Majority-efficiency and Competition-efficiency in a Binary Policy Model

Stefan Krasa   Mattias Polborn*

November 15, 2006

Work in progress. The most current version of this paper is available at
https://netfiles.uiuc.edu/polborn/www/.

Abstract

We introduce a general framework in which politicians choose a (possibly infinite) sequence of binary policies. The two competing candidates are exogenously committed to particular actions on a subset of these issues, while they can choose any policy to maximize their winning probability for the remaining issues. Citizens have general preferences over policies, and the distribution of preferences may be uncertain.

We also introduce two new normative concepts for political settings: A candidate's platform is majority-efficient if the candidate has no other feasible platform that is strictly preferred by a majority of voters. The equilibrium is competition-efficient if a social planner could not pick platforms that are different from the two candidates' ones and make a majority of the electorate better off.

We show that, while the standard Downsian model satisfies majority- and competition-efficiency, these properties are not satisfied in many other, slightly different, settings.

JEL Classification Numbers: C70, D72.
Keywords:

1 Introduction

The one-dimensional policy model based on the seminal contributions of Hotelling (1929) and Downs (1957) is the most widely used and successful model framework for a formal analysis of political equilibria. Yet, there are some tensions within the model, and between the model and some real-world observations.

First, in the one-dimensional spatial model, there is a strong tendency for candidates to converge to the same, moderate position that appeals to the "median voter" (mitigated only if the candidates care about policy and to the extent that the position of the median is uncertain); furthermore, all voters (including those with extreme preferences) are, in equilibrium, indifferent between the two candidates as they propose the same policy. Yet, in reality, candidates often run on considerably divergent policy platforms, and voters often intensely favor one candidate over the other.[1]

Second, while the model is formally one-dimensional and continuous, policy in reality is often multidimensional (there are many policy issues) and binary (e.g., candidates are either for withdrawing troops from Iraq or against it).[2] In fact, a widespread casual interpretation of policy in the one-dimensional model is based on multidimensional policies. For example, when we say that "Hillary Clinton used her support for the Iraq war to move towards the political center," we imply that her initial position (on other issues) is left-of-center, but by adopting a conservative position on a particular issue, she can move to the right on the policy line. More generally, some candidates' stand on particular issues may only be supported by a minority (e.g., support for state-provided health care) and they appear unable to change their positions on these issues directly. Instead, they have to use their positions on other issues to advance their electoral prospects.

Third, the informal multi-issue interpretation of the one-dimensional model is somewhat problematic in their treatment of moderates. Suppose, for example, that we accept the notion that support for state-provided health-care is a liberal position and support for the Iraq war is a conservative position, and suppose that these are the only two, and equally important, issues in our polity. Then both "Hillary Clinton" (with positions as described above) and a voter who opposes both state-provided health care and the Iraq war would be considered "moderates" with a position in the center in a one-dimensional model, suggesting that the voter is likely to support Hillary Clinton. Yet, it would appear quite plausible that the voter prefers another candidate with a "more extreme" position, say, someone who supports state-provided health care but opposes the Iraq war.

In this paper, we develop a model that directly treats policy as multidimensional and binary, with candidate positions exogenously given on some issues, while candidates are free to choose their positions on other issues. We find that, in this framework, adopting minority positions may sometimes be a strategy that *increases* a candidate's winning probability. This property of non-moderation of equilibrium policies makes it interesting to think more about notions of efficiency in

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[1]See, e.g., Bernhardt, Krasa, and Polborn (2006) for a documentation of polarized preferences in U.S. presidential elections.

[2]Even more nuanced positions are few in numbers and can usually be expressed by a small number of binary, "yes" or "no" answers.

our model.

In our model, a vector of binary variables describes a candidate's proposed positions on different issues. This binary model encompasses as special cases the standard Downsian model, the probabilistic voting model and a "weighted-issue model" (in which each citizen has a preference for a certain action on each issue, and a citizen's utility is a weighted sum of the issues on which the citizen agrees with the candidate). In particular, the weighted-issue model provides a new and tractable framework to deal with multidimensional policies. We consider the case of two office-motivated candidates, who are characterized by certain attributes and/or fixed positions on some subset of issues, while they are free to choose a position on the remaining issues. Fixed positions can be interpreted as characteristics of the candidate like party affiliation, incumbency, gender, race, experience in previous elected office, profession or rhetorical ability, to name just a few. Other fixed positions may correspond to political issues in which a candidate has taken a stand in the past and where a commitment to a different position is not credible and/or not helpful. For example, a candidate who took a strong pro-choice stand in his past legislative vote record may not be able to credibly commit to a pro-life platform, and therefore is essentially fixed to his previous position in the abortion issue. We show that it is generally important to consider fixed positions because they also influence the candidates' behavior in those issues where candidates are free to choose a policy platform.

Another contribution of our paper is a normative analysis of the equilibrium of the candidate positioning game. We define two concepts, *majority-efficiency* and *competition-efficiency* that are related to the notion that democracy (and, in particular, competition between office-motivated candidates) implements what is good for society, in the sense that at least a majority of citizens gets what they want. We define a proposed policy position to be *majority-efficient* if a majority of voters prefers the proposed policy to any other policy that candidate could choose, i.e., a majority efficient policy is a Condorcet winner among all the candidate's policies (taking fixed positions into account).[3] While such a Condorcet winner need not always exist, it does so in many interesting applications of our model framework. The concept of *competition-efficiency* is our second normative concept. The equilibrium is competition-efficient if a social planner could not pick two alternative platforms and make a majority of the electorate better off. Hence, rather than looking at the behavior of one candidate (like majority-efficiency), competition-efficiency deals with the two platforms proposed by the candidates and is of particularly interest when the distribution of preferences in society are unknown.

In a standard Downsian model where both candidates choose the median preferred policy as platform, policies are majority-efficient and the equilibrium is competition-efficient. In our more general framework, we characterize cases in which this property carries over and others where it does not. Failure to adopt majority-efficient policies does not necessarily have anything to do with

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[3]Majority-efficiency takes the sincere preferences of voters over policies as input. It may well be that, for example, a right-wing voter "prefers" (given the ensuing voting equilibrium) if the left candidate adopts a more liberal policy, because this decreases the left candidate's chance of winning the election. However, such strategic preferences are not useful for defining a notion of majority-efficiency.

Condorcet cycles. Indeed, we show that it can arise in horizontal differentiation frameworks or other settings where a Condorcet winner is guaranteed to exist in every state of the world.

If there is no uncertainty about voters' preferences, then, if both candidates choose majority-efficient policies, the equilibrium is competition-efficient; however, this is no longer true when there is uncertainty about the preference distribution. In this case, equilibrium candidate convergence may be excessive. As Myerson (1993) points out, the answer to this question is not at all obvious: "Many authors seem to accept Hotelling's view that convergence of candidates is an undesirable outcome, because this 'excessive sameness' gives voters no real choice. This view ignores some crucial differences between the economic and political interpretations of Hotelling's game. In the economic interpretation, when two shops are locating on Main Street, minimization of the consumers' total transportation cost requires separation of the two shops. In the political interpretation, however, every voter's utility is derived from the policy position of the winning candidate (rather than the policy position of the one for whom he votes), and so voters get no intrinsic utility from a diversity of options in the selection. Thus, candidate convergence in equilibrium does not necessarily cause any welfare loss." For example, while voters effectively have no choice in a horizontal differentiation framework without uncertainty (because both candidates offer the median voter's preferred policy), this equilibrium is still competition-efficient. However, when there is non-trivial uncertainty about the state of the world, we show that equilibria featuring candidate convergence are competition-inefficient, because a non-trivial difference between candidates' platforms would be valuable for a majority of voters.

1.1   Related Literature

We depart from the standard voting literature by using a binary description of a policy space. This description appears natural, because most political campaigns are focused on relatively few clearly defined issues, where the politician can only be on record as being in favor or against the position. For example, in the 2006 US midterm elections, the key issues of "whether or not to impose a timetable for the withdrawal of troops from Iraq"; "whether to support stem cell research"; or "whether to support a constitutional ban on gay marriage"; all can be answered by yes or no. Other issues such as raising the minimum wage are also more realistically thought of as encodable in binary form: Plausible position might be "support for raising the minimum wage by a dollar", by $1.50, by $2 dollars, or not at all, which is straightforward to encode as two yes-no positions. While a binary state space has been used in contract theory (e.g., Krasa and Williams (2006)), this paper is, to our knowledge, the first use of this approach in political economy. An advantage of the binary multidimensional model over multidimensional Euclidean models (Plott (1967), McKelvey (1976)) is that our model is relatively tractable: The set of preferences for which an equilibrium or a majority-efficient position exists has positive measure, and even if no pure strategy equilibrium exists, the mixed strategy equilibrium is straightforward to compute.

An alternative attempt to introduce multidimensionality of policies in a tractable form is the probabilistic voting model (henceforth PVM; see Persson and Tabellini (2000) and the references

therein). In the PVM, several voter groups have different preferences about policy, but also within each group, individuals differ in their "ideological" preferences for candidates. If the preference diversity within groups is sufficiently large, then a pure strategy equilibrium exists in which both candidates propose the same policy (because, in contrast to deterministic multidimensional policy models, a candidate who provides a group with a slightly higher utility than his competitor does not attract the whole group as voters). Moreover, in equilibrium, candidates cater more strongly to the views of swing voters, i.e., voters who are more likely to switch from one candidate to the other. While some effects in our model are similar, there are also substantial differences between the PVM and our general binary model.[4] In particular, in the PVM, both candidates converge to the same position. As a consequence, all voters are indifferent between the candidates, and swing voters are those voters who are ideologically indifferent between candidates. This need not be the case in our model: Candidates may find it attractive to cater to their hardcore supporters and hence choose substantially different policies from their opponent (so that very few voters are indifferent between candidates). Furthermore, "ideological" preferences enter only as random utility shocks to voters in the PVM, but are explicitly modeled as arising from fixed positions in our general model. We also show the crucial role of different fixed positions for the result that candidates may choose different positions in pledgeable issues.

Another point of departure from the previous literature is that we assume that candidates have some dimensions in which they are exogenously committed to certain positions, while there are other dimensions in which candidates are free to choose a position, aiming to increase their winning probability. There are two branches in previous literature related to the issue of candidate commitment. First, in the literature on the standard one-dimensional model pioneered by Hotelling (1929) and Downs (1957),[5] candidates are free to choose their position. Candidates may be office-motivated (in which case there is a strong tendency for their positions to converge) or policy motivated (in which case the convergence may be mitigated, if there is also uncertainty about the position of the median voter). Second, in the citizen candidate literature pioneered by Osborne and Slivinski (1996) and Besley and Coate (1997), candidates are policy motivated and cannot commit to any other position than their ideal one.[6] The argument given for the inability of candidates to commit is that it is impossible to write binding contracts with the electorate and that a promise to implement a policy different from the candidate's ideal point is not credible. Our model combines some dimensions on which candidates have no choice, either because these dimensions capture some innate characteristics of candidates, or because their preferences on some questions are well known and not credibly changeable, with other dimensions in which candidates are free to pick a position. In term of results, the differences between our model and the citizen candidate model become apparent when considering the scoring model, the analogue to the one-dimensional spatial

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[4]Note that the PVM can also be written in binary form, and is therefore a special case of our general model. Hence, the differences that we point out here are between the PVM and other special cases of our model like the weighted issue model.

