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# Semi-inclusive Deep Inelastic Scattering at Small-$x$

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## Abstract

We study the semi-inclusive hadron production in deep inelastic scattering at small-$x$. A transverse momentum dependent factorization is found consistent with the results calculated in the color-dipole framework in the appropriate kinematic region. The transverse momentum dependent quark distribution can be studied in this process as a probe for the small-$x$ saturation physics. Especially, the ratio of the quark distributions as functions of transverse momentum at different $x$ demonstrates strong dependence on the saturation scale. The $Q^2$ dependence of the same ratio is also studied by applying the Collins-Soper-Sterman resummation method.

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There have been compelling theoretical arguments and experimental evidence that the saturation physics [1, 2] plays a very important role in high energy hadronic scattering process, and an effective theory called color-glass-condensate emerges to describe the relevant physics [3, 4]. In particular, the parton distributions at small-$x$ ($x$ is the longitudinal momentum fraction of the hadron carried by the parton) and/or of large nucleus can be calculated from this effective theory, and they all demonstrate a saturation behavior. The rapidity ($Y = \ln 1/x$) evolution of these distributions are controlled by a nonlinear JIMWLK quation [5–7], which has been thoroughly studied in the last decade. By employing the saturation physics, the deep inelastic scattering (DIS) structure function measured by the HERA experiments can be very well described [8–10], as well as the diffractive structure functions [11–14] and vector-meson production [15–17]. Forward hadron suppression in $dA$ collisions at RHIC experiments also indicates the importance of the saturation physics in the small-$x$ region [18–21]. All these successes have encouraged rapid developments in the small-$x$ physics in the last few years [22].

One of the key predictions of this effective theory is the transverse momentum dependence of the parton distributions in big nucleus at small-$x$, especially the gluon distribution [3, 23, 24]. In the inclusive DIS process, the gluon distribution is convoluted into a dipole cross section, which only provides indirect probe. In this paper, we argue that the transverse momentum dependent parton distributions can be directly probed in the semi-inclusive processes, for example, in the semi-inclusive hadron production in DIS process (SIDIS) [25]. In this process, there are separate momentum scales: $Q^2$ the momentum transfer square for the virtual photon and transverse momentum $p_\perp$ for the final observed hadron. Because of the additional hard momentum scale $Q^2$, the final state hadron transverse momentum can be directly related to that of the parton distribution in nucleon/nucleus when $Q^2$ is much larger than $p_\perp^2$. The relevant QCD factorization theorem [26–28] has been rigorously studied for the leading power contribution to the differential cross section. In the following calculations, we will extend this factorization argument to the case that involves saturation physics, and we argue that the transverse momentum dependent factorization formula is still valid in the so-called geometric scaling regime [29–32], when $Q^2$ is much larger than the saturation scale $Q_s^2$, but saturation effects are still important. As an example, we will demonstrate this factorization for the semi-inclusive DIS at small-$x$.
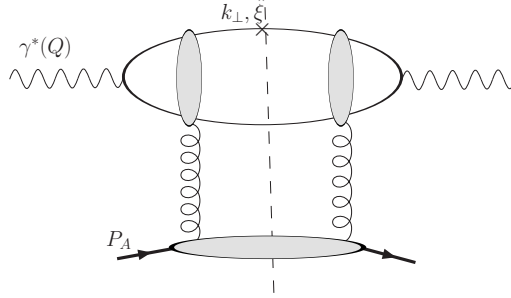
FIG. 1: *Semi-inclusive DIS at small-x, where the cross represents the quark fragmenting into final state hadron. The quark carries momentum fraction $\hat{\xi}$ of the virtual photon and transverse momentum $k_\perp$.*

In the semi-inclusive DIS process,

$$e + p(A) \rightarrow e' + h + X \ , \tag{1}$$

we observe the final state hadron with characteristic kinematic variables, such as the longitudinal momentum fraction $z_h$ of the virtual photon and transverse momentum $p_\perp$. The usual DIS kinematics variables are defined as $Q^2 = -q.q$, $x_B = Q^2/2P_A \cdot q$, $y = q \cdot P_A/\ell \cdot P_A$, and $z_h = P_h \cdot P_A/q \cdot P_A$, where $P_h$, $\ell$, $P_A$ and $q$ are momenta for the final state hadron, incoming lepton and nucleon (nucleus), and the exchanged virtual photon, respectively. The transverse momentum $p_\perp$ is usually defined in the center of mass frame of the virtual photon and the incoming hadron. In Fig. 1, we plot the schematic diagram for this process in the dipole framework at small-$x$ [24], where the virtual photon splits into a quark-antiquark dipole, and scatters off the nucleon/nucleus target before the quark (antiquark) fragments into a final state hadron. In the current fragmentation region (forward direction of the virtual photon) the quark-fragmentation contribution will dominate the cross section.

