Title

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Wayfinding and restructuring in a novel city: an insight problem solving task

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Abstract

Navigating in a novel environment can serve as an applied insight problem solving task, since many people gain a sudden, clear understanding (Aha-moment) of the spatial relations after being lost. With a unique design, we transformed the city center of a medieval German city into a virtual maze. The aim of the study was to test whether a spatial decision making task simulating real navigation would be feasible for investigating insight problem solving. Participants learned two pathways which they subsequently had to restructure to find their way to the navigation targets. We found evidence for the restructuring of participants' prior knowledge during the solution attempts. 73% of all problem solvers reported an Aha-moment and there was an error drop at the critical intersection by those who had insight. The slope of the learning curve was established as a measurement of insightful experiences.

Keywords: navigation; insight; virtual reality; restructuring

Introduction

In wayfinding, some people perform well effortlessly, while other people struggle tremendously. For the ones that struggle, the navigation in a novel environment is an applied insight problem-solving task: although they possess the same information as their proficient peers, they still cannot find out how to establish an accurate mental representation of the spatial relations. The Representational Change Theory (RCT) (Ohlsson, 1984, 1992) explains how an abrupt realization about the right path may occur. Initially, navigators have a misleading or incomplete mental representation about the spatial relations, which is misleading and thereby obstructs their solution attempts (i.e. finding the navigation target). After taking detours, people feel stuck at the problem and lost in space. The state of impasse may be resolved by cognitive changes, which evoke the sudden appearance of a fully crystallized solution idea. This 'Aha-moment' is an "abrupt change in mental perspective that leads one to the solution of an otherwise intractable problem" (Luo, Niki, & Phillips, 2004, p. 2013). Most of the problems may be dealt with using trial-and-error strategy as well; however, insightful solutions alleviate the solving process as they are elegant and sparse.

Critically, it must be understood how the cognitive change in the mental representation is achieved. Participants already possess the crucial information about the solution; nonetheless, the real difficulty lies in organizing the information in an efficient way, e.g. constructing an overarching map from separately learned routes. In the present study, an Aha-moment may occur while recognizing the spatial relationship of two trained routes and/or detecting a hidden rule for getting to the navigation targets. Experiencing an Aha-moment would mean that one has an understanding about the underlying structure, thus does not have to guess or contemplate about the answer anymore. If the participant has no or only minor insight, we predict the task to be rated subjectively difficult and a performance with high error rate. Whilst if participants reach a higher stage of understanding, their performance would peak, i.e. they would commit hardly any errors and they would rate the task being easy. Thus we expect participants' error rates to drop to (almost) zero after an Aha-moment and also to have significantly lower reaction times in selecting the proper navigation direction than the non-Aha group.

There are multiple ways of organizing the available information in a more advantageous form. One of them is restructuring, which helps to "see the problem in a new light" (Jung-Beeman, Bowden, Haberman, Frymiare, Arambel-Liu, Greenblatt, & Kounios, 2004, p. e97). Examples of restructuring include directing the attention to different knowledge segments as before, organizing the information in a different order, or stumbling upon new relations over the already established ones (Knoblich, Ohlsson, Haider, & Rhenius, 1999; Knoblich, Ohlsson, & Raney, 2001; Ohlsson, 1992). Applying the RCT to the spatial navigation domain, an Aha-moment may also occur if someone comes to an intersection that they know already from another angle. When they restructure and possibly extend their representation about the place, they realize that the two partial representations match and are fractions of the same view. This surprise can cause the reorganization of the spatial exploratory behavior.

In spatial exploratory behavior, the way how people mentally represent the external world shapes how efficient they can find their way and what kind of knowledge they form. They can memorize an algorithm of turns or landmarks to orient themselves, sometimes even establish a cognitive map filled with precise distance and direction estimations. Since pioneering work on cognitive maps (Tolman, 1948), which is an understanding of the spatial structure of the environment accessible from multiple orientations, a vast body of literature has been written on how spatial information is stored in the mind and what the neural background of navigation proficiency is. Navigation can be defined as coordinated and goal-directed movement through the environment, which consists of wayfinding and locomotion (Montello, 2005). In the present study, we controlled for locomotion in order to test wayfinding-related behavior and determine how it can be transformed into an insight task.