[5]For a survey of this literature, see Osborne (1995).

[6]While the citizen candidate model can, in principle, handle multiple policy dimensions, most papers in this literature only look at a standard one-dimensional framework.

voting model in our framework. We show that in general candidate's policies will converge in the sense that both candidates choose policies that maximizes the median voters payoff. In contrast, in citizen candidate models, there is in general no policy convergence.

A large literature exists that tries to explain, within the Downsian model, the empirical observation that candidates often propose considerably divergent policies. The first reason for divergence discussed in the literature is policy-motivation of candidates (see Calvert (1985)): Candidates may prefer to propose a non-moderate policy *even though* this decreases their winning probability (because they care about the implemented policy and have extreme preferences). A second reason may be that the two established parties differentiate their policies in order to deter entry by a third party (Palfrey (1984),Callander (2005)). Third, Aragones and Palfrey (2002) consider a models with uncertainty about the location of the median voter, in which a weaker candidate tries to differentiate himself from the stronger candidate, and hence policies do not converge. They prove that candidates' policies converge if uncertainty becomes small. Our model provides a novel explanation for policy divergence, which does neither require uncertainty about the distribution of voter preferences nor entry deterrence. In fact, purely office-motivated candidates may choose non-moderate policies in our model, thereby strictly increasing their probability of winning the election.

2 The Model

2.1 Setup

Two candidates, $j = 0, 1$, compete in an election. Candidates are office-motivated, i.e., receive utility 1, if elected, and utility 0, otherwise, independent of the implemented policy. There is an infinite number of issues, indexed by i. Candidate j, if elected, implements a policy described by $a^j = (a_i^j)_{i \in \mathbb{N}}$, where each a_i^j takes either the value 0 or 1 (0 can be interpreted as opposition to a particular proposal, and 1 as support of that proposal). There exists a subset of issues $S \subset \mathbb{N}$ on which the candidate can choose the policy freely, while on the remaining issues no commitment is possible. Thus, *candidate j's type* is given by $a = (a_i^j)_{i \notin S}$, while the *candidate's platform* is given by $a = (a_i^j)_{i \in S}$. The candidate's policy consists of the combination of type and platform. Let $(\bar{a}_i^j)_{i \notin S}$ be Candidate j's type. Then Candidate j's *set of feasible policies* is given by $A^j = \{(a_i)_{i \in \mathbb{N}} | a_i = \bar{a}_i^j$ for all $i \notin S$ and $a_i \in \{0, 1\}$ for $i \in S\}$.

The state of the world, $\omega = (\nu, \tau) \in \Omega$, is realized after candidates choose their platform and before the election. We let ν stand for a parameter that directly influences citizens' utility, for example, the candidates' valence level. In contrast, τ parameterizes the candidates' uncertainty about the frequency of different voter types.

Each citizen $\theta \in \Theta$ has preferences $u_\theta(a, \nu)$, where $a = (a_i)_{i \in \mathbb{N}} \in \{0, 1\}^{\mathbb{N}}$ is the implemented policy. The joint probability distribution over $\Theta \times \Omega$ is given by μ. Let μ_τ be the marginal distribution of θ given τ and let μ_Θ be the ex-ante distribution on Θ (i.e., $\mu_\Theta(S) = \mu(S \times \Omega)$ for all $S \in \Theta$).

The timing of the game is as follows:

Stage 1 Candidates $j = 1, 2$ simultaneously announce platforms $(a_i^j)_{i \in S}$ on a subset S of issues on which the candidates can commit. A mixed strategy by agent j consists of a probability distribution σ^j over $A_S^j = \{0, 1\}^S$.

Stage 2 Each citizen votes for his strictly preferred candidate, or abstains when he is indifferent between both candidates.[7] Candidate j wins in state ω if

$$\mu_\tau \left(\{\theta | u_\theta(a^j, \nu) > u_\theta(a^{-j}, \nu)\} \right) > \mu_\tau \left(\{\theta | u_\theta(a^j, \nu) < u_\theta(a^{-j}, \nu)\} \right).$$

In case of a tie between the two candidates, each of them wins with probability 0.5.

The assumption in our model that politicians have only a binary choice may, at first glance, seem restrictive, but it is possible to combine several issues in our formal framework to deal with issues that have more than just two possible positions. Suppose, for example, that possible positions concerning the legal drinking age could be 18, 19, 20, or 21 years. This can easily be represented by two binary positions, where the first position indicates whether $(0, 0)$ corresponds to 18, $(0, 1)$ to 19, $(1, 0)$ to 20, and $(1, 1)$ to 21. More generally, we can express any real number or any vector in $\mathbb{R}^n$ as a sequence of binary decisions. The multidimensional Euclidean voting model and the probabilistic voting model can therefore be represented as special cases in our framework.

2.2 Interpretation: Fixed issues

A key feature of our model is that candidates can commit to a policy on some issues, while they are fixed to an exogenously given position in other dimensions. Thus, our model combines the commitment assumption of the standard Downsian model (with respect to the first set of issues) with the assumption in citizen candidate models that no commitment to a policy other than the candidate's ideal point is possible. We believe that this is a reasonable convex combination of these two central models in the literature. In reality, candidates have commitment power on some issues. If a candidate makes a promise a central theme in his campaign (say, not to raise taxes, to end a war etc.), then breaking that promise is at least very costly for the candidate, and, counterexamples not withstanding, most candidates keep their central election promises. However, there are other dimensions in which candidates cannot easily commit to different positions. This is obviously true for characteristics of the candidate like gender, race, experience in previous elected office or profession, about which (at least some) voters plausibly care and which are fixed, at least in the short term that is the focus of our analysis. These characteristics can be interpreted as "fixed positions" in our framework.[8]

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[7]If a voter has a strict preference, then it is a weakly dominant strategy to vote for the preferred candidate. If an agent is indifferent, he could in principle vote for any candidate or abstain, but the assumption of abstention is quite natural, and none of the results in this paper depends critically on it.

[8]Note, however, that if we instead focus on the nominating behavior of parties, then fewer positions should be considered fixed than for any particular candidate. For example, while the party can choose whether to nominate a man or a woman, each candidate has his gender not as a choice variable.

Other fixed positions may correspond to political issues in which a candidate has taken a stand in the past and where a commitment to a different position is not credible and/or not helpful. For example, a candidate who took a strong pro-choice stand in his past legislative voting record may not be able to credibly commit to a pro-life platform, and therefore is essentially fixed to his previous position on the abortion issue.[9]

One important committed issue is party identity. When a candidate is running as a Democrat and wins, he is committed to support his fellow Democrats in committee appointments (i.e., even if the candidate chooses to run on a conservative platform, his seat counts for determining whether the Democrats are the majority party in Congress, with the associated privileges for possibly more liberal Democratic party leaders). This may make it difficult for a Democrat to win in very conservative districts, even if he adapted a very conservative platform otherwise. For example, in the 2006 elections, many Republican House candidates tried to tie their Democratic opponents to "liberal Nancy Pelosi", the prospective Speaker of the House in case of a Democratic majority. A related case in point is the 2006 Senate race in Rhode Island, which the incumbent, Senator Lincoln Chafee (a relatively liberal Republican) narrowly lost (47% to 53%) in spite of being personally very popular. In exit polls, 63% of voters approved of Chafee's job performance as U.S. Senator and 51% of voters said that Chafee's position on issues was "about right". However, by an overwhelming margin (63%-23%), voters stated that they wanted Democrats rather than Republicans in control of the Senate.[10]

Also, note that most senators from states that usually vote for Democrats in the presidential election are Democrats and vice versa. In a naive Downsian model without constraints on the policy platforms, candidates adopt the position of the median voter in their respective district, and win with 50% probability. Hence, while this model predicts that both Democratic and Republican candidates in conservative districts adopt more conservative positions than in liberal districts, it cannot explain why Republicans win significantly more of the conservative districts than Democrats and vice versa. Our framework allows a natural way of capturing such constraints on the policies to which candidates can commit.

2.3 Examples of Preferences: Scoring versus weighted-issue

We now introduce two different types of utility functions for citizens that are important in the following analysis. The *weighted-issue* utility function for a citizen θ is quite natural and given by

$$u_\theta(a) = -\sum_{i=1}^{\infty} \lambda_{i,\theta} |\theta_i - a_i|, \tag{1}$$

where $\theta = (\theta_i)_{i \in \mathbb{N}}$ denotes citizen θ's preferred position on policy i. The relative importance of each issue is determined by a weight $\lambda_{i,\theta}$. That is, we allow both preferred policy positions and their

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[9]See Kartik and McAfee (2006) for a model in which candidates want to appear inflexible, because those candidates who can easily change their position ("would say anything to be elected") are associated with having low valence.

[10]See http://www.cnn.com/ELECTION/2006//pages/results/states/RI/S/01/epolls.0.html for these exit poll results.

relative importance to differ among citizens. Clearly, this framework also accommodates settings with finitely many issues by fixing $\lambda_{i,\theta} = 0$ for all but a finite number of issues.

The second type of utility function, which we call the *scoring model*, is related to the standard one-dimensional Hotelling-Downsian model, in which policies and voter bliss points can be expressed as a number in the interval $[0, 1]$. Specifically, the utility of type θ is given by

$$u_\theta(a) = \left(\frac{1-\lambda}{\lambda} \sum_{i=1}^\infty a_i \lambda^i - \theta \right)^2, \tag{2}$$

where $\theta \in \mathbb{R}$ is the citizen's type, a candidate's policy is evaluated by first determining an overall, one-dimensional score, which can be interpreted as measure of how liberal or how conservative the candidate is, where 0 and 1 are the extreme points.[11] The citizens preferred policy is also given by a one dimensional parameter (θ), and he prefers the candidate who is closer to his ideal point. If we map each policy a into the "score" $x = \frac{1-\lambda}{\lambda} \sum_{i=1}^\infty a_i$, then the utility function becomes $u_\theta(x) = -(x - \theta)^2$. If all policies can be chosen freely, i.e., $S^c = \emptyset$, then we are in the standard Downsian model.

Note that x does not correspond to a unique policy a. For example, if $\lambda = 0.5$ then Candidate 0's policy $(0, 1, 1, 1, \dots)$ and Candidate 1's policy $(1, 0, 0, 0, \dots)$ result in the value of 0.5. While for $\lambda = 0.5$ these are the only two sequences that lead to an effective policy of 0.5, there is an infinite number of such sequences for $\lambda > 0.5$. The difference in actual policies does not matter for voters, because they only care about the weighted percentage of times that a candidate chooses liberal and conservative positions, respectively. As a consequence, two candidates can have the same effective policy position $(\frac{1-\lambda}{\lambda} \sum_{i=1}^\infty a_i)$, even if their policies (the vector a) differ.[12]

The key difference between utility functions (2) and (1) is the treatment of moderates. Suppose there are only two relevant issues and that both issue weights are 1. Then, in the scoring model, policies $(0, 1)$ and $(1, 0)$ are equivalent, and can both be considered as centrist. In contrast, in the weighted issue model, a voter with preference type $\theta = (0, 1)$ prefers policies $(1, 1)$ and $(0, 0)$ to the other supposedly centrist policy $(1, 0)$. In other words, if citizens care about the policy in specific issues, but we try to use a scoring model, then some moderates prefer a more partisan candidate (who agrees with them one issue) to a moderate one (who disagrees with them on both issues). Thus, using the scoring model to determine the polarization of the electorate would lead to underestimating the number of moderates.

