

UC San Diego

UC San Diego Electronic Theses and Dissertations

Title

Analysis of Acoustic Scattering from Seafloor Depressions

Permalink

<https://escholarship.org/uc/item/8k3011h5>

ISBN

9798191686387

Author

Doerner, Robert

Publication Date

2026-09-25

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA SAN DIEGO

Analysis of Acoustic Scattering from Seafloor Depressions

A Thesis submitted in partial satisfaction of the requirements
for the degree Master of Science

in

Engineering Sciences (Applied Ocean Science)

by

Robert Doerner

Committee in charge:

Professor Ying-Tsong Lin, Chair
Professor Andrew Lucas
Professor Eugene Pawlak

©

Robert Doerner, 2026

All rights reserved.

The Thesis of Robert Doerner is approved, and it is acceptable in quality and form for publication on microfilm and electronically.

University of California San Diego

2026

TABLE OF CONTENTS

THESIS APPROVAL PAGE	iii
TABLE OF CONTENTS	iv
LIST OF FIGURES	vii
ABSTRACT OF THE THESIS	ix
1 Introduction	1
1.1 Background	1
1.2 Literature Review	1
1.2.1 Seafloor Depressions	1
1.2.2 AUV-Based Observations and Regional Geoacoustic Context	2
1.2.3 Normal Modes, Coupled Modes, and Two-Way Parabolic Equations	4
1.2.4 Estimation and Parameterization of Sedimentary Properties	5
1.3 Present Knowledge Gap and Scope	5
2 Observation of a Seafloor Depression	7
2.1 Survey and Basic Data Processing	7
2.2 Observed Acoustic Features	9
2.3 Interpretation of Observations	11
3 Development of an Idealized Seafloor Depression Model	16
3.1 Incorporation of Observations	16
3.2 Normal Mode Formulation of the Idealized SFD Model	18
3.2.1 Complete Two-Edge Seafloor Depression Solution	22

4	Theoretical Analysis under the Coupled Normal Mode Framework	25
4.1	Effects of Perturbations to the Medium	25
4.2	Modal Susceptibility to the Seafloor Depression	30
4.3	Effects of Seafloor Depression Length	32
4.4	Far Field Limit	35
4.5	Short Seafloor Depression Limit	36
5	Numerical Analysis with Normal Mode Method	38
5.1	Development of a Normal Mode Program	38
5.1.1	General Information	38
5.1.2	Material-interface discretization	39
5.1.3	Eigenvalue and Eigenvector computation	42
5.1.4	Evaluation of the Range-Dependent Field	43
5.2	Primary Scenario	44
5.2.1	Description of Local Mode Basis	44
5.2.2	Structure of the Coupling Matrices	46
5.2.3	Incoherent full-field Scattering Response	49
5.2.4	Illustrative Full-Field Scattering Response	52
5.2.5	Comparison of Modeled and Observed Arrival Structure	54
5.3	Sub-Bottom Duct Scenario	56
5.3.1	Description of Local Mode Bases	57
5.3.2	Structure of the Coupling Matrices	58
5.3.3	Incoherent full-field Scattering Response	60
6	Conclusion	62
6.1	Discussion	62
6.2	Future Work	63
	APPENDICES	64

A Derivation of the Local Coupling Matrices	64
REFERENCES	66

LIST OF FIGURES

1.1	Context for Seafloor Depressions (SFDs). (a) Survey of the NESB with possible SFDs highlighted in red and green; courtesy of Doctor Jason Cheytor, USGS (private communication). (b) Pulse-compressed autonomous underwater vehicle towed array channel 0 record, showing a transect of one SFD.	2
2.1	Array processing techniques. (a) Pulse-compressed, coherently beamformed record. (b) Corresponding incoherently beamformed record. (c) Coherent and incoherent beampatterns. (d) Two-dimensional Fourier spectrum of the incoherent record; dashed lines delineate regions removed before reconstruction.	10
2.2	Filtered Data and simplified interaction model comparison	15
3.1	Idealized SFD geometry used in the numerical model.	17
5.1	Normal mode bases for the 250-Hz Primary scenario.	45
5.2	Normal mode bases for the 250-Hz Primary scenario Detail View.	46
5.3	Modal coupling at 250 Hz for the first 250 modes. (a) and (b) are the left-edge coupling matrices $\mathbf{C}_f^L$ and $\mathbf{C}_b^L$ shown in decibel scale, panels (d) and (e) are the corresponding right-edge matrices. (c) and (f) summarize the modal properties for incidence at each edge, including propagation angle, susceptibility and scattering. .	47
5.4	Broadband-smoothed full field.	51
5.5	Representative modal fields at 250 Hz.	52
5.6	Time domain reconstruction of the model results for transmit cycle 400 at selected pulse transit times. The source and array centroid locations are marked by green and magenta dots respectively.	53

5.7	Comparison of measured and modeled arrival structures. (a) Measured response from the REMUS 600 transect, pulse-compressed, incoherently beamformed, and directionally filtered. (b) Synthetic response evaluated at the moving towed-array centroid for the 250-Hz model.	54
5.8	Normal mode bases for the 250-Hz duct scenario.	57
5.9	Modal coupling for the duct scenario at 250 Hz for the first 250 modes. (a) and (b) are the left-edge coupling matrices $\mathbf{C}_f^L$ and $\mathbf{C}_b^L$ shown in decibel scale, (d) and (e) are the corresponding right-edge matrices. Panels (c) and (f) summarize the modal properties for incidence at each edge, including angle, susceptibility and scattering.	59
5.10	Duct-scenario broadband-smoothed full fields.	60
5.11	Representative duct-scenario modal fields at 250 Hz.	61

ABSTRACT OF THE THESIS

Analysis of Acoustic Scattering from Seafloor Depressions

by

Robert Doerner

Master of Science in Engineering Sciences (Applied Ocean Science)

University of California San Diego, 2026

Professor Ying-Tsong Lin, Chair

Acoustic observations collected by a REMUS 600 autonomous underwater vehicle on the New England Shelf Break show delayed, sloping, and diffuse returns associated with a seafloor depression and the disruption of otherwise persistent sub-bottom stratigraphy. To investigate mechanisms capable of producing the observed scattering features, a two-way coupled normal mode model is developed for an idealized seafloor depression with sloping edges and modified sediment structure. Modes with substantial support near the perturbed sub-bottom structure undergo the strongest redistribution, with strong non-adiabatic coupling including significant forward and backscattering across an array of output modes and angles. Water-column-confined modes generally interact

weakly with the leading edge, although they may undergo appreciable backscattering at the trailing bathymetric transition. Full-field calculations illustrate edge-generated forward and backscatter, strong coupling into sediment-associated propagation modes, and reradiation into the water column. Finally, comparison between the data recorded by the autonomous underwater vehicle and the model output confirms that the model has qualitatively demonstrated the mechanisms responsible for the patterns in the data.

CHAPTER 1

Introduction

1.1 Background

The present work investigates seafloor depressions on the New England Shelf Break (NESB), their interplay with the nearby stratigraphy, and the joint influence of the two on acoustic propagation. Figure 1.1a illustrates the distribution of seafloor depressions, or SFDs, identified within the NESB region. This image is provided by Dr. Jason Cheytor of USGS (private communication). Along the Southern New England portion of the U.S. Atlantic margin, these features are widespread; mapped depressions range from approximately 50 to 500 m in diameter and exhibit approximately 5–15 m of relief [1]. Three aspects of these features are particularly relevant to underwater acoustic propagation: the localized change in seafloor depth, the disruption of the prevailing sub-bottom structure, and the perturbation of sub bottom acoustic properties.

1.2 Literature Review

1.2.1 Seafloor Depressions

SFDs have been recorded in various continental shelf environments. They may be associated with past or present migration of subsurface fluids through unconsolidated marine sediments, though that has not been shown to be the case on the NESB. Their present morphology may also reflect erosion, bottom-current forcing, sediment transport, and partial infilling [2, 3]. Individual depressions range from meters to hundreds of meters in horizontal extent and may be accompanied by truncated or deformed sediment layers, disturbed material, and localized subsurface heterogene-

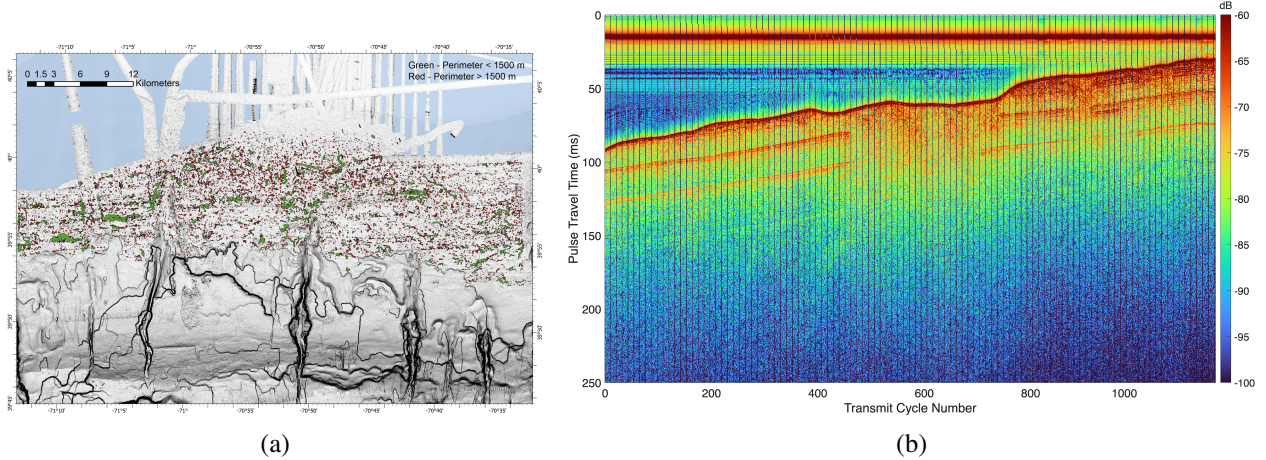


Figure 1.1: Context for Seafloor Depressions (SFDs). (a) Survey of the NESB with possible SFDs highlighted in red and green; courtesy of Doctor Jason Cheytor, USGS (private communication). (b) Pulse-compressed autonomous underwater vehicle towed array channel 0 record, showing a transect of one SFD.

ity [1,4]. These bathymetric and stratigraphic variations may introduce corresponding contrasts in sound speed, density, and attenuation.

Acoustic systems have been central to the discovery and characterization of SFDs. Side-scan and multibeam sonars resolve seabed morphology, while sub-bottom profilers reveal truncated or deformed stratigraphy and other subsurface structure [4,5]. Most of this literature uses sound primarily as a means of imaging the geological feature. Comparatively little attention has been given to the SFD as a component of the propagation environment itself.

1.2.2 AUV-Based Observations and Regional Geoacoustic Context

Autonomous underwater vehicles (AUVs) provide a means of collecting repeatable, close-to-bottom observations while carrying navigation, environmental, and acoustic instrumentation on a common platform. The REMUS 600 was developed as a modular, medium-depth AUV capable of supporting a range of scientific and survey payloads [6]. For acoustic experiments, such a platform can place a source, receiving array, and navigation sensors within a coordinated moving geometry. This permits dense sampling along a transect, although vehicle motion, uncertainty in the towed-array shape, and changing source–receiver geometry must be incorporated into the

subsequent processing and interpretation.

Previous geological surveys have identified numerous SFDs and laterally persistent sedimentary units along the southern New England margin [1]. Regional geoacoustic measurements provide additional context for the sediment properties encountered on the New England shelf and shelf break. Ballard et al. used an Acoustic Coring System to obtain in situ compressional-wave sound-speed profiles in the New England Mud Patch and Shelf Break regions. The analysis of Ballard et al. documented spatial and depth-dependent variation in sediment sound speed across these regions [7]. Core samples 18 through 20 provide measurements of bottom properties in the closest proximity to the site of the present study, though the nearest is approximately 5 km away and at least 100 meters up slope. These core samples document a sediment to water sound speed ratio that is between 1.04 and 1 within approximately 0.25 meters of the seabed, and between 1 and 0.96 otherwise. For a speed of sound of 1488 m/s in the water column, the deeper measurement corresponds to a sediment speed of sound of approximately 1460 m/s.

A complementary approach estimates sediment properties from the acoustic field itself. Knobles et al. inferred statistics of the sound-speed structure of fine-grained New England shelf sediment from sparse broadband measurements, illustrating both the information available in propagating fields and the uncertainty associated with an effective geoacoustic description [8]. More recently, Pfenninger and Lin developed an arrival-time-based Bayesian inversion for data collected using an AUV-mounted source and towed hydrophone array [9]. Multi-task Gaussian-process regression was used to estimate the mean and covariance of direct, seabed-reflected, and sub-bottom arrival times before Bayesian inversion. Their results show that arrival times can constrain range-dependent seabed properties, but also that sediment sound speed and layer thickness can be strongly coupled when the sub-bottom returns are variable. This tradeoff is especially relevant near an SFD, where disrupted stratigraphy and bathymetric range dependence weaken the assumptions of a locally range-independent inversion.

1.2.3 Normal Modes, Coupled Modes, and Two-Way Parabolic Equations

Normal mode theory represents the acoustic field in a range-independent waveguide as a sum of depth-dependent eigenfunctions, each having its own horizontal wavenumber and attenuation rate [10, 11]. This representation is particularly useful in a layered seabed because modes may be distinguished according to their interaction with the water column, sediment layers, and absorbing basement. Modes having little function amplitude $|\Psi(z)|$ near the seabed should be comparatively insensitive to a shallow perturbation, whereas sediment-interacting and leaky modes may be strongly affected by a change in bathymetry or sub-bottom structure.

In a range-dependent environment, the local modes and their horizontal wavenumbers vary with range. The adiabatic approximation assumes that each mode evolves independently, which can be appropriate for gradual environmental variation but may fail near rapid transitions [12]. Coupled-mode methods instead allow energy to transfer among the local modal bases. In a stepwise formulation, adjacent range-independent segments are connected by enforcing continuity of pressure and density-normalized normal velocity at each vertical interface between the segments [13, 14]. The resulting matrices describe forward transmission, conversion among different modes, and when both propagation directions are retained, backscatter.

Parabolic-equation methods provide an alternative treatment of range dependence by propagating a depth-dependent field on a spatial grid. Their standard one-way form is efficient for forward propagation but omits energy returned toward the source. Collins and Evans developed a two-way parabolic equation for ocean-acoustic backscattering [15], and Lingeitch et al. extended this approach to include repeated scattering by decomposing the field into outgoing and incoming components within successive range-independent regions [16]. Two-way PE and two-way coupled-mode methods can therefore both represent source-side returns and multiple interactions with a localized feature.

The coupled-mode formulation is adopted here because the redistribution of energy among physically meaningful modes is itself a primary diagnostic. The modal representation is more readily manipulated to find meaningful analytic results, and its coupling matrices expose which

incident modes interact with the SFD and where conversion into sediment-associated or backward-propagating components occurs. More detailed recreation of the data would be possible with a two-way PE model.

1.2.4 Estimation and Parameterization of Sedimentary Properties

Empirical relationships provide one means of estimating sediment properties that were not measured directly. Hamilton and Bachman compiled laboratory and in situ measurements of marine sediments and developed regression relationships among compressional-wave speed, wet bulk density, porosity, grain density, and grain size [17]. These relationships provide a practical basis for estimating the density associated with an acoustically inferred sediment speed.

Changes in porosity and density caused by burial or loading may instead be described using sediment-consolidation relationships. Burland developed an intrinsic compression framework for reconstituted clays in which the change in void ratio depends on effective stress and a normalized compression index [18]. O'Regan et al. applied this type of remolding and reconsolidation framework to seafloor cratering associated with fluid escape and demonstrated that sediment settlement can contribute appreciably to the resulting bathymetric relief [19].

Buckingham's viscous grain-shearing theory provides a complementary description of the acoustic consequences of sediment compaction [20]. In that model, the compressional and shear responses depend on porosity, grain properties, and the contact stiffness produced by effective overburden pressure. The theory therefore supplies a means of estimating how sound speed changes as a sediment is compacted, rather than prescribing an arbitrary sound-speed gradient.

1.3 Present Knowledge Gap and Scope

Acoustic systems have been used to image SFD morphology [4, 5]. Much less attention has been given to an SFD as a component of the propagation environment itself. A SFD may locally interrupt or modify sediment interacting propagation paths, potentially scattering energy between water-column-confined, sediment-interacting, leaky, and backward-propagating components. Nor-

mal mode theory provides a natural framework for examining these effects. The local acoustic field is represented as a sum of depth-dependent eigenfunctions with distinct horizontal wavenumbers and attenuation rates [10, 11]. In a range-dependent environment, the local modal basis changes with range; Coupled-mode methods connect the bases of adjacent segments by enforcing continuity of pressure and the density-normalized radial pressure gradient, thereby developing a solution which globally satisfies the governing equations and conditions [13, 14]. A two-way coupling formulation is required for the present problem because a SFD contains two edges and may generate appreciable complex scattering regimes involving multiple range-dependent interactions.

This work combines notable observations collected using an AUV with an idealized two-way coupled normal mode model. The observations are used to characterize delayed and backscattered returns associated with a large SFD and to identify the bathymetric and stratigraphic features that motivate the numerical environment. Modal susceptibility and scattering diagnostics are then used to determine which modal families are most susceptible to the SFD. Full-field calculations illustrate how the matrix-level coupling mechanisms appear in the resulting acoustic field. The field data were collected over a substantially higher frequency band than that used in the numerical model, and the modeled sub-bottom is deliberately idealized. The comparison therefore concerns broad features—including the interruption of otherwise available sedimentborne paths, and associated scattering of sound incident upon the SFD; but direct model to data comparison shows that the numerical model effectively captures the effects of interest.

CHAPTER 2

Observation of a Seafloor Depression and the Surrounding Seafloor Using a REMUS 600 AUV

The purpose of this chapter is first to characterize the geometry, features, and acoustic scale of the observed seafloor depression (SFD) environment, second, to establish that the sub-bottom structure and acoustic arrivals are sufficiently well resolved to motivate the coupled-mode analysis developed in later chapters, and third, to provide an estimate of the material properties of the feature for numerical modeling.

2.1 Survey and Basic Data Processing

In 2023, the NESB was surveyed using a REMUS 600 AUV operated by the Woods Hole Oceanographic Institution. The vehicle was equipped with an acoustic projector, an inertial measurement unit, an acoustic Doppler Velocity Logger (DVL) capable of providing bottom referenced navigation data, a 16-channel towed hydrophone array, and an AUV body mounted monitoring hydrophone. The source transmitted a 2.5–4.5-kHz linear frequency-modulated sweep with a 50-ms pulse duration and repetition rate of one transmission per second. The relevant survey mission was conducted near 39°54.7' N, 70°56.3' W. The transect considered here is approximately 2 km long and crosses an isolated SFD while the vehicle maintains a depth of approximately 475 m and a seabed-referenced ground speed of 1.36 m s^{-1} . The survey geometry and selected transect are shown in Figure 1.1a.

The recorded signal on each hydrophone was cross-correlated with the source-signal waveform to estimate the band-limited impulse response of the environment. The resulting data were

arranged by transmission number and relative travel time, allowing the direct arrival, seafloor return, and later sub-bottom arrivals to be tracked along the transect. A survey of the NESB showing estimated SFD distribution and the cross-correlation record of Channel 0 are shown in Figure 1.1b.

