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R. G. Boyd

July 1966

NUCLEATION OF SUPERCONDUCTIVITY AT A TUNNELING
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ABSTRACT

We present a calculation of the parallel critical field of a plane tunneling barrier in an otherwise homogeneous superconductor, valid for $T_c - T \ll T_c$. We have found a solution of the Ginzburg-Landau equation which satisfies the semi-phenomenological boundary conditions proposed by de Gennes. The critical field is significantly lower than H_{c3} when the tunneling current is of the same order of magnitude as the sheath current. When the ratio (tunneling current/sheath current) is small, it is proportional to $\xi(T)$. Since $\xi(T) \rightarrow \infty$ at T_c , in principle the critical field of a tunneling barrier of any transmissivity must go to H_{c2} as $T \rightarrow T_c$, although in practice this may be impossible to observe for small transmissivities due to the finite transition width at T_c . For the pure metal, specular barrier model at a fixed temperature T not too close to T_c , most of the drop in critical field from H_{c3} to H_{c2} occurs for transmissivities in the range .01 to .1.

The effect we have predicted, if experimentally confirmed, may be useful as a tool for investigating the electronic properties of grain boundaries in polycrystalline metal.

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I. INTRODUCTION

Consider a plane tunneling¹ barrier of negligible² thickness d with transmissivity t located at $x = 0$ in an infinite and otherwise homogeneous superconductor. We want to calculate the critical field for nucleation of superconductivity at the barrier in a magnetic field H parallel to the z -axis. Since the transition will be of second order the supercurrents can be arbitrarily small at nucleation, hence we may assume the field uniform. We shall restrict our attention to the temperature range $T_c - T \ll T_c$.

We know that for $t = 1$ (no barrier), the nucleation field $H_{||}$ is that for the bulk,³ $H_{||} = H_{c2}$, whereas for $t = 0$ (perfect insulation), the nucleation field is that for a free surface,⁴ $H_{||} = H_{c3} = 1.69 H_{c2}$. When $1 > t > 0$ supercurrents tunnel⁵ through the barrier. These currents are periodic along the barrier in the y -direction, and as $t \rightarrow 1$ ($H_{||} \rightarrow H_{c2}$) the order parameter and currents resemble those of a row of fluxoids along the barrier in the limit $H_{||} = H_{c2}$.

$H_{||}$ is substantially lower than H_{c3} when the currents tunneling through the barrier are of the same order of magnitude as the sheath currents along the barrier. When the tunneling current is small, both currents are proportional to the superfluid density $|\psi|^2$, but the proportionality constant is essentially independent of temperature for tunneling currents, and contains a factor $1/\xi(T)$ for sheath currents from the phase gradient. The ratio (tunneling current/sheath current) is therefore proportional to $\xi(T)$ when the ratio $\ll 1$. Since $\xi(T) \rightarrow \infty$ as $T \rightarrow T_c$, the ratio must increase until the tunneling current approaches the size of the sheath current, at which time the tunneling current becomes proportional to $|\psi|^2/\xi(T)$ rather than simply

$|\psi|^2$ and the ratio remains of order unity in the limit $T = T_c$. Hence in principle $H_{||}/H_{c2} \rightarrow 1$ at T_c for any transmissivity t , although in practice the steep decent of $H_{||}$ to H_{c2} near T_c will be lost (for small transmissivities) in the finite width of the temperature induced transition at T_c . For the pure metal, specular barrier model at a fixed temperature T not too close to T_c , most of the drop in critical field from H_{c3} to H_{c2} occurs for transmissivities in the range .01 to .1.

The linearized Ginzburg-Landau (GL) equation⁷ for the order parameter ψ near a second-order transition is valid for $T_c - T \ll T_c$ and for distances from the barrier⁸ greater than ξ_0 in the pure case and $\sqrt{\xi_0 l}$ in the dirty case. When the distance ξ_0 or $\sqrt{\xi_0 l}$ respectively is much smaller than the temperature-dependent coherence length $\xi(T)$, then the effect of the barrier is simply to impose boundary conditions⁸ on the solutions of the GL equation. These boundary conditions are obtained from the linear integral equation⁹ for Δ (or ψ) which is valid in the vicinity of a second order phase transition. We shall discuss the boundary conditions in Section II and solve the GL equation in Section III.

II. BOUNDARY CONDITIONS¹⁰

The order parameter¹¹ Δ at a phase transition of second order is a solution of the linear integral equation⁹

$$\Delta(\vec{s}) = \int d^3r K(\vec{s}, \vec{r}) \Delta(\vec{r}). \quad (1)$$

The range ξ_k of the kernel K in the bulk is ξ_0 in the pure case, $\sqrt{\xi_0 l}$ in the dirty case.

When $T_c - T \ll T_c$ the GL equation^{7,12} is also valid,

$$\frac{1}{\xi} \psi = (i\bar{\nabla} + \frac{2\pi}{\phi_0} \bar{A})^2 \psi, \quad (2)$$

except in a neighborhood of the barrier defined by the range ξ_k of the kernel K . Outside of this neighborhood both Eqs. (1) and (2) must have the same solutions.

Since Eq. (1) is more generally valid than Eq. (2), it would be better to solve Eq. (1). However, it is much easier to solve Eq. (2), so that is what we shall do. We cannot avoid the use of Eq. (1) in the neighborhood of the barrier, but we shall find that it is not necessary to find a complete solution, except to evaluate a single number which is probably best left as an adjustable parameter. The argument is based on symmetry properties of K .