As a more concrete example, consider the August 2006, primary between the incumbent Senator Lieberman and challenger Lamont. The candidates differed on a number of positions, of which the

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[11]Note that $(1-\lambda)/\lambda$ is chosen so that the score is in $[0, 1]$ for all policies a. With a little extra-notation and no qualitative change in results, we could also choose a weighting profile that is different from the geometrically declining one in (2).

[12]The determination of the relevant political position in this model works similarly to the calculation of legislative scores by lobby groups such as the ACLU and NRA for members of the U.S. congress. A score is the percentage the times (usually unweighted) that a politician voted for the group's preferred policy. A particular score (for example, in the middle) can be achieved by adopting an appropriate mix of voting for and against the interest group's preferred choice, and there are multiple voting vectors that give rise to the same score values, except for the extreme scores of 0 or 100.

question of support of the Iraq war by Lieberman was generally considered to be the most important one. In terms of other issues, Lieberman was more liberal than Lamont. For example conservative commentator Larry Kudlow (2006) writes that "Lamont may be slightly to the right of Lieberman on budget spending. In the CNBC interview with me, Lamont said he wanted to eliminate budget earmarks like the abusive transportation bill. Lieberman is a defender of earmarks." Similarly, one of Lieberman's first campaign commercials during the primary attacked Lamont for being supported by a former Republican Governor of Connecticut, and for voting with Republicans 80% of the time. Overall, both appear to be moderate candidates, but have different proposed policies on a number of issues. Thus, in a one-dimensional model, voters would be close to indifferent between these two candidates. In practice, however, there were intense preference differences among many voters. This can easily be captured by weighted-issue preferences but not by the scoring model.

Fixed issues have a different impact on candidates's policies in the scoring and weighted-issue models. Consider first the scoring model, and suppose that the liberal candidate is fixed to $a_1 = 0$, while the conservative candidate is fixed to $a_1 = 1$. Let θ_m be the position of the median voter (assuming there is not uncertainty). Then voters put more weight voters put on the liberal or conservative label of the candidate if their λ is smaller. For example, if $\lambda < 1/2$, then at least one of the candidates cannot obtain the ideal point of the median voter. In other words, one of the candidates is too far to the left, or too far to the right, and no feasible policy platform can remedy this problem. In contrast, if $\lambda > 1/2$, then candidate 0 can obtain all scores in $[0, \lambda]$ and Candidate 1 can obtain all scores in $[1 - \lambda, 1]$. Hence, if λ is sufficiently large the fixed position does not matter at all (e.g., a Democratic candidate can always move sufficiently far to the right by favoring sufficiently conservative policies on pledgeable issues). In other words, if citizens care sufficiently strongly about many issues (rather than just a few), then the party label becomes less relevant. Moreover, since only scores matter, both candidate can choose moderate positions (such as the policy that most appeals to the median voter), so that all citizens are indifferent.

Now compare this to the weighted issue model. In this model, citizens have (generically) strict preferences over the candidates, no matter which pair of platforms are chosen by the candidates, as candidates differ on some fixed issues. Also, while the scoring model in general induces candidates to converge, and become as similar as possible, this will not necessarily occur in the weighted issue model. For example, a Democrat in a Republican leaning district may surely lose if he chooses the same policies as the Republican candidate on pledgeable issues, but may have a chance if he differentiates himself from his opponent by choosing a different platform. We will explore this issue in section 4.

3 Majority Efficiency, Competition Efficiency and Existence of Equilibrium

3.1 Definition of the Concepts

We first define what it means for a majority of agents to prefer one policy over another policy.

Definition 1

1. *Let $a, a' \in A$. Then a is **majority preferred** to a' in state ω, denoted by $a \succeq_\omega a'$, if and only if $\mu_\omega\left(\{\theta | u_\theta(a, \omega) \geq u_\theta(a', \omega)\}\right) \geq \mu_\omega\left(\{\theta | u_\theta(a', \omega) \geq u_\theta(a, \omega)\}\right)$.*

2. *a is **strictly majority preferred** to a' in state ω, denoted by $a \succ_\omega a'$, if $a \succeq_\omega a'$ but not $a' \succeq_\omega a$.*

Our definition accounts for the fact that some citizens may be indifferent between a and a'. In other words, we cannot solely require that at least 50% of citizens find a at least as good as a', because it could easily be the case that at the same time more than 50% of citizens find a' at least as good as a, if some citizens are indifferent. Thus, our definition compares the number of citizens who find a at least as good as a' to the number of citizens who find a' at least as good as a.

We now introduce our normative concept of majority-efficiency.

Definition 2

1. *Candidate j's policy $a^* \in A^j$ is **majority-efficient in state ω** if and only if $a^* \succeq_\omega a$ for all $a \in A^j$.*

2. *Candidate j's policy $a^* \in A^j$ is **ex-ante majority-efficient** if and only if $\mu(\{\omega | a^* \succeq_\omega a\}) \geq \mu(\{\omega | a \succeq_\omega a^*\})$ for all $a \in A$.*

Intuitively, a policy is majority-efficient in state ω if the policy is a Condorcet winner among all of the candidate's feasible policies. Ex-ante majority-efficiency means that the policy is preferred in a majority of states by a majority of citizens to any other of the candidate's feasible policies.

To illustrate the concept of majority-efficiency, first consider the scoring model. Suppose the median voter is located at 0.5, and there is no uncertainty about his position. Then the policy that is closest to 0.5 among all feasible policies of the candidate is majority-efficient. For example, if the candidate is fixed to $a_1^j = 0$, and all other positions can be chosen freely, then for $\lambda < 1/2$, policy $(0, 1, 1, 1, \ldots)$ is majority-efficient as it places the candidate's score as close to the median voter's position as possible. For $\lambda \geq 1/2$ any policy with a score of 0.5—and there are in fact infinitely many—is majority-efficient.

A setting where a majority-efficient policy always exists is where each candidate can choose policy on one issue only (say, issue 1), while all other issues are fixed. Then choice 0 is majority-efficient if a majority of citizens prefers $(0, a_{-1})$ to $(1, a_{-1})$. Similarly, 1 is majority-efficient if a majority prefers $(1, a_{-1})$ to $(0, a_{-1})$. Note that this is true for arbitrary preferences that voters might have.

Next, we introduce our normative concept of competition-efficiency. Suppose the two candidates propose policies a^0 and a^1. Then the winning candidate's policy will be implemented, but who wins depends on state ω. If there is a tie, each candidate's policy is chosen with probability 0.5. The

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function $P(a^0, a^1, \omega)$ determines the probability that a^0 is implemented. Formally,

$$P(a, a', \omega) = \begin{cases} 1 & \text{if } a \succ_\omega a'; \\ 0 & \text{if } a' \succ_\omega a; \\ 0.5 & \text{otherwise.} \end{cases} \tag{3}$$

Using function P, we can map each pair of policy platforms proposed by the candidates into the policy implemented in equilibrium, and hence into utility allocations to voters. We now ask, whether a social planner could improve upon the status quo, by suggesting an alternative pair of policies. If such an improvement is not possible, then we say that the equilibrium is competition-efficient.

Let $Q(a^0, a^1, \hat{a}^0, \hat{a}^1, \omega)$ denote the percentage of citizens who prefer the policy that results if candidates choose platforms a^j, $j = 0, 1$ rather than $\hat{a}^j$, $j = 0, 1$. Formally,

$$\begin{aligned} Q(a^0, a^1, \hat{a}^0, \hat{a}^1, \omega) &= P(a^0, a^1, \omega)P(\hat{a}^0, \hat{a}^1, \omega)\mu_\omega \left(\{\theta | u_\theta(a^0, \omega) \geq u_\theta(\hat{a}^0, \omega)\} \right) \\ &+ (1 - P(a^0, a^1, \omega))P(\hat{a}^0, \hat{a}^1, \omega)\mu_\omega \left(\{\theta | u_\theta(a^1, \omega) \geq u_\theta(\hat{a}^0, \omega)\} \right) \\ &+ P(a^0, a^1, \omega)(1 - P(\hat{a}^0, \hat{a}^1, \omega))\mu_\omega \left(\{\theta | u_\theta(a^0, \omega) \geq u_\theta(\hat{a}^1, \omega)\} \right) \\ &+ (1 - P(a^0, a^1, \omega))(1 - P(\hat{a}^0, \hat{a}^1, \omega))\mu_\omega \left(\{\theta | u_\theta(a^1, \omega) \geq u_\theta(\hat{a}^1, \omega)\} \right). \end{aligned}$$

Definition 3

1. *Policy choices (a^0, a^1) by the two candidates are **competition-efficient in state ω** if and only if a majority of citizens prefer them to all other policy choices in state ω, i.e., $Q(a^0, a^1, \hat{a}^0, \hat{a}^1, \omega) \geq Q(\hat{a}^0, \hat{a}^1, a^0, a^1, \omega)$ for all $\hat{a}^j \in A^j$, $j = 0, 1$.*

2. *Policy choices (a^0, a^1) are **ex-ante competition-efficient** if and only if a majority of citizens prefer them ex-ante to all other policy choices in a majority of states, i.e., $\mu(\{\omega | Q(a^0, a^1, \hat{a}^0, \hat{a}^1, \omega) \geq Q(\hat{a}^0, \hat{a}^1, a^0, a^1, \omega)\}) \geq \mu(\{\omega | Q(\hat{a}^0, \hat{a}^1, a^0, a^1, \omega) \geq Q(a^0, a^1, \hat{a}^0, \hat{a}^1, \omega)\})$ for all $\hat{a}^j \in A^j$, $j = 0, 1$.*

To understand the relationship between our definition and Pareto efficiency, first suppose that ω is fixed. Then if (a^0, a^1) is not competition-efficient, we can find a policy $\hat{a}$ that is feasible for one of the candidates, and makes a majority of agents better off. If we replaced the statement "a majority of agents" by "all agents" we get the standard definition of Pareto efficiency. If there is uncertainty about ω, then the winning policy may depend on ω. As a consequence, we need to allow the planner to also propose a pair of policies $(\hat{a}^0, \hat{a}^1)$, such that depending on ω citizens can either choose $\hat{a}^0$ or $\hat{a}^1$. Thus, if we required an improvement to be acceptable to all agents in all states ω rather than only to a majority of agents in a majority of states, we would get again standard Pareto efficiency. As a consequence, competition-efficiency is stronger requirement than Pareto efficiency — competition-efficient policy choices are always Pareto efficient but the reverse implication is in general not true.

3.2   Discussion

We are not the first to attempt a normative analysis of political equilibria, but our normative concepts of majority-efficiency and competition-efficiency are novel, and warrant a more detailed discussion. Previous papers analyzing efficiency in political setups have applied one of two other approaches: (i) Pareto optimality and (ii) utilitarianism.

