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Impact of Finite Swept Wings on Leading-Edge Vortex
Structure and Stability under Impulsive Conditions

A thesis submitted in partial satisfaction
of the requirements for the degree Master of Science
in Aerospace Engineering

by

Henry Lamar Jones

ABSTRACT OF THE THESIS

Impact of Finite Swept Wings on Leading-Edge Vortex Structure and Stability under Impulsive Conditions

by

Henry Lamar Jones

Master of Science in Aerospace Engineering

University of California, Los Angeles, 2026

Professor Anya Jones, Chair

Wings encountering rapid changes in flow conditions, such as gusts, or those operating on small unmanned and rotary-wing aircraft, routinely experience unsteady separated flow in which the loads bear little resemblance to steady predictions. Much of the lift produced under these conditions comes from the leading-edge vortex, whose growth and eventual departure govern the transient force history. Leading-edge sweep is known to introduce spanwise flow along the vortex axis, but its influence on the transient loads and leading-edge vortex structure of a translating wing remains unsettled.

This work presents force measurements and dye flow visualization for a finite wing surging from rest, with sweep as the only geometric variable. A rectangular, untapered NACA 0012 planform of aspect ratio 4 was accelerated over one chord of travel and towed at constant velocity at a fixed angle of attack of 20 degrees, for leading-edge sweep angles of 0, 30 and 45 degrees. Lift, drag and pitching moment were measured at a Reynolds number of

100,000, and the flow was visualized at 5,000 in both chordwise and spanwise views. The measurements are interpreted alongside a low-order model in which the lift is decomposed into added mass, vortex growth and vortex motion contributions.

Lift peaks at approximately one chord of travel regardless of sweep, while its magnitude falls by nearly half between the unswept and most highly swept planforms. The peak is shown to exceed what added mass and attached-flow circulation together can supply, and is therefore attributed to circulation held in the leading-edge vortex. Normalizing by the velocity component perpendicular to the leading edge collapses the three peaks, consistent with the rate of convective vorticity flux normal to the freestream. The visualization indicates that sweep neither delays separation nor alters when three-dimensional structures first appear, but instead redirects the separated flow outboard towards the wing-tips rather than inboard, allowing the vortex to remain coherent near the surface for longer. This is reflected in the force histories, where the unswept wing retains roughly half of its peak lift after the vortex departs while the 45 degrees swept wing retains close to ninety percent.

The thesis of Henry Lamar Jones is approved.

Jeffrey D. Eldredge

Kunihiko Taira

Anya Jones, Committee Chair

University of California, Los Angeles

2026

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Chapter 1

Introduction

1.1 Motivation

Unsteady aerodynamics plays a crucial role in nearly every atmospheric flight vehicle. From changing flight conditions, environmental disturbances, or the nature of flight itself, unsteadiness presents itself and impacts critical engineering aerodynamic results such as forces and moments. One condition that presents highly unsteady flow is separated or detached flow, resulting from boundary layer separation due to an adverse pressure gradient [2, 27]. In the context of lifting bodies such as wings or propellers, separated flow is usually not preferred but sometimes unavoidable. It is for this reason that it is desirable to study some of the mechanisms and effects of separated flow on aerodynamic surfaces.

There are many forms of unsteadiness that lead to separation. At large length scales or Reynolds numbers, wind gusts and atmospheric turbulence are a prominent source of flow separation, even if only momentarily. Fixed-wing aircraft like airliners or Group 4/5 unmanned aircraft systems (UAS) face these challenges routinely, especially at operational flight altitudes. Vehicles with less inertia and smaller length scales, such as micro-air vehi-

cles (MAVs) are much more susceptible to gusts and disturbances. Flow separation can have dramatic performance impacts or loss of vehicle at the extreme. Unlike modern engineering, nature has shown to favor separated flow at certain Reynolds numbers as proof from birds and insects [7, 10, 21]. To accurately understand and model complete flight envelopes of aircraft and biological flight, the effects of flow separation must be understood.

The primary focus of this study is the formation of a leading-edge vortex (LEV), a phenomenon that occurs when flow separates at the leading edge due to rapid changes in angle of attack. The mechanism of the LEV is explained in greater detail in Section 2.5. Unfortunately, the LEV is a highly nonlinear, transient, and viscous effect, making it one of the most challenging aspects of fluid mechanics to study from a mathematical standpoint. Furthermore, the presence of a LEV couples inviscid lift from potential flow theory with viscous contributions, resulting in a transient lift force that current theory struggles to accurately model. For finite wings, there is also a 3D dependence that arises due to wing-tip vortices. With the addition of different wing configurations, such as sweep, spanwise flow also contributes to the structure and stability of the LEV. Collectively, the LEV is a complicated and parameter sensitive structure that requires further investigation.

1.2 Research Objectives

The literature reviewed in this study establishes that the leading-edge vortex governs the transient lift of a wing at high angle of attack accelerated from rest, and that its behavior on a finite wing is fundamentally three-dimensional. Wing sweep is known to introduce spanwise flow along the vortex axis, offering a route by which the vortex might retain or lose circulation that an unswept wing does not have, but its overall effect on the transient response is not settled. Existing studies on sweep’s impact to the LEV predominately focus on quasi-steady or periodic measurements of wake topology, lift distribution, stabilization, and sensitivity to varieties of kinematic motions such as rotation, plunging, and flapping

[4, 37, 38, 13, 6, 20, 19, 40]. The initial transient effect is either largely ignored or not presented for the surging kinematics presented in this study. Furthermore, the isolated effect sweep has on the development of the tip and leading-edge vortex structure largely remains an open question.

The central question of this study is therefore: *how does sweep influence the transient leading-edge vortex formation and structure of a surging finite wing, and does sweep aid in the stability of the leading-edge vortex hence providing enhancements in lift retention?* Answering this requires two independent signatures available in the present experiments: a change in the character of the measured force history, and the appearance and subsequent transport of spanwise structures in the flow visualization. Neither is a complete measure on its own, but taken together they pair qualitative and quantitative measures of the flow, and the extent to which they agree is itself an informative result.

Specifically, the objectives of this study are:

1. Characterize the transient forces produced by a finite wing surging from rest at a fixed angle of attack of $\alpha = 20^\circ$, and identify the contributions of added mass and leading-edge vortex circulation to the resulting force history.
2. Determine the effect of leading-edge sweep on the magnitude and timing of the transient lift response across sweep angles of $\Lambda = 0^\circ, 30^\circ$, and 45° .
3. Relate the measured force histories to the observed evolution of the leading-edge vortex and tip vortices using time-resolved dye flow visualization, establishing whether sweep alters timing and movement of three-dimensional structures.

The approach taken here is deliberately canonical. The wing is a rectangular, untapered NACA 0012 section of aspect ratio 4, accelerated from rest along a straight path at a fixed angle of attack of 20° , with sweep the only geometric parameter varied. This isolates the

effect of the swept leading edge from the pitching, plunging, and revolving kinematics that dominate much of the existing leading-edge vortex literature, and from the sectional and planform variations that complicate comparison between studies. Two features distinguish this work from prior investigations of swept wings. First, the force measurements resolve the full transient from the onset of motion rather than characterizing an established or quasi-steady state, which is the regime in which the majority of swept-wing separation studies have been conducted. Second, the force histories are paired with dye visualization of the same configurations, allowing features of the force record to be attributed to specific stages of vortex development rather than inferred from the force signal alone. Much of the canonical work in this area has employed sharp-edged flat plates, for which the separation point is fixed by the geometry; the rounded leading edge of the NACA 0012 permits the separation location to respond to the flow, which is representative of practical lifting surfaces and introduces a Reynolds number sensitivity absent from the flat-plate case.

The remainder of this thesis is organized as follows. Chapter 2 develops the aerodynamic background on which this study rests, beginning with the governing equations and steady thin-airfoil and finite-wing theory, proceeding to unsteady attached-flow theory and the decomposition of unsteady loads into circulatory and non-circulatory contributions, and concluding with the physics of separated flow, the leading-edge vortex, and the existing low-order model to represent the force decomposition. Chapter 3 describes the experimental methods, including the towing apparatus and motion control, the wing models, the force measurement and data reduction procedures, and the flow visualization setup. Chapter 4 presents the measured force histories and dye visualization across the sweep configurations tested and discusses them in the context of the theory of Chapter 2. Chapter 5 summarizes the principal findings and offers recommendations for future work.

Chapter 2

Aerodynamic Background

Aerodynamics is a rich field with advancements dating back to the 18th century and continues to evolve as evident by this study. While a concrete background is impractical to cover in depth, it is necessary to highlight the theory and research this work stems from and attempts to build on. This chapter also hopes to enlighten readers on the necessary background who do not come from an aerodynamic setting.

2.1 Governing Equations

2.1.1 Incompressible flow

In fluid mechanics, a flow field can usually be categorized into either *incompressible flow* or *compressible flow*. The formal definition of a compressible flow is the material derivative of density changes over time. Mathematically, this can be written as

$$\frac{D\rho}{Dt} = -\rho(\nabla \cdot \mathbf{u}), \quad (2.1)$$

where ρ is the fluid density, t is time, and $\mathbf{u}$ is the three-dimensional velocity vector. Typically, a flow is treated as compressible if its Mach number (M) is greater than 0.3, where

$$M = \frac{\|\mathbf{u}\|}{a}, \quad (2.2)$$

and a is the fluid's local speed of sound. Compressible flow has a wide range of applications, especially in aviation and many of the scenarios of separated flow presented in Section 1.1. However, for the context of this study and those similar, the flow is understood to be incompressible, meaning Eq. 2.1 reduces to

$$\nabla \cdot \mathbf{u} = 0, \quad (2.3)$$

and represents conservation of mass for an incompressible fluid.

2.1.2 Momentum conservation

Similar to mass conservation, a fluid also has governing equations for conservation of momentum. This comes in the form of the Navier-Stokes equations provided in Eq. 2.4, assuming Eq. 2.3 and the Newtonian law of viscosity [27] are satisfied.

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f} \quad (2.4)$$

Here, p represents the scalar pressure field, $\mathbf{f}$ are any external body forces per unit mass such as gravity, and ν is the kinematic viscosity of the fluid. These equations, along with flow specific boundary conditions, initial conditions, and thermodynamic property of state relations allow for the solution of the velocity and pressure fields of any flow. Once known, forces and moments can be computed, which is the common engineering value of interest from an aerodynamic analysis.

The Navier-Stokes equations can be reduced if the flow is assumed to be inviscid, irrotational and incompressible. This is known as the unsteady Bernoulli equation, and is expressed as

$$\frac{\partial \phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2} \|\mathbf{u}\|^2 + gz = B(t), \quad (2.5)$$

where ϕ is the velocity potential, g is the gravitational acceleration, z is elevation, and $B(t)$ is a function of time. Bernoulli majorly simplifies the conservation equations and can be applied in many practical cases, such as modeling lift produced by wings for attached flow.

2.1.3 Vorticity

In Eq. 2.4, the variables of interest are velocity and pressure. For many practical flows, pressure is difficult to both measure and compute. Additionally, real flows cannot neglect viscous effects which ultimately gives rise to vorticity ($\boldsymbol{\omega}$), defined as

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}. \quad (2.6)$$

Vorticity is a measure of a fluid's rotation and is a consequence of the fluid and surface velocity being equal at the interface, known as the no-slip boundary condition ($\mathbf{u} = \mathbf{U}$). Vorticity is generated during acceleration or through a tangential pressure gradient along the surface and forms a *boundary layer* [27]. These flows are known as *rotational*, and the vorticity transport equation shown below governs their dynamical behavior.

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{u} \cdot \nabla \boldsymbol{\omega} = \boldsymbol{\omega} \cdot \nabla \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} \quad (2.7)$$

Many times, particularly in this study, making use of Eq. 2.7 rather than Navier-Stokes can be beneficial due to avoiding pressure calculations and being able to theoretically isolate inviscid and viscous effects [27].

2.1.4 Nondimensionalization

A common approach done with Eq. 2.4 is to nondimensionalize the terms. In doing so, a critical parameter known as Reynolds number (Re) is revealed as

$$Re = \frac{U_{\infty} l}{\nu}, \quad (2.8)$$

where U_{∞} is the freestream velocity of the flow and c is the characteristic length. Reynolds number represents the ratio of inertial forces to viscous forces and very much dictates what the flow regime and behavior will be. Another important term resulting from nondimensionalization is convective time, expressed as

$$t^* = \frac{U_{\infty} t}{c}. \quad (2.9)$$

Convective time units explain how long it takes the flow to travel characteristic length c at characteristic velocity U_{∞} , and removes physical time dependency. Like Re , this helps gain quick insight into different flows and allows for comparisons without needing identical parameter matches.

2.2 Airfoil Aerodynamics

2.2.1 Airfoil nomenclature

While the main topic of discussion for this study revolves around 3D finite wings, the two-dimensional wing section, known as the airfoil, must first be examined. Airfoils come in all shapes and sizes and are used for general wings, flying wings, propellers, rotorcraft, and even some automobiles. The general profile and nomenclature of an airfoil is represented by Fig. 2.1. The chord (c) represents the distance from leading edge (LE) to trailing edge (TE), and is the characteristic length of an airfoil or wing. The angle of attack (α or

AOA) represents the angle between the chord line and the freestream velocity. The camber represents asymmetric curvatures on the upper and lower surface, which results in various 2D lift (L') values for different AOA's. An airfoil with no camber is known as a *symmetric airfoil*.

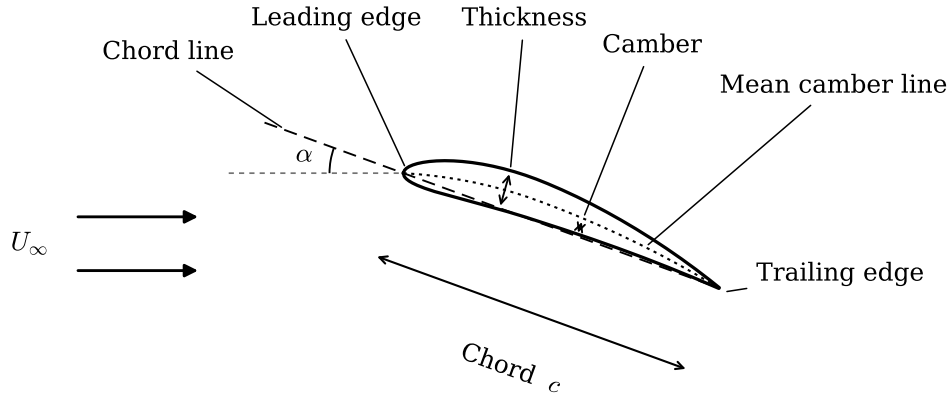


Figure 2.1: General airfoil nomenclature, adapted from [2].

A common family of airfoils stems from the National Advisory Committee for Aeronautics (NACA). NACA airfoils have a unique numbering system to correlate differences in camber and thickness. The four-digit series defines the profile by the following: The first digit describes the maximum camber of the airfoil as a percentage of the chord. The second digit indicates the location of maximum camber from the LE in tenths of the chord. The third and fourth digits express the maximum thickness of the profile as a percentage of the chord [1]. For example, the NACA 0012 airfoil means it has a symmetric profile with a thickness (t) of $0.12c$. This is also the airfoil used in this study, however the general equation for a symmetric NACA airfoil is given by

$$y_t = 5t[0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4] \quad (2.10)$$

where x is a position along the chord from 0 to 1 and y_t is the half thickness at a given x

position [1].

2.2.2 Thin airfoil theory

To calculate lift and moments on symmetric airfoils from a theoretical approach, 2D thin airfoil theory is used. The assumption that makes this theory valid are that the airfoil is sufficiently thin that the flow can be approximated as a vortex sheet placed on the chord line [2]. Underlying assumptions of this theory are that the flow is incompressible and inviscid, or more commonly known as *potential flow*. This inherently means the flow remains attached and is steady for this theory. Fig. 2.2 depicts the vortex sheet model for a symmetric airfoil. The induced velocity (w) from multiple point vortices like those in a vortex sheet can be

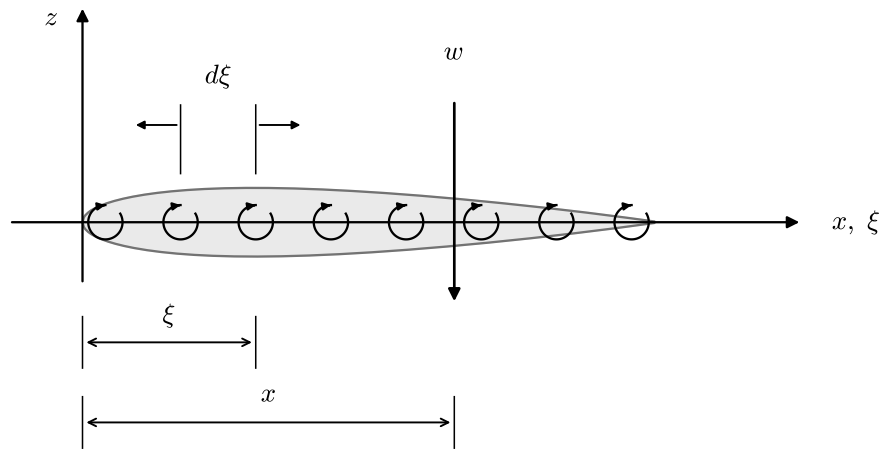


Figure 2.2: Schematic of a vortex sheet for induced velocity calculation on a symmetric airfoil, adapted from [2].

calculated. This gives rise to the *fundamental equation of thin airfoil theory*, which is given as

$$\frac{1}{2\pi} \int_0^c \frac{\gamma(\xi)d\xi}{x - \xi} = U_\infty \left(\alpha - \frac{dz}{dx} \right), \quad (2.11)$$

where γ is the strength of the vortex sheet which varies with ξ , a distance from the origin as shown in Fig. 2.2. The $\frac{dz}{dx}$ term describes the slope of the camber, however this is equal

to zero for a symmetric airfoil. For a prescribed α , Eq. 2.11 can be solved for the vortex sheet strength as a function of θ , a coordinate transformation of ξ , given by Eq. 2.12. This solution also satisfies the Kutta condition, stating $\gamma(TE) = 0$, and enforces the camber line as a streamline [2].

$$\gamma(\theta) = 2\alpha U_\infty \frac{(1 + \cos \theta)}{\sin \theta} \quad (2.12)$$

With the vortex sheet strength known, the total circulation (Γ) can be solved by integration as

$$\Gamma = \pi \alpha c U_\infty. \quad (2.13)$$

This result, along with

$$L' = \rho U_\infty \Gamma, \quad (2.14)$$

from the Kutta-Joukowski theorem yields the 2D lift coefficient, expressed as

$$c_l = \frac{L'}{\frac{1}{2} \rho U_\infty^2 c} = 2\pi \alpha. \quad (2.15)$$

The 2D lift slope (a_0) can therefore be stated as

$$\frac{dc_l}{d\alpha} = a_0 = 2\pi. \quad (2.16)$$

Eqs. 2.15 and 2.16 are the critical results 2D thin airfoil theory builds up and reveals a linear proportionality of lift and AOA. This theory can be validated experimentally through lift versus α measurements. An example of this is shown in Fig. 2.3 for an arbitrary symmetric airfoil. As seen, the linear regime for a moderate AOA aligns with the predicted 2π slope. At higher angles, the lift begins to taper off and decrease nonlinearly. This is known as a *stall* and occurs when the boundary layer of the flow separates. Thin airfoil theory cannot predict this, since it is a purely viscous effect and requires thorough computation or experimental analysis. More will be said about this behavior in Section 2.5.

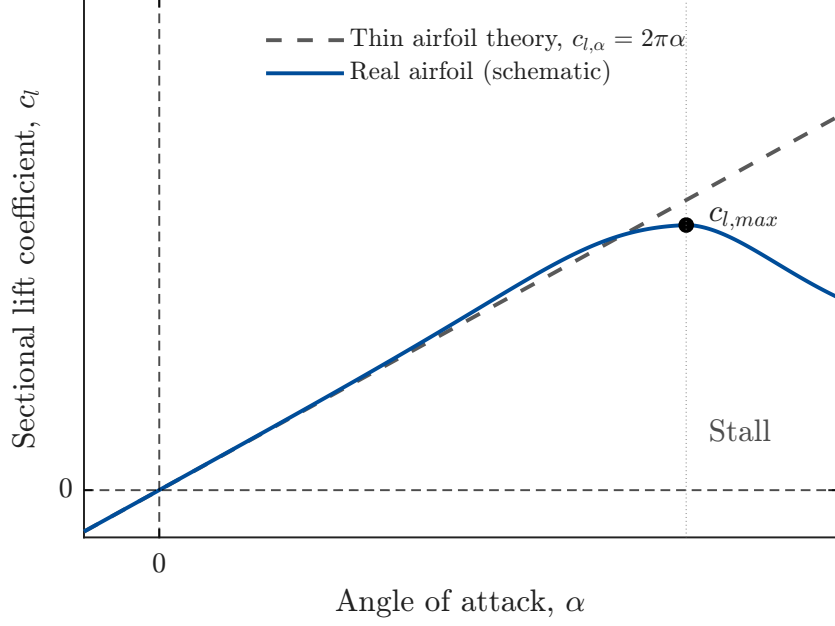


Figure 2.3: Sectional lift curve schematic for an arbitrary symmetrical airfoil.

With the vortex sheet defined for a symmetric airfoil, the 2D moment coefficient (c_m) about the LE can also be calculated as

$$c_{m,le} = -\frac{c_l}{4}, \quad (2.17)$$

and with basic moment correlations, the moment around the quarter-chord can be defined as

$$c_{m,c/4} = 0. \quad (2.18)$$

Equation 2.18 subtly says an important fact: the quarter-chord for a symmetric airfoil is both the center of pressure (CP) and aerodynamic center (AC), and can also be verified through experimental results [1]. Both the CP and AC have major implications in wing design in terms of lift distribution, loads, stability, and control. While many other airfoils have a shifting CP and an AC not exactly at the quarter-chord, the symmetric airfoil greatly simplifies this analysis.

2.3 Finite Wing Aerodynamics

2.3.1 Finite wing nomenclature

Now that the basic understanding of airfoils and the governing theory is covered, 3D finite wings can be covered in detail. Wings have taken many shapes and continue to evolve today. Fig. 2.4 depicts some common features of finite wings, including the span (b), sweep (Λ), taper ratio ($\lambda = \frac{c_t}{c_r}$), and wing area (S) denoted by the shaded section. A useful parameter often used is the aspect ratio (AR), defined as

$$AR = \frac{b^2}{S}, \quad (2.19)$$

and characterizes a wing's width relative to the length. This study will focus on untapered swept and rectangular wings exclusively.

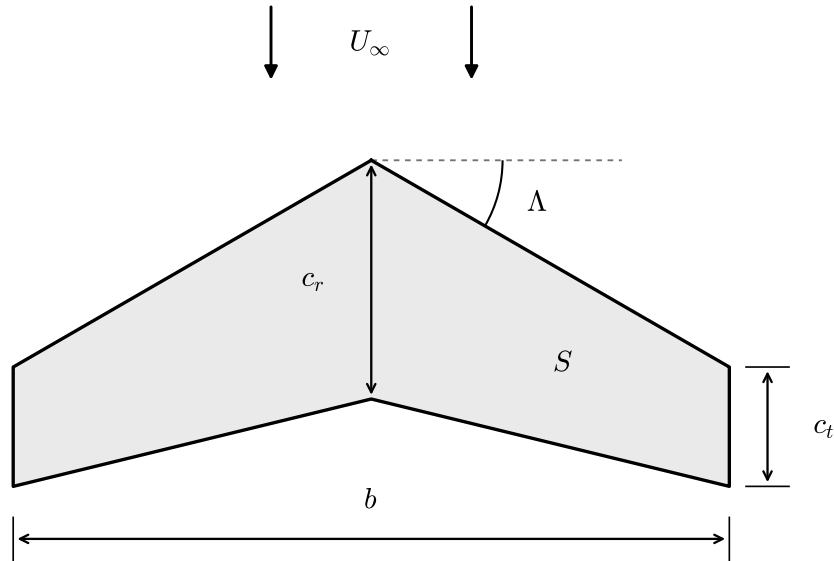


Figure 2.4: General finite wing planform nomenclature.

2.3.2 Wing-tip vortices

For a wing to produce an upward lift force it is clear there must be a lower pressure on the upper surface than the bottom. This does not have many other significant implications for an airfoil, but on a finite wing this pressure imbalance meets at the wing-tips and causes the high-pressure flow to curl outwards and around the tips to the low-pressure region. What forms is known as *wing-tip vortices*, and they are the primary cause for many of the three-dimensional effects finite wings possess [2]. Not only do these vortices concentrate at the tips, they bring in some of the surrounding flow and push it downwards. This is known as *downwash*, and it in turn creates *induced drag*. A general depiction of wing-tip vortices is shown in Fig. 2.5.