The differential cross section for the above process can be calculated in the dipole formalism [24] or in the classical Yang-Mills effective theory approach [23], and we readily have,

$$\frac{d\sigma(ep \rightarrow e'hX)}{d\mathcal{P}} = \frac{\alpha_{em}^2 N_c}{2\pi^3 x_B Q^2} \sum_f e_f^2 \int_{z_h} \frac{dz}{z} \frac{D(z)}{z^2} \int d^2b d^2q_\perp F(q_\perp, x_B) \times \mathcal{H}(\hat{\xi}, k_\perp) \ , \tag{2}$$

where $D(z)$ is the quark fragmentation function into the final state hadron, $F(q_\perp, x_B)$ the un-integrated gluon distribution defined below, $\hat{\xi} = z_h/z$, and the fragmenting quark's transverse momentum $k_\perp = p_\perp/z$. The phase space factor $d\mathcal{P}$ is defined as $d\mathcal{P} = dx_B dQ^2 dz_h dp_\perp^2$,

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and $\mathcal{H}$ reads as

$$\mathcal{H}(\hat{\xi}, k_\perp) = \left(1 - y + \frac{y^2}{2}\right)(\hat{\xi}^2 + (1 - \hat{\xi})^2)\left|\frac{k_\perp}{k_\perp^2 + \epsilon_f^2} - \frac{k_\perp - q_\perp}{(k_\perp - q_\perp)^2 + \epsilon_f^2}\right|^2$$

$$+ (1 - y)4\hat{\xi}(1 - \hat{\xi})Q^2\left(\frac{1}{k_\perp^2 + \epsilon_f^2} - \frac{1}{(k_\perp - q_\perp)^2 + \epsilon_f^2}\right)^2 , \tag{3}$$

where $\epsilon_f^2 = \hat{\xi}(1 - \hat{\xi})Q^2$. We have also taken the massless limit in the above formula for sim-
plicity, and the first term is the contribution from transversely polarized photons and second
one corresponds to longitudinally polarized photons. The unintegrated gluon distribution is
defined through the Fourier transform from the dipole-nucleon cross section,

$$F(q_\perp, x) = \int \frac{d^2 r}{(2\pi)^2} e^{-iq_\perp \cdot r}\left(1 - T_{q\bar{q}}(r, x)\right) , \tag{4}$$

where $T_{q\bar{q}}$ is the scattering amplitude, and is characterized by the saturation scale $Q_s^2$ which
depends on $x$. This unintegrated gluon distribution contains the saturation physics, which
diagrammatically represents the multiple scattering of the quark-antiquark dipole on nu-
cleon/nucleus target. When integrating over transverse momentum $p_\perp$ and the fragmen-
tation function using $\int dz D(z) = 1$, the above formula will reproduce the total DIS cross
section in $ep(A) \to eX$ [1].

In this paper, we are interested in the factorization property of the above differential
cross section in the kinematic region where $Q^2$ is much larger than the final state hadron
transverse momentum $p_\perp^2$. In the current fragmentation region, $z_h$ is in order of 1. Therefore
the quark transverse momentum $k_\perp$ is the same order of $p_\perp$. Furthermore, we assume that
$Q^2$ is also much larger than the saturation scale $Q_s^2$ where the transverse momentum $q_\perp$
of the unintegrated gluon distribution sets. Under these limits, we will be able to study
the transverse momentum dependent factorization, where we can separate the transverse
momentum dependence of the final state hadron into the incoming quark distribution and
fragmentation function and/or soft factor [26, 28]. An important advantage to utilize the
above limits is that we can apply the power counting to analyze the leading power contri-
bution, and neglect the higher order corrections in terms of $p_\perp^2/Q^2$ where $p_\perp$ stands for the
typical transverse momentum ($p_\perp \sim k_\perp \sim q_\perp$).

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[1] In the $k_t$-factorization at small-$x$, the gluon momentum fraction $x$ differs from $x_B$ because of the kinematic
constraints [33]. In the leading logarithmic ($\ln 1/x$) approximation, these two are consistent.

