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# QCD Evolution of the Transverse Momentum Dependent Correlations

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## Abstract

We study the QCD evolution for the twist-three quark-gluon correlation functions associated with the transverse momentum odd quark distributions. Different from that for the leading twist quark distributions, these evolution equations involve more general twist-three functions beyond the correlation functions themselves. They provide important information on nucleon structure, and can be studied in the semi-inclusive hadron production in deep inelastic scattering and Drell-Yan lepton pair production in  $pp$  scattering process.

Semi-inclusive hadronic processes have attracted much theoretical interest in recent years, where the so-called transverse momentum dependent (TMD) parton distributions and fragmentation functions can be studied [1–8]. These functions generalize the original Feynman parton picture, where the partons only carry longitudinal momentum fraction of the parenting (final state) hadron. They certainly provide further important information on nucleon structure, and are crucial to understand the novel spin phenomena, such as the single transverse spin asymmetry (SSA) [1–3, 9, 10].

Important aspects of the TMD parton distributions have been explored in the last few years, such as the gauge property and the crucial role of the initial/final state interaction for the nonzero Sivers quark distribution leading to the SSA in Semi-inclusive hadron production in deep inelastic scattering (SIDIS) and Drell-Yan lepton pair production processes [1–4]. Further study has shown that this mechanism is uniquely related to the twist-three quark-gluon correlation approach for the SSA phenomena [11–15]. In particular, these two approaches are unified to describe the same physics in the overlap region where both apply [16].

At the leading order, there are eight independent TMD quark distributions, depending on the polarizations of the nucleon and the quark [7, 8]. Three of them are called  $k_{\perp}$ -even distributions. After integrating over transverse momentum, they produce the leading-twist quark distributions, including the spin average, longitudinal polarized, and transversity quark distributions [17]. The rest five distributions are called  $k_{\perp}$ -odd distributions. Upon integral over the transverse momentum, they will vanish in the quark correlation matrix. In this paper, we are interested in four of these  $k_{\perp}$ -odd TMD quark distributions. They can be defined from the following matrix,

$$\mathcal{M}^{\alpha\beta}(x, k_{\perp}) = P^+ \int \frac{d\xi^-}{2\pi} e^{ix\xi^- P^+} \int \frac{d^2 b_{\perp}}{(2\pi)^2} e^{-i\vec{b}_{\perp} \cdot \vec{k}_{\perp}} \left\langle PS \left| \bar{\Psi}_v^{\beta}(0) \Psi_v^{\alpha}(\xi^-, 0, \vec{b}_{\perp}) \right| PS \right\rangle, \quad (1)$$

where  $x$  is the longitudinal momentum fraction of the proton carried by the quark and  $k_{\perp}$  is the transverse momentum. In the above definition we have chosen  $P = (P^+, 0^-, 0_{\perp})$  which is along the momentum direction of the proton,  $S$  is the polarization vector, and  $\Psi_v(\xi)$  is defined as  $\Psi_v(\xi) \equiv \mathcal{L}_v(-\infty; \xi) \psi(\xi)$ , with  $\mathcal{L}_v$  the gauge link. This gauge link contains the light-cone gauge link contribution and the transverse gauge link contribution at the spatial infinity [3]. We have chosen it goes to  $-\infty$ , indicating that we adopt the definition for the TMD quark distributions for the Drell-Yan process [1–3]. The four  $k_{\perp}$ -odd TMD quark

distributions can be obtained by the following expansion of the above quark correlation matrix [7, 8],

$$\begin{aligned} \mathcal{M} = \frac{1}{2M} & \left[ g_{1T}(x, k_\perp) \gamma_5 \mathcal{P}(k_\perp^\vec{\cdot} \cdot \vec{S}_\perp) + f_{1T}^\perp(x, k_\perp) \epsilon^{\mu\nu\alpha\beta} \gamma_\mu P_\nu k_\alpha S_\beta \right. \\ & \left. + h_{1L}(x, k_\perp) \lambda i \sigma_{\mu\nu} \gamma_5 P^\mu k_\perp^\nu + h_1^\perp(x, k_\perp) \sigma^{\mu\nu} k_\mu P_\nu + \dots \right] , \end{aligned} \quad (2)$$

