Title

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by
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Report to the Sponsors: Division of Highways, Department of Public Works, State of California, and the Bureau of Public Roads, Federal Highway Administration, United States Department of Transportation.

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Structures and Materials Research
Department of Civil Engineering
Division of Structural Engineering
and
Structural Mechanics

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to

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ABSTRACT

A method is presented for the analysis of orthotropic folded plate structures with eccentric stiffeners. The development is based on the derivation of a finite strip stiffness which couples the plate bending and the in plane action due to the eccentricity of the ribs. Harmonic analysis is utilized in conjunction with the direct stiffness method providing a very efficient computer program which can handle a variety of different loadings. At present the program is restricted to the analysis of prismatic folded plate structures which are simply supported at the two end diaphragms.

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LIST OF SYMBOLS

A list of often used symbols and their general meaning is summarized below. The notation distinguishes matrices which are denoted by straight brackets from vectors which are indicated by braces.

Latin Letters

a	Half span length of finite strip
A	Area of finite strip
A^x, A^y	Area of x, y stiffeners
b	Half width of finite strip
B	Width of finite strip
BW	Half band width
$[C]$	Elastic orthotropic plane stress material law with principal axes of orthotropy along x, y coordinates
C_{xx}, C_{xy}, C_{yy}, G_{xy}	Components of $[C]$
$[D]_P$, $[D]_R$	Material law relating stress resultants of plate and rib system to the strains and curvatures
$[D^N]$, $[D^M]$, $[D^{NM}]$	Submatrices of $[D]$
DOF	Number of degrees of freedom
e_x, e_y	Eccentricity of the centroid of x, y ribs to the mid surface of plate
E_x, E_y	Elastic moduli in x, y direction for plane stress material
E^x, E^y	Elastic moduli for x, y ribs
f	Body force; In two dimensional elasticity-surface loads
$\{f\}$	Nodal intensities of surface loads
f_u, f_v, f_w	Components of $\{f\}$
G_{xy}, G_R	Shear moduli for plate and rib

H^x, H^y	Rigidity of eccentric x, y stiffeners with closed cell cross-section which couples the twisting moment with the shear strain in the plate
I^x, I^y	Moment of inertia of x, y ribs about midsurface of plate
J^x, J^y	Torsional rigidity of x, y ribs
k_n	Constant = $n\pi/L$
$[k]$	Stiffness of finite strip in local x, y, z coordinates
$[k_{\varepsilon\varepsilon}]$, $[k_{\kappa\kappa}]$, $[k_{\varepsilon\kappa}]$	Submatrices of $[k]$
$[\bar{k}]$	Stiffness of finite strip in global X, Y, Z coordinates
$[K]$	Structural stiffness matrix; Assembly matrix
L	Span length of finite strip
$\{M\}$	Moment stress resultants (symmetric)
M_x, M_y, $M_{xy} = M_{yx}$	Components of $\{M\}$
$\{\bar{M}\}$	Moment stress resultants (a-symmetric)
M_x, M_y, $M_{xy} \neq M_{yx}$	Components of $\{\bar{M}\}$
MULSTR	Finite strip computer program for the analysis of orthotropic folded plates which are simply supported
MULTPL	Folded plate computer program for the analysis of isotropic folded plates which are simply supported
MUPDI	Folded plate computer program for the analysis of isotropic folded plates with interior diaphragms and supports
n	Number of harmonics
N	Number of harmonics times degrees of freedom = n*DOF
$\{N\}$	In plane stress resultants

N_x, N_y, $N_{xy} = N_{yx}$	Components of $\{N\}$
p	Surface loads; In two dimensional elasticity-joint loads
$\{p\}$	Nodal intensities of joint loads
p_u, p_v, p_w	Components of $\{p\}$
$\{r\}$	Global nodal displacements in X, Y, Z direction
r_{si}, r_{hi}, r_{vi}, $r_{\theta i}$	Components of $\{r\}$
$\{R\}$	Global nodal loads in X, Y, Z direction
R_{si}, R_{hi}, R_{vi}, $R_{\theta i}$	Components of $\{R\}$
$\{S\}$	Consistent nodal loads in local x, y, z direction
U_i, V_i, Q_i, M_i	Components of $\{S\}$
$\{\overline{S}\}$	Consistent nodal loads in global X, Y, Z direction
S_{si}, S_{hi}, S_{vi}, $S_{\theta i}$	Components of $\{\overline{S}\}$
s^x, s^y	Spacing of x, y ribs
S^x, S^y	Static moment of x, y ribs about midsurface of plate
$\{u\}$	Displacement field
u_o, v_o, w_o	Components of midsurface displacements
$\{v\}$	Approximation of displacement field
u, v, w	Components of $\{v\}$
$\{V\}$	Nodal displacement vector in local x, y, z direction
u_i, v_i, w_i, θ_i	Components of $\{V\}$
$\{\overline{V}\}$	Nodal displacement vector in global X, Y, Z direction
$\overline{V}_{si}$, $\overline{V}_{hi}$, $\overline{V}_{vi}$, $\overline{V}_{\theta i}$	Components of $\{\overline{V}\}$

x, y, z	Right handed local coordinates for finite strip
$\bar{x}, \bar{y}$	Normalized local coordinates < 1
$\bar{y}_i$	Normalized joint coordinate = ± 1
X, Y, Z	Right handed global coordinates for folded plate structure

Greek Letters

α, β	$\bar{x}$-distances from origin defining location of partial loading
$\bar{\delta}$	Normalized width of partial loading
$\{\epsilon\}$	Strain vector
$\epsilon_x, \epsilon_y, \gamma_{xy}$	Components of $\{\epsilon\}$
$\{\kappa\}$	Curvature vector
$\kappa_x, \kappa_y, \kappa_{xy}$	Components of $\{\kappa\}$
$[\Phi_v]$	Functional approximation of displacement field
Φ_u, Φ_v, Φ_w	Components of $[\Phi_v]$
$[\psi_f]$	Functional approximation of strip surface loads
ψ_u, ψ_v, ψ_w	Components of $[\psi_f]$
$[\psi_p]$	Functional approximation of joint loads
ψ_u, ψ_v, ψ_w	Components of $[\psi_p]$
$\pi(u)$	Total potential energy
$\pi(v)$	Approximation of total potential energy
$\{\sigma\}$	Stress vector
$\sigma_x, \sigma_y, \tau_{xy}$	Components of $\{\sigma\}$
$\bar{\xi}$	$\bar{x}$-distance to centroid of partial loading

1. INTRODUCTION

1.1 Objective

The objective of this investigation was the development of a general method of analysis for prismatic box girder bridges made up of orthotropic plates having closely spaced eccentric stiffeners or ribs. The study was restricted to the elastic analysis of bridges simply supported at the two ends. Ultimate goal of the investigation was the extension of the general computer programs MULTPL and MUPDI developed for the analysis of prismatic box girder bridges with isotropic plates to the case cited above.

1.2 General Remarks

In recent years bridges having cellular box girder cross-sections of various types have been proposed and used as economic and aesthetic solutions for the over-crossings, under-crossings, separation structures and viaducts found in today's modern highway system. The very large torsional rigidity of the box girder's closed cellular section provides structural efficiency, while its broad unbroken soffit, viewed from beneath, provides a pleasing appearance.

In California, the most widely used cellular type bridge is the reinforced or prestressed concrete box girder bridge, Fig. 1, which has a typical cross-section consisting of a top and bottom slab interconnected monolithically by vertical or sloping webs to form a cellular or box-like structure. Another type of cellular bridge is the composite steel-concrete box girder bridge, Fig. 2. This bridge consists of a concrete deck acting integrally with cellular steel boxes. The individual steel boxes are spaced uniformly over the width of the

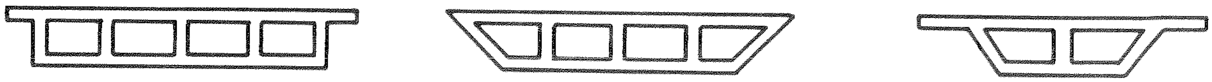


FIG. 1 TYPICAL CROSS-SECTIONS OF REINFORCED OR PRESTRESSED CONCRETE BOX GIRDER BRIDGES

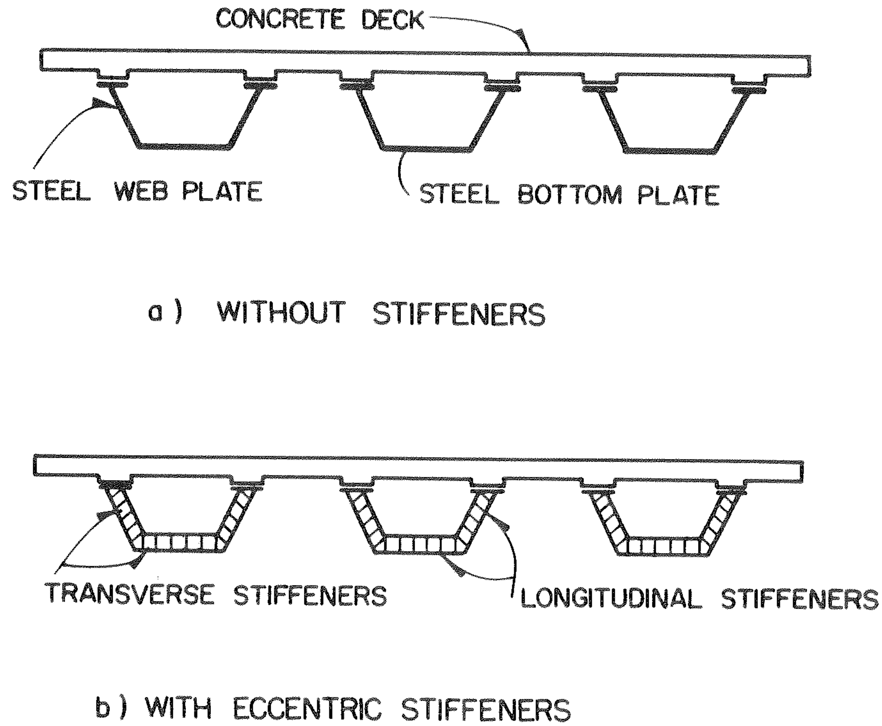


FIG. 2 TYPICAL CROSS-SECTIONS OF COMPOSITE STEEL-CONCRETE BOX GIRDER BRIDGES

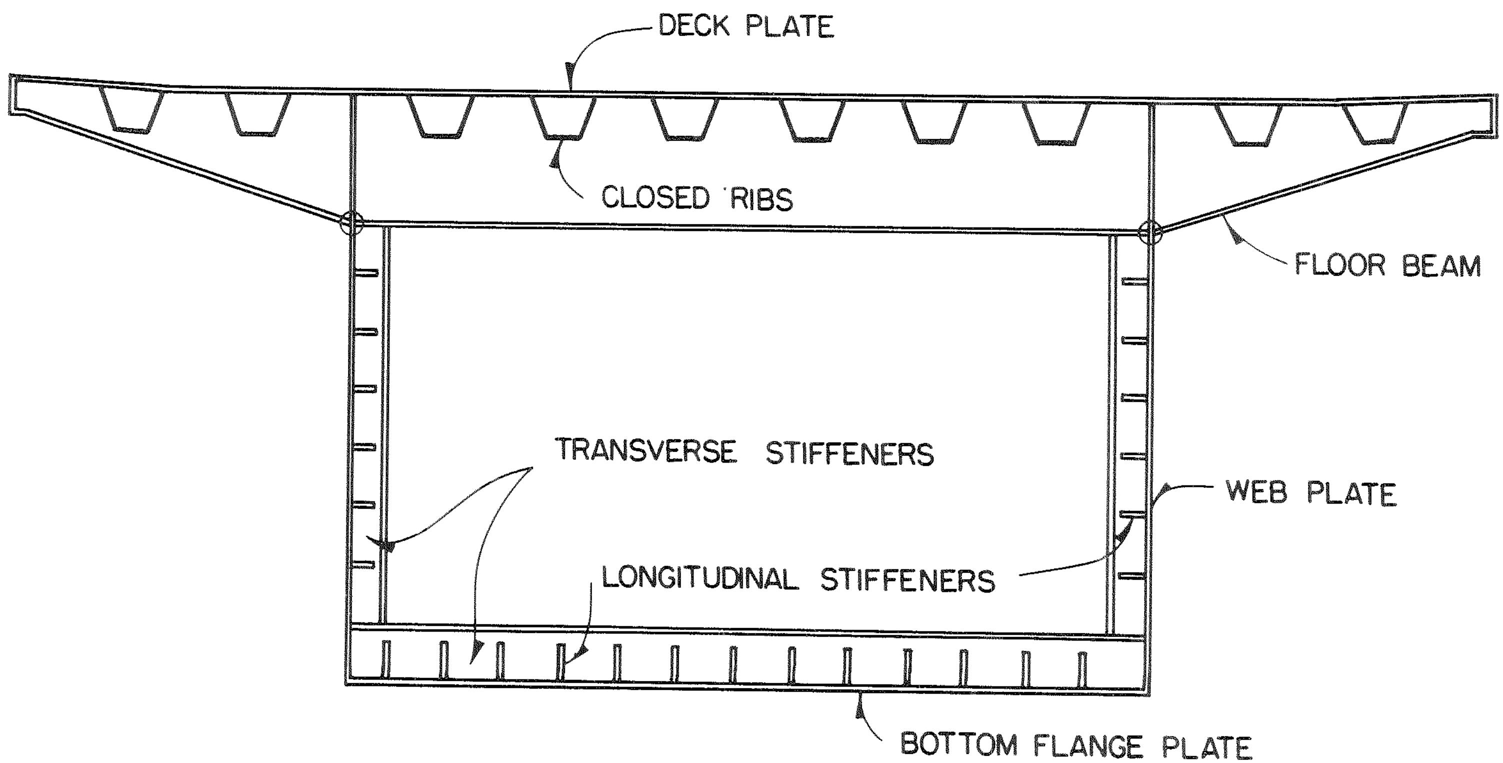


FIG. 3 TYPICAL CROSS-SECTION OF ORTHOTROPIC STEEL DECK BRIDGE

bridge. Each box consists of two narrow top flange plates welded to inclined web plates and a wide bottom flange plate connecting the two webs to form a steel box. In many cases eccentric transverse and longitudinal stiffeners are added to the web and bottom flange plates.

For long span bridges, orthotropic steel deck bridge systems, Fig. 3, have been used successfully in a number of cases. The bridge deck, stiffened by closed or open ribs and supported by transverse floor beams spaced at regular intervals longitudinally, is carried by one or more large steel box girder sections in which the web and bottom flange plates also have eccentric transverse and longitudinal stiffeners welded to them.

The accurate determination of internal stresses, forces, moments, and displacement in any of these box girder bridges requires the analysis of a highly indeterminate structure. Because of the complexity of these analyses, they must be programmed for solution by a digital computer to be of practical use.

1.3 Previous Studies

The present report is the fourth in connection with a continuing research program on box girder bridges at the University of California.

The first two reports [1, 2] dealt with the development of methods of analysis and general computer programs for the determination of internal forces, moments and displacements in simple and continuous multicelled box girder bridges made up of isotropic plates.

The third report [3] had the objective of studying wheel load distribution in concrete box girder bridges subjected to standard design truck loadings. A large number of cases were studied using the computer programs described in the first two reports [1, 2]. Based on these studies, improved design methods were presented for determining wheel load distribution in these bridges [3].

The present study extends the work previously done for box girder bridges with isotropic plates to bridges with orthotropic plates having closely spaced eccentric stiffeners or ribs.

No attempt will be made here to review the extensive literature on orthotropic plate bridges. Much of the theory for orthotropic plates used in this report is based on a formulation presented by Clifton, Chang, and Au [4]. Extensive discussions and lists of references on orthotropic plate bridges may be found in the publications prepared by Wolchuk [9] and by Troitsky [10].

In the present investigation, a direct stiffness solution similar to the harmonic analysis of folded plate theory [1, 2] is utilized in combination with a finite strip method for determining the plate element stiffnesses and consistent loadings. The finite strip method, which is a special form of the finite element method, has been described by Cheung [5, 6, 7] and by Powell [8].

1.4 Scope of Present Investigation

This investigation is concerned with the elastic analysis of prismatic box girder bridges made up of orthotropic plates having closely spaced eccentric stiffeners or ribs. Multicelled structures, simply supported at the two ends are considered.

In the present study a direct stiffness solution for box girder bridges using a folded plate harmonic analysis is utilized. This approach, briefly reviewed in Chapter 2, is the same as that used for bridges with isotropic plates previously reported [1, 2]. The key step in such a solution is the development of the stiffness matrix for the individual plates which make up the bridge cross-section. For isotropic plates this can be done directly using classical thin plate bending theory for loads normal to the plate (slab action) and two-dimensional plane stress theory for loads in the plane of the plate (membrane action). In the present case of orthotropic plates with eccentric ribs, the direct approach becomes too complex so that an alternative approach known as the finite strip method is used to develop the stiffness matrix and the corresponding consistent loadings for individual plates. This method, which is discussed in Chapter 3, may be thought of as a special form of the finite element method. It idealizes each plate by an assemblage of finite strips spanning in the longitudinal direction. Selected displacement patterns varying as harmonics longitudinally and as polynomials in the transverse direction represent the behavior of each strip in the total structure. As for all finite element methods, the finite strip method must be considered an approximate method in which the accuracy of the results obtained is dependent on the discretization used and the displacement patterns selected.

Once the stiffness matrix and the corresponding consistent loadings for individual strips have been derived, they may be used in the direct stiffness solution, which treats the structure as an assemblage of individual strips interconnected along the longitudinal joints. The development of general computer programs for box girder bridges made up of orthotropic plates with eccentric ribs follows the programs MULTPL and MUPDI which were developed for bridges with isotropic plates

[1, 2]. The new computer program named MULSTR is described in Chapter 4, and Appendices A and B contain both the input specifications and the FORTRAN IV listing for this program.

In order to check out the program developed several examples are considered in Chapter 5. In Examples 1 and 2, single isotropic plates under in-plane and normal loadings are analyzed. In Example 3, a general folded plate system consisting of several interconnected isotropic plates is studied. A horizontal plate with four eccentric vertical ribs is considered in Example 4. Results obtained by the finite strip method using the program MULSTR for Examples 1, 2, 3 and 4 are compared with those obtained by the elasticity method of folded plate theory using the program MULTPL. In Examples 5, 6 and 7, an orthotropic deck bridge example taken from a paper by Clifton, Chang and Au [4] is analyzed and the results are compared. Finally in Example 8, several cases of a single cell box are studied in which various amounts of transverse and longitudinal eccentric stiffeners are used. Results are compared and the effects of the stiffeners are briefly discussed.

2. ANALYSIS OF MULTICELL BOX GIRDER BRIDGES

2.1 General

A structure may be thought of as an assemblage of structural elements interconnected at joints or nodes. The size, type, and structural properties of the individual elements are dependent on the analytical model selected to idealize the actual structure. The problem to be solved in any structural analysis problem may be stated simply: given, a structure with known geometry, material properties, loading and boundary conditions; find the displacement of the joints and the internal forces in each of the structural elements. When such problems are solved with the aid of a digital computer, a direct stiffness method of solution is commonly employed. This method has been described in detail in previous reports [1,2] and consists of the following basic steps.

1. Derive the element stiffness k for each element in a local coordinate system.
2. Transform the element stiffnesses to a global coordinate system.
3. Assemble the structure stiffness K for the entire structure by properly adding the element stiffnesses.
4. Determine the load vector R.
5. Solve the equilibrium equation $R = Kr$ for the joint displacements r.
6. Compute the internal forces S in each element using the displacements r found in step 5.

2.2 Previous Analystical Models and Methods

For a box girder bridge a number of analytical models may be selected to idealize the structure. Three analytical models and methods of solution for prismatic bridges made up of isotropic plate elements have been discussed in detail in [2] and will be briefly reviewed here.

The first approach is the folded plate method which is restricted to bridges simply supported at the two ends. These end boundary conditions permit the use of a harmonic analysis utilizing Fourier series in the longitudinal direction. The basic structural element, Fig. 4, is a single plate having a width equal to the distance between longitudinal joints and a length equal to the overall length of the bridge. Element stiffnesses are determined by the elasticity method in which classical thin plate bending theory is used for loads normal to the plane of the plate (slab action) and two dimensional plane stress theory is used for loads in the plane of the plate (membrane action).

The second approach is the finite segment method which can be applied to bridges with arbitrary boundary conditions at the two ends. The basic structural element, Fig. 5, is a finite segment which is formed by dividing each plate element into a finite number of segments longitudinally. These finite segments each have a width equal to the transverse distance between the longitudinal joints. Nodal points are located at the midpoints of the four sides of the finite segments. Each finite segment has 14 degrees of displacement freedom and 14 corresponding forces. The relation between these forces and displacements are determined using elementary beam theory for in plane loads and transverse one way slab action for loads normal to the plane of the plate.

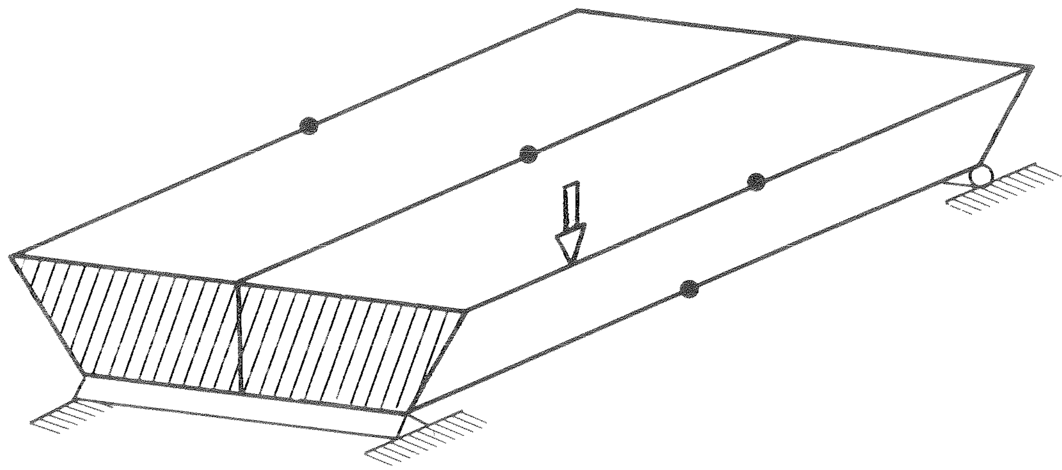


FIG. 4 FOLDED PLATE ANALYTICAL MODEL

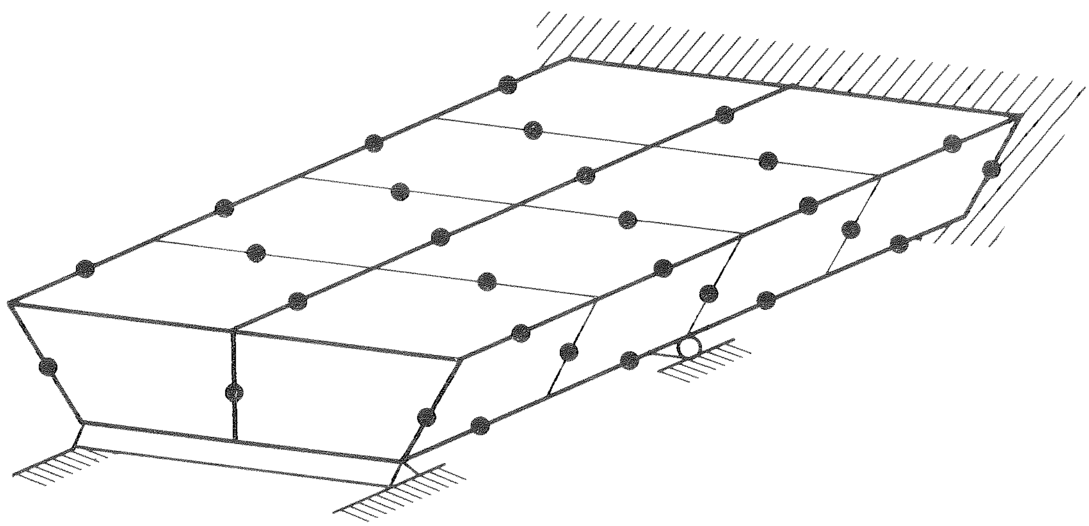


FIG. 5 FINITE SEGMENT ANALYTICAL MODEL

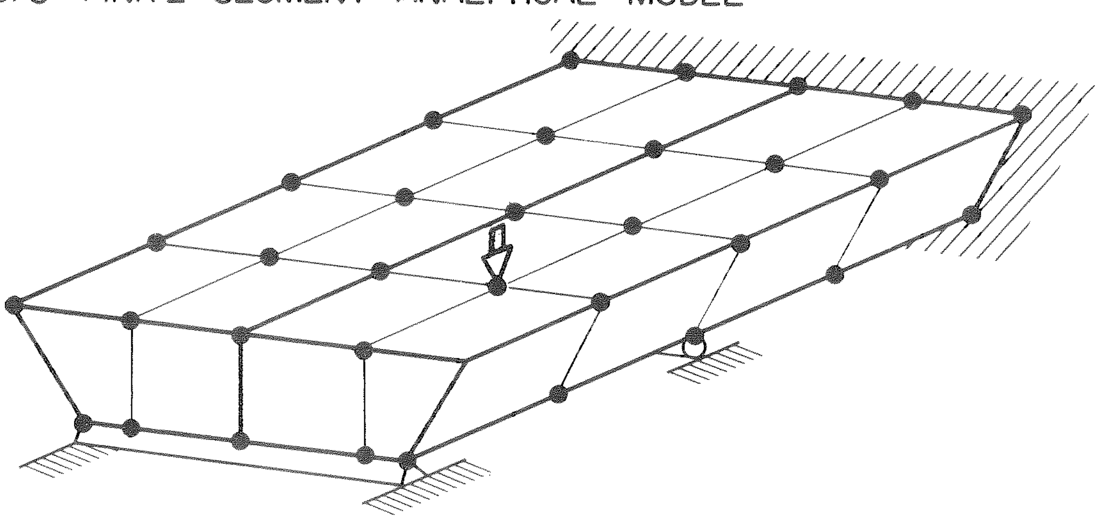


FIG. 6 FINITE ELEMENT ANALYTICAL MODEL

The third approach is the finite element method which can be applied to bridges with arbitrary boundary conditions. The basic structural element, Fig. 6, is formed by dividing each plate element transversely as well as longitudinally into an assemblage of smaller rectangular finite elements. The size, thickness and material properties of these rectangular finite elements can be varied as desired throughout the structure. The rectangular finite element used for prismatic box girder bridges [2] has nodes at the four corners only. Each node has 6 degrees of freedom making a total of 24 for each finite element. Element stiffnesses are determined using the principle of virtual work.

For bridges simply supported at the two ends composed of isotropic plates the folded plate method is greatly superior to the other two methods because it is an exact method of analysis and it requires the least amount of computer time and storage for a solution. Two general purpose computer programs MULTPL and MUPDI [1,2] have been developed using the folded plate method.

For bridges simply supported at the two ends, composed of orthotropic plates with eccentric ribs, the elasticity theory used to develop the element stiffnesses in the folded plate method becomes too complex and therefore a finite strip method is adopted for this purpose.

2.3 Finite Strip Method

In this method each plate is divided into a number of longitudinal finite strips, Fig. 7. The properties within each strip are taken as constant, however transverse variations in the properties of a

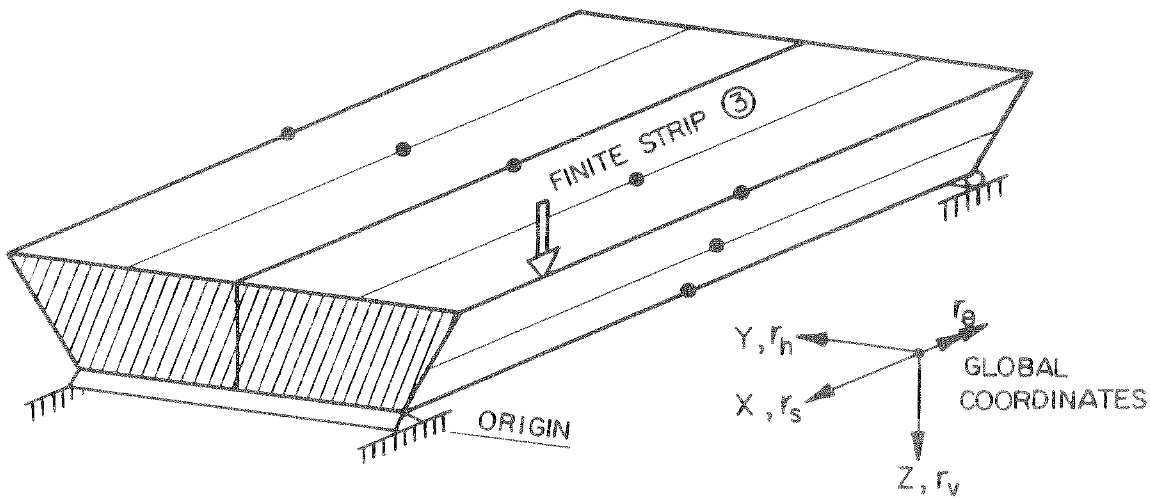


FIG. 7 FINITE STRIP ANALYTICAL MODEL

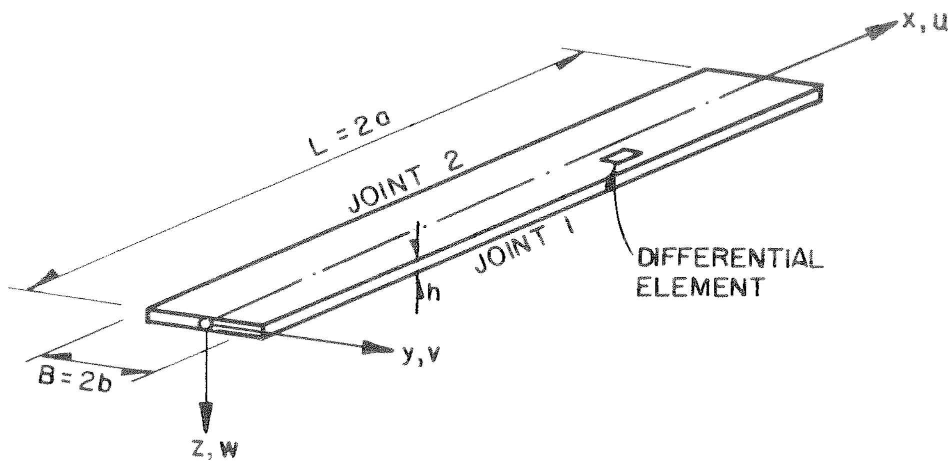


FIG. 8 DIMENSIONS AND LOCAL COORDINATE SYSTEM FOR FINITE STRIP ③

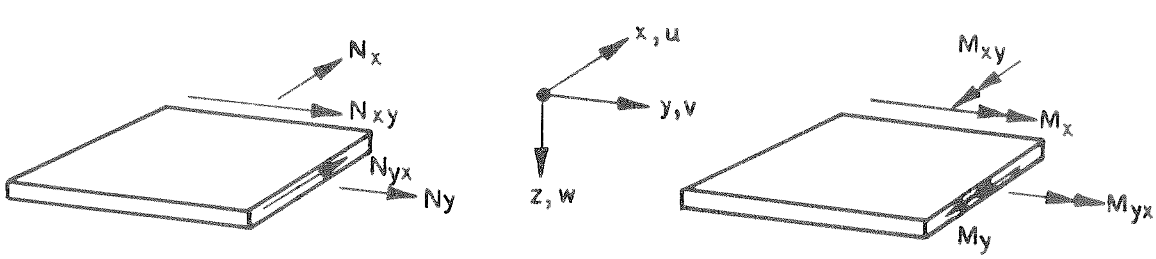


FIG. 9 POSITIVE DIRECTIONS OF INTERNAL FORCES ACTING ON A DIFFERENTIAL ELEMENT IN A FINITE STRIP

plate may be approximated by assigning different properties to each strip making up the plate. The stiffness matrix for each finite strip is derived in the same manner as that used in the finite element method. However, advantage is taken of the simple support conditions at the two ends of the strip. A harmonic analysis can be used such that all displacements, loadings, internal forces, etc., Figs. 8 and 9, can be expressed as harmonics of a Fourier series. Displacement functions varying as harmonics longitudinally and as polynomials transversely are used in deriving the stiffness matrix and the consistent loadings for each strip. Using this approach the nodal point forces S and the displacements V for each harmonic are as shown in Fig. 10a. Each nodal point has four degrees of displacement freedom and four corresponding forces. Once the element stiffness matrix k relating S to V has been derived, the direct stiffness method may be used to obtain the resulting displacements for each harmonic. The final solutions are obtained by summing the results for all of the harmonics used to represent the load. The sign convention and global coordinate systems for forces and displacements, which were used in developing the computer programs MULTPL and MUPDI, are also chosen in the present study. They are illustrated in Figs. 10b and 11.

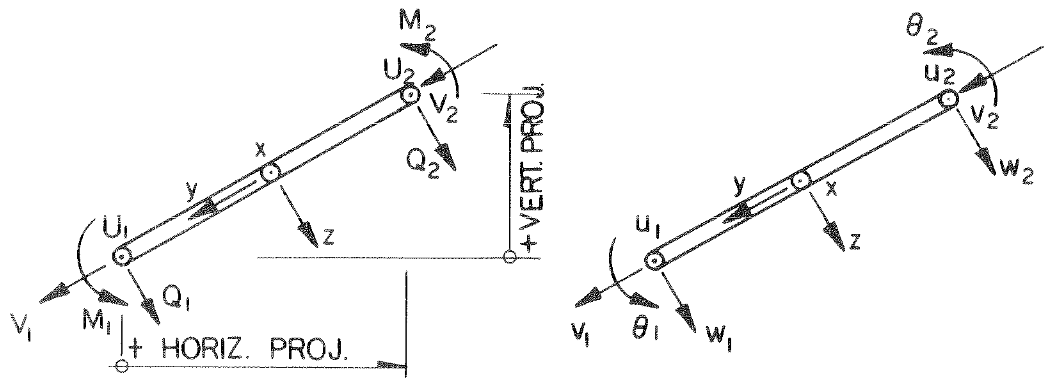


FIG. 10a NODAL POINT FORCES S AND DISPLACEMENTS V IN LOCAL STRIP COORDINATES

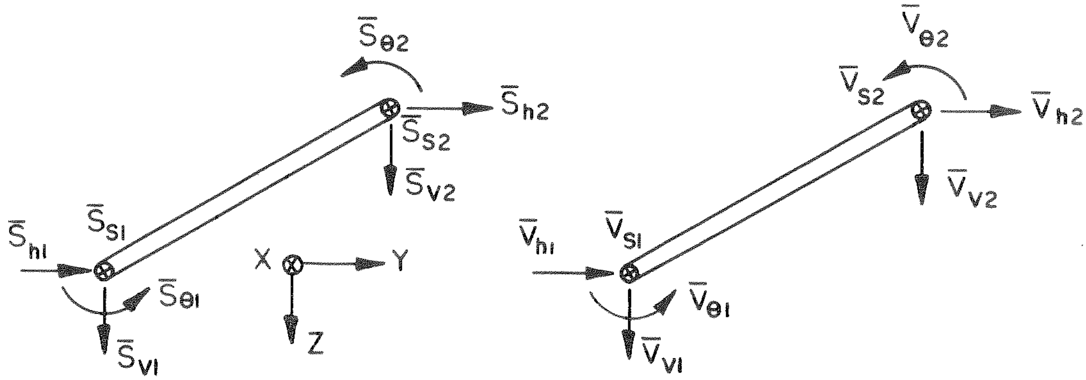


FIG.10b NODAL POINT FORCES $\bar{S}$ AND DISPLACEMENTS $\bar{V}$ IN GLOBAL COORDINATES

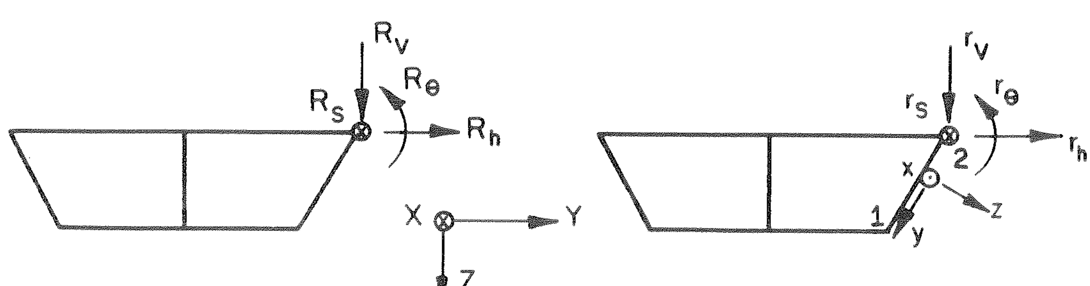


FIG.11 GLOBAL NODAL POINT FORCES R AND DISPLACEMENTS r LOOKING TOWARDS ORIGIN

3. FINITE STRIP ANALYSIS OF ORTHOTROPIC PLATE ELEMENTS WITH ECCENTRIC RIBS

3.1 General

Each finite strip is assumed to be made up of a deck-plate with closely spaced eccentric ribs or stiffeners in the longitudinal and transverse directions. For simplicity the combined plate-rib system is often referred to simply as an orthotropic plate. The properties of the orthotropic plate are assumed to be constant over the entire strip.