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More over, we notice that the integral of Eq. (2) is dominated by the end point contribution of $\hat{\xi} \sim 1$ where $\epsilon_f^2$ is in order of $k_\perp^2$ [24]. In order to extract the leading power term from this equation, we can introduce a delta function in Eq. (2): $\int d\xi \delta(\xi - 1/(1 + \Lambda^2/\epsilon_f^2))$ [2], where $\Lambda^2 = (1 - \hat{\xi})k_\perp^2 + \hat{\xi}(k_\perp - q_\perp)^2$, and integrate out $\hat{\xi}$ first. This delta function can be further expanded in the limit of $p_\perp^2 \ll Q^2$,

$$\delta(\xi - \frac{1}{1 + \frac{\Lambda^2}{\epsilon_f^2}}) = \frac{1 - \hat{\xi}}{\xi} \delta\left((1 - \xi)(1 - \hat{\xi}) - \frac{\Lambda^2}{Q^2}\right)$$

$$\rightarrow \frac{1 - \hat{\xi}}{\xi}\left(\frac{\delta(1 - \hat{\xi})}{(1 - \xi)} + \frac{\delta(1 - \xi)}{(1 - \hat{\xi})}\right) , \quad (5)$$

where a logarithmic term in the above expansion is power suppressed and has been neglected. The contribution from the second term is also power suppressed. To see this more clearly, we can substitute $\epsilon_f^2 = \xi\Lambda^2/(1 - \xi)$ into Eq. (3), and the hard coefficient $\mathcal{H}$ will have an overall factor $(1 - \xi)^2$. Combining this with the delta function expansion, we will find that the second term is the above expansion is power suppressed relative to the first one. Applying the delta function expansion in Eqs. (2) and (3), we will obtain the leading contribution to the differential cross section in the limit of $p_\perp \ll Q$,

$$\frac{d\sigma(ep \rightarrow e'hX)}{d\mathcal{P}}\Big|_{p_\perp \ll Q} = \frac{\alpha_{em}^2 N_c}{2\pi^3 Q^4} \sum_f e_f^2 \left(1 - y + \frac{y^2}{2}\right) \frac{D(z_h)}{z_h^2} \int \frac{d\xi}{x_B}$$

$$\times \int d^2b d^2q_\perp F(q_\perp, x_B) A(q_\perp, k_\perp) , \quad (6)$$

where

$$A(q_\perp, k_\perp) = \left|\frac{k_\perp |k_\perp - q_\perp|}{(1 - \xi)k_\perp^2 + \xi(k_\perp - q_\perp)^2} - \frac{k_\perp - q_\perp}{|k_\perp - q_\perp|}\right|^2 . \quad (7)$$

We noticed that the longitudinal photon contribution is power suppressed and has been dropped.

On the other hand, a transverse momentum dependent factorization can also be used to describe the SIDIS process when the hard scale $(Q^2)$ is much larger than the transverse momentum scale $p_\perp^2$. To leading power of $p_\perp^2/Q^2$, for example, we will have following factorization formula for the differential cross section for the semi-inclusive DIS [26–28],

$$\frac{d\sigma(ep \rightarrow e'hX)}{d\mathcal{P}} = \frac{4\pi\alpha_{em}^2}{Q^2}\left(1 - y + \frac{y^2}{2}\right) \int d^2k_\perp d^2p_{1\perp} d^2\lambda_\perp q(x_B, k_\perp; x_B\zeta) D(z_h, p_{1\perp}; \hat{\zeta}/z_h)$$

$$\times S(\lambda_\perp; \rho) H(Q^2, x_B, z_h; \rho)\delta^{(2)}(z_h k_\perp + p_{1\perp} + \lambda_\perp - p_\perp) , \quad (8)$$

