X_t$, and the function $T_t$ is the cooling schedule. We use the geometric decline

$$T_{t+1} = \frac{T_t}{1.01}.$$  

We accept the candidate sequence with probability $\alpha_{SA}$. If we accept $X^*$, then we set $X_{t+1} = X^*$ and return to step (ii). Otherwise we set $X_{t+1} = X_t$ and return to step (ii). We used 8 as $T_0$ and 20000 as $n$.

### 2.2.2 Parallel Tempering MCMC

While simulated annealing uses a single chain to find the global optimum, PTMCMC simultaneously simulates from multiple non-interacting MCMC chains, each one at a different temperature. That is, if the target distribution from which we aim to sample is given by

$$p(X) = \frac{e^{-\lambda(X)/T_0}}{Z}$$

for some small $T_0$, then we make an extended system:

$$p_{T_i}(X) = \frac{e^{-\lambda(X)/T_i}}{Z(T_i)}, \text{ for } i = 1, \ldots, n,$$

where $T_0 < T_1 < \cdots < T_n$. Higher temperatures have the effect of flattening the target distribution. Periodically during the iterations, two neighboring chains set at, say, temperatures $T_i$ and $T_{i+1}$, will swap their respective configurations $X_i$ and $X_{i+1}$ with probability defined by:

$$\alpha_{PT} = \min \left[1, e^{\left(\frac{1}{T_i} - \frac{1}{T_{i+1}}\right) \left(\lambda(X_i) - \lambda(X_{i+1})\right)}\right].$$

In this way parallel tempering uses several Markov chains to improve mixing and efficiently sample complicated landscapes. Note that in parallel tempering, two types of operations are used: the update operation, which generates a new sample for each chain using a Metropolis kernel, and an exchange operation, which swaps the samples between two neighboring chains. We used seven chains, i.e. $n = 6$. $T_0$ was chosen so that the acceptance probability of a move that increases the spectral envelope by 1% of the original spectrum at frequency $1/3$ is 1%. Succeeding temperatures were chosen as $T_i = 2^i T_0$. We used 10,000 simulations.
2.3 Protein domains and sequences

We obtained the annotated protein domains in human from PROSITE Release 20.81 [21] and converted them to coding DNA sequences by using the genome version HG19. Gene features and sequences in the human genome were obtained from GENCODE version 11 [22].

2.4 Position-specific frequency of nucleotides

Let $C(\alpha)$ be the set of codons coding for amino acid $\alpha$, and let $N_i$ be the nucleotide at position $i = 1, 2, 3$. Then, we can factorize the nucleotide frequency $P(N_i)$ at the $i$-th position as

$$P(N_i) = \sum_\alpha \sum_{C \in C(\alpha)} P(N_i|C)P(C|\alpha)P(\alpha),$$

(2)

where $P(N_i|C)$ is 1 if the $i$-th nucleotide in $C$ is $N_i$, and 0 otherwise. $P(C|\alpha)$ represents the codon usage bias for the amino acid $\alpha$, and $P(\alpha)$ represents the amino acid usage bias in the genome. We can rewrite this equation in a matrix form

$$N = CA,$$

(3)

where $N$ is a column vector of nucleotide frequencies $P(N_i)$, $A$ is a column vector of amino acid usage frequencies $P(\alpha)$, and $C$ is the remaining sum product in equation 2. It will be shown subsequently that there exist infinitely many combinations of amino acid and codon usage biases, $P(\alpha)$ and $P(C|\alpha)$, such that $P(N_i = A) = P(N_i = T) = 0.3$ and $P(N_i = G) = P(N_i = C) = 0.2$, for $i = 1, 2, 3$.

