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## COMPUTER AIDED DESIGN OF USEFUL SPHERICAL WATT I SIX-BAR LINKAGES

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### ABSTRACT

*This paper presents a software system for the kinematic synthesis of useful spherical Watt I six-bar linkages that can guide a body through five task positions. The design procedure begins with the specification of a spherical 3R open chain that reaches five specified task positions. The six-bar linkage is designed by constraining the 3R spherical chain to the topology of a Watt I spherical six-bar linkage. The CAD software SolidWorks is used to specify the 3R chain and the five spherical task positions. We describe the SolidWorks Add-In MechGen that reads the SolidWorks data and generates candidate linkages. Included in the task specification are tolerance zones that allow random adjustments to the task positions to search for defect-free linkages. An example is provided that demonstrates the five position synthesis of a useful spherical Watt I six-bar linkage.*

### INTRODUCTION

In this paper, a computer aided design system for the synthesis of spherical Watt I six-bar linkages is presented. The procedure verifies that the generated linkage is defect-free, which we term useful. This paper implements the synthesis procedure described by Soh and McCarthy [1], which designs a spherical six-bar linkage by computing constraints to a 3R spherical chain. The calculation of these constraints uses Burmester theory for spherical linkages described by Chiang [2]. In addition, this synthesis procedure implements search procedure introduced by Plecnik and McCarthy [3] that finds useful linkages for tasks

within a tolerance zone around the given task positions.

Our design procedure for a spherical Watt I six-bar linkage is implemented in a computer aided design system that integrates a Mathematica synthesis algorithm into the geometric modeling software SolidWorks as an Add-in using Visual Basic called *MechGen*. The organization of this computer-aided design system is demonstrated by an example synthesis of a spherical six-bar linkage.

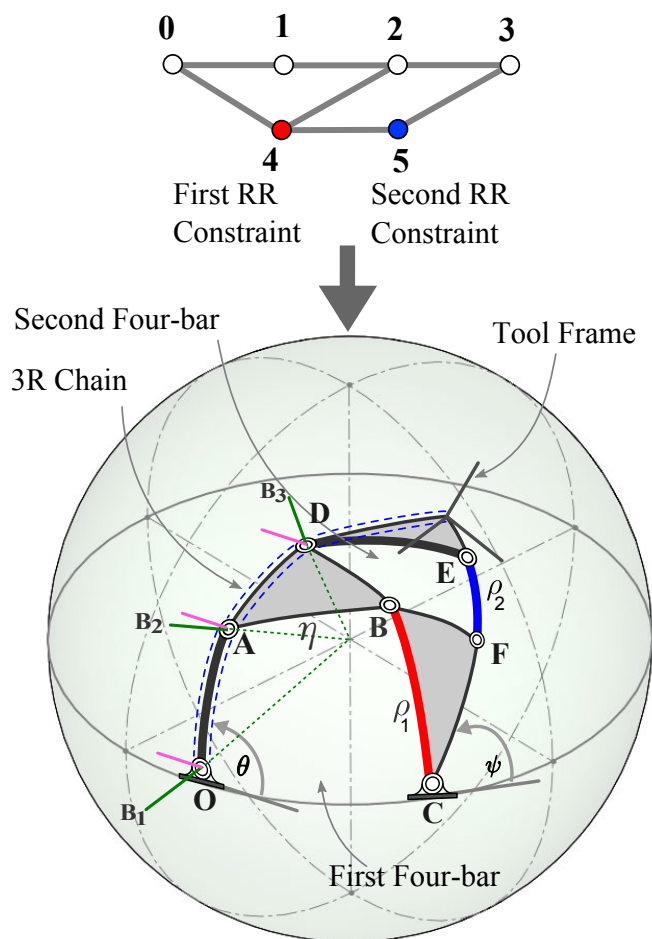
### LITERATURE SURVEY

The computer aided design of linkages that combines Burmester theory with interactive graphics originated with Kaufman [4, 5] who developed a system for the synthesis of planar four-bar linkages that guide a moving frame through a set of task positions. Also see Erdman [6, 7] and Waldron [8]. Ruth and McCarthy [9] extended this work to the design of spherical four-bar linkages, and Laroche and colleagues [10, 11] used Virtual Reality for the synthesis of both spherical and spatial four-bar linkages. Alvarez and Su [15] have extended this use of Virtual Reality to the conceptual design of linkages.

