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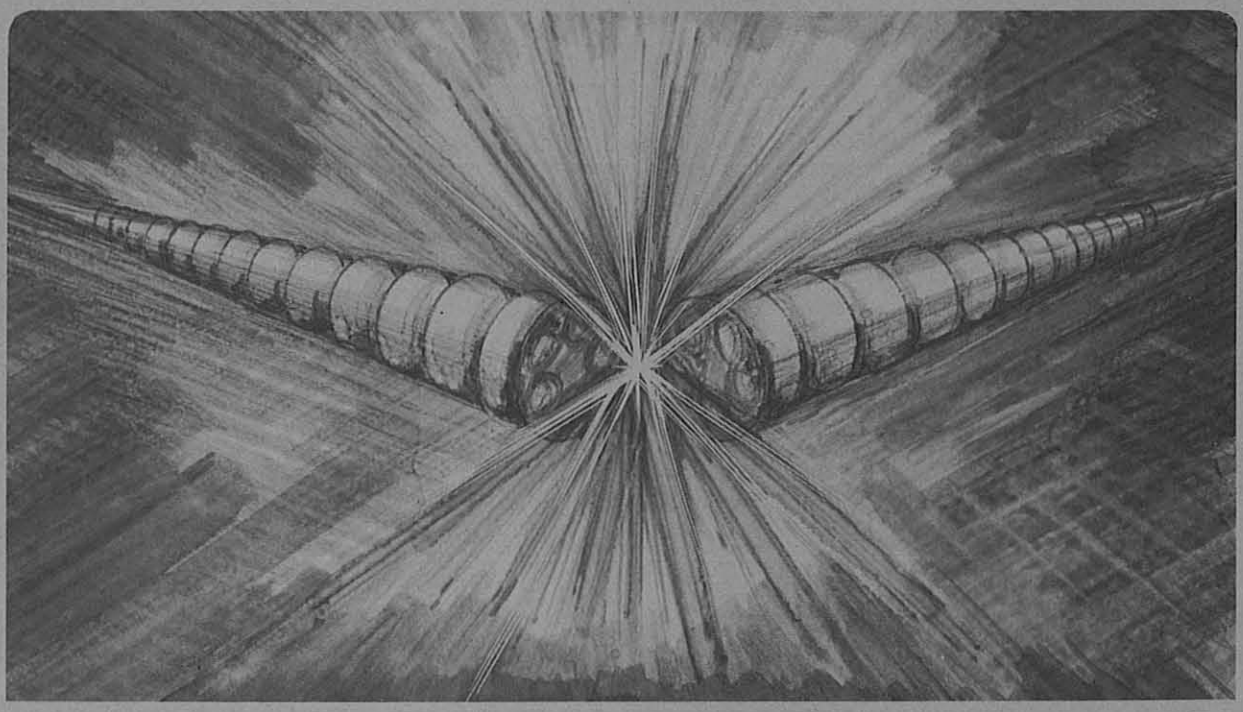
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Hourglass Effects for Asymmetric Colliders *

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Abstract

We give the expressions for the geometrical reduction factor of the luminosity and the geometrical beam-beam "aggravating factor" for the general asymmetric case, for tri-gaussian bunches colliding head-on. With these formulas we attempt a (limited) analytic understanding of the multiparticle tracking simulations carried out for the proposed SLAC/LBL/LLNL B factory [1] when parasitic crossings are ignored. We conclude the following: (a) the geometrical reduction in luminosity is $\sim$ 6% relative to the zero-bunch-length (nominal) value; (b) only the vertical beam-beam parameter of the LER is significantly altered by the hourglass effect: the geometrical enhancement of the central positron's vertical beam-beam parameter is $\sim$ 10% relative to the nominal value, and (c) the positrons at the head or tail of the bunch have vertical beam-beam parameters much larger than nominal. We discuss the electromagnetic disruption effect only qualitatively. This effect probably compensates (or overcompensates) the geometrical reduction of the luminosity, and it is possibly detrimental for the beam-beam parameters. This article summarizes Ref. [2].

I. Introduction

Although proposed B factories [1] call for designs that are asymmetric in energy, beam current and emittances, they also invoke to a greater or lesser degree a "transparency condition" by virtue of which the beam sizes are pairwise equal [3]. Because of the beam-beam interaction, however, the beams become different in size at least to some degree. Expressions available in the literature [4, 5, 6] for the hourglass factors for the luminosity and beam-beam parameters assume some sort of equality among the beam sizes or lattice functions. In this note we provide generalizations that are applicable to the most general asymmetric case, when the four beta-functions and the six rms beam sizes are arbitrary. With these formulas we attempt a (limited) analytic understanding of the multiparticle tracking results for the proposed SLAC/LBL/LLNL B factory [1] when parasitic collisions are ignored.

