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Modeling and Validation of Performance Limitations for the Optimal Design
of Interferometric and Intensity-Modulated Fiber Optic Displacement Sensors

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy

in

Structural Engineering

by

Erik Allan Moro

Committee in charge:

Michael Todd, Chair
Charles Farrar
William Hodgkiss
Francesco Lanza di Scalea
Mark Zumberge

2012

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Chair

University of California, San Diego

2012

Even youths grow tired and weary, and young men stumble and fall;

but those who hope in the Lord will renew their strength.

Isaiah 40:30-31

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LIST OF ABBREVIATIONS

ANOVA	Analysis of Variance
DAQ	Data Acquisition
DFT	Discrete Fourier Transform
DSP	Digital Signal Processing
DUT	Device under Test
EFPI	Extrinsic Fabry-Perót Interferometer
EM	Electromagnetic
FBG	Fiber Bragg Grating
FSR	Free Spectral Range
FWHM	Full-Width Half Maximum
GA	Genetic Algorithm
LP ₀₁	Primary, Linearly Polarized Optical Mode
OPL	Optical Path Length
PDF	Probability Density Function
PDL	Polarization Dependent Loss
PIRT	Phenomenon Identification Ranking Table
PSD	Power Spectral Density
RMSE	Root Mean Square Error
SLD	Superluminescent Diode
SNR	Signal to Noise Ratio
TEC	Thermoelectric Cooler

LIST OF SYMBOLS

a	Radius of single mode fiber core (m)
B	Cost function exponent
C	Cost (optimization cost function)
c	Speed of light in a vacuum ($=2.998 \times 10^8$ m/s)
D	Induced Doppler shift
E	Linear-modeling error, calculated for cost function (%)
E_{Rob}	Robustness error, calculated for cost function (%)
E_0	Electric field amplitude
$\tilde{E}_{circ}$	Circulating electric field (within Fabry-Perôt cavity)
$\tilde{E}_{inc}$	Incident electric field
$\tilde{E}_{inc,D}$	Doppler-shifted incident electric field
$\tilde{E}_{refl}$	Reflected electric field, as seen by photodetector
$\tilde{E}_{trans}$	Transmitted electric field
$\tilde{E}$	Electric Field at photodetector
$\tilde{E}_d$	Doppler-shifted electric field
$\tilde{E}_0$	Un-shifted electric field
e	Base of the natural logarithm ($=2.718$)
F	Fabry-Perôt cavity finesse
f	Frequency of propagating electromagnetic wave (Hz)
f_b	Beat frequency induced by Doppler-shifting (Hz)
f_{DAQ}	Sampling frequency of data acquisition system (Hz)
f_d	Doppler-shifted frequency (Hz)
f_{range}	Frequency range covered by tuned filter (Hz)
f_{sweep}	Sweep speed of tuned filter (Hz)
f_0	Un-shifted (nominal) frequency (Hz)
f_R	Time-domain frequency observed by the EFPI's photodetector that results directly from Fabry-Perôt resonance properties (Hz)
G	Fabry-Perôt cavity property
g_{rt}	Round trip Fabry-Perôt cavity gain
I	Electromagnetic intensity (W/m^2)
I_d	Electromagnetic intensity of Doppler-shifted signal (W/m^2)
I_0	Electromagnetic intensity of un-shifted signal (W/m^2)

I_{inc}	Incident electromagnetic intensity (W/m ²)
I_{refl}	Reflected electromagnetic intensity (W/m ²)
i	Positive integer
J_0	Bessel Function of the first kind (zeroeth order)
j	Imaginary unit ($=\sqrt{-1}$)
K	Discrete Fourier transform frequency bin
K_0	Modified Bessel function of the second kind (zeroeth order)
k	Wavenumber of electromagnetic field (rad/m)
L	Fabry-Perôt cavity length (m)
L_b	Cavity-length-bias caused by Doppler-induced beating
L_f	Tunable filter's (Fabry-Perôt) cavity length (m)
L_{max}	Discrete Fourier transform maximum displacement range (m)
L_{res}	Discrete Fourier transform displacement resolution (m)
N	Integer
n	Fabry-Perôt cavity refractive index
n_1	Refractive index of Surface 1
n_2	Refractive index of Surface 2
$NFFT$	Length of discrete Fourier transform
P_{in}	Optical lever input (source) power (mW)
P_{out}	Optical lever measured power (mW)
P_1	Measured power level of Group 1 (W)
P_2	Measured power level of Group 2 (W)
R	Fabry-Perôt cavity free spectral range (Hz)
(r, θ, z)	Polar coordinates
(r', θ')	Polar coordinates
(r_d, θ_d, z_d)	Polar coordinates
(r_1, t_1)	Amplitude reflection and transmission coefficients of Surface 1
(r_2, t_2)	Amplitude reflection and transmission coefficients of Surface 2
(r_f, t_f)	Amplitude reflection and transmission coefficients for tunable filter
S	Linear-model sensitivity, calculated for cost function (mW/mW)
t	Time coordinate (s)
u	Normalized transverse propagation constant
v	DUT velocity (m/s)
w	Normalized transverse attenuation constant

w_E	Optimization cost function weight
w_R	Optimization cost function weight
w_S	Optimization cost function weight
x	Position coordinate (Cartesian space) (m)
α_0	Fabry-Perôt cavity round trip attenuation constant
Δf	Frequency-range spanned by tunable filter
Δt	Time-duration per tunable filter sweep
ε_0	Permittivity of free space ($=8.854 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2$)
η	Coupling efficiency (normalized measured power)
λ	Wavelength of electromagnetic wave (m)
σ_{UP}	First quartile of uncertainty on measured power level (W)
τ	Fabry-Perôt cavity round trip travel time (s)
τ_S	Fabry-Perôt cavity $1/e$ storage travel time (s)
ψ	Propagating electromagnetic field
ψ_0	Amplitude of propagating electromagnetic field
ψ_{in}	Input (transmitted) electromagnetic field
ψ_{out}	Output (measured) electromagnetic field
ω	Frequency of electromagnetic field (rad/s)

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PUBLICATIONS

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Peer Reviewed Journal Articles

- *E. A. Moro, M. D. Todd, and A. D. Puckett, "Phase-diversity approach for simultaneous measurement of velocity and displacement using a white light Fabry-Perót interferometer," *Journal of Lightwave Technology*, In Review (2012).
- *E. A. Moro, M. D. Todd, and A. D. Puckett, "Dynamics of a non-contacting, white light Fabry-Perót interferometric displacement sensor," *Applied Optics*, doc. ID 162246 (posted 7 May 2012, in press).
- *E. A. Moro, M. D. Todd, and A. D. Puckett, "Using a validated transmission model for the optimization of bundled fiber optic displacement sensors," *Applied Optics*, 50 (35), 6526-6535 (2011).
- *E. A. Moro, M. D. Todd, and A. D. Puckett, "Experimental validation and uncertainty quantification of a single mode optical fiber transmission model," *Journal of Lightwave Technology*, 29 (6), 856-863 (2011).
- S. G. Taylor, K. M. Farinholt, E. B. Flynn, E. Figueiredo, D. L. Mascranas, E. A. Moro, G. Park, M. D. Todd, and C. R. Farrar, "A mobile-agent-based wireless sensing network for structural monitoring applications," *Measurement Science and Technology*, 20 (4), (2009).

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- E. A. Moro, M. D. Todd, and A. D. Puckett, "Uncertainty analysis of a full state estimator that utilizes white light Fabry-Perót interferometry."
- E. A. Moro, M. D. Todd, and A. D. Puckett, "Simultaneous measurement of displacement and velocity using white light extrinsic Fabry-Perót interferometry."

Conference Proceedings

- *E. A. Moro, M. D. Todd, and A. D. Puckett, "Performance limitations of a white light extrinsic Fabry-Perot interferometric displacement sensor," *Proc. SPIE 8370, 83700P: Fiber Optic Sensors and Applications IX*, Baltimore, MD, 2012; doi: 10.1117/12.914709.
- *E. A. Moro, M. D. Todd, and A. D. Puckett, "Performance optimization of bundled fiber optic displacement sensors," *Proc. SPIE 8346, 83460S: Smart Sensor Phenomena, Technology, Networks, and Systems 2012*, San Diego, CA, 2012; doi: 10.1117/12.882869.
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FIELDS OF STUDY

Major Field: Structural Engineering

Studies in Optical Fiber Sensing
Professor Michael Todd

Studies in Structural Dynamics
Professor Michael Todd

Studies in Digital Signal Processing
Professor Bill Hodgkiss

Studies in Advanced Structural Behavior
Professors Francesco Lanze di Scalea and Charles Farrar

ABSTRACT OF THE DISSERTATION

Modeling and Validation of Performance Limitations for the Optimal Design
of Interferometric and Intensity-Modulated Fiber Optic Displacement Sensors

by

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Doctor of Philosophy in Structural Engineering

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Optical fiber sensors offer advantages over traditional electromechanical sensors, making them particularly well-suited for certain measurement applications. Generally speaking, optical fiber sensors respond to a desired measurand through modulation of an optical signal's intensity, phase, or wavelength. Practically, non-contacting fiber optic displacement sensors are limited to intensity-modulated and interferometric (or phase-modulated) methodologies.

Intensity-modulated fiber optic displacement sensors relate target displacement to a power measurement. The simplest intensity-modulated sensor architectures are not robust to environmental and hardware fluctuations, since such variability may cause changes in the

measured power level that falsely indicate target displacement. Differential intensity-modulated sensors have been implemented, offering robustness to such intensity fluctuations, and the speed of these sensors is limited only by the combined speed of the photodetection hardware and the data acquisition system (kHz-GHz). The primary disadvantages of intensity-modulated sensing are the relatively low accuracy (μm -mm for low-power sensors) and the lack of robustness, which consequently must be designed, often with great difficulty, into the sensor's architecture. White light interferometric displacement sensors, on the other hand, offer increased accuracy and robustness. Unlike their monochromatic-interferometer counterparts, white light interferometric sensors offer absolute, unambiguous displacement measurements over large displacement ranges (cm for low-power, 5 mW, sources), necessitating no initial calibration, and requiring no environmental or feedback control. The primary disadvantage of white light interferometric displacement sensors is that their utility in dynamic testing scenarios is limited, both by hardware bandwidth and by their inherent high-sensitivity to Doppler-effects.

The decision of whether to use either an intensity-modulated interferometric sensor depends on an appropriate performance function (e.g., desired displacement range, accuracy, robustness, etc.). In this dissertation, the performance limitations of a bundled differential intensity-modulated displacement sensor are analyzed, where the bundling configuration has been designed to optimize performance. The performance limitations of a white light Fabry-Perót displacement sensor are also analyzed. Both these sensors are non-contacting, but they have access to different regions of the performance-space. Further, both these sensors have different degrees of sensitivity to experimental uncertainty. Made in conjunction with careful analysis, the decision of which sensor to deploy need not be an uninformed one.

Chapter 1

Overview of Optical Displacement Sensing Methodologies

1.1. Introduction

Currently available electromechanical displacement sensing approaches include linear variable differential transformers, potentiometers, capacitive displacement sensors, inductive eddy current displacement sensors, and various magnet- and encoder-based configurations, some of which are mature technologies that are widely, commercially available [1-3]. However, because of the way these sensor architectures couple mechanical motion to an electrical response, they necessitate bringing an electrical charge in close proximity to the device under test (DUT). This is true even for non-contacting magnetic (resistive or encoding) sensors, since an electrical current is ultimately required to measure the sensor's response to DUT motion. In applications where the DUT may be located in a hazardous environment, the introduction of spark sources to the test is unacceptable. Many optical displacement sensing approaches introduce only light (or light in addition to an inert optical conduit such as an optical fiber) to the area surrounding the DUT, thereby avoiding the issue of spark sources. This characteristic is reason enough to motivate the research, design, and implementation of optical sensors for use in

such applications. Optical fibers, in particular, separate bulky (electrically charged) opto-electrical hardware from the DUT, and when access to the DUT is limited, optical fibers may offer a convenient means for guiding the sensor's electromagnetic (EM) signal, permitting access around corners and through confined spaces.

In the last couple of decades, the availability of optical fiber and related hardware has increased dramatically while the corresponding costs have decreased. As a result of these trends, the implementation of optical fiber sensors has also increased, finding success in a variety of applications [4]. In addition to not acting as spark sources, optical fiber sensors are immune to EM interference [4], and because of the generally low-levels of opto-electrical noise present, they offer the potential for increased measurement resolution [4-6]. Non-contacting fiber optic sensors have also been proposed for kinematic sensing applications that require minimal invasiveness and/or access to hard-to-reach areas [6]. Generally speaking, fiber optic sensors respond to the measurand of interest from changes in the measured EM signal's wavelength, intensity, or phase. Non-contacting displacement sensors are limited to the latter two fields, whereas changes in EM signal wavelength require coupling between the DUT and an optical filter such as a fiber Bragg grating (FBG). The two most popular types of non-contacting fiber optic displacement sensors are interferometric (or phase-modulated) displacement sensors [7-9] and variations on the optical-lever-style intensity-modulated displacement sensor [10-12]. The goal of this dissertation is to compare thoroughly these two methodologies for use in fiber optic displacement sensing applications, generating an understanding of their fundamental performance limitations when subject to certain, unavoidable sources of uncertainty. The task of modeling sensor performance in an attempt to exploit high performance (or optimal) regions of a particular sensor's performance space is a necessary component of this decision. Claims regarding the superiority of one sensing methodology over another are only meaningful after

performance optimization and the limits on sensor performance have been considered. In other words, the goal is to answer the question: “Given a particular constrained test design, a particular set of performance needs, and particular sources of uncertainty, what is the best displacement sensing methodology to implement and why?”

1.2. Interferometric Displacement Sensors

1.2.1. Monochromatic Interferometers

In an interferometric displacement sensor, an optical source’s EM transmission is separated into two components that follow unique optical paths (Figure 1): a reference component and a measurement component. These two components are recombined prior to measurement, and they interfere with one another according to their optical path length (OPL) difference. The OPL difference, which is related to DUT displacement, may be demodulated from phase-changes in the interference pattern that forms when the two components are recombined and the resulting interferogram is detected at a square-law photodetector [13]. Typical interferometer sources transmit in either the ultraviolet, visible, near-infrared, or short-wavelength infrared ranges. As a result of the small scale of the transmission’s wavelength (less than $2\text{ }\mu\text{m}$), changes in phase are highly sensitive to changes in displacement, creating the possibility for extremely high-accuracy displacement sensing. Fabry-Perót interferometers in particular are useful, because the interferometer’s reference-path is entirely contained within the measurement-path. In other words, with Fabry-Perót interferometers, changes in the reference-path’s length do not affect the OPL difference (which, if they did, would falsely indicate DUT displacement). Monochromatic Fabry-Perót interferometers have been reported with sub-picometer resolution [14], constituting one of the highest-accuracy displacement sensing methodologies that is currently available. Similar sensor architectures have found applications

to gravitational wave sensing [15-18] and to metrology at the National Institute of Standards and Technology [14,19]. However, this level of accuracy comes at a cost, as the linear input/output relationship produced by these sensors only applies for displacements over a range of less than $\lambda/2$, with λ being the wavelength of the EM source. Outside of this region the sensor does not produce an unambiguous output, as the measured interferogram is periodic in nature (Figure 2). Monochromatic interferometers are typically demodulated using a single power measurement, and the reliability of this measurement to indicate DUT displacement hinges largely in feedback control of hardware and precise control of environmental conditions (e.g., ambient humidity, temperature, and pressure, all of which affect the ambient refractive index).

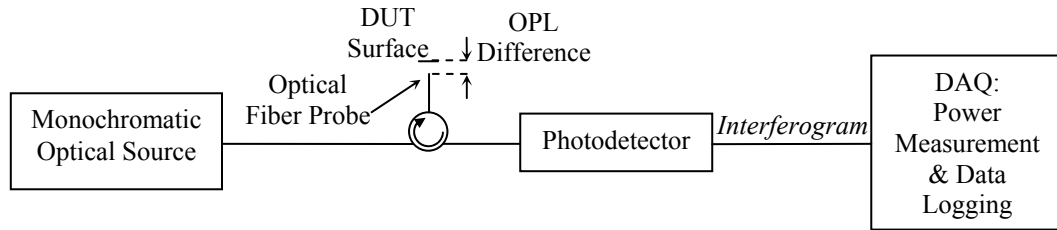


Figure 1: A monochromatic, fiber optic Fabry-Perot interferometer is shown. A partial reflection occurs at the optical fiber probe's tip (reference-component) and another reflection occurs at the DUT surface (measurement-component). In this case, the OPL difference is equal to DUT displacement.

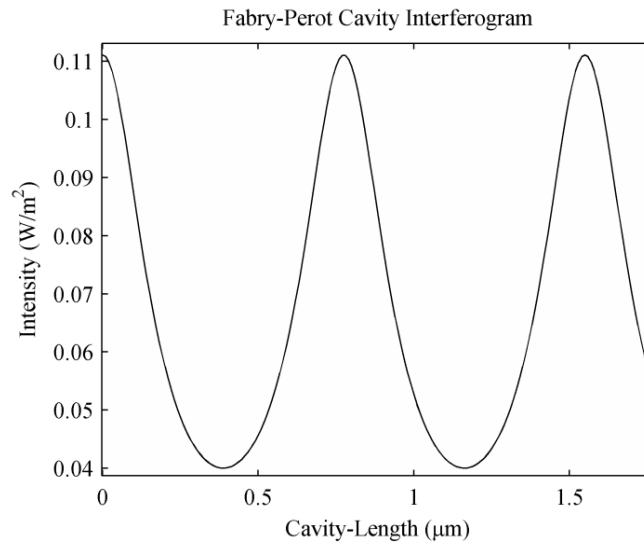


Figure 2: A typical interferogram is periodic in nature, with the period of the signal being equal to $\lambda/2$ (in this case $\lambda=1.55 \mu\text{m}$ and the period, consequently, is $0.775 \mu\text{m}$). Unambiguous sensor operation is limited to a region of less than $\lambda/2$.

1.2.2. White Light Interferometers

White light interferometers employ either a tuned narrowband source or a broadband source that is filtered by a tunable optical filter, and they measure the interferogram over a certain wavelength range [20-23]. DUT displacement is then demodulated from changes in phase as observed in the measured interferogram (and not directly from interferogram intensity-levels). Since phase demodulation is independent of the average measured power level, an additional advantage is that even the simplest white light interferometric displacement sensors are robust to fluctuations in measured power level that are not the result of changes in DUT displacement (e.g., fluctuations in source power level, optical loss mechanisms, surface reflectivity, etc.). White light interferometers also offer the capability of taking absolute (unambiguous) displacement measurements over a large displacement range. These performance advantages, however, come at a cost, since the demodulation process is nonlinear, and the digital signal processing (DSP) that is required to demodulate displacement from phase currently restricts the sensor's measurand bandwidth to the low-Hz range. In general, the accuracy of a particular white light interferometric displacement sensor is largely governed by the specific DSP techniques that are implemented during demodulation [9].

The *ultimate goal of this research* is to characterize non-contacting displacement sensors that offer *high-performance and robust operation* over a moderate (1-2 cm) axial displacement range. White light EFPI displacement sensors offer micrometer-level accuracy over a large axial displacement range, a high-degree of robustness to variability in environmental and hardware conditions, and absolute displacement measurement (requiring no initial calibration prior to use). For these reasons, the performance of a white light EFPI displacement sensor will be analyzed in this dissertation. In particular, the performance limitations and tradeoffs that exist in the presence of unavoidable levels of uncertainty will be

considered. More specifically, accuracy and displacement range are fundamentally related to the way that DUT displacement is demodulated from the interferogram. The *primary disadvantage* associated with this particular sensor is its operating speed and consequently its restriction to static (or quasi-static) testing. Although it was thought that sampling speed restrictions were primarily limited by the bandwidth of individual hardware components, it will be shown in this dissertation that DUT dynamics (even velocities as low as 0.1 mm/s) impart biases in DUT displacement measurements that are too large to ignore (on the order of tens of μm). This is of *particular interest* if these sensors are to be deployed as dynamic displacement sensors in addition to being static displacement sensors.

1.3. Intensity-Modulated Displacement Sensors

1.3.1. Sensor Architectures

Generally speaking, non-contacting intensity-modulated displacement sensors exploit the relationships between DUT displacement and the measured power level of an EM transmission, which reflects off the DUT and is detected at one or more fixed locations. The simplest non-contacting intensity-modulated optical displacement sensor architecture, referred to as the “optical lever” (Figure 3), was originally independently proposed by Frank in 1966 [10] and Kissinger in 1967 [11]. Although a displacement sensor based on the intensity-modulated optical-lever is *inherently less sensitive* to changes in displacement than (phase-modulated) interferometric displacement sensors, the intensity-modulated approach offers the advantage that *only the simplest design is required* to demodulate DUT displacement from measured optical power levels. As a result, large sampling rates, which are limited only by the bandwidths of the photodetector and the data acquisition hardware (on the order of hundreds of kHz through GHz), may be achieved.

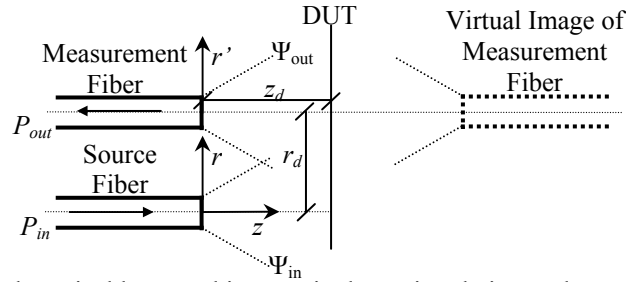


Figure 3: The simple optical lever architecture is shown in relation to the coordinates described in Section 2.2. The virtual image relationship is exploited in the way the experimental setup is used for testing (Section 2.3).

Aside from reductions in sensitivity and accuracy (which may or may not meet the demands of a particular sensing application), the *primary drawback* associated with intensity-modulated displacement sensors is that the reflected and measured power level at a single location is not robust to the previously mentioned environmental and hardware fluctuations, which may falsely indicate changes in DUT displacement. This undesirable characteristic has motivated the research of robust, differential intensity-modulated sensors that make use of power measurements at multiple locations, examples of which were reported in 1999 [24] and 2004 [25]. A general representation of a differential sensing architecture is shown in Figure 4, where receiving fibers have been arbitrarily assigned to one of two receiving groups. By taking the ratio of two measured power levels, collected over an arbitrary spatial range, fluctuations in the transmission power level that occur prior to collection by receiving fibers are eliminated from the final sensor output. Various differential intensity-modulated sensor architectures have been proposed [24-28], and these sensor architectures have been shown to be robust even to significant changes in the DUT's reflection coefficient [25]. However, these proposed sensor architectures make use of just a few receiving fibers, failing to take full advantage of the light reflected off the DUT.

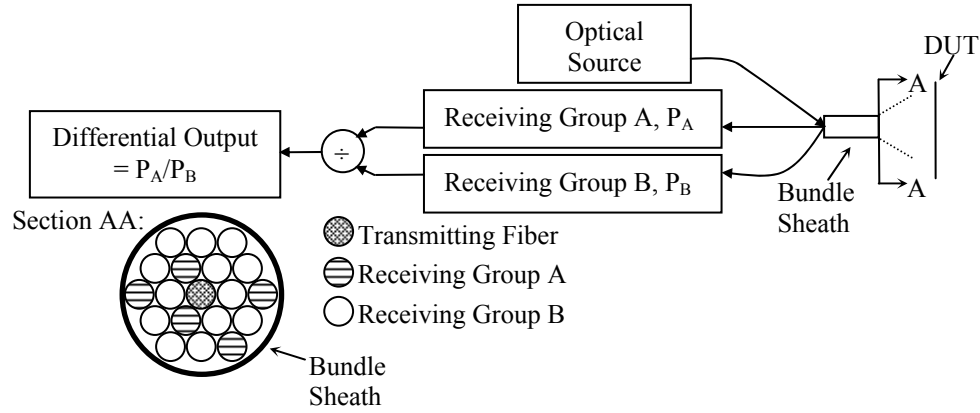


Figure 4: The bundled differential approach offers the potential for robust, high-performance non-contact optical displacement sensing.

Bundled intensity-modulated displacement sensor architectures, which make use of an increased number of receiving fibers, offer an *inherent performance advantage* by measuring a greater portion of the reflected light and thereby exhibiting increased sensitivity to changes in DUT displacement (Figure 4). Bundled intensity-modulated displacement sensors were first implemented in research environments in 1979 [12], have been implemented more recently with various design perturbations [27,28], and are also commercially available [29,30]. Although bundled architectures have been a topic of research since 1979; little consideration has been given to the possibility of combining the increased sensitivity of the bundled architecture with the robustness of the differential approach. Further, of the bundled displacement sensors that have been reported, it does not appear that anyone has used a validated model of the transmission from an optical fiber to accurately simulate the sensor's performance *a priori* or to guide the design of the bundle-configuration.

1.3.2. Simulating Sensor Performance

Initially, the performance of the optical-lever displacement sensor architecture was simulated using a ray-optic model; in an attempt to relate optical-lever geometry to

experimentally measured power levels [24,27]. This model demands little computation and generally captures the divergence behavior of an EM transmission propagating through space. However, implicit in this model is the assumption that the transmission's illumination is spatially uniform, rendering it incapable of describing the intensity distribution of a propagating transmission. The particular shortcomings of ray-optic models in a performance simulation role are discussed in more detail in [31]. More sophisticated models have been proposed, approximating the radial intensity distribution using a Gaussian function [28]. These approximations have been widely implemented, are relatively simple to compute, and are more accurate than the ray-optic models in describing the intensity-distribution of the LP_{01} mode, the only mode that propagates within a single mode optical fiber [32]. However, the accuracy of Gaussian approximations decreases off the axis of propagation, limiting their usefulness to conceptual visualization, component alignment, and the rough approximation of off-axis power levels. Consider the hexagonal close-packed bundle architecture shown in Figure 4, where the cores (or centers) of adjacent fibers are separated by at least the diameter of the fiber coating (250 μm is typical). This radial offset between cores is large enough to render the Gaussian approximation useless in determining the measured power levels at any of the receiving fibers. Transmission behavior off of the axis-of-propagation governs the sensor's performance when large lateral offsets (25-1500 μm) exist between transmitting and receiving fibers. *Perhaps the primary reason* that the relationship between fiber locations and assignments within a bundled architecture and sensor performance has not yet been used to guide sensor design is that the most widely implemented optical transmission models suffer from irreconcilable inaccuracies.

To the best of the author's knowledge, no one has made any published effort at characterizing the design space of differential, bundled fiber optic displacement sensors as part of a larger effort to optimize performance or, at the very least, achieve application-specific

levels of sensor performance. As its foundation, this effort necessarily requires an understanding of the EM signal's physics as it propagates from a transmitting fiber to any number of receiving fibers, and to date no one has gone so far as to validate a model for describing the spatial behavior of a signal emitting from a transmission fiber. In 2009 a physics-based transmission model was introduced in an effort to accurately characterize the transmission's intensity distribution and describe the spatial properties of a signal transmitted from one single mode optical fiber to another [33], although the model was not, nor has it since been, experimentally validated by the authors that originally introduced it.

In this dissertation, I will show the existence of multiple regions of linear sensitivity for various relative positions of transmitting and receiving fibers within a two-fiber optical lever, with the different two-fiber-configurations exhibiting distinct combinations of linear sensitivity and axial displacement range. The implication of this dependence is that the bundled displacement sensor's design space may contain optimal combinations of linear sensitivity and axial displacement range that meet the performance needs of a given sensing application. Specifically, the sensitivity, linearity, resolution, and displacement range of a sensor are functions of the relative positioning of the sensor's transmitting and receiving fibers, and accurate prediction of these parameters *a priori* necessitates the use of an accurate transmission model. By extension, a bundled intensity-modulated displacement sensor may be designed, as shown in Figure 4, with multiple groups of receiving fibers, such that high-performance and robust differential demodulation are both achieved. This approach is being proposed as a *generalized sensor design tool* that may be used to describe a differential, bundled intensity-modulated optical displacement sensor's *achievable performance metrics* in terms of the sources of modeling and experimental uncertainty that are present.

1.4. Additional Optical Sensing Methodologies

As alluded to earlier in this section, other optical sensing methodologies and other optical fiber sensing methodologies exist aside from the two that are being considered for analysis and comparison. Generally, these may be classified as intrinsic sensors (which necessitate physical contact with the DUT), laser sensors (LIDAR, laser vibrometers, etc.), or photodetector array-style sensors. Intrinsic sensors (such as white light polarization interferometers [34]) are not suitable for the desired application, since the goal of this dissertation lies in the development of a high-performance, *non-contacting* displacement sensor and these sensors necessitate physical contact with the DUT. Photodetector-array style sensors (e.g., charge-coupled devices or Shack-Hartman sensors) require positioning opto-electrical hardware in close proximity to the DUT, introducing a potential spark-source to the testing environment. Laser sensors are demodulated similarly to non-contacting optical fiber sensors, with the primary difference being that laser sensors do not use optical fibers as a conduit to guide the EM measurement signal. Nonetheless, in many applications it is necessary to take advantage of optical fibers to guide the measurement EM signal to the DUT (without having to achieve line-of-sight between bulky opto-electrical hardware and the DUT). With the stated objective being robust, non-contacting, non-spark-emitting sensor deployment, *none of these alternative optical sensing methodologies are preferred* over the white light EFPI approach or the differential, bundled intensity-modulated approach.

1.5. Summary

The *primary objective* of this research is supporting the development of a non-contacting displacement sensor that does not act as a spark source and does not require direct line-of-sight between bulky hardware and the DUT; this objective effectively removes

traditional electromechanical sensing methodologies from consideration. This also generally restricts the analysis to certain non-contacting, intensity-modulated fiber optic displacement sensors and non-contacting fiber optic interferometers. In this dissertation, for the reasons outlined in the previous three sub-sections, I will limit consideration to (1) non-contacting, differential bundled intensity-modulated fiber optic displacement sensors and (2) non-contacting fiber optic white light EFPI displacement sensors. *Secondarily*, it is also desirable that the sensor offer high-performance operation (accuracy, sensitivity, etc...) over an axial displacement range of up to a few centimeters, while being compact, simple-to-demodulate, easy to install, and robust to changes in surface reflectivity, environmental conditions, etc. This secondary goal necessitates an investigation of the fundamental relationships between sensor parameters, DSP and demodulation techniques, and performance metrics within a particular set of design constraints and requirements. *A third goal* is ultimately deploying this sensor in a dynamic testing scenario, bringing sensor bandwidth into consideration. Proper evaluation of these goals depends on the accurate generation of the sensor design space.

It is for these reasons, and to meet these specific objectives, that the scope of this dissertation is limited to a comparison of the performance limitations of optimized, differential, bundled intensity-modulated fiber optic displacement sensors and white light EFPI displacement sensors. In the intensity-modulated case, the bundle configuration must be optimized (or at the very least, deliberately designed with an understanding of performance-tradeoffs) in order to bring the robustness, accuracy, and displacement range capabilities anywhere near those of the white light EFPI displacement sensor. Chapter 2 sets the foundation for this design framework, with the introduction and validation of a physics-based EM transmission model. Chapter 3 follows the foundation set in Chapter 2, and uses the validated transmission model in conjunction with an optimization routine in an effort to design sensor

performance to suit application-specific needs. Similarly, the dynamic limitations of the white light EFPI displacement sensor must be properly considered, if the low-bandwidth limitation (or possibly its limitation to static and quasi-static only testing) is to be bypassed, enabling dynamic sensing. The performance of a static white light EFPI displacement sensor will be analyzed in Chapter 4, and the effects of dynamics on white light EFPI displacement sensors will be discussed in Chapter 5, with their direct effects on limiting sensor performance discussed and alternative sensing approaches also proposed in Chapter 5. These considerations will provide the dissertation's primary thrusts, as these two independent sensing methodologies are compared with regard to their feasibility and ultimate performance limitations in the presence of uncertainty. The dissertation will conclude with a side-by-side comparison of the findings and recommendations for the use of both sensor-types (Chapter 6).

