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People, Skills, and Resources: An Interaction Model for World Population Growth

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ABSTRACT

A model is set up that yields the equation followed by world population (P), past and present: $P = A/(D - t)^M$, where t is time and A , D , and M are constants. Cumulated historical population estimates confirm this hyperbolic pattern first noticed by Cailleux, Meyer, and Foerster. Technological indices should grow according to the same pattern; partial confirmation of this part of the model is presented. The growth acceleration crisis that we are now facing may have had a counterpart around 4500 B.C., during the neolithic agricultural revolution. World population projections up to 2000 A.D. are discussed in the light of the interaction model, with and without resource limitations.

Introduction

For many thousands of years world population has been growing faster than exponentially. In other words, the very *percent rate* of growth has been increasing [1]. This is highly unusual.

As a rule, growth processes start with a constant percent rate—this is exponential growth. The rate later decreases gradually to zero, so that growth stops. Only very rarely do we encounter the contrary “hyperexponential” pattern where the percent rate actually increases over time. Besides world population, the only example of which I can think is the price increase during the German 1923 inflation, during which mailing a letter came to cost 20 billion marks before a new currency started all over with 10-penny stamps. In mathematical language, a “singularity” occurred in the price-time function (i.e., a sudden discontinuity).

Past growth of the world population (P) over time (t) can be empirically fitted with an equation of quasihyperbolic form:

$$P = \frac{A}{(D - t)^M}, \quad (1)$$

where A , D , and M are constants. First noticed by Cailleux [2], followed by Meyer [3], this pattern has been rediscovered independently by several researchers who were initially unaware of previous work [4, 5]. Most of these data fits have yielded numerical constant values close to those of Meyer and Vallée [6], who observe purely hyperbolic growth ($M = 1$):

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$$P = \frac{200 \times 10^9}{(2026 \text{ A.D.} - t)}, \quad (2)$$

whereas Taagepera [5] proposes

$$P = \frac{50 \times 10^9}{(2005 \text{ A.D.} - t)^{0.74}}. \quad (3)$$

The Foerster et al. [4] best-fit line is

$$P = \frac{179 \times 10^9}{(2026.9 \text{ A.D.} - t)^{0.99}} \quad (4)$$

Quasihyperbolic equations have a mathematical singularity at $t = D$, where P becomes infinitely large and then shifts to negative or even imaginary numbers. Here we must distinguish between a theoretical model and the real situation it tries to approximate. Model singularities correspond in real physical systems to highly unstable crisis spots where the system undergoes drastic qualitative changes. Foerster et al. [7] document a number of examples from engineering practice. The “sound barrier” is a popularly known case. Simple modifications can be applied to Eq. (1) to avoid infinite growth, without altering the fit to past data [8].

It would thus be erroneous to deduce from Eq. (2) or (3) that world population would become infinite and then negative in 2026 or 2005 A.D. But a major crisis could, indeed, be projected. Taagepera [5] suggests that the neolithic agricultural revolution may have brought a previous population crisis of that type. Meyer ([9], p. 61) wonders whether the coming crisis may not mean a shift from energy-dominated to information-dominated technology. There is no reason to be alarmist about the implications of hyperbolic population growth unless one interprets it with naive rigidity. But the crisis warning should be used for preventive action. In order to do so we must first understand what causes this unusual hyperexponential growth.

Great technological advances are widely recognized as the cause of population explosion. Foerster et al. [4] have presented a game-theoretical argument for a shift from exponential to hyperbolic growth at high technology levels. Meyer [9] and Meyer and Vallée [6] have visualized the technology effect as a feedback system and have presented data to show that various technological indicators, too, have grown hyperexponentially. The rate of technology growth may be affected by population size because more people mean more potential inventors. Equations to express this reciprocal boosting effect have been formulated by Taagepera [5], and they lead to Eq. (1).

The present paper attempts to cast more light on the following questions, which emerge from previous work described:

1. Different data sets have been used by various researchers. Is the hyperbolic fit still confirmed when all these population estimates are combined?
2. Meyer has shown that various technology indicators grow faster than exponentially. But do they follow the hyperbolic pattern? If so, with which singularity dates?
3. Equations (2)–(4) all tend to disagree with population data prior to 4000 B.C. Was

there a previous neolithic population explosion comparable to ours? If so, how was it solved?

4. Can we formulate a more detailed technology-population interaction model that might avoid the unrealistic singularity of the hyperbolic approximation? Logically this question should come first, but it is left toward the end because of its mathematical complexity.
5. How does this work fit in with other demographic approaches and projections, and how could it be used for world demographic policy and action?

