

Subsidizing the Distribution Channel: Donor Funding to Improve the Availability of Malaria Drugs

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In countries that bear the heaviest burden of malaria, most patients seek medicine for the disease in the private sector. Because the availability and affordability of recommended malaria drugs provided by the private-sector distribution channel is poor, donors (e.g., the Global Fund) are devoting substantial resources to fund subsidies that encourage the channel to improve access to these drugs. A key question for a donor is whether it should subsidize the purchases and/or the sales of the private-sector distribution channel. We show that the donor should only subsidize purchases and should not subsidize sales. We characterize the robustness of this result to four key assumptions: the product's shelf life is long, the retailer has flexibility in setting the price, the retailer is the only level in the distribution channel, and retailers are homogeneous.

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1. Introduction

Malaria is estimated to have caused 660,000 deaths in 2010. The great majority of malaria cases and deaths are in sub-Saharan Africa, where the Democratic Republic of Congo and Nigeria alone account for more than 40% of all malaria deaths (World Health Organization 2012). In malaria-endemic countries, in part because public health clinics lack deep geographic reach especially into rural areas, the majority of people purchase malaria drugs from private-sector outlets such as drug shops (Laxminarayan et al. 2010, O'Connell et al. 2011). The private sector accounts for 74% of malaria drug volume in the Democratic Republic of Congo and 98% in Nigeria (O'Connell et al. 2011). Unfortunately, the private sector supply chain fails in providing high levels of availability for the drugs recommended to treat malaria, artemisinin combination therapies (ACTs). ACTs are the recommended first-line treatment for malaria because they are significantly more effective than previous generations of drugs to which the malaria parasite has developed resistance and because ACTs themselves are much less prone to encouraging the development of drug-resistant strains of malaria. In a study of availability of ACTs in sub-Saharan Africa, O'Connell et al. (2011) report that of private-sector outlets stocking malaria drugs, fewer than 25% had first-line quality-assured ACTs in stock. Further, the private-sector outlets priced ACTs 5 to 24 times higher than the previous generation, inferior malaria drugs.

A primary reason for the lack of affordable access to ACTs is that compared to the previous generation of drugs, ACTs are significantly more costly for private-sector outlets to acquire, largely because ACTs are more costly to produce (Arrow et al. 2004).

The lack of access to ACTs, and in particular the lack of access to ACTs at prices that are affordable to the poor, has motivated donors—bilateral donors such as the U.S. government; multilateral agencies such as the World Bank and the Global Fund to Fight AIDS, Tuberculosis and Malaria; large nongovernmental organizations such as the Clinton Health Access Initiative; and private philanthropic organizations such as the Bill and Melinda Gates Foundation—to intervene to improve access. Because of the important role played by the private-sector distribution channel, a primary way donors seek to achieve this objective is by designing and then funding product subsidies that encourage the channel to make decisions (e.g., stocking and pricing decisions) that improve the availability and affordability of the product to end consumers. Donors have committed a budget of \$216 million for ACT subsidies through the Affordable Medicines Facility—malaria (Adeyi and Atun 2010).

Bitran and Martorell (2009) emphasize that a fundamental issue that makes designing subsidies for ACTs different from designing subsidies for other products (e.g., non-health goods or preventative-health goods) is that demand for ACTs is uncertain in that it “arises

from an unpredictable event” (p. 16), namely infection, which triggers the need for treatment. The uncertainty in the number of infections is most pronounced at a narrow geographic level (e.g., the area served by a retail outlet). Alemu et al. (2012) report that the total number of malaria cases in a small geography (i.e., the area served by a single health clinic) varies significantly from year to year with no secular trend. Gomez-Elipse et al. (2007) report a similar finding at the geographic level of a province. Many factors contribute to the variability in the number of malaria cases: weather (rainfall, temperature), which impacts the population of the anopheles mosquitoes carrying the malaria parasite; the extent of parasite resistance to commonly used malaria drugs; the health conditions in the human population, which impact susceptibility to the malaria parasite; and the extent of malaria control and prevention activities such as the spraying of insecticide (Gomez-Elipse et al. 2007, Alemu et al. 2012). Each of these factors evolves in ways that can be challenging to predict, which contributes to uncertainty in the demand for malaria drugs.

A key question for donors in designing a subsidy is how it should be administered in the supply chain. One option is to reduce a firm’s cost of acquiring each unit via a *purchase subsidy*. A second option is to increase the revenue for each unit the firm sells via a *sales subsidy*. Voucher schemes have been used to implement sales subsidies at the retail level. The voucher provides a means by which the subsidy provider can verify a retailer’s sales to end consumers. A consumer presents a voucher when purchasing the product and receives a discount. For each redeemed voucher the retailer submits, the retailer receives a subsidy payment. In its report advising the global public health donor community on the design of subsidies for ACTs, the Institute of Medicine of the National Academies explicitly considers these two types of subsidies (Arrow et al. 2004).

The donor’s purpose in improving the availability and affordability of ACTs is ultimately to increase the consumption of ACTs by those afflicted by malaria. We formulate the donor’s problem as one of designing a purchase subsidy and sales subsidy to maximize consumer purchases subject to a constraint on the donor’s budget. Because donors are primarily interested in the availability and affordability of ACTs at the point where products are made available to consumers (Laxminarayan and Gelband 2009), we focus on the stocking and pricing decisions at the retail level. Our main finding is that the optimal subsidy consists solely of a purchase subsidy; i.e., the optimal sales subsidy is zero. This result, which we establish in the context of homogeneous retailers, extends when the retailers are heterogeneous. The result holds when the product’s shelf life is long relative to the replenishment interval,

as is typically the case for ACTs. The result breaks when the product’s shelf life is short; then it is optimal to offer a sales subsidy (in addition to a purchase subsidy) if and only if customer heterogeneity and the donor’s budget are sufficiently large. The result may also break when the retailer is not the only vertical layer in the distribution channel.

Several papers in the economics literature, starting with Pigou (1932), have examined the design of subsidies to encourage consumption of products with positive externalities. In this stream of literature, the donor’s utility depends on the recipient’s consumption of the product (e.g., Ben-Zion and Spiegel 1983). Daly and Giertz (1972) argue that if the donor’s utility depends on the recipient’s consumption choices, then the donor prefers a product subsidy to a cash transfer. In a product diffusion model, Kalish and Lilien (1983) examine how a donor should vary its product subsidy over time to accelerate consumer adoption. Our work differs from these papers in that they focus on the impact of subsidies on consumption levels, assuming product availability, whereas we focus on the impact of subsidies on product availability (and pricing), which in turn impacts consumption.

In an epidemiological model of malaria transmission, immunity, and drug resistance, Laxminarayan et al. (2006, 2010) study the impact of reductions in the retail price of ACTs on consumption. From simulation results under plausible parameters, they conclude that donor-funded price reductions are welfare enhancing. To capture the richness of disease progression and resistance, Laxminarayan et al. (2006, 2010) abstract away from the details of the distribution channel. We complement their macro-level approach with a micro-level approach that focuses on capturing these details: demand uncertainty, supply-demand mismatch, and the impact of subsidies on stocking and pricing decisions in the distribution channel.

Our micro-level approach is part of a stream of work in operations management and marketing that looks at the impact of incentives on the behavior of firms in a supply chain. Taylor (2002), Drèze and Bell (2003), Krishnan et al. (2004), and Aydin and Porteus (2009) examine a manufacturer who sets a per-unit purchase price and a rebate—a payment the manufacturer makes to the retailer for each unit the retailer sells to end consumers. From the retailer’s perspective, a rebate is similar to a sales subsidy in that both reward the retailer for sales, and a reduction in the purchase price is similar to a purchase subsidy in that both reward the retailer for purchasing. Taylor (2002), Drèze and Bell (2003), Krishnan et al. (2004), and Aydin and Porteus (2009) find that a manufacturer benefits by rewarding the retailer based on her sales. Drèze and Bell (2003) offer a sharper result: a manufacturer is better off rewarding the retailer based on her sales than her purchases. In

contrast, we show that it is optimal for a donor to reward the retailer not for sales, but only for purchases.

Two factors contribute to this divergence in results. First, the objectives of the incentive designers differ: the donor's objective is to maximize consumer purchases, whereas the manufacturer's objective is to maximize its profit. Second, the time scales of the incentives differ: Drèze and Bell (2003) consider short-term incentives (applying only during a promotion period), and Taylor (2002), Krishnan et al. (2004), and Aydin and Porteus (2009) consider incentives for a short shelf life product (one-period selling season); in contrast, we consider long-term incentives for a long shelf life product. Our finding that for a short shelf life product, the donor may benefit from rewarding the retailer based on her sales, similar to the finding in the contracting literature, indicates that the time scale of the incentive is important in driving the results.

Operations management researchers have examined subsidies for *short shelf life* products. Research on the influenza vaccine supply chain (e.g., Chick et al. 2008, Arifoğlu et al. 2012) identifies social-welfare enhancing interventions that influence manufacturer production decisions in settings where the production yield is uncertain and consumers' purchasing decisions are influenced by the fraction of the population that is vaccinated. Chick et al. (2008) shows that a properly designed supply-side intervention, namely a cost sharing contract, induces the manufacturer to produce the welfare-maximizing quantity. Arifoğlu et al. (2012) observe that combining a supply-side intervention with a demand-side intervention may be beneficial. In contrast, we find that for *long shelf life* products, it is unnecessary to intervene on the demand side (i.e., a purchase subsidy on the supply side is sufficient).

Cohen et al. (2013) and Ovchinnikov and Raz (2013) examine subsidizing a price-setting newsvendor retailer. Cohen et al. (2013) examine how demand uncertainty impacts the optimal sales subsidy. Ovchinnikov and Raz (2013) show that maximizing social welfare requires the use of both a purchase subsidy and a sales subsidy, where the sales subsidy is negative (a tax on consumption) unless the externality from consumption is small. This is directionally consistent with our finding that, for a short shelf life product, the optimal sales subsidy is strictly positive only if the donor's budget is large (which would tend to correspond to the case where the externality from consumption is large). We complement the literature on short shelf life products by focusing on long shelf life products and examining the impact of product shelf life length on the optimal subsidy.

2. Model Formulation and Preliminaries

We consider a donor (he) that offers subsidies to a retailer (she) that sells to end consumers. The donor

seeks to maximize sales to consumers subject to a budget constraint, whereas the retailer seeks to maximize her expected profit. Although, for simplicity, we model consumer sales as taking place through a single retailer, our results extend to the case with multiple, heterogeneous retailers, as described in §4.

The retailer sells to end consumers over a time horizon with an infinite number of time periods, indexed by $t = 1, 2, \dots$. In period t , consumer demand depends on the market condition M^t , where M^1, M^2, \dots are independent and identically distributed random variables with distribution F , density f , and continuous support on $[0, \infty)$.