Pareto optimality is the standard normative concept in economics, but it has little bite in political settings, because, very often, (almost) all policies are Pareto optimal. For example, in the scoring model, all policies located between the two most extreme voters' bliss points are Pareto optima. Similarly, in the weighted issue model, all policy vectors are Pareto optima, unless voters' preferences are extremely correlated. However, we usually would not think of policies that are supported only by a small minority as efficient. Majority efficiency also captures the spirit of democracy better, which is, after all, not about making no one worse off; democracy is about the majority of people deciding which policy they want.

The major alternative concept to Pareto efficiency used in the literature is utilitarianism. A policy is considered efficient if it maximizes the sum of the utilities of all voters. Typically, there is only one such policy. The concept thus generates a sharp benchmark against which we can measure the outcome of the political system. However, in order for the concept to make sense, the cardinal value of individual utilities must be meaningful, and we must be willing to trade-off the utility of different voters in a particular way.

Proponents of utilitarianism have argued that the right policy was that which would cause "the greatest happiness of the greatest number" of people, a quote usually ascribed to Jeremy Bentham. The obvious problem in formalising this theory is that choosing a policy that maximizes utility for all people is usually not possible. Utilitarianism has been interpreted as equivalent to maximizing the sum of voters' utilities, but it is not immediate that this is the only possible interpretation of achieving "the greatest happiness of the greatest number" of people. In some sense, majority-efficiency is quite close to the original quote in that it requires that a policy achieve a "greater happiness for the greater number of people" than any other policy.

Rather than looking at the behavior of one candidate (like majority-efficiency), competition-efficiency deals with the two platforms proposed by the candidates. It asks whether a social planner could choose the two platforms of the candidates in a way that makes a majority of the electorate better off than the platforms chosen in equilibrium by the candidates. If the distribution of voter preference types is known, competition-efficiency is very much related to majority-efficiency : We will show that, if both candidates choose their majority-efficient position, then the pair of platforms is competition-efficient. However, if there is uncertainty about the preference distribution, then the two concepts are not necessarily related. The reason is that the social planner has *two* instruments (the two platforms) for an optimal response to the uncertain preference distribution, and may find it optimal to choose platforms that are not ex-ante majority-efficient so as to diversify the choices that are available for the realized electorate. The concept of competition-efficiency therefore allows us, for example, to analyze whether equilibrium candidate convergence is excessive.

Finally, note that we can see majority-efficiency and Pareto efficiency as the two extreme end-points of a general concept that we could call α-efficiency, defined as follows: A policy a is α-efficient if the percentage of people who prefer a' to a (among all people with a strict preference) is not more than $1 - \alpha$, for all other policies a'. Clearly, as long as the α defined this way is positive for a policy (or $\geq 1/N$, in a finite electorate with N voters), then the policy is Pareto efficient. Majority efficiency corresponds to $\alpha = 1/2$. The higher α, the more stringent is the requirement that no other policy is preferred by a fraction $1 - \alpha$ of voters, so it is clear that the set of majority-efficient policies is always a subset of the set of Pareto optimal policies. More generally, the higher α, the fewer policies are α-efficient, and the more likely it is that no α-efficient policy exists. In contrast, for α sufficiently low, an α-efficient policy exists in almost all applications.[13] For example, note that the result of Caplin and Nalebuff (1988) can be interpreted as follows: In a multidimensional Euclidean voting model, under certain assumptions on the distribution of preferences (and with no fixed policies), $\frac{1}{e}$-efficient policies exist (where $e = 2.71\ldots$ is Euler's number).

4 The Model without Uncertainty

4.1 The Role of Fixed Positions

In our multi-issue model it is natural to assume that candidates' positions on certain issues are fixed. As explained earlier, the fixed position can include characteristics such as the candidate's previous experience, personal character traits, or party affiliation. In this section we will show that if fixed positions matter, candidates may choose non-majority-efficient policies on pledgeable issues. Interestingly, it turns out that candidates will always choose majority-efficient policies in the scoring model (if there is no uncertainty about ω), i.e., there is policy convergence in the sense that candidates choose policies with scores as close as possible to the median voter's preferred score. Thus, the results in this section that policy converges is a robust feature of the one-dimensional spatial voting model, but it is not robust in our generalized spatial voting model.

Our first results shows that there if candidates do not differ with respect to fixed positions, then both choose majority-efficient policies, and the equilibrium is competition-efficient.

Theorem 1 *Suppose that there is no uncertainty about ω and that $A^0 = A^1$.*

1. *Then (a^0, a^1) is a pure strategy equilibrium if and only if both a^0 and a^1 are ex-ante majority-efficient.*

2. *Suppose a majority-efficient policy exists. If (σ^0, σ^1) is a mixed strategy equilibrium, then almost every policy in the support of σ^0 and σ^1 is majority-efficient.*

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[13]A sufficient condition for the existence of a Pareto optimum is that there is at least one voter who has a bliss policy (a policy that he strictly prefers to all other policies) – that policy is a Pareto optimum. In contrast, suppose that all individuals prefer policy $(0, 0, \ldots)$ to policy $(1, 1, \ldots)$, and for all other policies, all voters prefer a policy a to a policy a', if policy a starts with a longer string of '1's before having the first 0. In this economy, no α-efficient policy exists for any $\alpha \geq 1/N$.

3. Suppose a majority-efficient policy exists. Then all equilibria are competition-efficient.

The following example indicates that the assumption that $A^1 = A^2$ is crucial. First, if one of the candidate's types is undesirable for sufficiently many citizens, then this candidate will never win. As a consequence, that candidate can choose an arbitrary position in equilibrium, because his choice is irrelevant. However, a counterexample exists even if both agents have a positive probability of winning.

Example 1 There are two issues and four different preference types. The following table gives the utility of each type from each policy, as well as the proportion of individuals of each type. (There is only one state of the world, so that this proportion does not depend on any ω.)

	(0,0)	(0,1)	(1,0)	(1,1)	Proportion of types
θ_1	6	8	5	2	40%
θ_2	2	5	8	6	40%
θ_3	8	0	0	0	10%
θ_4	0	0	0	8	10%

Table 1: Preference types and utilities

Candidate L is committed to 0 on the first issue (e.g., he is a Democrat), and candidate R is committed to 1 on the first issue (he is a Republican). Both are free to choose a positions on the second issue.

Note that these preferences have a spatial interpretation. Policy $(0,0)$ is the "left-most" policy and the most preferred choice by type θ_3. $(0,1)$, is the "moderate-left" policy. Note, however, that even "moderate Republicans" (type θ_2) prefer the "right-most" policy $(1,1)$ to this "moderate-left" policy (maybe, the first issue that candidates cannot change is the more important issue in the view of these voters).

Types θ_3 and θ_4 could be called left- and right-wing ideologues, respectively. These groups prefer a candidate from their party only if that candidate provides their preferred policy on both issues, otherwise they are indifferent and abstain from voting.

Types θ_1 and θ_2, i.e., 80% of the population, prefer policy $(0,1)$ to $(0,0)$. Because, candidate L's position on issue 1 is fixed, $(0,1)$ is his only majority-efficient policy. Similarly, $(1,0)$ is preferred by 60% of the population to $(0,0)$. However, the majority-efficient policies $a^L = (0,1)$ and $a^R = (1,0)$ are not an equilibrium.

In order to see this, note that only type θ_1 would vote for Candidate L when policies $a^L = (0,1)$ and $a^R = (1,0)$ are offered. If, instead, Candidate L switches to policy $(0,0)$ then type θ_3 will vote for him. Type θ_1 is made worse off by the policy change, however, these citizens still prefer $(0,0)$ to Candidate R's policy vector. Thus, 50% of the population votes for L, 40% for R, with θ_4 abstaining, and hence L will be elected with probability 1. Thus, Candidate L's deviation is optimal, implying that the majority-efficient policies are not an equilibrium.

In fact, it is easy to see that the only equilibrium is $a^L = (0,0)$ and $a^R = (1,1)$. It is optimal for candidates to choose more extreme policies in order to appeal to their ideological base (types θ_3 and θ_4, respectively) and ensure that those types turn out to vote. Types θ_1, and θ_2, would prefer majority-efficient policies, but still have a sufficient preference for issue 1 (the candidate's party) that they are not willing to switch to the other candidate when the more extreme policy is adopted. ∎

The result of example 1 that it may not be optimal for candidates to use majority-efficient policies strongly relies on the multi-dimensional structure of the policy space and the fact that different citizens care differently about individual policies. For example, suppose that issue 1 is the candidate's party affiliation (Republican versus Democrat) and issue 2 is the position on abortion. Type θ_3 could then be considered to be the Republican base (there strongest ideologues) and θ_4 the Democratic base. A type θ_3 would consider a Republican who chooses the majority-efficient policy $(0,1)$ to be a "RINO" (Republican in name only) and not turn out in the election. Appealing to the base by adopting more conservative policies ensures that the base supports the candidate. Moderate citizens dislike this pandering to the base, but they do not dislike it sufficiently to vote for a candidate from the other party.

In contrast, in a standard one-dimensional spatial voting model with single peaked preferences, it is never optimal to use policies that appeal to the base but that moderates dislike (we will show that formally in Theorem 2). The reason is that if one citizen of type θ prefers the conservative candidate, then so will all citizens $\theta' \leq \theta$. Promoting policies that appeal to the conservative base are therefore not beneficial, because they do not increase turnout by the base. Instead, the main objective is to use a policy that most appeals to the median voter.

In a comment written two days before the 2004 elections, Suellentrop (2004) writes: "The secret of Bill Clinton's campaigns and of George W. Bush's election in 2000 was the much-maligned politics of small differences: Find the smallest possible majority (well, of electoral votes, for both men) that gets you to the White House. In political science, something called the median voter theorem dictates that in a two-party system, both parties will rush to the center looking for that lone voter – the median voter – who has 50.1 percent of the public to the right (or left) of him. Win that person's vote, and you've won the election." In contrast, Suellentrop's anticipated that Bush's political strategist Karl Rove made a fatal mistake in the 2004 election, "Bush's campaign — and his presidency — have appealed almost entirely to the base of the Republican Party. . . . Rove has tried to use the Bush campaign to disprove the politics of the median voter. It was as big a gamble as any of the big bets President Bush has placed over the past four years." The results certainly indicated that Rove's policy of abandoning majority-efficient policies were successful. As in example 1 the strategy seems to have increased turnout by the base without alienating more moderate Republicans to the point where they would vote for a Democrat.

4.2 Existence of Majority Efficient Policies

We now provide conditions under which majority-efficient policies exist both in the scoring and the weighted-issue model. We first review the standard definition of single peakedness of preferences.

Definition 4 *Let $\theta \in \mathbb{R}$ and let $v_\theta \colon \mathbb{R} \to \mathbb{R}$. For every θ let $x(\theta)$ solve $\max_x v_\theta(x)$.*
*Then the collection of functions v_θ, $\theta \in \mathbb{R}$ is **single peaked** if and only if the following holds:*

- *$x(\theta)$ is monotone.*

- *$v_\theta(x') < v_\theta(x'')$ for all $x' < x'' < x(\theta)$, and $v_\theta(x') > v_\theta(x'')$ for all $x(\theta) > x' > x''$.*

The following results shows that if preferences are single peaked, then both candidates will adopt majority-efficient policies in the scoring model — and therefore in the Downsian model — even if candidates' choice of policies is restricted due to fixed issues.