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II. Luminosity

Consider two bunches moving in equal and opposite directions with speed c, with tri-gaussian particle distributions, such that the centers collide at the optical interaction point (IP, $s = 0$) with no displacement. We assume that the interaction region is a dispersionless drift section and that the IP is a symmetry point of the lattice. Then the transverse rms sizes σ_{x+}, σ_{y+}, σ_{x-} and σ_{y-} have an s dependence of the form $\sigma^2 = \sigma^{*2} \times (1 + s^2/\beta^{*2})$. The hourglass reduction factor for the luminosity is [2]

$$R(t_x, t_y) \equiv \frac{\mathcal{L}}{\mathcal{L}_0} = \int_{-\infty}^{\infty} \frac{dt}{\sqrt{\pi}} \frac{\exp(-t^2)}{\sqrt{(1 + t^2/t_x^2)(1 + t^2/t_y^2)}} \qquad (1)$$

where $\mathcal{L}$ is the actual luminosity and $\mathcal{L}_0$ is the luminosity in the zero-bunch-length limit,

$$\mathcal{L}_0 = \frac{f_b N_+ N_-}{2\pi\sqrt{(\sigma_{x+}^{*2} + \sigma_{x-}^{*2})(\sigma_{y+}^{*2} + \sigma_{y-}^{*2})}} \qquad (2)$$

where f_b is the bunch collision frequency, and t_x is defined by

$$t_x^2 = \frac{2(\sigma_{x+}^{*2} + \sigma_{x-}^{*2})}{(\sigma_{s+}^2 + \sigma_{s-}^2)\left(\sigma_{x+}^{*2}/\beta_{x+}^{*2} + \sigma_{x-}^{*2}/\beta_{x-}^{*2}\right)} \qquad (3)$$

with a corresponding expression for t_y. The superscript $*$ refers to the IP and $\sigma_{s\pm}$ are the rms bunch lengths. We exhibit $R(t_x, t_y)$ in Fig. 1.

It is easy to see from Eq. (1) that R is always < 1, except that $R(\infty, \infty) = 1$, as it should.

If both beams are flat, with $\sigma_{x\pm} \gg \sigma_{y\pm}$, then $t_x \gg 1$ and (1) can be expressed in terms of a modified Bessel function. This result is of similar form to that of Ivanov *et al.* [5] but is of more general validity because it does not assume $\beta_{y+}^* = \beta_{y-}^*$ or $\sigma_{s+} = \sigma_{s-}$ or $\sigma_{y+}^* = \sigma_{y-}^*$.

If the beams are such that $t_x = t_y$ (which may happen naturally in a round-beam design), then (1) can be expressed in terms of the complementary error function.

III. Beam-Beam Parameters

We focus on a single particle, say a positron, as it passes through the opposing electron bunch. In a first-order calculation we can assume that the particle follows a straight line trajectory with constant speed c. We assume that this particle is close to the collision axis and is displaced longitudinally by a finite distance z from the center of its

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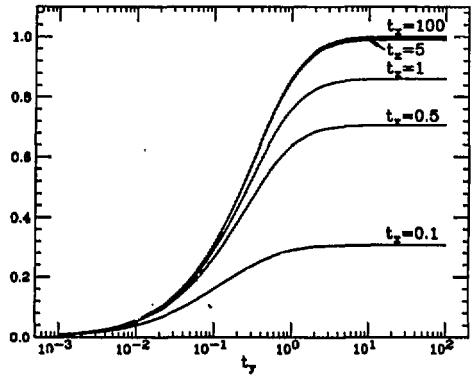


Figure 1: The luminosity reduction factor, Eq. (1), plotted *vs.* t_y for various values of t_x.