The sequence of events is as follows: First, the donor offers a per-unit purchase subsidy $a \geq 0$ and a per-unit sales subsidy $s \geq 0$. The retailer begins with zero inventory. In each period t , the retailer places and receives an order, incurring per-unit acquisition cost $c > 0$ less the per-unit purchase subsidy a . The market condition uncertainty is resolved, and the retailer observes the market condition $M^t = m$. The retailer sets the price $p \geq 0$, and demand $D(m, p)$ is realized. For each unit the retailer sells, the retailer receives the price p from the end consumer, and the donor pays the retailer the sales subsidy s . Unmet demand is lost, which is realistic in that demand is triggered by the immediate need for treatment that follows infection. Leftover inventory is carried over to the next period, incurring a per-unit holding cost that is normalized to zero without loss of generality. Our assumption that leftover inventory is carried over to the next period is motivated by the relatively long shelf life of ACTs, 24 to 36 months (Anthony et al. 2012).

Our assumption that between replenishment intervals the retailer has freedom to set the price in response to market conditions reflects that retailers in developing countries have substantial discretion in setting and adjusting prices. For example, in the eight countries where ACT subsidies have been piloted, retailers do not face regulatory restrictions in setting their prices (O'Meara et al. 2013). Such restrictions would be difficult to enforce in sub-Saharan Africa because regulatory enforcement capability is weak (Goodman et al. 2007, 2009). In addition, there is evidence that retailers exercise pricing power: retailer markups are high (Auton et al. 2008, Patouillard et al. 2010) and price dispersion within individual markets is significant (Auton et al. 2008, Goodman et al. 2009, O'Meara et al. 2013).

Such pricing power is enhanced when one or a small number of retailers dominates a market; for evidence of such a retail market, concentration see Goodman et al. (2009) and Patouillard et al. (2010). Finally, there is evidence that antimalarial prices exhibit temporal fluctuations (Fink et al. 2014), which is consistent with our assumption that retailers adjust prices over time.

Consumer demand $D(m, p) = y(p)m$. In the special case where $y(p) \leq 1$ for $p \geq 0$, the market condition m can be interpreted as the number of customers in need of the product (e.g., because of infection), and $y(p)$ can be interpreted as the fraction of potential customers that purchase the product under price p . We impose the following two mild assumptions on $y(p)$:

ASSUMPTION 1. $y(p)$ is continuous, twice differentiable, and strictly decreases in p on $p \geq 0$, with $y(\bar{p}) = 0$ for $\bar{p} \in (0, \infty)$.

ASSUMPTION 2. $y(p)/y'(p)$ and $y(p)y''(p)/[y'(p)]^2$ increase in p .

Assumptions 1 and 2 are satisfied by common demand functions studied in the literature, including $y(p) = a - bp^k$ ($a, b > 0$ and $k \geq 1$), $y(p) = (a - bp)^k$ ($a, b, k > 0$), and $y(p) = a - be^{kp}$ ($a, b, k > 0$) (see Song et al. 2009).

We next formulate the retailer’s ordering and pricing problem under any given subsidy (a, s) and then turn to the donor’s problem. Consider any given period. Let x denote the retailer’s inventory before ordering. The retailer makes ordering and pricing decisions to maximize her expected discounted profit under discount rate $\delta \in (0, 1)$. Specifically, the retailer chooses order quantity $z - x \geq 0$ so as to bring her inventory up to $z \geq x$ and incurs purchase cost $(c - a) \cdot (z - x)$. After observing the realized market condition m , the retailer chooses price p , which results in sales of $\min(y(p)m, z)$ and revenue $(p + s) \min(y(p)m, z)$. The leftover inventory $z - \min(y(p)m, z)$ is carried into the next period. Consequently, the retailer’s expected discounted profit is

$$V(x) = \max_{z \geq x} \left\{ -(c - a)(z - x) + E_m \left[\max_{p \geq 0} \{ (p + s) \cdot \min(y(p)m, z) + \delta V(z - \min(y(p)m, z)) \} \right] \right\}.$$

By adapting well-known arguments for the case with an exogenous price (e.g., Karlin 1958), one can show that a myopic ordering and pricing policy is optimal. All proofs are in the appendix.

LEMMA 1. *In each period, the retailer’s optimal decisions are to order so as to bring her inventory up to z^* and after observing the market condition m to set the price equal to $p^*(m, z^*)$, where z^* and $p^*(m, z^*)$ are the order quantity and price that maximize the retailer’s expected profit in a single period where each unsold unit has salvage value $\delta(c - a)$. That is, z^* and $p^*(m, z^*)$ are the solution to*

$$\max_{z \geq 0} \left\{ -(c - a)z + E_m \left[\max_{p \geq 0} \{ (s + p) \min(y(p)m, z) + \delta(c - a)(z - \min(y(p)m, z)) \} \right] \right\}. \quad (1)$$

Intuitively, the retailer’s optimal policy is stationary because the underlying parameters are stationary. In each period, the retailer orders up to z^* , so by carrying a unit of unsold inventory into a subsequent period, the retailer avoids the cost of purchasing that unit, $c - a$, in the subsequent period. From the perspective of the retailer’s ordering decision, these cost savings are discounted by δ because they occur in the subsequent period.

Now we turn to the donor’s problem, which is to maximize the retailer’s sales subject to a budget constraint on the donor’s subsidy payment, in a sense that will be made precise momentarily. It follows from the above characterization of the retailer’s problem that the quantity sold in each period is $\min(y(p^*(m, z^*))m, z^*)$, a random variable that is stationary over the time horizon. Consequently, maximizing expected discounted sales over the time horizon is equivalent to maximizing expected per-period sales, which is equivalent to maximizing average sales per period, i.e., $E_m[\min(y(p^*(m, z^*))m, z^*)]$. For concreteness, we assume the donor maximizes average sales per period, but our formulation admits either of the other two objectives. We now turn to the budget constraint. In each period $t \geq 2$, the retailer’s order quantity is equal to $\min(y(p^*(m^{t-1}, z^*))m^{t-1}, z^*)$ and the realized sales is $\min(y(p^*(m^t, z^*))m^t, z^*)$. Because m^{t-1} and m^t have identical distributions, the retailer’s purchase quantity and sales quantity have identical distributions. Consequently, from period 2 onward, in each period, the donor’s expected purchase subsidy payment is $aE_m[\min(y(p^*(m, z^*))m, z^*)]$ and his expected sales subsidy payment is $sE_m[\min(y(p^*(m, z^*))m, z^*)]$. This implies that the donor’s average subsidy payment per period over the infinite time horizon is $(a + s)E_m[\min(y(p^*(m, z^*))m, z^*)]$.

The donor’s problem is to choose the purchase subsidy a and the sales subsidy s to maximize the average per-period sales to consumers, subject to the constraint that the average per-period subsidy payment does not exceed the (finite) budget B :¹

$$(P) \quad \max_{a, s \geq 0} E_m[\min(y(p^*(m, z^*))m, z^*)] \\ \text{s.t. } (a + s)E_m[\min(y(p^*(m, z^*))m, z^*)] \leq B.$$

Because the donor is trying to influence two decisions of the retailer—her stocking decision and her pricing

¹ Our results extend to the case where the budget constraint is based on the donor’s expected discounted subsidy payment, provided that the donor’s discount factor is sufficiently large. The latter is realistic in the malaria drugs context because the interval between periods is relatively short (the retailer’s replenishment interval is typically on the order of weeks) and because donors invest committed funds conservatively prior to disbursement (consequently, donors’ value from postponing payments is relatively small).

decision—and because the two different subsidy types influence these decisions in different ways, it might be natural to conjecture that the donor’s optimal subsidy would consist of both subsidy types, i.e., $a^* > 0$ and $s^* > 0$, at least for some parameters. In the next section, we show this conjecture is false: the optimal subsidy consists solely of a purchase subsidy, i.e., $s^* = 0$.

Before proceeding, we note that the presence of uncertainty in the market condition M^t is crucial to our study. Without uncertainty, in each period, the retailer sells exactly the quantity she purchases. Consequently, the purchase and sales subsidies are equivalent, and the question of the optimal mix of subsidies does not arise.

3. Results

We begin by characterizing the retailer’s optimal pricing and ordering decisions under subsidy (a, s) . From Lemma 1, the retailer’s optimal price is the price that maximizes retailer expected profit in a single period where unsold units have salvage value $\delta(c - a)$, i.e., the maximand in (1). This holds for any order-up-to level z that the retailer employs because by carrying a unit of unsold inventory into a subsequent period, the retailer avoids the cost of purchasing that unit in that period. To understand how the donor’s subsidies impact the retailer’s decisions, it is useful to write the retailer’s one-period expected profit under inventory level z as

$$R(z) = -(1 - \delta)(c - a)z + E_m \left[\max_{p \geq 0} \{ [s + p - \delta(c - a)] \min(y(p)m, z) \} \right]. \quad (2)$$

The first term is the net cost of purchasing z units and then salvaging them at the end of period; this is the loss the retailer would incur if she sold zero units. The second term is the revenue from selling units, less the forgone salvage value. For each unit the retailer sells, she receives the price p and the sales subsidy s , but she gives up the value of carrying the unit into the next period $\delta(c - a)$. The retailer’s price is only influenced by the acquisition cost c and purchase subsidy a in that they impact the value of unsold units. Consequently, for any subsidy (a, s) , order-up-to level z and realized market condition m , the retailer’s optimal price is

$$p^*(a, s, m, z) = \arg \max_{p \geq 0} \{ [s + p - \delta(c - a)] \min(y(p)m, z) \}. \quad (3)$$

In (3), we generalize the notation for the retailer’s optimal price to reflect the dependence of the optimal price on the subsidy (a, s) . Let $\tilde{p}(x) \equiv \arg \max_{p \geq 0} (x + p - \delta c)y(p)$.

LEMMA 2. Under subsidy (a, s) , order-up-to level z , and realized market condition m , the retailer’s optimal price is

$$p^*(a, s, m, z) = \begin{cases} \tilde{p}(s + \delta a) & \text{if } m \leq z/y(\tilde{p}(s + \delta a)), \\ y^{-1}(z/m) & \text{if } m > z/y(\tilde{p}(s + \delta a)). \end{cases}$$

Intuitively, the retailer prices to sell her entire stock (i.e., she sells z units) when the market condition is strong:

$$m > z/y(\tilde{p}(s + \delta a)) \quad (4)$$

but prices to withhold stock (i.e., she sells $y(\tilde{p}(s + \delta a))m$, which is strictly less than her stock z) when the market condition is weak:

$$m < z/y(\tilde{p}(s + \delta a)). \quad (5)$$

Consequently, stocking a larger quantity (using a larger order-up-to level z) leads the retailer to price more aggressively, strictly so if and only if the market condition is strong (4). Both subsidies encourage the retailer to price more aggressively, but the sales subsidy is more effective in doing so. More precisely, for any given order-up-to level z and realized market condition m , the retailer’s optimal price $p^*(a, s, m, z)$ decreases more rapidly in the sales subsidy s than in the purchase subsidy a :

$$\partial p^*(a, s, m, z)/\partial s \leq \partial p^*(a, s, m, z)/\partial a \leq 0, \quad (6)$$

where the inequalities are strict if and only if the market condition is weak (5). We refer to this as the *pricing effect*. (When the market condition is strong (4), the retailer prices to sell out regardless of the subsidies. Consequently, marginal changes in either subsidy do not impact the retailer’s pricing decision.)