Alizade [12, 13] has developed a design methodology for spherical four-bar function generators. Hernandez et al. [14] have formulated the design of a spherical Stephenson six-bar linkage and Soh and McCarthy [1] use Burmester theory to design spherical six-bar linkages as constrained 3R spherical chains. This paper describes design system *MechGen* which is the first computer-aided design system that uses interactive graphics for the synthesis of useful spherical Watt I six-bar link-

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**FIGURE 1.** LINKAGE GRAPH AND SCHEMATIC OF A SPHERICAL WATT I SIX-BAR LINKAGE

ages.

## SYNTHESIS THEORY

Our synthesis procedure for the spherical Watt I six-bar linkage begins with the specification of a spherical 3R chain and a set of five task positions. The spherical six-bar is obtained by designing RR links that constraint the 3R chain to the Watt 1 topology, Fig.1. This synthesis procedure is described in McCarthy and Soh [16], and we summarize it here.

The task for the synthesis of the spherical Watt I six-bar linkage is prescribed by five task orientations which are described by the  $3 \times 3$  rotation matrices,  $[T_j]$ ,  $j = 1, \dots, 5$ , that define the orientation of the moving frame  $M$  relative to the ground frame  $F$ . In addition, the design specifies a 3R spherical chain defined by joint axes  $\mathbf{O}$ ,  $\mathbf{A}$  and  $\mathbf{D}$ , see Fig.1. Three link frames denoted as  $B_1$ ,  $B_2$  and  $B_3$  are attached to the three links of the 3R chain

such that they have their origins at  $\mathbf{O}$ ,  $\mathbf{A}$  and  $\mathbf{D}$  respectively, with their  $z$ -axes directed along the joint (radially outwards) and their  $y$ -axes perpendicular to the plane of the link as shown in Fig.1.

## Inverse kinematics of the 3R chain

Let  $[G]$  denote the transformation matrix that maps the base of the 3R chain to the fixed frame  $F$  and let  $[H]$  map the tool frame in the last link of the 3R chain. Then the kinematics equation for the 3R chain is given by

$$[K(\theta_1, \theta_2, \theta_3)] = [G][B_1(\theta_1)][B_2(\theta_2)][B_3(\theta_3)][H], \quad (1)$$

where  $\theta_i$ ,  $i = 1, 2, 3$  are the joint rotations.

With the knowledge of the kinematics equation  $[K(\theta_1, \theta_2, \theta_3)]$  for each of the five spherical 3R chains for the five task orientations  $[T_j]$ ,  $j = 1, \dots, 5$ , we can now use the inverse kinematics to compute the angles  $\theta_{i,j}$  such that,

$$[T_j] = [K(\theta_{1j}, \theta_{2j}, \theta_{3j})], \quad j = 1, \dots, 5. \quad (2)$$

## Synthesis of the first RR constraint

The joint values  $\theta_{i,j}$  allow us to compute the orientation of the link frame  $B_2$  relative to the ground frame in the five task orientations, that is

$$[C_j] = [G][B_1(\theta_{1j})][B_2(\theta_{2j})], \quad j = 1, \dots, 5. \quad (3)$$

This yields the rotations relative to the first task orientation, as

$$[R_{1j}] = ([G][B_1(\theta_{1j})][B_2(\theta_{2j})])([G][B_1(\theta_{11})][B_2(\theta_{21})])^T \quad j = 1, \dots, 5. \quad (4)$$

The five relative orientations  $[R_{1j}]$  of the link  $\mathbf{AD}$  are now used to design the RR chain  $\mathbf{CB}$ . The requirement that  $\mathbf{B}$  maintain a constant angle  $\rho_1$  relative to  $\mathbf{C}$  in the five orientations yields the five design equations

$$([R_{1j}]\mathbf{B}) \cdot \mathbf{C} = |\mathbf{B}||\mathbf{C}| \cos \rho_1, \quad j = 1, \dots, 5. \quad (5)$$

These equations can be solved numerically to yield as many six sets of coordinates for the link  $\mathbf{CB}$ , refer Chiang and McCarthy [2] [16].

