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ITD FORMULATION FOR CORNER DIFFRACTION

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1. INTRODUCTION

A high-frequency description of the scattering from complex structures that exhibits surface discontinuities such as edges and vertices, is of importance in a wide variety of practical applications. To this end , the Geometrical Theory of Diffraction (GTD) [1] and its uniform extension (UTD) [2] provided very effective tools for most engineering purposes. The practical applicability of this ray method depends on the solution of canonical problems that are used to locally approximate the actual configuration. Within this framework, an important canonical problem is that of a corner at the interconnection of two straight edges, joined by a plane angular sector.

It should be noted that for most practical purposes, the need for a corner diffraction coefficient in a UTD scheme mainly arises when the leading, edge diffracted field experiences a discontinuity, as it occurs when the diffraction point changes abruptly its location from one edge to the next adjacent edge. An even more serious impairment is encountered in applying GTD to RCS calculations, due to the fact that the leading, edge contributions are restricted to lie on the pertinent Keller cones. A ray field description of the contribution from a corner may provide an effective tool to remove this major limitation as well as other inaccuracies, that may occur even when standard GTD edge contributions are applicable.

In this paper, a first order, high-frequency solution for the scattering by a corner is presented. Our formulation is based on the Incremental Theory of Diffraction (ITD), which has been recently developed [3]. The procedure consists of three basic steps. a) First, a generalized Geometrical Optics (GGO) contribution is obtained: to this end, the incremental surface GO contributions (SGO) are integrated over the surface of the infinite plane angular sector, and the contribution from the linear integrations along the two semi-infinite edges of the incremental end-point GO fields (EGO) are extracted from the previous surface integral. b) Next, a diffraction contribution is obtained by a linear integration of the incremental diffraction coefficients (IDC) along the two semi-infinite edges. c) Finally the total scattered field is represented as the sum of the GGO field plus standard UTD edge diffracted fields and a diffracted field contribution from the tip.

For the sake of simplicity, the scalar case is treated herein after. The same basic procedure can be extended, without any significant difficulty, to treat the more general electromagnetic vector problem.

2. GENERALIZED GEOMETRICAL OPTICS CONTRIBUTION

The geometry at a corner interconnecting two edges is shown in Fig. 1. Let us denote by Σ the surface of the plane angular sector and by Ω the angle between the two edges. At each edge ($n=1,2$) it is useful to define a local coordinate system (x_n, y_n, z_n) with its origin at the vertex; the z_n-axis is chosen along the edge, and the y_n-axis is perpendicular to Σ. Accordingly a spherical coordinate system (r, β_n, ϕ_n) is also defined; a plane wave illumination is assumed, with a direction of incidence (β_n', ϕ_n'). Moreover, let us introduce a coordinate system (x_0, y_0, z_0) with the z_0 axis perpendicular to the surface Σ (Fig.1) and $x_0 \equiv z_1$; correspondingly, the spherical coordinates (r, Θ, Φ) and the coordinates (Θ', Φ') are used to denote the observation point and the incident field, respectively. The GGO field is represented by

$$\psi^{ggo} = \psi^{sgo} - \psi_1^{ego} - \psi_2^{ego} \tag{1}$$

in which ψ^{sgo} is the field contribution due to the integration of the SGO over the illuminated surface and ψ_n^{ego} ($n=1,2$) is the contribution due to the EGO integration along the edge n. The expressions of these contributions are

$$\psi^{sgo} = \int_0^\infty \int_0^{x_0 tan\Omega} F^{sgo}(x_0', y_0')\, e^{-j(x_0'u' + y_0'v')}\, dx_0'\, dy_0' \tag{2}$$

$$\psi_n^{ego} = \int_0^\infty F_n^{ego}(z_n')\, e^{-j z_n'w_n'}\, dz_n' \tag{3}$$

$$u' = -ksin\Theta'cos\Phi' \;;\; v'=-ksin\Theta'sin\Phi' \;;\; w_n'= kcos\beta_n' \;,\; (\Theta' < \tfrac{\pi}{2})$$

where $F^{sgo}(x_0', y_0')$ is the incremental SGO at the point (x_0', y_0') on the surface and $F_n^{ego}(z_n')$ is the incremental EGO at the point z_n' on the edge n. The spectral integral representations of the above contributions are obtained by applying the localization process defined in [3]. By introducing their expressions in (2) and (3) and by asymptotically evaluating them, ψ^{sgo} and ψ_n^{ego} are expressed as

$$\psi^{sgo} \sim \psi^{go} + \psi_{1s}^{sgo} + \psi_{2s}^{sgo} + \psi_0^{sgo} \tag{4}$$

$$\psi_n^{ego} \sim \psi_{ns}^{ego} + \psi_{n0}^{ego} \tag{5}$$

In (4) ψ^{go} is the standard GO contribution for the plane angular sector, which arises from the double stationary phase contribution, ψ_{ns}^{sgo} ($n=1,2$) are the stationary phase contributions from the boundary line of the integration domain, and ψ_0^{sgo} is the asymptotic end-point contribution associated with the tip. Similarly, in (5) ψ_{ns}^{ego} is the stationary phase contribution from the integration along the edge n, which asymptotically coincides with ψ_{ns}^{sgo}, and ψ_{n0}^{ego} is the end-point contribution from the same integration. In particular