Towed-array stability was verified from the direct and bottom-reflected arrivals using range-dependent ray tracing. At each transmit cycle, a least-squares fit was then used to estimate the array droop angle by jointly minimizing the forward and downward position residuals of all channels, assuming that the array formed a straight line attached to the AUV near the source location. The estimated droop angle ranged from approximately -1° to 3° , indicating the array was nearly horizontal and relatively stable, thus supporting array processing techniques. During most of the transect, the reconstructed hydrophone positions agreed with the fitted geometry to within an approximately 0.1-m root-mean-square error. Intervals of elevated fit error contained little differential arrival-time information across the array and may additionally reflect differences between the portions of the seafloor sampled by the DVL and the acoustic source. The DVL operates at a substantially higher frequency and consequently samples a smaller effective Fresnel zone, whereas the 2.5–4.5-kHz acoustic signal interacts with a broader area of the range-dependent seafloor [11]. This mismatch is most likely to become important where the bathymetry varies rapidly, such as near the sloping SFD edges.

Coherent and incoherent near-field delay-and-sum beamforming were applied to the channelwise cross-correlation functions using steering delays for a spherical incident wave, following standard array-processing methods [21]. For cross-correlation function $C_q(t)$ on channel q and focusing delay Δt_q , the coherent and incoherent outputs were computed in the forms

$$B_{\text{coh}}(t) = \left| \sum_{q=1}^{N_h} C_q(t + \Delta t_q) \right|, \quad (2.1)$$

$$B_{\text{inc}}(t) = \left[\sum_{q=1}^{N_h} |C_q(t + \Delta t_q)|^2 \right]^{1/2}, \quad (2.2)$$

where $N_h = 16$ denotes the number of hydrophone channels. The coherent sum retains interchannel

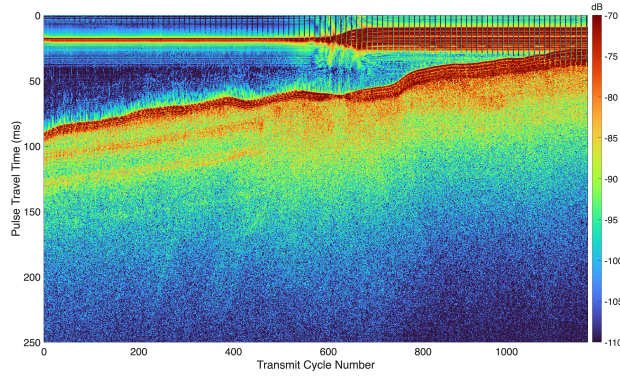
phase and therefore provides greater gain for arrivals that match the assumed steering model. The incoherent sum does not require interchannel phase consistency and is consequently useful for identifying extended, diffuse, or imperfectly modeled returns. The center-focused coherent and incoherent records are shown in Figure 2.1a and b, respectively. Representative normalized coherent and incoherent array responses are shown in Figure 2.1c.

Incoherent beamforming provided greater enhancement of several weak delayed features, whereas coherent beamforming produced stronger suppression of the local background surrounding the direct and seafloor arrivals. Both methods exhibit a mainbeam width of approximately 30° and no grating lobes; the incoherent method suppresses signals outside the main lobe by approximately -12 dB from peak; while the coherent method does so by approximately -24 dB from peak.

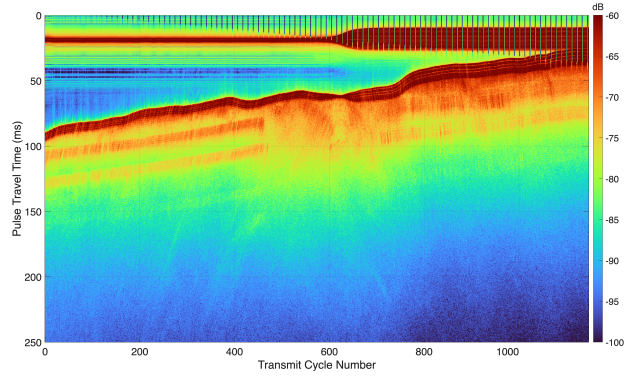
After the data was beamformed, weak features of interest were further enhanced using a directionally selective filter. The two dimensional Fourier spectrum was computed, and portions of the spectrum associated with stationary arrival structure, the 0 Hz transmit cycle bin, and with the AUV's missed transmit cycles were masked. The spectrum was re-synthesised into a beamformed, filtered, record. The array's beampattern, along with center-focused coherent and incoherent results and the two-dimensional Fourier spectrum with masked regions demarcated are shown in Figure 2.1. The re-synthesized, beamformed, filtered result is shown in Figure 5.7.

2.2 Observed Acoustic Features

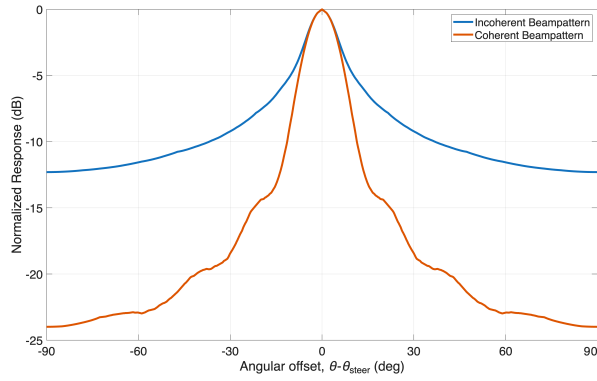
The unbeamformed cross-correlation data contain a strong direct arrival followed by the seafloor reflection and several later features. Two comparatively persistent sub-bottom returns occur approximately 20 and 40 ms after the seafloor return. These arrivals track the surrounding bathymetry but disappear near the leading edge of the SFD and reappear near its trailing edge. Their behavior is consistent with reflections from laterally continuous sub-bottom interfaces that are disrupted beneath the depression. The disappearance and reappearance of these returns define an SFD approximately 450 m in horizontal extent. The depression is approximately 5–10 m deeper than the surrounding seafloor, and its edges locally slope at approximately 20° relative to the



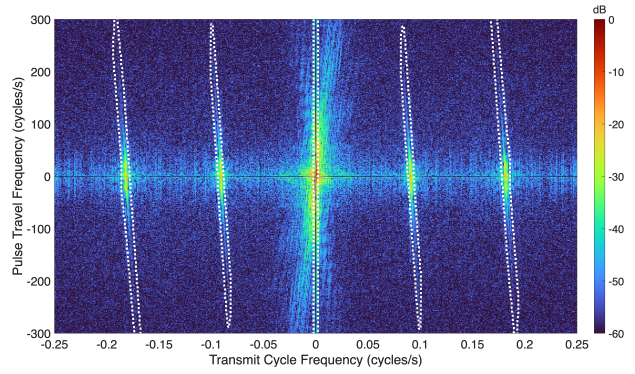
(a) Pulse-compressed and coherently beamformed record.



(b) Pulse-compressed and incoherently beamformed record.



(c) Normalized coherent and incoherent array responses.



(d) Two-dimensional spectrum and slope-selective regions.

Figure 2.1: Array processing techniques. (a) Pulse-compressed, coherently beamformed record. (b) Corresponding incoherently beamformed record. (c) Coherent and incoherent beampatterns. (d) Two-dimensional Fourier spectrum of the incoherent record; dashed lines delineate regions removed before reconstruction.

prevailing bathymetry. The observations therefore indicate that the feature combines an abrupt bathymetric depression with a loss of the well-defined stratigraphy present on either side.

The incoherent beamformed record is shown in Figure 2.1(b). When the array is focused beneath the central portion of the depression, incoherent beamforming reveals delayed sloping returns extending approximately 75–200 ms beyond the seafloor arrival. Their apparent slope differs substantially from that of the local seafloor reflection. In interval in which the persistent sub-bottom returns are absent, the received field contains more diffuse patches and steeply sloping strands.

Coherent beamforming retains some of the later-arriving structures but, several of these delayed features are less strongly reinforced by coherent than by incoherent beamforming. This indicates substantial phase variation across the array, consistent with non-specular scattering from sub-bottom structure, inaccuracy in the localization of array elements, or incorrect selection of steering direction. The coherent representation of the same interval is shown in Figure 2.1(a).

2.3 Interpretation of Observations

Two simplified ray models were overlaid on the measured travel-time images to aid interpretation of the observed arrivals. The first model represents incident energy that enters the sediment, propagates toward a fixed scattering feature, such as the SFD edge, as a guided wave, and then returns to the towed array along a refracted geometric path. The second model represents two independent geometric ray paths: a refracted path from the source to a fixed location which scatters the ray and a different refracted path to the towed array.

Both models assume locally planar bathymetry, homogeneous water and sediment sound speeds $c_{w,i}$ and c_s , and a fixed scattering event at along-track position x_{sc} and burial depth H . The offset T_0 accounts for timing not included in the geometric paths. For any refracted leg, let h be the endpoint height above the seafloor, D its separation from the scatterer parallel to the seafloor, and

θ_w and θ_s the water and sediment grazing angles. The refracted-leg model is

$$\alpha = \frac{\cos \theta_w}{c_{w,i}} = \frac{\cos \theta_s}{c_s}, \quad (2.3)$$

$$D = h \frac{\alpha c_{w,i}}{\sqrt{1 - (\alpha c_{w,i})^2}} + H \frac{\alpha c_s}{\sqrt{1 - (\alpha c_s)^2}}, \quad (2.4)$$

$$\tau_i^{\text{ref}}(D, h, H) = \frac{h}{c_{w,i} \sqrt{1 - (\alpha c_{w,i})^2}} + \frac{H}{c_s \sqrt{1 - (\alpha c_s)^2}}. \quad (2.5)$$

For each leg, Eq. (2.4) is solved numerically for α before evaluating the travel time.

The first model, shown diagrammatically in Figure 2.2c, incident wave is assumed to enter the sediment a fixed distance Δ_E ahead of the source, propagate to the scatterer with effective group speed c_g , and return to the array along a refracted ray. For a source grazing angle θ_E specified at reference transmission r ,

$$\Delta_E = h_{S,r} \cot \theta_E, \quad x_{E,i} = x_{S,i} + \Delta_E. \quad (2.6)$$

The modeled travel time is

$$\begin{aligned} T_i^{(g)} = T_0 &+ \frac{\sqrt{h_{S,i}^2 + \Delta_E^2}}{c_{w,i}} + \frac{x_{\text{sc}} - x_{S,i} - \Delta_E}{c_g} \\ &+ \tau_i^{\text{ref}}(x_{\text{sc}} - x_{A,i}, h_{A,i}, H). \end{aligned} \quad (2.7)$$

This path is associated with the diagonal return that becomes visible roughly 200 m before the identifiable layer traces terminate. The apparent slope of the modeled arrival is controlled principally by the AUV ground speed, the entry and exit geometry, and the horizontal propagation speed within the sediment layer. Figure 2.2b shows a plan view of the model and compares the filtered record with its predicted arrival time assuming a mean horizontal velocity of 1660 m/s.

A second group of returns becomes prominent closer to the layer-trace termination. These returns initially slope upward, then curve and merge before losing strength where the identifiable stratigraphy ends. They are likewise consistent with backscattered paths involving one or both

finite sediment layers. The approximately linear portions can be reproduced by the same guided-path model, whereas the strongest curvature occurs where the seafloor and inferred layer geometry change most rapidly. This correspondence suggests that the curvature is produced partly by the changing material properties and ray geometry near the SFD edge.

The second model uses independent refracted rays from the source to the scatterer and from the scatterer to the array. A diagram is shown in Figure 2.2d For the linear bathymetry $z_b(x) = mx + b$, define

$$s(x, z) = \frac{x + mz}{\sqrt{1 + m^2}}, \quad h^\perp(x, z) = \frac{z_b(x) - z}{\sqrt{1 + m^2}}, \quad (2.8)$$

and let

$$\begin{aligned} s_{sc} &= s(x_{sc}, z_b(x_{sc})), \\ D_{S,i} &= s_{sc} - s(x_{S,i}, z_{S,i}), & h_{S,i}^\perp &= h^\perp(x_{S,i}, z_{S,i}), \\ D_{A,i} &= s_{sc} - s(x_{A,i}, z_{A,i}), & h_{A,i}^\perp &= h^\perp(x_{A,i}, z_{A,i}). \end{aligned} \quad (2.9)$$

The modeled travel time is

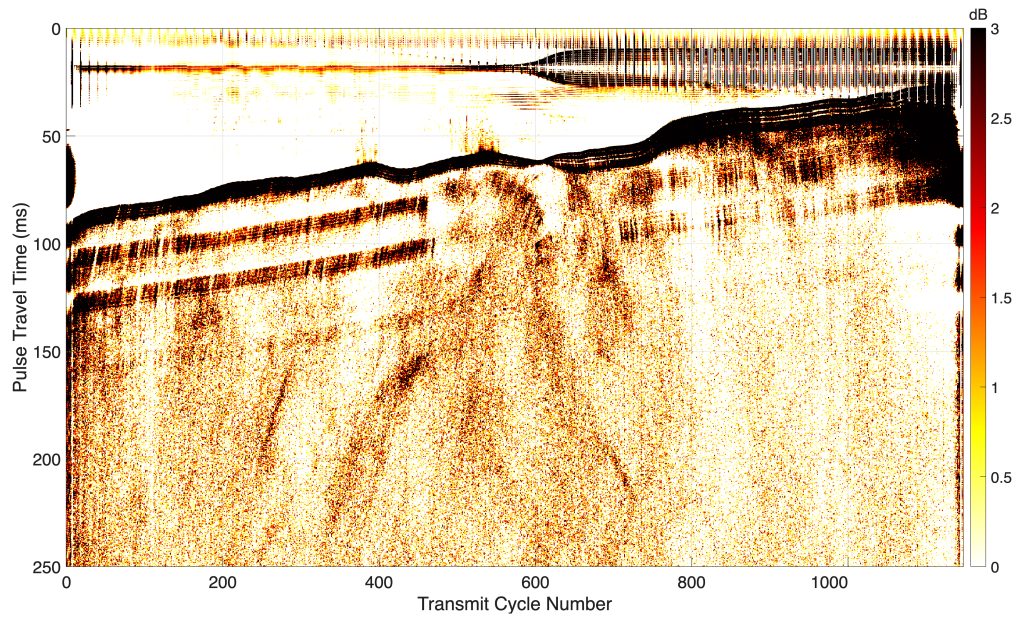
$$T_i^{(r)} = T_0 + \tau_i^{\text{ref}}(D_{S,i}, h_{S,i}^\perp, H) + \tau_i^{\text{ref}}(D_{A,i}, h_{A,i}^\perp, H). \quad (2.10)$$

The two refracted legs generally have different ray parameters $\alpha_{S,i}$ and $\alpha_{A,i}$.

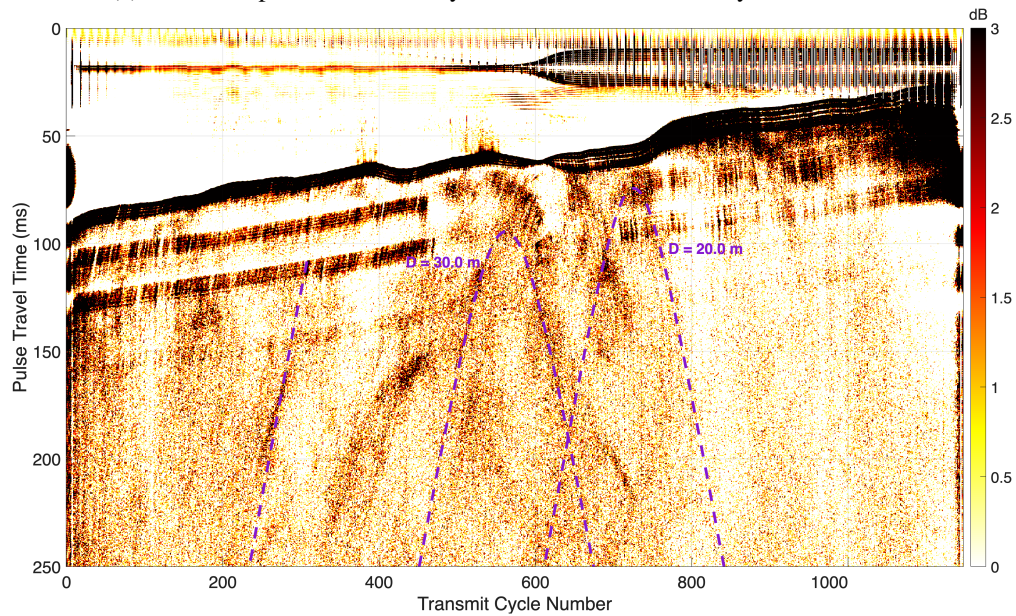
The diffuse returns beneath the SFD interior admit interpretation under this model. The resulting trajectory has its minimum travel time near the AUV's closest approach to the scattering location, while its curvature depends primarily on the interaction depth, sediment sound speed, and AUV ground speed. The acoustic data do not determine whether such an inhomogeneity represents compacted sediment, gas-bearing material, or another source of local impedance contrast. Figure 2.2b show a plan view of this model and compares the filtered record with two candidate arrival trajectories having assumed interaction with a sub-bottom scattering inducing structure, at

depths of 20 and 30 m below the seafloor with a uniform sedimentary sound speed of 1650 m/s.

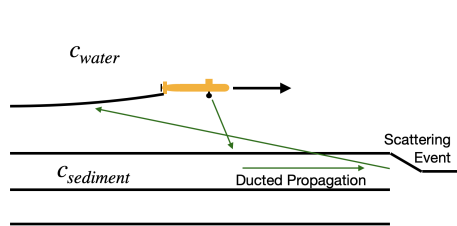
Agreement with the observed trajectories can be obtained over a range of parameter combinations. The kinematic overlays are broadly consistent with sediment propagation speeds of approximately $1400\text{--}1900\text{ m s}^{-1}$, and scattering interaction depths tens of meters below the seafloor. These models therefore establish the plausibility of sediment-associated and edge-backscattered propagation paths but do not provide unique estimates of the environment. The observations establish three features relevant to the analysis contained in this Thesis: persistent sediment-associated arrivals are interrupted within the SFD, delayed returns are consistent with backscattering from its sloping edges, and diffuse returns beneath its interior may result from localized material inhomogeneity. The coupled-mode model developed in Chapter 3 investigates mechanisms capable of producing these features.



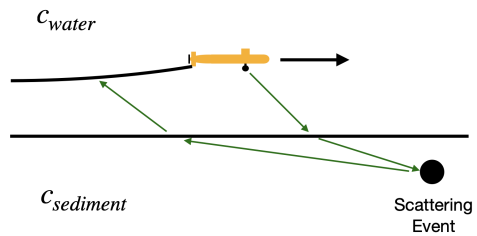
(a) Pulse-compressed, coherently beamformed, directionally filtered record.



(b) Pulse-compressed, coherently beamformed, directionally filtered record, with simplified bottom interaction model predicted travel times overlain.



(c) Plan diagram of scattering with pockmark edge.



(d) Plan diagram of scattering with heterogeneity under the sediment.

Figure 2.2: Filtered Data and simplified interaction model comparison

CHAPTER 3

Development of an Idealized Seafloor Depression Model

The environments developed in this Chapter are intended to investigate the acoustic mechanisms suggested by the observations rather than to reproduce the measured field directly. The simplifications employed preserve the principal elements of interest—a layered exterior waveguide, disruption of the stratigraphy beneath the seafloor depression (SFD), sloping SFD edges, and two-way conversion among water-column and sediment-interacting propagation paths—while permitting the individual scattering mechanisms to be examined directly. Consequently, the model permits a mechanistic study of SFD-induced scattering rather than as a quantitative prediction of the measured REMUS 600 field.

3.1 Incorporation of Observations

An idealized model of the seafloor and sub-bottom stratigraphy was developed to investigate the scattering mechanisms suggested by the observations. The model assumes that the material beneath the SFD is composed of the same general constituents as the surrounding seabed, although their distribution and fine-scale structure may have been altered by the SFD formation process.

The measurements of Ballard et al., summarized in Chapter 1, provide the best available direct constraint on the immediate seabed near the experiment, but their applicability to the propagation model is limited by both penetration depth and measurement location. The profiles characterize only the upper ~ 2 m of sediment and were approximately 5 km from the SFD of interest. They therefore do not directly determine the compressional-wave speed, density, or attenuation of the deeper layers associated with the SFD.