In the following, all growth patterns of the form of Eq. (1) are designated as hyperbolic although, strictly speaking, this term applies only to the special case when $M = 1$.

Evidence for Hyperbolic World Population Growth

Table 1 brings together all the various world population figures that seem to be based on independent estimates, as collected by Foerster [10], Meyer [9], and Taagepera [5]. Figure 1 shows these data plotted as $\log P$ versus $\log(D - t)$. Equation (1) yields a straight line on such a plot, provided that the right value of D is chosen. Figure 1 uses $D = 2005$ A.D. so that Eq. (3) corresponds to a straight line. This equation is seen to fit well most estimates for the periods 0–1300 A.D. and 1800–1975 A.D. It yields slightly too high values ($\leq 17\%$) for 1400–1700 A.D.—the period of bubonic plague and genocide in America. Equation (3) may agree with the spotty estimates for 7000–100 B.C., but it yields much too high values (by a factor of ≥ 10) for all prehistoric times prior to 7000 B.C.

The curve for Eq. (2) is also shown. [The curve for Eq. (4) is very similar.] It should be stressed that the downward curvature of this curve in Fig. 1 is an artifact due to using $\log(2005 \text{ A.D.} - t)$ on the horizontal axis. If $\log(2026 \text{ A.D.} - t)$ were used instead, then Eq. (2) would become a straight line and the Eq. (3) line would curve upward. Equation (2) fits the 1800–1975 period as well and the 1500–1750 period better than Eq. (3) does. For 0–1500 A.D., Eq. (2) tends to be at the low side of most estimates. Prior to 7000 B.C. it, too, tends to yield excessively high values, but much less so than does Eq. (3).

Whereas the two equations yield practically the same world population figures for 1900 to 1975, future projections differ appreciably; by 1985, Eq. (2) projects to 4.9 billion and Eq. (3), to 5.4 billion people. However, projections based on hyperbolic equations must be expected to err on the high side in the close vicinity of the singularity. For the past 2000 years both equations perform equally well, and for much earlier times both are inadequate, although Eq. (2) does a better job.

Combined data thus confirm that the world population growth has followed a hyperbolic pattern [Eq. (1)] at least for 2000 years, with remarkably few deviations. Due to the wide range of estimates for earlier times, there is some leeway in the choice of numerical parameters, as exemplified by Eqs. (2) and (3). Several qualitative issues are involved. Are some estimates based on firmer grounds than others? Can some time periods be given less weight than others because of known population catastrophies? Should the clearly deviant period prior to 7000 B.C. be included? Hence the best-fit problem cannot be solved at this point through a mechanical application of least-squares fit with all data points given equal weight.

Some Evidence for Hyperbolic Growth of Technology

According to the technology/population mutual stimulation model [5], the level of

TABLE 1

World Population Estimates

DATE B.C.	POPULATION ESTIMATES (MILLION) ^f			
10 ⁶	0.03 ^a	0.04 ^b	0.12 ^d	
800,000		0.1 ^c		
400,000	0.05 ^e			
300,000			1.0 ^d	
250,000	0.3 ^c			
70,000	0.7 ^c			
25,000	1.3 ^c	2.5 ^b	3.3 ^d	
8,000	5 ^{d,e}	7 ^f	8 ^a	10 ^b
7,000		10 ^c		60 ^g
6,500	7 ^h			
5,000	10 ⁱ			
4,000	19 ^b			86 ^d
2,400	30 ^c			
200			120 ^g	
DATE A.D.	POPULATION ESTIMATES (BILLION) ^f			
0	0.08 ^b	0.10 ^j	0.20 ⁱ	0.30 ^{a,e,h}
1		0.13 ^d	0.25 ^{f,g}	
350			0.25 ^f	
600			0.24 ^f	
800			0.26 ^f	
1,000	0.17 ^b	0.20 ^j	0.28 ^{f,k}	0.36 ^g
1,100			0.31 ^k	
1,200			0.35 ^k	0.38 ^f
1,300	0.26 ^b		0.38 ^{k,f}	0.40 ⁱ
1,400			0.37 ^k	
1,500	0.37 ^b		0.43 ^f	0.45 ^k
1,600			0.49 ^k	0.50 ^f
1,650	0.47 ^l	0.52 ^f	0.54 ^m	0.55 ⁿ
1,700		0.62 ^m	0.64 ^f	
1,750	0.69 ^l	0.73 ^{m,f}		0.79 ⁿ
1,800	0.87 ^j	0.89 ^f	0.91 ^m	0.92 ^l 0.98 ⁿ
1,850	1.09 ^{l,o}		1.18 ^m	1.26 ⁿ 1.32 ^h
1,900	1.55 ^o	1.57 ^l	1.61 ^m	1.65 ⁿ 1.67 ^{f,h}
1,920			1.81 ^p	
1,930		2.01 ^q		2.07 ^r
1,940		2.25 ^q		2.30 ^r
1,950	2.40 ^{j,o}	2.46 ^q		2.50 ^{r,b} 2.52 ⁿ
1,960		2.85 ^q		3.00 ^r
1,965		3.22 ^q		
1,970				3.63 ^r
1,975			3.97 ^s	

