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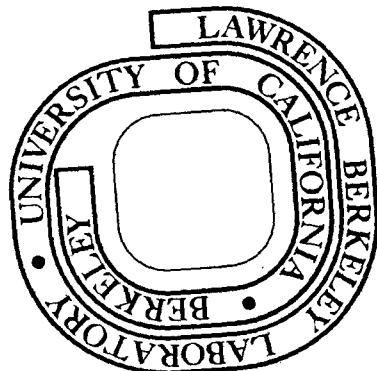
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QUARK PHYSICS WITHOUT QUARKS^{*†}
 A REVIEW OF RECENT DEVELOPMENTS IN S-MATRIX THEORY

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ABSTRACT

A review is given of the developments in S-matrix theory over the past five years which have made it possible to derive results characteristic of quark models without any need to postulate the existence of physical quarks. In the new approach, the quark patterns emerge as a consequence of combining the general S-matrix principles with the additional concept of order.

* Adapted from a series of lectures given by the author in Spring 1978 during his stay at Macalester College as Visiting Hubert H. Humphrey Professor.

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** Participating Guest.

I. INTRODUCTION

The belief that quarks and leptons are the fundamental constituents of matter is held by the majority of particle physicists and has become widely known in the physics community and by the public at large. An alternative view of matter, pursued by a small but growing minority of physicists, holds that quarks are not primary physical entities but merely patterns generated by the dynamics of the strong interactions. This view has arisen in the framework of S-matrix theory and of the bootstrap approach which does not accept any fundamental entities but tries to understand nature entirely through its self-consistency.

In the last five years, there have been several important developments in S-matrix theory which have brought extremely encouraging results. Starting from the dual models of the late sixties, topology was introduced into the S-matrix framework, and a further decisive advance was made when the notion of order was recognized as a new and important ingredient of hadron physics. The new approach culminated in the concept of the ordered S matrix which has made it possible to derive results characteristic of quark models without any need to postulate the existence of physical quarks. These results have generated great enthusiasm among S-matrix theorists, many of whom believe now that we shall be able, in the not too distant future, to go beyond the quark model; to do, as it were, quark physics without quarks.

The new developments in S-matrix theory have recently been reviewed by Chew and Rosenzweig¹ in a fairly technical paper. The purpose of our review is to communicate the recent results to a wider

audience. Our readers will be assumed to be familiar with quantum mechanics and special relativity but need not have any knowledge of S-matrix theory, nor will any detailed knowledge of particle physics be assumed. Those who wish to study the new S-matrix approach in greater detail are referred to Ref. 1 which will serve as our main guide and source of reference. Throughout this review, we shall hint at philosophical implications of the concepts and theories under discussion. Readers with a deeper interest in these philosophical questions are referred to Ref. 2, Chapters 16-18.

The paper is organized as follows. Section II reviews the concept of basic building blocks of matter from a historical perspective and presents the quark model as the most recent version of this age-old idea. The present state of the quark model and the difficulties inherent in the notion of physical quarks are discussed.

In Section III, the basic formalism and the underlying concepts of S-matrix theory are reviewed. They include the principles of Poincaré invariance, unitarity, and analyticity, the unified description of stable and unstable particles as poles of the S matrix, the concept of crossing, the notion of nuclear democracy, the Regge formalism, the bootstrap philosophy, and the dynamic re-formulation of the quark concept.

Section IV deals with the ordered S matrix. Starting from a brief description of duality, the notion of particle order is introduced; various categories of order are discussed and it is shown that the most general type of order consistent with the properties of the S matrix is the one represented by quark lines. The quark

patterns are thus seen to emerge as a consequence of combining the S-matrix principles with the notion of order.

Section V introduces the concepts of the planar S matrix and the topological expansion, which are necessary to make contact with experiment, and reviews some of the previously mysterious regularities observed in hadronic phenomena that can now be understood as manifestations of order.

Section VI concludes the review with a few speculative remarks about the meaning of order in fundamental physics.

II. QUARKS AS BASIC BUILDING BLOCKS

Physics in the twentieth century has been characterized by an ever-progressing penetration into the world of submicroscopic dimensions, down into the realms of atoms, nuclei and their constituents. This exploration of the submicroscopic world has been motivated by one basic question which has occupied and stimulated human thought throughout the ages: what is matter made of? Ever since the beginning of natural philosophy, men and women have speculated about this question, trying to find the "basic stuff" of which all matter is made. In fact, the philosophical questions that physicists have to confront today are just modern versions of problems that were discussed thousands of years ago in ancient Greece, in India and in China, as well as in many other cultures.

The school of thought that has had the strongest influence on modern physics was Greek atomism. The Greek atomists - Democritus, Leucippus, and others - saw matter as being made of several "basic building blocks". These were purely passive and intrinsically dead indivisible constituents, or atoms, which were too small to be seen,

but whose properties and behavior gave rise to all the physical phenomena observed in the everyday world.

In the twentieth century, atoms were actually observed and could be investigated experimentally. They were not indivisible but turned out to consist of smaller parts which, in turn, consisted of smaller parts. Penetrating deeper and deeper into matter, physicists applied, again and again, the approach suggested by Democritus over two thousand years ago: if you don't understand a material structure or a physical phenomenon, break it up into its constituents and try to understand it in terms of their properties and behavior!

In the past, this approach was extremely successful in explaining the physical world in terms of a few atoms; the structures of the atoms in terms of a few nuclei surrounded by electrons; and finally, the structures of the nuclei in terms of two nuclear "building blocks", the proton and the neutron. Thus atoms, nuclei and hadrons (i.e. protons, neutrons and other strongly interacting particles) were, in turn, considered to be "elementary particles". None of them, however, fulfilled that expectation. Each time, these particles turned out to be composite structures themselves, and each time physicists hoped that the next generation of constituents would finally reveal themselves as the ultimate components of matter.

On the other hand, the theories of atomic and subatomic physics made the existence of elementary particles increasingly unlikely. They revealed a basic interconnectedness of matter, showing that kinetic energy can be transformed into mass, and suggesting that particles are transient stages in an ongoing cosmic process. All these developments strongly indicated that the simple

mechanistic picture of basic building blocks had to be abandoned, and yet many physicists are still reluctant to do so. The age-old tradition of explaining complex structures by breaking them down into simpler constituents is so deeply ingrained in Western thought that the search for these basic components is still going on.

The most recent candidates for the basic building blocks of matter are the so-called quarks. The quark hypothesis was introduced in 1963 by Gell-Mann and Zweig in an attempt to account for the surprising regularities that had been discovered in the spectrum of hadrons. The properties characterizing the strongly interacting particles - their spin, electric charge and other, more abstract, "charges" called hypercharge, isospin, etc. - do not take arbitrary values but are restricted to integer or half-integer values (in certain units). These integers or half-integers are called quantum numbers in analogy to the quantum numbers in atomic physics. Each particle is characterized by a set of these quantum numbers which, in addition to its mass, specify its properties completely.

Quantum numbers are extremely useful to classify not only particles but also the interactions between them. This can be done because the quantum numbers characterizing hadrons are "conserved", i.e. they remain constant during an interaction. For example, in a situation where two particles, A and B, collide and three particles, C, D, and E, emerge from the collision, the total electric charge carried by the initial particles A and B will equal the total charge carried by the final particles C, D, and E; and the same is true for the particles' total isospin, hypercharge and other quantum numbers. Quantum numbers, then, are elements of constancy in the complex dance of subatomic matter and are thus ideal

to describe and classify the particle interactions.

When the particles are arranged according to the values of their quantum numbers, they are seen to fall into very neat patterns; hexagonal and triangular patterns known as octets and decuplets.³ Hadrons fall into two broad groups - mesons and baryons. Mesons are grouped in octets, baryons in octets and decuplets; mesons have integral values of spin, baryons have half-integral spins. Another difference between the two kinds of hadrons is that each meson octet contains particles as well as antiparticles, whereas for each baryon multiplet (octet or decuplet) there is a distinct multiplet containing the corresponding antibaryons.

The emergence of these patterns, known technically as $SU(2)$ and $SU(3)$ symmetries, was very surprising, and even more surprising was the discovery that they can be represented in a very simple way if one assumes that all hadrons are made of a small number of elementary entities which have so far eluded observation. These entities are the quarks introduced by Gell-Mann and Zweig. The two physicists succeeded independently in accounting for all the regularities mentioned above by assigning appropriate quantum numbers to three quarks and their antiquarks, and then putting these building blocks together in various combinations to form baryons and mesons whose quantum numbers are obtained simply by adding those of their constituent quarks. In this sense, baryons are said to consist of quarks, their antiparticles of the corresponding antiquarks, and mesons of a quark plus an antiquark.

The simplicity and efficiency of this model is striking, but it leads to severe difficulties if quarks are taken seriously as actual physical constituents of hadrons. So far, no hadrons have ever been broken up into their constituent quarks, in spite of bombarding them with the highest energies available, which means that quarks would have to be held together by extremely strong binding forces. According to our present understanding of particles and their interactions, these forces can only manifest themselves through the exchange of other particles, and consequently these other particles, too, would be present inside each hadron. If this were so, however, they would also contribute to the hadron's properties and thus destroy the simple additive scheme of the quark model.

In other words, if quarks are held together by strong interaction forces, these must involve other particles and the quarks must consequently show some kind of "structure", just like all the other strongly interacting particles. For the quark model, however, it is essential to have pointlike, structureless quarks. Because of this fundamental difficulty, it has so far not been possible to formulate the quark model in a consistent dynamic way which accounts for the observed hadron patterns as well as for the binding forces holding quarks together within the hadrons.