Ohlsson (1992) defined an insight task as a problem which provokes the essential representation for solving it with a low probability. It is really hard to determine the threshold of low probability, since there are massive individual differences with regard to the perception and representation of a given problem. There have been a few attempts of dividing insight from non-insight problems (Gilhooly, & Murphy, 2005; Weisberg, 1995). This process moved towards spotting the necessary demarcation line; still, some "insight problems" are only justified to be of that kind by the personal judgment of the researcher using them (Bowden, & Jung-Beeman, 2007). Determining whether a problem can be considered as an insight task depends on both the researcher and the problem solver. As a rule of thumb, one must always ask for some kind of self-report about the actual Aha-moment, since an Aha-experience has no behavioral marker (Danek, Fraps, von Müller, Grothe, & Öllinger, 2014). Luo & Knoblich (2007) defined several requirements for credible insight neuroimaging studies. Ideally, a good insight paradigm elicits restructuring and enables manipulations for hypothesis testing, the insight events can be iterated and are precisely time locked, and appropriate reference states can be defined to contrast the baseline and the target state, along with being able to test both internally and externally triggered insights. A paradigm which would satisfy all of these criteria is yet to be established.

To the best of our knowledge, no paper has been previously published about using human mazes as insight problems. We wanted to model how complex mental representations about spatial information are developed and organized on a higher cognitive level. A new environment, regardless of being an artificial maze or an unknown city, qualifies well as an insight problem solving task: depending on its complexity, the likelihood to reach the goal state during navigation may be quite low. Limiting free exploration and determining which parts of the surroundings become known by the navigators are likely to trigger a state of impasse. There is a need to restructure their mental map and/or realize a hidden rule in turns at the decision points to perform the task successfully. We created an ill-defined problem where there are discoverable rules with varying degrees of difficulty. Solving such a task informs us about the interplay between navigational strategy use and the restructuring of spatial information. By extending and re-interpreting the initially learned routes, one can see the relations in a new way and navigators can find an optimal solution to the task.

Methods

Participants

28 individuals living in Munich, Germany participated for a monetary reward (25€). Participants were aged between 21 and 27 years (M=24.66, SD=1.49, 12 females) with normal or corrected-to-normal visual acuity. The first 2 participants were excluded due to a change in the experimental paradigm. This resulted in an n=26 final sample size. All subjects were healthy and naive to the purpose of the experiment, 24 of them were right-handed. The study was approved by the Ethics Committee of the Ludwig-Maximilians University and conducted in accordance with the principles described in the Declaration of Helsinki. All subjects gave their informed written consent prior to the study.

The virtual environment

A photorealistic replica of the medieval city center of Tuebingen, Germany, a place not known by our participants, was used as a maze. We modeled everyday navigation as naturally as possible. Although artificial mazes are more controllable, wayfinding in a real-world city has a larger ecological validity (e.g. Spiers & Maguire, 2006). Nevertheless, in a complex, realistic environment, spatial learning remains poorer in the virtual as compared to a real, open-air setting (Richardson et al., 1999).

The virtual reality (VR) environment was created previously by Tobias Meilinger (Meilinger, Franz, & Bülthoff, 2008) with virtual reality modeling language. In the present study, it was implemented using Vizard 3.0 (Vizard Virtual Reality Software Toolkit, Worldviz, USA). The VR environment depicted the city center to the smallest details: streets, houses and intersections as well as shop signs, flower beds and the graffiti tags on the walls were all rendered into the model. Two main routes were configured (see Figure 1).

