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THE $K_1^o - K_2^o$ MASS DIFFERENCE IN S-MATRIX THEORY

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ABSTRACT

General considerations are presented on the perturbing effect of weak interactions on S-matrix dynamics specifying the neutral K-meson states. The perturbations due to weak-interaction amplitudes in the N/D dynamical scheme are illustrated for two models of the neutral K's:

(a) $K \equiv \sigma K$ bound state ($\sigma = \pi\pi$, $I = 0$, $J^\pi = 0^+$)

(b) $K \equiv \omega K$ bound state ($\omega = 3\pi$, $I = 0$, $J^\pi = 1^-$) .

The main result of examining these perturbations is the relation

$$m(K_2) > m(K_1) \quad .$$

Inadequacies of the (current) dynamical models permit only a crude estimate of the magnitude of the mass difference, namely between 10^{-5} and 10^{-6} electron volt.

INTRODUCTION

Recently, calculations have been performed, in the framework of S-matrix dynamical equations, which obtain strong and electromagnetic mass splittings[1,2,3] and also the symmetry-breakings of strong[4] and weak couplings[5,6] (the former due to both strong and electromagnetic effects[5]). In this note, we consider (via similar techniques) the remaining symmetry breaking effect operative for strongly interacting particles: weak mass splitting. The only case in which such effects generate observable consequences is that of the neutral K mesons;[7] our discussion thereof will fall into these categories:

(a) General examination of relevant S-matrix ideas.

(b) A discussion employing certain dynamical models to infer the sign of the mass difference.

(c) Comments on the absolute magnitude of the mass difference.

The main result of this study is

$$m(K_2^o) > m(K_1^o) \quad , \tag{1}$$

with the magnitude of the mass difference uncertain because of an inadequate dynamical model.

1. K MESONS AND RELEVANT SCATTERING CHANNELS

We start with a brief summary of the neutral K-meson properties.[8] The eigenstates of strangeness (except for weak-interaction effects) are $K^o(S = +1)$ and $\bar{K}^o(S = -1)$. The CP eigenstates are (usually) defined as

$$K_1^o = \frac{K_o - \bar{K}_o}{\sqrt{2}} \quad \text{with} \quad CP = + \quad ,$$

$$K_2^o = \frac{K_o + \bar{K}_o}{\sqrt{2}} \quad \text{with} \quad CP = - \quad . \tag{2}$$

The observed nonleptonic decays of these states, are

$$K_1 \to 2\pi \quad ,$$

$$K_2 \to 3\pi \quad . \tag{3}$$

(For simplicity, let us now omit the "zero" superscript.) If H is the total Hamiltonian for K's, then the K_2K_1 mass difference is given by

$$\delta_K \equiv m(K_2) - m(K_1)$$

$$= \langle K_2|H|K_2\rangle - \langle K_1|H|K_1\rangle \tag{4a}$$

$$\equiv 2\langle K^o|H|\bar{K}^o\rangle \quad . \tag{4b}$$

We thus see that a $\Delta S = 2$ interaction is responsible for δ_K. No such fundamental weak interaction is known; we therefore assume that a second-order (weak interaction) transition is involved. If we were to assume a fundamental $\Delta S = 2$ interaction of order g^2 (g being the weak-interaction coupling strength), then we could proceed no further except to estimate g^2 from experimental data on δ_K (since it is unlikely that experiment would otherwise study an order g^2 interaction). Our assumption then is that $\Delta S = 2$ arises from $(\Delta S = 1 , \Delta I = \frac{1}{2})$ to second order.

To elucidate the structure of the relevant S matrix, we consider those scattering channels connected with the K channel. In addition to the K channel, in which strong and weak interactions occur, there are channels accessible via $\Delta S = 1$ transitions, either PC violating, or PC conserving.

