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When High WMC Promotes Mental Set: A Model of the Water Jar Task

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Abstract

Differences in working memory capacity (WMC) relate to performance on a variety of problem solving tasks. High WMC is beneficial for solving analytical problems, but can hinder performance on insight problems (DeCaro & Beilock, 2010). One suggested reason for WMC-related differences in problem solving performance is differences in strategy selection, in which high WMC individuals tend toward complex algorithmic strategies (Engle, 2002). High WMC might increase the likelihood of non-optimal performance on Luchins' (1942) water jar task because high WMC solvers tend toward longer solutions, not noticing when shorter solutions become available. We present empirical data showing this effect, and a computational model that replicates the findings by choosing among problem solving strategies with different WM demands. The high WMC model used a memory-intensive strategy, which led to long solutions when shorter ones were available. The low WMC model was unable to use that strategy, and switched to shorter solutions.

Keywords: Working memory capacity; problem solving; strategy selection; computational modeling

Background

Problem solving, like cognitive processes generally, is bounded by resource limitations (Simon, 1972). In particular, the capacity of working memory (WMC) has repeatedly been found to be related to problem solving performance. In a major review, Wiley and Jarosz (2012) concluded that, "In analytical problem solving, the superior executive function associated with WMC seem to generally support more successful problem solving." (p. 219).

However, as Wiley and Jarosz point out, this conclusion may only hold for analytical problem solving. In this type of process, problems are solved by extrapolating from prior experience, and the problem solver makes steady, step by step progress towards the goal.

In contrast, creative problem solving is characterized by the need to override prior experience in order to identify solutions that do not confirm to or follow from that experience (Ohlsson, 1992, 2011). Evidence is accumulating that in creative problem solving, the relationship between WMC and performance works differently than the "more is better" relation observed in analytical problem solving. Several studies have documented a reversed relation, in which problem solvers with lower working memory capacity performing better on insight problems than solvers with greater capacity. For example, DeCaro, Van Stockum, and Wieth (2015) found that low WMC participants outperformed high WMC participants on match stick arithmetic problems (Knoblich et al., 1999) and insight word problems (Schooler, et al., 1993, Wieth & Burns, 2006). Similarly, other studies have found that if WMC is reduced through alcohol intoxication (Jarosz, Colflesh, & Wiley, 2012) or solving problems during one's non-optimal time of day (Wieth & Zacks, 2012), insight problem solving improved while analytical problem solving suffered. This outcome is counterintuitive and stands in need of explanation. Why is lower capacity associated with greater probability of reaching an insight solution?

One possible explanation is that WMC influences the types of strategies used during problem solving. Those with high WMC are better able to control attention, giving them an increased ability to suppress distracting information and process more information relevant to the task at hand (Engle, 2002; McCabe, Roediger, McDaniel, Balota, & Hambrick, 2010). Beilock and DeCaro (2007) suggested that individuals with lower WMC may lack the attentional control required to accurately use complex problem solving strategies and instead use associatively based strategies, whereas individuals with high WMC are able to use complex strategies. However, "sometimes high WMC participants may attempt to use complex strategies when simpler, more elegant, or more direct approaches are available" (Wiley & Jarosz, 2012, p. 210). Reliance on complex strategies are often beneficial on analytical problems which require the solver to hold multiple steps in working memory while progressing toward the goal (Jarosz, 2015). However, insight problems often trigger an inappropriate representation of the problem (Ohlsson, 1992), and using a complex strategy based on this representation will lead the solver toward impasse. The solution to an insight problem typically requires that complex yet familiar problem solving strategies are abandoned in favor of searching for a novel solution (Knoblich, Ohlsson, & Raney, 2001).

The Present Study

The purpose of the present paper is to describe a computational model of the classical findings on the water jar task. The latter was introduced into problem solving research by Luchins (1942). In this task, the solver is presented with a set of three jars of specified values, and is instructed to use only these jars to obtain a desired amount of water. The original problem set included a single practice problem to acquaint the solver with the task, followed by a set of ten problems (see Table 1). The practice problem is solved by filling jar A, then subtracting water with jar B three times (A - 3B). Next, problems 1-5 can be solved by filling jar B, then subtracting water with jar A once and jar C twice (B - A - 2C). Problems 6-7 are solvable using the previous *long formula*, but can also be solved using a *shorter formula* of filling jar A and either adding or subtracting jar C (A +/- C). Problem 8 can only be solved using the shorter formula, and problems 9-10 can once again be solved with either the long or short formula. The main finding of interest is that the participants showed a strong tendency to use the long path on the problems that could also be solved by the shorter and more elegant, single-step path. However, after being given a problem that could only be solved with the shorter solution, they then applied that shorter solution to the two subsequent problems that could be solved with either the short or the long solution. The tendency to continue using the more complicated, but previously successful solution was called the Einstellung effect by Luchins (1942), and is now more commonly referred to as mental set (Smith & Blankenship, 1991).