The scale of variation of GL solutions is given¹³ by

$$\begin{aligned} \xi(T) &= 0.74 \xi_0 [1 - T/T_c]^{-1/2} \text{ pure} \\ \xi(T) &= 0.85 \sqrt{\xi_0 l} [1 - T/T_c]^{-1/2} \text{ dirty.} \end{aligned} \quad (3)$$

When $T_c - T \ll T_c$ we have $\xi(T) \gg \xi_k$. We shall consider the problem of joining a solution of Eq. (2) - valid in the bulk - to a solution of Eq. (1) - valid near the barrier - in the asymptotic region $\xi(T) \gg x \gg \xi_k$ where both equations are valid. We see from Eq. (3) that we may locally approximate the GL solution in this region with the linear expression

$$\psi(\vec{r}) \approx \psi_0 + \bar{\nabla}\psi_0 \cdot \vec{r} \quad (4)$$

where $\psi_0, \bar{\nabla}\psi_0$ are evaluated at a point y_0 on the barrier. Therefore we shall study the solutions of Eq. (1) which are asymptotically linear.

Set $A = 0, T = T_c$ in Eq. (1); we will justify the omission of A later. We study linear solutions in the bulk first. From the property

$$\int d^3r K(\vec{s}, \vec{r}) = 1 \quad \text{at } T_c \quad (5)$$

it is evident that $\Delta = \text{constant}$ is a solution of Eq. (1). We may prove that $\Delta = \vec{q} \cdot \vec{r}$ is also a solution in the following manner: we use Eq. (5) to rewrite Eq. (1) in the form

$$0 = \int d^3r K(\vec{s}, \vec{r}) [\Delta(\vec{s}) - \Delta(\vec{r})] . \quad (6)$$

In the bulk where we have translational invariance, either because of perfect periodicity of the lattice or because a suitable average over the sites of impurity atoms has been performed, we have $K(\vec{s}, \vec{r}) = K(\vec{s} - \vec{r})$. Since $K(s, r) = K(r, s)$ when K is real ($A = 0$), we have $K(R) = K(-R)$. Hence on substitution of $\Delta = \vec{q} \cdot \vec{r}$ in Eq. (6) the integral vanishes, because K is even, $[\Delta(\vec{s}) - \Delta(\vec{r})]$ odd in the coordinate difference $\vec{s} - \vec{r}$.

Now we introduce the barrier at $x = 0$. The kernel K is still an even function of the y -coordinate difference $s_y - r_y$, so that there are solutions to Eq. (1) which are linear in y throughout the barrier region. On substituting the function $\Delta(x, y) = g(x)y$ in Eq. (1), we find that it is a solution if and only if $g(x)$ is a solution. We are therefore reduced to a study of the one-dimensional solutions $\Delta(x)$.

The kernel retains its inversion symmetry $K(\vec{r}, \vec{s}) = K(-\vec{r}, -\vec{s})$ in the presence of a symmetric barrier. Hence there are even and odd solutions $\Delta^e(x) = + \Delta^e(-x)$, $\Delta^o(x) = -\Delta^o(-x)$. In the absence of the barrier $\Delta^e = \text{constant}$, $\Delta^o \propto x$. In the presence of the barrier the asymptotic behavior of these solutions can be changed only to the extent of allowing the linear asymptote of Δ^o to have a finite intercept at the barrier,

$$\begin{aligned} \Delta^e_{\text{asymptotic}} &= \text{constant} \\ \frac{d}{dx} \ln \Delta^o_{\text{asymptotic}} &= \frac{1}{M} \end{aligned} \quad (7)$$

where M is a number which depends only on the properties of the barrier.

Any linear GL solution may be fitted in the asymptotic region $\xi(T) \gg x \gg \xi_k$ by the set of elementary solutions Δ^e , Δ^o , $\Delta^e y$, $\Delta^o y$. We shall assume the uniqueness of such a representation. This seems physically plausible, but the author is unable to make it mathematically rigorous by supplying a uniqueness theorem for the set of even and odd solutions of Eq. (1) which satisfy linear boundary conditions. Granted this assumption, we may summarize Eqs. (7) for general Δ in the one equation

$$\Delta_+ = \Delta_- + M \frac{d\Delta_-}{dx} \quad (8)$$

where Δ_+ , Δ_- are the linear asymptotes of Δ for $x > 0$, $x < 0$ evaluated at $x = 0+$, $0-$ respectively. This is the boundary condition which we shall require solutions of the GL equation to obey at the barrier.

In the presence of a magnetic field the kernel K may be approximated¹⁴ over distances small compared to $\xi(T)$ by the expression

$$K(\vec{s}, \vec{r}) \approx K_0(\vec{s}, \vec{r}) \exp \left\{ \frac{2ie}{\hbar c} [A(\vec{s}) \cdot \vec{s} - A(\vec{r}) \cdot \vec{r}] \right\}$$

where K_0 is the kernel in zero field. Distributing the two phase factors to the order parameters of the same argument, we find that locally

$$\Delta_0(\vec{r}) = \Delta(\vec{r}) \exp \left[- \frac{2ie}{\hbar c} A \cdot r \right]$$

is a solution of the zero field equation and must satisfy the same linear boundary conditions. Hence Eq. (8) is still valid for complex Δ when $A_x = 0$.