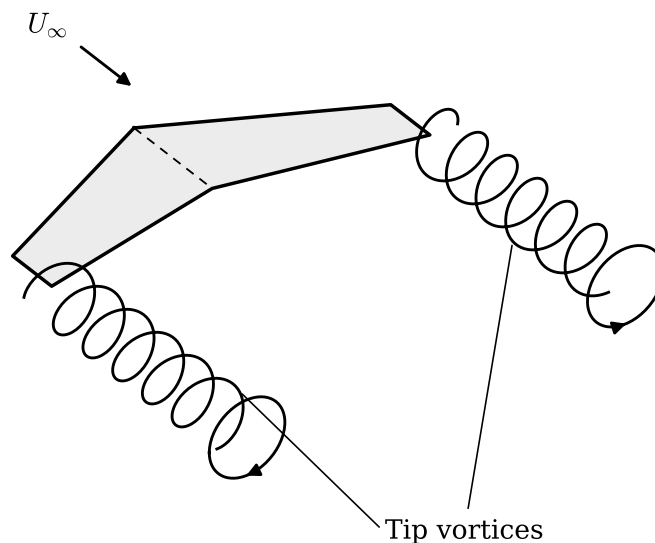


Figure 2.5: Schematic of wing-tip vortices.

2.3.3 Lifting-line theory

Recall the sectional lift described in Section 2.2.2 from the Kutta-Joukowski theorem in Eq. 2.14. The circulation, which was constant for steady flow on an airfoil, characterizes the lift produced by that 2D slice. For a 3D wing, lift can only be generated where circulation exists

on a finite wing, and it must fall to zero at the wingtips due to the pressure equalization [2]. This results in a spanwise distribution of circulation on the wing, $\Gamma(y)$, where y is the span axis. To say more about the circulation distribution, we must make use of Helmholtz's vortex theorems. These theorems state a vortex filament, which is essentially a 3D extrusion of a point vortex, has a constant circulation along the filament, and the filament cannot end in the fluid; it must form a closed path or extend to the boundaries of the fluid [27]. If the circulation truly does vary along the span, it is impossible for the wing to be represented as one vortex filament. Instead, it is superimposed of an infinite number of *horseshoe vortices*, each with their own constant strength vortex filament and trailing vortices as depicted in the schematic of Fig. 2.6. This is the logic behind Prandtl's lifting-line theory [29, 30].

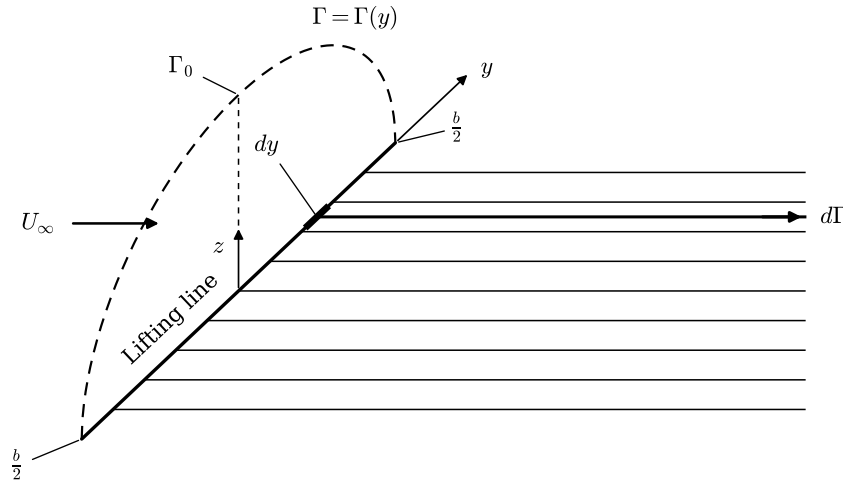


Figure 2.6: Superposition of horseshoe vortices as described by Prandtl's lifting-line theory, adapted from [2].

Prandtl theorized the lift on a finite rectangular, tapered, or elliptical wing, and specifically $\Gamma(y)$, can be represented by the induced velocities of the superimposed vortices through the Biot-Savart law, given as

$$\mathbf{du} = \frac{\Gamma}{4\pi} \frac{\mathbf{dl} \times \mathbf{r}}{\|\mathbf{r}\|^3}, \quad (2.20)$$

where $\mathbf{dl}$ is a segment of the vortex filament and $\mathbf{r}$ is the vector from $\mathbf{dl}$ to the induced

velocity element $\mathbf{du}$. Ultimately, this allows for the solution of $\Gamma(y)$ and the induced drag coefficient ($C_{D,i}$) on a finite wing, given by

$$C_{D,i} = \frac{C_L^2}{\pi e AR}. \quad (2.21)$$

The span efficiency factor (e) is the sole result of geometrical choices such as the airfoil and planform, and the 3D lift coefficient (C_L), given by

$$C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 S} \quad (2.22)$$

accounts for lift loss due to the wing-tip vortices. The induced drag is therefore the cost of this loss of lift. In other words, lift does not come free and a force, such as thrust, must overcome the drag. Along with induced drag, a 3D wing also produces a different lift slope (a), given as

$$\frac{dC_L}{d\alpha} = a = \frac{a_0}{1 + (a_0/(\pi AR))(1 + \tau)}, \quad (2.23)$$

where a_0 comes from Eq. 2.16 and τ , similar to e , depends on geometric choices.

Prandtl's lifting line theory is a great tool for most general wing geometries, but due to the singular lifting line representing the circulation around the wing, it fails to address the case of swept wings [2]. Sweep, as shown in Fig. 2.4, appears on many modern wing configurations and is a primary focus of this study. To address the potential flow theory suitable for swept wings, Prandtl's model is extended to include multiple lifting lines that span from the LE to TE . This creates a 3D vortex sheet on the wing, resulting in a *lifting surface* and a *wake* behind the wing. Numerically, this approach is known as the vortex lattice method (*VLM*). Separately, closed-form approximations to the lift slope for a swept wing have been developed [18], yielding

$$a = \frac{a_0 \cos \Lambda}{\sqrt{1 + [(a_0 \cos \Lambda)/(\pi AR)]^2 + (a_0 \cos \Lambda/(\pi AR))}}. \quad (2.24)$$

2.4 Unsteady Aerodynamics

Up to this point the aerodynamic background we have built up for airfoils and finite wings has assumed steady attached flow. To address the critical aspect of this study and explain how these steady results came to be, we must depart from these assumptions.

2.4.1 Generation of circulation

Unsteady or transient flow is one where the term $\frac{\partial \mathbf{u}}{\partial t}$ in Eq. 2.4 cannot be neglected. The velocity field cannot change instantaneously in incompressible flow, meaning changes in the flow from convective or viscous effects will force $\mathbf{u}$ to change over time. Section 2.2 explained the role circulation plays in an airfoil creating lift, but it did not cover how it was generated in the first place. The answer lies in *Kelvin's circulation theorem*, which mathematically states

$$\frac{D\Gamma}{Dt} = 0 \quad (2.25)$$

for inviscid flow. Eq. 2.25 means that for any closed curve of fluid elements, the change of circulation around that curve must be zero. So, if a lifting airfoil has a definite circulation, which is defined to be clockwise or positive, by Kelvin's theorem there must exist an equal and opposite circulation somewhere. This is exactly what a *starting vortex* is. An airfoil and fluid at rest undoubtedly do not possess any circulation. If the airfoil was brought into motion (or the flow in motion), a large region of vorticity would begin to form at the sharp trailing edge due to extreme velocity gradients and flow separation [2]. As vorticity grows in a counterclockwise manner, Kelvin states an equal clockwise circulation must form, known as the *bound vortex*. This process continues until the Kutta condition is satisfied and the shedding ceases, at which point both circulations remain constant thereafter. Fig. 2.7 helps to illustrate this process, and shows the total circulation in the system is zero.

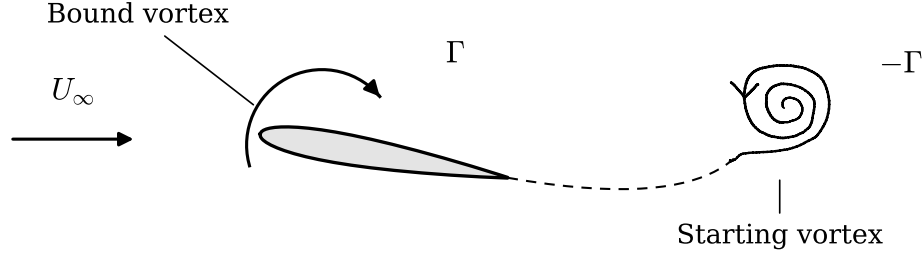


Figure 2.7: Schematic of the starting and bound vortex from Kelvin's circulation theorem, adapted from [2].

2.4.2 The Wagner function

The mechanism of how circulation is generated on an airfoil is now clear, but the question still remains of how to quantify and predict the process to steady circulation. From Eq. 2.15, the steady 2D lift coefficient a symmetric airfoil achieves is $2\pi\alpha$. At rest the solution is trivial: $c_l = 0$. So, how is a time varying 2D lift coefficient, $c_l(t)$, developed to connect the two? The Wagner function, $\Phi(t)$, developed by Herbert Wagner [36], bridges this gap.

In dynamic system terms, the Wagner function models the step response of an airfoil's circulation due to sudden changes in α . In this study, applications will be focused on U_∞ as the step input. This is also known as the *indicial response*. It achieves this through a rigorous mathematical background, including using a Joukowski transformation to model the airfoil using potential flow. The result is a model that accounts for changes in downwash as the starting vortex builds and sheds from the TE , which delays the build up of the bound vortex [15]. Ultimately, $\Phi(t)$ does not have a closed form solution, but it satisfies

$$c_l(s) = 2\pi\alpha\Phi(s), \quad (2.26)$$

where s is the reduced time in units of semi-chords and is given as

$$s = \frac{2U_\infty t}{c}. \quad (2.27)$$

The most common approximation of the Wagner function for practical calculations, given by R.T. Jones [16, 17], is

$$\Phi(s) = 1 - A_1 e^{-\lambda_1 s} - A_2 e^{-\lambda_2 s}, \quad (2.28)$$

where $A_1 = 0.165$, $A_2 = 0.335$, $\lambda_1 = 0.0455$, and $\lambda_2 = 0.3$. Fig. 2.8 shows the approximated Wagner function through 30 chords. An interesting result of Eq. 2.28 happens when $s = 0$:

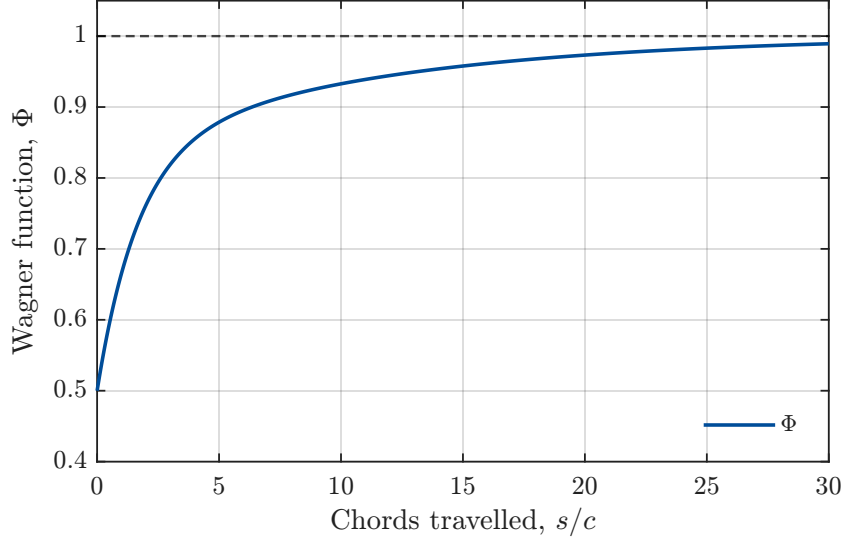


Figure 2.8: Approximated Wagner function through 30 chords.

Wagner's function is evaluated to be $\frac{1}{2}$, which raises a concern. If true, this is to say Eq. 2.26 yields $c_l(0) = \pi\alpha$. How is it possible to predict nonzero lift for zero chords traveled or time elapsed? Section 2.4.3 answers this.

2.4.3 Circulatory and noncirculatory forces

Circulatory forces are ones like the bound vortex and are responsible for most to all of the steady lift produced by wings. As now shown by the Wagner function, it takes time to develop and does not change instantaneously. Noncirculatory forces, on the other hand, come from the force it takes to move the surrounding fluid from rest due to inertia, and do appear instantaneously [27]. This force is often called *apparent* or *added mass*, because it is directly proportional to the acceleration of the motion. In theory, the total unsteady force

an airfoil experiences is the summation of circulatory and noncirculatory forces [35]. $\Phi(s)$ is taken to be a purely circulatory contribution, so Eq. 2.26 should be refined as

$$c_l(s) = 2\pi\alpha\Phi(s) + c_{l,nc}(s), \quad (2.29)$$

where the second term represents lift from apparent mass contributions. The $\frac{1}{2}$ ambiguity arises due to Jones's approximation already including the added mass for an impulsive change in AOA. For the case of an impulsive translation, or surge, of a 2D flat plate the noncirculatory lift coefficient is shown by [3] using potential flow to be

$$c_{l,nc} = \frac{\pi c \dot{U}}{4U_\infty^2} \sin 2\alpha, \quad (2.30)$$

where $\dot{U}$ is the time varying acceleration profile as true step inputs are impossible to physically achieve. As a dimensional force the added mass is $\frac{\pi}{8}\rho S c \dot{U} \sin 2\alpha$. Note that Eq. 2.30 does not depend on s , meaning the lift contributions are only added during acceleration or impulse stages of the kinematics.

2.4.4 Finite wing approximations

So far the unsteady theory has been built for the 2D airfoil case. To reflect 3D finite wings, as discussed in Section 2.3, modifications to the Wagner function by Jones [16] have been made. While studies like [5] have developed an unsteady lifting line theory, this will not be used in the present study. It should be noted the modifications of [16] model an elliptical wing subject to a unit step change in angle of attack. This study diverges from this planform and kinematics, but it allows for reasonable baseline predictions. The unsteady 3D lift coefficient slope can be approximated as

$$\frac{dC_L}{d\alpha}(s) = C_0 + C_1 e^{r_1 s} + C_2 e^{r_2 s}, \quad (2.31)$$

where the constants are given in Table 2.1. In the limit that $AR \rightarrow \infty$, the wing effectively

Table 2.1: Constant values for finite aspect ratio wings to be used in Eq. 2.31.

AR	C_0	C_1	r_1	C_2	r_2
∞	2π	-0.330π	-0.0455	-0.670π	-0.300
6	4.71	-1.740	-0.324	0	0
3	3.77	-1.070	-0.490	0	0

behaves as a 2D airfoil and the steady 2π lift slope is achieved. As aspect ratio decreases, 3D lift deficiencies become more prominent, but steady-state results are achieved much quicker than the infinite wing, as shown in Fig. 2.9. The latter may seem counterintuitive, but it is a result of wing-tip vortices reducing the influence of the starting vortex and wake [17].

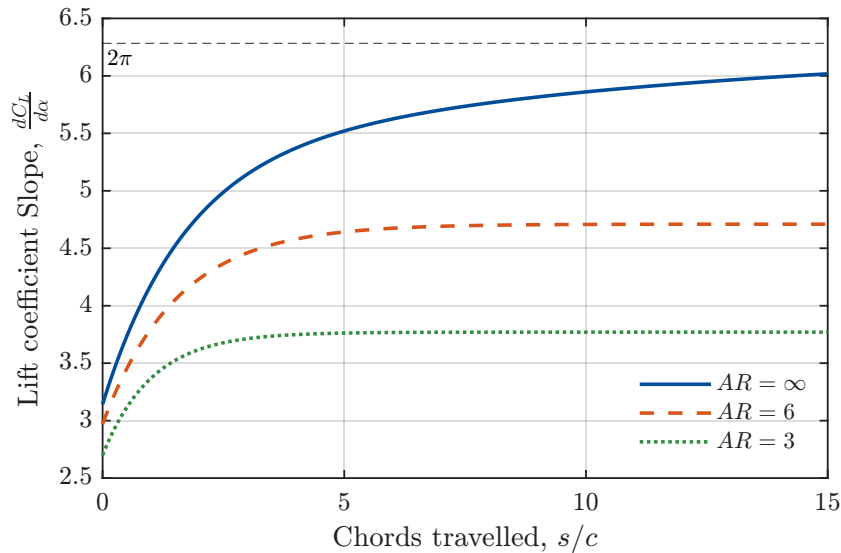


Figure 2.9: Unsteady lift coefficient slopes for finite aspect ratio wings over 15 chords of travel.

2.5 Separated Flow and the Leading-Edge Vortex

Section 2.4 dispensed with the assumption of steady flow, but attached flow was still assumed throughout. Wagner's function captures how circulation develops provided the Kutta condition can be enforced at the trailing edge and the flow tracks the surface everywhere

else. At the high angles of attack considered here, that assumption no longer holds, since the adverse pressure gradient near the leading edge overwhelms the boundary layer, which separates outright. This is the stall behavior already seen in Fig. 2.3. Potential flow offers no description of what follows, since the resulting structure is a product of viscosity, evolves strongly in time, and cannot be represented as a bound sheet along the chord line. In its place, the vorticity transport equation of Eq. 2.7 tracks the creation, transport, and fate of vorticity directly.

2.5.1 Leading-edge vortex mechanics

Section 2.1 established that vorticity arises from the no-slip condition at the surface. On an attached airfoil this vorticity is confined to a thin boundary layer and passes quietly into the wake. Once the flow separates at the leading edge, however, that vorticity is no longer swept downstream in a thin layer; it is shed directly into the outer flow as a free *shear layer*. Such a layer is unstable, and it rolls itself into a large, coherent concentration of vorticity sitting over the suction surface. This structure, the *leading-edge vortex* (LEV) shown forming in Fig. 2.10, is the central flow feature of this study. It remains a circulatory contribution to lift in the sense of Section 2.4.3, since its influence on the wing is still felt through Eq. 2.14, but it must not be confused with bound circulation. It is a free structure detached from the surface, free to grow, migrate, and ultimately depart the wing entirely [8, 28].

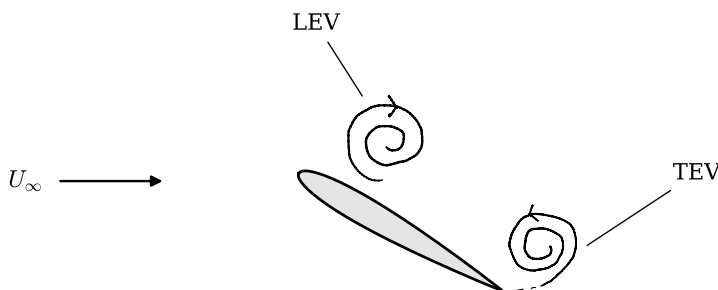


Figure 2.10: Schematic of leading-edge vortex in separated flow and trailing-edge vortex.

A vortex sustained entirely by a feeding shear layer has no fixed strength; its circulation grows or decays according to whatever balance of vorticity fluxes crosses its boundary. Drawing a control volume C around the vortex and accounting for those fluxes gives an evolution equation for the LEV circulation, following Eldredge and Jones [8],

$$\frac{\partial \Gamma(y, t)}{\partial t} = - \oint_C \mathbf{n} \cdot \mathbf{u} \omega_y dC + \nu \oint_C \frac{\partial \omega_y}{\partial n} dC + \oint_C \mathbf{n} \cdot \hat{\omega} v_y dC, \quad (2.32)$$

in which the three terms are, respectively, the convective flux of vorticity across the boundary, the diffusive flux, and the tilting of non-spanwise vorticity into the spanwise direction. Of these, the convective term dominates and is essentially the feeding shear layer itself; the diffusive term is small and confined near the wing surface; and the tilting term vanishes identically in two dimensions, entering only once three-dimensional structure is present [8]. Eq. 2.32 also gives a working definition of LEV *stability*, which holds wherever the right-hand side is approximately zero, so that the vortex neither gains nor loses circulation and can in principle sit fixed on the wing indefinitely [8].

2.5.2 Growth and shedding of vortices

With the third term of Eq. 2.32 absent in two dimensions, a two-dimensional LEV has no mechanism by which to shed the vorticity delivered to it, and so it can only grow, in both size and strength, for as long as the shear layer continues to feed it. At early times, while the vortex is still too small to sense the trailing edge, no length scale has yet been imposed on the problem and the growth is self-similar. $\Gamma/U_\infty c$ scales as $t^{1/3}$ and D/c as $t^{2/3}$, with D a characteristic vortex diameter [31]. Growth of this kind cannot proceed unchecked on a wing of finite chord. Once the vortex has grown to roughly a chord in size, $D/c \approx 1$, the enclosed recirculation region bursts, the feeding shear layer is severed, and the LEV separates into an independent structure that convects away into the wake [14], a chord-limited process referred to as *bluff-body shedding*. A second, competing mechanism has

also been proposed, in which secondary vorticity generated beneath the LEV accumulates until it chokes off the primary shear layer from underneath. Unlike bluff-body shedding, this mechanism is governed not by chord length but by a Reynolds number formed from the leading-edge geometry, $Re_{LE} = U_{\infty} L_{LE} / \nu$ [32], which is a useful reminder that two wings sharing a chord Reynolds number need not behave alike if their nose geometry differs, as between a sharp flat plate and a rounded airfoil [8]. In practice, detachment likely reflects some combination of the two mechanisms.

That LEV growth is finite in duration suggests a characteristic timescale governing when detachment occurs, captured by the *vortex formation number*,

$$\hat{T} = \frac{C\Gamma}{D\Delta U}, \quad (2.33)$$

where C is a configuration-dependent constant, typically unity for leading-edge vorticity production, D is taken as the chord, and ΔU is the velocity difference driving the feeding shear layer [8]. A number of vorticity-driven lift and propulsion problems have been found to share a common optimal formation time near $\hat{T} \approx 4$ [7, 12], though this universality does not hold in every case, with departures attributed to differences in the detachment mechanism discussed above [11]. Even where the precise value of $\hat{T}$ varies, the formation number remains a useful guide to when LEV growth, and the lift it sustains, should be expected to level off.

Vorticity is not shed only at the leading edge. As the LEV develops, the trailing edge continues to shed vorticity of opposite sign, giving rise to a *trailing-edge vortex* (TEV), likewise shown in Fig. 2.10. This is in fact the same structure as the starting vortex of Section 2.4.1, which for attached flow is the only vortex ever shed; once the flow separates, shedding instead continues alternately from both edges. In the early stages of the motion the LEV and TEV circulations are nearly equal in magnitude and opposite in sign, while the bound circulation on the wing itself stays small [8, 28], so that the lift produced is better

explained by added mass together with the LEV than by any bound vortex of attached-flow theory. This picture is reinforced by measurements showing that the LEV circulation exceeds what thin-airfoil theory would assign to bound circulation [28].

The LEV governs lift directly, by inducing a region of strong suction on the wing surface immediately beneath it, a region that grows in both size and strength as the vortex itself grows [28]. The resulting lift coefficient can therefore exceed anything achievable in steady, attached flow at the same incidence, but only for as long as the vortex remains close to the surface. Once it sheds and is carried downstream, its low-pressure footprint departs with it and the lift collapses. A wing accelerated from rest at high incidence should therefore show a lift history that rises to a pronounced peak within the first several chords of travel and then falls away, with the exact timing of that peak set by the acceleration profile rather than fixed [22, 3]. Beyond this initial peak, the continued alternating shedding of leading- and trailing-edge vortices produces a train of successively weaker peaks, and a genuine steady state is not reached until well past $t^* = 14$ [24].

2.5.3 The LEV on finite wings

Everything above describes a two-dimensional flow, in which the tilting term of Eq. 2.32 plays no role. A finite wing cannot be treated this way. Separation occurs simultaneously at the leading edge, trailing edge, and both wingtips from the very onset of motion, and the resulting vortex sheets join into a single closed loop [34]. This early wake topology depends only weakly on aspect ratio, but as the motion proceeds the tip vortices intensify and the wake develops a strong aspect-ratio dependence. On wings of very low aspect ratio the tip vortices can dominate the entire span and pin the leading-edge sheet against the surface; at higher aspect ratio their influence is limited to the outboard region, and the LEV instead bows away from the surface into an arch. On a rectangular planform, nonetheless, the sharp corners of the planform restrict spanwise vorticity transport between the LEV and the tip

vortices, so the two structures tend to remain distinct [8, 34, 39].

Sweeping the leading edge introduces something a straight wing does not have, a spanwise component of flow along the vortex axis, offering the LEV an outlet for vorticity that is otherwise unavailable. This is often likened to the delta wing, whose highly swept leading edge sustains a stable vortex well beyond the incidence at which an unswept wing would stall [19], and to the revolving wings of flapping insects, on which the LEV never sheds at all [21, 10]. The insect case, however, is not a clean analogue, since what stabilizes the LEV there is not spanwise flow in isolation but a Coriolis effect, arising from rotation, that tilts vorticity in a sense which cancels part of the LEV directly [21, 23]. A translating swept wing has spanwise flow but no rotation, and therefore no such Coriolis contribution. Where studies have isolated spanwise flow on its own, it has proven insufficient to reproduce the stabilization seen under rotation [4], and the broader literature on swept wings in translation report findings that sweep reshapes the wake topology and quasi-steady force measurements [38, 6, 20, 37, 13] but tell little of the initial LEV characteristics. Whether and how sweep governs the LEV structure and the transient force history of a wing accelerated from rest is therefore still unresolved, and it is this question the present study takes up.

