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THE I = 0 ππ s-WAVE AND BROKEN SCALE INVARIANCE*

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ABSTRACT

Renner and Staunton have shown that an $I = 0$ $\pi\pi$ s-wave with only an ϵ-resonance cannot satisfy the low energy theorems of broken scale invariance. Recent experimental results indicate that this partial wave amplitude also has a strongly inelastic $S^*(980)$ resonance. We show that if we also include contributions from this resonance in the calculation of Renner and Staunton the low energy theorems can be satisfied and imply that the dimension of the operator which breaks both scale and chiral invariance is one.

The assumption that the ϵ-meson is the Goldstone boson of broken scale invariance leads to a number of low energy theorems. In a recent paper[1] Renner and Staunton (hereafter referred to as RS) have sought to test those theorems, which involve the trace of the energy-momentum tensor, θ, between one pion states,[2] by using phenomenological $I = 0$ s-wave $\pi\pi$ phase shifts of both the up-down and down-up type. Despite the fact that this avoided the highly unrealistic narrow width approximation for the ϵ-pole used in previous treatments, RS found it impossible to satisfy these theorems with just ϵ-resonance contributions.

The purpose of this note is to show that these theorems can be satisfied if we include both ϵ- and S^*-resonance contributions. This is motivated by the results of a recent $\pi^+\pi^- \to \pi^+\pi^-$ scattering experiment which shows that the $I = 0$ s-wave amplitude not only has an ϵ-pole at roughly (600-1240) MeV but also has an S^*-resonance at 980 MeV, which couples strongly to the $K\bar{K}$ channel.[3] We shall follow almost exactly the calculations and approximations of the RS treatment (which should be seen for the details), but use this type of phase shift, which solves the up-down ambiguity in the ρ region, to test the low energy theorems. Since this $I = 0$ s-wave amplitude becomes highly inelastic beyond $K\bar{K}$ threshold we shall need to include, at least roughly, the effects of this inelasticity, by making a slight modification to the RS method.

Following RS we determine the matrix element $\langle\pi|\theta|\pi\rangle$ by using the Omnès equation but modified to include inelastic channels:

$$\langle\pi|\theta|\pi\rangle \equiv F(t) = 2m_\pi^2 \, P(t) \exp\left\{\frac{t}{\pi}\int_{4m_\pi^2}^{\infty} ds \, \frac{\phi(s)}{s(s-t)}\right\} \quad (1)$$

where $P(t)$ is a subtraction polynomial, which RS choose to be $(1 + \beta t)$ and the phase $\phi(s)$ is related to the $I = 0$ $\pi\pi$ s-wave phase shift $\delta_0^{\,0}(s)$ and its inelasticity factor $\eta(s)$. As derived in Ref. 4, ϕ is given by

$$\tan \phi = \frac{\eta \sin 2\, \delta_0^{\,0}}{1 + \eta \cos 2\delta_0^{\,0}} \qquad (2)$$

which gives just $\phi = \delta_0^{\,0}$ when the partial wave is elastic (i.e., $\eta = 1$). The function $\phi(s)$ we shall use is shown in Fig. 1 together with the data of Refs. (3) and (5). We shall further assume that for large s $\left(\phi(s) - \pi\right) \sim 1/s^2$, whilst η tends to 0. In Fig. 2 we show $|F(t)|$ (with $\beta = 0$) deduced from the phase $\phi(s)$ shown in Fig. 1. We see that $|F(t)|$ has a sharp peak in the S^* region as well as a much smaller bump around $t \simeq m_\epsilon^{\,2}$.