^a Ref. 25.^b Ref. 3; cf. Ref. 10.^c Ref. 41; cf. p. 28 of Ref. 9.^d Ref. 26.^e Ref. 27.^f Ref. 28.^g Ref. 29.^h Ref. 30.ⁱ Ref. 31.

(Table 1 cont.)

^jRef. 32; cf. Ref. 10.^kRef. 33.^lRef. 34.^mRef. 35.ⁿRef. 36.^oRef. 18.^pRef. 37.^qRef. 38; cf. Ref. 10.^rRef. 39.^sRef. 20.

^tEstimates are rounded off to one or two significant figures up to 1800 A.D. and to three later on. For a wide estimate range, the geometric average of extremes is shown, with one significant figure. Differing estimates for same date are arranged by increasing size.

technology (T) should also increase hyperbolically, with the same singularity-date (D) as for population in Eq. (1). A new problem that did not arise in connection with population is how to define and measure T . As long as various technology indicators (T_1, T_2 , etc.) are connected through a power relation (i.e., $T_2 = aT_1^b$, where a and b are constants), the problem can be circumvented: coefficients analogous to A and M in Eq. (1) would depend on the specific indicator used, but the hyperbolic form and the singularity date would be conserved in a shift from one indicator to another. But we have no evidence that relations are of the power type.

Meyer [9] has plotted a number of possible technology indicators against time on semilog graph paper where exponential equations appear as straight lines. He has shown that technology growth patterns curve upward, indicating that increase is faster than exponential. But is it hyperbolic? To test that, Figs. 2 and 3 show two of these indicators replotted on log/log scales; data are listed in Table 2.

Fig. 1. World population (log/log scale). [Eqs. (2), (3), and (7) superimposed to estimates in Table 1].

Fig. 2. Cumulative number of new tools (log/log scale) [Eq. (5) and data from Table 2].

The logarithm of the cumulative number of tools (T) of distinguishable form (and presumably with a different function) is seen to follow a straight line when plotted against $\log(2005 \text{ A.D.} - t)$ in Fig. 2. This line corresponds to

$$T = \frac{2 \times 10^5}{(2005 \text{ A.D.} - t)^{0.67}} \quad (5)$$

Since data for the most recent 3500 years are not given in the source, the relationship is actually quite imprecise. Any value of D between 1000 A.D. and 3000 A.D. would lead to a straight line. Equation (5) corresponds to 9000 distinguishable tools by 1900 A.D.; whether this is reasonable is left to the readers' judgment. Note that any tools made of perishable materials should be discounted, since they are not represented in the archaeological data. All that can be said as of now is that these data do not contradict the

Fig. 3. Blade length obtained from 1 kg of silex (log/log scale) [Eq. (6) and data from Table 2].

TABLE 2

**Cumulative Number of Tools of Distinguishable
Form, and Blade Length Obtained from 1 kg
of Silex^a**

Dates B.C.	Cumulative new tools	Blade length (meters)
1.8×10^6	—	0.4
600,000	25	1
300,000	—	2
120,000	—	4
100,000	95	—
40,000	200	—
30,000	—	8
15,000	—	70
10,000	350	—
5,000	550	120
1,500	900	—

^aSource: Ref. 40, as reported in pp. 39–40 of Ref. 9.

model-based expectation that the D value deduced from population-growth data should apply to technology growth as well.

The other technological indicator considered is the total length (L) of cutting edge that could be obtained from a kilogram of silex at a given technological level. This indicator could not be fitted with $D = 2005$ A.D. The general hyperbolic form still applies, but D has to be 5000–4000 B.C. Figure 3 shows the fit with

$$L = \frac{2 \times 10^4}{(4500 \text{ B.C.} - t)^{0.74}}, \quad (6)$$

where L is in meters and the exponent (0.74) happens to have the same value as in Eq. (3).