where  $M$  is the nucleon mass and the interpretations of the four  $k_\perp$ -odd TMD quark distributions are:  $g_{1T}$  and  $f_{1T}^\perp$  represent a longitudinal polarized and unpolarized quark distributions in a transversely polarized nucleon, respectively;  $h_{1L}$  and  $h_1^\perp$  represent transversely polarized quark distributions in a longitudinal polarized and unpolarized nucleon target, respectively. They are  $k_\perp$ -odd distributions, i.e., after integrating over the transverse momentum, the above expansion will vanish. However, if we weight the integral with transverse momentum, the above matrix will lead to a set of quark-gluon correlation functions at the twist-three level. These correlation functions can be calculated as transverse momentum-moment of the above four  $k_\perp$ -odd TMD quark distributions. The last  $k_\perp$ -odd TMD quark distribution  $h_{1T}^\perp$  represents a correlated transversely polarized quark distribution in a transversely polarized nucleon target, and is related to the twist-four quark-gluon correlation function. We will not discuss this function in this paper.

As mentioned above, the transverse-momentum-moment of the above four TMD quark distributions define the following transverse momentum dependent correlation functions in nucleon,

$$\int d^2 k_\perp \frac{\vec{k}_\perp^2}{2\pi M^2} f_{1T}^\perp(x, k_\perp) = T_F(x) , \quad \int d^2 k_\perp \frac{\vec{k}_\perp^2}{2M^2} g_{1T}^\perp(x, k_\perp) = \tilde{g}(x) , \quad (3)$$

$$\int d^2 k_\perp \frac{\vec{k}_\perp^2}{2\pi M^2} h_1^\perp(x, k_\perp) = T_F^{(\sigma)}(x) , \quad \int d^2 k_\perp \frac{\vec{k}_\perp^2}{2M^2} h_{1L}^\perp(x, k_\perp) = \tilde{h}(x) . \quad (4)$$

We emphasize again that the above TMD quark distributions follow their definitions in Drell-Yan process. If we choose those for the semi-inclusive DIS process, the above two equations associated with  $T_F(x)$  and  $T_F^{(\sigma)}(x)$  shall change signs. These correlation functions are related to more general quark-gluon correlation functions. For example,  $T_F(x)$  and  $T_F^{(\sigma)}(x)$  are diagonal parts of the general quark-gluon correlation functions  $T_F(x_1, x_2)$  and  $T_F^{(\sigma)}(x_1, x_2)$  which are responsible for the single spin asymmetry in hadronic process [12, 18]:  $T_F(x) \equiv T_F(x, x)$  and  $T_F^{(\sigma)}(x) \equiv T_F^{(\sigma)}(x, x)$ .<sup>1</sup> They can be defined through the following

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<sup>1</sup> For the convenience of our presentation, we have changed the normalizations for  $T_F(x_1, x_2)$  and

correlation matrix,

$$M_{F\alpha\beta}^\mu(x) \equiv \int \frac{dy^-}{2\pi} \frac{dy_1^-}{2\pi} e^{ixP^+y^-} \langle PS | \bar{\psi}_\beta(0) g F^{+\mu}(y_1^-) \psi_\alpha(y^-) | PS \rangle, \quad (5)$$

where  $\mu$  is a transverse index,  $F_{+\mu}$  the gluon field tensor, and the gauge link has been suppressed. Its decomposition contains the contribution from  $T_F(x)$  and  $T_F^{(\sigma)}(x)$ ,

$$M_{F\alpha\beta}^\mu(x) = \frac{M}{2} \left[ T_F(x) \epsilon_\perp^{\nu\mu} S_{\perp\nu} \not{p} + T_F^{(\sigma)}(x) i\gamma_\perp^\mu \not{p} \right]. \quad (6)$$

Similarly, we can calculate the other two correlation functions by [4],

$$\tilde{M}_{F\alpha\beta}^\mu(x) = \frac{M}{2} \left[ \tilde{g}(x) S_\perp^\mu \gamma_5 \not{p} + \tilde{h}(x) \lambda \gamma_5 \gamma_\perp^\mu \not{p} \right]. \quad (7)$$

where  $\tilde{M}_{F\alpha\beta}^\mu$  is defined as,

$$\tilde{M}_{F\alpha\beta}^\mu(x) = \int \frac{d\xi^-}{2\pi} e^{i\xi^- x P^+} \langle PS | \bar{\psi}_\beta(0) \left\{ iD_\perp^\mu(\xi^-) - \int_{\xi^-}^{-\infty} d\zeta^- g F^{+\mu}(\zeta^-) \right\} \psi_\alpha(\xi^-) | PS \rangle \quad (8)$$

Applying the time-reversal invariance, we find the above definition of  $\tilde{g}$  is the same as that in [14], except a normalization factor 2.