On the experimental side, there has been a fierce but, so far, unsuccessful "hunt for the quark" over the past decade. If single quarks exist, they should be quite conspicuous because the model requires them to possess some very unusual properties, like electric charges of $1/3$ and $2/3$ of that of the electron, which do not appear anywhere in the particle world. So far, no particles with

these properties have been observed in spite of the most intensive search. This persistent failure to detect them experimentally, plus the serious theoretical objections to their existence, have made the reality of quarks extremely doubtful.

On the other hand, the quark model continues to be very successful in accounting for the regularities found in the particle world, although it is no longer used in its original simple form.⁴ In the original model, all hadrons could be built from three kinds of quarks and their antiquarks, but in the mean time physicists have had to postulate additional quarks to account for the great variety of hadron patterns.

The three original quarks were denoted, rather arbitrarily, by u, d, and s for "up", "down" and "strange". The first extension of the model which emerged from the detailed application of the quark hypothesis to the full body of particle data was the requirement that each quark has to appear in three different varieties, or three different "colors". The use of the term color is, of course, quite arbitrary and has nothing to do with the usual meaning of color. According to the colored quark model, baryons consist of three quarks of different colors, whereas mesons consist of a quark plus an antiquark of the same color.

The introduction of color increased the total number of quarks to nine, and more recently an additional quark - again appearing in three colors - was postulated. With the physicists' usual penchant for fanciful names, this new quark was denoted by c for "charm". This brought the total number of quarks up to twelve -

four kinds, each of them appearing in three colors. To distinguish the different kinds of quarks from the different colors, physicists soon introduced the term "flavor" and now speak of quarks of different colors and flavors.

The latest attempts have been to add two new flavors to the model, denoted by t and b for "top" and "bottom" (or, more poetically, for "true" and "beautiful"), which brings the total number of quarks up to eighteen - six flavors and three colors. In this development, it is remarkable that more and more flavors seem to be necessary to account for the detailed hadron data, as new particles are discovered in collision experiments involving ever increasing energies, whereas the number of colors seems to be limited to three.

The current mathematical formulation of the quark model is known as QCD which stands for quantum-chromodynamics. It is a field theory and has been named in analogy with quantum-electrodynamics, or QED, the first and still most successful modern field theory, which describes the electromagnetic interactions between charged particles. Whereas electromagnetic interactions are mediated by the exchange of photons, the strong interactions between quarks are mediated, in QCD, by the exchange of "gluons". These are not real particles but some kind of quanta that "glue" quarks together to form mesons and baryons.

The main problem of the quark model is to explain why there are no free quarks. In the framework of QCD, this phenomenon has been given the name quark confinement, the idea being that quarks are,

for some reason, permanently confined within the hadrons and thus will never be seen. Several mechanisms have been proposed to account for quark confinement, but so far no consistent theory has been formulated.

This, then, is the present state of the quark model: to account for the observed patterns in the hadron spectrum, at least 18 quarks plus 8 gluons seem to be needed; none of these have ever been observed as free particles and their existence as physical constituents of hadrons would lead to severe theoretical difficulties; various mechanisms have been developed to explain their permanent confinement, but none of them represents a satisfactory dynamic theory. Yet, in spite of all these difficulties, most physicists still hang on to the idea of basic building blocks of matter which is so deeply ingrained in our Western scientific tradition.

There is an amusing analogy which illustrates the kind of basic conceptual error that may be involved in the idea of permanently confined building blocks. Suppose we have a vibrating string and we keep both of its ends fixed. The result is the well-known phenomenon of standing waves; wave patterns involving a limited number of well-defined shapes and, in particular, a limited number of nodes along the axis of the string; e.g. as shown in Fig. 1. The nodes are generated by the confinement of the string; they are a consequence of the detailed dynamics and boundary conditions of the phenomenon. It does not make sense to speak of "free nodes"; nodes only appear when the string is confined. They are not primary physical entities which are then confined but patterns generated by the confinement. Similarly, quarks may not be primary physical

entities which are confined within hadrons, but merely patterns generated by the detailed dynamics of the strong interactions. As it does not make sense to speak of free nodes, it may not make sense to speak of free quarks.

This analogy illustrates the view of quarks held in S-matrix theory, an approach to hadron physics which is in many ways complementary to the quantum-field-theory approach but is much less known outside the field of particle physics. The basic formalism and the main concepts underlying S-matrix theory will be discussed in the following section.

III. S-MATRIX THEORY

One of the major challenges of present-day physics is to describe the symmetries and regularities observed in the particle world in terms of a dynamic model, that is, in terms of the great variety of phenomena associated with the strong interactions: the continual transformation of hadrons into one another, their mutual interaction through the exchange of other hadrons, the formation of "bound states" of two or more hadrons, and their decay into various particle combinations. All these processes, which are usually given the general name "particle reactions" are essential features of the strong interactions and have to be accounted for in a dynamic model of hadrons.

S-matrix theory is the framework which seems to be most appropriate for the description of hadrons and their interactions. Its key concept, the S matrix, was originally proposed by Heisenberg in 1943 and has been developed, over the past two decades, into a complex mathematical structure which seems to be ideally suited to

describe the strong interactions. Many physicists have contributed to this development, but the unifying force and philosophical leader in S-matrix theory has been Geoffrey Chew,⁵ much in the same way as Niels Bohr was the unifying force and philosophical leader in the development of quantum theory half a century earlier.

The S matrix is a collection of probability amplitudes for all possible reactions involving hadrons. It derives its name from the fact that one can imagine the whole assemblage of possible hadron reactions arranged in an infinite matrix; the letter S is a remainder of the original name "scattering matrix" which refers to scattering processes, the majority of particle reactions.

To be more precise, suppose we perform an experiment of some kind and get a result α . This result is associated, according to the framework of quantum mechanics, with a state vector in Hilbert space. On the basis of that measurement, we then want to predict the probability of obtaining a certain result β when we do another experiment. This is done through the elements of the S matrix and is written

$$\langle \beta | S | \alpha \rangle \quad (\text{III.1})$$

where S is an operator connecting the states α and β . The absolute square of this matrix element will give the probability that the result α will be followed by the result β .

Any conceivable experimental results in hadron physics can be cast into this language. If we know the S matrix, we know everything that can be known about hadrons. However, this formalism cannot be used to describe the electromagnetic, weak, and gravitational

interactions, due to its inherent inability to deal with massless particles.

In hadron physics, a typical sequence of events would be a scattering process where two particles, A and B, collide and emerge from the collision as particles C and D. This can be illustrated by the diagram shown in Fig. 2a. In this example, the particles A and B constitute the initial state α , C and D the final state β , and the S-matrix element (III.1), squared, gives the transition probability between the two states.

More complicated processes are illustrated by the diagrams shown in Figs. 2b-d. It must be emphasized that these S-matrix diagrams are very different from the Feynman diagrams of quantum field theory. They do not picture the detailed mechanism of the reaction, but merely specify the initial and final particles in terms of their quantum numbers. Note also, that the experiments that determine the states α and β describe macroscopic experimental situations. This is an important point which was often emphasized by Bohr. Quantum mechanics represents a description of microscopic phenomena in terms of macroscopic experimental situations. In fact, Bohr's whole approach to atomic physics can be said to have been an S-matrix approach; of course, without the explicit introduction of the S matrix itself.

An important aspect of S-matrix theory is the shift of emphasis from objects to processes. Its basic concern is not with the particles, but with their reactions. Such a shift from objects to processes is required both by quantum mechanics and by relativity

theory. Quantum mechanics has made it clear that a subatomic particle can only be understood as a manifestation of the interaction between various processes of observation and measurement. It is not an isolated object but rather an occurrence, or event, which interconnects other events in a particular way.

Relativity theory has further influenced this picture by forcing us to conceive of particles in terms of space-time; as four-dimensional patterns, processes rather than objects. The S-matrix approach combines both of these viewpoints. Using the four-dimensional relativistic framework, it describes all properties of hadrons in terms of reactions - or, more precisely, in terms of reaction probabilities - and thus establishes an intimate link between particles and processes. Each reaction involves particles which link it to other reactions and thus build up a whole network of processes.

A neutron, for example, may participate in two successive reactions involving different particles; the first, say, a proton and a pion, the second a sigma and a kaon. The neutron thus interconnects these two reactions and integrates them into a larger process, as illustrated in Fig. 3. Each of the other particles will, in turn, be involved in other reactions, so that the original neutron is seen to be part of a whole network of interactions, e.g. as shown in Fig. 4, all described by the S matrix.

The interconnections in such a network cannot be determined with certainty but are associated with probabilities. Each reaction occurs with some probability which depends on the available energy and on the characteristics of the reaction, and these probabilities are given by the various elements of the S matrix.

This approach allows one to define the concept of "constituents" of a hadron in a thoroughly dynamic way. The neutron pictured in Fig. 3, for example, can be seen as a bound state of the proton and the pion from which it arises, and also as a bound state of the sigma and kaon into which it decays. Either of these hadron combinations, and many others, may form a neutron, and consequently can be said to be the neutron's "constituents".

A hadron, therefore, does not consist of a definite arrangement of constituent parts but shows tendencies to undergo various reactions, and these tendencies define the hadron's "constituents". This notion of constituents is in perfect agreement with the experimental facts. Whenever hadrons are broken up in high-energy collisions, they disintegrate into combinations of other hadrons; thus they can be said to "consist" potentially of all these hadron combinations. Each of the particles emerging from such a collision will, in turn, undergo various reactions, thus building up a whole network of events.

Although it is a matter of chance which network will arise in a particular experiment, each network is nevertheless structured according to definite rules. These are the conservation laws mentioned above. Only those reactions can occur in which a well-defined set of quantum numbers is conserved.

In S-matrix theory, the question of the structure of hadrons is reformulated in terms of the structure of hadron reactions. Hadrons "consist of" other hadrons or, rather, involve other hadrons in a dynamic way, and this dynamic way shows a certain structure.

