Title
Sub-additivity in combining infiltration with mechanical ventilation for single zone buildings

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Publication Date
2016-03-01

DOI
10.1016/j.buildenv.2015.12.020

Peer reviewed

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Sub-additivity in combining infiltration with mechanical ventilation

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Building Technologies and Urban Systems Division

October 2015

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ABSTRACT

In determining ventilation rates, it is often necessary to combine naturally-driven infiltration, with air flows from mechanical systems. When there are balanced mechanical systems, the solution is simple additivity, because a balanced system does not impact the internal pressure of the space or the air flows through the building envelope. Unbalanced systems, however, change internal pressures and therefore can impact natural ventilation non-linearly in such a way as to make it sub-additive. Several sub-additive approaches are found in the literature, but they are not robust across the full spectrum from tight to leaky buildings and ranges of mechanical ventilation air flow rates. There are two approaches for combining natural infiltration with mechanical ventilation that require different solutions. The *forward* problem is to find the total air flow when adding mechanical ventilation to natural infiltration, and this application has been investigated in previous studies. The *inverse* problem finds the required mechanical ventilation in order to meet a total ventilation rate given a known amount of natural infiltration. The inverse problem is required to be solved when designing mechanical ventilation systems to comply with ventilation standards. This article develops robust sub-additivity models for use with unbalanced systems appropriate for consensus standards and guidelines for both the forward and inverse problem. The improved models reduce errors to 1% or less and work across the air tightness spectrum.

KEYWORDS

Unbalanced ventilation, infiltration, standards, empirical models, superposition

1. INTRODUCTION

Most homes (and other single zone spaces) are ventilated by infiltration through leaks in the building envelope that are driven by wind and indoor-outdoor temperature differences. In order to decrease energy consumption, building envelopes are getting tighter. Combined with potential increases in pollutant sources in indoor living environments, this raises concerns for indoor air quality (IAQ). As a result, more houses are using a mechanical ventilation system to maintain a good air quality. There are standards for estimating the minimum total air exchange requirements for maintaining acceptable IAQ in homes. Some of these, e.g., ASHRAE 62.2-2013 give credit for infiltration towards the total air flow.

If a balanced ventilation system is installed, the impact on infiltration will not be significant, because the balanced system does not change the pressures across the building leaks. As a result, the total ventilation rate (Q_t) is simply the addition of the balanced fan flow and the natural infiltration.

Unbalanced mechanical ventilation systems modify the indoor pressure of the building, which interacts with the wind and stack induced flows, making the combination of the flows sub-additive. Exhaust fans depressurize the building, which increases the airflow in through the building envelope. The greater the fan flow, the higher proportion of the building envelope experiences inflow. The opposite effect occurs with supply-only systems.

The problem this article addresses is how to combine infiltration with mechanical ventilation in a single zone. This can be done using detailed mass balance physical and mathematical models to find the internal pressure that balances the incoming and outgoing mass flows. Such an approach is powerful, but requires many computational inputs and can be too time consuming for some purposes such as ventilation standards or simplified parametric modeling. Or it may be that the details of the mechanical side and the infiltration side are not available at the same time. An alternative is to use a simple empirical combinational model when the infiltration is determined using simplified approaches and use other methods to combine infiltration with mechanical ventilation.

A few models for combining infiltration and unbalanced mechanical ventilation were suggested and tested a few decades ago, but the results were sometimes contradictory. In this article we develop a new sub-additive approach for improving on previous relationships.

2. SUPERPOSITION BACKGROUND

The approach of combining a pre-calculated infiltration rate with a fixed mechanical ventilation rate to find the total air change (or ventilation) rate is called *superposition*. This is the *forward* problem of finding totals from individual pieces. (The opposite problem of sizing a mechanical ventilation system given infiltration is the *inverse* problem.) Analysis of the physical pressure-flow relationship (supported by measurements) shows that the totals will come out smaller than additivity because the unbalanced fan will impact the internal pressure which effectively reduces the amount infiltration contributes to the total.

The fundamental physics of air flow through leaks shows that these leaks have non-linear pressure-flow relationships, but the addition of pressures themselves is a simple linear phenomenon. Both individual leaks and the combinations of leaks found in building envelopes can be represented by a power law function (Walker et al. 1998), but the details of that are not important here except to realize there is nonlinear relationship between pressure and flow. The pressure difference depends on wind and indoor-outdoor temperature difference and an internal pressure shift that acts to balance the air flow in and out of the building. The operation of an unbalanced mechanical ventilation system changes this internal pressure and therefore the flow through each leak, and does so in a nonlinear way.

Sherman (1992) has shown for some configurations of driving forces and air leakage, superposition will not be symmetric with respect to supply and exhaust flows. Since we are not going to take leakage distribution into account in our modeling, we will ignore that potential asymmetry, in which case we can recast our modeling in terms of balanced and unbalanced flows:

$$\begin{cases} Q_m = Maximum(Q_{supply}, Q_{exhaust}) \\ Q_b = Minimum(Q_{supply}, Q_{exhaust}) \\ Q_u = Q_m - Q_b \end{cases} \qquad (1)$$

Where the “m” subscript denotes the total mechanical flow, the “b” subscript denotes the balanced portion, the “u” subscript the unbalanced portion and all flow rates are positive numbers. The supply and exhaust flows represent the total of all the individual supply and exhaust systems operating at that time.

