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Abstract*

Conservation knowledge and measurement abilities are two central components in quantitative development. Piaget's position is that conservation is a logical pre-requisite of measurement, while Miller's is the reverse. In this paper we illustrate how measurement is employed as an empirical tool in the construction of conservation knowledge. This account predicts the familiar pattern of conservation development from the limits on young children's measurement abilities. We present Q-Soar, a computational model that acquires number conservation knowledge by simulating children's performance in a published conservation training study. This model shows that measurement enables a verification process to be executed which is the basis of conservation learning.

Introduction

Two central conceptual attainments in the development of quantification abilities are conservation knowledge (understanding the behavior of quantities under transformation) and measurement skills (creating quantitative values for bodies of material). Yet, despite the centrality of these two aspects of quantification, relatively little attention has been paid to the developmental roles that they play. Inspection of the literature reveals two incommensurate positions. The view held by Piaget (Piaget, Inhelder and Szeminska, 1960) was that conservation is a logical pre-requisite to the ability to measure. He reasoned that, without an understanding of the essential nature of quantity, measurements in terms of those quantities would mean nothing and would be of no practical use. The opposing view is that measurement is the necessary precursor of conservation (Klahr & Wallace, 1976; Miller, 1984). Measurement is the empirical tool used to gather information about whether or not some dimension of a transformed entity has remained quantitatively invariant. Miller (1984) states that "practical measurement procedures appear not to be late-developing concomitants of a more general understanding of quantity. Instead, the measurement procedures of children embody their most sophisticated understanding of the domain in question. The limitations of these procedures constitute significant limits on children's understanding of quantity ... (p.221)".

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Such measurement is not always possible though. The limitations Miller speaks of determine what children can learn about quantity. They are responsible for the pattern in the development of conservation. Number, or discrete quantity, conservation is acquired first. Also, preconservers can reason successfully about transformations of small discrete quantities but not of large ones (Cowan, 1979; Fuson, Secada & Hall 1983; Siegler, 1981). Conservation of continuous quantities such as length, area and volume is acquired a year or two later (Siegler, 1981).

One type of limitation is on processes, i.e. on what kinds of things measurement procedures can be applied to. As Piaget et al (1960) state, "to measure is to take out of a whole, one ... unit, and to transpose this unit on the remainder of the whole". Thus, any material to be measured must afford the measurer some unit which can be used in that process. This characteristic is not present in continuous quantities. Beakers of water or pieces of string do not exhibit any evident sub-units. Only the employment of special tools such as rulers or measuring cylinders (and the knowledge of how to use them) can create sub-units that can be used for quantification. On the other hand, discrete quantities are defined by collections of individual sub-units of the quantity as a whole. No special tools are needed since quantification abilities are present to some extent in the heads of even the youngest children. Young children appear to be particularly sensitive to the fact that it is at the level of unitary objects, and not subparts of those objects, that quantification of collections should take place (Shipley & Shepperson, 1990). Thus, discrete quantities are clearly much easier to measure.

A second type of limitation is in the abilities of the children who are attempting to use measurement procedures. The children that need to carry out measurements to determine quantitative invariance are those below the age of five. However, their quantification skills are not well developed. They are efficient at subitizing: a fast, accurate perceptual quantification mechanism (Chi & Klahr, 1975; Svenson & Sjoberg, 1983). Subitizing, though, has a limit of about four objects (Atkinson, Campbell & Francis, 1976). Young children's counting is only reliable for collections of about the same size (Fuson, 1988).

The measurement-before-conservation view therefore predicts the learning events that enable the acquisition of quantitative invariance knowledge. It follows that, if measurement is needed to be able to reason about quantity, learning can occur only when the effects of transformations of small collections of objects are evaluated. These quantities will have to be discrete because young children are not capable of creating consistent sub-units from continuous quantities. Gelman (1977) has shown that one-year-olds can reason about some transformations when the number of objects involved is very small. The discrete quantity requirement is supported by Piaget et al's (1960) and Miller's (1984) findings that, given the task of dividing up an object such as a cookie into equal parts, young children created many arbitrarily-sized sub-units. These are unsuitable for quantification because counting them does not produce accurate absolute measures for a single entities or relative measures of multiple entities.