We have not attempted to calculate M for any model. P. G. de Gennes⁶ has calculated M for the pure metal, specular barrier model under the approximation $t \ll 1$. His result was

$$M = [1.1 \text{ NV}] \frac{\xi_0}{t} \quad (9)$$

The quantity in square brackets does not vary much, and is usually of order 0.3. Since ξ_0 appears here in its role as the range ξ_k of the kernel K , one suspects that in the more interesting dirty case ξ_0 will be replaced by $\sqrt{\xi_0 \ell}$. When $t \rightarrow 1$, Eq. (9) is no longer valid since M must approach zero. One expects M to be essentially independent of temperature in the GL region $T_c - T \ll T_c$ since it depends upon the essentially temperature independent quantity ξ_k . Any slow variation

of M with T in this region would be swamped out in any case by the much more rapid variation of $\xi(T)$ (which goes to ∞ as $T \rightarrow T_c$).

III. SOLUTION OF THE GINZBURG-LANDAU EQUATION

We choose the gauge $A_y = Hx$. For this gauge Eq. (2) has solutions with "even" and "odd" symmetry, $\psi^*(x) = \pm \psi(-x)$. We choose the even symmetry, $\psi^*(x) = \psi(-x)$. We write ψ in terms of its real and imaginary parts,

$$\psi = R + iI.$$

Then for the assumed symmetry, the boundary condition Eq. (8) can be written in terms of ψ_+ at $x = 0_+$ as

$$\frac{dR}{dx} = 0$$

$$\frac{dI}{dx} = \frac{2}{M} I. \quad (10)$$

We note that I is odd and R is even, therefore R , dR/dx , dI/dx are all continuous at the barrier, only I being discontinuous.

The real and imaginary parts of ψ are mixed in Eq. (2) only by the cross product term which is of the form $i \times \frac{\partial}{\partial y}$. If we take advantage of the periodicity of R and I to expand both in Fourier series, then cosine terms in the R series mix only with sine terms in the I series. We make the ansatz

$$R(x,y) = R(x) \cos ky$$

$$I(x,y) = I(x) \sin ky. \quad (11)$$

Then Eq. (2) becomes

$$\left[\frac{d^2}{dx^2} + \frac{1}{\xi^2} - k^2 - \left(\frac{2\pi H}{\phi_0} x \right)^2 \right] R + \frac{4\pi H}{\phi_0} kx I = 0$$

$$\left[\frac{d^2}{dx^2} + \frac{1}{\xi^2} - k^2 - \left(\frac{2\pi H}{\phi_0} x \right)^2 \right] I + \frac{4\pi H}{\phi_0} kx R = 0 \quad (12)$$

We may write two decoupled equations for the two quantities $R+I$,

$R-I$:

$$\left[\frac{d^2}{dx^2} + \frac{1}{\xi^2} - \left(k - \frac{2\pi H}{\phi_0} x \right)^2 \right] (R + I) = 0$$

$$\left[\frac{d^2}{dx^2} + \frac{1}{\xi^2} - \left(k + \frac{2\pi H}{\phi_0} x \right)^2 \right] (R - I) = 0 \quad (13)$$

These two equations can be reduced to the same standard form by making the following substitutions:

$$z = x \sqrt{\frac{4\pi H}{\phi_0}}$$

$$a = - \frac{\phi_0}{4\pi \xi^2 H} = - H_{c2}^2 / 2H$$

$$\kappa = k \sqrt{\frac{\phi_0}{\pi H}} = 2 \sqrt{|a|} \quad (k\xi) \quad (14)$$

$$R(x) + I(x) = u(z - \kappa)$$

$$R(x) - I(x) = \omega(z + \kappa)$$

Then $u(z)$ and $\omega(z)$ must both satisfy the same equation

$$\frac{d^2 y}{dz^2} = \left(a + \frac{z^2}{4} \right) y. \quad (15)$$

The two standard independent solutions of this equation are known as parabolic cylinder functions.¹⁵ They have the following asymptotic behavior for $x \gg |a|$:

$$\begin{aligned} U(a, z) &\sim e^{-\frac{1}{4} z^2} z^{-a-\frac{1}{2}} \\ V(a, z) &\sim e^{\frac{1}{4} z^2} z^{a-\frac{1}{2}}. \end{aligned} \quad (16)$$

Since V blows up, u and ω must both be proportional to U .

For $a = -\frac{1}{2}$, $H = H_{c2}$, the function

$$U\left(-\frac{1}{2}, z\right) = e^{-\frac{1}{4} z^2}; \quad (17)$$

U is finite in both limits $x \rightarrow \pm \infty$. For $a > -\frac{1}{2}$, $H > H_{c2}$ the function goes to $+\infty$ in the limit $x \rightarrow -\infty$; we display (in Fig. 1) its behavior near the origin for $a = -.4$, $H = 1.25 H_{c2}$. Since the GL boundary condition at a free surface is $\frac{du}{dx} = 0$, the maximum a (maximum H) for which the function has a zero derivative at some point yields the surface nucleation field H_{c3} . For this value of a the two points of zero derivative evident in Fig. 1 must coalesce to form a point of inflection, at which point the curvature must be zero, or $a + \frac{\pi^2}{4} = 0$.