Throughout this article we will refer to volumetric air flows rather than mass flows. This is for convenience because most, if not all, applications are in terms of volumetric air flow at standard density. In addition, volumetric air flow is related to mass flows by the indoor air density that varies very little (about 1%) over typical ranges of indoor air temperature.

Prior Work

Researchers have used both simplified physical and empirical approaches to develop superposition models since the late 70s, and many were reviewed by Li (1990). However, many of these are optimized for limited situations, such as the Palmiter and Bond (1991) method, referred to here as the half-fan model, which was developed for stack-only natural infiltration.

Li tested ten models by comparing them with a flow model over a range of wind speeds (0 to 8 m/s) and temperature differences (-20 to 20°C) with open and closed exterior doors and two different exhaust fan speeds. His conclusion was that the quadrature combination of natural and mechanical ventilation worked best. This result is in agreement with the earlier work of Modera and Peterson (1985), who also used a mass balance ventilation model.

Field tests with tracer gas measurements by Kiel and Wilson (1987) found that for strong exhaust mechanical ventilation (four times the natural rate), simple linear addition was the most acceptable method, but that from a theoretical point of view, a half-pressure addition and half-linear addition model had more appeal with similar results to the linear addition. (See Table 1 for model definitions.) Continuing this work, Wilson and Walker (1990) looked at a reduced fan flow rate that was approximately equal to the natural rate. The result was the same as Kiel and Wilson, where linear and half-linear/half-pressure addition were the closest to the measured and modeled combined rates. The above two studies looked at exhaust fans only, but over a wide range of natural infiltration driven by both wind and stack effects. Unlike Li, these studies showed large underpredictions using quadrature. This could be due to different building envelope leakage, weather conditions, leakage distributions and strengths of mechanical ventilation, but it mainly underlines the necessity of additional study.

Table 1 is a summary from the literature of published superposition models with some observations on their performance. The table gives the model functional form, the range over which it was evaluated, and if the comparisons were made to simulations (sim.) or experimental (exp.) data.

The most generally accepted of these models is quadrature. It is currently used in the ASHRAE Handbook of fundamentals. Previous versions of the Handbook used the half-fan model. Sherman (1992) reviewed the state of the art of superposition at the time and developed a model of quadrature, deriving some coefficient values based on leakage distribution properties, but this advanced quadrature is not typically used. In this study, our further investigation of previous models will be restricted to quadrature and half-fan as these are historically the most used.

Table 1: Summary of superposition models including source, basis and performance (Hurel, 2015)

Name/Ref	Model	Range	Comparison		
			Ref.	**Sim/Exp**	**Results**
Additivity	$Q_t = Q_u + Q_{inf}$	All	Kiel & Wilson	Exp.	best agreement
			Wilson & Walker	Exp.	overpredicts Q_t by 7%
			Li	Sim.	average error: 33%; max. error: 64%
Simple[1] Quadrature	$Q_t = \sqrt{{Q_u}^2 + {Q_{inf}}^2}$	All	Modera & Peterson	Sim.	good agreement: error on Q_t < 10%
			Kiel & Wilson	Exp.	underpredicts Q_t by 15-30%
			Wilson & Walker	Exp.	underpredicts Q_t by 20%
			Li	Sim.	good agreement: average error: 5%; max. error: 17%
			Palmiter & Bond	Exp.	underpredicts for $Q_{inf}<Q_f$; overpredicts for $Q_{inf}>Q_f$
Half-fan - Palmiter & Bond	$Q_t = \begin{cases} \frac{Q_u}{2} + Q_{inf} & \text{for } Q_u < 2Q_{inf} \\ Q_u & \text{for } Q_u \geq 2Q_{inf} \end{cases}$	Stack only	Palmiter & Bond	Exp.	good agreement
Levins (1982)	$Q_t = Q_{inf} + Q_u . \exp\left(-\frac{Q_{inf}}{Q_u}\right)$	All	Kiel & Wilson	Exp.	underpredicts Q_t by 15-30%
			Li	Sim.	good agreement: average error: 5%; max. error: 20%
Power Law	$Q_t = \left({Q_u}^{\frac{1}{n}} + {Q_{inf}}^{\frac{1}{n}}\right)^n$	All	Modera & Peterson	Sim.	bigger errors on Q_t than the quadrature model
			Kiel & Wilson	Exp.	underpredicts Q_t by 10-25%
			Li	Sim.	average error: 11%; max. error: 30%
Shaw (1985)	$Q_t = \begin{cases} Q_u & \text{for } {h_0}^2 > H \\ F\left({Q_{w-f}}^{\frac{1}{n}} + {Q_w}^{\frac{1}{n}}\right)^n & \text{for } h_0 < H \end{cases}$		Shaw	Exp.	in general within 25% of the measured values
			Kiel & Wilson	Exp.	underpredicts Q_t by 15-30%
Kiel	$Q_t = \sqrt{{Q_u}^2 + (2Q_{inf})^2}$	$Q_u \gg Q_{inf}$	Kiel & Wilson	Exp.	very spread data; overpredicts Q_t when Q_f<0.7Q_t ; otherwise mostly underpredicts Q_t
			Li	Sim.	average error: 56%; max. error: 100%
Li	$Q_t = \left({Q_u}^{\frac{1}{n}} + (2Q_{inf})^{\frac{1}{n}}\right)^n$	$Q_u \gg Q_{inf}$	Li	Sim.	average error: 98%; max. error: 160%
Kiel & Wilson	$Q_t = \sqrt{\left(\frac{Q_u}{2}\right)^2 + {Q_{inf}}^2} + \frac{Q_u}{2}$	All (Exhaust fan)	Kiel & Wilson	Exp.	underpredicts Q_t by 10-30%
			Li	Sim.	average error: 12%; max. error: 35%
			Palmiter & Bond	Exp.	overpredicts the fan efficiency
Wilson & Walker	$Q_t = \left(\left(\frac{Q_u}{2}\right)^{\frac{1}{n}} + {Q_{inf}}^{\frac{1}{n}}\right)^n + \frac{Q_u}{2}$	All (Exhaust fan)	Wilson & Walker	Exp.	underpredicts Q_t by 7%
			Li	Sim.	average error: 18%; max. error: 42%
Li	$Q_t = \frac{1}{2}\sqrt{{Q_u}^2 + (2Q_{inf})^2}$	$Q_u < Q_{inf}$	Li	Sim.	average error: 22%; max. error: 50%
Li	$Q_t = \frac{1}{2}\left({Q_u}^{\frac{1}{n}} + (2Q_{inf})^{\frac{1}{n}}\right)^n$	$Q_u < Q_{inf}$	Li	Sim.	average error: 21%; max. error: 50%