The main challenge of S-matrix theory is to use its thoroughly dynamic description of hadrons to account for the symmetries and regularities observed in hadronic phenomena. In such a theory, these regularities would be reflected in the mathematical structure of the S matrix in such a way that it would contain only elements which correspond to reactions allowed by the conservation laws. Furthermore, only those reactions would be possible which show a flow of quantum numbers exhibiting the patterns associated with quarks, i.e. the "two-ness" characteristic of mesons and the "three-ness" characteristic of baryons, together with the various flavors that can combine to form the quantum numbers of the observed hadrons. This would be the dynamic equivalent of the statement, made in the static quark model, that hadrons consist of quarks.

How can one construct a mathematical model of the S matrix which reflects all these patterns observed in the hadron world? At present, S-matrix theorists are trying to achieve this ambitious aim by postulating a few very general principles, all of which have been verified empirically, and which restrict the mathematical possibilities of constructing S-matrix elements and thus give the S matrix a definite structure. So far, three of these general principles have been established, all of which are related to our methods of observation and measurement.

Poincaré invariance

The first principle is suggested by relativity theory and by our macroscopic experience of space and time. It says that the reaction probabilities (and thus the S-matrix elements) must be

independent of displacements of the experimental apparatus in space and time, and independent of the state of motion of the observer. This principle is known as Poincaré invariance and reflects the homogeneity of space and time. The independence - or "invariance" - with respect to displacements in space means that we shall get the same results whether we perform an experiment in Los Angeles or in New York. In the formalism of quantum mechanics, this invariance implies the conservation of momentum. The invariance with respect to displacements in time means that particle reactions will occur in the same way on a Monday or on a Wednesday. This can be shown to imply the conservation of energy.

The invariance with respect to the state of motion of the observer - known as Lorentz invariance - is taken into account by expressing each S-matrix element as a function of Lorentz-invariant variables. To illustrate this, let us take a scattering process involving four particles. The kinematic variables in this process are the momenta and energies of these particles, as shown in Fig. 5, where we have adopted the relativistic notation

$$p = (\vec{p}, E/c) ; p^2 = m^2 c^2 \quad (\text{III.2})$$

When conservation of momentum and energy is taken into account, it turns out that only two independent invariants can be formed from the four momenta which are conventionally chosen as follows.

$$s = (p_1 + p_2)^2 \quad (\text{III.3})$$

$$t = (p_3 - p_1)^2 \quad (\text{III.4})$$

There is a third invariant which is useful,

$$u = (p_4 - p_1)^2, \quad (\text{III.5})$$

but it is not independent, since

$$s + t + u = \sum_i m_i^2. \quad (\text{III.6})$$

The variable s is called the invariant mass, squared; t is called the momentum transfer, squared. S-matrix elements, then, are expressed as a function of s and t , and this guarantees that they will be Poincaré invariant.

Unitarity

The second principle is suggested by quantum theory. It asserts that the outcome of a particular reaction can only be predicted in terms of probabilities and, furthermore, that superpositions of probability amplitudes associated with different experimental results correspond themselves to possible experimental results. This represents the essential quantum character of the S-matrix formalism. Finally, the principle states that the sum of the probabilities for all possible outcomes of a particle reaction - including the case of no interaction between the particles - must be equal to one. In other words, we can be certain that the particles will either interact with one another, or not. This seemingly trivial statement turns out to be, in fact, a very powerful principle. It is expressed, mathematically, by the requirement that the S matrix be unitary,

$$S^\dagger S = 1 \quad (\text{III.7})$$

The whole principle is therefore known under the name of unitarity.

The unitarity equation is usually written in a different form. Since the case where no particle interactions take place is

mathematically trivial, it is separated from the remainder of the S matrix by writing

$$\langle f|S|i \rangle = \langle f|i \rangle + i \langle f|T|i \rangle \quad (\text{III.8})$$

where the factor i is adopted for mathematical convenience. The unitarity equation can now be expressed in terms of the T matrix which contains the physically interesting reactions. With the notation

$$T_{fi} = \langle f|T|i \rangle, \quad (\text{III.9})$$

it reads

$$\text{Im } T_{fi}(s,t) = \frac{1}{2} \sum_n T_{fn}^\dagger(s,t) T_{ni}(s,t) \quad (\text{III.10})$$

where we have exhibited the dependence of the matrix elements on the kinematic variables s and t discussed before. We see that the unitarity requirement imposes a nonlinear condition on the probability amplitude. This condition severely restricts the possibilities of constructing S-matrix elements and forms the corner-stone of S-matrix theory.

Analyticity

The third and final principle is related to our notions of cause and effect and is known as the principle of macro-causality. It states that energy and momentum are transferred over macroscopic spatial distances only by particles, and that this transfer occurs in such a way that a particle can be created in one reaction and destroyed in another only if the latter reaction occurs after the former; furthermore, that the transfer of energy cannot occur with velocities exceeding the speed of light.

The mathematical formulation of the causality principle implies that the S matrix is an analytic function of the kinematic variables involved - in our example of the variables s and t - with certain isolated singularities. These singularities correspond to all the possible ways in which energy and momentum (plus other information) can be transferred over macroscopic distances. For example, for a reaction involving six particles, the singularities of the corresponding amplitude would include those represented by the diagrams shown in Figs. 6a-d. One special type of singularity that is of particular interest is the type shown in Fig. 6a in which a single particle transmits information over the macroscopic distance. The singularity represented by this diagram is a single pole. The position of the pole - i.e. the value of the energy variable at which the pole occurs - corresponds to the mass, squared, of the particle.

An important aspect of this formalism is the fact that it can accommodate unstable particles, i.e. the hadrons known as resonances. Unstable particles appear as poles at complex values of the energy variable. For example, in the case of our amplitude $T(s,t)$, the variable s is continued to complex values which can be represented in the complex plane (see Fig. 7). Only the values of s along the positive real axis are physical values, since an energy squared is always a positive real number, but the continuation of the energy variables to unphysical complex values is a very powerful mathematical tool.

A pole in this complex plane, corresponding to an unstable particle, may have, for example, the position shown in Fig. 8. The real part of the pole position gives the mass, squared, of the

particle; the imaginary part (Γ) is related to the inverse of the particle's lifetime. If we plot the probability for the reaction to occur as a function of the energy variable s , we shall find a peak in the probability as we pass the value of the pole. The resulting curve (see Fig.9) is a typical resonance curve, and this is why unstable hadrons are called resonances. In fact, the word "resonance" is a very appropriate term. It refers to a phenomenon characteristic of hadron reactions. The probability amplitude resonates when energy is transferred in a certain way. The resonance is therefore an event, an occurrence, rather than an object. This shows very clearly the close connection between particles and processes in S-matrix theory.

The width of the resonance curve (Γ), which is related to the inverse lifetime of the particle, corresponds to the displacement of the pole from the real axis. The closer the pole to the real axis, the sharper the peak will be; the longer the lifetime of the particle and the more important its contribution to the reaction.

Crossing

These, then, are the three principles which imply that the S matrix must exhibit the properties of Poincaré invariance, unitarity, and analyticity. Poincaré invariance, together with analyticity, implies another property of the S matrix which plays a crucial role in the theory; the property known as crossing.

To illustrate crossing, let us consider an S-matrix element represented by the diagram shown in Fig. 10a. The fact that the probability amplitude corresponding to this diagram is an analytic function of the particles' momenta (including their energies) means that it may be analytically continued to regions where some of these

momenta become negative. For example, we may continue the function to a region where the momentum of particle C is negative.

The crossing property, now, says that in this region - which is, of course, an unphysical region - there is an alternative interpretation of the amplitude. It can be interpreted as the probability amplitude for the reaction in which particle C , instead of being an ingoing particle, is an outgoing antiparticle. This can be illustrated by swinging the line representing C over to the other side and denoting the antiparticle by \bar{C} (see Fig. 10b). The crossing property thus interrelates two different processes: the amplitude for the process $A + B \rightarrow D + E + F + \bar{C}$ is the analytic continuation of the amplitude for the process $A + B + C \rightarrow D + E + F$ and vice versa. This can be carried on with the other particles; we can swing around the particle lines any way we like, changing ingoing particles into outgoing antiparticles, and vice versa, by analytic continuation of the amplitude from positive to negative momenta. The resulting processes, therefore, are not really independent but are merely different aspects, or "channels", of one and the same reaction. They are described by the same S-matrix element in different regions of the kinematic variables.

The concept of crossing is crucial for the picture of interparticle forces in S-matrix theory. As an illustration, let us consider the amplitude for the reaction $p\pi^+ \rightarrow p\pi^+$. An important contribution to this amplitude will come from the singularity pictured in Fig. 11a. In this diagram, the π^0 can be seen as the mediator of the interaction force between the proton and the π^+ . The two hadrons can be said to interact through the exchange of a

π^0 . On the other hand, the π^0 can also be seen as a bound state of a proton and antiproton in the "cross channel" which describes the reaction $p\bar{p} \rightarrow \pi^+\pi^-$, as shown in Fig. 11b. Thus, the particle that acts as the agent of a force in one channel, is a bound state in another channel.

In our example, the $p\pi^-$ system, too, can form a bound state. This will happen when the channel energy reaches the mass of the neutron, as illustrated in Fig. 11c. This neutron, then, can be seen as a composite of the proton and the π^- , and the binding force holding the composite together is associated, again, with the pion-pole in the cross channel. In terms of the kinematic variables introduced before, the neutron pole will be a pole in s , the pion pole a pole in t (see Fig. 12); the variable playing the role of momentum transfer in one channel plays the role of the channel energy in the cross channel and vice versa.