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[2] If we replace the gluon momentum fraction $x_B$ by $x = x_B/\xi$ in Eq.(2), we will reproduce the $k_t$-factorization formula [33] with this delta function.
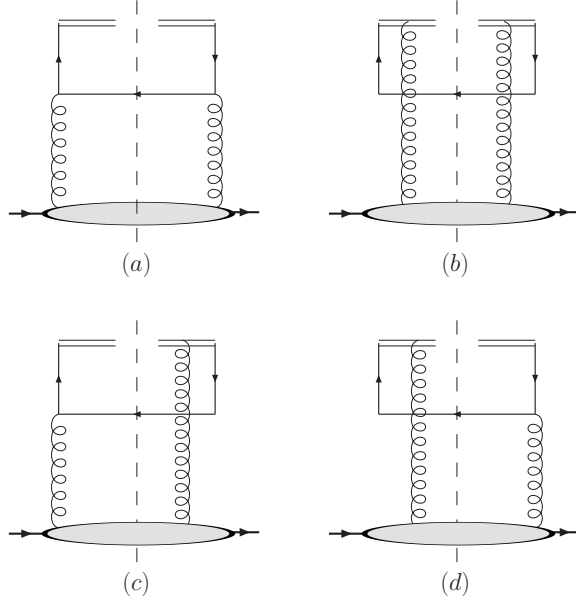
FIG. 2: *Transverse momentum dependent quark distribution calculated from small-x gluon splitting. The double line represents the gauge link contribution from the TMD quark distribution definition.*

where $q(x_B, k_\perp)$, $D(z_h, p_{1\perp})$, $S(\lambda_\perp)$, and $H$ are the transverse momentum dependent quark distribution, fragmentation function, soft factor, and hard factor, respectively. We emphasize that the above factorization is valid in the leading power of $p_\perp^2/Q^2$, and all power corrections have been neglected. The energy dependent parameter $\zeta$, $\hat{\zeta}$ and $\rho$ have been introduced to regulate the light-cone divergences in the associated functions. In a special frame, we can simplify them as $x_B^2\zeta^2 = \hat{\zeta}^2/z_h^2 = \rho Q^2$ [28]. The transverse momentum resummation can be performed by studying the evolution equation in terms of these variables.

The above factorization formalism was studied without considering the small-$x$ resummation effects [26–28]. Here, we assume that the factorization argument can still hold when the hard momentum scale $Q^2$ is much larger than the saturation scale $Q_s^2$ and we can use the power counting method to study the leading contribution in this process. On the other hand, if $Q_s^2$ is the same order as $Q^2$ (or even larger), the power counting used to argue the TMD factorization is no longer valid, and we will not have a TMD factorization. Similar studies for the heavy quark-antiquark production in $pA$ ($AA$) collisions have also been discussed in [36].

As an important check, in the following we will compare the prediction from the TMD formula Eq. (8) to the dipole result Eq. (6) in the same kinematic region, $Q^2 \gg p_\perp^2(Q_s^2)$. To

do that, we need to calculate the TMD quark distribution in nucleon/nucleus at small-$x$. This quark distribution is defined as [26]

$$q(x, k_\perp) = \frac{1}{2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^2} e^{-ixP^+\xi^- - ik_\perp \cdot \xi_\perp} \langle P|\bar{\Psi}(\xi)\mathcal{L}_\xi \gamma^+ \mathcal{L}_0 \Psi(0)|P\rangle \ , \qquad (9)$$

where $P$ is the momentum for the hadron, $x$ and $k_\perp$ are longitudinal momentum fraction of the hadron and transverse momentum carried by the quark. In the above equation, $\mathcal{L}$ is the gauge link introduced to guarantee the gauge invariance of the above definition [26, 28]. At this particular order, the gluon splitting contribution to the TMD quark distribution can be calculated in the $k_t$-factorization approach at small-$x$ limit. We plot the relevant Feynman diagrams in Fig. 2, where (b-d) diagrams come from the gauge link contributions. These diagrams have to be taken into account because the gauge field connecting to the hadron state (nucleon/nucleus) are dominated by the $A^+$ component in the $k_t$-factorization calculations. Their contributions are important to obtain a consistent and gauge invariant result. The derivation is straightforward, and we have,

$$q(x, k_\perp) = \frac{N_c}{8\pi^4} \int \frac{dx'}{x'^2} \int d^2b d^2q_\perp F(q_\perp, x') A(q_\perp, k_\perp) \ , \qquad (10)$$