We then calculate the two-pion contribution to the spectral function $\rho(s)$ in the generalized propagator

$$\Delta_{\theta\theta}(q^2) = -i \int dx^4\, e^{-iqx} \langle 0 \mid T\left(\theta(x),\ \theta(0)\right) \mid 0 \rangle$$

$$= \int_{4m_\pi^{\,2}}^{\infty} ds\ \frac{\rho(s)}{q^2 - s} \qquad (3)$$

where

$$\rho(s) \simeq \frac{3}{32\pi^2} \left(\frac{s - 4m_\pi^{\,2}}{s} \right)^{1/2} |F(s)|^2 \qquad (4)$$

and obtain the values for $\Delta_{\theta\theta}(0)$ and $\Delta'_{\theta\theta}(0)$. Following RS we introduce a cutoff in the integral for $\Delta_{\theta\theta}$ at $s = \Lambda$ to avoid the logarithmic divergence of that integral (Eq. 3) at large s when $\beta \neq 0$. We put $\Lambda = 100\ m_\pi^2$ so as to include both ϵ- and S^*-resonance effects. We then find (putting $m_\pi = 1$)

$$\Delta_{\theta\theta}(0) = -(0.48 + 33.9\,\beta + (724)_\Lambda\,\beta^2) \tag{5}$$

$$\Delta'_{\theta\theta}(0) = -(0.02 + 0.97\,\beta + (17)_\Lambda\,\beta^2) \tag{6}$$

where Λ denotes the results which are cutoff dependent. We note that our conclusions will not depend critically on the value of Λ chosen since if we increase Λ to $500\ m_\pi^2$ the two Λ-dependent results in Eqs. (5) and (6) change by less than 4% and under 1% respectively.

The low energy theorems[2] to be checked are

$$\Delta_{\theta\theta}(0) = d(4-d)\langle 0|\,u\,|0\rangle \tag{7}$$

$$\left(\frac{\Delta_{\pi\pi}(0)}{F_\pi\, m_\pi^2}\right)^2 \left(\frac{d}{dt}\ F_0(t=0)\right) = \Delta'_{\theta\sigma}(0) - \Delta'_{\pi\pi}(0)\ . \tag{8}$$

Let us define the various quantities in these two theorems: u is the chiral $SU_3 \times SU_3$ breaking operator which also breaks scale invariance and has dimension d. In the Gell-Mann, Oakes and Renner model[6] $u = u_0 - 1.25\,u_8$ where the u_i are the scalar components of a $(3, \bar{3}) \oplus (\bar{3}, 3)$ representation. Renner and Staunton then estimate the matrix element, $\langle 0|u|0\rangle$, and obtain

$$\langle 0|u|0\rangle \simeq -12.6\ m_\pi^4\ . \tag{9}$$

We shall assume d to be an integer and attempt to determine its value from this theorem and Eqs. (5) and (9). To do this we need a value for β, which we calculate from the second theorem. In Eq. (8) we have

$$\Delta_{\pi\pi}(q^2) = \frac{F_\pi^{\ 2} m_\pi^{\ 4}}{q^2 - m_\pi^{\ 2}} \tag{10}$$

where F_π is the pion decay constant ($F_\pi = 95$ MeV). Renner and Staunton suggest the approximation[1]

$$\Delta'_{\theta\sigma}(0) \simeq \frac{\Delta'_{\theta\theta}(0)}{4 - d} \tag{11}$$

which we shall use. Lastly $F_0(t)$ is the vertex $F(t)$ extrapolated off mass shell to where the pions are massless. We follow RS and make the approximation

$$\frac{d}{dt} F_0(t = 0) \simeq \frac{d}{dt} F(t = 0). \tag{12}$$

We are now in a position to use Eqs. (10), (11), (12), and (6) in Eq. (8) in order to calculate $\beta(> 0)$ for $d = 1, 2,$ and 3. We then use Eq. (7) to compute the value of $\Delta_{\theta\theta}(0)/d(4 - d)$ which is to be compared with the estimate of Eq. (9). Our results are shown in Table I together with those of Renner and Staunton for comparison.

TABLE I

For each value of $d = 1, 2, 3$ we show the values of β and $\Delta_{\theta\theta}(0)/d(4 - d)$ calculated from Eqs. (7) and (8). We also give the results obtained by Renner and Staunton using "down-up" and "up-down" phase shifts. Note we have set $m_\pi = 1$.