If the state we are interested in is a $J^\pi = 0^-$ state with a mixture of $S = \pm 1$ (as is the case for K_1 or K_2), then the relevant T matrix has the structure

$$T \equiv \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{12} & T_{22} & T_{23} \\ T_{13} & T_{23} & T_{33} \end{pmatrix} , \qquad (5)$$

where

T_{11} is a channel with $S = \pm 1$, $P = -$, strong + $(\text{weak})^2$ interactions,

T_{22} is a channel with $S = 0$, $P = -$, strong + $(\text{weak})^2$ interactions,

T_{33} is a channel with $S = 0$, $P +$, strong + $(\text{weak})^2$ interactions,

T_{12} represents $\Delta S = 1$, P conserving weak transitions,

T_{13} represents $\Delta S = 1$, P-nonconserving weak transitions,

T_{23} represents $\Delta S = 0$, P-nonconserving weak transitions, which therefore must be of second order in the weak interactions. This is because no first-order $\Delta S = 0$ nonleptonic weak interaction is known, and even leptonic transitions would have to both emit and reabsorb two leptons.

In general, if the K state is taken to occur in an n-channel system, then each T_{ij} is really an $n \times n$ submatrix.

2. DYNAMICAL MODELS

We shall now consider models for the K's which considerably simplfy the T matrix given by (5).

Model 1

There exists an $I = 0$ scalar particle, the σ ,[9] with mass around 400 MeV, and a $\pi\pi$ (strong) decay mode. The K's are $K\sigma$ bound states. K_1 decays into $\pi\pi$, and in this model K_2 decays (weakly) into $\sigma\pi$. This model, containing only K's , σ and π , can only have $K_1\sigma \to \pi\pi$ and $K_2\sigma \to \sigma\pi$, with single-particle exchange forces. The reason for this is simply that in the equation specifying the K bound states, off-diagonal transitions always occur at least as two multiplicative factors. These transitions are therefore negligibile if not of first order in the WI (we now denote weak interactions by WI). This implies the above restrictions (as examination of diagrams will make clear to the reader). The same argument rules out consideration of $K_1\sigma \leftrightarrow K_2\sigma$ transitions in this model.

In this model, we have only a two-channel T matrix, whose Born matrix elements are illustrated in Fig. 1, where an obvious diagrammatic notation is used. We have only the positive-parity $\pi\pi$ channel (assumed dominated by the σ) accessible to the K_1 , and only the negative-parity $\pi\sigma$ channel accessible to the K_2 .

We should now state clearly that in Model 1 we ignore couplings to vector-pseudoscalar (V - PS) channels, as well as to baryon-antibaryon (B - $\bar{B}$) channels, etc. The latter channels are probably important contributors to 0^- and 0^+ bound states, despite the high B - $\bar{B}$ threshold

energy.[10] This author admits to ignoring additional channels in the present models because of a desire to examine a simple model. However, it should also be pointed out that the K channels can communicate with both $J^{\pi} = 0^{-}$ and 0^{+} $B\bar{B}$ channels. At present, though, P wave baryon nonleptonic weak decays are ill-delineated by both theory and experiment, adding to the potential uncertainties of $B\bar{B}$ calculations. Another objection might well be that we should consider K bound states within an SU_3 scheme of, perhaps, meson scattering channels. The objection is valid, but unfortunately, current knowledge of $()_8()_8()_8$ mesonic weak interactions is too scant to permit such considerations.

Model 2

No σ exists and we assume that K's are predominantly $K\omega$ bound states. Note that an extension of this model to include more mesons would encounter the complication of a weak mass difference between the corresponding vector mesons K_1^* and K_2^*. The weak decays are now assumed to occur via the vertices $K_1\pi\pi$ and $K_2\rho\pi$. With the allowed strong coupling vertex being $\omega K_1 K_2$, the Born T matrix is as illustrated in Fig. 2.