1.6. Contributions of the Dissertation Work

The following aspects of this dissertation were made as unique contributions to the field of optical fiber sensing and metrology:

1. Validated a physics-based single mode optical fiber transmission model
2. Executed uncertainty analysis and sensitivity analysis of the transmission model, relating input parameter uncertainty to the degree of output variability
3. Proposed a framework that uses the validated transmission model to efficiently explore the differential bundled displacement sensor design space in search of optimally-designed bundle configurations
4. Prototyped two optimized bundled sensor-architectures and performed a sensitivity analysis relating performance to spatial (receiving-fiber-location) variability

5. Experimentally characterized a Doppler-induced displacement-bias that manifests itself in white light EFPI displacement sensors
6. Derived an analytical relationship between the magnitude (in the displacement-domain) of the Doppler-induced bias and a set of three swept-filter properties for a white light EFPI displacement sensor
7. Proposed a phase-diverse white light EFPI sensor architecture that exploits the (derived) relationship between the Doppler-induced displacement-bias and the filter parameters, ultimately resulting in unbiased measurements of a moving target's displacement
8. Successfully tested the phase-diverse white light EFPI displacement sensor, simultaneously measuring DUT velocity and unbiased DUT displacement
9. Performed a side-by-side comparison between (1) the differential bundled intensity-modulated and (2) the white light EFPI displacement sensing methodologies, taking the impact of uncertainty into account and describing the limitations that exist on sensor performance.

Chapter 2

Intensity-Modulated Sensor: Transmission Model

2.1. Background and Motivation

As stated in Chapter 1, a model that accurately characterizes measured optical power levels off of the transmission's axis of propagation was derived from the physics of a propagating EM wave, subject to the boundary conditions imposed by a step-index single mode optical fiber by Trudel and St-Amant in 2009 [33]. Although Trudel and St-Amant did an excellent job introducing the physics-based model and qualitatively discussing its spatial-behavior, they did not address model verification or uncertainty analysis, nor did they include a quantitative description of the error between the model and the reported experimental results. This chapter describes work that has been performed in an effort to verify the performance of, and analyze the effects of uncertainty on, this physics-based model. This chapter also describes the manner in which this model, once verified, may be implemented for exploiting a large design space of bundled sensor architectures, in an attempt to achieve high-performance while meeting application-specific performance requirements.

2.2. Model Formulation

The fundamental LP_{01} mode, the only mode that propagates within a single mode optical fiber, has a field function that is given by [32]

$$\Psi(r) = \begin{cases} \Psi_0 J_0\left(\frac{ur}{a}\right) & r < a \\ \Psi_0 \frac{J_0(u)}{K_0(w)} K_0\left(\frac{wr}{a}\right) & r \geq a. \end{cases} \quad (2.1)$$

Ψ_0 is the field magnitude, a is the diameter of the optical fiber core, J_0 and K_0 are the order-zero Bessel functions of the first and second kind, respectively, u is the normalized transverse propagation constant, and w is the normalized transverse attenuation constant. The propagating mode within the transmitting optical fiber ultimately exits the fiber tip where it diffracts into the surrounding air (Figure 3). After emitting through the tip of the transmitting fiber, the fundamental mode propagating through the air $\Psi_{out}(r, \theta, z)$ may be expressed as

$$\Psi_{out}(r, \theta, z) = \left(\frac{2j\pi}{\lambda z} \right) \int_0^\infty r' \exp\left(\frac{-j\pi}{\lambda z} (r'^2 + r^2) \right) J_0\left(\frac{2\pi r r'}{\lambda z} \right) \Psi(r') dr'. \quad (2.2)$$

Equation (2.2) is taken from the formulation for a cylindrically symmetric mode diffracting through a circular aperture [35]. In this formulation, the field $\Psi(r')$ propagates within the receiving optical fiber and the output field $\Psi_{out}(r, \theta, z)$ is the acceptance field of the receiving fiber. λ is the wavelength of the propagating light, $j = \sqrt{-1}$, (r, θ) are the coordinates of the input (transmitting fiber) frame, and (r', θ') are the coordinates of the output (receiving fiber) frame. The input and output coordinate frames are related by

$$r^2 = r'^2 + r_d^2 + 2r'r_d \cos(\theta' - \theta_d). \quad (2.3)$$

In Equation (2.3), r_d and θ_d constitute the translation and rotation between frames, respectively (Figure 3). For the purposes of this work, θ_d is always assumed to be zero, and r_d is referred to as the lateral (or radial) offset between the transmitting and a given receiving fiber. The coupling efficiency between the two fibers, or the fraction of the transmitted light that enters the receiving fiber, may be calculated using the normalized overlap integral [36], expressed in cylindrical coordinates as

$$\eta(r_d, z) = \frac{P_{out}}{P_{in}} = \frac{\left| \int_0^{2\pi} \int_0^\infty r \Psi_{in} \Psi_{out}^* dr d\theta \right|^2}{\left| \int_0^{2\pi} \int_0^\infty r \Psi_{in} \Psi_{in}^* dr d\theta \int_0^{2\pi} \int_0^\infty r \Psi_{out} \Psi_{out}^* dr d\theta \right|} \quad (2.4)$$

where, $\Psi_{in} = \Psi(r)$ as it is described in Equation (2.1), and $\Psi_{out} = \Psi_{out}(r, \theta, z)$ as it is described in Equation (2.2). By evaluating Equation (2.4), the measured optical power at any point in space may be calculated, given a particular set of transmitting and receiving field functions. Rational bounds need to be chosen for the limits of integration over the radial coordinate r in both the overlap integral (Equation (2.4)) and the diffraction integral (Equation (2.2)). After running numerous simulations, appropriate limits were established that differ significantly from the original values reported in [33]. For the level of numerical precision used during computation, an upper limit of $r' = 20 \mu\text{m}$ was sufficient for the calculation of Equation (2.2) at all of the axial displacements explored over the range $1000 \mu\text{m}$ to $50,000 \mu\text{m}$. For the same level of precision, the upper limits on Equation (2.4) ranged from $r = 400 \mu\text{m}$ at an axial displacement of $1000 \mu\text{m}$ to $r = 3900 \mu\text{m}$ at an axial displacement of $50,000 \mu\text{m}$. The LP_{01} mode is known to be smooth and continuous as either the radial or axial coordinate is changed, and integration limits that resulted in smooth continuous spatial behavior were considered to be sufficient. Consider the simulation results in Figure 5 for an example of the transmission model's behavior, and in

particular the smoothness of the transmitted power levels, across different limits of integration. Consider that the LP_{01} mode propagating through space is known *a priori* to exhibit smooth behavior [32].

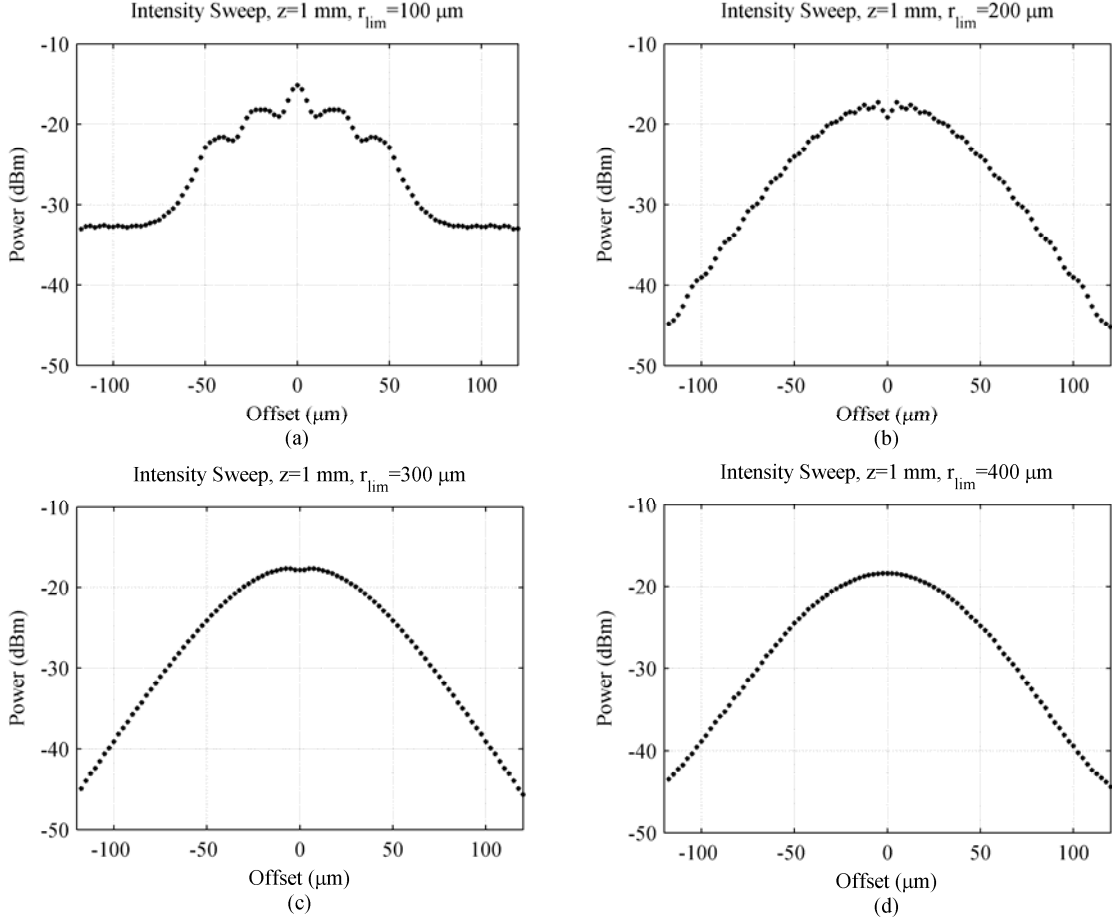


Figure 5: The limits of integration were varied until the result was a smooth spatial distribution that monotonically decreased from a central peak, as the LP_{01} mode propagating through space is known to behave in this way.

2.3. Experimental Investigation of Transmission Model

The experimental setup that was used to verify the accuracy of the model presented in Section 2.2 is shown in Figure 6. Optical power was sampled as a function of the relative

position between the transmitting and receiving fibers by positioning the receiving fiber at a given axial displacement $z=z_d$ and scanning across a range of lateral offsets r_d .

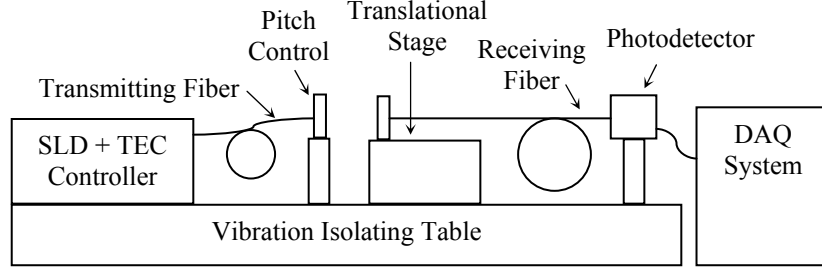


Figure 6: The test-bench experimental setup, used for experimentally measuring transmission power levels as a function of radial offset and axial displacement, is shown here.

The optical source used for this experiment was a 5 mW super luminescent diode (SLD) centered at 1528 nm (Covega Corp. SLD 1108) and controlled with a thermoelectric cooler (TEC) and laser driver (Covega Corp. LDC1300). Single mode step index optical fibers (Corning Inc. SMF-28e), a broadband InGaAs photodetector centered at approximately 1550 nm (Thorlabs Inc. DET01CFC), and a data acquisition module with an impedance of 1 M Ω (National Instruments Corp. NI PXI-4461) were used for reading the power level at the receiving fiber tip. A 3-axis translational stage (Newport Corp. 561D) and a 1-axis translational stage (Newport Corp. 443) were used for precise control of the receiving fiber position. The transmitting fiber was mounted on a device with pitch control (Thorlabs Inc. KM100T) so that alignment with the receiving fiber could be established.

During experimental testing, z_d was varied from 1000 μm to 50,000 μm . For larger axial displacements, the SLD's maximum rated output power of 5 mW was used in order to minimize the effects of quantization error. For smaller axial displacements, a lower output power was used to avoid saturation of the photodetector. In all cases the results were normalized by the transmitted power level, which was provided by the manufacturer as a function of input current and independently experimentally verified during testing. At a given $z=z_d$, r_d was incrementally increased by a fixed amount, creating a lateral offset to one side of the transmitting fiber. The

mean of 1000 photodetector power readings, sampled at 1 kHz, was calculated at each position. When the signal became sufficiently weak (between -50 dBm and -60 dBm), r_d was decreased and re-sampled at the same points, bringing the receiving fiber back to zero lateral offset. This procedure resulted in two samples at each point in space. Next, the same procedure was carried out in the other direction, creating a lateral offset to the other side of the transmitting fiber. Once again, these points were each sampled twice, once while increasing the offset and once while decreasing the offset. Scanning on both sides of the transmitting fiber tip was performed to ensure symmetry in the transmission's power distribution, which is a good indication of alignment between the transmitting and receiving optical fibers. These scans resulted in a partial view of the power distribution of the transmitted signal, a subset of which is shown in Section 2.4.4.

The noise floor of the test setup was estimated using the data acquisition module to measure the photodetector's output while no SLD transmission was being sent. As in the case of the previously described transverse scans, each power measurement was calculated as the mean of 1000 photodetector measurements sampled at 1 kHz. The mean of the noise data was the same if the photodetector was disconnected from the data acquisition module, suggesting that the observed noise floor is that of the data acquisition module. During testing, the noise floor was between -60 dBm and -50 dBm depending on the source power level (since experimental results were normalized by the source power level). The mean and variance of these noise samples were calculated so that the effect of the noise floor could be reproduced in the simulation results.

2.4. Model Verification and Uncertainty Quantification

2.4.1. Experimental Uncertainty

For the test-bench described in Section 2.3, experimental uncertainty results from a lack of knowledge regarding the initial distance calibration between fiber tips, the true changes in fiber tip position (limited by micrometer resolution), and optical loss mechanisms, and from random (noise) behavior in the optical source, photodetector, and data acquisition system. These sources of uncertainty may be analyzed in an effort to better understand the impact they have on the measured power level (the simulation output) and the corresponding implications on sensor design. The bias error that results from distance calibration is assumed to be less than $\pm 10 \mu\text{m}$. Using a first-order approximation, this error can be converted to units of dB by the derivative of the received power with respect to axial displacement, which itself is a function of axial position (Figure 7). Specifically, at a radial offset of $250 \mu\text{m}$ a bias measurement error of 0.1 dB would result from a $10 \mu\text{m}$ bias at an axial displacement of $1000 \mu\text{m}$, but the bias would be less than 0.01 dB assuming the same radial offset at an axial displacement of $50,000 \mu\text{m}$. The implication is that during testing the true axial displacement may have been offset from the estimated axial displacement by a constant value of as much as $10 \mu\text{m}$, resulting in a bias error of as much as ± 0.1 dB in the measured power level. User error during the transverse scanning process may result in a position discrepancy of up to $\pm 2.5 \mu\text{m}$, which is the resolution of the micrometers employed. Implementing the same first-order sensitivity approach discussed in the previous paragraph and replacing axial sensitivities with transverse sensitivities, $2.5 \mu\text{m}$ corresponds to 1.0 dB at a radial offset of $250 \mu\text{m}$ and an axial distance of $1000 \mu\text{m}$, and less than 0.01 dB at a radial offset of $250 \mu\text{m}$ and an axial distance of $50,000 \mu\text{m}$. The implication is that during testing, the lateral difference between the actual receiving fiber position and the assumed

receiving fiber position could have been as much as $\pm 2.5 \mu\text{m}$, which would have resulted in a resolution error of up to $\pm 1.0 \text{ dB}$ in the measured power level.

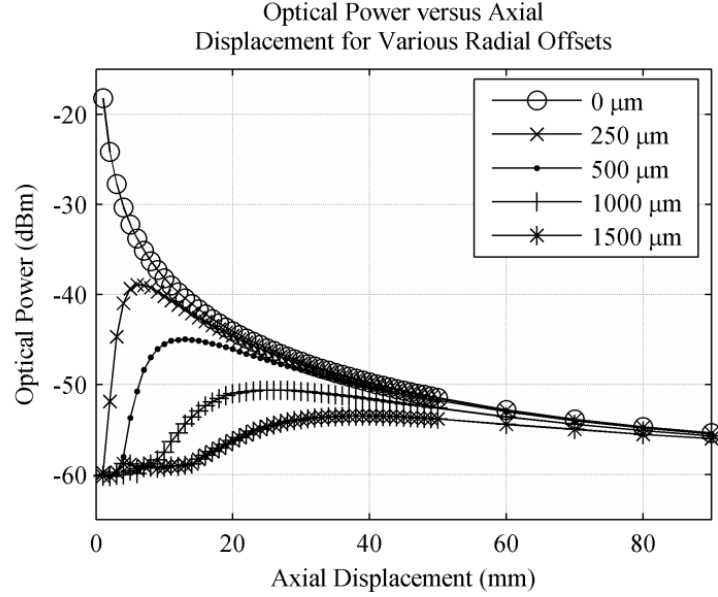


Figure 7: The relationship between the transmitted power-level and the axial displacement (or position) is shown for various radial offsets.

The effects of random noise on the SLD output, the photodetector, and the data acquisition system were measured, resulting in a combined deviation of $\pm 0.015 \text{ dB}$ from the mean power level at a given point in time. Additionally, the effects of long-term drift in the source power level (measured over 24 hours) were found to be less than 0.2 dB . Therefore the combined effects of these uncertainties on the measured power level are estimated to be as much as 0.215 dB . The estimated effects of this and the previously described sources of experimental uncertainty are summarized in Table I. The primary loss mechanisms in the test-bench experimental setup were polarization dependent loss (PDL) and coupler insertion losses. The rotation of the coiled optical fibers in the experimental setup was adjusted until it was concluded that PDL had been approximately minimized. According to component manufacturers, the total insertion loss in the experimental setup can be estimated as less than

0.65 dB. Assuming zero PDL, these loss mechanisms result in a wavelength-independent offset between experimental results and simulation results of less than 0.65 dB.

2.4.2. Modeling Uncertainty

Modeling uncertainty results from a lack of knowledge regarding the model's input parameters. Although no numerical approximation is perfectly accurate, it was assumed for this work that truncation errors resulting from finite computational precision are not significant compared to the effects of input parameter uncertainty. The input parameters in Equations (2.1)-(2.4) include a , λ , u , and w , in addition to E_0 , r , r_d , and z . Note that the uncertainty of E_0 and the uncertainty of the spatial coordinates r , r_d , and z were described in Section 2.4.1 as sources of experimental uncertainty.

A phenomenon identification ranking table (PIRT) may be used as a preliminary means of qualitatively assessing the significance of a particular parameter's uncertainty from the degree of the variability in that model input parameter and the sensitivity of the model output to that variability [37,38]. A PIRT analysis is shown in Table 1, where uncertainty designations were granted based on percent changes of anticipated uncertainty levels, and sensitivity designations were chosen based on the relationships described in Equations (2.1)-(2.4). The results of a PIRT analysis for this application (Table 1) showed that variability in λ is expected to have a relatively small effect on the simulation output, while variability in a , n_1 , and n_2 has the potential to significantly impact the simulation output. The value of and the tolerances on a are specified by the fiber manufacturer, but assumptions have to be made regarding the nature and the magnitude of the uncertainty on n_1 and n_2 . Recall that the effects of experimental uncertainty were directly observed as fluctuations in the measured power level. However, since these sources of modeling uncertainty affect the simulation output indirectly, the full ranges of possible values for uncertain input parameters must be propagated through the numerical model

in order to see the corresponding range of possible output values. This process is discussed in detail in the following sub-section.

Table 1: The PIRT analysis is used as a qualitative assessment of the effects of parameter uncertainty on a model's output.

Phenomenon	Symbol	Uncertainty	Sensitivity	Product
Transmission Wavelength	λ	Low	Low	Low
Axial Coordinate	z	Moderate	Low	Moderate
Radial Coordinate	r	Low	High	Moderate
Transmission Amplitude	E_0	Moderate	Low	Moderate
Core Radius	a	Moderate	High	High
Core, Cladding Indices of Refraction	n_1, n_2	Low	High	Moderate

2.4.3. Effect Screening and Uncertainty Propagation

After performing the PIRT analysis, effect screening was performed for the purpose of reducing the number of input parameters required in the uncertainty propagation stage [38]. Parameter down-selection is critical when simulation trials are computationally expensive and a reduction in the complexity of the uncertainty propagation process therefore results in significant time and computational cost savings. For this paper, a three-level full factorial design of experiments was used to test the effects that variability in E_0 , a , n_1 , and n_2 has on the measured optical power level at various points in (r, z) space, requiring $3^4=81$ trials. The design space explored for these input parameters is summarized in Table 2. Note that it is not necessary to assume probability density functions (PDFs) for these uncertain input parameters during the screening process. Rather, these input parameters were sampled uniformly over the entire range of possible values in order to assess the total potential impact of their variability on the variability of the model's output. A four-way analysis of variance (ANOVA) performed

between the four inputs (sampled over the design space) and the model's output provides insight into the impact each parameter's variability has on the variability of the model's output. R-squared values, calculated from the ANOVA, are shown in Figure 8, where a larger value indicates a stronger dependence between output variability and input parameter variability [38]. It may be deduced from these results, variability fluctuations on E_0 and a have relatively insignificant effect on output variability.

Table 2: Sources of experimental and modeling uncertainty are summarized. If applicable, the maximum estimated impact on the measured power level or the assumed probability distribution function is listed. Parameters that are listed in italics were found not to significantly impact output variability.

Source	Symbol	Nominal Value	Nominal Error	Error Range	Max. Impact	PDF
Tip Separation Distance	z	n/a	0 μm	$\pm 10 \mu\text{m}$	1 <u>dB</u>	Unif.
Micrometer Resolution	r_d	n/a	0 μm	$\pm 2.5 \mu\text{m}$	1 <u>dB</u>	Unif.
<i>Opto-Electrical Noise/Drift</i>	E_0	<i>n/a</i>	<i>0 dB</i>	<i>$\pm 0.215 \text{ dB}$</i>	<i>0.215 dB</i>	Gauss.
<i>Core Radius</i>	a	<i>4.1 μm</i>	<i>0 μm</i>	<i>$\pm 0.2 \mu\text{m}$</i>	<i>n/a</i>	Unif.
Core Index of Refraction	n_1	1.4492	0	± 0.0005	n/a	Gauss.
Cladding Index of Refraction	n_2	1.4440	0	± 0.0005	n/a	Gauss.

After effect screening, PDFs were assumed for describing the behavior of the uncertain input parameters (Table 2). Simulations were run while using a Latin hypercube technique to sample r_d , z , n_1 , and n_2 from their corresponding PDFs (listed in Table 2). This resulted in a distribution of output (measured power) values at the sampled points in space. The results of this uncertainty propagation are overlaid on the results of the experimental scans in Section 2.4.4, generally providing insight into the accuracy and the predictive capability of the model in the presence of well-described elements of uncertainty.

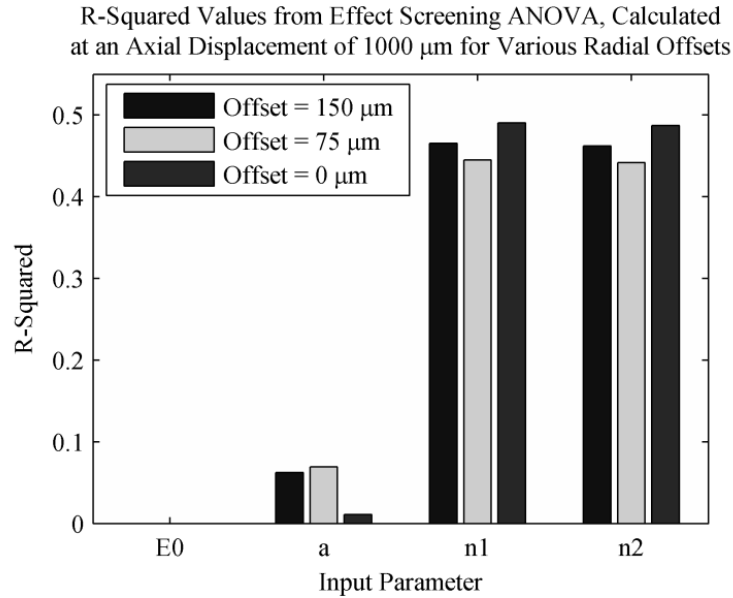


Figure 8: R-squared values, calculated from an ANOVA, are used to discriminate among those parameters whose variability significantly influences output variability and those whose influence is relatively insignificant.

2.4.4. Model Validation

Experimental results shown in Figure 9, illustrate the spatial behavior of the optical transmission. A close-up view of the transverse scan at an axial displacement of 1000 μm is shown in Figure 10. In these plots, each experimental data point represents the average of two measurements taken at approximately the same point in space, with the hope being that this average is less susceptible to any hysteresis effects the translational stage might exhibit (recall that each of the two, averaged points were sampled while the translational stage was being moved to one direction or to the other). The noise floor described in Section 2.3 and the 0.50 dB attenuation from the loss mechanisms described in Section 2.4.1 have been superimposed on the simulation results. Lines indicating the simulation results $\pm\sigma_{UP}$, one quartile of deviation calculated from the uncertainty propagated through the simulation, are shown to illustrate the level of agreement between experiment and simulation. As these results show, excellent

agreement between the experimental results and the simulation results has been achieved, with almost all of the experimental measurements lying within $\pm\sigma_{UP}$ of the simulation results.

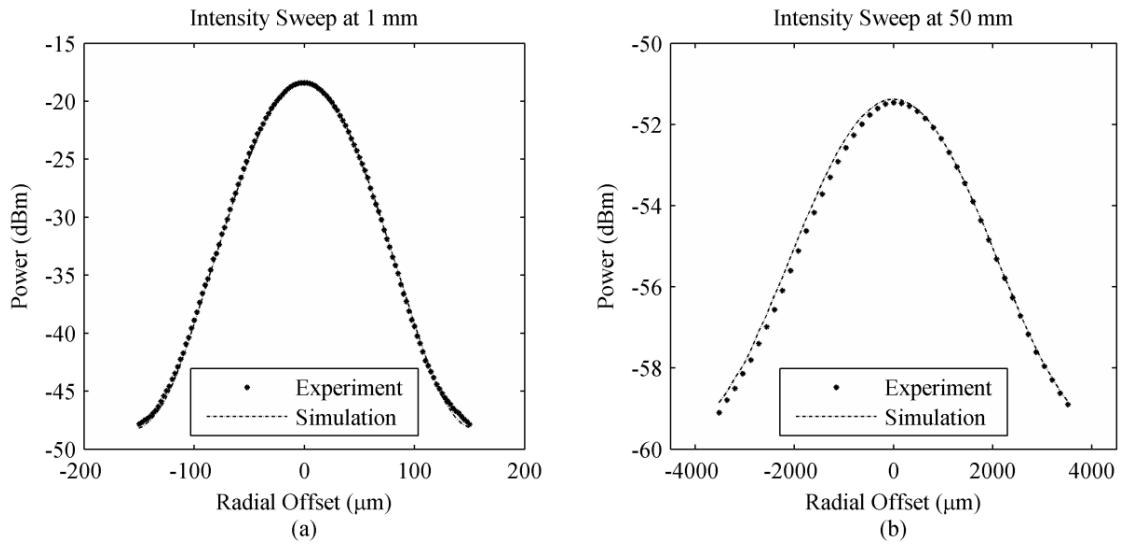


Figure 9: A comparison is shown between experimental and simulated power-distributions at axial displacements of (a) 1 mm and (b) 50 mm.

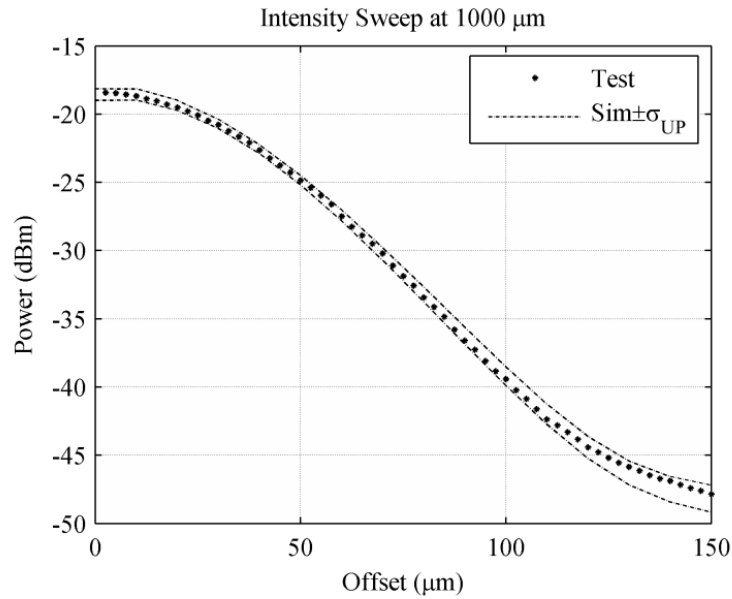


Figure 10: A close up view of Figure 9(a) provides a useful visual comparison between simulation and experimental results. The experimental data points fit within the simulation results $\pm\sigma_{UP}$.

Given this formal validation, two trends are observed to accompany an increase in axial displacement. First, the overall power level decreases, which is to be expected since power is

generally related to axial distance by an inverse-square relationship. Second, the modal field radius tends to increase as a result of divergence, causing the expected beam spreading of the transmission peak. These behaviors are captured well by the presented model. As expected, the power distribution is approximately Gaussian-shaped, which is the case for the LP_{01} mode. However, the agreement between the model and the experiment for large lateral offsets constitutes a noticeable improvement over the use of Gaussian approximations. Note that not only is there good agreement describing peak magnitude and distribution shape, but the power levels in the vicinity of the noise floor agree very well too, suggesting that the behavior of the observed noise floor, introduced in Section 2.3, is modeled accurately by the estimated noise floor.

The root mean square error (RMSE) between the experimental and simulated results is plotted at each experimentally tested axial displacement-value in Figure 11. This metric gives a clear representation of the absolute error between model and experiment for a given transverse scan, normalized by the number of points in that scan. The RMSE is observed to stay between 0.20 dB and 0.45 dB (Figure 11), which is on the order of the magnitude of σ_{UP} . While the agreement between the simulation results and the experimental results is strong, there is some error present with the received power as predicted by the simulation differing from the experimentally observed power level by as much as 0.1-1.0 dB at the tails of the transmission peak.

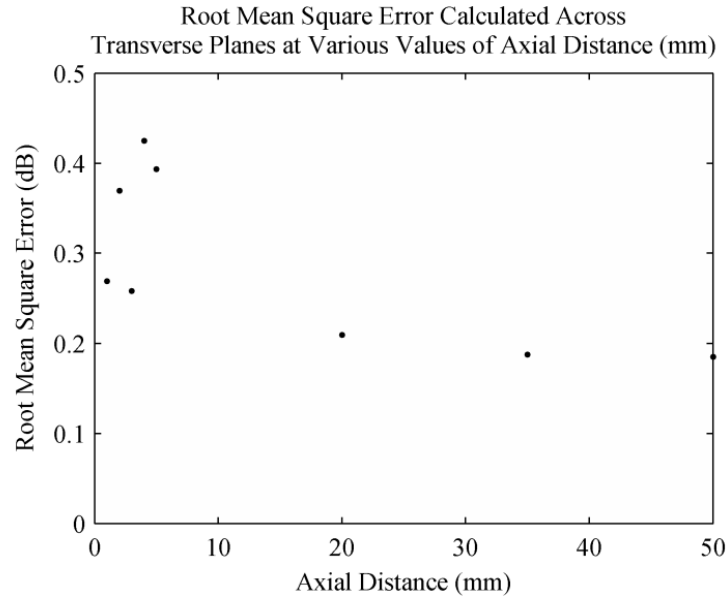


Figure 11: Root mean square error is calculated over a given transverse plane and plotted as a function of axial distance, providing insight into the discrepancy between simulation results and experimental results. The error is on the same order as σ_{UP} .

2.5. Discussion on Sensor Design Space

Similarly to the results shown in Figure 9 and Figure 10, experimentally-measured power values in Figure 12 help to illustrate the spatial behavior of the propagating optical transmission. By rearranging the axes, it is possible to view the direct relationships between measured power and axial displacement for a given radial offset (Figure 7). From these relationships, it is possible to infer the performance of a two-fiber optical lever displacement sensor.