The two basic types of eccentric ribs used are designated as torsionally soft ribs and torsionally stiff ribs (Figs. 12 and 13). The former consists of open slender sections that have little torsional resistance, whereas the latter includes open sections or closed box sections with considerable torsional resistance. A reference plane, $z = 0$, is selected at the mid-depth of the deck plate, and all internal forces and moments (stress-resultants) shown in Figs. 12 and 13 are taken with reference to this plane. The basic theory for orthotropic plates with either torsionally soft or torsionally stiff eccentric ribs loaded normal to their own plane has been presented by Clifton, Chang and Au [4]. This theory will be used and extended to include loads in the plane of the plate in the development of the element stiffness matrix and the consistent loadings for the finite strip analysis to be presented in this chapter.

The following assumptions are made for orthotropic plates with torsionally soft eccentric ribs.

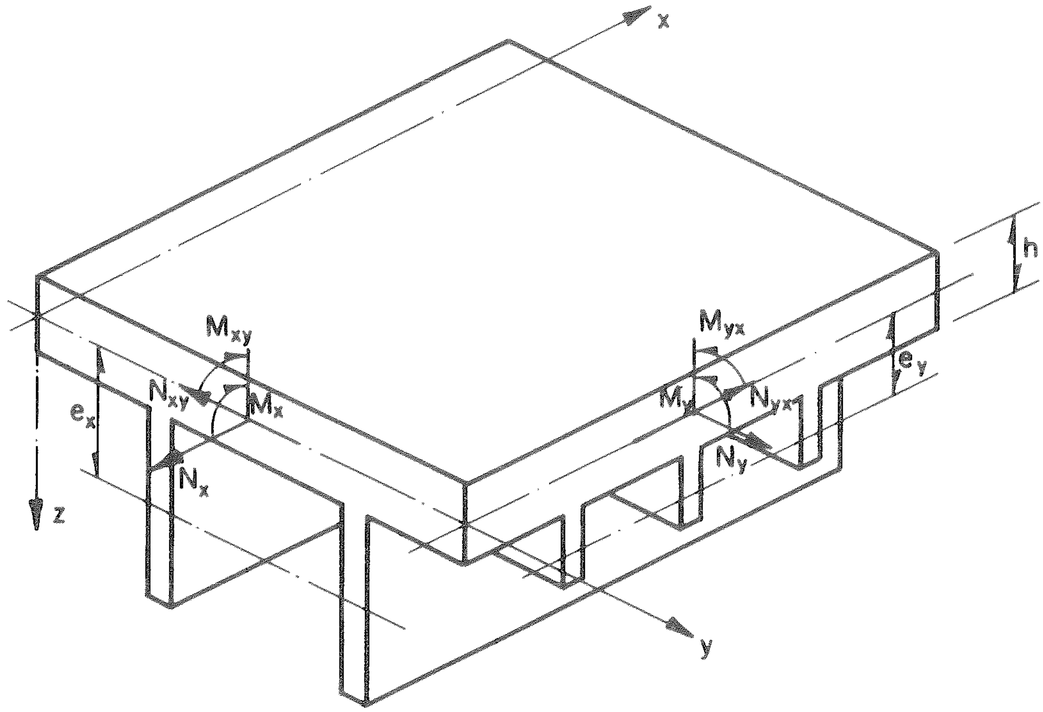


FIG. 12 TYPICAL ELEMENT OF TORSIONALLY SOFT ORTHOTROPIC PLATE

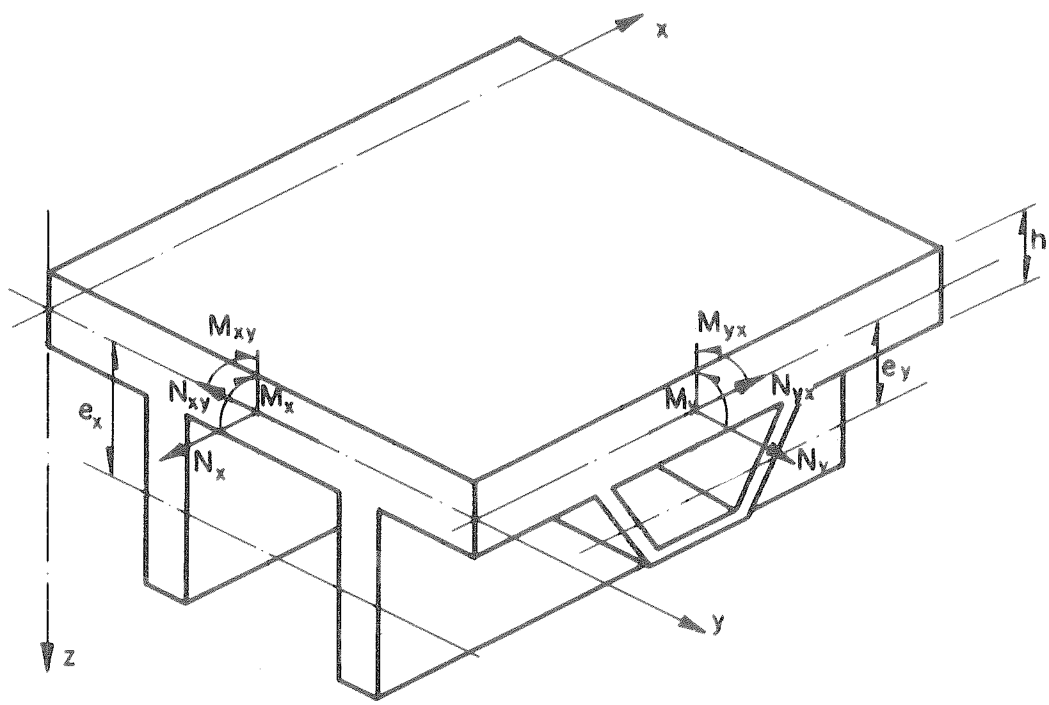


FIG. 13 TYPICAL ELEMENT OF TORSIONALLY STIFF ORTHOTROPIC PLATE

1. External loads are normal to or in the plane of the middle surface of the deck plate.
2. The orthotropic plate acts as a monolithic unit, therefore there is no relative movement between the deck plate and the ribs.
3. The deck plate is homogeneous, elastic, of constant thickness and has orthotropic properties in the longitudinal x, and transverse y direction.
4. The ribs in each direction are homogeneous, elastic, and isotropic and may have arbitrary cross-sections, that are repetitive and equally spaced in each direction. The spacing of the ribs is small in relation to the span length.
5. In the case of torsionally soft orthotropic plates, it is further assumed that the ribs consist of open sections which cannot resist torsion.
6. Plane sections initially perpendicular to the middle surface of the deck plate remain plane and perpendicular to the middle surface during slab bending.
7. Deflections are small in relation to the thickness of the orthotropic plate.

For orthotropic plates with torsionally stiff eccentric ribs the same assumptions are used except for the following modifications:

1. The deformation caused by the torsional warping is small, so that the assumption of plane sections remaining plane during bending may still be used.

2. The angle of twist per unit length of the closed box section is the same as that of the middle surface of the plate.
3. The torsional stiffness of a closed box section may be estimated by neglecting any restraint due to the warping of the cross-section.
4. The thickness of the rib forming a closed box section is constant and small compared to its length.

3.2 Kinematics

Displacements and deformations are assumed small, therefore, $\epsilon_z \ll 1$, $w_A = w_O$, $\gamma_{xz} = \gamma_{yz} = 0$ and $z' = z$, as illustrated in Fig. 14. For any point the displacements are

$$
\begin{aligned}
w &= w_0 \\
u &= u_0 + z\left(\frac{\partial u}{\partial z}\right)_0 = u_0 - z\,\frac{\partial w}{\partial x} \\
v &= v_0 + z\left(\frac{\partial v}{\partial z}\right)_0 = v_0 - z\,\frac{\partial w}{\partial y}
\end{aligned}
\tag{3.1}
$$

in which the subscript 0 indicates quantities at the midsurface of the deck plate, $z = 0$.

The linearized strain displacement relationships are

$$
\begin{aligned}
\epsilon_x &= \frac{\partial u}{\partial x} \\
\epsilon_y &= \frac{\partial v}{\partial y} \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
\end{aligned}
\tag{3.2}
$$

Plate curvatures are defined as

$$\varkappa_x = -\frac{\partial^2 w}{\partial x^2}$$
$$\varkappa_y = -\frac{\partial^2 w}{\partial y^2} \quad (3.3)$$
$$\varkappa_{xy} = -\frac{\partial w}{\partial x \partial y}$$

Substituting Eqs. (3.1) and (3.3) into (3.2)

$$\epsilon_x = \epsilon_x^0 + z\,\varkappa_x$$
$$\epsilon_y = \epsilon_y^0 + z\,\varkappa_y \quad (3.4)$$
$$\gamma_{xy} = \gamma_{xy}^0 + 2z\,\varkappa_{xy}$$

or in matrix form

$$\{\epsilon\} = \{\epsilon_0\} + z\,\{\varkappa\} \quad (3.4a)$$

where

$$\{\epsilon_0\} = \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \quad \text{and} \quad \{\varkappa\} = \begin{Bmatrix} \varkappa_x \\ \varkappa_y \\ 2\varkappa_{xy} \end{Bmatrix} \quad (3.5)$$

3.3 Constitutive Relationships

An orthotropic material law, with principal axes of orthotropy parallel to the x and y axes, describes the linearly elastic plane stress behavior of the deck plate in each finite strip

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_p = \begin{bmatrix} C_{xx} & C_{xy} & 0 \\ C_{yx} & C_{yy} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix}_p \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (3.6)$$

or simply

$$\{\sigma\}_p = [C]_p \{\varepsilon\} \tag{3.6a}$$

where it is assumed that

$$[C_p] = [C_p]^T \tag{3.7}$$

and

$$C_{xx} = \frac{E_x}{1 - \nu_{xy}\nu_{yx}}, \quad C_{yy} = \frac{E_y}{1 - \nu_{xy}\nu_{yx}}$$

$$C_{xy} = C_{yx} = \frac{E_y \nu_{yx}}{1 - \nu_{xy}\nu_{yx}} = \frac{E_x \nu_{xy}}{1 - \nu_{xy}\nu_{yx}} \tag{3.8}$$

which requires

$$E_y \nu_{yx} = E_x \nu_{xy} \tag{3.9}$$

ν_{xy} is defined as the ratio of the strain in the x direction to that in the y direction due to a uni-axial stress in the y-direction. ν_{yx} has a similar definition, only interchanging x and y. For an isotropic material

$$E_x = E_y = E$$

$$\nu_{xy} = \nu_{yx} = \nu \tag{3.10}$$

Soft eccentric ribs are subjected to a uniaxial state of stress and are assumed to have zero torsional stiffness. The rib stresses in the x and y direction are computed by simple beam theory with ν and $\tau_{xy} = 0$ and are related to the strains as follows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_R = \begin{bmatrix} E_x & 0 & 0 \\ 0 & E_y & 0 \\ 0 & 0 & 0 \end{bmatrix}_R \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{3.11}$$

or simply

$$\{\sigma\}_R = [C]_R \{\varepsilon\} \tag{3.11a}$$

3.4 Force-Displacement Relationships for Torsionally Soft Ribs

In the case of torsionally soft ribs the torsionally rigidity of the ribs is assumed to be zero. The stress resultants for the combined plate-rib system are shown in Fig. 12. These quantities are taken with reference to the middle surface of the plate, z = 0, and may be subdivided in the following sets of membrane forces and slab moments:

$$\{N\} = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \qquad \{M\} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} \tag{3.12}$$

Note that for the present case $N_{xy} = N_{yx}$ and $M_{xy} = M_{yx}$. Let

z = distance from middle surface of the plate

s = spacing of adjacent ribs

A = rib area excluding deck plate

h = plate thickness

For the membrane forces

$$\{N\} = \int_{-h/2}^{h/2} \{\sigma\}_p \, dz + \int_{A/s} \{\sigma\}_R \, dz$$

$$= \int_{-h/2}^{h/2} [C]_p (\{\varepsilon_0\} + z\{\kappa\}) dz + \int_{A/s} [C]_R (\{\varepsilon_0\} + z\{\kappa\}) dz$$

Note that

$$\int_{-h/2}^{h/2} z dz = 0; \qquad \int_{A/s} z dz \neq 0$$

therefore

$$\{N\} = [D]_P^N \{\varepsilon_0\} + [D]_R^N (\{\varepsilon_0\} + \{\kappa\}) \qquad (3.13)$$

For the slab moments

$$\{M\} = \int_{-h/2}^{h/2} \{\sigma\}_P \, zdz + \int_{A/s} \{\sigma\}_R \, zdz$$

$$= \int_{-h/2}^{h/2} [C]_P \, (z\{\varepsilon_0\} + z^2 \{\kappa\})dz + \int_{A/s} [C]_R \, (z\{\varepsilon_0\} + z^2 \{\kappa\})dz$$

$$\{M\} = [D]_P^M \{\kappa\} + [D]_R^M (\{\varepsilon_0\} + \{\kappa\}) \qquad (3.14)$$

or combining Eqs. (3.13) and (3.14)

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \left(\begin{bmatrix} D^N & 0 \\ 0 & D^M \end{bmatrix}_P + \begin{bmatrix} D^N & D^{NM} \\ D^{MN} & D^M \end{bmatrix}_R \right) \begin{Bmatrix} \varepsilon_0 \\ \kappa \end{Bmatrix} \qquad (3.15)$$

Note that $D_R^{MN} = D_R^{NM}$ couples the membrane and bending action.

By performing the necessary integrations in Eqs. (3.13) and (3.14), explicit expressions may be derived. The symbols used are defined as follows:

A^x, A^y = rib area of x, y ribs, see shaded area Fig. 15

s^x, s^y = spacing of x, y ribs

E^x E^y = elastic modulus of x, y ribs

S^x S^y = static moment of rib areas A^x, A^y about middle surface

I^x, I^y = Moment of inertia of rib areas A^x, A^y about middle surface

With the above definitions, the elements of the D matrix of Eq. (3.15) are defined as follows:

$$[D^N] = [D^N]_P + [D^N]_R$$

$$= \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} = \left[\begin{array}{c|c|c} (h\ C_{xx} + \frac{A^x\ E^x}{s^x}) & h\ C_{xy} & 0 \\ \hline h\ C_{yx} & (h\ C_{yy} + \frac{A^y\ E^y}{s^y}) & 0 \\ \hline 0 & 0 & h\ G_{xy} \end{array}\right] \quad (3.16)$$

$$[D^M] = [D^M]_P + [D^M]_R$$

$$= \begin{bmatrix} D_{44} & D_{45} & 0 \\ D_{54} & D_{55} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} = \left[\begin{array}{c|c|c} (\frac{h^3\ C_{xx}}{12} + \frac{I^x\ E^x}{s^x}) & \frac{h^3\ C_{xy}}{12} & 0 \\ \hline \frac{h^3\ C_{yx}}{12} & (\frac{h^3\ C_{yy}}{12} + \frac{I^y\ E^y}{s^y}) & 0 \\ \hline 0 & 0 & \frac{h^3\ G_{xy}}{12} \end{array}\right] \quad (3.17)$$

$$[D^{NM}] = [D^{NM}]_R$$

$$= \begin{bmatrix} D_{14} & 0 & 0 \\ 0 & D_{25} & 0 \\ 0 & 0 & 0 \end{bmatrix} = \left[\begin{array}{c|c|c} S^x\ E^x\ /s^x & 0 & 0 \\ \hline 0 & S^y\ E^y\ /s^y & 0 \\ \hline 0 & 0 & 0 \end{array}\right] \quad (3.18)$$

Note that for torsionally soft ribs the coupling matrix is symmetric:

$$D^{MN} = (D^{NM})^T = D^{NM} \quad (3.19)$$

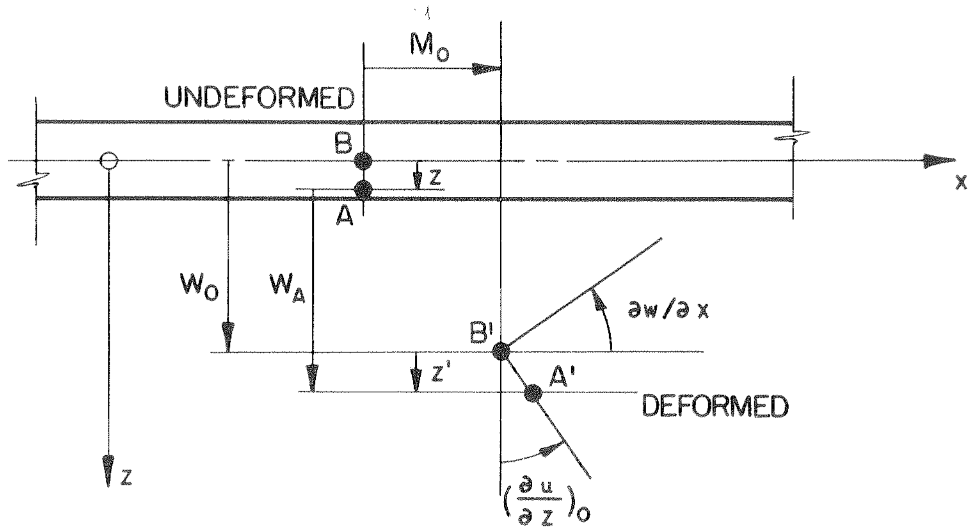
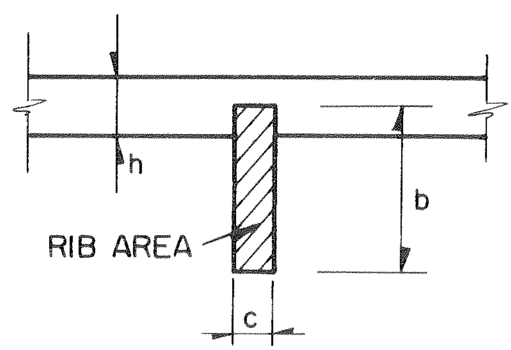
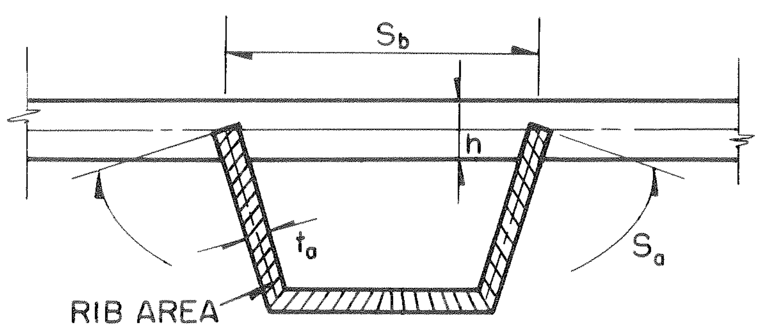


FIG. 14 KINEMATIC RELATIONSHIPS



a) OPEN SECTION



b) CLOSED SECTION

FIG. 15 DIMENSIONS FOR RIGIDITY OF TORSIONALLY STIFF RIBS

The final force displacement relationships can be written

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} D^{N} & D^{NM} \\ D^{MN} & D^{M} \end{bmatrix} \begin{Bmatrix} \epsilon_0 \\ \kappa \end{Bmatrix} = [D] \begin{Bmatrix} \epsilon_0 \\ \kappa \end{Bmatrix} \tag{3.20}$$

where Eqs. (3.2) and (3.3) define the strains and curvatures in terms of the primary field variables, the displacements.

3.5 Force-Displacement Relationships for Torsionally Stiff Ribs

The stress resultants for this case are shown in Fig. 13. The torsionally stiff ribs may have either an open or closed section, Fig. 15. In general, the rib properties differ in the x and y direction and thus $M_{xy} \neq M_{yx}$ in Eq. (3.12), while as before $N_{yx} = N_{yx}$ because thin ribs do not affect the shear stress resultants.

The torsional rigidities of the x, y ribs about their shear center will be defined as J^x and J^y. Using a strength of materials approach as proposed in [4], these quantities can be expressed in terms of the rib properties, illustrated in Fig. 15. For an open section

$$J = \frac{1}{3} bc^3 \, G_R \tag{3.21}$$

For a closed section

$$J = \frac{4A_a^2 \, h \, t_a}{s_a h + s_b t_a} G_R \tag{3.22}$$

in which A_a is the total area enclosed within the perimeter centerline of the cell. For closed rib sections, an additional contribution to the twisting moment M_{xy} arises due to the combined action of the shear force along the middle surface of the plate and the constant shear flow induced in the cell. The following coupling term H is derived in [4] and can be defined in terms of the closed rib properties by

$$H = \frac{2\, A_a\, t_a\, s_b}{s_b\, t_a + s_a\, h}\; G_R\, h \tag{3.23}$$

Note that in deriving H in reference [4], its contribution to the twisting moment is not accompanied by twisting of the closed rib.

The twisting moments can be defined separately in terms of the associated kinematic and material quantities

$$\begin{Bmatrix} M_{xy} \\ \\ M_{yx} \end{Bmatrix} = \begin{bmatrix} \frac{H^x}{s^x} & 0 \\ \\ 0 & \frac{H^y}{s^y} \end{bmatrix} \begin{Bmatrix} \gamma_{xy} \\ \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} (D_{66} + \frac{J^x}{2s^x}) & 0 \\ \\ 0 & (D_{66} + \frac{J^y}{2s^y}) \end{bmatrix} \begin{Bmatrix} 2\kappa_{xy} \\ \\ 2\kappa_{xy} \end{Bmatrix} \tag{3.24}$$

For a simple description of the energy density in Eq. (3.38) one can compact M_{xy} and M_{yx} in order to retain a square (6 X 6) matrix D defining the constitution of the orthotropic plate. The contribution to the twisting moment, Eq. (3.23), is based on purely statical considerations. Since no kinematic deformation accompanies this force quantity, its contribution to the energy can be omitted similar to the shear strain energy in simple beam theory. There remains only the torsional rigidity of the ribs given by either Eq. (3.21) or (3.22) to be accounted for in the energy consideration for torsionally stiff ribs. This is done simply by modification of the coefficient D_{66} in Eq. (3.17) to

$$\bar{D}_{66} = D_{66} + \frac{(J^x + J^y)}{4} \tag{3.25}$$

in which J^x and J^y are obtained from either Eq. (3.21) or (3.22).

The final force displacement relationships for torsionally stiff ribs have the same form as those for torsionally soft ribs, (see Eq. (3.20), the only difference being the definition of D_{66}.

3.6 Principle of Minimum Potential Energy

The principle of minimum potential energy will be used to derive the element stiffness and consistent loadings for a typical finite strip. The total potential energy $\pi(u)$ for a finite strip is equal to the sum of the strain energy stored in the strip and the potential energy of the external loads acting on the strip and may be written in matrix form for a general three-dimensional system as follows:

$$\pi(u) = \int_V \left(\tfrac{1}{2}\{\epsilon\}^T[C]\{\epsilon\} - \{f\}^T\{u\}\right) dV - \int_A \{p\}^T \{u_s\}\, dA \qquad (3.26)$$

The potential energy $\pi(u)$ is expressed in terms of the primary field variable, the displacement u only. The strain field ϵ is derived by the strain displacement relationships from u, and C describes the linearly elastic properties of the material. The body forces f and the surface loads p are associated to the conjugate displacements u and u_s while V and A denote the volume and surface area respectively. For plate type structures subjected to membrane and slab action, the three dimensional-problem may be reduced to a two-dimensional boundary value problem utilizing the assumptions given earlier which are those of the Poisson-Kirchoff theory for plates. For this theory it is assumed that ϵ_z, γ_{xz} and γ_{yz} do not contribute to the strain energy.

The first variation, $\delta\pi(u) = 0$, of Eq. (3.26) yields as the Euler equation, the differential equations of equilibrium in a form similar to that in [4], and as natural boundary conditions it gives the force boundary conditions. Because closed form solutions for these differential equations are complex, an approximate solution, based on a discretization of the structure, may be obtained by taking the first variation of the discretized potential energy in Eq. (3.26). In this approximate approach, one obtains a discrete number of equilibrium equations relating nodal point or generalized forces to nodal point or generalized displacements. Two discretization schemes may be adopted.

(a) Finite Element Method - a discretization using polynomial expansions for the description of the displacement field in the x and y direction in each element.

(b) Finite Strip Method - a discretization in which advantage is taken of the boundary conditions at the two ends of a finite strip such that the displacement field can be described by trigonometric functions or harmonics of a Fourier series in the longitudinal x-direction and by polynomials in the transverse y-direction.

In comparing (b) to (a), if the end boundary conditions are such that the finite strip method can be applied, the computational effort to solve the discrete set of equilibrium equations is vastly reduced. This is because for each harmonic, the number of nodal points and the band width of the equations to be solved are greatly decreased. A disadvantage of (b) compared to (a) is that roundoff errors in the computer impose a limit on the minimum width-length ratio of the strip used in the solution, and thus a decreasing mesh size (only the strip width decreases) will not necessarily give better answers if roundoff errors become large. Reasonable width-length ratios for each strip can generally be adopted to overcome this disadvantage.

In the rest of this chapter the finite strip method will be used for the discretization of Eq. (3.26). The element stiffness, consistent loadings, and stresses in the plate and rib for a typical finite strip will be derived.

3.7 Development of Element Stiffness for Finite Strip

3.7.1 Assumed Displacement Field

The assumed displacement field v for a typical finite strip can be expressed in terms of the eight nodal point displacements V, shown in Fig. 10 and also summarized in Fig. 16:

$$\{v\} = \sum_{n=1}^{\infty} [\Phi_v]_n \{V\}_n \tag{3.27}$$

in which the subscript n indicates the harmonic under consideration and Φ are the shape or interpolation functions. Considering a typical n^{th} harmonic and dropping the subscript n,

$$\underset{3X1}{\{v\}} = \underset{3X8}{[\Phi_v]} \; \underset{8X1}{\{V\}} \tag{3.28}$$

or expanding into components

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} \Phi_{ui} & 0 & 0 & 0 \\ 0 & \Phi_{vi} & 0 & 0 \\ 0 & 0 & \Phi_{wi} & \Phi_{\theta i} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_i \end{Bmatrix} \tag{3.28a}$$

The assumed interpolation functions Φ are shown in Fig. 17 and may be expressed in terms of the normalized coordinates $\bar{x} = x/a$ and

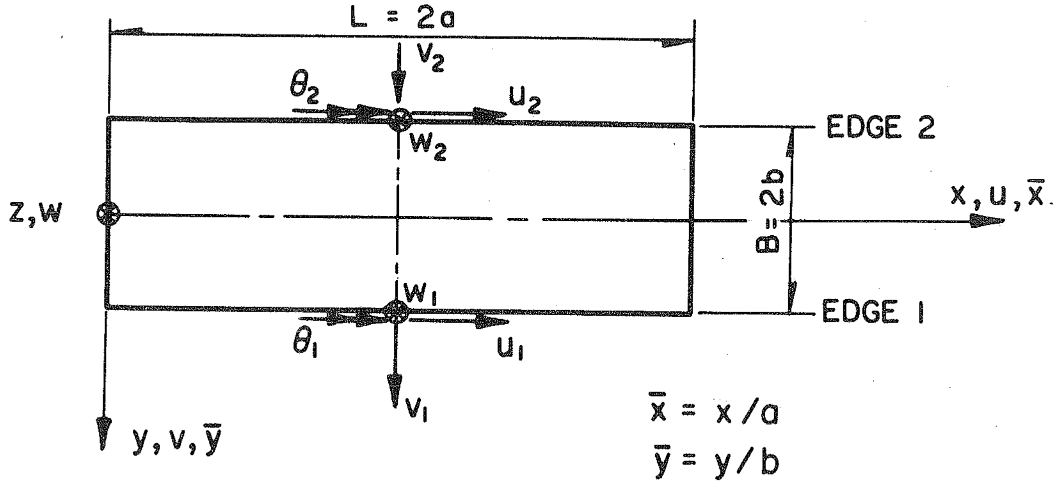


FIG. 16 POSITIVE NODAL POINT DISPLACEMENT AND LOCAL COORDINATE SYSTEM

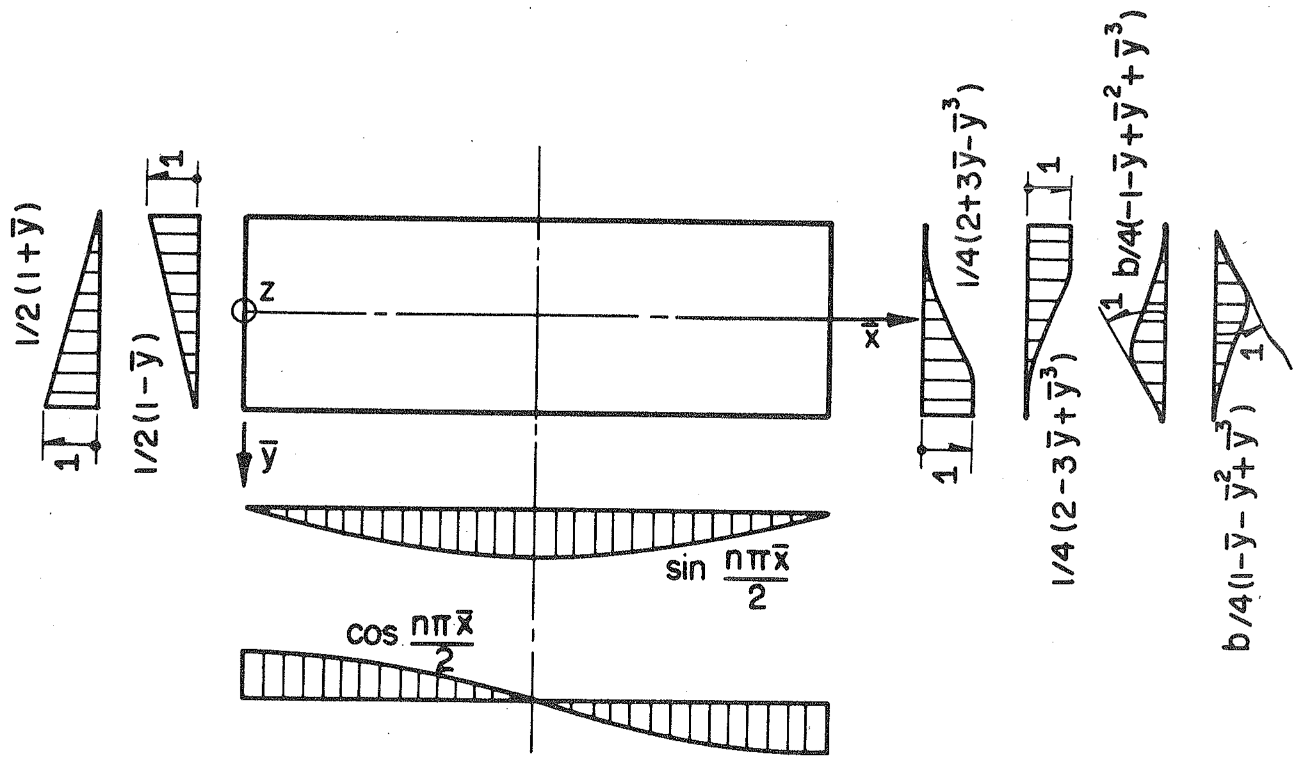


FIG. 17 DISPLACEMENT INTERPOLATION FUNCTION

$\bar{y} = y/b$ of Fig. 16 as follows

$$
\begin{aligned}
\Phi_{ui} &= \tfrac{1}{2}\,(1 + \bar{y}_i\,\bar{y})\,\cos\frac{n\pi\bar{x}}{2} \\
\Phi_{vi} &= \tfrac{1}{2}\,(1 + \bar{y}_i\,\bar{y})\,\sin\frac{n\pi\bar{x}}{2} \\
\Phi_{wi} &= \tfrac{1}{4}\,(2 + 3\bar{y}_i\,y - \bar{y}_i\,\bar{y}^3)\quad\sin\frac{n\pi\bar{x}}{2} \\
\Phi_{\theta i} &= \frac{b}{4}\,(-\,\bar{y}_i - \bar{y} + \bar{y}_i\,\bar{y}^2 + \bar{y}^3)\,\sin\frac{n\pi\bar{x}}{2}
\end{aligned}
\qquad (3.29)
$$

in which $\bar{y}_i = \pm 1$, depending on whether node 1 or 2 is subjected to a unit displacement with $i = 1,2$.

3.7.2 Strain Field

Denoting differentiation by (,) one can express the displacement gradients in terms of normalized coordinates

$$
\begin{aligned}
u_{,x} &= \frac{\partial u}{\partial x} = \frac{1}{a}\,\frac{\partial u}{\partial \bar{x}} \\
u_{,y} &= \frac{\partial u}{\partial y} = \frac{1}{b}\,\frac{\partial u}{\partial \bar{y}} \\
v_{,x} &= \frac{\partial v}{\partial x} = \frac{1}{a}\,\frac{\partial v}{\partial \bar{x}} \\
v_{,y} &= \frac{\partial v}{\partial y} = \frac{1}{b}\,\frac{\partial v}{\partial \bar{y}}
\end{aligned}
\qquad (3.30)
$$

and the curvatures

$$
\begin{aligned}
\kappa_{xx} &= -\,\frac{\partial^2 w}{\partial x^2} = -\,\frac{1}{a^2}\,\frac{\partial^2 w}{\partial \bar{x}^2} \\
\kappa_{yy} &= -\,\frac{\partial^2 w}{\partial y^2} = -\,\frac{1}{b^2}\,\frac{\partial^2 w}{\partial \bar{y}^2} \\
\kappa_{xy} &= -\,\frac{\partial^2 w}{\partial x \partial y} = -\,\frac{1}{ab}\,\frac{\partial^2 w}{\partial \bar{x}\partial \bar{y}}
\end{aligned}
\qquad (3.31)
$$

Considering a typical n^{th} harmonic, the strains may be expressed in terms of the nodal point displacements:

$$\{\varepsilon\} = [T]\,\{V\} \tag{3.32}$$

or expanding using Eqs. (3.4), (3.30) and (3.31)

$$\{\varepsilon\} = \{\varepsilon_0\} + z\,\{\kappa\}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} u,_x \\ v,_y \\ u,_y + v,_x \end{Bmatrix} - z \begin{Bmatrix} w,_{xx} \\ w,_{yy} \\ w,_{xy} \end{Bmatrix} = [T_\varepsilon] \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} + z\,[T_\kappa] \begin{Bmatrix} w_i \\ \theta_i \end{Bmatrix} \tag{3.33}$$

$$\{\varepsilon_0\} = \begin{bmatrix} T_{11} & 0 \\ 0 & T_{22} \\ T_{31} & T_{32} \end{bmatrix}_\varepsilon \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \tag{3.34}$$

$$\{\kappa\} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \\ T_{31} & T_{32} \end{bmatrix}_\kappa \begin{Bmatrix} w_i \\ \theta_i \end{Bmatrix} \tag{3.35}$$

The elements of the T matrices may be evaluated explicitly by substituting Eq. (3.28) into (3.33) and performing the necessary differentiations.