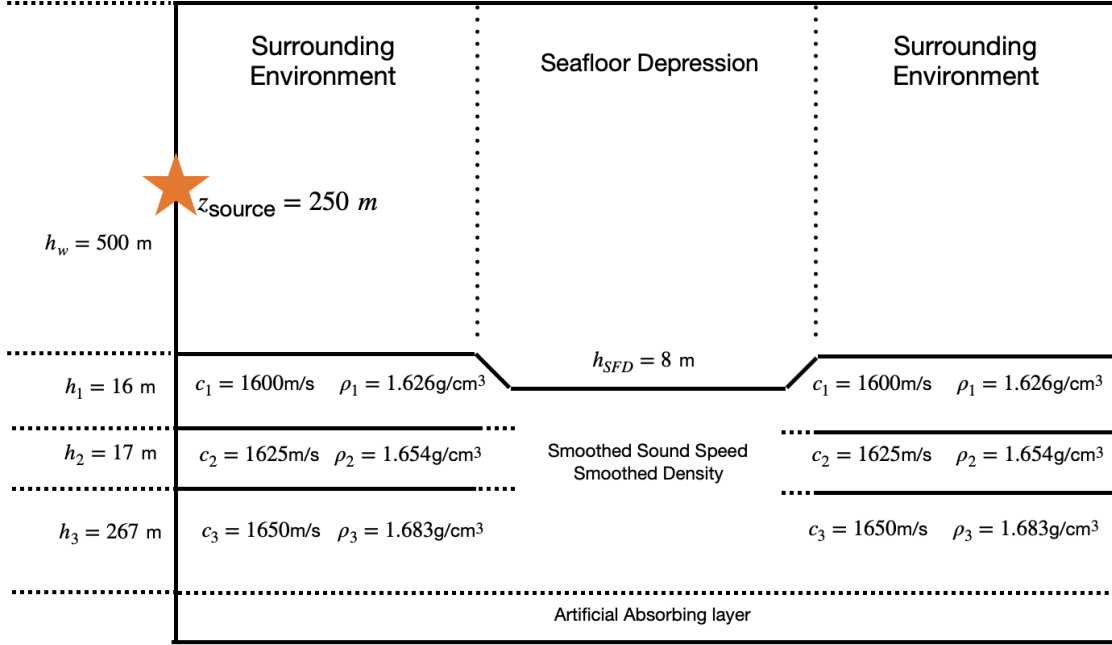


Figure 3.1: Idealized SFD geometry used in the numerical model.

Because the existing direct observations do not determine the geoacoustic structure sufficiently, representative values of compressional wave speed and layer thickness drawn from the simple ray model overlays are used to define the surrounding environment. The density of the undisturbed sediment is first estimated from this inversion-derived sound-speed profile using Hamilton's empirical velocity–density relationship for shallow marine sediment [22]:

$$\rho(z) = 1.135 \left(\frac{c_p(z)}{1000} \right) - 0.190, \quad (3.1)$$

Where $c_p(z)$ is the compressional wave speed as a function of depth.

The idealized geometry is shown in Figure 3.1. The model consists of identical surrounding source and receiver environments separated by an SFD interior and two sloping transition regions. The water column is 500 m deep with a homogeneous sound speed of 1488 m/s. The surrounding environment contains two sub-bottom layers, along with a basement which extends into an absorbing

region. The first of these layers has a uniform sound speed and density of 1600 m/s and 1.626 g/cm^3 , respectively, while the second layer has 1625 m/s and 1.654 g/cm^3 , and the basement 1650 m/s and 1.683 g/cm^3 . The upper layer has a thickness of 16 m, and the lower a thickness of 17 m. The SFD interior is 475 m across and its seafloor is 8 m deeper than the surrounding seafloor. A diagram of the model is shown in Figure 3.1. The SFD sub-bottom sound-speed and density profiles are obtained by smoothing the neighboring environment and are plotted in Figure 5.2.

Each edge is represented by many thin range-independent segments, each approximately 2.5 meters long and individually shorter than an acoustic wavelength. This ensures that each is small enough that it cannot be independently resolved by the frequencies in the model, and thus the error introduced by horizontal discretization of the domain is minimized. This progression of layers creates a seafloor that follows a 20° slope. The sub-bottom sound speed and density profiles are derived by progressively smoothing the neighboring environment providing a smooth lateral transition from surrounding environment to SFD interior. Localized sub-bottom heterogeneity is represented by spherical scatterers with randomized positions and sizes. The calculation assigns them a mean radius of 6 m, a sound speed of 2000 m/s, and a density of 2.0 g cm^{-3} . These are illustrative parameters rather than estimates of the observed inhomogeneities.

3.2 Normal Mode Formulation of the Idealized SFD Model

Consider the idealized environment described in section 3.1, in which the source and receiver regions have identical normal modes and modal wavenumbers. In Appendix A, we have carried out the standard stepwise coupled-mode formulation [11, 13, 14], enforcing the continuity of pressure and density normalized radial pressure gradient at interface j ; which gives two density-weighted

modal-overlap maps, $\widetilde{\mathbf{C}}^j$ and $\widehat{\mathbf{C}}^j$, with elements

$$\begin{aligned}\widetilde{C}_{\ell m}^j &= \int_0^\infty \frac{\Psi_\ell^{j+1}(z)\Psi_m^j(z)}{\rho_{j+1}(z)} dz, \\ \widehat{C}_{\ell m}^j &= \frac{k_{r,m}^j}{k_{r,\ell}^{j+1}} \int_0^\infty \frac{\Psi_\ell^{j+1}(z)\Psi_m^j(z)}{\rho_j(z)} dz.\end{aligned}\tag{3.2}$$

Where Ψ_ℓ^j is the normalized eigenfunction, ρ_j is the density, and $k_{r,\ell}^j$ is the modal radial wavenumber, equal to the square root of the eigenvalue which has a positive imaginary component, for mode ℓ in segment j . Half their difference,

$$\mathbf{C}_b^j = \frac{1}{2} (\widetilde{\mathbf{C}}^j - \widehat{\mathbf{C}}^j)\tag{3.3}$$

is the back-scattering induced by the simultaneous satisfaction of the interface conditions. This is termed the *backward coupling* matrix, it gives the unit amplitudes of the modes scattered back into the first segment with opposite propagation direction. Propagation into the adjacent segment with preserved direction is given by the projection of pressure into the next segment modified by $\mathbf{C}_b^j$,

$$\mathbf{C}_f^j = \widetilde{\mathbf{C}}^j - \mathbf{C}_b^j\tag{3.4}$$

which is called the *forward coupling* matrix.

$$\begin{bmatrix} \mathbf{a}^{j+1} \\ \mathbf{b}^{j+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{C}_f^j \mathbf{H}_1^j & \mathbf{C}_b^j \\ (\mathbf{H}_2^{j+1})^{-1} \mathbf{C}_b^j \mathbf{H}_1^j & (\mathbf{H}_2^{j+1})^{-1} \mathbf{C}_f^j \end{bmatrix}}_{\mathbf{R}^j} \begin{bmatrix} \mathbf{a}^j \\ \mathbf{b}^j \end{bmatrix}.\tag{3.5}$$

where each $\mathbf{a}^j$ and $\mathbf{b}^j$ are outgoing and incoming modal amplitudes, and $\mathbf{H}_n$ is a diagonal matrix of Hankel-function ratios evaluated at the radial bounds of a region.

In order to more accurately represent the SFD, we construct a symmetric feature with many small, locally range-independent segments connected by $2N$ interfaces. The length of each local

computational region is $\frac{1}{2}$ of the shortest acoustic wavelength utilized for any part of the model which needs to represent variation; this ensures that the defects introduced by discretization are smaller than can be resolved by the signal being studied. Interfaces indexed from 1 to N compose the left edge of the SFD, and those from $N + 1$ to $2N$ compose the right edge. The symmetry is such that interface j and interface $2N - j + 1$ connect segments with exactly reversed material properties. Let:

$$\mathcal{K}^j = \text{diag} (k^j, -k^j) \quad \mathbb{C}^j = \begin{bmatrix} \mathbf{C}_f^j & \mathbf{C}_b^j \\ \mathbf{C}_b^j & \mathbf{C}_f^j \end{bmatrix}. \quad (3.6)$$

Then block interface operators for an interface j may be written in terms of their symmetric pair j' :

$$\mathbb{C}^{j'} = (\mathcal{K}^j)^{-1} (\mathbb{C}^j)^T \mathcal{K}^{j+1} \quad (3.7)$$

where the superscript T denotes a standard transpose without complex conjugation, though its operands may be complex. The block form demonstrates a compact relationship between interfaces with reciprocal properties. Further, if the orthonormality of the modal basis is perfect and the underlying matrix is square, then $\mathbb{C}^j$ and $\mathbb{C}^{j'}$ are exactly each other's inverses. Following equation (3.5), each interface has a block transfer matrix of the form:

$$\mathbb{R}^j = \mathcal{H}_2^{j+1} \mathbb{C}^j \mathcal{H}_1^j \quad (3.8)$$

where

$$\mathcal{H}_1^j = \begin{bmatrix} \mathbf{H}_1^j & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \quad \mathcal{H}_2^j = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & (\mathbf{H}_2^j)^{-1} \end{bmatrix} \quad \mathcal{H}^j = \mathcal{H}_1^j \mathcal{H}_2^j = \mathcal{H}_2^j \mathcal{H}_1^j = \begin{bmatrix} \mathbf{H}_1^j & \mathbf{0} \\ \mathbf{0} & (\mathbf{H}_2^j)^{-1} \end{bmatrix} \quad (3.9)$$

The transfer operator associated with an arbitrary sequence of N interfaces on either side of a

symmetrically modeled feature is

$$\mathcal{R}^L = \prod_{j=1}^N \mathbb{R}^j = \prod_{j=1}^N \mathcal{H}_2^{j+1} \mathbb{C}^j \mathcal{H}_1^j = \mathcal{H}_2^{N+1} \prod_{j=2}^N [\mathbb{C}^j \mathcal{H}^j] \mathbb{C}^1 \mathcal{H}_1^1 \quad (3.10)$$

$$\mathcal{R}^R = \prod_{j=N+1}^{2N} \mathbb{R}^j = \prod_{j=N+1}^{2N} \mathcal{H}_2^{j+1} \mathbb{C}^j \mathcal{H}_1^j = \mathcal{H}_2^{2N+1} \prod_{j=N+2}^{2N} [\mathbb{C}^j \mathcal{H}^j] \mathbb{C}^{N+1} \mathcal{H}_1^{N+1} \quad (3.11)$$

For the left and right sides of the feature, respectively. Here the ordered product is defined as:

$$\prod_{j=a}^b A_j = A_b A_{b-1} \cdots A_a, \quad b > a.$$

As defined, $\mathcal{R}$ comprises the phase advance to reach the composite feature, then all coupling and phase advance inside the feature. The quantity primarily of interest here is what occurs inside the feature itself, neglecting the phase advance to reach it. Therefore:

$$\mathcal{R}^L = \mathcal{H}_2^{N+1} \mathbb{C}^L \mathcal{H}_1^1 \quad \mathcal{R}^R = \mathcal{H}_2^{2N+1} \mathbb{C}^R \mathcal{H}_1^{N+1} \quad (3.12)$$

$$\mathbb{C}^L = \prod_{j=2}^N [\mathbb{C}^j \mathcal{H}^j] \mathbb{C}^1 = \begin{bmatrix} \mathbf{C}_{ff}^L & \mathbf{C}_{fb}^L \\ \mathbf{C}_{bf}^L & \mathbf{C}_{bb}^L \end{bmatrix} \quad \mathbb{C}^R = \prod_{j=N+2}^{2N} [\mathbb{C}^j \mathcal{H}^j] \mathbb{C}^{N+1} = \begin{bmatrix} \mathbf{C}_{ff}^R & \mathbf{C}_{fb}^R \\ \mathbf{C}_{bf}^R & \mathbf{C}_{bb}^R \end{bmatrix} \quad (3.13)$$

where the blocks $\mathbf{C}_d^s$ represent the coupling that occurs on side s in direction d . Factoring the endpoint propagation matrices from these ordered products isolates the intrinsic composite-edge operators $\mathbb{C}^L$ and $\mathbb{C}^R$. Each composite operator contains all modal conversions at the many weak interfaces together with the phase accumulated in the thin segments between them. A gradual sloping edge can therefore be treated downstream as a single effective operator without losing the internal ordering of its constituent coupling and propagation events. Substituting the mirrored-interface relation into the right-side operator gives

$$\mathbb{C}^R = \prod_{j=N+2}^{2N} [\mathbb{C}^j \mathcal{H}^j] \mathbb{C}^{N+1} \quad (3.14)$$

$$= \prod_{j=N+2}^{2N} \left[\left(\mathcal{K}^{j+1} \right)^{-1} \left(\mathbb{C}^{j'} \right)^T \mathcal{K}^j \mathcal{H}^j \right] \left(\mathcal{K}^{N+2} \right)^{-1} \left(\mathbb{C}^N \right)^T \mathcal{K}^{N+1} \quad (3.15)$$

$$= \left(\mathcal{K}^{2N+1} \right)^{-1} \prod_{j=N+2}^{2N} \left[\left(\mathbb{C}^{j'} \right)^T \mathcal{H}^j \right] \left(\mathbb{C}^N \right)^T \mathcal{K}^{N+1} \quad (3.16)$$

$$= \left(\mathcal{K}^{2N+1} \right)^{-1} \left(\mathbb{C}^N \prod_{j=2N}^{N+2} [\mathcal{H}^j \mathbb{C}^{j'}] \right)^T \mathcal{K}^{N+1} \quad (3.17)$$

Because the feature is constructed symmetrically about its centerline, $\mathbb{C}^{j'} = \mathbb{C}^{2N-j+1}$, $\mathcal{H}^j = \mathcal{H}^{j'+1}$ and $\mathcal{K}^{2N+1} = \mathcal{K}^1$, thus:

$$\mathbb{C}^R = \left(\mathcal{K}^{2N+1} \right)^{-1} \left(\prod_{j=2}^N [\mathbb{C}^j \mathcal{H}^j] \mathbb{C}^1 \right)^T \mathcal{K}^{N+1} \quad (3.18)$$

$$= \left(\mathcal{K}^{2N+1} \right)^{-1} \left(\mathbb{C}^L \right)^T \mathcal{K}^{N+1} \quad (3.19)$$

$$= \left(\mathcal{K}^1 \right)^{-1} \left(\mathbb{C}^L \right)^T \mathcal{K}^{N+1} \quad (3.20)$$

The wavenumber factors and transpose symmetry prove that the entire right edge is the weighted signed transpose of the entire left edge, regardless of how many thin segments are used to represent the slope. It is then possible to derive a three-region solution which integrates complex, gradual transitions, which may be expressed in a form algebraically identical to one with simple interfaces.

3.2.1 Complete Two-Edge Seafloor Depression Solution

Collecting the source-, interior-, and receiver-region propagation factors as

$$\mathcal{H}_s = \mathcal{H}_1^1, \quad \mathcal{H}_p = \mathcal{H}_1^{N+1} \mathcal{H}_2^{N+1}, \quad \mathcal{H}_r = \mathcal{H}_2^{2N+1}. \quad (3.21)$$

where superscripts s , p , and r denote the source, SFD, and receiver regions, respectively. The complete two-edge SFD operator is

$$\begin{bmatrix} \mathbf{a}^r \\ \mathbf{b}^r \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{a}^s \\ \mathbf{b}^s \end{bmatrix}, \quad \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} = \mathcal{H}_r \mathbb{C}^R \mathcal{H}_p \mathbb{C}^L \mathcal{H}_s \quad (3.22)$$

Multiplying the block operators in Equation (3.22) from right to left gives

$$\begin{aligned} \mathbf{P}_{11} &= \left[\mathbf{C}_{ff}^R \mathbf{H}_1^p \mathbf{C}_{ff}^L + \mathbf{C}_{bf}^R (\mathbf{H}_2^p)^{-1} \mathbf{C}_{bb}^L \right] \mathbf{H}_1^s, \\ \mathbf{P}_{12} &= \mathbf{C}_{ff}^R \mathbf{H}_1^p \mathbf{C}_{bf}^L + \mathbf{C}_{bf}^R (\mathbf{H}_2^p)^{-1} \mathbf{C}_{fb}^L, \\ \mathbf{P}_{21} &= (\mathbf{H}_2^r)^{-1} \left[\mathbf{C}_{bb}^R \mathbf{H}_1^p \mathbf{C}_{ff}^L + \mathbf{C}_{fb}^R (\mathbf{H}_2^p)^{-1} \mathbf{C}_{bb}^L \right] \mathbf{H}_1^s, \\ \mathbf{P}_{22} &= (\mathbf{H}_2^r)^{-1} \left[\mathbf{C}_{bb}^R \mathbf{H}_1^p \mathbf{C}_{bf}^L + \mathbf{C}_{fb}^R (\mathbf{H}_2^p)^{-1} \mathbf{C}_{fb}^L \right]. \end{aligned} \quad (3.23)$$

Each block therefore contains one path through the outward-directed interior propagator $\mathbf{H}_1^p$ and one path through the inward-directed propagator $(\mathbf{H}_2^p)^{-1}$. The radiation condition requires that no field be incident from the receiver side, so $\mathbf{b}^r = \mathbf{0}$. The lower row of the complete block system is therefore

$$\mathbf{0} = \mathbf{P}_{21} \mathbf{a}^s + \mathbf{P}_{22} \mathbf{b}^s. \quad (3.24)$$

Assuming that $\mathbf{P}_{22}$ is invertible, the source-side reflected amplitudes are

$$\begin{aligned} \mathbf{b}^s &= -\mathbf{P}_{22}^{-1} \mathbf{P}_{21} \mathbf{a}^s \\ &\equiv \mathbf{R}_{\text{SFD}} \mathbf{a}^s. \end{aligned} \quad (3.25)$$

Substitution into the upper row gives

$$\begin{aligned}
\mathbf{a}^r &= \mathbf{P}_{11}\mathbf{a}^s + \mathbf{P}_{12}\mathbf{b}^s \\
&= \mathbf{P}_{11}\mathbf{a}^s - \mathbf{P}_{12}\mathbf{P}_{22}^{-1}\mathbf{P}_{21}\mathbf{a}^s \\
&= \left(\mathbf{P}_{11} - \mathbf{P}_{12}\mathbf{P}_{22}^{-1}\mathbf{P}_{21} \right) \mathbf{a}^s \\
&\equiv \mathbf{T}_{\text{SFD}}\mathbf{a}^s.
\end{aligned} \tag{3.26}$$

Thus $\mathbf{R}_{\text{SFD}}$ is the exact source-side reflection operator, while $\mathbf{T}_{\text{SFD}}$ is the Schur complement of $\mathbf{P}_{22}$ and gives the exact receiver side transmission operator, including the effects of an arbitrary number of backscatter interactions. The first term in $\mathbf{T}_{\text{SFD}}$ represents direct forward transmission, whereas the Schur-complement correction contains pathways that enter the backward channel and subsequently return to the forward field.

CHAPTER 4

Theoretical Analysis under the Coupled Normal Mode Framework

Equations (3.3), (3.4), (3.25) and (3.26), from Chapter 3 developed the coupled normal mode formulation used to calculate the complete acoustic response of the idealized SFD. The purpose of the present chapter is to extract physical interpretations from that formulation. The analysis proceeds in three stages. First, the transformations associated with each edge are decomposed to distinguish the effects of changes in modal structure, density, and horizontal wavenumber. Modal-susceptibility and scattering diagnostics are then introduced to identify the modal families most susceptible to the environmental perturbation. Finally, the two edges are considered together to determine how the phase and attenuation accumulated within the finite SFD control transmission, modal redistribution, and backscatter, including the limiting response of a very short feature.