This picture of particle interactions leads naturally to the notion that all hadrons have a more or less equivalent basis; a notion that Chew has called "nuclear democracy". We cannot say that any hadron is a constituent of any other in an absolute sense. Each hadron can play three roles: it is a composite, it may be the constituent of another hadron, and it may be exchanged between constituents and thus contribute to the binding forces holding hadrons together.

Each hadron, then, is held together by forces associated with other hadrons that appear as bound states in the cross channel, each of which is, in turn, held together by forces to which the first hadron makes a contribution. In this way, as Chew has put it, "each particle helps to generate other particles which, in turn, generate it."

Regge formalism

A particular advantage of the S-matrix framework is the fact that it is able to describe the exchange of entire groups, or "families", of hadrons in a natural way by associating them with a special type of poles. These groups are not the hadron multiplets mentioned in Section II, but another type of hadron families which was discovered in the nineteen-sixties.

All hadrons seem to fall into sequences whose members have identical properties, except for their masses and spins. The masses and spins increase with striking regularity within each sequence and the sequences seem to extend indefinitely. When the mass, squared, of each hadron is plotted against its spin, one obtains approximately linear patterns. For mesons the patterns look as shown in Fig. 13; baryons show similar patterns with spin sequences $J = 1/2, 3/2, 5/2,$ etc.

The way in which these patterns are incorporated into the S-matrix framework is based on a formalism proposed originally by Tullio Regge and the hadron sequences are therefore known as Regge trajectories. Over the past years, physicists have increasingly come to see the higher members of a Regge trajectory not as different particles, but merely as excited states of the member with the lowest mass, in analogy to the excited states in atomic physics. Like an atom, a hadron can exist in various short-lived excited states involving higher amounts of angular momentum (or spin) and energy (or mass). The incorporation of the Regge formalism into the S-matrix framework has thus been a major advance towards a unified description of hadron phenomena.

The formalism was developed by Regge for non-relativistic potential scattering and was then generalized to relativistic scattering and incorporated into the S-matrix framework.⁶ The starting point is the partial-wave expansion of the scattering amplitude in potential theory,

$$f(E, \cos \theta) = \sum_{\ell=0}^{\infty} (2\ell + 1) f_{\ell}(E) P_{\ell}(\cos \theta). \quad (\text{III.11})$$

By continuing the angular momentum ℓ to complex values, the partial-wave expansion can be written as an integral in the complex ℓ plane. Furthermore, it was shown by Regge that in the limit $\cos \theta \rightarrow \infty$ - which is, of course, unphysical - the amplitude can be written as a sum of poles in ℓ , the position of which depends analytically on E . These poles are called Regge poles, and the scattering amplitude reads

$$f(E, \cos \theta) \sim \sum_n \frac{\beta_n(E)}{\sin \pi \alpha_n(E)} P_{\alpha_n(E)}(\cos \theta) \quad (\text{III.12})$$

where the conventional change of notation $\ell \rightarrow \alpha(E)$ has been adopted.

All this remains highly unphysical in potential scattering but it acquires important physical relevance when it is generalized to relativistic scattering amplitudes. The partial-wave expansion for the amplitude $T(s, t)$ in the t channel reads

$$T(s, t) = \sum_{\ell=0}^{\infty} (2\ell + 1) T_{\ell}(t) P_{\ell}(\cos \theta) \quad (\text{III.13})$$

The essential new ingredient in the relativistic case is the crossing

property of the S-matrix. Whereas in potential scattering, the region in which Regge poles dominate the amplitude, namely $\cos \theta \rightarrow \infty$, is highly unphysical (the physical region running between +1 and -1), in relativistic scattering the scattering angle in one channel, for large values, becomes proportional to the energy variable in the cross channel,

$$\begin{aligned} \cos \theta_s &\sim t \\ \cos \theta_t &\sim s \end{aligned} \quad (\text{III.14})$$

The region $\cos \theta \rightarrow \infty$ corresponds therefore to high-energy scattering in the cross channel, and the application of Regge's technique makes it possible to write the amplitude in that region as a sum of Regge poles,

$$T(s,t) \underset{s \rightarrow \infty}{\sim} \sum_n \frac{\beta_n(t)}{\sin \pi \alpha_n(t)} s^{\alpha_n(t)}. \quad (\text{III.15})$$

In the expression (III.15), the functions $\alpha_n(t)$ represent the Regge trajectories. For each integer value of $\alpha(t_j) = j$, the amplitude will have a pole in t corresponding to the exchange of a particle with spin j and mass $m^2 = t_j$. The exchange of this whole family of particles is expressed by saying that a "reggeon", or Regge trajectory, $\alpha(t)$, is exchanged. This reggeon exchange determines the asymptotic behavior of the amplitude in the s channel (i.e. its behavior for $s \rightarrow \infty$) through the factor s^α .

A further consequence of crossing is the fact that Regge poles will appear not only in the s-channel amplitude but also in the u-channel amplitude. This so-called "exchange amplitude" has to be added to the s-channel amplitude, which has the effect of cancelling half of the particles on a Regge trajectory. Meson trajectories, for example, carry particles either at even values of spin ($j = 0, 2, 4, \text{etc.}$), or at odd values ($j = 1, 3, 5, \text{etc.}$); baryon trajectories carry particles either at $j = 1/2, 5/2, 9/2, \text{etc.}$, or at $J = 3/2, 7/2, 11/2, \text{etc.}$ These two types of trajectories are called trajectories of even and odd signature. Signature, then, is a new quantum number characteristic of reggeons.

In the early stages of Regge theory, Chew and others postulated that all poles of the S matrix are Regge poles or, in other words, that all particles lie on Regge trajectories. This hypothesis, as well as the resulting Regge behavior of scattering amplitudes for high energies ($s \rightarrow \infty$), was first met with a great deal of doubt, but has found striking experimental confirmation over the past decade. The incorporation of Regge theory into the S-matrix framework now represents a major advance in our understanding of hadron physics.

Bootstrap

The three basic properties of the S matrix - Poincaré invariance, analyticity and unitarity - are the mathematical consequences of general principles which are closely related to our methods of observation and measurement. All three of these principles are essential for the scientific approach to reality. Without them,

science as we know it would not be possible. Nevertheless, it has not been possible, so far, to construct a mathematical model of the S matrix that satisfies all three principles, i.e. an S matrix which is Poincaré invariant, analytic and unitary. The idea has therefore arisen that these three principles may be sufficient to determine the structure of the S matrix, and thus all the properties of hadrons, uniquely. This idea is known as the bootstrap hypothesis. Its originator and main advocate is Geoffrey Chew who has made the bootstrap approach the philosophical foundation of S-matrix theory and has also developed it into a more general "bootstrap" philosophy of nature.⁷

According to the bootstrap philosophy, nature cannot be reduced to fundamental entities, like fundamental building blocks of matter, but has to be understood entirely through self-consistency. All of physics has to follow uniquely from the requirement that its components be consistent with one another and with themselves.

This idea constitutes a radical departure from the traditional spirit of basic research in physics which had always been bent on finding the fundamental constituents of matter. At the same time, it is the culmination of the conception of the material world as an interconnected web of relations which has emerged from quantum theory. The bootstrap philosophy abandons not only the idea of fundamental building blocks of matter, but accepts no fundamental entities whatsoever - no fundamental laws, equations or principles. The universe is seen as a dynamic web of interrelated events.

None of the properties of any part of this web is fundamental; they all follow from the properties of the other parts, and the overall consistency of their mutual interrelations determines the structure of the entire web.

In the framework of S-matrix theory, the bootstrap approach attempts to derive all properties of hadrons and their interactions uniquely from the requirement of self-consistency. The only "fundamental laws" accepted are the three S-matrix principles which are required by our methods of observation and are thus essential parts of our scientific framework. Other properties of the S matrix may have to be postulated temporarily as "fundamental principles", but will be expected to emerge, eventually, as a necessary consequence of self-consistency.

If the bootstrap hypothesis is correct, its philosophical implications would be very profound. The fact that all the properties of hadrons are determined by principles closely related to our methods of observation would mean that the basic structures of the material world are determined, ultimately, by the way in which we look at this world. In other words, the observed patterns of matter are nothing but reflections of patterns of mind.

IV. THE ORDERED S MATRIX

How can we derive the structure of the S matrix, and in particular the quark structure of hadron interactions, from the general principles? The starting point is the unitarity equation (III.10) which can be represented graphically as shown in Fig. 14, where we have taken a $2 \rightarrow 2$ amplitude as an example. The unitarity equation imposes a very restrictive nonlinear condition on this amplitude, relating its imaginary part to an infinite sum over products of $2 \rightarrow n$ amplitudes. The aim of S-matrix theory is to construct mathematical models of the S matrix which satisfy this condition. Only the full physical S matrix will satisfy unitarity completely, but there may be approximations to the physical S matrix which satisfy the unitarity relations to some extent and bear sufficient resemblance to the observed hadron phenomena. One model of that kind, which has emerged from the developments of the past five years, is extremely promising, as it exhibits the desired quark structure.

Before discussing this model, let us remember the meaning of "quark structure" in the S-matrix framework. It means that hadron reactions proceed in such a way that the transfer, or flow, of the conserved quantum numbers exhibits certain patterns which may be called quark patterns; a certain "two-ness" associated with mesons and a certain "three-ness" associated with baryons. For example, in the reaction $p\pi^- \rightarrow p\pi^-$, the flow of quantum numbers can be exhibited as shown in Fig. 15. The "quark lines" in this diagram carry definite flavors and generate the $q\bar{q}$ structure of the mesons and the qqq structure of the baryons.

Quark-line diagrams can be used to exhibit the s-channel and t-channel poles simply by distorting the diagram, e.g. as shown in Fig. 16. The fact, known as duality, that s-channel and t-channel poles are equivalent representations of the amplitude was the starting point for the recent developments in S-matrix theory. The study of various "dual models"⁸ led to the model we shall now describe. The new model is quite distinct from the dual models, however, in that it recognizes unitarity from the beginning as an essential ingredient.