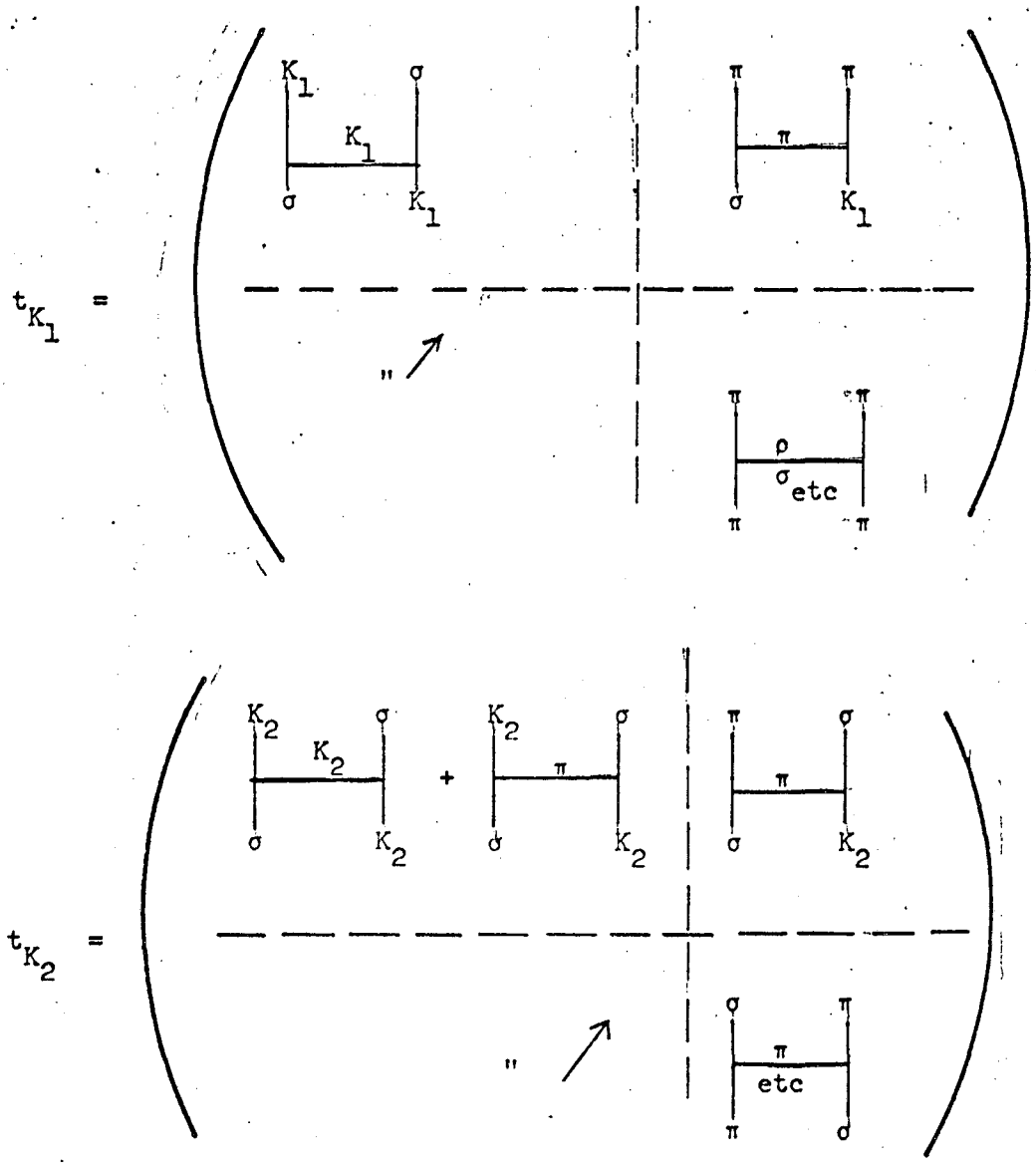

Fig. 1. Model 1--T Matrix for K_1, K_2.