4.1 Effects of Perturbations to the Medium

The coupling matrices derived in Chapter 3 depend on changes in density, modal structure, and horizontal wavenumber across an environmental interface. To separate these contributions, consider a single abrupt interface j as a proxy for the multiple interface composite edge operators

developed from equation (3.6). Define:

$$\delta k(z) = k_p(z) - k_s(z) \quad (4.1)$$

$$\delta \rho^{-1}(z) = \frac{1}{\rho_p(z)} - \frac{1}{\rho_s(z)} \quad (4.2)$$

$$\delta \Psi_\ell(z) = \Psi_\ell^p(z) - \Psi_\ell^s(z) \quad (4.3)$$

The interface condition matching matrices can be expressed in terms of these perturbations:

$$\begin{aligned} \tilde{C}_{\ell m} &= \int_0^\infty \frac{\Psi_\ell^p(z) \Psi_m^s(z)}{\rho_p(z)} dz \\ &= \int_0^\infty (\Psi_\ell^s(z) + \delta \Psi_\ell(z)) \Psi_m^s(z) \left(\frac{1}{\rho_s(z)} + \delta \rho^{-1}(z) \right) dz \\ &= \int_0^\infty \left[\frac{\Psi_\ell^s(z) \Psi_m^s(z)}{\rho_s(z)} + \Psi_\ell^s(z) \Psi_m^s(z) \delta \rho^{-1}(z) + \frac{\delta \Psi_\ell(z) \Psi_m^s(z)}{\rho_s(z)} + \delta \Psi_\ell(z) \Psi_m^s(z) \delta \rho^{-1}(z) \right] dz \\ &= \delta \ell_m + \int_0^\infty \left[\Psi_\ell^s(z) \Psi_m^s(z) \delta \rho^{-1}(z) + \frac{\delta \Psi_\ell(z) \Psi_m^s(z)}{\rho_s(z)} + \delta \Psi_\ell(z) \Psi_m^s(z) \delta \rho^{-1}(z) \right] dz \\ \hat{C}_{\ell m} &= \frac{k_{r,m}^s}{k_{r,\ell}^p} \int_0^\infty \frac{\Psi_\ell^p(z) \Psi_m^s(z)}{\rho_s(z)} dz \\ &= \frac{k_{r,m}^s}{k_{r,\ell}^p} \int_0^\infty \frac{(\Psi_\ell^s(z) + \delta \Psi_\ell(z)) \Psi_m^s(z)}{\rho_s(z)} dz \\ &= \frac{k_{r,m}^s}{k_{r,\ell}^p} \int_0^\infty \left[\frac{\Psi_\ell^s(z) \Psi_m^s(z)}{\rho_s(z)} + \frac{\delta \Psi_\ell(z) \Psi_m^s(z)}{\rho_s(z)} \right] dz \\ &= \frac{k_{r,m}^s}{k_{r,\ell}^p} \delta \ell_m + \frac{k_{r,m}^s}{k_{r,\ell}^p} \int_0^\infty \frac{\delta \Psi_\ell(z) \Psi_m^s(z)}{\rho_s(z)} dz \end{aligned}$$

where m is an incident mode and ℓ is an output mode. Substitution into the two overlap maps in Equation (3.5) gives the following exact relationships for an arbitrarily large density and mode shape change occurring at a single interface event.

$$\begin{aligned} (C_f)_{\ell m} &= \frac{1}{2}(1 + \kappa_{\ell m}^{s/p})A_{\ell m} + \frac{1}{2}(B_{\ell m} + M_{\ell m}), \\ (C_b)_{\ell m} &= \frac{1}{2}(1 - \kappa_{\ell m}^{s/p})A_{\ell m} + \frac{1}{2}(B_{\ell m} + M_{\ell m}). \end{aligned} \quad (4.4)$$

where:

$$\begin{aligned}
\kappa_{\ell m}^{s/p} &= \frac{k_{r,m}^s}{k_{r,\ell}^p}, \\
A_{\ell m} &= \delta_{\ell m} + \int_0^\infty \frac{\delta \Psi_\ell(z) \Psi_m^s(z)}{\rho_s(z)} dz, \\
B_{\ell m} &= \int_0^\infty \Psi_\ell^s(z) \Psi_m^s(z) \delta \rho^{-1}(z) dz, \\
M_{\ell m} &= \int_0^\infty \delta \Psi_\ell(z) \Psi_m^s(z) \delta \rho^{-1}(z) dz.
\end{aligned} \tag{4.5}$$

Here $A_{\ell m}$ contains the orthogonality baseline and mode-shape perturbation, $B_{\ell m}$ is the density contribution, and $M_{\ell m}$ is mixed and second order. Thus the leading term is diagonal in the reference basis, while off-diagonal coupling arises from environmental perturbations and modal-wavenumber mismatch. The two environments are identical throughout the water column, so:

$$\delta \rho^{-1}(z) = 0 \quad \text{for } z < z_{\text{sf}}.$$

Furthermore, sub-bottom densities are expected to be similar between the two environments, so the density perturbation is localized primarily to the interval of geometric change between the surrounding seafloor and the SFD floor. Density-driven coupling therefore contributes mainly where the modal amplitudes have support within this interval.

Similarly, the mode-shape perturbation $\delta \Psi_\ell$ is largest for modes whose structures overlap the environmental change. To make the sound-speed dependence of the modal projection in $A_{\ell m}$ explicit, we estimate the contribution of the change in medium wavenumber to the change in mode shape using further perturbation theory. The normal modes describing the acoustic field over the SFD satisfy

$$\rho_p \frac{d}{dz} \left[\frac{1}{\rho_p} \frac{d \Psi_\ell^p}{dz} \right] + \left[k_p^2 - \left(k_{r,\ell}^p \right)^2 \right] \Psi_\ell^p = 0. \tag{4.6}$$

Substituting Equations (4.3) into the perturbed modal equation and holding density fixed gives

$$\rho_s \frac{d}{dz} \left[\frac{1}{\rho_s} \frac{d(\Psi_\ell^s + \delta\Psi_\ell)}{dz} \right] + \left[(k_s + \delta k)^2 - (k_{r,\ell}^p)^2 \right] (\Psi_\ell^s + \delta\Psi_\ell) = 0. \quad (4.7)$$

subtraction of the Helmholtz equation governing the surrounding environment gives

$$\begin{aligned} 0 = & \rho_s \frac{d}{dz} \left[\frac{1}{\rho_s} \frac{d\delta\Psi_\ell}{dz} \right] + \left[k_s^2 - (k_{r,\ell}^s)^2 \right] \delta\Psi_\ell \\ & + \left\{ 2k_s\delta k - \left[(k_{r,\ell}^p)^2 - (k_{r,\ell}^s)^2 \right] \right\} \Psi_\ell^s \\ & + (\delta k)^2 \Psi_\ell^s + \left\{ 2k_s\delta k + (\delta k)^2 - \left[(k_{r,\ell}^p)^2 - (k_{r,\ell}^s)^2 \right] \right\} \delta\Psi_\ell. \end{aligned} \quad (4.8)$$

Multiplying Equation (4.8) by Ψ_m^s/ρ_s , and integrating over depth,

$$\begin{aligned} & \left[(k_{r,m}^s)^2 - (k_{r,\ell}^s)^2 \right] \int_0^\infty \frac{\Psi_m^s \delta\Psi_\ell}{\rho_s} dz + 2 \int_0^\infty \frac{\Psi_m^s \Psi_\ell^s}{\rho_s} k_s \delta k dz \\ & - \underbrace{\left[(k_{r,\ell}^p)^2 - (k_{r,\ell}^s)^2 \right] \int_0^\infty \frac{\Psi_m^s \Psi_\ell^s}{\rho_s} dz}_{\delta_{m\ell}} + O([\delta\Psi]^2) = 0. \end{aligned} \quad (4.9)$$

Neglecting higher order terms and re-organizing:

$$\int_0^\infty \frac{\delta\Psi_\ell(z) \Psi_m^s(z)}{\rho_s(z)} dz \approx \frac{(k_{r,\ell}^p)^2 - (k_{r,\ell}^s)^2}{(k_{r,\ell}^s)^2 - (k_{r,m}^s)^2} \delta_{\ell m} - \frac{2 \int_0^\infty \frac{\Psi_\ell^s(z) \Psi_m^s(z)}{\rho_s(z)} k_s(z) \delta k(z) dz}{(k_{r,\ell}^s)^2 - (k_{r,m}^s)^2}. \quad (4.10)$$

Finally, applying the perturbative approximation

$$\delta(k_{r,\ell}^2) \approx \int_0^\infty \frac{[\Psi_\ell^s(z)]^2}{\rho_s(z)} \delta[k_s^2(z)] dz. \quad (4.11)$$

yields:

$$\int_0^\infty \frac{\delta \Psi_\ell(z) \Psi_m^s(z)}{\rho_s(z)} dz \approx \begin{cases} 0, & m = \ell \\ \frac{2 \int_0^\infty \frac{\Psi_\ell^s(z) \Psi_m^s(z)}{\rho_s(z)} k_s(z) \delta k(z) dz}{(k_{r,\ell}^s)^2 - (k_{r,m}^s)^2}, & m \neq \ell \end{cases} \quad (4.12)$$

Then for a small change in sound speed, $A_{\ell m}$ can be approximated as

$$A_{\ell m} \approx \delta_{\ell m} + \frac{2(1 - \delta_{\ell m})}{(k_{r,\ell}^s)^2 - (k_{r,m}^s)^2} \int_0^\infty \frac{\Psi_\ell^s(z) \Psi_m^s(z)}{\rho_s(z)} k_s(z) \delta k(z) dz. \quad (4.13)$$

The diagonal therefore remains unity to first order; contributions to off-diagonal coupling are strongest when both modes overlap regions in which $k_s \delta k$ is appreciable and their squared radial wavenumbers are closely spaced. Modes with phase speeds comparable to the seabed sound speed can be particularly sensitive because their structures often have appreciable support within the perturbed seabed interval.

Under the additional assumption that the modal bases remain similar between the two environments, the strongest off-diagonal coupling is localized both in depth and modal index. Define the corresponding mode in the adjacent basis by

$$m^\star = \arg \max_\ell |\tilde{C}_{\ell m}| \quad (4.14)$$

The definition in Equation (4.14) directly selects the mode in the adjacent basis that is most nearly parallel to incident mode m . Evaluating Equation (4.4) at $\ell = m^\star$ gives the corresponding-mode contribution; the remaining rows quantify redistribution into less-parallel modes.

The structures of $\mathbf{C}_f$ and $\mathbf{C}_b$ determine how the environmental change modifies the transmitted modal amplitudes. Their perturbative forms suggest a blockwise interpretation. A subset of modes satisfies an approximately adiabatic interface relation: the corresponding entries of $\mathbf{C}_f$ remain concentrated near the corresponding output mode and those of $\mathbf{C}_b$ remain small, although the steep SFD edges still produce some nonadiabatic adjustment due to the projection of one mode basis onto the next. This subset consists primarily of water-column modes concentrated well above

the seafloor and deep sub-bottom modes lying below the interval of bathymetric change. For these modes, $\delta(1/\rho)$ is small and $\delta\Psi_\ell$ has a modest projection onto the incident mode shape, so diagonal coupling dominates. Central blocks of $\mathbf{C}_f$ and $\mathbf{C}_b$ contain modes concentrated in the region of greatest change, where density and mode-shape perturbations produce the strongest matrix structure. Blocks linking perturbed and weakly perturbed modal families then redistribute this response across dissimilar radial wavenumbers.

Because the diagonal entries of $\mathbf{H}_1^P$ are ratios of Hankel functions, transferring energy to a mode with a different wavenumber produces differential phase evolution; transferring it to a mode with a larger imaginary wavenumber produces differential attenuation. Modes with phase speeds comparable to the seabed sound speed are therefore expected to dominate the scattering response: they are strongly affected by the environmental change at the edge and remain sensitive during propagation through the perturbed segment.

4.2 Modal Susceptibility to the Seafloor Depression

The exact two-edge relation in Equation (3.26) suggests interpreting the SFD as a modal filter. Its response depends jointly on modal susceptibility to the medium perturbation and on the differential phase and attenuation accumulated within the perturbed segment. The m th column of $\mathbf{T}_{\text{SFD}}$ gives the complete receiver-side modal response to unit amplitude in incident exterior mode m , while the m th column of $\mathbf{R}_{\text{SFD}}$ gives the corresponding source-side backscattered response. At either edge, the m th columns of $\mathbf{C}_f^j$ and $\mathbf{C}_b^j$ resolve the local redistribution that contributes to these complete responses. A useful first predictor of susceptibility is the fraction of the integrated mode function contained within the perturbed region,

$$\mathbb{S} = \left| \int_{z_{\text{sf}}}^{z_{\text{pf}}} \frac{[\Psi_m^s(z)]^2}{\rho_s(z)} dz \right| \quad (4.15)$$

Where z_{sf} is the seafloor and z_{pf} is the SFD floor, equal to the sum of the seafloor and SFD depth. This quantity is termed the *susceptibility score*. The construction of this term is consistent with the

magnitude of the normalization of modes in an attenuating acoustic problem, evaluated only over the region of seafloor depth change. It admits that real and imaginary components of a mode could self cancel within the interval, just like the standard modal normalization. Modes with small $\mathbb{S}_S$ have little support in the modified region and remain largely unaffected, whereas modes with larger $\mathbb{S}_S$ are more strongly perturbed. The surrounding stratigraphy can support modes with substantial energy in the interval of bathymetric change. This construct predicts whether an incident mode samples the perturbed region, but it does not directly quantify the resulting redistribution. For a given interaction, define

$$\mathbb{S}_C^m = \frac{1}{M} \sum_{\ell=1}^M |(C_f)_{\ell m} - (C_{\text{adiab}})_{\ell m}| + \frac{1}{M} \sum_{\ell=1}^M |(C_b)_{\ell m}|, \quad (4.16)$$

$$(C_{\text{adiab}})_{\ell m} = \begin{cases} \exp(i \angle \tilde{C}_{m^* m}), & \ell = m^*, \\ 0, & \text{otherwise} \end{cases} \quad (4.17)$$

Here m^* is selected independently for each incident-mode column using Equation (4.14). Matrix columns m denote incident modes and rows ℓ denote output modes.

For the *Scattering Score*, $\mathbb{S}_C^m$, values near zero indicate that the corresponding-mode contribution dominates, while larger values indicate departure from the adiabatic condition as a function of the projection into the new basis and scattering induced by the interface conditions. $(C_{\text{adiab}})_{\ell m}$ is defined as shown in Equation 4.17 because for complex normal modes the mean phase angle of a given mode is arbitrary; so implementation of the adiabatic approximation would require evaluation of the mean phase alignment between corresponding modes.

The two scores describe complementary aspects of the interaction. Large susceptibility indicates that an incident mode has substantial support within the region of environmental change and is therefore susceptible to scattering. a large scattering score indicates redistribution away from the corresponding output mode, and two way propagation effects may be introduced. The vertical extent of the associated coupling-matrix column provides a more direct indication of whether that

redistribution spans a broad range of output wavenumbers and propagation angles, as would be expected when strong scattering occurs.

The SFD acts as an effective modal filter only when it is long enough for the modes inside it to accumulate distinguishable phase and attenuation. This will be explored in the subsequent sections. In addition, the incident frequency must be high enough to excite modes that resolve field variability within the sub-bottom strata. For a homogeneous layer with symmetric boundary conditions, this threshold is approximately $f_{\min} = c_{\text{layer}}/(2h_{\text{layer}})$. The surrounding stratigraphy must feature the layer structure which supports modes that sample the SFD geometry.

4.3 Effects of Seafloor Depression Length

We write the sum and difference of the diagonal Hankel function ratio matrices as $\mathbf{H}_1^p + (\mathbf{H}_2^p)^{-1}$ and $\mathbf{H}_1^p - (\mathbf{H}_2^p)^{-1}$. Rewriting the equations in terms of these is useful for illustrative purposes because when evaluated with a large argument they are approximated by multiples of cosine and sine, respectively. Continuing the first order approximation of each SFD edge as a single abrupt interface, we decompose each edge operator into the overlap matrices

$$\bar{C}_{nm}^{p/s} = \int_0^\infty \frac{\Psi_n^p(z) \Psi_m^s(z)}{\rho_s(z)} dz, \quad \bar{C}_{\ell n}^{s/p} = \int_0^\infty \frac{\Psi_\ell^s(z) \Psi_n^p(z)}{\rho_p(z)} dz. \quad (4.18)$$

Superscripts p/s and s/p identify the ordered pair of modal bases used in constructing an overlap, matching, or wavenumber-ratio operator. The slash serves as a compact separator between the two environment labels and does not indicate a direction of propagation. The two interface wavenumber ratios obey

$$\kappa^{p/s} \odot \left(\kappa^{s/p} \right)^T = \kappa^{s/s}, \quad (4.19)$$

where $\odot$ signifies element-wise multiplication. Therefore the intermediate SFD wavenumber

cancels from their product. Define the three independent propagation-channel operators as

$$\mathbf{G}_+ = \left(\overline{\mathbf{C}}^{p/s} \right)^T \left(\mathbf{H}_1^p + (\mathbf{H}_2^p)^{-1} \right) \left(\overline{\mathbf{C}}^{s/p} \right)^T, \quad (4.20)$$

$$\mathbf{G}_-^L = \left(\overline{\mathbf{C}}^{p/s} \right)^T \left(\mathbf{H}_1^p - (\mathbf{H}_2^p)^{-1} \right) \left(\boldsymbol{\kappa}^{s/p} \odot \overline{\mathbf{C}}^{p/s} \right), \quad (4.21)$$

$$\mathbf{G}_-^R = \left(\boldsymbol{\kappa}^{p/s} \odot \overline{\mathbf{C}}^{s/p} \right) \left(\mathbf{H}_1^p - (\mathbf{H}_2^p)^{-1} \right) \left(\overline{\mathbf{C}}^{s/p} \right)^T. \quad (4.22)$$

Because $\mathbf{H}_1^p + (\mathbf{H}_2^p)^{-1}$ is diagonal and the element-wise product of $\boldsymbol{\kappa}^{s/p}$ with a matrix X may be written as the product $[\text{diag}(k^p)]^{-1} X (\text{diag}(k^s))$, the remaining even-channel product simplifies to

$$\left(\boldsymbol{\kappa}^{p/s} \odot \overline{\mathbf{C}}^{s/p} \right) \left(\mathbf{H}_1^p + (\mathbf{H}_2^p)^{-1} \right) \left(\boldsymbol{\kappa}^{s/p} \odot \overline{\mathbf{C}}^{p/s} \right) = \boldsymbol{\kappa}^{s/s} \odot \mathbf{G}_+^T. \quad (4.23)$$

Using $\mathbf{C}_f = \frac{1}{2}(\widetilde{\mathbf{C}} + \widehat{\mathbf{C}})$ and $\mathbf{C}_b = \frac{1}{2}(\widetilde{\mathbf{C}} - \widehat{\mathbf{C}})$, the left-edge operators may be written directly as

$$\begin{aligned} \mathbf{C}_f^L &= \frac{1}{2} \left[\left(\overline{\mathbf{C}}^{s/p} \right)^T + \boldsymbol{\kappa}^{s/p} \odot \overline{\mathbf{C}}^{p/s} \right], \\ \mathbf{C}_b^L &= \frac{1}{2} \left[\left(\overline{\mathbf{C}}^{s/p} \right)^T - \boldsymbol{\kappa}^{s/p} \odot \overline{\mathbf{C}}^{p/s} \right]. \end{aligned} \quad (4.24)$$

The right-edge operators follow from the same construction after interchanging the s and p environments. Substitution into the two-edge operator gives

$$\begin{aligned} \mathbf{P}_{11} &= \frac{1}{4} \left[\mathbf{G}_+ + \boldsymbol{\kappa}^{s/s} \odot \mathbf{G}_+^T + \mathbf{G}_-^L + \mathbf{G}_-^R \right] \mathbf{H}_1^s, \\ \mathbf{P}_{12} &= \frac{1}{4} \left[\mathbf{G}_+ - \boldsymbol{\kappa}^{s/s} \odot \mathbf{G}_+^T - \mathbf{G}_-^L + \mathbf{G}_-^R \right], \\ \mathbf{P}_{21} &= \frac{1}{4} (\mathbf{H}_2^r)^{-1} \left[\mathbf{G}_+ - \boldsymbol{\kappa}^{s/s} \odot \mathbf{G}_+^T + \mathbf{G}_-^L - \mathbf{G}_-^R \right] \mathbf{H}_1^s, \\ \mathbf{P}_{22} &= \frac{1}{4} (\mathbf{H}_2^r)^{-1} \left[\mathbf{G}_+ + \boldsymbol{\kappa}^{s/s} \odot \mathbf{G}_+^T - \mathbf{G}_-^L - \mathbf{G}_-^R \right]. \end{aligned} \quad (4.25)$$

Equation (4.25) separates the response into four distinct channels. The operator $\mathbf{G}_+$ in equation 4.20 describes the two even paths which dominate when $\mathbf{H}_1^p + (\mathbf{H}_2^p)^{-1}$ is large and

$\mathbf{H}_1^p - (\mathbf{H}_2^p)^{-1}$ is small. On the other hand, The two odd operators $\mathbf{G}_-^L$ and $\mathbf{G}_-^R$ in equations 4.21 and 4.22 dominate when $\mathbf{H}_1^p + (\mathbf{H}_2^p)^{-1}$ is small and $\mathbf{H}_1^p - (\mathbf{H}_2^p)^{-1}$ is large. These channels are combined in different sum and difference combinations in the solutions for the various blocks of $\mathbf{P}$. The construction of the solutions is such that even channels and odd channels combine constructively in $\mathbf{P}_{11}$ and $\mathbf{P}_{22}$ but destructively in $\mathbf{P}_{12}$ and $\mathbf{P}_{21}$. The entries of the two even channels are such that they are likely to be similar in value and when the even channels dominate, a condition is created such that the entries of $\mathbf{P}_{12}$ and $\mathbf{P}_{21}$ may cancel, minimizing the Schur complement contribution to the total receiver side solution. On the other hand the two odd channels are differentially weighted by the modal wavenumbers such that if the two bases are different, they will not cancel.

