Subitizing and Mathematics Performance in Early Childhood

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Abstract

Subitizing is thought to be an important precursor to mathematical development in young children. However, evidence for this relationship is sparse. This study examined the relationship between subitizing range and mathematical performance in kindergarten-aged children. The large sample included typically-developing children from urban backgrounds. Findings indicated a significant link between subitizing ability and mathematical skills.

Keywords: math development; early childhood; subitizing.

Introduction

Subitizing ability is believed to underlie development of fundamental mathematics skills in early childhood and to support mathematics achievement. However, neither the development of subitizing nor its relationship to mathematics performance is well understood. Moreover, much of the existing research has been conducted with adults or special populations of children. The current study examines the relationship between subitizing ability and mathematics performance in a large sample of typically-developing children.

Subitizing is a fast small-number enumeration process different from counting. Subitizing in adults is estimated to take from 20-100 ms per item compared to counting, which is estimated to take 250-350 ms per item (Chi & Klahr, 1975; Trick & Pylyshyn, 1994; Wender & Rothkegel, 2000). Reports of the subitizable range among adults vary, but many studies agree on a range of 1-4 (Trick & Pylyshyn, 1994).

Children as young as 2 can subitize, but both children's subitizing speed and range may differ from that of adults. Children have been shown to be slower, taking an average of 100-200 ms per item when subitizing (Chi & Klahr, 1975; Svenson & Sjöberg, 1978). Estimates of children's subitizing range have varied from two to five items, but evidence from several studies suggests that the upper limit of the subitizable range is 3 or 4 for young children (e.g. Chi & Klahr, 1975; Gelman & Tucker, 1975).

Some evidence suggests that children's ability to subitize 3 objects or 4 is somewhat age-related. For example, Gelman and Tucker (1975) found differences in the subitizing accuracy of 3-, 4-, and 5-year-olds. For sets of three items, only about 58% of 3-year-olds were accurate, compared to 77% of 4-year-olds and 90% of 5-year-olds. The differences were even more dramatic for sets of four items, for which only about 19% of 3-year-olds were accurate, compared to 48% of 4-year-olds and 69% of 5-year-olds. Starkey and Cooper (1995) also found an Age x Set size interaction among children aged 2-5 for sets of three and four items. They suggested that the subitizing range may begin to gradually extend from 3 to 4 around the age of 3 ½.

Though such estimates are applicable across young children, in general, there are individual differences in subitizing ability among children. Svenson and Sjöberg

(1978), for instance, demonstrated wide variability in subitizing performance for 7- and 8-year-old children. Such individual differences, in concert with age-related differences, are of consequence because of the evidence that suggests that subitizing is a foundational skill for mathematical development.

Several recent studies have linked subitizing ability to counting, arithmetic, and number system knowledge. However, studies with young children have generally been small (e.g. Desoete & Grégoire, 2006; Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009; Penner-Wilger et al., 2007). For example, a study of 146 first-graders found that subitizing skill was related to both number system knowledge and calculation skills (Penner-Wilger et al., 2007). In a different study, Desoete and Grégoire (2006) demonstrated that subitizing skill is predictive of later mathematics performance. They administered a standardized mathematics assessment to 82 children at the end of kindergarten, and again in first grade and found that below-average subitizing skill in kindergarten significantly predicted low mathematics achievement in first grade. One larger study, with 375 participants, demonstrated that subitizing ability was related to arithmetic skills through high school, and that subitizing ability actually declined over time, but did not include children younger than 7 years old (Fischer, Gebhardt, & Hartnegg, 2008).

The Current Study

The current study also examined the link between subitizing ability and mathematics skill, but is unique in several ways. First, rather than measuring accuracy or speed of subitizing, this study investigated the upper limit of the subitizing range and its relationship to mathematical skill. Second, the subitizing data in this study were voice recordings of children performing a mathematics task rather than frequency counts or reaction times. Third, this study included a sample of typically-developing young children that is larger than the samples of any other known study of subitizing and early mathematics skill. Furthermore, the low-income, racially diverse, urban children in this sample comprise a population that has been neglected in much of the extant subitizing research.

Methods

As part of a larger study of a prekindergarten mathematics curriculum, 562 children were assessed in the spring of kindergarten using the Number and Geometry Assessments of the *Research-based Early Maths Assessment*, or REMA (Clements, Sarama, & Liu, 2008) and the Applied Problems and Quantitative Concepts assessments of the *Woodcock-Johnson III Achievement Battery* (WJ-III, Woodcock, McGrew, & Mather, 2001).

Mathematics Assessments

The REMA was developed to be used in conjunction with the *Building Blocks* (Clements & Sarama, 2007) prekindergarten mathematics curriculum. It is made up of two components: one that assesses number knowledge and one that assesses geometry skills. The items on the REMA Number Assessment measure the developmental progression of the following skills: recognition of number and subitizing, verbal counting and object counting, number comparison, number sequencing, number composition and decomposition, adding and subtracting, place value, and multiplication and division.