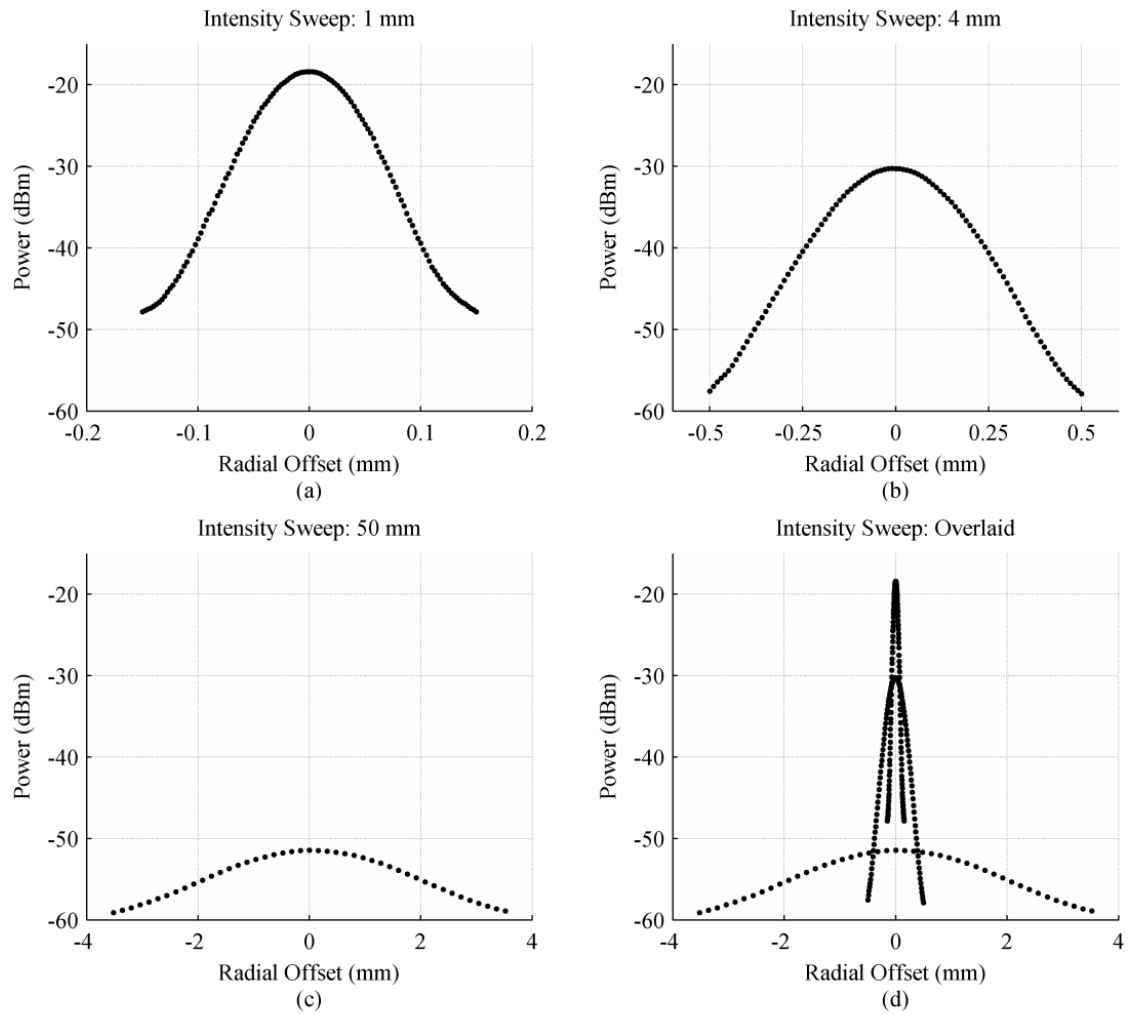


Figure 12: Optical power is shown as a function of radial offset for axial displacements of 1 (a), 4 (b), and 50 (c) mm. Note that the horizontal axes change between plots. The three intensity sweeps are also shown overlaid for easier comparison (d).

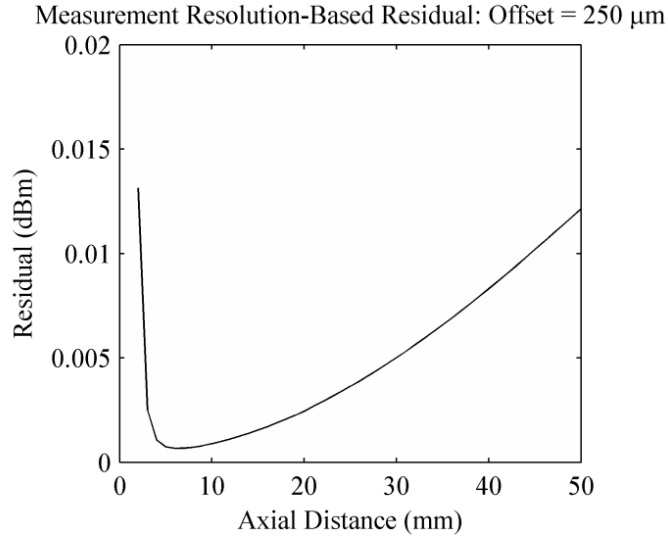


Figure 13: The difference between the measured signal (shown in Figure 7) and the measured signal minus the measurement resolution is used to calculate a sensor's displacement resolution.

A sensor's performance characteristics are governed by its specific architecture. As an example, consider the experimental results shown in Figure 7. First order, linear models were fit to the nearly-linear portions of each intensity curve, and the displacement range of a given architecture was determined by increasing the range of the linear model while keeping the maximum percent error between the linear model and the experimental data below 0.50%. The sensitivity was then calculated as the slope of the linear model. Most radial offsets exhibit two linear sensitivity regions, one with a positive sensitivity and one with a negative sensitivity. A sensor with a lateral radial offset of 250 μm between the transmitting and receiving fibers can exhibit either a sensitivity of 7.18 dBm/mm over an axial displacement range of 1 mm or a sensitivity of -0.10 dBm/mm over a displacement range of 40 mm, with a maximum linear model percent error of less than 0.50% in both cases. The performance of this and other architectures is summarized in Table 3.

Table 3: Two-fiber optical lever displacement sensor performance characteristics are listed in terms of the lateral radial offset between transmitting and receiving fibers.

Offset	Sensitivity (dBm/mm)	Axial Displacement (mm) Range [Min, Max]	Maximum Linear Model Error	Average Linear Model Error	Displacement Resolution (μm)
0 μm	-0.10	40 [50, 90]	0.45%	0.33%	122
250 μm	7.18	1 [2, 3]	0.0%	0.0%	2
250 μm	-0.10	40 [50, 90]	0.46%	0.33%	135
500 μm	2.67	2 [5, 7]	0.37%	0.25%	8
500 μm	-0.10	45 [45, 90]	0.50%	0.24%	198
1000 μm	0.92	7 [8, 15]	0.37%	0.20%	77
1000 μm	-0.09	55 [35, 90]	0.47%	0.14%	824
1500 μm	0.38	12 [13, 25]	0.38%	0.15%	179
1500 μm	-0.05	50 [40, 90]	0.23%	0.08%	1,280

The photodetector's maximum dark current is specified by the manufacturer as 2.5 nA. With a bandwidth of 160 kHz and a responsivity of 0.90 A/W at 1550 nm (both specified by the manufacturer as well), this dark current results in a maximum shot noise power of 12.5 pW. Experimentally, the noise behavior was measured by recording the photodetector's output while no transmission was being sent. To re-create the effect of measurement noise on the data shown in Figure 7, each power measurement once again was calculated as the mean of 1000 points sampled at 1 kHz. A histogram revealed that 800 such mean values were approximately Gaussian-distributed, and their standard deviation was calculated as 26 pW. Measurement resolution is estimated as three times this standard deviation, or 78 pW. The difference between the measured signal and the measured signal minus the measurement resolution is shown in Figure 13 for the 250 μm offset scenario, illustrating the direct effect the measurement noise has on the measured power level. In this particular case and over the explored displacement range, the measured power level could differ from the true power level by as much as 0.014 dBm. Given the 7.18 dBm/mm sensitivity of this architecture (Table 3), this discrepancy would result in a displacement error of as much as 2 μm . Generally, measurement resolution (in dBm) is related to displacement resolution (in μm) for each offset scenario by the corresponding sensitivity. In Table 3, resolution has been calculated in a similar way for each lateral offset

scenario. The noise floor of the experimental setup was observed to be roughly -60 dBm. This is the same as the noise floor of the data acquisition module while no input signal is provided.

Generally speaking, these results give evidence of a large design space, where an increase in sensitivity generally corresponds to a decrease in displacement range and an increase in resolution. With this knowledge of the fundamentals that govern this performance tradeoff, it may be possible to implement a specific offset between transmitting and receiving fibers in order to design a sensor, with application-specific performance characteristics explicitly given. Additionally, using an accurate model of the transmission's spatial behavior, it may be possible to explore a wide range of sensor architectures extending beyond the two-fiber approach. In particular, multiple receiving fibers in any number of spatial configurations may have any number of performance implications.

Portions of Section 2.4 appear in *Proc. SPIE: 21st International Conference on Optical Fiber Sensors 2011*, E. A. Moro, M. D. Todd, and A. D. Puckett, 2011. The title of this paper is "Performance characterization of an intensity modulated fiber optic displacement sensor". The dissertation author was the primary investigator and author of this paper.

2.6. Conclusions

In this chapter, a recently proposed single mode optical fiber transmission model was experimentally validated, and an uncertainty analysis was performed between input parameter uncertainty and output variability. Excellent agreement between model and simulation was established, with RMSE values of less than 0.5 dB, which is well within the 25% and 75% quantiles as defined during the uncertainty analyses. Such strong agreement between simulation and experiment is particularly encouraging, because it motivates the use of this physics-based transmission model for describing intensity-modulated displacement sensor performance. With

accurate knowledge of the spatial distribution of the sensor's EM transmission, it is straightforward to calculate the measured power level at a number of receiving fiber locations (assuming specular, or mirror-like, reflection off the DUT), and from this it is possible to simulate bundled displacement sensor performance *a priori*. Not only is performance simulation potentially useful for performance prediction, but it provides a path for optimization routines, which efficiently search the sensor's performance space in an effort to offer high-performance combinations of sensor performance metrics and thereby meet a particular set of application-specific performance needs.

This chapter has been published, in part, in *Journal of Lightwave Technology*, E. A. Moro, M. D. Todd, and A. D. Puckett, 2011. The title of this paper is "Experimental validation and uncertainty quantification of a single mode optical fiber transmission model". The dissertation author was the primary investigator and author of this paper.

A separate portion of this chapter has been published in *Proc. SPIE Smart Structures/NDE 7982*, E. A. Moro, M. D. Todd, and A. D. Puckett, 2011. The title of this paper is "Experimental verification of a model describing the intensity distribution from a single mode optical fiber". The dissertation author was the primary investigator and author of this paper.

Chapter 3 Intensity-Modulated Sensor: Design Optimization

3.1. Background and Motivation

A recently proposed model, based on the EM wave propagation equations of motion subject to the boundary conditions imposed by a step-index single mode optical fiber [33], has been shown to accurately characterize the optical transmission's power distribution both on and off the axis of propagation. In Chapter 2 this model was experimentally validated, and the effects of known sources of measurement uncertainty and modeling uncertainty were propagated into this model, ultimately resulting in a spatial distribution of (simulated) transmission power levels. Excellent agreement was achieved between this simulation and experimental measurements of the transmission's power levels, with errors of less than 1 dB over an axial displacement range of 1-50 mm and for radial offsets of up to 2 mm. In Chapter 2, the concept of designing a sensor with one of many selectable bundle configurations was also introduced as part of a larger effort to achieve desired performance specifications from within a span of achievable linear sensitivities and corresponding axial displacement ranges.

A yet unexplored area of research is the application of this validated transmission model to the design of a bundled optical displacement sensor. Unlike previously implemented

ray-optic or Gaussian-based models, this model is well-suited for the simulation of sensor performance and the task of sensor design, because of its accuracy in characterizing the transmitted power levels (which are the primarily-desired measurands of interest) that are measured by receiving fiber tips. The transmission model from Chapter 2 may be utilized for generating an accurate representation of the bundled sensor design space, for a range of bundle configurations, constrained by a set of assumptions and performance requirements. Accurate treatment of the design space is critical because the *specific assignment of receiving fibers in the bundle to one of two receiving groups for differential sensing dictates sensor performance*.

In this chapter, a bundled displacement sensor's sensitivity, linear modeling error, and axial displacement range are shown to be highly sensitive to the assignment of receiving fibers to one differential measurement group or another. High-performance arrangements of the receiving fibers in the bundled sensor are shown to exist, offering the potential to combine the robustness of the interferometric approach and the simple computation and high-bandwidth of the intensity-modulated approach. An *efficient exploration of the bundle configuration design space* may be achieved with the use of certain optimization approaches. Although not necessarily guaranteed to always find a globally optimal solution, a genetic algorithm (GA) may be particularly well-suited for the application in this dissertation because of its *efficiency in finding a globally optimal solution within a design space that contains many locally-optimal solutions* [39].

3.2. Comments Regarding Changes in Surface Reflectivity

It is instructive to discuss how differential bundled displacement sensors may exhibit robustness to changes in surface reflectivity, an observation that has been reported on multiple occasions [24,25]. The reflection of light off of any surface may be modeled as a combination

of specular and diffuse components [40,41], and for simplicity, the diffuse component is often assumed to be Lambertian. The apparent brightness reflecting from a Lambertian diffuser (or surface) is proportional to the cosine of the angle between the surface normal and the direction of the incident light [42]. The effect of a Lambertian diffuser is that the apparent reflected intensity is approximately the same when viewed from all directions, except for extremely oblique, grazing ones. There are shortcomings to modeling surface reflection as a combination of specular and Lambertian-diffuse components, as addressed in [41]. For example, this treatment does not model polarization or transmission well, and there is a certain arbitrary nature to the selection of specular and diffuse coefficients (i.e., just how specular and how diffuse is a particular surface?). However, for the purpose of analyzing the intensity of the reflected component of a signal, and for the purpose of explaining robustness to changes in surface in reflection, this treatment of surfaces will suffice.

If light reflecting off a surface is modeled as a combination of a specular reflection (mirror-like) and Lambertian-diffuse reflection (approximately directionally uniform), then the combined effect of these components may be thought of visually as a peak in noise (Figure 14). If changes from surface to surface are then modeled by changes in the relative magnitudes of the specular and diffuse components, then the reflected signal may vary as shown in Figure 14. What is important to glean from these figures is that the shape of the specular reflection is not affected by a change in surface reflectivity, but rather it is only made larger or smaller with respect to the diffuse component. In other words, the relative magnitude of the specular peak as a function of space is insensitive to changes in surface reflectivity. Intensity-modulated fiber optic displacement sensor performance hinges on the relative spatial magnitude of the specular reflection, and as long as a significant specular component is present (i.e., large enough that it

appears distinct from the diffuse component), then its relative spatial properties (or shape) will remain constant.

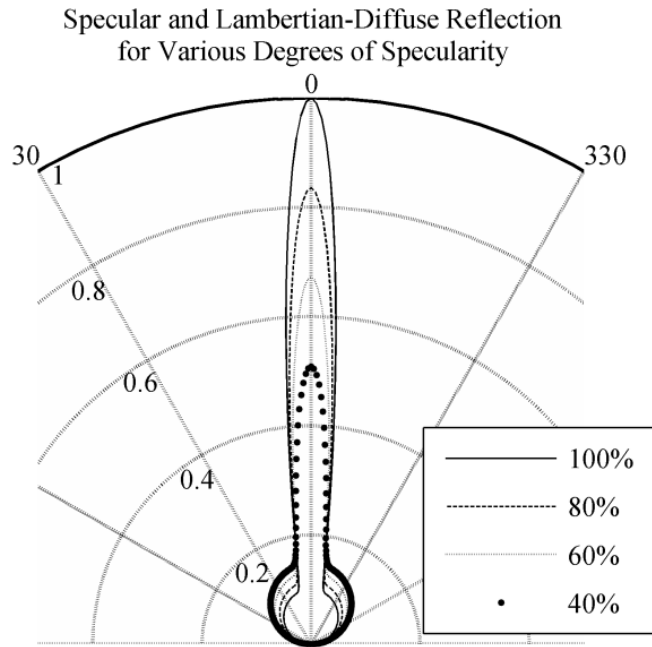


Figure 14: A polar graph of surface reflection shows the effects of changing the relative strengths of the specular and Lambertian-diffuse components. Conservation of energy is assumed, and the specular component strengths are indicated in the legend.

The reflection off the DUT may also be thought of as a “signal” (specular component) and a “noise floor” (diffuse component), and the ratio of the magnitude of the (specular) signal to the magnitude of the (diffuse) noise floor is the reflection’s signal to noise ratio (SNR). The diffuse reflection is spread over a large region (in terms of reflection angle), and as a result the magnitude of the diffuse “noise floor” is approximately constant, even with changes in surface reflection properties (Figure 14). Changes in the relative magnitudes of the specular and diffuse reflections therefore manifest themselves as apparent changes in the reflected signal’s SNR, as opposed to producing changes in the reflection’s shape or changes in the relative magnitudes of the specularly reflected component. Therefore, it is fitting to model reductions in the specular reflection and diffuse components as reductions in the transmission’s power level, or by

extension, *changes in the measured signal's SNR*. At a certain point, reductions in the magnitude of the specular reflection will drive the SNR sufficiently small that the specular signal is overcome by either actual measurement noise or by the diffuse “noise floor”, putting a limit on the differential sensor's robustness to changes in surface reflection.

Recall that differential optical displacement sensor architectures, similar to the sensor proposed in this chapter, have already been shown to be extremely robust to changes in surface reflectivity. For example, in [24] the differential sensor's output was the nearly unchanged when the target surface was changed from gold, to a hard disk, and then to videotape. The brief discussion in this subsection is simply intended to serve as a qualitative explanation for this observed phenomenon.

3.3. First-Iteration Optimization Approach

3.3.1. Optimization Strategy

The formulation of the optimization approach implemented in this section is based on a few foundational assumptions. The physical and optical properties of SMF-28e+ step-index single mode optical fibers (Corning, Inc.) are assumed. Operation over the axial displacement range of 6-8 mm is specifically targeted. The number of bundled fibers and their relative locations within the bundle are fixed, assuming a hexagonal close-packed structure; similar to the example shown in Figure 4 except that the candidate bundle for optimization contains 108 receiving fibers. It is also assumed that there is one centrally-located transmitting fiber and two groups of receiving fibers located around the bundle center (Figure 4). 108 receiving fibers were considered in the analysis because, assuming a cladding diameter of approximately 250 μm and hexagonal close-packing, additional fibers would not have measured an appreciable power level over the 6-8 mm axial displacement range. Complete, specular reflection at the DUT's surface

is assumed; however (as discussed in the Section 3.2) it has been reported that the performance of similar differential optical displacement sensors is robust to changes in surface reflectivity [25,26]. So while surface reflection could have been included as a random variable, the theoretical output insensitivity does not warrant that sophistication.

The measured power level at the location of each of the 108 receiving fibers is numerically calculated over the axial displacement range of interest using the physics-based transmission model (Chapter 2). Both the ratio of one power level to another and the ratio of the difference of two power levels to their sum were calculated, in addition to their reciprocals, resulting in a total of four distinct differential sensor outputs. Given this framework of parameters, constraints, and assumptions, a GA was used to determine which of the receiving fibers are assigned to each of the two differentially-interrogated receiving groups. The span of unique assignments of fibers to one of the two receiving groups constitutes the optimization design space. During this design iteration, a sensor that offers highly-sensitive linear performance over an application-specific axial displacement range of 6-8 mm shapes the target performance. While specific target performance objectives and constraints guide the approach used in this present work, the utility of this approach is that it *may be generalized to other sensing applications* by modifying these performance objectives and constraints (as shown in the following section), offering the potential for sensor designs that exhibit relatively high combinations of sensitivity and accuracy over any range of considered axial displacement ranges. In other words, the validated transmission model, used in conjunction with this optimization framework, may be utilized for designing displacement sensors for operation over small (micrometer) or large (millimeter) axial displacement ranges.

The GA started by describing a set of 100 randomly initialized architectures as binary strings. The strings were broken down into four-digit sections, each of which described the

number of fibers located at a particular radial offset. Numerically calculated power levels were then used to model the differential sensor outputs as a function of axial displacement, and a linear least-squares fit was calculated from this output data. The performance of the initialization configurations could then be evaluated using a weighted linear combination objective function of the form

$$C = w_S S - w_E E \quad (3.1)$$

where C is the function's cost (or score), S is the slope of the least-squares linear fit, which is also the sensor's analogue sensitivity in units of μm^{-1} , and E is the maximum absolute error between the linear fit and the simulated output levels. The values w_S and w_E are weights that set the relative significance of sensitivity and error in the calculation of a particular bundle architecture's cost. The goal of the GA is to maximize C for an application-specific ratio w_S/w_E , and *this is accomplished by a sensor-architecture with a relatively high sensitivity and a relatively low linear-modeling error over the prescribed axial displacement range.*

Candidate sensor architectures were ranked, based on their calculated costs, and were used to effectively breed a second generation of architectures, with the characteristics of the highest ranking architectures being the most likely to propagate into the next generation. This process was performed separately for each of the four differential approaches described, so high-performance architectures were produced for each differential approach. The process thus far was repeated 500 times, until the results converged and additional iterations resulted in no change in the population. The resulting 100 converged architectures were then input into another GA, which provided a small perturbation to each binary string in an attempt to ensure sufficient coverage of the design space, and continued with the approach detailed by the first

GA. This perturbed process was repeated until the results converged, indicated by a cost function leveling off at its largest value with no further population changes occurring.

3.3.2. Simulation Results

3.3.2.1. Optimized Bundle Configurations

The converged outputs of the GA were bundle configurations, according to each of the four differential outputs, that maximized Equation (3.1) over an axial displacement range of 6-8 mm and with a cost function weight ratio $w_S/w_E = 1.0$. This value for w_S/w_E was chosen for the fact that it produced reasonably low error levels with high sensitivity, and an analysis of the configurations that resulted from different values for w_S/w_E is given in the following subsection. Each bundle configuration was defined in terms of the number of optical fibers at a particular radial offset from the central (transmitting) fiber that is assigned to each of the two receiving groups; the diagrams of optimized configurations that are shown in this section are somewhat arbitrary, due to the radial symmetry of the optical transmission. The highest performing differential output according to Equation (3.1) was the ratio P_1/P_2 , whose optimized configuration is shown in Figure 15 (left). This sensor configuration has an analogue sensitivity of $-0.068 \mu\text{m}^{-1}$ and a maximum absolute output error of 3.8 mW/mW, resulting in a displacement measurement error of 56 μm and 97% linearity over the axial displacement range of 6-8 mm (Figure 16 (a)). As a basis for comparison, the optimized architecture of the second highest performing differential output of $(P_1+P_2)/(P_1-P_2)$ is also shown in Figure 15 (right); offering an analogue sensitivity of $0.067 \mu\text{m}^{-1}$ and a maximum absolute output error of 10.2 mW/mW, ultimately resulting in a displacement measurement error of 152 μm and 92% linearity over the axial displacement range of 6-8 mm (Figure 16 (b)). Within the class of differential bundled optical-lever displacement sensors, an analogue sensitivity of $0.0322 \mu\text{m}^{-1}$

is reported with 99% linearity over an axial displacement range of about 75-105 μm [24], and to the best of the author's knowledge, a higher-performing differential optical displacement sensor has not been reported prior to this. By comparison, the optimized configurations presented in this section each constitute a 2.1-fold increase in sensitivity over a much larger axial displacement range and 2% and 7% reductions in linearity, respectively.

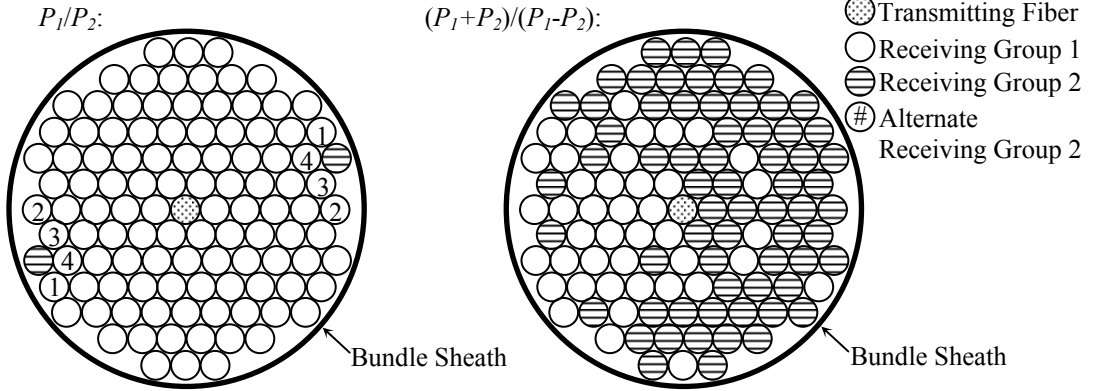


Figure 15: The configurations produced by the GA using Equation (3.1) are shown for $w_S/w_E=1.0$, with P_1/P_2 (left) and $(P_1+P_2)/(P_1-P_2)$ (right). The receiving fibers numbered one through four indicate the first through fourth nearest neighbors, of the receiving fibers in Group 2 (left).

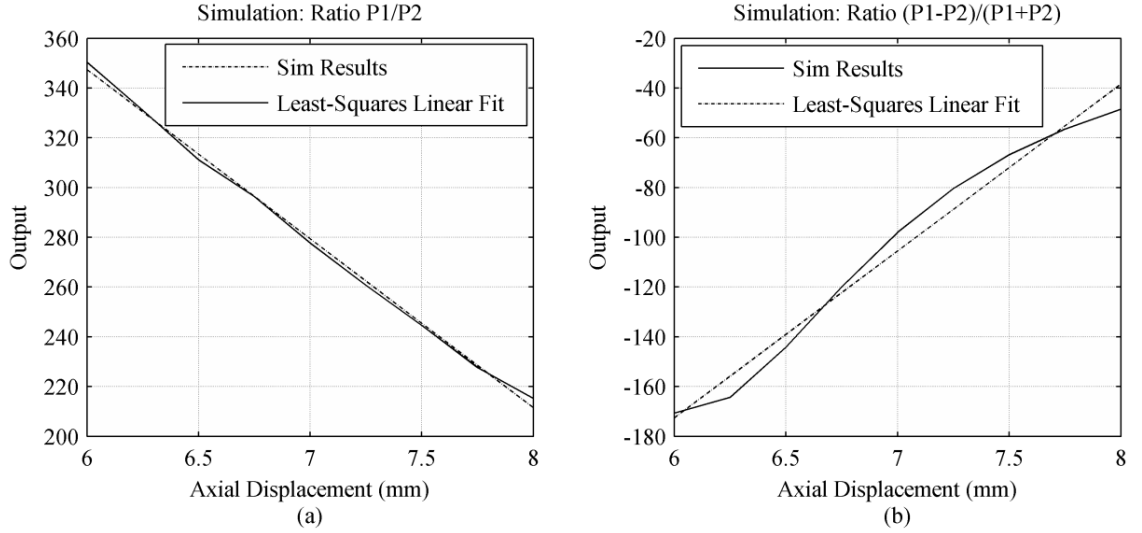


Figure 16: The output of the optimized sensor architecture corresponding to Figure 15 (left) is shown as a function of axial displacement (a). The output of the optimized architecture corresponding to Figure 15 (right) is shown as a basis for comparison (b).

From this point forward through the end of this section, analysis will be limited to the optimized architecture with the differential output of P_1/P_2 (Figure 15 (left)), since this sensor not only had the highest cost (Equation (3.1)) but was also relatively simple to prototype. This sensor's output is shown as a function of axial displacement in Figure 16(a), and the measurement's separate numerator and denominator terms (P_1 and P_2 , respectively) are shown as functions of axial displacement in Figure 17. This optimized result makes physical sense in that nearly the entire measured optical signal is grouped together, providing a relatively large power measurement P_1 , and this large measurement is scaled by the relatively small P_2 to produce a large sensitivity. The selection of the location of the fibers in Receiving Group 2 is appropriate (Figure 15 (left)), because P_2 changes approximately linearly with axial displacement over the 6-8 mm range while P_1 is approximately constant over the same range (Figure 17). In these ways, this optimized configuration is a natural response to the framework introduced by Equation (3.1).

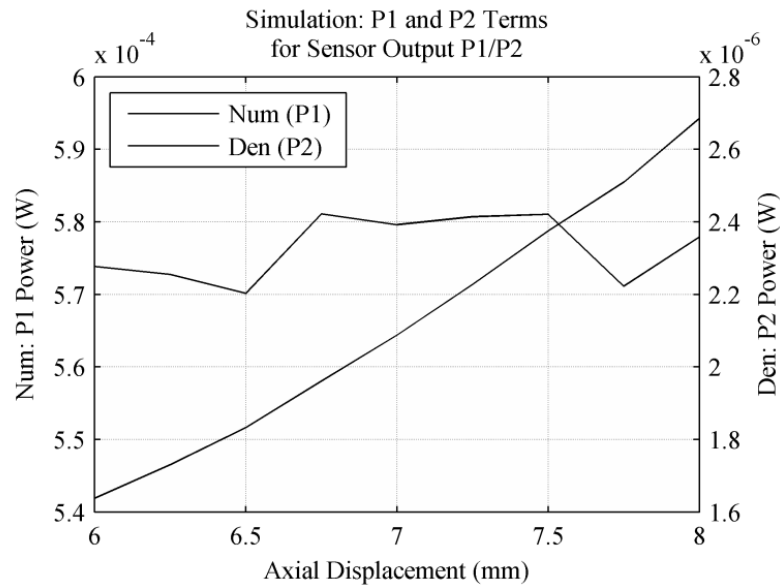


Figure 17: The separate numerator and denominator terms are shown here for the optimized sensor configuration for $w_S/w_E=1.0$ and with an output of P_1/P_2 .

3.3.2.2. Solution Space Analysis

As mentioned in the Section 3.3.2.1, the GA's output was sensitive to the particular cost function weights implemented in Equation (3.1), thereby allowing this design approach to be tuned to meet application-specific performance needs. These cost function weights set the relative importance of sensitivity and linear modeling error in ranking candidate bundle configurations. The optimized bundle configurations for $w_S/w_E = 0.03$ and $w_S/w_E = 10$, and for a sensor output of P_1/P_2 , are shown in Figure 18, assigning more weight to the sensor's error and sensitivity, respectively. The outputs of these configurations are shown as functions of axial displacement in Figure 19. Note that the use of $(P_1 - P_2)/(P_1 + P_2)$ as a sensor output resulted in a more variable design space, where changes in the cost function weights resulted in more dramatic changes in bundle configuration and sensor performance.

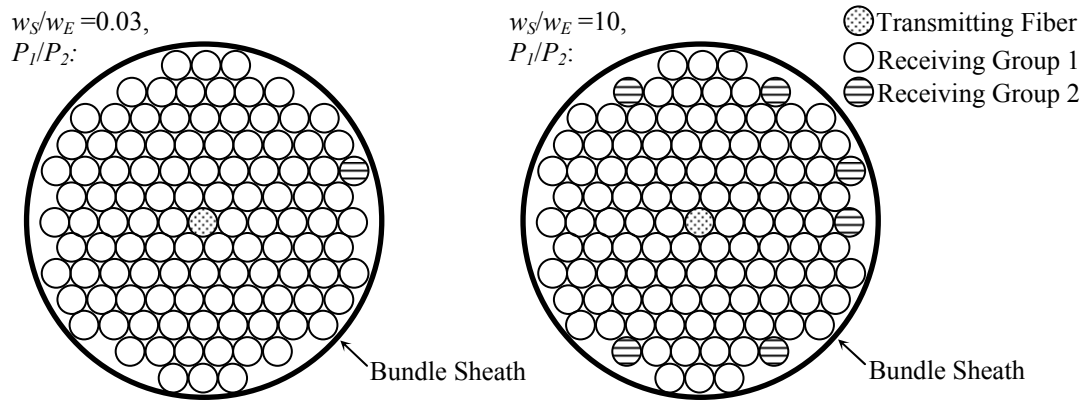


Figure 18: Optimized bundle configurations for a sensor output of P_1/P_2 are shown for $w_S/w_E = 0.03$ (left) and for $w_S/w_E = 10$ (right).