Participants navigated on ground-level from route perspective (Tversky, 1991) with a first person view. However, we restricted free navigation (1) to avoid spatial anxiety induced by getting lost, (2) because time boundaries are needed for designing a functional magnetic resonance imaging (fMRI) paradigm and (3) to prevent participants from wandering without orientation cues or reaching familiar locations. Thus a "semi-free navigation" procedure was applied, which reduced the navigation task to its most essential element: choosing how to turn at the intersections. Navigators traversed passively on pre-recorded route segments and actively had to make a choice at each turning point. We did not let anyone get lost; participants were forced to move on a pre-assigned "optimal route". If they did not select this optimal route, they did not move, but had to try again to make a better decision. Therefore, error rates and reaction times in the decision have been analyzed.

Since an actual city is only quasi-controllable as a maze, we specified the segregated routes according to the following criteria: (1) the two basic routes must be as parallel as possible (2) the connections between them ("bridges") should only be 1-2 intersections away (3) a hidden rule must be fulfilled, i.e. participants should be able to recognize that their movement pattern on the optimal route is fixed, they are always traversing with a zigzag-like pattern from one route to the other.

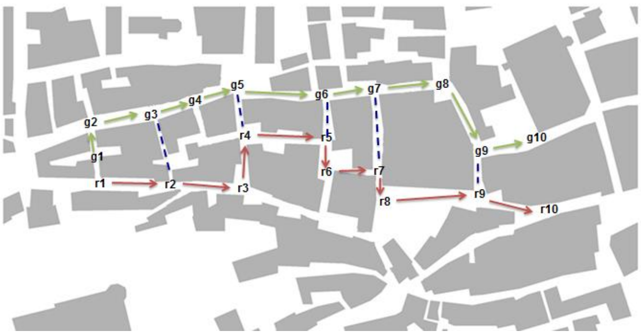

Figure 1 – structure of the maze; g and r stand for the respective stops of the two basic routes (green and red routes) and blue dashed lines sign the bridges.

The basic routes are not perfectly straight but they fulfill the 3 aforementioned criteria. 10 bridges were established (five connections, approached from two directions).

During the main experiment, an optimal navigation route was followed, consisting of a fixed pattern of four consecutive intersections. Thus between the starting point and the navigation target, a recurring pattern of task-related activities were carried out. The first intersection was already known by the observers. Therefore, choosing how to proceed was a simple recall: they have already seen the same location from the same angle during the training. Subsequently, they traveled this one stop on the previously learned route and arrived to the second intersection. The second one was the critical intersection where we expected the Aha-moment, since participants here always had to alternate from the path and look for a novel passage. From this passage, a "bridge" opened between the green and the red routes, thus after this stop, participants arrived to a known yet unknown place – to a location which they have already learned but have only seen from a different angle before. Hence they could only realize, if it even occurred to them, during or after arriving to the final intersection (which is also an already known location) that they were in fact coming from a stop of an already known route.

Apparatus

The present experiment is part of a bigger study investigating both navigation strategies and insightful moments linked to mental restructuring. Although we recorded the data in an fMRI scanner, we only expected to observe the neural correlates of spatial navigation strategies, memory recollection and the processing of contradicting information. There is no agreement yet on the neural correlate of insight (see Dietrich, & Kanso, 2010 for a review), therefore we did not hypothesized about neural correlates of insight at this stage of developing a new paradigm. We relied on verbal reports to verify that the task elicits Aha-moments and used neuroimaging to investigate spatial orienting and decision making behavior. From this perspective, the study is a behavioral experiment recorded in supine position.

Due to length restrictions, we only report the results regarding our insight problem solving paradigm and omit the technical details of the fMRI equipment. Visual stimuli were projected via a LCD display projector (Christie Digital Systems, Germany) and responses were collected by 2 two-button Lumina Response Pads (MR-compatible LU400-PAIR by Cedrus Corporation, USA). System times, as well as the selected path, reaction times and error numbers from the answers were recorded in a virtual log file.

Procedure

The procedure followed a within-subject design. Trials were randomized to control for sequence effects. Before attempting any of the tasks, subjects had to rate themselves on the Way-Finding Strategy Scale and the Spatial Anxiety Scale, both constructed by Lawton (1994).