The above four correlation functions are subsets of more general twist-three quark-gluon correlation functions [17, 19]:  $G_D(x, y)$ ,  $\tilde{G}_D(x, y)$ ,  $H_D(x, y)$  and  $E(x, y)$ . These twist-three functions and their contributions to the inclusive DIS and Drell-Yan lepton pair productions have been under intense investigations in the last two decades (see for example [17]). The above four correlation functions Eqs. (3,4), however, will enter in the transverse momentum weighted cross sections in the semi-inclusive hadron production in DIS and Drell-Yan lepton pair production in  $pp$  collisions [7, 8, 20]. They will provide additional information on the quark-gluon correlations in nucleon, and will be complementary to those studied in the inclusive DIS and Drell-Yan processes. Recent experimental developments will help to pin down these contributions, and build strong physics associated with these correlation functions [21].

One of the important questions remained to be answered is the scale evolution for these correlation functions. The evolution equation controls the energy dependence of the associated observables [22]. For example, with the evolution equations, we will be able to compare the single spin asymmetries coming from the same quark-gluon correlation function  $T_F(x)$

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$T_F^{(\sigma)}(x_1, x_2)$  by a factor of  $1/2\pi M$  as compared to those in [16, 18].

in hadronic processes at different energy experiments. General structure of the evolution equations for the twist-three quark-gluon correlation functions has been known in the literature [23]. However, the above correlation functions Eqs. (3,4) are special projections of the general twist-three quark-gluon correlations, and their evolutions are not directly available from the already known results [23]. Earlier attempts [24] have been made to derive the evolution equations for the correlation functions of Eqs. (3,4), but were not complete. On the other hand, from the large transverse momentum quark Sivers function calculated in [16], we would already obtain the evolution equation for  $T_F(x)$ , since the collinear divergence in that calculation will lead to the splitting function of  $T_F(x)$ . This splitting function was confirmed by a complete calculation of next-to-leading order QCD correction to the transverse-momentum weighted spin asymmetry in Drell-Yan lepton pair production [25]. More comprehensive evolution equations for  $T_F(x)$ , together with those for the three-gluon correlation functions which are relevant to the single spin asymmetry observables have recently been derived in [26]. In this paper, we will extend these studies to calculate the scale evolutions for the above four quark-gluon correlation functions. Their contributions to the azimuthal angle distributions in Drell-Yan lepton pair production in  $pp$  collisions, and the relevant QCD factorization analysis will be presented in a forthcoming publication [27].

In our calculations, we will choose the light-cone gauge:  $A^+ = 0$ . There are several advantages for this choice. First, the quark Sivers function was previously calculated in the covariant gauge [16]. Our calculation in the light-cone gauge will provide an important cross check for the results. Second, the light-cone gauge is more convenient to calculate the evolution equations for  $\tilde{g}$  and  $\tilde{h}$ . In particular, the evolution equations for  $T_F(x)$  and  $\tilde{g}(x)$  can be calculated simultaneously. The only difference is that for  $T_F(x)$  we have to take a pole contribution for some diagrams, whereas for  $\tilde{g}(x)$  we will not take the pole (see the discussions below). Third, we can further choose a particular boundary condition in the light-cone gauge [3], which will greatly simplify the derivation. We have also checked that the final results do not depend on the boundary condition. According to the quark distribution definition we have chosen above, it is convenient to choose the retarded boundary condition, i.e.,  $A_\perp(-\infty^-) = 0$ . With this choice, the gauge link associated with the TMD quark distributions in Eq. (1) becomes unit, and their contributions can be neglected [3]. From