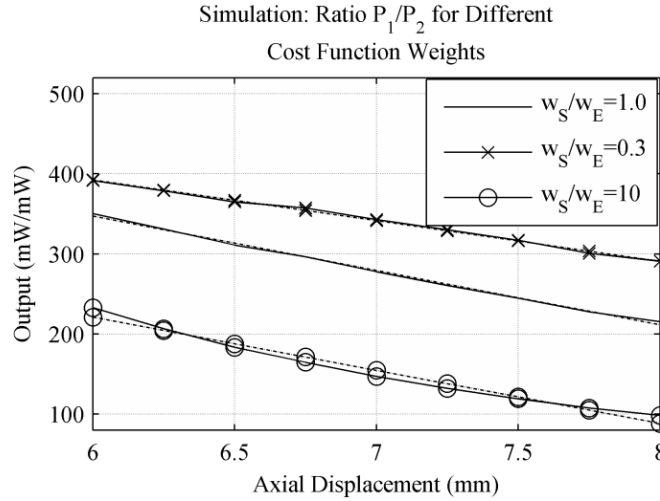


Figure 19: The outputs of optimized sensor architectures with an output of P_1/P_2 , calculated using $w_s/w_E = 0.03$ and $w_s/w_E = 10$, are shown as functions of axial displacement. The corresponding least-squares linear fits are shown as dotted lines.

The bundled displacement sensor's performance is also sensitive to the locations of the particular fibers assigned to each of the two receiving groups. To illustrate this relationship, the outputs of bundle configurations which differ slightly from the GA's optimized configuration are shown as a function of axial displacement in Figure 20. These alternate configurations substitute the fibers with the corresponding numbers in Figure 15 (left) for the optimized configuration's Receiving Group 2 fibers.

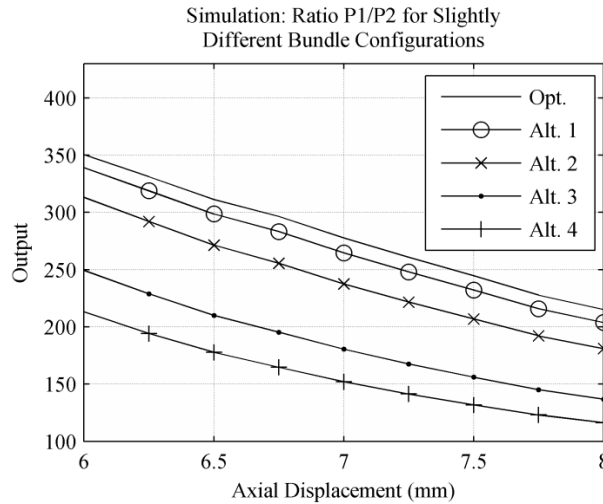


Figure 20: The outputs of bundle configurations that deviate slightly from the optimized bundle configuration are shown. The numbers of the alternate configurations correspond to the numbers of the nearest neighbor fibers in Figure 15 (left).

To further illustrate the broad nature of the sensor's design space, a histogram of cost values calculated from a 1,000,000-trial Monte Carlo analysis is shown in Figure 21, with Figure 21(b) providing a detailed view of the tail of the distribution shown in Figure 21(a). This Monte Carlo analysis was performed over a small subset of the sensor design space, in which the number of fibers at a particular radial offset that were assigned to Receiving Group 1 was necessarily at least three times larger than number of fibers at the same radial offset that were assigned to Receiving Group 2, thereby forcing the GA to converge to a high-performance result (Figure 15). 10,000,000-trial Monte Carlo analyses that were not limited to this subset of the sensor design space failed to produce the converged result that the GA produced after about 1,000 trials. The cost values shown in the histogram were calculated using Equation (3.1) with $w_S/w_E=1.0$ and with a sensor output of P_1/P_2 . The optimized architecture shown in Figure 15 (left) lies at the tail of the distribution, with a cost of 64.2.

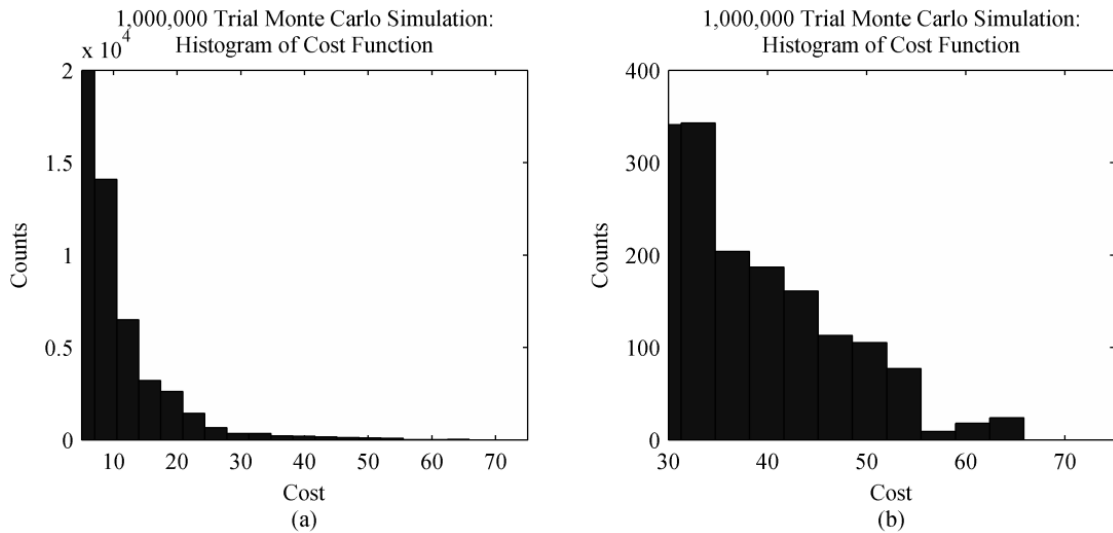


Figure 21: A histogram of cost values are calculated from a 1,000,000 trial Monte Carlo simulation using Equation (1) with $w_S/w_E=1.0$ and a sensor output of P_1/P_2 . The tail of the histogram in (a) is shown in more detail in (b).

3.3.3. Prototyping and Experimental Results

The optimized bundle configuration specified by the GA's output (Figure 15 (left)) was manufactured for the purpose of being experimentally tested, and the actual manufactured sensor, as bundled and grouped, is shown in Figure 22. Obviously the actual fiber bundling is not truly hexagonal close-packed, as was assumed during the optimization routine, but a sufficient amount of the bundle is hexagonal close-packed that the simulation and optimization are still applicable. Appropriate fibers were chosen for assignment to Receiving Group 2 (Figure 15 (left)) by selecting from hexagonal close-packed regions of the manufactured bundle. Recall that Group 1 constitutes an averaged measurement over the majority of the bundle-face and that the measurement P_I is approximately constant over the range of axial displacements from 6-8 mm (Figure 16(a)). For these reasons, it is expected that the slight deviations in the actual manufactured bundle from a true hexagonal close-packed structure will have a negligible effect on the behavior of P_I , and by extension on the sensor output.

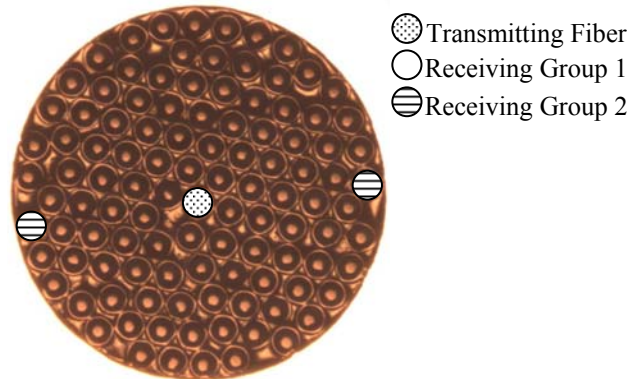


Figure 22: The actual bundle as manufactured and grouped differs slightly from the packing structure that was assumed during optimization. Compare this to the simulated, optimized architecture shown in Figure 15 (left).

The experimental setup used to test the sensor's performance is shown in Figure 23. As stated previously, the optical fibers within the bundle were exclusively SMF-28e+ single mode step-index optical fibers (Corning Inc.). The DUT was a front-surface protected silver coated

mirror (Thorlabs Inc. PF10-03-P01) with percent reflectivity at 1550 nm of about 98%. A pitch-control kinematic mount (Thorlabs Inc. KM100V) and a rotation platform (Thorlabs Inc. MSRP01) were used to control the alignment of the bundle and the DUT, respectively. A 3-axis translational stage (Newport Inc. 561D) was used for positioning the sensor probe relative to the DUT. A 4 mW broadband super-luminescent diode centered at 1528 nm (Covega Corp. SLD 1108) controlled by a thermoelectric cooler/controller (Covega Corp. LDC1300) acted as the optical source, and broadband photodetectors centered at 1550 nm (Thorlabs Inc. DET01CFC and DET50B) and a data acquisition module (National Instruments Inc. PXI-4461) were used for sampling and processing measurements of the received optical signal.

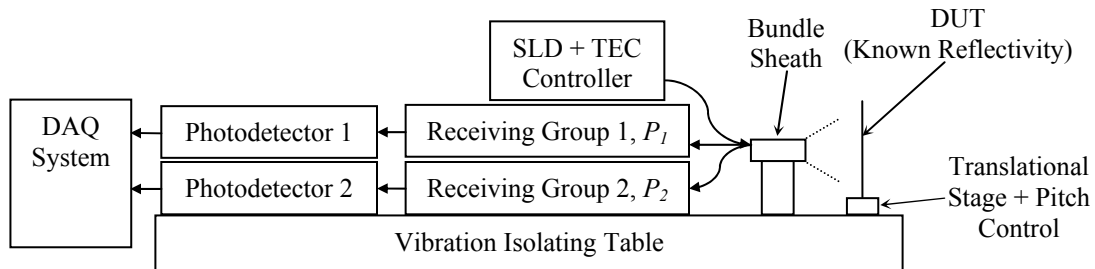


Figure 23: The experimental setup used to test the optimized bundle configuration is shown here.

With the bundled sensor clamped into place, alignment between the DUT and the sensor was established. The DUT was then translated relative to the sensor probe, in order to measure P_1 and P_2 as functions of axial displacement. Power measurements were made across a range of axial displacements and for a range of transmission power levels. From a preliminary analysis of the experimental results, the initial separation distance between the DUT was estimated to be 2.75 mm and the combined effect of all the optical loss mechanisms in the setup (back-reflection, optical coupling, etc.) was estimated to be 4.6 dB.

3.3.4. Sensor Characterization and Discussion

The experimentally measured sensor output is shown in comparison to the simulated output of the optimized sensor configuration in Figure 24, with good agreement between

simulation results and experimental results. The manufactured bundled displacement sensor is capable of measuring DUT displacement over a displacement range of 6-8 mm, with a sensitivity of $-0.066 \mu\text{m}^{-1}$ and an absolute maximum output error of 14.7 mW/mW. This compares to the simulated sensor performance, which predicted operation over a range of 6-8 mm with a sensitivity of $-0.068 \mu\text{m}^{-1}$ and an absolute maximum output error of 3.8 mW/mW. Sources of experimental and modeling uncertainty may very well be able to explain the increase in maximum output error in the experimental case. With this particular sensor, the accuracy over the axial displacement range of 6-8 mm is limited by the experimentally measured absolute maximum output error of 14.7 mW/mW, which corresponds to a displacement measurement error of 223 μm and 89% linearity.

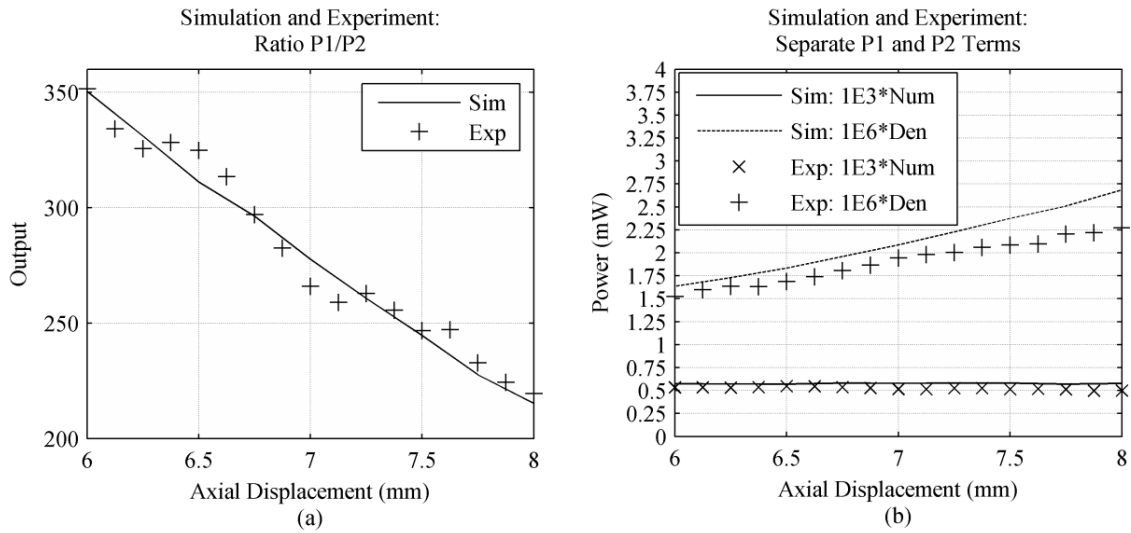


Figure 24: A comparison between experimental results and simulation results shows reasonable agreement.

The robustness of the differential sensing approach was evaluated by changing the source power level during testing, in accordance with the discussion on surface reflectivity in Section 3.2. The sensor output of P_1/P_2 changed notably while decreasing the SLD output power from 4.0 mW to 3.3 mW (Figure 25). In other words, contrary to the expectation, the

differential sensor was not robust to changes in the transmission power level, and by extension it may be assumed that this implies a lack of robustness to changes in DUT surface reflectivity. Since Section 3.2 discussed the anticipated robustness to changes in DUT surface reflectivity, this discrepancy is worth analyzing in greater detail.

Experimental P_I and P_2 measurements are shown as functions of axial displacement for various transmission power levels in Figure 26(a) and (b), respectively. The noise floor of the data acquisition module is also shown in relation to P_2 (Figure 26(b)). The 4.0 mW data is also shown, scaled by 95% and 82.5%, as a basis for comparison, since 3.8 mW and 3.3 mW constitute 95% and 82.5% of 4.0 mW, respectively; ideally a reduction in the transmission power level should correspond to the same reduction in measured power level. Note that for P_I the 3.8 mW data are reasonably close to 95% of the 4.0 mW data, and the 3.3 mW data are reasonably close to 82.5% of the 4.0 mW data (Figure 26(a)), while for P_2 no such agreement exists (Figure 26(b)). The lack of sensor robustness is attributed to the fact that P_2 is close in magnitude to the experimental noise floor (Figure 26(b)), and therefore is not scaled by changes in transmission power level in the same way that P_I is. It is expected that this differential sensing architecture would be robust to environmental and hardware fluctuations, if the measured power levels P_I and P_2 were both to remain much larger than the experimental noise floor over the prescribed axial displacement range.

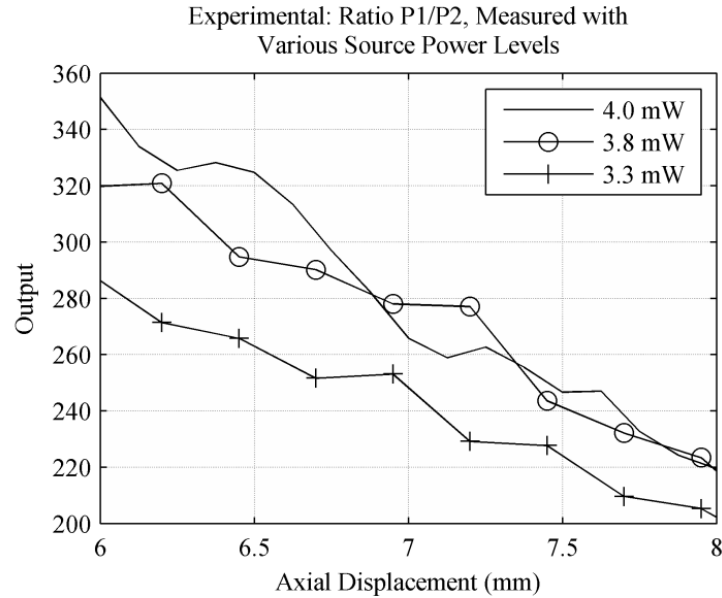


Figure 25: The sensor output is shown as a function of axial displacement for varying transmission power levels. Note that the relationship between input and output is not robust to these changes in power level.

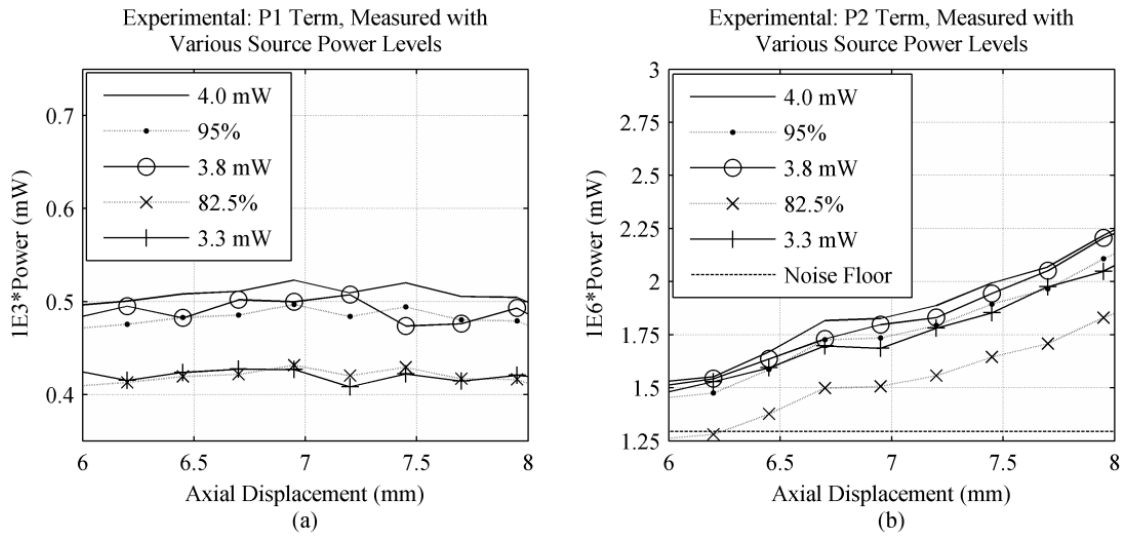


Figure 26: P_1 (a) and P_2 (b) are shown while reducing the transmission power level from 4.0 mW to 3.3 mW. The noise floor of the DAQ system is also shown in relation to P_2 (b). 95% of the 4.0 mW experimental data and 82.5% of the 4.0 mW experimental data are also plotted in both graphs.

Section 3.3 has been published, in part, in *Applied Optics*, E. A. Moro, M. D. Todd, and A. D. Puckett, 2011. The title of this paper is “Using a validated transmission model for the optimization of bundled fiber optic displacement sensors”. The dissertation author was the primary investigator and author of this paper.

3.4. Second-Iteration Optimization Approach

3.4.1. Revised Optimization Strategy

In response to *strong overall agreement* between simulation and experiment, but also the *poor robustness to changes in DUT surface reflectivity* that the first-iteration prototype exhibited (Section 3.3), a second-iteration optimization design was attempted. The differential bundled displacement sensor's architecture is shown in Figure 27, and the 54 receiving fibers are directed to one of two measurement groups for differential displacement measurement. For this design-iteration, an axial-displacement range of 3.5-5.5 mm is specifically targeted, and with a low-power source (<5 mW) additional receiving fibers (beyond the 54 fibers shown in Figure 27) will not measure an appreciable signal level at these ranges. As in the previous subsection, four differential sensor outputs were considered during optimization: P_1/P_2 , P_2/P_1 , $(P_1-P_2)/(P_1+P_2)$, and $(P_1+P_2)/(P_1-P_2)$.

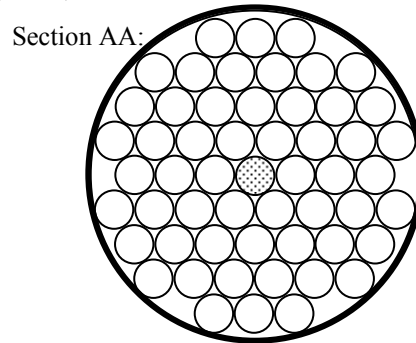


Figure 27: The centrally-located fiber designated as the transmitting fiber, and the remaining 54 fibers are assigned to one of two receiving groups in an effort to optimize sensor performance. “Section AA” refers to the cross-section indicated in Figure 4.

Once again, a GA was employed for searching the bundle's design space, since in the previous effort the GA was quite capable of efficiently exploring its design space to maximize the cost function (Equation (3.1)). For the current design-iteration, however, the GA will search the design space for configurations that maximize the cost function

$$C = w_S |S| - (w_E E)^B - w_R E_{Rob}. \quad (3.2)$$

S is the sensor's sensitivity and is calculated as the slope of the output versus axial displacement curve (calculated in mm^{-1}), E is the linear-modeling error (in %) and is calculated as the maximum absolute error (normalized by the output range) between the sensor output and a linear least-squares fit of the output data, and E_{Rob} is the root-mean-square error (in %) between the nominal linear least-squares fit and a linear least-squares fit calculated when the transmitted power level is reduced to 40% of nominal. 60% simulated reductions in the transmitted power level account for significant reductions in either the transmission level or (more likely) changes in DUT surface reflectivity, and it was shown in during the previous design-iteration (Section 3.3) that sensor robustness hinges on P_1 and P_2 being affected identically by changes in transmission power levels. The cost function weights $w_S=5$, $w_E=200$, $w_R=50$ and $B=3$ in Equation (3.2) were chosen by trial-and-error, with the goal of maximizing sensitivity and keeping linear modeling error and robustness error each below 5%. In other words, although high-sensitivity was desired, its utility is limited if the sensor output's deviation from linearity is too large (in this case, defined as greater than 5%).

3.4.2. Simulation Results, Prototyping, and Experimental Results

The resulting optimized configuration is shown in Figure 28 with the differential sensor output being measured as $(P_1+P_2)/(P_1-P_2)$. This design is intuitive: the numerator is large, the denominator is small, and since P_1 and P_2 are close in magnitude they both scale similarly with reductions in the transmitted (or reflected) power level. This sensor was prototyped by a third party, and a photograph of the actual, prototyped bundle is shown in Figure 29. Obvious discrepancies exist between the assumed hexagonal close-packing (Figure 28) and the actual packing (Figure 29), and the financial burden associated with manufacturing a true hexagonal

close-packed prototype was significantly higher than that required to prototype a less-than-perfect attempt at hexagonal close-packing. The impact of this discrepancy is discussed in detail in the following subsections.

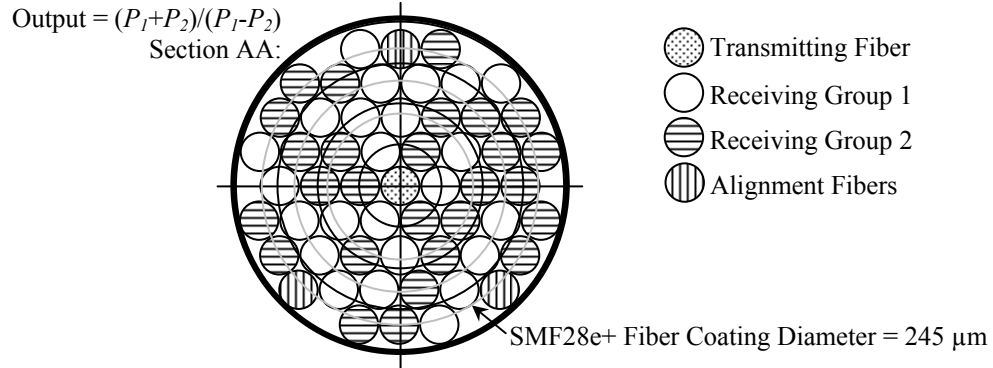


Figure 28: The simulation's optimized configuration is shown. Concentric circles show the radii at which receiving fibers are located. "Section AA" refers to the cross-section in Figure 4.

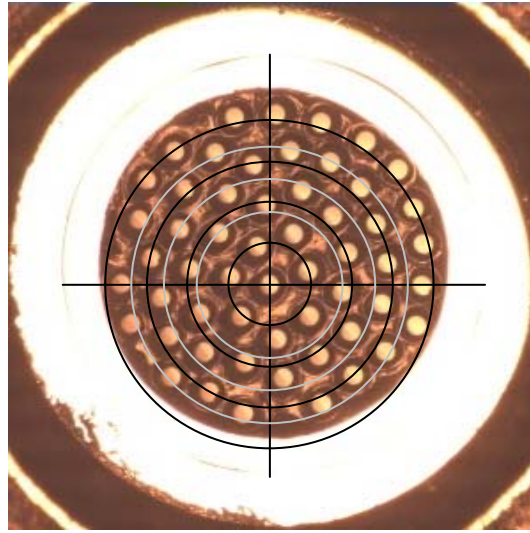


Figure 29: The photograph of the actual prototype bundle is shown here. For reference, axes and concentric circles (corresponding to those shown in Figure 28) are drawn centered at the transmission fiber.

The simulated performance of this second-iteration sensor configuration is shown in Figure 30. This sensor offers a relatively large sensitivity ($0.043 \mu\text{m}^{-1}$), with a low linear modeling error (3.1%), over the prescribed 3.5-5.5 mm axial displacement range. Further, a 60% reduction in the transmitted power level results in only slight (1.1%) deviations to the

linear least-squares fit, predicting that the sensor should have high levels of robustness to changes in surface reflectivity of up to 60%. This robustness, in theory, is derived from the notion that changes in the transmitted power level and changes in surface reflectivity will ultimately be manifested in the same way in both the numerator term (P_1+P_2) and denominator term (P_1-P_2). This compares to the first-iteration optimized configuration which offered a sensitivity of $-0.066 \mu\text{m}^{-1}$ with 11% nonlinearity error over an axial displacement range of 6-8 mm, but which offered no robustness to changes in surface reflectivity, since the measured power levels P_1 and P_2 were of such different scales.

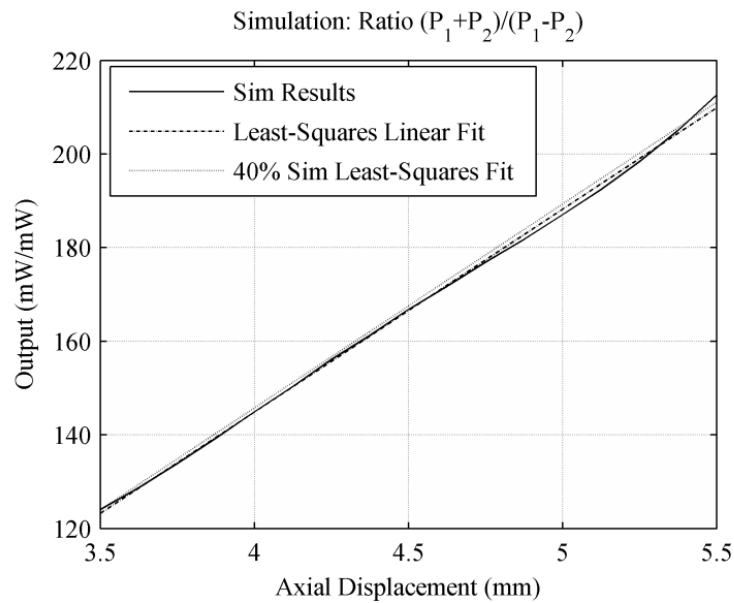


Figure 30: The simulated sensor output is shown for the optimized bundle configuration as a function of axial displacement. A linear fit of the simulated data provides an indication of the output's linearity, and a linear fit is generated for a 60% reduction in the transmission's power level.

In order to characterize the relationship between sensor output and DUT displacement, the optimized prototype was clamped and aligned with the DUT (Figure 31). The optical source was a thermo-electrically controlled 5 mW superluminescent diode, whose broadband transmission is centered near 1528 nm (Covega Corp. SLD 1108). The DUT was a front-surface silver coated mirror (Thorlabs Inc., PF10-03-P01) with a percent reflectivity at 1550 nm of

about 98%. The DUT position was controlled using a tri-axial linear stage with micrometer accuracy (Newport Corp. 561D). Single mode step index fibers (Corning, Inc. SMF-28e+) were used in the bundled sensor. Broadband InGaAs photodetectors with large (1 mm^2) detection areas (Thorlabs Inc. DET10C) and a data acquisition system (National Instruments Corp. NI PXI-4461) converted the measured optical signals to digitized, electrical signals. This system had a maximum sampling rate of 200 kHz, limited by the particular data acquisition system employed. With a load resistance of $10 \text{ k}\Omega$, the photodetectors have a bandwidth of 400 kHz, and this may be increased by decreasing the photodetectors' load resistance. 4 dB of attenuation was estimated (experimentally) between the optical source and the data acquisition system. The coupler between the source and the bundled probe has 2 dB of attenuation in and of itself, and it is not unrealistic to attribute 2 dB of attenuation to the bundled package.

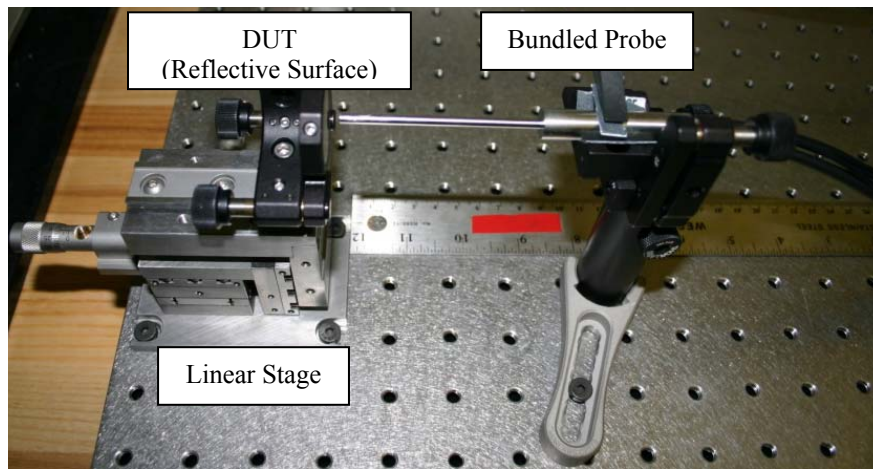


Figure 31: A photograph of the experimental setup is shown here.

3.4.3. Results and Discussion

3.4.3.1. Comparison between Experimental and Simulation Results

A comparison of simulated and experimental power measurements P_1 and P_2 (as designated in Figure 28 and Figure 29) is shown in Figure 32, with the experimental data being

sampled at a rate of 1 kHz. Two points of agreement between experimental data and simulation data may be observed — (1) P_1 is generally larger than P_2 and (2) both P_1 and P_2 tend to increase as the axial displacement of the DUT is increased from 3.5 mm to 5.5 mm. However, there are also two distinct discrepancies between experimental and simulated results that should be discussed. Firstly, and most significantly, the magnitude of the simulated power measurements is as much as 125% of the experimentally observed magnitude over the 3.5-5.5 mm range. Judging from the discussion in Section 2, this is likely attributed to discrepancy between the predicted bundle packing (true hexagonal close-pack shown in Figure 28) and the actual bundle packing (a less-than-perfect attempt at hexagonal close-packing shown in Figure 29). Deviation from hexagonal close-packing necessarily results in a *reduction in fiber density*, and this is readily apparent from the photograph of the actual packing of the prototype (Figure 29). Consequently, one can, at the very least, expect the experimentally measured power levels to be less than simulated (anticipated) ones (particularly at close ranges). A sensitivity analysis is performed in the following subsection in an effort to further characterize the relationship between uncertainty in receiving fiber locations and variability in the observed sensor output. The second readily apparent discrepancy between experiment and simulation is the noise level on P_1 and P_2 . While the magnitude of the noise floor was assumed during the optimization routine (Section 2), the effects of random noise on the SNR of the measured data were not considered. Noise on P_1 and P_2 introduces a peaky behavior, which is especially large in comparison to the magnitude of the difference $P_1 - P_2$. When this difference term is put in the denominator of the sensor output, the peaks are amplified, resulting in erratic behavior (Figure 33).