Because the later problems in the water jar task can be solved using two different solutions, it is a particularly interesting task for examining how strategy use varies with WMC. Switching to the shorter solution is an insight-like process because it requires the solver to change their representation of the problem. In contrast, continuing to use the longer solution suggests that the solver may be using complex algorithmic strategies which tend to be better for analytical problem solving. Based on previous problem solving research, it would then be expected that low WMC individuals may be more likely to switch to the shorter solution than high WMC individuals, and high WMC individuals are more likely to experience mental set.

Two recent studies using the water jar task have found this effect (Beilock & DeCaro, 2007; Van Stockum & DeCaro, 2015). Our model provides a possible explanation why less working memory capacity might be associated with higher probability of finding and using the short solution on water jar problems.

Table 1: Classic water jar problem set (Luchins, 1942).

Problem	Formula(s)	Jar A	Jar B	Jar C	Goal
0	Practice	29	3	0	20
1	Long	21	127	3	100
2	Long	14	163	25	99
3	Long	18	43	10	5
4	Long	9	42	6	21
5	Long	20	59	4	31
6	Long, Short	23	49	3	20
7	Long, Short	15	39	3	18
8	Short	28	76	3	25
9	Long, Short	18	48	4	22
10	Long, Short	14	36	8	6

Problem solving typically relies on prior knowledge and experience, and successful problem solutions and task performances are generated by extrapolating that prior experience and applying it to a new problem or situation. Such analytical problem solving necessarily involves potentially complex interactions between working memory and long-term memory, because the latter has to be engaged in locating the relevant prior experience, retrieve it, and adapt it to the task at hand.

However, insight problems are characterized by the need to override prior experience and engage other processes than memory retrieval in order to achieve a novel solution path. The latter might include so-called *weak methods,* problem solving strategies of high generality (Laird & Newell, 1983). If the cognitive load required to access long-term memory is greater than the cognitive load imposed by those alternative processes, then working memory capacity might be one of the factors that impacts the probability that a problem solver will cease trying to use prior experience and instead engage problem solving processes that are abstract and local and hence might find a different solution than the one implied by prior experience.

It is plausible that the participants store information about the longer path in memory during the initial five problems, and that they draw upon that information when solving each successive problem. However, memory encoding is seldom perfect and similar items, such as steps in the water jar problem, are subject to interference. Applying what is remembered from a previous problem might impose significant cognitive load.

However, people also possess general or abstract problem solving processes. A common type of strategy is to compare the current state of the problem at hand with the desired or designated goal state, and be guided by how or in what respect they differ. Strategies of this sort are commonly referred to as means-ends strategies (Newell & Simon, 1972). The latter type of computation can be performed on information that is visible to the participants (current water levels in the jars and the desired goal state), and so do not impose high cognitive load nor require operations on long-term memory. The participants might prefer to work a problem by extrapolating prior experience, and only switch to an abstract and local difference-reduction strategy when prior experience imposes too high a cognitive load or turns out to be unsuccessful.

A Computational Model

Our model assumes that people have multiple strategies for solving water jar problems that vary with respect to the WMC they require. WMC interacts with problem solving by impacting strategy choice. The current version of the model utilizes three different problem solving strategies. This was not meant to be an exhaustive list of all possible strategies, but to exemplify plausible strategies that a human solver might use for the water jar task. One strategy was *solving from memory*. In this strategy, if there are steps from a previously successful solution path stored in memory, the model can follow the path to check if that will solve the next problem. This strategy could be considered a type of case-based reasoning (Riebeck & Schank, 1989).

A second strategy is a *difference reduction strategy*, which is a simplified form of a means-ends analysis (Newell & Simon, 1972).. This strategy finds a starting value by evaluating which two operations will put the solver closest to the goal value and will pick the first operation of the two. For each subsequent step, the solver picks a value to add or subtract from the current state that will bring the current state closest to the goal value.