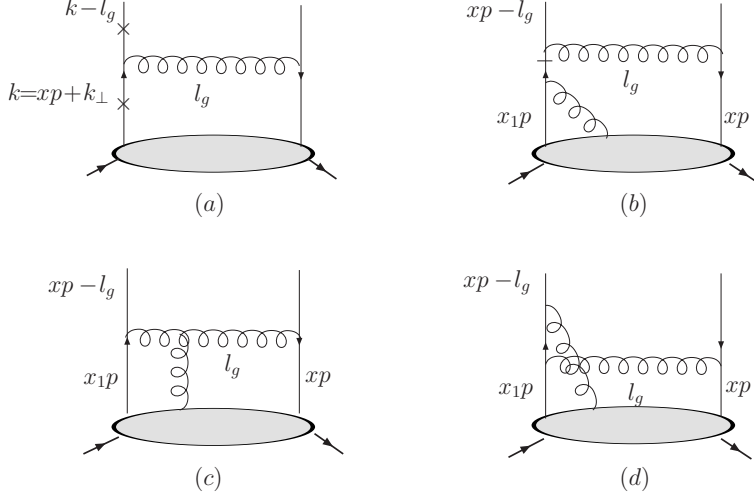


FIG. 1: Real gluon radiation contribution to the evolution equations for the twist-three quark-gluon correlation functions  $T_F(x)$ ,  $\tilde{g}(x)$ ,  $T_F^{(\sigma)}(x)$ ,  $\tilde{h}(x)$ .

this, we can re-write the quark-gluon correlation functions  $T_F(x)$  and  $\tilde{g}$  as

$$T_F(x) = \int \frac{dy^-}{8\pi^2 M} e^{ixP^+y^-} \langle PS | \bar{\psi}(0) \not{n} \epsilon_{\perp}^{\nu\mu} S_{\perp\nu} i\partial_{\perp\mu} \psi_{\alpha}(y^-) | PS \rangle, \quad (9)$$

$$\tilde{g}(x) = \int \frac{dy^-}{4\pi M} e^{ixP^+y^-} \langle PS | \bar{\psi}_{\beta}(0) \gamma_5 \not{n} S_{\perp\mu} i\partial_{\perp}^{\mu} \psi_{\alpha}(y^-) | PS \rangle, \quad (10)$$

in the light-cone gauge with retarded boundary condition. Similar expressions hold for other two correlation functions,  $T_F^{(\sigma)}$  and  $\tilde{h}$ . In the following calculations, we will focus on the derivation for the evolution functions for  $T_F$  and  $\tilde{g}$ , especially for  $T_F$ , and those for  $T_F^{(\sigma)}$  and  $\tilde{h}$  can be obtained accordingly.

To calculate the splitting function for the above two functions, we have to take into account the contributions from the operators  $(\bar{\psi}\partial_{\perp}\psi)$  and  $(\bar{\psi}A_{\perp}\psi)$  [19], because they are at the same order. Especially, because of the contribution from  $A_{\perp}$ , the evolution of the above correlation functions will involve more general twist-three functions:  $G_D$  and  $\tilde{G}_D$  or  $T_F(x_1, x_2)$  and  $T_F^{(\sigma)}(x_1, x_2)$ . This is an important feature for the scale evolution of the higher-twist distributions, such as that of the  $g_T$  structure function [23].

We plot the Feynman diagram contributions from the real gluon radiations in Fig. 1, where (a) is the contribution from the partial derivative on the quark field, and (b – d) are those from  $A_{\perp}$  contributions. The virtual corrections only contribute to partial derivative part, and they are easy to carry out.

We will perform the collinear expansion for the hard scattering part to calculate the

contribution from Fig. 1(a). The linear  $k_\perp$  expansion term combining with the quark field will lead to the quark-gluon correlation function  $T_F(x)$  and  $\tilde{g}(x)$  in Eqs. (9,10). In the collinear expansion in terms of  $k_\perp$ , we can fix the transverse momentum of the probing quark ( $l_q$ ) or the radiated gluon ( $l_g$ ), because of momentum conservation and we are integrating over them to obtain  $T_F(x)$  and  $\tilde{g}(x)$ . We have also checked that they will generate the same result. In the following calculations, we choose  $l_g$  being fixed in the collinear expansion. This will avoid the collinear expansion of the on-shell condition for the radiated gluon, and simplify the derivations.