The key element of the new approach is the notion of order as a new aspect of the S matrix. In the context of S-matrix theory, order means a definite structure of interconnectedness between particles, to be defined more precisely below. When this concept of order is incorporated into the S-matrix framework, only a few special categories of ordered relationships turn out to be consistent with the well-known properties of the S matrix. These categories of order can be identified with the quark patterns observed in hadronic phenomena. Thus, the quark structure emerges as a manifestation of order without any need to postulate quarks as physical constituents of hadrons.

What, then, is the definition of order in S-matrix theory? In a general diagram picturing a particle reaction, the ingoing and outgoing particles are not placed in any particular order. For example, the three diagrams pictured in Fig. 17 are all equivalent. Order can be introduced in the simplest way by placing particles in a linear sequence, associating each particle with a definite predecessor and a definite successor, as shown in Fig. 18a. In such an ordered

diagram, each particle is "connected" to two other particles. Order, then, represents a definite connectedness structure. In the physical S matrix, this structure is not observed, but it turns out that the ordered S matrix represents an important approximation to the physical S matrix. We shall now describe the properties of the ordered S matrix⁹ and shall then, in Section V, discuss the experimental evidence justifying the assumption that it is a good approximation to the physical S matrix.

Sequential order is not the only possible connectedness structure. A general representation of particle order may be given in terms of connected graphs whose vertices are associated with particles and are interconnected through a number of edges. In this notation, the diagram of Fig. 18a turns into the "ring diagram" shown in Fig. 18b, with particles represented by 2-vertices. More complicated types of order are represented by graphs in which vertices are interconnected by more than two edges, e.g. as shown in Fig. 19a. The vertices representing the particles are ordered in the sense that each has a unique set of neighbors and occupies a unique position in the graph. Note, however, that these graphs merely describe the connectedness structure and are not to be understood as projected on a plane. The graph shown in Fig. 19a for example, could also be drawn as shown in Fig. 19b. Each hadron reaction, of course, is part of a larger network of reactions, so that the connectedness structure represented by a particular order refers to the connectedness of the whole network.

In order to see how the properties of the S matrix restrict the categories of particle order, it will be useful to distinguish between amplitude graphs, picturing the whole amplitude, and channel graphs containing only the ingoing or outgoing particles. For example, the amplitude graph shown in Fig. 19a can be divided into the two channel graphs shown in Fig. 20. The S matrix is defined as an operator connecting ingoing and outgoing channels in such a way that the corresponding probability amplitude is uniquely defined by specifying the two channels. Therefore, the two channel graphs uniquely define the amplitude graph. In other words, there must be a unique prescription for sewing the two channel graphs together to form the amplitude graph. Conversely, all channel graphs must be obtainable by cutting the amplitude graph into two connected pieces.

When these cutting and resewing conditions are combined with the other properties of the S matrix, it is found that the only allowed ordered amplitudes are the ones represented by graphs that can be obtained from a ring by successively adding "bubbles" on any of the edges. Examples of such graphs, exhibiting "bubbles" and "bubbles within bubbles", are shown in Fig. 21. Furthermore, 2-vertices may be placed on any of the edges. These graphs are called fully reducible because they can be reduced to a ring by successively eliminating all 2-vertices and all bubbles.

In order to be able to resew all possible channel graphs of an amplitude graph in a unique way, their "dangling" edges have to be distinguished. For example, the structure of the channel graphs shown in Fig. 22 (obtained from the amplitude graph shown in Fig. 21c) allows us to distinguish the edge *c* from the others, but

the edges a , b , d , and e cannot be distinguished from one another unless we label, or "color", them. (Remember that these graphs merely describe connections and are not to be understood as projected on a plane). In order to satisfy the resewing condition, then, we need graphs with colored edges. The term "color" is used here in the way it is used by mathematicians in graph theory, e.g. in the four-color theorem, but will turn out to be related to the color concept of quantum chromodynamics.

When one studies the most economic way of coloring amplitude graphs, it turns out that all edges of any graph can be completely distinguished in the following way. Graphs are colored with three colors in such a manner that the three edges meeting at any 3-vertex show different colors, whereas the two edges meeting at a 2-vertex have the same color. An example of a channel graph, colored in this way is shown in Fig. 23a, where the colors have been denoted by 1, 2, 3. This coloring procedure still leaves a two-fold ambiguity which can be removed by assigning an "orientation" to each vertex, e.g. by labeling it with a (+) or (-) label, as shown in Fig. 23b. All fully reducible graphs colored and labeled in this manner will satisfy the cutting and resewing conditions.

An easily derived property of such graphs is the fact that (ignoring 2-vertices) their edges connect 3-vertices of opposite orientation, e.g. as shown in Fig. 24a. In other words, the orientation of neighboring 3-vertices alternates. This fact allows us to introduce a different notation for the two kinds of vertices. Instead of labeling them by (\pm), we may assign a direction to their edges, as shown in Fig. 25. The (\pm) labels and directed edges are completely equivalent notations. No new information is added by

putting arrows on the edges. In the new notation, the graph of Fig. 24a takes the form shown in Fig. 24b.

This, then, is the final result: the most general type of order consistent with the properties of the S matrix is the one represented by fully reducible graphs, colored with three colors and containing 2-vertices and 3-vertices connected by directed edges in the way described above. In these graphs, the directed edges can now be identified with quark lines, and the vertices associated with hadrons in the way shown in Fig. 26. Reversing the direction of the arrows in a graph corresponds to interchanging particles and antiparticles. Contact with the conventional quark-line notation can be made by "opening up" the 2-vertices and 3-vertices in a graph, e.g. as shown in Fig. 27.

The quark structure, then, and in particular the 3-color structure of hadrons, is a property of the ordered S matrix. It emerges as a consequence of combining the S -matrix principles with the notion of order. The role of flavor in the ordered S matrix is not yet understood, but work on this problem is in progress.¹⁰

V. THE PLANAR S MATRIX AND THE TOPOLOGICAL EXPANSION

Ordered S -matrix elements cannot be compared to experiment, since physical hadron reactions do not exhibit particle order. However, by summing over all possible ordered amplitudes for a given reaction, one can construct an S -matrix element which does not depend on particle order and may be compared to experiment. S -matrix elements constructed in this way make up the so-called planar S matrix. The meaning of the term "planar" will be clarified below.

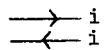
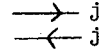
Although the elements of the planar S matrix are independent of particle order, they exhibit several striking regularities because of their linear connection with the elements of the ordered S matrix. These regularities have been observed to hold approximately in nature and have been the main motivation for introducing the concept of order into S-matrix theory. They indicate that the planar S matrix provides a reasonably accurate description of hadronic phenomena.

To go beyond the planar approximation, an expansion of the physical S matrix has been developed in which the planar S matrix constitutes the leading term. Successive levels of the expansion are characterized by increasing degrees of disorder. The language of topology is used to distinguish order from disorder, and the expansion is therefore known as the topological expansion. This approach has been carried out successfully for reactions involving mesons and is presently being extended to include baryons. In the following, we shall limit the discussion to phenomena involving mesons only.

As an example, let us study the process $A + B \rightarrow C + D$. The planar amplitude for this reaction will consist of the terms shown in Fig. 28, where the diagram representing the planar amplitude, which does not exhibit any definite ordering of the particles $A - B$ and $C - D$, has been given the label P , whereas the diagrams representing the ordered amplitudes have been given labels R to indicate that they are ring diagrams exhibiting definite particle order.

The quark-line diagrams corresponding to the ring diagrams of Fig. 28 are of the form shown in Fig. 29. They are planar diagrams, i.e. they can be drawn on a plane without any quark lines crossing.

All elements of the planar S matrix can be represented by such planar diagrams.

Among the approximate regularities observed in hadronic phenomena, a selection rule known as the Okubo-Zweig-Iizuka (OZI) rule represents a particularly striking manifestation of order. The rule states that meson states represented by quark lines  do not communicate with states represented by quark lines . This leads to the suppression of certain reactions which are not forbidden by any conservation law, such as the reaction $\phi \rightarrow \rho\pi$. The quark-line structure of the mesons participating in this reaction is shown in Fig. 30. One sees immediately that no connected planar diagram can be drawn to represent the amplitude for this process. The reaction is therefore forbidden at the planar level. At higher levels of the topological expansion, however, the reaction will occur, as we shall show below.

Another striking regularity which is exact at the planar level and is broken at levels of increasing disorder is the fact that meson Regge trajectories of opposite signature coincide to form degenerate pairs - a property known as exchange degeneracy. Examples of such pairs are the $\rho - A_2$ trajectories and the $\omega - f$ trajectories whose approximate degeneracy is a well-known experimental fact. It can readily be shown that the existence of exchange-degenerate Regge trajectories is a requirement of the planar S matrix.

The planar S matrix exhibits most of the properties to be satisfied by the physical S matrix, e.g. Poincaré invariance, analyticity and crossing. However, it is not unitary. This can

easily be seen by imposing the unitarity condition, as represented in Fig. 14, on the planar amplitude shown in Fig. 28. On the right-hand side of the unitarity equation, the products of $2 \rightarrow n$ amplitudes will include sixteen terms exhibiting two-particle intermediate states which can be represented as shown in Fig. 31, where the connections $a - a$ and $b - b$ have to be made to produce the appropriate intermediate states. It is evident that these products will involve terms of the two kinds shown in Fig. 32. The corresponding quark-line diagrams (with arrows suppressed) are shown in Fig. 33.

Diagrams of the type shown in Fig. 33b are clearly nonplanar and represent the first degree of disorder introduced by unitarity. They are inevitably generated from ordered amplitudes by the formation of "nonplanar" products in the unitarity equation. Their existence implies that unitarity cannot be imposed on the planar amplitude. Products involving more particles in the intermediate states will include terms of increasing disorder. For example, among the diagrams involving three-particle intermediate states, there will be terms of the form shown in Fig. 34.