The two subsidies encourage the retailer to price more aggressively, but for different reasons. The sales subsidy encourages the retailer to price more aggressively because the retailer receives not only the price but also the sales subsidy for each unit she sells, which makes it attractive for the retailer to reduce the price so as to increase the volume of units that are eligible for the subsidy. The reason why the purchase subsidy encourages more aggressive pricing is more subtle. The purchase subsidy, by reducing the cost of acquiring units in the subsequent period, reduces the value to the retailer of carrying unsold units into the next period. Consequently, the retailer prices more aggressively to clear out her existing inventory. Because the sales subsidy impacts the retailer’s profit immediately, whereas the purchase subsidy impacts the retailer’s profit in the next period, the donor must offer a more generous purchase subsidy in order to have the same impact on the retailer’s price. Specifically, to have the same impact on the retailer’s pricing decision as sales subsidy s requires purchase subsidy $a = s/\delta$. Although sales subsidy s and purchase subsidy $a = s/\delta$ have the

same impact on the retailer's pricing decision under a fixed order-up-to level, we will see below that the two subsidies have different impacts on the retailer's optimal order-up-to level.

We now turn to the retailer's optimal ordering decision. As noted above, the retailer's optimal order-up-to level is the order quantity that maximizes retailer expected profit in a single period where each unsold unit has salvage value $\delta(c - a)$. Embedding the optimal price from Lemma 2, we can write the retailer's one-period expected profit under subsidy (a, s) and inventory level z more explicitly as

$$\begin{aligned} R(a, s, z) = & \int_0^{z/y(\tilde{p}(s+\delta a))} [s + \tilde{p}(s + \delta a) - \delta(c - a)] \\ & \cdot y(\tilde{p}(s + \delta a))mf(m) dm \\ & + \int_{z/y(\tilde{p}(s+\delta a))}^{\infty} [s + y^{-1}(z/m) - \delta(c - a)] \\ & \cdot zf(m) dm - (1 - \delta)(c - a)z. \end{aligned}$$

The next lemma characterizes the retailer's optimal order-up-to level $z^*(a, s)$.

LEMMA 3. *The retailer's one-period expected profit $R(a, s, z)$ is strictly concave in the order-up-to level z . Its unique maximizer $z^*(a, s)$ is the unique solution to*

$$\begin{aligned} & \int_{z/y(\tilde{p}(s+\delta a))}^{\infty} (s + H(z, m) - c + a)f(m) dm \\ & = \int_0^{z/y(\tilde{p}(s+\delta a))} (1 - \delta)(c - a)f(m) dm, \quad (7) \end{aligned}$$

where $H(z, m) \equiv d\{y^{-1}(z/m)z\}/dz$.

The retailer's one-period expected profit under inventory level z consists of two elements: the cost of stocking and not selling units (i.e., the net cost of purchasing and then salvaging z units) and the contribution (i.e., revenue less purchase cost) from selling units. The retailer chooses an order-up-to level that equates the marginal contribution from stocking and selling a unit (the left-hand side of (7)) with the marginal cost of stocking and not selling a unit (the right-hand side of (7)).

Expression (7) illuminates the distinct mechanisms by which each subsidy increases the retailer's optimal order-up-to level $z^*(a, s)$. An incremental dollar of sales subsidy has the same impact as an incremental dollar of purchase subsidy on the marginal contribution from stocking and selling a unit. However, only the purchase subsidy impacts the marginal cost of stocking and not selling a unit. Therefore, the purchase subsidy is more effective in boosting the retailer's order-up-to level. Formally, the retailer's optimal order-up-to level $z^*(a, s)$ increases more rapidly in the purchase subsidy a than in the sales subsidy s :

$$\partial z^*(a, s)/\partial a > \partial z^*(a, s)/\partial s > 0. \quad (8)$$

We refer to this as the *quantity effect*.

From Lemmas 2 and 3, the average per-period sales to consumers is

$$\begin{aligned} \mathcal{S}(a, s) = & E_m[\min(y(\tilde{p}(s + \delta a))m, z^*(a, s))] \\ = & \int_0^{z^*(a, s)/y(\tilde{p}(s+\delta a))} y(\tilde{p}(s + \delta a))mf(m) dm \\ & + \int_{z^*(a, s)/y(\tilde{p}(s+\delta a))}^{\infty} z^*(a, s)f(m) dm, \quad (9) \end{aligned}$$

where the retailer's optimal order-up-to level $z^*(a, s)$ satisfies (7).

The donor's problem is to choose the purchase subsidy a and the sales subsidy s to maximize the average per-period sales to consumers, subject to the constraint that the average per-period subsidy payment does not exceed the budget B :

$$\begin{aligned} \text{(P)} \quad & \max_{a, s \geq 0} \mathcal{S}(a, s) \\ & \text{s.t. } (a + s)\mathcal{S}(a, s) \leq B. \end{aligned}$$

Increasing each subsidy causes the retailer to increase her stock level and to price more aggressively, both of which further the donor's objective of increasing sales to consumers. However, increasing each subsidy also increases the donor's subsidy payment. How should the donor optimally choose the mix of purchase subsidy a and sales subsidy s ?

To build intuition about the donor's optimal mix of subsidies, it is useful to examine how each subsidy impacts the volume of sales to consumers. Both subsidies increase the retailer's sales volume $\mathcal{S}(a, s)$ not only by encouraging the retailer to stock more ex ante (the quantity effect) but also by inducing the retailer to price more aggressively ex post (the pricing effect). However, the magnitude of these two effects differs under the two subsidies. As noted in (6), the sales subsidy is stronger in encouraging the retailer to price more aggressively (for any given order-up-to level z). In contrast, as noted in (8), the purchase subsidy is stronger in encouraging the retailer to stock more aggressively. Let (4') and (5') denote (4) and (5), where $z = z^*(a, s)$. When the market condition is strong (4'), the pricing effect is irrelevant because the retailer prices to sell her entire stock; consequently, only the quantity effect impacts sales to consumers. In contrast, when the market condition is weak (5'), the quantity effect is irrelevant because the retailer prices to withhold stock; consequently, only the pricing effect impacts sales to consumers.

To summarize, the sales subsidy is more effective in increasing sales when the market condition is weak, but the purchase subsidy is more effective in increasing sales when the market condition is strong. Lemma 4 establishes that, averaging across market condition realizations, the latter effect dominates the former: the purchase subsidy is more effective in increasing average per-period sales to consumers. This is plausible in

that when the market condition is weak (which can be interpreted as there being few customers in the market), the amount by which the subsidies impact the magnitude of sales to consumers is limited. Because the pricing effect (which favors the sales subsidy) is only at work when the market condition is weak, the magnitude of this effect is limited—in a way that the magnitude of the quantity effect (which favors the purchase subsidy) is not.

LEMMA 4. *Average per-period sales to consumers $\mathcal{P}(a, s)$ increase more rapidly in the purchase subsidy a than in the sales subsidy s :*

$$\partial \mathcal{P}(a, s) / \partial a > \partial \mathcal{P}(a, s) / \partial s > 0$$

for any $s \geq 0$ and $a \geq 0$.

The observation that, regardless of the subsidy levels (a, s) , a marginal increase in the purchase subsidy is more effective than a marginal increase in the sales subsidy in increasing sales to consumers (Lemma 4) is the key insight that drives the donor's optimal mix of subsidies (Proposition 1). To see this, consider any subsidy (a, s) where the sales subsidy $s > 0$. From Lemma 4, the same average per-period sales to consumers can be achieved by eliminating the sales subsidy and increasing the purchase subsidy a by an amount that is strictly smaller than s . Clearly, this modification strictly reduces the average per-period subsidy payment. Consequently, any subsidy (a, s) with $s > 0$ cannot be optimal. This establishes the paper's main result.

PROPOSITION 1. *The donor's optimal subsidy consists solely of a purchase subsidy, i.e., the optimal sales subsidy $s^* = 0$.*

Proposition 1 is consistent with industry practice in that the current subsidy for ACTs, as provided through the Affordable Medicines Facility—malaria, is solely a purchase subsidy—a subsidy that reduces the distribution channel's acquisition cost (Adeyi and Atun 2010). Although Proposition 1 is consistent with practice, it does not provide the only explanation for why donors chose a purchase subsidy. An important factor that donors consider in designing a subsidy is the administrative costs to implement the subsidy, and these costs will tend to be higher for the sales subsidy (Arrow et al. 2004, Bitran and Martorell 2009). Our contribution is to show that even when such administrative costs are ignored, when one considers the operational elements of market uncertainty and supply-demand mismatch, the purchase subsidy is superior to the sales subsidy.

4. Extensions

In this section, we consider the extent to which our main result—the donor should offer a purchase subsidy but not a sales subsidy—is robust to four of our key

assumptions: the donor designs his subsidy with a single retailer in mind, the retailer has flexibility in setting the price, the product has a long shelf life such that leftover inventory can be sold in a subsequent period, and the retailer is the only level in the distribution channel. We show that the result is robust to the first two assumptions (Propositions 2 and 3). For a perishable product, we find that the result carries through provided that the donor's budget is small, customer heterogeneity in valuations is small, or the product's shelf life is sufficiently long. When these conditions are all reversed, the result is reversed: the optimal subsidy consist of both subsidies (Proposition 4). The result may also be reversed when there is a price-setting intermediary in the supply chain.

More generally, this section provides a more complete picture of how various factors influence the design of subsidies to improve consumer access to public health goods. Because these factors (e.g., product shelf life) differ depending on the product and the nature of the distribution channel, these insights broaden the applicability of the findings.

4.1. Heterogeneous Retailers

The formulation with a single retailer informs the donor's decision when a particular type of retailer (e.g., a drug shop) is the primary means by which consumers access the product in the region where the subsidy is offered, and retailers of this type are relatively homogeneous. Then the donor can design his subsidy with this representative retailer in mind. The formulation is also appropriate when the donor is able to tailor his subsidy to each retailer (or to each type of retailer, in the case that retailers fall into categories). Proposition 1 implies that when the donor facing N retailers is able to offer a different subsidy (a_i, s_i) to each retailer $i \in \{1, \dots, N\}$, it is optimal to set each retailer i 's sales subsidy $s_i^* = 0$ and to only offer purchase subsidies.

However, offering different subsidies to different retailers entails administrative costs and introduces the threat of product diversion. Consequently, a donor may be compelled to offer a uniform subsidy (a, s) to a heterogeneous pool of retailers. This subsection shows that the result from the tailored-subsidy case carries over to the uniform-subsidy case: the optimal uniform subsidy (a^*, s^*) has sales subsidy $s^* = 0$.

Specifically, consider N retailers, each facing a common subsidy (a, s) . Retailer i 's per-unit acquisition cost is c_i . The market condition for retailer $i \in \{1, \dots, N\}$ in period $t \in \{1, 2, \dots\}$, M_t^i , has distribution F_t and density f_t . Under market condition $M_t^i = m_t$ and price p_t , retailer i 's demand is $D_i(m_t, p_t) = y_i(p_t)m_t$.