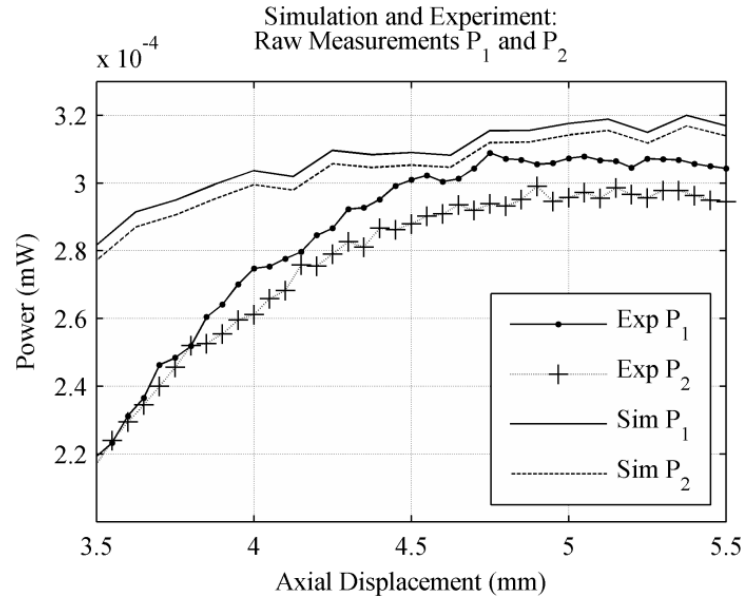


Figure 32: Simulated and experimental power measurements (P_1 and P_2) have a similar shape, although the simulated power levels are generally larger in magnitude.

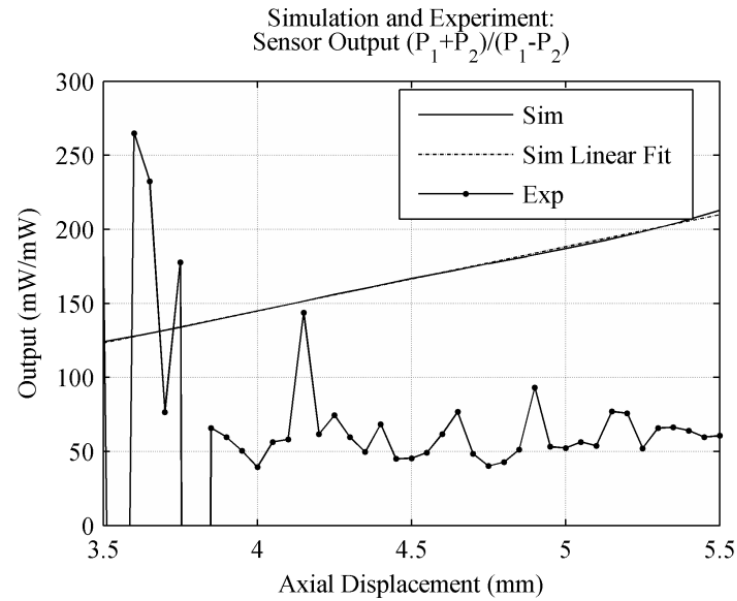


Figure 33: A comparison of experimental and simulated sensor output data shows poor agreement, which can be attributed to reduced fiber density within the bundle and low SNR in P_1 and P_2 .

Recall from Section 2 that this sensor was designed to exhibit high sensitivity between its output and changes in DUT displacement. In actuality, this sensor exhibits high-sensitivity to any fluctuations in P_1 and P_2 , and it was previously assumed that these fluctuations would

primarily be displacement related. Unfortunately, detector noise and data acquisition system noise ultimately result in a measurement signal with a low SNR, which fundamentally limits the performance of this low-power displacement sensor. The optimization routine (Equation (3.2)) did what was “asked” of it; in that, a sensor with *high-sensitivity* was designed with an *allowable degree of nonlinearity* over a *prescribed axial displacement range*. The magnitude of the noise floor was considered as it pertained to the magnitudes of P_1 and P_2 , since *robustness hinges on these terms responding in the same way to variability* (e.g., changes in surface reflection, transmitted power level, etc.). However, the optimization routine did not consider overall SNR along with the fact that high sensitivity to axial displacement is in reality a high sensitivity to perturbations on the sensor input (i.e., measured power level). The end result in this case is *high sensitivity to noise* on the measured power levels (manifested either in the photodetectors or the data acquisition system (Figure 4)). To rectify this issue, the optimization cost function needs to be restructured to consider the final SNR of measured signals in addition to the magnitude of the noise floor as it pertains to large (e.g., 40%) reductions in surface reflectivity.

3.4.3.2. Sensitivity Analysis

A sensitivity analysis of the transmission model for a signal emitting from a step-index, single mode optical fiber was performed in Chapter 2, and in that analysis variability in the radial offset between a transmitting fiber and a receiving fiber was shown to largely impact the measured power level. In particular, for the second-iteration optimized prototype sensor, small changes in the output’s denominator term ($P_1 - P_2$) may result in large changes in the measured sensor output, thereby amplifying the already sensitive relationship between P_1 and P_2 and the true positioning of receiving fibers.

A *sensitivity analysis* is performed by perturbing a model's input parameter (in this case, the radial offset between transmitting and receiving fibers) in such a way as to thoroughly cover a realistic parameter space. The variability that results on an output parameter is then observed and related to the degree of input parameter variability that was induced. In this case, the received power levels at particular radial offsets were allowed to vary (according to a uniform distribution) between 80% of nominal and 120% of nominal. This 40% range realistically accounts for deviations in the actual location of receiving fibers as well as the reduced density of receiving fibers, and the assumption of a uniform distribution assumes no prior knowledge (thereby acting as a conservative estimate of location uncertainty). The input parameter variability in this case admittedly will not accurately reflect the actual variability seen in the prototype, but it is useful for showing the highly-sensitive relationship between input parameter variability and sensor output variability. The results of this analysis are shown in Figure 34, where the 25%-quantile and the 75%-quantile of the variable sensor output data are shown in comparison to the observed, experimental sensor output data. Note that these quantiles are calculated assuming no additional noise, and therefore are the direct result of variability in the receiving fiber locations.

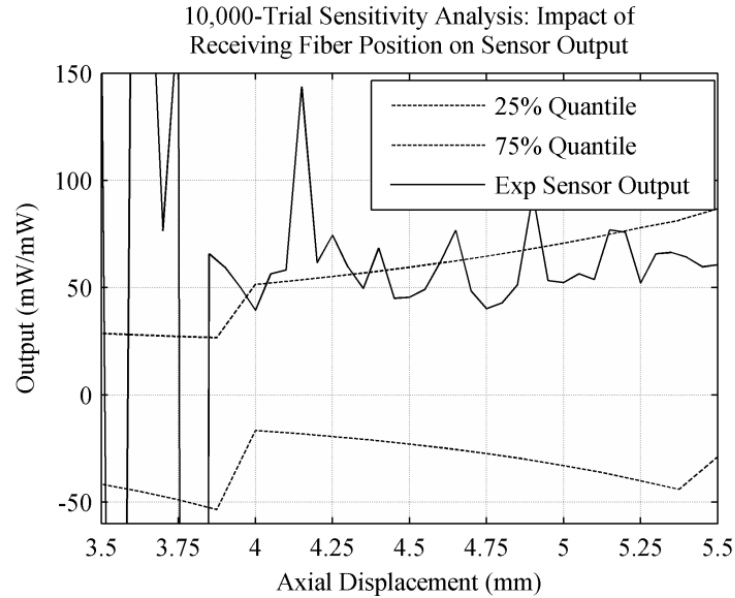


Figure 34: Sensitivity analyses show the dependence that the sensor's output has on the particular location (and bundling) of receiving fibers. In this analysis, variability in fiber position was simulated using variations on the measured power levels, uniformly distributed between 80% and 120% of nominal.

Section 3.4 has been published, in part, in *Proc. SPIE Smart Structures/NDE 8346*, E. A. Moro, M. D. Todd, and A. D. Puckett, 2012. The title of this paper is "Performance optimization of bundled fiber optic displacement sensors". The dissertation author was the primary investigator and author of this paper.

3.5. Conclusions

An optimization framework was proposed for use in designing bundled fiber optic displacement sensors whose specific bundling configurations are designed for *optimal performance over a prescribed, application-specific displacement range and with an allowable degree of error*. This method employs a recently validated optical transmission model to accurately characterize a given sensor configuration's transmitted and measured power levels, thereby simulating sensor performance *a priori*; allowing one to take advantage of high-performance regions of a sensor design space. The use of specific, optimized bundling strategies

were employed in hopes of achieving high-overall combinations of sensitivity, accuracy, and linear displacement range, while maintaining high sampling-rates that intensity-modulated sensors offer.

The first iteration differential bundled displacement sensor offered a high-performance combination of sensitivity and maximum absolute error over an axial displacement range of 6-8 mm; with the experimentally measured sensitivity of $-0.066 \mu\text{m}^{-1}$ comparing well to the simulated sensitivity of $-0.068 \mu\text{m}^{-1}$. The experimental displacement measurement error of $223 \mu\text{m}$ was inferior to the simulated displacement measurement error of $56 \mu\text{m}$, and this discrepancy may be attributed to various sources of experimental and modeling uncertainty that were not accounted for during optimization, and which may have been amplified by an unstable sensor design. The result of the GA was shown to depend on the weights of the cost function, which set the relative significance of sensor error and sensitivity, and the utility of this approach is in *generating a sensor design space* over an axial displacement range of interest and achieving high-sensitivity with an acceptable level of displacement measurement error. Unfortunately, this optimized sensor *was not robust to changes in the transmission power-level*, suggesting a lack of robustness to changes in DUT surface reflectivity.

The second iteration of the optimized sensor was designed in an effort to address the first-iteration prototype's lack of robustness to changes in DUT surface reflectivity. The primary shortcoming of the particular sensor investigated in this paper was the difference between the assumed (hexagonal close-packed) bundle structure and the actual prototyped bundle structure. A sensitivity analysis showed that the sensor's differential output is highly-sensitive to variability in the true location of receiving fibers within the bundle. The other major shortcoming of this particular prototyped sensor is the low-SNR of the measured signals. The highly-sensitive nature of the optimized bundle configuration resulted in a sensor that was

highly sensitive to input variations of every type, whether they be noise or actual DUT displacement. In future work, some metric of SNR must be used in any effort to optimize sensor performance, in addition to a consideration of the noise floor magnitude as it relates to sensor robustness (as discussed regarding the first iteration). Practically the power level of the differential, measured signals may be simulated using the previously described transmission model, and the noise level of the optical source, photodiodes, and any digitizing hardware may be estimated from hardware specifications. Thus, with means for estimating these two levels *a priori*, the SNR may be determined for a particular differential bundle arrangement, and the cost function weights associated with the SNR may then be established to meet appropriate degrees of sensor stability and robustness. Finally, after the GA has completed and as a further investigation of sensor robustness and stability, it would make sense to take a particular optimized architecture and simulate its performance in the presence of varying degrees of measurement noise and changes in surface reflectivity.

With both of the designed and manufactured prototypes failing to exhibit robustness to significant changes in the transmitted power level, it is not possible to provide experimental evidence supporting the notion of robustness that these differential sensors exhibit to changes in DUT surface reflectivity. The fact remains that without a sufficient SNR in each of the distinct power measurements, it cannot be assumed that each of those power measurements will be affected by fluctuations in the same way. In spite of these shortcomings, this design framework *still holds promise*, as the first-iteration optimized prototype showed *strong agreement between model and experiment*. The lessons learned in this research must be considered in any subsequent implementations of this design framework. Further, this approach offers the potential for *facilitating high-performance sensor operation* that suits application-specific

needs, offering non-contacting displacement measurement with a non-spark emitting source at high (kHz-MHz) sampling rates.

Chapter 4 Extrinsic Fabry-Perót Interferometric Displacement Sensor: Static Performance

4.1. Background and Motivation

As mentioned in Section 1.2, EFPI displacement sensors measure the absolute displacement of some DUT by relating the interferometer's OPL difference to interference in the measured interferogram [20-23]. White light EFPIs use either a tunable source or a broadband source transmitted through a tunable filter [23] to measure several interference fringes over a range of wavelengths (or frequencies) (Figure 35), and the OPL difference may be calculated from a peak in the interferogram's PSD estimate. A discrete Fourier transform (DFT) is often utilized for this task of PSD estimation, and in [9] M. Han presents an excellent analysis of different techniques utilized for EFPI PSD estimation. White light EFPI displacement sensors offer absolute (unambiguous) displacement measurements over relatively large (cm) displacement ranges with moderate-accuracy (μm), and are robust to fluctuations in the measured power level (cause by variation in the transmission, surface reflectivity, loss mechanisms, or environmental conditions), since displacement is measured directly from phase in the interferogram (and not from intensity in the interferogram). In some applications, this

combination of absolute-displacement measurement and robust operation is highly desirable, since experimental uncertainty can be costly to eliminate.

The primary shortcomings of the white light sensing approach are that, as of this writing they have been limited to sampling rates in the low-Hertz range and they have not achieved the sub-nanometer accuracy-levels of less-robust, feedback-controlled, monochromatic EFPI displacement sensors (which are typically deployed in highly-controlled testing environments) [14]. Nonetheless, white light EFPI sensors constitute a significant improvement in accuracy, robustness, and displacement range over attempts at performance-optimization of intensity-modulated sensing architectures (as described in Chapter 3). In this chapter, the performance relationships between displacement resolution, maximum displacement range, and various EFPI input parameters will be reviewed, providing the basis for a final performance comparison between optimally designed intensity-modulated displacement sensors and interferometric ones.

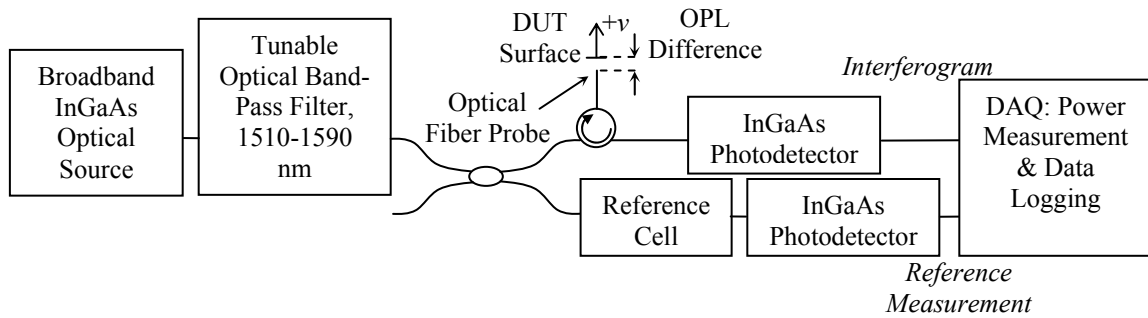


Figure 35: A diagram of a typical swept-filter, white light EFPI displacement sensor is shown here. This configuration, with components that operate in the 1510-1590 nm range, is representative of the one researched in this paper.

4.2. Performance Relationships in the Static Case

As described in Section 4.1, DUT displacement is calculated from the PSD estimate of a white light EFPI's interferogram. Consider the formulation for the ratio of the intensity reflected by a Fabry-Perót cavity I_{refl} to the intensity incident on that cavity I_{inc} [35]

$$\frac{I_{refl}}{I_{inc}} = \frac{G \sin^2(2\pi f L n / c)}{1 + G \sin^2(2\pi f L n / c)}, \quad (4.1)$$

where the function G is defined as

$$G = \frac{4r_1 r_2}{(1 - r_1 r_2)^2}. \quad (4.2)$$

In Equations (4.1) and (4.2), f is the frequency of the propagating EM wave, L is the Fabry-Perót cavity's length (or the OPL difference), n is the refractive index within the cavity, $c=2.998 \times 10^8$ m/s is the speed of light in a vacuum, and r_i is the amplitude-reflection coefficient of the i^{th} cavity surface. Note that, as a result of the periodicity of Equation (4.1), the frequencies f at which I_{refl} is maximized must satisfy the relationship

$$f = \frac{Nc}{2nL} \quad (4.3)$$

for integer N . Values for f that solve Equation (4.3) are referred to as the Fabry-Perót cavity's resonant frequencies, and the space between adjacent resonant frequencies R , also referred to as the cavity's FSR, is calculated in units of Hz as

$$R = \frac{c}{2nL}. \quad (4.4)$$

The FSR is constant as long as L and n remain unchanged, and in the static case, DUT displacement is calculated from the Fabry-Perót cavity's FSR by rearranging Equation (4.4) to yield

$$L = \frac{c}{2nR}. \quad (4.5)$$

By sweeping over a frequency range, a series of intensity fringes are produced whose spacing is constant and equal to the cavity's FSR. The task of estimating the FSR is related to the well-studied problem of estimating the frequency of a sinusoid. An example of a typical white light EFPI interferogram is shown in Figure 36, where the horizontal axis has been described in terms of wavelength units, since these values are more familiar. The spacing between adjacent peaks is approximately constant in units of nm over the range shown in Figure 36, but it is absolutely constant in units of Hz (according to Equation (4.4)), where frequency and wavelength are related to one another by c . This constant spacing is the inverse of the signal's apparent time domain frequency. A thorough overview of the calculation of the white light EFPI's FSR is given in [9], and a more general overview of the estimation of a sinusoid's frequency as it pertains to DSP is given in [43].

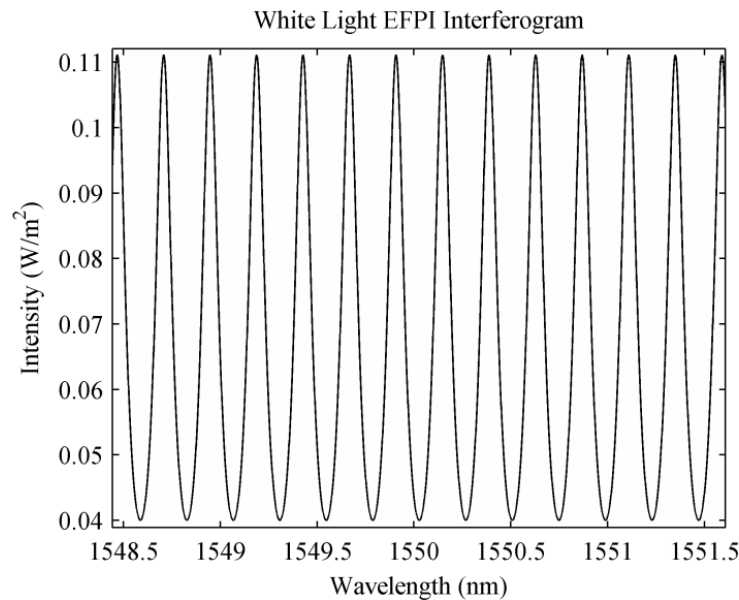


Figure 36: A typical white light EFPI interferogram is shown here, with the spacing between resonant peaks being approximately constant over this wavelength range.

The calculation of the DFT is one of the simplest ways to estimate a sinusoid's frequency, and it has been widely implemented for the task of white light EFPI displacement sensing specifically [9,21-23,44-45]. The DFT approach to frequency-estimation offers the

advantage that it is easy to characterize its performance capabilities, and the properties of the DFT are well understood. The DFT operation is also fast and computationally simple to perform. Finally, according to [9], while the DFT approach does not achieve the level of accuracy attained by various curve-fitting approaches, this is shown to be less of an issue over larger (mm-cm) axial displacement ranges. In other words, at an axial displacement that is less than or near the resolution of the DFT operation, it is obvious that the sensor would have trouble characterizing DUT displacement. The DUT displacement (or OPL difference) is calculated from the DFT using the relationship

$$L = \frac{cK}{2nNFFT|f_1 - f_2|} \quad (4.6)$$

In Equation (4.6), K is the frequency bin in which the DFT's FSR-based peak-value is located, $NFFT$ is the length of the DFT, and f_1 and f_2 are adjacent frequency values taken from the interferogram. When this approach is used, the fundamental performance limitations of the quasi-static displacement sensor are governed by the limitations of the Fourier analysis. Without any additional DSP, the displacement resolution may be calculated from the length of the DFT

$$L_{res} = \frac{c}{2nNFFT|f_1 - f_2|} \quad (4.7)$$

and since K is at a maximum $NFFT/2-1$ according to the Nyquist-Shannon sampling theorem, the maximum displacement range that is calculated without aliasing is calculated approximately

$$L_{max} = \frac{c}{4n|f_1 - f_2|} \quad (4.8)$$

From Equation (4.7) and Equation (4.8), it is apparent that L_{max} depends almost entirely on the spacing between adjacent filter frequencies, whereas L_{res} depends on this as well as the

length of the DFT. In fact, an increase in $NFFT$ that is accompanied by a decrease in $|f_1 - f_2|$ may not have any impact in L_{res} , and this relationship is made more obvious by describing L_{res} directly in terms of EFPI hardware input parameters. Consider that the parameter $NFFT$ may be described directly in terms of the sampling rate of the DAQ system (f_{DAQ}) and sweep rate of the swept filter (f_{sweep})

$$NFFT = \frac{f_{DAQ}}{f_{sweep}}, \quad (4.9)$$

and the space between adjacent frequency values may be described using these rates along with the frequency range covered by the swept filter

$$|f_1 - f_2| = \frac{f_{range}}{NFFT} = \frac{f_{range} f_{sweep}}{f_{DAQ}}. \quad (4.10)$$

Equations (4.9) and (4.10) may be substituted into Equation (4.7) to yield

$$L_{res} = \frac{c}{2nNFFT} \left(\frac{f_{DAQ}}{f_{range} f_{sweep}} \right) = \frac{c}{2n} \left(\frac{1}{f_{range}} \right), \quad (4.11)$$

Interestingly, L_{res} depends only on the input parameter f_{range} , which makes sense in light of the relationship between $NFFT$, f_{DAQ} , and f_{sweep} . Similarly, Equation (4.8) may be rearranged and described as

$$L_{max} = \frac{c}{4n} \left(\frac{f_{DAQ}}{f_{range} f_{sweep}} \right). \quad (4.12)$$

Equations (4.11) and (4.12) describe the displacement-resolution and displacement-range performance limitations of a white light EFPI directly in terms of EFPI tunable filter input parameters.

It is conceivable that these performance limitations may be improved upon using DSP techniques, such as filtering, averaging (either prior to or following the DFT), and zero-padding. In particular, although zero-padding does not increase the actual resolution of the signal being measured, it does decrease the width (or spacing) of frequency bins in the DFT, and it has been shown to enhance DFT performance [43]. Averaging prior to the DFT operation is referred to as coherent averaging of spectra, and care must be taken during this process to ensure that the signals to be averaged are not out of phase with one another, or interference may occur and the signal content may be lost [46]. Incoherent averaging is performed after the DFT, and it eliminates the risk of signal loss due to phase changes. Also, coherent averaging, when done properly, offers faster convergence than incoherent averaging [46]. In either case, averaging may be used to enhance the SNR of the PSD-estimate, possibly enabling the measurement of DUT displacement at large ranges or in the presence of extremely low DUT surface-reflectivities. Averaging does not, however, have any impact on L_{res} or L_{max} , although it may affect the overall accuracy of a white light EFPI displacement sensor system, particularly when the sensor is deployed in a low SNR testing environment.

4.3. Conclusions

As described in Section 1.2, the white light EFPI offers basic performance advantages within the category of non-contacting fiber optic displacement sensors: namely the absolute displacement measurement capabilities (requiring no calibration) and the robustness to power fluctuations that is inherent to the PSD (sinusoidal frequency-estimation) problem (as introduced in Section 1.2.2). With the use of Equations (4.11) and (4.12), it is possible to begin to compare the performance limitations of a white light EFPI, demodulated using a DFT, to the performance capabilities of other displacement-sensing methodologies. However, what has not

been discussed in this chapter is the impact of dynamics on the EFPI measurement system. Not only is the dynamic-performance-capability of a white light EFPI limited by the DAQ system's sampling rate and by the speed at which the swept filter sweeps, but, as will be shown in the following chapter, its performance is also limited more fundamentally by the distorting effects that DUT motion imparts on the signal circulating within a Fabry-Perót cavity.

Portions of this chapter have been submitted to *Journal of Lightwave Technology*, E. A. Moro, M. D. Todd, and A. D. Puckett, 2012. The title of this paper is "Phase diversity approach for simultaneous measurement of velocity and displacement using a white light Fabry-Perót interferometer". The dissertation author was the primary investigator and author of this paper.

Chapter 5 Extrinsic Fabry-Perót Interferometric Displacement Sensor: Dynamic Performance

5.1. Background and Motivation

If a white light EFPI is to be implemented as a dynamic displacement sensor, the effects of Fabry-Perót cavity dynamics on the measurement signal must be considered. At the very least, two questions must be taken into account. Firstly, if the DUT's position changes during the course of the white light EFPI's frequency sweep, how does this motion impact the measured interferogram for different velocities and sensor sampling rates? And second, to what degree does a Doppler-shift, imparted when the signal reflects off a moving DUT, impact the sensor's displacement measurement?

Excellent analyses of the dynamic responses of Fabry-Perót interferometers were performed by Rakhmanov *et al.* [15, 16] and by Redding *et al.* [17] with the particular goal of cavity-length feedback control in gravity-field measurement applications where the interferometer's sensing cavity is tens to thousands of meters in length and where sensor-implementation takes place near one of the cavity's optical resonances. An analysis of the effects of other dynamically changing EFPI parameters (e.g., refractive indices) is given in [47].

Another excellent analysis by Lawrence *et al.* [18] is based on the same fundamental formulations and considered the effects of small deviations in the source wavelength and the sensing cavity length; specifically with regard to modeling high-finesse cavities that have highly reflective surfaces and consequently allow a signal to oscillate within the cavity for relatively long storage times prior to being attenuated below the $1/e$ field amplitude-level. Each of these analyses, however, focused on the time-domain response (or impulse response) of a long, high-finesse Fabry-Perôt cavity that experiences a parameter perturbation. To the best of the author's knowledge, no one has performed such an analysis on the effects of cavity dynamics for a low-finesse white light EFPI, where the cavity has a relatively short $1/e$ storage-time, but the dynamic response of the interference spectrum is nonetheless impacted by a Doppler-shift in reflections off a moving surface.

It should also be noted that previous conclusions published in [15] and [18] describe the critical target surface velocity (at which point Doppler induced shifting perceptibly impacts the measured signal) in terms of the cavity's $1/e$ storage-time (or finesse). In this research, these critical velocity thresholds are shown not to apply to the low-finesse cavity of interest, which experiences significant Doppler induced shifting with target surface velocities that are orders of magnitude below calculated critical velocities. While the critical velocities proposed in [15] and [18] were likely meaningful for time-domain analyses, they appear less meaningful for white light, spectral analyses. A low-finesse cavity may be formed between a low-reflectivity optical fiber tip and the high- or low-reflectivity surface of a nearby DUT (Figure 35), and this is a simple and practical way to implement a non-contacting fiber optic displacement sensor. Further, in the case of a dynamic white light EFPI, not only does the cavity length change as the DUT displaces, but the source wavelength also changes periodically as the source's wavelength is tuned across a particular range.

In this chapter, the effects that Fabry-Perót cavity dynamics have on the spectrum of the sensor's measured interferogram are analyzed using two modeling approaches. With the first approach (investigated in Section 5.2), a swept-filter white light EFPI displacement sensor is modeled as two Fabry-Perót cavities in series: a quasi-static cavity for the swept optical filter and a low-finesse, dynamic cavity for the gap between the sensor and the DUT. Doppler shifting imparted on the signal circulating in the dynamic cavity will be discussed as it manifests itself in the measured sensor output. With the second approach (Section 5.3), the system's frequency-behavior is analyzed in the context of signal mixing and the beat-frequencies that result when two high-frequency oscillations are combined.

5.2. Fabry-Perót Cavity Dynamics: Field Based Model

5.2.1. Field Equations

The model implemented in this section is similar to the one implemented in [18], which in turn is fundamentally based on the propagating EM wave formulations in [35]. For a Fabry-Perót cavity, as shown in Figure 37, there are four EM fields that may be considered – the incident field $\tilde{E}_{inc}$, the circulating field $\tilde{E}_{circ}$, the transmitted field $\tilde{E}_{trans}$, and the reflected field $\tilde{E}_{refl}$. As Figure 37 applies to the setup in Figure 35, Surface 1 is the optical fiber probe tip and Surface 2 is the DUT target surface. The relationships between these fields are well established [35] and have been used in the previously reported dynamic cavity analyses. The circulating field within the cavity, $\tilde{E}_{circ}$, is a combination of the current value of $\tilde{E}_{inc}$ transmitted through the Surface 1 and the previous value for $\tilde{E}_{circ}$, and it may be described as

$$\tilde{E}_{circ}(t) = jt_1\tilde{E}_{inc}(t) + g_{rt}(t)\tilde{E}_{circ}(t - \tau), \quad (5.1)$$

where $\tau=2L/c$ is the Fabry-Perót cavity's round-trip travel time and g_{rt} is the cavity's round-trip gain as a function of time

$$g_{rt}(t-(i-1)\tau) = r_1 r_2 \exp \left[-2\alpha_0 L(t-i\tau/2) - \frac{j2\omega(t-i\tau)L(t-i\tau/2)}{c} \right]. \quad (5.2)$$

In Equation (5.2) i is a positive integer, t is time, L is the cavity's length (given as a function of time), ω is the frequency of the propagating EM wave (given as a function of time in units of radians per second), α_0 is the cavity's attenuation factor, $c=2.998 \times 10^8$ m/s is the speed of light in a vacuum, and r_i and t_i are the amplitude-reflection and amplitude-transmission coefficients, respectively, of the i^{th} surface in Figure 37. Note that the current value for g_{rt} depends on the value for ω one round trip earlier and the value for L one half of a round trip earlier.

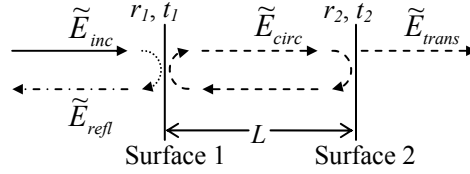


Figure 37: This diagram illustrates the relationships between the different fields present in a Fabry-Perót cavity, which is bounded by Surface 1 and Surface 2. $\tilde{E}_{refl}$ is a combination of a partial reflection of $\tilde{E}_{inc}$ and a partial transmission of $\tilde{E}_{circ}$, and $\tilde{E}_{trans}$ is a partial transmission of $\tilde{E}_{circ}$.

Using Equation (5.1), the current value for $\tilde{E}_{circ}$ immediately prior to reflection/transmission at Surface 1 is put directly in terms of N -previous values for $\tilde{E}_{inc}$ and g_{rt}

$$\begin{aligned} \tilde{E}_{circ}(t) = & jt_1 \{ g_{rt}(t) \tilde{E}_{inc,D}(t-\tau) \\ & + g_{rt}(t) g_{rt}(t-\tau) \tilde{E}_{inc,D}(t-2\tau) + \dots \\ & + g_{rt}(t) g_{rt}(t-\tau) \dots g_{rt}(t-(N-1)\tau) \tilde{E}_{inc,D}(t-N\tau) \} \end{aligned} \quad (5.3)$$

$\tilde{E}_{inc,D}$, as it appears Equation (5.3), is the incident field ($\tilde{E}_{inc}$ in Figure 37) after it has transmitted through Surface 1 and reflected off of Surface 2, where the subscript “D” indicates a Doppler

shift that takes place when the field reflects off Surface 2, whose velocity is nonzero. The resulting form of $\tilde{E}_{inc,D}$ is given by

$$\tilde{E}_{inc,D}(t-i\tau) = E_0 \exp[jD(t-i\tau/2)(k(t-i\tau)x - \omega(t-i\tau)(t-i\tau))], \quad (5.4)$$

which is the electric field component of the incident EM field, oscillating as a function of both position and time. E_0 is the field amplitude (assumed constant in this case), k is the wavenumber ($k=\omega/c$), and x is the position coordinate. D is the Doppler shift that results from L changing with time

$$D(t-i\tau/2) = 1 + 2 \frac{v(t-i\tau/2)}{c}, \quad (5.5)$$

and it changes the frequency of the circulating field as it reflects off a moving surface, according to Equation (5.4). In Equation (5.5), v is the time rate-of-change of L (or the DUT's velocity). When there is no reflection off a moving surface (as in the case of the initial transmitted field, $\tilde{E}_{inc}$ in Equation (5.1)) $\tilde{E}_{inc,D}$ is calculated for $D=1$.