For the  $A_\perp$  contribution, we notice that  $F^{+\mu} = \partial^+ A_\perp^\mu$  in the light cone gauge. Therefore, one can relate the corresponding soft matrix to the correlation function  $T_F(x, x_1)$  in the following way,

$$\begin{aligned} & \frac{i}{x - x_1 + i\epsilon} \int \frac{dy^- dy_1^-}{4\pi} e^{ix_1 P^+ y^-} e^{i(x-x_1)P^+ y_1^-} \langle PS | \bar{\psi}_\beta(0^-) \not{\epsilon}_\perp^{\nu\mu} S_{\perp\nu} g F^+{}_\mu(y_1^-) \psi_\alpha(y^-) | PS \rangle \\ &= \int \frac{dy^- dy_1^-}{4\pi} P^+ e^{ix_1 P^+ y^-} e^{i(x-x_1)P^+ y_1^-} \langle PS | \bar{\psi}_\beta(0^-) \not{\epsilon}_\perp^{\nu\mu} S_{\perp\nu} g A_{\perp\mu}(y_1^-) \psi_\alpha(y^-) | PS \rangle . \end{aligned} \quad (11)$$

In the above formula, the soft gluon pole appears in the first line comes from the partial integration. The prescription of this pole has been determined because we have chosen the retarded boundary condition. For the same reason, we have to regulate the light cone propagator in a consistent manner, and the gluon propagator in Fig. 1(c) in the light cone gauge with retarded boundary condition is given by [3],

$$D^{\alpha\beta}(l) = \frac{-i}{l^2 + i\epsilon} \left( g^{\alpha\beta} - \frac{l^\alpha n^\beta + n^\alpha l^\beta}{l \cdot n + i\epsilon} \right) , \quad (12)$$

where  $l$  is the gluon propagator momentum entering the quark-gluon vertex in Fig.1(c).

Adding the contributions from the partial derivative and  $A_\perp$ , we reach our final formula for the  $T_F(x)$  splitting calculation,

$$\begin{aligned} T_F^{(1)}(x_B) &= \int dx d^2 l_{g\perp} \frac{\partial}{\partial k_\perp^\mu} \left\{ [\hat{H}(k, l_g) \not{p}] \times l_{q\perp}^\mu \right\} |_{k_\perp=0} T_F(x, x) \\ &+ \int dx dx_1 d^2 l_{g\perp} \left\{ [\hat{H}_\mu(xP, x_1P, l_g) \not{p}] \times l_{q\perp}^\mu \right\} \frac{1}{\pi} \frac{i}{x - x_1 + i\epsilon} T_F(x, x_1) , \end{aligned} \quad (13)$$

where the transverse spin vector has been integrated out, and the transverse index  $\mu$  is not meant to be summed up.  $l_{q\perp}$  is the probing quark transverse momentum. In the above equation,  $\hat{H}(k, l_g)$  represents the hard partonic part in Fig. 1(a) with transverse momentum dependence on  $k_\perp$ , and  $\hat{H}_\mu(xP, x_1P, l_g)$  the hard part for Figs. 1(b-d) with transverse



polarized gluon  $A_{\perp\mu}$  insertion where all momenta are collinear. We have to include both contributions to obtain a complete result.

We have similar expression for  $\tilde{g}(x)$  splitting. The only difference is to replace  $\not{p}$  with  $\gamma_5 \not{p}$  and similar replacement in the hard parts in the above two terms. More over, because of simple Dirac algebra, the first term is the same for both  $T_F(x)$  and  $\tilde{g}(x)$  which comes from Fig. 1(a). Let us first discuss this contribution. In the calculations, we have to perform the collinear expansion in terms of  $k_{\perp}^{\mu}$ . Because of the momentum conservation,  $l_q^{\mu} = k_{\perp}^{\mu} - l_g^{\mu}$ , we can separate the contribution from the explicit dependence on  $k_{\perp}^{\mu}$ ,

$$T_F^{(1)}|_{\text{Fig.1(a)}} = \int dx T_F(x, x) d^2 l_{g\perp} \left\{ \left[ \hat{H}(k, l_g) \not{p} \right] |_{k_{\perp}=0} - l_{g\perp}^{\mu} \frac{\partial}{\partial k_{\perp}^{\mu}} \left[ \hat{H}(k, l_g) \not{p} \right] |_{k_{\perp}=0} \right\}. \quad (14)$$