In order to prevent infinite loops of adding and subtracting the same value, the model uses a form of the no-loop heuristic (Atwood & Polson, 1976). If the solver is about to undo a previous step by adding the same value that was subtracted in the previous step or vice versa, it will instead randomly pick a jar.

The third problem solving strategy is *guided random solving*. For this strategy, at each step the solver determines whether a value needs to be added or subtracted in order to get closer to the goal, and then randomly selects a jar to perform the operation.

One potential way in which the model can deviate from human performance is that unless a limit is placed onto the model, the difference reduction and random solving strategies are both capable of performing an infinite number of steps until a solution is reached, whereas a human solver would only be capable of performing a limited number of steps. In order to resolve this problem, the number of steps that could be taken by either of these strategies was limited to a maximum of seven steps. A limit of seven steps was chosen because people on average can hold about seven or fewer items in memory depending on the type of information being stored (Baddeley & Hitch, 1974; Cowan, 2010; Miller, 1956).

The model was implemented in python 3.4 and is approximately 300 lines of code. Working memory was a fixed size storage for which the capacity could be specified. If a problem successfully solved a problem using any strategy, the solution path was saved into the model's long term memory, which then was available to be used by the memory strategy. For the memory strategy, the number of steps that can be saved is limited by WMC. This means that if the solution path was four steps long, but WMC only allows memory for three steps, only the first three steps will be saved. The steps were saved as a list of steps for which each step had an operation (add or subtract), and the jar used to perform the operation (A, B, or C), For example, the path for the longer solution would appear in WM as: ((add, B), (sub, A), (sub, C), (sub, C)). This solution path would remain in WM until a problem is solved using a different solution, and then the new solution would be saved into WM.

The model selects strategies in order from highest to lowest WM demand. It will first attempt the solving from memory strategy, followed by the difference reduction strategy, followed by the guided random strategy. This order was chosen because studies of WMC and strategy choice in problem solving have found that those with higher WMC tend to use more demanding strategies, sometimes instead of using less demanding, but valid strategies. This suggests that if a more demanding strategy can be used, it is more likely that it will be used (Beilock & DeCaro, 2007; Wiley & Jarosz, 2012).

However, because human solvers do not always select strategies in such a deterministic way, there is some noise in the strategy selection so that ten percent of the time it will skip the solve from memory strategy, and if it skips the first strategy, in ten percent of those cases, it will also skip the difference reduction strategy and go straight to the random solving strategy. We have not yet found a way to ground this parameter in the empirical data.

Empirical Study

Participants were 67 undergraduate students who were enrolled in an introductory psychology course and received credit for participation in this study.

Materials and Measures

Working Memory Capacity. WMC was measured using the automated symmetry span task (aSymspan; Redick, et al., 2012), and the automated running span task (aRunspan; Broadway & Engle, 2010). The aSymspan is a computer-based complex span task in which a memory task and processing task are interleaved.

In the *aSymspan*, participants judge whether an image is symmetrical across a vertical axis followed, and are then presented with a red square located in a 4x4 grid. After 2-5 trials, participants are then shown a grid and asked to click on the locations of the red squares in the order they were presented. Participants complete 12 sets of trial, 3 of each length. A participant's score is the number of red squares correctly remembered, and can range from 0-42.

The *aRunspan* is a computer-based simple span task in there is not a separate processing and memory component of the task. Participants are told to remember the last specified amount of letters in a string (3-7). Then participants are shown a string of letters of unknown length one at a time. Participants are then shown a screen with letters and are asked to click on the specified letters in the order they appeared. Score is the number of letters correctly remembered, and can range from 0-75. The WMC measures took approximately 5-10 minutes each to complete.

Problem Solving. Participants completed the water jar problems shown in Table 1 in the order presented. Problems were presented on paper with one problem per page. Participants first received an instruction page which included a completed example problem, and the practice problem (problem 0). Once participants correctly solved the example problem, they were given the rest of the task to complete. This task took approximately 15-20 minutes to complete.

Modeling Results

In order to explore whether the model replicates WMC-related differences in performance on the water jar task, the model was run 20 times, 10 with high WMC and 10 with low

WMC. For high WMC, the model was capable of remembering five steps, and for low WMC, the model was capable of remembering three. The classic Luchins (1942) problem set was used, and problems were performed in the order listed in Table 1.