where $A$ has been defined in Eq. (7). This is the quark distribution calculated in the $k_t$-factorization. In order to compare to the results we obtain above in the color-dipole formalism, we need to extrapolate in the leading logarithmic approximation at small-$x$, i.e., replacing the unintegrated gluon distribution $F(q_\perp, x')$ by $F(q_\perp, x)$ in the above equation. Following this replacement, we will reproduce the differential cross section Eq. (6) calculated in the dipole framework at the leading order of $p_\perp^2/Q^2$. Therefore, we have demonstrated that the small-$x$ calculation of the differential cross section for the SIDIS process is consistent with the TMD factorization at this particular order. At even higher order, we will have to take into account the contributions from the fragmentation function and soft factor. At this order, they are trivial: $D(z_h, p_{1\perp}) = D(z_h)\delta^{(2)}(p_{1\perp})$ and $S(\lambda_\perp) = \delta^{(2)}(\lambda_\perp)$ where $D(z_h)$ is the integrated fragmentation function. We further argue that the TMD factorization will work at higher orders as well, because the power counting is valid when $Q^2 \gg p_\perp^2 (Q_s^2)$ as we mentioned above.

In the leading logarithmic approximation at small-$x$, we can further integrate out $\xi$ in Eq. (10),

$$xq(x, k_\perp) = \frac{N_c}{4\pi^4} \int d^2b d^2q_\perp F(q_\perp, x) \left(1 - \frac{k_\perp \cdot (k_\perp - q_\perp)}{k_\perp^2 - (k_\perp - q_\perp)^2} \ln \frac{k_\perp^2}{(k_\perp - q_\perp)^2}\right) \ , \qquad (11)$$

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which is consistent with the result calculated before [23]. A number of interesting features of this quark distribution have been discussed in the literature [23, 24]. For example, at small $k_\perp$ limit, the quark distribution saturates: $xq(x, k_\perp)|_{k_\perp \to 0} \propto N_c/4\pi^4$; at the large $k_\perp$ limit, it has power behavior $xq(x, k_\perp)|_{k_\perp \gg Q_s} \propto Q_s^2/k_\perp^2$. These two features will be manifested if we employ the GBW model for the unintegrated gluon distribution from saturated dipole cross section: $F(q_\perp, x) = e^{-q_\perp^2/Q_s^2}/\pi Q_s^2$, where $Q_s^2$ is parameterized as $Q_s^2 = (x/x_0)^\lambda GeV^2$ with $x_0 = 3 \cdot 10^{-4}$ and $\lambda = 0.28$ [8]. Note that while the large $q_\perp$ behavior of the unintegrated gluon distribution $F(q_\perp, x)$ is incorrect in the GBW model (it falls exponentially instead of a power law), this bad feature does not translate to the TMD quark distribution: the convolution with the splitting kernel in Eq. (7) insures the proper leading-twist behavior. In Fig. 3 (left panel), we show the ratio of the TMD quark distribution $xq(x, k_\perp)$ relative to that at $x = 10^{-2}$ as a function of $k_\perp$ for $x = 10^{-4}$ and $x = 10^{-3}$, respectively. From this figure, we can clearly see that the ratio remains unchanged when $k_\perp$ goes to 0, whereas the ratio is proportional to the ratio of $Q_s^2$ at different $x$ when $k_\perp$ is large. This clearly demonstrates that the transverse momentum dependence provides an important information on the saturation physics. We have shown that these TMD quark distributions can be studied in semi-inclusive DIS process.

Furthermore, the transverse momentum dependence is also sensitive to the QCD dynamics in the small-$x$ evolution. In the above example, we took the simple parameterization from the GBW model [8]. This result shall be modified by the nonlinear evolution. For example, at large $q_\perp$, the un-integrated gluon distribution scales as $(q_\perp^2/Q_s^2)^{-\lambda_c}$ where $\lambda_c$ is the anomalous dimension [18, 30, 34, 35]. In the DGLAP domain, we have $\lambda_c = 1$ whereas in the BFKL domain it is $\lambda_c = 0.5$. By solving the BK equation, it was found that $\lambda_c = 0.63$ when the rapidity $Y = \ln 1/x$ goes to infinity [30]. With this modification, the ratio of the TMD quark distribution at large $k_\perp$ will approach $(Q_s^2)^{\lambda_c}$ instead of $Q_s^2$.