Training Two basic routes were trained. Participants were explicitly instructed to establish a mental map of the environment. The routes did not cross or intersect. Each route consisted of 10 intersections, i.e. subjects had to make decisions how to turn 8 times when walking along the path. Each route was learned separately. First, the subjects were passively guided throughout the relevant part of the environment once. Locomotion was constant (running speed). They had to observe the path carefully in order to reproduce the steps themselves afterwards. During the reproduction phase, they were asked at each intersection which direction they need to go to continue along the route. It was possible to turn left, right or go straight, but not backwards. If they selected the right direction, they were passively moved to the next intersection. If they did not select the correct turn, they received a short feedback message stating "It's not the optimal route!" and they could try again. They went through both routes until making no errors.

Main experiment Participants had to search for 10 objects (non-native animals in the city) in the VR environment while lying in supine position. The arbitrarily selected animals were free online models loaded into the virtual city; they floated above the streets and revolved around their own vertical axis to enhance their saliency (Figure 2). Subjects had to collect each animal 3 times across 30 trials (20+10), which were separated by an anatomical scan. They were navigating via responding to direction questions appearing on the screen. Each presentation of an intersection question was jittered between 1 and 2.25 s with a step of 0.25 s.

Figure 2 - A typical navigation target: an animal (giraffe) floating above the street, turning around its own center.

Participants started out at any intersection along the learned routes and were asked to find an animal using the optimal route. Unlike in the training, participants have not seen what the optimal route is but they had to figure out by themselves. The animal always appeared at the end of the route, at a stable position. We were interested in (1) how many attempts participants needed to find the bridges between the pre-defined routes and (2) whether they would realize the hidden rule of the pathway. To obtain an indication about the participants' current orientation, each trial began with two questions: "Do you know where you are?" and "Do you know which direction you would go to continue along the learned route?". We assumed that the answers may differ to these two questions, and also expected to get more yes answers with increasing the familiarity with the city.

Following these two questions, participants attempted locating the animal announced on the screen by choosing a direction at each intersection. Through trial-and-error, they learned the location of each animal. After having known the animal locations, we provided a limited number of choices in finding the correct direction to avoid guessing. If navigators made more than one error per intersection, they were informed on the screen they have exceeded the error limit and had to repeat the trial. The repetition aided the consolidation of the correct route, as well as it deterred from guessing because of the tediousness of repetitions. Participants could not see the animal earlier than their last movement decision. Finally, they had to respond to the exact same two questions as at the beginning of the trial in order to register whether they noticed that they switched to the other route. Once the desired object was found, they proceeded to the next trial and were teleported to a new location. Circa 50 minutes were spent in the scanner.

After participants finished the experiment, we interviewed them shortly to learn more about their insight experience and the navigation strategies they used. We examined their route knowledge by asking to describe the spatial relations between the pre-learned routes, as well as to draw the routes on an outline map of the city center.

Results

Evaluation of the paradigm

First, we looked at the subjective difficulty of our paradigm. Participants rated the task to be rather easy (M=4.87, SD=2.26 on a 10-point Likert scale, where 1 stands for extremely easy and 10 for impossible). A Pearson-correlation between the average numbers of errors committed at intersections (M=0.68, SD=0.32) and the self-report difficulty ratings were not related significantly (r=.34, ns.).

Insight

To see whether participants had a sudden, explicit realization of any hidden rule, we looked at their reaction times and error rates by each trial repetition. In addition, we asked people whether they had any Aha-moment during the completion of the task, and prompted to freely recall what they realized. In a different question, we asked them to indicate if they identified any hidden rules.

We split our participants into two groups: one group consisted of the people who experienced any kind of insights (n=19) and the other contained those who did not (n =7). Then we looked at the differences in reaction times and error rates of the two groups.

Reaction times After the outlier correction (n=5; cut-off value=20s), a 3 (trial repetition) x 2 (insight) one-way repeated-measures ANOVA was conducted. It revealed a main effect of repetition [$F(2,46)=63.2$, $p<.001$, $\eta^2_{partial}=.733$], as there was a large drop in the reaction times (shown in Figure 3). It is a general learning effect, since the difference between those who declared to have insight (n=19) and those who did not report any Aha-moment (n=7) was non-significant [$F(1,23)=.052$, ns.].