Stone tool technology reached its ultimate perfection around 4500 B.C. and could not develop any further. [Again, the infinite value of L resulting for that date from Eq. (6) must not be taken literally.] Crisis was solved by a shift to metal tools. In the case of the majority of world population, the ability to work silex probably dropped rapidly. I doubt whether any of the readers could obtain 0.4 m of cutting edge out of one kilogram of silex: as far as this once vital technology is concerned, we are back to where we started 2 million years ago! But the silex crisis of around 4500 B.C. brings us to the question of a simultaneous neolithic population crisis.

The Neolithic Singularity

A unique surge in world population during 8000–4000 B.C. has been claimed, for example, by Braidwood [11] and Trewartha [12]. Returning to Fig. 1, we see unusually wide disagreements over the population size during that period, suggesting rapid change and instability.

We already have observed a hyperbolic singularity around 4500 B.C., in a technology growth curve. The interaction model suggests a population singularity at the same time.

The scattering of paleolithic population estimates is so wide that hyperbolic equations with widely different constants could be fitted. Faced with this choice, we might pick the same exponent 0.74 that occurs in Eq. (3) for later population growth, on the grounds of uniformity. With D and M [in the general Eq. (1)] thus determined, the best fit is close to

$$P = \frac{3 \times 10^9}{(4500 \text{ B.C.} - t)^{0.74}} \quad (7)$$

as shown in Fig. 1. In conjunction with Eq. (6), this leads to proportionality between the average population density (d , per square mile) of the world and the efficiency of silex technology as expressed by L (in meters per kilogram):

$$d = 0.0027 L \quad (8)$$

for human prehistory up to 4500 B.C.

The visible reason for the neolithic population explosion is the shift from gathering and hunting to agriculture. But was agriculture the cause or the result of the crisis? Origins of agriculture are currently placed much further back than 8000 B.C.—even as far as 20,000 B.C. [13]. During the same time improved tools made hunting ever more efficient, boosting the human population but possibly depleting the game populations [14]. Crisis could arise from the hunting technology thus destroying its own basis and forcing people to develop the up-to-then secondary agricultural alternative [16]. The other possibility is, of course, that development of agriculture itself brought a crisis by boosting the population size while destroying the most valuable farmland, before more conservation-minded agricultural practices were developed.

Two traditional calendars count their years starting from the presumed creation of the world. The Jewish calendar places this event at 3761 B.C. and the Byzantine one, at 5508 B.C. Although many other reasons for these choices could be offered, it might be worth noting that the dates straddle the period of the possible neolithic singularity. Was it felt that a new world was created out of a *Götterdämmerung*?

Our evidence for a neolithic singularity is by no means conclusive. It merely underlines the importance of a more thorough study of the period 8000 to 1000 B.C. This period just may offer the answer to the question of how humankind has previously handled a major technological-demographic crisis. How rapidly did world population grow prior to 4500 B.C.? What peak size did it reach? Was there any appreciable decrease in population after 4500 B.C.? If so, how suddenly did it occur? What were the population trends for 3000–500 B.C.? Note that we have only one population estimate for this long period, whereas estimates for the preceding millennia are much more numerous.

People, Skills, and Resources: An Extended Interaction Model

It is time now to explain and extend the technology-population interaction model to which frequent reference has been made in the preceding sections. This necessarily mathematical section has been delayed so as not to lose in the very beginning those readers who do not appreciate the sober beauty of differential equations. Such readers are advised to skim this section lightly and proceed to the discussion.

The thorough review of long-range population growth research by Meyer and Vallée concludes that the hyperbolic equation fits the data but

... does not result from an underlying theoretical model. Hence it is not as easily understandable as the exponential or logistic models. It is true that the above-mentioned rationalizations (positive feedback, two-person game, relay principle) do justify self-accelerating functions. However, they do not necessarily lead to a hyperbolic function, which remains without a theoretical and epistemological foundation in a simple model [6].

Such a model should consist of differential equations analogous to the basic exponential and logistic equations:

$$\begin{aligned} \text{Exponential: } \frac{dP}{dt} &= kP \\ \text{Logistic: } \frac{dP}{dt} &= k(1 - P/S)P, \end{aligned} \tag{9}$$

which both express the rate of change of P in time (dP/dt) as a function of P itself; k is a rate constant, and S is the saturation size of P beyond which growth cannot continue. When P is much smaller than S , the logistic equation is equivalent to the exponential one.