Another important QCD dynamics effects is the transverse momentum resummation [27], which will affect the $Q^2$ dependence of the $k_\perp$ spectrum. In the results we plotted in the left panel of Fig. 3, this effect was not considered, which correspond to the low $Q^2$ results, for example, at $Q^2 = Q_0^2 = 10 GeV^2$. This effect can be studied by applying the Collins-Soper-Sterman resummation method [27]. There have been great applications of this method to various high energy processes, in particular, in the semi-inclusive DIS at HERA [37] where important effects have been observed. To demonstrate this effect in the transverse
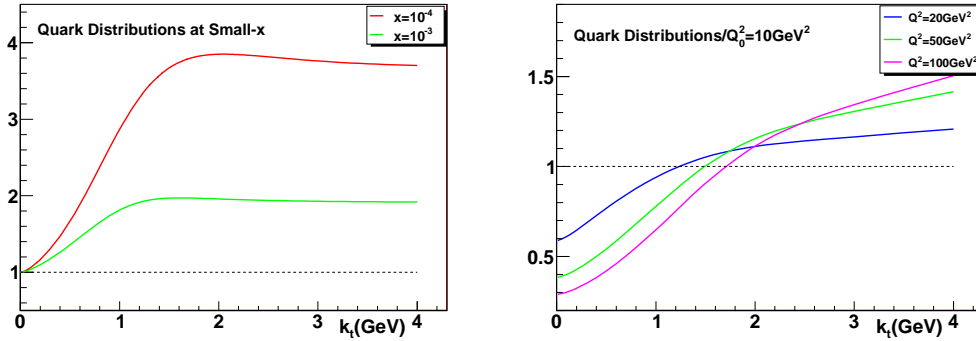
FIG. 3: *The transverse momentum dependent quark distributions at small-x: (left) at $x = 10^{-4}$ and $10^{-3}$ as ratios relative to that at $x = 10^{-2}$ for fixed $Q^2 = Q_0^2 = 10 GeV^2$ where the transverse momentum resummation effect is not important; (right) at different $Q^2$ relative to that at $Q^2 = 10 GeV^2$ for fixed $x = 3 \cdot 10^{-4}$ with $Q_s^2 = 1 GeV^2$ in the GBW model.*

momentum dependent quark distribution at small-$x$ we calculated above, we take the double leading logarithmic approximation (DLLA) to solve the evolution equation for the quark distributions. Under this approximation, we can write down quark distribution at higher $Q^2$ in terms of that at lower $Q_0^2$ [38, 39][3],

$$q(x, k_\perp; Q^2) = \int \frac{d^2 r}{(2\pi)^2} e^{ik_\perp \cdot r} e^{-S(Q^2, Q_0^2, r)} \int d^2 k'_\perp e^{-ik'_\perp \cdot r} q(x, k'_\perp; Q_0^2) , \qquad (12)$$

where the Sudakov form factor at the DLLA is defined by

$$S(Q^2, Q_0^2, r) = \ln \frac{Q^2}{Q_0^2} \left[ \frac{\alpha_s C_F}{4\pi} \ln(Q^2 Q_0^2 r^4) + c_0 r^2 \right] , \qquad (13)$$

where we have also included a non-perturbative form factor contribution $c_0 r^2 \ln Q^2/Q_0^2$ [40]. This resummation effect will shift the transverse momentum distribution to higher end when $Q^2$ increases. As an example, in Fig. 3 (right panel) we show the typical changes for the quark distributions at $Q^2 = 20, 50, 100 GeV^2$ as compared to the lower $Q_0^2 = 10 GeV^2$, with the following parameters: fixed coupling $\alpha_s = 0.3$, and $c_0 = 0.1$ for the non-perturbative input for the form factor [37]. From this plot, we can see that indeed, the transverse momentum distribution becomes harder when $Q^2$ is larger.

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[3] Here, we approximate the energy dependent parameter $\zeta$ in the TMD quark distribution by $Q$ assuming they are in the same order [28].

In conclusion, we have studied the semi-inclusive DIS processes at small-$x$, and found that the quark distribution studied can be used as a probe for the saturation physics. Especially, the ratio of the quark distributions is crucially depending on the saturation scale. We also studied the quark distribution at different $Q^2$, and found that the resummation effects shift the distribution to larger $k_\perp$ with larger $Q^2$. An ideal place to study this physics will be an electron-ion collider in the near future [41], where large nucleus target will provide an addition direction to study the saturation physics. Meanwhile, we notice that the ratios plotted in Fig. 3 qualitatively agree with the experimental data from HERA [42]. Of course, in order to compare to these data, we have to take into account the fragmentation contributions to calculate the differential cross sections. We also notice that an extension to a study on the gluon transverse momentum distributions [43] will have to consider both small-$x$ and transverse momentum resummations. The result from this paper shall provide us confidence to carry out these important studies.

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