Theorem 2 *Suppose that citizens utility is of the form $u_\theta(a) = v(\theta, f(a))$, where f is real valued and continuous. Let $v(\theta, x)$ be single peaked and let θ_m be the median voter. Then*

1. *a^j is majority-efficient if and only if a^j solves $\max_{a^j \in A^j} u_{\theta_m}(a^j)$.*

2. *There exists an equilibrium in which all candidates choose majority-efficient policies.*

3. *If there exists an equilibrium in which one candidate i chooses a non-majority-efficient position, then one of the candidate wins with probability 1. The payoffs to candidates are equivalent to an equilibrium where both candidates choose majority-efficient policies.*

The theorem characterizes a majority-efficient position in the scoring model to be the position that is most preferred by the median voter. Hence, if both candidates have a chance to win, all policy choices are targeted to attract the median voter, and the median voter is indifferent between the candidates. The restriction to a fixed issue, not present in the Downsian model, generates the possibility that one of the candidate is sufficiently inferior that he cannot win even if the other candidate does not choose a majority-efficient policy. Also, even in this setting, an equilibrium in which both candidates choose majority-efficient policies still exists. Unlike in Example 1, it is not optimal to choose policies that are attractive to the "base" because in the one-dimensional setting of citizen types, θ, the base always supports the candidate and any policy that appeals more to the base loses voters in the center and is therefore counterproductive.

We now show existence of majority-efficient policies in the weighted issue model, if the preferences over each issue are stochastically independent.

Theorem 3 *Suppose that $(\lambda, \theta) \in \Theta = T \times \{0, 1\}^{\mathbb{N}}$, and that $u_\theta(a) = \sum_{i=1}^{\infty} \lambda_{i,t} |\theta_i - a_i|$. Let μ be independent across θ_i and T. Then, for each candidate j, there exists a policy $a^j \in A^j$ that is majority-efficient in state ω.*
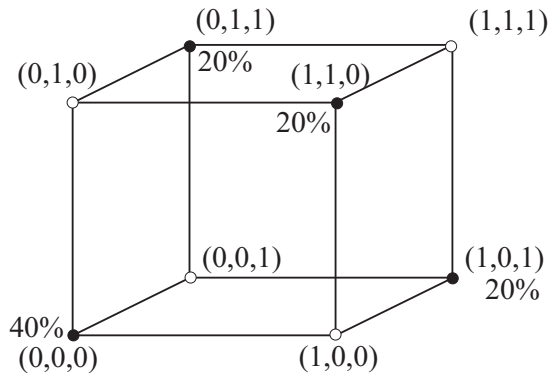
Figure 1: Non-Existence of a Moderate Policy

Theorem 3 uses independence, but it is straightforward to show that the result generalizes if there is some (sufficiently small) correlation in the distribution of types. However, if there is too much correlation, then no majority-efficient position may exist as the following example indicates.

Example 2 Suppose $u_\theta(a) = -\sum_{i=1}^{3}(a_i - \theta_i)^2$, i.e., there are three issues that enter utility with the same weight. The are no fixed issues for the candidate. The distribution of voter types is given in Figure 1. Note that a citizen's disutility is given by the distance, measured along the edges of the cube, between the voter's location and the proposed policy.

It is easy to see that no majority-efficient position exists. For example, $(0,0,0)$ is not majority-efficient, because $(1,1,1)$ is majority preferred to $(0,0,0)$: For citizens located at $(0,1,1)$, $(1,0,1)$ and $(1,1,0)$, who constitute 60% of the population, $(1,1,1)$ is closer than $(0,0,0)$. However, $(1,1,1)$ is also not majority-efficient. In particular, policy $(1,0,1)$ is preferred by the citizens located at $(0,0,0)$ and those located at $(1,0,1)$, again a 60% majority. Next, policy $(1,0,0)$ dominates policy $(1,0,1)$ because it is closer to the ideal point of the citizens at $(0,0,0)$ and those at $(1,1,0)$. However policy $(0,0,0)$ is now preferred by citizens at $(0,0,0)$ and at $(0,1,1)$, but we have already shown that policy $(0,0,0)$ is itself dominated. By symmetry it follows that all remaining policies can also be dominated. ∎

It is interesting to note that if there only two issues that can be chosen and if all citizens have the same weights (λ) in their respective utility functions, then majority-efficient policies always exist. Thus, the above example presents the simplest situation in which non-existence of majority-efficient policies occurs.

We now show that a majority-efficient policy need not be an optimal choice for a candidate, even with independently distributed preferences of the type considered in Theorem 3, and with every citizens putting the same weight on each issue.

Example 3 Suppose that there are three utility relevant issues. Let $\Theta = (\theta_1, \theta_2, \theta_3)$, with $\mu(\theta_1 = 1) = 0.4$, $\mu(\theta_2 = 1) = 0.2$ and $\mu(\theta_3 = 1) = 0.4$. The realizations of the θ_i are independent. The policies on issues 1 and 2 are fixed, and only the policy on issue 3 can be chosen freely.

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Candidate 0 and 1's position on the fixed issues are $(1,0)$ and $(0,1)$, respectively. The citizens' utility functions are $-|a_1 - \theta_1| - 0.8|a_2 - \theta_2| - 0.5|a_3 - \theta_3|$.

First, note that $a_3^* = 0$ is the unique majority-efficient policy, because 60% of citizens prefer $a_3 = 0$ to $a_3 = 1$. However, we now show that there does not exist an equilibrium in which at least one of the candidates always chooses the majority-efficient policy. Table 2 shows the net-payoffs of citizens from choosing Candidate 0 over Candidate 1. Clearly, a citizen will vote for the Candidate 0 if and only if this net-payoff is positive.

Citizen's type	Percent of citizens	Net Benefit from candidate 0, when $a_3^0 = a_3^1$.	Net Benefit from candidate 0, when $a_3^0 = 0, a_3^1 = 1$.	Net Benefit from candidate 0, when $a_3^0 = 1, a_3^1 = 0$.
$(0,0,0)$	28.8	0.2	0.7	-0.3
$(0,0,1)$	19.2	0.2	-0.3	0.7
$(0,1,0)$	7.2	1.8	2.3	1.3
$(0,1,1)$	4.8	1.8	1.3	2.3
$(1,0,0)$	19.2	-1.8	-1.3	-2.3
$(1,0,1)$	12.8	-1.8	-2.3	-1.3
$(1,1,0)$	4.8	-0.2	0.3	-0.7
$(1,1,1)$	3.2	-0.2	-0.7	0.3
Candidate 0's vote share	60	45.6	34.4	

Table 2: Citizen Preferences and Vote Shares

Candidate 0's vote share in the bottom line of the table indicates that Candidate 0 wins whenever both candidates choose the same policy, while Candidate 1 wins whenever he can distinguish himself by choosing a different policy. Thus, we have a "matching pennies" game in which each candidate randomizes with equal probability over the two policies. Therefore, always choosing the majority-efficient policy $a_3^j = 0$ is not an equilibrium strategy for both the candidates, and the majority-efficient policy will only be implemented 50% of the time. ∎

The intuition for Example 3 is as follows. Suppose first that both candidates choose the majority-efficient policy. Citizens with the same preferences over the first two issues (i.e., either $(0,0)$ or $(1,1)$) on the first two issues are the potential swing voters, because their utility difference between the candidates is the smallest among all voters. Because Candidate 1 is slightly inferior to Candidate 0, there are more swing voters who favor Candidate 0, if both candidates adopt the same policy on the third issue. More precisely, 48% of the population's preferred position is "0" on the first two issue, and these voters have a utility benefit of 0.2 of voting for Candidate 0, if both candidates choose the same policy on the third issue. Only 8% of the population, prefer "1" on the first two issues, and these agents are the potential swing voters that favor Candidate 1, if both candidates choose the same policy on the third issue. If Candidate 1 chooses a policy on issue 3 that differs from his opponent's policy, then these potential swing voters can be moved. If,

for example, 40% of the swing voters preferred Candidate 1's position to Candidate 0's position on issue 3 (because Candidate 1 chooses policy 1 on the third issue, while Candidate 0 continues to choose policy 0), then 40% of the swing voters in Candidate 0's camp will switch, and Candidate 1 receives the votes of an additional 19.2% of the population. Of course, at the same time Candidate 1 loses 60% of his own swing voters, which, however, constitute only 4.8% of the population. Thus, the net-gain is still 14.4% which is sufficient to win the election.

If we call citizens with stronger preference intensities (1.8 or −1.8 in the example) as the candidate's base, then Candidate 1 has one key advantage over Candidate 0: He has a much stronger support among his base. As a consequence, he can get enough swing voters to win elections even if he chooses an unpopular position on issue 3, as long as Candidate 0 sticks to the popular issue. Unlike Example 1, this unpopular position is not geared toward energizing the base, but rather toward peeling away enough swing voters from the other candidate. For example, President Bush's decision to subsidize the U.S. steel industry, may not have been popular with the majority of citizens, but it may certainly have been effective in convincing some Democrats to vote Republican. While the tariff was unpopular with the pro-business part of his party's base, it was not important enough for the Republican base to change voting behavior.

It should be quite clear that it is not necessary for swing voters to split 50-50 in favor of Candidate 1's new position on issue 3 in order to get a sufficient gain of votes to win the election. All that is needed with respect to the preference distribution on the third issue above is that at least 20% prefer policy 1 on issue 3. We should also emphasize that this result is *not* due to preference intensity: Voters who prefer policy 1 on issue 3 have exactly the same weight on this issue in their utility function as voters who prefer policy 0 on issue 3. The example therefore shows that catering to a (clear) minority of voters may even be attractive for a candidate if that minority does not feel more strongly about its position than the majority. Finally, note that a similar result cannot be obtained in the Downsian model, because of its one-dimensional structure. Also, the result differs from the result in the probabilistic voting model that minorities may get the special interest policy they want, provided that they feel sufficiently more strongly about an issue than the majority.

If we abandon the independence assumption, it is easy to provide robust example with a pure strategy equilibrium in which the winning candidate adopts a policy that is only favored by a minority.

Example 4 Suppose that there are two issues, where the first issue (e.g., the party) is fixed by both candidates. There are four types (θ_1, θ_2), $\theta_i \in \{0, 1\}$. Types $(0, 0)$ and $(1, 0)$ have utility functions $u_\theta(a) = -0.5|\theta_1 - a_1| - 0.2|\theta_2 - a_2|$. Types $(0, 1)$ and $(1, 1)$ have utility functions $u_\theta(a) = -0.5|\theta_1 - a_1| - |\theta_2 - a_2|$. Suppose that θ_1 and θ_2 are stochastically independent. In particular, suppose that $\theta_2 = 1$ for 20% and $\theta_1 = 1$ for $p \in (37.5\%, 62.5\%)$.