2.5 Area-preserving 2D projection of the scaling function

As previously mentioned, we use the convention where $\beta(T) = 0$ and $(x, y, z) = (\beta(G), \beta(C), \beta(A))$ lies on the upper unit hemisphere $S^2_+$, where $z \geq 0$. To compute the density of $\beta$ at frequency $1/3$, we would like to project the distribution of $\beta$ onto a plane by using an area preserving map. Note that neither the usual stereographic projection nor the map $(x, y, z) \mapsto (x, y)$ preserves area and can lead to density
artifacts. A simple calculation shows that

\[(x, y, z) \mapsto \left( \frac{\sqrt{2(1-z)} \, x}{\sqrt{x^2 + y^2}}, \frac{\sqrt{2(1-z)} \, y}{\sqrt{x^2 + y^2}} \right) \] (4)

is an area preserving projection of the upper hemisphere onto a disk of radius \(\sqrt{2}\).

2.6 Annotation of CpG islands

28691 CpG islands in the human genome (HG19) were obtained from http://genome.ucsc.edu/. To search for CpG islands that do not contain coding exons, we removed CpG islands that overlap UCSC known genes, human mRNAs from GenBank, expressed sequence tag (EST), or Yale pseudogenes (Release 60), also obtained from http://genome.ucsc.edu/. This filtering resulted in 1819 non-coding CpG islands.

2.7 Influenza A virus sequences

Nucleotide sequences of 4222 human, 411 swine, and 89 avian H1N1 influenza A viruses were obtained from the NCBI Influenza Virus Resource [23] (http://www.ncbi.nlm.nih.gov/genomes/FLU/).

3 RESULTS

3.1 Spectral envelope of protein domains

We calculated the spectral envelope of 20491 DNA sequences corresponding to human protein domains from PROSITE. To test for significance of periodicity at frequency 1/3 for each protein motif, we permuted the corresponding DNA sequence 100 times, calculated the spectral envelope for each permutation, and calculated the resultant mean envelope. For each protein motif at frequency 1/3, we counted how many permuted spectral envelopes were greater than the spectral envelope of the original DNA sequence to determine an empirical \(p\)-value. We found that 3381 (16.5\%) DNA sequences had a spectral envelope at frequency 1/3 that was greater than the spectral envelope of all 100 permuted sequences. We have defined the corresponding 3381 protein domains to have a significant period-3 property at \(p\)-value cutoff of 0.01. Supplementary Figure S1 (a) shows a histogram of the calculated \(p\)-values for all the analyzed protein domains.
The periodic protein domains were significantly longer than the protein domains that lacked the period-3 property (Wilcoxon Rank Sum Test $p$-value $< 10^{-16}$). The median length of periodic protein domains was 57 amino acids, while that of non-periodic protein domains was 31. In addition, the periodic protein domains were enriched for the fork head domain (binomial $p$-value $= 9.7 \times 10^{-4}$) and the basic-leucine zipper (bZIP) domain (binomial $p$-value $= 1.7 \times 10^{-2}$).

To assess the contribution of codon bias to periodicity, we uniformly sampled 100 synonymous DNA sequences for each protein motif and calculated the spectral envelope, as well as the resultant mean envelope. We found that 3839 (18.7%) sequences had a spectral envelope value at frequency 1/3 that was greater than the spectral envelope of all 100 synonymous sequences. Supplementary Figure S1 (b) shows a histogram of the $p$-values calculated from this simulation. To determine the fraction of spectral envelope at frequency 1/3 that can be explained by codon usage bias, we computed the $N$-statistic defined as:

$$N = (\lambda - \bar{\lambda}_{\text{codon}})/\lambda,$$

where $\lambda$ is the spectral envelope at frequency 1/3 and $\bar{\lambda}_{\text{codon}}$ is the mean envelope from 100 corresponding synonymous DNA sequences. Figure 1(a) shows the distribution of the $N$-statistic for significant period-3 protein domains. The mean $N$-statistic is 0.46, with a confidence interval of [0.44, 0.48].