## Analyze the four-bar chain

The joint  $\mathbf{A}$  of the original 3R chain and the new joint  $\mathbf{B}$  are part of the same link, therefore their coordinates satisfy the

constraint equation

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos \eta, \quad (6)$$

where  $\eta$  is the angle between the joint axes  $\mathbf{A}$  and  $\mathbf{B}$ , refer Fig.1. Expanding this constraint equation in terms of the input angle  $\theta_1$  and the angle  $\psi$  at  $\mathbf{C}$ , we obtain an equation of the form

$$A(\theta_1) \cos \psi + B(\theta_1) \sin \psi = C(\theta_1), \quad (7)$$

where the  $A(\theta_1)$ ,  $B(\theta_1)$  and  $C(\theta_1)$ , are defined in terms of the dimensions of the spherical chain  $\mathbf{OABC}$  refer McCarthy [16].

The solution of Eqn. (7) for each of the values  $\theta_{1j}$ , yields the angles  $\psi_j$ ,  $j = 1, \dots, 5$ . The orientation of the link  $\mathbf{CB}$  in each of the task orientations is given by

$$[D_j] = [G_C][B_4(\psi_j)], \quad j = 1, \dots, 5, \quad (8)$$

where  $[G_C]$  is the transformation from the base frame  $F$  to the joint  $\mathbf{C}$  and  $[B_4(\psi_j)]$  is the rotation about  $\mathbf{C}$ .

### Synthesis of the second RR constraint

We now use the known orientations of link  $B_3$  of the 3R chain and the link  $\mathbf{CB}$  to design a second RR constraint  $\mathbf{FE}$ . From the inverse kinematics of the 3R chain, we can compute the relative positions  $[R_{1j}]$  of the end-link,

$$\begin{aligned} [\bar{R}_{1j}] &= ([G][B_1(\theta_{1j})][B_2(\theta_{2j})][B_3(\theta_{3j})] \\ &\quad \times ([G][B_1(\theta_{11})][B_2(\theta_{21})][B_3(\theta_{3j})])^T \\ &\quad j = 1, \dots, 5. \end{aligned} \quad (9)$$

Similarly, we can compute the relative positions of link  $\mathbf{CB}$ ,

$$[S_{1j}] = ([G_C][B_4(\psi_j)])([G_C][B_4(\psi_1)])^T, \quad j = 1, \dots, 5. \quad (10)$$

The second RR chain constrains a joint  $\mathbf{E}$  in the end-link to maintain a constant angle relative to the joint  $\mathbf{F}$  in the link  $\mathbf{CB}$ , in each of the task orientations. This yields the five design equations,

$$([\bar{R}_{1j}]\mathbf{E}) \cdot ([S_{1j}]\mathbf{F}) = |\mathbf{B}||\mathbf{C}| \cos \rho_2, \quad j = 1, \dots, 5. \quad (11)$$

These equations can be solved to obtain as many as six sets of values for the coordinates of the joints  $\mathbf{E}$  and  $\mathbf{F}$ . The result is a set of spherical Watt I six-bar linkages that guide an end-effector through five task orientations, see McCarthy [16].