Miller (1989) has further demonstrated the interaction between the use of measurement procedures and the acquisition of quantitative knowledge. Miller tested three- to ten-year-olds on a modified equivalence-conservation task. A variety of transformations were applied to different materials to test number, length and area conservation. He predicted that the effects of transformations would be easy to determine when relevant measurement procedures provided good cues to the actual quantity and vice versa. For example, counting is a good way to determine the resulting quantity of a transformation like spreading objects out. Thus, Miller predicted that the number task would be easier than length or area because it is easiest to measure. But enumeration is a bad method for evaluating the effect of changing the objects' size, since it does not assess their total mass. For this transformation, Miller predicted the number task would produce the worst performance. The results were as predicted, showing that what quantitative knowledge one can learn depends on what measurement procedures one uses.

Our theory (Simon, Newell & Klahr, 1991) follows Klahr (1984) in stating that it is measurement of collections of discrete objects that provides information upon which knowledge about quantitative invariance is built. Conservation knowledge is acquired in situations where invariance can be empirically verified. In other words, learning events occur when the materials allow children to use their measurement capabilities to obtain a numerical measurement for a collection of objects before and after it has been transformed. The two measurements can then be compared and the result attributed to the transformation as its effect.

If the difference is zero, the quantity is unchanged and the transformation is deemed to have a non-quantitative effect for the dimension in question, e.g. it "conserves number". If some difference is detected, the transformation is found to be non-conserving. Such differences can be simply detected by means of discriminations based on subitizing. With sufficient domain knowledge, the direction and magnitude of the change can also be determined. Thus we conclude that the initial learning experiences for invariance knowledge will be based on measurements of small collections of discrete objects within the subitizing range.

Q-Soar

To show that measurements are the stuff of which conservation knowledge is made, we built a computational model, based on the above theory. Q-Soar's foundation is the Soar architecture for intelligent behavior (Laird, Newell & Rosenbloom, 1987) and associated cognitive theory (Newell, 1990) which involves performance organization in terms of problem spaces and goal-oriented, experienced-based learning in terms of chunking. Q-Soar simulated the acquisition of number conservation demonstrated in a training study by Gelman (1982) and thus is the first demonstration that chunking can account for developmental transitions. Gelman's study contained two training conditions of interest; experimental and cardinal-once.

The experimental condition was an equivalence-conservation task where one of two rows of three or four objects was spread out or compressed, leaving the original one untouched. Before and after each transformation, children were required to count each row and state the absolute and relative numerosities of the rows. Gelman designed this condition to help children make use of one-to-one correspondence matching. The cardinal-once condition was an identity conservation condition, so called because the single row involved just one before and after count. The numerical comparison required was between the pre- and post-transformation quantities of the same row. In this condition there was no means of using one-to-one matching and so Gelman predicted that this group would not benefit from training.

Results showed that three- and four-year-olds learned conservation from the experimental condition, since they solved large number tests. The cardinal-once condition produced no learning in three-year-olds but it did benefit the four-year-olds, though they performed less well than their experimental peers. The no-cardinal control group, who saw no transformations, failed the tests.

Q-Soar began with the ability to simulate the pre-training competence of the three- and four-year old children in Gelman's study. The precise details of the model can be found in Simon et al (1991). There were two variants of the model; Q-Soar-3 and Q-Soar-4 (which modelled the three- and four-year-olds respectively). They consisted of a set of problem spaces enabling the execution of the behaviors required in Gelman's conditions. The problem spaces also enabled the model variants to execute a *verification process*. This involved comparing pre- and post-transformation measurements of the numerical aspect of the arrays to determine the effect of the transformation. Thus, after training, the effect of chunking over the verification processing was that Q-Soar now knew the numerical effects of the transformations rather than having to determine them empirically. Untrained versions of each model faced with Gelman's larger number post-tests failed in the characteristic manner. Their quantification abilities were not sufficient to measure the arrays and so they resorted to estimation based on the lengths of the rows. The result was that the wrong answer was always given, just the same as children in Gelman's control group.

The only difference between the two model variants was that Q-Soar-4 always executed the verification process (unless chunks fired to provide the effect of a familiar transformation) whereas Q-Soar-3 did not. The following observations support this. First, Gelman (1977) among others, has shown that young children assume that a set of objects that has undergone no visible transformation will not undergo any alteration in quantity. Second, many experiments (see Donaldson, 1978) have shown that young children assume that the quantitative value of a set of objects will change if it undergoes an obvious visible transformation. These lead to two theoretical assumptions. First, three-year-olds will not attempt to verify the assumed quantitative change resulting from a visible transformation unless presented with conflicting evidence which suggests that the quantity has remained unchanged. Second, four-year-olds will always verify the quantitative effect of a transformation, irrespective of the post-transformation perceptual information.