While previous research has found that the three-nucleotide periodicity is attributable to the uneven distribution of base compositions at different codon positions, to the best of our knowledge, the possible contribution of amino acid positions have not yet been characterized. For example, the DNA sequences CCT AAA CCT GCT TGG CCT GAT CCT GAG CCT TAA CCT and CCT CCT CCT CCT AAA GCT TGG GAT AGT TAA GAG each have the same numbers of amino acid types, and even the same base compositions at the three codon sites, yet the spectrum for the second sequence at frequency 1/3 is 0.1695 while the spectrum for the first is 0.1321. To determine the fraction of the spectral envelope at frequency 1/3 that can be explained by the amino acid position, we computed the $A$-statistic defined as:

$$A = (\lambda - \bar{\lambda}_{aa})/\lambda,$$

where $\lambda$ is the spectral envelope at frequency 1/3, and $\bar{\lambda}_{aa}$ is the mean envelope from 100 DNA sequences each with permuted amino acid locations. Figure 1(b) shows the distribution of the $A$-statistic for
significant protein domains. The mean A-statistic is 0.059, with a confidence interval of [0.055, 0.063].

To study the nucleotide content of periodic sequences, we projected the scaling functions for the periodic protein domains onto a disk by using an area-preserving map. Figure 2 shows the projection onto the GC plane, and indicates that for periodic protein domains, the scaling function predominantly maps nucleotides C and G to similar values. Supplementary Figures S2 and S3 show the projection of the distribution of the scaling functions for periodic domains onto the AC and AG planes, respectively; these figures further show that the scaling function frequently maps the nucleotide A to values near 0. Recalling that the scaling function maps T to 0 by definition, these particular assignments suggest that a strong-weak signal is the dominant signal at frequency 1/3. The GC content was 56.6%, 39.2%, 77.8% at the three positions of the consensus codon for periodic protein domains, demonstrating the prevalence of NWS triplets.

3.2 Spectral envelope of full-length proteins

To examine the three-base periodicity globally in the human genome, we computed the spectral envelope for 75979 GENCODE transcripts that contain a coding DNA sequence (CDS). We removed the UTR regions in the computation. To assess the significance of the spectral density, we also computed the spectral envelope for 100 random permutations of each original transcript; a transcript was then considered to possess significant three-base periodicity if its spectral envelope at frequency 1/3 is greater than the maximum of the envelopes of the 100 permuted sequences at frequency 1/3. According to this definition, 18200 (24.0%) transcripts did not show significant periodicity. Periodic sequences were significantly longer than non-periodic sequences (Wilcoxon Rank Sum Test \( p \)-value \(< 10^{-324} \)): the median length of amino acids (AA) was 416 for periodic and 122 for non-periodic transcripts. Gene ontology analysis showed that the non-periodic genes were significantly involved in alternative splicing, mitochondrial parts, and ncRNA metabolic processes [24].

Compared to periodic sequences, non-periodic sequences had a 5.4-fold enrichment for transcripts that are being targeted for nonsense mediated decay (Fisher’s Exact Test \( p \)-value \(< 10^{-324} \)): the difference in distribution of AA lengths was still present between the two classes even after removing those transcripts: median AA length was 432 for periodic and 147 for non-periodic protein-coding sequences (Wilcoxon Rank Sum Test \( p \)-value \(< 10^{-324} \)).

The Pearson correlation between the log ratio of observed over expected spectral envelopes at fre-
frequency 1/3 and the log length of protein was 0.74, where the expected spectral envelope was obtained by averaging the envelopes for 100 permuted sequences. Figure 3 shows the scatter plot of the log ratio and the log CDS length for each GENCODE transcript.

To assess whether a particular codon position can dominate in creating periodicity, we performed three sets of controlled permutations of GENCODE sequences having by fixing the nucleotides at codon position 1, 2 or 3, and permuting the remaining nucleotides. Compared with the original sequences, permutations fixing the first position nucleotides led to a median decrease of 64% in the spectral envelope at frequency 1/3. By contrast, permutations fixing the second position or the third position nucleotides led to a median decrease of only 26% or 33%, respectively. These results suggest that the first codon position may be much less important than the second and third positions for creating periodicity.