## EVALUATION OF GENERATED LINKAGES

The candidate linkages generated from the synthesis process may suffer from circuit and branch defects, as described by Chase et al. [17]. In order to filter these linkages for defects, MechGen uses the direct kinematics equation Eqn. (7) to analyze each of the two spherical four-bar linkages using the theory presented in McCarthy [16]. Solving this equation yields two values for the output crank angle for a given input crank angle as shown in the Eqn. (12) below. The two angles result from the fact the coupler link and the output crank can be assembled in two configurations namely elbow up and elbow down for the same input crank angle, which corresponds to the plus and minus values in the equation,

$$\psi(\theta) = \arctan\left(\frac{B}{A}\right) \pm \arccos\left(\frac{C}{\sqrt{A^2 + B^2}}\right). \quad (12)$$

This equation also gives us a formula to find the bounds on the range of the input crank angle, through the argument of the arccos term, which is subjected to the bounds  $-1$  and  $+1$ . If this is not satisfied then the linkage cannot be assembled for the specified input crank angle. MechGen uses this Eqn. (12) to filter both the first four-bars and second four-bars. The two checks used to filter defective linkages are:

### 1. *The orientation of the output crank with respect to the coupler that is the elbow up or elbow down configuration*

The elbow of the output crank with respect to the coupler should be either up or down for all the five task positions. This avoids circuit change when the input crank is a crank (range  $0$  to  $2\pi$ ), branch change when it is a 0-rocker or  $\pi$ -rocker or just rocker, see McCarthy [16].

### 2. *The limits on the input crank*

The 5 crank angles for the five task positions should lie in the range bounded by the two values obtained from the arccos argument in Eqn. (12). This defect occurs in the case, when the input crank is a rocker. The linkage might have elbow up or down configuration same for all the task positions, that is all configurations may seem to lie on the same branch but they might be on different circuits. So this check prevents circuit change when input crank is rocker.

These two checks are sufficient to take care of circuit or branch defects and the resulting spherical Watt I six-bar linkages are usable linkages.

## OUTLINE OF THE LINKAGE DESIGN ALGORITHM

The flowchart for linkage design algorithm is shown in Fig. 2. The input for the linkage design algorithm consists of the five task positions (orientations) of the moving body with respect to the fixed body, the spherical serial 3R chain data that

guides the end effector through the five task positions, and the number of iterations and tolerances.

Using the task positions and 3R chain data, the algorithm first synthesizes all the possible first four-bar **OABC** candidates, refer Fig.1, and then analyzes them for defects. If the algorithm does not find a single useful linkage, it breaks the control sequence, randomizes the task positions within the tolerance range and starts the process again. If some useful linkages are found, then the algorithm pushes them to the next step of synthesizing the second four-bars **BDEF**, for each these first four-bars. After analyzing each of the second four-bar, if the algorithm again does not find a useful linkage, it breaks control sequence, randomizes the task positions within the tolerance range and starts the process again. If some useful linkages are found, then the spherical Watt I six-bar linkage as a whole is saved. This process repeats itself till the counter increments to the number of iterations. Following that, the saved useful spherical Watt I six-bar linkages are ranked on a criteria, which is the ratio of length of the longest link to the length of the shortest link, as described by Soh [18].

### MECHGEN DESIGN PROCESS

The paper uses the *car door opening example* for explaining the design process.

#### Step 1: Motion Requirement

The user's requirement for the motion of the car door with respect to the car body as five orientations is shown in Fig.3. Since MechGen can do only five position synthesis, the motion requirement has to be discretized into five door positions.

#### Step 2: Specifying five Task Positions

Due to the inherent complexity in specifying the spherical task positions (orientations), MechGen has a part file embedded in it, which is referred to as the *Environment*. This part file has two sketches built in; one for the five task positions and one for the 3R serial chain. In this step MechGen software is opened and Environment part file is imported in the working assembly as shown in Fig.6. The user edits the sketch for task positions to get a movement that is close to the requirement. Five doors are attached to the task positions to confirm this as shown in Fig.4.

#### Step 3: Specifying the five 3R chains

In this step the user edits the 3R chain sketch in the Environment file to specify the backbone serial chain as shown in Fig.5.