Topology can be used to classify these diagrams, according to their degree of order, by noting that nonplanar diagrams can be embedded in two-dimensional closed surfaces of increasing "genus" or number of "handles". (A sphere has zero handles; a torus can be pictured as a sphere with one handle; a double torus as a sphere with two handles, etc.). For example, the diagrams shown in Figs. 33b and 34 can be embedded in a sphere and a torus, respectively, as shown in Fig. 35.

The topological expansion, then, is an expansion of the unitarity equation into a sum of diagrams of increasing disorder, with the diagrams making up the planar S matrix as the leading term. It can be shown that the nonplanar products are smaller than the planar products and that their values decrease as the diagrams become increasingly complex, so that the expansion converges. Unitarity is restored by these successive steps, and the disorder introduced in this way is measured in terms of topological parameters.

The first correction to the planar S matrix is known as the cylinder correction, since the corresponding diagrams can be embedded in a cylinder (which is, of course, topologically equivalent to a sphere). It has the effect of shifting the positions of the Regge poles, so that the exchange degeneracy of Regge trajectories is lifted. For example, the $\omega - f$ trajectories look as shown in Fig. 36. This breaking of exchange degeneracy has been worked out quantitatively without the need of introducing any arbitrary parameters.

The cylinder correction also breaks the OZI rule. The reaction $\Phi \rightarrow \rho\pi$ for example, can be represented by the diagram shown in Fig. 37a. Again, the topological expansion gives a quantitative account of the extent to which the OZI rule is broken. The accuracy of the rule is directly related to the accuracy of exchange degeneracy. It is also directly related to the $\Phi - \omega$ mixing in $SU(3)$ symmetry, as can be seen by redrawing the diagram of Fig. 37a in the way shown in Fig. 37b. The situation where the Φ is a pure $s\bar{s}$ state is known as ideal mixing. It corresponds to the planar level where the OZI rule and exchange degeneracy hold exactly.

The observed $\phi - \omega$ mixing angle of about 4° shows that the correction introduced by the cylinder diagrams is small. Thus the OZI rule and exchange degeneracy are found to be observed in nature quite accurately.

In a similar way, various other regularities in hadronic phenomena can be understood in the framework of the ordered S matrix and the topological expansion. Note, in particular, that this approach enables one to relate symmetry patterns, such as the quark structure of hadrons, or the $\phi - \omega$ mixing, to the dynamics of the strong interactions, as expressed by the OZI rule, exchange degeneracy, or the resulting asymptotic behavior of scattering amplitudes. All these regularities are seen as different facets of a single principle of order.

VI. THE MEANING OF ORDER

The significance of the notion of order in hadron physics is still mysterious, and the extent to which it can be incorporated into the S-matrix framework is not yet fully known. Order and unitarity appear to be opposed principles. Disorder is an inevitable consequence of unitarity which prevents the physical S matrix from being ordered. It is therefore unlikely that the concept of order will play the role of a new S-matrix principle, similar in status to Poincaré invariance, analyticity and unitarity.

However, it is intriguing to note that, like those three principles, the notion of order plays a very basic role in the scientific approach to reality and is a crucial aspect of our methods of observation. The ability to recognize order seems to be an essential property of the rational mind; every perception of a

pattern is, in a sense, a perception of order. The clarification of the concept of order in a field of research where patterns of matter and patterns of mind are increasingly being recognized as reflections of one another promises thus to open fascinating new frontiers of knowledge.

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REFERENCES

1. G. F. Chew and C. Rosenzweig, Lawrence Berkeley Laboratory Report LBL-6783, to be published in Physics Reports.
2. F. Capra, The Tao of Physics (Shambhala, Berkeley, 1975; Bantam, New York, 1977).
3. See, for example, D. B. Lichtenberg, Unitary Symmetry and Elementary Particles (Academic Press, New York, 1970).
4. For a recent, more detailed, review of the quark model, see S. Glashow, *Scient. American* 233, 4, 38 (Oct. 1975).
5. See G. F. Chew, The Analytic S Matrix (Benjamin, New York, 1966).
6. For an introduction to Regge theory, see P. D. B. Collins, An Introduction to Regge Theory and High Energy Physics (Cambridge U. P., New York, 1977).
7. G. F. Chew, *Science* 161, 762 (1968); *Phys. Today* 23, 23 (Oct. 1970).
8. See M. Jacob (Ed.), Dual Theory (Physics Reports Reprint Book Series, North-Holland, 1974).
9. G. F. Chew, J. Finkelstein, J-P. Sursock and G. Weissmann, Lawrence Berkeley Laboratory Report LBL-7237; to be published in Nuclear Physics B. Our description, throughout the remainder of this section, will closely follow this paper.
10. There are indications that flavor may be a necessary consequence of topological color. If this turns out to be true, it would open up the exciting possibility of deriving the symmetry patterns characteristic of the hadron spectrum - SU(2) symmetry and broken SU(3) symmetry - from the basic properties of the ordered S matrix; G. F. Chew, private communication.

FIGURE CAPTIONS

- Fig. 1. Standing-wave pattern.
- Fig. 2. Diagrams picturing various scattering processes.
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- Fig. 4. Network of interconnected hadron reactions.
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- Fig.16 Quark-line diagrams for the reaction shown in Fig.15, exhibiting (a) the s-channel pole, (b) the t-channel pole.

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- Fig.20. Channel graphs obtained from amplitude graph shown in Fig.19a.
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- Fig.23. Example of a channel graph with (a) colored edges, (b) colored edges and oriented vertices.
- Fig.24. Example of an amplitude graph with edges connecting vertices of opposite orientation, vertices being distinguished (a) by (\pm) labels, (b) by directed edges.
- Fig.25. Relation between (\pm) labels and directed edges.
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- Fig.27. Relation between graph notation and quark-line notation.
- Fig.28. Diagram representing the planar amplitude for the reaction $A + B \rightarrow C + D$, pictured as the sum of diagrams representing ordered amplitudes.
- Fig.29. Planar quark-line diagram.
- Fig.30. Quark-line structure of mesons participating in the reaction $\Phi \rightarrow \rho\pi$.
- Fig.31. Unitarity products involving two-particle intermediate states, generated by imposing unitarity on the planar amplitude for the reaction $A + B \rightarrow C + D$.

- Fig.32. Planar and non-planar contributions to the unitarity products shown in Fig.31.
- Fig.33. Diagrams of Fig.32 in quark-line notation.
- Fig.34. Non-planar diagram showing a three-particle intermediate state.
- Fig.35. Embedding of diagrams shown in Figs.33b and 34 in a sphere and a torus, respectively.
- Fig.36. Breaking of exchange degeneracy for $\omega - f$ trajectories.
- Fig.37. Cylinder diagrams representing (a) breaking of OZI rule for the reaction $\Phi \rightarrow \rho\pi$, (b) relation between OZI rule and $\Phi - \omega$ mixing.

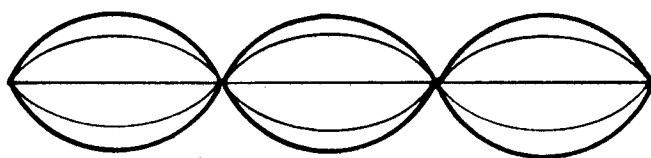


Fig. 1 XBL786-1031

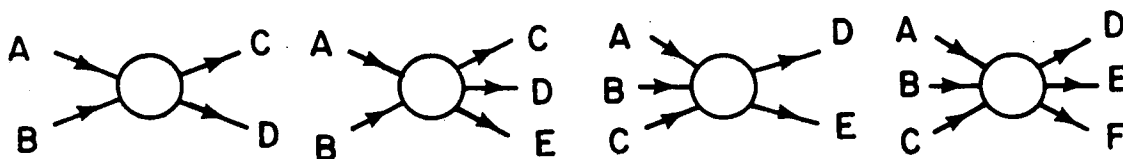
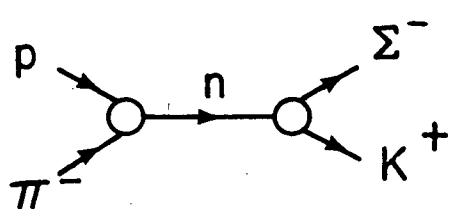