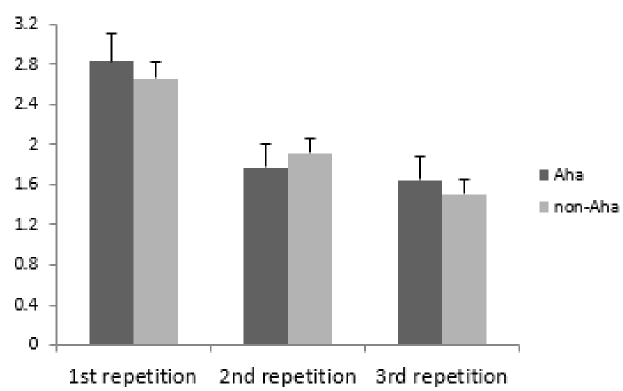

Figure 3 - average reaction times (in seconds) at each repetition of a trial. Error bars express the standard error of the mean. *Aha: participants who have experienced Aha-moment(s); non-Aha: participants who have not reported any Aha-moment.*

Error rates A 3 (repetition) x 3 (intersection) repeated-measure ANOVA with a between-subjects factor (reporting

Aha-moment) was conducted to assess the overall effect of the experimental manipulations in the error rates. The values are Greenhouse-Geisser corrected. We obtained main effects by the factor repetition [$F(1.95, 445.6) = 27.66$, $p < .001$, $\eta^2_{partial} = .108$] and intersection [$F(1.9, 439.1) = 36.84$, $p < .001$, η2partial = .139], as well as by the interaction of repetition and intersection [$F(3.4, 779) = 16.51$, $p < .001$, η2partial = .067], thus the error rates have indeed dropped significantly (see Figure 4). What is more, experiencing insight yielded a tendency for showing significant difference $F(1,229)=3.6$, $p=0.59$, $\eta^2_{partial}=.015$].

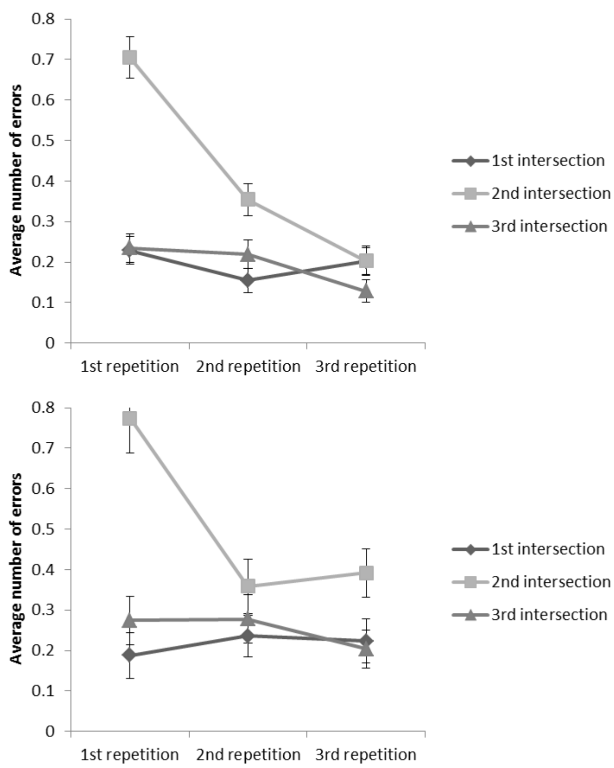

Figure 4 – average error rates by each intersection at each repetition of a trial. The upper panel illustrates participants who have experienced an Aha-moment, while the lower panel shows data of those who have not reported any Aha-moment. Error bars depict the standard error of the mean.