For exterior modes m and ℓ coupled through SFD mode n , the two even paths carry wavenumber factors 1 and

$$\frac{k_{r,n}^p k_{r,m}^s}{k_{r,\ell}^s k_{r,n}^p} = \frac{k_{r,m}^s}{k_{r,\ell}^s}, \quad (4.26)$$

so the intermediate SFD wavenumber cancels from their relative weighting. For identical source and receiver environments, these factors are exactly balanced for the diagonal response, $\ell = m$. The odd paths instead carry the reciprocal factors $k_{r,m}^s/k_{r,n}^p$ and $k_{r,n}^p/k_{r,\ell}^s$, whose relative scaling is

$$\frac{k_{r,n}^p/k_{r,\ell}^s}{k_{r,m}^s/k_{r,n}^p} = \frac{(k_{r,n}^p)^2}{k_{r,\ell}^s k_{r,m}^s}. \quad (4.27)$$

These paths balance only when the intermediate wavenumber is near the geometric mean of the incident and outgoing wavenumbers. Otherwise their reciprocal weighting exposes the modal-wavenumber mismatch as a path contrast.

As the decompositions for $\mathbf{C}_f$ and $\mathbf{C}_b$ show, the first-order density term $\mathbf{B}$ enters both operators with the same sign and coefficient. Density contrast therefore contributes equally to the local direction-preserving and direction-reversing coupling and is carried into both the even and odd two-edge channels. As a result, a local perturbation to material density contributes to scattering regardless of the dominant propagation regime that a mode experiences across the SFD.

4.4 Far Field Limit

The approximation to the Hankel function for large arguments

$$H_v^{(1)}(z) \approx \sqrt{\frac{2}{\pi z}} e^{i(z - \frac{v\pi}{2} - \frac{\pi}{4})}, \quad (4.28)$$

$$H_v^{(2)}(z) \approx \sqrt{\frac{2}{\pi z}} e^{-i(z - \frac{v\pi}{2} - \frac{\pi}{4})}. \quad (4.29)$$

is generally applicable for a given mode m when $k_{r,m}r \gg 1$, where r is the distance from the source. Separating the wavenumber into real and imaginary components with $k_{r,m}^p = \beta_m + i\alpha_m$, the approximation gives

$$\begin{aligned} [\mathbf{H}_1^p + (\mathbf{H}_2^p)^{-1}]_{mm} &\approx 2 \left(1 + \frac{L_p}{r_p}\right)^{-1/2} e^{-\alpha_m L_p} \cos(\beta_m L_p), \\ [\mathbf{H}_1^p - (\mathbf{H}_2^p)^{-1}]_{mm} &\approx 2i \left(1 + \frac{L_p}{r_p}\right)^{-1/2} e^{-\alpha_m L_p} \sin(\beta_m L_p). \end{aligned} \quad (4.30)$$

where L_p denotes the length of the SFD interior and r_p denoted the radial distance from source to the near SFD edge. This simplification converts the even and odd propagation combinations into cosine and sine channels. Both acquire the same cylindrical-spreading factor and the same attenuation factor $e^{-\alpha_m L_p}$; attenuation therefore controls the overall strength of the response but not whether a mode occupies the constructive or destructive channel. That partition is selected solely by the phase advance $\beta_m L_p$, so changing SFD length or modal horizontal wavenumber moves a mode continuously between transmission-like and scattering-prone regimes.

If $\beta_m L_p \approx n\pi$, then the contributions from $\mathbf{H}_1^p$ and $(\mathbf{H}_2^p)^{-1}$ add constructively, and $\mathbf{G}_+ \gg \mathbf{G}_-^{L/R}$. This corresponds to a transmission-like regime, in which the mode passes through the SFD with minimal scattering. On the other hand, if $\beta_m L_p \approx \left(n - \frac{1}{2}\right)\pi$, then $\mathbf{G}_-^{L/R} \gg \mathbf{G}_+$ and modal scattering can potentially become significant if density and modal wavenumber perturbations are large enough. To explain this from a more intuitive perspective, if the mode as it exits the SFD is in phase with its amplitude when it entered, then the mode is exactly aligned to contribute to the new

environments modes in the exact same proportion that they contributed to it at the start of the SFD, and no scattering occurs at the right edge, though energy may be attenuated. On the other hand if the exiting phase is in quadrature with its position at the start of the SFD then its incompatibility with the surrounding environment is maximized; leading to maximum scatter and re-distribution.

4.5 Short Seafloor Depression Limit

For a SFD in the far field shorter than mode m 's radial wavelength,

$$\beta_m L_p \ll 1 \quad \mathbf{H}_1^p + (\mathbf{H}_2^p)^{-1} \rightarrow 2\mathbf{I}, \quad \mathbf{H}_1^p - (\mathbf{H}_2^p)^{-1} \rightarrow \mathbf{0}. \quad (4.31)$$

and thus the odd channel vanishes, while the even channel simplifies,

$$\mathbf{G}_-^L \rightarrow \mathbf{0}, \quad \mathbf{G}_-^R \rightarrow \mathbf{0}, \quad \mathbf{G}_+ \rightarrow 2 \left(\overline{\mathbf{C}}^{s/p} \overline{\mathbf{C}}^{p/s} \right)^T, \quad (4.32)$$

And when the orthonormality of both bases is perfect,

$$\begin{aligned} \left[\overline{\mathbf{C}}^{s/p} \overline{\mathbf{C}}^{p/s} \right]_{\ell m} &= \sum_n \left[\int_0^\infty \frac{\Psi_\ell^s(z) \Psi_n^p(z)}{\rho_p(z)} dz \right] \left[\int_0^\infty \frac{\Psi_n^p(z') \Psi_m^s(z')}{\rho_s(z')} dz' \right] \\ &= \int_0^\infty \int_0^\infty \Psi_\ell^s(z) \left[\sum_n \frac{\Psi_n^p(z) \Psi_n^p(z')}{\rho_p(z)} \right] \frac{\Psi_m^s(z')}{\rho_s(z')} dz' dz \\ &= \int_0^\infty \int_0^\infty \Psi_\ell^s(z) \delta(z - z') \frac{\Psi_m^s(z')}{\rho_s(z')} dz' dz \\ &= \int_0^\infty \frac{\Psi_\ell^s(z) \Psi_m^s(z)}{\rho_s(z)} dz = \delta_{\ell m}. \end{aligned} \quad (4.33)$$

These completeness identities establish the inverse property of the overlap operators that constitute the interface coupling transformation. In a complete modal basis, applying the corresponding reversed-interface operator exactly undoes the forward-interface projection. This supplies

the explicit inverse relation used in the reversed-edge formulation of Section 3.2.1. Consequently,

$$\mathbf{P}_{11} \approx \mathbf{H}_1^s, \quad \mathbf{P}_{12} \approx \mathbf{0}, \quad \mathbf{P}_{21} \approx \mathbf{0}, \quad \mathbf{P}_{22} \approx (\mathbf{H}_2^s)^{-1}, \quad (4.34)$$

$$\mathbf{a}^r \approx \mathbf{H}_1^s \mathbf{a}^s, \quad \mathbf{b}^s \approx \mathbf{0}. \quad (4.35)$$

Therefore a SFD that is short relative to the reciprocal of the real portion of the modal wavenumber produces no net mode conversion and no backscatter. Each edge may individually generate substantial modal mixing, but when essentially no differential phase accumulates between the edges, completeness makes the reverse projection exactly undo the first. In a typical normal mode problem only a finite number of normal modes are of relevance to the far field; in this if the SFD is short relative to the steepest trapped mode, then there is essentially no effect on the far field.

CHAPTER 5

Numerical Analysis with Normal Mode Method

The theoretical analysis proposed in Chapter 4 was verified and illustrated using a custom normal mode program. A center frequency of 250 Hz was selected to resolve the modeled stratigraphy while keeping the number of waveguide modes computationally manageable. The local spectra contain water-column modes, modes associated primarily with individual sediment layers, and strongly leaky modes. Because these families are interleaved in a global wavenumber ordering, the susceptibility score is used together with the scattering score to identify the behavior of each mode. Coupling matrices were computed between each adjacent pair of modal bases, and the many local transformations within each sloping SFD edge were combined with their two-way propagation factors to form a composite edge operator. This section describes the numerical procedures used to perform these operations, and interprets their results.

5.1 Development of a Normal Mode Program

5.1.1 General Information

The normal mode program developed for this work implements the range-independent modal representation and range-dependent field construction described in standard treatments of computational ocean acoustics [11]. Its local modal solver draws on established finite-difference methods for ocean-acoustic eigenvalue problems, including the methods developed by Porter and Reiss [23, 24]. Several extensions were introduced here to support layered, attenuating, and leaky waveguides and the two-way range-dependent calculations required by the present problem. For each unique range-independent environmental profile and frequency, the depth-separated acoustic

eigenproblem is assembled as a generalized tri-diagonal matrix using a Numerov-type discretization [25]. At acoustic half-spaces, the exterior derivative is replaced by the outgoing vertical admittance γ/ρ , where $\gamma = (k_r^2 - k_{\text{hs}}^2)^{1/2}$ is evaluated on the radiation branch. This makes the boundary row nonlinear in the eigenvalue and requires branch-cut management, derivative-aware orthogonalization, and nonlinear corrections to problem matrix while evaluating the solution. If desired, the problem is solved on multiple nested grids to facilitate extrapolating out numerical truncation error using the Richardson method [26].

5.1.2 Material-interface discretization

The acoustic Helmholtz equation may be written as

$$\frac{d^2\Psi}{dz^2} + (k^2 - k_r^2)\Psi = 0, \quad k = \frac{\omega}{c}. \quad (5.1)$$

Let an abrupt interface lie at $z = z_i$, and define the neighboring modal samples and grid spacings by

$$\Psi_{i-1} = \Psi(z_i - h_{i-1}), \quad \Psi_i = \Psi(z_i), \quad \Psi_{i+1} = \Psi(z_i + h_i). \quad (5.2)$$

Pressure is represented by the single value Ψ_i at the interface, thereby imposing its continuity. Continuity of density weighted pressure gradient requires

$$\frac{1}{\rho_i^{(-)}} \frac{d\Psi}{dz} \Big|_{z_i^{(-)}} = \frac{1}{\rho_i^{(+)}} \frac{d\Psi}{dz} \Big|_{z_i^{(+)}} , \quad (5.3)$$

where the minus and plus subscripts denote the limiting properties above and below the interface, respectively.

To retain a higher leading order numerical truncation error than a conventional second-order interface row, the two one-sided derivatives are corrected using the Helmholtz equation. Integrating the derivative over each adjacent interval and linearly interpolating it between the interval endpoints

gives

$$\begin{aligned}\mathcal{D}_i^{(-)} &= \frac{\Psi_i - \Psi_{i-1}}{h_{i-1}} + \frac{h_{i-1}}{6} \left[\Psi''_{i-1} + 2\Psi'' \left(z_i^{(-)} \right) \right], \\ \mathcal{D}_i^{(+)} &= \frac{\Psi_{i+1} - \Psi_i}{h_i} - \frac{h_i}{6} \left[2\Psi'' \left(z_i^{(+)} \right) + \Psi''_{i+1} \right].\end{aligned}\quad (5.4)$$

A Taylor expansion about the interface shows that

$$\begin{aligned}\mathcal{D}_i^{(-)} &= \Psi' \left(z_i^{(-)} \right) + \frac{h_{i-1}^3}{24} \Psi^{(4)} \left(z_i^{(-)} \right) + \mathcal{O}(h_{i-1}^4), \\ \mathcal{D}_i^{(+)} &= \Psi' \left(z_i^{(+)} \right) - \frac{h_i^3}{24} \Psi^{(4)} \left(z_i^{(+)} \right) + \mathcal{O}(h_i^4).\end{aligned}\quad (5.5)$$

The interface-flux approximation is therefore locally third-order accurate. Substitution into Equation (5.4), followed by enforcement of $\frac{\mathcal{D}_i^{(-)}}{\rho_i^{(-)}} = \frac{\mathcal{D}_i^{(+)}}{\rho_i^{(+)}}$, produces one row of the generalized eigenproblem,

$$a_{i-1} \Psi_{i-1} + a_i \Psi_i + a_{i+1} \Psi_{i+1} = k_r^2 (b_{i-1} \Psi_{i-1} + b_i \Psi_i + b_{i+1} \Psi_{i+1}), \quad (5.6)$$

$$\begin{aligned}a_{i-1} &= \frac{1}{\rho_{i-1}} \left(\frac{1}{h_{i-1}} + \frac{h_{i-1} k_{i-1}^2}{6} \right), & b_{i-1} &= \frac{h_{i-1}}{6\rho_{i-1}}, \\ a_i &= \frac{1}{\rho_i^{(-)}} \left(-\frac{1}{h_{i-1}} + \frac{2h_{i-1} k_i^{2(-)}}{6} \right) + \frac{1}{\rho_i^{(+)}} \left(-\frac{1}{h_i} + \frac{2h_i k_i^{2(+)}}{6} \right), & b_i &= \frac{2h_{i-1}}{6\rho_i^{(-)}} + \frac{2h_i}{6\rho_i^{(+)}} \\ a_{i+1} &= \frac{1}{\rho_{i+1}} \left(\frac{1}{h_i} + \frac{h_i k_{i+1}^2}{6} \right), & b_{i+1} &= \frac{h_i}{6\rho_{i+1}}.\end{aligned}\quad (5.7)$$

Thus, each off-diagonal contribution carries a weight of $1/6$, while each side contributes a weight of $2/6$ to the interface diagonal. The diagonal is split because the upper and lower fluxes have different grid spacings, densities, and potentially different sound speeds. For a smoothly varying sound speed, the corresponding endpoint values of k^2 are used in Equation (5.4); the generalized row remains tridiagonal.

A conventional, Helmholtz equation corrected, one-sided finite-difference estimate uses only the interface second derivative and has a leading error of order h^2 . Using both endpoint second derivatives in Equation (5.4) cancels that term and leaves an order- h^3 interface-flux error. By comparison, the smooth-layer Numerov identity is

$$\Psi_{i-1} - 2\Psi_i + \Psi_{i+1} = \frac{h^2}{12} (\Psi''_{i-1} + 10\Psi''_i + \Psi''_{i+1}) + \mathcal{O}(h^6), \quad (5.8)$$

which has an order- h^6 defect. The compact interface row is therefore more accurate than the conventional second-order interface treatment, but it does not retain the accuracy of a global Numerov discretization without interfaces.

Within a smooth layer, the implementation also permits density to vary with depth. In that case the modal equation is written as

$$\frac{d^2\Psi}{dz^2} + g(z)\frac{d\Psi}{dz} + (k^2 - k_r^2)\Psi = 0, \quad g(z) = -\frac{d}{dz} \ln \rho(z). \quad (5.9)$$

The values of g are evaluated independently within each smooth layer and incorporated into modified Numerov coefficients while retaining a generalized tridiagonal eigenproblem. These coefficients reduce identically to the standard Numerov coefficients when $g = 0$. At a material discontinuity, the density jump is not differentiated: the smooth-layer row is replaced by Equation (5.6), using the limiting densities on the two sides. The adjacent upper-layer row is likewise evaluated using the upper limiting properties so that its stencil does not sample through the discontinuity.

Before the generalized eigenproblem is assembled, each smooth interior row is multiplied by the local factor $\frac{h_i}{\rho_i}$. The interface equation already contains separate contributions proportional to $\frac{h_{i-1}}{\rho_i^{(-)}}$ and $\frac{h_i}{\rho_i^{(+)}}$ from the two adjoining layers. Because its unscaled off-diagonal weights are $\frac{1}{6}$, the entire interface row is multiplied by $\frac{1}{2}$. After scaling its off-diagonal entries in $\mathbf{B}$ are therefore $\frac{h_{i-1}}{12\rho_i^{(-)}}$ and $\frac{h_i}{12\rho_i^{(+)}}$, exactly matching the corresponding entries contributed by the neighboring Numerov rows. Consequently, $\mathbf{B}$ remains symmetric to numerical precision across both smooth regions and material interfaces. The off-diagonal entries of $\mathbf{B}$ contain only the density-weighted quadrature

coefficients, whereas those of $\mathbf{A}$ combine the finite-difference derivative term, the local k^2 contribution, and, for varying-density layers, additional density-gradient corrections. These components do not generally transform in the same proportion, so no single row scaling is generally capable of symmetrizing both matrices simultaneously. Preserving the symmetry of $\mathbf{B}$ is more important because it defines the discrete modal weighting and normalization. The remaining normal mode calculations are therefore implemented without assuming that $\mathbf{A}$ is symmetric.

5.1.3 Eigenvalue and Eigenvector computation

For real-valued problems, the discrete spectrum is first isolated using Sturm counts and interval bisection, after which inverse iteration is used to compute the associated eigenvectors, while Rayleigh quotient iteration refines the eigenvalues. When an analytic half-space makes the boundary condition nonlinear in k_r^2 , the corresponding eigenvalue refinement is instead performed using Newton's method. These methods have previously been applied to ocean-acoustic normal mode calculations [23, 24]. The present implementation additionally uses matrix balancing and scaling, refinement to enforce a high level of complex symmetric correct linear independence, and Rayleigh–Ritz projection. These are established numerical-linear-algebra operations [27, 28]; their particular integration with radiation boundary conditions was developed for this program. If an adjacent frequency bin has already converged, perturbation theory is used to extrapolate k_r^2 in frequency using $d(k_r^2)/d\omega$, providing initial guesses for inverse iteration [?]. If no adjacent-frequency guess is available, the corresponding lossless problem is solved first, after which material attenuation is introduced perturbatively to estimate the complex eigenvalues [24]. These complex guesses are inverse-iterated directly, and every converged mode is immediately inserted into a deflation list. In practice, this continuation strategy greatly reduces the number of expensive two-dimensional contour searches required.