The first term is easy to derive, and its contribution will be

$$\frac{\alpha_s}{2\pi} \int \frac{dx}{x} \frac{dl_{g\perp}^2}{l_{g\perp}^2} C_F \left( \frac{1+z^2}{1-z} \right) T_F(x, x). \quad (15)$$

where  $z = x_B/x$ , and the well-known splitting kernel appears. This splitting kernel contains the end-point divergence, which should be canceled out by the virtual diagram contributions. After taking into account the virtual contribution, the end-point will be regulated by the plus function,

$$\frac{\alpha_s}{2\pi} \int \frac{dx}{x} \frac{dl_{g\perp}^2}{l_{g\perp}^2} C_F \left( \frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right) T_F(x, x), \quad (16)$$

where the plus function follows the definition of [22]. To calculate the second term of Eq. (14), we will do the collinear expansion of the hard scattering part  $\hat{H}(k, l_g)$  with  $l_{g\perp}$  fixed. The transverse momentum  $k_{\perp}$  flow can go through the quark line in Fig. 1(a), for which we label as ‘‘cross’’ in the diagram. We can further simplify the derivation by using the following identity,

$$\frac{\partial}{\partial k^{\alpha}} \frac{i}{\not{k}} = \frac{i}{\not{k}} i\gamma^{\alpha} \frac{i}{\not{k}}, \quad (17)$$

which essentially represents the application of the Ward identity. Applying the above identity, we can relate the  $k_{\perp}$  expansion in the quark propagator and quark line to that with a transverse polarized gluon insertion with zero momentum attachment. These contributions shall be combined with those from Figs. 1(c-d). We will discuss them below.

As we mentioned above, the contributions from Fig. 1(a) are the same for the evolutions of  $T_F(x)$  and  $\tilde{g}(x)$ . Therefore, the above results apply for that of  $\tilde{g}(x)$  too. However, the

contributions from Figs. 1(b-d) are different for  $T_F(x)$  and  $\tilde{g}(x)$ . This is because for this part, we have to take pole contribution to obtain the splitting for  $T_F(x)$ , whereas for  $\tilde{g}(x)$  we do not take pole contribution. We first discuss their contributions to the evolution of  $T_F(x)$  correlation function. Depending on the value of  $x_g = x - x_1$  when we take the pole, these poles are called soft ( $x_g = 0$ ) and hard ( $x_g \neq 0$ ) poles, respectively. The hard pole contribution only comes from the light-cone propagator in Fig. 1(c), and its contribution is easy to calculate. For this, we obtain

$$T_F^{(1)}|_{\text{Fig.1(c)}\text{hp}} = \frac{\alpha_s}{2\pi} \int \frac{dx}{x} \frac{dl_{g\perp}^2}{l_{g\perp}^2} \frac{C_A}{2} \left( \frac{1+z}{1-z} \right) T_F(xz, x). \quad (18)$$

We emphasize that for the hard pole contribution the explicit factor  $1/(x - x_1)$  has been included in the above result, which is finite because  $x \neq x_1$ .

On the other hand, the soft pole contribution comes from the explicit pole in Eq. (13) which leads to a delta function  $\delta(x_1 - x)$ . Because this pole results into zero gluon momentum insertion to the diagrams, we can combine these contributions with the second term in Eq. (14) as we mentioned above. Therefore, we can add them together,

$$- \int dx T_F(x, x) d^2 l_{g\perp} l_{g\perp}^\mu \left\{ \frac{\partial}{\partial k_\perp^\mu} \left[ \hat{H}(k, l_g) \not{p} \right] |_{k_\perp=0} - \left[ \hat{H}_\mu(xP, xP, l_g) \not{p} \right] \right\}. \quad (19)$$

From this equation, we find that the contribution from Fig. 1(b) cancels out that from the  $k_\perp$  expansion on the quark line with momentum “ $k$ ”, because they have the same color-factor but opposite signs. The Fig. 1(d) and the  $k_\perp$  expansion on the quark propagator “ $k - l_g$ ” are also the same but with different color-factor: color-factor for Fig. 1(d) is  $-1/2N_c$  whereas that for Fig. 1(a) is  $C_F$ . Their total contributions will add up to a color-factor  $C_A/2$ . The same color-factor  $C_A/2$  appears for Fig. 1(c). Thus, the final result for this contribution will be proportional to  $C_A/2$ . By applying the identity of Eq. (17) again, we can re-write this part of contribution as