Interaction of population with technology (T) was formulated by Taagepera [5] as an extension of the simple exponential model:

$$\begin{aligned} \frac{dP}{dt} &= (kT^n)P \\ \frac{dT}{dt} &= (hP^m)T \end{aligned} \tag{10}$$

where k , h , n , and m are constants. The parentheses in Eqs. (10) express the idea that both P and T grow in a basically exponential way but that the rate constants increase as a power of the other variable. Eliminate time by member-by-member division of these equations, and integrate:

$$(k/n)T^n = (h/m)P^m + \text{const.} \tag{10a}$$

The constant is zero, if we assume that $T = 0$ when $P = 0$. Introduce the value of T from Eq. (10a) into the first of Eqs. (10). Integration of this equation leads to the hyperbolic equation [like Eq. (1)]. The power index m is connected to that in Eq. (1) as $M = (1/m)$. Imperfect interaction between potential inventors would be expected to lead to a value of m between 1 and 2. Equation (3) in fact yields $m = (1/0.74) = 1.35$.

Although Eqs. (10) do lead to the observed hyperbolic growth of population [Eq. (2) or (3)] and of some technology indicators [Eqs. (5) and (6)], they are not satisfactory conceptually, except for very low P and T levels. The rate constant for population cannot increase indefinitely as a power of T because of biological and ecological limitations. Such limitations are built into the logistic equation (through S), but not into the exponential one. The rate of technology increase should become independent of population size for very large P , since then a larger number of potential inventors might just mean more duplication of discoveries. A large population at a high level of technology would use up nonrenewable resources (R) at a considerable rate, thus further reducing population

growth in spite of continuing advances of technology. These conceptual objections tie in with the impossibility of an infinite population growth suggested by the simple hyperbolic curve.

A further oversimplification involved in Eqs. (10) is disregard of time lag between birth and maturity; dP/dt is, strictly speaking, not proportional to the total population (P) but to the adult population only. This detail can be neglected when the population increase rate is slow, that is, when the population doubling time is much larger than the biological lag time (which is ca. 20 years). But if Eq. (1) applies, then doubling time decreases linearly with time until, at the date D , a zero doubling time is reached [10, 15]. The present world population doubling time is already down to about 35 years, so that the 20-year maturation lag can no more be neglected. It will make the actual population figures fall below those projected from Eq. (2) or (3), and will prevent an infinite population increase. A somewhat similar lag time takes place with respect to research and development and to technology diffusion throughout the world, except that increasing technology tends to reduce its own diffusion lag time. It may have taken thousands of years for a neolithic invention to reach most of the world population, while nowadays it is often only a matter of a few years.

In the following model resource depletion and various saturation effects are taken into account. Lag times will be neglected because they complicate the model considerably. But whenever very large increase rates occur, we should remember that lag times are important under such conditions.

Consider an isolated population with negligible maturation and diffusion lag times, in surroundings that can support a maximum population S . Assume that the population will tend to increase up to this saturation point, following the logistic Eq. (9). However, for a technologically developing system, the rate constant k and the saturation level S may also be changing.

Howland [7] has suggested that S might increase linearly with time: $S = at + S_0$. But direct dependence on time is hard to justify conceptually. It is more reasonable to assume that S increases with technology, possibly as a constant power of some technology indicator T . Depletion of nonrenewable resources (R) should reduce S roughly proportionately, but this effect would enter only when resources are depleted below a critical level. (The term "nonrenewable resources" is used here in a very general sense; it includes not only raw materials but also space for living and for disposal of waste products.) These considerations can be formulated as

$$S = \frac{gT^a R}{R + C}, \quad (11)$$

where g is a proportionality constant and C is a critical resource level: if R is much larger than C , then $R/(R + C) = 1$ so that S does not depend on nonrenewable resources; but if R becomes much smaller than C , then $R/(R + C) = (R/C)$ so that S becomes directly proportional to R .

Austin and Brewer [17] have investigated the case where S remains constant but k grows as an inverted exponential of some power of population: $k = k_0 [1 - \exp(-aP^n)]$. However, it is not conceptually clear why sheer population size should directly affect the growth rate. It is more reasonable to assume that k is affected by technology that reduces child and young adult mortality and thus enables more people to reach their full reproductive potential. Thus:

$$k = k_0 [1 - \exp(-aT^n)]. \quad (12)$$

For very low T , the rate constant k grows as a power of T : $k = k_0 a T^n$. For very high T , k approaches a ceiling of k_0 . Combining Eqs. (11) and (12) with the logistic Eq. (9) leads to

$$\frac{dP}{dt} = k_0 [1 - \exp(-aT^n)] [1 - (R + C)P/(gT^q R)]P \quad (13)$$

Since resources are decreasing, this model can lead to eventual population decrease, in contrast to the Howland model, which implies indefinite growth, and to the Austin and Brewer model, which leads to eventual stabilization.