2.5.4 Theoretical low-order model

The mechanisms developed above can be assembled into a low-order model for the lift of a wing accelerated from rest. The version adopted here follows Babinsky et al. [3] and Stevens et al. [33] as part of the AVT-202 comparison [26]. It is not offered as an accurate predictor of the measured loads, but as a way of decomposing the total lift into contributions whose physical origin is already understood, so that the force history can be tied back to the kinematics producing it, which is the same purpose those authors intended it for.

The total lift is taken as the sum of a noncirculatory and a circulatory part,

$$C_L = C_{L,nc} + C_{L,circ}, \quad (2.34)$$

a decomposition which, as touched on in Section 2.4, is mathematically rigorous within potential flow but not experimentally separable; the two contributions are distinguished here by their dependence on the kinematics rather than by any measurement.

The noncirculatory contribution is simply the added mass of Eq. 2.30. It should be noted the noncirculatory lift coefficient is derived for a 2D flat plate via potential flow and therefore finite wing or sweep corrections are not applied to this term.

The circulatory contribution, by contrast, is built from the impulse carried by the vortex system rather than from any bound circulation. Three simplifying assumptions underlie it, that the leading- and trailing-edge vortices form as a pair of equal and opposite strength, consistent with Kelvin’s theorem, that essentially all of the lift-producing circulation resides in the LEV, and that the bound circulation itself is negligible [3]. This last assumption agrees with the small early-time bound circulation reported in Section 2.5.2 for separated flow. A counter-rotating pair of circulation Γ and separation d carries a hydrodynamic impulse of $\rho\Gamma d$, and since force is the rate of change of impulse, differentiating this product produces two distinct contributions,

$$C_{L,circ} = \frac{2}{U_\infty^2 c} \left[\underbrace{(u_{LEV} - u_{TEV}) \Gamma_{LEV}}_{\text{vortex motion}} + \underbrace{(x_{LEV} - x_{TEV}) \dot{\Gamma}_{LEV}}_{\text{vortex growth}} \right], \quad (2.35)$$

each with a distinct physical role [3].

Of the two, *vortex growth* is the easier to picture. As the feeding shear layer strengthens the LEV, Γ_{LEV} climbs, and a pair of fixed separation but growing strength carries a growing impulse. It is largest immediately after the wing sets in motion, while $\dot{\Gamma}_{LEV}$ is at its

greatest, and fades as the vortex approaches its limiting strength. This behavior is precisely what the Wagner function of Section 2.4.2 describes, under the assumption that LEV and bound circulation may be grouped together with the latter taken as negligible [3], and that Γ_{LEV} tends toward the steady value $\Gamma_{ss} = \frac{1}{2}acU(s)\alpha$, though the true circulation is likely somewhat below this [3].

Vortex motion is the less intuitive of the two, and it is what keeps the lift from vanishing once growth has run its course. Even with Γ held fixed, a pair whose members are drawn apart carries an impulse that grows with their separation, and the rate of that growth is itself a force. Separation increases because the LEV and TEV do not travel at the same speed. The LEV, still close to the wing, is carried along with it, while the TEV, once shed into the wake, is left behind. The resulting lift scales with the relative speed between the two, and because that relative speed persists for as long as both vortices exist, this term tends toward a constant rather than decaying away. It is therefore the vortex motion term that fixes the level to which the force history eventually settles, with vortex growth responsible instead for the peak that comes before it [3].

Three quantities must still be supplied to close the model, whether from experiment or from estimation. These are the core-to-core separation of the LEV and TEV, their relative advection velocity, and the circulation of the LEV along with its growth rate [26]. The separation is taken here as one chord, $x_{LEV} - x_{TEV} = c$, and the relative advection velocity as $u_{LEV} - u_{TEV} = 0.5U_\infty$, values reported for kinematics comparable to those of this study [26]. The circulation itself follows from lagging its steady value behind the motion, using the indicial response of Section 2.4.2 written as a Duhamel convolution,

$$\Gamma(s) = \int_0^s \frac{d\Gamma_{ss}}{d\sigma} \Phi(s - \sigma) d\sigma, \quad (2.36)$$

with Φ being Wagner's function [16]. Since the wing begins from rest, Γ_{ss} vanishes at the

onset of motion, and the circulatory lift accordingly builds up from zero with no need to alter Φ itself. Setting $a = a_0 = 2\pi$ and taking Φ as the two-dimensional Wagner approximation of Eq. 2.28 yields the two-dimensional model, plotted together with its individual contributions in Fig. 2.11. There, the lift peak near the end of the acceleration phase is seen to be governed largely by the noncirculatory and vortex-growth terms together, while the steady-state asymptote is set by vortex motion alone.

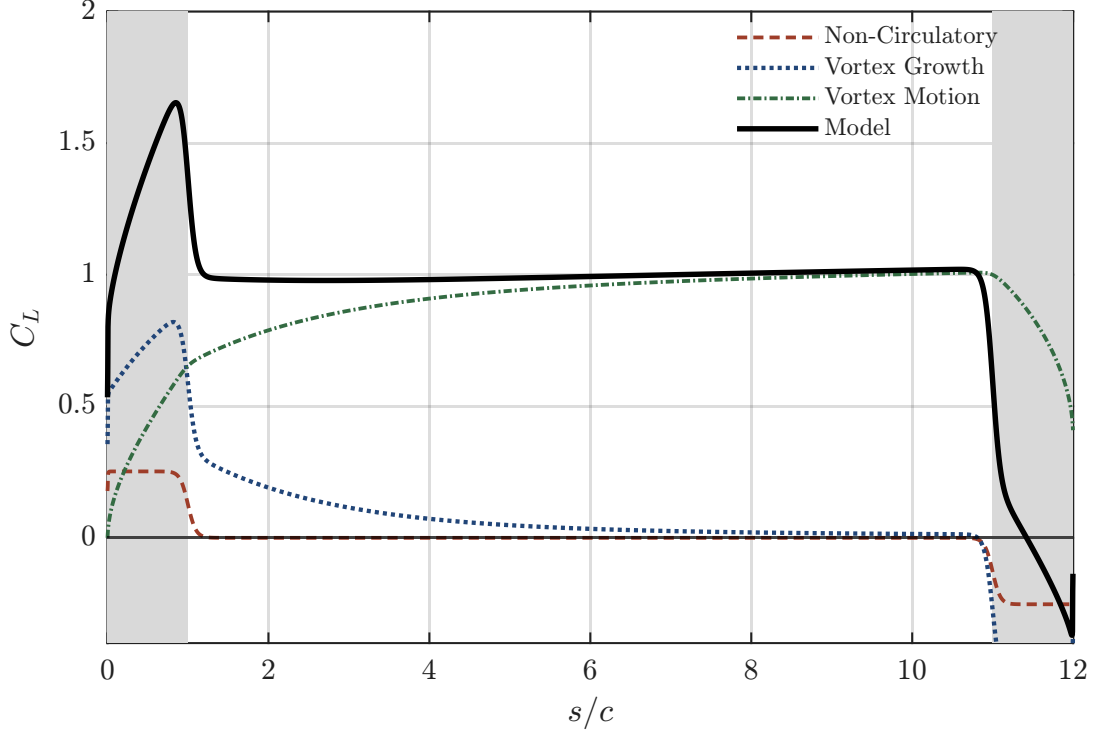


Figure 2.11: Low-order model prediction of 2D lift coefficient for a surging flat plate.

Two adjustments bring the model closer to the finite, swept wings considered here. The first replaces the two-dimensional Wagner function with the finite-aspect-ratio indicial function of Jones [16], using the Table 2.1 constants interpolated to $AR = 4$. The second replaces the 2D lift-curve slope with the swept finite-wing approximation of Eq. 2.24 [18]. The noncirculatory term is unaffected by either change, since the kinematics themselves do not vary between configurations. Fig. 2.12 shows the resulting model for $AR = 4$ at $\Lambda = 0^\circ$, 30° , and 45° alongside the unmodified two-dimensional case. The dominant effect of the

three-dimensional corrections is a substantial reduction in overall force, with both the lift peak and the steady-state level falling well below the two-dimensional prediction, as expected. Sweep itself proves less consequential than the finite-aspect-ratio correction, though a further, smaller deficiency is still visible in progressing from the unswept case to 45° . Both forms of the model are compared against the measured force histories in Chapter 4.

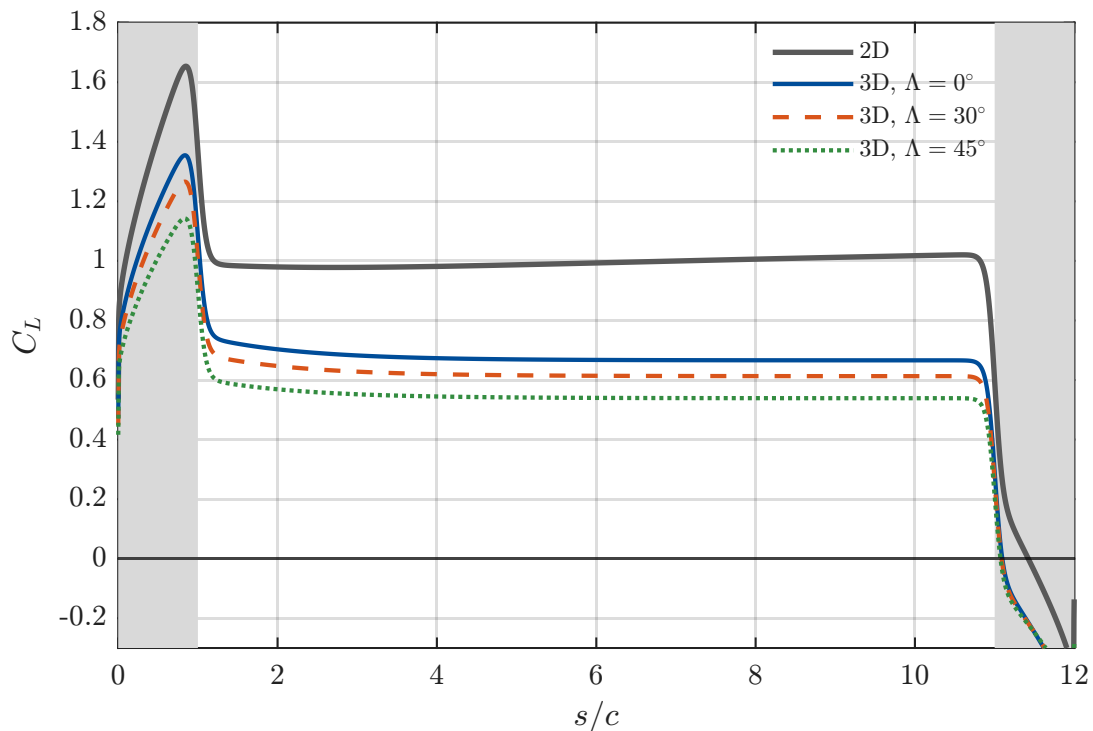


Figure 2.12: Low-order model prediction of lift coefficients for surging wing of $AR = 4$ with swept wing lift slope approximations.

2.6 Chapter Summary

This chapter developed the theoretical framework needed to interpret the results of Chapter 4, beginning from the governing equations and progressing through steady and unsteady attached-flow theory to the separated, three-dimensional flow which characterizes the present experiments. The inviscid treatment establishes the baseline against which departures are measured. Thin-airfoil and lifting-line theory give the lift-curve slope and its finite-wing

correction, Wagner’s function and the added-mass relations describe how circulation and non-circulatory loading develop under unsteady motion, and the low-order model of Section 2.5.4 combines these with a simple vortex-impulse argument to give a first prediction of the transient lift, including its dependence on aspect ratio and sweep. None of this machinery, however, has anything to say once the leading edge separates, which it does at the angle of attack tested in this study.

The viscous, separated-flow picture developed in Section 2.5 fills that gap. The leading-edge vortex is shown to be a self-sustaining but finite-lived structure, fed by a shear layer whose strength is set by the vorticity flux across its boundary, and capable of enhancing lift well beyond what attached flow could produce for as long as it remains close to the surface. Its eventual detachment, whether through bluff-body shedding or shear-layer choking, sets a natural timescale for the transient captured by the formation number, and its interaction with the trailing-edge vortex and, on a finite wing, the tip vortices governs the three-dimensional evolution of the flow. Sweep introduces a spanwise route for this vorticity that a straight wing does not have, but whether that mechanism meaningfully alters the LEV or the resulting forces remains unresolved in the literature, and answering that question for the present configuration is the objective of the results which follow.

Chapter 3

Experimental Methodology

This chapter describes the experimental apparatus and procedures used to obtain the results of Chapter 4. All experiments were conducted in a towing tank, in which a wing model was accelerated from rest along a straight path at a fixed angle of attack. Two separate campaigns were carried out. The first measured the transient aerodynamic loads on the wing using a six-axis force transducer, and the second visualized the evolution of the flow using dye injected through the wing surface.

3.1 Towing Tank Apparatus

3.1.1 Tank

Experiments were performed at the University of Southern California in their towing tank, a glass-walled tank measuring $1 \times 1 \times 2.5$ m with a free surface. The tank is filled with tap water and can easily be drained and filled. A photograph of the facility is shown in Fig. 3.1.

The wing spans 400 mm, which is 40% of the tank width and leaves three chord lengths from

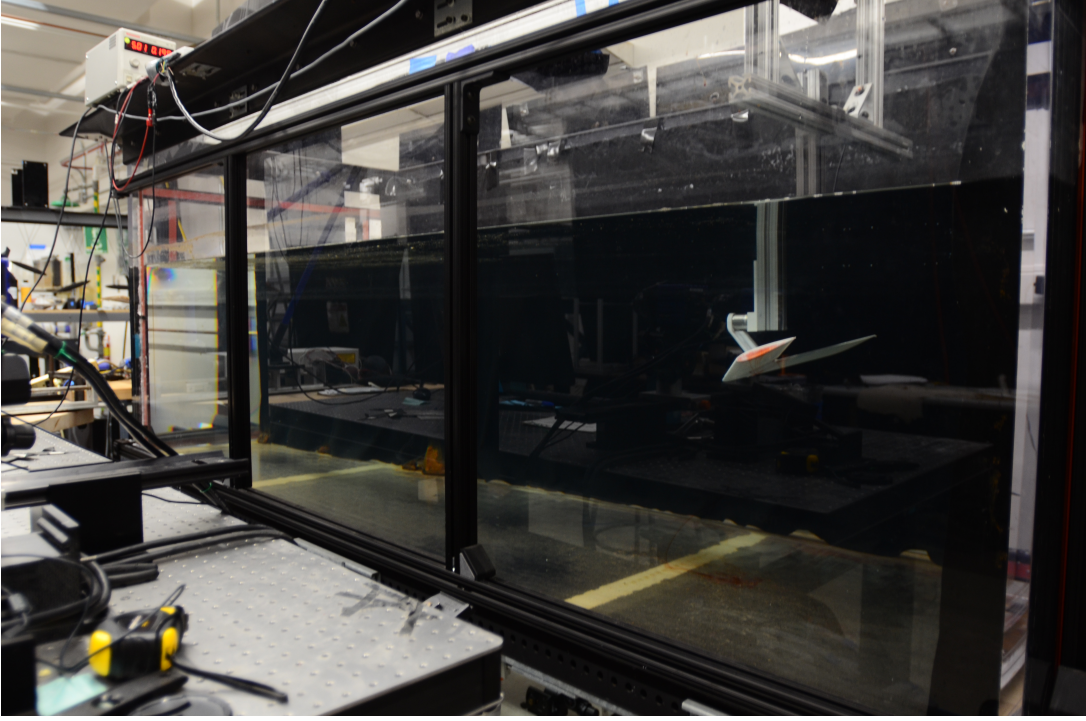


Figure 3.1: Photo of USC towing tank.

tip to wall. This is a substantial fraction of the span direction, but the relevant measure of interference is the frontal area presented to the flow. At the angle of attack tested, the projected area of the wing is $S \sin \alpha = 0.0137 \text{ m}^2$ against a tank cross-section of 1 m^2 , giving a blockage ratio of 1.37%. No blockage correction was applied. Additionally, it was assumed the tip vortex did not have sufficient time to grow and interact with the wall for the measurements taken. The wing was submerged with its midspan approximately 4 chords below the free surface and 5 chords above the tank floor.

Because the tow velocity required to achieve a given Reynolds number depends on the kinematic viscosity of the water, and the water temperature in the tank varied between test sessions, fluid properties were determined at the start of each session. Water temperature and density were measured using a Mettler Toledo, the dynamic viscosity was obtained from standard tables at the measured temperature, and the kinematic viscosity was computed as $\nu = \mu/\rho$. The commanded tow velocity was then recalculated as $U_\infty = Re \nu/c$ so that the

Reynolds number was held fixed across sessions. Over the course of the test campaign the water temperature ranged from 21 to 27°C.

3.1.2 Towing and motion control

The wing was mounted to a carriage traversing the length of the tank on a linear rail, shown in Fig. 3.2. Motion was commanded by a Parker ACR7000 motion controller driving a Parker IPA04-HC servo drive and an MPJ0922C1E-NPSN brushless AC servo motor. The traverse resolution was manually measured to be 135.159 encoder counts per millimeter of carriage travel. The drive was powered from a 120 VAC supply against a rated 240 VAC input, which halves the available bus voltage and correspondingly limits the peak torque available during acceleration. This constraint is the principal limit on the maximum Reynolds number attainable with the present wing, and is discussed further in Section 3.3.2. Motion

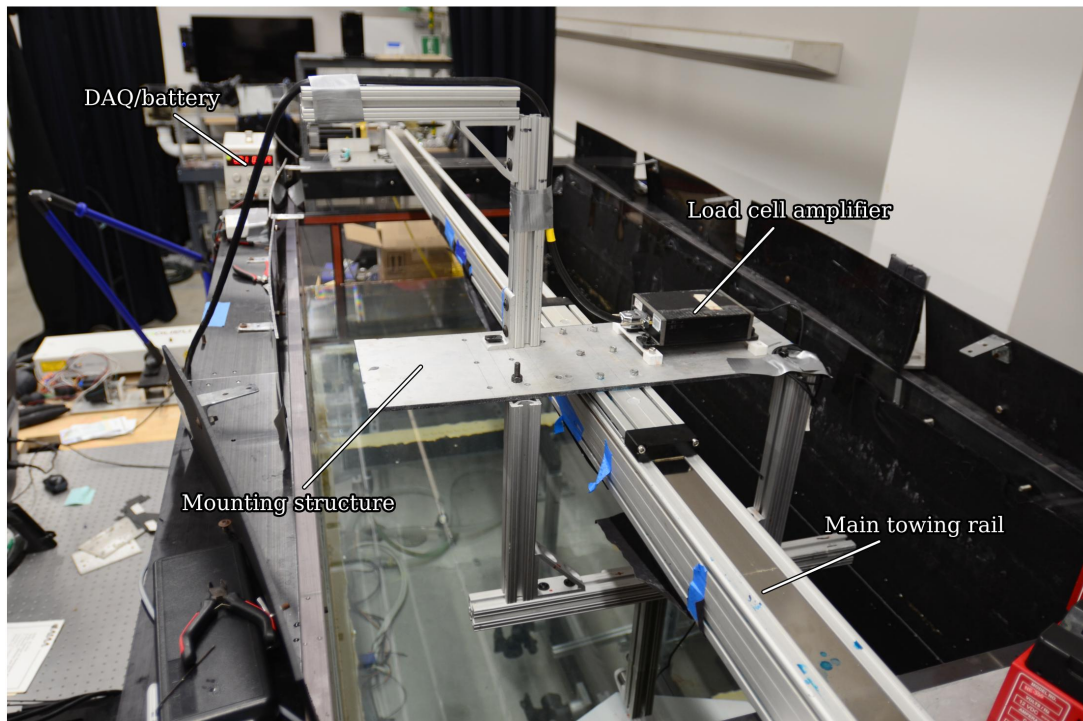


Figure 3.2: USC towing rail and electronics.

was programmed in AcroBasic using the controller’s electronic cam functionality rather than a canned trapezoidal move. The cam approach was chosen because it allows an arbitrary

position profile to be prescribed point by point, which is necessary to realize the smoothed velocity profile described in Section 3.3. The position table was generated in MATLAB, uploaded to the controller as four cam segments corresponding to the acceleration, cruise, deceleration, and hold phases of the motion, and clocked against the controller’s internal 1 kHz timebase.

Initial testing at the highest Reynolds numbers revealed a position-tracking error which grew during the acceleration phase. This was traced to torque saturation of the drive consistent with the supply voltage limitation noted above. The servo gains were retuned to reduce the tracking error, with the proportional gain increased and the integral gain reduced relative to the original settings, and a velocity feedforward term introduced. The final gain settings are listed in Table 3.1. Following tuning, the peak position error during the motion relative to the desired trajectory was reduced to approximately ± 10 mm.

Table 3.1: Servo loop gain settings following tuning, used for all runs reported in this study.

Parameter	Value
Proportional gain, K_P	1.2×10^{-2}
Integral gain, K_I	4.0×10^{-3}
Derivative gain, K_D	1.0×10^{-4}
Velocity feedforward, K_{FF}	2.2×10^{-5}

Because the convective time axis used throughout Chapter 4 is constructed from the prescribed position profile, it was necessary to confirm that the commanded motion was actually achieved. The controller was configured to sample the actual position, commanded position, following error, and torque command at 1 ms intervals for the duration of each run. These records were compared against the analytic velocity profile in convective time, as shown in Fig. 3.6b. Data analysis of the encoder position over the duration of the motion confirmed a 5.7 mm RMS error between the analytical profile. Agreement between the measured and commanded profiles confirms the validity of the s/c mapping used in Chapter 4.

3.2 Wing Models

The wing is an untapered planform of NACA 0012 section with a chord of $c = 100$ mm and an aspect ratio of $AR = 4$, giving a span of 400 mm and a planform area of $S = 0.04$ m². Three sweep configurations were tested, $\Lambda = 0^\circ$, 30° , and 45° . Because the planform is untapered, the leading edge, trailing edge, and quarter-chord lines are all swept by the same angle, and Λ is quoted as the leading-edge sweep throughout. All models were tested at a fixed geometric angle of attack of $\alpha = 20^\circ$. The three planforms are shown in Fig. 3.3. To better illustrate the geometrical effects of sweep and the relative positioning of the wing-tips, Fig. 3.4 overlays all planforms with the root chord leading edge aligned.

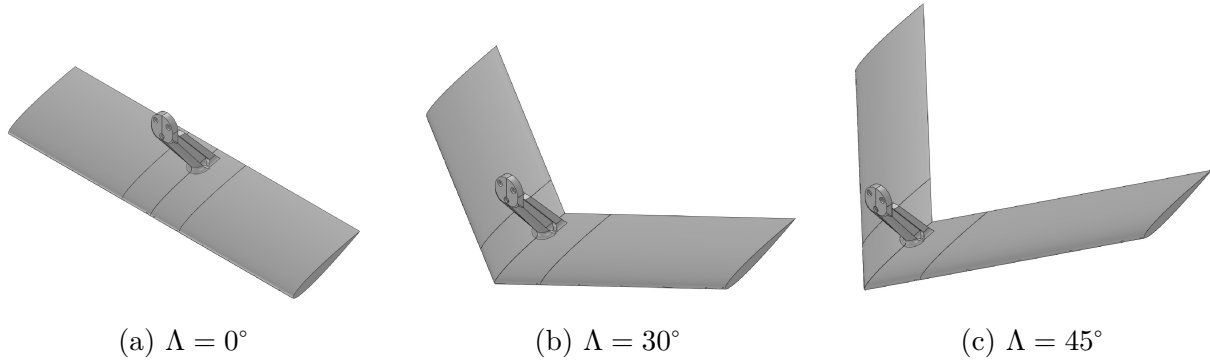


Figure 3.3: Wing model designs of all sweep configurations.

The models were designed using CAD and additively manufactured with Hatchbox PLA Pro 1.75 mm filament using a Creality CR-10S Pro V2 3D printer. Each wing was printed in three spanwise sections, joined by wooden dowels seated in the mating faces of the central section. Segmenting the model in this way was necessary to accommodate the build volume of the printer, and had the additional benefit of providing access to the internal dye passages described in Section 3.5.1. After assembly the surfaces were sanded to remove print layer lines and sealed with liquid rubber Flex Seal to prevent dye leaks.