Since a majority of citizens prefers $a_2 = 0$, it follows immediately that $a_2 = 0$ is majority efficient. However, $(0, 0)$ and $(1, 0)$ are not equilibrium policies. First, suppose that $p \le 0.5$, i.e., in equilibrium Candidate 1 wins at most with probability 0.5. If Candidate 1 deviates and offers $(1, 1)$, then types $(0, 1)$ will now vote for Candidate 1. Thus, Candidate 1 receives a vote proportion

of $p + (1-p)0.2 > 0.5$, so that he wins with probability 1, a contradiction. Similarly, if $p \geq 0.5$ it follows that Candidate 0 wins if he chooses $a_2 = 1$. ∎

Example 4 formalizes the insight why politicians may support minority views in their platforms. For example, in the U.S. some form of gun control is favored by a majority of citizens. However, politicians have learned that supporting gun control is a losing strategy in elections. If sufficiently many gun control opponents are easily willing to switch party lines to vote for a candidate who supports their view, and the remaining voters do not feel too strongly about this issue to cross party lines, then it is optimal for politicians to support the minority view that opposes gun control. Note, that opposing gun control remains optimal even if a majority of the electorate favors the candidate's party.

Another case in which a minority opinion is successfully adopted by a candidate, is the case of immigration in the 2006 US elections. As stated in the New York Times of November 10, 2006, incumbent representative Hayworth, whose platform was strongly in favor of sealing the borders and ejecting illegal immigrants lost against Democratic challenger Gifford, who had a somewhat more pro immigrant platform. Similarly, Republican Randy Graf, who is a Minuteman border vigilante, lost (in a previously Republican district) against a Democrat who favored citizenship to illegal immigrants in some cases. However, results on Arizona ballot initiatives indicates that a majority of the electorate favored measures against illegal immigrants. For example, Propositions 102 and 300, which banned illegal immigrants from receiving punitive damages in civil lawsuits and from certain public services, passed in Arizona with a large majority of more than 70%.

5   Valence

An important model in the literature that introduces some multidimensionality into the standard Downsian model is a one-dimensional policy model in which voters care also about a candidate's "valence". Valence is interpreted as a characteristic of the candidate that all voters appreciate (like competence or honesty). While most models in the literature simply assume that valence enters as a linearly additive shock into all voters' utility functions, the following definition is somewhat more general.

Definition 5 *The variable $\nu \in \mathbb{R}$ is a net-valence shock for Candidate 1 if and only if*

$$u_\theta(a, \nu) - u_\theta(a', \nu) > u_\theta(a, \nu') - u_\theta(a', \nu') \tag{4}$$

for all $\nu > \nu'$, for all policies $a \in A^1$, $a' \in A^0$, and for all citizens θ.

Intuitively, if the valence shock increases from ν' to ν, the set of types θ who prefer Candidate 1 over Candidate 0 cannot shrink.

Under which conditions will candidates in a valence model choose majority-efficient policies? It is clear that this will not generally be the case, even in models without uncertainty about the type distribution. For example, introducing a small linearly-additive valence shock in Example 1 will not

change the equilibrium platforms chosen by the candidates. It turns out that a sufficient condition for candidates to choose a majority-efficient policy is that society is "polarized" over policy.

Let $\Theta_1(a', a, \nu) := \{\theta | u_\theta(a, \nu) \geq u_\theta(a', \nu)\}$ be the set of voter types who find Candidate 1 to be at least as good as Candidate 0, given that Candidate 0 chooses a', Candidate 1 chooses a and the state is ν. Let $\Theta_0(a', a, \nu)$ be defined analogously. Let m^0 be a majority-efficient policy for Candidate 0, and let m^1 be a majority-efficient policy for Candidate 1. Let $P^1(p, m^1, \nu) := \{\theta | u_\theta(p, \nu) \geq u_\theta(m^1, \nu)\}$ be the set of voters who prefer that candidate 1 implements policy p rather than policy m. Let $P^0(p, m^0, \nu)$ be defined analogously.

Definition 6 *A society of voters is polarized if there exists an ordering of voter types such that the following two conditions hold:*

1. $\Theta_0(a^0, a^1, \nu)$ *is either an upper or lower interval (i.e., of the form $\{\theta_1, ..., \theta_c\}$ or $\{\theta_c, ..., \theta_N\}$[14]) for all policies $a^j \in A^j$*

2. $P^j(a, m^j, \nu)$, $j = 0, 1$ *is either an upper or a lower interval for all policies $a^j \neq m^j$ with $a^j \in A^j$.*

Theorem 4 *If society is polarized and ν is a valence shock, then there is an equilibrium in which both candidates choose their majority-efficient policies m^0 and m^1, respectively.*

We now show that the valence shocks may affect the equilibrium policy if the conditions of Definition 6 are not satisfied.

Example 5   Consider again example 1, but now we add a net-valence shock. In particular, as indicated in table 3 the valence shock $v \in \{-1, 0, 1\}$ and enters additively (hence, Definition 5 is obviously satisfied).

			$v = -1$		$v = 0$		$v = 1$		
	(0,0)	(0,1)	(1,0)	(1,1)	(1,0)	(1,1)	(1,0)	(1,1)	Proportion of types
θ_1	6	8	4	1	5	2	6	3	40%
θ_2	2	5	7	5	8	6	9	7	40%
θ_3	8	0	−1	−1	0	0	1	1	10%
θ_4	0	0	−1	7	0	8	1	9	10%

Table 3: Preference types and utilities

To see that Definition 6 is violated, suppose that both candidates choose majority-efficient policies. Then for the low valence shock $v = -1$, Candidate 1 is only supported by θ_2, whereas for $v = 1$, Candidate 0 is only supported by type θ_1. This would imply that θ_1 and θ_2 are the extremists in the ordering of types. Now consider the second part of Definition 6. Only type θ_3

---

[14]The cutoff θ_c can depend on the policies a and a' and on the realization of the shock ν.

prefers a movement from the majority-efficient policy $(0, 1)$ to the extreme policy $(0, 0)$. Thus, θ_3 would also have to be an extremists, which shows that there is no ordering of types that satisfies Definition 6.

Let p_l, p_m, and p_h be the probabilities for the the low valence shock $v = -1$, the intermediate $v = 0$ and the high valence shock $v = 1$, respectively. Each candidate has two choices: the extremist policy, denoted by E, which is $(0, 0)$ for Candidate 0 and $(1, 1)$ for Candidate 1; and the majority-efficient policy, denoted by M. The payoff matrix for the two candidates is then given by

	E	M
E	0.5, 0.5	$p_l + p_m$, p_h
M	p_l, $p_m + p_h$	$p_l + 0.5p_m$, $0.5p_m + p_h$

Table 4: The payoff matrix

Thus, both candidates choosing the extreme policy is an equilibrium if and only if $p_l, p_h \le 0.5$. Note that M, M is never an equilibrium. If, $p_h > 0.5$ then the only equilibrium is E, M. Similarly, if $p_l > 0.5$ then the unique equilibrium is M, E. ∎

The intuition for Example 5 is that type θ_3 and θ_4, the parties base will reliably vote for their party's candidate only if the extreme policy is adopted. If a candidate chooses a majority-efficient position, then the base will only for the candidate if the opponent has lower valence. In contrast, citizens θ_1 and θ_2, the moderates in the election will switch, to the other candidate, if that candidate takes a majority-efficient position and has superior valence. Thus, if a candidate is expected to have a higher valence than his opponent, then it is attractive to be majority-efficient. In contrast, if candidate's valence is low, the candidate must ensure himself of the support of the party's base, and therefore adopt the extreme position.

6 Type Uncertainty

We now analyze the impact of uncertainty about the distribution of voters on the efficiency of electoral outcomes. The main result of this section indicates that majority inefficient positions may only be adopted in the scoring model if candidates are restricted from selecting the median voter's most preferred policy. If it is feasible, both candidates choose the median voter's most preferred policy, and we get policy convergence. However, Theorem 6 below shows that there is in fact too much convergence. More formally, the theorem proves that electoral outcomes are not competition-efficient if there is uncertainty over the position of the median voter.

In the following a voter's type is given by $\theta \in \mathbb{R}$. The distribution of θ depends on a state τ. As explained in section 2 the marginal distribution of τ is given by μ_τ. We assume there is no uncertainty over ν, the parameter that directly enters citizens' preferences.

The key determinant of equilibrium policies is the position of the "ex-ante median voter." In particular, θ_m is the ex-ante median if the realized median voter $\theta_m(\tau) \ge \theta_m$ with probability 0.5

and $\theta_m(\tau) \leq \theta_m$ with probability 0.5. We assume that there is sufficient uncertainty about μ_τ such that there is a unique ex-ante median θ_m.

Theorem 5 *Suppose that citizens utility is of the form $u_\theta(a) = v(\theta, f(a))$, where f is real valued and continuous. Let $v(\theta, x)$ be single peaked, and let θ_m be the unique ex-ante median voter. Suppose that the distribution over τ is continuous. Then*

1. *Suppose that $\theta_m \in f(A^j)$ for $j = 0, 1$. Then (a^0, a^1) is an equilibrium if and only if a^0 and a^1 are ex-ante majority-efficient.*

2. *Suppose that $f(A^j) \leq \theta_m \leq f(A^{-j})$ for $j = 0, 1$. Then there exists an equilibrium in which both candidates's policies are ex-ante majority efficient.*

3. *Let $\theta_m \in f(A^j)$. Suppose there exists an $x \in f(A^j)$ such that either $x_m < x < \operatorname{conv} f(A^{-j})$ or $\theta_m > x > \operatorname{conv} f(A^{-j})$. Further, suppose that Candidate $-j$ can implement the most preferred policy for a set of realized median voters that has positive probability. Then Candidate j's equilibrium policy is not ex-ante majority-efficient.*
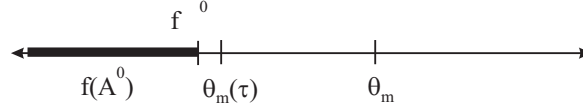

Figure 2: Majority Efficient Equilibrium

Theorem 5 shows that ex-ante majority inefficient outcomes occur in the scoring model only if exactly one of the candidates is unable to choose the ex-ante median voter's most preferred policy. The intuition can easily be seen in figure 2. Because of fixed positions Candidate 0 can only offer policies that are the most preferred policies for citizens in $f(A^0)$. If Candidate 1 offered the policy that most appeals to θ_m, then in state τ median voter $\theta_m(\tau)$ will vote for Candidate 0, because $f(a^0)$ is closer to his ideal point. If, instead, Candidate 1 offers the most preferred policy of voter $\theta_m(\tau)$, then $\theta_m(\tau)$ will vote for Candidate 1. Thus, Candidate 1 can increase his winning probability by offering a majority-inefficient policy.

We now show that if there is policy convergence, then in general there is too much convergence from an efficiency perspective.

Theorem 6 *Suppose that citizens utility is of the form $u_\theta(a) = v(\theta, f(a))$, where f is real valued and continuous. Let $v(\theta, x)$ be single peaked, and let $x_m = \max_x v(\theta_m, x)$, where θ_m is the unique ex-ante median voter and suppose that $x_m \in f(A^j)$, $j = 0, 1$,. Then the equilibrium is competition-efficient if and only if $\theta_m = \theta_m(\tau)$ a.e., (i.e., there is no uncertainty about the median voter).*

The intuition for the result is that if one agent offered a policy that is most preferred by some voter θ' with, for example, $\theta' < \theta_m$ then efficiency of the electoral outcome is always strictly

improved when the realized median voter $\theta_m(\tau) \leq \theta'$. In other words, if the position of the median voter is not known, then efficiency can be increased if candidates offer different platforms. However, for strategic reasons, offering differing platforms is not in the interest of the candidates in a scoring model.