We express the boundary conditions (Eqs. (10)) in the new symbols:

$$\begin{aligned} \left. \frac{du}{dz} \right|_{-\kappa} &= \frac{\xi \sqrt{|a|}}{M} [u(-\kappa) - \omega(\kappa)] \\ \left. \frac{d\omega}{dz} \right|_{\kappa} &= - \frac{\xi \sqrt{|a|}}{M} [u(-\kappa) - \omega(\kappa)]. \end{aligned} \quad (18)$$

Let

$$\begin{aligned} u(z) &= U(a, z) \\ \omega(z) &= \alpha U(a, z). \end{aligned} \quad (19)$$

Adding Eqs.(18), we get

$$\alpha = - \frac{\left. \frac{dU}{dz} \right|_{-K}}{\left. \frac{dU}{dz} \right|_K}. \quad (20)$$

Eliminating α from Eqs. (18), we get

$$\left. \frac{d}{dz} \ln U \right|_K \cdot \left. \frac{d}{dz} \ln U \right|_{-K} = \frac{\xi \sqrt{|a|}}{M} \left[\left. \frac{d}{dz} \ln U \right|_K + \left. \frac{d}{dz} \ln U \right|_{-K} \right]. \quad (21)$$

We use the relation¹⁶

$$\left. \frac{d}{dz} \ln U \right|_K + \left. \frac{d}{dz} \ln U \right|_{-K} = - \frac{\sqrt{2\pi}}{\Gamma(\frac{1}{2} + a) U(K)U(-K)} \quad (22)$$

to reduce the boundary condition to the form

$$F_K(a) = - \frac{\Gamma(\frac{1}{2} + a)}{\sqrt{2\pi|a|}} \left. \frac{dU}{dz} \right|_K \left. \frac{dU}{dz} \right|_{-K} = \frac{\xi(T)}{M}. \quad (23)$$

Here the magnetic field appears only on the left in the dimensionless ratio $a = -H_c/2H$. The right hand side depends only on the properties of the barrier (through M) and the temperature (through ξ , which $\rightarrow \infty$ at T_c).

We need to choose K to maximize H , because the maximum H for which a solution can be found to match the boundary conditions will be the

critical field $H_{||}$. Since $F_K(a)$ must be positive, κ must be in the finite interval within which $\left. \frac{dU}{dz} \right|_{-\kappa}$ is positive. Since M goes from 0 to ∞ as $H_{||}$ goes from H_{c2} to H_{c3} , $F_K(a)$ must be a decreasing function of a ; this is compatible with the fact that $\Gamma(\frac{1}{2} + a) = \infty$ at H_{c2} while $\left. \frac{dU}{dz} \right|_{-\kappa} = 0$ at H_{c3} . Therefore we shall choose κ to maximize $F_K(a)$, so that Eq. (23) will be satisfied for maximum a . The derivative U' reaches extrema when $U'' = 0$, which occurs when $a + \frac{z^2}{4} = 0$. Hence¹⁷

$$\kappa = 2\sqrt{|a|} \quad (24)$$

maximizes $F_K(a)$, which we write hereafter as

$$F(a) = - \frac{\Gamma(\frac{1}{2} + a)}{\sqrt{2\pi|a|}} \left. \frac{dU}{dz} \right|_{2\sqrt{|a|}} \left. \frac{dU}{dz} \right|_{-2\sqrt{|a|}} = \frac{\xi(T)}{M}. \quad (25)$$

We have tabulated $F(a)$ in Table I. This table may be used to generate curves giving $H_{||}/H_{c2}$ as a function of T for any fixed barrier via the relation

$$F(a) = \frac{\gamma}{\sqrt{1 - T/T_c}}. \quad (26)$$

In Fig. 2 we present such curves for several values of the adjustable parameter γ .

The order parameter ψ at $H = H_{||}$ for $x > 0$ is given by

$$\begin{aligned} \psi(x, y) = & U(a, \sqrt{2}(x/\xi - 1)) e^{iy/\xi} \\ & + \alpha U(a, \sqrt{2}(x/\xi + 1)) e^{-iy/\xi}. \end{aligned} \quad (27)$$

α must be calculated from Eq. (20). In Fig. 3 we display $R(x)$ and $I(x)$ for $a = -.4$.

In the limit $H_{\parallel} = H_{c_2}$ (no barrier) the order parameter ψ may be written quite simply as the sum of two bulk Gaussian GL solutions⁷

$$\begin{aligned} \psi = & \exp [iy/\xi - (x - \xi)^2/2\xi^2] \\ & + \exp [-iy/\xi - (x + \xi)^2/2\xi^2] \end{aligned} \quad (28)$$

centered at $x = \pm \xi$. The interference of the two exponentials produces zeroes in ψ at intervals of $\pi\xi$ along the barrier, separated alternately by maxima and minima in ψ (ψ is real at $x = 0$ when $H = H_{c_2}$). ψ can be thought of as a row of fluxoids in this limit, formed in the same way as the bulk fluxoid array from a superposition of Gaussians.³

The barrier tunneling current is given by

$$j_x \Big|_0 = \frac{4e\hbar}{m} \frac{IR}{M} \Big|_0 \quad (29)$$

for the symmetry we have chosen. As $M \rightarrow 0$, $I \rightarrow 0$ also in such a way that the ratio $I/M \propto 1/\xi$ in the limit $M = 0$, $H_{\parallel} = H_{c_2}$ (neglecting the y dependence and normalization of ψ). This may be shown by calculating j_x for the limiting solution ψ given in Eq. (28).