$[T_\varepsilon]$:

$$\begin{aligned} T_{11} &= -\frac{n\pi}{2a}\,(1 + \bar{y}_i\,\bar{y})\,\sin\frac{n\pi\bar{x}}{2} \\ T_{31} &= \frac{\bar{y}_i}{2b}\,\cos\frac{n\pi\bar{x}}{2} \\ T_{22} &= \frac{\bar{y}_i}{2b}\,\sin\frac{n\pi\bar{x}}{2} \\ T_{32} &= \frac{n\pi}{2a}\,(1 + \bar{y}_i\,\bar{y})\,\cos\frac{n\pi\bar{x}}{2} \end{aligned} \tag{3.36}$$

$[T_\varkappa]$:

$$T_{11} = \frac{n^2\pi^2}{16a^2}\,(2 + 3\,\bar{y}_i\,\bar{y} - \bar{y}_i\,\bar{y}^3)\,\sin\frac{n\pi\bar{x}}{2}$$

$$T_{21} = \frac{3}{2b^2}\,\bar{y}_i\,\bar{y}\,\sin\frac{n\pi\bar{x}}{2}$$

$$T_{31} = -\frac{3n\pi}{8ab}\,(\bar{y}_i - \bar{y}_i\,\bar{y}^2)\,\cos\frac{n\pi\bar{x}}{2}$$

$$T_{12} = \frac{n^2\pi^2 b}{16a^2}\,(-\bar{y}_i - \bar{y} + \bar{y}_i\bar{y}^2 + \bar{y}^3)\,\sin\frac{n\pi\bar{x}}{2} \tag{3.37}$$

$$T_{22} = -\frac{1}{2b}\,(\bar{y}_i + 3\bar{y})\,\sin\frac{n\pi\bar{x}}{2}$$

$$T_{32} = -\frac{n\pi}{8a}\,(-1 + 2\bar{y}_i\bar{y} + 3\bar{y}^2)\,\cos\frac{n\pi x}{2}$$

in which $\bar{y}_i = \pm 1$ for the edges $i = 1, 2$ respectively.

3.7.3 Stress Field

Considering a typical n^{th} harmonic, the stress resultants of Eq. (3.20) may be expressed in terms of the nodal point displacements by substituting Eq. (3.33) into (3.20).

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} D^N & D^{NM} \\ D^{MN} & D^M \end{bmatrix} \begin{Bmatrix} \epsilon_0 \\ \varkappa \end{Bmatrix} \tag{3.20}$$

$$= \begin{bmatrix} D^N & D^{NM} \\ D^{MN} & D^M \end{bmatrix} \begin{bmatrix} T_\epsilon & 0 \\ 0 & T_\varkappa \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_i \end{Bmatrix} \tag{3.38}$$

or subdividing

$$\underset{3\times1}{\{N\}} = \underset{3\times3}{[D^N]}\;\underset{3\times4}{[T_\epsilon]}\;\underset{4\times1}{\begin{Bmatrix} u_i \\ v_i \end{Bmatrix}} + \underset{3\times3}{[D^{NM}]}\;\underset{3\times4}{[T_\varkappa]}\;\underset{4\times1}{\begin{Bmatrix} w_i \\ \theta_i \end{Bmatrix}} \tag{3.39}$$

$$\underset{3\times1}{\{M\}} = \underset{3\times3}{[D^{MN}]}\;\underset{3\times4}{[T_\epsilon]}\;\underset{4\times1}{\begin{Bmatrix} u_i \\ v_i \end{Bmatrix}} + \underset{3\times3}{[D^M]}\;\underset{3\times4}{[T_\varkappa]}\;\underset{4\times1}{\begin{Bmatrix} w_i \\ \theta_i \end{Bmatrix}} \tag{3.40}$$

3.7.4 Evaluation of Finite Strip Stiffness Matrix

The discretized form of the total potential energy for a typical finite strip can now be expressed as follows:

$$\pi(v) = \sum_{n}^{\infty} \sum_{m}^{\infty} \{ \int_{A} (\tfrac{1}{2} \{V\}_n^T [T]_n^T [D] [T]_m \{V\}_m - \{V\}_n^T [\Phi_v]_n^T [\psi_f]_m \{f\}_m) \, dA - \int_{S} \{V\}_n^T [\Phi_v]_n^T [\psi_p]_m \{p\}_m \, ds \} \quad (3.41)$$

in which n and m are harmonic numbers. The body and surface forces are described through the interpolation functions ψ using a Fourier expansion in the x direction and a polynomial expansion in the y-direction and by their nodal intensity vectors $\{f\}$ and $\{p\}$. Equation (3.41) is of quadratic form in the generalized coordinates V. These are the nodal amplitudes of the displacement components, which vary as harmonics in the x-direction.

When the integrations of Eq. (3.41) are performed the orthogonality of the trigonometric functions is preserved since the integrands appear only in the form

$$\int_0^2 \sin \frac{n\pi\bar{x}}{2} \sin \frac{m\pi\bar{x}}{2} \, dx \qquad \text{or} \qquad \int_0^2 \cos \frac{n\pi\bar{x}}{2} \cos \frac{m\pi\bar{x}}{2} \, dx$$

both of which equal zero for $n \neq m$ and equal 1 for $n = m$. Therefore, in Eq. (3.41) only a single summation over n is necessary and the subscript m may be dropped. This orthogonality is a very important property. Instead of having to solve a single set of N X N equations, where N is the number of degrees of freedom (DOF) times the number of harmonics (n), it is only necessary to solve n independent sets of DOF X BW equations, each set of which has a very narrow band width (BW).

Since the solution of the equations is proportional to the square of the band width the computational effort is reduced by the factor n^2.

In essence, the orthogonality permits the analysis to be carried out for all of the loading components of each particular harmonic independently. The final results are obtained by summing the results for all n harmonics used to represent the load. Once the solution technique, which involves extensive computations, has been developed for a single harmonic it can be reused for any harmonic and thus the approach is well suited to the application of the digital computer.

Taking the first variation of the total potential $\pi(v)$ the solution of $\delta\pi(v) = 0$ yields an upper bound for the discretized energy $\pi(v)$ to the true minimum $\pi(u)$ because of the positive definite nature of the stiffness matrix $\delta^2\pi(v) > 0$. The discrete set of equilibrium equations is obtained from $\delta\pi(v) = 0$:

$$\sum_{1}^{n} \left(\int_A [T]_n^T [D] [T]_n \{v\}_n \, dA - \int_A [\Phi_v]_n^T [\psi_f]_n \{f\}_n \, dA - \int_s [\Phi_v]_n^T [\psi_p]_n \{p\}_n \, ds \right) = 0 \tag{3.42}$$

Dropping the subscript n of the n^{th} harmonic the stiffness matrix $[k]$ and the consistent nodal point forces $\{S\}$ are defined from Eq. (3.42) as follows:

$$[k] = \int_A [T]^T [D] [T] \, dA \tag{3.43}$$

$$\{S\} = \int_A [\Phi_v]^T [\psi_f] \{f\} \, dA + \int_s [\Phi_v]^T [\psi_f] \{p\} \, ds \tag{3.44}$$

The element stiffness matrix [k] for a finite strip can be obtained by explicitly performing the integration indicated in Eq. (3.43) using the previously derived expressions for [D] given in Eqs. (3.15), (3.16), (3.17), (3.18), (3.25) and for [T] in Eqs. (3.33), (3.34), (3.35), (3.36), (3.37). Positive directions of nodal point forces and corresponding displacements are given in Figs. 10 and 16. The ordering of the element stiffness matrix is as follows:

$$\begin{Bmatrix} U_1 \\ U_2 \\ V_1 \\ V_2 \\ \hline Q_1 \\ Q_2 \\ M_1 \\ M_2 \end{Bmatrix} = \left[\begin{array}{c|c} \begin{matrix} k_{\epsilon\epsilon} \\ 4\times4 \end{matrix} & \begin{matrix} k_{\epsilon\varkappa} \\ 4\times4 \end{matrix} \\ \hline \begin{matrix} k_{\varkappa\epsilon} \\ 4\times4 \end{matrix} & \begin{matrix} k_{\varkappa\varkappa} \\ 4\times4 \end{matrix} \end{array}\right] \begin{Bmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \\ \hline w_1 \\ w_2 \\ \theta_1 \\ \theta_2 \end{Bmatrix} \qquad (3.45)$$

Values for each term in the element stiffness matrix are given on the following pages in which $k_n = n\pi/L$; $B = 2b$ and $L = 2a$ are defined in Fig. 16 and values of D are given in Eqs. (3.16), (3.17), (3.18), (3.25).

Elements of 4 X 4 Matrix $[k_{\epsilon\epsilon}]$

	u_1	u_2	v_1	v_2
U_1	$k_n^2 \frac{LB}{6} D_{11} + \frac{L}{2B} D_{33}$	$k_n^2 \frac{LB}{12} D_{11} - \frac{L}{2B} D_{33}$	$k_n \frac{L}{4} (-D_{12} + D_{33})$	$k_n \frac{L}{4} (-D_{12} - D_{33})$
U_2		$k_n^2 \frac{LB}{6} D_{11} + \frac{L}{2B} D_{33}$	$k_n \frac{L}{4} (-D_{12} - D_{33})$	$k_n \frac{L}{4} (-D_{12} + D_{33})$
V_1	SYMM.		$k_n^2 \frac{LB}{6} D_{33} + \frac{L}{2B} D_{22}$	$-k_n^2 \frac{LB}{12} D_{33} + \frac{L}{2B} D_{22}$
V_2				$k_n^2 \frac{LB}{6} D_{33} + \frac{L}{2B} D_{22}$

(3.46)

Elements of 4 X 4 Matrix $[k_{\kappa\epsilon}] = [k_{\epsilon\kappa}]^T$

	u_1	u_2	v_1	v_2
Q_1	$-k_n^3 \frac{7LB}{40} D_{14}$	$-k_n^3 \frac{3LB}{40} D_{14}$	0	0
Q_2	$-k_n^3 \frac{3LB}{40} D_{14}$	$-k_n^3 \frac{7LB}{40} D_{14}$	0	0
M_1	$k_n^3 \frac{LB^2}{40} D_{14}$	$k_n^3 \frac{LB^2}{60} D_{14}$	$- \frac{L}{2B} D_{25}$	$- \frac{L}{2B} D_{25}$
M_2	$-k_n^3 \frac{LB^2}{60} D_{14}$	$-k_n^3 \frac{LB^2}{40} D_{14}$	$\frac{L}{2B} D_{25}$	$\frac{L}{2B} D_{25}$

(3.47)

Elements of 4 X 4 Matrix $[k_{\varkappa\varkappa}]$

	w_1	w_2	θ_1	θ_2
Q_1	$k_n^4 \frac{13LB}{70} D_{44} + \frac{6L}{B^3} D_{55} + k_n^2 \frac{3L}{5B} D_{aa}$	$k_n^4 \frac{9LB}{140} D_{44} - \frac{6L}{B^3} D_{55} - k_n^2 \frac{3L}{5B} D_{aa}$	$-k_n^4 \frac{11B^2L}{420} D_{44} - \frac{3L}{B^2} D_{55} - k_n^2 \frac{L}{20} D_{bb}$	$k_n^4 \frac{13LB^2}{840} D_{44} - \frac{3L}{B^2} D_{55} - k_n^2 \frac{L}{20} D_{aa}$
Q_2	SYMM.	$k_n^4 \frac{13LB}{70} D_{44} + \frac{6L}{B^3} D_{55} + k_n^2 \frac{3L}{5B} D_{aa}$	$-k_n^4 \frac{13LB^2}{840} D_{44} + \frac{3L}{B^2} D_{55} + k_n^2 \frac{L}{20} D_{aa}$	$k_n^4 \frac{11LB^2}{420} D_{44} + \frac{3L}{B^2} D_{55} + k_n^2 \frac{L}{20} D_{bb}$
M_1			$k_n^4 \frac{LB^3}{210} D_{44} + \frac{2L}{B} D_{55} + k_n^2 \frac{LB}{15} D_{aa}$	$-k_n^4 \frac{LB^3}{280} + \frac{L}{B} D_{55} - k_n^2 \frac{LB}{60} D_{aa}$
M_2				$k_n^4 \frac{LB^3}{210} D_{44} + \frac{2L}{B} D_{55} + k_n^2 \frac{LB}{15} D_{aa}$

(3.48)

where $D_{aa} = 2D_{45} + 4D_{66}$ and $D_{bb} = 12D_{45} + 4D_{66}$

3.8 Consistent Loadings

The consistent nodal point forces $\{S\}$ of Eq. (3.44) are force quantities which provide the same energy as the body forces f and surface forces p in going through the chosen displacement patterns corresponding to unit values of each of the corresponding nodal point displacements, $\{V\}$.

Considering first, only body forces $f(x,y)$ for a typical n^{th} harmonic.

$$\underset{8\times1}{\{S\}} = \int_A \underset{8\times3}{[\Phi]^T}\ \underset{3\times6}{[\psi_f]}\ \underset{6\times1}{\{f\}}\ dA \tag{3.49}$$

in which ψ_f are the interpolation functions defining the distribution of the body forces throughout the finite strip for unit values of the load vector $\{f\}$ whose components are the load intensities at each nodal joint of the finite strip under consideration.

$$[\psi_f] = \begin{bmatrix} \psi_u & 0 & 0 \\ 0 & \psi_v & 0 \\ 0 & 0 & \psi_w \end{bmatrix} \quad ; \quad \{f\} = \begin{Bmatrix} f_u \\ f_v \\ f_w \end{Bmatrix} \tag{3.50}$$

The shape functions ψ approximate the functional variation in the x and y direction of each body force component. They are determined by a standard Fourier analysis in the longitudinal x-direction and are assumed to vary linearly in the y-direction. Thus, for a longitudinal load variation $f(\bar{x})$ from $\bar{x} = \alpha$ to $\bar{x} = \beta$ with a linear variation in the $\bar{y}$ direction the interpolation functions ψ are for the n^{th} harmonic

$$\psi_u = \frac{\int_{\alpha}^{\beta} f_u(\bar{x}) \cos \frac{n\pi \bar{x}}{2} \, d\bar{x}}{\int_{0}^{2} \cos^2 \frac{n\pi \bar{x}}{2} \, d\bar{x}} \cos \frac{n\pi \bar{x}}{2} \quad \frac{1}{2} (1 + \bar{y}_i \; \bar{y})$$

$$\psi_v = \frac{\int_{\alpha}^{\beta} f_f(\bar{x}) \sin \frac{n\pi \bar{x}}{2} \, d\bar{x}}{\int_{0}^{2} \sin^2 \frac{n\pi \bar{x}}{2} \, dx} \sin \frac{n\pi \bar{x}}{2} \quad \frac{1}{2} (1 + \bar{y}_i \; \bar{y}) \qquad (3.51)$$

$$\psi_w = \frac{\int_{\alpha}^{\beta} f_w(\bar{x}) \sin \frac{n\pi \bar{x}}{2} \, d\bar{x}}{\int_{0}^{2} \sin^2 \frac{n\pi \bar{x}}{2} \, d\bar{x}} \sin \frac{n\pi \bar{x}}{2} \quad \frac{1}{2} (1 + \bar{y}_i \; \bar{y})$$

For a uniform load over the entire longitudinal span the load interpolation functions reduce to

$$\psi_u = \frac{4}{n\pi} \cos \frac{n\pi \bar{x}}{2} \quad \frac{1}{2} (1 + \bar{y}_i \; \bar{y})$$

$$\psi_v = \frac{4}{n\pi} \sin \frac{n\pi \bar{x}}{2} \quad \frac{1}{2} (1 + \bar{y}_i \; y) \qquad (3.52)$$

$$\psi_w = \frac{4}{n\pi} \sin \frac{n\pi \bar{x}}{2} \quad \frac{1}{2} (1 + \bar{y}_i \; y)$$

With the displacement shape functions of Eq. (3.28) the consistent load vector $\{S\}$ can be easily determined for various load distributions by performing the appropriate integrations in Eq. (3.49).

The consistent loads for the following four body forces cases are listed in Eq. (3.54) for unit intensities of load components in the y and z directions. In all cases the loads are assumed to be uniformly distributed across the width of the strip:

1) Uniform load distribution over the entire finite strip

2) Uniform load over the total strip width and over a partial length at an arbitrary longitudinal position.

3) Uniform line load across the width of the strip at midspan.

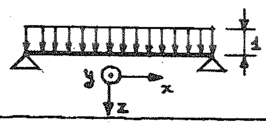
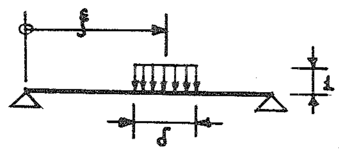
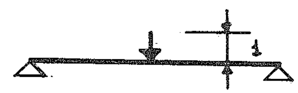
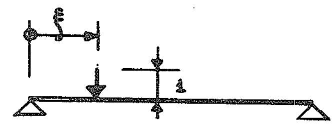
4) Uniform line load across the width of the strip at an arbitrary longitudinal position.

Define by $\bar{\xi} = \xi/a$ the $\bar{x}$ distance from the origin to the centroid of the distributed body force and by $\bar{\delta} = \delta/a$ the length of the partial loading in the $\bar{x}$ direction. The following factors modify the uniform load distribution of the basic case (1) to any one of the other load cases treated:

$$
\begin{aligned}
C_1 &= 1 \\
C_2 &= \sin\frac{n\pi\bar{\xi}}{2}\sin\frac{n\pi\bar{\delta}}{4} \\
C_3 &= \frac{n\pi}{2L}(-1)^{\frac{n-1}{2}} \qquad\qquad n = 1,3,5 \\
C_4 &= \frac{n\pi}{2L}\sin\frac{n\pi\bar{\xi}}{2} \\
C_5 &= \cos\frac{n\pi\bar{\xi}}{2}\cos\frac{n\pi\bar{\delta}}{4} \\
C_6 &= \frac{n\pi}{2L}\cos\frac{n\pi\bar{\xi}}{2}
\end{aligned}
\tag{3.53}
$$

Exactly the same procedure applies to the determination of consistent surface loads which are line loads along a longitudinal joint of the finite strip. The consistent loads for the same four load cases as in the case of body forces are listed in Eq. (3.55) for unit intensities of load components in the x, y and z direction and for a transverse joint moment M_y along joint 1.

CONSISTENT LOADS FOR BODY FORCES ALL HAVING A UNIFORM DISTRIBUTION IN Y-DIRECTION

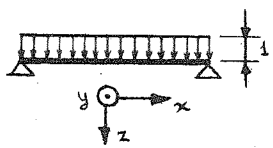
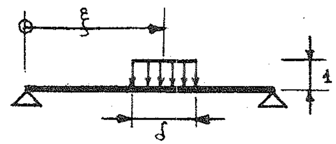
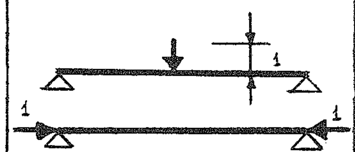
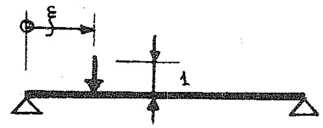
LOAD CASE	BODY FORCE DISTRIBUTION OVER FINITE STRIP AREA							
	(1) UNIFORM LOADING 		(2) PARTIAL STRIP LOADING 		(3) TRANSVERSE LINE LOADING AT MIDSPAN 		(4) TRANSVERSE LINE LOADING 	
S	Direction		Direction		Direction		Direction	
	Y	Z	Y	Z	Y	Z	Y	Z
U_1	0	0	0	0	0	0	0	0
U_2	0	0	0	0	0	0	0	0
V_1	B/k_n	0	$C_2\ B/k_n$	0	$C_3\ B/k_n$	0	$C_4\ B/k_n$	0
V_2	B/k_n	0	$C_2\ B/k_n$	0	$C_3\ B/k_n$	0	$C_4\ B/k_n$	0
Q_1	0	B/k_n	0	$C_2\ B/k_n$	0	$C_3\ B/k_n$	0	$C_4\ B/k_n$
Q_2	0	B/k_n	0	$C_2\ B/k_n$	0	$C_3\ B/k_n$	0	$C_4\ B/k_n$
M_1	0	$-B^2/6k_n$	0	$-C_2\ B^2/6k_n$	0	$-C_3\ B^2/6k_n$	0	$-C_4\ B^2/6k_n$
M_2	0	$B^2/6k_n$	0	$C_2\ B^2/6k_n$	0	$C_3\ B^2/6k_n$	0	$C_4\ B^2/6k_n$

(3.54)

(1) $L = 2a,\ B = 2b,\ k_n = \frac{n\pi}{L} = \frac{2n\pi}{a}$

(2) The constants C_i, with i = 1,..6, are defined on page 41.

CONSISTENT LOADS FOR SURFACE LINE LOADS AT JOINT 1

LOAD CASE	LINE LOADS ALONG JOINT 1 OF FINITE STRIP														
	(1) UNIFORM LINE LOADING 			(2) UNIFORM PARTIAL LINE LOADING 				(3) CONCENTRATED LOADING AT MIDSPAN 				(4) CONCENTRATED LOADING 			
S	Direction			Direction				Direction				Direction			
	Y	Z	My	X	Y	Z	My	X	Y	Z	My	X	Y	Z	My
U_1	0	0	0	$2C_5/k_n$	0	0	0	2	0	0	0	C_6	0	0	0
U_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
V_1	$2/k_n$	0	0	0	$2C_2/k_n$	0	0	0	1	0	0	0	C_4	0	0
V_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Q_1	0	$2/k_n$	0	0	0	$2C_2/k_n$	0	0	0	1	0	0	0	C_4	0
Q_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M_1	0	0	$2/k_n$	0	0	0	$2C_2/k_n$	0	0	0	1	0	0	0	C_4
M_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(3.55)

(1) $L = 2a$, $B = 2b$, $k_n = \frac{n\pi}{L} = \frac{2n\pi}{a}$

(2) The constants C_i, with $i = 1,\ldots 6$, are defined on page 41.

(3) In case 2 and 4, the total longitudinal force in the x-direction shown must be balanced by another force of the same magnitude somewhere along the same joint.

(4) In case 3, equal and opposite unit longitudinal forces in the x-direction are assumed to be acting.

3.9 Direct Stiffness Method

This procedure is recapitulated only briefly since it has been described extensively in [1,2].

The individual strip stiffness matrices for the n^{th} harmonic are transformed into the global coordinate system and are then added into the appropriate places of the global assembly matrix [K]. Similarly the global load vector [R] is formed by combining all consistent load contributions of the n^{th} harmonic. After imposing the geometric boundary conditions the resulting system of equations is solved by direct Gauss elimination for the unknown nodal displacements of the n^{th} harmonic:

$$[K]\{r\} = \{R\} \tag{3.56}$$

The structural stiffness matrix [K] is of the size (DOF × BW) where the total number of degrees of freedom DOF equals four times the number of joints in the structure, and the band width BW equals four times the sum of maximum nodal joint difference of any finite strip in the structure plus one. Hence, in comparison to any finite element scheme the computational effort is vastly reduced even if Eq. (3.56) is solved n-times, where n is the total number of harmonics considered necessary for the Fourier expansion of the loading. Using the solution of Eq. (3.56) for the unknown nodal displacements the displacement variation is obtained within each finite strip by Eq. (3.28). The contribution of each harmonic is accumulated to yield the final displacement field, Eq. (3.27).

3.10 Determination of Internal Forces

The internal forces are evaluated by accumulation of each harmonic contribution to a specific stress resultant similar to the displacement field. Three cases can be distinguished depending on which portions of the material law are used for the determination of the internal forces: One can obtain the stress resultants of the combined plate-rib system, of the plate system alone and of the rib system alone. In order to capture the difference in the twisting moments $M_{xy} \neq M_{yx}$ of a plate-rib system with torsionally stiff ribs, it is necessary to modify the moment-displacement relationships of Eq. (3.40) by treating M_{xy} and M_{yx} individually. Previously the following was obtained

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} D^{N} & D^{NM} \\ D^{MN} & D^{M} \end{bmatrix} \begin{bmatrix} T_{\epsilon} & 0 \\ 0 & T_{\varkappa} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_i \end{Bmatrix} \qquad (3.38)$$

Redefine the moment relationships in the following way:

$$\underset{4\times1}{\{\bar{M}\}} = \underset{4\times3}{[\bar{D}^{MN}]}\ \underset{3\times4}{[T_{\epsilon}]}\ \underset{4\times1}{\begin{Bmatrix} u_i \\ v_i \end{Bmatrix}} + \underset{4\times3}{[\bar{D}^{M}]}\ \underset{3\times4}{[T_{\varkappa}]}\ \underset{4\times1}{\begin{Bmatrix} w_i \\ \theta_i \end{Bmatrix}}$$

where

$$\{\bar{M}\} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \\ M_{yx} \end{Bmatrix} \qquad (3.57)$$

$$[\bar{D}^{MN}] = [\bar{D}^{MN}]_R = \begin{bmatrix} S^x E^x / s^x & 0 & 0 \\ 0 & S^y E^y / s^y & 0 \\ 0 & 0 & H^x / s^x \\ 0 & 0 & H^y / s^y \end{bmatrix} \qquad (3.58)$$

$$[\bar{D}^M] = [D^M]_P + [D^M]_R =$$

$$= \begin{bmatrix} \left(\frac{h^3 C_{xx}}{12} + \frac{I^x E^x}{s^x}\right) & \left(\frac{h^3 C_{xy}}{12}\right) & 0 \\ \left(\frac{h^3 C_{yx}}{12}\right) & \left(\frac{h^3 C_{yy}}{12} + \frac{I^y E^y}{s^y}\right) & 0 \\ 0 & 0 & \left(\frac{h^3 G_{xy}}{12} + \frac{J^x}{2s^x}\right) \\ 0 & 0 & \left(\frac{h^3 G_{xy}}{12} + \frac{J^y}{2s^y}\right) \end{bmatrix} \quad (3.59)$$

3.10.1 Internal Forces in the Combined Plate-Rib System

The plate and rib contributions are contained in the material law. The contribution of the n^{th} harmonic to the combined stress resultants are now expressed in terms of the nodal displacements by

$$\begin{Bmatrix} N \\ \bar{M} \end{Bmatrix} = \begin{bmatrix} D^N & D^{NM} \\ \bar{D}^{MN} & \bar{D}^M \end{bmatrix} \begin{bmatrix} T_\epsilon & 0 \\ 0 & T_\varkappa \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_i \end{Bmatrix} \quad (3.60)$$

3.10.2 Internal Forces in the Plate System Alone

Only the plate contributions are retained in the material law. The contribution of the n^{th} harmonic to the plate stress resultants are

$$\begin{Bmatrix} N \\ M \end{Bmatrix}_P = \begin{bmatrix} D^N & D^{NM} \\ D^{MN} & D^M \end{bmatrix}_P \begin{bmatrix} T_\epsilon & 0 \\ 0 & T_\varkappa \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_i \end{Bmatrix} \quad (3.61)$$

3.10.3 Internal Forces in the Rib System Alone

Only the rib contributions are retained in the material law. Either the smeared or the local contributions of the n^{th} harmonic to the rib stress resultants are obtained by including or excluding the rib spacing in the material law

$$\begin{Bmatrix} N \\ M \end{Bmatrix}_R = \begin{bmatrix} D^N & D^{NM} \\ \bar{D}^{MN} & \bar{D}^M \end{bmatrix}_R \begin{bmatrix} T_\epsilon & 0 \\ 0 & T_\varkappa \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_i \end{Bmatrix} \tag{3.62}$$

3.10.4 Fiber Normal Stresses in Plate or Ribs

Once the internal forces in the deck plate alone are known from Eq. (3.61), the fiber stresses in plate may be found as follows:

$$\begin{aligned} (\sigma_x)_P &= \left(\frac{N_x}{h}\right)_P \pm \left(\frac{12\, M_x\, z}{h^3}\right)_P \\ (\sigma_y)_P &= \left(\frac{N_y}{h}\right)_P \pm \left(\frac{12\, M_y\, z}{h^3}\right)_P \end{aligned} \tag{3.63}$$

In a similar manner, with the internal forces being known from Eq. (3.62), the fiber stresses in the ribs may be found as follows:

$$\begin{aligned} (\sigma_x)_R &= \left(\frac{N_x\, s^x}{A^x}\right)_R \pm \left(\frac{M_x\, s^x\, z}{I^x}\right)_R \\ (\sigma_y)_R &= \left(\frac{N_y\, s^y}{A^y}\right)_R \pm \left(\frac{M_y\, s^y\, z}{I^y}\right)_R \end{aligned} \tag{3.64}$$

3.11 Interpretation and Significance of Results Obtained

When interpreting the results obtained from the analysis, one should keep in mind the assumptions made in developing the analytical model. In essence the deck plate with discrete eccentric ribs was replaced by an equivalent combined plate-rib system in which the ribs were assumed to be spread uniformly across the width of the finite strip. With this type of assumption one cannot expect the analysis to yield accurate values for localized plate moments and torques between ribs or for localized deflections due to concentrated loads between ribs. On the other hand the analysis should yield accurate values for displacements along rib lines, for fiber stresses in plate and ribs along rib lines, and most important for the magnitude and distribution of internal forces in the combined plate-rib system. These latter results can be utilized in design to check the overall adequacy of a typical repeating width of the deck-rib section.

4. COMPUTER PROGRAM "MULSTR"

4.1 General

A general computer program has been written to perform the finite strip analysis described in Chapter 3. The program, entitled MULSTR, was written in FORTRAN IV language for the CDC 6400 computer. Modern features, such as dynamic storage allocation and an automatic field length reduction, are incorporated to adjust the required storage to the data under consideration. Detailed descriptions of the input, output, sign conventions and restrictions of this program are given in Appendix A. The listing of the source program is presented in Appendix B.

4.2 Input, Output

A brief description of the program is given below.

a) Input Data

1. Geometry of the structure and its idealization in terms of the span, number of strips, joints and the number of harmonics considered for the Fourier representation of the loading.
2. Dimensions and material properties of each strip which is made up of a deck plate and possible eccentric rib stiffeners.
3. Nodal joint array including magnitudes and locations of surface loads.
4. Displacement and force boundary conditions along the longitudinal joints.

5. Magnitudes and locations of additional concentrated joint loads.

6. Desired locations for final results in output.

b) Output Data

1. The echo of the input data is printed as a check.

2. Resulting global joint displacements are given at specified locations along the span.

3. For each strip all internal forces and displacements are printed for each transverse section specified across the plate width and at the x-coordinates along the plate length.

4.3 Limitation Regarding Application

Since the required storage is allocated in accordance to the data there are no restrictions on the maximum number of strips, joints, material properties or harmonics considered. The use of the automatic field length reduction program RFL and LWA written in COMPASS language enables one to determine the variable storage requirements and to reserve automatically the amount of storage needed for the particular problem analyzed. In Appendix A expressions are given for the hand calculation of the required field length during execution.

Since the finite strip analysis provides stiffness matrices for each harmonic which have a very narrow band width there is no need to use an out of core solver. Hence, a direct in core band solver is utilized to solve the set of equations taking advantage of symmetry and the band structure. If one has access to computers with a very limited core storage only, resort can be taken to a band solver which divides the set of equations into blocks using peripheral units, such as tapes or disks.

It should be emphasized again that during the development of the eccentrically stiffened strip stiffness the rib properties are assumed to be uniformly distributed or "smeared" over the strip. Hence one cannot expect that this method provides valuable information regarding the local behavior of the plating between ribs. Different examples in the next chapter will illustrate that this smearing of closely spaced ribs yields excellent results of the overall behavior while structures with widely spaced beams exhibit the limitations of this method. A study was made if an eccentrically stiffened strip could degenerate to a discrete beam spanning in the longitudinal direction. The best results were obtained by assuming an orthotropic material law for the plate with fictitious zero stiffness in the direction of the stiffener and with the actual material properties in the transverse direction. The rib properties of the strip were those of the actual beam about its top fiber which was assumed to lie at the midsurface of the strip plate. The results of this investigation are not recorded in this report. An analogous type of idealization was used for the eccentrically stiffened plate of example 4 where the discrete beams were approximated by finite strips of the same width. Unfortunately, this attempt to capture the local effects of discrete longitudinal girders did not improve the results obtained from the standard smearing procedure.

5. EXAMPLES

5.1 General Remarks

Several examples of gradually increasing complexity have been chosen to illustrate the application of the computer program MULSTR based on the finite strip method. Whenever possible, the results obtained are compared with values obtained by other independent solutions.

Examples 1 and 2 deal with a single isotropic plate subjected to edge loads or a distributed surface load. These results can be compared directly with those obtained by the folded plate method using the MULTPL program which may be considered "exact" for the purpose of comparison.

Example 3 is taken from the paper by DeFries-Skene and Scordelis [11]. It deals with the analysis of a prismatic folded plate structure consisting of a number of isotropic plate components. This structure is simply supported at two ends and is subjected to joint loads uniformly distributed in the longitudinal direction. For this case also the results can be compared to those obtained using MULTPL.

Example 4 consists of a deck-plate with eccentric open ribs in one direction only which is subjected to edge or distributed surface loads. For this case also the results can be compared to those obtained using MULTPL.

Examples 5, 6, and 7 are taken from the paper by Clifton, Chang and Au [4] and involve a deck plate with eccentric ribs in two directions. These examples cannot be solved using MULTPL, however results can be compared with the exact solution given in [4].

Example 8 consists of a single cell box subjected to symmetric and antisymmetric concentrated loads at midspan. Several cases are solved: (a) no ribs; (b) longitudinal ribs only; (c) transverse ribs only; and (d) both longitudinal and transverse ribs. Results from the first case are compared with those obtained by MULTPL and the other cases are used to discuss the effect of rib stiffness on the behavior of the structure.

5.2 Isotropic Plate Structures

5.2.1 Example 1 - Single Plate Under Edge Loads (Fig. 11)

The single isotropic plate shown in Fig. 18 is analyzed by MULSTR using one (FS-1) and then four (FS-4) finite strips to represent the entire width of the plate. Results are compared in Table 1 with those obtained by the folded plate method (FP) using MULTPL. The edge loads at point "a" consist of two concentrated midspan loads of 1 kip, one transverse and one in the plane of the plate.