The wing was mounted in an inverted orientation, with the suction surface facing the floor of the tank, so that the flow over the suction surface and the developing leading-edge vortex

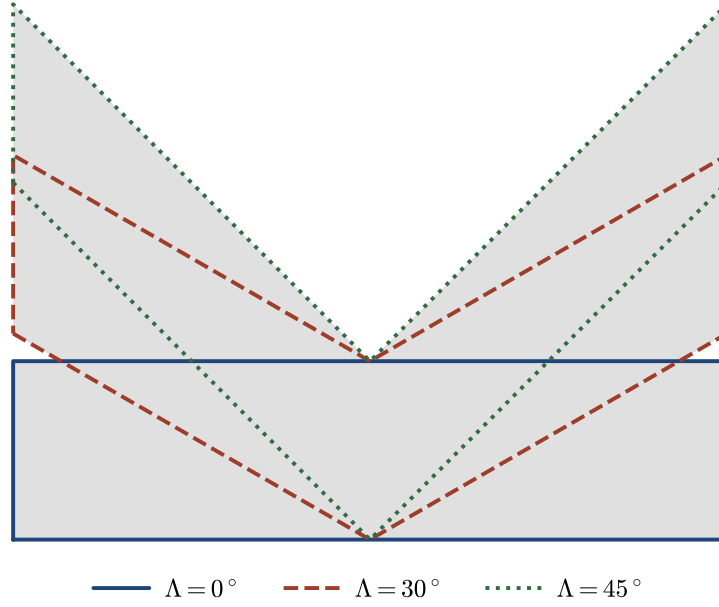
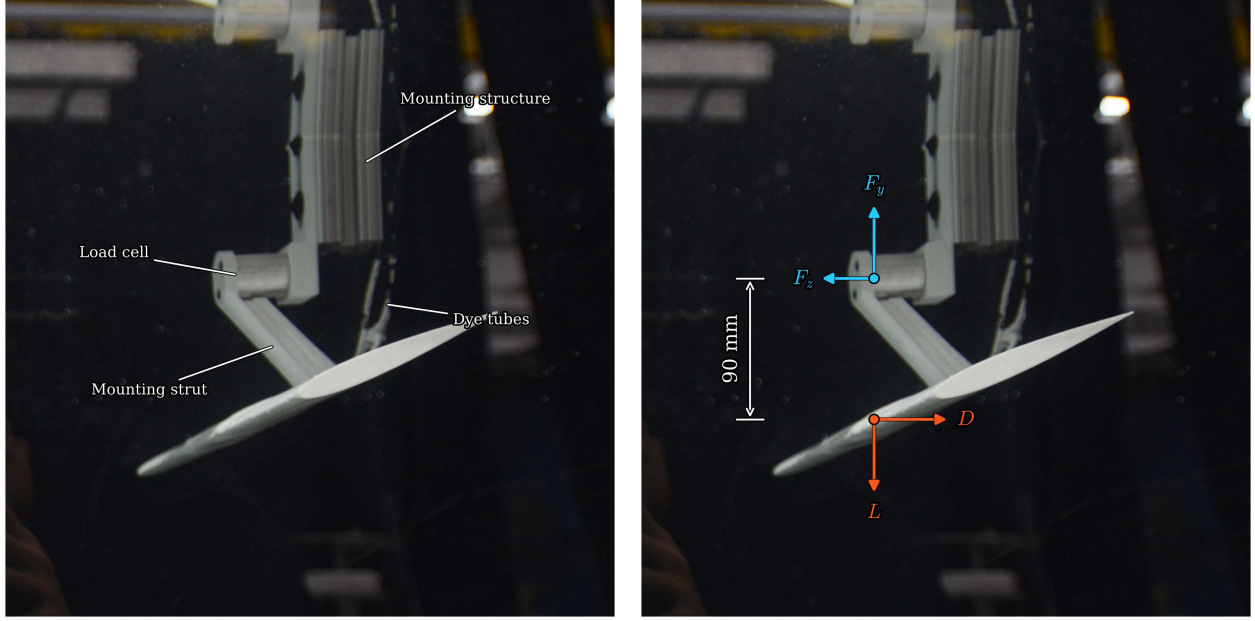


Figure 3.4: Swept wing planform overlay.

were not influenced by the wake of the supporting structure and allowed for undistorted imaging. The model was attached to the force transducer through a single strut. The strut ensures the transducer origin lies at the quarter-chord of the wing in order to minimize the aerodynamic moment transmitted, as described in Eq. 2.18. For all models, the vertical spacing between the wing root chordline and the load cell origin is exactly 90 mm. The mounting arrangement is shown in Fig. 3.5a.

Because the wing is mounted inverted, the transducer origin axes are not aligned with the conventional aerodynamic axes. The sensor $+x$ direction opposes lift and the sensor $+z$ direction opposes drag. The measured components are therefore related to the aerodynamic loads by $L = -F_x$ and $D = -F_z$. The sensor and wing axis systems are shown in Fig. 3.5b.



(a) Photo of model mounting.

(b) Coordinate system of wing and load cell.

Figure 3.5: Wing model mounting and coordinate system.

3.3 Kinematics

3.3.1 Velocity profile

The commanded motion is a trapezoidal velocity profile whose corners have been smoothed using the smoothing function of Eldredge et al. [9]. A pure trapezoid has discontinuous acceleration at the start and end of each ramp, which excites structural ringing in the model and support and produces a discontinuous added mass force. The smoothing function replaces these corners with a continuous transition while preserving the overall shape of the profile. The smoothed profile is constructed from

$$G(t) = \ln \left[\frac{\cosh(k(t - t_1)) \cosh(k(t - t_4))}{\cosh(k(t - t_2)) \cosh(k(t - t_3))} \right], \quad k = \frac{a_s U_\infty}{c}, \quad (3.1)$$

where t_1 through t_4 mark the start of acceleration, end of acceleration, start of deceleration, and end of deceleration respectively, and a_s is a smoothing parameter which controls the

sharpness of the corners. The velocity profile follows as

$$V(t) = U_{\infty} \frac{G(t)}{\max[G(t)]}, \quad (3.2)$$

which is normalized so that the profile reaches the intended cruise velocity. A smoothing parameter of $a_s = 10$ was used throughout. The resulting velocity profile is shown in Fig. 3.6a with the acceleration portion of the movement shaded in gray, along with the encoder verification of the position profile. Although the encoder largely verified the kinematics, it was discovered after the testing was complete that the point by point cam motion programmed to the controller did not include the initial points to command a smooth acceleration on the first corner. The result is a brief discontinuity in acceleration during the motion onset. The missing section of the profile is illustrated in Fig. 3.7. Unfortunately this error resides in the following results, however the initial jerk was deemed to have minimal impact on the overall flow structure and measurements.

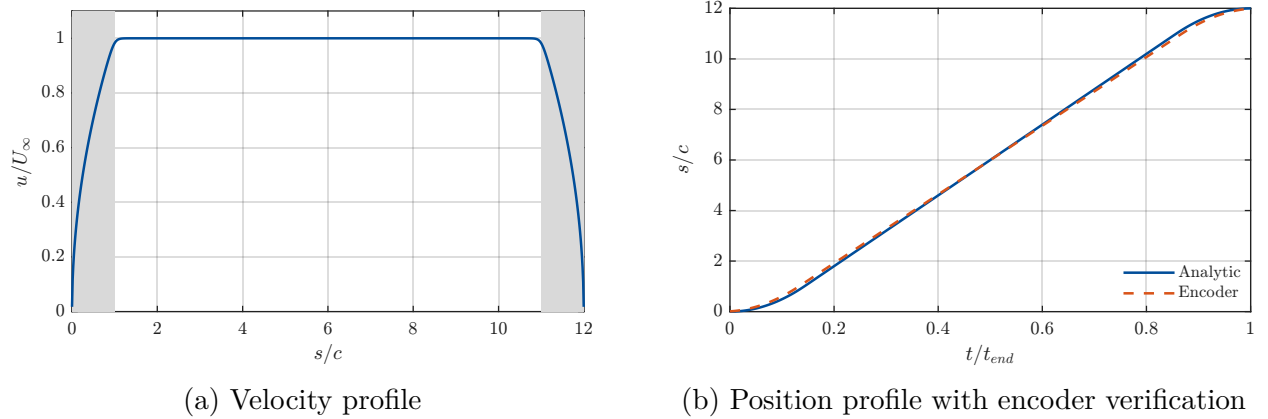


Figure 3.6: Kinematics of surging wing accelerating over 1 chord.

The motion consists of a one-chord acceleration, a constant-velocity cruise, and a one-chord deceleration. Prescribing the acceleration in terms of chords traveled rather than in terms of time ensures that the acceleration phase occupies the same portion of the convective time axis regardless of Reynolds number, so that the added-mass contribution is confined to

$s/c < 1$ in every case. For the force measurements the cruise phase was 10 chords, giving a total travel of 12 chords or 1.2 m. This leaves the model well clear of the tank end walls throughout the motion and allows the transient to be largely complete before deceleration begins. For the flow visualization the cruise phase was shortened to 6 chords, giving a total travel of 8 chords, for reasons described in Section 3.5.2.

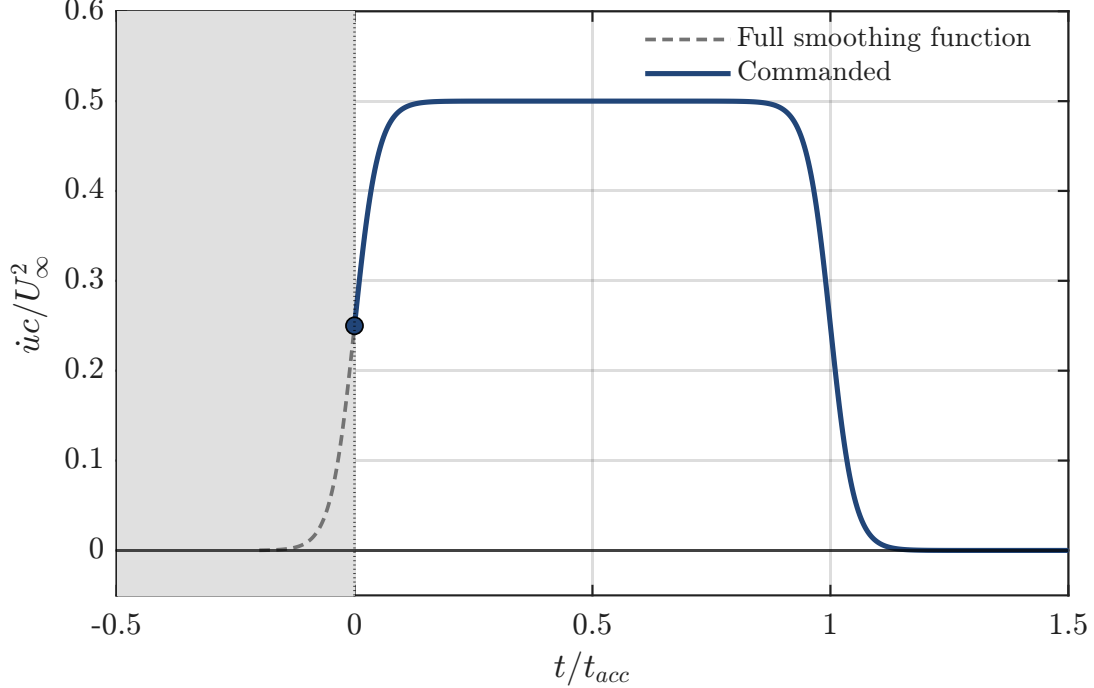


Figure 3.7: Acceleration deficit missing from programmed motion control.

3.3.2 Test conditions

Force measurements were acquired at $Re = 100,000$ and dye visualization at $Re = 5,000$. The separation of the two campaigns in Reynolds number was a deliberate consequence of the competing requirements of the two measurement techniques.

The force measurements were run at the highest Reynolds number the facility could achieve with the present model. The hydrodynamic loads scale with dynamic pressure, and the Nano25 transducer used here is a general-purpose six-axis sensor with a considerably larger full-scale range than the purpose-built balances employed in several comparable studies; a

higher dynamic pressure is therefore required to produce a signal that is large relative to the resolution and noise floor of the transducer. The attainable Reynolds number is bounded above by the tow velocity the drive can reach within the torque limitation described in Section 3.1.2, and by the tank length available for the acceleration and cruise phases. $Re = 100,000$ represents the practical maximum under these constraints.

The dye visualization requires the opposite compromise. At Reynolds numbers of order 10^5 the dye convects and diffuses too rapidly to remain a coherent marker of the vortex structure over the several chords of travel of interest, and the resulting structures move too quickly to be captured cleanly at the available shutter speed and frame rate. Reducing the Reynolds number much below 5,000 is equally unsatisfactory. The acceleration phase becomes so slow that the dye cannot be injected at a rate which marks the flow without disturbing it, and the total duration of the run exceeds the number of frames the camera can capture in a continuous sequence. $Re = 5,000$ is the compromise at which the dye remains coherent, the motion is slow enough to photograph, and several chords of travel can be captured within a single burst.

The consequence of this separation is that the force and visualization data are not simultaneous, and the flow structures observed at $Re = 5,000$ are used to interpret rather than to explain the loads measured at $Re = 100,000$. This is justified by the observation that the early development of the leading-edge vortex is relatively insensitive to Reynolds number over this range [26], though the comparison remains a qualitative one and is treated as such throughout Chapter 4.

3.4 Force Measurement Procedure

3.4.1 Transducer and Acquisition

Aerodynamic loads were measured using an ATI Nano25 IP68 six-axis force/torque transducer, serial number FT12919, with an SI-125-3 calibration. The calibrated ranges are ± 125 N in F_x and F_y , ± 500 N in F_z , and ± 3 N·m in all three moment components. The effective resolution in the x and y axis are $1/48$ N and $1/1320$ N·m. The z axis is $1/16$ N and $1/2640$ N·m, and it is for this reason the transducer was mounted in the configuration shown in Fig. 3.5b to produce a cleaner lift signal as the dominant force of interest. The manufacturer quotes a measurement uncertainty at 95% confidence of 1.00% of full scale for the force components and 1.25% for the moments. The calibration is traceable to NIST and was performed on 14 March 2014; the transducer has not been recalibrated since, and the quoted uncertainties are therefore treated as a lower bound in Section 3.6.

Transducer output was recorded through a National Instruments USB-6210 data acquisition device using six differential analog input channels configured for a ± 5 V range at 16-bit resolution. Data were sampled at 10 kHz and decimated by a factor of ten to an effective rate of 1 kHz using a finite impulse response filter. Oversampling and decimating in this manner provides anti-alias protection and reduces the broadband noise on each channel relative to sampling directly at 1 kHz, while the linear-phase FIR filter preserves the timing of the transient features of interest. Acquisition was controlled from MATLAB.

The six gauge voltages were converted to forces and moments using the 6×6 calibration matrix ($\mathbf{C}$) supplied with the transducer,

$$\mathbf{F} = \mathbf{C} (\mathbf{V} - \mathbf{V}_{\text{bias}}), \quad (3.3)$$

where $\mathbf{V}_{\text{bias}}$ is the bias voltage vector determined as described below.

3.4.2 Taring

Before each run the transducer was tared by recording two seconds of data with the model stationary and at rest, and averaging to obtain the bias voltage vector. Each record was additionally inspected against the calibrated load limits to confirm that no channel approached saturation, and the unbiased force traces were monitored across a session to check for drift in the transducer output.

Ten runs were recorded for each sweep angle. Consecutive runs were separated by 3 – 5 minutes to allow residual motion in the tank to decay. The ten records for each configuration were ensemble averaged after alignment as described in Section 3.4.3, and the run-to-run scatter is reported as a shaded band about the mean in applicable figures of Chapter 4.

It should be noted there was an attempt to record the force measurements in air with the wing raised above the water line. In theory, this should help isolate the hydrodynamic contribution of force by subtracting the air record from the water. However, the increased structural noise present in air ultimately provided little to no benefit to the final measurements, with less than a 1% difference in water only records. Therefore, the measurements shown in Chapter 4 are water only records. Additionally, any hydrodynamic contribution of the mounting strut shown in Fig. 3.5 is not accounted for and represents the inherent error that exists in experimental studies.

3.4.3 Synchronization and filtering

Each force record begins with the model stationary, and the mean of this pre-motion interval is subtracted from the record to remove any residual offset. The instant at which motion begins must then be identified in each record so that runs can be aligned with one another and mapped onto a common convective time axis. Onset is detected by searching forward through the record for the first crossing of a fixed threshold of 0.5 N in the lift trace. The threshold itself is not a sensitive parameter: the noise floor of the stationary record is

approximately 0.02 N, while the force during the acceleration phase is of order 5 N, so any threshold spanning two orders of magnitude between these values returns the same instant to within a few milliseconds. As a check on the detection, the interval between the detected onset and the subsequent lift peak is compared against the acceleration time $t_{acc} = 2c/U_\infty$. Across all thirty records this interval was 0.209 to 0.219 s against a nominal $t_{acc} = 0.207$ s, confirming that the onset is correctly identified in every run.

The records show a small, repeatable disturbance in the tens of milliseconds preceding the detected onset. It appears simultaneously in the lift and drag channels, is oscillatory rather than monotonic, and is coherent across runs. These characteristics identify it as the structural response of the model and support to the motion of the carriage rather than as an aerodynamic load, and it is not treated as such in what follows. Additionally, the mapping from physical time to s/c stretches this portion of the movement, causing a 10 ms feature to appear large before the $s/c = 0$ mark.

Aerodynamic loads were obtained by ensemble averaging the ten records acquired for each configuration after alignment. Averaging N records whose fluctuations are uncorrelated between runs reduces the variance of the mean by a factor of N ,

$$\text{Var} \left(\frac{1}{N} \sum_{i=1}^N x_i \right) = \frac{1}{N^2} \sum_{i=1}^N \text{Var}(x_i) = \frac{\sigma^2}{N}, \quad (3.4)$$

so that the standard deviation of the mean is smaller than that of a single record by $\sqrt{N}$. Content which repeats identically from run to run is unaffected by averaging, so the reduction actually achieved indicates how much of the fluctuation is random. In the present records the high-frequency content falls by close to the full factor of $\sqrt{10}$, confirming that it is dominated by run-to-run variability of the flow rather than by a deterministic response to the motion. A large part of this random content is likely electrical noise, confirmed by 60 Hz spikes seen in PSD plots of signal measurements for each axis. The scatter band reported

with the averaged histories in Chapter 4 therefore represents genuine variability of the flow between nominally identical runs, and not measurement uncertainty.

A residual oscillation remains in the averaged records, arising from the compliance of the mounting structure, the material of the model, and vibration of the towing rail. The frequency of this mode was identified from the power spectral density of the lift signal, estimated using Welch’s method, and lies near 27 Hz for the swept models and 55 Hz for the unswept model. A second-order Butterworth low-pass filter with a cutoff of 15 Hz was applied to remove it. The cutoff was selected to lie above the frequency content of the transient itself, which occupies roughly 1.5 s and therefore has no significant content above approximately 10 Hz, while remaining below the lowest structural frequency observed. The filter was applied in the forward and reverse directions so that the net phase response is zero and the temporal locations of features in the force history are preserved, which is necessary given that the timing of the measurements is a principal result of this study. The effects of averaging and of filtering are illustrated in Fig. 3.8.

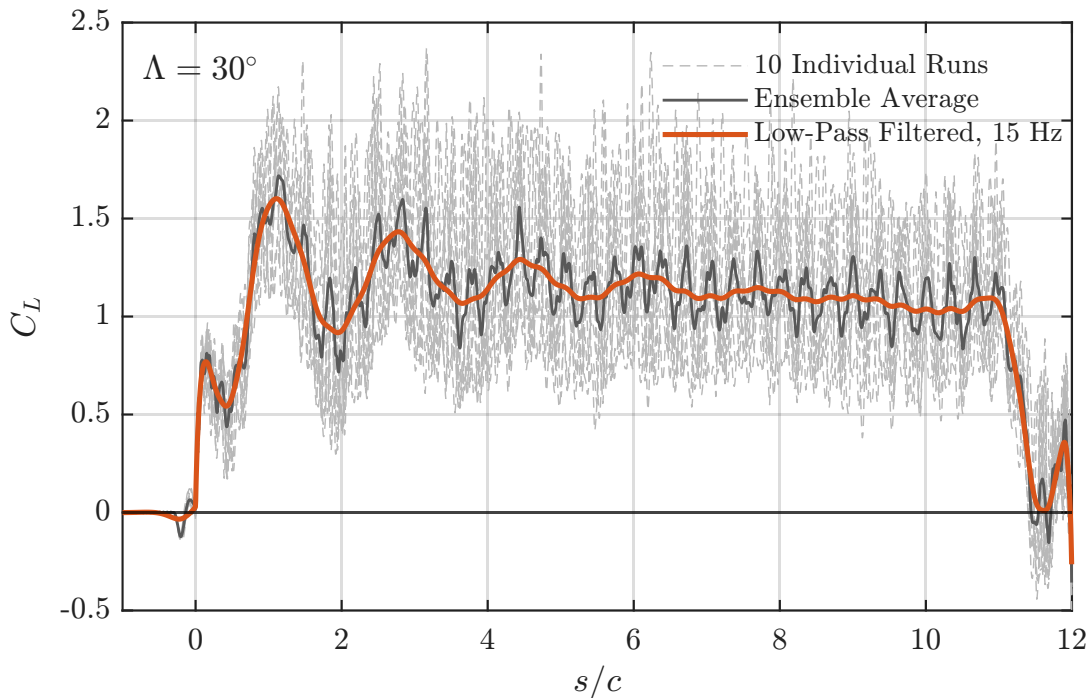


Figure 3.8: Example of unfiltered, ensemble averaged, and filtered lift signature for $\Lambda = 30^\circ$.

3.4.4 Nondimensional mapping

The force records are mapped from physical time onto the convective time axis s/c using the prescribed position profile. The position is obtained by numerical integration of the velocity profile of Eq. 3.2, and each sample in the aligned force record is assigned the distance traveled at that instant. The records are then interpolated onto a uniform s/c grid spanning the interval from one chord before onset to twelve chords after, so that all runs and all configurations share a common abscissa. Distances are quoted in chords throughout, as distinct from the semi-chord convention of Section 2.4.2. This convention was chosen over t^* since that implies a uniform freestream velocity throughout.

Lift, drag, and pitching moment coefficients are formed in the conventional manner,

$$C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 S}, \quad C_D = \frac{D}{\frac{1}{2}\rho U_\infty^2 S}, \quad C_m = \frac{M}{\frac{1}{2}\rho U_\infty^2 S}, \quad (3.5)$$

Normalizing by the cruise velocity means that the coefficients are directly comparable between configurations and across the whole of the motion, but it also means that the coefficients during the acceleration phase should not be interpreted as conventional aerodynamic coefficients, since the instantaneous dynamic pressure during this phase is lower than the reference value. Dimensional forces are therefore reported alongside coefficients where the acceleration phase is under discussion.

3.5 Flow Visualization

3.5.1 Dye injection system

Dye was introduced into the flow through orifices distributed along the leading edge of the wing, fed by internal passages formed during printing. A separate internal channel supplies a second set of orifices near the wing tip, allowing dye of a different color to be released there.

Marking the tip region independently of the inboard span makes it possible to distinguish the tip vortex from the leading-edge vortex in the resulting images, and to observe any spanwise transport between them. Blue dye was used for the inboard leading-edge ports and red for the tip channel. Five inboard orifices exist on each semi-span, separated equally by 17 mm. The tip orifice is located 1 mm inwards from the wing tip. This results in the outermost inboard orifice and tip orifice being separated by 14 mm. Due to the presence of pressure differentials and air bubbles, it was decided to purposely block the three most inward orifices, leaving two open ports for the blue dye. While this limits the full spanwise LEV structure that is visible, it forces dye to be injected towards the wing-tip whose interaction is of interest for this study. A wireframe view of the $\Lambda = 30^\circ$ model is shown in Fig. 3.9 to reveal the internal dye channels and spacing for the wooden dowels.

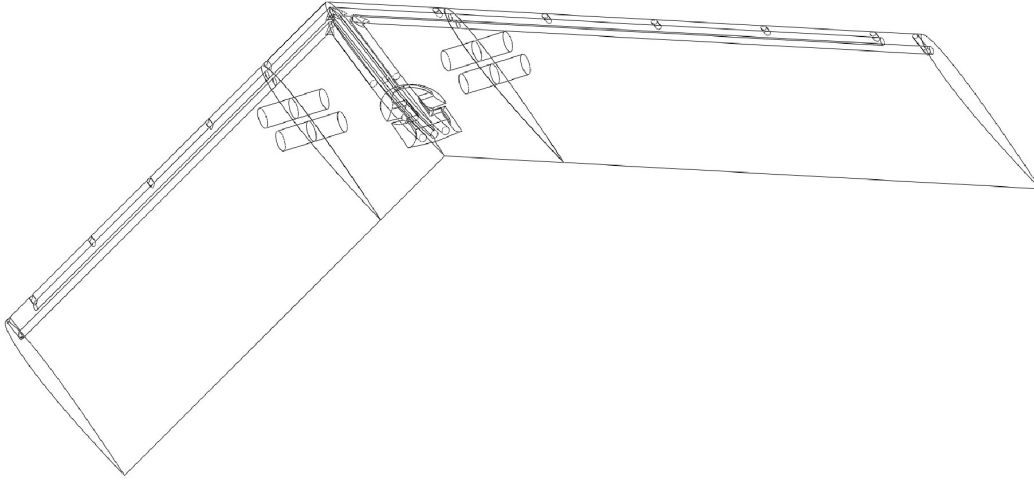


Figure 3.9: Wireframe view of $\Lambda = 30^\circ$ model to reveal internal dye structure.