IV. CONCLUSIONS

The essential result of this paper is contained in Eq. (25) and Table I, which determine the parallel critical field ratio $H_{||} / H_{c_2}(T)$ near T_c for tunneling barriers which yield boundary conditions on GL solutions of the type given in Eq. (8). We have said nothing yet about the construction and experimental investigation of tunneling barriers with transmissivities in the right range for observation of interesting results.

Obviously our paper is not relevant to the conventional high resistance metal-oxide-metal tunneling barriers formed by evaporation, since in all practical cases these composites must have $H_{||} = H_{c_3}$. There do exist plane defects in polycrystalline metals which may have transmissivities in the right range for $H_{||}$ to have an observable intermediate value between H_{c_2} and H_{c_3} ; we have in mind the grain boundaries in metals containing oxygen as an impurity. It seems probable that supercurrents carried by such internal sheathes have already been observed.¹⁸

For the conditions of our problem to be satisfied, a barrier need only have reasonable uniformity on the scale ξ_k extending over a distance greater than $\xi(T)$. For the purpose of observation of a sheath current, a continuous superconducting path must connect two terminals. The sheath current will follow a path of maximum barrier resistivity, and will be dominated by the point on the path which has lowest $H_{||}$ (a saddle point in resistivity). The (integrated) resistivity of the barrier to normal currents is unlikely to be related to the barrier resistivity seen by a sheath current (unless the barrier is perfectly uniform), since normal currents pick out areas of minimum barrier

resistivity. It is incorrect to conclude that a window screen is not a metallic conductor simply because it transmits light!

In any experiment designed to test our results, the effect of the free surface of the experimental specimen must be eliminated either geometrically or by an appropriate coating of normal metal.

Since H_{11}/H_{c2} is especially sensitive to temperature very close to T_c , it might seem that measurements in this temperature range will be the most crucial test of the validity of our theory. For this reason one should choose a material with a narrow transition width. However, we must sound a warning: if the composition of the metal near the barrier differs from that of the bulk, it may have a slightly different critical temperature, in which case it is possible that our results may not be strictly applicable for temperatures extremely close to T_c . For any given alloy system, it would be wise to choose a composition at which T_c has an extremum, so that small variations in composition will not appreciably affect T_c .

We wish to point out that the effect we have predicted may be useful as a tool for the experimental investigation of the electronic properties of grain boundaries in polycrystalline metal.

ACKNOWLEDGEMENTS

I wish to express my gratitude to Kenneth Ralls for suggesting this problem.

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1. We have used the word "tunneling" throughout this paper, simply because we have been unable to think of a convenient alternative. We realize the word has generally been applied in the past to barriers of very high resistivity, and that the word is likely to call to mind theories of tunneling which are only applicable in this limit. On the other hand, we have used a "theory of tunneling" due to de Gennes which is applicable for all barrier transmissivities: P. G. de Gennes, Phys. Letters 5, 22 (1963).
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10. This entire section is based upon Chapter 7 of P. G. de Gennes' excellent book, Ref. 6. We acknowledge our indebtedness here to avoid sprinkling too many footnotes throughout.

11. $\Delta \propto \psi$. Since all equations considered here are linear and homogeneous in Δ or ψ , the constant of proportionality is irrelevant to us. We use Δ in Eq. (1), ψ in Eq. (2), which is the usage sanctioned by custom.
12. Here ϕ_0 is the flux quantum $hc/2e$, $\xi = \xi(T)$ is the GL coherence length, and we use standard cgs units.
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15. NBS "Handbook of Mathematical Functions", edited by M. Abramowitz and I. A. Stegun, now pub. by Dover Publications, Inc., 1965; Chapter 19.
16. In Ref. 15, differentiate 19.4.2, use 19.4.1 to eliminate V' , then use 19.4.2 again to simplify result. Remember that

$$\left. \frac{df(x)}{dx} \right|_{-c} = - \left. \frac{df(-x)}{dx} \right|_c .$$
17. We note that our choice of κ has the curious consequence that $\kappa^{-1} = \xi(T)$. The period of ψ along the barrier at nucleation is independent of the nucleation field ratio $H_{||}/H_{c2}$.
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FIGURE CAPTIONS

Fig. 1 This is the parabolic cylinder function $U(a, z)$ for $a = -.4$, $-3 \leq z \leq +3$. All the functions of interest to us have this same general appearance, except in the two limits $a = -\frac{1}{2}$ (where the valley goes to $-\infty$ and we get a Gaussian) and $a \approx -.295$ (where the valley becomes a point of inflection and disappears). The two vertical bars indicate the two points of zero curvature, $z = \pm 2\sqrt{|a|}$.

Fig. 2 We display $H_{||}/H_{c2}$ as a function of T/T_c for several values of the adjustable parameter γ (defined in Eq. (26)). For the pure metal-specular barrier model,⁶ $\gamma = (.025, .25)$ corresponds to transmissivities $t \approx (.01, .1)$. We note that the slope of $H_{||}/H_{c2}$ is infinite at T_c since

$$(H_{||} - H_{c2})/H_{||} \propto [1 - T/T_c]^{1/2}$$

near T_c . However, $dH_{||}/dT \rightarrow dH_{c2}/dT$ at T_c .