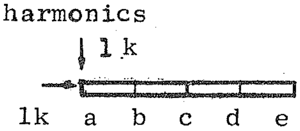
For FS-4, values at each point were obtained by averaging the values at the edges of the two finite strips on either side of the joint. Values of u, v, w, N_x, M_x and M_{xy} obtained for both, FS-1 and FS-4, agree very well with those of FP. The use of a finer mesh in FS-4 as compared to FS-1 results in an improvement of the agreement of the values of N_y, N_{xy} and M_y with those found by FP. Observe that the values of N_y violate considerably the zero force boundary conditions along the free edge due to the effect of Poisson's ratio. Values at the center of each strip are more meaningful to represent the distribution of the N_y quantity.

Table 1. COMPARISON OF RESULTS FOR EXAMPLE 1 (FIG. 18)

QUANTITY	METHOD [1]	y(ft)→	0	2.5	5.0	7.5	10.0
		x(ft)↓	a	b	c	d	e
u (ft. × 10^{-5})	FP	0	8.15	4.05	0	-4.07	-8.17
	FS-1	0	7.96	3.98	0	-3.99	-7.98
	FS-4	0	8.14	4.04	0	-4.06	-8.16
v (ft. × 10^{-4})	FP	50	-5.58	-5.57	-5.57	-5.57	-5.56
	FS-1	50	-5.44	-5.44	-5.43	-5.43	-5.43
	FS-4	50	-5.56	-5.56	-5.56	-5.55	-5.54
w (ft. × 10^{-2})	FP	50	5.53	5.48	5.43	5.39	5.36
	FS-1	50	5.54	5.48	5.43	5.39	5.36
	FS-4	50	5.54	5.48	5.43	5.39	5.36
N_x (k/ft. × 10°)	FP	50	-1.57	-0.71	0.02	0.72	1.47
	FS-1	50	-1.47	-0.73	0.00	0.73	1.46
	FS-4	50	-1.55	-0.69	0.02	0.72	1.47
N_y (k/ft × 10^{-1}	FP	50	-1.96	-1.46	-0.77	-0.23	0.00
	FS-1	50	-2.92	-1.82	-0.72	0.38	1.48
	FS-4	50	-2.31	-1.31	-0.77	-0.34	0.46
N_{xy} (k/ft. × 10^{-2})	FP	0	0.00	-5.32	-7.37	-5.58	0.00
	FS-1	0	-4.59	-4.72	-4.85	-4.98	-5.11
	FS-4	0	-2.40	-4.91	-6.84	-5.03	-2.93
M_x (k-ft/ft. × 10^{-2})	FP	50	2.71	2.53	2.42	2.35	2.31
	FS-1	50	2.70	2.53	2.42	2.35	2.30
	FS-4	50	2.71	2.53	2.42	2.35	2.31
M_y (k-ft/ft. × 10^{-2})	FP	50	0.00	-7.21	-7.63	-5.16	0.00
	FS-1	50	-5.98	-6.43	-6.03	-5.01	-3.61
	FS-4	50	-1.03	-7.72	-7.81	-5.36	-0.26
M_{xy} (k-ft/ft. × 10^{-1})	FP	0	-1.52	-1.39	-1.24	-1.08	-0.92
	FS-1	0	-1.52	-1.38	-1.24	-1.08	-0.92
	FS-4	0	-1.52	-1.39	-1.24	-1.08	-0.92

1. FP = Folded plate method - MULTPL computer program
   FS = Finite strip method - MULSTR computer program
   n = 19 harmonics

2. Loading:



5.2.2 Example 2 - Single Isotropic Plate Under Uniform Dead Load (Fig. 19)

The single isotropic plate shown in Fig. 19 is analyzed by MULSTR using one (FS-1) and then four (FS-4) finite strips to represent the entire width of the plate. Results are compared in Table 2 with those obtained by the folded plate method (FP) using MULTPL. The loading consists of a uniform dead load of 1.414 ksf acting over the entire plate, which is inclined 45° with the horizontal. This loading then produces both membrane and slab action in the plate.

For FS-4, values at each point were obtained by averaging the values at the edges of the two finite strips on either side of the joint. Values of u, v, w, N_x, and M_{xy} obtained for both FS-1 and FS-4 agree very well with those of FP. The use of a finer mesh in FS-4 as compared to FS-1 results in an improvement of the agreement of the values of N_{xy} and M_y with those found by FP. Values of N_y obtained by both FS-4 and FS-1 compare very poorly with FP values at the free edges due to the effect of Poisson's ratio. Again only the values at the center of each strip are a meaningful representation of the N_y quantity.

5.2.3 Example 3 - Prismatic Folded Plate Structure Under Uniform Joint Loads (Fig. 20)

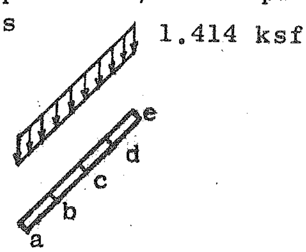
The folded plate structure from Reference [11] consists of three isotropic plates which are joined at the longitudinal joints "b" and "c" and is symmetrical about joint "d." Each individual plate has a span of L = 30 feet and is simply supported at the end diaphragms. The structure is subjected to line loads uniformly distributed in the direction of the longitudinal joints.

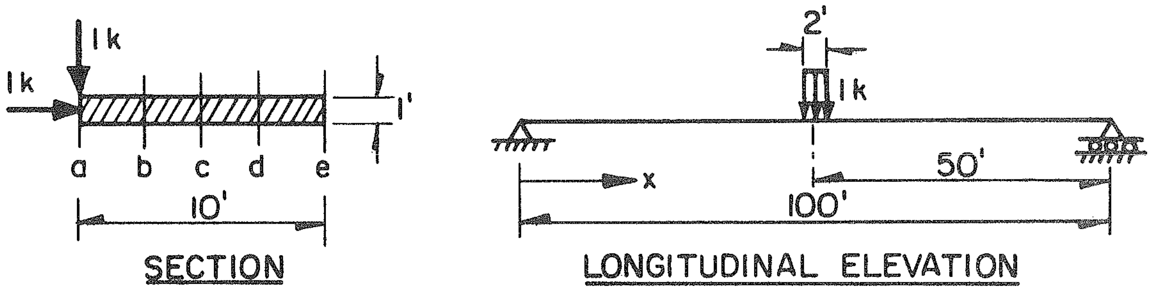
Table 2. COMPARISON OF RESULTS FOR EXAMPLE 2 (FIG. 19)

QUANTITY	METHOD[1]	y (ft) →	0	2.5	5.0	7.5	10.0
		x (ft) ↓	a	b	c	d	e
u (ft. × 10^{-2})	FP	0	-5.46	-2.71	0	2.71	5.46
	FS-1	0	-5.31	-2.66	0	2.66	5.31
	FS-4	0	-5.43	-2.69	0	2.69	5.43
v (ft. × 10^{-1})	FP	50	3.47	3.47	3.47	3.47	3.47
	FS-1	50	3.38	3.38	3.38	3.38	3.38
	FS-4	50	3.45	3.45	3.45	3.45	3.45
w (ft. × 10^{1})	FP	50	3.40	3.40	3.40	3.40	3.40
	FS-1	50	3.40	3.39	3.39	3.39	3.39
	FS-4	50	3.40	3.40	3.40	3.40	3.40
N_x (k/ft. × 10^{2})	FP	50	7.52	3.74	0	-3.74	-7.52
	FS-1	50	7.50	3.75	0	-3.75	-7.50
	FS-4	50	7.54	3.73	0	-3.73	-7.54
N_y (k/ft. × 10^{-1})	FP	50	0.00	9.22	0	-9.20	0.00
	FS-1	50	1120.	562.	0	-562.	-1120.
	FS-4	50	294.	3.20	0	-3.70	-294.
N_{xy} (k/ft. × 10^{1})	FP	0	0.00	5.51	7.30	5.51	0.00
	FS-1	0	4.90	4.90	4.90	4.90	4.90
	FS-4	0	2.92	4.46	6.75	4.46	2.92
M_x (k-ft/ft. × 10^{3})	FP	50	1.25	1.25	1.25	1.25	1.25
	FS-1	50	1.25	1.25	1.25	1.25	1.25
	FS-4	50	1.25	1.25	1.25	1.25	1.25
M_y (k-ft/ft. × 10°)	FP	50	0.00	2.43	3.25	2.43	0.00
	FS-1	50	2.35	2.14	2.07	2.14	2.35
	FS-4	50	0.15	2.59	3.40	2.59	0.15
M_{xy} (k-ft/ft. × 10^{1})	FP	0	-3.02	-1.48	0	1.48	3.02
	FS-1	0	-3.00	-1.50	0	1.50	3.00
	FS-4	0	-3.02	-1.47	0	1.44	2.92

1. FP = Folded plate method; MULTPL
   FS-1 = Finite strip method; 1 strip for total width; MULSTR
   FS-4 = Finite strip method; 4 strips for total width; MULSTR
   n = 19 harmonics

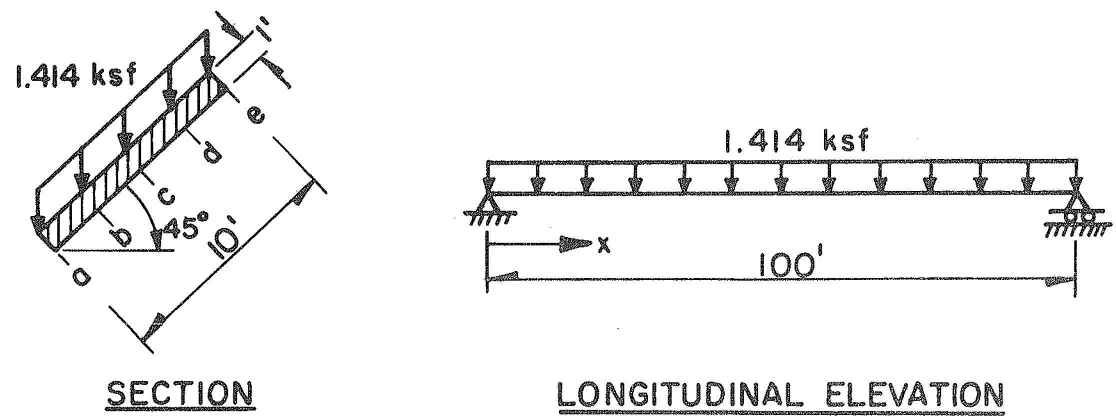
2. Loading:





E = 460,000 ksf; ν = 0.15; n = 19 harmonics

FIG. 18 DATA FOR EXAMPLE 1



E = 460,000 ksf; ν = 0.15; n = 19 harmonics

FIG. 19 DATA FOR EXAMPLE 2

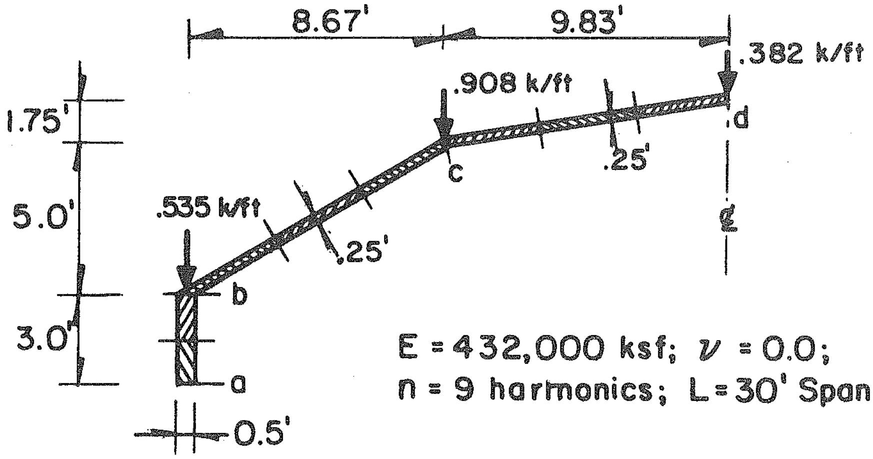


FIG. 20 DATA FOR EXAMPLE 3

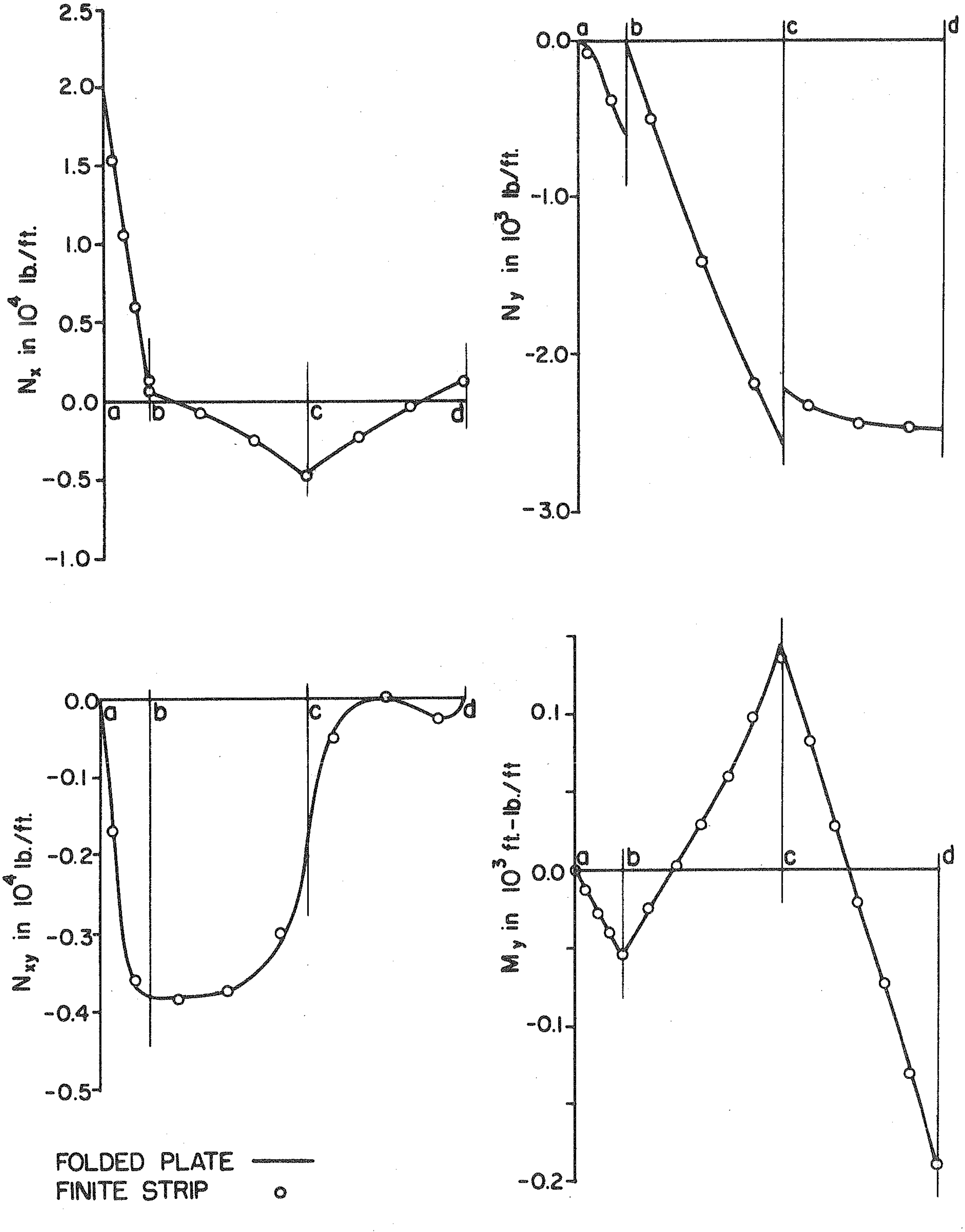


FIG. 21 RESULTS FOR EXAMPLE 3 — SPAN L = 30'

The structure is analyzed by MULSTR using two finite strips to idealize the vertical plate and three finite strips for each sloping plate. The results are compared pictorially in Fig. 21 with those obtained by the folded plate method using MULTPL. The values at each point are either averaged values at the edges of two finite strips or are output at the center of each finite strip. Observe the excellent agreement of all quantities, but especially of the transverse quantities N_y and M_y if the center values in each strip are used.

5.2.4 Example 4 - Plate With Eccentric Open Ribs in One Direction Only, Torsionally Stiff (Fig. 22)

The system shown in Fig. 22 is analyzed for the loading cases of Fig. 26 using three different approaches.

First, it is analyzed by MULTPL (FP) using the nodal point layout shown in Fig. 23. Centerline dimensions are used to establish the two element types, which are a rib element [1] and a deck plate element [2]. Modulus of elasticity E = 30,000 ksf and $\nu = 0.15$ are assumed for all elements.

Second, it is analyzed by MULSTR using 10 finite strips (FS-10) with the nodal point layout shown in Fig. 24. Here the overall width dimension of the deck is used and two element types occur. Element type [1] consists of a plate plus rib combination, in which the plate has a cross-section of 0.50 X 0.50 ft. and the rib has a cross-section of 0.50 X 2.25 ft. Note that the rib area extends to the mid-surface of the plate, thus overlapping a portion of the deck plate. As mentioned in Chapter 4, extensive numerical studies have indicated this assumption for the rib area yields the best results if the following orthotropic material properties are used for the plate:

$E_x = 900$ ksf and $E_y = 30{,}000$ ksf while $\nu_{xy} = \nu_{yx} = 0$. The longitudinal rib has an elastic modulus of $E_x = 30{,}000$ ksf. Element type [2] consists only of the isotropic deck plate and has a cross-section of 1.67 X 0.50 ft. with $E_x = E_y = 30{,}000$ ksf and $\nu = 0.15$. Torsional stiffness of the ribs was included.

Third, it is analyzed by MULSTR using 6 finite strips (FS-6) with the nodal point layout shown in Fig. 25. The overall width is taken from center to center of the outside ribs, thus giving a slightly smaller width than that used in FS-10. All elements are assumed to have the same width of 1.67 ft. Exterior element type [1] consists of the deck plate with a full thickness rib distributed over the width of the strip, and interior element type [2] consists of the deck plate with a half-thickness rib distributed over the width of the strip. Torsional stiffness of the ribs was included.

It is evident from the above description that in cases where only a few ribs exist, such as is true here, a variety of assumptions can be made. The example chosen is a severe test of MULSTR since the theory is predicated on there being a large number of closely spaced ribs in the system rather than a few isolated ones.

Results for u, v, w, σ_x at the plate mid-surface, and σ_x at the bottom fiber of the ribs are given in Tables 3A through 3E for the loading cases shown in Fig. 26. Results for loads normal to the deck, examples 4A and 4B, obtained by MULSTR compare favorably with those found by MULTPL. Results for loads parallel to the plane of the deck, 4C and 4D, and for an edge moment, 4E, compare less favorably due to the reason cited above.

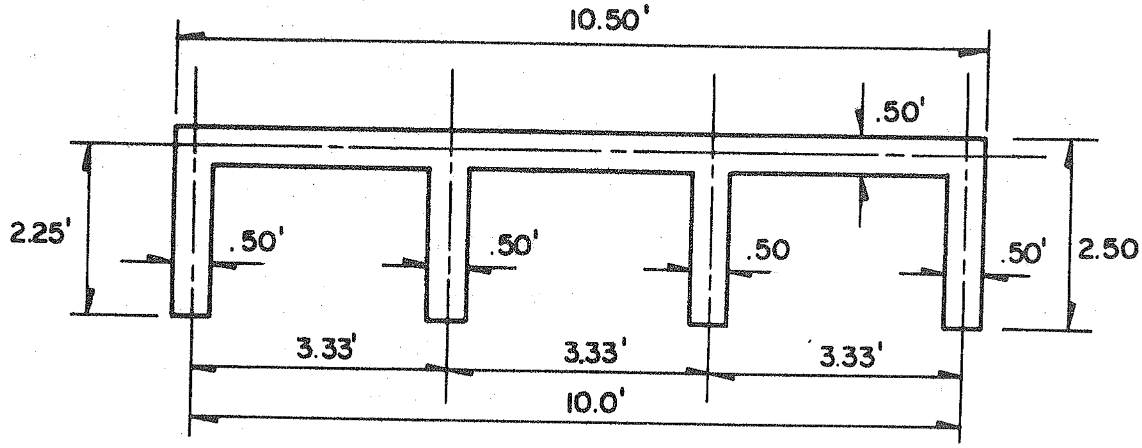


FIG. 22 DATA FOR EXAMPLE 4

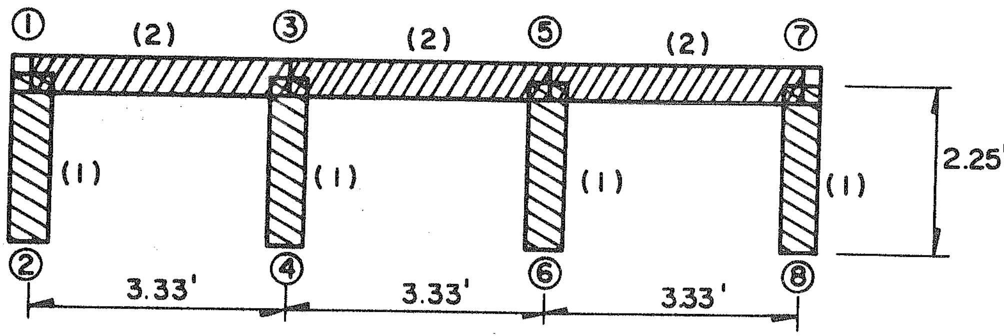


FIG. 23 PLATE IDEALIZATION FOR MULTPL SOLUTION (FP)

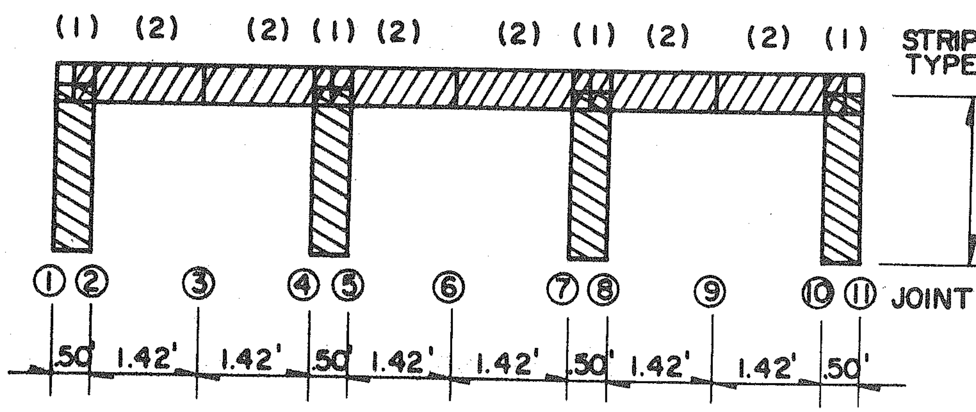


FIG. 24 FINITE STRIP IDEALIZATION FOR DISCRETE MULSTR SOLUTION (FS-10)

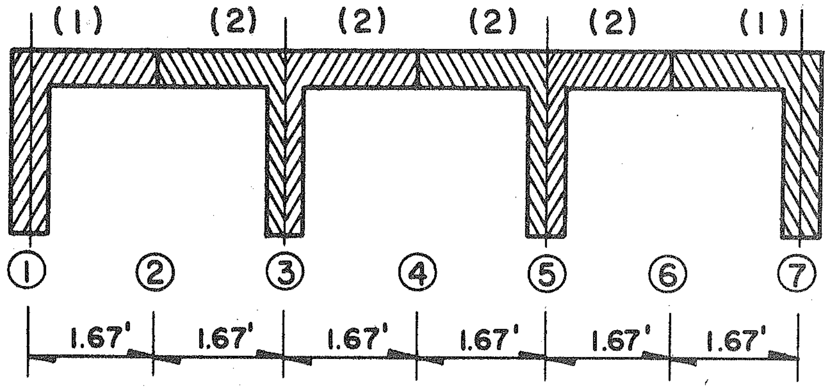


FIG. 25 EXAMPLE 4 – FINITE STRIP IDEALIZATION FOR SMEARED MULSTR SOLUTION (FS-6)

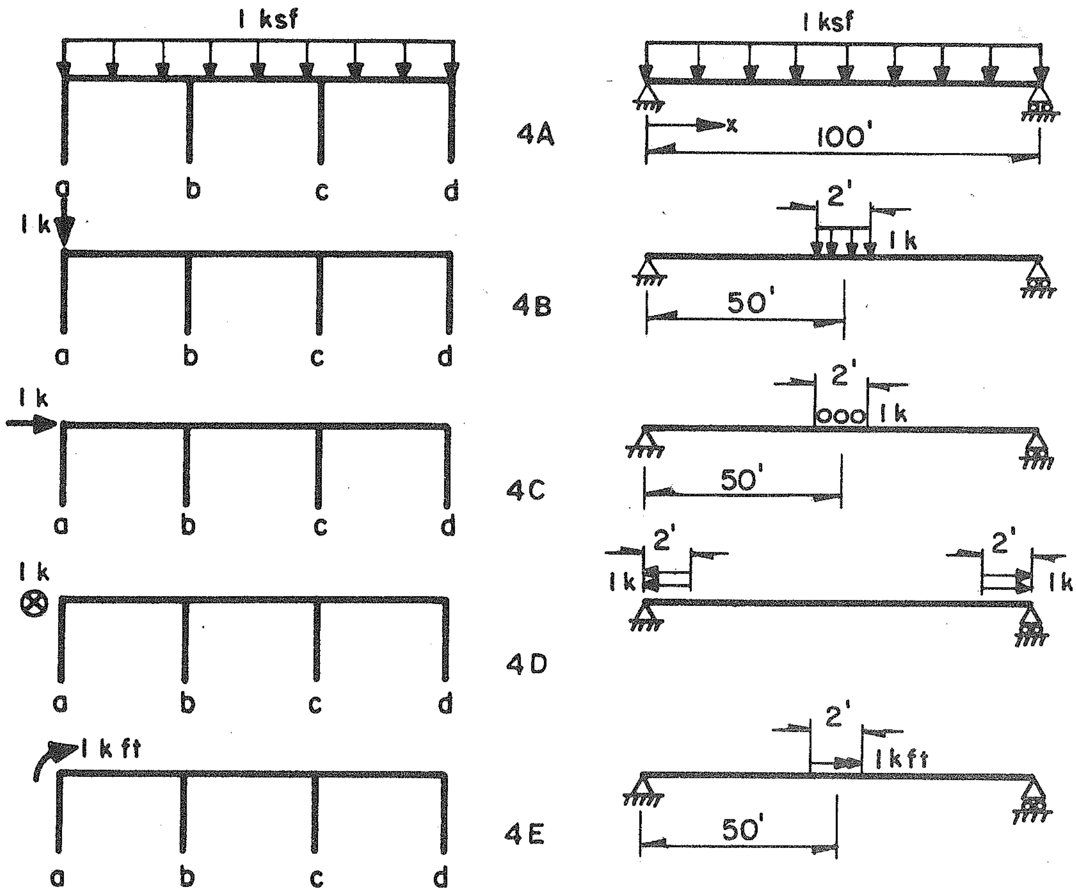


FIG. 26 EXAMPLE 4 LOAD CASES AND OUTPUT POINTS

Table 3A. COMPARISON OF RESULTS FOR EXAMPLE 4A (FIG. 26)

QUANTITY	METHOD [1]	y (ft) →	0	3.33	6.66	10.00
		x (ft) ↓	a	b	c	d
u (ft. X 10^{0})	FP	0	1.49	1.48	1.48	1.49
	FS-10	0	1.52	1.47	1.47	1.52
	FS-6	0	1.48	1.47	1.47	1.48
v (ft. X 10^{-2})	FP	50	3.28	1.09	-1.09	-3.28
	FS-10	50	3.25	1.07	-1.07	-3.25
	FS-6	50	3.30	1.09	-1.09	-3.30
w (ft. X 10^{1})	FP	50	8.71	8.70	8.70	8.71
	FS-10	50	8.69	8.69	8.69	8.69
	FS-6	50	8.67	8.66	8.66	8.67
Plate σ_x at mid surf. (ksf X 10^3)	FP	50	-1.34	-1.33	-1.33	-1.34
	FS-10	50	-1.36	-1.32	-1.32	-1.36
	FS-6	50	-1.33	-1.33	-1.33	-1.33
Rib σ_x at bot. fiber (ksf X 10^3)	FP	50	4.29	4.29	4.29	4.29
	FS-10	50	4.28	4.30	4.30	4.28
	FS-6	50	4.29	4.28	4.28	4.28

1. FP = Folded plate method, MULTPL, see Fig. 23
   FS-10 = Finite strip method, MULSTR, see Fig. 24
   FS-6 = Finite strip method, MULSTR, see Fig. 25
   n = 19 harmonics

2. Loading:

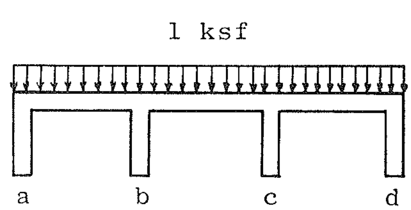


Table 3B. COMPARISON OF RESULTS FOR EXAMPLE 4B (FIG. 26)

QUANTITY	METHOD[1]	y (ft) → / x (ft) ↓	0 / a	3.33 / b	6.66 / c	10.00 / d
u (ft. × 10^{-2})	FP FS-10 FS-6	0 0 0	3.01 3.21 2.92	2.48 2.52 2.45	1.96 1.89 1.98	1.45 1.31 1.51
v (ft. × 10^{-2})	FP FS-10 FS-6	50 50 50	-5.96 -6.36 -5.47	-6.01 -6.41 -5.53	-6.04 -6.44 -5.56	-6.06 -6.46 -5.58
w (ft. × 10^{0})	FP FS-10 FS-6	50 50 50	1.85 1.93 1.86	1.54 1.56 1.53	1.24 1.21 1.23	.95 .87 .94
Plate σ_x at mid surf. (ksf × 10^{1})	FP FS-10 FS-6	50 50 50	-5.00 -5.76 -4.84	-3.06 -3.06 -3.02	-2.00 -1.92 -2.02	-1.02 - .86 -1.06
Rib σ_x at bot. fiber (ksf × 10^{1})	FP FS-10 FS-6	50 50 50	13.57 14.19 16.77	9.19 9.30 9.06	6.71 6.50 5.55	4.91 4.48 4.68

1. FP = Folded plate method, MULTPL, see Fig. 23
   FS-10 = Finite strip method, MULSTR, see Fig. 24
   FS-6 = Finite strip method, MULSTR, see Fig. 25
   n = 49 harmonics

2. Loading:

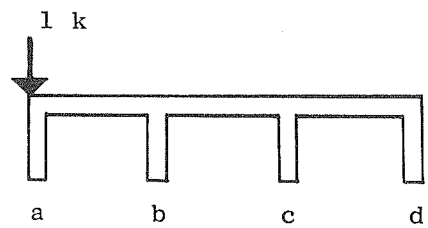


Table 3C. COMPARISON OF RESULTS FOR EXAMPLE 4C (FIG. 26)

QUANTITY	METHOD[1]	y(ft)→	0	3.33	6.66	10.00
		x(ft)↓	a	b	c	d
u (ft. X 10^{-2})	FP FS-10 FS-6	0 0 0	1.11 1.00 1.25	.36 .32 .41	- .37 - .32 - .41	-1.11 -1.00 -1.25
v (ft. X 10^{-2})	FP FS-10 FS-6	50 50 50	-7.93 -7.30 -8.88	-7.88 -7.25 -8.84	-7.86 -7.22 -8.82	-7.85 -7.21 -8.80
w (ft. X 10^{-2})	FP FS-10 FS-6	50 50 50	5.96 6.37 5.47	1.94 2.06 1.78	-2.07 -2.20 -1.92	-6.06 -6.46 -5.58
Plate σ_x at mid surf. (ksf X 10^0)	FP FS-10 FS-6	50 50 50	-18.08 -15.34 -19.80	-4.04 +2.74 -4.42	4.60 4.00 5.02	14.00 13.44 15.72
Rib σ_x at bot. fiber (ksf X 10^0)	FP FS-10 FS-6	50 50 50	-5.66 -4.35 -8.07	-2.11 -1.48 -2.92	1.87 1.04 2.43	6.39 5.12 8.50

1. FP = Folded plate method, MULTPL, see Fig. 23
   FS-10 = Finite strip method, MULSTR, see Fig. 24
   FS-6 = Finite strip method, MULSTR, see Fig. 25
   n = 49 harmonics

2. Loading:

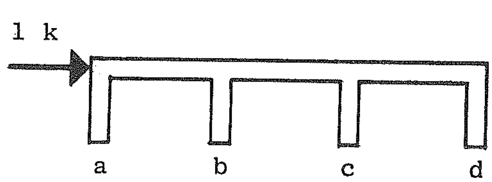


Table 3D. COMPARISON OF RESULTS FOR EXAMPLE 4D (FIG. 26)

QUANTITY	METHOD[1]	y (ft) →	0	3.33	6.66	10.00
		x (ft) ↓	a	b	c	d
u (ft. $\times 10^{-3}$)	FP	0	-7.89	-3.97	-1.03	1.86
	FS-10	0	-9.88	-3.89	-0.91	1.98
	FS-6	0	-8.51	-4.12	-0.85	2.40
v (ft. $\times 10^{-2}$)	FP	50	2.21	2.21	2.22	2.22
	FS-10	50	2.08	2.09	2.09	2.09
	FS-6	50	2.50	2.50	2.51	2.51
w (ft. $\times 10^{-2}$)	FP	50	-6.01	-4.97	-3.93	-2.90
	FS-10	50	-6.59	-5.25	-3.92	-2.59
	FS-6	50	-5.84	-4.90	-4.43	-3.49
Plate σ_x at mid surf. (ksf $\times 10^{0}$)	FP	50	4.10	2.44	0.80	-0.86
	FS-10	50	4.28	2.50	0.74	-1.02
	FS-6	50	4.48	2.56	0.68	-1.24
Rib σ_x at bot. fiber (ksf $\times 10^{0}$)	FP	50	1.42	-0.03	-1.51	-2.99
	FS-10	50	1.47	-0.01	-1.52	-3.05
	FS-6	50	1.86	0.11	-1.65	-3.41

1. FP = Folded plate method, MULTPL, see Fig. 23
   FS-10 = Finite strip method, MULSTR, see Fig. 24
   FS-6 = Finite strip method, MULSTR, see Fig. 25
   n = 49 harmonics

2. Loading:

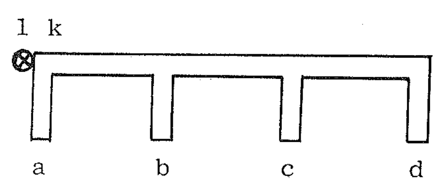


Table 3E. COMPARISON OF RESULTS FOR EXAMPLE 4E (FIG. 26)

QUANTITY	METHOD [1]	y (ft) → / x (ft) ↓	0 / a	3.33 / b	6.66 / c	10.00 / d
u (ft. × 10^{-3})	FP	0	-1.57	-0.53	0.50	1.54
	FS-10	0	-1.66	-0.56	0.53	1.65
	FS-6	0	-1.42	-0.48	0.46	1.40
v (ft. × 10^{-2})	FP	50	1.19	1.20	1.20	1.19
	FS-10	50	1.11	1.11	1.11	1.11
	FS-6	50	1.11	1.10	1.10	1.10
w (ft. × 10^{-1})	FP	50	0.95	0.26	-0.31	-0.85
	FS-10	50	0.98	0.26	-0.32	-0.87
	FS-6	50	1.01	0.28	-0.31	-0.87
Plate σ_x at mid surf. (ksf × 10^0)	FP	50	-5.56	-0.32	1.04	2.82
	FS-10	50	-9.04	-0.28	1.02	2.76
	FS-6	50	-6.32	-0.48	1.02	2.74
Rib σ_x at bot. fiber (ksf × 10^0)	FP	50	13.37	-3.08	-2.19	-5.12
	FS-10	50	10.90	-2.38	-3.01	-4.89
	FS-6	50	67.94	-1.34	-2.39	-5.46

1. FP = Folded plate method, MULTPL, see Fig. 23
   FS-10 = Finite strip method, MULSTR, see Fig. 24
   FS-6 = Finite strip method, MULSTR, see Fig. 25
   n = 49 harmonics

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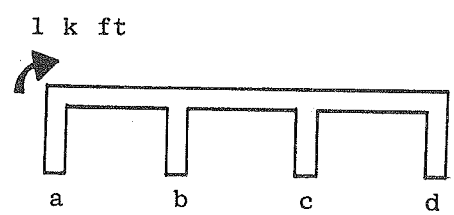


5.3 Orthotropic Deck Bridge (Fig. 27)

An isotropic deck plate with three different arrangements of closely spaced eccentric ribs is analyzed and compared with the analytical results of Reference [4]. This deck which is simply supported on all four edges is illustrated in Fig. 27. The boundary conditions allow the application of the trigonometric expansion of MULSTR in either the x- or the y-direction. Thus, two types of analyses are performed to find the solution:

First, 95 harmonics are used to describe the trigonometric variation in the x-direction, while the width of the plate is idealized by 20 finite strips.