3.3 Amino acid usage bias is insufficient to create periodicity

To assess the extent to which human coding sequences have evolved to maximize or minimize the spectrum at frequency 1/3, we used MCMC optimization techniques of simulated annealing and PTMCMC to search for synonymous DNA sequences that have the maximum or minimum spectral envelope at frequency 1/3. For 3380 of the 3381 protein domains with significant periodicity, using either simulated annealing or PTMCMC, we found a synonymous DNA sequence whose spectrum at frequency 1/3 is lower than the minimum spectrum at frequency 1/3 of 100 random permutations by nucleotide. Thus, given nearly any protein motif, we have shown that we can construct a synonymous DNA sequence that does not have the typical period-3 property; that is, we can almost always choose codons that destroy the peak at frequency 1/3.

DV TITT LRDSGTFTCIASN AAGEA TAPVEVC is the protein domain with significant periodicity which MCMC optimization techniques fail to find a synonymous DNA sequence whose spectrum at frequency 1/3 is lower than the minimum spectrum at frequency 1/3 of 100 random permutations. However, upon inspecting the sequence one can see that each of the amino acids in the second block of the domain (TITT) are encoded by nucleotide triplets beginning with A. Similarly, all amino acids in the fourth block (AAGEA) are encoded by nucleotide triplets beginning with G. Because of this close positioning of the same nucleotide in the same codon position, synonymous DNA sequences will not have a spectrum at frequency 1/3 that is lower than that of random permutations by nucleotide. Indeed, if we randomly switch amino acids between block 2 and block 4, and then perform PTMCMC for this resulting DNA
sequence, we are able to find a synonymous sequence whose spectrum at frequency 1/3 is lower than the minimum spectrum at frequency 1/3 of 100 random permutations by nucleotide.

This shows that the amino acid usage bias itself is not sufficient to create a significant periodicity. In addition, either by PTMCMC or by simulated annealing, we found a synonymous DNA sequence for each protein domain whose spectrum at frequency 1/3 is higher than the maximum spectrum of 100 random permutations by nucleotide. In this regard it is interesting to see that the natural DNA sequence coding for a particular protein motif does not exhibit the maximum spectral envelope value for frequency 1/3.

As an example, Figure 4 shows the spectral envelope calculation for the protein motif PS00108 (‘Serine/Threonine protein kinases active-site signature’). Supplementary Figures S4 and S5 show additional spectral envelope plots for other protein motifs.

3.4 The absence of stop codons contributes to periodicity

Figure 3 shows that the spectrum at frequency 1/3 in coding regions depends on the length of sequence. By contrast, intronic and intergenic sequences have position-independent multinomial distributions of nucleotides (Table 2) and thus do not possess three-base periodicity even as the length increases, as shown in Figure 5(a). However, Figure 5(b) shows that simply removing stop codons from intronic sequences in a fixed reading frame is sufficient to create fictitious periodicity, and the spectrum at 1/3 also increases with the length of intron. This artificial periodicity results from the fact that all stop codons (TAA, TAG, TGA) begin with T and that two of them have A in the second and third positions. Removing stop codons thus breaks the symmetry in the multinomial distribution of nucleotides across the three positions, as shown in Table 2, and this asymmetry is able to create three-base periodicity for sufficiently long sequences. This finding is consistent with the previous analysis that attributed the periodicity in coding regions to the differential multinomial probability [17]. It also highlights the fact that the absence of stop codons alone may be sufficient to create periodicity in long coding sequences.