#### Step 4: Import the task position and 3R chain information in MechGen

In this step the user selects the 3R chain sketch in Environment and clicks on the "Import From SolidWorks" button. MechGen captures all the information and displays it to the user, along with several verification checks to display errors if any, as shown in Fig.7.

#### Step 5: Generate linkage solutions

In this step the user specifies the number of iterations and the

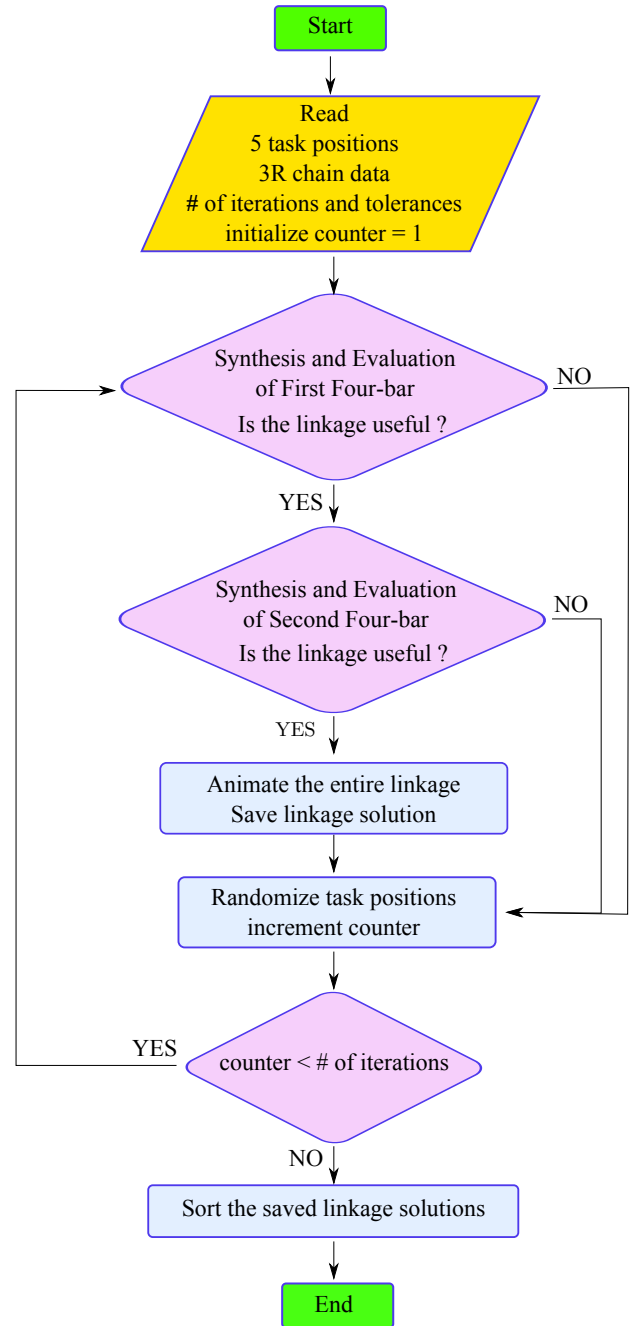
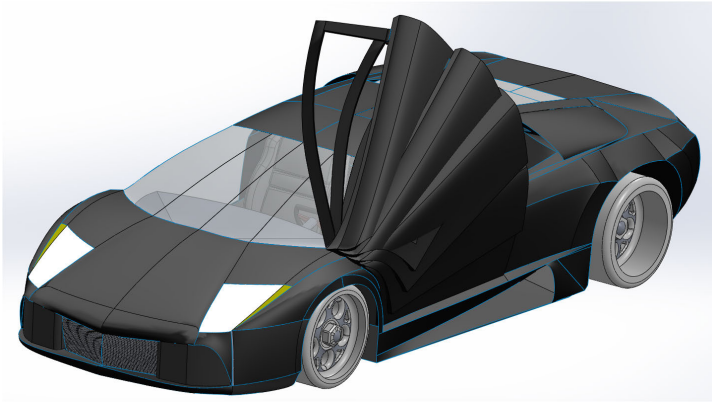
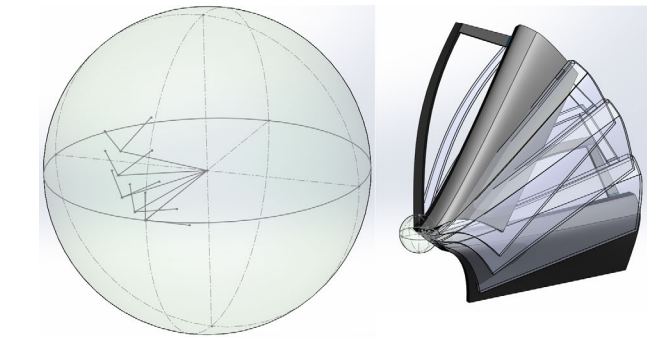