Second, 15 harmonics are used to describe the trigonometric variation in the y-direction, while the length of the plate is represented by 80 finite strips. The difference of the number of harmonics considered originates in the change of rate of convergence caused by the large difference in load distribution due to the different spans.

The structure is subjected to a 1 kip loading at the center of the deck which is distributed uniformly over an area of 15 X 15 in. Due to symmetry, only half of the structure has to be analyzed using odd harmonics only. The following quantities at the center of the plate are compared with the analytical results of Clifton, Chang, and Au [4]: the transverse displacements w, the top fiber stresses in the deck plate, and the bottom fiber stresses in the individual ribs. Three different types of closely spaced eccentric ribs are considered:

5.3.1 Example 5 - Open Rib System, Torsionally Soft

The proportions of the ribs are illustrated on top of Fig. 28. The open rib sections are spaced 12 in. on center in both directions.

The torsional rigidity of the stiffness is not considered. Table 4A presents a comparison of the finite strip results using harmonic expansions in the x- or in the y-direction with the exact results obtained from Reference [4]. The agreement is excellent.

5.3.2 Example 6 - Open Rib System, Torsionally Stiff

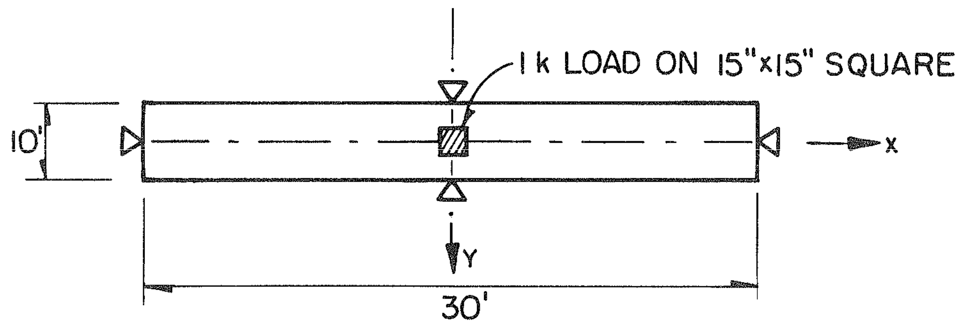
The proportions of the ribs are illustrated in the middle of Fig. 28. The open ribs are spaced 12 in. on center in both directions. The torsional rigidity of the open sections is included. Table 4B presents a comparison of the finite strip results using harmonic expansions in the x- or in the y-direction with the exact results obtained from Reference [4]. Again the agreement is excellent.

5.3.3 Example 7 - Closed Rib System, Torsionally Stiff

The properties of the ribs are illustrated at the bottom of Fig. 28. The closed ribs in the y-direction are spaced 24 in. on center while the open ribs in the longitudinal x-direction are spaced 12 in. on center. The torsional rigidity of both the open and the closed sections are considered. Table 4C presents a comparison of the finite strip results using harmonic expansions either in the x- or in the y-direction with the exact results obtained from Reference [4]. Again the agreement is excellent even for the case where the harmonics are expanded in the longitudinal x-direction, along which the structure is much more flexible than in the transverse y-direction which has a considerably shorter span and much larger stiffeners.

5.3.4 Comparison of Results

These examples indicate again that the stress resultants in the direction of the harmonic expansion are considerably better than



(1) PLATE MATERIAL   E = 30,000 ksi; ν = 0.30

(2) RIB MATERIAL   E = 30,000 ksi; G = 15,000 ksi

FIG. 27 PLAN DIMENSIONS AND LOADING FOR EXAMPLES 5, 6, 7

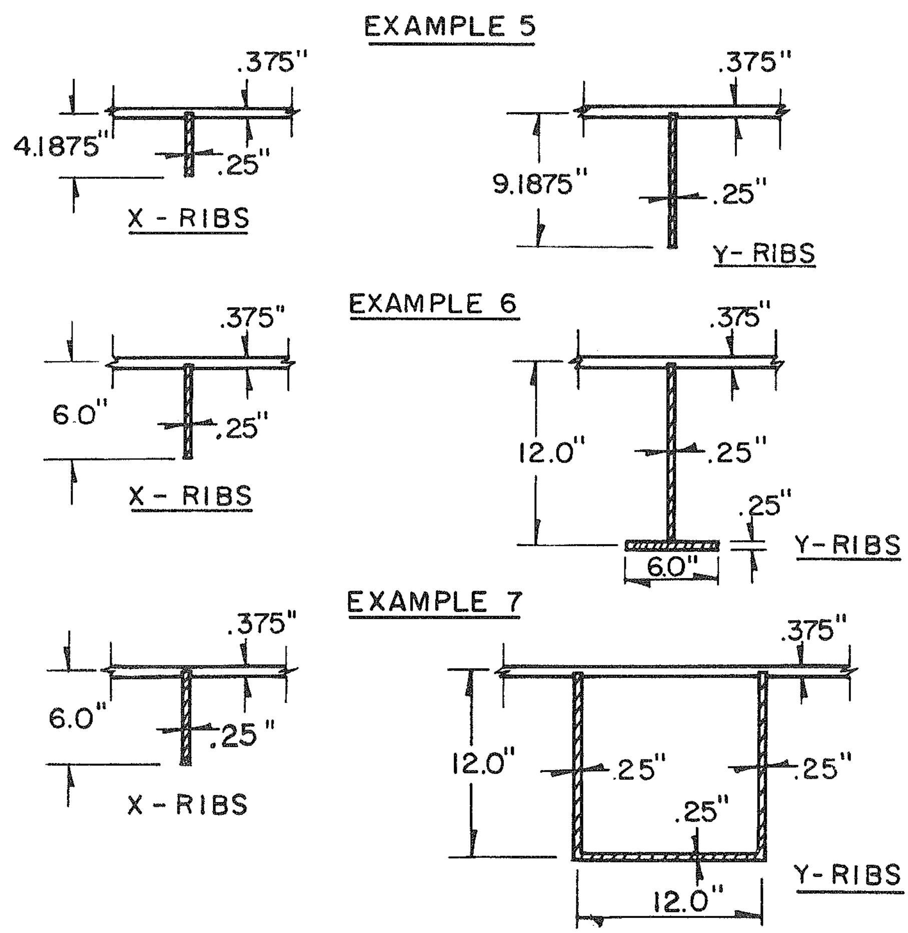


FIG. 28 RIBS IN X AND Y DIRECTIONS FOR EXAMPLES 5, 6, 7

Table 4. COMPARISON OF RESULTS AT CENTER FOR EXAMPLES 5, 6, 7 (FIG. 27)

METHOD OF ANALYSIS [1]		DEFLECTION w (in)	STRESSES AT TOP OF PLATE (psi)		STRESSES AT BOTTOM OF STIFFENER (psi)	
			σ_x	σ_y	σ_x	σ_y
TABLE 4A EXAMPLE 5	CLIFTON, CHANG & AU [4]	.00451	-163.	-230.	1059.	1092.
	FS M-10	.00456	-163.	-228.	1076.	1116.
	FS M-40	.00449	-160.	-232.	1117.	1090.
TABLE 4B EXAMPLE 6	CLIFTON, CHANG & AU [4]	.00101	-71.6	-123.	414.	273.
	FS M-10	.00110	-73.0	-125.	444.	294.
	FS M-40	.00108	-70.8	-128.	459.	286.
TABLE 4C EXAMPLE 7	CLIFTON, CHANG & AU [4]	.00078	-53.0	-99.0	294.	211.
	FS M-10	.00081	-49.9	-94.9	292.	214.
	FS M-40	.00080	-49.1	-98.6	304.	211.

1. CLIFTON, CHANG & AU [4] = Exact solution of orthotropic plate formulation

FS M-10 = Finite strip method; Mesh: harmonic expansion in the longitudinal x-direction with 10 strips idealizing the half width (Fig. 27) - program MULSTR.

FS M-40 = Finite strip method; Mesh: harmonic expansion in the transverse y-direction with 40 strips idealizing the half length (Fig. 27) - program MULSTR.

those in the direction of the polynomial expansion. This fact becomes obvious if one recalls that the trigonometric expansion does satisfy the force boundary conditions at the simple supports in addition to the displacement boundary conditions.

The results obtained from the examples 6 and 7 illustrate the beneficial effect of the large torsional rigidity of the closed ribs in comparison to the open ribs. Recall that the effective moments of inertia are identical for both types of stiffeners, only the torsional rigidities differ. The use of closed ribs reduces the center deflections by 20% while the top fiber stresses in the plate and the bottom fiber stresses of the ribs decrease by 20% to 30%.

5.4 Orthotropic Box Girder

A single cell box with different arrangements of eccentric stiffeners is analyzed. For the case of no stiffeners the results can be compared with the exact ones obtained from folded plate analysis. The effect of eccentric ribs on the structural response is studied by cases of only longitudinal x-, only transverse y-, or both longitudinal x- and transverse y-stiffeners.

5.4.1 Example 8 - Single Cell Box With and Without Eccentric Stiffeners

The overall dimensions of the single cell box are given in Fig. 29. The structure is subjected to two symmetric or antisymmetric loadings at midspan and is simply supported at two opposite ends. Nineteen harmonics are chosen to describe the trigonometric variation in the longitudinal x-direction. Taking advantage of symmetry only the odd harmonics need to be used. The deck plates of the single cell box are idealized by 5 finite strips while the web plates are represented by 3 finite strips, see Fig. 30. Four arrangements of stiffeners are chosen to study their effect on the structural response of the single cell box:

1) The case of no stiffeners, illustrated in Fig. 32 at the left, allows one to assess the accuracy of the finite strip results by comparing them with the exact results obtained from folded plate analysis. All plate components have isotropic material properties. The deck plates are 1.5 in. thick while the web plates are 0.75 in., exhibiting very little bending stiffness.

2) The case of longitudinal x-stiffeners, illustrated in Fig. 32 at the right, increases considerably the inertia moment of the section

ERRATA

"Analysis of Orthotropic Folded Plates with Eccentric Stiffeners"

K. J. Willam and A. C. Scordelis, Structural Engineering and Structural Mechanics Report No. SESM 70-2, U.C. Berkeley, February 1970.

(1) Owing to an error in the input data, the cross-section of the structure actually analyzed in example 8 on page 73 is as shown below in Figure 1, instead of Figure 2 as included in the report.

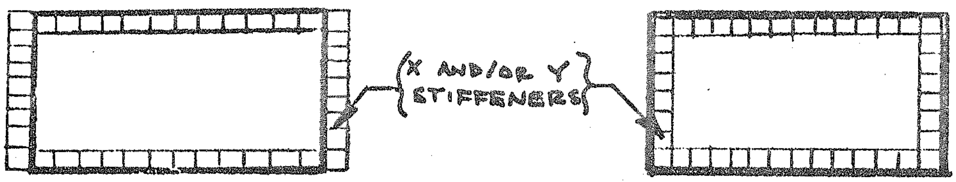
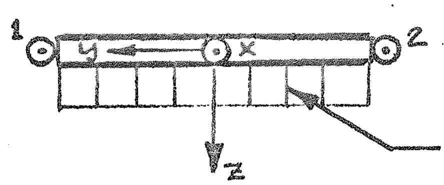


Figure 1 Figure 2

As explained at the top of page A6 of the report, the eccentricity of the rib is positive if it lies in the positive z-direction of the local strip coordinates (see Figure 3 below).



stiffener ribs with positive eccentricity

Figure 3

In the original analysis given in the report, wrong signs were input for the SMX, SMY, ERX and ERY in the strip type cards for the webs (page A5, paragraph 4.5 of the input description given in report), therefore resulting in the cross-section shown in Figure 1 above.

(2) Also in the report, on page A5, strip type cards, third card, the following corrections should be made:

"Col. 41 to 50"	should read	"Col. 21 to 30"
"Col. 51 to 60"	should read	"Col. 31 to 40"
"Col. 61 to 70"	should read	"Col. 41 to 50"
"Col. 71 to 80"	should read	"Col. 51 to 60"

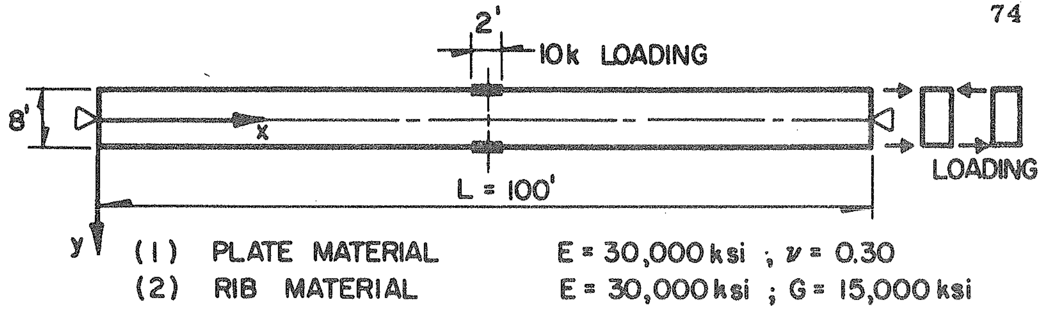


FIG. 29 DATA FOR EXAMPLE 8

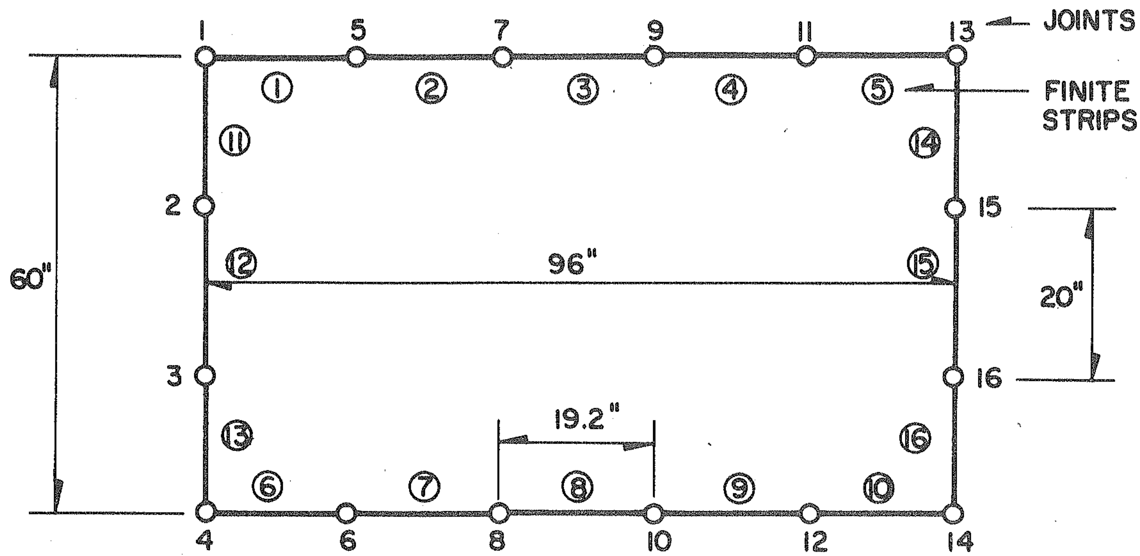


FIG. 30 EXAMPLE 8 - FINITE STRIP IDEALIZATION OF CROSS-SECTION (FS-5-3)

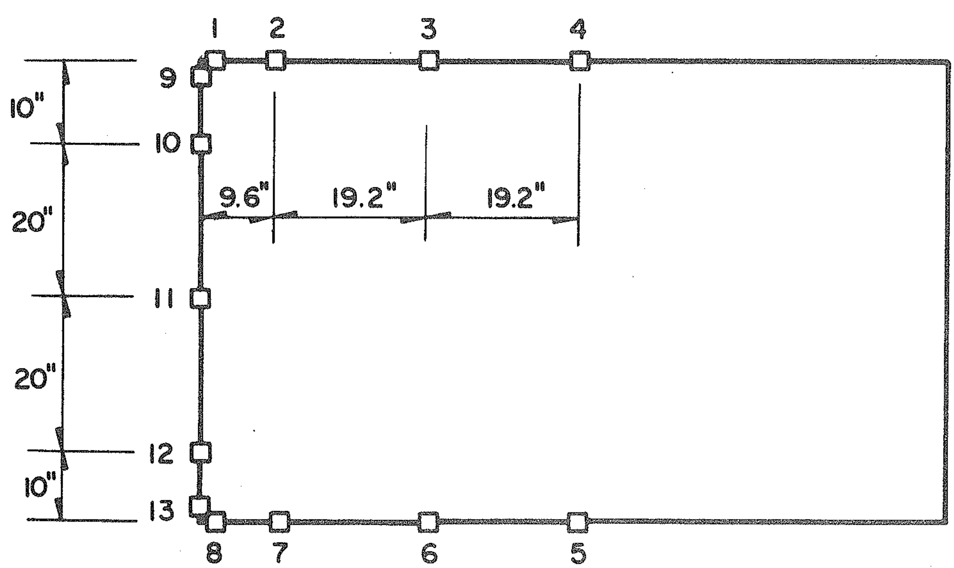


FIG. 31 EXAMPLE 8 - LOCATION OF OUTPUT QUANTITIES AT CROSS-SECTION

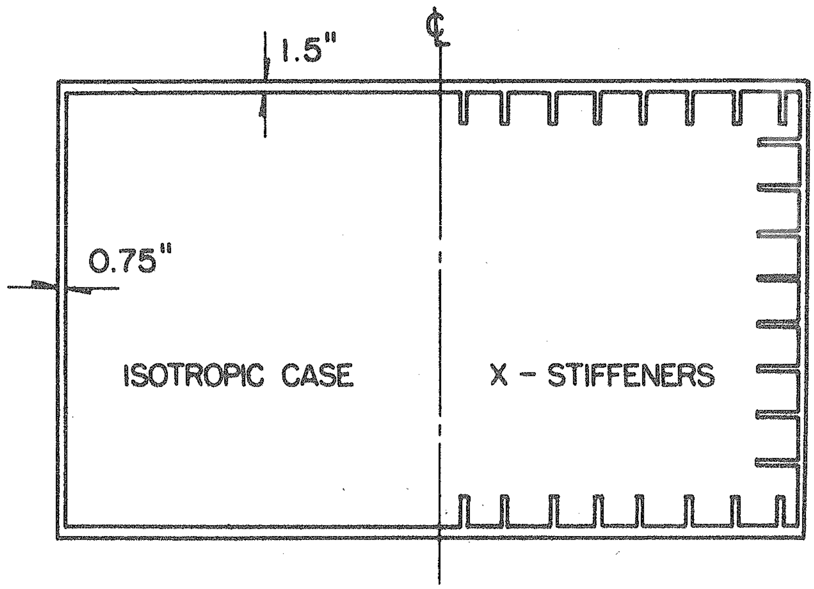


FIG. 32 EXAMPLE 8 - ISOTROPIC CASE AND CASE OF X- STIFFENERS ONLY

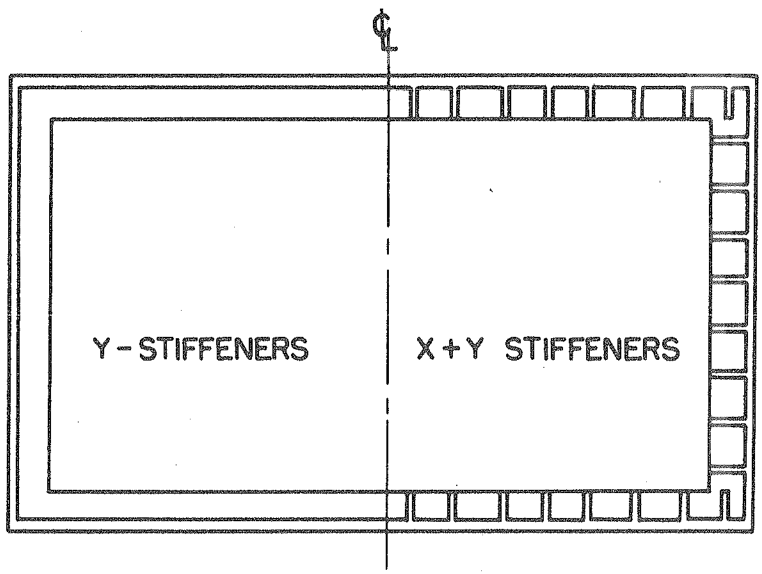


FIG. 33 EXAMPLE 8 - CASE OF Y-STIFFENERS AND CASE OF X+Y STIFFENERS

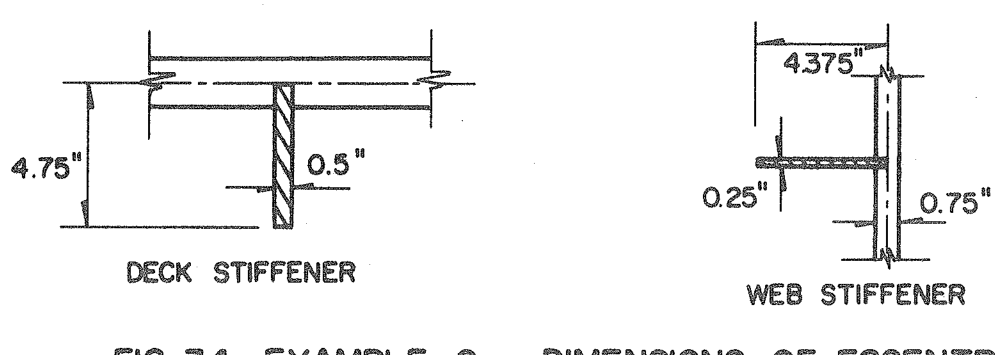


FIG. 34 EXAMPLE 8 - DIMENSIONS OF ECCENTRIC RIBS

without changing the transverse plate stiffness or the overall torsional rigidity. The ribs which are spaced 6 in. on center are illustrated in Fig. 34. Their torsional rigidity is neglected.

3) The case of transverse stiffeners, illustrated in Fig. 33 at the left demonstrates that the torsional rigidity of the box section is of minor importance if compared with the effect of transverse ribs which reduce sharply cross sectional distortions. The ribs which are spaced 6 in. on center, are illustrated in Fig. 34. Their torsional rigidity is neglected.

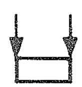
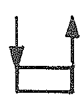
4) The case of transverse and longitudinal stiffeners, illustrated in Fig. 33 at the right, clearly combines the effects of both types of ribs described in 2) and 3). Their proportions are illustrated in Fig. 34. All of them are spaced 6 in. on center and their torsional rigidity is neglected.

5.4.2 Comparison of Results

Tables 5A, 5B and 5C present a comparison of the vertical deflections, axial stress resultants N_x and transverse moments M_y for the different arrangements of longitudinal and transverse stiffeners described in the previous section. Note that stress resultants and moments are those of the combined rib plate system. The locations of output which are positioned at the center of each strip except at the corners of the box section are illustrated in Fig. 31 at the left. These midspan values are given for both load cases, symmetric bending and antisymmetric torsion.

A comparison of the results for the isotropic box without stiffeners illustrates the excellent agreement between the exact folded

Table 5A. COMPARISON OF VERTICAL DISPLACEMENTS (in $\times 10^{-2}$) FOR EXAMPLE 8 (FIG. 29)

LOADING	STIFFENER TYPE	ANALYSIS METHOD [1]	TOP DECK				BOTTOM DECK				WEB				
			1	2	3	4	5	6	7	8	9	10	11	12	13
 SYMM.	No	FP	9.04	9.03	9.01	9.01	8.99	9.00	9.01	9.02	9.04	9.04	9.04	9.03	9.02
	No	FS-5-3	9.03	9.02	9.00	8.99	8.99	8.98	9.00	9.01	9.03	9.03	9.03	9.02	9.01
	X-Stiff	FS-5-3	7.51	7.51	7.52	7.52	7.51	7.51	7.50	7.50	7.51	7.51	7.51	7.50	7.49
	Y-Stiff	FS-5-3	8.95	8.93	8.90	8.89	8.83	8.84	8.89	8.93	8.95	8.95	8.95	8.93	8.93
	X+Y Stiff	FS-5-3	7.46	7.44	7.41	7.40	7.36	7.37	7.41	7.44	7.46	7.46	7.45	7.44	7.44
 A-SYMM.	No	FP	4.59	3.88	2.07	0.0	0.0	2.07	3.86	4.57	4.59	4.59	4.58	4.58	4.57
	No	FS-5-3	4.58	3.87	2.07	0.0	0.0	2.06	3.85	4.56	4.58	4.58	4.58	4.57	4.56
	X-Stiff	FS-5-3	4.37	3.69	1.98	0.0	0.0	1.97	3.68	4.35	4.37	4.37	4.36	4.36	4.35
	Y-Stiff	FS-5-3	0.69	0.60	0.33	0.0	0.0	0.20	0.58	0.67	0.69	0.69	0.68	0.68	0.67
	X+Y Stiff	FS-5-3	0.67	0.58	0.32	0.0	0.0	0.31	0.56	0.66	0.67	0.67	0.67	0.66	0.66

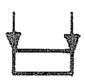
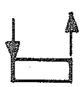
1. FP = Folded plate method - MULTPL, see Fig. 31

   FS-5-3 = Finite strip method - MULSTR, see Fig. 31

   x = L/2 at midspan

   n = 19 number of harmonics

Table 5B. COMPARISON OF N_x (k/in × 10^{-1}) FOR EXAMPLE 8 (FIG. 29)

LOADING	STIFFENER TYPE	ANALYSIS METHOD[1]	TOP DECK				BOTTOM DECK				WEB				
			1	2	3	4	5	6	7	8	9	10	11	12	13
 SYMM.	No	FP	-9.80	-9.39	-8.93	-8.80	8.87	9.00	9.46	9.86	-5.39	-3.42	-0.02	3.26	4.93
	No	FS-5-3	-9.79	-9.39	-8.93	-8.80	8.88	9.01	9.48	9.88	-5.43	-3.48	0.01	3.27	4.98
	X-Stiff	FS-5-3	-10.01	-9.55	-9.00	-8.85	8.92	9.07	9.64	10.12	-5.58	3.58	0.02	3.37	5.11
	Y-Stiff	FS-5-3	-9.84	-9.42	-8.94	-8.81	8.89	9.02	9.49	9.89	-5.33	-3.41	0.00	3.23	4.94
	X+Y Stiff	FS-5-3	-10.06	-9.58	-9.01	-8.86	8.93	9.27	9.64	10.10	-5.48	-3.51	0.02	3.33	5.07
 A-SYMM.	No	FP	-7.59	-5.83	-2.77	0.0	0.0	2.84	5.95	7.73	-4.28	-2.66	0.04	2.53	3.86
	No	FS-5-3	-7.65	-5.83	-2.77	0.0	0.0	2.84	5.97	7.83	-4.28	-2.72	0.03	2.55	3.87
	X-Stiff	FS-5-3	-8.39	-6.39	-3.02	0.0	0.0	3.08	6.52	8.57	-4.73	-3.02	0.03	2.85	4.30
	Y-Stiff	FS-5-3	-3.16	-2.37	-1.04	0.0	0.0	1.10	2.47	3.29	-1.58	-0.91	0.03	0.78	1.26
	X+Y Stiff	FS-5-3	-3.36	-2.48	-1.06	0.0	0.0	1.12	2.57	3.47	-1.76	-1.02	0.04	0.88	1.40

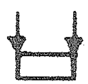
1. FP = Folded plate method - MULTPL, see Fig. 31

FS-5-3 = Finite strip method - MULSTR, see Fig. 31

x = L/2 at midspan

n = 19 number of harmonics

Table 5C. COMPARISON OF My (k-in/in × 10^{-4}) FOR EXAMPLE 8 (FIG. 29)

LOADING	STIFFENER TYPE	ANALYSIS METHOD[1]	TOP DECK				BOTTOM DECK				WEB				
			1	2	3	4	5	6	7	8	9	10	11	12	13
 SYMM.	No	FP	7.60	2.63	-1.74	-2.73	-0.89	-0.15	2.79	5.82	-7.60	-4.57	-0.33	3.40	5.82
	No	FS-5-3	6.70	2.85	-1.63	-2.65	-0.83	-0.08	2.88	5.26	-7.04	-4.53	0.34	3.38	5.44
	X-Stiff	FS-5-3	-5.65	6.66	26.0	33.1	44.0	40.3	25.2	11.4	11.7	7.90	-6.29	-7.73	1.06
	Y-Stiff	FS-5-3	886.	846.	785.	765.	433.	435.	438.	440.	-889.	-669.	-228.	217.	439.
	X+Y Stiff	FS-5-3	811.	727.	593.	547.	370.	375.	379.	374.	-812.	-633.	-267.	142.	362.
 A-SYMM.	No	FP	1748.	1390.	691.	0.0	0.0	691.	1389.	1744.	1748.	1162.	0.0	1160.	1744.
	No	FS-5-3	1742.	1389.	690.	0.0	0.0	691.	1388.	1740.	1748.	1161	0.0	1159.	1740.
	X-Stiff	FS-5-3	1660.	1334.	670.	0.0	0.0	681.	1352.	1678.	-1650.	-1097.	-6.49	1097.	1663.
	Y-Stiff	FS-5-3	4039.	3219.	1596.	0.0	0.0	1376.	2760.	3454.	-4043.	-2794.	-296.	2203.	3455.
	X+Y Stiff	FS-5-3	3885.	3088.	1519.	0.0	0.0	1347.	2688.	3351.	-3882.	-2692.	0.8	2108.	3337.

1. FP = Folded plate method - MULTPL, see Fig. 31

FS-5-3 = Finite strip method - MULSTR, see Fig. 31

x = L/2 at midspan

n = 19 number of harmonics

plate analysis with the finite strip method. All values exhibit a relative error of less than 1% even underneath the loading except for the transverse moments which are more sensitive due to their small size. Note that the displacements for the case of antisymmetrical loading are about one half of those compared with the case found for the case of symmetrical loading.

Table 5A indicates that longitudinal x-stiffeners reduce considerably the vertical displacements for the case of symmetrical loading but do not alter the displacements significantly under antisymmetric loading. Transverse y-stiffeners do not change the structural response under symmetrical loading but reduce sharply the vertical displacements under antisymmetrical loading.

Table 5B verifies that the longitudinal stress resultants N_x yield the same statical moment for all cases under symmetric loading. The x-stiffeners increase the longitudinal stress resultants slightly but still satisfy statics within 2%.

Table 5C compares the transverse moment distribution M_y which varies greatly for the different cases of stiffeners and loadings. In the symmetric load case the y-stiffeners vastly increase the M_y moments, while for no y-stiffeners the M_y moments are negligible. Obviously, in the case of antisymmetric loading the transverse moments are much larger resisting distortions of the cross section and increase with the amount of transverse stiffeners.

6. CONCLUSIONS

A method, ideally suited for computer application, was presented for the analysis of orthotropic folded plates with eccentric stiffeners. The computer program MULSTR, developed in this investigation, is restricted to the analysis of prismatic structures which are simply supported at two end diaphragms.

The derivation of the finite strip stiffness forms the basis for the harmonic analysis of these structures. Additional coupling of the in plane and plate bending action is provided in the case eccentric stiffeners are present. These rib properties are assumed to be distributed uniformly over the strip area. The exact theory for eccentrically stiffened plates does not lend itself to the analytical derivation of the stiffness properties. Hence, the approximate finite strip method is utilized representing the displacement field by trigonometric expansions in the longitudinal direction and by polynomial expansions in the transverse direction. The loading is expressed in terms of a Fourier series decoupling the load-displacement relationship of different harmonics due to orthogonality of the trigonometric functions. Hence, the total assembly matrix consists of stiffness matrices with very narrow bandwidths which are isolated for each harmonic. The computer program MULSTR takes advantage of these properties similar to the computer program MULTPL which was developed earlier for the analysis of isotropic folded plates [1]. It requires very little computational effort reducing the analysis of these complex structures to the trivial task of preparing input data.

The accuracy and efficiency of this program was tested on a variety of examples. The results of the finite strip analysis of isotropic plates, isotropic sheets, isotropic folded plates and eccentrically stiffened plate structures were compared with exact solutions. In addition a single cell box was analyzed to study its structural response using varying amounts of longitudinal and transverse stiffeners.

All these examples indicate that the finite strip method provides a very efficient tool to determine the overall behavior and the internal forces and moments in a combined plate-rib system. However, localized plate bending stresses between ribs in the actual structure cannot be predicted due to the assumption used in the analysis that the ribs are spread uniformly across the width of the finite strip.

At present, it is contemplated that the program will be extended to the analysis of multispan folded plate structures with eccentric stiffeners. A further improvement of the in plane strip behavior could be attained by incorporating an additional node at the centroid without affecting the connectivity of the strip. Furthermore, a numerical integration scheme could be chosen to determine the stiffness coefficients for strips with variable thickness in the transverse direction.

7. ACKNOWLEDGEMENTS

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Mr. G. D. Mancarti, Assistant Bridge Engineer, and Mr. R. E. Davis, Senior Bridge Engineer, of the Research and Development Section, provided close liaison from the Bridge Department, Division of Highways, State of California.

The support of the Computer Center at the University of California, Berkeley, is gratefully acknowledged for providing its facilities.

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11. DeFries-Skene, A. and Scordelis, A. C., "Direct Stiffness Solution for Folded Plates," Journal of Structural Division, ASCE, Vol. 90, No. ST4, August 1964.

APPENDIX A

Description of Computer Program MULSTR for the Analysis of Orthotropic Folded Plates with Eccentric Stiffeners

UNIVERSITY OF CALIFORNIA
Berkeley, California
February 1970

Department of Civil Engineering
Division of Structural Engineering
and Structural Mechanics

CDC 6400 Computer Program for the Analysis of Orthotropic Folded Plates with Eccentric Stiffeners

1.0 IDENTIFICATION

1.1 Program Name: MULSTR - Computer program for the analysis of simply supported orthotropic folded plates with eccentric ribs by the finite strip method.

1.2 Programmed by: Kaspar Willam, Junior Research Specialist.

1.3 Faculty Investigator: A. C. Scordelis, Professor of Civil Engineering.

1.4 References:

a) Willam, K. J. and Scordelis, A. C., "Analysis of Orthotropic Folded Plates with Eccentric Stiffeners," Structures and Materials Research Report, Division of Structural Engineering and Structural Mechanics, Department of Civil Engineering, University of California, Berkeley, SESM 70-2, February 1970.

b) Scordelis, A. C., "Analysis of Simply Supported Box Girder Bridges," Structures and Materials Research Report, Division of Structural Engineering and Structural Mechanics, Department of Civil Engineering, University of California, Berkeley, SESM 66-17, October 1966.