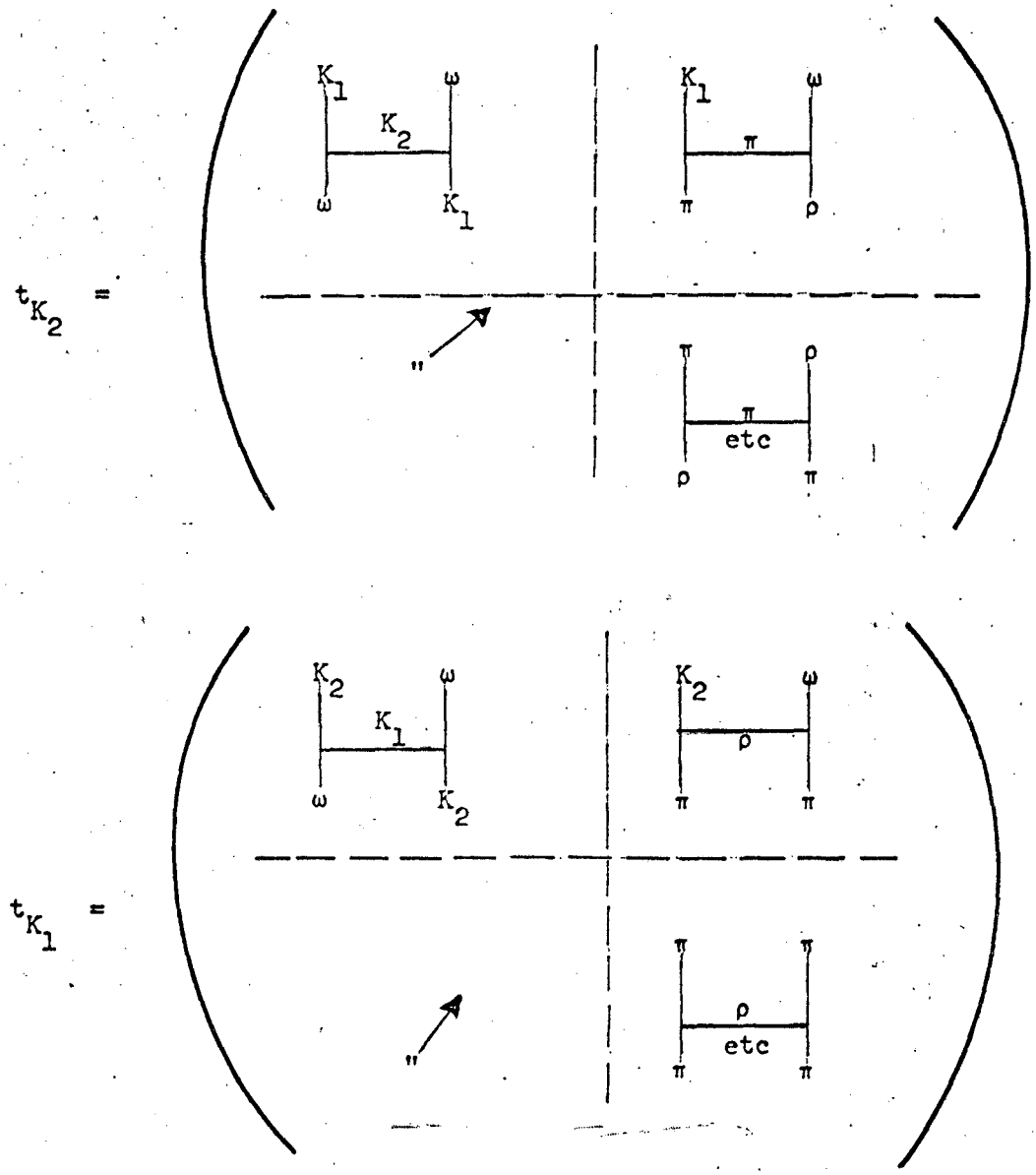

Fig. 2. Model 2--t-matrix for K_1, K_2.