3.5 Codon and amino acid biases are insufficient to create periodicity

In equation 2, set $P(A_i) = P(T_i) = 0.3$ and $P(G_i) = P(C_i) = 0.2$, for $i = 1, 2, 3$, which are the expected nucleotide frequencies in the human genome; we shall demonstrate here that there exist continuous families of codon and amino acid biases which lead to these nucleotide frequencies and thus do not yield three-base periodicity in coding sequences. We simulated the codon bias frequencies 10000 times as follows: for
each amino acid $\alpha$ with $n_\alpha$ distinct codons, we sampled the order statistics $X_{(1)}, X_{(2)}, \ldots, X_{(n_\alpha-1)}$ of $n_\alpha - 1$ standard uniform random variables, and defined $X_{(k)} - X_{(k-1)}$ to be the $k$-th codon bias $P(C_k | \alpha)$, where $X_{(0)} = 0$ and $X_{(n_\alpha)} = 1$. Using these values, equation 2 gives 12 linear equations in 20 unknowns $P(\alpha)$. For each simulated codon bias matrix $C$ in equation 3, we performed constrained least square optimization with the constraint $0 < P(\alpha) < 1$, for all amino acid $\alpha$, and solved for $A$. Supplemental Figure S6 shows the distribution of the relative residuals $\|N - CA\|_2 / \|N\|_2$ and shows that for 48.2% of randomly chosen codon bias matrices, we can find amino acid frequencies $P(\alpha)$ such that the relative residual is less than $10^{-7}$. The null space of $C$ has dimension 10, so the solutions form a 10 dimensional affine space. Thus, there are infinitely many instances of codon and amino acid usage biases that nevertheless have the same multinomial nucleotide frequencies $P(N_i)$ at all three codon positions $i = 1, 2, 3$, and that do not lead to three-base periodicity in coding regions. An example is given in Supplementary Table 1.

### 3.6 Periodicity in non-coding CpG islands

We now give examples of some other applications of the spectral envelope. Periodicity in non-coding regulatory regions might also indicate repeating units of information associated with their evolutionary origin and function. We computed the spectral envelope for 1819 non-coding CpG islands. As described in Methods, we removed any CpG island that has previously annotated coding potential. A CpG island was considered to possess statistically significant periodicity if its spectral envelope was greater than all spectral envelopes for 100 permuted sequences, corresponding to an empirical $p$-value cutoff of 0.01. We found 89 CpG islands to possess significant spectrum at frequency 1/3. Out of 89, 39 CpG islands consisted of tandem repeat sequences and had several spectral peaks; see Figure 6(a). Tandem repeat CpG islands tend to be methylated and lie in heterochromatin; some of them also regulate imprinted genes [25]. At least 11 of the remaining 50 CpG islands arose from segmental duplications of genomic loci that harbor protein coding genes. For example, Figure 6(b) shows the spectral envelope for a CpG island that had several segmental duplications on chromosome 11. A significant peak can be seen at frequency 1/3. This observation points towards the exonic origin of some CpG islands, which might have lost their coding potential after duplication through evolution and now function as regulators other nearby genes.
3.7 Spectral evolution of H1N1 influenza A virus

Influenza viruses affect millions of people every year and continue to outwit our immune system by evolving through mutations and reassortment of genetic information among related strains. From a disease control perspective, the two major properties of an influenza virus are its pathogenicity and its transmissibility, either from person to person or from species to species. The recent 2009 H1N1 pandemic virus has segments of human, avian and swine origin; this strain was in fact first detected in 2005, but was dormant between 2006 and 2008. It is highly transmissible between humans, but its pathogenicity is not high. On the other hand, the avian H5N1 virus is highly virulent, but it does not transmit easily to humans. It is thus important to monitor the evolution of these viruses and detect changes associated with their adaptation and newly acquired transmissibility or pathogenicity. We here show that we can use our projection map of the scaling function at frequency $1/3$ to trace the evolution of H1N1 virus and detect the triple reassortment event that was first found in a 2005 strain and that subsequently led to the 2009 pandemic.

The 2009 pandemic H1N1 virus has the PA protein of avian origin and NP protein of classical swine origin. Figure 7(a,b) traces the changes in the projected scaling functions for these two proteins during the past 80 years. A sudden jump is clearly seen in year 2005 for both proteins. Interestingly, the scaling functions revert back to those from previous years in years 2006 through 2008, agreeing with the fact that the reassortant H1N1 was dormant during this period. From 2009 onward, the reassortant virus became the prevalent H1N1, and this pattern is well reflected in Figure 7(a,b). Note that $\beta(A)$, $\beta(C)$, and $\beta(G)$ are all positive for these viral sequences; as a result, their distribution on the unit sphere is invariant under changing the constraint on the scaling function from

$\beta(A) > 0$ to either $\beta(C) > 0$ or $\beta(G) > 0$, and the above separation of avian and swine sequences from human is not an artifact of our choice of convention.