We end this section, by considering a situation with both uncertainty over the median voter and preference (valence) shocks. Valence shocks may prevent policy convergence, because a weaker candidate may only be able to win if he can differentiate himself from his opponent. For simplicity, we assume that only Candidate 1 can make a policy choice. We get policy convergence, whenever the expected valence of the Candidate 1 is above a cutoff value ν_0, and policy divergence otherwise.

Example 6   Consider an economy in which there is only one issue. Candidate 0 is exogenously committed to 0, while candidate 1 can choose either 0 or 1. With probability π, a majority of the population prefers policy 1, while with probability $1 - \pi$, a majority prefers policy 0.[15] The utility difference of a type $\theta \in \{0, 1\}$ citizen, between candidate 1 and candidate 0, is

$$\theta - |\theta - a| + \nu, \tag{5}$$

where $a \in \{0, 1\}$ is the policy chosen by Candidate 1 and ν is the valence shock for Candidate 1 relative to Candidate 0. We assume that ν is normally distributed with mean v_0 and standard deviation 1, and let $\Phi(\cdot)$ denote the cumulative distribution function of the standard normal $N(0, 1)$ distribution.

Clearly, policy 1 is ex-ante majority-efficient if and only if $\pi \geq 0.5$. We now determine the optimal policy of Candidate 1.

First, suppose that $\pi > 0.5$. If $a^1 = 0$, then candidate 1 wins with probability $1 - \Phi(-v_0) = \Phi(v_0)$ (independent of the realized preference distribution, because both candidates propose the same policy). If Candidate 1 chooses policy $a^1 = 1$ and a majority prefers that policy, Candidate 1 wins whenever $\nu > -1$, which occurs with probability $1 - \Phi(-v_0 - 1) = \Phi(v_0 + 1)$. If Candidate 1 chooses policy 1 and the majority prefers policy 0, Candidate 1 wins whenever $\nu > 1$, i.e., with probability $1 - \Phi(-v_0 + 1) = \Phi(v_0 - 1)$. Thus, Candidate 1 chooses policy $a^1 = 1$ if

$$\pi \Phi(v_0 + 1) + (1 - \pi)\Phi(v_0 - 1) \geq \Phi(v_0). \tag{6}$$

Because $\pi \geq 0.5$, Φ is convex for negative arguments and $\Phi(x) = 1 - \Phi(-x)$, it follows that (6) is always satisfied when $v_0 \leq 0$. However, for positive arguments, $\Phi(\cdot)$ becomes concave, so that it depends on π and v_0 whether (6) is satisfied. Note that (6) is equivalent to

$$\frac{\pi}{1 - \pi} \geq \frac{\int_{v_0 - 1}^{v_0} \phi(t)dt}{\int_{v_0}^{v_0 + 1} \phi(t)dt}. \tag{7}$$

It is straightforward to show that the term on the right-hand side of (7) goes to infinity as $v_0 \to \infty$, and converges to 0 as $v_0 \to -\infty$. Thus, if Candidate 1 has considerably lower expected valence

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[15]Note that, for our purposes, it does not matter how large the majority is that prefers either policy. Furthermore, to simplify the presentation of results, we assume that the probability that exactly 50% prefer each policy is 0.

than candidate 0, then he will always choose policy 1. Conversely, if Candidate 1 has considerably higher expected valence than Candidate 0, then he will always choose policy 0. How likely it is that a majority prefers policy 1 (i.e., π) matters, but is not the only determinant of Candidate 1's policy choice. ∎

The candidate with the lower expected valence ν can be interpreted as the weaker candidate. What drives the example is the weaker candidate's attempt to differentiate himself from his opponent. If $\pi > 0.5$, then differentiating is also beneficial policy wise, because $a^1 = 1$ is majority efficient. If $\pi < 0.5$, then differentiating is costly because it involves choosing a majority inefficient policy. However, since there is a benefit from differentiating himself from the opponent, Candidate 1 will choose (the now inefficient) policy 1 if v_0 is sufficiently low.

How would a social planner choose policy for Candidate 1? It is clear that, for π close to $1/2$ and v_0 close to 0, a social planner would choose $a^1 = 1$, in order to differentiate the candidates. In contrast, if $\pi < 1/2$ and v_0 very large, the planner expects that almost certainly Candidate 1 is the better candidate for the majority of voters (because of his higher valence). Hence, it is optimal for the planner to set $a^1 = 0$, which is more likely to be the preferred policy of the majority. Hence, while the social planner also sometimes chooses to differentiate candidates, the parameter combinations when this is socially optimal are quite different from the ones that induce the candidate to differentiate himself. A more formal analysis of this case is in progress.

7   Conclusion

In this paper, we have developed a binary model of political decisions and introduced new normative criteria to analyze political outcomes. We believe that both components, joint or separately, will be useful for future research.

The binary policy model provides a useful and intuitive framework to think about multidimensional policy choice for candidates. While the binary nature of decisions takes away the ability for candidates to take nuanced positions, this is probably a more realistic feature for real-life campaigns than the infinitely many distinct possible positions in the standard Downsian model. Besides realism, another advantage of the binary model is tractability. Often, a pure strategy equilibrium or majority efficient position exists, and even if this is not the case, the mixed strategy equilibrium is very tractable, if the number of pledgeable issues is not too large.

We assume that, if candidates can choose their position on multiple issues, then any combination of policies on the different issues is feasible. We have also assumed that the platform choice is made by the candidates themselves. Alternatively, one could think of candidates as citizen candidates who cannot commit to a policy that is different from their own ideal point; however, parties can commit to a particular policy, through the choice of the person whom they nominate as candidate. In this interpretation of the model, however, the choice set of the party may be a strict subset of the product set of $\{0, 1\}$ on all pledgeable issues. For example, there may only be two viable potential candidates, and they may have a choice between a particular man who opposes and a woman who

favors gun control. While both the position on gun control and the gender of the candidate are in principle choosable issues (from the point of view of the party), only two out of four positions are feasible options for the party.

We analyze the normative properties of equilibrium, assuming that candidates are office-motivated.[16] Clearly, our normative concepts can also be applied using different frameworks (e.g., more than two candidates, runoff rule, proportional representation) and assumptions about the motivations of candidates. We leave these topics to future research.

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[16]We chose the framework with office-motivated candidates because it is the best case scenario for candidate moderation, and so examples in which office-motivated candidates do not choose moderate policies are particularly surprising.

8 Appendix

Proof of Theorem 1. Suppose that (a^0, a^1) is an equilibrium. First, note that each candidate must win with probability 0.5. Otherwise, if one candidates, say Candidate 1, always loses, then he can improve by choosing $\tilde{a}^1 = a^0$. Policy $\tilde{a}^1$ is feasible because $A^0 = A^1$. Let $\hat{a}$ be an arbitrary feasible policy. If $\hat{a} \succ^* a^0$, then Candidate 1 could win (with probability 1) by offering policy $\hat{a}$. Thus, $a^0 \succeq^* \hat{a}$. Similarly, we can conclude that $a^1 \succeq^* \hat{a}$. Since $\hat{a}$ is arbitrary this establishes that both a^0 and a^1 are majority-efficient.

Now suppose that a^0 and a^1 are majority-efficient. We must show that (a^0, a^1) is an equilibrium. Since a^0 is majority-efficient we get $a^0 \succeq^* a^1$. Similarly, because a^1 is majority-efficient , $a^1 \succeq^* a^0$. Thus, candidates 0 and 1 each get exactly 50% of the votes, and therefore win with probability 0.5 each. Moderation implies that $a^0 \succeq^* \hat{a}$ and $a^1 \succeq^* \hat{a}$. Thus, none of the candidates can profitably deviate by offering an alternative policy, which implies that (a^0, a^1) is an equilibrium.

Now consider a mixed strategy equilibrium (σ^0, σ^1). Each candidate must win with probability 0.5. Otherwise, the candidate who wins with the lower probability would deviate by using the same strategy as his opponent, thereby increasing his winning probability to 0.5. Furthermore, in order for mixing to be optimal, almost every policy in the support of σ^j must give agent j a winning probability of 0.5. Now, assume by way of contradiction that the support of σ^j contains a set B with $\mu(B) > 0$, such that no policy in B is majority-efficient. Then policies in B only win if Candidate $-j$ also selects a non-majority-efficient policy. Because the winning probability must be 0.5, this implies that the opponent uses a non-majority-efficient strategy with strictly positive probability. Let $\tilde{a}^j$ be a majority-efficient policy. Suppose that Candidate j uses the alternative strategy $\tilde{\sigma}^j$ which uses $\tilde{a}^j$ whenever a policy in B is selected under σ^j and corresponds to σ^j, otherwise. Then $\tilde{a}^j$ wins whenever the opponent selects a non-majority-efficient policy and ties whenever the opponent uses a majority-efficient policy. Thus, Candidate j's winning probability is strictly increased, a contradiction. Thus, almost every policy in the support of σ^j is majority-efficient.

Because any equilibrium uses majority-efficient policies, there do not exist alternative policies $(\hat{a}^0, \hat{a}^1)$ that improve upon (a^0, a^1) or (σ^0, σ^1). Thus, all equilibria are competition-efficient. ∎

Proof of Theorem 2. We first prove statement 1. Suppose that a^j is ex-ante majority-efficient , but that there exists $\tilde{a}^j$ with $u_{\theta_m}(\tilde{a}^j) > u_{\theta_m}(a^j)$. Let $\tilde{x} = f(\tilde{a}^j)$ and $x = f(a^j)$ Without loss of generality we can assume that $\tilde{x} < x$. Thus, there exists $\tilde{\theta}$ with $\tilde{\theta} < \theta_m$ such that $u_\theta(\tilde{a}^j) > u_\theta(a^j)$ for all $\theta \geq \tilde{\theta}$. Because, $\tilde{\theta}$ is less than the expected median, it follows that $\mu(\{\theta | u_\theta(\tilde{a}^j) > u_\theta(\hat{a}^j)\}) > 0.5$, contradicting that a^j is ex-ante majority-efficient.

To prove the reverse implication of statement 1, let a^j solve $\max_{a^j \in A^j} u_{\theta_m}(a^j)$. Let $\tilde{a}^j$ be arbitrary. Without loss of generality we can assume that $f(\tilde{a}^j) = \tilde{x} < x_m$. Then there exists $\tilde{\theta} < \theta_m$ such that all citizens $\theta \geq \tilde{\theta}$ prefer a^j to $\tilde{a}^j$. Because, θ_m is the expected median, this implies that a majority of citizens is expected to prefer a^j to $\tilde{a}^j$. Thus, a^j is ex-ante majority-efficient.