FIGURE 2. DESCRIPTION OF THE SPHERICAL WATT I SIX-BAR LINKAGE SYNTHESIS ALGORITHM IN MECHGEN.

tolerances and then clicks on the "Generate Linkages" button to generate solutions. The generated solutions are displayed as shown in Fig.8. The user can select a linkage from the solution box and generate its SolidWorks assembly by clicking on the "Start" button. Figure 9 show four example linkages gener-



**FIGURE 3.** THE DESIGN TASK IS SPECIFIED AS A SET OF ORIENTATIONS OF THE CAR DOOR RELATIVE TO THE CAR BODY



**FIGURE 4.** THE TASK ORIENTATIONS READ BY MECHGEN.

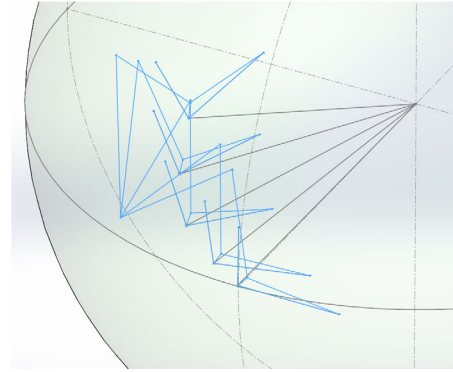
ated by MechGen in SolidWorks.

**Step 6: Integrating desired linkage in user assembly**

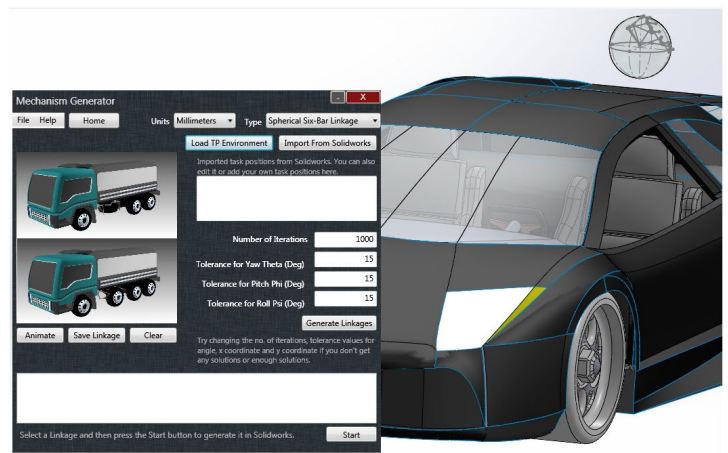
In this step the user imports the desired linkage solution in to the car assembly and integrates it. Figure 10 shows the final car door assembly along with the spherical Watt I six-bar linkage that guides the door through the five task positions. A video of this linkage in action can be found on <http://mechanicaldesign101.com>.