[1] See (Sherman, 1992) for general/advanced quadrature

[2] Height of neutral level compared to ceiling

3. CASES OF INTEREST

When selecting superposition models it can be important to consider the difference between short-term (typically hourly) flows and long-term (typically annual average flows.) Even if the mechanical system flow rate were constant, infiltration varies in time due to changes in weather. Since flows through leaks typical of infiltration are non-linear, there is no reason to believe that the best superposition model for hourly air flows is the best one for annual average air flows. Sherman & Wilson (1986) show that for IAQ purposes providing the effective average ventilation is also a non-linear process because internally generated pollutant concentrations are inversely proportional to the ventilation rate.

Superposition addresses the *forward* problem of finding the total when the infiltration and mechanical ventilation are known. Sometimes one wishes to solve the *inverse* problem of determining the mechanical ventilation required to meet a desired total—for example, when trying to meet a minimum ventilation requirement such as ASHRAE Standard 62.2. Although not always easy to do, the equations in Table 1 could be mathematically inverted. Because of the non-linearities mentioned above, however, that would not necessarily lead to the best approach for solving the inverse problem.

To be thorough we need to look at four related but slightly different applications depending on whether the time-period of concern is short or long and whether it is a forward or inverse application. Specifically we consider the following:

- Hourly, Forward Case: for the hourly air change rate prediction that is useful for estimating energy loads and is needed for relative exposure calculations (that have applications in dynamic ventilation systems (Sherman and Walker (2011)). This application is used, for example, in the ASHRAE Handbook of Fundamentals and is the canonical superposition problem.
- Annual, Forward Case: predicting the annual effective ventilation given the effective infiltration and a fixed (or effective) fan flow; for IAQ purposes.
- Hourly, Inverse Case: when one wants to vary the fan size each hour to compensate for varying hourly infiltration in order to keep the total ventilation constant. This is not much used, but could be in future smart ventilation system controls.
- Annual, Inverse Case: for finding the fixed fan size that will combine with effective infiltration to produce a desired total ventilation; useful for building codes/standards applications such as ASHRAE Standard 62.2 (ASHRAE 2013).

4. SUB-ADDITIVY APPROACH

We can define a general approach to combining total air flows (Q_t), mechanical air flows (Q_m) and infiltration air flows (Q_{inf}) that includes a sub-additivity function, ϕ. For *forward* cases the sub-additivity model predicts Q_t:

$$Q_{t,predicted} = Q_m + \phi Q_{inf} \tag{2}$$

and for *inverse* cases the model predicts Q_m:

$$Q_{m,predicted} = Q_t - \phi Q_{inf} \tag{3}$$

The sub-additivity coefficient, ϕ, will depend on system details and may have a different functional form for different situations. Note that for purely balanced mechanical ventilation then ϕ is unity. Our interest is often in the unbalanced case, but we will solve the problem for the general case where there could be both balanced and unbalanced mechanical ventilation at the same time.

Previous studies have selected functional forms for superposition and then tested them against their limited data. Field data of the quality necessary to determine a functional form for the sub-additivity coefficient over a broad range of factors has never existed, thus limiting its probative value. We have elected to use a simulation approach so that all the important parameters of interest can be exercised and that calculations of the combined effects of infiltration and mechanical ventilation can be determined. Simulation allows the calculation of Q_{inf} and Q_t for exactly the same house and weather conditions – this simultaneous data is not available from field studies as one can only measure the total and mechanical air flow rates directly and the co-incident infiltration cannot be measured. The simulations used the single-zone mass-balance multi-leak program REGCAP to generate minute-by-minute combined air flows; four different envelope leakage levels (form 0.6 ACH50 to 10 ACH50); three different ventilation systems (supply, exhaust, and balanced); one two, and three stories single-zone buildings (that give different envelope leakage distributions); three different foundation types (slab, crawlspace, basement) using weather data from eight different climates (Miami, Houston, Memphis, Baltimore, Chicago, Burlington, Duluth, Fairbanks). More details of the simulation procedure can be found in Hurel (2015).