2.0 GENERAL DESCRIPTION

2.1 Nature of Program: This program is capable of analyzing orthotropic folded plates with eccentric stiffeners which are prismatic and simply supported by diaphragms at the two ends. These structures can be subjected to a variety of surface loads, joint loads and concentrated loads. Each plate component of the folded plate structure is idealized by a number of finite strips which are interconnected along the longitudinal joints by four degrees of freedom. Each finite strip consists of an orthotropic plate with eccentric stiffeners and exhibits in plane and flexural stiffness. The properties of longitudinal and transverse ribs are distributed uniformly over the area of each strip and are accounted for in the analysis.

The input data is so arranged that only the properties of a typical cross-section need to be specified. All final nodal displacements and internal forces within each finite strip are printed out at points selected by the user.

2.2 Definitions:

Finite Strip - a rectangular plate component whose location is defined by its two longitudinal joints 1 & 2. The strip is assumed to be simply supported by the diaphragms at the two ends as illustrated in Fig. A1.

Joint - a longitudinal line of junction interconnecting two or more finite strips.

Finite Strip Type - defined by the geometry which is described in terms of the horizontal and vertical projections, the thickness and possible rib dimensions and by the material properties which are defined by an elastic orthotropic material law for the plate and an elastic isotropic material law for the stiffeners.

2.3 Sign Conventions: These are given in Figs. A1 to A8. Reference is made to two right hand coordinate systems. The global structural system X, Y, Z defines the positive directions of external loads, joint displacements and the horizontal and vertical projections of a finite strip. The local strip system x, y, z defines the orientation of the element for the interpretation of the positive directions of internal forces and strip displacements.

2.4 Method of Solution: The solution is based on a standard harmonic analysis as described in reference cited in 1.4.b. The finite strip method is utilized to derive the stiffness matrix of a strip with eccentric ribs for the harmonic under consideration. These individual strip contributions are assembled with the help of the direct stiffness method to obtain a complete solution. A detailed description of the method of solution can be found in the reference cited in 1.4.a.

2.5 General Capabilities and Restrictions:

a) The program is restricted to the analysis of eccentrically stiffened folded plate structures simply supported at the two end diaphragms.

b) The material and rib properties must be distributed uniformly over the area of a finite strip.

c) The smearing of the rib properties provides an excellent insight into the overall response but cannot yield information on the local stress distribution of the plating between ribs.

d) No restrictions to the number of strips, joints, etc., are imposed since the program features a dynamic storage allocation coupled with an automatic field length reduction to optimize automatically the storage requirements. An explicit formula for the hand calculation of the required field length is given at the end of this appendix.

e) Restrictions as to the maximum number of strip types, intermediate printouts and output locations are given under the input data.

f) Only one load case can be treated in each problem.

g) The program contains an option for the integration of stress resultants to obtain a check of the gross moment about the neutral axis of a particular cross-section. Moreover, the moments of each individual girder, assembled from a specified number of strips, are given to provide some information on the overall load distribution.

3.0 PROGRAM STRUCTURE

3.1 Computer System and Language: This program is written for a CDC 6400 computer in FORTRAN IV language.

3.2 Program Decks: The program MULSTR contains the following decks which need not be in sequence since no overlay system is used:

PROGRAM	MULSTR
SUBROUTINE	STIFF
SUBROUTINE	FORCE
SUBROUTINE	STRIP
SUBROUTINE	BANSOL
SUBROUTINE	PINVAL
SUBROUTINE	OPRINT
SUBROUTINE	MOMPER
SUBROUTINE	ADDMOM
SUBROUTINE	FL (in COMPASS language)

The purpose of subroutine FL is twofold. It retrieves the last word address of the program during execution if called CALL LWA(N) or it resets the field length dynamically if called CALL RFL (N). This program is not a standard FORTRAN IV capability but its equivalent should be available at any computer center. Otherwise a fixed amount of storage has to be calculated by hand, as shown at the end of this appendix, and has to be reserved in the area of blank COMMON.

3.3 Tapes Used: Tape Unit 1 is used for temporary storage of the joint displacements for each harmonic.

4.0 INPUT SPECIFICATIONS

The input data is key punched on cards as specified below. The sequential order of the input cards must be strictly adhered to and consistent units must be used throughout a problem.

4.1 Title Card (12A6)

Col. 1 to 72 - TITLE (12), title of the problem to be printed with output for identification

4.2 Control Card (F10.0, 7I4, 11I2)

Col. 1 to 10 - SPAN, span length
Col. 11 to 14 - NPL, number of types of finite strips, maximum 50
Col. 15 to 18 - NEL, number of elements
Col. 19 to 22 - NJT, number of joints
Col. 23 to 26 - NXP, number of points along x-axis at which results are desired
Col. 27 to 30 - MHARM, maximum Fourier series limit
Col. 31 to 34 - NCHECK, check on odd or even harmonics
+ 1 to work on odd harmonics only (symmetry)
0 to include all harmonics
- 1 to include even harmonics only (anti-symmetry)
Col. 35 to 38 - NXBAND, estimate of bandwidth equalling the (maximum difference of joint numbers in any strip + 1)*4. This estimate is checked internally and reset if necessary.
Col. 39 to 40 - INTPRT, number of harmonics for which intermediate results are desired, maximum 20.
Col. 41 to 42 - NSURL, number of partial surface loads
Col. 43 to 44 - NCONL, number of partial joint loads
Col. 45 to 46 - LA, option for output of fiber stresses
Col. 47 to 48 - LB, option for output of internal forces in combined rib-plate system
Col. 49 to 50 - LC, option for output of internal forces in plate alone
Col. 51 to 52 - LD, option for output of internal forces in smeared ribs alone.
Col. 53 to 54 - LE, option for output of internal strip displacements
Col. 55 to 56 - MCHECK, moment integration option
Col. 57 to 58 - NOXMP, number of sections at which moment integration is desired (subset of NXP)
Col. 59 to 60 - NGIR, number of girders considered in moment integration

Selection of option: 0 - option is not calculated and output
1 - option is calculated and output

4.3 X-Coordinate Card (10F7.3)

XP(I) - x-coordinates at which results are desired.

4.4 Intermediate Result Card (20I4)

INTP(I) - harmonic numbers at which results to be output. Omitted if no intermediate result desired, subset of MHARM.

4.5 Strip Type Cards

Three cards for each type of finite strips.
First card - properties of plate (I10, 7F10.0)
Col. 1 to 10 - I, type number
Col. 11 to 20 - H(I), horizontal projection of strip
Col. 21 to 30 - V(I), vertical projection of strip
Col. 31 to 40 - TH(I); thickness of plate
Col. 41 to 50 - EPX(I), plate modulus of elasticity in the x-direction
Col. 51 to 60 - EPY(I), plate modulus of elasticity in the y-direction
Col. 61 to 70 - GP(I), plate shear modulus
Col. 71 to 80 - FNU(I), plate Poisson's ratio equals ν_{xy}, the ratio of the x-strain to the y-strain due to a uniaxial stress in the y-direction
Second card - smeared rib properties (per unit width of strip), left blank if no stiffeners (8F10.0)
Col. 1 to 10 - ARX(I), area of x-stiffeners
Col. 11 to 20 - ARY(I), area of y-stiffeners
Col. 21 to 30 - SMX(I), first moment of x-stiffeners about the midsurface of the plate
Col. 31 to 40 - SMY(I), first moment of y-stiffeners about the midsurface of the plate
Col. 41 to 50 - TMX(I), second moment of x-stiffeners about the midsurface of the plate
Col. 51 to 60 - TMY(I), second moment of y-stiffeners about the midsurface of the plate
Col. 61 to 70 - AJX(I), torsional rigidity of x-stiffeners
Col. 71 to 80 - AJY(I), torsional rigidity of y-stiffeners
Both AJX and AJY must have the shear modulus incorporated
Third card - material properties of ribs, left blank if no stiffeners (8F10.0)
Col. 1 to 10 - ERX(I), modulus of elasticity for x-ribs
Col. 11 to 20 - ERY(I), modulus of elasticity for y-ribs
Col. 41 to 50 - DX (I), distance from plate midsurface to fiber of x-rib at which stress is desired (positive in local z-direction)
Col. 51 to 60 - DY(I), distance from plate midsurface to fiber of y-rib at which stress is desired (positive in local z-direction)
Col. 61 to 70 - HX (I), additional rigidity coupling the twisting moment M_{xy} with the shear strain for x-ribs having a closed cross-section

Col. 71 to 80 - HY (I), additional rigidity coupling the twisting moment M_{yx} with the shear strain for y-ribs having a closed cross-section

Note that both the first moments of inertia, SMX and SMY, and the distances to the rib fibers, DX and DY, can have a positive or negative sign depending on the eccentricity of the rib. The eccentricity is positive if it lies in the positive z-direction of the local strip coordinates.

It is recommended to use the shaded areas of Fig. A9 for the definition of the rib properties.

All rib properties are those of the equivalent distributed (smeared) rib structure per unit width. Hence, they must incorporate the spacing between adjacent ribs.

4.6 Strip Array Cards (5I4, 3F10.0) - one card for each finite strip. Uniform loads given below exist over entire strip area.

Col. 1 to 4 - I, strip number
Col. 5 to 8 - NP1(I), 1 joint number
Col. 9 to 12 - NP2(I), 2 joint number
Col. 13 to 16 - KPL(I), type of strip used
Col. 17 to 20 - NSEC(I), number of transverse sections for internal forces and displacement output, maximum 4, if NSEC = 0 no internal forces or displacements will be output
Col. 31 to 30 - DL(I), dead load, force in vertical Z-direction per unit surface area
Col. 31 to 40 - HL(I), uniform horizontal load, force per unit vertical projected area
Col. 41 to 50 - VL(I), uniform vertical load, force per unit horizontal projected area

4.7 Joint Cards (I10, 4F10.0, 4I2) - one card for each joint.

Col. 1 to 10 - I, joint number
Col. 11 to 20 - AJFOR (1,I), applied horizontal joint force or displacement
Col. 21 to 30 - AJFOR (2,I), applied vertical joint force or displacement
Col. 31 to 40 - AJFOR (3,I), applied joint moment or rotation
Col. 41 to 50 - AJFOR (4,I), applied longitudinal joint force or displacement
Col. 52 - LCASE (1,I), index for horizontal force or displacement, (can be 0,1,2 or 3)
Col. 54 - LCASE (2,I), index for vertical force or displacement, (can be 0,1, or 3)

Col. 56 - LCASE (3,I), index for moment or rotation, (can be 0,1,2, or 3)
- 0 - for given zero force
- 1 - for uniformly distributed force, input uniform force/unit length for AJFOR
- 2 - for concentrated force at midspan, input total force for AJFOR
- 3 - for given zero displacement

Col. 58 - LCASE (4,I), index for longitudinal force or displacement, (can be 0,2, or 3)
- 0 - for given zero force
- 2 - for prestress P at each end, input total force at one end for AJFOR, positive away from midspan
- 3 - for given zero displacement

4.8 Partial Surface Load Cards

Surface load cards (I10, 4F10.0) - one card for each partial surface load. No cards required if NSURL = 0. Loads given below are uniform over plate width and have a length equal to that given under SURDEL. (P equals the total load, V and H equal the vertical and horizontal strip projections).

Col. 1 to 10 - LEL, strip number

Col. 11 to 20 - SURHL, horizontal load, P/V-area, P/V-length if transverse line load is applied

Col. 21 to 30 - SURVL, vertical load, P/H-area, P/H-length if transverse line load is applied

Col. 31 to 40 - SURXI, location from left support to center of distributed length

Col. 41 to 50 - SURDEL, distributed length in x-direction, for line load equals zero
If SURDEL $\neq$ 0, input SURHL and SURVL as force/unit area
If SURDEL = 0, input SURHL and SURVL as force/unit width

4.9 Partial Joint Load Cards

Joint load cards (I10, 6F10.0) - one card for each partial joint-load. No cards required if NCONL = 0. More than one location along a joint may be loaded, but each location requires a separate card.

Col. 1 to 10 - LJT, joint number

Col. 11 to 20 - CONHL, total horizontal force

Col. 21 to 30 - CONVL, total vertical force

Col. 31 to 40 - CONM, total moment

Col. 41 to 50 - CONS, total longitudinal force P (Note - it must be balanced by one -P somewhere along the same joint)

Col. 51 to 60 - CONXI, location from left support to center of load

Col. 61 to 70 - CONDEL, distributed length in x-direction (=0 for concentrated load)

4.10 <u>Girder Moment Integration Data</u>

X-Section Card (10F7.3) - X(I), subset of XP(I)

Next cards (3I4, 3F10.0) - one card for each finite strip

Col. 1 to 4 - I, strip number

Col. 5 to 8 - NGIEL (I,1), girder which joint 1 of strip I belongs to.

Col. 9 to 12 - NGIEL (I,2), girder which joint 2 of strip I belongs to, leave blank if contribution only to girder of NGIEL (I,1).

Col. 13 to 22 - DNA1(I), vertical distance from neutral axis to joint 1, downward is positive.

Col. 23 to 32 - DNA2(I), vertical distance from neutral axis to joint 2, downward is positive

Col. 33 to 42 - XDIV(I), horizontal distance from node 1 to the dividing line if the finite strip belongs to two girders.

The same set of data cards are repeated for the next problem.

Two blank cards are added at the end of the data deck to terminate execution.

5.0 OUTPUT DESCRIPTION

First, the input data is printed for an echo check. The final results consist of the joint displacement in the global coordinate direction and the internal forces and displacements in the local strip coordinates at locations specified by the user. Options cited in Paragraph 4.2 may be used to select desired output.

5.1 <u>Input Check Printout:</u> The complete input data is properly labeled and printed out for an echo check.

5.2 <u>Final Joint Displacements:</u> The four displacement components in the global coordinates r_h, r_v, r_θ, r_s are printed successively for each joint at x-coordinates specified in input.

5.3 <u>Internal Forces:</u> The stress resultants N_x, N_y, N_{xy} and the moment resultants M_x, M_y, M_{xy}, M_{yx} are printed out at specified locations of each finite strip. All these internal forces of each finite strip are given for the combined rib-plate, the plate alone and the ribs alone with the rib properties assumed to be smeared. The output of the results for the combined system and for the rib system is omitted if the smeared area of respective ribs equals zero. Moreover, the fiber stresses in the ribs are given at the midsurface and at a specified distance from the mid-surface of the plate. This distance can differ from the x- and y-ribs and is positive in the positive z-direction of the local strip coordinate system. Furthermore, the outside fiber stresses of the plate are given at the same locations.

5.4 Strip Displacements: The local strip deflections u, v, w are printed at the same locations specified by the user.

5.5 Moment Integration: The girder moments are determined by numerical integration of the stress resultants and moments providing an excellent insight into the load distribution.

5.6 Execution Time: The execution time for the solution of the problem is printed out in seconds with the number of degrees of freedom, number of harmonics and the band width.

6.0 REMARKS

a) Select joint numbering so as to minimize band width, which is a function of the maximum absolute difference between joint numbers for any finite strip.

b) The execution time can be estimated by the formula below:

$$T = \alpha * N * BW^2 + \beta * NEL$$

with

$$\alpha \sim 0.000033$$

$$\beta \sim 0.25$$

T - the total time in seconds for a CDC 6400 computer using the FUN compiler
N - four times the number of joints times the number of harmonics considered
BW - the half band width equaling four times the maximum difference of joint numbers at one finite strip plus one
NEL - total number of finite strips
α - a coefficient which depends on the efficiency of the equation solver
β - a coefficient which depends on the efficiency of the program determining the internal forces and displacements. To determine β it was assumed that NSEC(I) = 2, MHARM = 25 and INTPRT = 0.

This estimate is based on the execution times obtained from a limited number of runs. Hence, it has to be treated with caution.

c) The storage requirement for a specific problem is determined and allocated automatically within the program. The following formula is useful to determine the required field length in case it is impossible to retrieve the last word address of the program and to reset the field length during execution. This estimate is based on experience with a CDC 6400 computer using the FUN compiler.

$$ST = FIX + VAR$$

where ST is the maximum storage required for a specific problem. FIX is the fixed storage area used by each set of data and VAR is the variable storage area which depends on the problem being solved. There are two subroutines, STIFF and FORCE, which require a minimum storage area for their blank COMMON; the larger determines the size of VAR.

for STIFF:

$$VAR = 7\ NEL + NXP\ (2\ MM + 4\ NJT + 1) + 4\ NJT + (3 + NXBAND) + 72\ NPL + 5\ NSURL + 7\ NCONL$$

for FORCE:

$$VAR = 9\ NEL + MM\ (120 + 2\ NXP) + 153\ NXP + NOXMP\ (2 + 3\ NGIR) + 120$$

and

$$FIX = 12{,}000 \text{ words}$$

with the following definitions

- NPL - number of strip types
- NEL - number of strips
- NXP - number of sections
- NJT - number of joints
- MM - number of harmonics considered
- NSURL - number of surface loads
- NCONL - number of joint loads
- NXBAND - band width
- NOXMP - number of sections at which moment integration required
- NGIR - number of girders considered for moment integration

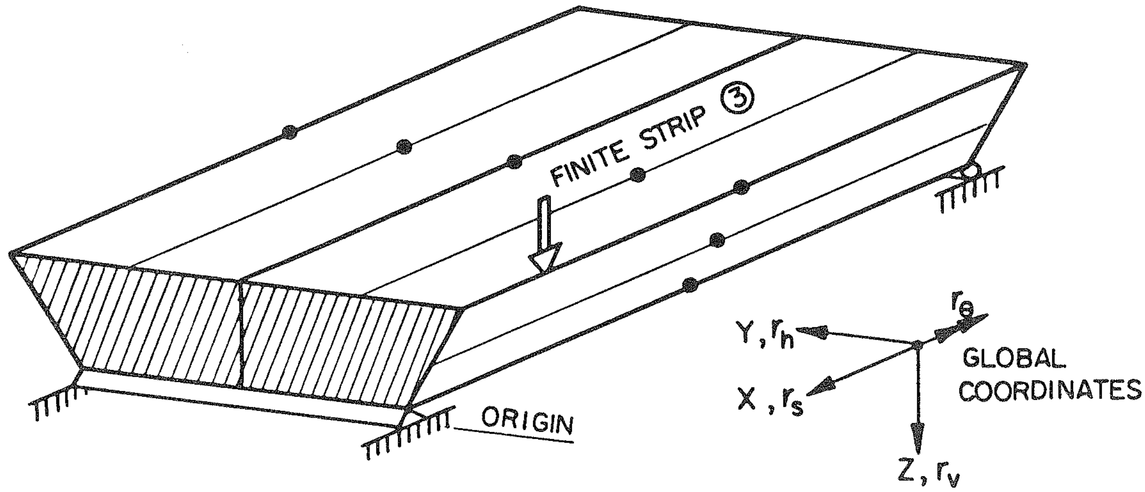


FIG. A1 FINITE STRIP ANALYTICAL MODEL

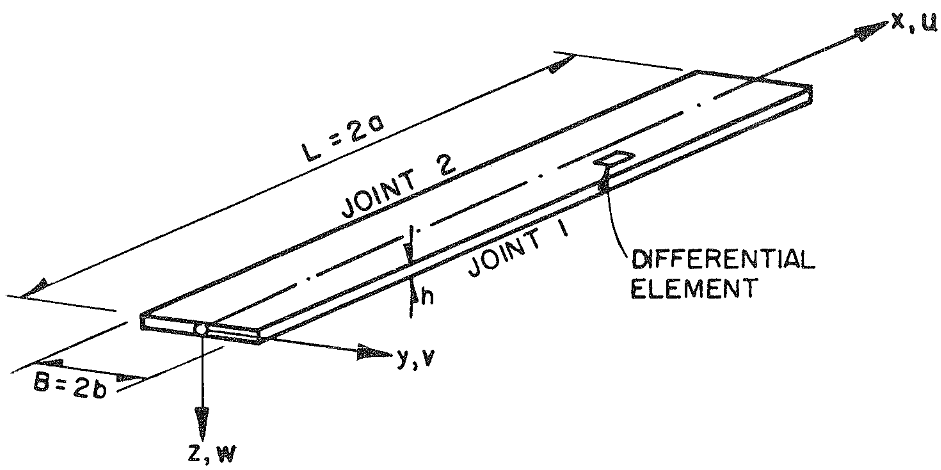


FIG. A2 DIMENSIONS AND LOCAL COORDINATE SYSTEM FOR FINITE STRIP ③

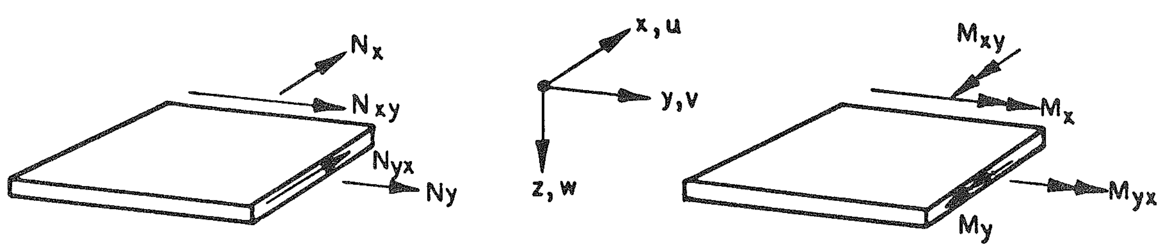


FIG. A3 POSITIVE DIRECTIONS OF INTERNAL FORCES ACTING ON A DIFFERENTIAL ELEMENT IN A FINITE STRIP

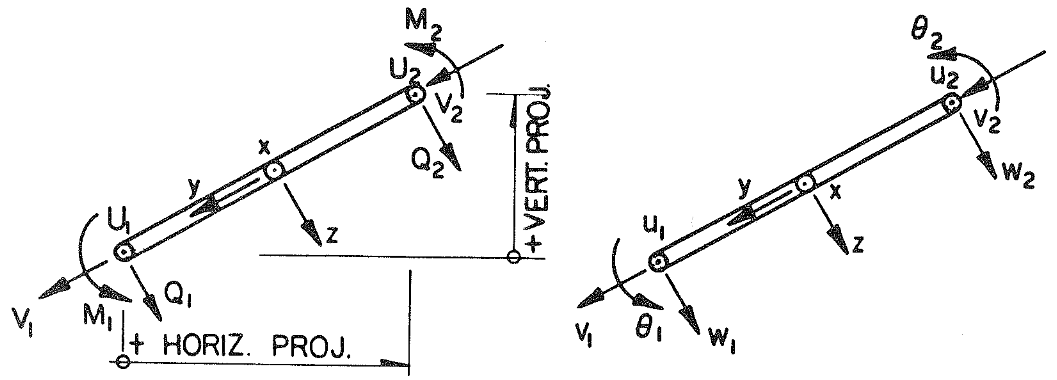


FIG. A4 NODAL POINT FORCES S AND DISPLACEMENTS V IN LOCAL STRIP COORDINATES

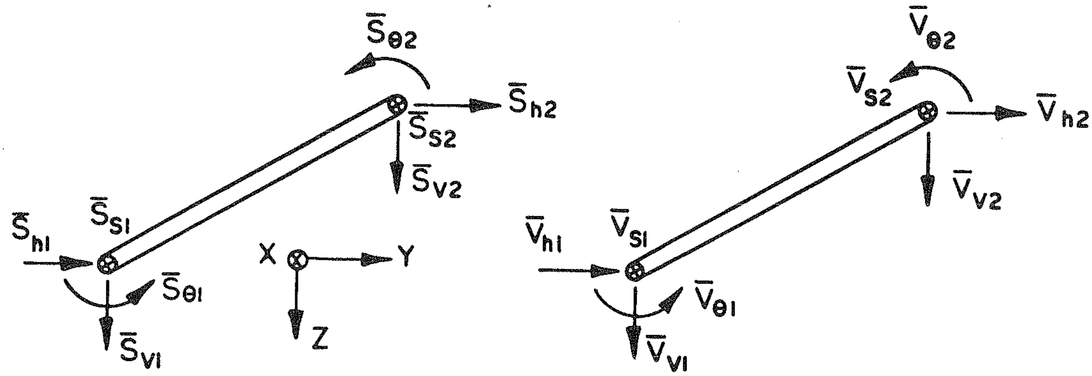


FIG. A5 NODAL POINT FORCES $\bar{S}$ AND DISPLACEMENTS $\bar{V}$ IN GLOBAL COORDINATES

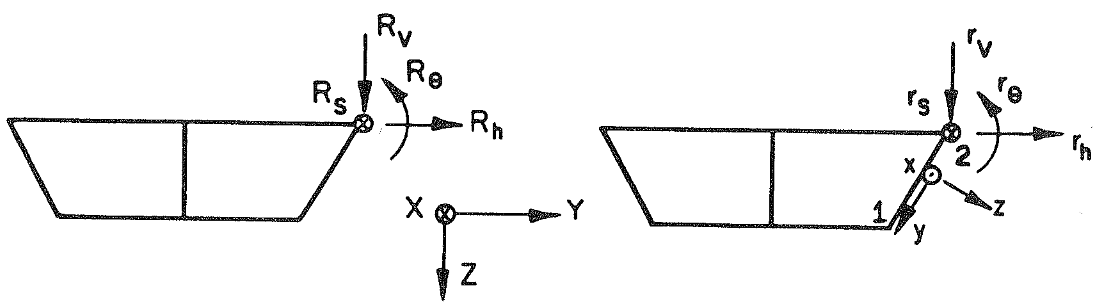


FIG. A6 GLOBAL NODAL POINT FORCES R AND DISPLACEMENTS r LOOKING TOWARDS ORIGIN

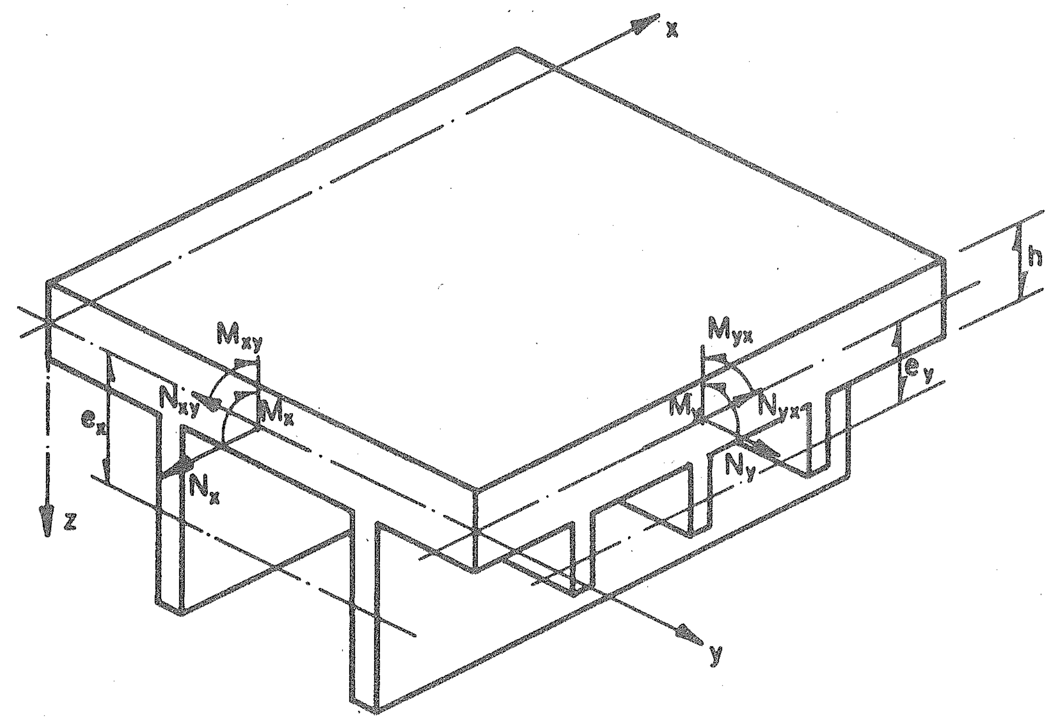


FIG. A7 TYPICAL ELEMENT OF TORSIONALLY SOFT ORTHOTROPIC PLATE

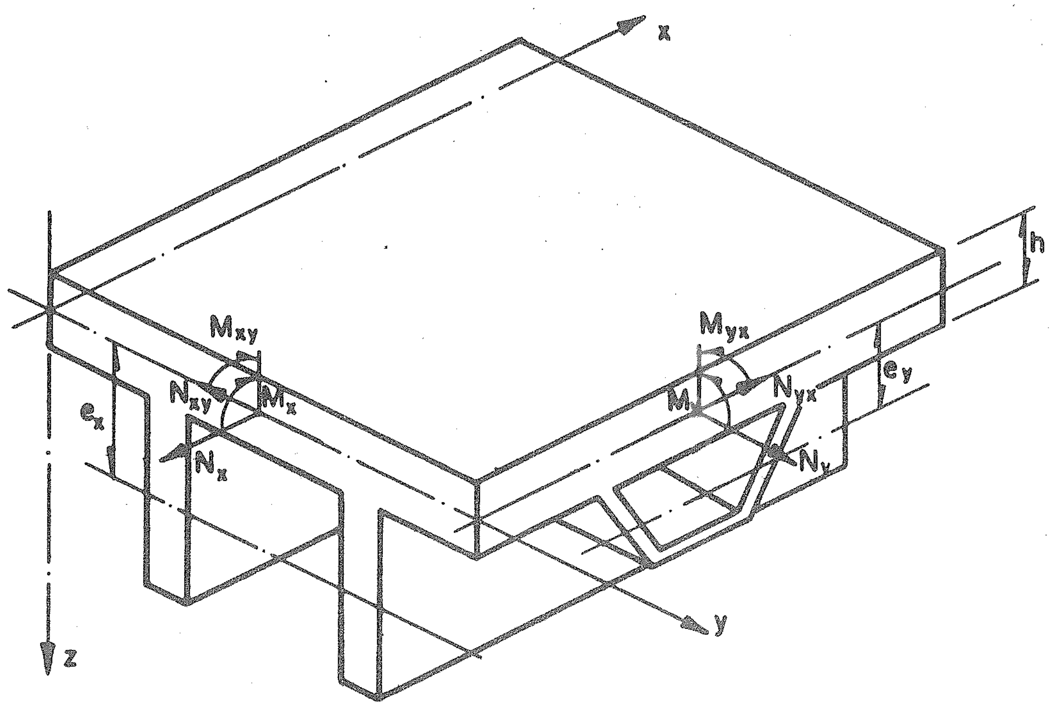
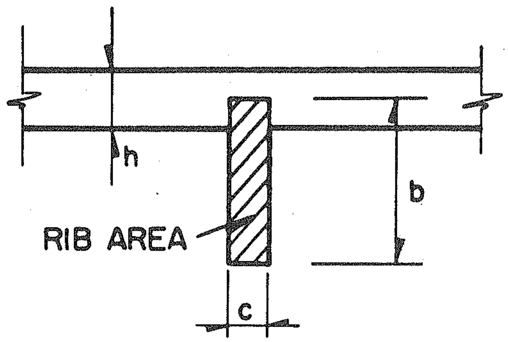
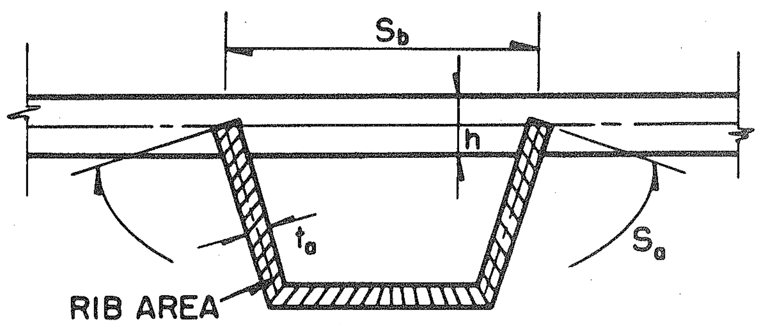