3. DYNAMICAL CALCULATIONS

We assume that $\underset{\sim}{N}/\underset{\sim}{D}^{-1}$ expressions are obtained for the partial-wave amplitudes of interest. Here $\underset{\sim}{N}$ and $\underset{\sim}{D}$ are 2 × 2 matrices, and the condition specifying the energy of a bound state is

$$\text{determinant } \underset{\sim}{D} \equiv |\underset{\sim}{D}| = 0 \quad , \tag{6a}$$

where

$$\underset{\sim}{D} = \underset{\sim}{1} - \frac{s - s_t}{\pi} \int_0^\infty \frac{\underset{\sim}{X}\,\underset{\sim}{N}}{(s' - s)(s' - s_t)}\, ds' \quad , \tag{6b}$$

$\underset{\sim}{X}$ is a diagonal matrix whose elements are $\delta_{ij}\theta(s - \underline{i}\text{th threshold energy})$ ($\underline{i}$th channel phase space factor ρ_i) ; s_t is a subtraction point. We adopt a determinantal model for simplicity, wherein

$$\underset{\sim}{N} \equiv \underset{\sim}{t}(s)\big|_{\text{Born}} \quad . \tag{6c}$$

Now, using the fact that

$$\left| D_{K_1(K_2)}(s = m^2(K_1)(m^2(K_2)) \right| = 0 \quad , \tag{7}$$

we perform a Taylor expansion of $|D_{K_2}(m_{K_2})|$ about the parameters of $|D_{K_1}(m_{K_1})|$ to order g^2. In other words, we express $|D_{K_2}(m_{K_2})| - |D_{K_1}(m_{K_1})| \equiv 0$ as a power series (to order g^2) in mass differences, and "force" differences. This is the basic perturbation[1-6] approach providing expressions for δ_K. In the following, let $\frac{\partial D_{ij}}{\partial m^2}$ be the derivative with respect to the mass of a single external or exchanged particle.

Let us first consider Model 1. The above technique furnishes the relation, evaluated at $s = m^2(K)$,

$$\left(2 \frac{\partial D_K}{\partial m^2(K)} + \frac{\partial D_{K\sigma}}{\partial m^2(K)]_{\text{exchange}}} + \frac{\partial D_{K\sigma}}{\partial s} \right) \delta_K$$

$$+ (D_{K\sigma}]_{\pi \text{ exchange}} - 1) - \frac{D^2_{\pi\sigma\to\sigma K}]_{\pi \text{ exchange}}}{D_{\pi\sigma}}$$

$$+ \frac{D^2_{\pi\pi\to\sigma K}]_{\pi \text{ exchange}}}{D_{\pi\pi}} = 0 \quad . \tag{8}$$

Actually, the D matrix is not symmetric, and what we have denoted by D^2_{ij} is really $D_{ij}D_{ji}$. Nevertheless, for $i \neq$ or $=$ to j , $D_{ij}D_{ji}$ is a positive quantity here. An additional approximation here is to ignore δG effects in the perturbations, i.e., we ignore possible terms proportional to $(g^2_{\sigma K_1 K_1} - g^2_{\sigma K_2 K_2})$.

Scale invariance of D_{ij} is the statement that because each D_{ij} is dimensionless, if we increase all masses by a factor β , then the output bound-state mass is similarly increased by a factor β . The consequence of this for Eq. (8) is that the first parentheses contain an expression equal to

$$\frac{-2\partial D_{K\sigma}}{\partial m^2(\sigma)} \quad , \tag{9a}$$

where we neglect the $K - \sigma$ mass difference as a tolerable approximation. We observe that

$$t_{K\sigma})_{Born} \equiv \frac{G^2}{4q^2} \ln\left(1 + \frac{4q^2}{m^2(K)}\right) , \qquad (9b)$$

where the strong coupling constant G^2 has dimensions of $(\text{mass})^2$ and plausibly $\equiv \bar{G}^2 \times m^2(\sigma)$. Thus, (even without some implicit mass dependence in G^2)

$$\frac{\partial t_{K\sigma}}{\partial m^2(\sigma)} > 0 \quad , \text{ so that } \quad \frac{-\partial D_{K\sigma}}{\partial m^2(\sigma)} > 0 \quad . \qquad (9c)$$