Motivated by this finding, we trained a support vector machine (SVM) using radial basis kernels on scaling functions from avian and human PA sequences as well as those from swine and human NP sequences. All avian H1N1 sequencing data from NCBI were used. Because H1N1 was the predominant strain in swine before 1998, when H3N2 subtypes began infecting pigs, we restricted to pre-1998 swine sequences in order to avoid contamination of reassorted sequences. For human, pre-reassortant H1N1 sequences prior to 2005 were used. Figure 7(c) shows the decision boundaries for the trained SVM. We then tested the predictor on 3276 post-reassortment human H1N1 sequences from 2005 and 2009-11,
and 430 sequences from pre-2005. Our SVM predicts 99.3% human for pre-2005 and 92.4% avian for post-reassortment PA sequences. It predicts 99.5% human for pre-2005 and 92.4% classical swine for post-reassortment NP sequences.

4 DISCUSSION

This study uses the rigorous method of spectral envelope to comprehensively analyze the period-3 property of protein-coding DNA sequences. The method selects the optimal mapping of categorical data to real numbers at each frequency and allows us to test the statistical significance of periodicity in the entire human genome. We provide a thorough quantitative investigation of sequence properties that contribute to the phenomenon. The two main questions that we consider are: 1) How can we quantify different sources of the base-three periodicity? 2) What is a minimal set of attributes sufficient for periodicity?

It has been previously reported that the three-base periodicity depends only on the occurrence frequencies of the nucleotides in each of the 3 codon positions of a DNA sequence [26]. However, another possible source of 3-base periodicity is the particular ordering of amino acids; for example, two DNA sequences that have exactly the same nucleotide distributions across the three codon positions, but with different orderings of amino acids, can have different spectral envelopes as we have illustrated. Here we report that, on average, the fraction of the spectral envelope at frequency 1/3 that can be attributed to the amino acid ordering is about 6% for human protein domains.

Previous studies have implicated amino acid usage bias, i.e. the prevalence of a few of amino acids, as a dominant source of periodicity [7]. This bias creates a difference in nucleotide frequencies across codon positions. For instance, all but three amino acids have the same first base in their synonymous codons, so the preponderance of a single amino acid could yield a periodic appearance of the first-position nucleotide. In this study, however, we found that amino acid usage bias cannot be the sole explanation for the three-base periodicity in human structural protein domains. MCMC optimization showed that for any DNA sequence corresponding to a protein domain, one can construct a synonymous DNA sequence that destroys any existing three-base periodicity. Thus, even though most protein-coding sequences consist of only a few amino acids, the extent of amino acid usage bias in human protein domains is insufficient to create a significant periodicity.

Furthermore, contrary to previous reports [15], we found that the synonymous codon usage bias in
human protein structural motifs is likewise insufficient to generate significant three-base periodicity. On average, we have determined that the fraction of the spectral envelope at frequency 1/3 that can be explained by the codon bias is about 46% for protein structural domains. Our method of projecting the scaling function onto a two-dimensional disk provides an intuitive way of assessing the sequence content of periodic nucleotides and demonstrates the prevalence of NWS triplets, in contrast to the previous suggestions of RNY [3] or GHN repeats [5]. Our genome-wide analysis thus does not support the hypothesis that the present-day genes have evolved from an ancestral form of RNY repeats.