FIG. A7 TYPICAL ELEMENT OF TORSIONALLY STIFF ORTHOTROPIC PLATE



a) OPEN SECTION



b) CLOSED SECTION

FIG.A9 DIMENSIONS FOR RIGIDITY OF TORSIONALLY STIFF RIBS

APPENDIX B

FORTRAN IV Listing of Computer Program MULSTR

Considerable time, effort and expense have gone into the development of this computer program. It is obvious that it should be used only under the conditions and assumptions for which it was developed. These are described in the report. Although the program has been extensively tested by the authors, no warranty is made regarding the accuracy and reliability of the program and no responsibility is assumed by the authors or the sponsors of this research project.

```
      PROGRAM MULSTR (INPUT,OUTPUT,TAPE1)
C
C*****************************************************************************
C         FINITE STRIP PROGRAM FOR THE ANALYSIS OF ECCENTRICALLY
C         STIFFENED FOLDED PLATES WHICH ARE SIMPLY SUPPORTED.
C
C         PROGRAMMED ON THE CDC 6400 BY KASPAR J. WILLAM
C         UNIVERSITY OF CALIFORNIA, BERKELEY, FEBRUARY 1970
C*****************************************************************************
C
      COMMON A(1)
      COMMON / SETUP / SPAN,NPL,NEL,NJT,NXP,MHARM,NCHECK,MM,NXBAND,
     *                 INTPRT,MCHECK,NSURL,NCONL,MX,PI,N1,N2,II,IJ,IL,
     *                 LA,LB,LC,LD,LE,INTP(21),NOXMP,NGIR
      DIMENSION TITLE(12)
      LOGICAL EVEN
C
C         READ AND PRINT CONTROL INFORMATION
C
      NFL = 0
  101 CALL SECOND (TO)
      CALL LWA (NNN)
      READ 10, (TITLE(I),I=1,12)
      READ 12, SPAN,NPL,NEL,NJT,NXP,MHARM,NCHECK,NXBAND,INTPRT,NSURL,
     *          NCONL,LA,LB,LC,LD,LE,MCHECK,NOXMP,NGIR
      IF (SPAN.EQ.0.0) GO TO 999
      PRINT 15
      PRINT 11, (TITLE(I),I=1,12)
      PRINT 17
      PRINT 16,SPAN,NPL,NEL,NJT,NXP,MHARM,NCHECK,NXBAND,INTPRT,NSURL,
     *          NCONL,LA,LB,LC,LD,LE,MCHECK,NOXMP,NGIR
      PRINT 18
      IF(MHARM-(MHARM/2)*2)100,110,100
  100 EVEN=.FALSE.
      GO TO 112
  110 EVEN=.TRUE.
  112 IF (NCHECK) 103,105,104
  103 PRINT 13
      IF(EVEN)GO TO 105
  120 MHARM=MHARM-1
      PRINT 45,MHARM
      GO TO 105
  104 PRINT 14
      IF(EVEN)GO TO 120
  105 MM=MHARM/2+1
      IF(NCHECK.EQ.0)MM=MHARM
C
C         DETERMINE REQUIRED STORAGE FOR STIFF SUBROUTINE
C
      N4   = 4*NJT
      MX   = N4
      L1   = 1
      L2   = L1   + NEL
      L3   = L2   + NEL
      L4   = L3   + NEL
      L5   = L4   + NEL
```

```
      L6  = L5  + NXP
      L7  = L6  + MM*NXP
      L8  = L7  + MM*NXP
      L9  = L8  + NSURL
      L10 = L9  + NSURL
      L11 = L10 + NSURL
      L12 = L11 + NSURL
      L13 = L12 + NSURL
      L14 = L13 + NCONL
      L15 = L14 + NCONL
      L16 = L15 + NCONL
      L17 = L16 + NCONL
      L18 = L17 + NCONL
      L19 = L18 + NCONL
      L20 = L19 + NCONL
      L21 = L20 + NEL
      L22 = L21 + NEL
      L23 = L22 + NEL
      L24 = L23 + N4
      L25 = L24 + N4
      L26 = L25 + N4*NXP
      L27 = L26 + 64*NPL
      L28 = L27 + 8*NPL
      L29 = L28 + N4
      L30 = L29 + N4*NXBAND
C
C        DETERMINE REQUIRED STORAGE FOR FORCE SUBROUTINE
C
      NA  = 5*NXP
      N9  = L8  + NXP
      N10 = N9  + NXP
      N12 = N10 + 8*15
      N13 = N12 + NA
      N14 = N13 + NA
      N15 = N14 + NA
      N16 = N15 + NA
      N17 = N16 + NA
      N18 = N17 + NA
      N19 = N18 + NA
      N20 = N19 + NA
      N21 = N20 + NA
      N21 = N20 + NA
      N22 = N21 + NA
      N23 = N22 + NA
      N24 = N23 + NA
      N25 = N24 + NA
      N26 = N25 + NA
      N27 = N26 + NA
      N28 = N27 + NA
      N29 = N28 + NA
      N30 = N29 + NA
      N31 = N30 + NA
      N32 = N31 + NA
      N33 = N32 + NA
      N34 = N33 + NA
      N35 = N34 + NA
```

```
      N36 = N35 + NA
      N37 = N36 + NA
      N38 = N37 + NA
      N39 = N38 + NA
      N40 = N39 + NA
      N41 = N40 + NA
      N42 = N41 + 8*MM*15
      N43 = N42 + 2*NEL
      N44 = N43 + NEL
      N45 = N44 + NEL
      N46 = N45 + NEL
      N47 = N46 + NOXMP
      N48 = N47 + NOXMP
      N49 = N48 + NOXMP*NGIR
      N50 = N49 + NOXMP*NGIR
      N51 = N50 + NOXMP*NGIR
C
C         RESET FIELDLENGTH
C
      NNM = NNN+L30
      NNP = NNN+N42
      IF (MCHECK.NE.0) NNP=NNN+N51
      IF (NNP.GT.NNM) NNM=NNP
      IF (NNM.GT.(NFL-1000).AND.NNM.LT.NFL) GO TO 200
      NFL = NNM
      IF (NFL.LT.140000B) GO TO 220
      PRINT 500, NFL
      GO TO 999
  220 CALL RFL (NFL)
C
  200 CALL SECOND (T1)
      CALL STIFF (A(L1),A(L2),A(L3),A(L4),A(L5),A(L6),A(L7),A(L8),A(L9),
     *   A(L10),A(L11),A(L12),A(L13),A(L14),A(L15),A(L16),A(L17),A(L18),
     *   A(L19),A(L20),A(L21),A(L22),A(L23),A(L23),A(L24),A(L24),A(L25),
     *   A(L26),A(L27),A(L28),A(L28),A(L29),A(L29),A(L23),N4,MM,NNM)
      CALL SECOND (T2)
      CALL FORCE (A(L1),A(L2),A(L3),A(L4),A(L5),A(L6),A(L7),A(L8),A(N9),
     *   A(N10),A(N10),A(N12),A(N12),A(N12),A(N13),A(N14),A(N15),A(N16),
     *   A(N17),A(N18),A(N19),A(N20),A(N21),A(N22),A(N23),A(N24),A(N25),
     *   A(N26),A(N27),A(N28),A(N29),A(N30),A(N31),A(N32),A(N33),A(N34),
     *   A(N35),A(N36),A(N37),A(N38),A(N39),A(N40),A(N41),A(N42),A(N43),
     *   A(N44),A(N45),A(N46),A(N47),A(N48),A(N49),A(N50),NXP,MM,NOXMP)
      CALL SECOND (T3)
      TA = T2 - T1
      TB = T3 - T2
      TC = T3 - T0
      PRINT 300, TA,TB,TC
      PRINT 400, MX,NXBAND,MM
C
C     FORMAT STATEMENTS
C
   10 FORMAT (12A6)
   11 FORMAT (1H1,12A6)
   12 FORMAT (F10.3, 7I4, 11I2)
   13 FORMAT (41HOCALCULATIONS SKIP ALL ODD FOURIER SERIES)
   14 FORMAT (42HOCALCULATIONS SKIP ALL EVEN FOURIER SERIES)
```

```
   15 FORMAT (1H1)                                                          MULS 169
   16 FORMAT (/// 39H SPAN LENGTH  ......................  F10.3/          MULS 170
     *            40H NUMBER OF PLATE TYPES ..............  I4/             MULS 171
     *            40H NUMBER OF FINITE STRIPS ............  I4/             MULS 172
     *            40H NUMBER OF LONGITUDINAL JOINTS ......  I4/             MULS 173
     *            40H NUMBER OF X-LOCATIONS FOR OUTPUT ....  I4/            MULS 174
     *            40H MAXIMUM HARMONIC CONSIDERED .........  I4/            MULS 175
     *            40H TYPE OF HARMONICS (EVEN, ALL, ODD) ..  I4/            MULS 176
     *            40H BANDWIDTH =(MAX JOINT DIFF + 1)*4 ...  I4/            MULS 177
     *            40H NUMBER OF INTERMEDIATE RESULTS ......  I4/            MULS 178
     *            40H NUMBER OF PARTIAL SURFACE LOADS .....  I4/            MULS 179
     *            40H NUMBER OF PARTIAL JOINT LOADS .......  I4/            MULS 180
     *            40H OUTPUT STRESSES IN PLATE AND RIBS ...  I4/            MULS 181
     *            40H OUTPUT INT FORCES IN COMB PLATE RIB .  I4/            MULS 182
     *            40H OUTPUT INT FORCES IN PLATE ALONE ....  I4/            MULS 183
     *            40H OUTPUT INT FORCES IN RIB ALONE ......  I4/            MULS 184
     *            40H OUTPUT STRIP DISPLACEMENTS ..........  I4/            MULS 185
     *            40H GIRDER MOMENT INTEGRATION INCLUDED ..  I4/            MULS 186
     *            40H NUMBER OF SECTIONS FOR MOMENT INTEGRAT I4/            MULS 187
     *            40H NUMBER OF GIRDERS AT CROSS SECTION ..  I4)            MULS 188
   17 FORMAT (/// 30H DATA CONTROL INFORMATION         )                    MULS 189
   45 FORMAT (35H0  NUMBER OF HARMONICS SET EQUAL TO I4)                    MULS 190
   18 FORMAT (/   20X, 25H ZERO DENOTES .FALSE.      /                      MULS 191
     *            20X, 25H ONE  DENOTES .TRUE.      //)                     MULS 192
  300 FORMAT (/// 15H TIMING STIFF  F10.4/                                  MULS 193
     *            15H TIMING FORCE  F10.4/                                  MULS 194
     *            15H TOTAL TIME    F10.4)                                  MULS 195
  400 FORMAT (///30H NUMBER OF DEGREES OF FREEDOM  I4/                      MULS 196
     *           30H BANDWIDTH                     I4/                      MULS 197
     *           30H NUMBER OF TERMS IN HARM ANAL  I4)                      MULS 198
  500 FORMAT (// 44H MAX FIELDLENGTH OF 140000 OCTALS EXCEEDED    I10//)    MULS 199
      GO TO 101                                                             MULS 200
  999 STOP                                                                  MULS 201
      END                                                                   MULS 203
```

```
      SUBROUTINE STIFF (NP1,NP2,KPL,NSEC,XP,SINKX,COSKX,LEL,SURHL,SURVL,
     *   SURXI,SURDEL,LJT,CONHL,CONVL,CONM,CONS,CONXI,CONDEL,HL,VL,DL,
     *   AJFOR,AJP,LCASE,LIND,RJDIS,SMALLK,P,PTOT,DISP,BIGK,EDP,NPDIF,
     *   N4,MH,NNM)
C
C***********************************************************************
C        DATA IS INPUT AND PRINTED, STRUCTURAL STIFFNESS AND LOAD VECTORS
C        ARE FORMED FOR EACH HARMONIC AND THE SET OF EQUATIONS IS SOLVED
C        FOR THE UNKNOWN JOINT DISPLACEMENTS
C***********************************************************************
C
      COMMON / SETUP / SPAN,NPL,NEL,NJT,NXP,MHARM,NCHECK,MM,NXBAND,
     *                 INTPRT,MCHECK,NSURL,NCONL,MX,PI,N1,N2,II,IJ,IL,
     *                 LA,LB,LC,LD,LE,INTP(21),NOXMP,NGIR
      COMMON / SPROP / H(50),V(50),TH(50),PWTH(50),EPX(50),EPY(50),
     *                 GP(50),FNU(50),ARX(50),ARY(50),SMX(50),SMY(50),
     *                 TMX(50),TMY(50),AJX(50),AJY(50),ERX(50),ERY(50),
     *                 DX(50),DY(50),HX(50),HY(50)
      DIMENSION SERIES(2)
      DIMENSION NP1(1),NP2(1),KPL(1),NSEC(1),HL(1),VL(1),DL(1),NPDIF(1),
     *          LEL(1),SURHL(1),SURVL(1),SURXI(1),SURDEL(1),LJT(1),
     *          CONHL(1),CONVL(1),CONM(1),CONS(1),CONXI(1),CONDEL(1),
     *          AJFOR(4,1),AJP(1),LCASE(4,1),LIND(1),RJDIS(N4,1),
     *          SMALLK(8,8,1),P(8,1),PTOT(1),DISP(1),BIGK(N4,1),EDP(1),
     *          XP(1),SINKX(MH,1),COSKX(MH,1)
C
C       EQUIVALENCED ARRAYS HAVING THE SAME FWA
C        (DL,NPDIF), (LCASE,LIND), (PTOT,DISP), (BIGK,EDP), (AJFOR,AJP)
C
C       READ AND PRINT INPUT DATA
C
      PRINT1000
      READ 1001, (XP(I),I=1,NXP)
      PRINT1002, (XP(I),I=1,NXP)
      IF (INTPRT.EQ.0) GO TO 104
      READ 1004, (INTP(I),I=1,INTPRT)
      PRINT1005, (INTP(I),I=1,INTPRT)
  104 INTP(INTPRT+1) = 0
      DO 106 N=1,NPL
      READ 1021, I,H(I),V(I),TH(I),EPX(I),EPY(I),GP(I),FNU(I)
      READ 1022, ARX(I),ARY(I),SMX(I),SMY(I),TMX(I),TMY(I),AJX(I),AJY(I)
  106 READ 1023, ERX(I),ERY(I),DX(I),DY(I),HX(I),HY(I)
      PRINT1020
      PRINT1025, (I,H(I),V(I),TH(I),EPX(I),EPY(I),GP(I),FNU(I),I=1,NPL)
      PRINT1026
      PRINT1027, (I,ARX(I),ARY(I),SMX(I),SMY(I),TMX(I),TMY(I),AJX(I),
     *            AJY(I),I=1,NPL)
      PRINT1028
      PRINT1029, (ERX(I),ERY(I),DX(I),DY(I),HX(I),HY(I),I=1,NPL)
      PRINT1030
      READ 1031, (I,NP1(I),NP2(I),KPL(I),NSEC(I),DL(I),HL(I),VL(I),
     *            I=1,NEL)
      PRINT1032, (I,NP1(I),NP2(I),KPL(I),NSEC(I),DL(I),HL(I),VL(I),
     *            I=1,NEL)
      PRINT1060
      DO 108 I=1,NJT
```

```
  108 READ 1061, N, (AJFOR(J,N),J=1,4), (LCASE(J,N),J=1,4)
      DO 109 N=1,NJT
  109 PRINT1062, N,(AJFOR(J,N),LCASE(J,N),J=1,4)
      IF (NSURL.EQ.0) GO TO 110
      PRINT1050, NSURL
      READ 1051, (LEL(I),SURHL(I),SURVL(I),SURXI(I),SURDEL(I),I=1,NSURL)
      PRINT1052, (LEL(I),SURHL(I),SURVL(I),SURXI(I),SURDEL(I),I=1,NSURL)
  110 IF (NCONL.EQ.0) GO TO 111
      PRINT1040, NCONL
      READ 1041, (LJT(I),CONHL(I),CONVL(I),CONM(I),CONS(I),CONXI(I),
     *               CONDEL(I),I=1,NCONL)
      PRINT1042, (LJT(I),CONHL(I),CONVL(I),CONM(I),CONS(I),CONXI(I),
     *               CONDEL(I),I=1,NCONL)
  111 CONTINUE
C
C        INITIALIZATION
C
      PI = 3.14159265358979
      MA = MX*NXP
      DO 121 I=1,MA
  121 RJDIS(I) = 0.0
C
C        DETERMINE PLATEWIDTH AND SET H=H/PWTH, V=V/PWTH
C
      DO 125 I=1,NPL
      PWTH(I)=SQRT(H(I)**2+V(I)**2)
      H(I)=H(I)/PWTH(I)
  125 V(I)=V(I)/PWTH(I)
C
C        MODIFY SURFACE LOADS AND CHECK FOR MAXIMUM BANDWIDTH
C
      NBAND = 0
      DO 130 I=1,NEL
      J=KPL(I)
      VL(I)=VL(I)*ABS(H(J))+DL(I)
      HL(I)=HL(I)*ABS(V(J))
      ZL=VL(I)*H(J)+HL(I)*V(J)
      YL=VL(I)*V(J)-HL(I)*H(J)
      VL(I)=ZL
      HL(I)=YL
      NPDIF(I) = NP2(I)-NP1(I)
      K=IABS(NPDIF(I))
      IF (NBAND-K) 126,127,127
  126 NBAND = K
  127 NP1(I)=NP1(I)*4-4
  130 NP2(I)=NP2(I)*4-4
      MAXJTD = NBAND
      NBAND = NBAND*4+4
      NDIF = NXBAND - NBAND
      IF (NDIF.GE.0) GO TO 129
      NNP = IABS(NDIF)*MX
      NNM = NNM + NNP
      CALL RFL(NNM)
      PRINT 2010, NXBAND,NBAND
 2010 FORMAT (// 30H ERROR IN INPUT OF BANDWIDTH    ///5X,
     *             30H SPECIFIED BANDWIDTH          I5/5X,
```

```
     *              30H CORRECTEC BANDWIDTH           I5)
      NXBAND = NBAND
C
C         MODIFY PARTIAL SURFACE LOADS
C
  129 IF (NSURL) 135,135,132
  132 DO 133 I=1,NSURL
      K=LEL(I)
      J=KPL(K)
      SURVL(I)=SURVL(I)*ABS(H(J))
      SURHL(I)=SURHL(I)*ABS(V(J))
      ZL=SURVL(I)*H(J)+SURHL(I)*V(J)
      YL=SURVL(I)*V(J)-SURHL(I)*H(J)
      SURVL(I)=ZL
  133 SURHL(I)=YL
C
C         MODIFY LCASE (LIND) MATRIX AND PRESTRESS FORCES
C
  135 DO 136 I=1,MX
  136 LIND(I)=LIND(I)+1
      DO 138 I=1,NJT
      IF (LCASE(4,I)-3) 138,137,138
  137 LCASE(4,I)=LCASE(4,I)+2
      AJFOR(4,I)=AJFOR(4,I)*4./SPAN
  138 CONTINUE
C
C         INITIATE CYCLE FOR EACH HARMONIC
C
      REWIND 1
      MA=0
      IF (NCHECK) 140,141,142
  140 N1=2
      GO TO 143
  141 N1=1
      N2=1
      GO TO 144
  142 N1=1
  143 N2=2
C
  144 DO 700 NN=N1,MHARM,N2
      MXB = MX*NXBAND
      DO 145 I=1,MXB
  145 BIGK(I) = 0.0
C
C         INITIALIZE BIGK MATRIX
C
      FN=NN
      FK=FN*PI/SPAN
      MA=MA+1
C
C         DETERMINE HARMONIC AND FOURIER MULTIPLIERS
C
      DO 150 I=1,NXP
      XX=FK*XP(I)
      SINKX(MA,I)=SIN(XX)
  150 COSKX(MA,I)=COS(XX)
```

```
      FSTRP=0.5*SPAN
      N3=(-1)**NN
      IF (N3) 152,155,155
  152 SERIES(1)=4./(FN*PI) * FSTRP
      SERIES(2)=2./SPAN*(-1.)**((NN+3)/2) * FSTRP
C
C          STRIP STIFFNESS IS DETERMINED FOR EACH STRIP TYPE
C
  155 CALL STRIP (FK,SMALLK,P,NPL)
C
C          ASSEMBLE STRUCTURAL STIFFNESS MATRIX  BIGK
C
      DO 210 L=1,NEL
      K=KPL(L)
      M=NP1(L)
      N=NP2(L)
      DO 201 I=1,4
      II=M+I
      IJ=N+I
      IK=I+4
      DO 201 JB=I,4
      J = JB - I + 1
      BIGK(II,J)=BIGK(II,J)+SMALLK(I,JB,K)
  201 BIGK(IJ,J)=BIGK(IJ,J)+SMALLK(IK,JB+4,K)
      IF (NPDIF(L)) 205,202,2C2
  202 IK=N-M-4
      DO 203 I=1,4
      II=M+I
      DO 203 J=5,8
      IJ = IK + J - I + 1
  203 BIGK(II,IJ)=BIGK(II,IJ)+SMALLK(I,J,K)
      GO TO 210
  205 IK=M-N
      N=N-4
      DO 206 I=5,8
      II=N+I
      DO 206 J=1,4
      IJ = IK + J - I + 5
  206 BIGK(II,IJ)=BIGK(II,IJ)+SMALLK(I,J,K)
  210 CONTINUE
C
C          COMPUTE ANC ASSEMBLE JCINT FCRCES FGR UNIFORM SURFACE LOADS
C
      DO 215 I=1,MX
  215 PTOT(I)=0.0
      IF (N3) 211,221,221
  211 DO 220 L=1,NEL
      K=KPL(L)
  220 CALL FIXFCR (H(K),V(K),FL(L),VL(L),NP1(L),NP2(L),P,NPL,PTOT,MX,K)
C
C          CCMPUTE ANC ASSEMBLE JCINT FCRCES FCR PARTIAL SURFACE LOADS
C
  221 IF (NSURL) 231,231,222
  222 DO 230 I=1,NSURL
      L=LEL(I)
      K=KPL(L)
```

```
      IF (SURDEL(I)) 223,224,223
  223 C=SIN(FK*SURXI(I))*SIN(FK*SURDEL(I)/2.)
      GO TO 225
  224 C=SIN(FK*SURXI(I))*FK/2.
  225 EQH=SURHL(I)*C
      EQV=SURVL(I)*C
  230 CALL FIXFCR (H(K),V(K),EQH,EQV,NP1(L),NP2(L),P,NPL,PTOT,MX,K)
C
C        DETERMINE INPUT JOINT LCADS ANC ASSEMBLE INTO LOAD VECTOR
C
  231 IF (N3) 232,239,239
  232 DO 238 I=1,MX
      K=LIND(I)
      GO TO (233,234,235,238,236),K
  233 PTOT(I)=-PTOT(I)
      GO TO 238
  234 PTOT(I)=AJP(I)*SERIES(1)-PTOT(I)
      GO TO 238
  235 PTOT(I)=AJP(I)*SERIES(2)-PTOT(I)
      GO TC 238
  236 PTOT(I)=AJP(I)*FSTRP-PTCT(I)
  238 CONTINUE
      GO TO 241
  239 DO 240 I=1,MX
  240 PTOT(I)=-PTCT(I)
C
C        ADC CCNCENTRATED JOINT LOADS
C
  241 IF (NCCNL) 251,251,242
  242 DO 250 I=1,NCCNL
      J=LJT(I)*4-4
      C=FK*CONXI(I)
      IF (CONDEL(I)) 244,244,243
  243 XX=FK*CCNCEL(I)/2.
      EQH=2./(XX*SPAN)*SIN(XX)
      EQS=EQH*CCS(C)*FSTRP
      EQH=EQH*SIN(C)*FSTRP
      GO TC 245
  244 XX=2./SPAN
      EQH=XX*SIN(C)*FSTRP
      EQS=XX*COS(C)*FSTRP
  245 PTOT(J+1)=PTOT(J+1)+EQH*CONHL(I)
      PTOT(J+2)=PTOT(J+2)+EQH*CONVL(I)
      PTOT(J+3)=PTOT(J+3)+EQH*CCNM(I)
  250 PTOT(J+4)=PTOT(J+4)+EQS*CONS(I)
C
C        IMPOSE DISPLACEMENT BCUNDARY CCNDITIONS
C
  251 DO 260 J=1,NJT
      DO 260 I=1,4
      IF (LCASE(I,J).NE.4) GO TC 260
      IL = (J-1)*4 + I
      DO 253 L=1,NXBAND
      BIGK(IL,L) = 0.0
      IJ = IL-L+1
      IF (IJ.LE.0) GC TO 253
```

```
      BIGK(IJ,L) = 0.0
  253 CONTINUE
      PTOT(IL)=0.0
  260 CONTINUE
C
C        SOLVE SYSTEM OF EQUATIONS FOR UNKNOWN GLOBAL JOINT DISPLACEMENT
C
      CALL BANSOL (MX,NXBAND,MX,BIGK,PTOT,0)
C
C
C        ACCUMULATE GLOBAL JOINT DISPLACEMENTS AT SPECIFIED POINTS
C
  500 DO 510 II=1,NXP
      C=COSKX(MA,II)
      S=SINKX(MA,II)
      DO 510 L=4,MX,4
      I=L-3
      J=L-1
      DO 505 K=I,J
  505 RJDIS(K,II)=RJDIS(K,II)+DISP(K)*S
  510 RJDIS(L,II)=RJDIS(L,II)+DISP(L)*C
C
C        DETERMINE LOCAL EDGE DISPLACEMENTS FOR EACH STRIP AND STORE
C        ON TAPE 1
C
      N=0
      DO 600 L=1,NEL
      K=KPL(L)
      I=NP1(L)
      J=NP2(L)
      C=H(K)
      S=V(K)
      EDP(N+1)= DISP(I+3)
      EDP(N+2)= DISP(J+3)
      EDP(N+3)= S*DISP(I+1)+C*DISP(I+2)
      EDP(N+4)= S*DISP(J+1)+C*DISP(J+2)
      EDP(N+5)=-DISP(I+4)
      EDP(N+6)=-DISP(J+4)
      EDP(N+7)=-C*DISP(I+1)+S*DISP(I+2)
      EDP(N+8)=-C*DISP(J+1)+S*DISP(J+2)
  600 N=N+8
      WRITE (1) (EDP(I),I=1,N)
  700 CONTINUE
      END FILE 1
C
C        PRINT RESULTING GLOBAL JOINT DISPLACEMENTS
C
      DO 710 I=1,NJT
      J=4*I
      LIND(J)=I
      LIND(J-1)=I
      LIND(J-2)=I
  710 LIND(J-3)=I
      IF (NXP-7) 720,720,721
  720 II=NXP
      IL=1
```

```
      GO TO 730                                                         STIF 337
  721 II=7                                                              STIF 338
      IJ=NXP                                                            STIF 339
      IL=(NXP-1)/7 + 1                                                  STIF 340
  730 PRINT 40                                                          STIF 341
      CALL PINVAL (LIND,RJDIS,MX,NXP,XP,MX,II,IJ,IL,1)                  STIF 342
      PRINT 41                                                          STIF 343
      CALL PINVAL (LIND,RJDIS,MX,NXP,XP,MX,II,IJ,IL,2)                  STIF 344
      PRINT 42                                                          STIF 345
      CALL PINVAL (LIND,RJDIS,MX,NXP,XP,MX,II,IJ,IL,3)                  STIF 346
      PRINT 43                                                          STIF 347
      CALL PINVAL (LIND,RJDIS,MX,NXP,XP,MX,II,IJ,IL,4)                  STIF 348
C                                                                       STIF 349
C         FORMAT STATEMENTS FOR INPUT AND ECHC                          STIF 350
C                                                                       STIF 351
   27 FORMAT (I10,4F10.0,4I2)                                           STIF 352
   28 FORMAT (39H1INPUT LOADS OR DISPLACEMENTS AT JOINTS//86H JOINT     STIF 353
     *  HORIZONTAL IH          VERTICAL IV        ROTATIONAL IM     LONGITSTIF 354
     *UDINAL IS)                                                        STIF 355
   29 FORMAT (I6,4(E17.6,I3))                                           STIF 356
   30 FORMAT (//37H IH,IV,IM,IS = 0 FOR GIVEN ZERO FORCE/44H            STIF 357
     *   1 FOR UNIF. DISTRIBUTED FORCE/81H                  2 MEANS CONC. FSTIF 358
     *ORCE AT MIDSPAN FOR IH, IV, IM AND PRESTRESS FOR IS/44H           STIF 359
     *    3 FOR GIVEN ZERO DISPLACEMENT)                                STIF 360
   40 FORMAT (14H1FINAL RESULTS/26H2FINAL JOINT DISPLACEMENTS////10X,25HSTIF 361
     * HORIZONTAL DISPLACEMENTS)                                        STIF 362
   41 FORMAT (////10X,23H VERTICAL DISPLACEMENTS)                       STIF 363
   42 FORMAT (////10X,10H ROTATIONS)                                    STIF 364
   43 FORMAT (////10X,27H LONGITUDINAL DISPLACEMENTS)                   STIF 365
   46 FORMAT (A6)                                                       STIF 366
 1000 FORMAT (////46H PRINT RESULTS AT CROSS-SECTIONS OF X EQUAL TO //) STIF 367
 1001 FORMAT (10F7.3)                                                   STIF 368
 1002 FORMAT (1CF12.2)                                                  STIF 369
 1004 FORMAT (20I4)                                                     STIF 370
 1005 FORMAT (///40H PRINT INTERMEDIATE RESULTS AT HARMONICS//20I5)     STIF 371
 1020 FORMAT (132H1 STR TYPE         H-PROJ.          V-PROJ.       THISTIF 372
     *CKNESS          E-MOD EPX          E-MOD EPY          G-MOD GP    STIF 373
     *POISS-R VXY           /)                                          STIF 374
 1021 FORMAT (I10,7F10.3)                                               STIF 375
 1022 FORMAT (8F10.3)                                                   STIF 376
 1023 FORMAT (6F10.3)                                                   STIF 377
 1025 FORMAT (I8,2E18.5,E16.5,3E18.5,F13.3)                             STIF 378
 1026 FORMAT (/// 45H SMEARED MATERIAL PROPERTIES OF STIFFENERS     //  STIF 379
     * 125H  STR TYPE          X-AREA          Y-AREA        X-FMOM      Y-FSTIF 380
     *MOM          X-SMOM        Y-SMOM         X-TORS R        Y-TORS R      )STIF 381
 1027 FORMAT (I7,3X,2F14.3,4F13.3,2E15.5)                               STIF 382
 1028 FORMAT (/// 100H      E-MOD ERX          E-MOD ERY        BOT X-DIS   STIF 383
     *    BOT Y-DIS        CLOSED X-RIB       CLOSED Y-RIB       )       STIF 384
 1029 FORMAT (E13.5,E16.5,F14.3,F17.3,8X,E13.5,E18.5)                   STIF 385
 1030 FORMAT (66H1  ELE       I      J     PL   NSEC         DL        UNIF HL STIF 386
     *    UNIF VL/)                                                     STIF 387
 1031 FORMAT (5I4,3F10.0)                                               STIF 388
 1032 FORMAT (5I6,3F12.3)                                               STIF 389
 1040 FORMAT (38H1NUMBER OF CONCENTRATED JOINT LOADS = I3//102H JOINT   STIF 390
     *         H-LOAD            V-LCAC             MCMENT      LONG. FORCE     STIF 391
     *    LOCATICN       LOAD WIDTH/)                                   STIF 392
```

```
 1041 FORMAT (I10,6F10.0)                                                   STIF 393
 1042 FORMAT (I6,6E16.6)                                                    STIF 394
 1050 FORMAT (35H1NUMBER OF PARTIAL SURFACE LOADS = I3//70H   ELE           STIF 395
     *   H-LOAD          V-LOAD          LOCATION       LOAD WIDTH)         STIF 396
 1051 FORMAT (I10,4F10.0)                                                   STIF 397
 1052 FORMAT (I6,4E16.6)                                                    STIF 398
 1060 FORMAT (39H1INPUT LOADS OR DISPLACEMENTS AT JOINTS  //88H JOINT       STIF 399
     *      HORIZONTAL IH         VERTICAL IV       ROTATIONAL IM      LONSTIF 400
     *GITUDINAL IS      )                                                   STIF 401
 1061 FORMAT (I10,4F10.0,4I2)                                               STIF 402
 1062 FORMAT (I6,4(E17.6,I3))                                               STIF 403
C                                                                           STIF 404
      RETURN                                                                STIF 405
      END                                                                   STIF 406
```

```
      SUBROUTINE FORCE (NP1,NP2,KPL,NSEC,XP,SINKX,COSKX,SKX,CKX,DI,DIS, FORC
     *   XA,XAA,XN,XNP,XNR,YN,YNP,YNR,XYN,XM,XMP,XMR,YM,YMP,YMR,XYC,YXC,FORC
     *   XYM,XYR,YXR,XBP,XBR,YBP,YBR,XTP,XTR,YTP,YTR,UD,VD,WD,D,NGIEL,   FORC
     *   XDIV,DNA1,DNA2,X,MOPT,GIRMOM,TENS,COMP,NX,MH,NOP)               FORC
C                                                                       FORC
C***********************************************************************FORC
C        INTERNAL FORCES AND DISPLACEMENTS ARE DETERMINED FOR EACH STRIPFORC
C        AND ARE ACCUMULATED FOR ALL HARMONICS. INTERNAL FORCES FOR THE FORC
C        COMBINED PLATE RIB SYSTEM, THE PLATE SYSTEM ALONE AND THE RIB  FORC
C        SYSTEM ALONE CAN BE OUTPUT.  MOREOVER, THE TOP AND BOTTOM FIBERFORC
C        STRESSES SEPARATED FOR PLATES AND RIBS CAN BE PRINTED AT       FORC
C        POINTS SELECTED BY THE USER                                    FORC
C***********************************************************************FORC
C                                                                       FORC
      COMMON / SETUP / SPAN,NPL,NEL,NJT,NXP,MHARM,NCHECK,MM,NXBAND,     FORC
     *                 INTPRT,MCHECK,NSURL,NCONL,MX,PI,N1,N2,II,IJ,IL,  FORC
     *                 LA,LB,LC,LD,LE,INTP(21),NOXMP,NGIR               FORC
      COMMON / SPROP / H(50),V(50),TH(50),PWTH(50),EPX(50),EPY(50),     FORC
     *                 GP(50),FNU(50),ARX(50),ARY(50),SMX(50),SMY(50),  FORC
     *                 TMX(50),TMY(50),AJX(50),AJY(50),ERX(50),ERY(50), FORC
     *                 DX(50),DY(50),HX(50),HY(50)                      FORC
      DIMENSION NP1(1),NP2(1),KPL(1),NSEC(1),HL(1),VL(1),DL(1),COSKX(MH,FORC
     *          1),SINKX(MH,1),CKX(1),SKX(1),DI(1),DIS(8,1),XN(NX,1),XNPFORC
     *          (NX,1),XNR(NX,1),YN(NX,1),YNP(NX,1),YNR(NX,1),XYN(NX,1),FORC
     *          XM(NX,1),XMP(NX,1),XMR(NX,1),YM(NX,1),YMP(NX,1),YMR(NX,1FORC
     *          ),XYC(NX,1),YXC(NX,1),XYM(NX,1),XYR(NX,1),YXR(NX,1),XBP(FORC
     *          NX,1),XBR(NX,1),YBP(NX,1),YBR(NX,1),XTP(NX,1),XTR(NX,1),FORC
     *          YTP(NX,1),YTR(NX,1),UD(NX,1),VD(NX,1),WD(NX,1),D(8,1),  FORC
     *          NGIEL(2,1),XDIV(1),DNA1(1),DNA2(1),X(1),MOPT(1),GIRMOM( FORC
     *          NOP,1),TENS(NOP,1),COMP(NOP,1),XP(1),XA(1),XAA(1)       FORC
      DIMENSION D11(3),D22(3),D12(3),D33(3),D44(3),D55(3),D45(3),D56(3),FORC
     *          D65(3),D14(3),D25(3),D67(3),D76(3)                      FORC
      DIMENSION FNX(3),FNY(3),FNXY(3),FMX(3),FMY(3),FMXY(3),FMYX(3)     FORC
C                                                                       FORC
C        EQUIVALENCED VECTORS ASSIGNING THE SAME FWA FOR (DI,DIS)        FORC
C                                                                       FORC
      LOGICAL S1,S2                                                     FORC
C                                                                       FORC
C        INPUT AND ECHO OF GIRDER MOMENT DATA                           FORC