Fig. 3 We display the real and imaginary parts of $\psi = r + iI$ for $a = -.4$, $H_{||} = 1.25 H_{c2}$. R and I are continued to negative z by the prescription R even, I odd. The vertical scale is arbitrary. We omit the y dependence.

TABLE I

-a	F(a)	-a	F(a)	-a	F(a)
.29	-.01665	.37	.353	.45	1.681
.30	+.01745	.38	.427	.46	2.205
.31	.0538	.39	.512	.47	3.08
.32	.0928	.40	.613	.48	4.81
.33	.1350	.41	.735	.49	10.00
.34	.1812	.42	.885	.50	∞
.35	.2322	.43	1.076		
.36	.2890	.44	1.329		

The variable $a = -H_{c2}/2H_{||}$. The function $F(a)$ is defined in Eq. (25), which determines $H_{||}(T)/H_{c2}(T)$ for fixed M . The error in the tabulation probably does not exceed one per cent. H_{c3} corresponds to $F(a) = 0$; H_{c2} corresponds to $F(a) = \infty$.

-19-

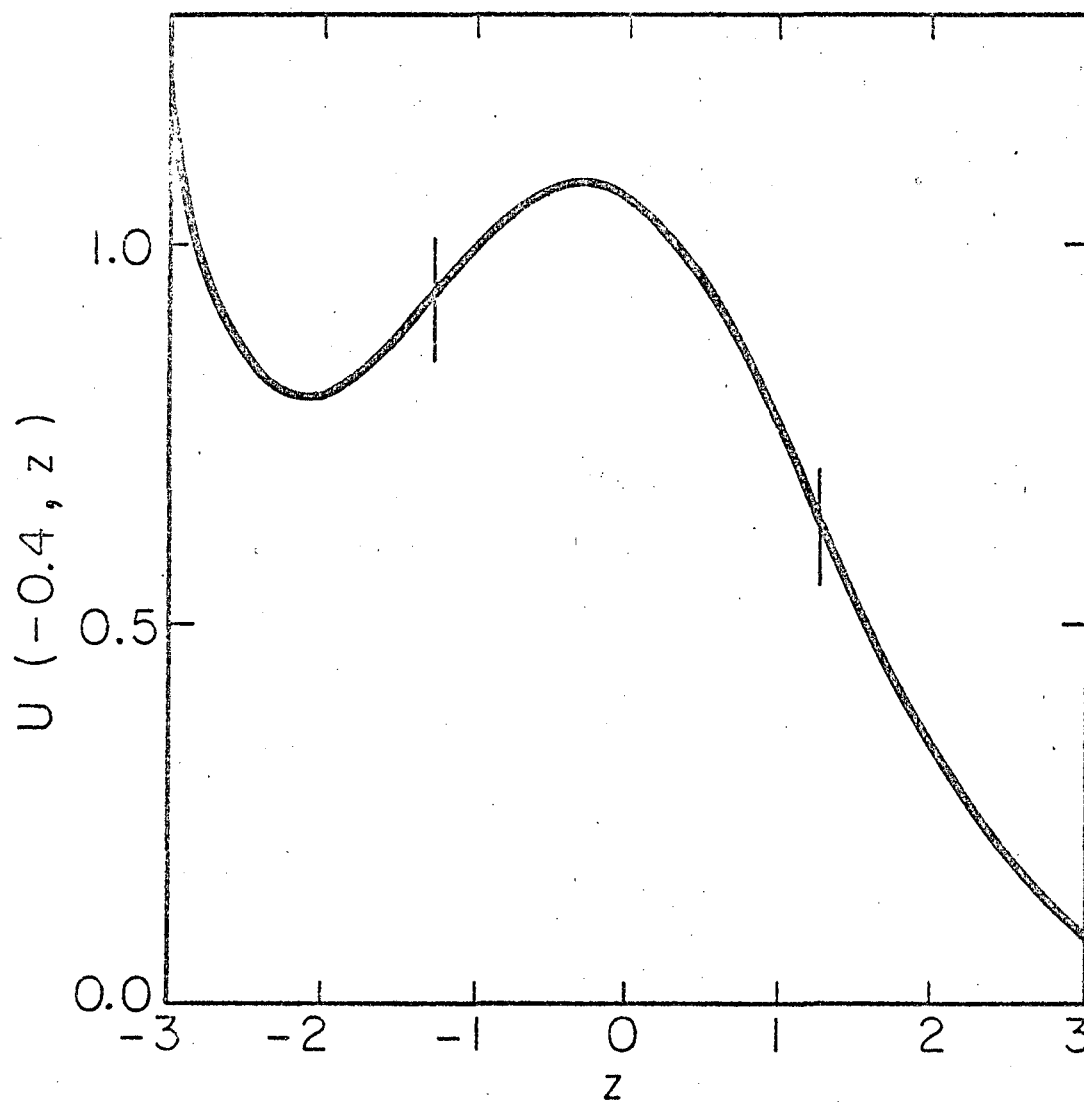


FIGURE 1

-20-

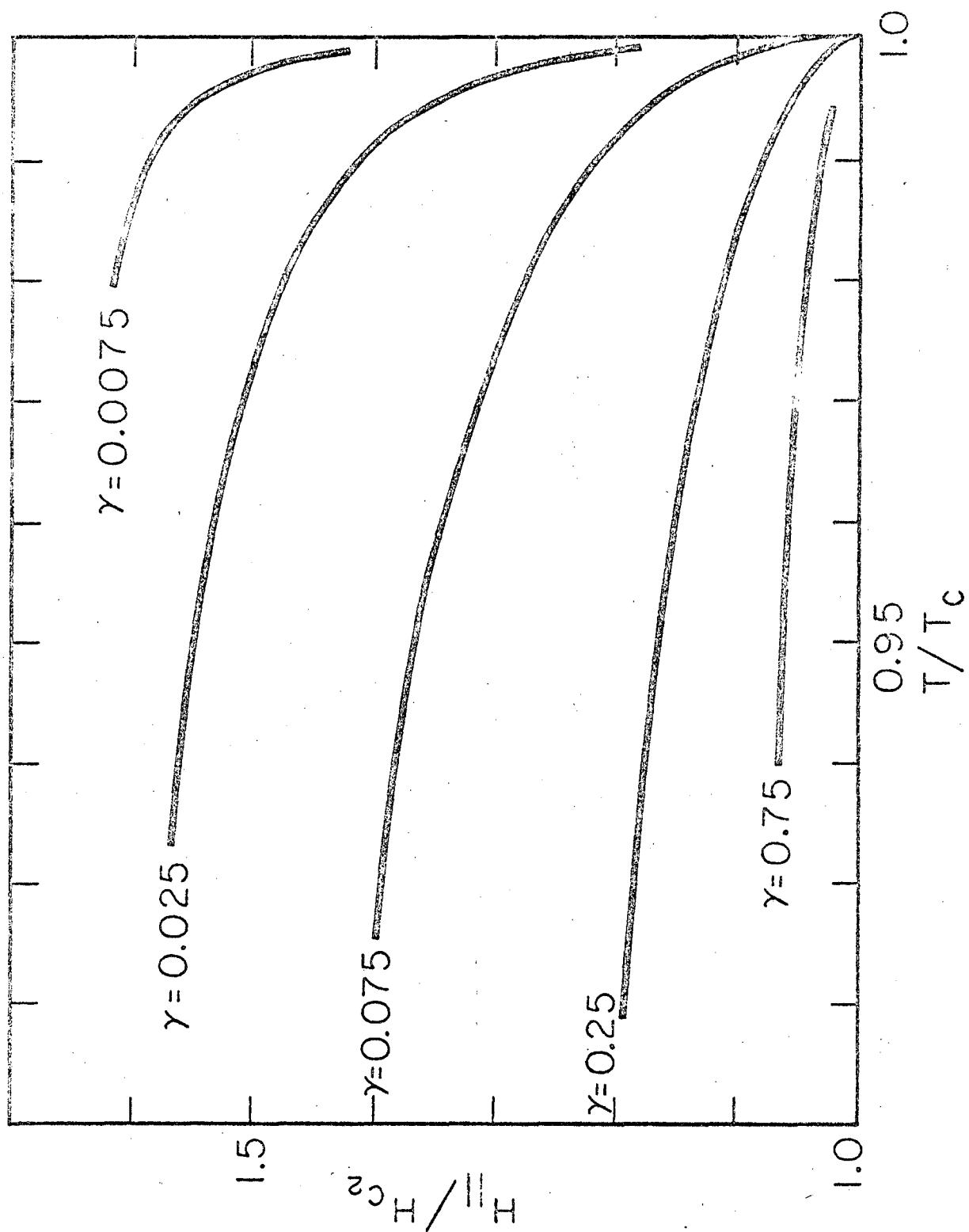
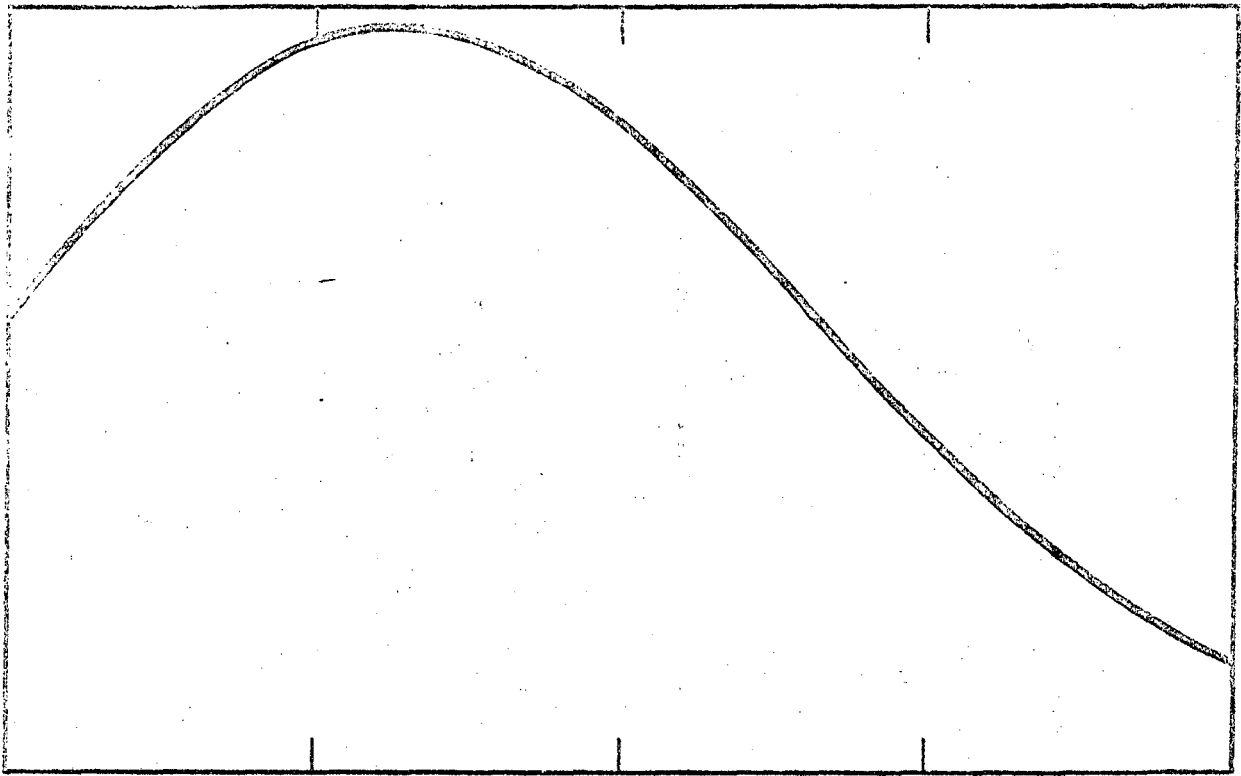