A significant difference was found in the error rates between the first and the second repetition [$t(24)=7.32$, $p<.001$, CI 95% [0.15, 0.26]], likewise between the first and the third repetition [$t(23)=8.1$, $p<.001$, CI 95% [0.19, 0.33]]. Besides, significant differences were observed among all of the intersections [$t_{1\text{-}2}(24)=-11.9$, $p<.001$, CI 95% [-0.64, -0.45]; $t_{1\text{-}3}(24)=-10.8$, $p<.001$, CI 95% [-0.82, -0.56]; $t_{2\text{-}3}(24)=-3.8$, $p<.001$, CI 95% [-0.23, -0.07]].

The second intersection was the critical one for the insight. An independent sample T-test showed a significant difference between the Aha and non-Aha third repetition errors at this intersection [$t(88.4)=2.47$, $p=.016$, Cohen's $d=0.53$, CI 95% [0.03, 0.34]].

Discussion

The aim of this study was to test whether a spatial decision making task simulating real navigation would be feasible for an insight problem solving paradigm. We designed a navigation task in a complex, naturalistic VR environment and manipulated participants' understanding about the spatial relations. They were encouraged to restructure the information available to them, e.g. convert the route perspective into survey perspective, and thereby experience an insight while getting to the navigation target. Participants had to colligate segments of two routes into a new pathway by discovering connections between them.

More than 70% of our participants reported to have experienced insight during the course of the experiment. Although we have not obtained a significant difference in the reaction times between the two groups (people with and without an Aha-moment), we have found a difference in error rates in favor of the Aha-group. To avoid a floor effect in task difficulty, we could increase the complexity of our task. Enhancing the working memory load by introducing more target objects or prolonging the optimal route with an extra intersection would require more chance to get disorientated while building the corresponding mental map.

In each trial, the second intersection was the stop where participants had to leave the learned pathway and look for a new direction in order to find the navigation target. There has been a drop in error rates from the first to the second repetition of the trial due to general learning effects. However, by those who declared an Aha-moment, a further decrease was noted in the error rates: the average number of errors was reduced to zero or very close to zero. Moreover, the error rates have increased from the second to the third repetition by the group whose members have not restructured their knowledge.

Notably, a disadvantage of the current design is that participants cannot report their Aha-moments already in the scanner. Any indication incorporated to the paradigm (e.g. a button press) would only give an approximate measure, since such reports are always delayed. However, we are searching for tools to correct this delay. Here, we relied on self-reports instead of any kind of online measurements to discern the insightful incidents. Although Jäkel & Schreiber (2013) argue for the importance of introspection data in cognitive processes, we are not sure how much we can trust the acquired self-reports. An online and direct measurement of "Aha" would be desirable but hard to establish.

Since there are only very few problems which are exclusively solvable with insight, if participants do not report their experience, insightful and non-insightful solutions get mixed together, which reduces the power of the effects (Bowden, & Jung-Beeman, 2007). We hypothesize that if the groups with and without Aha-moments would not be determined on a participant but on a trial basis, the effects would get even more pronounced. This is why we are developing the paradigm further to fulfill all of the criteria outlined by Luo & Knoblich (2007). In terms of restructuring, which was in the focus of our study, the task proved to be successful. It was absolutely necessary to build new relations (the bridges) between the elements of the problem, to re-interpret the learned segments of the route and to assemble novel pathways from them (Ohlsson, 1992). The re-combination of the parts gave meaning to a new "whole" (Wertheimer, 1959).

During the task, it was possible to experience multiple insight events, but the number of Aha-moments is not specified. The controlling for accurate onset times was the weakest aspect of the paradigm. We could not secure the occurrence of an insight.

In terms of hypothesis testing, we are confident that our task can be modified in numerous ways to enable a more fine-grained analysis of the involved brain regions as well as to experimentally manipulate various factors and thereby inform the underlying theories. What is more, when it comes to the problem of reference states, it is favorable that our task can be solved both with and without insight. If we could segregate the insightful events accurately, we could easily contrast them with the non-insightful solutions and sort out the arising differences.

The paradigm models a natural way of obtaining insight. Thus, it does well in internally provoking Aha-moments. Nevertheless, in order to control for the exact onset time of such a moment, we could complement our design with external solution cues pointing to the relevant part of recognition participants should make to achieve a deeper understanding of the task.