Type of phase shift	d	β	$\frac{\Delta_{\theta\theta}(0)}{d(4-d)}$	$\langle 0 \|u\| 0 \rangle$ Eq. (7)
Between-down-S^* case	1	0.19	-10.6	-12.6
	2	0.16	- 5.8	-12.6
	3	0.11	- 4.3	-12.6
Down-up case (RS)	1	0.18	- 7.1	-12.6
	2	0.15	- 3.9	-12.6
	3	0.10	- 2.9	-12.6
Up-down case (RS)	1	0.25	- 4.2	-12.6
	2	0.21	- 2.3	-12.6
	3	0.14	- 1.4	-12.6

We see that assuming an integer value for the dimension-parameter d that $d = 1$ is strongly favored. We are strengthened in this conclusion by the fact that we have considered several other possible forms for the phase $\phi(s)$ beyond $s = 1$ $(\text{GeV}/c)^2$ and all have given a dimension of one within the approximations used.

We conclude by remarking that Renner and Staunton have tested the low energy theorems of broken scale invariance and shown that they cannot be satisfied by an ϵ-resonance contribution alone. They suggested that the contributions from higher states are required. We have shown that if we add the inelastic S^* resonance contribution to that of the ϵ, as recent $\pi\pi$ phase shifts indicate we should, we find the theorems can in fact be satisfied (at least to within 16%) and imply that the scale dimension of the operator which breaks both chiral and scale invariance is one.

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FOOTNOTES AND REFERENCES

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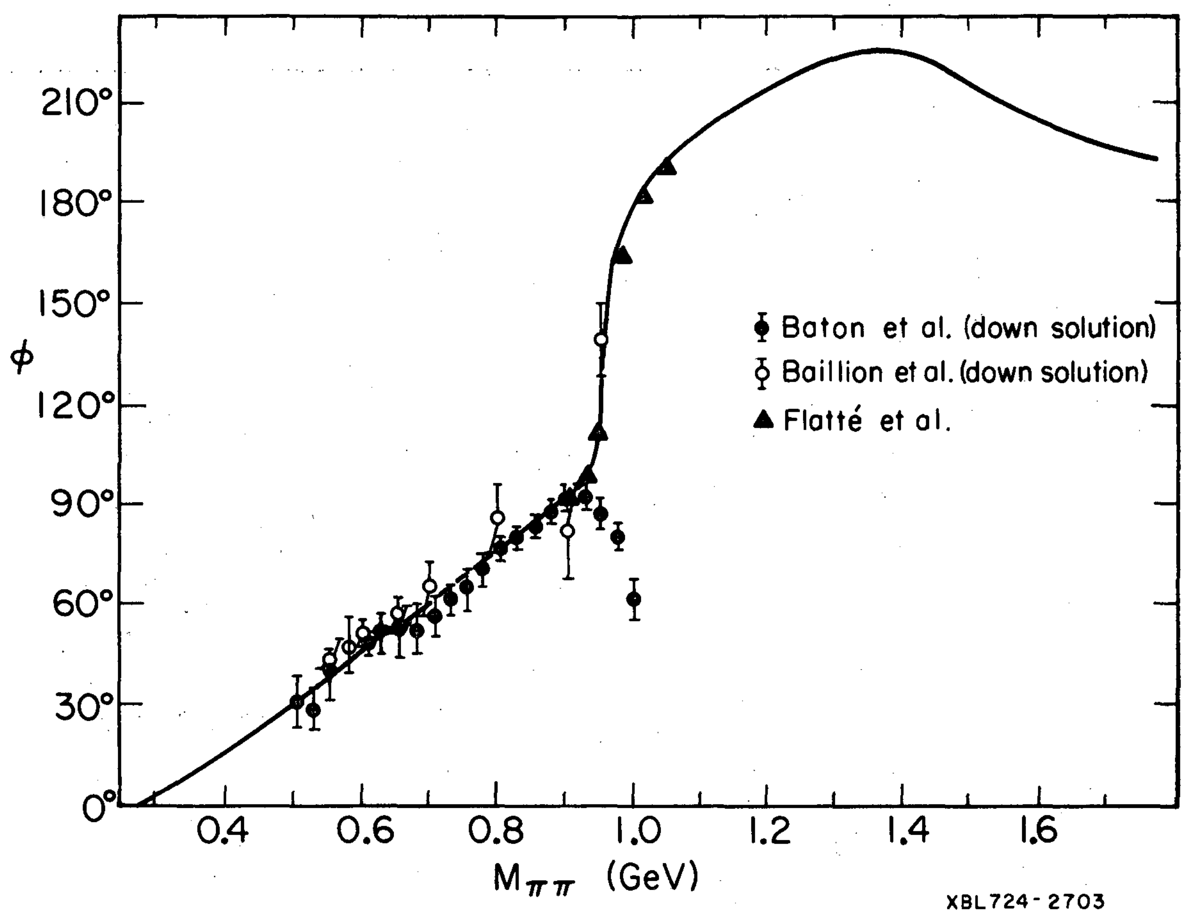
FIGURE CAPTIONS