Overall, the high WMC version of the model had higher accuracy in solving the water jar problems than the low WMC solver. The proportion correct was .97 for high WMC and .80 for low WMC. However, when broken down by problem, it can be seen that the differences in accuracy are driven by a few specific problems (see Table 2). More specifically, the low WMC solver failed to solve problem 5 all ten times, problem 3 nine times, and problem 4 three times. In contrast, the high WMC solver failed problem 5 in two instances, but successfully solved problems 3 and 4 every time. Errors occurred on these problems when the solver used the difference reduction or random strategies.

Table 2. Proportion correct on each water jar problem as a function of WMC.

	Low WMC		High WMC	
Problem	Model	Human	Model	Human
0	1.00	1.00	1.00	1.00
1	1.00	0.91	0.90	0.95
2	1.00	0.91	1.00	1.00
3	0.10	0.91	1.00	0.82
4	0.70	1.00	1.00	1.00
5	0.00	1.00	0.80	0.95
6	1.00	0.95	1.00	0.91
7	1.00	1.00	1.00	1.00
8	1.00	0.68	1.00	0.59
9	1.00	1.00	1.00	0.95
10	1.00	1.00	1.00	0.95
Total	0.80	0.94	0.97	0.92

Strategy use also varied by WMC (see Table 3 for a summary). The low WMC solver was unable to successfully use the solving from memory strategy, and instead used the difference reduction strategy, and rarely the random strategy to successfully solve the problems. The high WMC solver successfully solved from memory on half of the problems, and solved using the difference reduction strategy on just under half of the problems. For the high WMC, when the solver did not solve from memory, the majority of these instances were problems in which the most recent successfully solved problem had a different solution formula, so there was not a relevant path stored in memory.

Table 3. Proportion of strategy use as a function of WMC

Strategy	Low	High
Memory	0.00	0.50
Dif. Red.	0.79	0.47
Random	0.01	0.00
Fail	0.20	0.03

The main question is whether the model replicated the finding that high WMC solvers are more likely than low WMC solvers to use the long solution on problems 6-7 and 9-10. The model behaved in exactly this way (see Figure 1). The low WMC solver always used the short solution formulas instead of the long, whereas the high WMC solver used the long solution formula just under half the time (see Figure 2). When specifically examining problems 6-7, the high WMC solver had an even higher tendency toward using the longer solution, using it 90% of the time. On problems 9-10, because problem 8 was solved using a short solution, the high WMC solver overcame Einstellung and used the short solution.

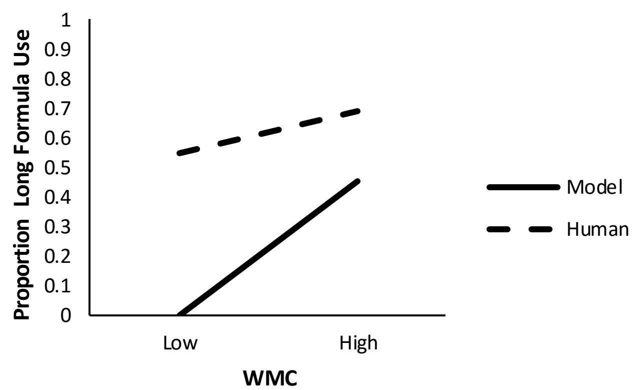

Figure 1. Proportion of long formula use on problems 6-7 and 9-10 as a function of WMC.

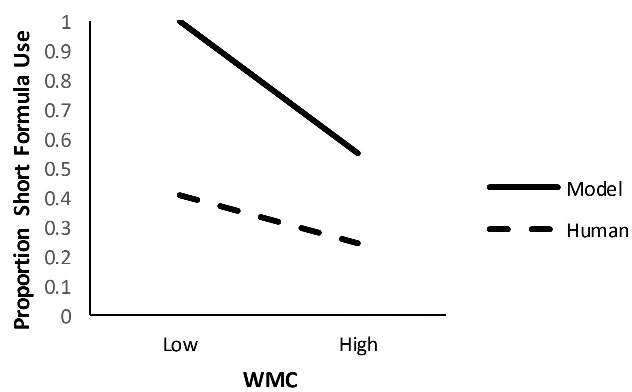

Figure 2. Proportion of short formula use on problems 6-7 and 9-10 as a function of WMC.