Furthermore, the π exchange force in the $K\sigma$ channel is attractive, so that $1 - D_{K\sigma}]_{\pi \text{ exchange}} > 0$. We thus have

$$\left(\frac{-2\partial D_{K\sigma}}{\partial m_\sigma^2} > 0\right) \times \delta_K = (1 - D_{K\sigma}]_{\pi \text{ exchange}} > 0)$$

$$+ \frac{D^2_{\pi\sigma\to\sigma K}}{D_{\pi\sigma}} - \frac{D^2_{\pi\pi\to\sigma K}}{D_{\pi\pi}} \quad . \qquad (10)$$

Now, the $\pi\pi$ channel contains the σ pole; therefore (with our sign convention, attractive forces produce positive amplitudes) at $s = m^2(K) > m^2(\sigma)$, $D_{\pi\pi} < 0$. The final quantity to be examined is $D_{\pi\sigma}$. As this should have a pole at $s = m^2(\pi)$, it is also negative, but one expects that

$$D^{-1}_{\pi\sigma} \gtrsim \frac{m^2(K) - m^2(\sigma)}{m^2(K) - m^2(\pi)} \times D^{-1}_{\pi\pi}$$

$$\gtrsim \frac{1}{3} D^{-1}_{\pi\pi} \quad \text{for} \quad m(\sigma) \gtrsim 400 \text{ MeV} \quad . \qquad (11a)$$

This merely states that the "slopes" are expected to be about the same for $D_{\pi\sigma}$ and $D_{\pi\pi}$. In addition, one expects the $\pi\sigma$ force to be greater than the $\pi\pi$ force, since $m(\pi) < m(\sigma)$. This has the effect of replacing 1/3 by a smaller number. Finally, one can estimate $g^2_{K\sigma\pi}/g^2_{K\pi\pi}$ from the K decay model of Reference 4, which assumes $K_2 \to \sigma\pi \to 3\pi$. With their assignment $\frac{g^2_{\sigma\pi\pi}}{4\pi} \not\ll 1$, $m(\sigma) \approx 400$ MeV, and a crude three particle phase-space estimate, one finds $g^2_{K\sigma\pi}$ smaller than $g^2_{K\pi\pi}$. All these arguments serve to demonstrate that the final two terms of Eq. (10) have a positive sum. We consequently infer that $\delta_K > 0$.

Now let us consider Model 2. The result of applying perturbation techniques as previously is the analogue of Eq. (8), namely

$$-2\delta_K \left(\frac{\partial D_{K\omega} \times r}{\partial m^2(\omega)]_{\text{external}}} + \frac{2\partial D_{K\omega}}{\partial m^2(K)]_{\text{external}}} \right)$$

$$= \frac{D^2_{K\omega \leftrightarrow \pi\rho}}{D_{\pi\rho}} - \frac{D^2_{K\omega \leftrightarrow \pi\pi}}{D_{\pi\pi}} \qquad (12)$$

where

$$r = \frac{m^2(\omega)}{m^2(K)} .$$

As argued before, if the σ occurs in the $\pi\pi$ channel, the right-hand side of Eq. (12) is positive. In this event, though, the lighter-mass σK channel might be more important.

If no σ occurs, we have the right-hand side of (12) negative. Examination of the $K\omega$-channel Born amplitude indicates that the coefficient of δ_K is negative, and therefore, again δ_K is positive!