REGCAP was first used to calculate Q_{inf} for all combinations of these parameters. REGCAP was then run with the added mechanical ventilation to obtain Q_t for the same parameter combinations. The output of the simulation was roughly 500 million minute-by-minute data points that we reduced to 7.5 million hourly averaged points, representing over 850 different building/climate configurations. For each parameter combination we have Q_t, Q_m and Q_t. These allow for the evaluation of different sub-additivity coefficients by comparing $Q_{t,predicted}$ or $Q_{m,predicted}$ from Equations 2 and 3 to those achieved/used in the simulations.

Sub-Additivity Model Evaluation

In evaluating different models, it is important that they work over a wide span of air leakage values/infiltration values and mechanical ventilation air flow rates. The range of methods used in previous research has shown that the selecting the optimum model can depend critically on the relative contribution of infiltration. We therefore will explore the performance of the models as a function of the infiltration fraction, α.

$$\alpha = \frac{Q_{inf}}{Q_t - Q_b} \tag{4}$$

This parameter is used in our figures and can be calculated for all the cases of interest using simulation data, but in real applications it can only be easily calculated for inverse problems. There is an alternative definition for infiltration fraction which is useful in the *forward* problem, but is numerically a bit different particularly when the unbalanced flows and the infiltration are roughly the same:

$$\alpha' = \frac{Q_{inf}}{Q_{inf} + Q_u} \tag{5}$$

We selected the error in the flow rate predicted by the model relative to the total flow as the metric we wish to minimize in evaluating any model. For the *forward* case the model error is as follows:

$$\varepsilon_t = \frac{Q_{t,predicted}}{Q_t} - 1 \tag{6}$$

And for the *inverse* case the model error is as follows (note that in our evaluation we only considered unbalanced mechanical ventilation so Equation 7 is presented in both forms):

$$\varepsilon_m = \frac{Q_{m,predicted} - Q_m}{Q_t} = \frac{Q_{u,predicted} - Q_u}{Q_t} \tag{7}$$

For the same data these two metrics have opposite signs—reflecting the fact that an over-prediction of the impact of infiltration will cause a positive bias in the prediction of the total, but a negative bias in the prediction of the necessary mechanical ventilation

Figure 1 displays these errors for two of the four cases using the 3 most commonly used superposition models (the first three models in Table 1). In this figure there are 864 points plotted for the annual plot. Since there are over 7.5 million hourly points for each plot, we have binned the data into a box and whisker plot every α increment of 0.05, where the box contains the middle 50% of the data and the mean of the data is shown by the horizontal bar.

Of the models presented in Figure 1, Additivity—which works perfectly for balanced flows—is the poorest when considering unbalanced flows. This is, of course, why people were motivated to develop superposition models. As the forward model shows infiltration is always sub-additive, but how sub-additive it is depends on the relative contribution of the infiltration, with the largest errors when infiltration and mechanical ventilation are roughly equal.

Both simple quadrature and the half-fan model are improvements over additivity, but each demonstrates systematic deviations as a function of infiltration fraction and has some areas of notably poor performance. These models are traditionally applied to the forward problem and qualitatively similar with each having superior performance in different infiltration regimes. The large dataset enables us to develop improved sub-additivity models with far less systematic deviations.

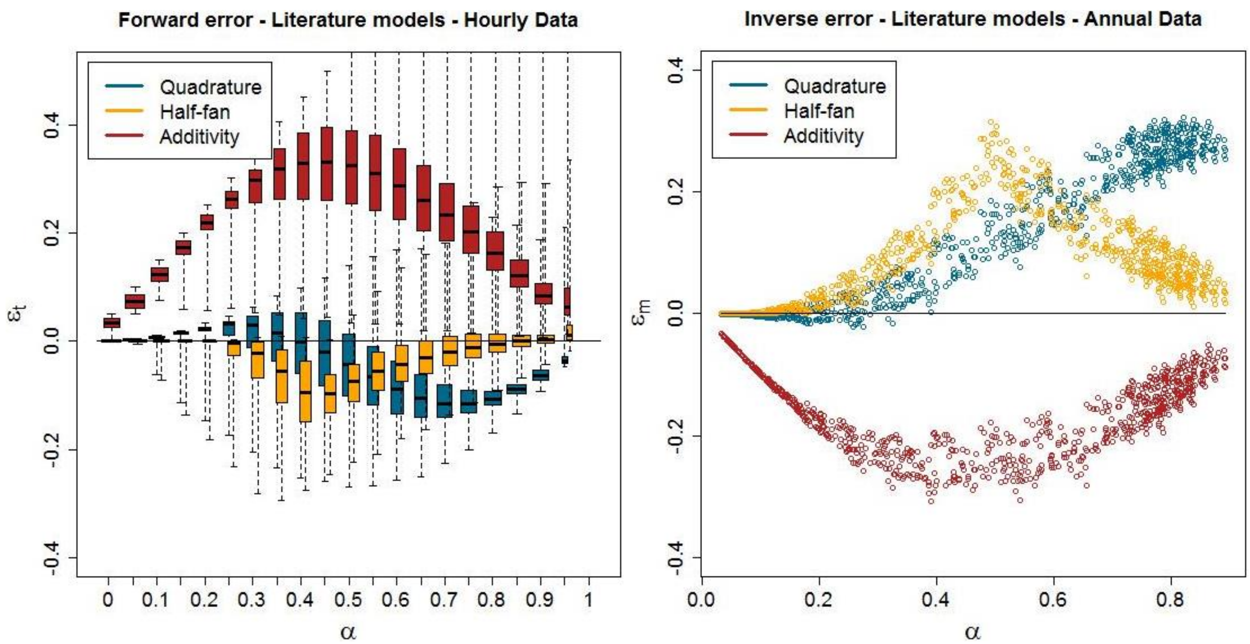

Figure 1: Error in unbalanced flows from 3 basic superposition models a) error in forward, hourly model, b) error in inverse annual model.