$$\begin{aligned} & -\frac{\alpha_s}{2\pi} \frac{C_A}{2} \int dx T_F(x, x) d^2 l_{g\perp} \left[ \frac{\partial}{\partial l_{g\perp}^\mu} \hat{H}_0(xP, l_{g\perp}) \right] \times (-l_{g\perp}^\mu) \\ &= -\frac{\alpha_s}{2\pi} \frac{C_A}{2} \int dx T_F(x, x) d^2 l_{g\perp} \hat{H}_0(xP, l_{g\perp}) \\ &= -\frac{\alpha_s}{2\pi} \frac{C_A}{2} \int dx \frac{dl_{g\perp}^2}{l_{g\perp}^2} \left( \frac{1+z^2}{1-z} \right) T_F(x, x), \end{aligned} \quad (20)$$

where  $\hat{H}_0$  represent the hard scattering part without color factor and we have made use of the fact that the hard part  $\hat{H}_0(xP, l_{g\perp}) \propto 1/l_{g\perp}^2$ . Again, the same splitting kernel appears.

Finally, there is also contribution from  $\tilde{T}_F(x_1, x_2)$ , which only comes from the hard pole diagram Fig. 1(c). Summing up all contributions, we obtain the scale evolution equation for the diagonal part of the quark-gluon correlation function  $T_F(x_1, x_2)$ ,

$$\begin{aligned} \frac{\partial}{\partial \ln \mu^2} T_F(x_B, \mu^2) &= \frac{\alpha_s}{2\pi} \int \frac{dx}{x} \left[ C_F \left\{ \frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right\} T_F(x, x) \right. \\ &\quad \left. + \frac{C_A}{2} \left\{ \frac{1+z}{1-z} T_F(xz, x) - \frac{1+z^2}{1-z} T_F(x, x) + \tilde{T}_F(xz, x) \right\} \right], \quad (21) \end{aligned}$$

which is consistent with that in Ref. [25, 26]. The complete evolution equation for  $T_F(x)$  shall also contain contributions from the three-gluon correlation functions, which have been calculated in [26].

As we mentioned above, the contributions from Figs. 1(b-d) to the evolution of  $\tilde{g}(x)$  are different from that of  $T_F(x)$ . For  $\tilde{g}(x)$  splitting, we do not take pole contributions from these diagrams. For example, we will not have cancelation between diagrams Fig. 1(b) and collinear expansion of quark line “ $k$ ” of Fig. 1(a). More over, without taking pole there will be an additional integral variable in the splitting function, similar to that for the evolution of  $g_T$  structure function [23]. The  $A_\perp$  contribution from Figs. 1(b-d) can be transformed into  $T_F$  and  $\tilde{T}_F$ , or to  $G_D$  and  $\tilde{G}_D$  [17]. Because we do not take a pole for the scattering amplitudes, the calculations for these diagrams are straightforward. The partial derivative contribution from Fig. 1(a) is similar to that for  $T_F(x)$  calculation. This part depends on  $\tilde{g}(x)$ . After adding all these contributions together, we obtain the evolution equation for  $\tilde{g}(x)$ ,

$$\begin{aligned} \frac{\partial \tilde{g}(x_B, \mu^2)}{\partial \ln \mu^2} &= \frac{\alpha_s}{2\pi} \int \frac{dx dy}{x} \left\{ \tilde{g}(x) \delta(y-x) \left[ C_F \left( \frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right) - \frac{C_A}{2} \frac{1+z^2}{1-z} \right] \right. \\ &\quad \left. + \tilde{G}_D(x, y) \left[ C_F \left( \frac{x_B^2}{x^2} + \frac{x_B}{y} - \frac{2x_B^2}{xy} - \frac{x_B}{x} - 1 \right) + \frac{C_A}{2} \frac{(x_B^2 + xy)(2x_B - x - y)}{(x_B - y)(x - y)y} \right] \right. \\ &\quad \left. + G_D(x, y) \left[ C_F \left( \frac{x_B^2}{x^2} + \frac{x_B}{y} - \frac{x_B}{x} - 1 \right) + \frac{C_A}{2} \frac{x_B^2 - xy}{(y - x_B)y} \right] \right\}, \quad (22) \end{aligned}$$