Each channel was supplied by its own NE-1000 syringe pump, permitting the two flow rates to be set independently. Flexible silicone tubing connected the pumps to the model, and the junctions were sealed with a liquid rubber sealant to prevent leakage and ensure dye reached the orifices. Due to the presence of air bubbles and pressure differences, the three inboard orifices closest to the root were purposely sealed to achieve dye flow towards the outer wing

and tip region of interest. Flow rates in the range of 8 to 18 mL/min for the leading-edge channel and 1 to 5 mL/min for the tip channel were tested; the working values were selected as the lowest rates which produced a clearly visible dye filament without the momentum of the injected dye visibly disturbing the flow. The system is shown in Fig. 3.10.

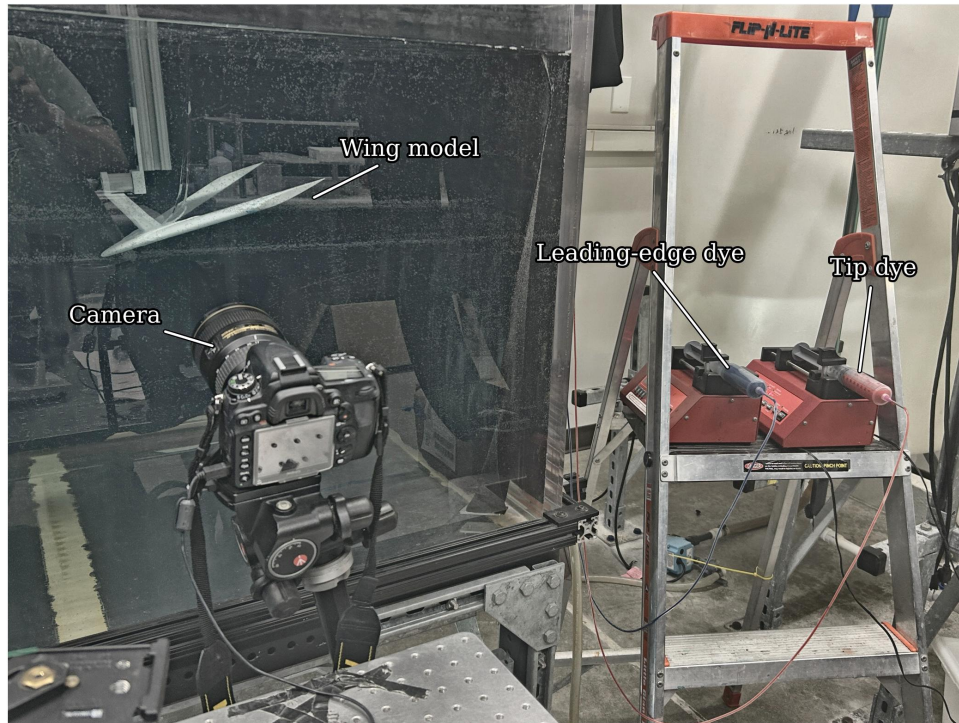


Figure 3.10: Flow visualization setup.

The dye is a mixture of food coloring, whole milk, isopropyl alcohol, and water in a volumetric ratio of approximately 1:5:1:8. The food coloring provides the hue which distinguishes the two channels and contrasts against the wing surface and the tank background. The milk helps decrease viscous diffusion and keep dye structures coherent. The isopropyl alcohol reduces the density of the mixture, and the ratio was tuned so that the dye is approximately neutrally buoyant in the tank water. Residual dye was neutralized with bleach between sessions to keep the working fluid clear.

3.5.2 Imaging

Images were captured with a Nikon D700 digital camera in two configurations. In the side-view configuration the camera views the tank through the glass side wall, giving a chordwise view of the flow over the suction surface to track vortex growth and separation points. In the bottom-view configuration the camera views the model from beneath the tank, giving a spanwise view suited to identifying spanwise structure and the interaction of the leading-edge and tip vortices. In both cases the aperture was set to $f/2.8$ and the shutter to $1/30$ s, this being the shortest exposure which admitted sufficient light at the available illumination while permitting a continuous burst of the required length. Lighting was provided by positioned flashlights, with the background lights arranged differently in the two configurations to maximize contrast between the dye and the background. Lighting was intended to be kept consistent between sessions, but slight differences were observed in some of the photos presented in Chapter 4.

The cruise phase for the visualization runs was shortened to six chords, for a total travel of eight chords, because the camera field of view spans approximately 4 chords in either configuration and the structural cross-members of the tank frame restrict the usable portion of that view. Capturing travel beyond this distance is therefore not possible without translating the camera, and the leading-edge vortex development of interest occurs well within the available window.

Image capture was initiated manually relative to the start of each run, and the first frame of each sequence was recorded so that subsequent frames could be assigned a convective time. Images were processed in Python to improve legibility. A calibration was applied to each frame to relate pixel coordinates to physical distance, using the wing apex as the reference point, so that a convective time could be assigned to each frame and a s/c axis overlaid. In doing so it is assumed the camera is approximately parallel to the wing and no major diffraction occurs. Also, due to the concentrated FOV it is assumed the linear

translation of the wing is also linear in frames and there is minimal perspective distortion. The selected frames to display were equally cropped about the point of interest to best display the dye features. Finally, the cropped frames were post-processed by increasing brightness and sharpness to better display and contrast the dye.

3.6 Uncertainty

The dominant contribution to the uncertainty in the measured loads is the calibration of the force transducer. The manufacturer quotes a measurement uncertainty at 95% confidence of 1.00% of full scale for the force components and 1.25% for the moments, applied to each axis at its own calibrated range. Because the axes have different ranges, the resulting uncertainty differs considerably between components. On the lift axis, 1.00% of the ± 125 N range is 1.25 N, which is 3.4% of the peak lift measured at $\Lambda = 0^\circ$ and 5.9% at $\Lambda = 45^\circ$. The drag is measured on the axis calibrated to ± 500 N, so the same percentage corresponds to 5.00 N; since the drag force is smaller than the lift, this is 37% of the peak drag at $\Lambda = 0^\circ$ and 54% at $\Lambda = 45^\circ$. The moment uncertainty of 0.038 N·m is between 2.4% and 4.1% of the peak measured moment. The absolute magnitude of the drag coefficient, and of any quantity derived from it such as the lift-to-drag ratio, is therefore subject to a much larger uncertainty than the lift. As noted in Section 3.4, the calibration was performed in 2014 and the transducer has not been recalibrated, so these figures are treated as a lower bound.

The random contribution is quantified directly by the repeatability of the ten runs recorded for each configuration. After alignment and filtering, the standard deviation between runs is 1.3 to 1.5% of the peak lift, rising to between 1.6 and 2.2% when averaged over the whole of the motion. This scatter increases with convective time, from approximately 0.01 in C_L at $s/c = 0.5$ to 0.03 at $s/c = 10$, reflecting the fact that the early transient is highly repeatable while the developed wake is not. As discussed in Section 3.4.3, this band represents genuine run-to-run variability of the flow rather than measurement uncertainty, the transducer noise

floor being some thirty to forty times smaller.

Uncertainty in the convective-time axis arises from two sources. The closed-loop position error of ± 10 mm corresponds to 0.10 chords, and the identification of the motion onset is uncertain by approximately 12 ms, or 0.12 chords, judged by the departure of the measured onset-to-peak interval from the acceleration time. Combining these in quadrature gives an uncertainty in s/c of approximately 0.15 chords. This is small relative to the separation between the features of interest, but it is comparable to the difference in peak location between sweep configurations, and the peak locations reported in Chapter 4 should be interpreted accordingly.

The tow velocity is set from the water temperature measured at the start of each session, as described in Section 3.1, so that the Reynolds number is held fixed with respect temperature changes. The residual uncertainty in Re is that of the temperature measurement and of the tabulated viscosity, and is well below the sensitivity of the measured loads to Reynolds number over the range considered here.

Chapter 4

Results and Discussion

4.1 Overview

In this chapter results are presented and discussed. Transient force measurements for all configurations are presented in Section 4.2 with comparisons to the theory outlined in Chapter 2. Flow visualization images are presented in Section 4.3 and depict the dynamical evolution of the flow for each configuration. When comparing the two methods it is important to recall that forces were collected at $Re = 100,000$ while images were taken at $Re = 5,000$. While studies like [26] have argued there is a Re independency especially in the early transients, it is possible some flow features differ across the methods.

4.2 Force Measurements

4.2.1 Lift

The transient lift coefficient for all sweep configurations is shown in Fig. 4.1. Vertical dashed lines at $s/c = 1$ and $s/c = 11$ indicate the acceleration corners, and shaded bands represent $\pm 1\sigma$ variability between filtered runs. As expected, the lift peak for all planforms

occurs at the end of the initial acceleration at approximately 1 chord of travel. This is the added mass contribution described by Eq. 2.30 along with circulatory vortex growth terms. For all measurements an initial spike occurs immediately after the motion onset, approximately at $s/c = 0.1$ and can be explained by the reasoning in Section 3.3. Due to the commanded motion profile missing the smoothing function after the onset, the wing experiences a high impulse momentarily and consequently causes a premature spike in force. Each planform also shows a secondary peak near $s/c = 3$. The secondary peak is suspected to be attributed to reattachment or stability of the LEV and Section 4.3 attempts to verify this through flow visualization comparisons. Near the end of the motion all lift histories converge and begin to settle near $C_L = 1$, reaffirming sweep invariance for quasi-steady lift measurements. However, due to the confined stroke length of the translation set by the tank, the measurements at $s/c = 9$ and onward are likely still exhibiting bluff-body shedding and should not be treated as quasi-steady. This is further confirmed by comparing the quasi-steady lift coefficients in Zhang et al. [38] which produced C_L between 0.4 and 0.5 for similar planforms and kinematics.

Table 4.1: Peak lift coefficient and its convective-time location for each sweep configuration.

Λ [deg]	$C_{L,peak}$	s/c
0	1.979	1.03
30	1.601	1.12
45	1.125	1.09

Some key distinctions in the planforms are the magnitude of the lift peaks. Table 4.1 gives the precise location and magnitude for each sweep, which decreases by nearly 50% from $\Lambda = 0^\circ$ to 45° . This can be explained by the simple geometrical effects of sweep. The C_L coefficient in Fig. 4.1 is normalized by the freestream velocity (U_∞) which has a smaller component contributing to lift as sweep increases. If instead the lift force was normalized by the velocity perpendicular to the leading edge given as $U_\perp = U_\infty \cos \Lambda$, the peak difference is minimal and the behavior towards the end of the motion contains the sweep dependence.

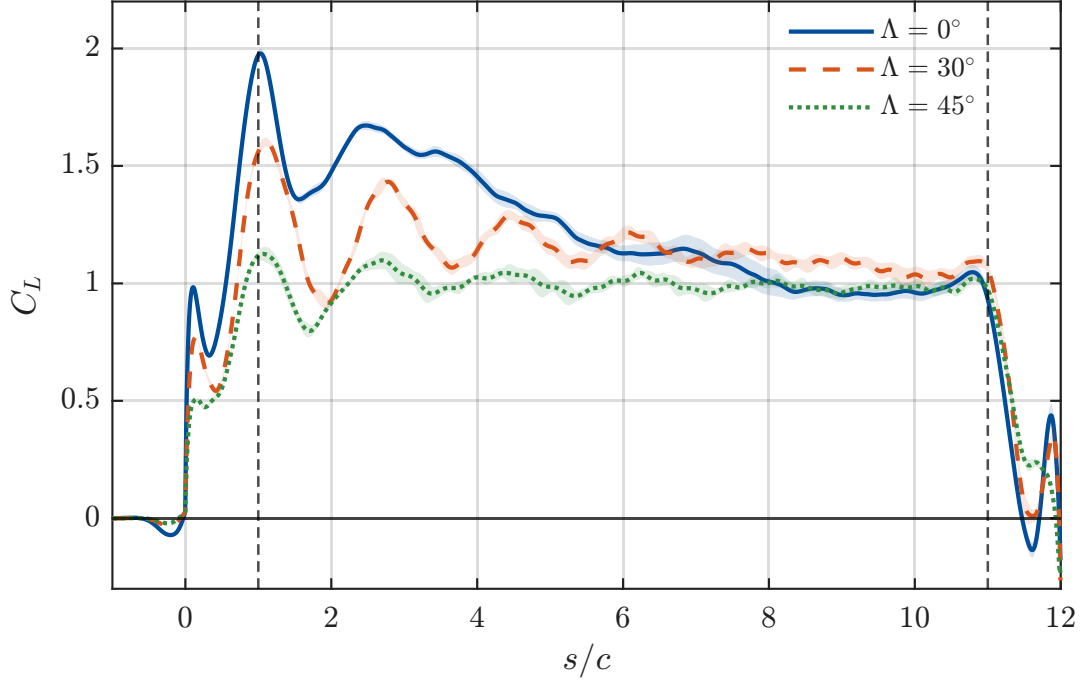


Figure 4.1: Lift coefficients for surging swept wings normalized by U_∞ .

This is shown in Fig. 4.2 and more accurately portrays the effective sweep dependency in lift. However, the raw lift measurements follow the trend of Fig. 4.1 and the freestream normalization is the common convention that will be used unless otherwise stated.

The region of $s/c = 1$ to 8 is also of interest and reveals differing transient behavior. The unswept planform has an abrupt and steady loss of lift after the secondary peak. The 45° planform has a much more steady behavior and both initial and secondary lift peaks are comparable. The 30° planform exhibits an oscillatory behavior with gradual decay and loss of lift. An immediate explanation of this behavior could be explained by the lift generation of the LEV. As sweep increases, lift contribution from the LEV is likely to be more stable. Oscillatory behavior and no sharp decline may indicate the LEV is able to stay in close proximity to the wing and produce lift for longer times due to the influence of spanwise flow that exists for swept wings. Conversely, the oscillation may represent quicker shedding of the LEV and TEV, which prevents a steep loss of lift but limits the attributable circulation and lift magnitude. Additionally, the present oscillations may be a byproduct of the structural

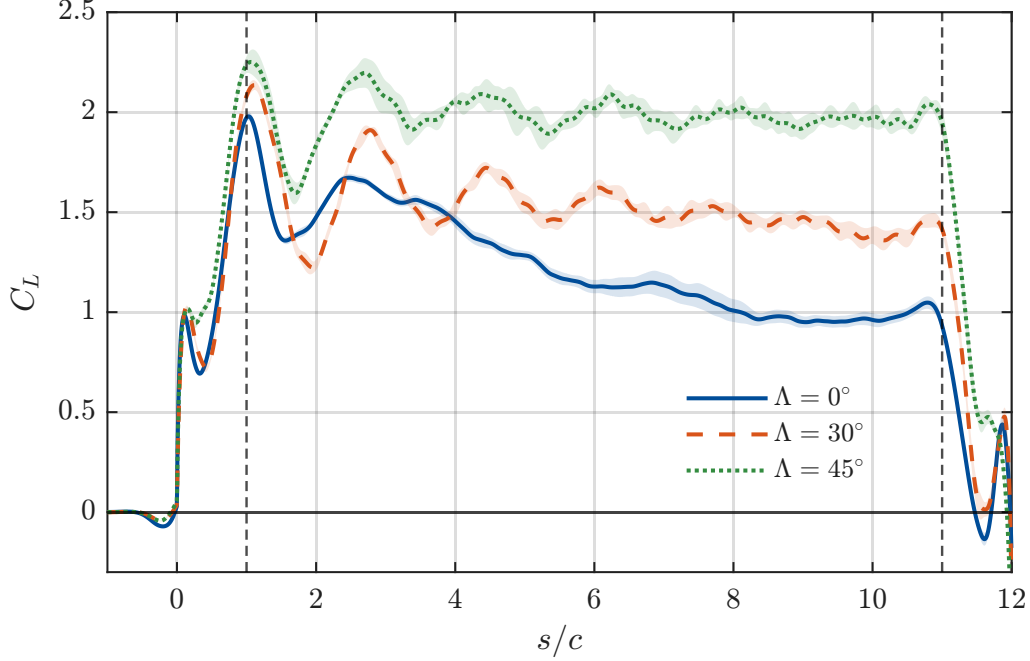


Figure 4.2: Lift coefficients for surging swept wings normalized by $U_{\perp}$.

response of the model. While the frequency of the peaks in Fig. 4.1 is approximately 6.2 Hz which could be indicative of vortex shedding, this is post filtering. When analyzing the unfiltered ensemble averaged data, the dominant oscillation is 30.1 Hz which corresponds with the measured structural mode. It is therefore difficult to isolate LEV mechanisms off the force histories alone.

The $\Lambda = 45^\circ$ case reaches a “steady” value of $C_L = 1$ rather quickly and without much overshoot or oscillation. This likely supports a stable LEV that does not greatly impact the lift signature. The unswept behavior suggests a LEV that detaches and sheds from the wing in a quicker fashion, leading to the dramatic circulation and lift loss. However, due to the absence of spanwise flow from sweep effects, it appears the initial vortex growth is able to obtain a higher strength and contributes to the increase in lift seen in the first and secondary peaks, along with the noncirculatory contributions.

When comparing the results of Fig. 4.1 to similar studies of accelerating flat plates at high angles of attack [25, 26, 33], the $\Lambda = 0^\circ$ planform exhibits the expected behavior. It is

difficult to make direct comparisons as most translational surge studies were measured with an $AR = 4$ flat plate and $\alpha = 45^\circ$. In such studies with similar kinematics, a secondary peak was often observed in lift histories around $s/c = 7$ to 8 and was attributed to secondary LEV formations or regions of recirculation after initial separation and shedding that form at very high angles of attack. In the current study the secondary peak is seen to be closer to $s/c = 3$ as mentioned. Section 4.3 will help determine if this is for similar reasons as the $\alpha = 45^\circ$ cases or the structural reasons mentioned. While the swept planforms exhibit largely different transient behavior, the unswept case does have a much smaller third peak observed close to the $s/c = 7$ mark, which would be similar to results of [26]. Unfortunately, this regime is far past what can be observed in Section 4.3, so no thorough explanation can be deduced via flow imaging.

Comparisons to the low-order theoretical model presented in Section 2.5.4 are presented in the following figures. Beginning with the 2D flat plate model without finite-wing or sweep corrections, Fig. 4.3 shows a somewhat surprising correlation to the measured results. Using $0.5U_\infty$ as the relative LEV-TEV velocity, the model asymptotes approximately at $C_L = 1$ and agrees with the measured data. This choice can be verified though the quasi-steady lift measurements if it is assumed $\Gamma_{LEV} \rightarrow \Gamma_{ss}$ and $\dot{\Gamma}_{LEV} \rightarrow 0$ of Eq. 2.35 by the $s/c = 10$ point. This yields the quasi-steady lift coefficient as $C_{L,qs} = Ka\alpha$, where $K = \frac{u_{LEV} - u_{TEV}}{U_\infty}$. From the data with configurations at approximately $C_L = 1$ at $s/c = 10$ and $a = 2\pi$, K is near 0.5 for all cases. The peak, defined by the added mass and Wagner's vortex growth curve, aligns almost identically with the $\Lambda = 30^\circ$ case, but quickly asymptotes and then tracks the $\Lambda = 45^\circ$ case very well after the $s/c = 3$ mark. Neither secondary peaks nor oscillations are present in the model. This is due to the vortex motion term assuming an almost constant force after the acceleration phase and does not account for LEV reattachment or continuous shedding that may be occurring, or the structural responses inherent to only the experimental data.

The correlation between the model and measurements, specifically the swept cases, in Fig.

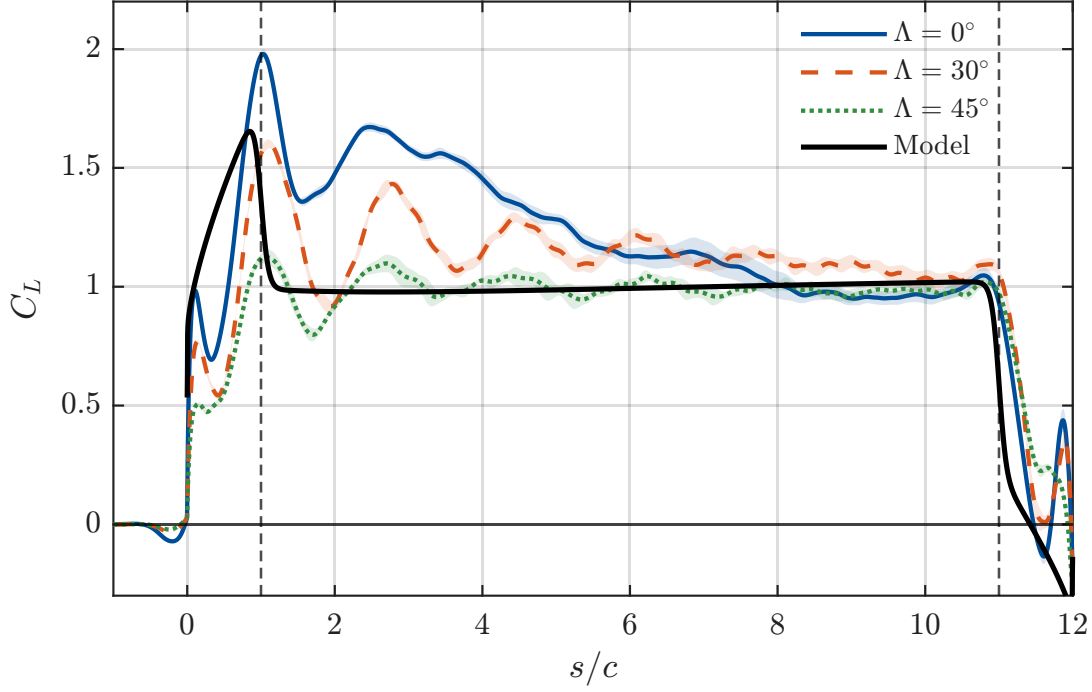


Figure 4.3: Lift coefficient low-order model prediction compared with experimental data.

4.3 begs the question of why. As stated, the model is intentionally a low-order prediction that represents a 2D flat plate and does not consider finite-wing or sweep effects. To address the distinctions, Fig. 4.4 applies corrections to the 2D Wagner function, interpolating Table 2.1 for $AR = 4$ and uses Eq. 2.24 to estimate the 3D lift slope for swept wings. This noticeably impacts the vortex growth and motion terms as shown in Fig. 2.12, while the added mass contribution remains unchanged since it should only depend on the kinematic acceleration which stays constant throughout. Figure 4.4a uses $0.5U_\infty$ as the relative LEV-TEV velocity, and the distinctions between the model and measurements are vast. The models now undershoot the measurements, particularly in the post-acceleration phase. Since vortex motion is the only term contributing to lift in this phase, it appears either the model does not represent the true relative vortex velocities, the vortex growth and non-circulatory terms incorrectly limit the achievable lift, or the measurements are inaccurate. To address the vortex motion comparison, Fig. 4.4b uses $0.8U_\infty$. The choice of $K = 0.8$ instead of 0.5 is the result of using the appropriate 3D lift slopes as opposed to 2π when comparing the

data at $s/c = 10$. While it is not likely the LEV advects at a speed comparable to the TEV at early times, this choice did agree with the data more strongly. Ultimately, disagreement between model and measurement can simply be due to the fact the model is not made to predict any real viscous flow effects like subsequent vortex shedding, reattachment, or diffusion.

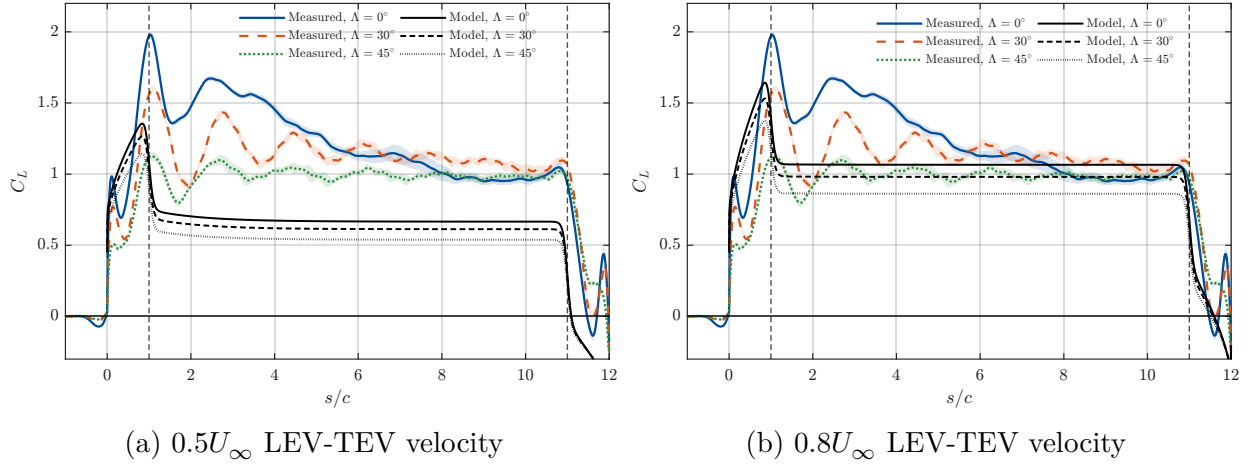


Figure 4.4: Lift coefficient finite wing low-order model predictions compared with experimental data.