The remaining complex roots are found using a deflated winding-integral procedure in the complex k_r^2 plane. This procedure applies the argument principle to count and locate roots of the nonlinear characteristic determinant [28, 29]. The admissible search domain is first divided

into rectangles that do not cross square-root branch cuts associated with the acoustic half-space admittances. Each rectangle is counted by evaluating the winding number of the determinant around its boundary after subtracting the contribution of roots that have already converged through inverse iteration. Before the costly recursive two-dimensional bisection is started, the code also traces the real and imaginary zero-contours of the determinant inward from the rectangle boundaries. A Newton-step contour-following scheme advances along the $\Re [Det(k_r^2)] = 0$ and $\Im [Det(k_r^2)] = 0$ curves, and locations where the complementary determinant component changes sign are treated as candidate roots [?, ?]. These candidates are inverse-iterated; if they converge, they are added to the deflation set and removed from the remaining winding count. Thus for most frequency bins, the neighboring bin extrapolation, perturbative guesses, or traced determinant-zero intersections locate nearly all complex roots before the two-dimensional bisection stage is required.

Although the solver also supports analytic acoustic half-spaces, the calculations reported below use the finite absorbing layer described in Chapter 3, as this provided the best numerical stability. A pressure-release boundary at the bottom of the numerical grid lies below the absorbing layer, serving as a stable terminal condition; this imitates the radiation boundary condition. The plotted and interpreted fields are restricted to the physical water-column and sediment domain above the absorber.

5.1.4 Evaluation of the Range-Dependent Field

After the local modes have been computed, the program constructs the range-dependent solution by reusing the unique modal bases along the ordered propagation path. At each environmental interface, coupling matrices are computed between the modal bases of adjacent range segments, and the modal amplitudes are marched iteratively in both directions until the newly generated forward or backscattered energy falls below a specified tolerance. The radial dependence is represented with cylindrical Hankel functions. The program also includes discrete spherical scatterers embedded within range segments. These objects may be vacuum/pressure-release spheres, perfectly rigid spheres, or acoustic fluid spheres with arbitrary radius, density, sound speed, and

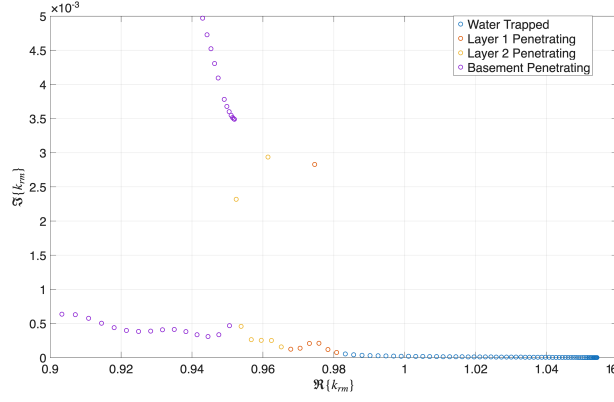
attenuation. Their contribution is represented through modal scattering matrices derived from the classical spherical plane-wave scattering solutions for spheres in a waveguide [30,31]. This allows the same two-way marching framework to include both environmental range dependence and multiple scattering from finite-sized acoustic sediment inhomogeneity, after which the outgoing and incoming modal amplitudes are reconstructed on the requested range–depth grid to obtain the final pressure field.

5.2 Primary Scenario

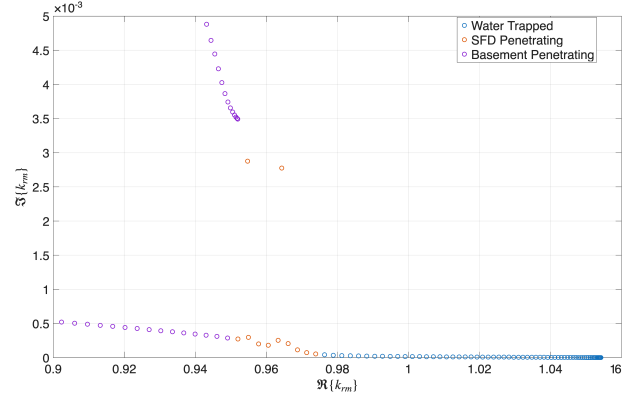
The environment defined in Chapter 3 is treated throughout the following analysis as the *primary scenario*. Its sediment sound speed increases with depth, and the associated modal basis contains families with progressively greater penetration into successive sedimentary strata. A second configuration, referred to as the *duct scenario*, is included later as a sensitivity study rather than as an alternative reconstruction of the observed site.

5.2.1 Description of Local Mode Basis

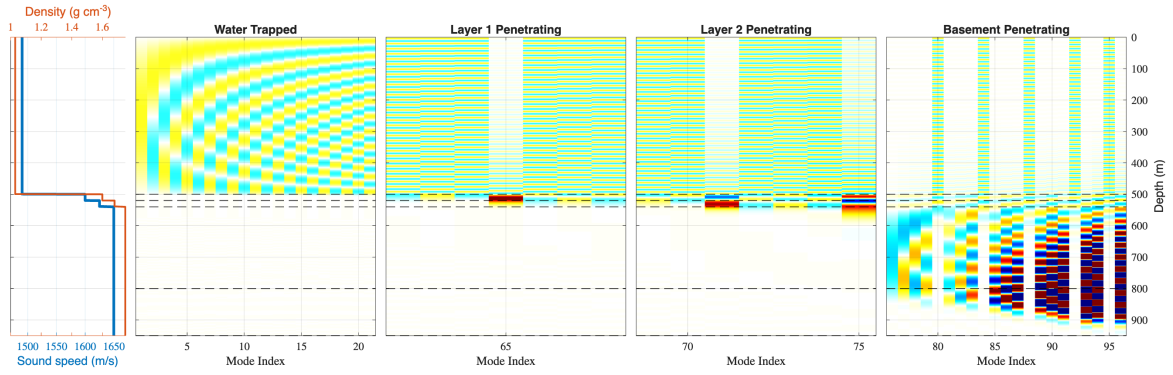
The normal modes are displayed in Figure 5.2 sorted by decreasing $\Re[k_r]$. The normal modes form a coherent global sequence, and yet certain modes can be associated with specific layers within the model. In specific, mode 65 of the surrounding environment effectively forms the ‘first mode’ of variability in the first acoustic layer, while 71 is the first of the second layer. Modes 76 and 77 of the surrounding environment and SFD, respectively, are the first modes that can penetrate the basement. Because the basement continues smoothly into a strong absorbing sponge, these modes and all sediment associated modes after have growing amplitude through the basement until the sponge is reached, then they rapidly decay. The sediment portion of these modes is accompanied by a large imaginary component (not plotted); this and the growth into the basement region mimic the behavior of a radiation boundary condition at the interface of the second layer and the basement.



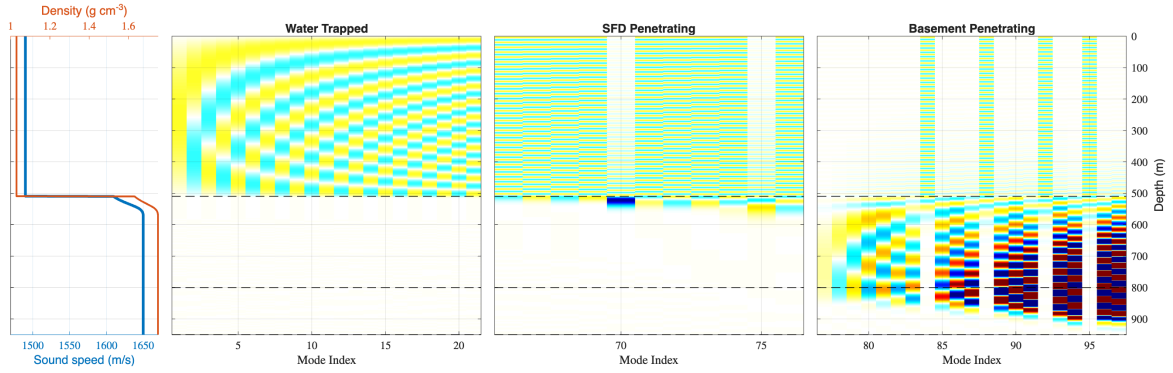
(a) Surrounding-stratigraphy modal spectrum.



(b) SFD modal spectrum.



(c) Surrounding-environment mode groups.



(d) SFD mode groups.

Figure 5.1: Normal mode bases for the 250-Hz primary scenario. (a) and (b) show the modal spectra in the complex horizontal-wavenumber plane for the surrounding and SFD environments, respectively. (c) and (d) show the corresponding sound-speed profiles and the first 20 modes from each identified group.

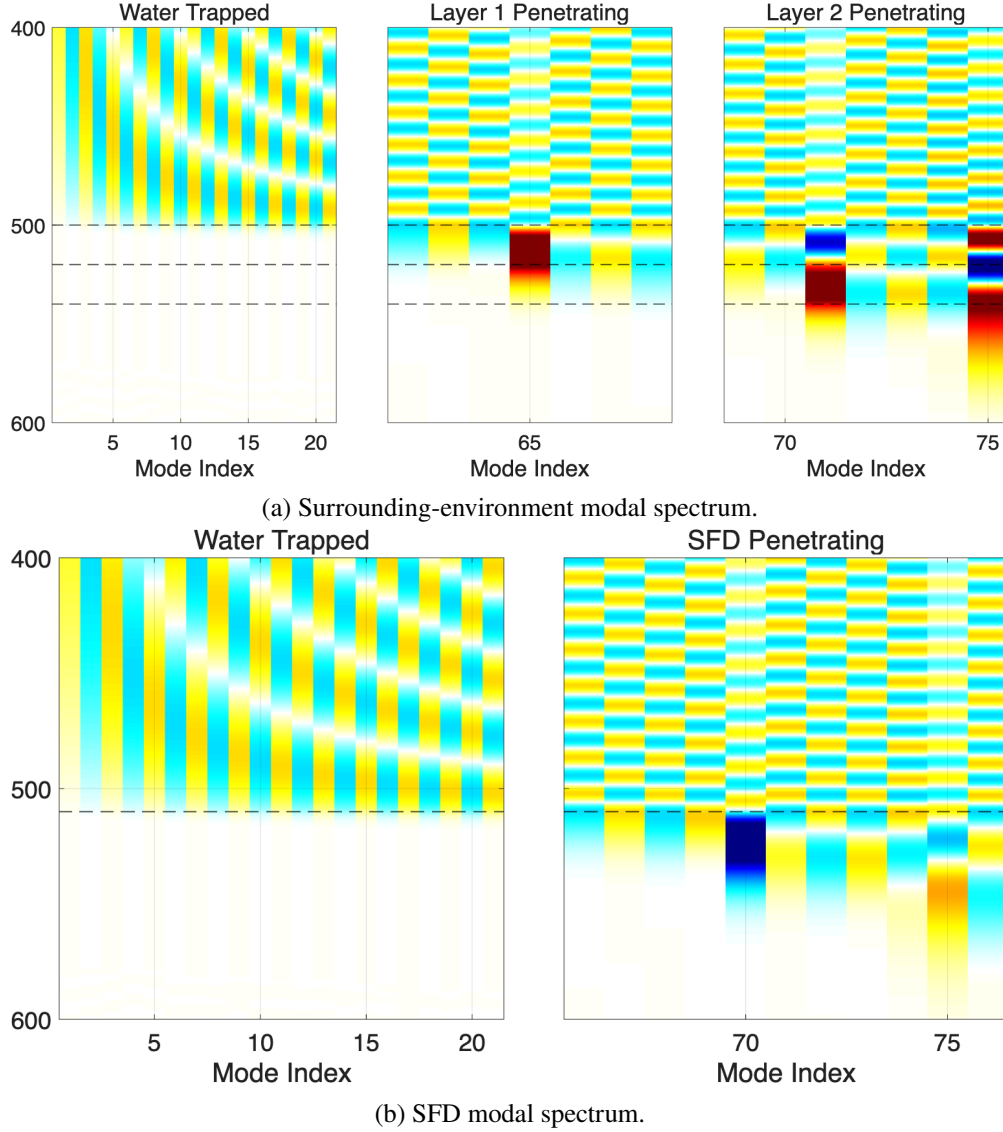


Figure 5.2: Normal mode bases for the 250-Hz primary scenario detail view. (a) shows the first 20 modes from each identified group for the surrounding environment, (b) is the corresponding plot for the SFD.

5.2.2 Structure of the Coupling Matrices

The modal coupling matrices exhibit structural features consistent with the perturbative decomposition of section 4.1. Figure 5.3 shows the coupling matrices for the first 250 modes together with the susceptibility and scattering diagnostics. The forward coupling matrices exhibit a diagonal backbone arising from the leading-order orthogonality term, for which each incident mode maps primarily to its corresponding mode. Strong off-diagonal structures are organized into approximate blocks associated with distinct modal families. These blocks coincide with modes

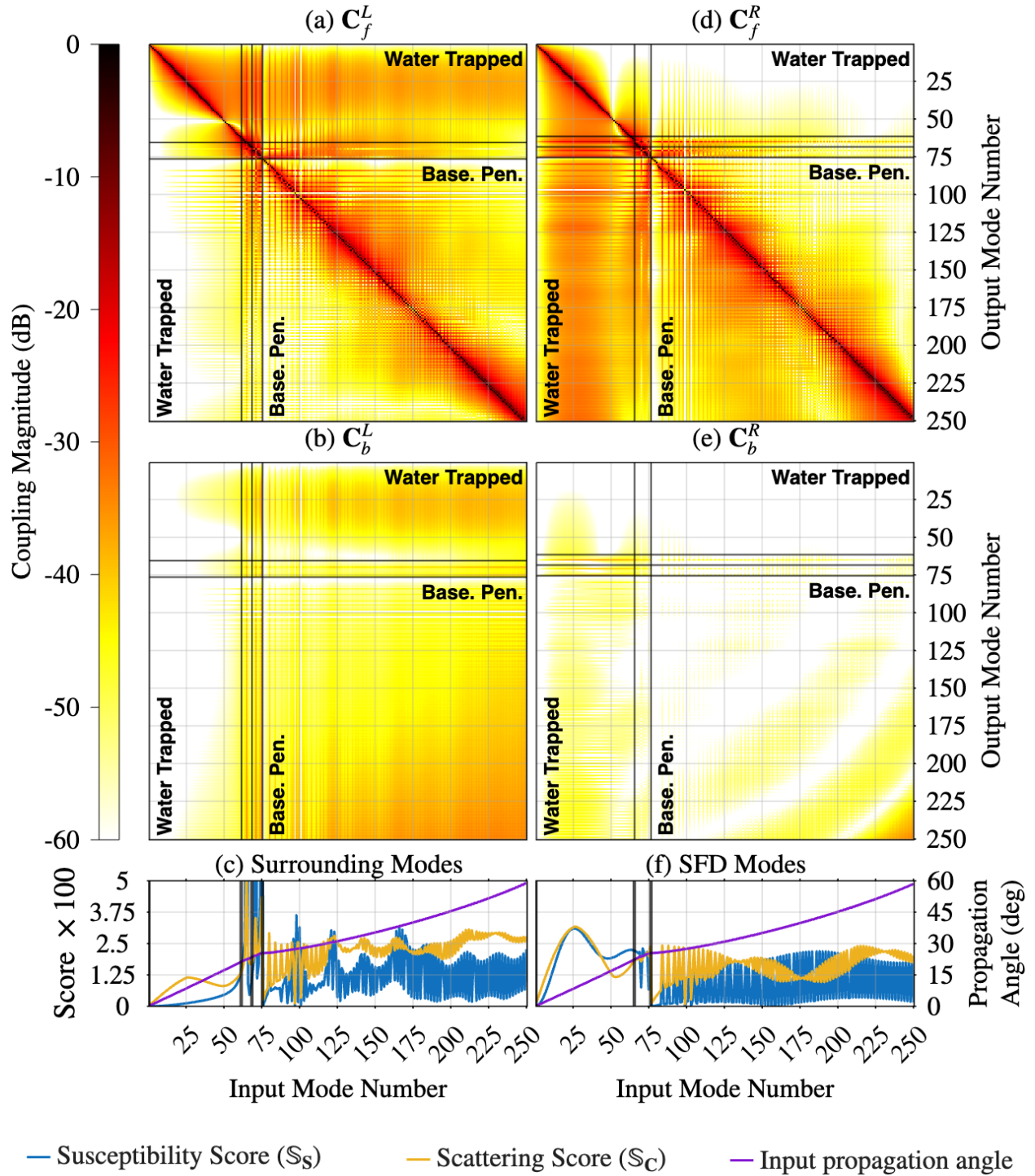


Figure 5.3: Modal coupling at 250 Hz for the first 250 modes. (a) and (b) are the left-edge coupling matrices C_f^L and C_b^L shown in decibel scale, panels (d) and (e) are the corresponding right-edge matrices. (c) and (f) summarize the modal properties for incidence at each edge, including propagation angle, susceptibility and scattering.

having appreciable susceptibility and as a function of their aspect ratio represent redistribution across a spectrum of output wavenumbers or conversely, ducting of a broad spectrum of angles into a single output channel depending on the block orientation. The backward coupling matrices are more selective. Their strongest columns correspond to modes with large susceptibility scores and extend vertically over broad ranges of output mode number. The two edges also approximately exhibit the weighted transpose relationship in Equation (3.20). Although each matrix shown in Figure 5.3 is a composition of many interfaces and very short propagation segments to achieve the effect of a sloping transition, the resulting operators retain the reciprocal structure of the underlying modal projections. Overall, coupling behavior is localized in depth, organized by modal family, and has the most dramatic effect for modes whose support overlaps the environmental change.

Modes trapped in the water column have negligible sub-bottom support apart from an evanescent tail and therefore interact only weakly with the left SFD edge. The corresponding region of $\mathbf{C}_f^L$ in Figure 5.3(a) is dominated by its diagonal backbone, along with a redistribution among modes confined to the water column, driven by the change in seafloor depth and modal wavenumbers. The respective entries of $\mathbf{C}_b^L$ in Figure 5.3(b) remain weak.

At the right edge, the entirety of the water incidence blocks of the coupling matrices $\mathbf{C}_f^R$ and $\mathbf{C}_b^R$ in Figures 5.3(d) and (e) both have elevated entries. For $\mathbf{C}_b^R$ this indicates that all watercolumn trapped modes induce scattering, in particular across the entire steeper modal spectrum; this is driven by the upward bathymetry step which ensures these modes receive significant contributions via $\delta\rho$ and $\delta\Psi$. For $\mathbf{C}_f^R$, scattering augments projection into the shallower confined mode space, while Horizontal bands in the water-column-incidence portions of $\mathbf{C}_f^R$ indicate coupling into the first sedimentary stratum.

Modes associated with field variability in the sedimentary layers have substantial support in the seabed, as evidenced by elevated susceptibility scores. Their interaction of primary interest occurs at the left edge; both $\mathbf{C}_f^L$ and $\mathbf{C}_b^L$ have dominant columns in the blocks representing modes with phase speeds comparable to the sedimentary layer speeds. This shows that one incident mode can excite numerous outgoing modes. Because these outputs span a broad range of horizontal

wavenumbers, the response includes both forward- and backward-propagating fields over a broad range of propagation angles. Modes with steeper angles undergo a similar, but slightly less powerful interaction. At the right edge, significant interactions with sediment interacting sound occur only for very steep angles which are more sensitive to the upward slope of the SFD wall.

