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On activity-based network design problems

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Abstract

This paper examines network design where OD demand is not known *a priori*, but is the subject of responses in household or user itinerary choices that depend on subject infrastructure improvements. Using simple examples, we show that falsely assuming that household itineraries are not elastic can result in a lack in understanding of certain phenomena; e.g., increasing traffic even without increasing economic activity due to relaxing of space-time prism constraints, or worsening of utility despite infrastructure investments in cases where household objectives may conflict. An activity-based network design problem is proposed using the location routing problem (LRP) as inspiration. The bilevel formulation includes an upper level network design and shortest path problem while the lower level includes a set of disaggregate household itinerary optimization problems, posed as household activity pattern problem (HAPP) (or in the case with location choice, as generalized HAPP) models. As a bilevel problem with an NP-hard lower level problem, there is no algorithm for solving the model exactly. Simple numerical examples show optimality gaps of as much as 5% for a decomposition heuristic algorithm derived from the LRP. A large numerical case study based on Southern California data and setting suggest that even if infrastructure investments do not result in major changes in itineraries the results provide much higher resolution information to a decision-maker. Whereas a conventional model would output the best set of links to invest given an assumed OD matrix, the proposed model can output the same best set of links, the same OD matrix, and a detailed temporal distribution of activity participation and travel, given a set of desired destinations and schedules.

Keywords: activity based model; network design; location routing problem; HAPP; pickup and delivery problem; bi-level problem

1. Background

Network design problems (NDPs) are a class of optimization models related to strategic or tactical planning of resources to manage a network (Magnanti and Wong, 1984). In general, NDPs assume static demand (elastic or not) at a node or trip-based origin-destination demand, even for purposes of improving road networks for commuters (Yang and Bell, 1998) despite the complexity of traveler choices (Recker, 2001). While this assumption is sufficient in many applications, there is increasing recognition that explicit consideration of travelers' schedules, choices, and time dimension is needed. This need has grown in parallel to three related research trends in network design in the last few years: (operational) network design with dynamic assignment considerations when considering only peak period effects, (tactical) service network design with schedule-based demand under longer periods of activity, and (planning) facility location problems that explicitly consider the effects they have on routing and scheduling of vehicles for a decision-maker. At the planning level, these NDPs have often been based on private firm decisions, rather than on household-based urban transportation planning considerations.

The rationale behind dynamic network design problems is rooted in bi-level NDPs that feature congestion effects. These NDPs operate primarily in civil infrastructure systems, as other types of networks do not generally share the same "selfish travelers" assumptions. In this paradigm, the performance of infrastructure improvements is assumed to depend primarily on the route choices of travelers (primarily the commuter) during peak periods of travel, which in turn depend on the choices of other travelers. The dynamic component further allows modelers to assess intelligent transportation systems (ITS) that require more realistic modeling of traffic propagation obeying physical queuing constraints and information flow. Some examples include the stochastic dynamic NDP from Waller and Ziliaskopoulos (2001), Heydecker's (2002) NDP with dynamic user equilibrium (DUE), the linear DUE-NDP from Ukkusuri and Waller (2008), dynamic toll pricing problem with route and departure time choice (Joksimovic *et al.*, 2005), and the reliability maximizing toll pricing problem with dynamic route and departure time choice (Li *et al.*, 2007). Although these NDPs are especially useful for ITS evaluation and operational strategies, they focus primarily on choices made over a single trip.

Tactical level NDPs tend to place more emphasis on time use and scheduling over congestion effects. Tactical service NDPs (Crainic, 2000) are a specific class used to manage fleets of vehicles with such temporal decision variables as service frequency. However, most of these NDPs focus on the schedules of the service being provided, rather than on incorporating the demand-side schedules of the travelers/users as endogenous elements of the design. Despite the incorporation of temporal effects, most service NDPs assume trip-based demand. There has been a surge of research in schedule-based transit assignment (as opposed to NDP), where travelers' departure time choices are handled explicitly. Tong and Wong (1998) formulated such a model with heterogeneous traveler values of time. Poon *et al.* (2004) presented a dynamic equilibrium model for schedule based transit assignment. Hamdouch and Lawphongpanich (2008) developed a schedule-based transit assignment model that accounts for individual vehicle capacities, and proposed one of the few schedule-based service network design problems, in the form of a transit congestion pricing problem that models passengers' departure time choices (Hamdouch and Lawphongpanich, 2010). Their model uses a time-expanded network and considers fare pricing to optimize the distribution of travelers within specific capacitated transit vehicles. The origin-destination (OD) demand remains fixed, and not as a linked itinerary.

Despite having the greatest need for such consideration, at the planning level there are no NDP models that consider routing and scheduling choices of travelers. . It has long been acknowledged that models of traveler activities and time use are much more accurate than statistical trip-based approaches (Recker, 2001; Pinjari and Bhat, 2011). Activity consideration can bring about a tighter integration of infrastructure investment with land use planning and demand management strategies. Activity-based models can capture realistic impacts on travelers that are not limited to single trips but rather to chains of trips and activities forming detailed daily itineraries. Historically, the bulk of activity-based models have been designed primarily as econometric models that do not account for network routing and scheduling mechanisms. The emerging trend in seeking to integrate network characteristics has been to force an interaction with a dynamic traffic assignment problem (e.g. Lin *et al.*, 2008; Konduri, 2012), which extends the planning model toward operational applicability. However, this approach still ignores the network constraints present in scheduling and selection of activities for a household. There have been two primary exceptions to this approach. The first is the disaggregate activity route assignment model (HAPP) pioneered by Recker (1995), with subsequent studies on dynamic rescheduling/rerouting of those itineraries (Gan and Recker, 2008), and calibration of the activity route assignment models (Recker *et al.*, 2008; Chow and Recker, 2012). The second is the aggregate time-dependent activity-based traffic assignment model (Lam and Yin, 2001). Both modeling frameworks address the issue of activity scheduling, although Lam and Yin's model gives up disaggregate itinerary route choices and trip chains in favor of capturing congestion effects.

Although the transportation planning field has not seen any significant NDP research that models traveler routing and scheduling, the private logistics field has. One such model is the location routing problem (LRP), formulated and solved by Perl and Daskin (1985). The LRP is a set of inter-related problems that includes a facility location problem. What distinguishes LRPs from other NDPs is that it doesn't assume that demand for the nodes is given in terms of round trips to demand nodes. Instead, a lower level vehicle routing problem is embedded in the model to satisfy demand nodes in the most

efficient manner, subject to where the facilities are located. In essence, it is an integrated NDP that accounts for responsive routing and scheduling. Numerous studies have been conducted in variants of the problem or applications in industry. Several literature reviews have been published, including one from Min *et al.* (1998) and a more recent contribution by Nagy and Salhi (2007). Problem types developed over the years that may be applicable to activity-based network design in transportation planning include: stochastic LRP (Laporte and Dejax, 1989), where there is more than one planning horizon and customer locations and demands change over time; LRP with a mixed fleet (Wu *et al.*, 2002) for multimodal network consideration; location-routing-inventory (Liu and Lee, 2003) for modeling activity types as inventory-based needs that are fulfilled periodically; and LRP with nonlinear costs (Melechovsky *et al.*, 2005) that may provide means to incorporate congestion effects at link or activity node level. Readers are referred to Nagy and Salhi's paper for further details. One direct application of LRP with truck fleet replaced by household travelers is shown in Kang and Recker (2012a). They use HAPP as a routing subproblem in a hydrogen fuel cell refueling station LRP that allows households to respond to located facilities to refuel, which can reflect the behavioral impacts of siting decisions.

Given the increasing realization that transportation planning needs to reflect travelers' preferences at the activity level, we make a parallel observation to Perl and Daskin—that in the transportation planning field there is also a need for integrated NDPs that feature explicit consideration of travelers' tour patterns including trip chaining, scheduling, time windows and even destination choice. At the activity-based level, we are concerned more with tactical and planning level policies, and less so with such operational technologies as ITS and information flow (hence foregoing congestion effects for now). In essence, we propose to change the conventional NDP, with a given OD matrix, to a new class of activity-based NDPs. This new problem accounts for a population of travelers with demand for activities at particular locations and at particular times, which are fulfilled via calibrated activity routing models. Like the LRP, the activity-based NDP is a set of integrated models. Unlike the conventional NDP, the OD matrix is not given *a priori*, since it depends on the scheduling choices of households which in turn depend on travel impedances. The solution of this set of models is a set of infrastructure link investments as well as the resulting optimal itineraries decided by the households in response to the changes. The itineraries can then be aggregated to obtain the final OD matrix resulting from the NDP.

In Section 2, several examples and insightful paradoxes are used to illustrate why an activitybased approach is necessary at the tactical and planning level NDP. Section 3 introduces the formulation as a bi-level structure with shortest path allocation and disaggregated subproblems per household. While the inspiration of the formulation is from Perl and Daskin's LRP, key differences are also noted. An alternative model with activity/destination choice is also provided. Since the problem is nonconvex and NP-hard, Section 4 presents a heuristic solution method and suggestions for meta-heuristics, using a simple test network to demonstrate the method and the sensitivity of underlying assumptions. Section 5 presents a larger-scale case study of the Orange County, California region as a test network to demonstrate the model's practical application to systematic improvement.