Empirical Results

Participants' WMC score was based on a factor score created by calculating the shared variance between aSymspan score and aRunspan score. In order to compare model data to human data, participants were split into low, medium, and high WMC groups. The low and high WMC groups were used to compare to the model.

High and low WMC groups did not differ in overall accuracy, $t(42) = 0.84$, $p = .41$ (Table 2.). Additionally, there were no WMC differences in accuracy on any individual problem. The low WMC human solvers had higher accuracy than the low WMC model, and the high WMC human solvers had slightly lower accuracy than the high WMC model

Although not significant, we found that the high WMC group had a higher rate of using the long solution formula on problems 6-7 and 9-10 compared to the low WMC group, $t(42) = -1.19$, $p = .24$, and a lower rate of using the short solution formula, $t(42) = 1.52$, $p = .14$. When analyzed as a correlation across all participants, there is a negative correlation between WMC and using the short solution formula, $r(65) = -.28$, $p = .02$, and a marginal positive correlation between WMC and using the long solution formula $r(65) = .22$, $p = .08$. Compared to the model, both low and high WMC human solvers showed a higher tendency of using the long solution.

Discussion

This model demonstrated how WMC influences strategy use on the water jar task and how strategy selection in turn affects the likelihood of experiencing mental set. By placing WMC limits on the memory strategy, the model was able to simulate the finding that high WMC solvers are more likely to use the long solution. When WMC was high, like human solvers, the model was more likely to continue using the long solution on problems 6-7, even though the short solution was available. The high WMC solver generally did not switch to the short solution until it failed to solve problem 8 from memory and used difference reduction to search for a new solution. When WMC was low, the solver was not able to store the full four step solution of the long formula, and was incapable of solving from memory using the long formula. The low WMC solver instead used the difference reduction and guided random solving strategies.

Even though the model was able to simulate WMC differences in formula use, compared to human performance, the model under predicted the likelihood of continuing to use the long solution once the short solution becomes available. There are a couple possible explanations for this finding. One possibility is that people resist changing strategies in a way that this model does not account for. Another possibility is that there are more strategies that could lead to using the long solution. Other possible strategies could include the undershoot or overshoot strategies used by Lovett on her model of the building sticks task, which is an isomorph of the water jar problem (Lovett, 1998; Lovett & Anderson, 1996).

One limitation of the model is that the low WMC solver failed to solve certain problems (problems 3-5) at a much higher rate than low WMC human solvers. This may also be because humans were using problem solving strategies not included in the model. One possible future direction would be to perform a think aloud study in order to learn what strategies people are using to solve this problem. Any new strategies that are learned could be incorporated into a future iteration of this model.

Another limitation to this model is that it selects its strategies in a pre-specified order: memory, difference reduction, and then random search. The only variation is that it sometimes skips an earlier strategy. People are not likely to move down a list of strategies in a particular order, especially if a strategy has not proven to be successful on previous problems. Another future direction could be to have the model randomly select a strategy based on the weighted utility of the strategy. The utility could be updated on success or failure of the strategy. If WMC determines which strategies can be used, then it would be expected that high and low WMC versions of the model would give higher utility to different strategies, with high WMC giving higher utility to more memory-intensive strategies.

Our results support the theory that strategy selection in problem solving is influenced by WMC limits. High WMC problem solvers are better able to make use of memory-intensive strategies such as remembering entire solution paths or algorithms. Low WMC problem solvers are less likely to use these strategies because they may require storing more information in working memory than the solver is able, and have to rely on strategies with lower memory demand. On the water jar task, the difference in strategy use meant that the high WMC solver had higher overall accuracy than the low WMC solver, but it was also more likely to use a non-optimal solution when a shorter possible solution was introduced. The model explains the Einstellung effect as a consequence of the interaction between the structure of the task environment and the boundaries on human cognitive capacity. When the task environment supports extrapolating from prior experience and the extrapolation imposes low cognitive load, people will tend to respond on the basis of memory, with Einstellung, ruts, and mindlessness as consequences. But when extrapolation is capacity demanding and the environment allows a strategy that is based on perceptually available information and hence imposes low cognitive load, solutions that go beyond prior experience become possible. Hence, the counterintuitive beneficial effect of low WMC on insightful problem solving.

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