**Generating Solid Geometries in SolidWorks**

The development of this module in MechGen began with recording macros of simple shapes drawn in the SolidWorks user interface. When saved, these macros provided the Visual Basic commands, parameters and code sequence required to produce these simple shapes. Various macros were then altered and combined into a simple Visual Basic (VB6) program. With further development, MechGen became capable of producing SolidWorks assembly for the planar six-bar linkage using primitive shapes. To take advantage of the .NET Framework, the development of the software was later shifted the to VB.NET.



**FIGURE 5.** 3R SPHERICAL CHAIN READ BY MECHGEN



**FIGURE 6.** SPECIFY NO. OF INTERACTIONS AND TOLERANCE ZONES IN MECHGEN

During the linkage generation process, MechGen creates a separate part file for each spherical link. A link is created using lines and arcs to form a closed loop and then extruded to give it a three dimensional structure. Once all the part documents are created, an assembly document is made and the parts are imported into the assembly model space. The software then selects the origin of a link in a part document and mates it to the origin of the assembly document. The process is repeated until all the part document origins are coincident with the assembly document origin. Next, the software applies a collinear mate between the joint axis of the connecting links. Each link is connected in this manner until all joint axes are constrained. This completes the assembly for the spherical Watt I six-bar linkage and rotating the input link will cause the end-effector to move through the five task positions.



FIGURE 7. IMPORT DESIGN DATA INTO MECHGEN AND EXECUTE SYNTHESIS ROUTINE

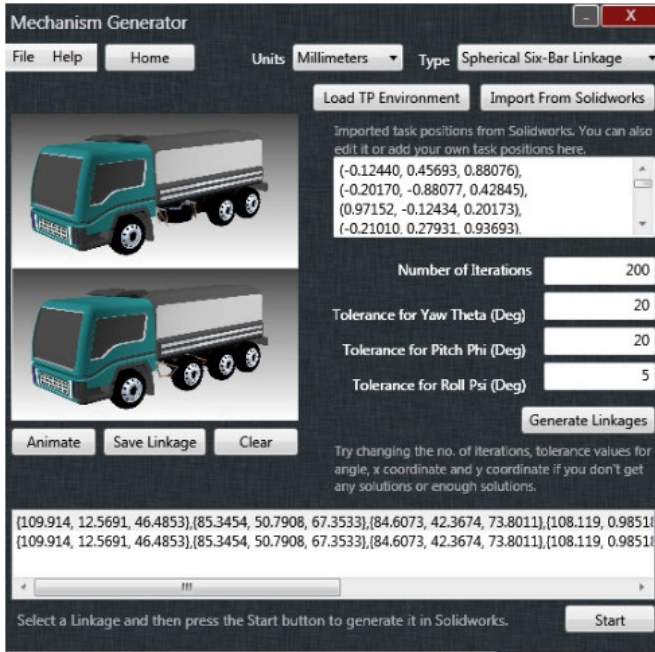


FIGURE 8. SPHERICAL WATT I SIX-BAR LINKAGE DESIGN DATA RETURNED BY MECHGEN.

## CONCLUSIONS

This paper describes the linkage design system *MechGen* that reads a SolidWorks part file that contains a user-defined 3R spherical chain, a five orientation task, and a set of tolerances for acceptable variations of the task. *MechGen* returns a set of spherical Watt I six-bar linkages that pass through acceptable task orientations without defect. This combines six-bar linkage synthesis techniques based on constraining a 3R open chain with ran-