Retailer i 's optimal order-up-to level and price under any subsidy (a, s) are given in Lemmas 2 and 3. The donor's problem is to choose the purchase subsidy a and the sales subsidy s to maximize the average per-

period sales to consumers across all retailers, subject to the constraint that the average per-period subsidy payment across all retailers does not exceed the budget B :

$$\begin{aligned} \max_{a, s \geq 0} \quad & \sum_{i=1}^N \mathcal{S}_i(a, s) \\ \text{s.t.} \quad & (a + s) \sum_{i=1}^N \mathcal{S}_i(a, s) \leq B, \end{aligned}$$

where $\mathcal{S}_i(a, s)$ is retailer i 's average per-period sales to consumers. Our main result, Proposition 1, extends when the donor faces multiple retailers.

PROPOSITION 2. *Suppose the donor faces heterogeneous retailers. The donor's optimal subsidy consists solely of a purchase subsidy, i.e., the optimal sales subsidy $s^* = 0$.*

The intuition is that for each individual retailer, a marginal increase in the purchase subsidy is more effective than a marginal increase in the sales subsidy in increasing sales to consumers (Lemma 4). Consequently, the purchase subsidy strictly dominates the sales subsidy in effectiveness.

4.2. Pricing Flexibility

We have assumed that the retailer has considerable flexibility in setting the price: In each period t , the retailer sets the price *after* observing the market condition $M^t = m^t$. This is most realistic when a retailer is well informed about current demand conditions and can easily adjust the price in response to these conditions. In this subsection, we consider two scenarios in which the retailer has less pricing flexibility. First, if a retailer lacks complete freedom to adjust the price and lacks a strong understanding of market conditions at the time when she must commit to the price, then a more fitting assumption is that in each period t the retailer chooses the price *before* observing the market condition $M^t = m^t$. Second, in the extreme case, the retailer lacks any pricing flexibility: the price p is exogenous (e.g., the price is dictated by regulation). Our main result, Proposition 1, extends to both of these settings.

PROPOSITION 3. *Suppose either (a) in each period t , the retailer chooses the price p prior to observing the market condition M^t , or (b) the price p is exogenous. The donor's optimal subsidy consists solely of a purchase subsidy, i.e., the optimal sales subsidy $s^* = 0$.*

In all three scenarios of pricing flexibility, the quantity effect (which favors the purchase subsidy) outweighs the pricing effect (which favors the sales subsidy). When the level of pricing flexibility is reduced, the magnitude of the pricing effect diminishes, which furthers the dominance of the purchase subsidy over the sales subsidy.

4.3. Product Shelf Life

We have assumed that the product has a sufficiently long shelf life that leftover inventory can be sold in a

subsequent period. The mostly commonly used ACTs have a shelf life of 24 months from the time of manufacturer, although some have a shelf life of 36 months (Anthony et al. 2012). If the time from manufacturer to the time of retailer receipt of the product is not too long (measured in months rather than years) and the retailer's replenishment interval is not too long (weekly or monthly rather than annually), then this assumption is a reasonable approximation. However, for some retailers that are remotely located, where transportation is difficult and costly, these assumptions may not be realistic and the perishability of the product may be a real concern.

To address this issue, we extend our base model by assuming that in each period a deterministic fraction $\gamma \in [0, 1]$ of leftover inventory perishes. Perishability $\gamma = 1$ represents the case where the *product has a short shelf life*: the product's remaining shelf life at the time of retailer receipt is sufficiently short relative to the retailer's replenishment interval, such that unsold inventory cannot be sold in a subsequent period. At the other extreme, the base model with $\gamma = 0$ represents the case where the *product has a long shelf life*. We label perishability $\gamma \in (0, 1)$ as representing the case where the *product has a moderate shelf life*, acknowledging that this is only an approximation (a richer model would explicitly keep track of the remaining shelf life of each unit of inventory).

We begin by establishing that Lemmas 1–4 hold for $\gamma \in [0, 1]$. Recall that in the base model ($\gamma = 0$), by Lemma 1 the retailer's optimal order quantity and price in each period are the solutions to the single-period problem in which each unsold unit has salvage value $\delta(c - a)$. When a fraction γ of the unsold units perishes, the same result holds, except that each unsold unit has salvage value $\delta(1 - \gamma)(c - a)$. Consequently, the retailer's optimal price $p_\gamma^*(a, s, m, z)$ and order quantity $z_\gamma^*(a, s)$ are as stated in Lemmas 2 and 3, where δ is replaced by $\delta(1 - \gamma)$. (We abuse notation by using the subscript to denote dependence on the perishability γ .) Consequently, the average per-period sales to consumers $\mathcal{S}_\gamma(a, s)$ is given by (9), where δ is replaced by $\delta(1 - \gamma)$. Further, inequalities (6) and (8), as well as Lemma 4, continue to hold.

In each period, the donor pays the purchase subsidy a on each unit the retailer purchases and the sales subsidy s on each unit the retailer sells. The donor's average per-period subsidy payment is

$$a\{\mathcal{S}_\gamma(a, s) + \gamma[z_\gamma^*(a, s) - \mathcal{S}_\gamma(a, s)]\} + s\mathcal{S}_\gamma(a, s), \quad (10)$$

which is equivalent to the donor's expected subsidy payment in a single period. To understand the first term in (10), note that in each period, to bring her inventory up to $z_\gamma^*(a, s)$, the retailer purchases not only the quantity she sold in the previous period (on average,

$\mathcal{S}_\gamma(a, s)$) but also the quantity that perished at the end of that period (on average, $\gamma[z_\gamma^*(a, s) - \mathcal{S}_\gamma(a, s)]$). Therefore, the donor’s problem is

$$(P_\gamma) \quad \max_{a, s \geq 0} \mathcal{S}_\gamma(a, s) \\ \text{s.t. } a\gamma z_\gamma^*(a, s) + [a(1 - \gamma) + s]\mathcal{S}_\gamma(a, s) \leq B.$$

The remainder of this subsection is organized as follows. First, we focus on the case where the product has a short shelf life $\gamma = 1$ and establish analytical results regarding the donor’s optimal subsidy (a^*, s^*) (Proposition 4). Second, we turn to the case where the product has a moderate shelf life $\gamma \in (0, 1)$ and report numerical results.

Recall that when the product has a long shelf life, the donor’s optimal subsidy consists solely of a purchase subsidy; i.e., the optimal sales subsidy $s^* = 0$ (Proposition 1). To what extent does this result continue to hold when the product has a short shelf life?

The donor’s optimal mix of subsidies is driven by three effects, the first two of which carry over from the long shelf life product setting. First, the pricing effect (6) favors the sales subsidy: for any inventory level z , the sales subsidy is more effective in encouraging the retailer to price aggressively. Second, the quantity effect (8) favors the purchase subsidy: the purchase subsidy is more effective in encouraging the retailer to stock aggressively. However, a key feature of the setting where the product has a long shelf life—that the purchase subsidy has the effect of reducing the value of unsold inventory—disappears when the product has a short shelf life. A consequence is that the product’s having a short shelf life makes both the pricing effect and the quantity effect more pronounced: the product’s having a short shelf life makes the sales subsidy even more effective relative to the purchase subsidy in encouraging the retailer to price aggressively but even less effective in encouraging the retailer to stock aggressively.

The product having a short shelf life introduces a third effect, which we label the *payment effect*. When the product has a short shelf life, the sales subsidy is more attractive to the donor in that the donor only pays the sales subsidy on units the retailer sells, which are fewer than the units the retailer purchases. (In contrast, when the product has a long shelf life, in each period, the quantity to which the sales subsidy is applied and the quantity to which the purchase subsidy is applied have identical distributions; see §2.)

Because Lemma 4 extends to the case where the product has a short shelf life, we know that, in terms of increasing average per-period sales, the purchase subsidy is more effective than is the sales subsidy (i.e., the quantity effect dominates the pricing effect). Therefore, we are left with two countervailing forces. The purchase subsidy is more effective than the sales subsidy in increasing the donor’s objective. However,

the purchase subsidy is also more costly to the donor (the payment effect). How do these effects interact to determine the optimal mix of donor subsidies?

Intuition suggests that the donor’s optimal subsidy will include a sales subsidy $s^* > 0$ only when one or both of the factors that favor the sales subsidy are strong: the payment effect and the pricing effect. We now argue that the payment effect tends to be stronger when budget is large. When the budget (and hence subsidy level) is small, the retailer will tend only to purchase units she is quite confident she will sell. Because the quantity to which the sales subsidy applies (sold units) differs little from the quantity to which the purchase subsidy applies (purchased units), the payment effect is small. When the budget (and hence subsidy level) is large, the retailer will tend to be willing to stock aggressively, recognizing that for some realizations of the market condition she will be left with unsold inventory. That is, the gap between the (average) quantity to which the sales subsidy applies and the quantity to which the purchase subsidy applies is significant; consequently, the payment effect will tend to be large. In sum, the payment effect becomes increasingly important as the budget increases.

Intuition suggests that the pricing effect becomes more important as the customers become more heterogeneous in their willingness to pay. If customers are completely homogeneous in their willingness to pay (i.e., they share a common valuation), then the pricing effect is irrelevant because the retailer will set the price equal to the common valuation regardless of the subsidy. As customers become more heterogeneous in their willingness to pay, the retailer’s pricing decision becomes more important and is more influenced by the subsidies.

All this suggests that the donor’s optimal subsidy will include a sales subsidy $s^* > 0$ when the budget is large and customers are sufficiently heterogeneous in their willingness to pay but will not include a sales subsidy $s^* = 0$ when these factors are not present. We now turn to formally establishing this conjecture. For convenience, we adopt a specific functional form for the demand curve where customer heterogeneity is captured in a single parameter:

$$y(p) = \begin{cases} 1 & \text{if } p \leq \mu - \Delta, \\ (\mu + \Delta - p)/2\Delta & \text{if } p \in [\mu - \Delta, \mu + \Delta], \\ 0 & \text{if } p \geq \mu + \Delta. \end{cases} \quad (11)$$

The case with linear demand (11) corresponds to the case where customers in need of the product are uniformly distributed in their willingness to pay over the interval $[\mu - \Delta, \mu + \Delta]$; $\Delta \in [0, \mu]$ is a measure of the degree of heterogeneity in the customers’ willingness to pay. (Although (11) deviates from Assumption 1 when $\Delta < \mu$, the four lemmas leading up to Proposition 1

continue to hold. The only exception is that results parallel to Lemmas 2 and 3—see Lemmas 2A and 3A in the appendix—characterize the retailer’s optimal decisions.)

PROPOSITION 4. *Suppose the product has a short shelf life (perishability $\gamma = 1$) and that demand is linear (11). There exists $\underline{B} \in (0, \infty)$ such that (a) if the donor’s budget $B < \underline{B}$ or customer heterogeneity $\Delta \leq \mu/3$, then the donor’s optimal subsidy consists solely of a purchase subsidy, i.e., the optimal sales subsidy $s^* = 0$. (b) Otherwise, the donor’s optimal subsidy consists of both subsidies, i.e., the optimal purchase subsidy $a^* > 0$ and the optimal sales subsidy $s^* > 0$.*

Lemma 5 in the appendix characterizes how the optimal subsidies (a^*, s^*) change as the budget B increases. For the interesting case where customer heterogeneity $\Delta > \mu/3$, a^* and s^* weakly increase in the budget. As the budget increases from zero, initially only the purchase subsidy a^* strictly increases, then only the sales subsidy s^* strictly increases, and then finally only the purchase subsidy a^* increases.