The reflected field, then, just prior to photodetection, is determined from Figure 37 to be

$$\tilde{E}_{refl}(t) = r_1 \tilde{E}_{inc}(t) + j \frac{t_1}{r_1} \tilde{E}_{circ}(t), \quad (5.6)$$

where it consists of an immediately-reflected component and a circulating- and transmitted-component. By substituting Equations (5.3) and (5.4) into Equation (5.6) and choosing an arbitrary position $x=0$, without destroying generality a more thorough representation of the reflected field results in

$$\tilde{E}_{refl}(t) = r_1 E_0 \exp[-j\omega(t)] - \frac{t_1^2}{r_1} \sum_{i=1}^{10\tau_s/\tau} [g_{rt}(t - (i-1)\tau) E_0 \exp[-jD^i(t - i\tau/2)\omega(t - i\tau)(t - i\tau)]] \quad (5.7)$$

The reflected signal's intensity as measured by a square-law photodetector is thus

$$I_{refl}(t) = \frac{1}{2} c \epsilon_0 \tilde{E}_{refl} \tilde{E}_{refl}^* \quad (5.8)$$

where $*$ denotes a complex conjugate and ϵ_0 is the permittivity of free space. In Equation (5.7), τ_s refers to the cavity's storage-time, at which point the normalized circulating field amplitude drops below $1/e$, and is calculated from Equation (5.1) as

$$\tau_s = -\frac{\tau}{\ln(r_1 r_2 \exp(-2\alpha_0 L))} \quad (5.9)$$

The upper limit in the summation in Equation (5.7) is set to $10\tau_s/\tau$ simply because terms as far back as 5-6 storage times (τ_s) contributed noticeably to the simulated interferogram, and the previously proposed upper limit of τ_s/τ [18] is shown in this work not to fully capture the dynamics of the reflected and measured signal. $\tilde{E}_{circ}$ has a periodic component in the complex exponential term (g_{rt}), and the frequencies at which g_{rt} is maximized are the cavity's resonant frequencies. The resonant relationship is calculated from Equation (5.2) as $2\omega L/c = 2\pi N$ for integer values N . The spacing between adjacent resonant frequencies, commonly referred to as the cavity's FSR, is calculated in units of radians per second by multiply Equation (4.4) by 2π . This periodic behavior manifests itself in the reflected measurement (Equation (5.7)) and the relationship between R and L provides the basis for displacement sensing, via spectral analysis, with white light Fabry-Perot interferometry.

The tunable optical band-pass filter (Figure 35) may be modeled as a quasi-static Fabry-Perot cavity, where the transmitted intensity level I is calculated using the well-known Airy function [35]

$$I(t) = \frac{t_f^4}{1 + r_f^4 - 2r_f \cos(2\omega(t)L_f(t)/c)}. \quad (5.10)$$

L_f is the filter's cavity length, and it is assumed that both cavity boundaries have identical amplitude-transmission and amplitude-reflection coefficients t_f and r_f , respectively. The filtered field's normalized intensity and frequency are calculated as a function of time using Equation (5.10), and the filtered intensity and filtered frequency are used as inputs for Equations (5.7) and (5.8) to then describe the time-dependent reflected EM field intensity as it is measured at the photodetector. The physical parameters assumed during this analysis are listed in Table 4, and the specific hardware that was used will be described later in Section 3.

Table 4: The physical parameters assumed during this analysis are listed.

Parameter	Value	Method
r_1	0.184	Calculated from refractive indices using Fresnel Equations
r_2	0.980	Estimated for polished aluminum plate
t_1	0.983	Calculated from r_1 assuming lossless fiber-to-air boundary
t_2	0.000	Estimated for polished aluminum plate
α_0	130	Estimated from experimental PSD peak magnitudes; equates to ~23% attenuation for per mm traveled
r_f	0.9996	Estimated from filtered signal
t_f	0.0283	Estimated from filtered signal

The filter's transmitted narrow-band spectrum looks like a single peak in the power versus frequency domain, with the peak's exact full-width half maximum (FWHM) being governed by the filter's finesse. With a FSR of about 120 nm and a FWHM of about 0.03 nm, the filter's finesse is calculated as about 4000 (or the ratio of the FSR to the FWHM). The finesse F of the dynamic sensing cavity may be estimated more directly from the reflectivity of the two cavity boundaries (Table 4), and using the relationship

$$F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}, \quad (5.11)$$

the sensing cavity's finesse is found to be about 1.6. This low finesse may be related back to the sensing cavity having a short storage-time (Equation (5.9)), since both characteristics result from low-reflectivity cavity boundaries. The ratio of the sensing cavity's storage time (Equation (5.9)) to the sensing cavity's round trip travel time is calculated for the sensing cavity as $\tau_s/\tau=0.522$ for $L=1$ mm and $\tau_s/\tau=0.269$ for $L=10$ mm (assuming $\alpha_\theta=100$). In other words, the time it takes the field $\tilde{E}_{circ}$ to drop below the $1/e$ field-amplitude level is less than the time it takes $\tilde{E}_{circ}$ to traverse the sensing cavity once (Figure 37). According to [18], this should render any Doppler effects in $\tilde{E}_{refl}$ and I_{refl} (Equations (5.7) and (5.8)) imperceptible; however, Doppler induced shifting is shown in Section 4 to significantly impact the measurement signal.

The length range of the swept filter's Fabry-Perot cavity is approximately $0.36 \mu\text{m}$, and assuming that this range is covered in 0.1 seconds, then the surface velocity of the cavity boundary is at a maximum $3.6 \mu\text{m/s}$. Although the ratio τ_s/τ is estimated to be 279 for the filter (which is large as a result of the high surface reflectivities), Doppler shifts that result from velocities of $3.6 \mu\text{m/s}$ will be negligible. For this reason, the quasi-static model is suitable for the filter cavity, even though a fully-dynamic model is required to characterize the behavior of the sensing cavity.

5.2.1.1. Filtered and Measured Fields

It is instructive to consider how the propagating EM signal is modified as it travels from the optical source and filter to the photodetector and data logging hardware in Figure 35. As shown in Figure 38, the filtered signal is not truly monochromatic, although it is narrowband; at any point in time the filter's output has a FWHM on the order of 0.03 nm near 1550 nm (or about 3.75 GHz). The photodetector's output is proportional to the sum of the power values (over all frequencies) at a particular point in time, and it is measured by the data logging hardware, along with the filter's center frequency, as a function of time.

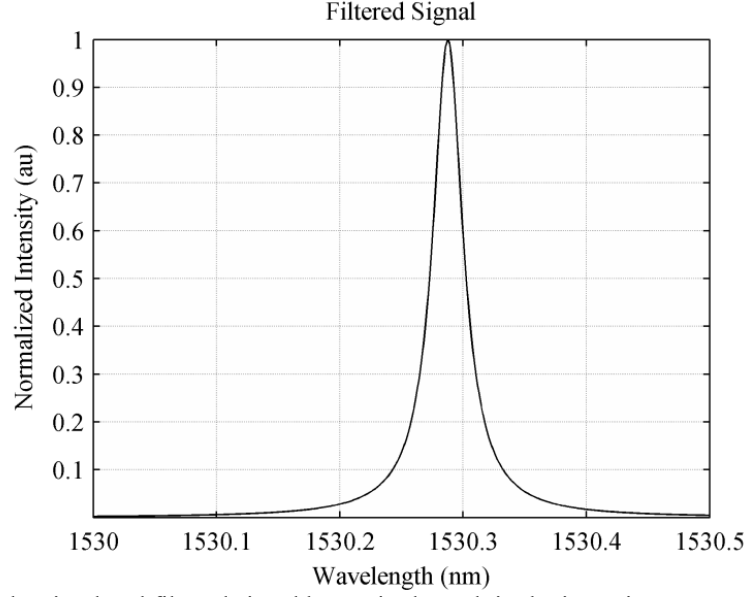


Figure 38: The simulated filtered signal has a single peak in the intensity versus wavelength (or frequency) domain.

As the filter sweeps over a range of pass-band frequencies, the measurement signal exhibits interference fringes according to the sensing cavity's FSR. A representative signal, as seen by the photodetector, as the filter's peak transmission-frequency is swept across a frequency range is shown in Figure 39. A Fourier transform may be used to estimate the measurement signal's power spectral density (PSD), from which the cavity length (or DUT displacement) is calculated. The combination of cavity spectra results in a deterministic noise-floor behavior (Figure 40), even though at this point zero noise has been added to the model. These observations indicate that it would be inappropriate to assume that the filtered source (and $\tilde{E}_{inc}$ in Figure 37) is truly monochromatic.

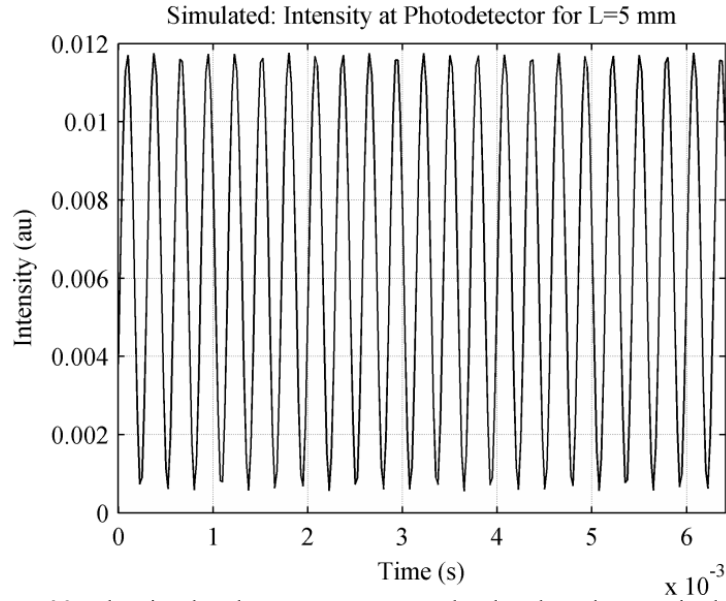


Figure 39: The simulated measurement seen by the photodetector is shown here.

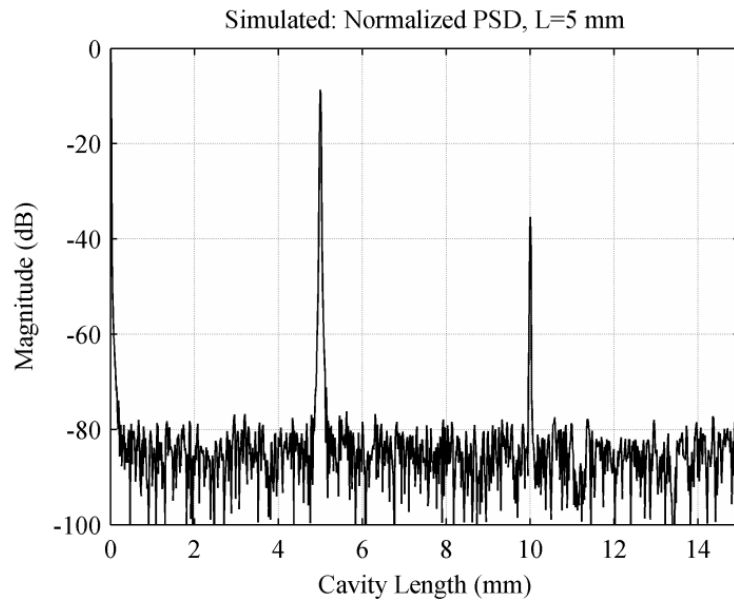


Figure 40: Fourier analysis of photodetector's simulated, measured signal (Figure 39:) reveals the cavity's FSR, which may be related back to the cavity length. The peak at 5 mm corresponds to the cavity's true length, while the peak at 10 mm is a harmonic of the primary peak.

5.2.2. Experimental Setup

The hardware used to perform experiments is shown in Figure 41. Together, the optical interrogator (National Instruments Inc. PXIe-4844) and its chassis (National Instruments Inc.

PXIe-1082) housed the optical source, the swept band-pass filter, the circulator, and any referencing, photodetection, and data logging hardware (compare to Figure 35). This system sweeps the entire 1510-1590 nm wavelength range at a rate of 10 Hz, providing the entire spectrum of measured power versus frequency (or wavelength) data every 0.04 seconds. As explained in Section 1, DUT displacement is then calculated from the spectrum of this power versus frequency data. In practice, a Fourier transform of the measured spectrum is performed real-time using LabVIEW™ development software (National Instruments Inc.), and a peak detection algorithm determines and logs the location of the dominant peak in the Fourier domain along with the corresponding cavity length.

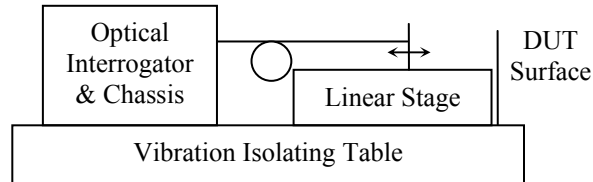


Figure 41: The experimental setup is shown here. The optical source, swept filter, circulator, referencing hardware, photodetectors, and data logging hardware are all included within the optical interrogator and its chassis.

The relative positioning of the sensor probe and the DUT surface was controlled using a motorized linear stage (Newport Corp. ILS200CCL), which has an absolute accuracy of 4.8 μm . The motorized linear stage is capable of moving from one absolute position to another over a range of 0-200 mm at a prescribed velocity of up to 100 mm/s. During testing, the velocity and position of the sensor probe were controlled, and the stage's absolute position data was logged in addition to the reflected and measured signal's spectrum. The controlled laboratory environment allowed for direct comparison of actual displacement to measured displacement as calculated from a distorted (Doppler-shifted) measured spectrum.

5.2.3. Results and Discussion

PSDs of the measured power versus frequency data may be estimated using a DFT. Figure 42 offers a comparison of simulated and experimental PSD estimates for the static cavity case at varying cavity lengths. The model is able to predict the peak amplitude and location in these examples where the DUT is motionless. PSD estimates from simulated and experimental dynamic cavities are shown in Figure 43 for a range of DUT positions and velocities. A Doppler induced length bias of approximately $-0.70v$ (in mm; where v is the DUT velocity in mm/s) manifests itself in the PSD estimates. In other words, the Doppler bias is linearly related to DUT velocity over the entire input-space explored, and it occurs in the opposite direction of the DUT motion. Intuitively this makes sense, since a negative velocity (or a shortening of the cavity) will increase the frequency of any interference fringes in $\tilde{E}_{refl}$ (Figure 37), *thereby decreasing the measured FSR and falsely giving the indication of an increased cavity length*. The opposite is true for a lengthening cavity (i.e., a positive velocity). The model does not appear to capture a spectral broadening of the cavity-length peak that takes place in the dynamic scenario (Figure 43).

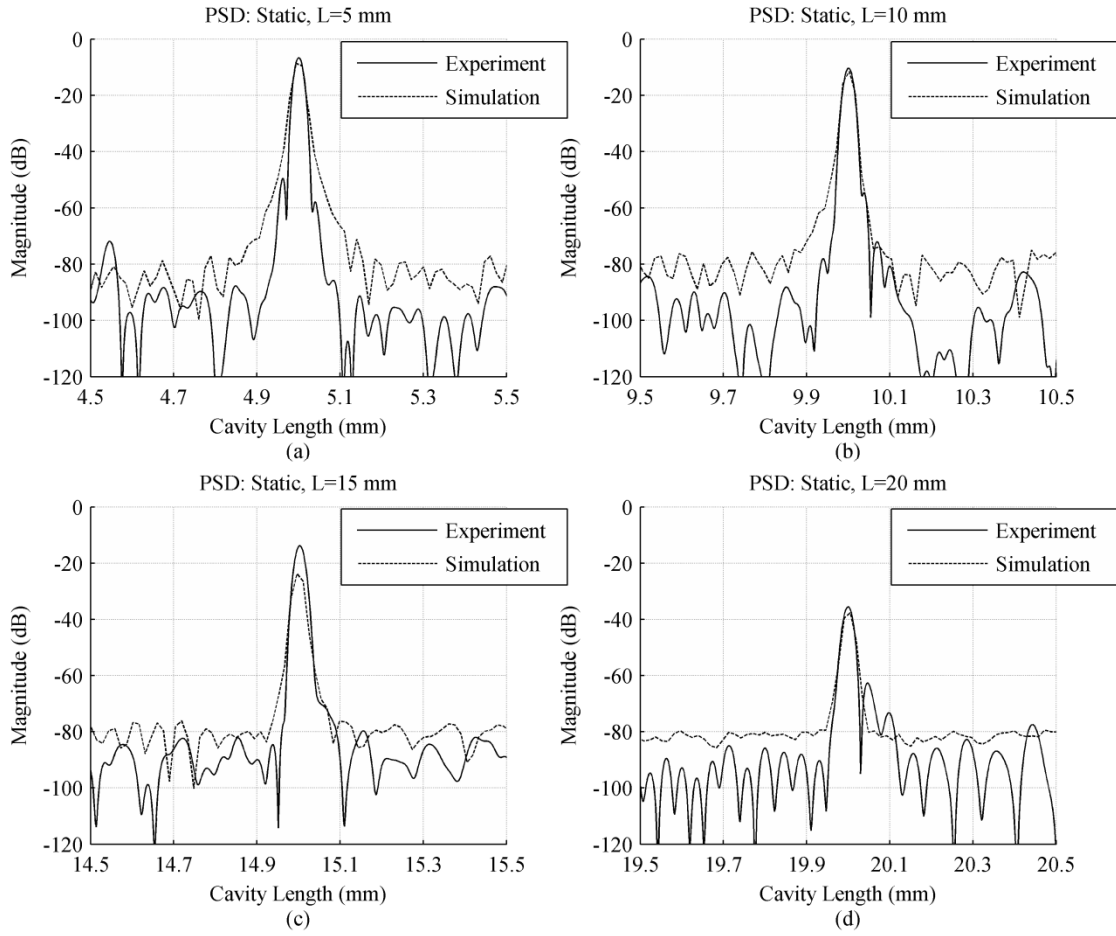


Figure 42: Experimental and simulated spectra are shown for the static case, with the simulation results indicated by dashed lines. The locations and magnitudes of the simulated and experimental peaks are in agreement.

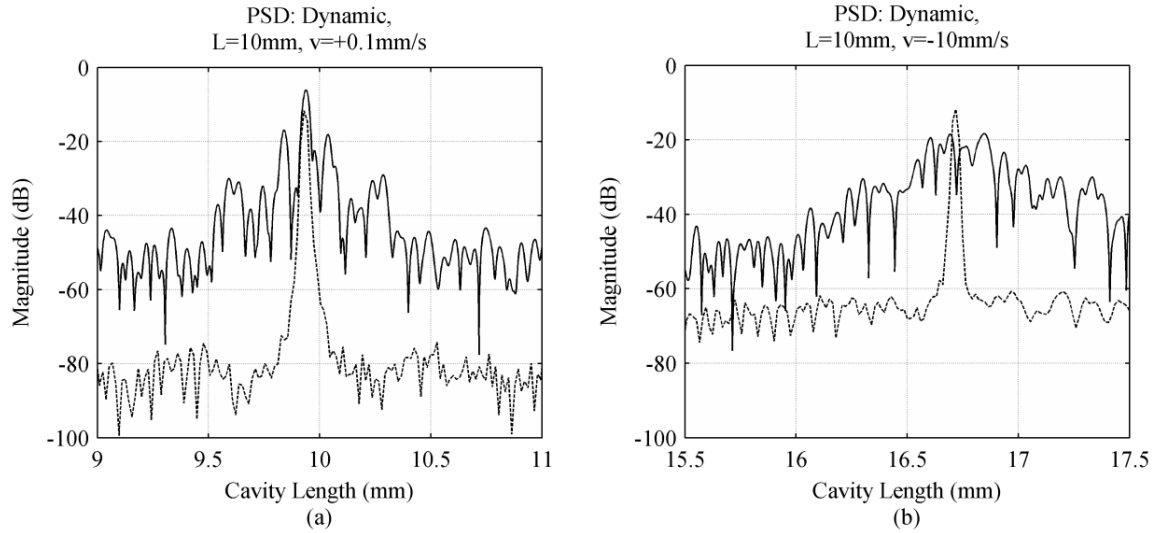


Figure 43: Experimental and simulated spectra are shown for the dynamic case. The locations of the simulated and experimental peaks are in agreement. The Doppler induced shift results in a cavity length bias of approximately $-0.70v$ (in mm), where v is the DUT velocity in mm/s.

A simple peak detection algorithm was implemented immediately following the calculation of the discrete Fourier transform in LabVIEWTM, in an effort to automate the measurement of the cavity length. Peak location data is shown as a function of time for a range of DUT positions and velocities in Figure 44 and Figure 45. In these figures, the linear stage was moved back and forth (at constant velocities) between two fixed locations, while taking a few seconds to pause prior to changing direction. Horizontal regions in Figure 44 and Figure 45 indicate a DUT at rest, while data points connecting adjacent horizontal sections are indicative of DUT motion. As soon as the stage starts moving, the Doppler-induced bias makes it look like it quickly moves to one direction for a fraction of a second, then proceeds at a constant velocity in the opposite direction. The apparent quick initial movement is an artifact of the Doppler shift, and in reality the stage moves in one direction at a fixed velocity. Once again, it is apparent that the Doppler-induced bias occurs in the opposite direction of the DUT motion and in a magnitude of approximately $-0.70v$. Note that this linear relationship holds true for the setup employed, over the entire position- and velocity-space explored.

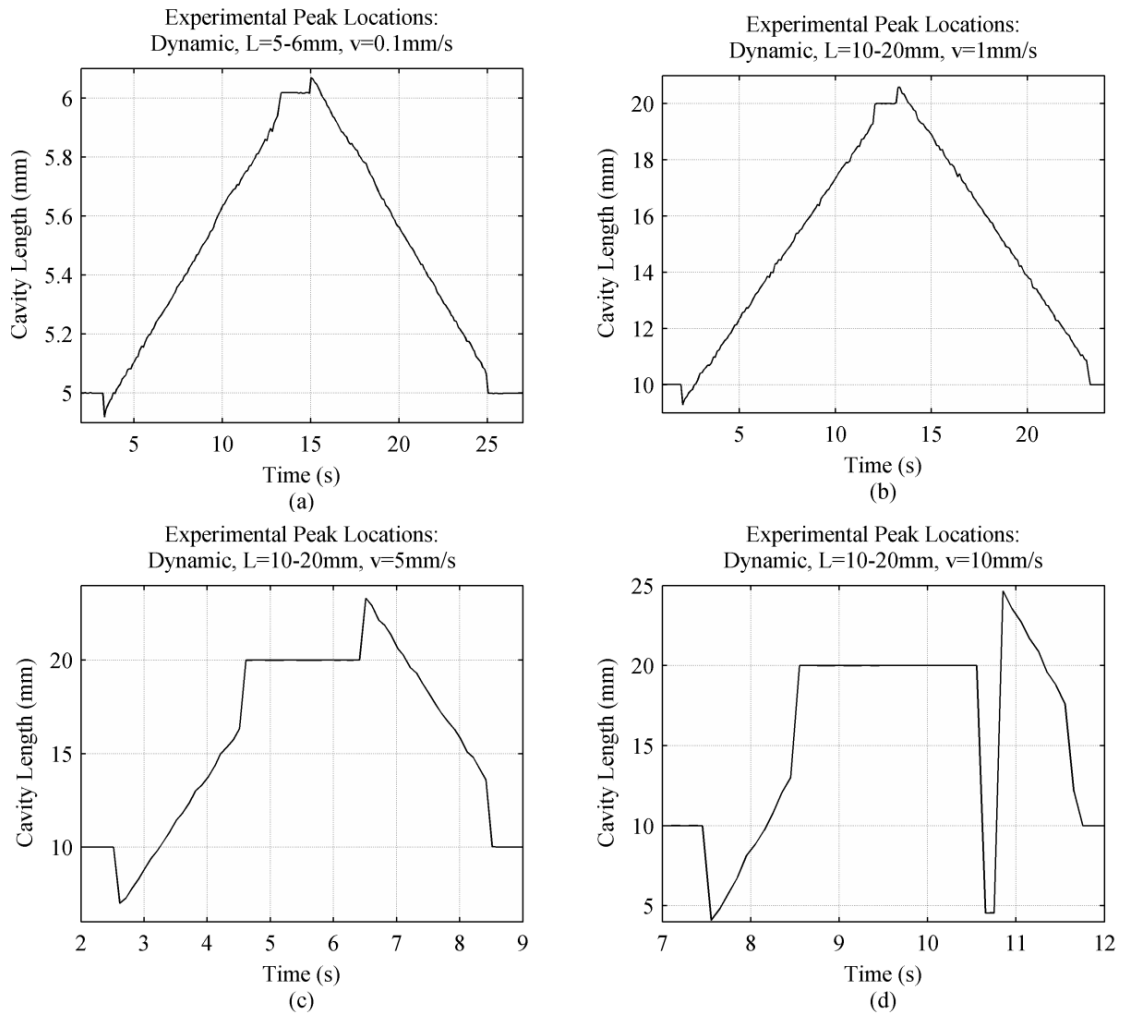


Figure 44: Doppler shifted peak location data is shown (a) for $L=5-6\text{ mm}$ with a velocity of 0.1 mm/s and (b)-(d) for $L=10-20\text{ mm}$ with various velocities over the range $1-10\text{ mm/s}$. The anomaly at $t=10.6\text{ s}$ in (d) occurred when the peak detection algorithm lost track of the true, cavity length peak.

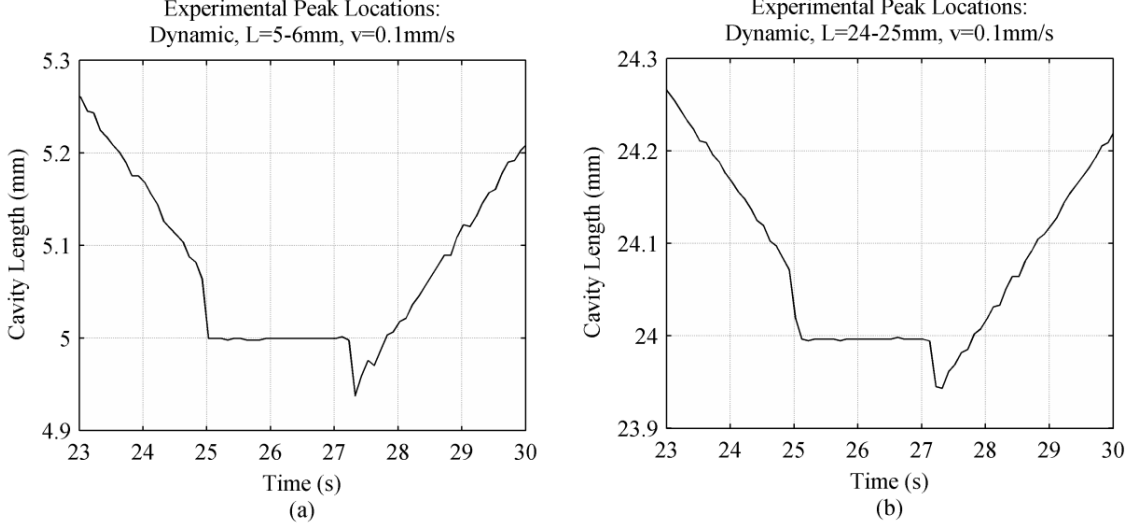


Figure 45: Experimental peak location data is shown for $L=5-6$ mm (a) and $L=24-25$ mm (b), where in both cases the position was changing at a rate of 0.1 mm/s. The Doppler induced bias may be seen when the linear stage stops moving (about 25 s) and starts moving again (about 27 s).

5.2.4. Summary

At this point it may be inferred that the proportionality factor results from the way that the propagating signal's Doppler shift (imparted to $\tilde{E}_{circ}$ in Equations (3)-(5) as shown in Figure 37) is related to the bias that manifests in the estimated PSD (see Figure 43). Although these two signals are quite related, Doppler effects manifest themselves differently in each of them, particularly because the PSD is estimated from power versus frequency data taken while ω , L , and t are all changing. Recall that the frequency of $\tilde{E}_{circ}$ is on the order of 10^{14} Hz (for EM waves propagating around 1550 nm), while the time-domain frequency of the PSD estimate is based on the number of FSRs scanned per second (which for this setup is on the order of 1000-10,000 Hz, with a filter sweep rate of 10 Hz and $L=1-10$ mm). These two (raw and PSD) Doppler shifts are ultimately related by frequencies of these two signals, or more specifically by the filter's center frequency and by the number of FSRs that the sensor covers per second.

Previously, it was generally assumed that white light EFPI displacement sensors were restricted to low sampling rates by hardware limitations such as the mechanical bandwidth of

the swept filter, the width and traceability of reference cell fringes, and the optoelectric bandwidths of the photodetectors; all of these design factors combine to present a performance tradeoff between operation speed and accuracy. This tradeoff makes deployment in high-frequency applications (e.g., shock testing, which requires kHz sampling) impractical. With the results from this analysis, however, it is readily apparent that DUT velocities as low as 1 mm/s distort the measured spectra and induce an approximately 0.70 mm displacement measurement bias. This same bias would persist even if other widely used demodulation techniques (e.g., curve-fitting, wavelength-tracking, etc.) were implemented, since the bias manifests itself directly in the measured signal's FSR. In [9], Han provides a scrupulous review and comparison of such signal processing techniques for white light EFPI demodulation. Therefore, this white light EFPI setup is fundamentally limited by the dynamics of the Fabry-Perôt sensing cavity, even in the presence of low DUT speeds. If this sensor were deployed in the hopes of achieving micrometer accuracy (or nanometer accuracy as has been reported in [9]), this Doppler induced bias would be too large to ignore. For a white light EFPI to be successfully deployed as a dynamic displacement sensor, it must utilize a signal demodulation scheme that simultaneously measures DUT position and velocity. With simultaneous knowledge of distorted spectrum and the induced Doppler shift, one could back out the values for the true (unbiased) DUT displacement and velocity.

In spite of their advantages, it was shown in this section that white light EFPI displacement sensors are highly-sensitive to DUT velocity, where any DUT motion imparts a Doppler shift on the signal circulating within the Fabry-Perôt sensing cavity (Figure 46). This Doppler shift distorts the sensor's measured interferogram, and during demodulation it results in a biased calculation of the DUT displacement. Recently, the relationship between DUT velocity and displacement-bias was experimentally observed in this section to be approximately

$$L_b \approx -0.70v, \quad (5.12)$$

where v is the DUT velocity (in m/s) and $bias$ is calculated in m. In other words, for a DUT velocity as low as 0.1 mm/s, a bias of about 70 μm would be present in the final calculation of displacement. Errors of this magnitude severely limit sensor performance, since in the static testing case the very same sensor offers an accuracy of about 10 μm over an axial displacement range of up to 5 cm. Bear in mind that with no prior knowledge of DUT velocity (and using a setup similar to the one shown in Figure 35), no process currently exists for determining whether or not the DUT is moving, much less to what degree and in what direction it may be moving. Although Doppler velocimetry sensors have been successfully implemented [48-50], they are not capable of making absolute-measurements of DUT displacement, and without added complexity they lack the ability to discern the direction of travel. The goal of this research is to contribute to the future implementation of white light EFPI displacement sensors in dynamic testing environments, in an effort to take full-advantage of their robust and absolute measurement capabilities.

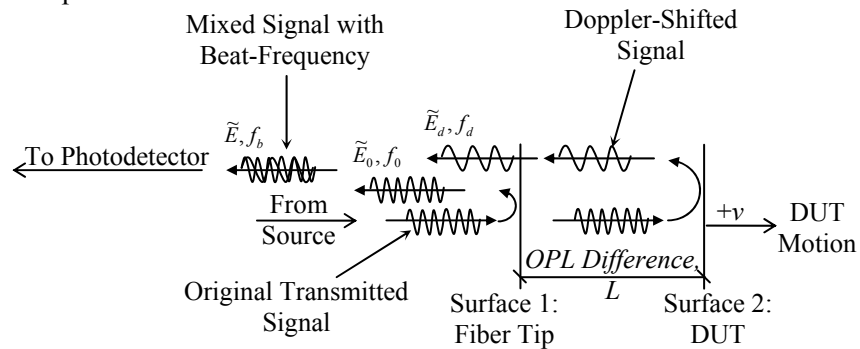


Figure 46: The relationship between DUT velocity, Doppler-shifting, and the resultant beat-frequency is illustrated here.

Section 5.2 in part has been published in *Applied Optics*, E. A. Moro, M. D. Todd, and A. D. Puckett, 2012. The title of this paper is “Dynamics of a non-contacting, white light Fabry-

Perót interferometric displacement sensor”. The dissertation author was the primary investigator and author of this paper.