C                                                                       FORC
      IF (MCHECK.EQ.0) GO TO 25                                         FORC
      READ 1000, (X(I),I=1,NOXMP)                                       FORC
      READ 1004, (I,NGIEL(1,I),NGIEL(2,I),DNA1(I),DNA2(I),XDIV(I),      FORC
     *            J=1,NEL)                                              FORC
      PRINT 1001                                                        FORC
      PRINT 1002, (X(I),I=1,NOXMP)                                      FORC
      PRINT 1005                                                        FORC
      PRINT 1006, (I,NGIEL(1,I),NGIEL(2,I),DNA1(I),DNA2(I),XDIV(I),     FORC
     *             I=1,NEL)                                             FORC
      DO 110 I=1,NOXMP                                                  FORC
      DO 120 J=1,NXP                                                    FORC
      IF (X(I).NE.XP(J)) GO TO 120                                      FORC
      MOPT(I) = J                                                       FORC
      GO TO 110                                                         FORC
  120 CONTINUE                                                          FORC
      PRINT 1010, X(I)                                                  FORC
```

```
      MOPT(I) = 0
  110 CONTINUE
C
C         INITIALIZE MOMENT INTEGRATICN ARRAYS
C
      NA = 3*NGIR*NCXMP
      DO 230 I=1,NA
  230 GIRMOM(I) = 0.0
C
C         READ LCCAL JOINT DISPLACEMENTS FROM TAPE 1
C
   25 NELINC = 14
      NEL2 = 0
   30 NEL1=NEL2+1
      IF (NEL1-NEL) 31,31,10C
   31 NEL2=MINO((NEL1+NELINC),NEL)
      NDI=NEL2*8
      REWINC 1
      L = 0
      DO 36 I=1,MM
      READ (1) (DI(J),J=1,NDI)
      DO 35 J=NEL1,NEL2
      L=L+1
      DO 35 K=1,8
   35 D(K,L)=DIS(K,J)
   36 CONTINUE
C
C         FOR EACH FINITE STRIP
C
      NDI=NEL2-NEL1+1
      DO 99 IE=NEL1,NEL2
      FN=NSEC(IE)
      IF (FN) 99,99,38
   38 NUMY=NSEC(IE)+1
      L   = KPL(IE)
      S1 = .FALSE.
      S2 = .FALSE.
      IF (ARX(L).NE.0.0) S1=.TRUE.
      IF (ARY(L).NE.0.C) S2=.TRUE.
C
C         INITIALIZATICN
C
      NA = 29*5*NXP
      DO 40 I=1,NA
   40 XN(I) = 0.0
C
C         FORMATION OF MATERIAL LAW FOR PLATE, FCR RIB AND FOR
C         CCMBINED PLATE RIB SYSTEM
C
      U   = FNU(L)*SQRT(EPX(L)/EPY(L))
      B   = PWTH(L)
      TN   = TH(L)
      TM   = TN**3/12.0
      DIFY = B/FN
      EX = EPX(L)/(1.0-U**2)
      EY = EPY(L)/(1.0-U**2)
```

```
      D11(2) = TN*EX
      D22(2) = TN*EY
      D12(2) = TN*EX*FNU(L)
      D33(2) = TN*GP(L)
      D44(2) = TM*EX
      D55(2) = TM*EY
      D45(2) = TM*EX*FNU(L)
      D56(2) = 2.0*TM*GP(L)
      D65(2) = D56(2)
      D11(3) = ARX(L)*ERX(L)
      D22(3) = ARY(L)*ERY(L)
      D44(3) = TMX(L)*ERX(L)
      D55(3) = TMY(L)*ERY(L)
      D56(3) = AJX(L)
      D65(3) = AJY(L)
      D14(3) = SMX(L)*ERX(L)
      D25(3) = SMY(L)*ERY(L)
      D67(3) = FX(L)
      D76(3) = FY(L)
      D11(1) = D11(2) + D11(3)
      D22(1) = D22(2) + D22(3)
      D12(1) = D12(2)
      D33(1) = D33(2)
      D44(1) = D44(2) + D44(3)
      D55(1) = D55(2) + D55(3)
      D45(1) = D45(2)
      D56(1) = D56(2) + D56(3)
      D65(1) = D65(2) + D65(3)
      D14(1) = D14(3)
      D25(1) = D25(3)
      D67(1) = D67(3)
      D76(1) = D76(3)
      D14(2) = 0.0
      D25(2) = 0.0
      D67(2) = 0.0
      D76(2) = 0.0
      D12(3) = 0.0
      D33(3) = 0.0
      D45(3) = 0.0
C
C         FOR EACH HARMONIC
C
      N=0
      KJK=1
      DO 90 NN=N1,MHARM,N2
      N=N+1
      N3=(-1)**NN
   49 FM=NN
      SC1=FM*PI/SPAN
      SC2=SC1**2
      SC3=SC1**3
   51 I=NDI*(N-1)+(IE-NEL1+1)
      DISP1 = D(1,I)
      DISP2 = D(2,I)
      DISP3 = D(3,I)
      DISP4 = D(4,I)
```

```
      DISP5 = C(5,I)
      DISP6 = D(6,I)
      DISP7 = C(7,I)
      DISP8 = C(8,I)
      DO 52 J=1,NXP
      CKX(J)=COSKX(N,J)
   52 SKX(J)=SINKX(N,J)
C
C         DETERMINATION OF DISPLACEMENTS AT TRANSVERSE SECTIONS
C
   82 DO 80 IY=1,NUMY
      IJK=(IY-1)*(IY-NUMY)
      FI=IY-1
      FI=FI*DIFY
      Y=B/2.-FI
   84 B3 = B**3
      B2 = B**2
      Y3 = Y**3
      Y2 = Y**2
      TA = (0.5*B+Y)/B
      TB = (0.5*B-Y)/B
      RA = (B3/4.0+C.75*B2*Y-Y3)*2.0/B3
      RB = (B3/4.0-C.75*B2*Y+Y3)*2.0/B3
      SA = (-B3/8.0-C.25*B2*Y+C.5*B*Y2+Y3)/B2
      SB = ( B3/8.0-C.25*B2*Y-C.5*B*Y2+Y3)/B2
      FUD = TA*CISP5 + TB*DISP6
      FVD = TA*CISP7 + TB*DISP8
      FWD = RA*CISP3 + RB*DISP4 + SA*DISP1 + SB*DISP2
C
C         DETERMINATION OF INTERNAL FORCES AT TRANSVERSE SECTIONS
C
      TA1 = 1.0/B
      TB1 =-1.0/B
      RA1 = 6.0*(-B2/4.0+Y2)/B3
      RB1 =-RA1
      SA1 = (B2/4.0-B*Y-3.0*Y2)/B2
      SB1 = (B2/4.0+B*Y-3.0*Y2)/B2
      RA2 = 12.C*Y/B3
      RB2 =-RA2
      SA2 = 2.0*(-0.5*B-3.0*Y)/B2
      SB2 = 2.0*( 0.5*B-3.0*Y)/B2
      RA3 = 12.C/B3
      RB3 =-RA3
      SA3 =-6.0/B2
      SB3 = SA3
C
C         STRAINS ANC CUVATURES
C
      FSX = -SC1*(TA*DISP5+TB*DISP6)
      FSY =  TA1*DISP7+TB1*DISP8
      FSXY=  TA1*DISP5+TB1*DISP6+SC1*(TA*DISP7+TB*DISP8)
      FKX =  SC2*(RA*DISP3+RB*DISP4+SA*DISP1+SB*DISP2)
      FKY =  RA2*DISP3+RB2*DISP4+SA2*DISP1+SB2*DISP2
      FKXY=  SC1*(RA1*DISP3+RB1*DISP4+SA1*DISP1+SB1*DISP2)
C
C         INTERNAL FORCES
```

```
C
      DO 123 IA=1,3
      FNX(IA) = D11(IA)*FSX + D12(IA)*FSY + D14(IA)*FKX
      FNY(IA) = D22(IA)*FSY + D12(IA)*FSX + D25(IA)*FKY
      FNXY(IA)= D33(IA)*FSXY
      FMX(IA) = D44(IA)*FKX + D45(IA)*FKY + D14(IA)*FSX
      FMY(IA) = D55(IA)*FKY + D45(IA)*FKX + D25(IA)*FSY
      FMXY(IA)= D56(IA)*FKXY+ D67(IA)*FSXY
      FMYX(IA)= D65(IA)*FKXY+ D76(IA)*FSXY
  123 CONTINUE
C
C         ACCUMULATE  INTERNAL  STRIP  DISPLACEMENTS
C
   70 DO 75 I=1,NXP
      T1=SKX(I)
      T2=CKX(I)
      UD(I,IY)   = UD(I,IY)   + FUD*T2
      VD(I,IY)   = VD(I,IY)   + FVD*T1
      WD(I,IY)   = WD(I,IY)   + FWD*T1
C
C         ACCUMULATE  INTERNAL  STRIP  MOMENTS
C
      XM(I,IY)   = XM(I,IY)   + FMX(1)*T1
      XMP(I,IY)  = XMP(I,IY)  + FMX(2)*T1
      XMR(I,IY)  = XMR(I,IY)  + FMX(3)*T1
      YM(I,IY)   = YM(I,IY)   + FMY(1)*T1
      YMP(I,IY)  = YMP(I,IY)  + FMY(2)*T1
      YMR(I,IY)  = YMR(I,IY)  + FMY(3)*T1
      XYC(I,IY)  = XYC(I,IY)  + FMXY(1)*T2
      YXC(I,IY)  = YXC(I,IY)  + FMYX(1)*T2
      XYM(I,IY)  = XYM(I,IY)  + FMXY(2)*T2
      XYR(I,IY)  = XYR(I,IY)  + FMXY(3)*T2
      YXR(I,IY)  = YXR(I,IY)  + FMYX(3)*T2
C
C         ACCUMULATE  INTERNAL  STRIP  STRESS  RESULTANTS
C
      XN(I,IY)   = XN(I,IY)   + FNX(1)*T1
      XNP(I,IY)  = XNP(I,IY)  + FNX(2)*T1
      XNR(I,IY)  = XNR(I,IY)  + FNX(3)*T1
      YN(I,IY)   = YN(I,IY)   + FNY(1)*T1
      YNP(I,IY)  = YNP(I,IY)  + FNY(2)*T1
      YNR(I,IY)  = YNR(I,IY)  + FNY(3)*T1
      XYN(I,IY)  = XYN(I,IY)  + FNXY(1)*T2
C
C         ACCUMULATE  RIB  STRESSES
C
      XTR(I,IY)  = XTR(I,IY)  + FSX*ERX(L)*T1
      XBR(I,IY)  = XBR(I,IY)  + (FSX+FKX*DX(L))*ERX(L)*T1
      YTR(I,IY)  = YTR(I,IY)  + FSY*ERY(L)*T1
      YBR(I,IY)  = YBR(I,IY)  + (FSY+FKY*DY(L))*ERY(L)*T1
   75 CONTINUE
   80 CONTINUE
  200 IF(INTP(KJK).LE.0) GO TO 204
      IF(NN-INTP(KJK))204,206,202
  202 KJK=KJK+1
      GO TO 200
```

```
  204 IF(NN.NE.MHARM) GO TO 90
C
C         DETERMINE FIBER STRESSES IN RIBS AND PLATES
C
  206 DO 86 IY=1,NUMY
      DO 88 I=1,NXP
  127 XTP(I,IY) = XNP(I,IY)/TN - 0.5*TN*XMP(I,IY)/TM
      XBP(I,IY) = XNP(I,IY)/TN + 0.5*TN*XMP(I,IY)/TM
      YTP(I,IY) = YNP(I,IY)/TN - 0.5*TN*YMP(I,IY)/TM
      YBP(I,IY) = YNP(I,IY)/TN + 0.5*TN*YMP(I,IY)/TM
   88 CONTINUE
   86 CONTINUE
C
C         OUTPUT OF INTERNAL FORCES AND DISPLACEMENTS FOR EACH STRIP
C
      I=NP1(IE)/4+1
      J=NP2(IE)/4+1
      PRINT 10, IE,I,J,NN
C
C         OUTPUT OF FIBER STRESSES IN PLATES AND RIBS
C
      IF (LA.EQ.0) GO TO 310
      IF (.NOT.S1) GO TO 410
      PRINT 137
      PRINT 114
      CALL OPRINT (XTR,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 116
      CALL OPRINT (XBR,NXP,NUMY,XP,NUMY,II,IJ,IL)
  410 IF (.NOT.S2) GO TO 420
      PRINT 138
      PRINT 115
      CALL OPRINT (YTR,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 117
      CALL OPRINT (YBR,NXP,NUMY,XP,NUMY,II,IJ,IL)
  420 PRINT 136
      PRINT 114
      CALL OPRINT (XTP,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 116
      CALL OPRINT (XBP,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 115
      CALL OPRINT (YTP,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 117
      CALL OPRINT (YBP,NXP,NUMY,XP,NUMY,II,IJ,IL)
C
C         OUTPUT OF INTERNAL FORCES OF COMBINED RIB PLATE SYSTEM
C
  310 IF (LB.EQ.0) GO TO 320
      PRINT 132
      PRINT 16
      CALL OPRINT (XN ,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 17
      CALL OPRINT (YN ,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 18
      CALL OPRINT (XYN,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 11
      CALL OPRINT (XM ,NXP,NUMY,XP,NUMY,II,IJ,IL)
```

```
      PRINT 12
      CALL OPRINT (YM  ,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 13
      CALL OPRINT (XYC,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 14
      CALL OPRINT (YXC,NXP,NUMY,XP,NUMY,II,IJ,IL)
C
C        OUTPUT OF INTERNAL FORCES OF PLATE SYSTEM ALONE
C
  320 IF (LC.EQ.0) GO TO 330
      PRINT 133
      PRINT 16
      CALL OPRINT (XNP,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 17
      CALL OPRINT (YNP,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 18
      CALL OPRINT (XYN,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 11
      CALL OPRINT (XMP,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 12
      CALL OPRINT (YMP,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 13
      CALL OPRINT (XYM,NXP,NUMY,XP,NUMY,II,IJ,IL)
C
C        OUTPUT OF INTERNAL FORCES OF SMEARED RIBS ALONE
C
  330 IF (LC.EQ.0) GO TO 340
      IF (.NOT.S1) GO TO 335
      PRINT 134
      PRINT 16
      CALL OPRINT (XNR,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 11
      CALL OPRINT (XMR,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 13
      CALL OPRINT (XYR,NXP,NUMY,XP,NUMY,II,IJ,IL)
  335 IF (.NOT.S2) GO TO 340
      PRINT 135
      PRINT 17
      CALL OPRINT (YNR,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 12
      CALL OPRINT (YMR,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 14
      CALL OPRINT (YXR,NXP,NUMY,XP,NUMY,II,IJ,IL)
C
C        OUTPUT OF INTERNAL STRIP DISPLACEMENTS
C
  340 IF (LE.EQ.0) GO TO 350
      PRINT 131
      PRINT 19
      CALL OPRINT (UC  ,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 20
      CALL OPRINT (VC  ,NXP,NUMY,XP,NUMY,II,IJ,IL)
      PRINT 21
      CALL OPRINT (WC  ,NXP,NUMY,XP,NUMY,II,IJ,IL)
C
C        DETERMINATION OF GIRDER MOMENTS BY STRESS INTEGRATION
```

```
C
  350 IF (MCHECK.EQ.0) GO TO 90
      IF (NN.NE.MHARM) GO TO 90
      S = DNA1(IE)
      C = DNA2(IE)
      HH = H(L)
      VV = V(L)
      I = NGIEL(1,IE)
      J = NGIEL(2,IE)
      XX = XDIV(IE)
      CALL MOMPER (XN,XM,B,IE,NUMY,I,J,S,C,HH,VV,XX,NXP,X,MOPT,
     *             GIRMOM,TENS,CCMP,NOXMP)
   90 CONTINUE
   99 CONTINUE
      GO TO 30
C
C        DETERMINATION OF GIRDER MOMENT PERCENTAGES
C
  100 IF (MCHECK.LE.0) GO TO 500
      DO 510 I=1,NOXMP
      PA = 0.0
      PB = 0.0
      PC = 0.0
      PD = 0.0
      DO 540 J=1,NGIR
      PB = PB+GIRMOM(I,J)
      PC = PC+TENS(I,J)
  540 PD = PD+CCMP(I,J)
      PRINT 141, X(I)
      IF (PB.EQ.0.0) GO TO 510
      DO 520 J=1,NGIR
      PE = GIRMOM(I,J)/PB*100.0
      PA = PA + PE
  520 PRINT 143, J,GIRMOM(I,J),PE,TENS(I,J),CCMP(I,J)
      PRINT 145,PB,PA,PC,PD
  510 CONTINUE
C
C        FORMAT STATEMENTS
C
   10 FORMAT (76H1INTERNAL FORCES PER UNIT LENGTH AND INTERNAL DISPLACEM
     *ENTS FOR ELEMENT NO. I4,17H  BETWEEN JOINTS I3,5H AND I3,6H   N =
     *  I4)
   11 FORMAT (/// 10X,5H M(X))
   12 FORMAT (/// 10X,5H M(Y))
   13 FORMAT (/// 10X,6H M(XY))
   14 FORMAT (/// 10X,6H M(YX))
   16 FORMAT (/// 10X,5H N(X))
   17 FORMAT (/// 10X,5H N(Y))
   18 FORMAT (/// 10X,6H N(XY))
   19 FORMAT (/// 10X,2H U)
   20 FORMAT (/// 10X,2H V)
   21 FORMAT (/// 10X,2H W)
  114 FORMAT (/// 10X,15H SIGMA-X TOP    )
  115 FORMAT (/// 10X,15H SIGMA-Y TOP    )
  116 FORMAT (/// 10X,15H SIGMA-X BOT    )
  117 FORMAT (/// 10X,15H SIGMA-Y BOT    )
```

```
  131 FORMAT (/// 35H DISPLACEMENTS OF MIDSURFACE          )
  132 FORMAT (/// 49H STRESS RESULTANTS OF THE COMBINED RIB PLATE SYST )
  133 FORMAT (/// 35H STRESS RESULTANTS IN THE PLATE       )
  134 FORMAT (/// 38H STRESS RESULTANTS IN SMEARED X-RIBS     )
  135 FORMAT (/// 38H STRESS RESULTANTS IN SMEARED Y-RIBS     )
  136 FORMAT (/// 40H NORMAL FIBER STRESSES IN PLATE            )
  137 FORMAT (/// 40H NORMAL FIBER STRESSES IN X-RIBS           )
  138 FORMAT (/// 40H NORMAL FIBER STRESSES IN Y-RIBS           )
  141 FORMAT (58H2 GIRDER MOMENT AND AXIAL STRESS RESULTANTS AT SECTION
     * X=    F8.2 /// 65H GIRDER NO    MOMENT    PERCENTAGE     TENSION
     *       COMPRESSION      /)
  143 FORMAT (I6,E16.6,F9.2,2E16.6)
  145 FORMAT (//6H TOTAL  E16.6,F9.2,2E16.6)
 1000 FORMAT (10F7.3)
 1001 FORMAT (//// 50H1 DETERMINE GIRDER MOMENTS AT SECTIONS X EQUAL TO
     *          /)
 1002 FORMAT (10F12.2)
 1004 FORMAT (3I4,3F10.0)
 1005 FORMAT (//// 75H    STRIP      1ST GIRDER   2ND GIRDER        DNA1
     *       DNA2          XDIV        /)
 1006 FORMAT (I5,2I13,3X,3F13.3)
 1010 FORMAT (//// 16H X-SECTION COORD  F7.2, 50H IS DISREGARDED SINCE I
     *T IS NOT CONTAINED IN XP(I)        )
C
  500 RETURN
      END
```

```
      SUBROUTINE STRIP (FK,SM,P,NP)
C
C***********************************************************************
C         FOR GIVEN HARMONIC THE LOCAL STIFFNESS AND CONSISTENT LOAD
C         ARE DETERMINED FOR A FINITE STRIP WITH ECCENTRIC STIFFENERS
C         AND ARE TRANSFORMED INTO GLOBAL COORDINATES
C***********************************************************************
C
      DIMENSION SM(8,8,1),P(8,1),SK(8,8),SKA(8,8)
      COMMON / SETUP / SPAN,NPL,NEL,NJT,NXP,MHARM,NCHECK,MM,NXBAND,
     *                 INTPRT,MCHECK,NSURL,NCONL,MX,PI,N1,N2,II,IJ,IL,
     *                 LA,LB,LC,LD,LE,INTP(21),NOXMP,NGIR
      COMMON / SPROP / H(50),V(50),TH(50),PWTH(50),EPX(50),EPY(50),
     *                 GP(50),FNU(50),ARX(50),ARY(50),SMX(50),SMY(50),
     *                 TMX(50),TMY(50),AJX(50),AJY(50),ERX(50),ERY(50),
     *                 DX(50),DY(50),HX(50),HY(50)
C
C         INITIALIZATION FOR GIVEN HARMONIC
C
      PL   = SPAN
      WPI  = FK
      WPI2 = WPI**2
      WPI3 = WPI**3
      WPI4 = WPI**4
C
C         FORMATION OF ORTHOTROPIC MATERIAL LAW
C         FOR EACH FINITE STRIP TYPE
C
      DO 100 L=1,NPL
      HD = H(L)
      VD = V(L)
      U  = FNU(L)*SQRT(EPX(L)/EPY(L))
      TA = TH(L)
      TB = TA**3/12.0
      WI = PWTH(L)
      EX = EPX(L)/(1.0-U**2)
      EY = EPY(L)/(1.0-U**2)
      D11 = TA*EX + ARX(L)*ERX(L)
      D22 = TA*EY + ARY(L)*ERY(L)
      D12 = TA*EX*FNU(L)
      D33 = TA*GP(L)
      D44 = TB*EX + TMX(L)*ERX(L)
      D55 = TB*EY + TMY(L)*ERY(L)
      D45 = TB*EX*FNU(L)
      D66 = TB*GP(L)*4. + AJX(L)+AJY(L)
      D14 = SMX(L)*ERX(L)
      D25 = SMY(L)*ERY(L)
C
C         LOCAL STIFFNESS MATRIX FOR FINITE STRIP WITH
C         ECCENTRIC STIFFENERS
C
C         STIFFNESS OF PLATE BENDING ACTION
C
      SK(1,1) = WPI4*PL*WI**3*D44/210.0 + 2.0*PL*D55/WI +
     X          WPI2*PL*WI/15.0*(2.0*D45+D66)
      SK(1,2) =-WPI4*PL*WI**3*D44/280.0 + PL*D55/WI -
```

```
     X          WPI2*PL*WI/6C.0*(2.0*C45+D66)
      SK(3,3) = 13.0*WPI4*PL*WI*C44/7C.C + 6.0*PL*D55/WI**3 +
     X          3.0*WPI2*PL/(5.0*WI)*(2.0*D45+D66)
      SK(3,4) =-9.0*WPI4*PL*WI*D44/140.C + 6.C*PL*D55/WI**3 +
     X          3.0*WPI2*PL/(5.C*WI)*(2.0*D45+D66)
      SK(1,3) = 11.0*WPI4*PL*WI**2*D44/420.C + 3.0*PL*D55/WI**2 +
     X          WPI2*PL/20.0*(12.0*D45+D66)
      SK(1,4) =-13.0*WPI4*PL*WI**2*D44/840.C + 3.0*PL*D55/WI**2 +
     X          WPI2*PL/20.0*(2.C*D45+D66)
      SK(2,2) = SK(1,1)
      SK(4,4) = SK(3,3)
      SK(2,4) = SK(1,3)
      SK(2,3) = SK(1,4)
C
C        STIFFNESS OF COUPLING BETWEEN IN PLANE AND PLATE BENDING
C        ACTION DUE ECCENTRICITY OF STIFFENERS
C
      A15     =-WPI3*PL*WI**2*D14/40.0
      A16     =-WPI3*PL*WI**2*D14/60.0
      A17     =-PL*D25/(2.0*WI)
      A35     =-7.0*WPI3*PL*WI*D14/4C.0
      A36     =-3.0*WPI3*PL*WI*D14/4C.0
      SK(1,5) = A15
      SK(1,6) = A16
      SK(1,7) = A17
      SK(3,5) = A35
      SK(3,6) = A36
      SK(2,6) =-SK(1,5)
      SK(2,5) =-SK(1,6)
      SK(1,8) = SK(1,7)
      SK(2,7) =-SK(1,7)
      SK(2,8) = SK(2,7)
      SK(4,6) =-SK(3,5)
      SK(4,5) =-SK(3,6)
      SK(3,7) = 0.0
      SK(3,8) = C.0
      SK(4,7) = 0.0
      SK(4,8) = C.0
C
C        STIFFNESS OF IN PLANE ACTION
C
      SK(5,5) = WPI2*PL*WI*D11/6.C + PL*D33/(2.C*WI)
      SK(5,6) = WPI2*PL*WI*D11/12.0 - PL*D33/(2.0*WI)
      SK(7,7) = PL*D22/(2.0*WI) + WPI2*PL*WI*D33/6.0
      SK(7,8) = PL*D22/(2.0*WI) - WPI2*PL*WI*D33/12.0
      SK(5,7) = WPI*PL/4.0*(D12-D33)
      SK(5,8) = WPI*PL/4.0*(D12+D33)
      SK(6,6) = SK(5,5)
      SK(8,8) = SK(7,7)
      SK(6,8) = SK(5,7)
      SK(6,7) = SK(5,8)
      DO 220 I=1,8
      DO 220 J=I,8
  220 SK(J,I) = SK(I,J)
C
C        DETERMINE CCNSISTANT LOAD VECTCRFOR UNIFORM
```

```
C         TRANSVERSE LOAC ZL=1 ANC IN PLANE LOAC YL=1
C
      P(1,L) = WI**2/(6.0*WPI)
      P(2,L) =-P(1,L)
      P(3,L) = WI/WPI
      P(4,L) =-P(3,L)
      P(5,L) = C.0
      P(6,L) = C.0
      P(7,L) =-WI/WPI
      P(8,L) =-P(7,L)
C
C         TRANSFCRMATIUN OF STRIP STIFFNESS INTC GLOBAL COORDINATES
C
  300 DO 10 I=1,8
      SKA(I,1)=-SK(I,3)*VD - SK(I,7)*HD
      SKA(I,2)=-SK(I,3)*HD + SK(I,7)*VD
      SKA(I,3)= SK(I,1)
      SKA(I,4)= SK(I,5)
      SKA(I,5)= SK(I,4)*VD + SK(I,8)*HD
      SKA(I,6)= SK(I,4)*HD - SK(I,8)*VD
      SKA(I,7)= SK(I,2)
   10 SKA(I,8)= SK(I,6)
C
      DO 20 I=1,8
      SM(1,I,L)=-SKA(3,I)*VD-SKA(7,I)*HD
      SM(2,I,L)=-SKA(3,I)*HD+SKA(7,I)*VD
      SM(3,I,L)= SKA(1,I)
      SM(4,I,L)= SKA(5,I)
      SM(5,I,L)= SKA(4,I)*VD+SKA(8,I)*HD
      SM(6,I,L)= SKA(4,I)*HD-SKA(8,I)*VD
      SM(7,I,L)= SKA(2,I)
   20 SM(8,I,L)= SKA(6,I)
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BANSOL (NN,MM,NDIM,A,B,KKK)
C
C**********************************************************************
C          IN CORE BAND SOLVER
C
C          NN    NO OF EQUATIONS
C          MM    HALFBANDWIDTH + 1
C          NDIM  NO OF ROWS IN DIMENSION STATEMENT OF A
C          A     SHIFTED HALFBAND OF SYMMETRIC POSITIVE DEF. COEFF MATRIX
C                REDUCED MATRIX IS OVERWRITTEN
C          B      VECTOR  -  SOLUTION VECTOR IS OVERWRITTEN
C          KKK = 0   REDUCTION OF A AND B AND BACKSUBSTITUTION
C          KKK.LE.1  REDUCTION OF A
C          KKK.GT.1  REDUCTION OF B AND BACKSUBSTITUTION
C**********************************************************************
C
      DIMENSION A(NDIM,1), B(1)
C
      NR = NN - 1
      IF (KKK.GT.1)  GO TO 300
C
C         REDUCTION OF BAND MATRIX  A
C
      DO 200  N = 1,NR
      PIVOT = A(N,1)
      IF (PIVOT.EQ.0.0) GO TO 200
      M = N - 1
      MR = MIN0 (MM,NN-M)
      DO 190  L = 2,MR
      C = A(N,L)/PIVOT
      IF (C.EQ.0.)  GO TO 190
      I = M + L
      J = 0
      DO 180  K = L,MR
      J = J + 1
  180 A(I,J) = A(I,J) - C*A(N,K)
      A(N,L) = C
  190 CONTINUE
  200 CONTINUE
      IF (KKK.NE.0) GO TO 500
C
C         REDUCTION OF VECTOR  B
C
  300 DO 360  N = 1,NR
      IF (A(N,1).EQ.0.0) GO TO 360
      M = N - 1
      MR = MIN0 (MM,NN-M)
      C = B(N)
      B(N) = C/A(N,1)
      DO 350  L = 2,MR
      I = M + L
  350 B(I) = B(I) - A(N,L)*C
  360 CONTINUE
      IF (A(NN,1).LE.0.0) GO TO 380
      B(NN) = B(NN)/A(NN,1)
C
```

```
C          BACKSUBSTITUTION
C
  380 DO 400 K=2,NN
      M = NN - K
      N = M + 1
      MR = MIN0 (MM,K)
      DO 400  L = 2,MR
      I = M + L
  400 B(N) = B(N) - A(N,L)*B(I)
  500 RETURN
      END
```

```
      SUBROUTINE FIXFOR (HD,VD,YL,ZL,NPI,NPJ,P,M,PTOT,N,K)              FIXF
C                                                                       FIXF
C***********************************************************************FIXF
C         DETERMINATION AND ASSEMBLAGE OF JOINT FORCES INTO LOAD VECTOR FIXF
C***********************************************************************FIXF
C                                                                       FIXF
      DIMENSION P(8,M),PTOT(N)                                          FIXF
C                                                                       FIXF
      PTOT(NPI+1)=PTOT(NPI+1)-VD*ZL*P(3,K)-HD*YL*P(7,K)                 FIXF
      PTOT(NPI+2)=PTOT(NPI+2)-HD*ZL*P(3,K)+VD*YL*P(7,K)                 FIXF
      PTOT(NPI+3)=PTOT(NPI+3)+ZL*P(1,K)                                 FIXF
      PTOT(NPI+4)=PTOT(NPI+4)+YL*P(5,K)                                 FIXF
      PTOT(NPJ+1)=PTOT(NPJ+1)+VD*ZL*P(4,K)+HD*YL*P(8,K)                 FIXF
      PTOT(NPJ+2)=PTOT(NPJ+2)+HD*ZL*P(4,K)-VD*YL*P(8,K)                 FIXF
      PTOT(NPJ+3)=PTOT(NPJ+3)+ZL*P(2,K)                                 FIXF
      PTOT(NPJ+4)=PTOT(NPJ+4)+YL*P(6,K)                                 FIXF
      RETURN                                                            FIXF
      END                                                               FIXF
```

```
      SUBROUTINE OPRINT (A,M,N,X,NY,K1,K2,NCYC)                         OPRI
C                                                                       OPRI
C***********************************************************************OPRI
C          PRINTING SUBROUTINE FOR MATRICES OF STRESS RESULTANTS        OPRI
C***********************************************************************OPRI
C                                                                       OPRI
      DIMENSION A(M,N),X(M),N1(2),N2(2)                                 OPRI
C                                                                       OPRI
    1 FORMAT (I6,1P7E16.7)                                              OPRI
    2 FORMAT (6HOSECT.,7(6H    X =F10.3)/)                              OPRI
      DATA N1(1),N1(2)/1,8/                                             OPRI
      N2(1)=K1                                                          OPRI
      N2(2)=K2                                                          OPRI
      DO 10 K=1,NCYC                                                    OPRI
      J1=N1(K)                                                          OPRI
      J2=N2(K)                                                          OPRI
      PRINT 2, (X(I),I=J1,J2)                                           OPRI
      DO 10 I=1,NY                                                      OPRI
   10 PRINT 1,  I,(A(J,I),J=J1,J2)                                      OPRI
      RETURN                                                            OPRI
      END                                                               OPRI
```

```
      SUBROUTINE PINVAL (IND,D,M,N,X,MX,K1,K2,NCYC,L)
C
C***********************************************************************
C         PRINTING SUBROUTINE FOR GLOBAL JOINT DISPLACEMENTS
C***********************************************************************
C
      DIMENSION IND(M),D(M,N),X(N),N1(2),N2(2)
C
    1 FORMAT (I6,1P7E16.7)
    2 FORMAT (6H0JOINT,7(6H    X =F10.3)/)
      DATA N1(1),N1(2)/1,8/
      N2(1)=K1
      N2(2)=K2
      DO 10 K=1,NCYC
      J1=N1(K)
      J2=N2(K)
      PRINT 2, (X(I),I=J1,J2)
      DO 10 I=L,MX,4
   10 PRINT 1,  IND(I),(D(I,J),J=J1,J2)
      RETURN
      END
```

```
      SUBROUTINE MOMPER (XN,XM,W,I,NY,N1,N2,DI,DJ,H,V,XDIV,NX,X,MOPT,
     *                   GIRMCM,TENS,COMP,NCP)
C
C*************************************************************************
C         SUMMATICN CF STRIP CCNTRIBUTIONS TC THE MOMENTS OF GIDER
C         N1 ANC N2 WHICH IT BELONGS TO
C*************************************************************************
C
      CCMMON / SETUP / SPAN,NPL,NEL,NJT,NXP,NHARM,NCHECK,MM,NXBAND,
     *                 INTPRT,MCHECK,NSURL,NCONL,MX,PI,NA,NB,II,IJ,IL,
     *                 LA,LB,LC,LD,LE,INTP(21),NOXMP,NGIR
      DIMENSION XN(NX,1),XM(NX,1),X(1),MOPT(1),GIRMOM(NOP,1),
     *          TENS(NOP,1),CCMP(NOP,1)
C
      NSC=NY-1
      SC=NSC
      DEL=W/SC
      DEV=(CJ-DI)/SC
      IF(DEV.EQ.0.) GO TO 5
      XDIV=ABS(XCIV)
      DEH=ABS(CEV*H/V)
      GO TO 7
    5 DEH=DEL
    7 DO 100 J=1,NCXMP
      IT=MOPT(J)
      IF(IT.EQ.0) GC TO 100
      X1=DI
      IF(N2.GT.C) GC TO 20
C
C         STRIP CONTRIBUTES ONLY TO ONE GIRDER
C
      DO 10 NN=1,NSC
      X2=X1+DEV
      CALL ADDMCM(J,N1,X1,X2,CEL,DEH,XN(IT,NN),XN(IT,NN+1),
     *            XM(IT,NN),XM(IT,NN+1),GIRMCM,TENS,COMP,NOP)
   10 X1=X2
      GO TO 100
C
C         STRIP CCNTRIBUTICN TC FIRST OF THE TWC GIRDERS
C
   20 NN=1
      HH=0.
   30 HH=HH+DEH
      IF(HH.GT.XDIV) GO TO 4C
      X2=X1+DEV
      CALL ADDMCM(J,N1,X1,X2,CEL,DEH,XN(IT,NN),XN(IT,NN+1),
     *            XM(IT,NN),XM(IT,NN+1),GIRMCM,TENS,COMP,NOP)
      X1=X2
      NN=NN+1
      GO TO 30
   40 FA=(XCIV+CEH-HH)/DEH
      XL=FA*DEL
      XH=FA*DEH
      X2=X1+FA*CEV
      XN2=XN(IT,NN)+FA*(XN(IT,NN+1)-XN(IT,NN))
      XM2=XM(IT,NN)+FA*(XM(IT,NN+1)-XM(IT,NN))
```

```
      CALL ADDMCM(J,N1,X1,X2,XL,XH,XN(IT,NN),XN2,XM(IT,NN),XM2,GIRMOM,
     *            TENS,CCMP,NOP)
C
C        STRIP CCNTRIBUTICN TC SECOND OF THE TWC GIRDERS
C
      X3=X1+DEV
      XL=DEL-XL
      XH=DEH-XH
      CALL ADDMCM(J,N2,X2,X3,XL,XH,XN2,XN(IT,NN+1),XM2,XM(IT,NN+1),
     *            GIRMCM,TENS,COMP,NOP)
      X1=X3
   50 NN=NN+1
      IF(NN.GT.NSC) GO TO 10C
      X2=X1+DEV
      CALL ADDMCM(J,N2,X1,X2,CEL,DEH,XN(IT,NN),XN(IT,NN+1),XM(IT,NN),
     *            XM(IT,NN+1),GIRMOM,TENS,CCMP,NOP)
      X1=X2
      GO TO 50
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ACCMCM (J,N,X1,X2,XL,XH,XN1,XN2,XM1,XM2,GIRMOM,TENS,
     *                   COMP,NCP)
C
C*****************************************************************************
C         NUMERICAL INTEGRATICN CF CONTRIBITICN OF STRESS RESULTANTS
C         TC THE MOMENT
C*****************************************************************************
C
      CCMMON / SETUP / SPAN,NPL,NEL,NJT,NXP,MHARM,NCHECK,MM,NXBAND,
     *                 INTPRT,MCHECK,NSURL,NCONL,MX,PI,N1,N2,II,IJ,IL,
     *                 LA,LB,LC,LD,LE,INTP(21),NOXMP,NGIR
      DIMENSION GIRMCM(NOP,1),TENS(NOP,1),CCMP(NOP,1)
      F1=.5*XN1*XL
      F2=.5*XN2*XL
      XM=(F1*(X2+2.*X1)+F2*(X1+2.*X2))/3.
      F=F1+F2
      XM=XM+.5*(XM1+XM2)*XH
      GIRMCM(J,N)=GIRMCM(J,N)+XM
      IF(F.LT.0.) GC TO 10
      TENS(J,N)=TENS(J,N)+F
      GO TO 20
   10 COMP(J,N)=COMP(J,N)+F
   20 RETURN
      END
```