2. Motivating Examples

The argument that we provide here, much like Perl and Daskin (1985) did for locating warehouses, is that the choice of which element of a network to improve can have a significant impact on how impacted households set their itineraries each day. Trip-based (even dynamic ones) or fixed schedules ignore changes that each driver/household makes according to the changes made in the network, such as departure time, sequence of activities, or routing. The following three cases demonstrate the influence that network designs can have on a household, which would be unaccountable under trip-based circumstances. For these examples, the utility maximization framework from Recker (1995) is assumed: households are multi-objective decision-makers with their own sets of objectives with respective weights that dictate how they choose to schedule and route their activities. This has been demonstrated empirically in Chow and Recker (2012) where a population of households were fitted with heterogeneous sets of

objective weights and desired arrival times to activities such that each of their observed itineraries were considered optimal to them.

2.1. Departure Time Choice and Itinerary Re-Sequencing

Assume a household has one household member and one vehicle, and two activities to perform for the day: a work activity and a grocery shopping activity. Specifications of start and completion time windows and activity duration are shown in Table 1, in units of hours¹. Assume also that the household objective is solely to minimize the length of their itinerary, $\min Z = \sum (T_{2n+1}^{\nu} - T_0^{\nu})$ $\sum_{v \in V}$ ⁽¹2n $Z = \sum (T_{2n+1}^{\nu} - T_{\nu})$ $=\sum_{v\in V} (T_{2n+1}^v-T_0^v)$.

Household	Location	a_u^h,b_u^h	a_{n_h+u} , b_{n_h+u}	
Home	Node 0	a_0^h, b_0^h $= [6, 21]$	$a_{2n_h+1}^h, b_{2n_h+1}^h$ $=[10, 22]$	NA
work activity	Node 3	[9,9]	$\left[10,22\right]$	
Grocery Shopping activity	Node 1	[5, 20]	6,22	

Table 1. Case 1 household characteristics

Assume a grid network with four nodes, and network connections as shown in Figure 1-(a). Travel time for each link is 0.5 hours. Figure 1-(b) shows the optimal pattern if no investment is made.

(a) Current Network (b) Optimal Household Activity Pattern

 \overline{a}

 1 Here and throughout, the notation used in Recker (1995) is followed.

(c) $t_{03} = 0.7 \rightarrow$ Time Adjustment (d) $t_{30} = 0.7 \rightarrow$ Activity Sequence and Time Adjustment **Figure 1: The Optimal Household Activity Patterns for Case 1**

Even in this simplest case, two types of schedule responses can be observed for standard link investments which would be ignored in conventional NDPs. If link {0,3} is constructed with travel time of 0.7 hours as shown in Figure 1-(c), the household would now be able to delay its departure time from 8AM to 8:18AM. Alternatively, if link {3,0} is instead constructed with travel time of 0.7 hours as shown in Figure 1-(d), the optimal itinerary results in a re-sequence of activities as well as an adjustment in departure times.

2.2. Trip Chaining Trade-Offs

A paradoxical consequence of considering elastic itineraries in network design is that it is possible to evaluate a *link investment that generates traffic without any increase in economic activity*. Traditionally, the argument made with elastic demand considerations is that improving infrastructure may result in additional trips made to fulfill latent demand between an OD pair. However, exceptions can also exist if travel is viewed as a way of achieving objectives while constrained within a space-time prism. By relaxing some of those constraints through network improvements, we may observe only increased trips due to untangling of less desired travel patterns within the tighter constraints. This can result in more trips made if it improves the overall objective of the household but would not contribute in any way to economic demand because the household may be reconfiguring the same itinerary without adding new destinations to visit. This occurrence can be best illustrated with a household with activities that have very strict time windows.

We consider the same activity agenda as in the previous section, but with both activities having strict constraining start time windows as in Table 2. Both activities require the household member to be at the respective locations at a specific time, which is often quite a realistic assumption. Assume also that this particular household has two potentially conflicting objectives: to minimize the travel time with weight β_T , and to minimize delay from returning home after an activity, with weight β_C . The delay from returning home objective represents the desire of the household to minimize the duration of any particular activity period away from home, as discussed in Recker (1995) and calibrated empirically in Chow and Recker (2012). The higher the weight of this objective relative to travel time, the more likely it is that a household would not want to trip chain. Then the objective function becomes:

$$
\min Z = \beta_T \cdot \sum_{v \in V} \sum_{w \in N} \sum_{u \in N} t_{uw} \cdot X_{uw}^v + \beta_C \cdot \sum_{u \in N} (T_{u+n} - T_u)
$$

where the weights are assumed to be $\beta_T = 1$ and $\beta_C = 1$. The optimal solution on the base network is shown in Figure 2-(a), with the objective function travel disutility of 14.25 and a total of three trips made. Due to the time windows, the household traveler is constrained to trip chain from the work activity to the social activity.

Household	Location a_{μ} , b_{μ}		a_{n+u} , b_{n+u}	S_u
Home	Node 0	$ a_0, b_0 = [6, 21]$	$ a_{2n+u}, b_{2n+u} = [10, 22]$	NA
work activity	Node 3	[9,9]	[10, 22]	
social activity	Node 1	[18.25, 18.25]	[18.5, 22]	

Table 2. Case 2 household characteristics

Figure 2: The Optimal Household Activity Patterns for Case 2

Now consider a link addition {3,0} with travel time of 0.7 hours. Because the household can now return home immediately after work and still make the social activity in time, they do so for an improved travel disutility of 12.9. The result is not only a change in trip ODs (due to re-sequence in a tour), but one extra trip is also created as shown in Figure 2-b (4 trips). Essentially a trip has been added without adding a new non-home destination to visit, but the household sees an improvement in travel disutility because of the relaxation of spatial-temporal constraints that were binding before the network improvement. A conventional trip-based approach, or even a fixed schedule approach, would miss such a response altogether.

2.3.Increasing Travel Disutility

If we consider a continuous link improvement (in which a route travel time is improved), then another counterintuitive situation can occur. Consider the household in Table 2 again, but in this case let's assume that the household seeks to minimize idle time. Idle time is defined as the extent of the travel day that is not used in performing activities or traveling—such tradeoffs are similar to studies comparing values of in-vehicle travel time against out-of-vehicle access or idle/wait time. The potential for conflict values of in-vehicle travel time against out-of-vehicle access or idle/wait time. The potential for conflict
between the two objectives is not immediately apparent; however, in the presence of strict time windows
it is po it is possible that improving travel times can result in increasing idle time.
 $\min Z = (B_x - B_y) \cdot \sum \sum \sum t_i \cdot X^{\nu} + B_y \cdot \sum (T^{\nu}) \cdot -T^{\nu}$

$$
\min Z = (\beta_T - \beta_W) \cdot \sum_{v \in V} \sum_{w \in N} \sum_{u \in N} t_{uw} \cdot X_{uw}^v + \beta_W \cdot \sum_{v \in V} (T_{2n+1}^v - T_0^v)
$$

Where $\beta_T = 1$ and $\beta_W = 1.5$. The durations of the activities s_u are not included because they are constant and drop out. In the base case shown in Figure 2-(a), the disutility under this new objective is 16.625 instead of 14.25.

If a continuous improvement is made to link $\{3,1\}$ such that travel time improves from 0.5 hours to 0.25 hours (e.g. repaving, lane expansion), then due to time window constraints there are no other alternative routes and the household would still have to follow the same schedule. However, this results in a direct trade-off between travel time and idle time. *If a household values idle time minimization more than travel time minimization, then such an improvement can result in a paradoxically higher disutility, even without considering congestion effects*. The travel time improvement simply results in a decrease in travel time objective of 0.25 but a direct increase in idle time of 0.25. Since $\beta_W > \beta_T$, the disutility actually increases from 16.625 to 16.75. Effects such as this would be completely ignored if NDPs were applied without considering their effect on household scheduling. However, explicitly incorporating household scheduling mechanisms into the NDP allow paradoxes such as this to be avoided.

We have presented three scenarios that can arise from network improvements when realistically considering the effects they have on household scheduling and planning. Network changes can cause significant reshaping of temporal /spatial constraints for households which result in changes in their trip patterns. We argue that these effects should not be ignored when considering NDPs at the tactical or planning level.