FIGURE 2

$I(z)$ (arbitrary units)



$R(z)$ (arbitrary units)

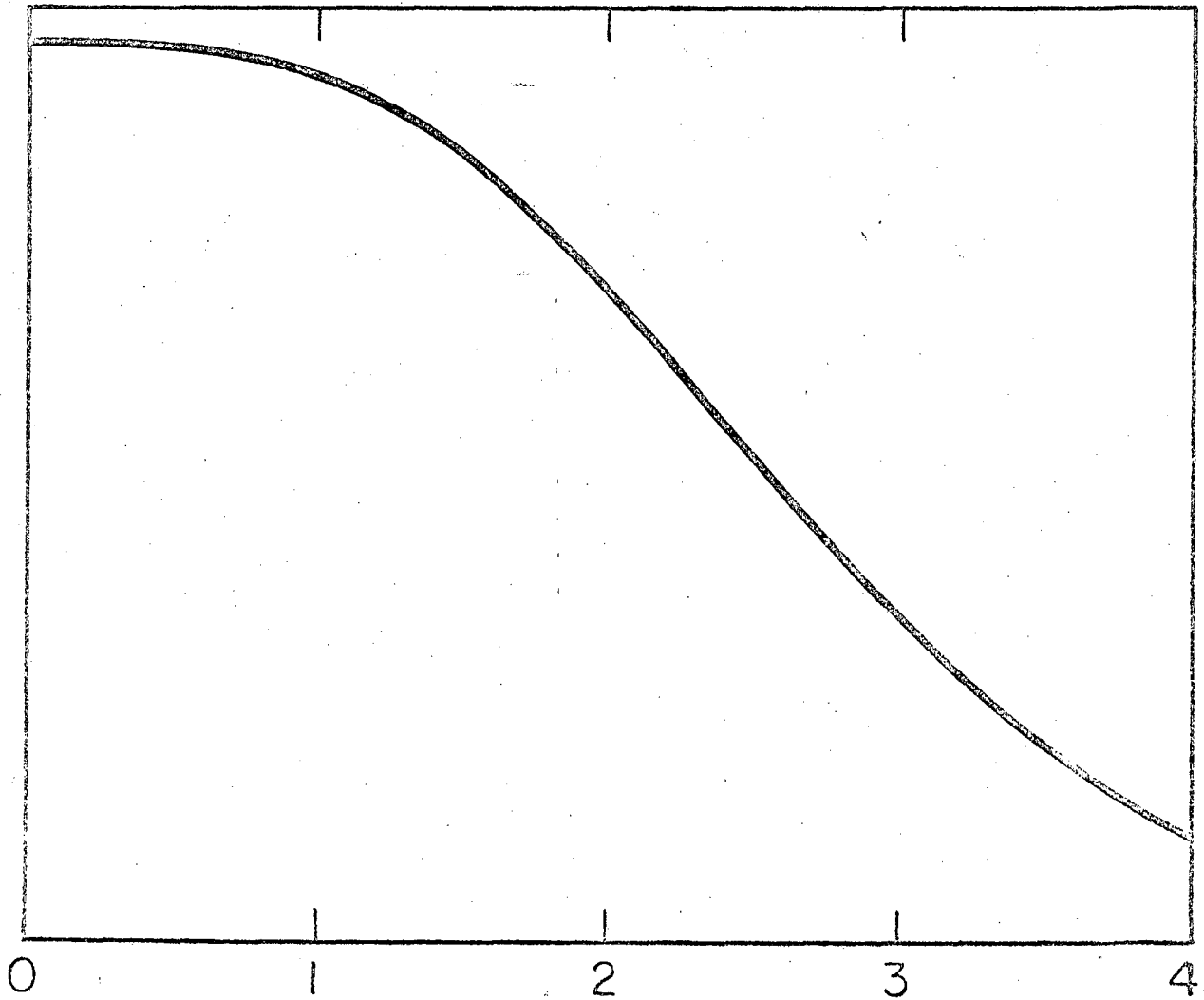


FIGURE 3

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