We conclude this section with an indication of how we may crudely estimate the magnitude of δ_K in Model 1. To begin with, we have an upper bound

$$|\delta_K| < \left| \frac{D^2_{\pi\pi-\sigma K}}{D_{\pi\pi}} \times \left(-2 \frac{\partial D_{K\sigma}}{\partial m^2(\sigma)} \right)^{-1} \right| . \tag{13a}$$

We write $D^2_{\pi\pi-\sigma K} \simeq 3 G_{K\pi\pi} G_{\sigma\pi\pi} J^2$, where all mass differences are ignored in estimating the integrals encountered. The factor 3 arises from the fact that the $\pi\pi$'s are in an $I = 0$ state. From the total K_1 decay rate we estimate

$$\frac{G_{K\pi\pi}}{m^2(K)} \simeq 2 \times 10^{-4} . \tag{14}$$

A rough estimate of $\partial D_{K\sigma}/\partial m^2(\sigma)$ is obtained by assuming that

$$t_{K\sigma} \approx \bar{G}_{K\sigma} m^2(\sigma) \times \left(\frac{1}{4q^2} \ell n \text{ (etc.)} \right) \tag{15a}$$

and

$$\therefore \quad \frac{\partial D_{K\sigma}}{\partial m^2(\sigma)} \simeq \frac{D_{K\sigma}}{m^2(\sigma)} \simeq \frac{G_{KK\sigma}}{m^2(\sigma)} J . \tag{15b}$$

Thus,

$$|\delta(m(K))| \equiv \left| \frac{\delta_K}{2m(K)} \right| = \frac{3}{4} \frac{G_{K\pi\pi} G_{\sigma\pi\pi}}{G_{KK\sigma}} \frac{m^2(\sigma)}{m(K)} \frac{J}{D_{\pi\pi}} . \tag{16}$$

We next estimate

$$D_{\pi\pi}(m^2(K)) \approx (m^2(K) - m^2(\sigma))(-G_{\sigma\pi\pi}) \times \frac{\partial J}{\partial s} \quad . \tag{17a}$$

This assumes that σ exchange dominates the scalar $\pi\pi$ channel. A crude estimate of the derivative in (17a) is

$$\frac{\partial J}{\partial s} \approx \frac{J}{s(\text{threshold})} \approx \frac{J}{4m^2(\pi)} \quad . \tag{17b}$$

We estimate $G_{\sigma\pi\pi} \approx G_{\sigma KK}$ (as in an SU_3 scheme) and find now that

$$|\delta(m(K))| \approx \frac{3G_{K\pi\pi}}{G_{\sigma\pi\pi}} \frac{m^2(\sigma)}{m(K)} \frac{m^2(\pi)}{m^2(K) - m^2(\sigma)} \quad . \tag{18}$$

For a 100-MeV width of the σ (which could be considered an upper estimate), $G_{\sigma\pi\pi} \approx 5m^2(\pi)$, taking $m(\sigma) \approx 400$ MeV. Thus we finally estimate

$$|\delta(m(K))| \approx 2 \times 10^{-14}\, m(K) \approx 10^{-5} \text{ eV} \quad . \tag{19}$$

Of course, this estimate is extremely rough, and certainly cannot be considered as anything but an order-of-magnitude estimate. As such, Eq. (19) is not in the least surprising. We have presented the procedure above only to indicate which parameters would be extracted from dynamical calculations, and which from external data or assumptions.

DISCUSSION

An amusing situation has been suggested recently, namely that the (CP = -) particle might decay entirely into 2π, violating CP-invariance, while appropriate properties of the interaction symmetries might cause the (CP = +) state to decay primarily into 3π. This effectively reverses the CP assignments of the states responsible for a given decay. In the above dynamical scheme, the effect is to reverse the sign of $\delta(K)$ although the quantity (mass of 3π-decaying entity) - (mass of 2π-decaying entity) is independent of the CP assignments of the neutral K states.[11]

Finally, we remark that the present statement $\delta_K > 0$ agrees with the results of References 12 and 13, where a rather different approach is employed.

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