5.3. Fabry-Perót Cavity Dynamics: Simplified Beat-Frequency Model

5.3.1. Introduction

The high-sensitivity to DUT velocity observed in Section 5.2.3 is particularly noteworthy, since results in [15] and [18] describe critical DUT velocities (at which point Doppler-shifting becomes significant) as being orders of magnitude higher than those observed in Section 5.2.3. While the rules-of-thumb in [15] and [18] may have been perfectly suitable for describing the impulse response behavior of monochromatic EFPI displacement sensors (i.e., the intended application in [15] and [18]), *the fact remains that the white light EFPI displacement sensor is much more sensitive to DUT velocity than was originally assumed.* In order for white light EFPI displacement sensors (or similar sensor architectures) to be successfully implemented in dynamic testing environments, this fundamental relationship between DUT velocity and displacement-bias must be understood.

In this section, a direct relationship between three tunable filter input parameters and the magnitude of the Doppler-induced bias in the displacement calculation is derived. It is shown that with two interferograms corresponding to two known sets of filter-tuning parameters, it would be straightforward to calculate the DUT velocity (both the magnitude and direction) and displacement. While hardware-based limitations on sensor bandwidth still exist, which currently limit sensor performance to tens of Hz, without first addressing this Doppler phenomenon it would have been fruitless to attempt to achieve high-speed, dynamic sensing performance.

5.3.2. Doppler-Induced Displacement-Bias

Refer back to Equation (4.1) – Equation (4.5) in Chapter 4. Practically, the Fabry-Perôt cavity's FSR is calculated directly from the frequency of the interferogram as it is measured by the photodetector (Figure 35). In the static case the FSR is constant, and the interferogram looks much like a sinusoid, whose period is related to the FSR. Therefore, f_R , the frequency that is observed by the interferometer's photodetector in the time-domain as the tuned filter sweeps over multiple FSRs, is related to the tuning range and speed of the swept filter (Figure 35), and is given by

$$f_R = \frac{1}{R} \frac{\Delta f}{\Delta t}. \quad (5.13)$$

In Equation (5.13), the variables Δf and Δt refer to the frequency range and the time-duration, respectively, per filter-sweep. The term $\Delta f / R$ may be described as the number of FSRs through which the interferometer's filter passes per sweep (or the number of cycles through which the interferogram oscillates per sweep). The number of completed cycles divided by Δt yields the observed frequency of the interferogram in units of Hz. During testing, the cavity length is calculated from the time-domain interferogram, by solving Equation (5.13) for FSR and substituting it into Equation (4.5), as

$$L = \frac{c}{2n} f_R \frac{\Delta t}{\Delta f}. \quad (5.14)$$

It is instructive now to consider the anatomy of the interferogram. The signal measured by the photodetector in a Fabry-Perôt interferometer is comprised of two interfering, high-frequency EM waves. For example, for an optical source whose transmission is centered at 1550 nm the corresponding EM wave frequency is 1.935×10^{14} Hz. A moving DUT surface imparts

Doppler shifts on any EM waves that reflect off of it, and the combination of the Doppler-shifted EM wave with the un-shifted EM wave results in a beating signal (Figure 46). The frequency of the beating signal f_b is calculated as

$$f_b = f_0 - f_d = -2\left(\frac{v}{c}\right)f_0, \quad (5.15)$$

where v is the DUT velocity, with positive values indicating motion away from the sensor (as shown in Figure 35 and Figure 46). In Equation (5.15), f_0 and f_d are the un-shifted and Doppler-shifted signal frequencies (in Hz), respectively.

Similarly to the term f_R in Equations (5.13) and (5.14), the beat-frequency f_b describes a time-domain signal that is measured by the photodetector. Without prior knowledge of the DUT's trajectory, however, it is not obvious from the measured signal whether the interferogram's frequency is caused entirely by the Fabry-Perót cavity's resonant properties (f_R) or whether it may be partly attributed to DUT motion, Doppler-shifting, and beating (f_b). Consider that, if f_b is used in place of f_R in Equation (5.14), the length associated with the beat-frequency is

$$L_b = -v \left(f_0 \frac{\Delta t}{\Delta f} \right). \quad (5.16)$$

During a static test, L is accurately calculated from f_R , but during a dynamic test, the influence of f_b is misinterpreted by the sensor, according to Equation (5.16), resulting in a measurement bias. In other words, the frequency f_R calculated in (5.13) results exclusively from tuning the swept filter, and the frequency f_b calculated in (5.15) results exclusively from DUT motion, and when filter tuning and target motion occur simultaneously, it is not readily apparent which of these frequency components is manifested in the measured interferogram. In fact, it will be

shown in the following subsection that, for a moving DUT, the Doppler-induced displacement-bias that occurs in the sensor's output is exactly equal to L_b as it is calculated using Equation (5.16).

It is worth noting that in a limiting sense, the filter may be set to a fixed wavelength (i.e., $\Delta f_0=0$, for a monochromatic source), and the modulation experienced by that particular sensor path will be due entirely to target motion (and not due to sweeping over multiple FSR's). This, in effect, is the same architecture used in photon Doppler velocimetry, which has the advantage of being capable of measuring velocities greater than 1 km/s [48]. However, as mentioned earlier, the disadvantage of this approach is that most Doppler velocimetry techniques are not able to measure unambiguously the direction of target motion, and the directionality only begins to become apparent when filter sweeping is introduced.

5.3.3. The Measured Interferogram

The electric field at the photodetector ($\tilde{E}$) is the sum of the un-shifted ($\tilde{E}_0$) and the Doppler-shifted ($\tilde{E}_d$) electric fields (Figure 46)

$$\tilde{E} = \tilde{E}_0 + \tilde{E}_d = E_0 \exp(-j2\pi f_0 t) + E_d \exp(-j2\pi f_d(t - \tau)) \quad (5.17)$$

with

$$\tau = \frac{2Ln}{c}. \quad (5.18)$$

In Equations (5.17) and (5.18), $j = \sqrt{-1}$ and τ is the Fabry-Perót cavity's round-trip travel-time.

The intensity of the signal measured by the photodetector is therefore described as

$$I = \frac{1}{2} c \epsilon_0 \tilde{E} \tilde{E}^* = I_0 + I_d + 2\sqrt{I_0 I_d} \cos\left(2\pi f_d t + \frac{4\pi f_d L n}{c}\right), \quad (5.19)$$

In Equation (5.19), $*$ denotes a complex conjugate, I is the intensity of the signal at the photodetector, I_0 and I_d are the intensities of the un-shifted and Doppler-shifted fields, respectively, and $\epsilon_0 = 8.854 \times 10^{-12} \text{ m}^3 \text{ kg}^{-1} \text{ s}^4 \text{ A}^2$ is the permittivity of free space. While I_0 may be constant, I_d is itself a function of the EM wave's frequency f_0 and is modulated by the Fabry-Perot cavity according to Equation (4.1). Equation (5.19) may therefore be re-written as

$$I(t) = I_0 + I_d \frac{G \sin^2(2\pi f_d(t)L(t)n/c)}{1 + G \sin^2(2\pi f_d(t)L(t)n/c)} + 2\sqrt{I_0 I_d} \frac{G \sin^2(2\pi f_d(t)L(t)n/c)}{1 + G \sin^2(2\pi f_d(t)L(t)n/c)} \cos\left(2\pi f_b t + \frac{4\pi f_d L n}{c}\right). \quad (5.20)$$

Note that for $G \approx 1$, the right side of Equation (5.20) is well approximated by $\frac{1}{2} \sin^2(2\pi f_d(t)L(t)n/c)$ and in this case (as will be shown in the following section), G is approximately equal to 1.1. Using this simplification along with well-known trigonometric identities, Equation (5.20) may be broken down into a combination of terms at different frequencies

$$I(t) = I_0 + \frac{I_d G}{2} - \frac{I_d G}{2} \cos\left(\frac{4\pi f_d L n}{c}\right) + \sqrt{I_0 I_d} G \left[\cos\left(\frac{4\pi f_d L n}{c} + 2\pi f_b t\right) - \frac{1}{2} \cos\left(\frac{8\pi f_d L n}{c} + 2\pi f_b t\right) - \frac{1}{2} \cos(2\pi f_b t) \right]. \quad (5.21)$$

Equation (5.21) separates the different tones that are present in the photodetector's measured interferogram. These tones are summarized in Table 5, where Doppler-induced (beat) tones are distinguished from resonant-cavity (FSR) tones. The description of the dominant tone (i.e., the one with the largest magnitude) is indicated by bold-faced text.

Table 5: The tones present in the interferogram and their respective magnitudes are listed here. The dominant tone is indicated by bold-faced text in the Description column.

Frequency	Magnitude	Description
$\frac{4\pi f_d L n}{c}$	$-\frac{I_d G}{2}$	Fabry-Perót Cavity Resonance
$2\pi f_b t$	$-\frac{\sqrt{I_0 I_d} G}{2}$	Doppler-Induced Beat-Frequency
$\frac{4\pi f_d L n}{c} + 2\pi f_b t$	$\sqrt{I_0 I_d} G$	Fabry-Perót Cavity Resonance Plus Doppler-Induced Beat-Frequency
$\frac{8\pi f_d L n}{c} + 2\pi f_b t$	$-\frac{\sqrt{I_0 I_d} G}{2}$	Harmonic of Fabry-Perót Cavity Resonance Plus Doppler-Induced Beat-Frequency

As the tunable filter is swept across its frequency range, the “Fabry-Perót Cavity Resonance” terms in Table 5 (see also Equations (4.1)-(4.3)) produce an interferogram whose frequency (f_R) is calculated using Equation (5.13). The dominant tone in Table 5, therefore, oscillates at a frequency calculated as the sum: $f_R + f_b$, where f_R constitutes an unbiased displacement-dependent component and f_b is a velocity-dependent bias. The sensor’s displacement-bias L_b is calculated using Equation (5.16), and with a nominal frequency $f_0 = 1.935 \times 10^{14}$ Hz (for $\lambda_0 = 1550$ nm), a frequency range $\Delta f = 9.993 \times 10^{12}$ Hz (for λ over the range 1510-1590 nm), and a filter-sweep time-duration $\Delta t = 0.040$ s, L_b is calculated as $-0.77v$ (or about $77 \mu\text{m}$ for $v = 0.1$ mm/s). This result is *very similar* to the approximation that was reported in Equation (5.12), as should be expected, since the physical parameters used in this example are borrowed from the hardware that was used. This example stands as *excellent agreement* between previously published experimental results and the relationship predicted by Equation (5.16) and Table 5.

5.4. Phase-Diversity Approach for Full-State Estimation and Compensation of Doppler-Shift

5.4.1. Phase-Diverse Sensor Architecture

With Table 5 and Equation (5.16) together describing the EFPI displacement sensor's response to DUT dynamics, it stands to reason that the dependence of L_b on three specific tunable filter parameters, all of which are easily controlled in an experimental setting, may be exploited to enhance sensor performance. For example, I propose a phase-diversity approach where two unique paths to the DUT exist, and where each path is subject to a distinct set of known tuning parameters (Figure 47). The DUT velocity may be determined exactly by calculating the difference between two observed, biased PSD peak-locations

$$(L + L_{b1}) - (L + L_{b2}) = -v \left(f_{01} \frac{\Delta t_1}{\Delta f_1} \right) - \left(-v \left(f_{02} \frac{\Delta t_2}{\Delta f_2} \right) \right) = v \left(f_{02} \frac{\Delta t_2}{\Delta f_2} - f_{01} \frac{\Delta t_1}{\Delta f_1} \right), \quad (5.22)$$

where f_0 , Δt , and Δf are known for each filter prior to testing. In Equation (5.22) the subscripts $_1$ and $_2$ indicate one of two swept filters, and $(L + L_{bi})$ constitutes the biased displacement measurement calculated from the i^{th} interferogram and its PSD. Once v has been calculated using Equation (5.22), it is straightforward to calculate either L_{b1} or L_{b2} using Equation (5.16). The actual DUT displacement is then taken as the difference between a raw measured value and its corresponding biased value (e.g., $(L + L_{bi}) - L_{bi}$). Note that for $v=0$, $L_{b1}=L_{b2}$, and the sensor's output is unbiased. Therefore, with knowledge of the DUT velocity, the DUT displacement may be calculated from either of the two (distorted) PSD estimates. This proposed architecture (Figure 47) is marginally more complicated than a single-path white light EFPI displacement sensor (Figure 35), requiring only an additional filter, photodetector, and referencing

mechanism, though it offers significant performance-enhancements in that it may operate accurately in the presence of DUT motion.

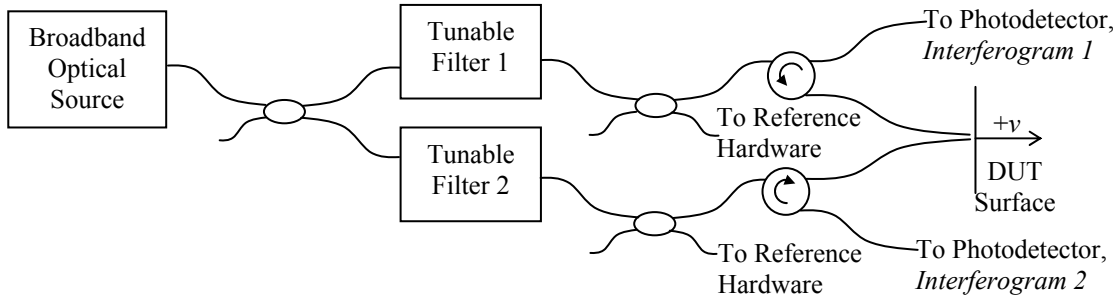


Figure 47: The proposed phase-diversity approach takes advantage of two tunable filters, each with its own distinct set of tuning parameters.

5.4.2. Experimental Setup

The experimental setup used to evaluate the utility of the phase-diverse EFPI included two commercially-available tuned-filter systems (sm125 from Micron Optics Inc. and PXIe-4844 from National Instruments Corp.); each containing its own optical source, swept-filter, optical circulator, wavelength referencing hardware, and photodetector. The nominal wavelengths and wavelength-ranges of these two systems are identical (1550 nm and 1510-1590 nm, respectively), and their filter-sweep time-durations were experimentally measured as 0.40 s (sm125) and 0.040 s (PXIe-4844). Knowledge of these filter parameters is what enables velocity measurement and compensation for the Doppler-induced displacement-bias. Both of these systems were connected to a single mode optical fiber (SMF-28e Corning, Inc.) that was directed toward the DUT, and a feedback controlled motorized linear stage with an absolute accuracy of 4.8 μm (ILS200CCL, Newport Corp.) enabled precise control of the DUT's position with respect to the sensors. LabVIEWTM development software was used to measure interferograms from the sm125 and the PXIe-4844, perform Fourier transforms (to estimate PSDs), and locate the (biased) PSD-peaks corresponding to the DUT displacement. The sampling rates of these systems were 1 Hz for the sm125 and at 10 Hz for the PXIe-4844. Since

the slowest EFPI path (the sm125) operates at 1 Hz, the dynamic capabilities (simultaneous velocity and unbiased-displacement measurement) are limited to this rate. Dynamic tests were run, translating the DUT to prescribed positions at known velocities, and simultaneously measuring interferograms from both EFPIs. During post-processing, the filters' tuning parameters were used to calculate the DUT velocity, and with knowledge of the DUT velocity, either of the EFPIs could be used to calculate the unbiased DUT displacement.

5.4.3. Results and Discussion

Raw peak location data is shown in Figure 48(a), where the subscripts F_1 and F_2 correspond to the sm125 system and the PXIe-4844 system, respectively. Since these peak locations are calculated directly from biased interferograms, any DUT motion is expected to impart a Doppler-induced bias. Horizontal regions indicate that the DUT is at rest, while sloped data indicate DUT motion. The DUT position and velocity were controlled using the motorized stage (Section 3), where less-steep slopes (before $t=250$ s) are indicative of a prescribed velocity of ± 0.1 mm/s and steeper slopes (after $t=250$ s) indicate a prescribed velocity of ± 0.5 mm/s. Note that motion in a particular direction is immediately preceded by an instantaneous jump in the opposite direction and is immediately followed by a corrective jump (Figure 48(a)). The PXIe-4844 data ("Raw $_{F_2}$ ") shows less of a Doppler-induced bias than the sm125 data ("Raw $_{F_1}$ "), which is consistent with Equation (5.21), since Δt_2 is 10-times less than Δt_1 , and consequently L_{b2} is 10-times less than L_{b1} . The difference between the two biased measurements is shown in Figure 48(b), corresponding to the left side of Equation (5.22).

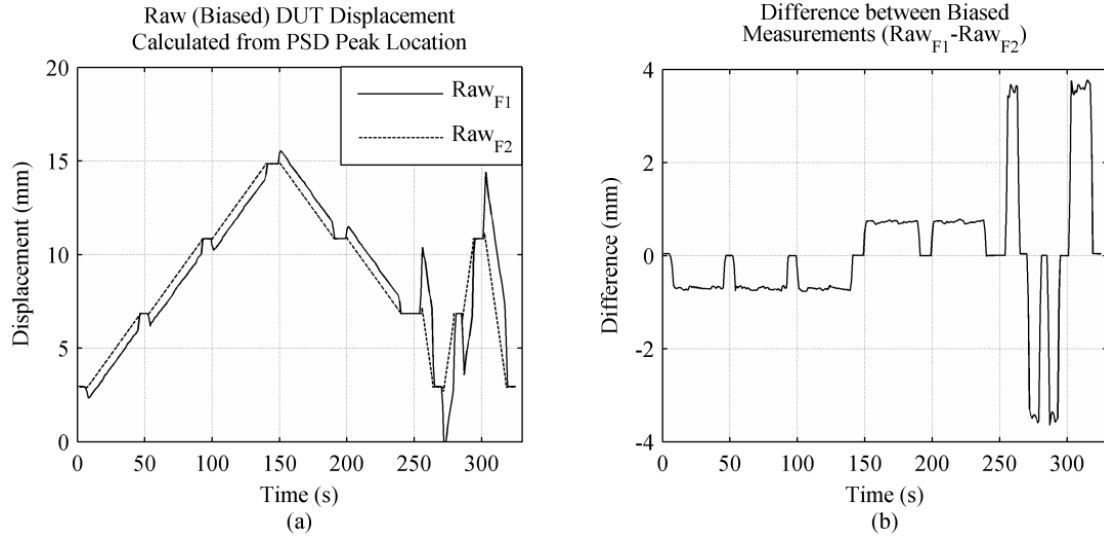


Figure 48: Raw (biased) DUT displacement data measured from both filters is shown (a) along with the difference between these measurements (b).

Using the difference between the biased measurements (Figure 48(b)), it is possible to rearrange Equation (5.22) and calculate the DUT's velocity v as a function of time. The calculated velocity data (Figure 49) corresponds well to the experimentally implemented velocities of 0.1 and 0.5 mm/s. The maximum absolute velocity error in Figure 49 is 0.01 mm/s for $v=0.1$ mm/s (or 10%) and 0.02 mm/s for $v=0.5$ mm/s (or 4%), and these values are governed primarily by the interferogram's signal to noise ratio (or the magnitude of the PSD peak relative to other PSD content). However, fundamental performance limitations exist as a result of the particular approach used to estimate the interferogram's PSD. The PSD-estimate's displacement-resolution-limit is determined by the tuned-filter's frequency range to be $15\text{ }\mu\text{m}$ (Equation (4.11)); resulting in a velocity-resolution of $2.2\text{ }\mu\text{m/s}$ (based on Equation (5.22)). The sensor's maximum displacement range is a function of the PSD's displacement-resolution-limit and the length of the PSD-estimate, and for this setup it is calculated as 12 cm. Any Doppler-shifted (or biased) displacements outside of this range will alias in the PSD, putting a limitation on the combinations of DUT displacement and velocity that may be unambiguously measured using this phase-diverse EFPI setup.

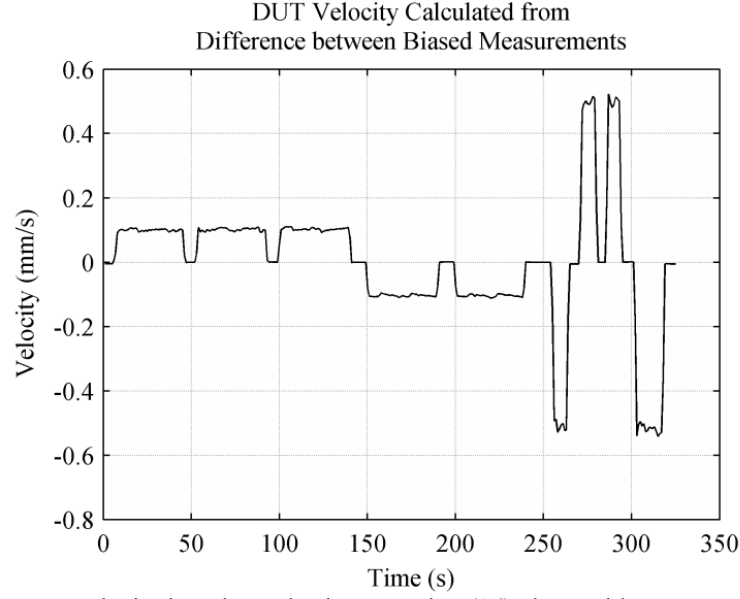


Figure 49: The DUT velocity is estimated using Equation (16) along with two sets of filter parameters.

With velocity data, it is possible to calculate either L_{b1} or L_{b2} directly using Equations (5.16) and (5.22). In Figure 50, the velocity-compensated DUT displacement (“Comp_{F1}”) is calculated as the difference between the biased peak location and the Doppler-induced bias (or $(L+L_{b1})-L_{b1}$). Note from Figure 50 that the velocity-compensated DUT displacement does not show the same jumpy, Doppler-shifting behavior that the raw displacement measurements show, providing a smoother record of the DUT’s trajectory and eliminating the effects of even the 65 μm bias. These results show excellent agreement to the derivation presented in this paper, validating the derivation and showing the capability to measure, simultaneously, DUT velocity and an unbiased (Doppler-shift-compensated) DUT displacement.

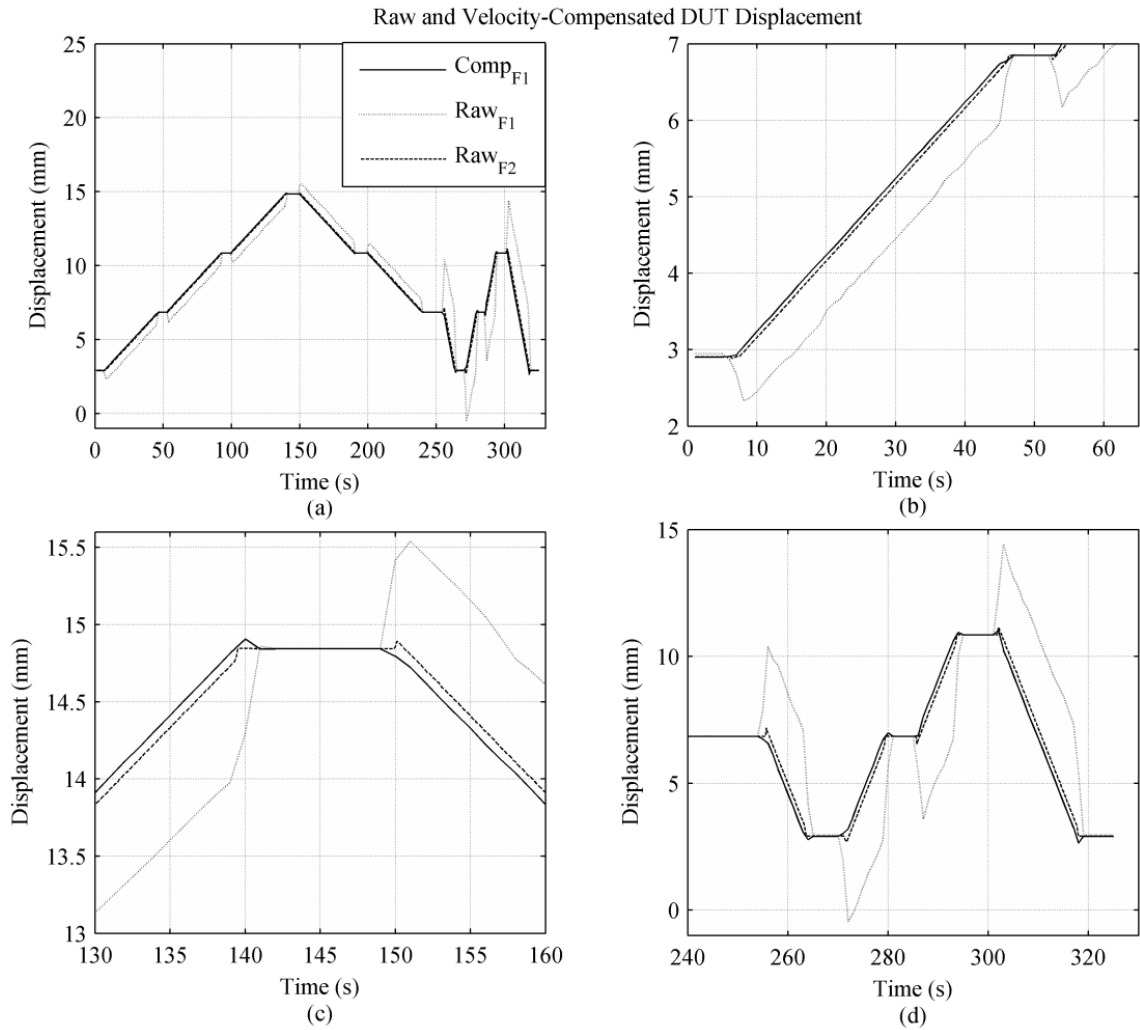


Figure 50: Using an estimate of the DUT velocity (Figure 49) it is possible to remove the Doppler-induced displacement bias from the raw results shown in Figure 48. (a) provides the global perspective of a test while detailed views are shown in (b), (c), and (d).

In addition to the PSD-based limitations on performance, it should be noted that even with the capability for simultaneous velocity and displacement measurement, the proposed phase-diversity architecture is still bandwidth-limited, simply based on the fact that the EFPI's electro-optical components are themselves bandwidth-limited. For the setup described in Section 3, the final (unbiased displacement measuring) sampling rate is limited to 1 Hz. However, if one were to build a phase diverse EFPI displacement sensor where both filter-paths

use technology similar to the PXIe-4844, there is no reason that a sampling rate of at least 10 Hz could not be obtained (for these tests, I simply used the most readily available hardware). Tuned filters with mechanical bandwidths near 1 kHz are commercially available, photodetectors and data acquisition systems with bandwidths in the MHz-GHz range are available, and the limiting factor is generally going to lie in the speed at which any wavelength referencing scheme can reliably operate.

5.5. Uncertainty Analysis

The uncertainty characteristics of the parameters listed in Equation (5.21) are summarized in Table 6. Refer to Chapter 2 for a more in-depth discussion of analysis techniques used to propagate modeling uncertainty and measurement uncertainty through an optical transmission model. For this analysis, I_0 and I_d were normalized for simplicity, and variation on these parameters was measured experimentally, which is results from noise in the source, photodetector, and digitizer. The nominal value and error range for n were calculated using the Edlén Equation [53] for a wide range of ambient laboratory conditions in Los Alamos, NM, USA (where the displacement sensor is implemented). r_1 was calculated using Fresnel's Equations, using a nominal value of 1.4492 for optical fiber core's index of refraction and varying that value between 1.4487 and 1.4497. r_2 was calculated for a polished aluminum surface whose percent (power) reflectivity is in the range of 95-98%. Variation on the parameter L is assumed to be zero for this analysis, since the DUT's actual displacement is deterministic. Variation on f_d and f_b is based on the 5 pm absolute wavelength accuracy (± 2.5 pm) given by the manufacturer of the system employed. Generally speaking for this analysis, nominal errors of zero indicate that there is no reason to assume that the parameter has an intrinsic bias, and

uniform distributions imply no prior knowledge (or no central tendency), as is the case with resolution-limited errors (or quantization errors).

Table 6: The uncertainty characteristics of input parameters, which govern the EFPI's measured output, are summarized here.

Parameter	Description	Nominal Value	Nominal Error	Error Range	Distribution
I_0	Un-shifted signal intensity	1 Wm ⁻²	0 Wm ⁻²	± 0.05 Wm ⁻²	Gaussian
I_d	Doppler-shifted signal intensity	1 Wm ⁻²	0 Wm ⁻²	± 0.05 Wm ⁻²	Gaussian
n	Fabry-Perót cavity refractive index	1.00019	0	± 0.00003	Gaussian
r_1	Amplitude reflection coefficient of Fabry-Perót cavity Surface-1	0.1833	0	± 0.0002	Uniform
r_2	Amplitude reflection coefficient of Fabry-Perót cavity Surface-2	0.9800	0	± 0.01	Uniform
L	Fabry-Perót cavity length	varies	0 μ m	0 μ m	Gaussian
f_d	Doppler-shifted filter passband frequency	varies	0 GHz	± 0.31 GHz	Uniform
f_b	Frequency of beat signal	varies	0 GHz	± 0.31 GHz	Uniform

With each of the parameters in Table 6 being treated as a random variable, a Monte Carlo approach is used to sample from their distributions and generate a distribution of outputs for the static case ($f_b=0$) of Equation (5.21). 100 random initializations were used at each simulated DUT position. The behavior of the output distribution impacts the ultimate accuracy of the white light EFPI displacement sensor, as a sensor output that is highly variable and easily influenced by input parameter variability will ultimately result in reduced accuracy and stability. However, as will be discussed in the following chapter, the displacement resolution L_{res} calculated for the static testing case and with example parameters (e.g., a tuned filter that

sweeps the 1510-1590 nm wavelength range) is 15 μm , and levels of measurement uncertainty at or below the levels in Table 6 will not significantly impact static sensor accuracy at close ranges.

5.6. Conclusions

White light EFPI displacement sensors were shown both experimentally and using a field equation model of the Fabry-Perót cavity to be highly sensitive to DUT motion, and the effects of DUT dynamics on sensor performance were considered. DUT motion was shown to cause Doppler-shifting that manifests itself in a beat-frequency, and the final, measured interferogram was shown to be the combination of a component at this beat-frequency and a component that oscillates according to the EFPI's resonant-properties. It was also demonstrated that this signal mixing ultimately results in a biased displacement measurement whenever the DUT exhibits a non-zero velocity. For the first time according to the author's knowledge, an analytical relationship was derived that describes this displacement-bias directly in terms of three swept-filter tuning-parameters.

A phase-diverse approach that employs two swept-filters with unique tuning-parameters was proposed and experimentally tested. The filters' tuning-parameters were shown to govern, as predicted, the magnitude and direction of the Doppler-induced displacement-bias, thereby validating the results of the derivation. Experimental tests successfully demonstrated simultaneous measurement of DUT velocity and displacement, eliminating any displacement-biases while measuring DUT velocity with absolute maximum velocity errors of 10%.

The properties of the PSD-estimate (used during demodulation) imposed displacement- and velocity-resolutions of 15 μm and 2.2 $\mu\text{m/s}$, respectively. The PSD-estimate's length and displacement-resolution also imposed a maximum displacement range of 12 cm, beyond which

any (biased or unbiased) displacement measurements will result in aliasing. With an understanding of the effects of Doppler-shifting on displacement measurement bias, future advances in component-bandwidth may enable the use of white light interferometers as dynamic displacement sensors, though the achievable performance space will fundamentally be governed by component bandwidth and by the properties of the PSD estimate. Future work will include an investigation of these performance tradeoffs along with efforts to integrate high-speed components.

Portions of this chapter have been submitted to *Journal of Lightwave Technology*, E. A. Moro, M. D. Todd, and A. D. Puckett, 2012. The title of this paper is “Phase diversity approach for simultaneous measurement of velocity and displacement using a white light Fabry-Perót interferometer”. The dissertation author was the primary investigator and author of this paper.

Separate portions of this chapter have been published, in part, in *Proc. SPIE Defense, Security, and Sensing 8370*, E. A. Moro, M. D. Todd, and A. D. Puckett, 2012. The title of this paper is “Performance limitations of a white light extrinsic Fabry-Perót interferometric displacement sensor”. The dissertation author was the primary investigator and author of this paper.