We conclude that a donor subsidizing a short shelf life product should only offer a sales subsidy when his budget and customer heterogeneity in willingness to pay are sufficiently large. The health condition of a patient prior to contracting malaria impacts the health consequences of contracting the disease (e.g., a person with HIV tends to suffer more severe health consequences from malaria), which impacts the value the patient attaches to effective treatment. Consequently, heterogeneity in willingness to pay will tend to be most pronounced when the customer population exhibits significant heterogeneity in health conditions and income.

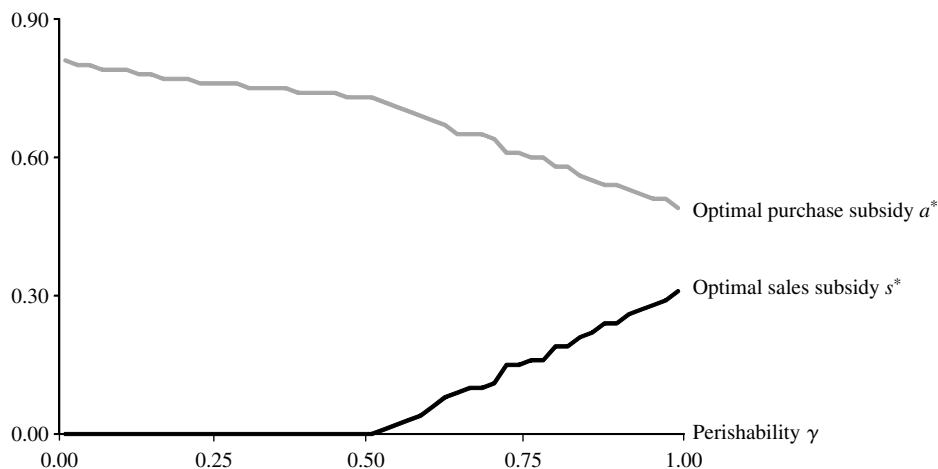
To obtain insights when the product has a moderate shelf life $\gamma \in (0, 1)$, we conducted a numerical study. We assume demand is linear (11) and $M \sim \text{Uniform}(0, 1)$. We consider the 9,180 combinations

of the following parameters: $\mu = 1$, $\Delta \in \{0.25, 0.50, 0.75, 1.00\}$, $c \in \{0.2(\mu + \Delta), 0.5(\mu + \Delta), 0.8(\mu + \Delta)\}$, $\delta \in \{0.2, 0.5, 0.8\}$, $B \in \{0.1\bar{B}, 0.3\bar{B}, 0.5\bar{B}, 0.7\bar{B}, 0.9\bar{B}\}$, where \bar{B} is the budget under $a = c$ and $s = 0$, and $\gamma \in \{(0.00, 0.02, 0.04, \dots, 1.00)\}$. There are 99 combinations of Δ , c , δ , and B for which the donor’s budget $B < \bar{B}$ or customer heterogeneity $\Delta \leq \mu/3$; there are 81 combinations for which $B \geq \bar{B}$ or customer heterogeneity $\Delta > \mu/3$.

In each instance in which the donor’s budget $B < \bar{B}$ or customer heterogeneity $\Delta \leq \mu/3$, the optimal sales subsidy is $s^* = 0$. This suggests that Proposition 4(a)’s insight that donor’s optimal subsidy consists solely of a purchase subsidy if either the donor’s budget or customer heterogeneity are small extends when $\gamma \in (0, 1)$. In each instance in which the budget $B \geq \bar{B}$ and customer heterogeneity $\Delta > \mu/3$, the optimal sales subsidy s^* is zero for perishability $\gamma \in [0, \bar{\gamma}]$ and then is strictly positive and increasing in perishability γ on $\gamma \in (\bar{\gamma}, 1]$ for some $\bar{\gamma} \in (0, 1)$. Figure 1 depicts a representative example. The intuition is that as perishability γ increases, the payment effect—which favors the sales subsidy—strengthens, so that the donor’s optimal mix of subsidies shifts by increasing the sales subsidy s^* and decreasing the purchase subsidy a^* .

The numerical study provides evidence that the insights from Propositions 1 and 4 are not driven by only the extreme cases of perishability $\gamma \in \{0, 1\}$. Across the instances in the large budget and customer heterogeneity regime, the median perishability threshold $\bar{\gamma}$ is 0.54 (the average threshold $\bar{\gamma}$ is 0.52). That the median perishability thresholds $\bar{\gamma}$ is well above zero suggests that the insights from Proposition 1 are not driven by the limiting assumption that $\gamma = 0$. That the median perishability threshold’s $\bar{\gamma}$ is well below unity suggests that the insights from Proposition 4 are not driven by the limiting assumption that $\gamma = 1$.

Figure 1 Donor’s Optimal Subsidy as Function of Perishability



Note. Parameters are $\mu = 1$, $\Delta = 1$, $c = 0.5(\mu + \Delta)$, $\delta = 0.5$, and $B = 0.7\bar{B}$.

4.4. Endogenous Retailer Acquisition Cost: Price-Setting Intermediary

Because donors are primarily interested in the availability and affordability of ACTs at the point where products are made available to consumers (Laxminarayan and Gelband 2009), it is essential that a model include the retailer’s stocking and pricing decisions. However, in focusing on the retail level, we have ignored vertical layers above in the distribution channel (e.g., wholesaler). In designing a subsidy for ACTs, a concern for donors is how much of the subsidy will be passed through to consumers (Arrow et al. 2004). Our model addresses this issue in that the retailer passes through only a portion of the subsidy to consumers. In practice, retailers’ markups for malaria drugs are larger than those at wholesale levels (Patouillard et al. 2010), which provides support for focusing on the retail level. Goodman et al. (2009) point to the retail level as being of central concern with respect to the issue of pass through because of market concentration and pricing power there. Our model captures the setting where the wholesale markup—or more precisely, the retailer’s acquisition cost c —is not influenced by the subsidy (a, s) . This may be plausible when wholesale prices are regulated (because there are fewer wholesalers than retailers, it is easier to regulate wholesale prices than retail prices) or when the wholesaler applies a common markup across a set of products. However, in other settings the retailer’s acquisition cost c will be influenced by the subsidy (a, s) , which will affect how much of the subsidy is passed through to consumers.

To address this issue, we extend our base model to include a supplier that produces to order, incurring cost k per unit. Before period $t = 1$, the donor sets the subsidy (a, s) , and then the supplier sets the wholesale price, which, with some abuse of notation, we denote as c because it represents the retailer’s per-unit acquisition cost. The supplier sets the wholesale price c to maximize her average per-period profit, $(c - k)\mathcal{S}(a, s, c)$, where we generalize the notation \mathcal{S} , the retailer’s average per-period sales to consumers, to reflect its dependence on the wholesale price c . Let $c^*(a, s)$ denote the supplier’s optimal wholesale price. To obtain insights we conducted a numerical study. We assume demand is linear (11) and $M \sim \text{Uniform}(1 - \eta, 1 + \eta)$. We consider the 6,250 combinations of the following parameters: $\mu = 1$, $\Delta \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, $\eta \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, $k \in \{0.1(\mu + \Delta), 0.3(\mu + \Delta), 0.5(\mu + \Delta), 0.7(\mu + \Delta), 0.9(\mu + \Delta)\}$, $\delta \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, and $B \in \{0.1\bar{B}, 0.2\bar{B}, \dots, 1.0\bar{B}\}$, where \bar{B} is the budget under $a = k$ and $s = 0$.

We highlight two observations from the numerical results. First, the optimal sales subsidy s^* is strictly positive in some instances. Second, the donor loses very little by restricting his subsidy to be only a purchase subsidy. Let \hat{a} denote the donor’s optimal purchase subsidy under sales subsidy $s = 0$. Across the 6,250 instances,

the median loss in average per-period sales under the optimal purchase subsidy relative to the optimal purchase and sales subsidy $[\mathcal{S}(a^*, s^*, c^*(a^*, s^*)) - \mathcal{S}(\hat{a}, 0, c^*(\hat{a}, 0))]/\mathcal{S}(a^*, s^*, c^*(a^*, s^*))$ is 0.36% (the average loss is 0.43% and the maximum loss is 2.22%).

Each observation maps to a managerial message. First, the result that the optimal sales subsidy $s^* = 0$ need not hold when the retailer’s acquisition cost is endogenous. The presence of a price-setting intermediary increases the relative attractiveness of the sales subsidy because of what we label the *pass-through effect*. Because the purchase subsidy pushes down the retailer’s cost of purchasing a unit, the supplier can directly capture a portion of the subsidy by raising the wholesale price; i.e., the supplier can limit the fraction of the subsidy that is passed through to the retailer. The sales subsidy has a less direct effect on the retailer’s profit, so the supplier tends to allow a larger fraction of the subsidy to be passed through to the retailer. This makes the sales subsidy comparatively more attractive to the donor. This is consistent with Institute of Medicine’s report advising donors, which speculates that the issue of pass through may be more pronounced for a purchase subsidy than a sales subsidy (Arrow et al. 2004). However, the fact that the purchase subsidy alone performs well indicates that the magnitude of the pass-through effect tends to be small—at least relative to the quantity effect, which favors the purchase subsidy. This leads into the next message.

Second, the managerial recommendation that donors should focus on offering purchase subsidies instead of sales subsidies appears robust to the presence of a price-setting intermediary. For a donor that is offering a purchase subsidy, the gain from adding a sales subsidy is minimal and is likely to be outweighed by the substantial administrative cost involved in administering such a subsidy.

5. Conclusion

This paper provides guidance to donors designing purchase and sales subsidies to improve consumer access to a product in the private-sector distribution channel. Specifically, we characterize analytically how the product’s characteristics (short versus long shelf life), customer population (degree of heterogeneity), and the size of the donor’s budget impact the donor’s subsidy design decision. It is always optimal to offer a purchase subsidy. For short shelf life products, it is optimal to offer a sales subsidy (in addition to a purchase subsidy) if and only if the customer heterogeneity and the donor’s budget are sufficiently large. In contrast, for long shelf life products (e.g., ACTs typically), donors should only offer a purchase subsidy.

Although we have focused on ACTs, our results could inform donor subsidy decisions for other products. As with ACTs, for other medicines, in much of the developing world the private sector is the primary way

patients access treatment (Prata et al. 2005, International Finance Corporation 2007), yet private-sector supply chains fail in providing high levels of availability of medicines (Cameron et al. 2009). For example, oral rehydration salts (ORS) are the first-line treatment for childhood acute diarrhea, the second-leading cause of child mortality worldwide. Most treatment for childhood acute diarrhea is accessed in the private sector, but only 30% of children with diarrhea in high burden countries receive ORS. The United Nations’ newly launched Commission on Life-Saving Commodities for Women’s and Children’s Health is examining ways to increase access to essential medicines such as ORS (Sabot et al. 2012). One option, which has received limited testing, is to subsidize ORS (MacDonald et al. 2010, Gilbert et al. 2012).