4.2.2 Pitching moment

As mentioned, the Nano25 is a 6 DOF transducer which allowed for the measurements of both forces and moments. As shown in Fig. 3.5b, the load cell origin is offset by $h = 0.9c$ vertically from the root quarter-chord, but horizontally aligned. This means raw moment measurements include contributions from both lift and drag forces. To isolate the pitching moment about the root quarter-chord due to lift, Eq. 4.1 is applied,

$$C_{m, c_r/4} = C_m + \frac{h}{c} C_D, \quad (4.1)$$

where C_m is the uncorrected measured moment coefficient.

The resulting pitching moment coefficient is shown in Fig. 4.5. There is a large sweep

dependency as expected due to wing area producing lift aft of the root quarter-chord reference point. There is also transient behavior associated with C_m . Recall from Section 2.2.2 that 2D steady attached flow predicts a constant moment coefficient of 0 at the quarter chord. Even for the unswept planform, the pitching moment does not settle to 0, indicating either transients still active throughout the motion, 3D and separation effects causing deviations from the theory, or measurement error. The second is highly likely, as flow separation and a shedding LEV will naturally move the CP aft of the quarter-chord, resulting in a positive and nonzero pitching moment coefficient.

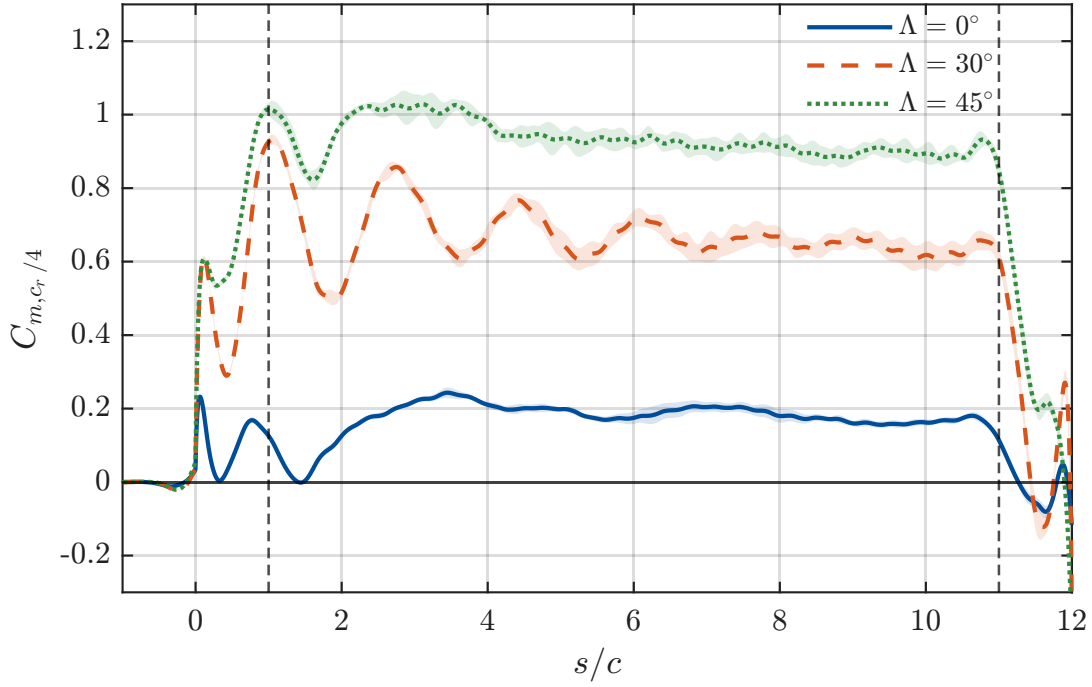


Figure 4.5: Pitching moment coefficients about the root quarter-chord for surging swept wings.

Perhaps a more useful metric is the location of the center of pressure (x_{cp}), given in Eq. 4.2 where $x_{ref} = 0.25c$ and is the chordwise location of measurement.

$$\frac{x_{cp}}{c} = \frac{x_{ref}}{c} + \frac{C_{m,c_r/4}}{C_L}. \quad (4.2)$$

The center of pressure, known to be at the quarter-chord for symmetric airfoils under steady

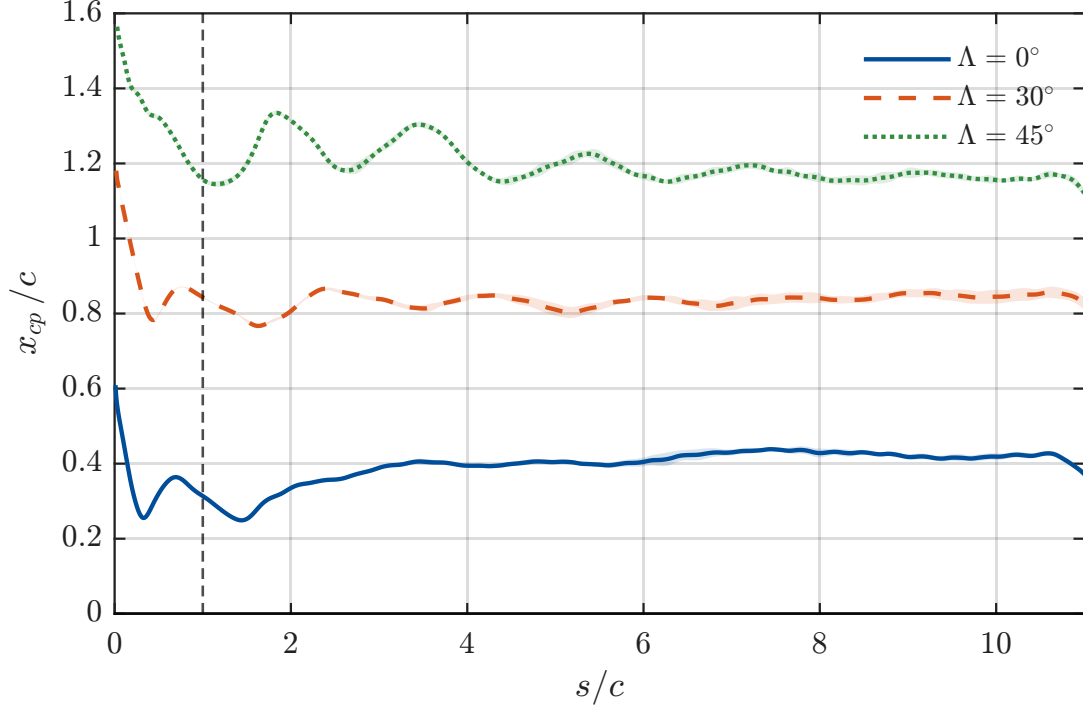


Figure 4.6: Center of pressure relative to leading edge root chord.

attached conditions, is an indication of where lift acts as a point force. A shifting x_{cp} is the result of unsteady behavior such as a LEV causing the concentration of lift to move up- or downstream along the wing. As seen in Fig. 4.6, sweep impacts the trend of x_{cp} much the same as it does for the pitching moment, but results show far more consistent transient behavior. After 2 chords of movement, the center of pressure is essentially fixed for $\Lambda = 30^\circ$ and drifts slightly downstream for $\Lambda = 0^\circ$. The $\Lambda = 45^\circ$ case does show more oscillatory behavior but still reaches an approximately constant position near the end of the movement. Only 11 chords of movement are shown due to numerical blow-up from dividing by C_L which passes 0 in this regime.

4.2.3 Drag

Drag measurements are shown in Fig. 4.7. Note that the drag axis resolution and uncertainty is much greater than that of lift as described in Section 3.6.

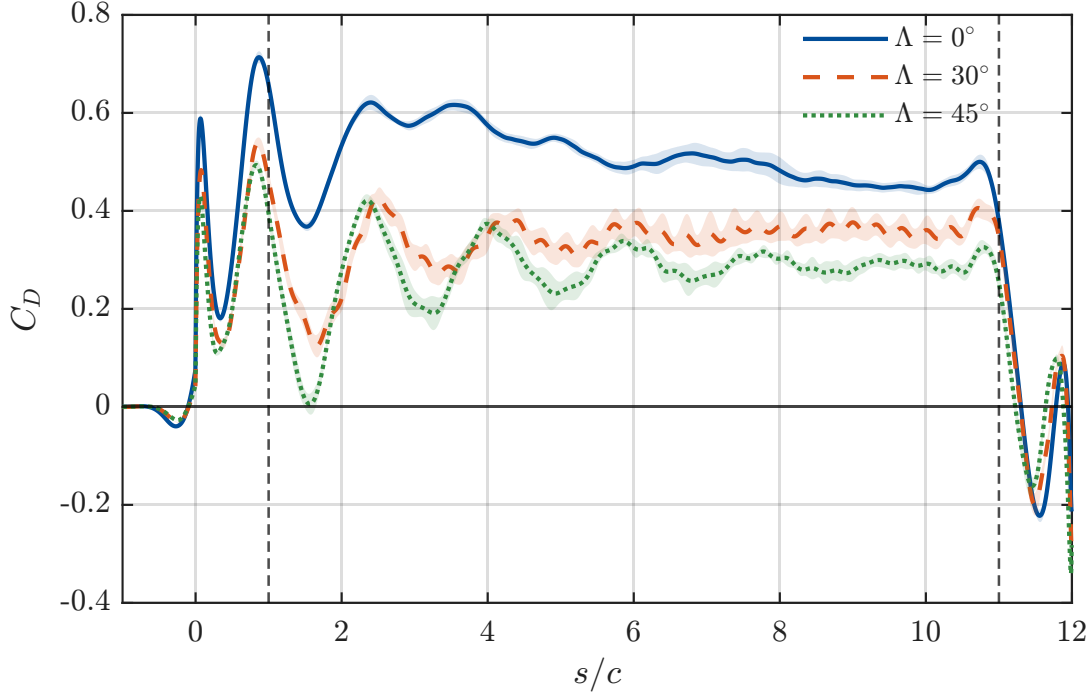


Figure 4.7: Drag coefficients for surging swept wings.

It can be seen that drag peaks around $s/c = 0.85$, slightly before the acceleration phase is complete. Additionally, the end behavior differs from what was observed in the lift histories. The three configurations converge to unique values at $s/c = 9$ and onward of roughly $C_D = 0.45$, 0.35 , and 0.28 for $\Lambda = 0^\circ$, 30° , and 45° respectively. This suggests the trend of sweep decreasing drag. The transients from $s/c = 1$ to 8 show heavy oscillations in drag, especially in $\Lambda = 45^\circ$. This is interesting and may be better explained by pairing flow imaging results. Another possible reason is the structural response as described for the oscillations in the lift signature.

Aerodynamic efficiency is often not quoted as single lift or drag values but the ratio of the lift to drag coefficient. This is shown in Fig. 4.8. As seen, the swept cases exhibit a larger ratio than the unswept case. This could indicate sweep efficiency gains in the separated flow regime if the drag measurements are taken to be faithful. The unswept ratio is noticeably almost constant compared to the swept cases whose oscillation periods are nearly identical. This is likely due to the structural response as sweep adds a larger moment

to the model. The oscillation magnitude and frequency appears too extreme to resemble physical vortex shedding. However, the end behavior flattens out and still supports the case of sweep enhancing aerodynamic efficiency for separated flow.

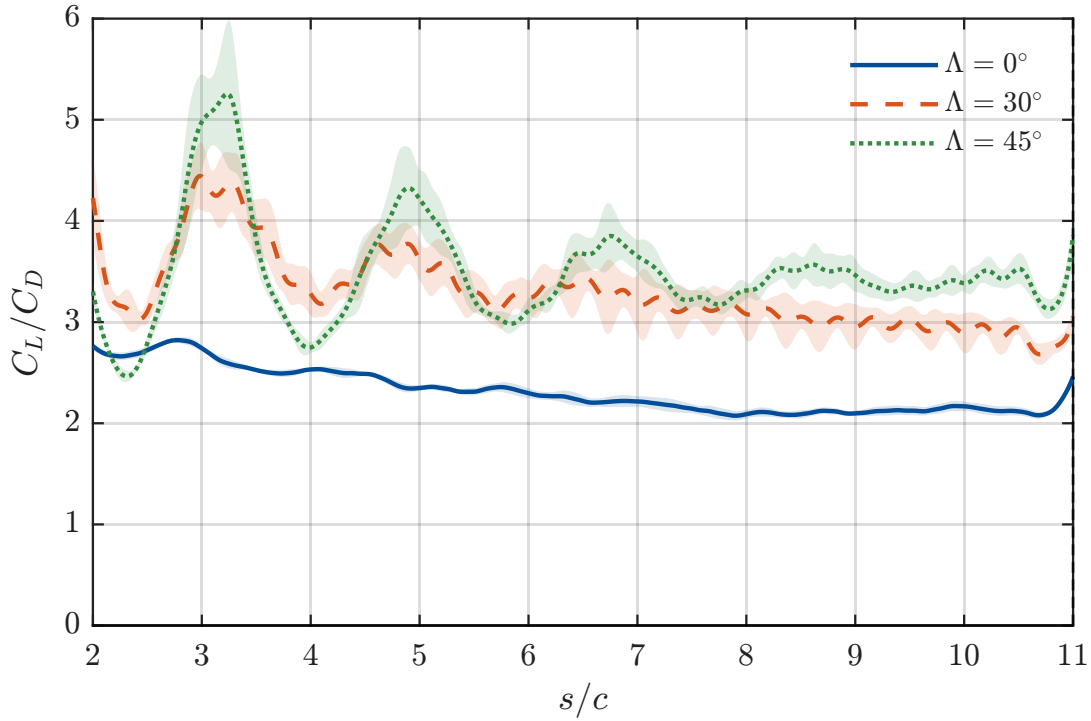


Figure 4.8: Lift over drag coefficients for surging swept wings.

4.3 Flow Visualization

This section displays the flow visualization results via dye injection. As described by Section 3.5.1, blue dye was injected for the inboard orifices spanning across the leading edge and aim to represent the LEV while red dye was used at the wing-tips in an attempt to visualize the wing-tip vortices. It should be noted that dye injection visualizes the flow-field as a *streakline*, which is a line of fluid particles whom originate through a specific point over time. Therefore, the images shown contain the history of the flow and are not to be taken as instantaneous streamlines. In some configurations the dye did not inject out of the proper orifices or did not get fully entrained into the LEV or wing-tip especially after separation.

This may give a slightly inaccurate representation of the full flow-field and is why vortex growth, motion, separation and other metrics can only be qualitatively described.

While hundreds of images were captured for each translational stroke, only a handful can realistically be presented and are of interest. Due to the dye having to physically advect some distance to visualize the flow, very early points during the acceleration phase cannot be displayed. Presenting images near the end of the camera’s FOV also presents challenges due to dye diffusing with the surrounding water along with the natural turbulent and instability structures that form after separation occurs. Therefore, the images presented were concentrated from the $s/c = 0.5$ to 4 phase of the translation. While this does not cover the full range of the force history, it does provide insight into the formation, separation, and three-dimensional transition of the leading-edge vortex. Figure 4.9 shows the lift coefficient for each sweep configuration with markings labeled “a” to “l” to indicate the wing position in the subsequent images.

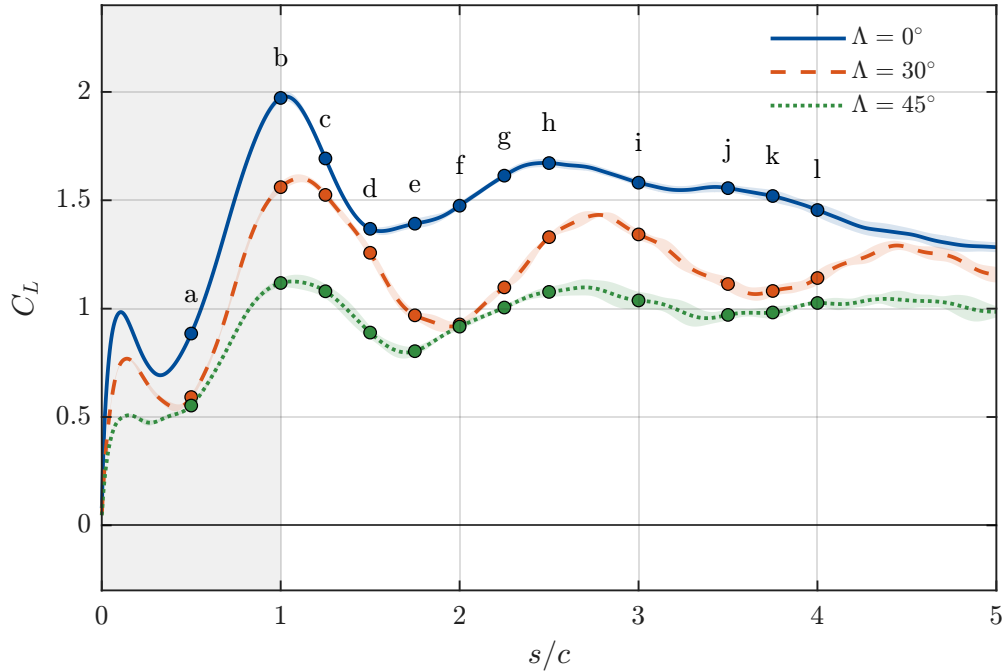


Figure 4.9: Lift coefficients normalized by U_∞ with circular markings indicating the wing position in the following flow-visualization images.

To clarify the wing orientation in the flow, Fig. 4.10 depicts the freestream direction, angle of attack, and sweep for the chordwise and spanwise view. The negative α indicates the wing being inverted in the lab reference frame.

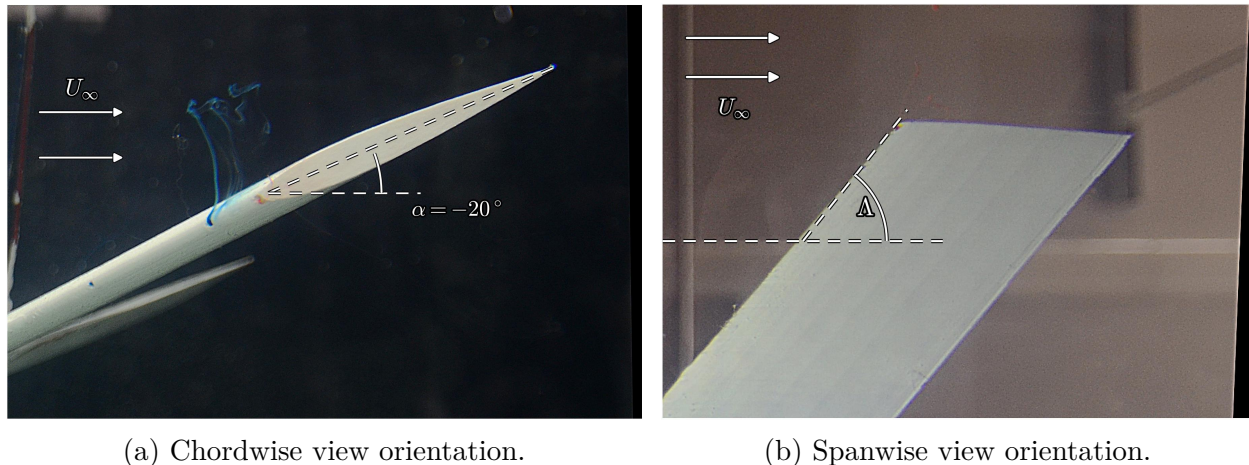


Figure 4.10: Wing orientation for flow visualization imaging.

4.3.1 Chordwise view

The presence of sweep and three-dimensional flow effects provides the need to present the flow as an orthographic projection. The chordwise or side view images are shown in this section with the spanwise or top view images presented in Section 4.3.2. The chordwise view is necessary to resolve mechanisms like the LEV formation, separation point, vortex shedding, and wing-tip structure. Only 3 chords of travel are shown in this configuration due to imaging FOV limits.

Beginning with the unswept planform, Fig. 4.11 shows the chordwise view of the flow progression. The blue dye is predominately injected through the first inboard orifice, with faint traces seen from the outermost orifice. The red dye is seen towards the wing tip. At $s/c = 1$ the beginnings of LEV formation can be seen, indicated by the growing concentration of blue dye. The red dye noticeably does not initially concentrate and proceeds to advect downstream towards the trailing edge. By $s/c = 1.5$ the flow is fully separated and the

LEV begins to advect and form a coherent structure which is fully visible by $s/c = 1.75$. Meanwhile, the red dye has also begun to concentrate around the quarter-chord point and move inboards as influenced by the higher pressure region on the opposite wing side. The coherent LEV structure (blue) largely collapses at $s/c = 2$ and onward. Dye diffusion begins to dominate at this point and a turbulent wake starts to form for the remainder of the translation. The wing-tip red dye has almost turned to flow completely in the spanwise direction by $s/c = 3$ and will be better depicted in the spanwise view of Fig. 4.14.

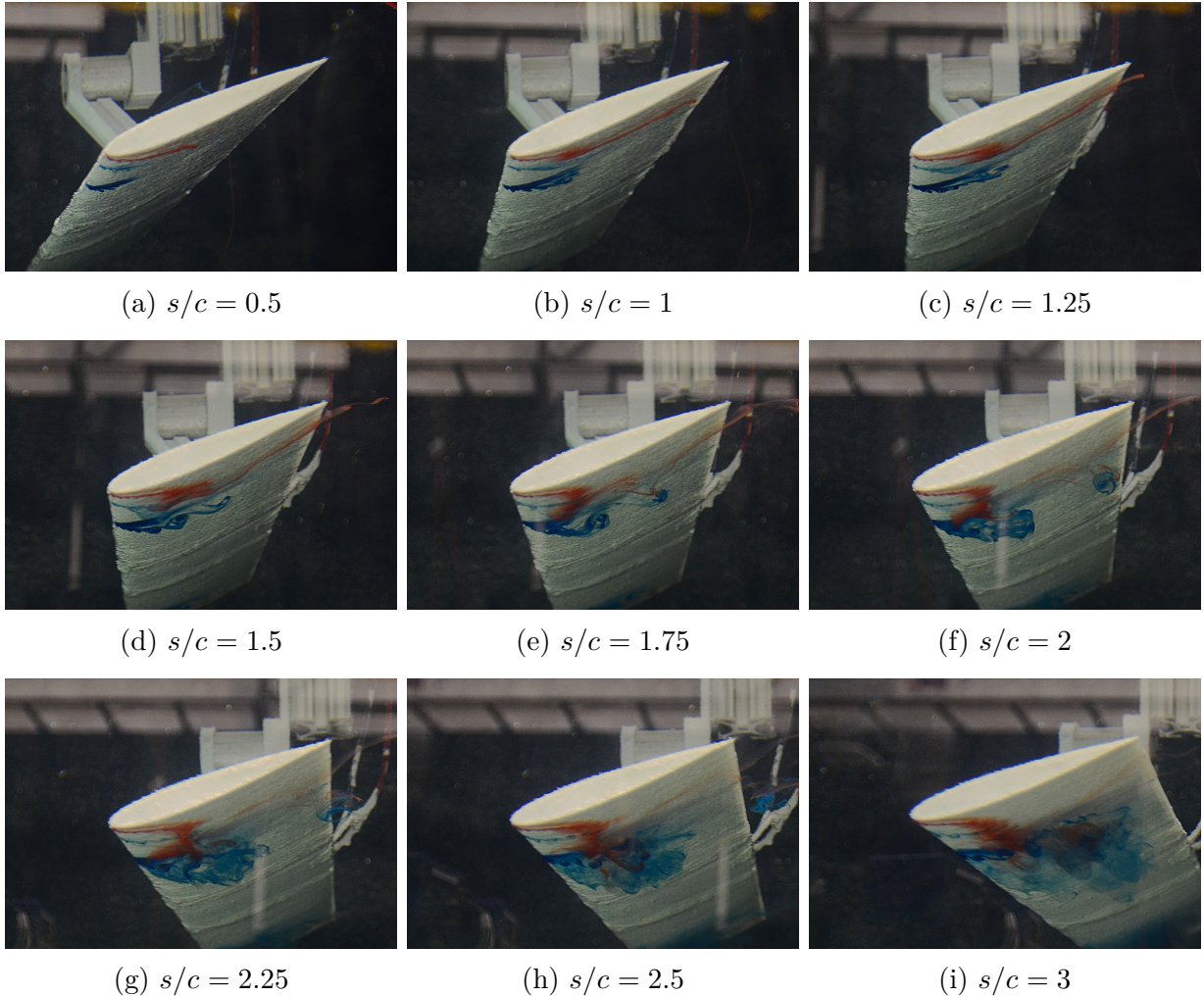


Figure 4.11: $\Lambda = 0^\circ$ flow visualization chordwise view.