Interestingly, removing stop codons from long introns in a fixed reading frame forces them to lose their position-independent multinomial distributions of nucleotides and creates significant 3-base periodicity. This simulation suggests that the absence of stop codons within long coding sequences is sufficient to create periodicity, as demonstrated in Figure 3. Thus, even though our observations indicate that amino acid bias and codon bias are each insufficient to create base-three periodicity in human protein domains, the asymmetry in the multinomial distribution of nucleotides across the three codon positions is enough to create this phenomenon, at least in long protein sequences. Note that periodicity can depend on other biases besides this asymmetry; as noted above, factors such as amino acid position can also contribute.

Using rigorous mathematics to supplement our spectral envelope approach, we have additionally shown that there are infinitely many pairs of codon and amino acid usage biases that will lead to the expected background nucleotide frequencies. Our work thus demonstrates that the presence of codon and amino acid biases in itself is not sufficient to generate three-base periodicity. That is, in theory, it would be possible to have a genome with both codon and amino acid usage biases, but no period 3 periodicity. Thus, even though the codon and amino acid usage biases observed in biological sequences do contribute to the periodicity, the true source of periodicity lies not in the fact that the biases exist, but in that the observed biases lead to unequal multinomial distributions of nucleotides across codon positions. This subtle difference has not been previously appreciated.

In addition to facilitating the analysis of periodicity, spectral envelope also provides a novel technique for studying evolution. Traditional phylogenetic analyses require sequence alignments and do not readily yield a summary statistic that can effectively capture genome-wide sequence changes. This paper demonstrated how to use spectral envelope to detect residues of coding potential in CpG islands that arose through segmental duplication from protein-coding regions. These select CpG islands retained a significant peak at frequency 1/3 even though they no longer coded for a functional protein. CpG islands
consisting of tandem repeats could be also distinguished by their regular spectral peaks. We further introduced the concept of spectral evolution, which represents genome-wide changes in sequences as changes in the scaling function that maximizes periodicity. This approach is unique in that it does not require sequence alignments and succinctly summarizes genome-wide sequence evolution in two dimensions. We applied this concept to the H1N1 influenza A virus by mapping the evolution of the scaling function at frequency 1/3 during the past 80 years. The projection method illustrates both random drift and abrupt changes in the viral genome. The gradual change in scaling function is similar to the previously observed random drift from G,C to A,U in human influenza H1N1 virus, which may reflect a species-specific mutational bias [27]. Our approach captures a higher order pattern in sequence evolution, corresponding to changes in genome-wide periodicity. Importantly, our method is also able to detect reassortment events that may correspond to inter-species jumps resulting in sudden changes in pathogenicity and transmissibility. The method of spectral evolution thus provides an efficient new tool to monitor the evolutionary trends in influenza and other viruses.

5 CONCLUSION

Understanding the structure and function of genetic information encoded in diverse genomes represents a major challenge. Traditional local alignment and sequence motif analyses have fundamental limitations and fail to answer several important problems in biology. There is thus an emerging need for a paradigm shift in sequence analysis that goes beyond alignment-based information. Here, we have proposed one alternative approach of spectral envelope to gain new insight into the human three-base periodicity and the spectral evolution of the H1N1 influenza A viral genome. This paper highlights the possibility that nonlocal properties of DNA relying on long-distance sequence patterns may mark distinct functional sites and also help summarize genome-wide sequence evolution.

References


### 6 Tables

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<th>Amino acid</th>
<th>%</th>
<th>Amino acid</th>
<th>%</th>
<th>Amino acid</th>
<th>%</th>
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**Table 1.** Amino acid usage in the human genome.

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<th>Nucleotide</th>
<th>Position 1 (%)</th>
<th>Position 2 (%)</th>
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**Table 2.** Position-specific nucleotide frequency in the human genome. For whole genome, all triplets in the human genome were enumerated, and the nucleotide frequencies were derived from these triplet frequencies.
Figure 1. (a) Histogram of the $N$-statistic (quantification of the contribution of codon bias to periodicity) for protein domains with significant three-nucleotide periodicity. (b) Histogram of the $A$-statistic (quantification of the contribution of amino acid position to periodicity) for protein domains with significant three-nucleotide periodicity.
**Figure 2.** Area-preserving 2D projection of the scaling function of protein domains with significant period-3 property.