The increase rate of technology might be expected to be proportional to some power of population size [as in Eqs. (10)] for small P , and independent of P for large P , as discussed earlier. A simple way to formulate this condition (and the basic exponential relation to T itself) would be

$$\frac{dT}{dt} = h \left[\frac{P}{U + P} \right]^m T \quad (14)$$

where U is a characteristic population size much below that P affects dT/dt and much above that it does not.

The rate of depletion of nonrenewable resources could be expected to be proportional to the number of people and their level of technology. However, the effect of technology might be different in the future; instead of aiming at consuming more and more resources, future technology may be directed at preserving dwindling resources. If complete recycling is eventually possible, so that dwindling of nonrenewable resources stops, the relation might be expressed as

$$\frac{dR}{dt} = \frac{-fVTP}{(V + T)^2}, \quad (15a)$$

where f is a positive proportionality constant and V is a critical technology level. For $T \ll V$, the resource-depletion rate ($-dR/dt$) increases proportionately to T ; this rate reaches a maximum at $T = V$ (provided that P is constant) and becomes very small when $T \gg V$. This is likely to be overly optimistic; the best we might be able to reach is a stabilization of the per capita depletion rate:

$$\frac{dR}{dt} = -\frac{fTP}{V + T} \quad (15b)$$

Here depletion rate again increases first proportionately to T , but it eventually stabilizes for $T \gg V$, at $(dR/dt) = -fP$.

Equations (13), (14) and either Eq. (15a) or (15b) form a closed system in the sense that the rates of change in time of all the variables considered (P , T , and R) are expressed in terms of these same variables only. In principle, the system can be solved; in practice, it might be feasible only by iteration processes. Further features (e.g., lag times) could be easily introduced, rendering the model even more lifelike conceptually—and more unmanageable mathematically. Without aiming at complete solution, let us see what this model leads to, under special conditions.

THE EARLY PHASE OF TECHNOLOGICAL DEVELOPMENT

Initially, humankind was:

$$\begin{aligned} \text{Low in population: } P &\ll U; \\ \text{Low in technology: } T &\ll V \text{ and } T^n \ll \frac{1}{a} \\ \text{Rich in resources: } R &\gg C. \end{aligned} \tag{16}$$

Under these conditions, Eqs. (13)–(15) are simplified into

$$\begin{aligned} \frac{dP}{dt} &= (k_0 a) T^n P \\ \frac{dT}{dt} &= \left(\frac{h}{U^n} \right) P^m T \\ \frac{dR}{dt} &= - \left(\frac{f}{V} \right) T P, \end{aligned} \tag{17}$$

provided that we assume that the population-saturation level $S = gT^q$ is constantly raised much above the actual population size through technology expansion. The first two of these equations are equivalent to the previous Eqs. (10), which lead to hyperbolic population and technology growth. We still may be in that phase. Subsequent pattern depends on which of the three inequalities (16) reverses itself first. There is actually likely to be some simultaneous change, but let us consider the “pure” extreme possibilities.

RESOURCE-CONSERVATION ALTERNATIVE

If technology switches to resource recycling [$T \gg V$ in Eq. (15a)] before the other inequalities (16) lose validity, hyperbolic growth would continue until the other inequalities switch. If per capita resource consumption is stabilized instead [$T \gg V$ in Eq. (15b)], the same result obtains.

TECHNOLOGY SLOW-DOWN ALTERNATIVE

If population becomes so large that technological expansion no longer depends on the world population size [$P \gg U$ in Eq. (14)], then T continues to grow purely exponentially ($T = T_0 \exp ht$). If at the same time resources are still plentiful ($R \gg C$) and the ceiling on k is far away ($T^n \ll \frac{1}{a}$), then Eq. (13) yields a population growth pattern that is still hyperexponential but falls short of the hyperbolic extreme:

$$P = P_0 \exp[He^{Kt}], \quad (18)$$

where P_0 , H , and K are constants. This "doubly exponential" equation does not contain a crisis date [like D in Eq. (1)]. With proper choice of constant P_0 , it results in a straight line when $\log(P/P_0)$ is plotted on semilogarithmic graph paper. (This should not be confused with plotting P itself on semilog paper for testing of exponentiality.) For $P_0 = 1$ billion, world population data since 1850 yields, indeed, a straight line in such a plot (see Fig. 4) corresponding to