Chapter 6 Performance Comparison of Intensity-Modulated and White Light Interferometric Sensing Methodologies

6.1. Summary of Intensity-Modulated Sensor Performance

In addition to offering high-bandwidth displacement sensing capabilities, differentially interrogated bundled optical displacement sensors have been previously shown [24-26] to provide robustness to fluctuations in the measured power level that are independent of DUT motion (e.g., source instability or changes in surface reflectivity). Performance-optimization of bundled differential intensity-modulated displacement sensors was introduced as a framework for sensor design and implemented in Chapter 3. It was shown not only that the design space of differential bundled displacement sensor architectures is complicated and varied, but also that it is possible to employ a search algorithm for analyzing the sensor's design space in an effort to produce a bundling configuration that *offers high-accuracy monochromatic linear operation over a prescribed axial displacement range and with an allowable degree of error*. With this approach, the combination of the optical transmission model and GA was used to generate *high-performance sensor architectures*, and although these architectures made intuitive, physical sense, their specific design would not necessarily have been obvious without the aid of this

design framework. Although the model that was implemented was experimentally verified and shown to do an excellent job of simulating the spatial distribution of the transmission signal, and even though the GA was able to efficiently locate high-performance bundling configurations in the sensor's design space, it is clear that *the experimental performance of this sensing methodology is very sensitive to, and fundamentally limited by, measurement noise.*

Sensor performance was limited by noise in at least two distinct ways. Firstly, the SNR of the experimentally measured differential signals limited the ultimate accuracy of the first optimized sensor prototype to hundreds of micrometers (Section 3.3). In the case of the second optimized architecture (Section 3.4), the sensor's output stability was limited by the measurements' SNRs in that the highly-sensitive sensor architecture was destabilized by measurement noise. In other words, it was observed that a high-sensitivity sensor architecture is not only highly-sensitive to changes in DUT displacement, but it is also sensitive to other changes to the sensor's measurement signals (i.e., measurement noise). As sensor accuracy is ultimately determined by the relative size of the measurement noise. Secondly, sensor robustness was shown to be governed by the degree to which the multiple, differential measurements were affected by changes in the transmitted/reflected power level. If the multiple power measurements were significantly enough larger than the noise floor, then they were likely to be affected by changes in surface reflectivity in the same way (i.e., a 40% reduction in surface reflectivity yields a 40% reduction in each of the measured power levels). But if one or both of the differential power measurements existed too close in magnitude to the noise floor, then there was no guarantee that the same robustness to changes in power level may be expected. In these ways, sensor performance hinges ultimately on the SNR of the measured signals. The GA and optimization routine generated a result in response to the scenario presented by the cost functions (Equations (3.1) and (3.2)), but the performance of the output of

an optimization approach is only as good as the ability of the problem statement to accurately reflect the needs and desires expected of the optimized output. In this case, it was concluded that *some notion of overall measurement signal SNR must be incorporated into the cost function* in order to ensure (1) sensor robustness, (2) sensor accuracy, and (3) overall sensor stability.

As an example, consider the optimized bundle configurations from Chapter 3, with simulated analogue sensitivities of $-0.068 \mu\text{m}^{-1}$ and $0.043 \mu\text{m}^{-1}$ over axial displacement ranges of 6-8 mm and 3.5-5.5 mm, respectively. If these optimized sensors were used in conjunction with photodetection and digitizing systems that were fundamentally limited by a noise effective power of $1.5 \times 10^{-15} \text{ WHz}^{-1/2}$ (Thorlabs Inc. DET01CFC; at 1550 nm), then sampling at 1.2 GHz would result in a noise magnitude of 5.2 pW. Considering still the optimized architectures from Chapter 3, the resulting accuracy limitations would be 9.1 μm and 1.4 μm , respectively. Practically, with the digitizer that was used (National Instruments PXI-4461) the experimentally measured noise was on the order of 0.1 nW, resulting in a measurement error of 100-200 μm (Section 3.3). Therefore, with the experimental setup that was available for this research, the sensor's performance was seriously limited by the noise characteristics of the digitizer. This is indicative of an overarching issue that must be carefully accounted for during the implementation of intensity-modulated displacement sensors: that intensity-modulated sensors are highly sensitive to the specific photodetection and digitizing hardware that are used to convert optical measurements to observable electric signals.

Other architectures, or other bundling configurations, offer different combinations of sensitivity, accuracy, and axial displacement range, but the architectures described here have been generated in an effort to optimize the combination of sensitivity and linearity over their respective axial displacement ranges. It is therefore fair to assume that the optimized configurations discussed here represent the upper end of performance capabilities, in terms of

high-sensitivity operation and correspondingly high-accuracy capabilities. This assumption and the description of the success of the GA to yield high-performance sensor architectures are in agreement with the observation that sensor architectures that were previously described as being “high-sensitivity” [24] offer lower sensitivity than the optimized architectures described in this dissertation. This constitutes a reasonable scenario, therefore, for describing upper limits on sensor performance, and accuracy limits of $9.1\text{ }\mu\text{m}$ and $1.4\text{ }\mu\text{m}$ constitute an idealized scenario, since photodetector shot noise is often less than the noise levels in a typical digitizer. Therefore, these numbers may be considered as upper limits on the sensor’s accuracy, for bundling configurations that have been optimized for performance over their respective axial displacement ranges, assuming photodetector shot noise is the limiting factor. With these performance metrics acting as representative limits on performance, and assuming robust operation and fast sampling, the example described here is indicative of the performance limitations of the differential intensity-modulated optical-lever-style displacement sensor, when an optimization routine is used to search the sensor’s design space.

The design framework proposed in this dissertation for simulating the sensor’s design space and using a GA for efficiently searching that design space hold value. Obviously, the sensor’s highest-achievable sensitivity is determined by the bundling configuration and the axial-displacement range of interest, and the utility of the GA optimization approach described here is that it enables one to achieve a relatively high value for analogue sensitivity, given a particular operational axial displacement range.

6.2. Summary of Interferometric Sensor Performance

White light EFPI displacement sensors offer robust performance along with absolute measurement of DUT displacement over a large axial displacement range. As a result, they do

not require calibration, making them relatively simple to implement. Their primary disadvantage is in their limited dynamic sensing capabilities, where they are hardware limited to low sampling rates, and Doppler-shifting, imparted by DUT motion, distorts the sensor's measured interferogram. In Section 5.4, a phase-diverse white light EFPI architecture was proposed for the simultaneous measurement of DUT velocity and displacement. This full state-estimator was shown to successfully remove the effects of Doppler-induced distortion, allowing for unbiased measurement of DUT displacement in the presence of DUT motion.

Consider that L_{max} is the largest displacement value that may be measured from the PSD estimate without aliasing, according to the data sampling rates. It is worth pointing out that since any DUT velocity imparts a bias L_b on the length measurement, the property L_{max} is not simply an upper limit on the true DUT displacement L , but it is a limit on the biased displacement $(L+L_b)$. Therefore, L_{max} puts an upper limit on the combined values of DUT displacement and velocity

$$0 < \left(L - f_0 \frac{\Delta t}{\Delta f} v \right) \leq \frac{c}{4n} \left(\frac{f_{DAQ}}{f_{sweep} \Delta f} \right), \quad (6.1)$$

where the biased displacement measurement given by Equation (5.16) is related to the right side of Equation (4.12) and the biased displacement measurement must be greater than zero to avoid ambiguity. Since L_{res} depends only on Δf (Equation (4.11)), *a logical sequence for establishing the hardware frequencies* is to start by establishing Δf to meet a particular application's resolution requirement. In practice, Δf is ultimately constrained by the EFPI hardware (optical source, fibers photodetectors, tuned filters, etc.), but values as high as 9990 GHz are easily achieved with off-the-shelf hardware (e.g., 1510-1590 nm is a typical range for InGaAs based hardware such as the sm125 optical interrogator module from Micron Optics Inc.). Since f_{DAQ} is

likely fixed for a given experimental setup in the MHz-GHz range (typical sampling properties for photodiodes and data acquisition hardware), the displacement range L_{max} is then set by selecting an appropriate value for f_{sweep} ; where larger values decrease L_{max} and increase the sensor's operating speed (the converse is also true).

The rate at which the sensor acquires displacement measurements is identical to f_{sweep} , since one complete sweep of the tuned-filter (i.e., one complete interferogram) is required to support the calculation of DUT displacement. In this sense, the operating speed is limited by how large one can make f_{sweep} . In order to maintain a constant value for $NFFT$, an increase in f_{sweep} should be accompanied by the same increase in f_{DAQ} . Practically, obtaining large values for f_{DAQ} tends not to be difficult, since tuned filter mechanical bandwidths are on the order of 1 kHz or less, and the sample rates of photodiodes and data acquisition systems often exist in the MHz-GHz range. The upper limit on f_{sweep} is therefore determined by the ability to determine a relationship between the tuned filter's passband frequency and time. Pressurized gas reference cells are common for the task of frequency referencing, but they necessitate being able to distinguish frequency lines in the reference spectrum. If, for example, $N_{ref}=1024$ data points are required to characterize the reference spectrum, f_{sweep} is limited to less than or equal to $f_{DAQ}/1024$ (obtained by rearranging Equation (4.9) and substituting the term N_{ref} for $NFFT$). The upper limit on f_{sweep} is either governed by this relationship or by a mechanical limitation within the tuned-filter whereby a stable input/output relationship no longer exists between the filter's drive voltage and its passband frequency.

As an example of phase diverse white light EFPI performance, consider a data acquisition system with $f_{DAQ}=200$ MHz (e.g., NI PXI-5124 available from National Instruments Corp.), a photodiode whose bandwidth is larger than this, and a tuned filter frequency-range $\Delta f=9990$ GHz (corresponding to wavelengths of 1510-1590 nm). If the sensor's sampling rate

were set to $f_{sweep}=20$ kHz, the resulting parameters would include: $NFFT=10,000$, $L_{res}=15$ μm , and $L_{max}=7.5$ cm. If f_{sweep} were reduced to 10 kHz, then L_{res} would remain unchanged and the maximum displacement range would be increased to $L_{max}=15$ cm. Consider that Δf and L_{res} are set to meet an application-specific requirement, and the remaining performance-tradeoff is between L_{max} and f_{sweep} . Accordingly, for a fixed tuned filter frequency-parameters f_0 and Δf , a decrease in Δt (or an increase in f_{sweep}) results in reduced Doppler-effects (or a smaller L_b) according to Equation (5.16). In other words, not only does a large f_{sweep} enable higher-bandwidth sensing, but it also reduces the magnitude of L_b , relaxing the maximum displacement requirement on L (if the sum $(L+L_b)$ is to remain within the constraint imposed in Equation (5.22)). For $f_{sweep}=10$ kHz, the proportionality term relating v and L_b in Equation (5.16) becomes -0.002 s, and a velocity $v=20$ m/s imparts a bias of 4 cm (making it a manageable speed for $L_{max}=15$ cm and a nominal (standoff) displacement value of $L=7.5$ cm). By easing the resolution requirement, or by using a data acquisition system with a faster sampling rate f_{DAQ} , it is possible to enhance this performance space, increasing L_{max} or the displacement and velocity pairs which may be measured within L_{max} . However, these values are representative of performance that may be achieved with commercially available off-the-shelf components. The remaining consideration to make is regarding the displacement and velocity resolution values as they relate to experimental uncertainty.

The effects of uncertainty and the limitations of displacement resolution are shown for the static case in Figure 51. $\Delta f=9990$ GHz (1510-1590 nm) and $NFFT=10,000$ were assumed for this analysis. From these results it is apparent that the sensor being investigated here is fairly robust to the levels of uncertainty that were simulated. This stands to reason, since a primary advantage of this particular sensor-architecture is its robustness to uncertainty that manifests itself in fluctuations in measured power levels. As a result, the sensor's accuracy is primarily

governed by the displacement resolution limit L_{res} imposed by the parameter Δf . This behavior is further characterized by the error calculated between the true DUT displacement and the displacement calculated from the interferogram's DFT (Figure 51(b)). If Δf were increased, and L_{res} were decreased, it is possible that the DFT imposed displacement resolution would become small enough that measurement uncertainty would have more influence on determining the sensor's accuracy. Displacement resolution is related to velocity resolution using Equation (5.22), and for the hardware parameters assumed, a velocity resolution of $2.2 \mu\text{m/s}$ results. The measurement of DUT velocity is only as susceptible to measurement uncertainty as the measurement of displacement is—in this case being primarily governed by L_{res} . The velocity resolution changes if either L_{res} or the proportionality relationship between L_b and v changes. For enough large values of displacement, the interferogram's signal to noise ratio drops enough that the ability to measure displacement is lost.

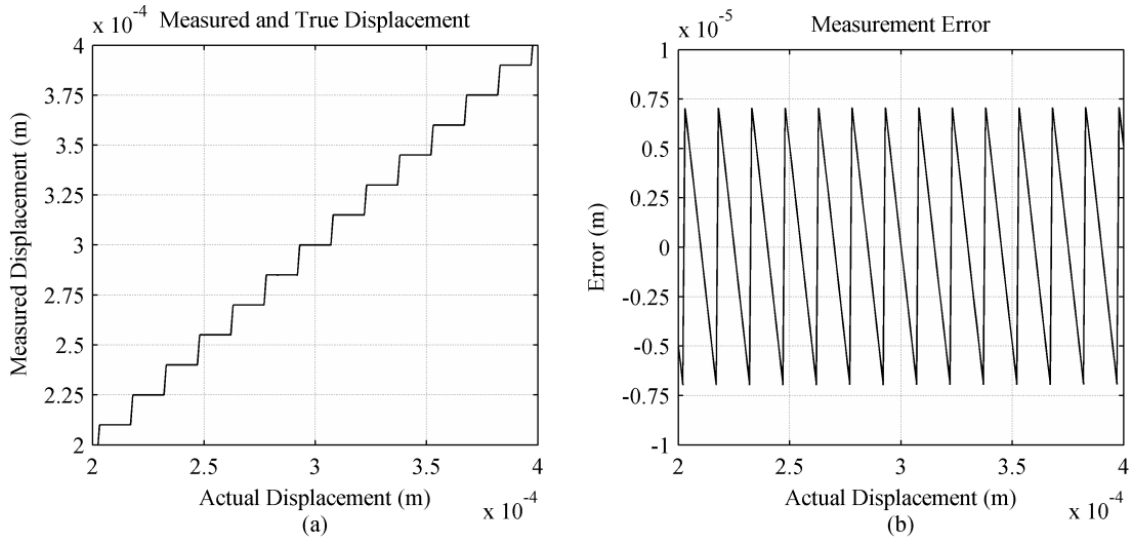


Figure 51: The performance-limiting effects of DFT resolution are shown.

The combination of L and v that may be unambiguously measured using a DFT are constrained by the inequality relationship shown in Equation (6.1). The upper and lower velocity limits have been plotted as a function of L for various values of f_{DAQ} and f_{sweep} in Figure

52, with the filter's range assumed to be fixed at $\Delta f = 9990$ GHz. In these plots the horizontal axis upper limits indicate L_{max} . It is evident from Figure 52 that the ultimate limits on the combination of v and L are imposed by L_{max} , which itself is a function of f_{DAQ} and f_{sweep} . An increase in f_{DAQ} results in an increase in L_{max} and consequently it results in an increase in the magnitude of the velocity limits. An increase in f_{sweep} does not affect the upper and lower velocity limits, although it does result in an increase in L_{max} .

Upper and Lower Velocity Limits, Shown as Functions of Displacement for Various Sampling Parameters

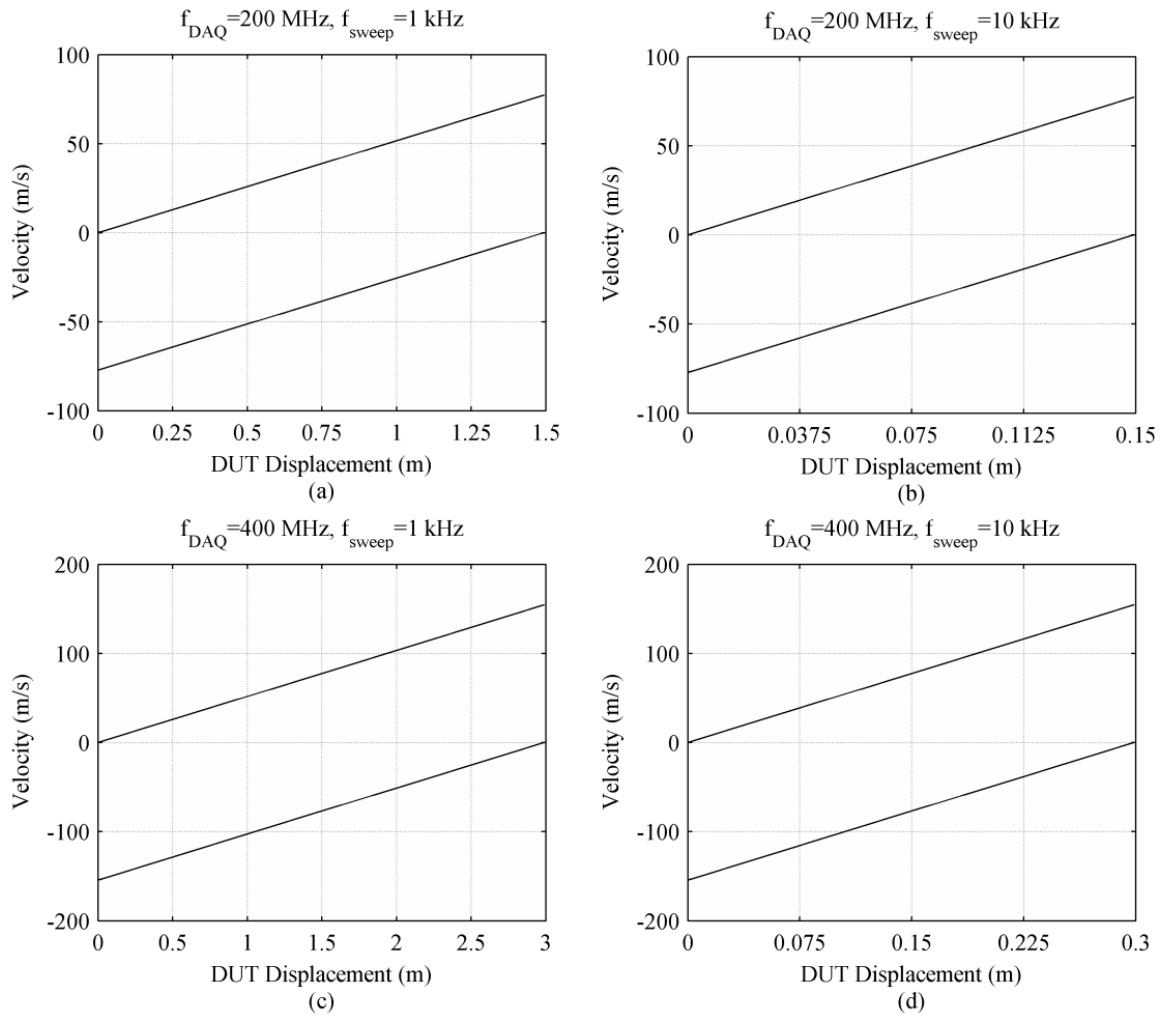


Figure 52: The upper and lower limits on DUT velocity are shown as functions of f_{DAQ} and f_{sweep} .

The filter's tuned frequency range Δf governs L_{res} according to Equation (4.11), but it also governs L_{max} according to Equation (4.12) (Figure 53). However, as shown in Figure 53, changes in Δf do not impact the ultimate upper and lower velocity limits, since the term drops out when the inequality relationship in Equation (6.1) is solved for v . An increase in Δf drives an increase in both L_{res} and L_{max} , and this may be thought of as compressing the DFT's content (with a constant value for $NFFT$) over a smaller range, thereby providing a denser sampling. The opposite effect is observed when Δf is decreased, spreading out the DFT's content over a larger range and decreasing the displacement resolution. These observations describe a tradeoff that exists between L_{res} and L_{max} , which is governed by Δf .

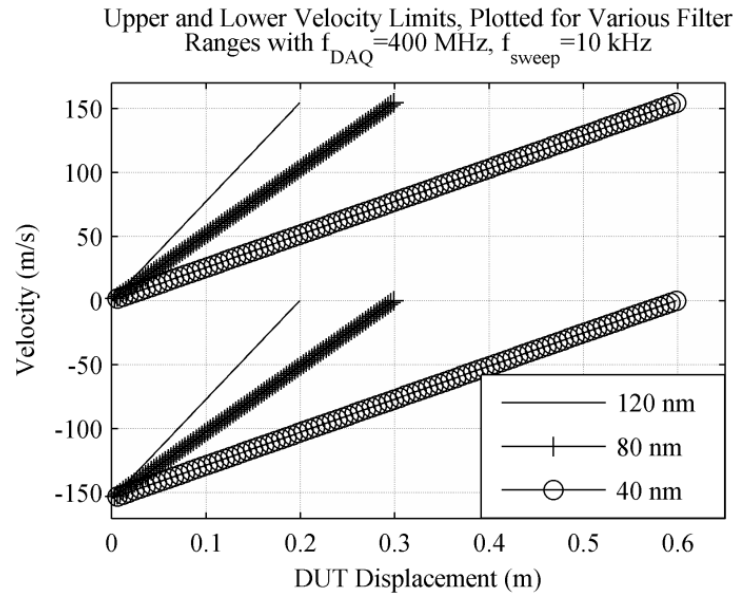


Figure 53: The upper and lower limits on DUT velocity are shown for various values of Δf .

This section has been published, in part, in Proc. *SPIE Defense, Security, and Sensing* 8370, E. A. Moro, M. D. Todd, and A. D. Puckett, 2012. The title of this paper is “Performance limitations of a white light extrinsic Fabry-Perot interferometric displacement sensor”. The dissertation author was the primary investigator and author of this paper.

6.3. Comparison and Recommendations

A comparison is drawn between the two displacement sensing methodologies assuming both sensors are subject to similar levels of measurement uncertainty and similar degrees of hardware sophistication in Table 7. A conclusion that may be drawn from the table is that the phase-diverse white light EFPI operates with increased accuracy and over a larger axial displacement range with the convenience of the absolute measurement of displacement (i.e., requiring no calibration), although its sampling speeds are limited. Intensity-modulated sensors are more susceptible to experimental uncertainty, both in their sensitivity to noise and in the sensitivity of sensor performance to the spatial properties of the actual bundling configuration, and the robustness of differential intensity-modulated sensors was shown also to depend largely on the SNR of measurement components.

Table 7: The performance characteristics of intensity-modulated optical-lever-style displacement sensors are compared to those of phase-diverse white light EFPI displacement sensors.

Performance Metric	Differential Bundled Intensity-Modulated Sensor	Phase-Diverse White Light EFPI Sensor
Axial Displacement Range	μm -mm for high-sensitivity performance (can do cm at lower sensitivity and accuracy)	μm -cm
Accuracy	μm for typical photodiode; worse if digitizer introduces noise; generally increases with displacement range	μm with standard components (e.g., source, filter, and photodiode)
Sampling Bandwidth	kHz-GHz	10's of Hz (possible hardware advances may enable kHz-sampling)
Robustness	Theoretically yes; depends largely on SNR of differential power measurements	Yes, extremely; performance is limited by DFT properties as opposed to measurement uncertainty

The performance space of the white light EFPI displacement sensor, as well as displacement and velocity resolutions and upper and lower sensing limits, are all fairly predictable with the use of analytically-derived parametric relationships, and opportunities for

enhancing performance may exist through various DSP techniques. The performance space of the intensity-modulated sensor is less straightforward to characterize. That being said, bandwidth is straightforward to determine from hardware limitations. Sensor accuracy is determined by the combination of sensor noise characteristics (SNR) and sensitivity, and sensitivity and displacement range are loosely related. Most applications will demand performance over a particular displacement range, from which a maximum sensitivity may be determined using the GA optimization approach. With a value for sensitivity and an idea of the hardware to be utilized, it is then straightforward to determine the achievable accuracy for a particular, robust configuration.

Overall, there is no way the intensity-modulated approach can compete favorably with the white light EFPI if high-accuracy (μm) is desired over axial displacement ranges of more than 1-2 mm. Another way of stating this is that, for the same source power level and measurement noise level (or the same hardware and SNR), the static capabilities of the white light EFPI are better than those of the optimized differential bundled intensity-modulated optical displacement sensor. If the high-speed capabilities of the intensity-modulated approach are desired for a sensing application, simultaneously maximizing the source's power level and minimizing photodetector and DAQ (measurement) noise levels will help achieve high overall performance. These steps for improving the sensor's hardware limited bandwidth characteristics, in conjunction with transmission modeling and optimization approach like the one proposed in this dissertation, will be the user's best shot at high performance.

In the case where high-speed sampling is desired over small axial displacements (<1 mm, and greater than $\lambda/2$), then the overall combination of achievable accuracy and axial displacement range of the intensity-modulated approach may be useful. By comparison, the white light EFPI does not get large performance advantages by decreasing the axial

displacement range, aside from relaxing the sampling requirements (this, however, does not directly enhance displacement resolution, which is otherwise limited by the tuned-filter's sweep range). So it may be possible that this niche application presents a marked advantage in the implementation of the intensity-modulated sensor. However, as discussed, this has motivated the investigation of the full state estimating phase-diverse white light EFPI architecture. Also, it should be pointed out that the practicality of the optimized differential bundled displacement sensor hinges on minimizing uncertainty, both in terms of the spatial positioning of receiving fibers and in terms of measurement uncertainty. Part of the appeal to the white light EFPI displacement sensor is its robustness, which translates directly into the ease with which the sensor is installed, set-up, and implemented. It may therefore be stated with a reasonable degree of certainty that the white light EFPI offers a serious advantage with regard to simplicity of deployment and ease of use.

Chapter 7 Summary and Future Work

7.1. Summary

7.1.1. Overview

Non-contacting optical displacement sensing methodologies were specifically researched, according to the design constraints imposed by a particular set of displacement sensing test scenarios and driven primarily by the qualities of *minimal invasiveness* and *not acting as a spark source*. Within the class of non-contacting optical displacement sensors, two specific sensing architectures were analyzed in this dissertation. Differential bundled fiber optic displacement sensors and white light EFPI were introduced and motivated as the *best-suited sensing methodologies for use in a non-contacting fiber optic sensing application where, in addition to the previously described advantages and performance capabilities, robustness is desired*. These sensors are generally capable of offering μm (or sometimes sub- μm) accuracy over axial displacement ranges on the order of μm -cm.

A phase-diverse approach was proposed for implementing the white light EFPI displacement sensor, in an effort to enhance the sensor's dynamic capabilities. Generally speaking, the white light EFPI offers increased accuracy, axial displacement range, and

robustness over the intensity-modulated approach, while the intensity-modulated approach has the potential to measure DUT motion with sampling rates in the GHz range. The performance-spaces of these two sensing approaches were discussed as they pertain to practical implementation in a non-contact displacement sensing role, where certain unavoidable or irreducible sources of measurement uncertainty exist.

7.1.2. Intensity-Modulated Displacement Sensors

A recently proposed optical transmission model was validated, and uncertainty and sensitivity analyses were performed on this model in an effort to characterize its overall performance. A new design framework was proposed for designing a differential bundled fiber optic displacement sensor, employing this recently validated optical transmission model to accurately characterize a given sensor configuration's transmitted and measured power levels, thereby simulating sensor performance *a priori*. The utility of this approach is in generating a sensor design space over an axial displacement range of interest and achieving high-sensitivity with an acceptable level of displacement measurement error.

From an analysis of the bundling configuration's design space, it was concluded that the optimized configurations analyzed in this dissertation constitute extrema of the GA's cost function, outperforming the rest of the design space and offering high-performance over application specific axial displacement ranges of 6-8 mm and 3.5-5.5 mm. The sensors' performance was shown to be sensitive to the particular arrangement of receiving fibers in the bundle, and the result of the GA was shown to depend on the weights of the cost function, which set the relative significance of sensor error and sensitivity. In the 6-8 mm scenario, the GA was shown to outperform a random walk-through (Monte Carlo simulation), converging to the optimized configuration in about 1,000 trials, whereas a 10,000,000-trial Monte Carlo analysis of the entire sensor design space failed to produce similarly high-performance results.

The second iteration optimized bundle configuration, proposed for use over the 3.5-5.5 mm range (Section 3.4) showed evidence of extremely high sensitivities between the locations of receiving fibers within the bundle. These results also indicated that achieving truly robust differential displacement sensing may be, in some cases, easier said than done, and robustness is related to measurement signal SNR. With the use of a low-power SLD source and relatively inexpensive off-the-shelf photodetectors, it was difficult to achieve a large-enough SNR to achieve truly robust sensing. Additionally, sensor architectures that are highly-sensitive to DUT motion are also highly sensitive to input noise, making the SNR metric of the utmost importance.

While the potential does exist for optimized bundle configurations that offer high-sensitivity and accuracy over a prescribed axial displacement range, extreme care must be taken during optimization to ensure that the SNR is large enough to enable stable sensing, high-accuracy, and robustness to sources of measurement noise. More generally, this sensing methodology is particularly sensitive to measurement uncertainty, both in the measured signals and in the actual location of fibers within the bundling configuration. These sensors have the potential to find niche application to scenarios where (1) high measurement SNR (either high source-power, low photodetector and DAQ system noise, or a combination of the two) is practical and (2) where high-speed measurement is desired with moderate accuracy over axial displacement ranges of approximately 1 mm or less (down to axial displacement ranges of $\lambda/2$, where monochromatic interferometers offer superb performance).

7.1.3. White Light Interferometric Displacement Sensors

By way of comparison, a phase-diverse white light EFPI displacement sensor was also researched. The performance limitations of this sensor were discussed for the static case in the context of various (tuned filter) input parameters, and performance was shown largely to be

governed by the properties of the Fourier transform used during demodulation. Contrary to previously held notions about the impact of Doppler-shifting in a Fabry-Perót cavity, and contrary to the previously held belief that white light EFPI displacement sensors are limited to low-bandwidths simply on account of hardware limitations, it was shown theoretically and experimentally that the impact of Doppler-shifting within the Fabry-Perót cavity has significant impacts on the measurement of DUT displacement. In other words, the white light EFPI was shown to be particularly sensitive to the effects of DUT motion during sensing. A relationship was derived between the Doppler-induced measurement displacement bias and three tuned filter (input) parameters, and this relationship was experimentally verified. With knowledge of this relationship, it was shown that a phase-diverse approach may be implemented, with two unique swept-filters (and correspondingly, two unique sets of tuning parameters) in order to simultaneously measure DUT velocity and *unbiased* DUT displacement. This contribution is particularly exciting, because it allows for *full-state estimation using a single sensor*, and it measures a raw velocity and a raw displacement directly.

With an understanding of the dynamic characteristics of a white light EFPI displacement sensor, a further elaboration on its performance limitations was made, and an uncertainty analysis and a sensitivity analysis were performed. Sensor performance was found to be largely governed by the properties of the DFT, where measurement uncertainty played a particularly small role in dictating the final displacement resolution and velocity resolution. Combined upper and lower limits on DUT displacement and velocity were described in terms of hardware sampling parameters, and the possibility for high-performance, dynamically deployed phase diverse white light EFPI displacement sensors was discussed.

7.1.4. Final Comparison and Analysis

A side-by-side comparison between the differential bundled intensity-modulated approach and the phase-diverse white light EFPI approach was made. It was shown that the performance limitations on the white light EFPI are “hard”, and if one needs to work around these limitations (e.g., if high-bandwidth sensing is simply required as part of the sensing application) then it makes sense to implement an intensity-modulated displacement sensor. The performance of the intensity-modulated displacement sensor is ultimately governed by its SNR, since this sensor was not robust to power-fluctuations. In other words, the intensity-modulated sensor is governed by the power of the source and the measurement noise.

7.2. Future Work

Future work on the optimized bundled displacement sensor will focus on incorporating the differential measurement component SNRs into the optimization routine’s cost function. Proper accounting of the SNR may be used to ensure sensor robustness, stability, and accuracy. With this future effort, and along with the design framework proposed in this dissertation, it may be possible to develop a self-contained package for deployment and remote sensing, that would measure DUT displacements in the kHz-GHz range and output simple voltage measurements.

Future work on the phase-diverse white light EFPI displacement sensor will be in the way of prototyping and developing the sensor’s dynamic capabilities. To the best of the author’s knowledge, the proposed use of side-by-side white light EFPI is novel and has the potential to open the door to an entirely new way of non-contact displacement and velocity sensing. With knowledge of the performance-limiting relationships described in this dissertation the task of addressing the sensor’s performance bottlenecks is straightforward. In particular, the speeds of

the filtering and referencing operations were shown to limit sensor bandwidth. With the full state-estimating capability proposed in this dissertation, and with advances in filter speed and wavelength referencing, it may soon be possible to simultaneously measure velocities of 10's to 100's of m/s along with axial displacements of up to 10's of cm with micrometer accuracy and with overall sampling rates in the kHz range. Such sensors would find broad applicability, driven largely by the simultaneous measurement capability along with the fact that calibration is not required to make absolute displacement and velocity (both magnitude and direction) measurements.

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