To prove the remaining statements, let (a^0, a^1) be majority-efficient. Then according to the first part of the proof, each policy a^j, $j = 0, 1$ is the policy that the median voter most prefers in A^j. Suppose the election ends in a tie. Suppose Candidate 0 deviates to $\tilde{a}^0$ then the median voter will strictly prefer a^1. Hence Candidate 1 wins, and the deviation is not profitable. Now assume without loss of generality that Candidate 0 wins. The Candidate 1 cannot win by changing to a majority-efficient policies (a^1 is not majority-efficient). Similarly, Candidate 0 would remain the winner if he chooses a majority-efficient position. ∎

Proof of Theorem 3. Let p_i be the percentage of citizens with $\theta_i = 1$. For $j \in S$ we choose $a_j^* = 1$ if $p_i \geq 0.5$ and 0 otherwise. Let a^* be the resulting policy (which combines the platform and the type). Let X_i be a random variable with

$$
X_i(\theta) = \begin{cases} 1 & \text{if } |\theta_i - a_i^*| = 0; \\ 0 & \text{if } |\theta_i - a_i^*| = 1. \end{cases}
$$

Let a be an arbitrary alternative policy. Let D be the set of issues for which $a_i^* \neq a_i$. Then policy a^* is at least as good as policy a for agent θ if and only if

$$
\sum_{i \in D} \lambda_{i,t} X_i(\theta) \geq \sum_{i \in D} \lambda_{i,t}(1 - X_i(\theta)). \tag{8}
$$

Then (8) is equivalent to

$$
\sum_{i \in D} \lambda_{i,t} X_i(\theta) \geq 0.5 \sum_{i \in D} \lambda_{i,t}. \tag{9}
$$

Note that each random variable $X_i = Y_i + Z_i$ where Y_i assumes values 0 and 1 with probability 0.5 each, and where $Z_i \geq 0$. Thus,

$$
\mu\left(\left\{\theta \,\Big|\, \sum_{i \in D} \lambda_{i,t} X_i(\theta) \geq 0.5 \sum_{i \in D} \lambda_{i,t}\right\} \,\Big|\, t\right) \geq \mu\left(\left\{\theta \,\Big|\, \sum_{i \in D} \lambda_{i,t} Y_i(\theta) \geq 0.5 \sum_{i \in D} \lambda_{i,t}\right\} \,\Big|\, t\right). \tag{10}
$$

Because the X_i are independent it follows that the random variables Y_i are independent. Next, note that the distribution of $\sum_{i \in D} \lambda_i^t Y_i$ is symmetric for fixed t, because each Y_i has a symmetric distribution, and the Y_i are independent. Furthermore, $E\left[\sum_{i \in D} \lambda_{i,t} Y_i | t\right] = \sum_{i \in D} \lambda_i^t 0.5$. Because the median and the mean coincide for symmetric distributions, it follows that the right-hand side of (10) is 0.5. Thus, $P(\{\theta | u_\theta(a) \geq u_\theta(a^*)| t\}) \geq 0.5$. Similarly, it follows that $P(\{\theta | u_\theta(a^*) \geq u_\theta(a)\} | t) \leq 0.5$. Therefore,

$$
P(\{\theta | u_\theta(a) \geq u_\theta(a^*)\} | t) \geq P(\{\theta | u_\theta(a^*) \geq u_\theta(a)\} | t). \tag{11}
$$

Integrating both sides of (11) with respect to t implies that a^* is majority preferred to a. Since a was arbitrary, this proves that a^* is majority-efficient. ∎

Proof of Theorem 4. Let θ_m be the median voter type with respect to the ordering of types. Then $\mu(\{\theta_1, ..., \theta_{m-1}\}) < 0.5 \leq \mu(\{\theta_1, ..., \theta_m\})$. Consider first the platform profile (m^0, m^1) and

suppose that there exists a shock ν^* such that $u_{\theta_m}(m^1, \nu^*) = u_{\theta_m}(m^0, \nu^*)$. Candidate 0 wins whenever $\nu \leq \nu^*$ and Candidate 1 wins whenever $\nu > \nu^*$. Since m^1 is majority-efficient and society is polarized, $\theta_m \notin P^1(p, m^1, \nu^*)$, otherwise p would be majority preferred to m^1. Furthermore, let ν' be the cutoff shock such that Candidate 1 wins if he proposes instead policy p and the shock satisfies $\nu > \nu'$: $u_{\theta_m}(p, \nu') = u_{\theta_m}(m^0, \nu')$. Clearly, $\nu' \geq \nu^*$, which implies that the winning probability of Candidate 1 cannot be bigger when choosing policy p instead of m^1. ∎

Proof of Theorem 5.

Proof of Statement 1. Let (a^0, a^1) be a choice of feasible policies by the two candidates with $f(a^j) = \theta_m$. Then each candidate wins with probability 0.5. Now suppose that one of the candidates, say Candidate 0, deviates to $\hat{a}^0$ with $f(\hat{a}^0) \neq \theta_m$. Without loss of generality suppose that $f(\hat{a}^0) < \theta_m$. Then there exists $\hat{\theta} < \theta_m$ such that all citizens $\theta < \hat{\theta}$ prefer policy $\hat{a}^0$ to a^1 and all citizens $\hat{\theta} > \theta_m$ strictly prefer Candidate 1's policy. Thus, in order for Candidate 0 to win the election, the realized median voter $\theta_m(\tau) < \hat{\theta}$. If $\theta_m(\tau) = \hat{\theta}$ then the election ends in a tie. But $\hat{\theta} < \theta_m$ and the fact θ_m is unique, implies that $\theta_m(\tau) < \hat{\theta}$ with a probability strictly less than 0.5. Thus, the deviation would lower the agents payoff, a contradiction.

Now suppose that (a^0, a^1) is an equilibrium but that one candidate's policy, e.g., Candidate 0's, $f(a^0) \neq \theta_m$ but that $f(a^1) = \theta_m$. Then the above argument implies that Candidate 0's winning probability is strictly less than 0.5. If, instead, Candidate 0 would offer a policy $\tilde{a}^0$ with $f(\tilde{a}^0) = x_m$ then his winning probability would increase to 0.5, a contradiction.

Next, suppose that $f(a^j) \neq \theta_m$ for both candidates. Note that both candidates winning probability must be $1/2$. Otherwise, an candidate could improve by deviating to a policy $\tilde{a}^j$ with $f(\tilde{a}^j) = \theta_m$. Without loss of generality suppose that $f(a^0) < \theta_m$. Then, using the above argument, Candidate 1's winning probability would be strictly greater than 0.5 if he offered $\tilde{a}^j$ with $f(\tilde{a}^j) = \theta_m$, a contradiction.

We now show that any policy with $f(a^j) = \theta_m$ is majority efficient. Let $\tilde{a}^j$ be arbitrary. Without loss of generality we can assume that $f(\tilde{a}^j) < \theta_m$. Let $\hat{\theta}$ be the citizen who is indifferent between $\tilde{a}^j$ and a^j. Clearly, $\hat{\theta} < \theta_m$. Then $a^j \succeq_\tau^* \tilde{a}^j$ if the realized median voter $\theta_m(\tau) \geq \hat{\theta}$, which occurs with a probability 0.5. Similarly, $a^j \preceq_\tau^* \tilde{a}^j$ with probability strictly less than 0.5. Thus, a^j is ex-ante majority efficient.

Proof of Statement 2. Without loss of generality let $f(A^0) \leq \theta_m \leq f(A^1)$. Let $\bar{x}_0 = \max f(A^0)$ and $\bar{x}_1 = \min f(A^1)$. Note that $\bar{x}_i$, $i = 0, 1$ exist, because A^j, $j = 0, 1$ is compact by Tychonoff's theorem, and because f is assumed to be continuous. Thus, $f(a^0) = \bar{x}_0 \leq \theta_m \leq \bar{x}_1 = f(a^1)$. We show that (a^0, a^1) is an equilibrium. Suppose that one of the candidates, say candidate 0 deviates to $\tilde{a}^0$ with $f(\tilde{a}^0) < \bar{x}_0$. Let θ and $\tilde{\theta}$ be the citizens who are indifferent between policies $\bar{x}_1$ and $\bar{x}_0$, and between $\bar{x}_1$ and $f(\tilde{a}^0)$, respectively. Then $\tilde{\theta} < \theta$. Under the original policy Candidate 0 wins if the realized median voter $\theta_m(\tau) < \theta$. Under the new policy, $\theta_m(\tau) < f(\tilde{a}^0)$ must hold. Thus, Candidate 0's winning probability decreases, and the deviation is not optimal.

It remains to prove that a^0 and a^1 are majority efficient. Let $\tilde{a}^0 \in A^0$ be an abitrary policy.

Then $f(\tilde{a}^0) \leq f(a^0)$. All citizens $\theta \geq f(a^0)$ prefer a^0 to $\tilde{a}^0$. Since $f(a^0) \leq \theta_m$ this means that with a probability of at least 0.5, a^0 is majority preferred to $\tilde{a}^0$. Similarly, it follows that $\tilde{a}^0$ is majority preferred to a^1 with probability less than 0.5. Thus, a^0 is ex-ante majority efficient. The proof for a^1 is similar.

Proof of Statement 3. Suppose by way of contradiction that $f(a^j) = \theta_m$. Without loss of generality we can assume that $f(A^{-j}) > \theta_m$. The first part of the prove implies that it is optimal for Candidate $-j$ to choose the position that is as close as possible to θ_m, i.e. let $\hat{x} = \min f(A^{-j})$. Then any policy a^{-j} with $f(a^{-j}) = \hat{x}$ is optimal. By assumption there exist a policy $\tilde{a}^j$ with $\theta_m < f(\tilde{a}^j) < \hat{x}$. Thus, $\tilde{a}^j$ increases the probability that candidate j wins, a contradiction to the assumption that a^j is the equilibrium strategy. Finally, since $f(a^j) \neq \theta_m$, agent j's policy is not ex-ante majority-efficient. ∎

Proof of Theorem 6. Theorem 5 implies that in equilibrium $f(a^j) = x_m$ for $j = 0, 1$. Suppose that $\theta_m(\omega) < \theta_m$ with positive probability. Let $\hat{x}$ be marginally smaller than x_m. Then in states $\omega \in \tilde{\Omega}$ a majority of voters prefers $\hat{x}$, implying that the equilibrium is not competition efficient.

Conversely, suppose that $\theta_m(\omega) = \theta_m$ for a.e. ω, but that the equilibrium (a^0, a^1) is not competition-efficient. Let $\hat{a}^j \in A^j$, $j = 0, 1$ be arbitrary. If $f(\hat{a}^1) = f(\hat{a}^2)$ then citizens are indifferent between the two policies. Furthermore, if $x_m \neq f(\hat{a}^j)$ then a majority of citizens in state ω prefer (a^1, a^2) to $(\hat{a}^1, \hat{a}^2)$, i.e.,

$$Q(a^1, a^2, \hat{a}^1, \hat{a}^2, \omega) \geq Q(\hat{a}^1, \hat{a}^2, a^1, a^2, \omega). \tag{12}$$

If $f(\hat{a}^1) \neq f(\hat{a}^2)$ then Candidate j wins if $u_{\theta_m(\omega)}(\hat{a}^j) \geq u_{\theta_m(\omega)}(\hat{a}^{-j})$. Without loss of generality suppose that $f(\hat{a}^j) > x_m$. Then there exists $\tilde{\theta} > \theta_m$ such that all agents $\theta < \tilde{\theta}$ strictly prefer a^j. Since, $\theta_m(\omega) = \theta_m$, for a.e. ω, this means that a majority of citizens prefer a^j to $\hat{a}^j$, i.e., (12) holds. Thus, (a^1, a^2) is competition-efficient. ∎

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