$$P = 10^9 \exp [0.46e^{0.015(t-1900)}]. \quad (19)$$

We thus may already have switched from the doomsday-bound hyperbolic growth to the merely madly accelerating doubly exponential growth that projects to $P = 5.1$ billion in 1985, similarly to Eq. (2). From Eqs. (3) and (19), the numerical value of U is found to be 2.3 billion. This would mean that population in excess of 2 billion would not contribute much to the speed of technical development, according to the simple model discussed. As k approaches its ceiling (k_0), P will start falling below the level shown in Eq. (19), and would eventually slow down to exponential increase.

RESOURCE-DEPLETION ALTERNATIVE

If nonrenewable resources are depleted to the point where $R \ll C$, then the saturation population level in Eq. (11) becomes $S = (gT^a R/c)$, an expression that may increase or decrease depending on whether the resource-depletion or technology-expansion factor prevails. The situation is too complex to be described here, but an eventual population decrease is likely.

Apart from these simplistic special cases, the complete model [Eqs. (13)–(15)] is not analyzed here. It has been seen that this model agrees with the observed hyperbolic population growth of the past but predicts a future slow down which may already have started, according to Fig. 4. It thus avoids the singularity inherent in the simpler purely

Fig. 4. World population ("doubly logarithmic" scale) [Eq. (19) and range of estimates from Table 1].

hyperbolic model [Eq. (10)]. Does this mean that crisis projections for the period 2005–2030 A.D. become invalid? I do not think so. On the contrary, they become more credible.

The major feature altered by the extended model is infinite population growth—an unrealistic aspect of the purely hyperbolic model that has invited criticism. The shift away from the early hyperbolic phase [Eqs. (16) and (17)] still represents a crisis, and the D -date of the hyperbolic submodel still may be a good indicator of the midpoint of this crisis. This is the more likely since we have come to within 30–50 years of D without yet having clearly deviated from the hyperbolic mood—the swerving away from it, therefore, must come quite suddenly.

Discussion and Projections

A major assumption of the model presented is that the world is a single interacting system, demographically and technologically. How good is this assumption in face of obvious past and present inhomogeneities? Is not population growth already slowing down in many countries? As more and more countries gradually undergo this transition, would not a world-wide crisis be drawn out in time and thus blunted? This brings us to another objection: Highest population growth rates presently do not occur in countries with the most advanced technology, which is in apparent contradiction to Eqs. (10) and (17).

The answer to the latter objection is simple; what is true for the isolated system need not be true for its nonisolated parts. The modern world is an interacting whole. Population explosion in Ceylon or Brazil is indirectly caused by technology explosion in Europe and North America. Nothing in the system-wide model says that P and T increases must occur in the same geographic location. In the long run, technologically advanced populations certainly have crowded out the less advanced ones. But in the short run, population tends to increase most rapidly in areas where technology level *changes* the most rapidly. In nonisolated subsystems this can happen through technology import—a feature not expressed by the world model, since the total system must generate all its technology internally.

Can the world system be considered homogenous? Not even the simplest physical system is homogenous at any given instant. However, we are not concerned with instant states but with the average state over an observation or usage interval. The length of interval used depends on the objectives. For our purposes the world can be considered homogenous, if new technology can diffuse and reach most people in the world before world population changes appreciably. It is a question of the characteristic technology diffusion time versus the population doubling time. During the neolithic era, both were of the order of millenia. Now both are of the order of decades and, for some aspects of technology, of the order of years. Our instantaneous impressions are often misleading in this respect. Most technologically underdeveloped nations are trailing most developed nations not by centuries but only by decades. The isolated countries and tribes that fall further behind are outnumbered and carry negligible weight in the total world population. In the perspective of one world-population doubling time, the majority of this population has been and still is fairly homogenous technologically. When the population-technology-resources interaction crisis comes, it will rapidly diffuse throughout the world, although the specific symptoms are likely to differ between Canada and India. No country will be an island by itself.

Macro and micro views of the same phenomenon are often different. Uniform stretches of sandy beach are not uniform under the microscope. Toynbee's grand systematics of history may seem irrelevant to a specialist of 15th-century Estonia. The generalist

who considers average trends of world population growth over millenia usually has few contact points with the demographer who projects population trends a few decades ahead. The micro and macro trends observed may be different without being in conflict—changes over a few decades are random noise in the millennial perspective, and millennial average changes are of no concern when one wants to project the transport or housing needs of a region during the next decade. This is usually the case. But not today.