**Figure 3.** The ratio of observed over expected spectral envelope at frequency 1/3 is highly correlated with the length of protein.
**Figure 4.** Spectral envelope calculations for the protein motif PS00108 which has a significant period-3 property. Black line is the spectral envelope of the original DNA sequence; blue, spectral envelope of synonymous DNA sequence found by MCMC minimization at frequency 1/3; red, spectral envelope of the synonymous DNA sequence found by MCMC maximization; dashed green, average spectral envelopes of 100 random synonymous sequences; solid green, average spectral envelope of 100 random permutations.

**Figure 5.** Spectral envelope of intronic sequences. (a) Period 3 is not present in 1000 random introns of length 100 bp, 1 kb, 2 kb and 3kb. (b) Removing the stop codons (TAA, TGA, TAG) in a fixed frame from the sequences in (a) increases the spectrum at period 3.
Figure 6. Spectral envelope for CpG islands at (a) chr4:3565311-3567185 consisting of 78-mer tandem repeats and (b) chr11:89713070-89713801 having several segmental duplications on chromosome 11. The coordinates are in HG19.
Figure 7. Evolutionary trace of the scaling function for the human H1N1 (a) PA protein and (b) NP protein. Each dot corresponds to the median projected scaling functions of all sequences from the indicated year. (c) Support vector machine decision boundaries and training data points are shown. Support vectors are indicated as $\times$ and other training data points are indicated as circles. Human data points are in black and avian or swine data points are in red.
Supplementary Table 1: The amino acid usage bias and codon usage bias described in the table will not create periodicity at 3 nucleotides. There are infinitely many such combinations of biases that do not create periodicity.

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Figure S1: (a) Histogram of $p$-values based on nucleotide permutations of protein motif sequences. 3381 (16.5%) protein domains have a $p$-value less than 0.01, and 6855 (33.5%) have a $p$-value less than 0.05. (b) Histogram of $p$-values based on uniformly sampling synonymous sequences. 3839 (18.7%) protein domains have a $p$-value less than 0.01, and 6713 (32.8%) have a $p$-value less than 0.05.
Figure S2: Area-preserving 2D projection of the scaling function of protein domains with significant period 3 property.
Figure S3: Area-preserving 2D projection of the scaling function of protein domains with significant period 3 property.
Figure S4: Spectral envelope calculations for the protein motif PS51450 (Leucine-rich repeat profile”), which is defined to have significant 3-base periodicity. Blue - Spectral envelope of synonymous DNA sequence found by MCMC optimization to maximize spectral envelope at frequency 1/3. Dashed green - The average of the spectral envelopes of 100 random synonymous sequences. Solid green - The average of the spectral envelope of 100 random permutations. Black - Spectral envelope of the original DNA sequence. Red - Spectral envelope of the synonymous DNA sequence found by MCMC optimization to maximize spectral envelope at frequency 1/3.
Figure S5: Spectral envelope calculations for the protein motif PS00027 ("Homeobox domain signature"), which is defined to not have significant 3-base periodicity. Blue - Spectral envelope of synonymous DNA sequence found by MCMC optimization to maximize spectral envelope at frequency 1/3. Dashed green - The average of the spectral envelopes of 100 random synonymous sequences. Solid green - The average of the spectral envelope of 100 random permutations. Black - Spectral envelope of the original DNA sequence. Red - Spectral envelope of the synonymous DNA sequence found by MCMC optimization to maximize spectral envelope at frequency 1/3.
Figure S6: Distribution of relative residuals, $-\log_{10}(\|N - CA\|_2/\|N\|_2)$, for constrained least square optimization solutions to the equation $N = CA$ with the constraints $0 < P(A) < 1$, for all amino acid $A$. 

$\|N\|_2$ is the Euclidean norm of the vector $N$, and $\|N - CA\|_2$ is the Euclidean norm of the difference between $N$ and $CA$. The bars in the diagram represent the number of simulations for each bin of relative residual values.
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