Fig. 2

XBL786-1032



XBL786-1034

Fig. 3

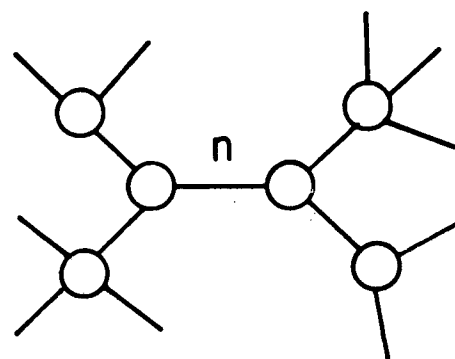


Fig. 4

XBL786-1033

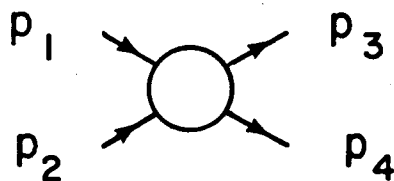


Fig. 5 XBL786-1018

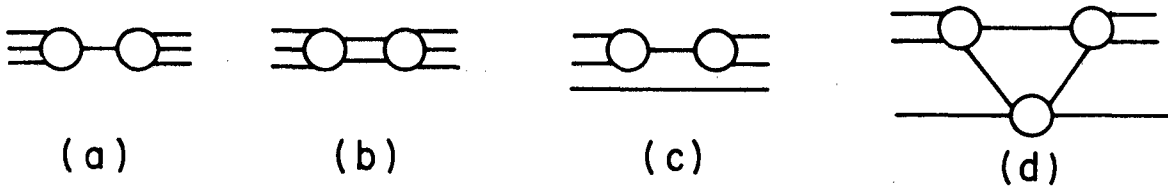


Fig. 6

XBL786-999

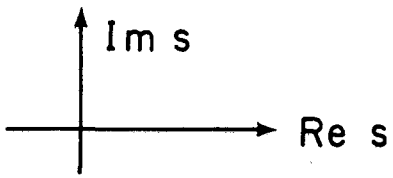


Fig. 7

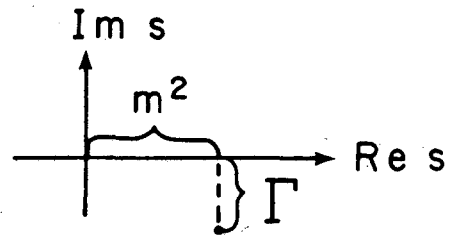


Fig. 8

XBL786-1011

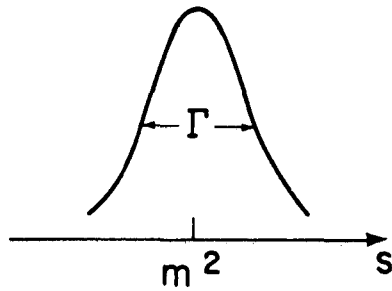
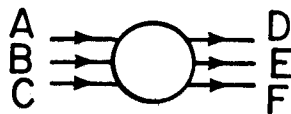
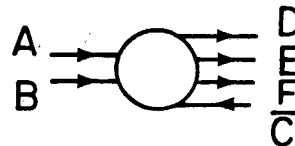


Fig. 9

XBL786-1012



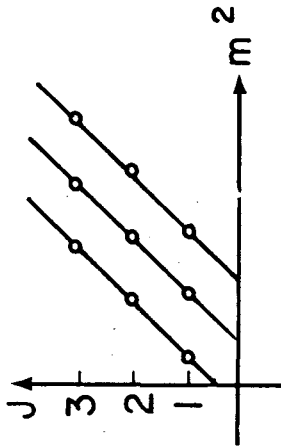
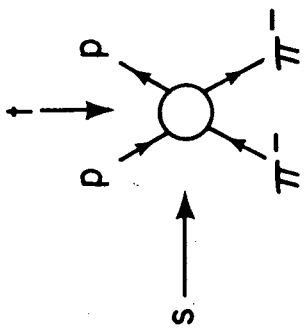
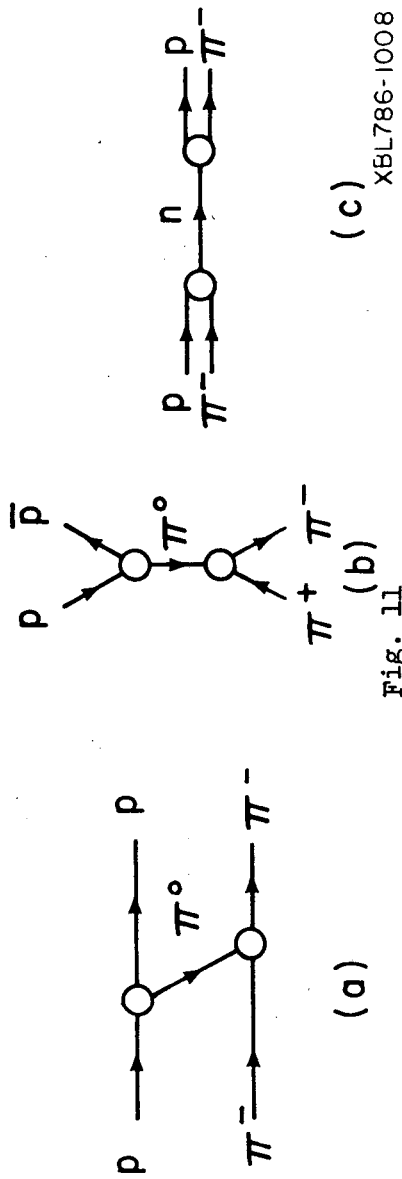
(a)



(b)

Fig. 10

XBL786-1013



$$Im \text{ (diagram)} = \sum_n \text{ (diagram)}$$

Fig. 14 XBL786-1005

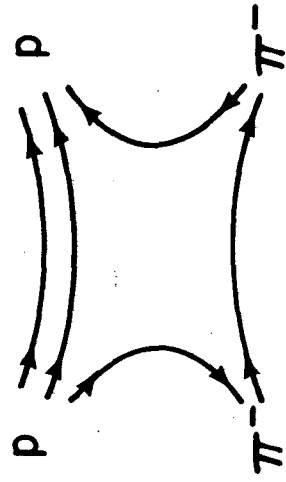
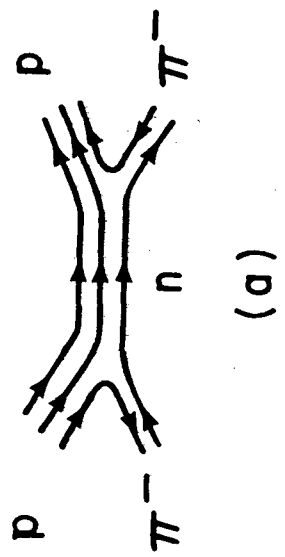
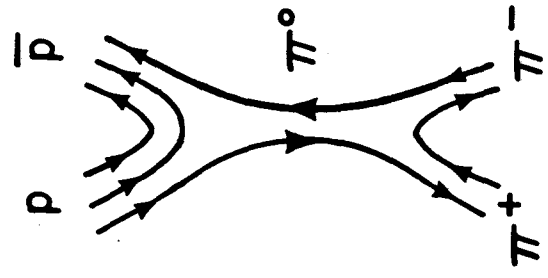


Fig. 15 XBL786-1006



(a)



(b)

Fig. 16 XBL786-1007

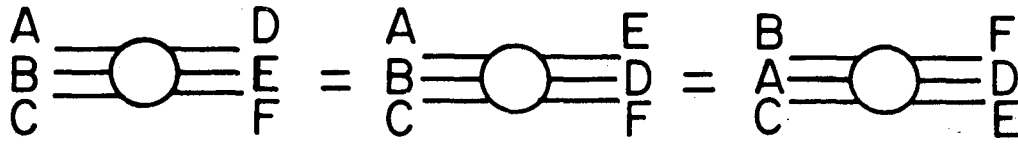
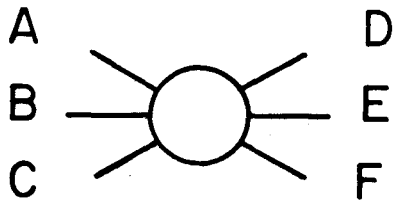
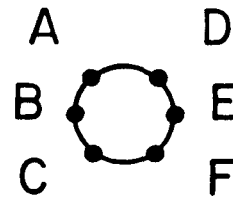


Fig. 17

XBL786-1002



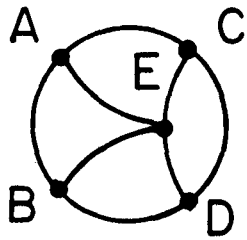
(a)



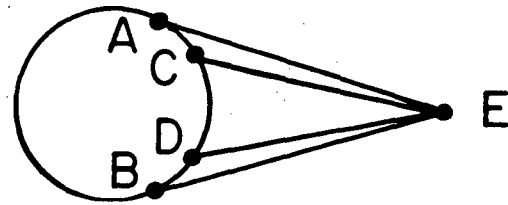
(b)

Fig. 18

XBL786-1003



(a)



(b)

Fig. 19

XBL786-1004

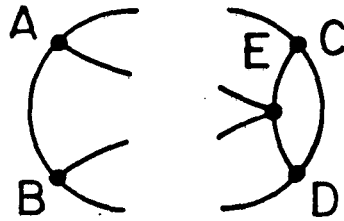
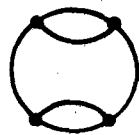


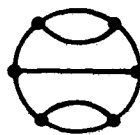
Fig. 20 XBL786 - 1027



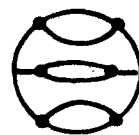
(a)



(b)



(c)



(d)

XBL786 - 1028

Fig. 21

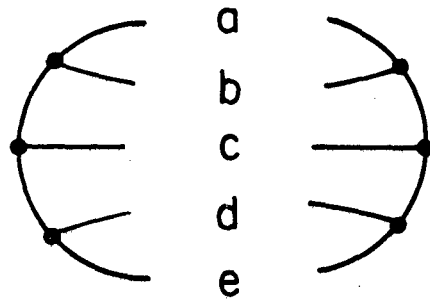
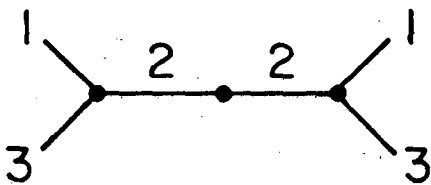
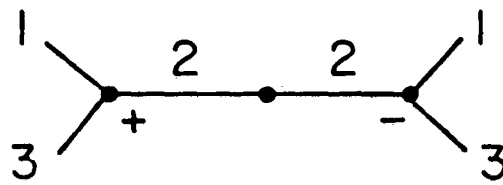


Fig. 22 XBL786 - 1029



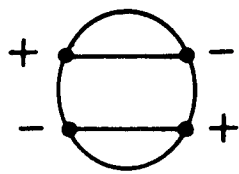
(a)



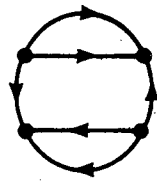
(b)

Fig. 23

XBL786 - 1030



(a)