3. Proposed NDP-HAPP Model

3.1.Definitions

As a kernel activity-based NDP, the NDP-HAPP is formulated using the simplest structure. Essentially, the activity-based NDP using HAPP subproblems to address household schedule response to network changes is here designated as NDP-HAPP. More complex formulations that explore link capacities, vehicle and household member interactions, multimodal networks, or congestion effects will be explored in future research. The kernel formulation is first presented as a set of multiple subproblems, and then further modified to consider activity choice in cases with non-compulsory activities. There are two distinct types of networks in this problem: an infrastructure network where changes can actively be made, and a responsive activity network that represents the routing and scheduling decisions made at the household level. Assume an infrastructure network layer L_I , and the following parameters for the infrastructure network system:

- **N** set of all nodes in the analysis
- **E** set of all direct links in the analysis
- F_{ii} fixed arc design costs

 c_{ii} operational per unit link routing costs

- *B* total budget for the network system
- t_{ii} travel time between the direct link from node *i* to node *j*
- $c_{ij}^{\nu,h}$ personal travel cost for vehicle *v* of household *h* , between the direct link from node *i* to node *j*

Variables concerning the infrastructure network system are:

 f_{ij} flow on direct link (i, j)

 z_{ii} binary decision variable that indicates whether or not link (i, j) is chosen as part of the network's design

Assume also an activity layer L_{A_2} and the following parameters for the activity network system

- **P** set of all activity nodes in the analysis. It is a subset of the node set from the infrastructure network, **N** .
- $(u, w),$ $u, w \in \mathbf{P}$ route from activity point u to activity point w . Its connectivity is derived from L_{I} .

 $\mathbf{H} = \{h_1, h_2, ..., h, ..., |\mathbf{H}|\}$ set of households using on the activity nodes **P** in the analysis.

Although their physical locations are the same, the two sets of networks operate in a bi-level fashion. This bi-level property of NDP-HAPP, together with it unfolding in the time-space dimension, can be conceptually depicted in Figure 3. Such separation of networks, a supernetwork approach, has been used widely in activity-based transportation networks, mainly concerning various modal choices and their specific networks (e.g., TRANSIMS, 2012; Arentze and Timmermans, 2003). However, an optimizationbased routing and scheduling procedure has never been applied to the activity layer in response to infrastructure changes.

Figure 3. Bi-level interactions between the infrastructure and activity networks

Following the notation of Recker (1995), we define the following sets and parameters that are specific for each household, $h \in \mathbf{H}$:

 ${\boldsymbol{\beta}}_h = {\beta_h^a, \beta_h^b, ...\}$ the set of relative weights for different travel disutility terms for household *h* A_h the set of out-of-home activities to be completed by travelers in household, h . V_h : the set of vehicles used by travelers in household h to complete their scheduled activities. $n_h = |A_h$ the number of activities to be performed by household h $P_h^+ \subset \mathbf{P}$ the set designating location at which each assigned activity is performed for household, h . Each activity and the physical location is different for each household. $P_h^- \subset \mathbf{P}$: the set designating the ultimate destination of the "return to home" trip from out-of-home activities to be completed by travelers in household, h . (Note: the physical location of each element of P_h^{A-} is "home".) μ^h _u, b_μ^h $\left[a_{u}^{h},b_{u}^{h}\right]$: the time window of available start times for activity *I* for household *h* . $b_{n_h+u}^h,b_{n_h}^h$ $\left[a_{n_h+u}^h, b_{n_h+u}^h \right]$: the time windows for the "return home" arrival from activity *I* of household *h*. (Note: b_i^h must precede $b_{n_h}^h$ $b_{n_h+u}^h$ by an amount equal to or greater than the duration of the activity) $\left[\,a_0^h, b_0^h\,\right]$: the departure window for the beginning of the travel day for household *h* . $\frac{h}{2n_h+1}, b_{2n_h+1}^h$ *h h* $\left[a_{2n_h+1}^h, b_{2n_h+1}^h \right]$: the arrival window by which time all members of the household h must complete their travel.

h s_u^h : the duration of activity *u* of household *h* . *h* t_{uw}^h the travel time from the location of activity u to the location of activity w . ,*h* $c_{uw}^{v, \prime}$: travel cost for household *h* , from location of activity *u* to the location of activity *w* by vehicle ν . B^h_C : : the travel cost budget for household *h* . $B_{T}^{\nu,h}$ the travel time budget for the household h 's member using vehicle v . $\mathbf{P}_h = \mathbf{P}_h^+ \cup \mathbf{P}_h^-$: the set of nodes comprising completion of the activities of household *h* . $\mathbf{Q}_h = \{0, \mathbf{P}_h, 2n_h + 1\}$: the set of all nodes for household h, including those associated with the initial departure and final return to home. This is a subset of **P** .

Here i, j are used to refer to nodes in the infrastructure layer, and link (i, j) refers to the direct link connecting those two nodes. Notation u, w are used to refer to activity nodes in the activity layer, and it is not necessarily a direct infrastructure link but rather a path between u, w . Path information such as travel time and travel cost are passed onto the activity layer from the infrastructure layer, but the connectivity data of the path needs to be drawn from the infrastructure layer.

The household-specific decision variables are:

, $X_{uv}^{v,h}$, $u, w \in \mathbf{Q}_h$, $v \in V_h$, $h \in \mathbf{H}$ binary decision variable equal to one if vehicle ν travels from activity *u* to activity *w*, and zero otherwise. T_u^h , $u \in P_h$, $h \in \mathbf{H}$ time at which participation in activity u of household h begins. $T_0^{v,h}, T_{2n_h+1}^{v,h}, \quad u \in P_h, h \in \mathbf{H}$ times at which vehicle ν from household h first departs from home and last returns to home, respectively Y_u^h , $u \in P_h$, $h \in \mathbf{H}$ total accumulation of either sojourns or time (depending on the selection of D and d_u) of household h on a particular tour immediately following completion of activity *u*.

Variables connecting the infrastructure network, L_I , and the activity network system, L_A , are:

 $\delta_{uw,ij} = \delta_{uw,ij}(\mathbf{z})$ binary indicator variable whether route *uw* in the activity network uses link *ij* in *LI* . Assuming the shortest cost path is used between two activity nodes, the design variables determine the connectivity of nodes in L_i . If link *ij* is not constructed, $z_{ij} = 0$, $\delta_{uw,ij}$ is automatically 0, and otherwise, it can be identified by solving a shortest path problem between the origin and the destination, *uw*. $\begin{aligned} w. & 0 & z_{ij} = 0 \end{aligned}$

$$
\delta_{uw,ij} = \delta_{uw,ij}(\mathbf{z}) = \begin{cases} 0 & z_{ij} = 0\\ \left(\delta_{uw,ij}\right)^* & z_{ij} = 1 \end{cases}
$$

where $(\delta_{uwii})^*$ $(\delta_{uw,ij})^*$ is the solution of a shortest path problem for each activity link u, w , i.e. **Shortest Path Allocation Problem**

 $\overline{1}$

$$
\min \sum_{(i,j)\in \mathbf{E}} t_{ij} \cdot \delta_{uw,ij} \tag{1}
$$

Subject to

Subject to
\n
$$
\sum_{j \in \mathbb{N}} \delta_{uw,ji} - \sum_{j \in \mathbb{N}} \delta_{uw,ij} = \begin{cases} 1 & i = u \\ 0 & i \neq u, w \\ -1 & i = w \end{cases}
$$
\n(2)

$$
\delta_{\mathit{uw},ji} \in (0,1) \tag{3}
$$

The problem is defined for all households and their activity routes, $u, w \in \mathbf{Q}_h$, $v \in V_h$, $h \in \mathbf{H}$.

 $t_{uw} = t_{uw}(\mathbf{z})$: : travel time from the location of activity *u* to the location of activity *w*. It is a function of the decision variable vector **z**, and the given network (**N**, **E**) since the connectivity
decision variables of z_{ij} determine the travel times.
 $t_{uw} = t_{uw}(z) = \sum_{j \in \mathbf{E}} \sum_{i \in \mathbf{E}} t_{ij} \cdot \delta_{uw,ij}(\mathbf{z}), \quad u, w \in \mathbf{Q}_h, h \in$ decision variables of z_{ij} determine the travel times.
 $t_{uw} = t_{uw}(z) = \sum_{n} \sum_{n} t_{ij} \cdot \delta_{uw,ij}(\mathbf{z}), \quad u, w \in \mathbf{Q}_h, h$

ables of
$$
z_{ij}
$$
 determine the travel times.
\n
$$
t_{uw} = t_{uw}(z) = \sum_{j \in \mathbf{E}} \sum_{i \in \mathbf{E}} t_{ij} \cdot \delta_{uw,ij}(\mathbf{z}), \quad u, w \in \mathbf{Q}_h, h \in \mathbf{H}
$$
\n(4)

 $c_{uw}^{v,h} = c_{uw}^{v,h}(\mathbf{z})$ travel cost from the location of activity *u* to the location of activity *w* for vehicle *v* of household h . It is a function of the decision variable vector z , and the given network household *h*. It is a function of the decision variable vector **z**, and the given i