The $\Lambda = 30^\circ$ planform chordwise view of the flow visualization is shown in Fig. 4.12. There is a noticeably minimal amount of dye for this configuration particularly in the blue ports.

Unfortunately due to session-to-session variances and inevitable trapped air bubbles, the dye was largely injected on the opposite semi-span side and could not be properly imaged. However, some observations can still be made. The faint blue trace can be seen to similarly concentrate as opposed to the red wing-tip initially and by $s/c = 1$ to 1.25 the LEV can be seen. The behavior from $s/c = 1.5$ to 2 is interesting and shows the LEV forming a recirculation zone along with the wing-tip vortex near the leading edge. The blue dye appears to resemble a stagnation point near the leading edge. The red dye fully separates at $s/c = 2$ and proceeds to diffuse while being entrained into a vortex. The blue dye in the later position frames are too faint and little can be deduced, but the red dye remains visible and stays largely towards the wing-tips as opposed to Fig. 4.11.

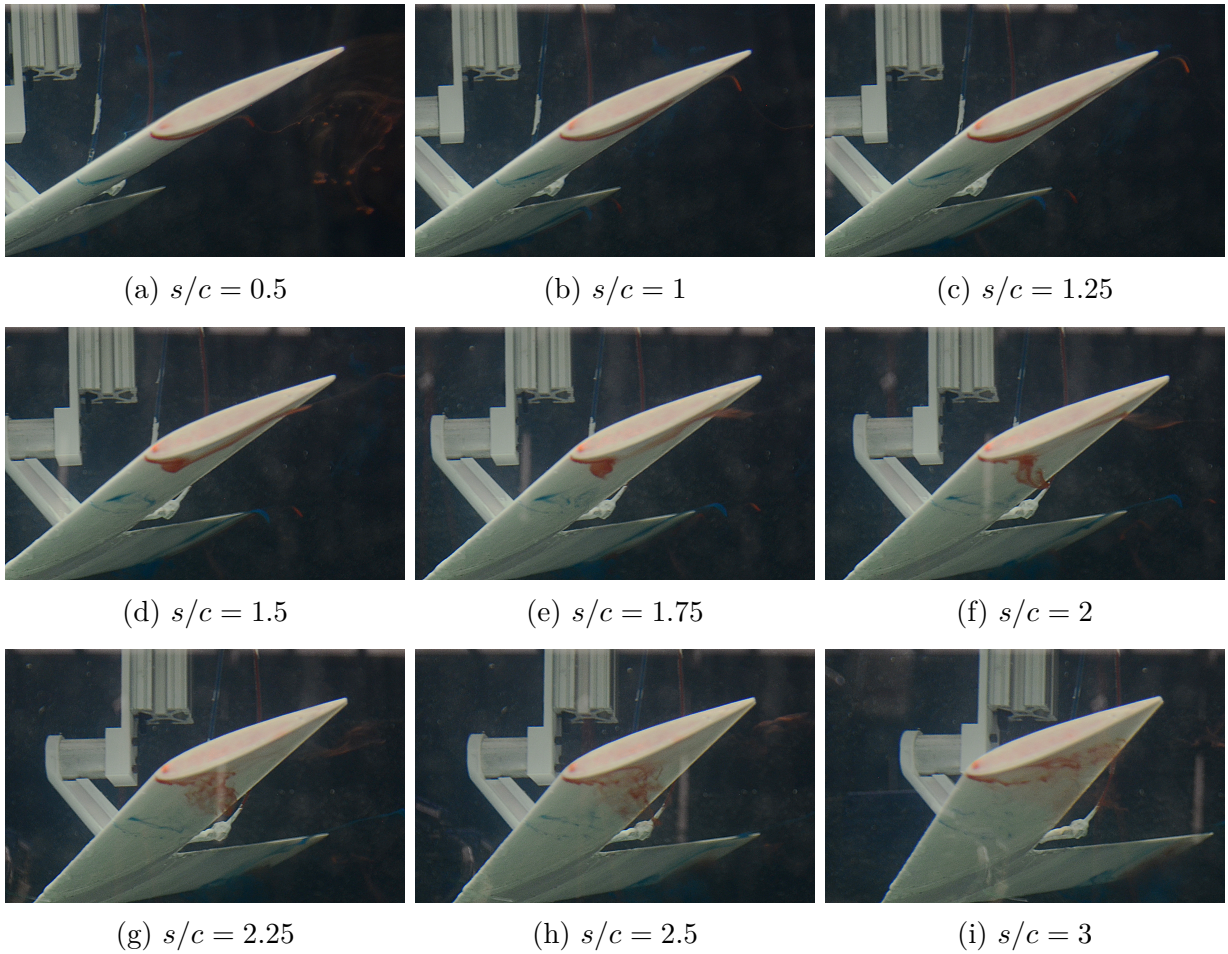


Figure 4.12: $\Lambda = 30^\circ$ flow visualization chordwise view.

The final chordwise view for the flow visualization is shown in Fig. 4.13 for the $\Lambda = 45^\circ$ planform. This planform produced excellent dye flow and provides much more insight than the 30° case. At the beginning of the motion the LEV is growing and attached up to $s/c = 0.5$. At $s/c = 1$ the blue dye is still completely attached. By $s/c = 1.25$ both blue streaklines have separated from the wing surface. The red dye appears to never get fully entrained in the boundary layer and is likely influenced by the high pressure region that pushes the trace inboards and against the normal direction early on. At $s/c = 1.5$ large regions of recirculation are seen for the LEV and the wing-tip vortex begins to form in the red dye. From $s/c = 1.75$ to 2.25 the LEV appears to detach and begin to advect downstream. During this process a secondary LEV may be forming as well which is seen as a new recirculation zone at $s/c = 2.25$. After this point much of the dye loses coherency and does not get entrained close to the wing suction surface to see any new vortex development or shedding events.

4.3.2 Spanwise view

This section provides the spanwise or top view of the suction surface for all wing planforms. The spanwise view provides insight on LEV and wing-tip interactions, three-dimensional transitions, and unsteady vorticity transport along the span. While meant to be paired with the chordwise results of the previous section, the images presented are from independent runs and flow variability is likely to exist. Only one wing-tip is shown resulting in around 25% of the total wing planform in view due to resolving flow features and antisymmetric dye injections. The spanwise view was able to capture 4 chords of travel.

The spanwise flow visualization of the unswept planform is shown in Fig. 4.14. Unfortunately for this case flow was not achieved from the orifice closer to the wing-tip for the blue dye. Nonetheless, an interesting flow structure can be seen to develop. Immediately at $s/c = 0.5$ the wing-tip streakline begins to drift inboards as it advects downstream from the high

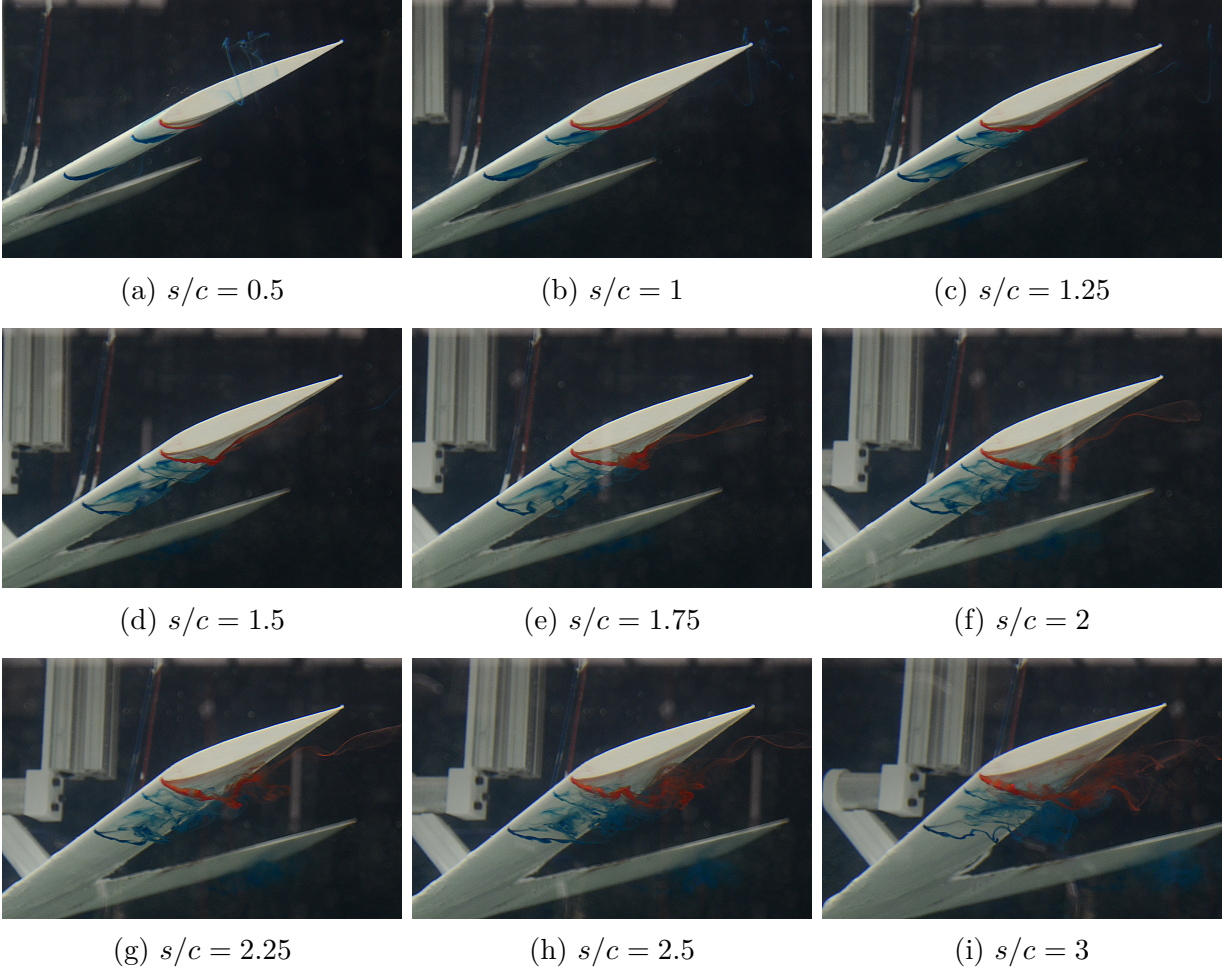


Figure 4.13: $\Lambda = 45^\circ$ flow visualization chordwise view.

pressure region curling around the tip, suggesting that the dye does not get entrained by the tip vortex, but remains inboard of it. At $s/c = 1$ the LEV in the blue dye is concentrated around the quarter-chord and begins to spread inboards as well. By $s/c = 1.25$ the LEV recirculation region can be seen and the red dye is pushed further inboards along the span. Between $s/c = 1.5$ and 1.75 the initial LEV appears to detach and advect towards the root rather than the trailing edge. The red dye also begins to make a sharp turn towards the root and dye stops being entrained downstream along the wing-tip. This process continues from $s/c = 2$ to 4 with the red dye running parallel to the leading edge and concentrated approximately at the quarter-chord. The blue dye begins to diffuse but continues to drift inboards as the LEV is shed and develops a turbulent wake. While only 4 chords of travel could be imaged, it was visually confirmed the red dye did not get entrained back towards the wing-tip until over 6 chords of travel. The blue dye closest to the injection point appears to run parallel with the wing chord for the duration of the translation and does not exhibit spanwise effects.

The $\Lambda = 30^\circ$ planform spanwise flow visualization is shown in Fig. 4.15. This configuration did produce dye from both blue orifices. The LEV is predominately seen to form and cause recirculation on the root side blue streakline. The wing-tip red dye follows similar behavior as the unswept case up to $s/c = 1.25$. As this point the LEV begins to detach and, unlike the unswept case, advects outboards towards the wing-tip. As the flow progresses, the red and blue dye begin to interact at $s/c = 1.75$. Perhaps due to this interaction, the blue dye towards the leading edge of the tip orifice begins to form a streakline angled towards the wing-tip rather than parallel with the chord and freestream. This spanwise flow from the leading edge appears to force the red dye back towards the wing-tip from $s/c = 2$ to 4 . By $s/c = 3$ the red dye runs parallel with the chord at the wing-tip from the injection point and both blue streaklines are angled towards the tip. Additionally, the shed LEV at $s/c = 2$ mixes with the vortex shed from the red dye around $s/c = 2.5$ and collectively advects downstream. By $s/c = 4$ the red dye is angled outside the wing-tip completely. These

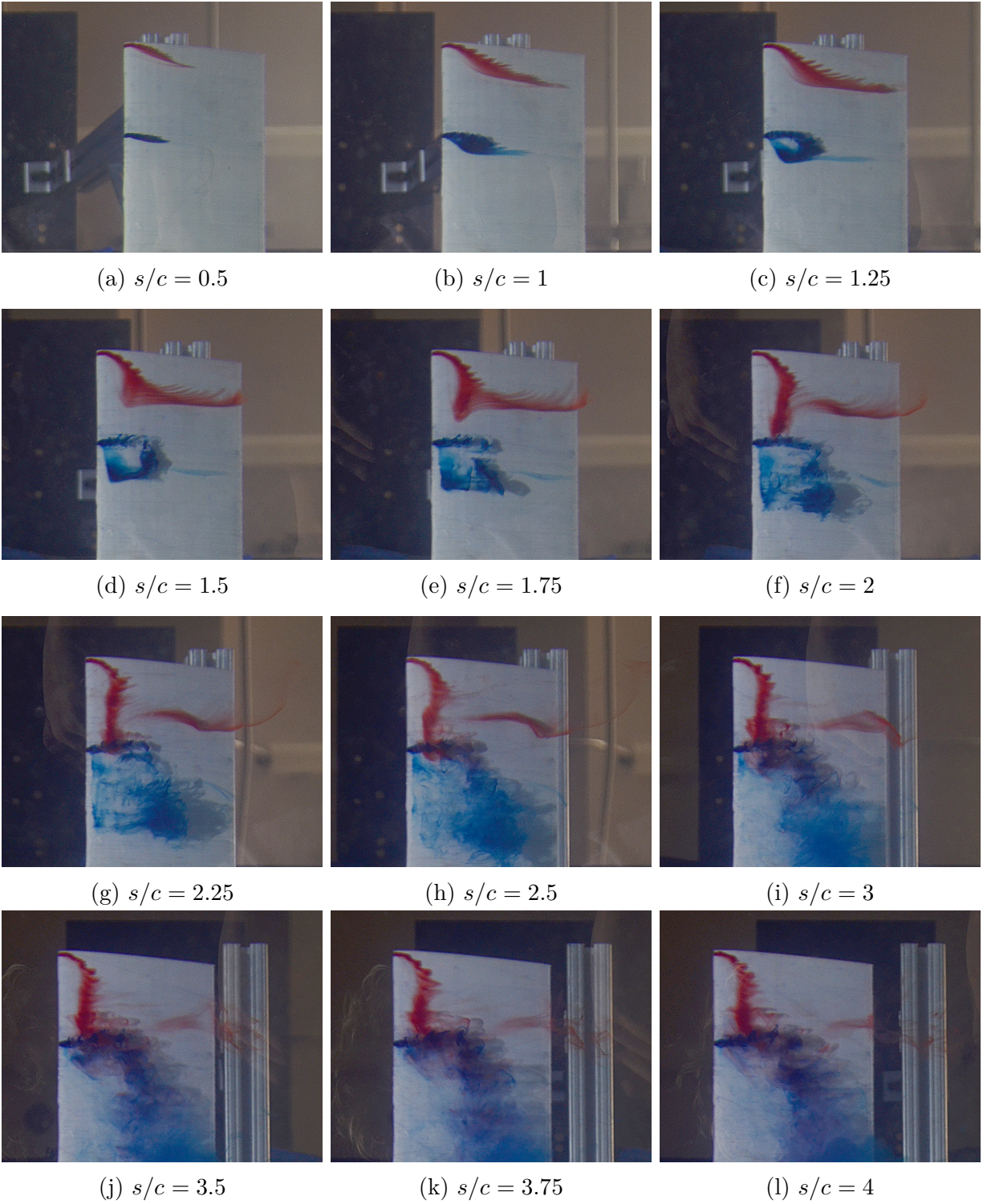


Figure 4.14: $\Lambda = 0^\circ$ flow visualization spanwise view.

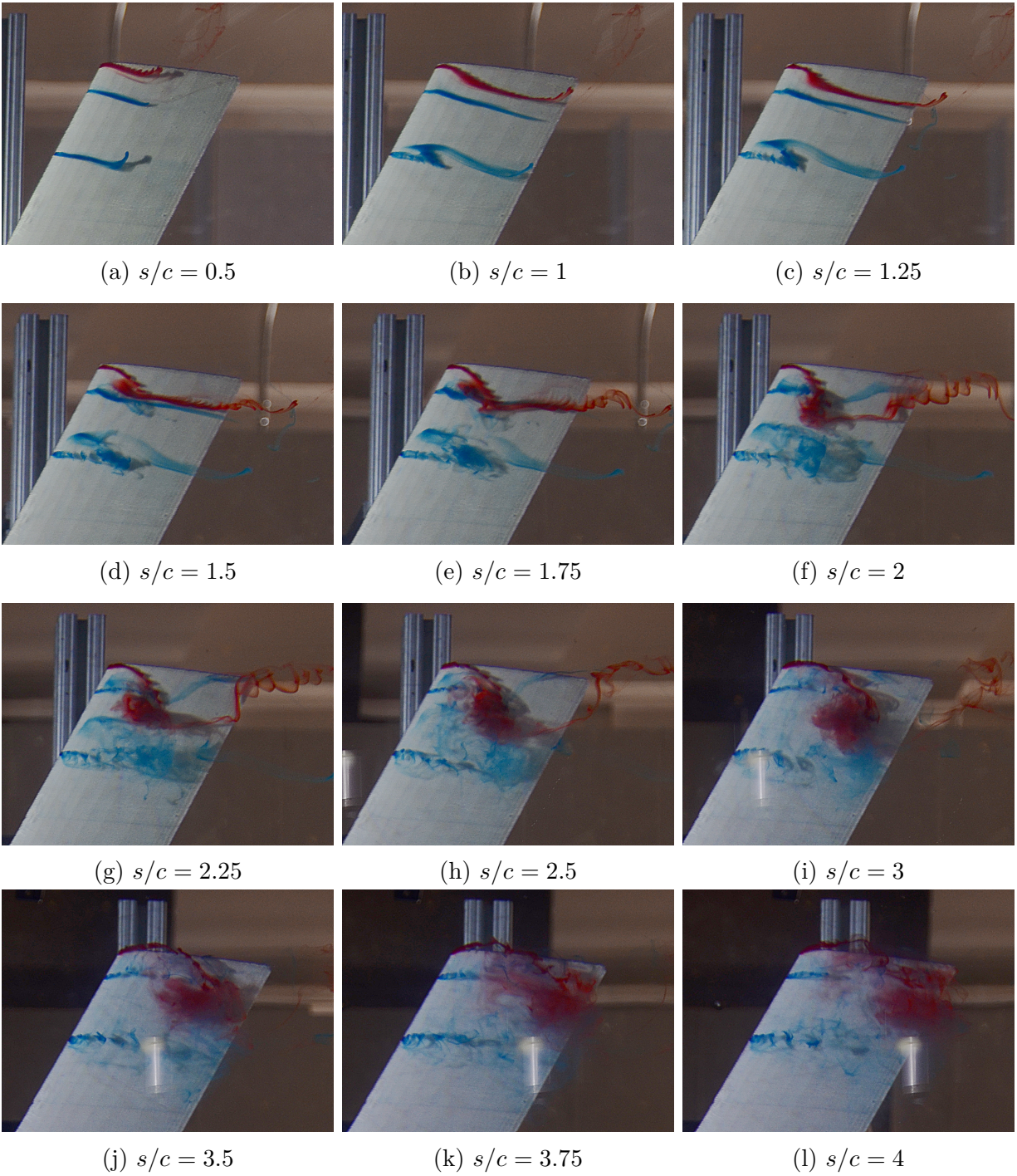


Figure 4.15: $\Lambda = 30^\circ$ flow visualization spanwise view.

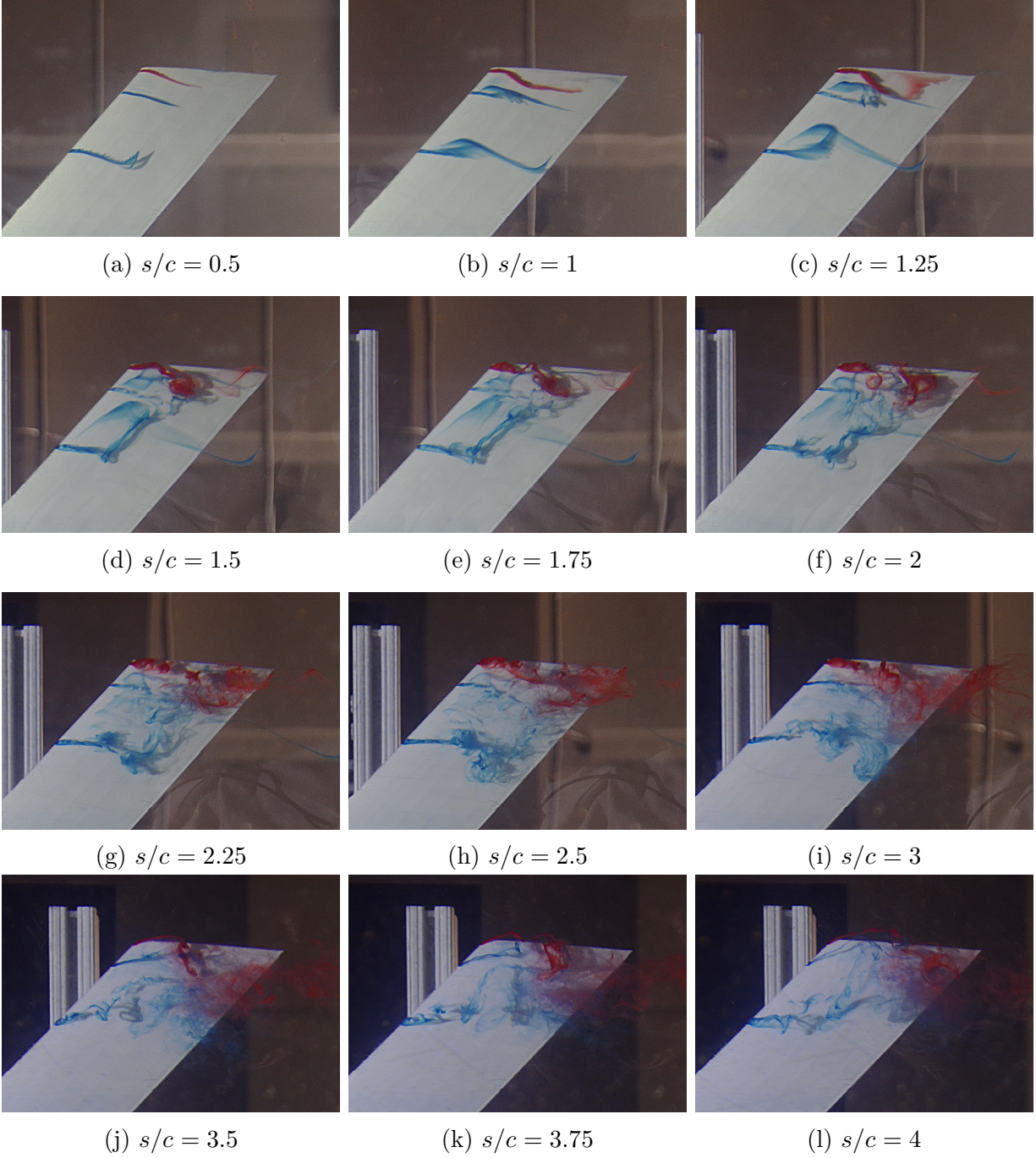


Figure 4.16: $\Lambda = 45^\circ$ flow visualization spanwise view.

observations point towards the spanwise flow and advection of the LEV fighting against the natural inboard flow of the wing-tip vortex, particularly during the early transients.

The final flow visualization images taken are shown in Fig. 4.16 for the $\Lambda = 45^\circ$ planform spanwise view. The overall structure and flow patterns immediately appear similar to the 30° case. The LEV formation is seen at $s/c = 1$, with separation and spanwise advection of it occurring from $s/c = 1.25$ to 2. Particularly, at $s/c = 1.5$ and 1.75 a coherent vortex core is seen to travel in the spanwise direction towards the wing-tip and nearly parallel with the leading edge. The presence of spanwise advection of the LEV, specifically outboards against the wing-tip advection, is an inherent trait of sweep. This structure also supports the conclusion that sweep keeps the LEV near the surface and working as a circulatory contribution of lift for extended periods of time. This can also be considered as stabilizing the LEV. The red dye, which is initially pushed inboards from the wing-tip vortex as seen before, is met by the opposing spanwise flow at $s/c = 1.5$. The resulting wing-tip vortex that proceeds to develop from the red streakline is then confined to the wing-tip region from $s/c = 1.75$ and onward. While the blue streaklines continue to deposit flow outboards towards the wing-tip, it is not until $s/c = 3$ that the leading edge streaklines form an angle towards the tip and break their parallel orientation to the chord and freestream. This angle continues to grow by the $s/c = 4$ position where the tip blue and red dye are angled well towards the wing-tip. Additionally, from $s/c = 3.5$ to 4 a new vortex core can be seen advecting from the inboard blue dye in the spanwise direction. This gives more reason to both support the idea the LEV fights and eventually overpowers the inboard flow from the wing-tip vortex at large sweep angles and that the LEV stays in close proximity to the surface to contribute to lift.