The quasihyperbolic overacceleration of world population growth has compressed millennial trends into specific projections for the next few decades. The generalist has invaded the demographer's home turf. Not surprisingly, this intrusion is resented.

The intrusion is the more galling because the demographer's own well-proven tools of trade have been floundering in world population projections, always on the low side. In 1951 the United Nations mean projection for the world population in 1980 was 3.3 billion [18]—a figure actually reached as early as 1965. The 1958 projections for 1975 ranged from a low of 3.59 to a high of 3.86 billion [19]; the actual mid-1975 figure was 3.97 billion [20]. The basically exponential approach that would have been adequate for short-term projections in any other century is failing in the close vicinity of the crisis date of the quasihyperbolic trend.

When the generalist steps in to fill the underestimation gap, the demographer may respond in various ways. He may attack conceptual flaws in the hyperbolic model, studiously ignoring the question as to why the "wrong" model agrees with data while the "right" model fails to do so [7, 21, 22]. He may also ignore the hyperexponential growth and hope that it will go away. This silence treatment is moderately effective; when formulating the simple interaction model [5], I scanned demographic literature extensively but failed to unearth any reference to Meyer's or Foerster's work, which I discovered only through a letter in *Science* [23]. If demographers manage to maintain this cover-up long enough, then the hyperbolic growth will go away, indeed, as the assumption of plentiful resources in Eq. (16) starts breaking down. Meanwhile, decision makers may continue to receive low projections, at a critical time when we most need realistic ones.

The most fruitful option for the demographer is to engage in a constructive dialogue with the general systems approach in order to work out a synthesis and develop realistic world population projections. What looks realistic as of now?

The highest projection is given by the quasihyperbolic equation with an early

TABLE 3
World Population Projections (Billion)

Date	Ref. 18	Ref. 19	Ref. 20	Exponential with 2% growth per year	Doubly exponential [Eq. (19)]	Foerster et al. [Eq. (4)]	Meyer & Vallée [Eq. (2)]	Taagepera [Eq. (3)]
1975	—	3.6 to 3.9	4.0	4.0	4.1	3.6	3.9	4.0
1980	3.3	—	—	4.4	4.5	4.0	4.4	4.6
1985	—	—	—	4.9	5.1	4.5	4.9	5.4
1990	—	—	—	5.4	5.8	5.1	5.6	6.7
1995	—	—	—	5.9	6.7	5.8	6.4	9.1
2000	—	4.9 to 6.9	6.2	6.5	7.8	6.9	7.7	15.2

Fig. 5. World population projections (from Table 3).

singularity [i.e., Eq. (3)]. It is bound to become an overestimate by 2005 A.D. The projection of steady exponential growth at the present rate of 2% per year may turn out to be an underestimate, until technology stops affecting the growth constant [k in Eq. (12)], or until resource depletion starts affecting the maximum sustainable population [S in Eq. (11)] in a major way. The hyperbolic Eq. (2) with a more distant singularity date and the doubly exponential Eq. (19) offer intermediate projections and may well prove to be the closest to actual values during the next quarter century. All four projections are shown in Table 3 and Fig. 5, up to 2000 A.D. For the later crisis period, I dare not project any world population figures. The alternatives possible on the basis of the extended model [Eqs. (13)–(15)] are similar to those considered by the Club of Rome [24]. The main conceptual difference is that they take the present state of the world (including the present first derivatives) as the only given data set for forward projection. The model presented here uses fewer variables but has more historical depth. It thus takes into account the higher derivatives of the variables, that is, the long-term rates of change of the rates of change themselves. Using recent data, our model could be projected not only forward but also backward into history—and the results would agree with known facts as far back as 4000 B.C. An attempt to run the Club of Rome model backward yielded a population of 4 billion in 1880, followed by sudden decrease [42], and the model's proponents agreed that their model cannot be run backwards [43].

How could we influence future growth to soften the impact of the impending crisis? First, the hyperexponential growth mechanism must be understood; the work reviewed and extended here is only a beginning. Next, the results have to find their way into the thinking of the decision-makers' demographic advisors and into the thinking of the general public whose life would be strongly affected by any crisis-prevention measures. In this

respect the beginning has not yet even begun. Specific crisis-prevention measures will depend on what degree of insight will be achieved and accepted before the last precrisis century runs its full course.

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