(**N**, **E**) since the connectivity decision variables of z_{ij} determine the travel times.
 $c_{uw}^{v,h} = c_{uw}^{v,h}(z) = \sum_{j \in \mathbf{E}} \sum_{i \in \mathbf{$

$$
c_{uw}^{\nu,h} = c_{uw}^{\nu,h}(z) = \sum_{j \in \mathbf{E}} \sum_{i \in \mathbf{E}} c_{ij}^{\nu,h} \cdot \delta_{uw,ij}(\mathbf{z}), \quad u, w \in \mathbf{Q}_h
$$
 (5)

$$
f_{ij} = f_{ij}(X)
$$
link flow on direct link *ij*. It is a function of the household activity decision variable vector, *X*, and connects the path flow on layer *L_A* to the link flow on layer *L_I*. E given network (**N**,**E**) since the connectivity decision variables of *z_{ij}* determine the travel times.
$$
f_{ij}(X) = \sum_{h \in \mathbf{H}} \sum_{u \in \mathbf{Q}_h} \sum_{v \in \mathbf{Q}_h} \sum_{v \in \mathbf{V}^h} \delta_{uw,ij}(\mathbf{z}) \cdot X_{uw}^{v,h}, \quad (i, j) \in \mathbf{E}
$$
 (6)

$$
f_{ij}(X) = \sum_{h \in \mathbf{H}} \sum_{u \in \mathbf{Q}_h} \sum_{w \in \mathbf{Q}_h} \sum_{v \in V^h} \delta_{uw,ij}(\mathbf{z}) \cdot X_{uw}^{v,h}, \quad (i, j) \in \mathbf{E}
$$
 (6)

3.2.Decomposed Formulation of NDP-HAPP

Typically, the LRP formulation includes three parts: location, routing, and allocation. This property applies to the NDP-HAPP as well, where the upper level "location" is the network design variables and the lower level routing part is the HAPP model. Allocation refers to assignment of the activity link impedance from the shortest path problem in the infrastructure network, already shown in Equation $(1) - (3)$. The objective function of the upper problem in the LRP is to minimize the overall cost, which is comprised of depot cost and vehicle cost.

Similarly, NDP-HAPP in the most basic form is decomposed into two models solved as a bi-level problem: NDP (upper) and HAPP (lower). There are two sets of decision makers, so the solution can be classified as a leader/follower Stackelberg equilibrium, as described in Yang and Bell (1998). Instead of a traffic equilibrium lower level problem, the NDP-HAPP has a set of household scheduling problems in the lower level, where each household decides on its activity pattern. Considering the network design problem as the upper level decision and the household activity/scheduling/routing decisions (HAPP) as reactions to the network design, we can express the problem most generally in Equations (7).

$$
\min_{z,f} G(z, f(X(z)), T(z)) = \varphi_{dNDP}(z)
$$
\nsubject to\n
$$
H(z, f(X(z)), T(z)) \le 0
$$
\nwhere\n
$$
\min_{x, \tau} g(X, T(\delta(f))) = \varphi_{dHAPP}(X, T(\delta(f)))
$$
\nsubject to\n
$$
h(z, X, T(\delta(f))) \le 0
$$
\n(7b)

where G is the objective function, z is the decision vector, and H is the constraint set of the upper level problem. In the lower level problem, g is the objective function, X, T is the decision vector, and h is the constraint set.

The kernel network design problem we present is simply a modified version of the unconstrained multicommodity case of the formulation in Magnanti and Wong (1984). The formulation minimizes the design cost while satisfying the given flow demands at origin and destination nodes. The formulation is in terms of direct links and link flows only, whereas the integrated NDP-HAPP includes path flows which are connected to direct link flows, f_{ij} , by $\delta_{uw,ij}$. In order for the OD pairs to be assigned to sequences of direct links, we treat each OD pair as a commodity as in the case of multicommodity flow problems, i.e., we define a single commodity f_{ij}^{uw} , $\forall (u, w) \in \mathbf{K}$ where (u, w) *uw ij* - ∠ J*ij u w* $f_{ii} = \sum f$ $=\sum_{(u,w)\in{\bf K}}$, where **K** is the set of all OD

 (u, w) pairs.

We formulate this decomposed NDP (dNDP) in terms of direct link flows only, and each OD pair is represented as a commodity. The demand values are calculated as shown in Equation (14). They take household sequence decisions and aggregate them into origin-destination pairs.

Upper Level NDP (dNDP)
\n
$$
\min \varphi_{dNDP}(z, f) = \sum_{(i,j)\in \mathbf{E}} F_{ij} \cdot z_{ij} + \sum_{(i,j)\in \mathbf{E}} c_{ij} \cdot f_{ij}
$$
\n(8)

Subject to:

$$
\min \varphi_{dNDP}(z, f) = \sum_{(i,j)\in \mathbf{E}} F_{ij} \cdot z_{ij} + \sum_{(i,j)\in \mathbf{E}} c_{ij} \cdot f_{ij}
$$
\nSubject to:

\n
$$
\sum_{j\in \mathbf{N}} f_{ji}^{uw} - \sum_{l\in \mathbf{N}} f_{il}^{uw} \ge D^{uw}, \quad \forall i = u \in \mathbf{N}, \forall (u, w) \in \mathbf{K}
$$
\n(9)

Subject to:
\n
$$
\sum_{j\in \mathbb{N}} f_{ji}^{uw} - \sum_{l\in \mathbb{N}} f_{il}^{uw} \ge D^{uw}, \quad \forall i = u \in \mathbb{N}, \forall (u, w) \in \mathbb{K}
$$
\n
$$
\sum_{j\in \mathbb{N}} f_{ij}^{uw} - \sum_{l\in \mathbb{N}} f_{li}^{uw} \ge D^{uw}, \quad \forall i = u \in \mathbb{N}, \forall (u, w) \in \mathbb{K},
$$
\n
$$
\sum_{j\in \mathbb{N}} f_{ji}^{uw} - \sum_{j\in \mathbb{N}} f_{ij}^{uw} = 0, \quad \forall i \in \mathbb{N}, i \ne u, i \ne w, \forall (u, w) \in \mathbb{K}
$$
\n(11)

$$
\sum_{j\in \mathbf{N}} f_{ji}^{uw} - \sum_{j\in \mathbf{N}} f_{ij}^{uw} = 0, \qquad \forall i \in \mathbf{N}, i \neq u, i \neq w, \forall (u, w) \in \mathbf{K}
$$
\n(11)

$$
\sum_{j \in \mathbb{N}} J_{ji} \sum_{j \in \mathbb{N}} J_{ij} \qquad \forall (i, j) \in \mathbb{E}, (u, w) \in \mathbb{K}
$$
\n
$$
f_{ij}^{uw} \le D^{uw} \cdot z_{ij}, \qquad \forall (i, j) \in \mathbb{E}, (u, w) \in \mathbb{K}
$$
\n
$$
(12)
$$

$$
z_{ij} \in (0,1), \qquad (i,j) \in \mathbf{E} \tag{13}
$$

where

where
\n
$$
D^{uv} = \sum_{h \in \mathbf{H}} \sum_{v \in V^h} X_{uw}^{v,h}, \qquad w = i \in \mathbf{N}, \forall (u, w) \in \mathbf{K}
$$
\n(14)

Equations (9) – (10) require each path $(u, w) \in K$ to satisfy the given OD demand. Equations (11) simply show the conservation of flows for intermediate nodes. Equation (12) constrains flow variables to be on the links that are built in a manner that does not exceed the capacity. Because we do not consider cases in which the capacity of links is exceeded in this paper, only the shortest path will be loaded with flows. As such, the shortest path information is provided directly by the f_{ij}^{uw} variable. We can implicitly obtain the shortest path variables for each OD pair as shown in Equation (15) instead of having to solve Equations $(1) - (3)$ separately.

Equations (1) – (3) separately.
\n
$$
\delta_{uw,ij} = \begin{cases}\n0 & f_{ij}^{uw} = 0 \\
1 & \text{otherwise}\n\end{cases} \quad \forall (i, j) \in \mathbf{E}, (u, w) \in \mathbf{K}, v \in V_h, h \in \mathbf{H}
$$
\n(15)

The decomposed lower-level HAPP (dHAPP) problem is shown in Equations $(16) - (19)$. It is composed of the set of constraints in the Appendix which would be equivalent to the original constraints from Case 1 in Recker (1995) if travel time/cost factors are not functions of the allocated shortest path. Also, each household can be treated separately since all constraints as well as the objective functions are separable by households. With constant travel times/costs, i.e., without congestion effects, each household's dHAPP is solved separately. In the treated separately since all constraints as well as the objective cholds. With constant travel times/costs, i.e., without congestion is solved separately.