5.2.3 Incoherent full-field Scattering Response

When considering the effect of the SFD on an associated pressure field as a function of a single frequency, it is important to note that the largest intensity changes will be fundamentally connected to shifts in modal phase rather than amplitude. To suppress the outsize effect that a small shift in modal phase has with only a single frequency, it is important to consider a broadband signal; where the phase shifting will be different across the frequency band so that the associated intensity variations will be smoothed when considering the incoherent broadband; thus allowing the more important effects of the SFD to be more easily visualized.

For this section only, the coupling matrices were evaluated at a representative center frequency rather than recomputed throughout the full signal bandwidth. Incoherent broadband behavior within each range-independent segment was approximated using the relationship between frequency-averaged and range-averaged modal intensity developed by Harrison and Harrison [32]. This procedure suppresses the fine interference structure associated with a single-frequency solution while avoiding repeated full coupled-mode calculations across the entire band. Its application here assumes that the dominant coupling structure varies slowly over the bandwidth of interest. Because the original approximation was developed for range-independent propagation, its use within the segmented range-dependent calculation was checked against an explicit broadband calculation. The range dependent extension of Harrison and Harrison's method offered great advantages in terms of computational efficiency.

Full-field results where all modes are allowed to be excited by the source except those that can penetrate the lowest sediment layer, are shown in Figure 5.4.

While the coupling matrices provide a compact representation of how energy is redistributed

between modal families, examination of the resulting full acoustic fields permits a more direct physical interpretation of the scattering process. To isolate the behavior associated with specific modal families, simulations were conducted in which only a single incident mode was excited at the source. The resulting pressure fields therefore visualize how energy associated with a particular mode evolves as it propagates across the SFD. The modes selected for individual excitation were chosen to represent distinct propagation regimes within the waveguide. Modes 1, 65, and 85 sample progressively more sediment-interacting modal families. Modes whose energy is naturally distributed between multiple strata generally exhibit behavior analogous to a superposition of the layer-confined cases discussed below.

Modes confined primarily to the water column interact comparatively weakly with the leading SFD edge. The full-field simulations show that the dominant water-column component of the incident field is only weakly modified. This behavior is consistent with the quasi-diagonal structure of the corresponding portion of $\mathbf{C}_f^L$. For these modes, the second edge produces substantially stronger scattering. In the full-field simulations, waterborne energy incident on the upward bathymetric transition generates clear forward scatter and backscatter in the surrounding water column. This behavior is consistent with the nonzero entries in the waterborne portions of $\mathbf{C}_f^R$ and $\mathbf{C}_b^R$, which extend to high-order output modes. Some energy also couples into the first stratum, consistent with the horizontal band in $\mathbf{C}_f^R$.

Energy associated with the first sedimentary layer strongly samples the region of bathymetric and stratigraphic perturbation, and interaction with the SFD produces strong scattering into up and downward propagation angles at the left edge. The forward-propagating sound splits into two primary components: one that propagates through the SFD wall and is upward refracted, and another that remains trapped within the sediment beneath the SFD, as shown in Figure 5.5. Energy associated with the second sedimentary layer experiences comparatively weak scattering at the left edge, however, the modified sound speed structure in the SFD permits refraction out.

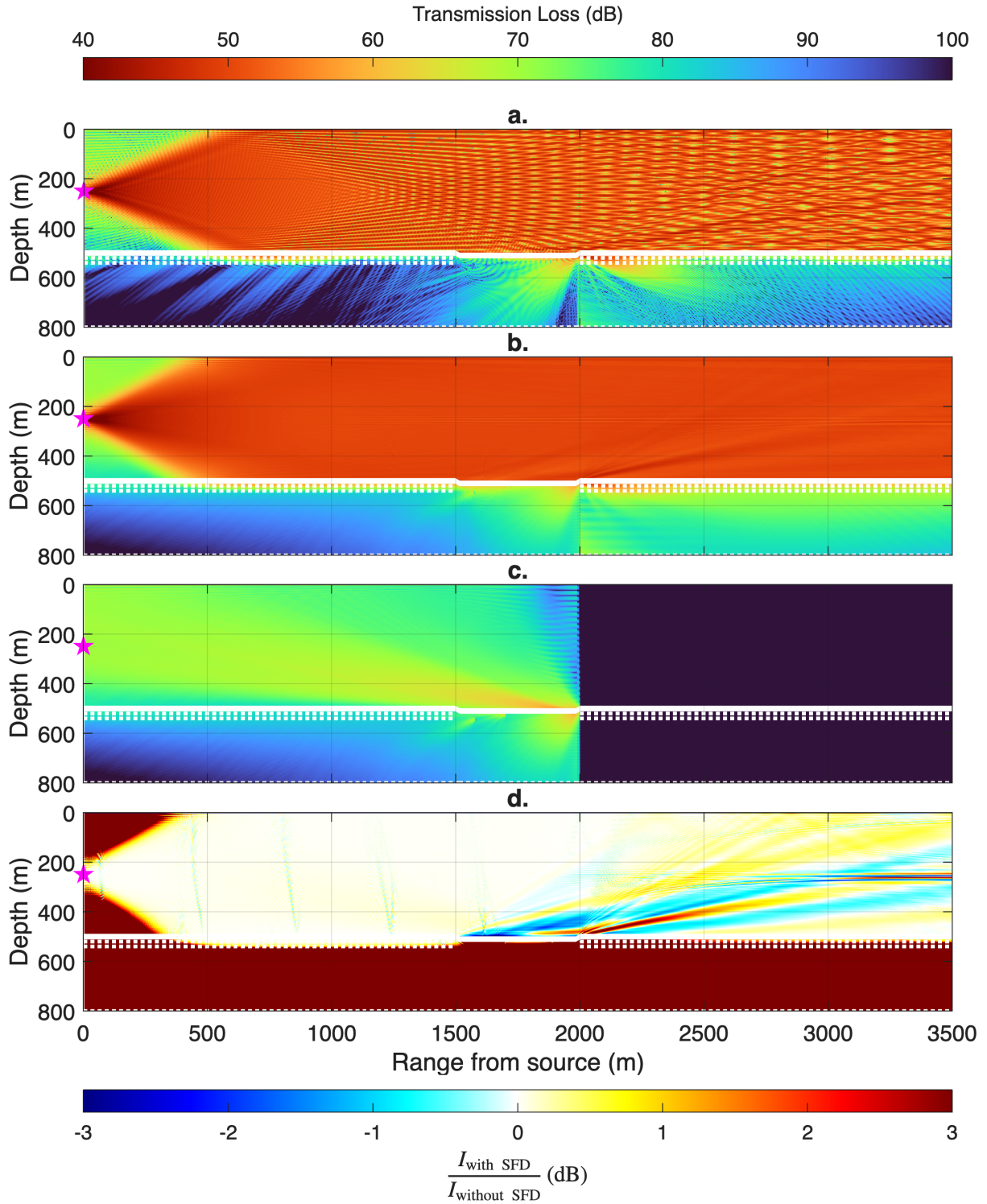


Figure 5.4: Broadband-smoothed full-field results at a 250-Hz center frequency and 300-Hz smoothing bandwidth. Panels (a)–(c) show the total two-way field, forward-going field, and backscattered field, respectively. Panel (d) shows the intensity-level change relative to the no-SFD reference field, $\Delta L_I = 10 \log_{10}(I_{\text{SFD}}/I_{\text{ref}})$.

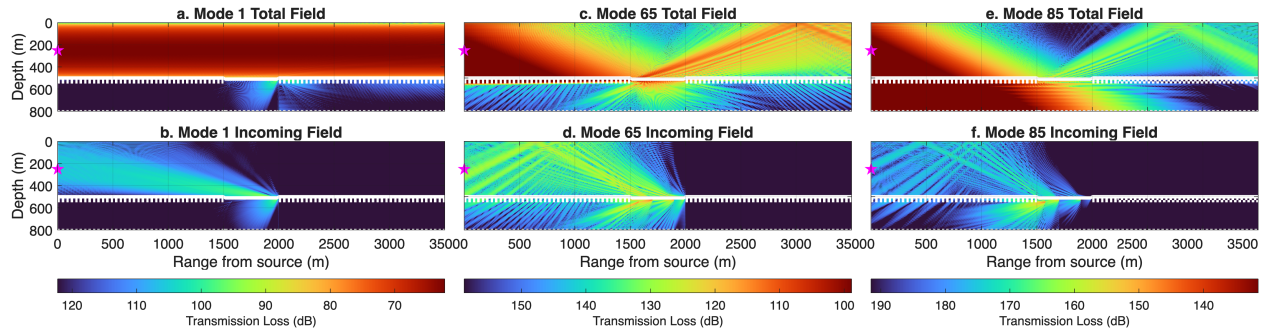


Figure 5.5: Total and backscattered fields for selected source-excited modes. Panels (a) and (b) show the total and backscattered fields for mode 1, panels (c) and (d) show the corresponding fields for mode 65, and panels (e) and (f) show the corresponding fields for mode 85.

5.2.4 Illustrative Full-Field Scattering Response

In this section and section 5.2.5 a true broadband calculation was performed, evaluating the normal modes and coupling matrices at every frequency bin. This calculation included modes with a very steep propagation angle and slow group speed to help resolve the nearly vertical propagation paths involved, but since computational resources were limited some significant time wrapping effects are evident, in particular near the source. The time-domain calculation summarized in Fig. 5.6 shows that, as the incident pulse reaches each SFD edge, that edge temporarily behaves as a point-source-like scatterer. The newly generated field initially forms an approximately circular wavefront centered on the edge. This wavefront contains components scattered back into the water column, transmitted through the SFD, and coupled downward into the sediment. This is consistent with the coupling matrices: first-layer penetrating modes scatter over a broad range of angles at the left edge, while waterborne energy has a similar interaction at the right edge.

The downward-coupled component propagates through sediment whose compressional sound speed exceeds that of the water column and therefore advances ahead of the waterborne portion of the pulse. As it propagates, the sediment-borne field continuously reradiates energy into the water column, producing a characteristic head wave contribution [10, 11]. Separate head-wave branches originate at the left and right SFD edges. The resulting field is a superposition of direct waterborne propagation, edge scattering, and faster sediment-borne paths that return energy to the water column at later ranges.

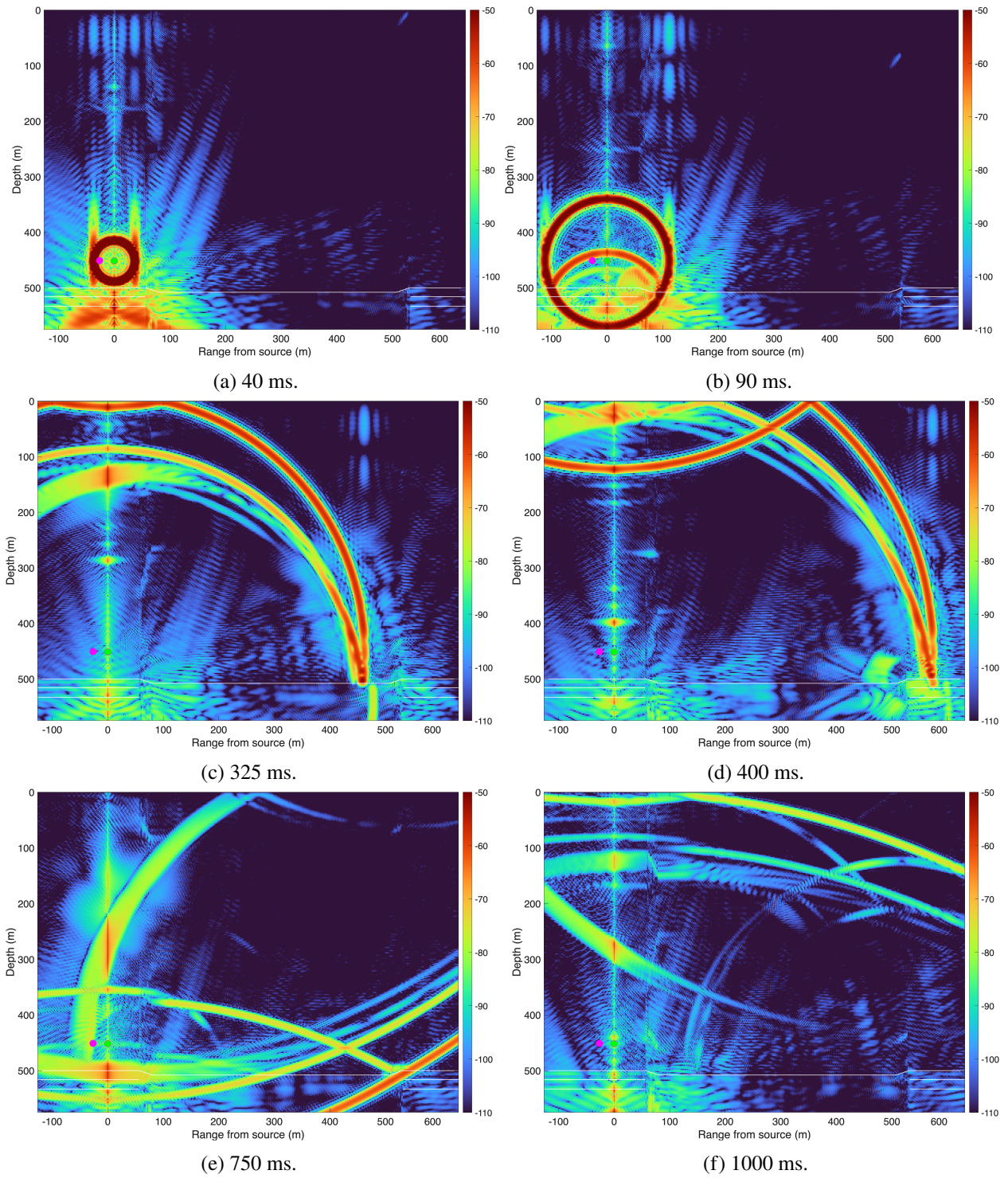


Figure 5.6: Time domain reconstruction of the model results for transmit cycle 400 at selected pulse transit times. The source and array centroid locations are marked by green and magenta dots respectively.

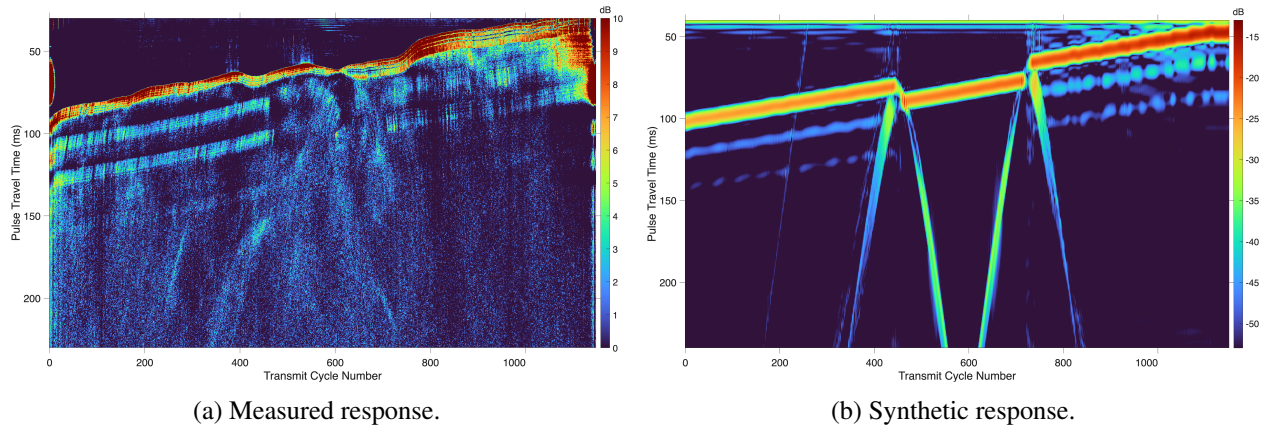


Figure 5.7: Comparison of measured and modeled arrival structures. (a) Measured response from the REMUS 600 transect, pulse-compressed, incoherently beamformed, and directionally filtered. (b) Synthetic response evaluated at the moving towed-array centroid for the 250-Hz model.

5.2.5 Comparison of Modeled and Observed Arrival Structure

To compare the modeled field directly with the experimental data, the broadband calculation was sampled using the motion of the AUV–towed-array system. At each modeled transmission time, the source and receiver were advanced along the survey trajectory and a separate field solution was evaluated at the instantaneous position of the towed-array centroid. In the model the AUV altitude is reduced with increasing transmit cycle; this simulates the sloping bottom in the data without requiring many additional range independent regions and coupling computations. The resulting time-domain responses were assembled by pulse travel time and transmit cycle, producing a synthetic record with the same basic coordinates as the measured cross-correlation data. Scattering from a fixed location can therefore be followed as the source and receiver approach, pass, and recede from that location. The measured and modeled records are compared in Figure 5.7.

Figure 5.7(a) is a beamformed, directionally filtered record, which reveals the backscattered arrival structure most plainly, but removes nearly stationary arrival structure. As a result, the direct source to array path is distorted, and the displayed intensity level is not indicative of the intensity actually measure by the array. The figures of Chapter 2 may be referenced to see a representation of the received acoustic intensity and the undistorted direct path arrival; the following analysis is primarily concerned with gross structure rather than exact replication of the data. The discrete

scattering spheres used throughout the rest of the present thesis were omitted to ease interpretation. Figure 5.7(b) contains several instances of time wrapped arrivals attributable to simulating the problem at an insufficient number of frequencies. Specifically, the two groups of arrivals that slope down and to the right are wrapped arrivals of surface bounces, and the horizontal arrival around 125 ms is the wrapped arrival of a surface and bottom interaction.

Several distinct arrivals can be identified directly in both the measured and modeled records. The earliest is the direct source-to-array path, followed by the seafloor-reflected arrival. Their identification is unambiguous from their travel times, continuity across the transect, and response to the changing source–receiver geometry. The model reproduces both arrival classes and their broad evolution as the AUV crosses the SFD. The model further resolves arrivals associated with the sub-bottom material property interfaces. Outside the SFD, these returns follow the layered seafloor and occur at delays determined by the depths and sound speeds of the individual layers, terminating where the interfaces are truncated by the SFD edges.

More importantly, the model contains diagonally sloping delayed traces that follow a parabolic trajectory, passing through a minima in pulse travel time at the SFD edges. These traces are generated when incident acoustic energy interacts with a sloping SFD edge and is scattered back toward the moving array. The trace slope results from the pulse-to-pulse change in the total source–edge–receiver path as the AUV approaches or recedes from the fixed interaction point. In the data, this feature is strongly identifiable only in the region below the interfacial discontinuity returns. Because the modeled traces originate at the known edge locations and exhibit the same approximate orientation as the corresponding measured traces, they can be identified as manifestations of the same mechanism. These scattering events are modeled in Figure 5.6(b) and (d) for the left and right edges, respectively.