All in all, in the present study the research fields of spatial navigation and problem solving were connected to demonstrate that simulating navigation behavior in a virtual environment is an appropriate scanner task for investigating insight problem solving. Inspecting wayfinding in a novel city leads to both a more profound knowledge about the interplay of spatial navigation strategies and their mental representation and can contribute to the quest for the neural correlate of insight.

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References

Bowden, E. M., & Jung-Beeman, M. (2007). Methods for investigating the neural components of insight. *Methods, 42*(1), 87-99.

Danek, A. H., Fraps, T., von Müller, A., Grothe, B., & Öllinger, M. (2014). It's a kind of magic—what self-reports can reveal about the phenomenology of insight problem solving. *Frontiers in psychology, 5*.

Dietrich, A., & Kanso, R. (2010). A review of EEG, ERP, and neuroimaging studies of creativity and insight. *Psychological bulletin, 136*(5), 822.

Gilhooly, K. J., & Murphy, P. (2005). Differentiating insight from non-insight problems. *Thinking & Reasoning, 11*(3), 279-302.

Jäkel, F., & Schreiber, C. (2013). Introspection in Problem Solving. *The Journal of Problem Solving, 6*(1), 4.

Jung-Beeman, M., Bowden, E. M., Haberman, J., Frymiare, J. L., Arambel-Liu, S., Greenblatt, R., & Kounios, J. (2004). Neural activity when people solve verbal problems with insight. *PLoS biology, 2*(4), E97.

Knoblich, G., Ohlsson, S., Haider, H., & Rhenius, D. (1999). Constraint relaxation and chunk decomposition in insight problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 25*(6), 1534.

Knoblich, G., Ohlsson, S., & Raney, G. E. (2001). An eye movement study of insight problem solving. *Memory & Cognition, 29*(7), 1000-1009.

Lawton, C. A. (1994). Gender differences in way-finding strategies: Relationship to spatial ability and spatial anxiety. *Sex roles, 30*(11-12), 765-779.

Luo, J., & Knoblich, G. (2007). Studying insight problem solving with neuroscientific methods. *Methods, 42*(1), 77-86.

Luo, J., Niki, K., & Phillips, S. (2004). Neural correlates of the 'Aha! reaction'. *Neuroreport, 15*(13), 2013-2017.

Meilinger, T., Knauff, M., & Bülthoff, H. H. (2008). Working memory in wayfinding—A dual task experiment in a virtual city. *Cognitive Science, 32*(4), 755-770.

Montello, D. R. (2005). Navigation. In P. Shah & A. Miyake (Eds.) *The Cambridge handbook of visuospatial thinking*. Cambridge: Cambridge University Press.

Ohlsson, S. (1984). Restructuring revisited: II. An information processing theory of restructuring and insight. *Scandinavian Journal of Psychology, 25*, 117–129.

Ohlsson, S. (1992). Information-processing explanations of insight and related phenomena. In M. Keane & K. J. Gilhooly (Eds.) *Advances in the psychology of thinking*. London: Harvester-Wheatsheaf.

Richardson, A. E., Montello, D. R., & Hegarty, M. (1999). Spatial knowledge acquisition from maps and from navigation in real and virtual environments. *Memory & cognition, 27*(4), 741-750.

Spiers, H. J., & Maguire, E. A. (2006). Thoughts, behaviour, and brain dynamics during navigation in the real world. *Neuroimage, 31*(4), 1826-1840.

Tolman, E. C. (1948). Cognitive maps in rats and men. *Psychological Review, 55*(4), 189-208.

Tversky, B. (1991). Spatial mental models. *Psychology of Learning and Motivation*, 27, 109-145.

Weisberg, D. S., Keil, F. C., Goodstein, J., Rawson, E., & Gray, J. R. (2008). The seductive allure of neuroscience explanations. *Journal of Cognitive Neuroscience, 20*(3), 470-477.

Wertheimer, M. (1959). *Productive thinking.* M. Wertheimer (Ed.). New York: Harper & Row.