P (**dHAPP) for Each Household**
 $= \sum_{h \in \mathbf{H}} \sum_{v \in V^h} \beta$

Lower Level HAPP (dHAPP) for Each Household

household's dHAPP is solved separately.
\nLower Level HAPP (dHAPP) for Each Household
\n
$$
\min \varphi_{dHAPP}(X,T) = \sum_{h \in H} \sum_{v \in V^h} \beta_h^T \cdot (T_{2n_h+1}^{v,h} - T_0^{v,h}) + \sum_{h \in H} \sum_{u \in Q_h} \sum_{w \in Q_h} \sum_{v \in V^h} \beta_h^C \cdot c_{uw}^{v,h} \cdot X_{uw}^{v,h}
$$
\n(16)

Subject to $(A1) - (A26)$

Where

Where

$$
t_{uw}(z) = \sum_{(i,j)\in \mathbf{E}} t_{ij} \cdot \delta_{uw,ij}, \quad u, w \in \mathbf{Q}_h, h \in \mathbf{H}
$$
 (17)

$$
c_{uw}^{v,h}(z) = \sum_{(i,j)\in\mathbf{E}} c_{ij}^{v,h} \cdot \delta_{uw,ij}, \quad u, w \in \mathbf{Q}_h
$$
\n(18)

$$
c_{uw}^{v,h}(z) = \sum_{(i,j)\in \mathbf{E}} c_{ij}^{v,h} \cdot \delta_{uw,ij}, \quad u, w \in \mathbf{Q}_h
$$

\n
$$
\delta_{uw,ij} = \begin{cases} 0 & f_{ij}^{uw} = 0 \\ 1 & \text{otherwise} \end{cases} \forall (i, j) \in \mathbf{E}, (u, w) \in \mathbf{K}, v \in V_h, h \in \mathbf{H}
$$
\n(19)

As discussed in other HAPP model studies, the objective shown in Equation (16) is just one example multiobjective problem. Others can be specified and estimated using the method from Chow and Recker (2012). The process of specifying the multiple objectives and calibrating their coefficients with desired arrival times can be thought of as a confirmatory, normative modeling process that seeks to fit a hypothesis of how household travelers behave onto a data set. Fitness of an objective is determined by the significance of its estimated coefficient relative to other objectives. For example, a data set might reveal that Equation (16) results in a length of day coefficient (first term) equal to 0.0001 relative to a weight of 1 for the travel cost objective. In that case, it would suggest that the first objective is not very important in the travelers' scheduling choices.

NDP-HAPP as presented in Section 3.2 differs conceptually from the LRP in two primary ways. First, the LRP has a single decision-maker involved in both planning and tactical strategic design, whereas the NDP-HAPP has a single decision-maker involved in planning and multiple household decision-makers responding to the plan at a tactical level. Second, the node demand for the upper level problem in the LRP is known *a priori*, but the cost of delivering service to the demand node is not known. Instead, it is derived from the output of the VRP. Alternatively, the NDP-HAPP does not have OD demand known *a priori*, but costs between each node are given. The OD demand is derived from the output of the HAPP.

3.3.Generalized NDP-HAPP (NDP-GHAPP)

The NDP-HAPP model is extended to include the capability for households to choose locations for non-primary activities, such as grocery shopping and refueling. This is simply done by relaxing the condition in the HAPP that each household needs to visit each specifically designated location, but rather visits one candidate location from a cluster of such service types. This is similar to the generalized traveling salesman problem (e.g. the E-GTSP in Fischetti *et al.*, 1997) and generalized vehicle routing problem (Ghiani and Improta, 2000) in the logistics literature, where visits to nodes are modified to visits to single nodes from each cluster. The generalized HAPP (GHAPP) has been formulated and applied (Kang and Recker, 2012a; Kang and Recker, 2012b), and a variation of this approach with exogenously defined activity utilities and time windows was developed for activity-based traveler information systems (Chow and Liu, 2012).

In GHAPP, the constraints in Equation (A1) are modified to Equation (A1-1). Instead of requiring each node to have a flow, the generalized formulation instead requires one node from a cluster of nodes to be visited. Compulsory activity types would only have one node in the cluster, whereas non-primary

activity of the activity of the activity of the activity. The energy of the energy is given by:

\n
$$
\sum_{u \in \mathbf{P}_{A_a}^+} \sum_{v \in \mathbf{V}_h} \sum_{v \in \mathbf{Q}_h} X_{uw}^{v,h} = 1, \quad A_a \in \mathbf{A}, h \in \mathbf{H}
$$
\n(A1-1)

where

$$
A = \{A_1, A_2, ..., A_a, ..., A_m\}
$$
 the set of *m* different activity types with unspecified locations

$$
P_{A_a}^+
$$
 the set designating "potential" locations at which activity A_a may be performed

Integrated with NDP, GHAPP becomes infeasible if one or more candidate nodes are not connected to the network; constraints in (A7), (A11) also need to be modified to be conditional such that the temporal constraints are imposed only when there is a visit to that candidate location. This allows
having one or more of unconnected candidate nodes, which have infinite travel times.
 $\sum_{v \in V_h} \sum_{w \in Q_h} X_{wu}^{v,h} = 1 \Rightarrow$

having one or more of unconnected candidate nodes, which have infinite travel times.
\n
$$
\sum_{v \in V_h} \sum_{w \in \mathbf{Q}_h} X_{wu}^{v,h} = 1 \Longrightarrow T_u^h + s_u^h + t_{uw}^h(\mathbf{z}) = T_{n+u}^h, \quad u \in P_h^+, h \in \mathbf{H}
$$
\n(A7-1)

$$
\sum_{v \in V_h} \sum_{w \in \mathbf{Q}_h} X_{wu}^{v,h} = 1 \Longrightarrow T_u^h + s_u^h + t_{uw}^h(\mathbf{z}) = T_{n+u}^h, \quad u \in P_h^+, h \in \mathbf{H}
$$
\n(A7-1)\n
$$
\sum_{v \in V_h} \sum_{w \in \mathbf{Q}_h} X_{wu}^{v,h} = 1 \Longrightarrow a_u^h \le T_u^h \le b_u^h, \quad u \in P_h, h \in \mathbf{H}
$$
\n(A11-1)

Similarly, when the objective function involves time variables, the time variables for the

unvisited activity nodes need to be constrained. For example:
\n
$$
\sum_{v \in V_h} \sum_{w \in \mathbf{Q}_h} X_{wu}^{v,h} = 0 \Rightarrow T_u^h = 0, \quad u \in P_h, h \in \mathbf{H}.
$$
\n(A7-2)

3.4.Decomposition Solution Algorithm

l

There are many different types of solution algorithms developed for LRPs (Nagy and Salhi, 2007), and they can potentially be adopted for NDP-HAPP. However, the iterative method proposed here decomposes the problem into several blocks that actually represent each decision maker's rationale in this complex problem. Additionally, this kind of decomposition does not necessarily require the problem to be formulated in the structure of mathematical optimization as long as the drivers' response to the network design is captured and updated. This means different types of integrated activity-based approaches can be used to model individuals' routing/scheduling behavior. Because the majority of these activity-based models are based on discrete-choice models or simulation-based (e.g., Bowman and Ben-Akiva, 2000; Bhat *et al.*, 2004; Balmer *et al.*, 2006), the suggested decomposition method is highly adaptable to different types of activity-based models.

The decomposed problems remain computationally challenging, particularly the NP-hard HAPP. Because these problems are widely studied, there are various methods available. Geoffrion and Graves (1974) are referred for network design problems, and Cordeau and Laporte (2003) are referred for a survey of algorithms for the Pickup and Delivery Problem with Time Windows (PDPTW), which the simplest HAPP is based on. The decomposition proposed here is comparable to Perl and Daskin (1985) in the context of Location Routing Problems, and the Iterative Optimization Assignment (IOA) algorithm in Yang and Bell (1998) in the context of bi-level Network Design Problems. Perl and Daskin (1985) used three decomposed models to tackle the warehouse location routing problem: the complete multi-depot vehicle-dispatch problem (MDVDP), the warehouse location-allocation problem (WLAP), and the multidepot routing-allocation problem (MDRAP). The location-allocation and muti-depot routing allocation blocks are in parallel with dNDP and dHAPP. For NDP, the iterative optimization-equilibrium in Friesz and Harker (1985) includes similar blocks of Equilibrium Assignment Program and Design Optimization, in line with dHAPP and dNDP. Since there is no congestion in the dHAPP model, the issue of having IOA converge to a Cournot-Nash equilibrium is not relevant here.²

An iterative solution algorithm for the NDP-HAPP is depicted in Figure 4. First, the initial network decision solution is assumed to use all links, $z_{ij}^0 = 1$, $(i, j) \in \mathbf{E}$. Then, $\delta_{uv, ij}^0$ can be derived from

 z_{ij}^0 using the standard shortest path problem—for example, Floyd's Algorithm can be used to efficiently update the travel time matrix. Based on the updated travel times, dHAPP is solved independently for each household since no congestion effect is present. Hypothetically speaking, if congestion is incorporated in future research (perhaps through integration with Lam and Yin's (2001) framework), this framework should still be feasible. After the travel decisions are made by each household, supply and demand are updated from Equations (15). dNDP can then be solved as the conventional NDP. The proposed iterative process continues until there is no improvement in the objective function. The implicit shortest path allocation from the upper level problem and the path-link conversion conditions in Equations $(4) - (6)$ are maintained throughout this iterative process. The same algorithm can be applied to NDP-GHAPP.

 2^2 For the examples and case studies presented in this paper, the overall processes are coded in Java calling a CPLEX library for dHAPP and dNDP problems.