5. SUB-ADDITIVITY MODEL DEVELOPMAENT AND ANALYSIS

Before developing any specific models it is instructive to examine how the sub-additivity coefficient varies as a function of the infiltration fraction based solely on the simulation data. Figure 2 presents that data for the four cases of interest for unbalanced systems. We can see that the supply fans and exhaust fans have slightly but noticeably different curves. It is beyond the scope of this work, but in principle better fits would be obtained if treating the two separately.

Each of the four plots is slightly different, suggesting that we will likely need four different models, but there are some commonalities to note: 1) the additivity coefficients are all monotonically increasing from zero to unity as the infiltration fraction increases from zero to unity; 2) there is a value of infiltration fraction (roughly 10-20%) below which infiltration does not contribute in any significant way—the mechanical ventilation and the total are functionally the same.

Hurel et al. (2015) found that advanced quadrature models were superior to the 3 basic superposition models. However, advanced superposition relationships are complex (particularly to the inverse problem) and did not give as good a result as the Exponential model so we did not include the more general quadrature approach in this paper.

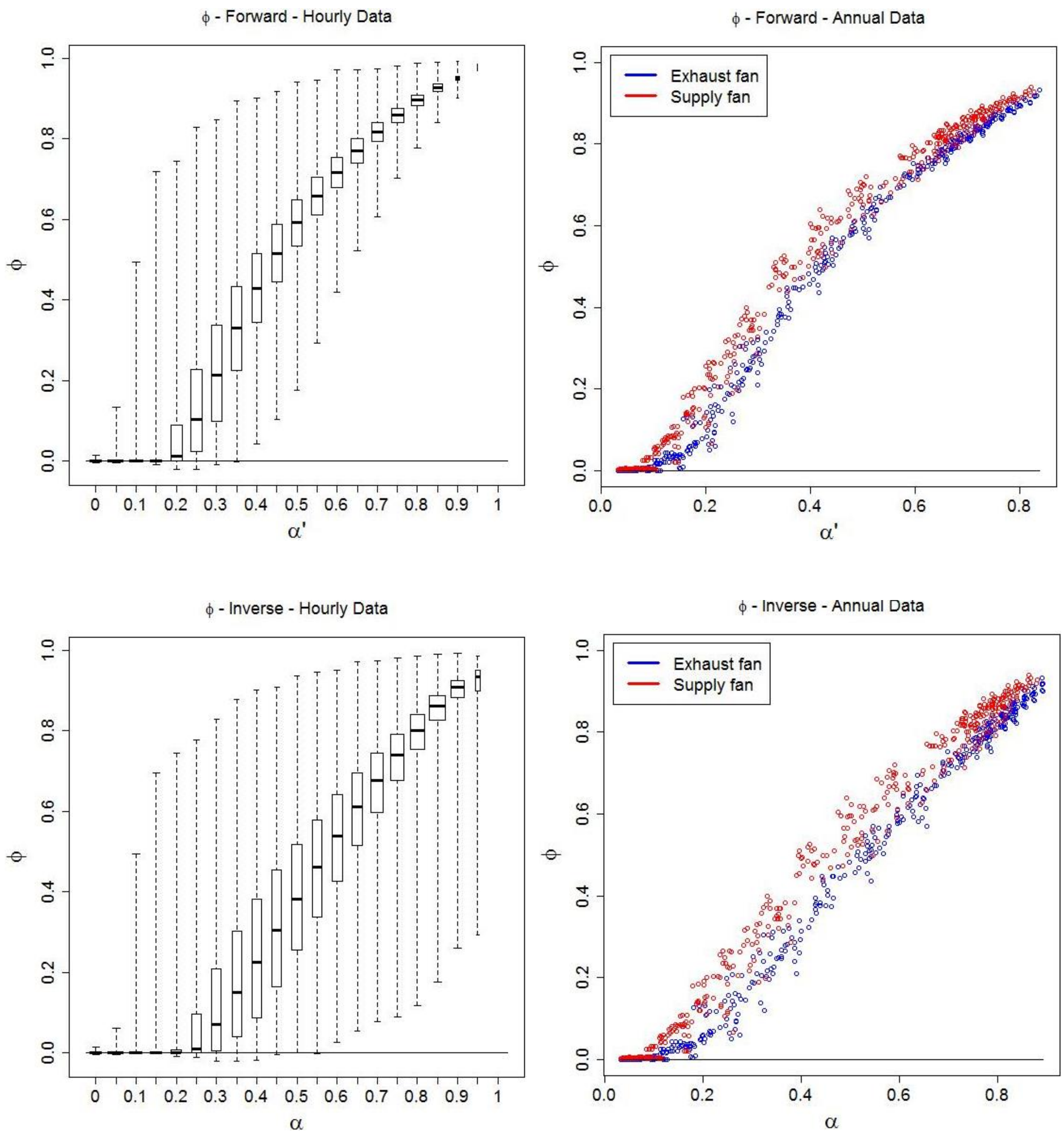

Figure 2: Sub-additivity coefficient (Φ) as a function of infiltration fraction for the four applications of interest

Simple Model

These observations suggest that we should explore some models of a different functional form from the superposition models of the literature. The simplest thing to do that visually looks like it will match the data well is a *simple* linear (in infiltration) model. For the *forward* case we can express this as

$$\phi = \frac{Q_{inf}}{Q_u + Q_{inf}} = \alpha' \tag{8}$$

For the *inverse* case we can express this as

$$\phi = \frac{Q_{inf}}{Q_t - Q_b} = \alpha \tag{9}$$

Exponential Model

The simple model does not take into account the low infiltration fraction behavior observed in the data. To take this into account we need a more complex model. We have developed an *exponential* model that will allow the "knee" in the curve that is observed in the data.