own bunch. We assume that z is constant during the collision process (or during several successive collisions), which is a very good approximation in practice. Then the "aggravating factor" [4] for the positron's vertical beam-beam parameter is [2],

$$R_{y+}(z) \equiv \xi_{y+}(z)/\xi_{0y+} =$$

$$\int_{-\infty}^{\infty} \frac{dt}{\sqrt{\pi}} \frac{(1+t^2/t_1^2)\exp(-(t-t_0)^2)}{\sqrt{1+t^2/t_2^2}\left(v\sqrt{1+t^2/t_2^2}+h\sqrt{1+t^2/t_3^2}\right)} \quad (4)$$

where $h = \sigma_{x-}^*/(\sigma_{x-}^* + \sigma_{y-}^*)$, $v = \sigma_{y-}^*/(\sigma_{x-}^* + \sigma_{y-}^*)$, $t_0 = z/\sqrt{2}\sigma_{s-}$, $t_1 = \sqrt{2}\beta_{y+}^*/\sigma_{s-}$, $t_2 = \sqrt{2}\beta_{y-}^*/\sigma_{s-}$ and $t_3 = \sqrt{2}\beta_{x-}^*/\sigma_{s-}$. The nominal (zero-bunch-length) vertical beam-beam parameter ξ_{0y+} of the central positron is

$$\xi_{0y+} = \frac{r_0 N_- \beta_{y+}^*}{2\pi\gamma_+\sigma_{y-}^*(\sigma_{x-}^* + \sigma_{y-}^*)} \quad (5)$$

where r_0 is the classical electron radius and γ_+ is the usual relativistic factor. The expressions for the remaining three beam-beam parameters ξ_{x+}, ξ_{x-} and ξ_{y-} are obtained from Eqs. (4) and (5) by the substitutions $x \leftrightarrow y$ and/or $+ \leftrightarrow -$ in h, v and the t_i's.

It should be noted that the aggravating factors can be > 1 or < 1, as opposed to the luminosity reduction factor, which is always < 1. However, if $\beta_{x+}^* = \beta_{y+}^* = \beta_{x-}^* = \beta_{y-}^*$ we obtain

$$R_{x+}(z) = R_{y+}(z) = R_{x-}(z) = R_{y-}(z) = 1 \quad (6)$$

for all z regardless of the beam sizes. This allows, in principle, for the possibility of designing the lattice so that there is no hourglass effect on the beam-beam parameters.

If the beams are flat such that $\sigma_{x+} \gg \sigma_{y+}$, $\sigma_{x-} \gg \sigma_{y-}$, $\beta_{x+}^* \gg \beta_{y+}^*$ and $\beta_{x-}^* \gg \beta_{y-}^*$, Eq. (4) yields, for the central particle, $R_{x+}(0) \simeq R_{x-}(0) \simeq 1$, and

$$R_{y+}(0) \simeq \frac{t_2}{2\sqrt{\pi}} e^{t_2^2/2}\left[(2-\rho)K_0(t_2^2/2) + \rho K_1(t_2^2/2)\right] \quad (7)$$

where $\rho \equiv (t_2/t_1)^2$ and K_0, K_1 are Bessel functions. $R_{y-}(0)$ is obtained from (7) by exchanging $t_1 \leftrightarrow t_2$.

If $t_2 = t_3$ for both beams (such as for round beams), then $\beta_{x+}^* = \beta_{y+}^*$ and $\beta_{x-}^* = \beta_{y-}^*$; we allow, however, for the possibility that $\beta_{x+}^* \neq \beta_{x-}^*$ and we assume nothing about the six rms beam sizes. Then we find that

$$R_{x+}(0) = R_{y+}(0) = \rho + (1-\rho)\sqrt{\pi}\, t_2 \exp(t_2^2)\,\mathrm{erfc}(t_2) \quad (8)$$

where $\mathrm{erfc}(x)$ is the complementary error function. $R_{x-}(0) = R_{y-}(0)$ is obtained by exchanging $t_1 \leftrightarrow t_2$.

For particles away from the bunch center, Eq. (4) implies that $\xi(z) = \xi(-z)$ for each of the four beam-beam parameters. This means that the particles at the head and the tail of the bunch suffer the same beam-beam tune shift. This property follows from the assumed lattice symmetry about the IP and the assumed lack of bunch disruption.

The aggravating factors saturate [2] to a limit when $z \to \infty$. In practice this limit applies to particles with $|z| \gg \beta^*$, where β^* is here any of the four beta-functions at the IP; therefore this limit may or may not be sensibly reached in specific machine designs. Furthermore, this property follows from a first-order calculation; it may not hold in higher orders if $\xi(\infty)$ is large.

IV. The SLAC/LBL/LLNL B Factory

For nominal parameters of the APIARY 6.3-D design for the proposed SLAC/LBL/LLNL B factory [1] we obtain $t_x = 47.43$ and $t_y = 1.897$, so that $R = 0.945$. This implies that the luminosity is 5.5% smaller than the zero-bunch-length estimate.