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Appendix

PROOF OF LEMMA 1. Taking the term $(c - a)x$ outside of the maximization, we can rewrite $V(x)$ as $V(x) = (c - a)x + \max_{z \geq x} J(z)$, where

$$J(z) = -(c - a)z + E_m \left[\max_{p \geq 0} \{ (p + s) \min(y(p)m, z) + \delta V(z - \min(y(p)m, z)) \} \right].$$

Let z^* be any maximizer of $J(z)$ over $z \geq 0$. Next we will show that

$$V'(x) \leq c - a, \quad \text{where the inequality holds with equality if } x \leq z^*. \quad (12)$$

For $x \leq z^*$, $V(x) = (c - a)x + J(z^*)$ and thus $V'(x) = c - a$. Consider any $x_1 > x_2 \geq z^*$. Then,

$$\begin{aligned} V(x_1) &= (c - a)x_1 + \max_{z \geq x_1} J(z) \\ &= (c - a)x_2 + \max_{z \geq x_1} J(z) + (c - a)(x_1 - x_2) \\ &\leq (c - a)x_2 + \max_{z \geq x_2} J(z) + (c - a)(x_1 - x_2) \\ &= V(x_2) + (c - a)(x_1 - x_2), \end{aligned}$$

which implies that $V'(x) \leq c - a$ for $x \geq z^*$. This establishes (12). Define $R(z)$ to be the maximand in (1). Note that $R(z)$ differs from $J(z)$ only in the last term, where $V(x)$ is replaced with $(c - a)x$. It is straightforward to verify that $R(z)$ is strictly unimodal and has a unique maximizer. It follows from (12) that $J'(z) \leq R'(z)$, where the inequality holds with equality if $z \leq z^*$. Because z^* is a maximizer of $J(z)$, $J'(z^*) = 0$. Therefore, $R'(z^*) = 0$, which together with the fact that $R(z)$ is strictly unimodal, implies that z^* is also the maximizer of $R(z)$.

Hence, $R'(z) < 0$ for $z > z^*$, which together with the result that $J'(z) \leq R'(z)$, implies that $J'(z) < 0$ for $z > z^*$. Consequently, the retailer’s optimal ordering policy is the following: if the starting inventory x is less than z^* , order $z^* - x$ to bring the inventory up to z^* ; otherwise, ordering nothing.

Because the retailer’s starting inventory in period 1 is zero, under the above optimal ordering policy, the starting inventory in any period will not exceed z^* and thus the inventory after ordering in any period is z^* . This implies that the retailer’s optimal pricing decision depends only on the realized market condition m :

$$p^*(m, z^*) = \arg \max_{p \geq 0} \{ (p + s) \min(y(p)m, z^*) + \delta V(z^* - \min(y(p)m, z^*)) \},$$

which together with the earlier result that $V'(x) = c - a$ for $x \leq z^*$, implies that

$$p^*(m, z^*) = \arg \max_{p \geq 0} \{ (p + s) \min(y(p)m, z^*) + \delta(c - a)(z^* - \min(y(p)m, z^*)) \}. \quad \square$$

PROOF OF LEMMA 2. Without loss of generality, we can add the constraint $y(p)m \leq z$ to the retailer’s pricing problem because the retailer’s profit strictly improves by increasing p if $y(p)m > z$. Thus, we can rewrite $p^*(a, s, m, z)$ as follows:

$$\begin{aligned} p^*(a, s, m, z) &= \arg \max_{p \geq 0} \{ [s + p - \delta(c - a)]y(p)m \} \\ &\text{s.t. } y(p)m \leq z. \end{aligned} \quad (13)$$

The objective function $[s + p - \delta(c - a)]y(p)$ is unimodal with a unique maximizer because its first-order derivative with respect to p is

$$y(p) + [s + p - \delta(c - a)]y'(p) = y'(p)[y(p)/y'(p) + s + p - \delta(c - a)],$$

which changes sign at most once, by Assumptions 1 and 2. Because $y(p)$ strictly decreases in p , constraint (13) can be rewritten as $p \geq y^{-1}(z/m)$. This together with the result that the objective function is unimodal implies that $p^*(a, s, m, z) = \tilde{p}(s + \delta a)$ (i.e., the maximizer of the unconstrained problem) if $\tilde{p}(s + \delta a) \geq y^{-1}(z/m)$ or equivalently $m \leq z/y(\tilde{p}(s + \delta a))$, and $p^*(a, s, m, z) = y^{-1}(z/m)$ otherwise. \square

PROOF OF LEMMA 3. Note that

$$\begin{aligned} \frac{\partial R(a, s, z)}{\partial z} &= \int_{z/y(\tilde{p}(s+\delta a))}^{\infty} [s + H(z, m) - \delta(c - a)]f(m) dm \\ &\quad - (1 - \delta)(c - a), \end{aligned}$$

where $H(z, m) = d\{y^{-1}(z/m)z\}/dz = y^{-1}(z/m) + z/[my'(y^{-1}(z/m))]$. Hence,

$$\frac{\partial^2 R(a, s, z)}{\partial z^2} = \int_{z/y(\tilde{p}(s+\delta a))}^{\infty} \frac{\partial H(z, m)}{\partial z} f(m) dm.$$

Note that in deriving the above equality, we have used the following result:

$$\begin{aligned} &s + H(z, m) - \delta(c - a)|_{m=z/y(\tilde{p}(s+\delta a))} \\ &= s + \tilde{p}(s + \delta a) + y(\tilde{p}(s + \delta a))/y'(\tilde{p}(s + \delta a)) - \delta(c - a) \\ &= 0, \end{aligned}$$

where the last equality follows from the first-order condition that must be satisfied by the maximizer $\tilde{p}(s + \delta a)$.

By definition of $H(z, m)$, we have the following:

$$\begin{aligned} \frac{\partial H(z, m)}{\partial z} &= \frac{2}{y'(y^{-1}(z/m))m} - \frac{y''(y^{-1}(z/m))}{[y'(y^{-1}(z/m))]^3} \frac{z}{m^2} \\ &= \frac{y(x)}{y'(x)z} \left[2 - \frac{y''(x)y(x)}{[y'(x)]^2} \right], \end{aligned} \quad (14)$$

where $x = y^{-1}(z/m)$. In (14), the term in square brackets is strictly positive because for $x \in [0, \bar{p}]$, $y''(x)y(x)/[y'(x)]^2 \leq y''(\bar{p})y(\bar{p})/[y'(\bar{p})]^2 = 0$, where the inequality follows from Assumption 2 and the equality follows from Assumption 1. In (14), the first term is negative because, by Assumption 11, $y'(x) < 0$. This implies $\partial H(z, m)/\partial z < 0$, which, in turn, implies that $\partial^2 R(a, s, z)/\partial z^2 < 0$. Therefore, $R(a, s, z)$ is strictly concave in z and its unique maximizer $z^*(a, s)$ is determined by the first-order condition (7). \square

PROOF OF LEMMA 4. Because $z^*(a, s)$ is the unique solution to (7), we have

$$\frac{\partial z^*(a, s)}{\partial a} = \frac{\int_{z/y(\tilde{p}(s+\delta a))}^{\infty} f(m) dm + (1-\delta) \int_0^{z^*/y(\tilde{p}(s+\delta a))} f(m) dm}{-\int_{z/y(\tilde{p}(s+\delta a))}^{\infty} (\partial H(z, m)/\partial z) f(m) dm} \Big|_{z=z^*(a, s)} \quad (15)$$

$$\frac{\partial z^*(a, s)}{\partial s} = \frac{\int_{z/y(\tilde{p}(s+\delta a))}^{\infty} f(m) dm}{-\int_{z/y(\tilde{p}(s+\delta a))}^{\infty} (\partial H(z, m)/\partial z) f(m) dm} \Big|_{z=z^*(a, s)}. \quad (16)$$

By (9),

$$\begin{aligned} \partial \mathcal{S}(a, s)/\partial a - \partial \mathcal{S}(a, s)/\partial s &= -(1-\delta) \int_0^{z^*(a, s)/y(\tilde{p}(s+\delta a))} y'(\tilde{p}(s+\delta a)) \tilde{p}'(s+\delta a) m f(m) dm \\ &\quad + \int_{z^*(a, s)/y(\tilde{p}(s+\delta a))}^{\infty} \left[\frac{\partial z^*(a, s)}{\partial a} - \frac{\partial z^*(a, s)}{\partial s} \right] f(m) dm \\ &= -(1-\delta) \int_0^{z^*(a, s)/y(\tilde{p}(s+\delta a))} y'(\tilde{p}(s+\delta a)) \tilde{p}'(s+\delta a) m f(m) dm \\ &\quad + (1-\delta) \frac{\int_0^{z^*(a, s)/y(\tilde{p}(s+\delta a))} f(m) dm \int_{z^*(a, s)/y(\tilde{p}(s+\delta a))}^{\infty} f(m) dm}{-\int_{z/y(\tilde{p}(s+\delta a))}^{\infty} (\partial H(z, m)/\partial z) f(m) dm} \Big|_{z=z^*(a, s)}, \end{aligned} \quad (17)$$

where the last equality follows from (15) and (16).

By (14) and Assumption 2 that $-y(x)/y'(x)$ decreases in x and $y''(x)y(x)/[y'(x)]^2$ increases in x where $x = y^{-1}(z/m)$ increases in m , $-\partial H(z, m)/\partial z$ decreases in m . Hence,

$$\begin{aligned} &-\int_{z/y(\tilde{p}(s+\delta a))}^{\infty} \frac{\partial H(z, m)}{\partial z} f(m) dm \Big|_{z=z^*(a, s)} \\ &< -\int_{z/y(\tilde{p}(s+\delta a))}^{\infty} \frac{\partial H(z, z/y(\tilde{p}(s+\delta a)))}{\partial z} f(m) dm \Big|_{z=z^*(a, s)} \\ &= \left[-\frac{y(\tilde{p}(s+\delta a))}{y'(\tilde{p}(s+\delta a))z^*(a, s)} \right] \left[2 - \frac{y''(\tilde{p}(s+\delta a))y(\tilde{p}(s+\delta a))}{[y'(\tilde{p}(s+\delta a))]^2} \right] \\ &\quad \times \int_{z^*(a, s)/y(\tilde{p}(s+\delta a))}^{\infty} f(m) dm. \end{aligned} \quad (18)$$

Recall that $\tilde{p}(s + \delta a) = \arg \max_{p \geq 0} (s + \delta a + p - \delta c)y(p)$. It follows from the first-order condition that

$$(s + \delta a + \tilde{p}(s + \delta a) - \delta c)y'(\tilde{p}(s + \delta a)) + y(\tilde{p}(s + \delta a)) = 0.$$

Taking the derivative of both sides of the above equation with respect to s , we obtain

$$(1 + 2\tilde{p}'(s + \delta a))y'(\tilde{p}(s + \delta a)) + (s + \delta a + \tilde{p}(s + \delta a) - \delta c) \cdot y''(\tilde{p}(s + \delta a))\tilde{p}'(s + \delta a) = 0,$$

which together with the first-order condition $s + \delta a + \tilde{p}(s + \delta a) - \delta c = -y(\tilde{p}(s + \delta a))/y'(\tilde{p}(s + \delta a))$, implies that

$$2 - \frac{y''(\tilde{p}(s + \delta a))y(\tilde{p}(s + \delta a))}{[y'(\tilde{p}(s + \delta a))]^2} = -\frac{1}{\tilde{p}'(s + \delta a)}.$$