The REMA Geometry Assessment measures development in the areas of shape recognition, shape composition and decomposition, congruence, construction of shapes, spatial imagery (turns), geometric measurement, and patterning using geometric shapes. These assessments are comprehensive measures of mathematics development, and so are used as the designated measures of mathematics performance in this study. The REMA can be used with children from 3 to 8 years old and is a scripted assessment that is administered in a one-on-one interview format.

The WJ-III is a standardized achievement assessment made up of many subtests. Two mathematics subtests, Applied Problems and Quantitative Concepts, were employed to measure mathematical reasoning. The WJ-III subtests are also scripted and administered individually.

Based on prior evidence, subitizing was expected to be related to performance on the REMA Number Assessment but not on the REMA Geometry Assessment or either of the WJ-III subtests because subitizing has been presumed to be related specifically to numeracy skills.

Subitizing Assessment

For the basal item on the REMA *Number Assessment,* children are shown a picture of a bear, below which are three green dots representing grapes, and a picture of a cat, below which are four green dots. Children are required to compare the two sets and to explain how they determined whether or not the sets are the same in quantity.

Children's responses were audio-recorded and transcribed. The inter-rater reliability rate between two raters was 98.53%. Responses were categorized into one of

Table 1: Subitizing groups and sample responses.

Subitizing Group	Definition of Score	Characteristic Response
0	Can subitize neither 3 nor 4	"Because the bear has one, two, three, and the cat has one, two, three, four."
1	Can subitize 3 but not 4	"Because the bear has three, and the cat has one, two, three, four, four."
2	Can subitize both 3 and 4	"Because the bear has three, and the cat has four."
999	Did not indicate subitizing	"Because the bear has more." or "Because I'm smart."

three groups, based on the indicated subitizing range. In Group 0, children provided responses that indicated that they subitized neither 3 nor 4. Typically, children in Group 0 counted aloud both sets by ones. Children in Group 1 counted only the set of 4, having subitized 3. Finally, children in Group 2, subitized both sets and did not have to count.

In all 562 cases included in these analyses, children's responses began immediately after the assessor's question and did not include any pauses during which children may have counted inaudibly. The current sample excludes 122 children whose responses were not indicative of subitizing range. Table 1 includes examples of such excluded responses.

Results

Over 77% of the analysis sample was African American, 93% was receiving free-and-reduced meals. Overall, age ranged from 66 months to 90 months, with a mean of 72.61 months. Group 0 was comprised of the fewest children (11%) and Group 2 included the greatest number of children (70.3%), (X^2 = 350.246, *df* = 2, *p* < .001). A between-group age comparison found there were no significant differences among the subitizing groups (*F* (2, 555), *F*=2.384, *p* <.093).

Results from an ANOVA demonstrated significant differences among subitizing groups in performance on the REMA *Number Assessment* (*F* (2,559), *F*=48.919, *p* <.001), REMA *Geometry Assessment* scores (*F* (2,559), *F*=24.644, *p* <.001), WJ-III *Applied Problems* subtest (*F* (2,559), *F*=52.224, *p* <.001), and WJ-III *Quantitative Concepts* subtest (*F* (2,559)), *F*=32.073, *p* <.001). Post-hoc pairwise comparisons revealed significant differences among all groups on all assessments, with Group 0 scoring the lowest and Group 2 scoring the highest (see Table 2).

Table 2: Mean scores on mathematics assessments by subitizing group.

Subitizing Group	n	REMA Number		REMA Geometry		WJ Applied Problems		WJ Quantitative Concepts	
		M	*(SD)*	*M*	*(SD)*	*M*	*(SD)*	*M*	*(SD)*
0	62	25.23	(10.3)	9.23	(3.85)	418.00	(17.40)	437.53	(14.28)
1	105	31.65	(8.9)	10.76	(3.33)	430.03	(14.20)	444.76	(11.61)
2	395	37.53	(10.12)	12.26	(3.45)	437.23	(13.91)	449.23	(10.51)
Overall	562	35.07	(10.74)	11.65	(3.62)	433.76	(15.65)	447.10	(11.80)

Linear mixed modeling was employed to take into account the hierarchical nature of the data. Children were nested within classrooms and classrooms nested within schools. Each mathematics outcome measure was modeled separately. Demographic variables, school system, age, and pretest scores were entered into each model as covariates. There were no significant effects of age, gender, ethnicity, or free-and- reduced-priced meal program status (used as a proxy for socioeconomic status). For all outcome measures, over and beyond pretest, subitizing group membership had a significant unique relationship with mathematics performance. Compared to children who could subitize neither 3 nor 4 (Group 0), children who could subitize 3 but not 4 (Group 1) performed significantly better on *REMA Number*, *WJ-III Applied Problems*, and *WJ-III Quantitative Concepts* assessments. Children who could subitize both 3 and 4 (Group 2) performed significantly better than children who could not subitize on all measures. The effect sizes between the scores of a child who can subitize neither 3 nor 4 and those of a child who can subitize both 3 and 4 were *d* = 0.67 for the WJ-III *Applied Problems* subtest, *d* = 0.62 for the REMA *Number Assessment*, *d* = 0.52 for the WJ-III *Quantitative Concepts* subtest, and *d* = 0.48 for the REMA *Geometry Assessment*.