For the *forward* case the model is of the following form:

$$\phi = e^{-kQ_u/Q_{inf}} = e^{-k(1/\alpha\prime - 1)} \tag{10}$$

For the *inverse* case it is as follows

$$\phi = e^{-k((Q_t - Q_b)/Q_{inf} - 1)} = e^{-k(1/\alpha - 1)} \tag{11}$$

Unlike the simple model, the exponential model has an adjustable parameter, k, in it. We used the calculated sub-additivity coefficient data to fit for the best value of the adjustable parameter. The results are in Table 2.

Table 2: Best fit values of adjustable parameter, k. (Hurel 2015)

	Forward	Inverse[3]
Hourly	2/3	1
Annual	4/9	2/3

6. MODEL COMPARISON

We can use the error metrics in Equations 6 and 7 to evaluate the performance of the models. For each simulated point and for each model we can construct an error. The average of these errors for any one

[3] If only balanced ventilation is used then Q_m=Q_b=Q_t - Q_{inf}

case of the model gives the bias. The bias tells us if the model is overall high or overall low; we prefer a zero bias.

The root-mean-square (RMS) error, tells us how far off a single application of the model is likely to be. It is always positive and always larger than (the absolute value of) the bias. Table 3 lists the bias and RMS errors for the original three superposition models and the two new sub-additivity models for both hourly (hr) and annual (yr) calculations.

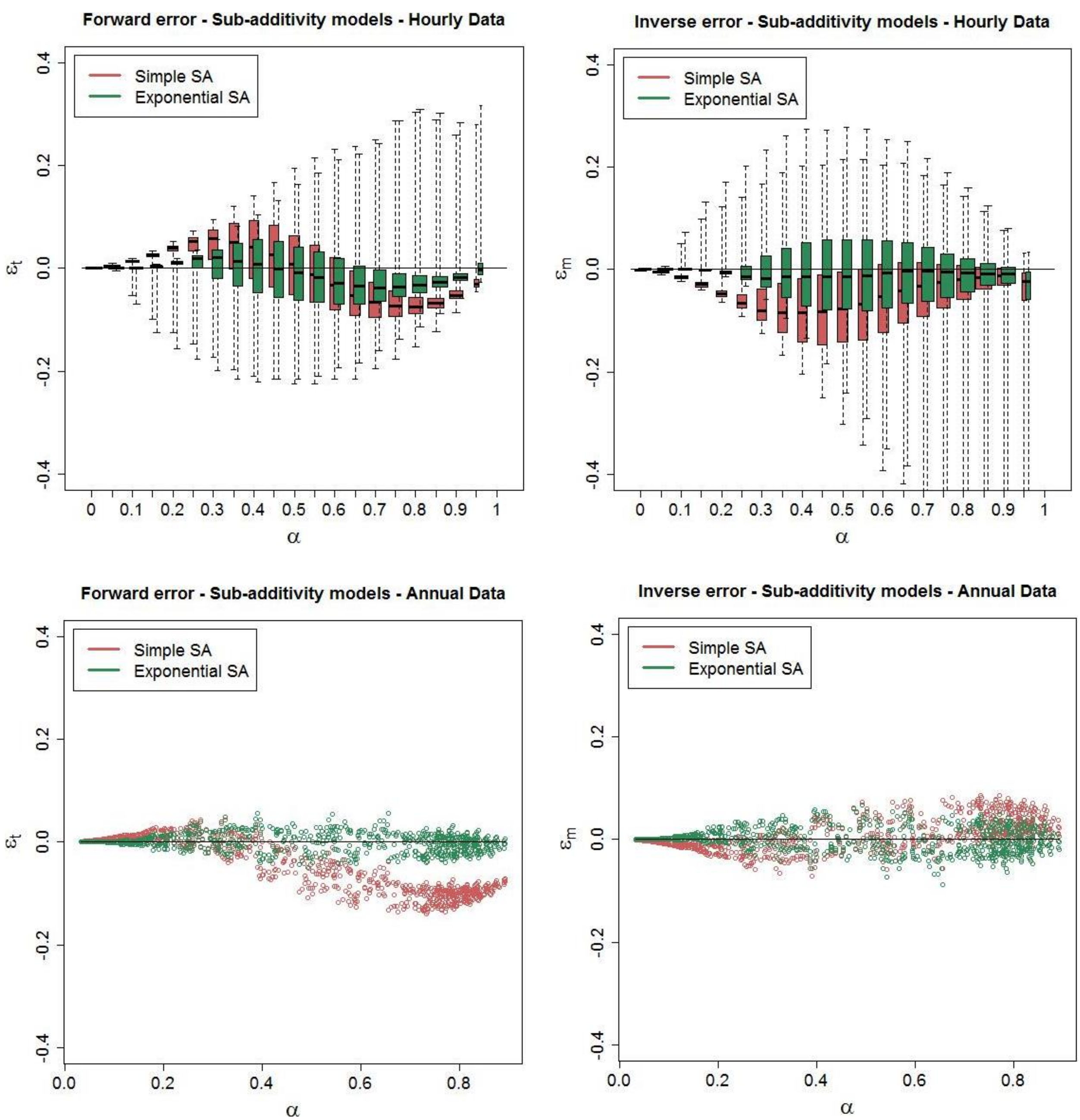

Figure 3: Error in unbalanced flows from the sub-additivity models for the four applications of interest