This together with (18) implies that

$$\begin{aligned} &-\int_{z/y(\tilde{p}(s+\delta a))}^{\infty} \frac{\partial H(z, m)}{\partial z} f(m) dm \Big|_{z=z^*(a, s)} \\ &< \frac{y(\tilde{p}(s + \delta a))}{y'(\tilde{p}(s + \delta a))z^*(a, s)} \frac{1}{\tilde{p}'(s + \delta a)} \int_{z^*(a, s)/y(\tilde{p}(s+\delta a))}^{\infty} f(m) dm, \end{aligned}$$

implying that

$$\begin{aligned} &\int_0^{z^*(a, s)/y(\tilde{p}(s+\delta a))} y'(\tilde{p}(s + \delta a)) \tilde{p}'(s + \delta a) m f(m) dm \\ &\quad \times \left[-\int_{z/y(\tilde{p}(s+\delta a))}^{\infty} \frac{\partial H(z, m)}{\partial z} f(m) dm \right] \Big|_{z=z^*(a, s)} \\ &< \int_0^{z^*(a, s)/y(\tilde{p}(s+\delta a))} y'(\tilde{p}(s + \delta a)) \tilde{p}'(s + \delta a) \frac{z^*(a, s)}{y(\tilde{p}(s + \delta a))} f(m) dm \\ &\quad \times \frac{y(\tilde{p}(s + \delta a))}{y'(\tilde{p}(s + \delta a))z^*(a, s)} \frac{1}{\tilde{p}'(s + \delta a)} \int_{z^*(a, s)/y(\tilde{p}(s+\delta a))}^{\infty} f(m) dm \\ &= \int_0^{z^*(a, s)/y(\tilde{p}(s+\delta a))} f(m) dm \int_{z^*(a, s)/y(\tilde{p}(s+\delta a))}^{\infty} f(m) dm. \end{aligned}$$

This together with (17) establishes that $\partial \mathcal{S}(a, s)/\partial a - \partial \mathcal{S}(a, s)/\partial s > 0$. \square

PROOF OF PROPOSITION 1. The result follows from the arguments preceding the proposition. \square

PROOF OF PROPOSITION 2. The result follows from the fact that $\partial \mathcal{S}_i(a, s)/\partial a - \partial \mathcal{S}_i(a, s)/\partial s > 0$ for $i = 1, 2, \dots, N$ and from the arguments preceding Proposition 1. Consider any subsidy (a, s) where the sales subsidy $s > 0$. It follows from Lemma 4 that $\partial \mathcal{S}_i(a, s)/\partial a - \partial \mathcal{S}_i(a, s)/\partial s > 0$ for $i = 1, 2, \dots, N$ and thus $\partial(\sum_{i=1}^N \mathcal{S}_i(a, s))/\partial a - \partial(\sum_{i=1}^N \mathcal{S}_i(a, s))/\partial s > 0$, implying that the same average per-period aggregated sales over the N retailers can be achieved by eliminating the sales subsidy and increasing the purchase subsidy a by an amount that is strictly smaller than s . Clearly, this modification strictly reduces the average per-period subsidy payment. Consequently, any subsidy (a, s) with $s > 0$ cannot be optimal. \square

PROOF OF PROPOSITION 3. (a) It follows from Federgruen and Heching (1999) that Lemma 1 holds when in each period t , the retailer chooses the price p prior to observing the market condition M^t . The retailer's optimal order-up-to level z^* and price p^* are the solution to

$$\max_{z, p \geq 0} \{-(1-\delta)(c-a)z + [s+p-\delta(c-a)]E_m \min(y(p)m, z)\}.$$

It is useful to introduce a change in variables. Let $X = z/y(p)$ and $Y = y(p)$. The retailer’s problem can be rewritten as

$$\max_{X, Y \geq 0} \{-(1 - \delta)(c - a)XY + [s + y^{-1}(Y) - \delta(c - a)]YE_m \min(m, X)\}. \quad (19)$$

Let (X^*, Y^*) denote the solution to (19). Because (X^*, Y^*) satisfies the first-order condition with respect to X

$$-(1 - \delta)(c - a)Y + [s + y^{-1}(Y) - \delta(c - a)]Y\bar{F}(X) = 0,$$

it follows that $\partial X^*/\partial a - \partial X^*/\partial s$ has the same sign as $(1 - \delta)Y^* + \delta Y^*\bar{F}(X^*) - Y^*\bar{F}(X^*)$; the latter is strictly positive. Similarly, because (X^*, Y^*) satisfies the first-order condition with respect to Y

$$-(1 - \delta)(c - a)X + [s + d(y^{-1}(Y)Y)/dY - \delta(c - a)]E_m \min(m, X) = 0,$$

it follows that $\partial Y^*/\partial a - \partial Y^*/\partial s$ has the same sign as $(1 - \delta)X^* + \delta E_m \min(m, X^*) - E_m \min(m, X^*)$; the latter is strictly positive. Therefore, we have shown that

$$\partial X^*/\partial a - \partial X^*/\partial s > 0, \quad (20)$$

$$\partial Y^*/\partial a - \partial Y^*/\partial s > 0. \quad (21)$$

The average per-period sales to consumers under the retailer’s optimal decisions is

$$\begin{aligned} \mathcal{P}(a, s) &= E_m \min(y(p^*)m, z^*) \\ &= Y^* E_m \min(m, X^*), \end{aligned}$$

which increases in both X^* and Y^* . This together with (20) and (21) implies that $\partial \mathcal{P}(a, s)/\partial a > \partial \mathcal{P}(a, s)/\partial s$. Thus, Lemma 4 continues to hold, implying that it is optimal for the donor not to offer any sales subsidy. This completes the proof for part (a).

(b) Under the fixed retail price p , the retailer’s optimal order-up-to level z^* is the solution to

$$\max_{z \geq 0} \{-(1 - \delta)(c - a)z + [s + p - \delta(c - a)]E_m \min(y(p)m, z)\}.$$

Because $\mathcal{P}(a, s) = E_m \min(y(p)m, z^*)$ which depends on a and s only via z^* and because $\mathcal{P}(a, s)$ increases in z^* , to prove Lemma 4 that $\partial \mathcal{P}(a, s)/\partial a > \partial \mathcal{P}(a, s)/\partial s$, it suffices to show that $\partial z^*/\partial a - \partial z^*/\partial s > 0$. Because z^* satisfies the first-order condition

$$-(1 - \delta)(c - a) + [s + p - \delta(c - a)]\bar{F}(z/y(p)) = 0,$$

it follows that $\partial z^*/\partial a - \partial z^*/\partial s$ has the same sign as $1 - \delta + \delta\bar{F}(z^*/y(p)) - \bar{F}(z^*/y(p))$; the latter is strictly positive. This completes the proof of part (b). \square

The remainder of the appendix addresses the case in which the product has a short shelf life and demand is linear (11). Arguments parallel to those in the proofs of Lemmas 2 and 3 establish the retailer’s optimal pricing and ordering decisions, which we state in Lemmas 2A and 3A, respectively.

LEMMA 2A. *Suppose the product has a short shelf life (perishability $\gamma = 1$) and that demand is linear (11). Under subsidy (a, s) , order-up-to level z , and realized market condition m , the retailer’s optimal price is*

$$p_1^*(m, z) = \max((\mu + \Delta - s)/2, \mu - \Delta, \mu + \Delta - 2z\Delta/m).$$

LEMMA 3A. *Suppose the product has a short shelf life (perishability $\gamma = 1$) and that demand is linear (11). The retailer’s one-period expected profit is strictly concave in the order quantity z . Its unique maximizer $z_1^*(a, s)$ is the unique solution to*

$$\begin{aligned} &\int_{\max(4z\Delta/(s+\mu+\Delta), z)}^{\infty} (s + \mu + \Delta - 4z\Delta/m - c + a)f(m) dm \\ &= \int_0^{\max(4z\Delta/(s+\mu+\Delta), z)} (c - a)f(m) dm. \end{aligned}$$

Let \underline{B} and \bar{B} be the donor’s average per-period subsidy payment under the subsidy $(a, s) = (c/2, 0)$ and the subsidy $(a, s) = (c/2, \max(3\Delta - \mu, 0))$, respectively. Clearly, $\bar{B} \geq \underline{B}$.

LEMMA 5. *Suppose the product has a short shelf life (perishability $\gamma = 1$) and that demand is linear (11). If $3\Delta \leq \mu$, then $s^* = 0$; further, as B increases over $[0, \infty)$, a^* strictly increases from 0 to c . If $3\Delta > \mu$, then (i) as B increases over $[0, \underline{B}]$, $s^* = 0$ and a^* strictly increases from 0 to $c/2$; (ii) as B increases over (\underline{B}, \bar{B}) , s^* strictly increases from 0 to $3\Delta - \mu$ and $a^* = c/2$; (iii) as B increases over $[\bar{B}, \infty)$, $s^* = 3\Delta - \mu$ and a^* strictly increases from $c/2$ to c .*

PROOF OF LEMMA 5. From Lemma 2A, the average per-period sales to consumers

$$\begin{aligned} &E_m[\min(y(p_1^*(m, z_1^*(a, s)))m, z_1^*(a, s))] \\ &= E_m[\min((s + \mu + \Delta)m/(4\Delta), m, z_1^*(a, s))]. \end{aligned}$$

This expression, as well as the expression for the retailer’s optimal order quantity $z_1^*(a, s)$, simplifies based on whether $\mu + s \leq 3\Delta$ or $\mu + s \geq 3\Delta$. Therefore, we can rewrite the donor’s problem (P₁) for these two cases:

$$\begin{aligned} \text{(P}_1\text{)} \quad &\max_{a, s \geq 0} \int_0^{4z_1^*(a, s)\Delta/(s+\mu+\Delta)} [(s + \mu + \Delta)/(4\Delta)]mf(m) dm \\ &+ \int_{4z_1^*(a, s)\Delta/(s+\mu+\Delta)}^{\infty} z_1^*(a, s)f(m) dm \\ \text{s.t.} \quad &az_1^*(a, s) + s \left[\int_0^{4z_1^*(a, s)\Delta/(s+\mu+\Delta)} [(s + \mu + \Delta)/(4\Delta)] \right. \\ &\cdot mf(m) dm + \left. \int_{4z_1^*(a, s)\Delta/(s+\mu+\Delta)}^{\infty} z_1^*(a, s)f(m) dm \right] \leq B \\ &\int_{4z_1^*(a, s)\Delta/(s+\mu+\Delta)}^{\infty} (s + \mu + \Delta - 4z_1^*(a, s)\Delta/m)f(m) dm \\ &- (c - a) = 0 \\ &\mu + s \leq 3\Delta, \end{aligned}$$

and

$$\begin{aligned} \text{(\bar{P}}_1\text{)} \quad &\max_{a, s \geq 0} \int_0^{z_1^*(a, s)} mf(m) dm + \int_{z_1^*(a, s)}^{\infty} z_1^*(a, s)f(m) dm \\ \text{s.t.} \quad &az_1^*(a, s) + s \left[\int_0^{z_1^*(a, s)} mf(m) dm \right. \\ &\left. + \int_{z_1^*(a, s)}^{\infty} z_1^*(a, s)f(m) dm \right] \leq B \\ &\int_{z_1^*(a, s)}^{\infty} (s + \mu + \Delta - 4z_1^*(a, s)\Delta/m)f(m) dm \\ &- (c - a) = 0 \\ &\mu + s \geq 3\Delta. \end{aligned}$$

The donor's optimal subsidy is then the solution to either (\underline{P}_1) or (\bar{P}_1) , whichever has a greater objective value.