Figure 4. A decomposition solution method for NDP-HAPP

4. Numerical Examples

4.1.Simple Example with Known Optimal Solution

Assume a grid network with nine nodes, with possible link construction as Figure 5. When constructed, travel time for each link is 0.5. Construction cost for each link is 3, and operating cost per link per flow is 0.5, i.e. $\min \varphi_{dNDP}(z, f) = \sum_{(i,j) \in \mathbb{E}} 3 \cdot z_{ij} + \sum_{(i,j) \in \mathbb{E}}$ for each link is 0.5. Construction cost for ϵ
min $\varphi_{dNDP}(z, f) = \sum_{ij} 3 \cdot z_{ij} + \sum_{ij} 0.5 \cdot f_{ij}$ $\sum_{(i,j)\in\mathbf{E}}$ $3 \cdot z_{ij} + \sum_{(i,j)}$ *x* k with nine nodes, with possible link constant in the seach link is 0.5. Construction cost for each $\varphi_{dNDP}(z, f) = \sum_{(i,j)\in E} 3 \cdot z_{ij} + \sum_{(i,j)\in E} 0.5 \cdot f_{ij}$.

Assume two households, $\mathbf{H} = \{h_1, h_2\}$, with one vehicle each, $V_1 = \{1\}$, $V_2 = \{1\}$, and their activities $A_1 = \{work, grocery shopping\}$, $A_2 = \{work, general shopping\}$ to perform. These activities' locations in the infrastructure layer are shown in Figure 5, and their activity start/end time windows, activity durations are all shown in Table 3. Except for the activity start time windows of work activities, time windows are not necessarily constraining, leaving some room to explore different path sequences. Both households' objective functions are assumed to be minimizing total travel cost only sequences. Both households' objective functions are assumed to be minimizing total travel consider the travel costs 1 for all direct links), $\min \varphi_{dHAPP}^h(X) = \sum_{u \in Q_h} \sum_{v \in V^h} \sum_{v \in V^h} c_{uw}^{v,h} \cdot X_{uw}^{v,h}$ to be minimizing total travel cos
 h^{h} (X) = $\sum \sum \sum c^{v,h} \cdot X^{v,h}$ $\lambda_{dHAPP}^{h}(X) = \sum_{w} \sum_{w} \sum_{w} c_{uw}^{v,h} \cdot X_{uw}^{v,h}$ $\sum_{u \in \mathbf{Q}_h} \sum_{w \in \mathbf{Q}_h} \sum_{v \in V}$ d to be minimizing total travel $\varphi_{dHAPP}^h(X) = \sum \sum \sum \sum c_{uw}^{v,h} \cdot X$ com to explore different
 $\text{minizing total travel cost}$
 $= \sum_{u \in \mathbb{Q}_h} \sum_{v \in \mathbb{Q}^h} \sum_{v \in V^h} c_{uw}^{v,h} \cdot X_{uw}^{v,h}.$

	Location on L_{I}	$\left[a_{\mu}^h, b_{\mu}^h\right]$	$ a_{n_h+u}^h, b_{n_h+u}^h $	S_u^h
h_1 home	Node 0	$ a_0^h, b_0^h = [6, 21]$ $\left[a_{2n_h+1}^h, b_{2n_h+1}^h\right] = [10, 24]$		NA
h_1 work activity	Node 2	[9, 9.5] [10, 22]		8
h_1 grocery shopping activity	Node 5	[10, 22] [5, 22]		
h_2 home	Node 5	$ a_0^h, b_0^h = [6, 21]$	$\left[a_{2n_h+1}^h, b_{2n_h+1}^h\right] = [10, 22]$	NA
$h2$ work activity	Node 6	[8.5, 9]	[10, 22]	8
$h2$ general shopping activity	Node 8	[5, 21]	[10, 22]	

Table 3. Simple Example Household Characteristics

Figure 5. Supernetwork depiction.

Because the NDP-HAPP is not a simple problem to check for optimality, all possible combinations of household decisions are enumerated and given to dNDP, and its objective value combined with the objective value of corresponding household decision combination is used to derive the true optimal solution value. Figure 6 shows the solution from the proposed method $(6-(a), 6-(b))$ and the actual optimal solution (6-(c), 6-(d)). The decomposition solution converged after one iteration and is 5%

worse than the actual optimal solution, 40, for this simple example. Detailed illustration of the computational process is available in Table 4.

Figure 6. NDP-HAPP Decomposition Solution Comparison to Enumerated Exact Solution.

	Iteration 1	Iteration 2	
	Path ³ : Home (0) \rightarrow work (2) \rightarrow grocery shopping	Path: Home (0) \rightarrow work (2) \rightarrow grocery	
dHAPP1	$(5) \rightarrow$ home (0)	shopping $(5) \rightarrow$ home (0)	
	Objective Value: 3	Objective Value: 3	
	Path ⁴ : Home (5) \rightarrow work (6) \rightarrow general shopping	Path: Home $(5) \rightarrow$ work $(6) \rightarrow$ general	
dHAPP2	$(8) \rightarrow$ home (5)	shopping $(8) \rightarrow$ home (5)	
	Objective Value: 3	Objective Value: 3	
dNDP	Network Design Decisions: Z01, Z12, Z25, Z30, Z36, Z43, Z54, Z36, Z78, Z85 dNDP objective value: 36 HH1 Paths link Flows: • Home $(0) \rightarrow (1) \rightarrow Work(2)$ • Work $(2) \rightarrow$ Grocery Shopping (5) • Grocery Shopping $(5) \rightarrow (4) \rightarrow (3) \rightarrow$ Home (0) HH2 Paths link Flows: • Home $(5) \rightarrow (4) \rightarrow (3) \rightarrow Work(6)$ • Work $(6) \rightarrow (7) \rightarrow$ General Shopping (8) • General Shopping $(8) \rightarrow$ Home (5) Update each dHAPP objective values: HH1: 3 HH2: 3	NA ⁵	
Final Objective	42	42	

Table 4. Detailed Computational Illustration of the basic NDP-HAPP Example

4.2.Simple Example: Generalized HAPP

Using the generalized model allows us to include behavioral changes in destination choice as well as routing/scheduling of activities with respect to network design decisions. Following the example in Section 4.1, assume that there are two grocery shopping locations, node 1 and node 5, $P_{A_a=GroceryShopping}^+$ = {1,5} $_{\text{GroceryShopping}} = \{1, 5\}$, and two general shopping locations, node 3 and node 8, $P_{A_a = \text{GeneralShopping}}^+ = \{3, 8\}$ $=$ GeneralShopping $=$ —each household is required to visit one, and only one, of the candidate locations to perform the shop activity.

Here, NDP-GHAPP optimality is checked in the same way as the previous example, i.e., by comparing to the results of dNDP for all possible combinations of household decisions, including the destination choice as well as path sequence decisions and arrival time decisions to return. The solution from the iterative method reached the true optimal value after three iterations, shown in Figure 7. The intuition is that the flexibility introduced by NDP-GHAPP allows the method to search for many different options. Detailed illustration of the computational process of the proposed algorithm is shown in Table 5.

l

 3 These paths are based on the assumption that all links are available.

⁴ These paths are based on the assumption that all links are available.

 $⁵$ No changes in variables and objective function value. Therefore aborted after this iteration.</sup>

In this simple example, changes in activity sequence, link level flow in dNDP, and dNDP network design decisions are shown.

Figure 7. NDP-SHAPP Example Enumerated Optimal Solution

	Iteration 1	Iteration 2	Iteration 3	Iteration 4
dHAPP1	Path ⁶ : Home (0) \rightarrow grocery shopping (1) \rightarrow work (2) \rightarrow home (0) Objective Value: 2	Path: Home $(0) \rightarrow$ work $(2) \rightarrow$ grocery shopping $(1) \rightarrow$ home (0) Objective Value: 2	Path: Home $(0) \rightarrow$ grocery shopping (5) \rightarrow work (2) \rightarrow home (0) Objective Value: 4	Path: Home (0) \rightarrow grocery shopping (5) \rightarrow work $(2) \rightarrow$ home (0) Objective Value: 3
dHAPP2	Path ⁷ : Home (5) \rightarrow work $(6) \rightarrow$ general shopping $(8) \rightarrow$ home (5) Objective Value: 3	Path: Home $(5) \rightarrow$ work $(6) \rightarrow$ general shopping $(8) \rightarrow$ home (5) Objective Value: 3	Path: Home $(5) \rightarrow$ work $(6) \rightarrow$ general shopping $(3) \rightarrow$ home (5) Objective Value: 4	Path: Home (5) \rightarrow work (6) \rightarrow general shopping $(3) \rightarrow$ home (5) Objective Value: 4
dNDP	Network Design Decisions: Z01, Z10, Z12, Z21, Z58, Z67, Z76, Z78, Z85, Z87	Network Design Decisions: Z03, Z10, Z21, Z36, Z52, Z67, Z78, Z85 dNDP objective	Network Design Decisions: Z03, Z10, Z21, Z34, Z36, Z45, Z52, Z63 dNDP objective	NA^8

 6 These paths are based on the assumption that all links are available.

l

 $⁷$ These paths are based on the assumption that all links are available.</sup>

 8 No changes in variables and objective function value. Therefore aborted after this iteration.

4.3.Large Network Example: NDP-HAPP

This case study focuses on a major roadway system located in Orange County, a subsystem of the Los Angeles metropolitan roadway network, to compare the NDP-HAPP with the conventional NDP. The base network with household locations and their activities throughout the day are shown in Figure 8-(a). We assume that the network design decision maker is a public agency from Orange County, and its goal is to provide the best mobility for Orange County residents, where the mobility is expressed in terms of total travel times. Hypothetically suggested candidate improvements on the network system are extensions of SR 39, SR 57, SR 55, SR 22, SR 261, and SR 241 as seen in dashed red lines in Figure 8- (b).