4.3.3 Sweep comparison

To better compare the effect sweep has for the flow visualization shown above it is useful to observe the different planforms in the same figure. Positions at $s/c = 1$, 1.75, and 2.5 are shown for the chordwise view and $s/c = 4$ included for the spanwise view.

The chordwise sweep comparison is presented in Fig. 4.17. The effect of sweep on the LEV formation seen at $s/c = 1$ appears to have little effect. All configurations appear to have a concentrated LEV forming yet still attached to the wing surface at $s/c = 1$, though the $\Lambda = 30^\circ$ case is difficult to say with confidence due to the minimal dye. Referencing the individual chordwise figures, flow separation at the leading edge occurs around $s/c = 1.25$ for all planforms. By simultaneously referencing the spanwise view in Fig. 4.18, the unswept wing has red dye being entrained towards the blue dye and the LEV largely being advected inboard and downstream while the swept cases show their LEV lingering at $s/c = 1.75$. By $s/c = 2.5$ the red dye has been entrained into the wing-tip vortex and is making its way towards the tip edge for the swept cases. The red dye on the unswept wing continues to advect inboard and the bulk flow collectively approaches the root. Therefore, the dominant impact sweep has on the flow appears to be the spanwise advection after separation has occurred. This is better illustrated in the spanwise sweep comparison presented in Fig. 4.18.

The spanwise view sweep comparison presented in Fig. 4.18 reveals the profound impact sweep has on the spanwise flow structure. At $s/c = 1$ the red dye looks similar across all configurations which should be expected. With identical wing-tips aligned parallel to the freestream, the influence felt by the tip vortex at early times should not be impacted by sweep. The LEV formation does resemble different characteristics at $s/c = 1$. The unswept wing concentrates all the injected blue dye around the quarter-chord point while the swept cases show a similar but less intense concentration point with the initial dye advected to the trailing edge. Although the dye properties and injection point could influence

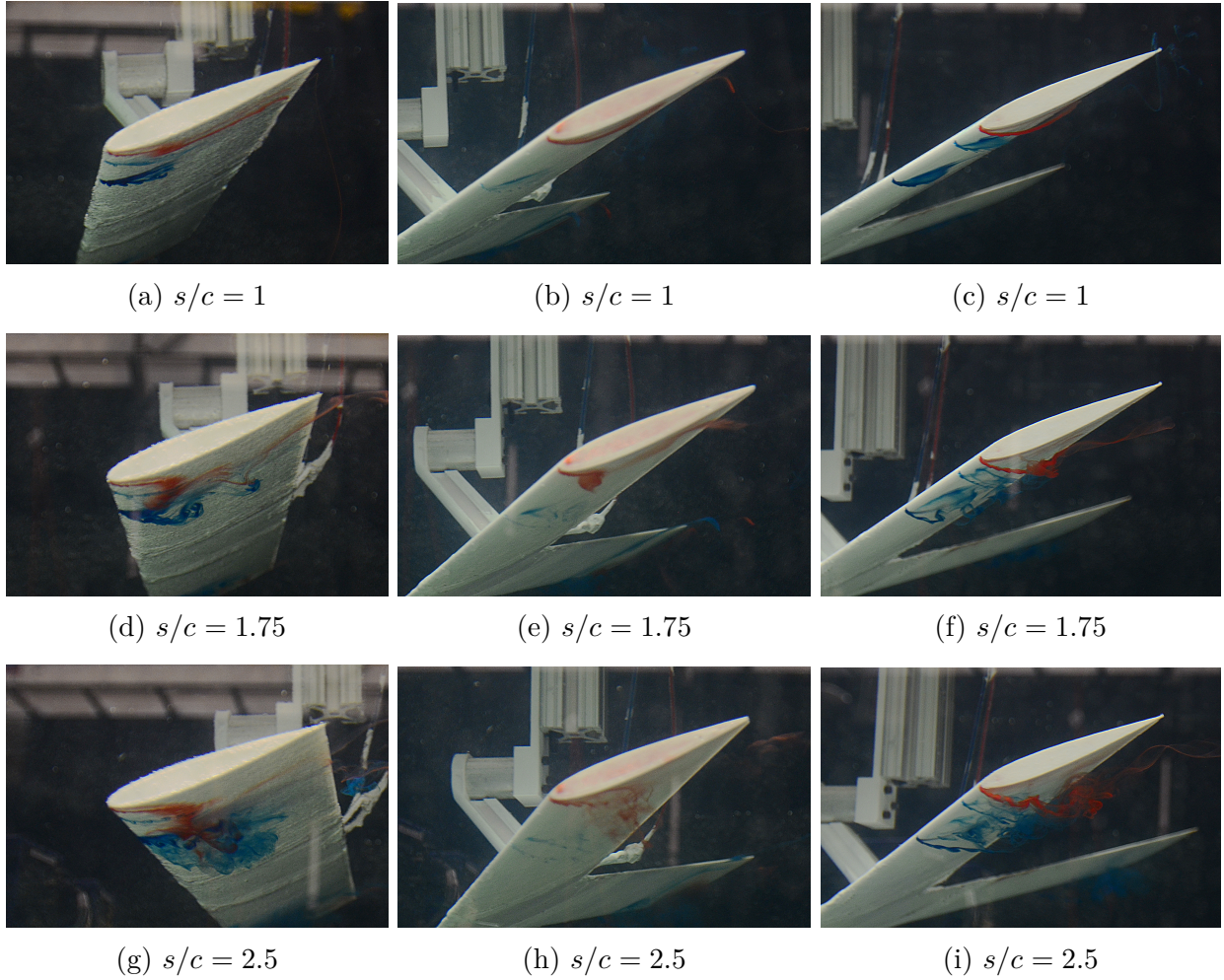


Figure 4.17: Sweep comparison of flow visualization for chordwise view. $\Lambda = 0^\circ$: (a), (d), (g). $\Lambda = 30^\circ$: (b), (e), (h). $\Lambda = 45^\circ$: (c), (f), (i).

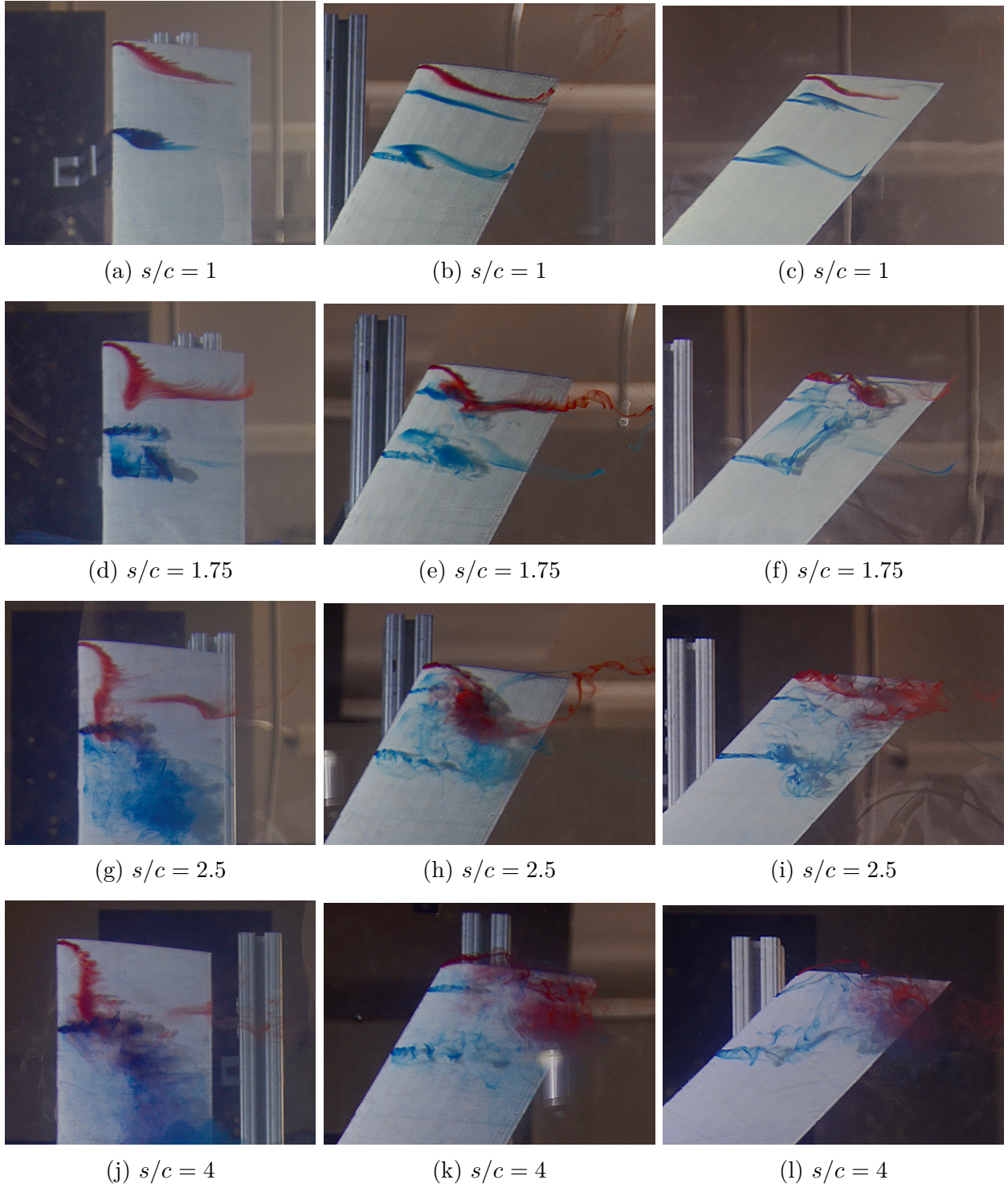


Figure 4.18: Sweep comparison of flow visualization for spanwise view. $\Lambda = 0^\circ$: (a), (d), (g), (j). $\Lambda = 30^\circ$: (b), (e), (h), (k). $\Lambda = 45^\circ$: (c), (f), (i), (l).

these streaklines, it appears the unswept case may form a stronger initial LEV. In other words, sweep decreases the amount of circulation produced in the LEV. Mathematically this is reasonable and can be explained by the convective flux term in Eq. 2.32 given as $-\oint_C \mathbf{n} \cdot \mathbf{u} \omega_y dC$. This term is directly proportional to $U_\perp^2$, which was shown to decrease with sweep in Section 4.2.1. Therefore, at early times when the flow is largely two-dimensional and the diffusive flux is relatively small, $\frac{d\Gamma}{dt} \propto U_\perp^2 = U_\infty^2 \cos^2 \Lambda$. While quantitative measurements of the LEV strength are needed to confidently confirm this, the reasoning would assist in explaining the lift peak differences across sweeps in Fig. 4.1. After the flow separates and large scale spanwise structures form, sweep dependency largely dominates as seen at $s/c = 1.75$. A LEV that travels inboard for $\Lambda = 0^\circ$ and outboard aligned with the leading edge for $\Lambda = 45^\circ$ can only be explained by the spanwise effect sweep has. As sweep increases so does the spanwise transport of the LEV, evident by the $\Lambda = 45^\circ$ case that shows a coherent vortex core traveling towards the wing-tip. The images at $s/c = 1.75$ and 2.5 also suggest the spanwise LEV strength relative to the wing-tip vortex strength grows with sweep. The red dye is limited in how inboard it initially is advected for 45° and is quickly forced towards the tip from the LEV. At $s/c = 4$ the red dye is angled outboard of the tip, indicating the spanwise component of the LEV overpowers the inboard wing-tip vortex. This process is seen to take longer for $\Lambda = 30^\circ$ and does not occur at all for $\Lambda = 0^\circ$.

4.4 Chapter Summary

Force measurements and dye flow visualization were presented for the surging wing at $\Lambda = 0^\circ$, 30° and 45° . The lift history for every configuration peaks near $s/c = 1$, at the end of the acceleration phase, with the peak magnitude falling by nearly half from the unswept to the most highly swept case, while a secondary peak appears near $s/c = 3$ for all three. Each configuration settles toward a common $C_L \approx 1$ near the end of the recorded motion. Normalizing by $U_\perp$ rather than U_∞ collapses the three peaks close together. Drag follows

a distinct trend, peaking earlier than lift and settling to different values with sweep. The low-order model of Section 2.5.4 was compared against the measurements in both its two-dimensional and finite-wing corrected forms, with the two-dimensional version tracking the settled lift and the corrected version undershooting it, and the value of K required to restore agreement was discussed.

The flow visualization showed leading-edge separation and the onset of large-scale spanwise structure occurring at a similar convective time across all three sweep angles, but with the direction of subsequent spanwise motion depending strongly on sweep. Dye marking the leading-edge vortex was observed traveling inboard for the unswept wing and outboard along the leading edge for the swept configurations, with the outboard motion becoming pronounced enough by $s/c = 4$ to overcome the tip vortex for $\Lambda = 45^\circ$. Chapter 5 draws these force and visualization results together and discusses what they support regarding the influence of sweep on the transient lift and the leading-edge vortex.

Chapter 5

Conclusions

The transient aerodynamic loads on a finite wing surging from rest have been measured across three leading-edge sweep angles, and the accompanying flow development recorded using dye visualization. The wing was an untapered NACA 0012 planform of aspect ratio 4 held at a fixed angle of attack of 20° , accelerated over one chord of travel and then towed at constant velocity. Force measurements were acquired at $Re = 100,000$ and visualization at $Re = 5,000$, for $\Lambda = 0^\circ$, 30° and 45° . Section 5.1 collects the conclusions which the two data sets support, and states where the evidence is weaker than the argument would like it to be. Section 5.2 states the major limitations of this study and provides some suggestions for future work.

5.1 Summary and Conclusions

The lift history of a wing surging from rest at high incidence is dominated by a single transient peak occurring at the end of the acceleration phase. Peak lift coefficients of 1.979, 1.601 and 1.125 were measured for $\Lambda = 0^\circ$, 30° and 45° , in every case between $s/c = 1.03$ and 1.12. The location of the peak is therefore very nearly independent of sweep while its

magnitude falls by nearly half, a difference far outside the $\pm 1\sigma$ run-to-run variability of the ensemble. Sweep changes how much lift the transient produces and not when it is produced. Normalizing instead by the velocity perpendicular to the leading edge, $U_{\perp} = U_{\infty} \cos \Lambda$, very nearly collapses the three peaks onto one another, which indicates that the peak scales with the effective dynamic pressure the swept wing experiences rather than with the freestream. The same normalization moves the sweep dependence to the end of the motion, where the freestream-normalized histories had converged near $C_L = 1$ for all three planforms. Drag peaks earlier than lift, near $s/c = 0.85$, and unlike lift it settles to distinct values of roughly 0.45, 0.35 and 0.28 with increasing sweep. The resulting lift-to-drag ratio is larger for the swept planforms, which suggests that sweep offers an efficiency benefit in the separated regime, though the drag axis carries the larger uncertainty of the two and this should be treated accordingly.

The origin of the lift peak deserves comment, since it is not accounted for by either of the two contributions which can be accurately predicted by potential flow models. The non-circulatory contribution of Eq. 2.30 is confined to the acceleration phase and is small relative to the measured peak, so the load is not dominated by added mass. Neither can the remainder be supplied by attached-flow theory. The finite-aspect-ratio indicial response of Section 2.4, interpolated to $AR = 4$, is essentially complete within the first one to two chords of travel and rises monotonically toward its steady value, and a monotonic function cannot produce a peak followed by a decay. The measured peak also exceeds the steady lift the same wing eventually settles to by a factor approaching two for the unswept case. What remains is circulation held in the separated flow itself, that is, lift enhancement from the leading-edge vortex while it remains close to the surface. This cannot be demonstrated directly from the force measurements, since bound and vortex circulation are not separable in a force record, but it is the only one of the three candidate mechanisms which is capable of producing a transient in excess of the steady value and then losing it.

The transient behavior between $s/c = 1$ and 8 differs markedly with sweep, and the amount of lift retained after the peak is the clearest way to express it. Taking the settled value of $C_L \approx 1$ common to all three planforms, the unswept wing retains only about half of its peak lift, the 30° planform roughly 62%, and the 45° planform close to 90%. The unswept history shows an abrupt and sustained loss after the secondary peak, the 45° history reaches its settled value quickly and with comparable initial and secondary peaks, and the 30° history oscillates with a gradual decay. It is tempting to read the oscillation as evidence of periodic vortex shedding, and the frequency of the peaks in the filtered histories is approximately 6.2 Hz which would be plausible for that. However the dominant oscillation in the unfiltered ensemble is 30.1 Hz, which corresponds to the measured structural mode of the model and support. The oscillation visible in the filtered force histories cannot therefore be attributed to shedding on the strength of the force data alone, and no such claim is made here.

Comparison with the low-order model of Section 2.5.4 is instructive in two opposite directions. In its two-dimensional form, using $K = 0.5$ for the relative LEV–TEV advection velocity, the model reproduces the settled lift of all three configurations and produces a peak which aligns closely with the $\Lambda = 30^\circ$ measurement, tracking the 45° case well beyond $s/c = 3$. Applying the finite-wing and swept lift-slope corrections, however, causes the model to undershoot the measurements at the peak and throughout the post-acceleration phase, and recovering agreement requires $K = 0.8$. Without quantifying measurements of the LEV and TEV velocities K cannot be known for certain and a value over 0.5 is realistically unlikely. The more meaningful observation is that the corrected model departs from the measurements where the lower-order 2D model prediction does not. The shortfall is better attributed to 3D corrections effectively taking away lift that is modeled for attached flow but does not consider the sustained lift the LEV provides in separated 3D flow. While more test points are needed to validate, a reasonable 2D model for separated flow on a wing can prove to be useful for quick force or moment calculations, but does not address the differences observed due to sweep.

The flow visualization, performed at $Re = 5,000$, indicates that sweep has little influence on when the flow separates. Leading-edge separation occurs near $s/c = 1.25$ for all three planforms. This is expected as the Reynolds number based on $U_\perp$ falls only modestly across the range of sweep tested, and the flow at this incidence separates at the leading edge in all cases regardless. Additionally, flow visualization confirms large scale 3D spanwise structures forming near $s/c = 1.25$ as well. Formation of 3D structures aligning with the flow separation position logically makes sense as the free shear layer in separated flows is more susceptible to spanwise movement. What sweep alters is not the onset of separation or formation of three-dimensionality but the direction in which the separated fluid subsequently travels, and the images show this clearly. By $s/c = 1.75$ the unswept leading-edge vortex has largely advected downstream while the swept vortices linger near the surface and travel outboard. By $s/c = 2.5$ the tip dye has been entrained into the tip vortex and carried outboard for the swept planforms while the unswept dye continues inboard toward the root. By $s/c = 4$ the tip dye for $\Lambda = 45^\circ$ is angled outboard of the tip entirely, indicating that the spanwise component associated with the swept leading edge has overcome the inboard influence of the tip vortex. This transition takes longer at 30° and does not occur at all for the unswept wing. It should be stressed throughout that dye marks streaklines and carries the history of the flow, so these are qualitative observations of vortex position and coherence and not measurements of vorticity.

Two conclusions require both data sets. The first concerns the magnitude of the lift peak. The spanwise images at $s/c = 1$ show the unswept planform concentrating its injected dye into a tighter and more intense region than either swept planform, suggesting a stronger leading-edge vortex formation at the instant the lift peaks. This is consistent with the convective flux term of Eq. 2.32 dominating the circulation budget at early times, before large-scale three dimensional structure has developed and while the diffusive flux remains small. That term scales as $U_\perp^2 = U_\infty^2 \cos^2 \Lambda$, so the rate at which circulation is supplied to the vortex, and hence the circulation accumulated by a given convective time, should

fall as $\cos^2 \Lambda$. The force measurements support this independently. It is precisely the $U_\perp$ normalization of Fig. 4.2 which collapses the peaks. The dye observation and the force scaling therefore point to the same mechanism from different directions, although a direct measurement of LEV circulation would be needed to confirm it.

The second concerns the retention of lift after the peak. The images show that the swept leading-edge vortices remain coherent and close to the surface for longer, traveling outboard along the leading edge rather than being drawn inboard, and the force histories show correspondingly smaller losses of lift after the peak. The most defensible explanation is kinematic. On a finite unswept wing the presence of tip vortices induces a velocity field with an inboard component over the suction surface, which advects the leading-edge vortex toward the root and breaks its spanwise uniformity, and the images show exactly this bulk inboard motion for $\Lambda = 0^\circ$. A swept leading edge imposes an outboard spanwise component which opposes that induced flow, so the vortex retains its spanwise coherence and its associated suction for longer. In this sense sweep may reasonably be said to stabilize the leading-edge vortex in the present experiments, not by preventing its formation or delaying separation, but by preserving the coherence of the vortex once separation has occurred and keeping it close to the wing surface.

5.2 Limitations and Future Work

The results presented in this study had several limitations that led to some less than ideal results. Beginning with the force measurements, the model and structure lacked stiffness relative to the experienced dynamic pressures due to the 3D printed manufacturing process. While chosen for budget and design robustness, which allowed for the internal dye ports, a structure manufactured to increase rigidity would result in reduced noise and structural interference. This is critical to distinguish structural oscillations opposed to those from vortex shedding. Similarly, the error in implementing the initial smoothing function to

the motion control led to deviations from the referenced canonical kinematics and caused the premature spike in the force histories. This also occurred during a critical phase of the motion, since the acceleration profile directly correlates added mass and total vorticity generation. This error likely influenced the tracking error experienced by the motor controller as well.

The flow visualization technique via dye injection had several drawbacks worth discussing. Firstly, the present dye injection method yielded inconsistent results at times. As seen in the chordwise view of $\Lambda = 30^\circ$ and spanwise view of $\Lambda = 0^\circ$, dye was not always successfully injected through all the orifices or equally across the span. The primary fault for this was the presence of trapped air bubbles within the internal ports that resulted in a large pressure difference the dye flow rate could not overcome. A more preferable design would be only introducing dye ports to one wing-tip that was being imaged. This would avoid the dye choosing the path of least resistance if air bubbles were present. Secondly, even with ideal injection through the ports, dye was not always entrained to visualize the flow areas of interest. This proved high sensitivity of the injection location of the orifices, and particularly those of the wing-tip. With more time, iterations could be tested on different injection locations to yield the best visualization results. Lastly, quantitative assessments about the flow field cannot be made from the images presented. For that, PIV measurements are necessary at a minimum to construct the velocity field, and hence the vorticity and circulation. This study could have greatly benefited from pressure measurements as well to explicitly measure the low pressure region the LEV contributes and how sweep impacts this. These measurements undoubtedly introduce much more complexity and time that could realistically be achieved.

A notable challenge of this study was the timing of different systems. The DAQ system, motion control, camera shutter, and dye pumps were all triggered independently for their respective runs. This resulted in a statistical onset detection for the forces and a trial and

error procedure for the flow visualization. The system introduced inherent human error into the measurements and made the formation of a shared s/c axis a post-processing procedure. With a more sophisticated infrastructure, measurements sharing a trigger system would lead to both higher reliability and ease of repeatability of results. Another challenge unique to this study was the constrained time available to conduct research at the USC Stratified Fluids Lab. While the opportunity to use this facility is greatly appreciated and allowed for a much higher degree of results otherwise available, the physical constraint of travel and time made testing a finite resource.

Future work aiming to address the mentioned limitations and further expand the work of this study can be done. The impact of sweep in unsteady separated aerodynamics and its complete influence on the leading-edge vortex, and trailing-edge vortex for that matter, remains an open question in some regards. Studies including the quantitative measurement techniques mentioned along with broader kinematic motions would aid to answer this question. This study did not address the additional geometric parameter spaces of forward sweep, wing taper, varying angle of attack or aspect ratio, wing-tip orientation, thickness, or cambered airfoils. All of these are likely to have a profound impact on the overall flow-field and are worth consideration to deduce more practical results as it pertains to wing design. Additionally, wing sweep and its contribution in heavily unsteady flight conditions such as gusts is an area of interest this study hopes to lay the foundations for. From a practical standpoint, active flow control methods aimed to exploit the lift due to the LEV and the contribution due to sweep are of high interest. Introducing control surfaces to test effectiveness under separated flow conditions would be of practical importance, especially if paired with flow control techniques. Ultimately, future work is required to fully understand the mechanisms, modeling, and applications.

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