Table 3: Prediction Errors in Unbalanced Flows for Superposition and Sub-Additivity Models

Model	Forward error				Inverse error			
	Bias(hr)	RMS(hr)	Bias(yr)	RMS(yr)	Bias(hr)	RMS (hr)	Bias(yr)	RMS(yr)
Additivity	21.2%	22.1%	17.6%	17.8%	-21.2%	21.1%	-17.6%	17.8%
Quadrature	-3.7%	6.4%	-7.3%	7.7%	7.7%	10.9%	11.5%	12.0%
Half-fan	-2.6%	4.6%	-6.2%	6.4%	3.9%	7.4%	9.6%	9.9%
Simple	-0.8%	5.7%	-4.31	5.4%	-4.4%	7.3%	0.5%	2.8%
Exponential	-0.9%	4.0%	-0.1%	1.6%	-0.9%	5.6%	-0.1%	2.1%

The models are listed roughly in the order from worst to best. Additivity serves at the scale by which to judge the usefulness of the other models with typical errors of 20% on hourly data and 17% on annual data. The Exponential model is the best having biases typically below 1% and RMS errors at least a factor of four (and sometimes an order of magnitude) better than additivity.

Visually the Exponential and Simple models look much better than the extant approaches. Figure 3 displays the errors for the four cases on the same scale as Figure 1. Neither of these models exhibit the areas of poor performance or systematic trends shown by the superposition models from the literature. While the Exponential model is arguably the best one to use from an uncertainty point of view, there may be applications where other factors, such as ease of use, may suggest the simple approach.

7. APPLICATIONS

1. Estimating total airflow

The traditional use of superposition models is to combine hourly infiltration—often calculated with a simplified physical model—with a specific mechanical ventilation system. Equation 2 can be used in this case, where the mechanical ventilation is the total mechanical ventilation (i.e. the sum of balanced and unbalanced flows), but the sub-additivity coefficient must be based only on the unbalanced part of the mechanical ventilation. Using our Exponential model this would be:

$$\phi = e^{-2Q_u/3Q_{inf}} = e^{-2(1-\alpha')/3\alpha'} \tag{12}$$

For time series data (e.g., hourly infiltration rates for a year and/or time varying mechanical system air flows) this calculation can be repeated for each time interval.

If superposition were desired for long-term, such as annual averages, the Exponential model should still be used but with slightly different values.

$$\phi = e^{-4Q_u/9Q_{inf}} = e^{-4(1-\alpha')/9\alpha'} \tag{13}$$

The Simple model is almost as good as the Exponential model and may be more desirable for practical reasons in which case Equation 8 can be used for ϕ, and Equation 2 becomes:

$$Q_t = Q_m + \frac{Q_{inf}^2}{Q_u + Q_{inf}} \quad (14)$$

and we do not distinguish between long term and short term calculations.

2. Fan Sizing

The inverse model is used when determining the role of infiltration in sizing a mechanical ventilation system. A typical application is the problem that occurs when one is trying to meet a target total ventilation rate for some standard, guideline or program, such as ASHRAE Standard 62.2.

Up through the 2013 version of Standard 62.2 the mechanical ventilation was sized by subtracting the effective annual infiltration from the total requirement. From Figure 1, it is clear this could undersize an unbalanced ventilation system substantially (by about 17%). Using Equation 3 and the Exponential sub-additivity values for ϕ,

$$\phi = e^{-2(Q_t - Q_b - Q_{\text{inf}})/3Q_{inf}} = e^{-2(1-\alpha)/3\alpha} \quad (15)$$

reduces the RMS error to about 2%. Note that for the ASHRAE 62.2 application we used the annual coefficients in ϕ because the infiltration rate is calculated as an effective annual average in the standard. If real-time fan flow rate calculations were being performed then the following would be used:

$$\phi = e^{-(Q_t - Q_b - Q_{\text{inf}})/Q_{inf}} = e^{-(1-\alpha)/\alpha} \quad (16)$$

Almost as good is the Simple model of Equation 9. The uncertainty of this model is arguably the same but the implementation is easier. It is also the easiest to describe: When you are talking about sizing with an unbalanced system, the fraction of infiltration that be used to reduce the fan size is the fraction infiltration compared to the desired total.

$$Q_m = Q_t - \frac{Q_{inf}}{Q_t - Q_b} Q_{inf} = Q_t - \frac{Q_{inf}^2}{Q_t - Q_b} \quad (17)$$

Equation 17 works for the case where there is either unbalanced mechanical ventilation or both balanced and unbalanced. For the case of balanced only $Q_m=Q_b=Q_t$. The Exponential model has different numerical values depending on whether the model is being applied to short-term (hourly) or long-term (annual) problems. The Simple model does not, which makes it a bit easier to apply. The Simple model works best for forward-hourly and inverse-annual calculations that are the ones most usually used. In the forward-annual and inverse-hourly cases the Simple model is noticeably less accurate, but those two cases are rarely as much interest.