Specifications of each candidate link are in Appendix B. The speed is drawn from the average speed for all links on the same facility, and construction cost for each link is assumed to be proportional to both average speed and distance.

(a) Sample households and their activity locations (b) Extracted network

Figure 8. Large-Scale Case Study Area

A sample of 60 single-member, single-vehicle households residing in Orange County drawn from the California Travel Survey (2001) is used to reflect fairly realistic trip patterns of this class of households. The objective function for dNDP is to minimize the total travel times for the system, $\sum_{(i,j)}$ mouseholds. The objective function
min $\varphi_{dNDP}(z, f) = \sum t_{ij} \cdot f_{ji}$ *i j* enotas. The objective function $\varphi_{dNDP}(z, f) = \sum_{(i,j)\in E} t_{ij} \cdot f_{ji}$,

 $\sum_{(i,j)\in\mathbf{E}} t_{ij} \cdot f_{ji}$, and the objective function for each dHAPP is to minimize its own travel
xample, individual household's travel disutility is defined by the linear combination of
the day, the travel times, and t disutility. For this example, individual household's travel disutility is defined by the linear combination of the total extent of the day, the travel times, and the delay of return home caused by trip chaining for each of out-of-home activities by the individual weights of such measurements, β_h^E , β_h^T , β_h^D . disutility. For this example, individual household's travel disutility is defined by the linear combination of
the total extent of the day, the travel times, and the delay of return home caused by trip chaining for each
o *E* individual weights of such measurements, β_h^E , β_h^T , β_h^D :
 β_h^E \cdot $(T_{2n_k+1}^{v,h} - T_0^{v,h}) + \sum_{h} \sum_{h} \beta_h^D \cdot (T_{w+n_k}^{h} - T_w^{h}) + \sum_{h} \sum_{h} \sum_{h} \sum_{h} \beta_h^T \cdot t_{uw}^{v,h} \cdot X_{uw}^{v,h}$ ility. For this example, individual household's travel disutility is defined by the linear combination of otal extent of the day, the travel times, and the delay of return home caused by trip chaining for each *t*t-of-hom

$$
\min \varphi_{dHAPP}(X,T) = \sum_{h \in \mathbf{H}} \sum_{v \in V^h} \beta_h^E \cdot (T_{2n_h+1}^{v,h} - T_0^{v,h}) + \sum_{h \in \mathbf{H}} \sum_{w \in \mathbf{P_h^*}} \beta_h^D \cdot (T_{w+n_h}^h - T_w^h) + \sum_{h \in \mathbf{H}} \sum_{u \in \mathbf{Q}_h} \sum_{w \in \mathbf{Q}_h} \sum_{v \in V^h} \beta_h^T \cdot t_{uw}^{v,h} \cdot X_{uw}^{v,h}
$$

The weights of these 60 households are individually estimated from the inverse optimization calibration process in Chow and Recker (2012). For the households in the sample, the estimated results have the process in Chow and Recker (2012). For the households in the sample, the estimated results have the values of $\beta_h^E = 0.84$, $\beta_h^D = 0.74$, $\beta_h^T = 3.45$, which means that on average these household decision makers value a minute of travel time savings about 4 times more than a minute of total extent of the day savings, and about 5 times more than a minute delay in returning home caused by trip chaining from outof-home activities. The values were based on having the same set of arrival time penalties for all activity types, with 0.613 early penalty and 2.396 late penalty, similar to Chow and Recker (2012). The correlations from the 60 samples were close to zero for $\rho_{E,T}$ and $\rho_{D,T}$, although the correlation between extent of day savings and return home delay was $\rho_{E,D} = 0.248$. Time windows of activities are separately estimated using the methodology from Kang and Recker (2012b), which adopted the method from Recker and Parimi (1999) with slight modifications.

In Table 6, results of NDP-HAPP are compared to conventional NDP solutions that take the O/D matrix derived from the optimal HAPP results with current network as an input. Six different budget limits are tested. The results indicate that both dNDP and dHAPP objective function values improved

with increasing budget limits, together with more households benefiting from the improvements. These households experience shorter travel times, but given the coarse geographic network, these improvements are not sufficiently large for the sample households to change their activity sequences. The O/D table stays the same, and therefore the conventional NDP delivers what appears to be the same results as NDP-HAPP. However, a view of the time of day distribution of all activity participation reveals changes that can be captured as a result of the NDP-HAPP, as shown in Figure 9.

Figure 9: Comparison of Activity Arrival Time Histograms

As shown in Figure 9, the schedules of most households did not change much towards the evening, but shifts in arrival times can be seen as a consequence of changes in the network. There is a noticeable shift, particularly in the morning periods, as a result of the network improvements and the structure of the time windows defined for the households' activities.

Table 6. Large-Scale NDP-HAPP Results

5. Conclusion

Given the arguments for considering activity behavior in transportation planning, it is logical to consider the applicability of activity scheduling in network design problems. Conventional NDPs studied previously focused on congestion issues, such as Braess' Paradox. This research takes a step toward gaining a better insight to NDPs where OD demand is not known *a priori*, but rather is the subject of responses in household itinerary choices that depend on the infrastructure improvements. Using simple examples, we show that falsely assuming that household itineraries are not elastic can result in a lack of understanding in certain phenomena; e.g., increasing traffic even without increasing economic activity due to relaxing of space-time prism constraints, or worsening of utility despite infrastructure investments in cases where household objectives may conflict.

An activity-based network design problem is proposed using the location routing problem as inspiration. The kernel problem is a bilevel formulation that includes an upper level network design and shortest path problem while the lower level includes a set of disaggregate household itinerary optimization problems, posed as HAPP (or in the case with location choice, as generalized HAPP) models. As a bilevel problem with an NP-hard lower level problem, there is no algorithm for solving the NDP-HAPP exactly. Nonetheless, the simple numerical examples demonstrate the sufficient accuracy of the decomposition heuristic algorithm derived from the LRP. The large numerical example based on Southern California data and setting suggest that even if infrastructure investments do not result in major changes in itineraries (or any, in this particular example), the results provide much higher resolution information to a decision-maker. Whereas a conventional NDP would output the best set of links to invest in given an assumed OD matrix, the NDP-HAPP can output the same best set of links, the same OD matrix, and a detailed temporal distribution of activity participation and travel.

Beyond the most obvious extensions and future research applicable to this work (improved heuristics, adding uncertainty, dynamic policies, etc.), there are a number of important issues that need further study. Congestion effects certainly fall among the top of that list. The kernel NDP-HAPP currently handles planning and tactical considerations, but expansions of the problem are needed include to operational design strategies such as optimal toll pricing, ramp metering, or signal timing. There are actually two levels of congestion for consideration. The first is the effect on the infrastructure layer, which is what Lam and Yin (2001) or a dynamic traffic assignment integration could achieve. Congested links in the upper level problem would result in multiple paths between each pair of nodes, which means some weighting of travel times is needed to translate over a single perceived travel time matrix for the lower level household scheduling problems. The other congestion effect is at the activity layer, and more generally speaking refers to both negative (congestion) and positive (bandwagon) effects. For example, the time-dependent utility of some activities may depend heavily on how popular they are with multiple individuals. Another effect that can be incorporated is the link capacity in the upper level problem. Since only the shortest path between all nodes is being allocated to the households, adding capacity would require some weighted average path travel times similar to the link congestion effect.

Another important consideration is the number of new types of NDPs that can benefit from having activity or itinerary response, not just from transportation planning perspective. In transportation planning there are many design problems where demand is not simply a single trip from origin to destination. One example is in public transit design, where station location and design is a significant determinant of fleet schedules and operations, which in turn have an effect on household travel itineraries.