We also obtain $R_{y+}(0) = 1.093$, so that the vertical beam-beam parameter of the central positron is 9.3% larger than the nominal value. The other three aggravating factors are slightly smaller than unity [2]. For particles away from the center of the bunch, R_{y+} grows almost linearly with z. Fig. 2 shows ξ_{y+} as a function of the positron's longitudinal distance away from the center of the bunch. The remaining three aggravating factors deviate significantly from unity only for $z \gtrsim 10\sigma_s$. Fig. 3 shows all four aggravating factors as a function of the particle's distance away from the center of the bunch.

A qualitative estimate of the electromagnetic bunch disruption can be obtained from results for multiparticle simulations for single-pass, symmetric, beam collisions [7]. For flat bunches that are uniform in x and gaussian in y and s, one obtains, from Chen's empirical fit, that the disruption is $H_D = 0.998$ with an estimated accuracy of $\pm 10\%$, for nominal APIARY 6.3-D parameters ($A = 0.53$ and $D = 0.20$). Since H_D takes into account both the geometrical and the electromagnetic disruption effects, we conclude, to this accuracy, that the geometrical reduction in luminosity is compensated by the pinching effect. This

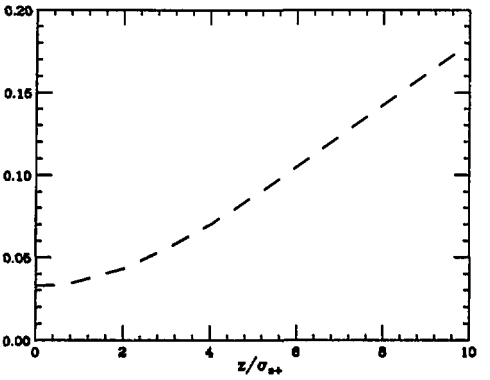


Figure 2: The vertical beam-beam parameter for the positron, $\xi_{y+}(z)$, for nominal APIARY 6.3-D parameters. The nominal value is $\xi_0 = 0.03$ for all four beam-beam parameters. The almost linear rise is explained in [2].

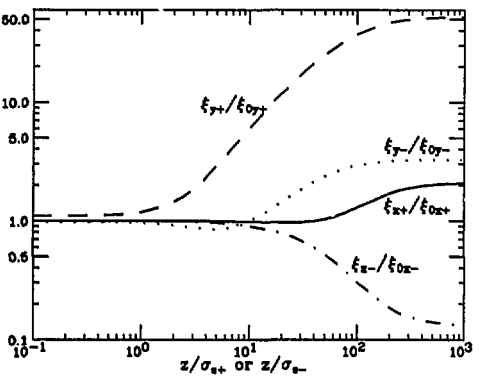


Figure 3: The aggravating factors for both beams for nominal APIARY 6.3-D parameters. The saturation property is seen at (unphysically) large values of z ($\sigma_{z\pm} = 1$ cm).

result is consistent with the multiparticle tracking simulation results for the SLAC/LBL/LLNL B factory when parasitic collisions are ignored [1]. One should keep in mind, however, that since Chen's results apply to single-pass collisions, a potentially important dependence on the tune of the machine may be missed in this interpretation.

V. Conclusions

As in the symmetric case, the luminosity reduction factor is a sensitive function of β^*/σ_z. Unlike the symmetric case, however, this factor depends explicitly on the transverse bunch sizes in addition to the bunch lengths and beta-functions.

A numerical application to the SLAC/LBL/LLNL B factory shows a 5.5% geometrical reduction of the luminosity and a 9.3% geometrical enhancement of the central positron's ξ_{y+} relative to the nominal values.

Positrons with $x \simeq y \simeq 0$ at the head or tail of the bunch have higher ξ_{y+} than the central positron due to the fact that they sample, on average, a higher β^*_{y+} during the collision process ($\beta^*_{y+} = 1.5$ cm is the smallest of the four β^*'s). Positrons with $|z| > 6\sigma_z$ have $\xi_{y+} > 0.1$; this number can be made smaller, however, by a modest increase in β^*_{y+} [2].

We estimate the electromagnetic pinching effect to be small, since it modifies the results of the geometrical calculations by $\sim \pm 10\%$. It is probably beneficial for the luminosity, and it is probably detrimental for the beam-beam parameters.

VI. Acknowledgments

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VII. References

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