First, suppose $3\Delta \leq \mu$. It suffices to consider (\bar{P}_1) because $s = 0$ at any feasible solution to (\underline{P}_1) , if any exists. Consider any solution to (\bar{P}_1) , denoted by (a^*, s^*) . We will prove, by contradiction, that $s^* = 0$. Suppose instead that $s^* > 0$. Consider the subsidy $(a', s') = (a^* + s^*[1 - F(z_1^*(a^*, s^*))], 0)$. The subsidy (a', s') results in the same order quantity for the retailer as under (a^*, s^*) because the left-hand side of the second constraint of (\bar{P}_1) is

$$\begin{aligned} & \int_{z_1^*(a^*, s^*)}^{\infty} (s' + \mu + \Delta - 4z_1^*(a^*, s^*)\Delta/m) f(m) dm - (c - a') \\ &= \int_{z_1^*(a^*, s^*)}^{\infty} (s^* + \mu + \Delta - 4z_1^*(a^*, s^*)\Delta/m) f(m) dm - (c - a^*) = 0, \end{aligned}$$

and further (a', s') leads to a strictly lower average per-period subsidy payment relative to that under (a^*, s^*) because the left-hand side of the first constraint under (a', s') is

$$\begin{aligned} & a' z_1^*(a^*, s^*) + s' \left[\int_0^{z_1^*(a^*, s^*)} m f(m) dm + \int_{z_1^*(a^*, s^*)}^{\infty} z_1^*(a^*, s^*) f(m) dm \right] \\ &= a^* z_1^*(a^*, s^*) + s^* [1 - F(z_1^*(a^*, s^*))] z_1^*(a^*, s^*) \\ &< a^* z_1^*(a^*, s^*) + s^* \left[\int_0^{z_1^*(a^*, s^*)} m f(m) dm \right. \\ & \quad \left. + \int_{z_1^*(a^*, s^*)}^{\infty} z_1^*(a^*, s^*) f(m) dm \right] \\ &\leq B. \end{aligned} \tag{22}$$

The subsidy (a', s') also satisfies the third constraint because $\mu \geq 3\Delta$ and $s' = 0$. Therefore, (a', s') is a feasible solution to (\bar{P}_1) . Because the objective value of (\bar{P}_1) is the same under (a', s') and (a^*, s^*) , (a', s') must also be an optimal solution to (\bar{P}_1) . However, from (22), the average per-period subsidy payment under (a', s') is strictly less than the budget B . Therefore, one can increase a' by a small amount and by doing so strictly increase the retailer's order quantity and the objective value of (\bar{P}_1) without violating any constraint of (\bar{P}_1) . This contradicts that (a', s') is an optimal solution to (\bar{P}_1) . We conclude that the optimal sales subsidy $s^* = 0$.

Second, suppose $3\Delta > \mu$. This implies that the third constraint must be binding at any optimal solution to (\bar{P}_1) . If not, then one can construct a strictly better solution to (\bar{P}_1) by following the same arguments as above. Therefore, (\underline{P}_1) dominates (\bar{P}_1) , and it suffices to solve (\underline{P}_1) . Note that the first constraint of (\underline{P}_1) must be binding at the optimal solution because otherwise the donor can strictly increase the average sales per-period by increasing the purchase subsidy by a sufficiently small amount without violating any constraint of (\underline{P}_1) . We can use the second constraint to replace the decision variable a with z and rewrite (\underline{P}_1) as follows:

$$\begin{aligned} (\underline{P}'_1) \quad & \max_{s, z \geq 0} X(s, z) \\ \text{s.t.} \quad & Y(s, z) = B \\ & \mu + s \leq 3\Delta \\ & c - \int_{4z\Delta/(s+\mu+\Delta)}^{\infty} (s + \mu + \Delta - 4z\Delta/m) f(m) dm \geq 0, \end{aligned}$$

where

$$\begin{aligned} X(s, z) &= \int_0^{4z\Delta/(s+\mu+\Delta)} [(s + \mu + \Delta)/(4\Delta)] m f(m) dm \\ & \quad + \int_{4z\Delta/(s+\mu+\Delta)}^{\infty} z f(m) dm \\ Y(s, z) &= cz + \int_0^{4z\Delta/(s+\mu+\Delta)} [(s + \mu + \Delta)/(4\Delta)] s m f(m) dm \\ & \quad - \int_{4z\Delta/(s+\mu+\Delta)}^{\infty} (\mu + \Delta - 4z\Delta/m) z f(m) dm. \end{aligned}$$

We next derive the optimal solution to the relaxed version of (\underline{P}'_1) wherein the third constraint is dropped. It is straightforward to verify that the optimal solution to the relaxed problem satisfies the third constraint and thus is also optimal for (\underline{P}'_1) .

Note that $Y(s, z)$ is strictly increasing in z . Let $\hat{z}(s) = \{z \mid Y(s, z) = B\}$ and $A(s) = X(s, \hat{z}(s))$. Then, (\underline{P}'_1) can be further simplified to

$$\begin{aligned} (\underline{P}'_1) \quad & \max_{s \geq 0} A(s) \\ \text{s.t.} \quad & 0 \leq s \leq 3\Delta - \mu. \end{aligned}$$

Hence, the optimal solution to (\underline{P}'_1) , denoted by (a^*, s^*) , must satisfy one of the following three conditions: (1) $s^* = 0$ and $(d/ds)A(s)|_{s=s^*} \leq 0$; (2) $0 < s^* < 3\Delta - \mu$ and $(d/ds)A(s)|_{s=s^*} = 0$; (3) $s^* = 3\Delta - \mu$ and $(d/ds)A(s)|_{s=s^*} \geq 0$.

Note that

$$\begin{aligned} (d/ds)A(s) &= (\partial/\partial s)X(s, z)|_{z=\hat{z}(s)} + (\partial/\partial z)X(s, z)|_{z=\hat{z}(s)}(d/ds)\hat{z}(s) \\ &= \left[(\partial/\partial s)X(s, z) - \frac{(\partial/\partial z)X(s, z)(\partial/\partial s)Y(s, z)}{(\partial/\partial z)Y(s, z)} \right] \Big|_{z=\hat{z}(s)} \\ &= \frac{1}{4\Delta} \left\{ \left[c - 2 \int_{4z\Delta/(s+\mu+\Delta)}^{\infty} (s + \mu + \Delta - 4z\Delta/m) f(m) dm \right] \right. \\ & \quad \times \left(\int_0^{4z\Delta/(s+\mu+\Delta)} m f(m) dm \right) \\ & \quad \left. / \left(c - \int_{4z\Delta/(s+\mu+\Delta)}^{\infty} (\mu + \Delta - 8z\Delta/m) f(m) dm \right) \right\} \Big|_{z=\hat{z}(s)}. \end{aligned}$$

Because the term on the last line immediately above is strictly positive, $(d/ds)A(s)|_{s=s^*} \leq 0$ implies that

$$c - 2 \int_{4\hat{z}(s^*)\Delta/(s^*+\mu+\Delta)}^{\infty} (s^* + \mu + \Delta - 4\hat{z}(s^*)\Delta/m) f(m) dm \leq 0,$$

which together with the second constraint of (\underline{P}_1) , i.e.,

$$\int_{4\hat{z}(s^*)\Delta/(s^*+\mu+\Delta)}^{\infty} (s^* + \mu + \Delta - 4\hat{z}(s^*)\Delta/m) f(m) dm - (c - a^*) = 0,$$

implies that $a^* \leq c/2$. Similarly, $(d/ds)A(s)|_{s=s^*} = 0$ implies that $a^* = c/2$, and $(d/ds)A(s)|_{s=s^*} \geq 0$ implies that $a^* \geq c/2$. Hence, conditions (1)–(3) can be rewritten as (1) $s^* = 0$ and $a^* \leq c/2$; (2) $0 < s^* < 3\Delta - \mu$ and $a^* = c/2$; (3) $s^* = 3\Delta - \mu$ and $a^* \geq c/2$.

CLAIM 1. If $B \in [0, B]$, then condition (1) holds.

PROOF OF CLAIM 1. If either condition (2) or condition (3) holds, then $z_1^*(a^*, s^*) \geq z_1^*(c/2, 0)$ and thus the donor's average per-period subsidy payment under subsidy (a^*, s^*) is strictly

larger than that under the subsidy $(a, s) = (c/2, 0)$, i.e., \underline{B} . This contradicts that $B \in [0, \underline{B}]$. Therefore, condition (1) must hold, and thus $s^* = 0$. By the budget constraint of (\underline{P}_1) , $a^* = \{a \mid az_1^*(a, 0) = B\}$, which strictly increases from 0 to $c/2$ as B increases over $[0, \underline{B}]$. This completes the proof of part (i).

CLAIM 2. If $B \in (\underline{B}, \bar{B})$, then condition (2) holds.

PROOF OF CLAIM 2. If condition (1) holds, then $z_1^*(a^*, s^*) \leq z_1^*(c/2, 0)$ and thus the donor's total average per-period subsidy payment under (a^*, s^*) is no more than that under the subsidy $(a, s) = (c/2, 0)$, i.e., \underline{B} , which is strictly less than the donor's budget B . This contradicts the earlier observation that under the optimal subsidy, the donor's budget constraint binds. Hence, condition (1) cannot hold. If condition (3) holds, $z_1^*(a^*, s^*) \geq z_1^*(c/2, 3\Delta - \mu)$ and thus the donor's average per-period subsidy payment under (a^*, s^*) is no less than that under the subsidy $(a, s) = (c/2, 3\Delta - \mu)$, i.e., \bar{B} . This contradicts that $B < \bar{B}$. Hence, condition (3) cannot hold. Therefore, condition (2) must hold. This completes the proof of part (ii).

CLAIM 3. If $B \in [\bar{B}, \infty)$, then condition (3) holds.

PROOF OF CLAIM 3. If either condition (1) or condition (2) holds, then $z_1^*(a^*, s^*) \leq z_1^*(c/2, 3\Delta - \mu)$ and thus the donor's average per-period subsidy payment under (a^*, s^*) is strictly less than that under the subsidy $(a, s) = (c/2, 3\Delta - \mu)$, i.e., \bar{B} , which is less than the donor's budget B . This contradicts the earlier observation that under the optimal subsidy, the donor's budget constraint binds. Hence, condition (3) must hold. This completes the proof of part (iii). \square

PROOF OF PROPOSITION 4. The result follows from Lemma 5. \square

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