8. SUMMARY & CONCLUSIONS

We have shown that when dealing with the problem of combining unbalanced mechanical ventilation and infiltration, the concept of infiltration sub-additivity considerably reduces errors. The concept is quantified by the ***sub-additivity coefficient*** that reduces the contribution of infiltration. This coefficient is unity for purely balanced mechanical ventilation flows. For unbalanced mechanical flows the coefficient goes monotonically from zero to unity as a function of the fractional infiltration.

The functional form of the coefficient is not determinable in general from first principles. Instead we empirically determined appropriate functions using a large set of simulated data. The resulting models were then evaluated by comparing them to the simulated data set. This process resulted in models which were substantially improved over the ones in the literature and generally reduce the error from superposition well below the errors associated with other factors common in such modeling. Table 4 displays the best solutions we found.

Table 4: Best sub-additivity models

	Forward	Inverse
Short-term	$Q_t = Q_m + Q_{inf} e^{-2Q_u/3Q_{inf}}$	$Q_m = Q_t - Q_{inf} e^{-(Q_t - Q_b - Q_{inf})/Q_{inf}}$
Long-term	$Q_t = Q_m + Q_{inf} e^{-4Q_u/9Q_{inf}}$	$Q_m = Q_t - Q_{inf} e^{-(2/3)(Q_t - Q_b - Q_{inf})/Q_{inf}}$

The solution for the forward problems (i.e. that of determining combined flow rate of infiltration and mechanical ventilation) is similar but a bit different from that of inverse problem (i.e. that of finding what mechanical ventilation is necessary to achieve a given total with a given infiltration) because different variables are known and the process of air leakage is non-linear.

The solution for short-term and long-term is also a bit different because of the way variable ventilation rates average over time with respect to indoor air quality. The short-term result was based on hourly data and is reasonable for time basis of the order of the turn-over time or shorter. The long-term result was derived from annual data and is reasonable when the time basis is much longer than the turn-over time.

The issue of time basis is a complicating factor and exponentials may also be inconvenient to use. The Simple model (See Table 5) may be used to avoid these practical or aesthetic issues with only a small increase in uncertainty.

Table 5: Simple models

Forward	Inverse[4]
$Q_t = Q_m + \frac{Q_{inf}^2}{Q_{inf} + Q_u}$	$Q_m = Q_t - \frac{Q_{inf}^2}{Q_t - Q_b}$

[4] If only balanced ventilation is used then $Q_m = Q_b = Q_t - Q_{inf}$

Because these models were empirically determined, one does not know their range of applicability beyond the data used to confirm them. While the data was reasonably broad with respect to single-family, detached homes, it did not include attached or multifamily configurations and so it is not clear how natural and unbalanced ventilation would combine in such circumstances.

9. REFERENCES

ASHRAE Handbook of Fundamentals, ASHRAE. (2013). Atlanta, GA: ASHRAE.

ASHRAE Standard 62.2-2013, Ventilation and Acceptable Indoor Air Quality in Low-Rise Residential Buildings. (2013). Atlanta, GA: ASHRAE.

Hurel, N., Sherman, M. H., & Walker, I. S. (2015). *Simplified Methods for Combining Natural and Mechanical Ventilation.* Lawrence Berkeley National Laboratory Report 184001.

Kiel, D., & Wilson, D. (1987). Influence of natural infiltration on total building ventilation dominated by strong fan exhaust. *ASHRAE Transactions vol. 93 part 2*.

Levins, W. P. (1982). Measured Effect of Forced Ventilation on House Infiltration Rate. *ASHRAE/DOE Thermal Performance of the Exterior Envelopes of Buildings II Conference.* Las Vegas.

Li, Y. (1990). Simplified Method of Combining Natural and Exhaust Ventilation. *Climate and Buildings vol.2*, 29-35.

Modera, M., & Peterson, F. (1985). *Simplified Methods for Combining Mechanical Ventilation and Natural Infiltration.* Lawrence Berkeley National Laobratory report no. LBL-18955.

Palmiter, L., & Bond, T. (1991). Interaction of mechanical systems and natural infiltration. *12th AIVC Conference - Air Movement and Ventilation Control Within Buildings*, (pp. 285-295). Ottawa, Canada.

Shaw, C. Y. (1985). *Methods for Estimating Air Change Rates and Sizing Mechanical Ventilation Systems for Houses.* Division of Building Research, National Research Council of Canada, Building Research Note 237.

Sherman, M. H. (1992). Superposition in Infiltration Modeling. *Indoor Air*, *2*(2), 101-114.

Sherman, M. H., & Wilson, D. J. (1986). Relating Actual and Effective Ventilation in Determining Indoor Air Quality. *Building and Environment, 21 (3/4)*, 135-144.

Walker, I. S., Forest, T. W., & Wilson, D. J. (2005). An Attic-Interior Infiltration and Interzone Transport Model of a House. *Building and Environment*, 701-718.

Wilson, D., & Walker, I. S. (1990). Combining Air Infiltration and Exhaust Ventilation. *IndoorAir '90*, (pp. 467-472). Toronto, Canada.

10. ACKNOWLEDGEMENTS

Funding was provided by the U.S. Dept. of Energy under Contract No. DE-AC02-05CH11231, by the French National Research Agency (ANR) through its Sustainable Cities and Buildings program (MOBAIR project n°ANR-12-VBDU-0009) and by the Région Rhône-Alpes.