When the Q-Soar-4 variant undergoes either of the Gelman training conditions it carries out the verification processing which has the effect of allowing it to learn the effects of spreading and compressing transformations. When Q-Soar-3 experiences the cardinal-once training condition, the post-transformation array appears totally consistent with its assumption of quantitative change. There is a single row that is longer or shorter than it was before. There is no reason to check what appears to be an obvious result, i.e. that transforming the row has altered the number of objects in it. Thus Q-Soar-3 in cardinal-once

Gelman, 1982 (% correct)	Experimental	Cardinal-Once	No-Cardinal
3-year-olds	71	9	6
4-year-olds	70	46	15
Q-Soar (response)			
Q-Soar -3	correct	incorrect	incorrect
Q-Soar-4	correct	correct	incorrect

Table 1. Test responses from Gelman's subjects and Q-Soar

does not execute the verification procedure and so does not provide for itself the chance to learn about the conservation effects on number of spreading and compressing transformations.

However, when Q-Soar-3 experiences the experimental condition there are two post-transformation arrays, both within the subitizing range. The transformed row is now assumed to be quantitatively different, but two conflicting types of perceptual input are available. The length of the rows appears to confirm the assumption. However, based on subitizing evidence, the two rows still maintain their original numerical values. This conflict leads Q-Soar-3 to execute the verification procedure and, like Q-Soar-4, learn about the conservation effects of the transformations. This suggests that the conflict is what persuades the three-year-olds to use the verification process when they seek to determine quantitative invariance. Presumably, once stimulated to employ the verification process by such conflict, these three-year-olds will eventually automatically do so, as is the case for four-year-olds.

Results

Q-Soar-3 and -4 underwent the same training and testing as Gelman's subjects. Comparison of test responses is presented in Table 1. Similar performance is evident for Q-Soar and human subjects in the experimental and no-cardinal conditions. The cardinal-once condition produced no learning in Gelman's three-year-olds, as she predicted, but the older children clearly did benefit from the training. The precise reasons for their variable performance are not clear at present. Nevertheless, this counters the prediction that cardinal-once offers no opportunity for conservation learning due to the fact that correspondence matching is not possible. Though Q-Soar-4 learned rather too well from the training, the result suggests that its processes provide the means of learning conservation knowledge.

Conclusions

In this paper we have presented Q-Soar, a computational model, which simulates the acquisition of conservation knowledge in a published training study. Q-Soar implements and extends Klahr's theory that discrete quantities are foundational to conservation learning. It also demonstrates that Soar's chunking mechanism can account for significant developmental acquisitions such as number conservation. Although chunking was originally constructed to model practice effects over many trials (Rosenbloom & Newell, 1986), its application has been extended to a wide range of cognitive tasks (Lewis et al, 1990). This suggests that, as a goal-directed, experience-based learning mechanism within a problem space architecture, chunking may be a sufficient account of human learning.

Q-Soar also predicted learning events that were not consistent with Gelman's theory. Our work shows that conservation knowledge is acquired when young children apply their limited measurement capabilities to empirically verify the quantitative effect of transforming a collection of objects. The result is then bound to the type of transformation observed as its general quantitative effect on the quantity concerned. In other words, measurement enables conservation judgments to be made, while verification enables conservation knowledge to be learned.

This reverses the logical relationship between conservation and measurement in Piaget's formulation, making the empirical process of measurement a prerequisite for necessity judgments about conservation. It also limits the scope of such "logical" conservation judgments from a general principle about all quantities in Piaget's case, to a domain-specific generalization in our case. Number conservation must be learned first, by determining the effects of transformations in terms of number. The transfer to continuous quantities appears to require transfer via representation change, which is presumably why it takes so long after the initial acquisition. We have not yet addressed this issue in detail. Another issue arising from this work is the need to identify the exact processes required to learn conservation knowledge. Q-Soar could complete Gelman's task in a number of ways, not all of which would learn what the children in her experiment did. The data show that the children's learning is not all or none as Q-Soar's is. A parametric analysis of Q-Soar variants is being undertaken. From this we expect to discover the range of necessary knowledge and processes that can be employed to acquire conservation knowledge from this task in the way that human subjects do.

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