Discussion

The current analyses found a significant relationship between subitizing range and mathematics performance in kindergarten-aged children. Children who demonstrated ability to subitize both 3 and 4 had higher scores on all four mathematics outcome measures compared to children who could subitize 3 but not 4 or children who could subitize neither 3 nor 4. These findings provide further support that subitizing skill may be foundational for mathematics development in early childhood, and that the ability to subitize 4 might be an important milestone. Furthermore, all four measures of mathematical performance were significantly correlated, suggesting that mathematical understanding in young children may be domain general and not specific to particular aspects of mathematics. Subitizing appears to be linked to overall mathematical skill rather than only to numeracy.

However, there is an important difference in the age range of the children in the current study from that of children in previous studies. Other studies have reported that children can generally subitize 3 by age 3 (Benoit et al., 2004; Starkey & Cooper, 1995), and that older 3-year-olds and 4- and 5-year-olds were generally able to subitize 4 (Starkey & Cooper, 1995). However, almost a third (29.72%) of the children in this study, though much older with ages ranging from 5 ½ to 7 ½, did not subitize 4. Because the low-income, urban population from which the current sample was drawn has not been extensively studied in regards to subitizing ability, it is possible that the present findings are indicative of differences specifically linked to the low-income, urban backgrounds of children who attend government-funded Head Start or public prekindergarten programs and qualify for the free-and- reduced-priced meal program. This is a possibility that warrants further investigation in future studies.

Educational Implications

Based on the findings in this study, subitizing range is a strong indicator of students' mathematical abilities, and subitizing skill assessment could convey important information to a teacher about a child's mathematical development. It may be useful to consider subitizing as a screening tool that could provide a rough estimate of children's mathematical abilities, an idea that has been suggested by others (Desoete, Ceulemans, Roeyers, & Huylebroeck, 2009).

In prior studies, poor subitizing skills have been linked to mathematics learning disability (MLD) (Desoete & Grégoire, 2006; Fischer, Gebhardt, & Hartnegg, 2008; Fischer, Köngeter, & Hartnegg, 2008; Koontz & Berch, 1996), and it is a fact that, in the current study, the students who fell into the lowest subitizing group scored significantly lower than their peers on all mathematics assessments. Though it would be impossible to link the subitizing abilities of children in the current sample with MLD, one may wonder if some of the students who did not effectively subitize 3 might be identified as having MLD at a later age. It is possible that a short measure of subitizing could serve as a screen for identifying children who, without intervention, may be at risk of developing MLD, or mathematics difficulties, later on. Early identification of risk for MLD followed by focused mathematical intervention may attenuate initial delays in mathematical development due to poor subitizing skills.

There are, in fact, several studies that have been able to successfully produce changes in subitizing ability with teaching or coaching (Fischer, Köngeter, & Hartnegg, 2008; Wilson, Revkins, Cohen, Cohen, & Dehaene, 2006). However, the children in these studies were older and had already been identified as having MLD. Thus, currently, there is very little evidence that subitizing can be taught to young children, or that doing so would interrupt the development of later mathematical difficulties or MLD.

Limitations

There are several limitations to the current study. It is important to note that the mathematical task used to determine subitizing range was not intended as a measure of subitizing. The audio recordings of the children's responses provided incidental data on children's subitizing ability. A more intentional measure may have prevented 17% of the sample from being excluded from the analyses.

Related to this point, it could be argued that children's subitizing range was inferred from children's responses rather than directly measured. However, the speed and confidence with which children responded when subitizing contrasted with the slowness and deliberateness when counting clearly distinguished the two processes in all included cases. Furthermore, there were no pauses in the recording during which children might have been counting sub-vocally.

Finally, children's subitizing range was assessed based on a response to one item. Therefore, it is important to point out that these data represent only a single snapshot of children's subitizing ability. Again, intentionality in designing future studies would remedy this limitation.

Conclusion

Despite its limitations, this study provides important evidence for a relationship between subitizing ability, as defined by range, and mathematics skills in early childhood. Having included typically developing children from low-income urban backgrounds in a sample much larger than those of previous studies contributes uniquely to the extant research. Understanding the link between subitizing and mathematical development in diverse populations of children has important implications for both researchers and practitioners.

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