$$\psi_0^{sgo} = \epsilon\, \frac{e^{-jkr}}{2\pi jkr}\, \frac{sin\Omega sin\beta_1 |sin\phi_1|}{(cos\beta_1 - cos\beta_1')(cos\beta_2 - cos\beta_2')}\, T(\tilde\delta_1, \delta_1, \tilde\delta_2, \delta_2) \tag{6}$$

and

$$\psi_{n0}^{ego} = \epsilon \frac{e^{-jkr}}{2\pi jkr} \frac{|\sin\phi_n|}{(\cos\beta_n - \cos\beta_n')(\cos\phi_n + \cos\phi_n')} \mathcal{F}\left[kr\bar{\delta}_n^2(\beta_n' = \beta_n)\right] \mathcal{F}\left[kr\delta_n^2\right] \quad (7)$$

where either $\epsilon = -1$ or $\epsilon = \text{sgn}(z_0)$ for either soft or hard boundary conditions, respectively, T is the transition function defined in [4], which involves the generalized Fresnel integral, and $\mathcal{F}$ is the UTD transition function; their arguments are

$$\delta_n = \sqrt{2}\sin\left(\frac{\beta_n' - \beta_n}{2}\right) \quad ; \quad \bar{\delta}_n = \sqrt{2\sin\beta_n \sin\beta_n'}\cos\left(\frac{\phi_n + \epsilon_1\phi_n'}{2}\right) \quad (8)$$

in which $\epsilon_1 = \text{sgn}(z_0)$.

3. DIFFRACTION CONTRIBUTIONS

Next, in order to introduce appropriate diffracted field contributions that lead to a continuous and well-behaved description of the scattering phenomenon, incremental edge diffracted contributions $F_n^d(z_n')$ are defined according to the ITD procedure. They are distributed and integrated along the two semiinfinite edges of the corner. This process yields two diffracted fields ψ_n^d (n=1,2) that are given by

$$\psi_n^d = \int_0^\infty F_n^d(z_n') e^{-j z_n' w_n'} dz_n' \quad (9)$$

Its asymptotic evaluation provides a representation of the field as the sum of two contributions

$$\psi_n^d \sim \psi_{ns}^d + \psi_{n0}^d \quad (10)$$

where ψ_{ns}^d, which arises from stationary phase contribution, is the standard UTD edge diffracted field when it exists, and

$$\psi_{n0}^d = -\frac{e^{-jkr}}{2\pi jkr} \frac{2P(\phi_n, \phi_n')}{(\cos\beta_n - \cos\beta_n')(\cos\phi_n + \cos\phi_n')} \mathcal{F}\left[kr\delta_n^2(\beta_n' = \beta_n)\right] \mathcal{F}\left[kr\delta_n^2\right] \quad (11)$$

is the end-point contribution, in which

$$P(\phi_n, \phi_n') = \begin{cases} \sin\frac{\phi_n'}{2}\left|\sin\frac{\phi_n}{2}\right| & \text{soft b.c.} \\ -\cos\frac{\phi_n'}{2}\cos\frac{\phi_n}{2} & \text{hard b.} \end{cases} \quad (12)$$

Finally, the diffracted field contribution from the tip of the corner is represented by

$$\psi_0 = \psi_0^{sgo} - \psi_{10}^{ego} - \psi_{20}^{ego} + \psi_{10}^d + \psi_{20}^d \quad . \quad (13)$$

When the above field is summed to the UTD edge diffracted fields ψ_{ns}^d and to GO reflected field ψ^{go}, a uniform, high frequency solution is obtained for the total scattered field.

4. CONCLUDING REMARKS.

A uniform diffraction coefficient has been presented for the scattering in the near zone by a corner at the interconnection between two straight edges, when it is illuminated by a plane wave. The solution is obtained by applying the Incremental Theory of Diffraction. The high-frequency expressions provide the desired continuity of the total scattered field at those aspects where the leading ray-field contributions experience a jump discontinuity.

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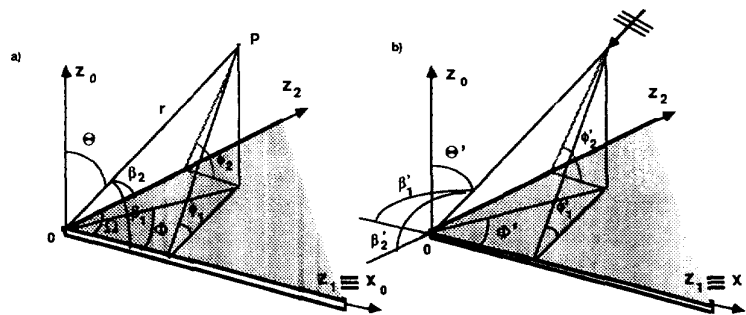
Fig. 1. Geometry at a plane angular sector: a) observation aspects; b) incidence aspects.