where again  $z = x_B/x$ , and the definitions of  $G_D$  and  $\tilde{G}_D$  follow that in [17]. The end-point singularity from  $\tilde{g}(x)$  with color factor  $C_A/2$  at right hand side of the equation is canceled out by that from  $\tilde{G}_D$  at the second line. We further notice that we can replace  $G_D$  and  $\tilde{G}_D$  with  $T_F$  and  $\tilde{T}_F$  at the right hand side by using the relations between them [15, 19]. However, we still have the  $\tilde{g}(x)$  term at the right hand side of equation. Although we can re-write  $\tilde{g}(x)$  in terms of  $\tilde{G}_D$  and  $\tilde{T}_F$  [14], that will not eliminate its dependence completely

and the right hand side will depend on  $\tilde{G}_D$ ,  $\tilde{T}_F$  and  $G_D$  instead. Therefore, the evolution of  $\tilde{g}(x)$  depends on three functions:  $\tilde{g}(x)$ ,  $G_D(x, y)$  and  $\tilde{G}_D(x, y)$ . This feature is different from that for  $T_F(x)$ , where it only depends on  $T_F$  and  $\tilde{T}_F$ . It may indicate the nontrivial QCD dynamics associated with the evolution of the correlation function  $\tilde{g}(x)$ . This has also been shown in its contribution to the Drell-Yan dilepton azimuthal asymmetry in  $pp$  scattering. We leave this study in a future publication.

Since the derivation follows the similar procedure, we skip the technique details and only list the final result for the evolution equation of correlation functions  $T_F^{(\sigma)}(x, x)$ ,  $\tilde{h}(x)$ . For  $T_F^{(\sigma)}$ , we have

$$\begin{aligned} \frac{\partial}{\partial \ln \mu^2} T_F^{(\sigma)}(x_B, \mu^2) = & \frac{\alpha_s}{2\pi} \int \frac{dx}{x} \left[ C_F \left\{ \frac{2z}{(1-z)_+} + 2\delta(1-z) \right\} T_F^{(\sigma)}(x, x) \right. \\ & \left. + \frac{C_A}{2} \left\{ \frac{2}{1-z} T_F^{(\sigma)}(xz, x) - \frac{2z}{1-z} T_F^{(\sigma)}(x, x) \right\} \right], \quad (23) \end{aligned}$$

which is consistent with the large transverse momentum Boer-Mulders function  $h_1^\perp(x, k_\perp)$  calculated in [18]. Accordingly, we obtain the evolution equation for  $\tilde{h}$ ,

$$\begin{aligned} \frac{\partial \tilde{h}(x_B, \mu^2)}{\partial \ln \mu^2} = & \frac{\alpha_s}{2\pi} \int \frac{dx dy}{x} \left\{ \tilde{h}(x) \delta(y-x) \left[ C_F \left( \frac{2z}{(1-z)_+} + 2\delta(1-z) \right) - \frac{C_A}{2} \frac{2z}{1-z} \right] \right. \\ & \left. + H_D(x, y) \left[ C_F \frac{2(x-y-x_B)}{y} + \frac{C_A}{2} \frac{2x_B(x_Bx + x_By - x^2 - y^2)}{(x_B-y)(x-y)y} \right] \right\} \quad (24) \end{aligned}$$

where the twist-three function  $H_D(x, y)$  has been introduced in the Ref. [17]. Similar to that of  $\tilde{g}(x)$ , the evolution of  $\tilde{h}$  depends on  $\tilde{h}$  and  $H_D$ .

In conclusion, we have derived the scale evolution for the transverse momentum dependent quark-gluon correlation functions associated with the four  $k_\perp$ -odd TMD quark distributions. We have performed our calculations in light-cone gauge with a particular boundary condition for the gauge potential, and we have checked that our results do not depend on these boundary conditions. Our result on the evolution of  $T_F(x)$  confirms recent calculations [25, 26]. The scale evolution for  $\tilde{g}$  and  $\tilde{h}$  reveals nontrivial QCD dynamics. We hope this will stimulate further theoretical studies.

Meanwhile, we notice that the scale evolution for the general twist-three operators have been calculated in the literature [23]. It will be interested to compare the evolution equations for the correlation functions studied in this paper with these well-known results. Especially, the evolution of the twist-three distribution  $g_T(x)$  and its contribution to semi-inclusive

processes deserve further investigations. We will address these issues in the forthcoming papers.

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