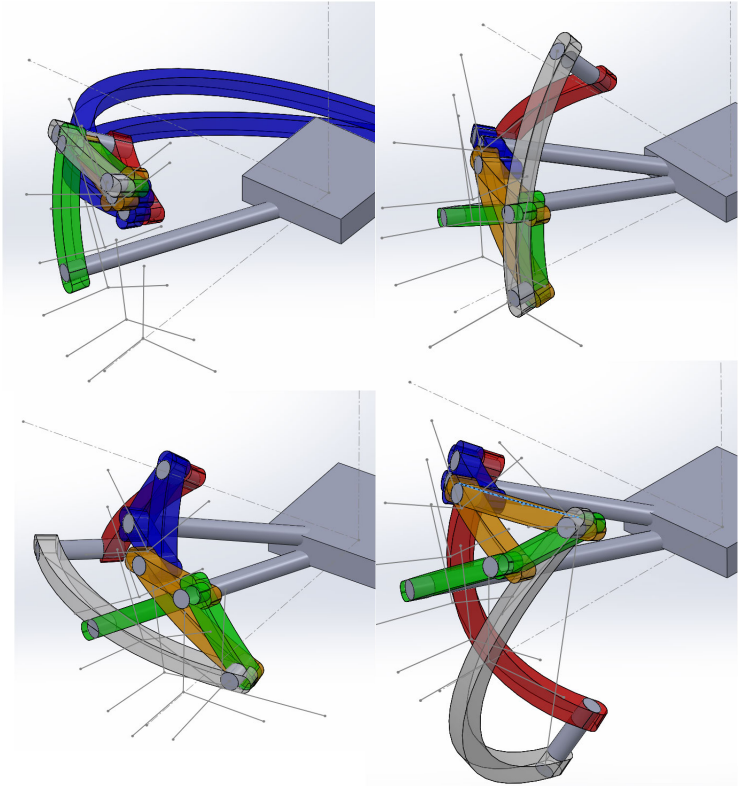


FIGURE 9. MECHGEN GENERATES A SOLID MODEL ASSEMBLY

domized search in tolerance zones to yield an effective procedure for the design of useful spherical Watt I linkages.

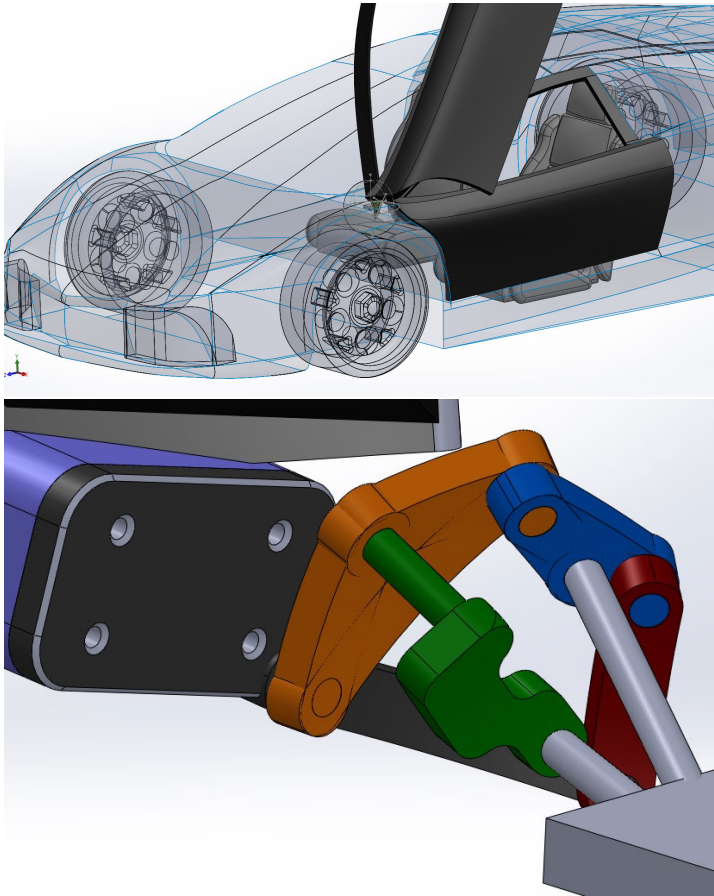
As an example, the synthesis of a spherical Watt I linkage that guides the opening of a cardoor in a combination of a Lamborghini and gull-wing style is presented. This software is available on request from the authors.

## ACKNOWLEDGMENT

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**FIGURE 10.** THE MECHGEN SPHERICAL WATT I SIX-BAR LINKAGE ASSEMBLED INTO THE CAR AND CAR DOOR

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