(b)

Fig. 24 XBL786-1022



(+)



(-)



Baryon Antibaryon Meson

Fig. 25 XBL786-1023

Fig. 26

XBL786-1024

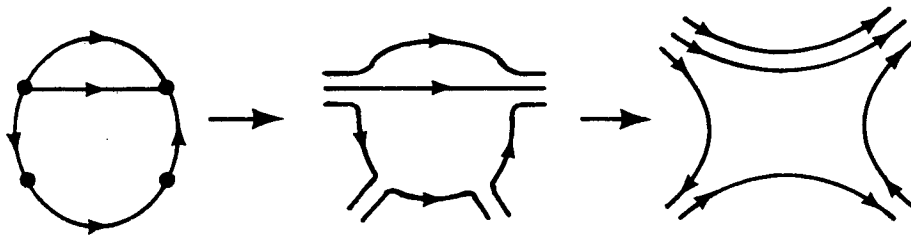


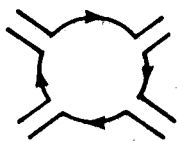
Fig. 27

XBL786-1025

$$\begin{array}{c} A \\ \diagdown \\ \textcircled{P} \\ \diagup \\ B \end{array} \begin{array}{c} C \\ \diagup \\ \textcircled{P} \\ \diagdown \\ D \end{array} = \begin{array}{c} A \\ \diagdown \\ \textcircled{R} \\ \diagup \\ B \end{array} \begin{array}{c} C \\ \diagup \\ \textcircled{R} \\ \diagdown \\ D \end{array} + \begin{array}{c} A \\ \diagdown \\ \textcircled{R} \\ \diagup \\ B \end{array} \begin{array}{c} D \\ \diagup \\ \textcircled{R} \\ \diagdown \\ C \end{array} + \begin{array}{c} B \\ \diagdown \\ \textcircled{R} \\ \diagup \\ A \end{array} \begin{array}{c} C \\ \diagup \\ \textcircled{R} \\ \diagdown \\ D \end{array} + \begin{array}{c} B \\ \diagdown \\ \textcircled{R} \\ \diagup \\ A \end{array} \begin{array}{c} D \\ \diagup \\ \textcircled{R} \\ \diagdown \\ C \end{array}$$

Fig. 28

XBL786-1026



XBL786-1018

Fig. 29

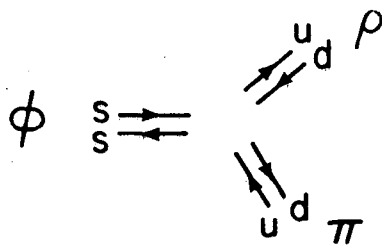


Fig. 30 XBL786-1014

$$\begin{aligned}
 & \left(\begin{array}{c} A \\ \text{---} \\ \text{---} \\ B \end{array} \begin{array}{c} a \\ \text{---} \\ \text{---} \\ b \end{array} + \begin{array}{c} A \\ \text{---} \\ \text{---} \\ B \end{array} \begin{array}{c} b \\ \text{---} \\ \text{---} \\ a \end{array} + \begin{array}{c} B \\ \text{---} \\ \text{---} \\ A \end{array} \begin{array}{c} a \\ \text{---} \\ \text{---} \\ b \end{array} + \begin{array}{c} B \\ \text{---} \\ \text{---} \\ A \end{array} \begin{array}{c} b \\ \text{---} \\ \text{---} \\ a \end{array} \right) \\
 & \times \left(\begin{array}{c} a \\ \text{---} \\ \text{---} \\ b \end{array} \begin{array}{c} C \\ \text{---} \\ \text{---} \\ D \end{array} + \begin{array}{c} b \\ \text{---} \\ \text{---} \\ a \end{array} \begin{array}{c} C \\ \text{---} \\ \text{---} \\ D \end{array} + \begin{array}{c} a \\ \text{---} \\ \text{---} \\ b \end{array} \begin{array}{c} D \\ \text{---} \\ \text{---} \\ C \end{array} + \begin{array}{c} b \\ \text{---} \\ \text{---} \\ a \end{array} \begin{array}{c} D \\ \text{---} \\ \text{---} \\ C \end{array} \right)
 \end{aligned}$$

Fig. 31

XBL786-1015

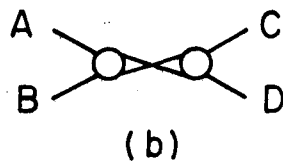
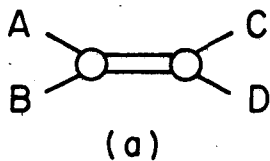


Fig. 32

XBL786-1016

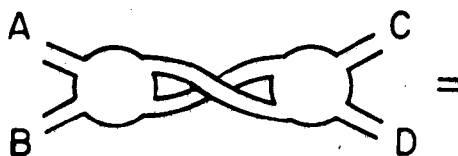
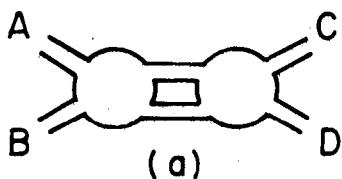
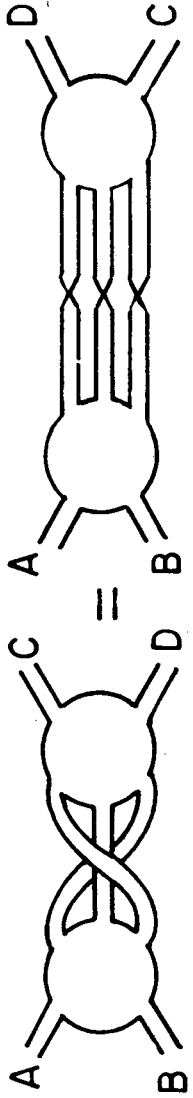


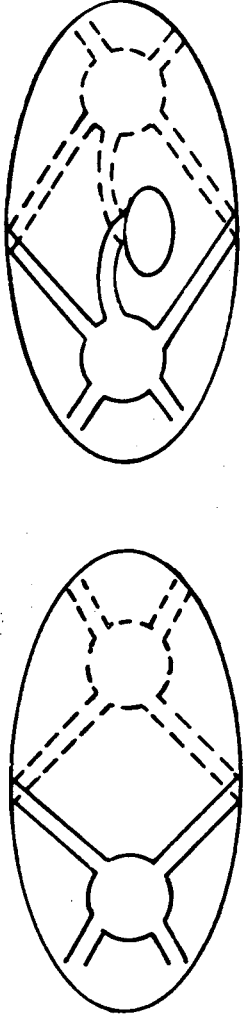
Fig. 33

XBL786-1017



XBL786-1019

Fig. 34



XBL786-1020

Fig. 35

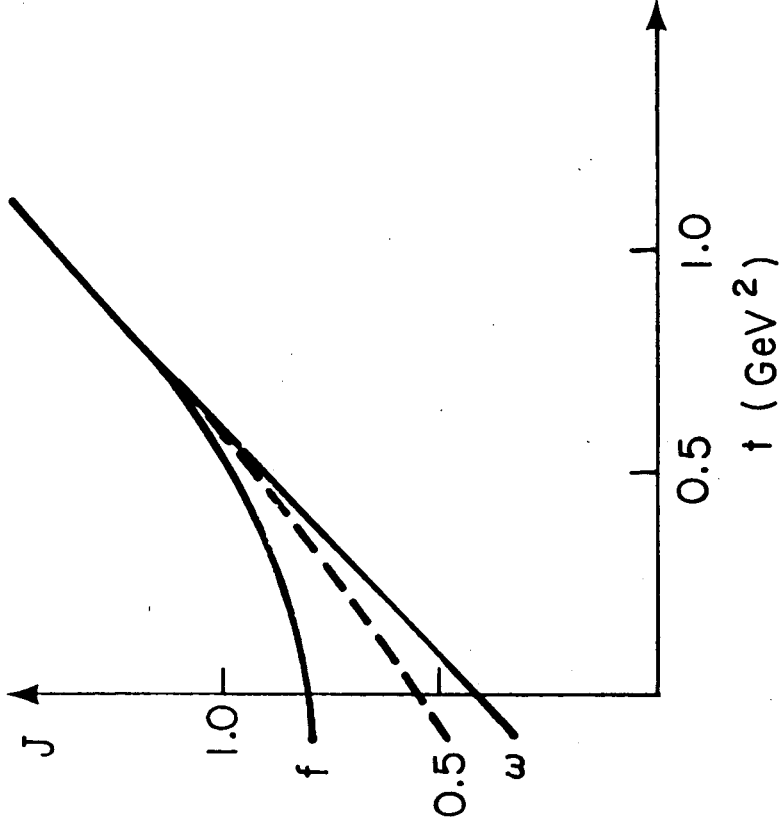
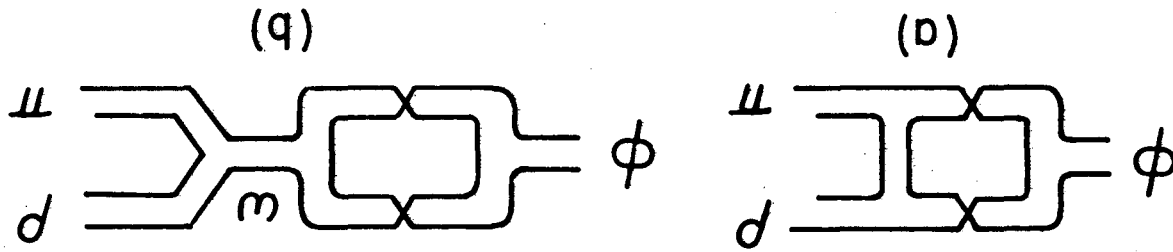


Fig. 36 XBL786-1021

XBL786-1000

FIG. 37



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