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Appendix A: dHAPP Constraints (same as original HAPP constraints in Recker, 1995)
\n
$$
\sum_{v \in V_h} \sum_{w \in Q_h} X_{uw}^{v,h} = 1, \quad u \in P_h^+, h \in H
$$
\n(A1)

$$
\sum_{v \in V_h} \sum_{w \in \mathbf{Q}_h} X_{uw}^{v,h} = 1, \quad u \in P_h^+, h \in \mathbf{H}
$$
\n
$$
\sum_{w \in \mathbf{Q}_h} X_{uw}^{v,h} - \sum_{w \in \mathbf{Q}_h} X_{wu}^{v,h} = 0, \quad u \in P_h, v \in V_h, h \in \mathbf{H}
$$
\n(A2)

$$
\sum_{w \in P_h^+} X_{0w}^{v,h} = 1, \quad v \in V_h, h \in \mathbf{H}
$$
\n(A3)

$$
\sum_{w \in P_h^+} X_{0w}^{v,n} = 1, \quad v \in V_h, h \in \mathbf{H}
$$
\n
$$
\sum_{u \in P_h^-} X_{u,2n+1}^{v,h} - \sum_{u \in P_h^+} X_{0,u}^{v,h} = 0, \quad v \in V_h, h \in \mathbf{H}
$$
\n
$$
\sum_{w \in \mathbf{Q}_h} X_{wu}^{v,h} - \sum_{w \in \mathbf{Q}_h} X_{w,n+u}^{v,h} = 0, \quad u \in P_h^+, v \in V_h, h \in \mathbf{H}
$$
\n(A5)

$$
\sum_{u \in P_h^-} A_{u,2n+1} \sum_{u \in P_h^+} A_{0,u}^v = 0, \quad v \in V_h, n \in \mathbf{H}
$$
\n
$$
\sum_{w \in \mathbf{Q}_h} X_{wu}^{v,h} - \sum_{w \in \mathbf{Q}_h} X_{w,n+u}^{v,h} = 0, \quad u \in P_h^+, v \in V_h, h \in \mathbf{H}
$$
\n(A5)

$$
\overline{\mathbf{u}}_{\mathbf{u}}^{\mathbf{b}} + \mathbf{S}_{\mathbf{u}}^{\mathbf{b}} + \mathbf{t}_{\mathbf{u}\mathbf{w}}^{\mathbf{b}} = \overline{\mathbf{T}}_{n+u}^{\mathbf{h}}, \quad u, w \in P_h^+, h \in \mathbf{H}
$$
\n(A6)

$$
T_{u}^{h} + S_{u}^{h} + t_{uw}^{h} = T_{n+u}^{h}, \quad u, w \in P_{h}^{+}, h \in \mathbf{H}
$$

\n
$$
X_{uw}^{v,h} = 1 \Rightarrow T_{u}^{h} + S_{u}^{h} + t_{uw}^{h} = T_{w}^{h}, \quad u, w \in P_{h}, v \in V_{h}, h \in \mathbf{H}
$$

\n
$$
X_{0w}^{v,h} = 1 \Rightarrow T_{0}^{v,h} + S_{u}^{h} + t_{0w}^{h} = T_{w}^{h}, \quad w \in P_{h}^{+}, v \in V_{h}, h \in \mathbf{H}
$$

\n(A8)

$$
X_{0w}^{\nu,h} = 1 \Longrightarrow T_0^{\nu,h} + s_u^h + t_{0w}^h = T_w^h, \quad w \in P_h^+, v \in V_h, h \in \mathbf{H}
$$
\n(A8)

$$
X_{uw} = 1 \Rightarrow T_u^{v,h} + S_u^{h} + t_{uw}^{h} = T_w^{h}, \quad u, w \in T_h, v \in V_h, h \in \mathbf{H}
$$

\n
$$
X_{0w}^{v,h} = 1 \Rightarrow T_0^{v,h} + S_u^{h} + t_{0w}^{h} = T_w^{h}, \quad w \in P_h^{+}, v \in V_h, h \in \mathbf{H}
$$

\n
$$
X_{u,2n+1}^{v,h} = 1 \Rightarrow T_u^{h} + S_u^{h} + t_{u,2n+1}^{h} = T_{2n+1}^{h}, \quad u \in P_h^{-}, v \in V_h, h \in \mathbf{H}
$$

\n(A9)

$$
a_u^{n} \le T_u^n \le b_u^n, \quad u \in P_h, h \in \mathbf{H}
$$
\n(A10)

$$
a_0^h \le T_0^{\nu, h} \le b_0^h, \quad v \in V_h, h \in \mathbf{H}
$$
\n(A11)

$$
a_0 \leq I_0 \leq b_0, \quad v \in V_h, h \in \mathbf{H}
$$

\n
$$
a_{2n+1}^h \leq T_{2n+1}^{v,h} \leq b_{2n+1}^h, \quad v \in V_h, h \in \mathbf{H}
$$
\n(A12)

$$
a_{2n+1}^{h} \leq T_{2n+1}^{v,h} \leq b_{2n+1}^{h}, \quad v \in V_h, h \in \mathbf{H}
$$

\n
$$
X_{uw}^{v,h} = 1 \Rightarrow Y_u^h + d_w = Y_w^h, \quad u \in P_h, w \in P_h^+, v \in V_h, h \in \mathbf{H}
$$

\n
$$
X_{uw}^{v,h} = 1 \Rightarrow Y_w^h = 0, \quad u \in P_h, w \in P_h^-, v \in V_h, h \in \mathbf{H}
$$

\n(A14)

$$
X_{uw}^{\nu,h} = 1 \Longrightarrow Y_w^h = 0, \quad u \in P_h, w \in P_h^-, v \in V_h, h \in \mathbf{H}
$$
\n(A14)

$$
X_{uw}^{\nu,h} = 1 \Longrightarrow Y_w^h = 0, \quad u \in P_h, w \in P_h^-, v \in V_h, h \in \mathbf{H}
$$

\n
$$
X_{0w}^{\nu,h} = 1 \Longrightarrow Y_0^h + d_w = Y_w^h, \quad w \in P_h^+, v \in V_h, h \in \mathbf{H}
$$

\n(A15)

$$
X_{0w}^{n} = 1 \Rightarrow Y_0^n + d_w = Y_w^n, \quad w \in P_h^+, v \in V_h, h \in \mathbf{H}
$$

\n
$$
Y_0^h = 0, \quad 0 \le Y_0^n \le D, \quad u \in P_h^+, h \in \mathbf{H}
$$
\n(A16)

$$
Y_0^h = 0, \quad 0 \le Y_0^h \le D, \quad u \in P_h^+, h \in \mathbf{H}
$$

$$
\sum_{v \in V_h} \sum_{u \in \mathbf{Q}_h} \sum_{w \in \mathbf{Q}_h} c_{uw}^{v,h} \cdot X_{uw}^{v,h} \le B_C^{v,h}, \quad h \in \mathbf{H}
$$

(A17)

$$
\sum_{v \in V_h} \sum_{u \in \mathbf{Q}_h} \sum_{w \in \mathbf{Q}_h} c_{uw}^{v,h} \cdot X_{uw}^{v,h} \leq B_C^{v,h}, \quad h \in \mathbf{H}
$$
\n(A17)\n
$$
\sum_{v \in V_h} \sum_{u \in \mathbf{Q}_h} \sum_{w \in \mathbf{Q}_h} t_{uw}^h \cdot X_{uw}^{v,h} \leq B_T^{v,h}, \quad v \in V_h, h \in \mathbf{H}
$$

$$
\mathcal{L} \mathbf{v}_h \, u \in \mathbf{Q}_h \, w \in \mathbf{Q}_h
$$

$$
\sum_{\nu \in V_h} u \in \mathbf{Q}_h \, w \in \mathbf{Q}_h \tag{A19}
$$
\n
$$
X_{uv}^{\nu, h} \in (0, 1), \quad u, w \in \mathbf{Q}_h, v \in V_h, h \in H
$$

ID A node	B node	Facility	Distance	Travel Time	Avg Speed	Cost	
				(miles)	(minutes)	(MPH)	
3779	37	79	SR 39	6.03	13.13	27.56	166.19
7937	79	37	SR 39	6.03	13.13	27.56	166.19
7917	79	17	SR 39	5.5	11.98	27.56	151.58
1779	17	79	SR 39	5.5	11.98	27.56	151.58
6086	60	86	SR 57	6.36	6.50	58.75	373.65
8660	86	60	SR 57	6.36	6.50	58.75	373.65
8667	86	67	SR 57	4.77	4.87	58.75	280.24
6786	67	86	SR 57	4.77	4.87	58.75	280.24
4839	48	39	SR 55	12.27	15.68	46.96	576.20
3948	39	48	SR 55	12.27	15.68	46.96	576.20
6162	61	62	SR 22	5.29	6.71	47.28	250.11
6261	62	61	SR 22	5.29	6.71	47.28	250.11
6587	65	87	SR 261	4.6	4.30	64.20	295.32
8765	87	65	SR 261	4.6	4.30	64.20	295.32
8788	87	88	SR 261	2.53	2.36	64.20	162.43
8887	88	87	SR 261	2.53	2.36	64.20	162.43
8889	88	89	SR 261	4.07	3.80	64.20	261.29
8988	89	88	SR 261	4.07	3.80	64.20	261.29
7875	78	75	SR 241	5.65	5.21	65.09	367.76
7578	75	78	SR 241	5.65	5.21	65.09	367.76

Appendix B: Case Study Link Improvement