Fig. 1. The phase $\phi(s)$ of Eq. (2). The data are from Refs. 3 and 5, and the solid line is the phase used to calculate $F(t)$.

Fig. 2. The scalar form factor, $|F(t)|$, calculated from Eq. (1).



210°
180°
150°
120°
90°
60°
30°
0°
φ
0.4
0.6
0.8
1.0
1.2
1.4
1.6
$M_{\pi\pi}$ (GeV)
Baton et al. (down solution)
Baillion et al. (down solution)
Flatté et al.
XBL724-2703

Fig. 1

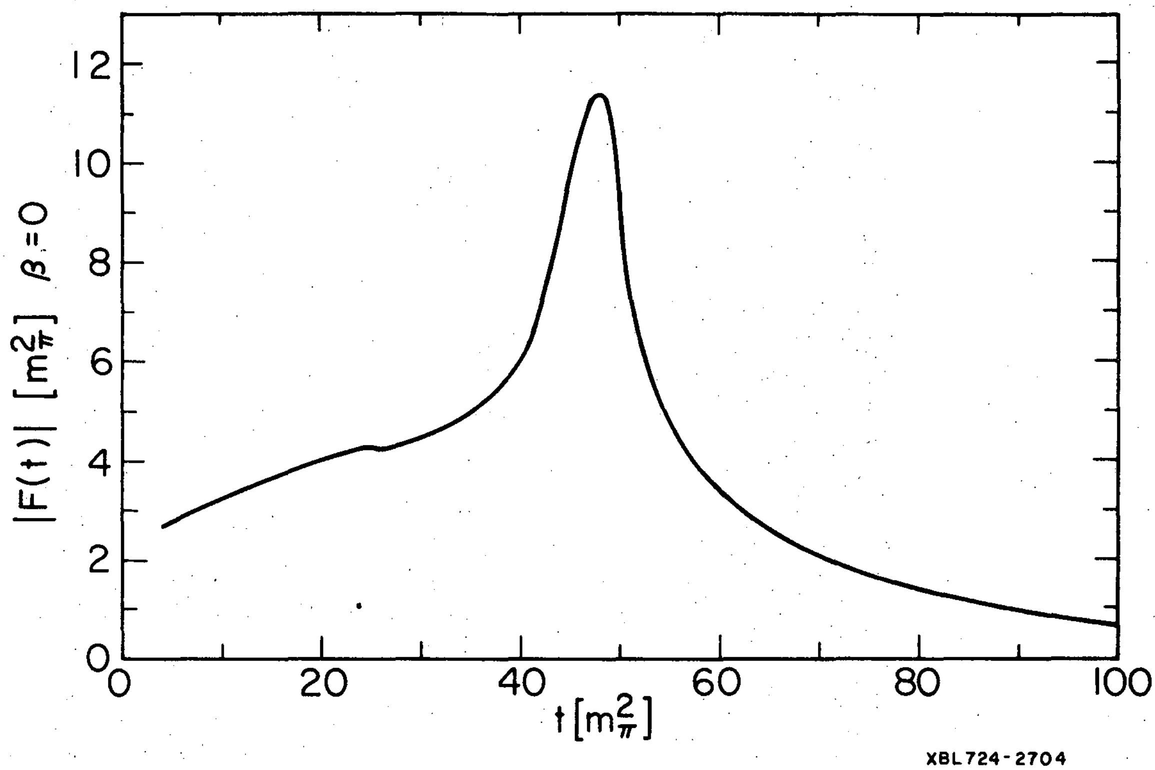


Fig. 2

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