Finally, the model contains a pair of traces which slope at a more gentle angle, each angled toward the opposite wall side and dropping strongly in intensity at around 150 ms pulse time, nearly coincident with the SFD edge. These arrivals can be attributed to a path that leaves the source, backscatters off the far SFD edge into a steep angle, reflects off the sea surface, and returns to the

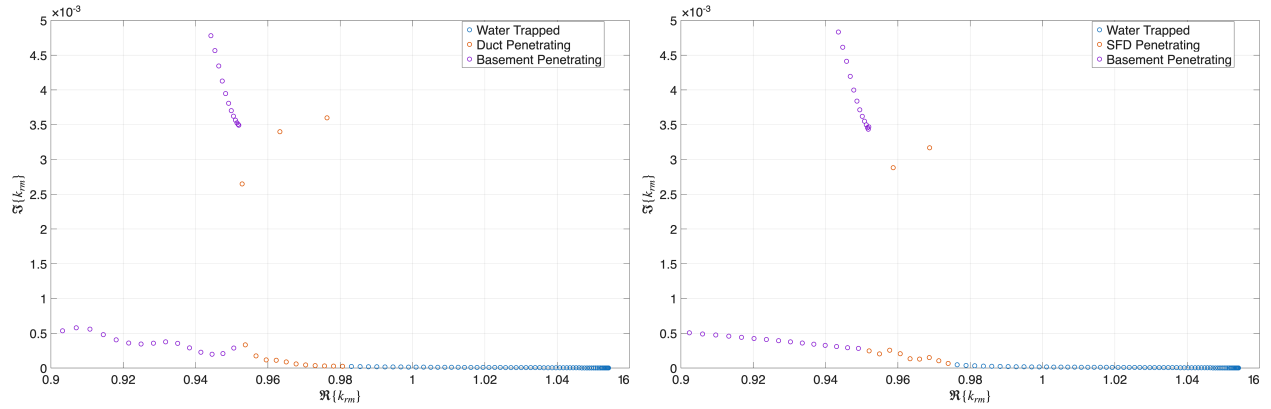
array. This provides one possible explanation for the traces in the data with the same approximate position and orientation, dropping out nearly entirely at the SFD edge. This surface reflected scatter is modeled for transmit cycle 400 in Figure 5.6(e) and (f), shown advancing toward the source before arriving at approximately 1150 ms pulse time and then being displayed on the next cycle's record at approximately 150 m/s in Figure 5.7(b) due to the 1 seconds pulse repetition rate.

Thus, the comparison does more than establish a general resemblance between the two fields. It connects specific measured arrivals to specific mechanisms in the numerical environment: direct waterborne propagation, seafloor reflection, reflection or propagation associated with persistent sub-bottom interfaces, and backscatter generated where those interfaces intersect the SFD edges.

5.3 Sub-Bottom Duct Scenario

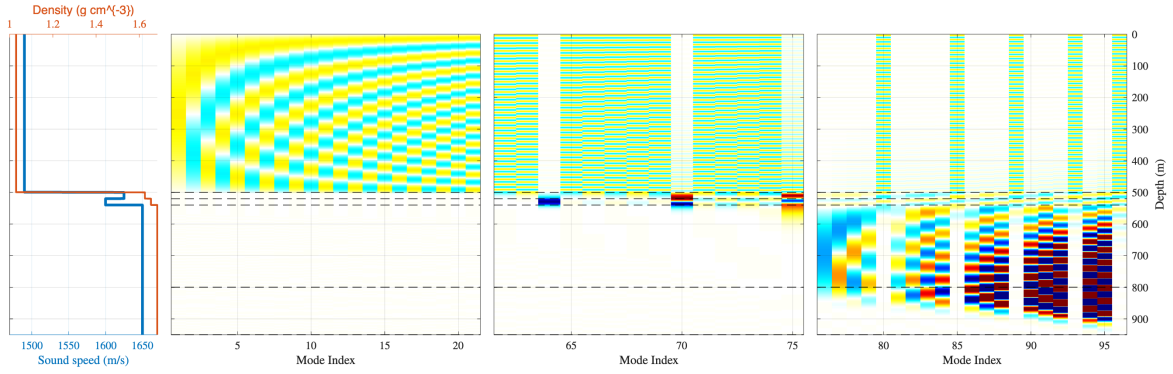
The second environmental configuration, referred to as the *duct scenario*, is included as a sensitivity study. Relative to the primary scenario, the ordering of the first two sediment sound speeds is reversed, creating a local sound-speed minimum within the second sedimentary stratum. This structure supports a more distinct ducted modal family, while modes associated primarily with variability in the faster stratum above it are more weakly confined or leaky. The duct scenario is used to test whether the scattering mechanisms identified in the primary scenario persist when sediment-interacting energy is organized into a more clearly isolated ducted family. The primary scenario provided the chief analysis, and the duct scenario is considered where its response differs notably.

5.3.1 Description of Local Mode Bases

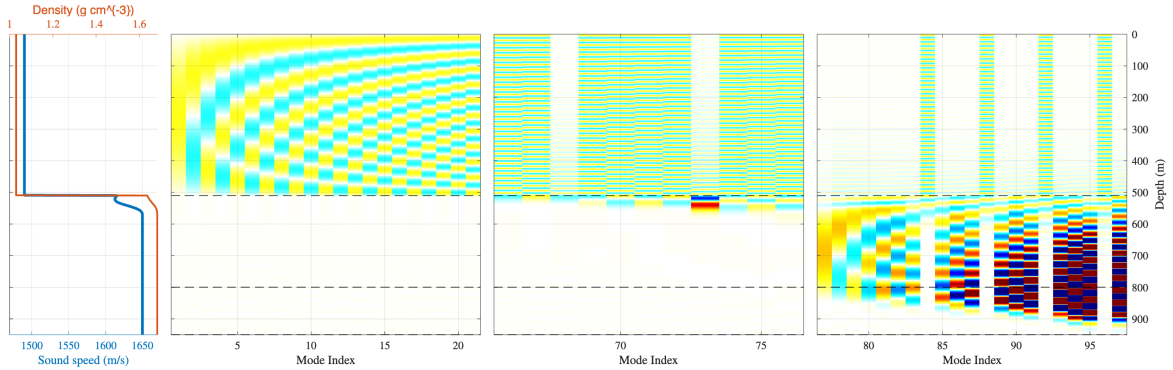


(a) Surrounding-stratigraphy modal spectrum.

(b) SFD modal spectrum.



(c) Surrounding-stratigraphy mode groups.



(d) SFD mode groups.

Figure 5.8: Normal mode bases for the 250-Hz duct scenario. (a) and (b) show the modal spectra in the complex horizontal-wavenumber plane for the surrounding and SFD environments, respectively. (c) and (d) show the corresponding sound-speed profiles and the first 20 modes from each identified group, together with their phase velocity, group velocity, and propagation angle.

The normal modes are displayed in Figure 5.8. In the duct scenario, as k_r goes up, sound is first able to propagate in the duct itself rather than the first layer. Accordingly, mode 64 is the first mode of the duct in the surrounding region, and mode 70 is the second. Modes firmly associated with variability in the first layer are interleaved with modes that are duct associated and higher order, leaky modes.

5.3.2 Structure of the Coupling Matrices

Like the primary scenario, in the duct scenario the modal coupling matrices exhibit structural features consistent with the perturbative decomposition. Figure 5.9 shows the coupling matrices for the first 250 modes together with the susceptibility and scattering diagnostics. The duct scenario coupling matrices are not substantially different from the primary scenario in the water column incidence region. In the region of layer borne sound incidence, less extreme scattering scores and less accentuated columns are seen. This is consistent with the duct scenario allowing sediment penetrating sound to become primarily entrained in a region that does not encounter the bathymetric step at the left edge; in other words the duct is underneath the step. Elevated scattering and susceptibility are seen in certain portions of the steeper, leaky incidence region, corresponding to higher order modes which are strongly associated with the duct. Response of the right edge is not substantively different from the primary scenario.

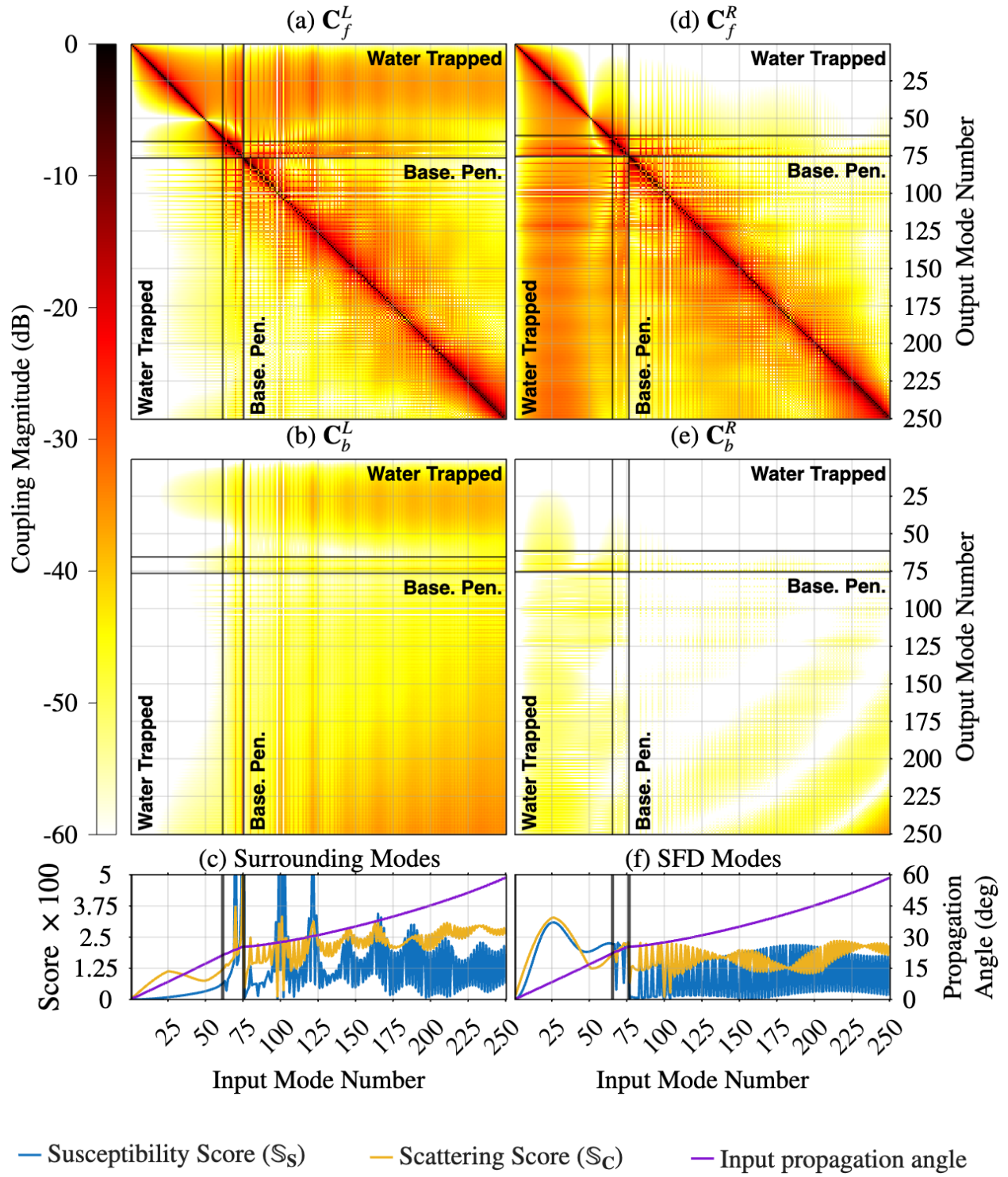


Figure 5.9: Modal coupling for the duct scenario at 250 Hz for the first 250 modes. (a) and (b) are the left-edge coupling matrices C_f^L and C_b^L shown in decibel scale, (d) and (e) are the corresponding right-edge matrices. Panels (c) and (f) summarize the modal properties for incidence at each edge, including angle, susceptibility and scattering.

5.3.3 Incoherent full-field Scattering Response

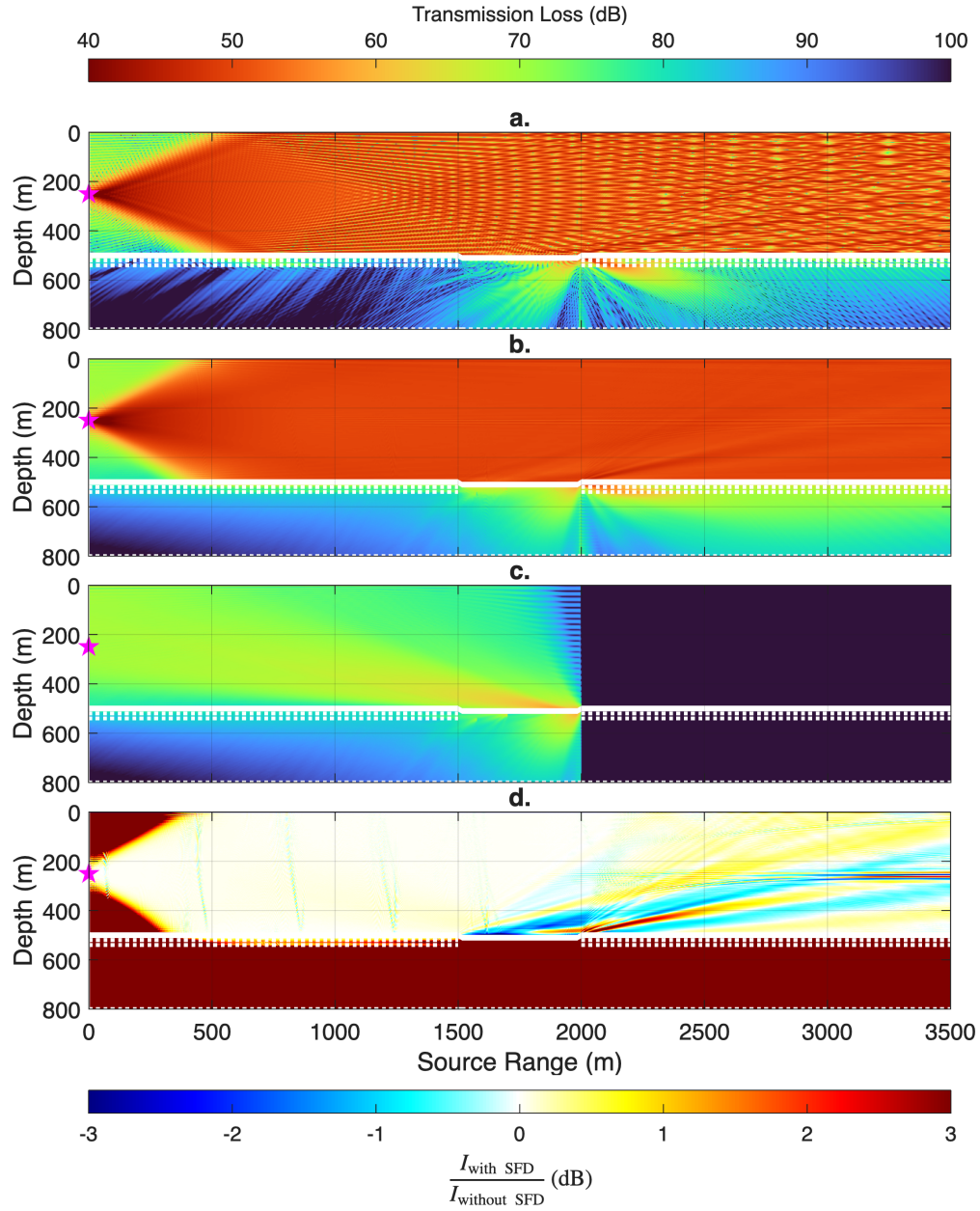


Figure 5.10: Broadband-smoothed full-field results for the duct scenario at a 250-Hz center frequency and 300-Hz smoothing bandwidth. Panels (a)–(c) show the total two-way field, forward-going field, and backscattered field, respectively. Panel (d) shows the intensity-level change relative to the no-SFD reference field, $\Delta L_I = 10 \log_{10}(I_{\text{SFD}}/I_{\text{ref}})$.

For this section, like the corresponding section for the primary scenario, the problem was evaluated utilizing a range dependent implementation of the smoothed modal intensity model

developed by Harrison and Harrison for results using the full spectrum of trapped source modes [32].

Energy associated with the second sedimentary layer is best showcased by the duct scenario. It experiences comparatively weak scattering at the left edge, however, the modified sound speed structure in the SFD permits refraction out. At the right edge, the sound field of this mode becomes re-ducted into the surrounding environment with relatively little additional scattering as shown in Figure 5.11.

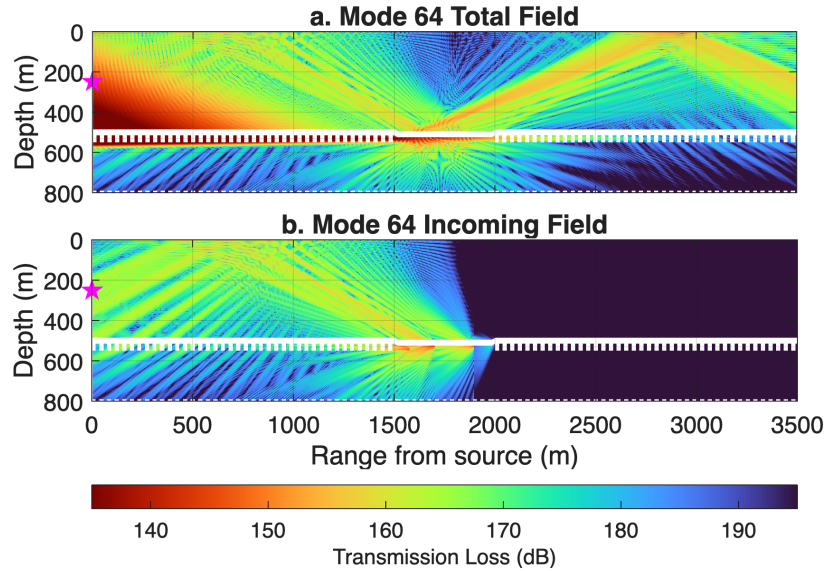


Figure 5.11: Duct-scenario total and backscattered fields for source-excited mode 64 in panels (a) and (b), respectively.

CHAPTER 6

Conclusion

6.1 Discussion

REMUS 600 observations reveal anomalous sub-bottom and waterborne returns localized near a large SFD on the NESB. Several arrivals approach or terminate where the surrounding stratigraphy is disrupted, indicating that the SFD modifies sediment-interacting propagation paths. A two-way coupled-mode model based on an idealized version of the observed geometry shows that the sloping edges and interrupted sedimentary layers can convert sediment-guided energy into waterborne, leaky, and backward-propagating components.

Modal susceptibility is governed primarily by the extent to which an incident mode samples the perturbed sub-bottom. Modes with low susceptibility undergo a coupling event which could be primarily described as simple projection of their energy onto the new basis, whereas sediment-interacting modes exhibit larger susceptibility and greater scattering evidenced by vertically extended, higher valued, coupling-matrix columns. The two edges need not affect a modal family in the same way: the leading edge strongly redistributes sediment-associated energy, while the trailing upward transition can scatter water-column energy that crossed the depression with comparatively little modification. The total response is controlled jointly by the bathymetric transition, the change in sub-bottom stratigraphy, and the length of the feature.

Because the field measurements and numerical calculations use substantially different frequencies and the modeled environment is deliberately idealized, the comparison is mechanistic rather than quantitative. Nevertheless, both the calculations demonstrate the processes which cause the scattering evident in the real acoustic field.

6.2 Future Work

The relative timing and strength of waterborne, sediment-guided, and backscattered arrivals from a pulse transmitted at a SFD may constrain its width, wall slope, depression depth, sediment-layer thickness, compressional sound speed, density, and attenuation. Because many of these parameters are correlated and the observed environment is only partially known, a probabilistic inversion would be preferable to a single best-fit estimate. Bayesian methods could combine prior geological information with measured arrival times, beamformed scattering location estimates, measured modal energy redistribution, and frequency-dependent loss to quantify both the details of the sediment stratigraphy and the geometry of the SFD which interrupts it, along with the associated uncertainty of such estimates.

The framework may also be extended beyond an isolated SFD. Closely spaced SFDs could be treated as a distributed scattering medium in which repeated mode conversion produces cumulative attenuation, modal diffusion, and loss of coherence over range; this may permit definition of an effective modal scattering mean free path for a SFD field. Understanding the net effect of repeated interaction with SFDs could be important for characterizing the acoustic response of regions densely populated with these features, such as continental shelves.

APPENDIX A

Derivation of the Local Coupling Matrices

In a range-dependent environment, the acoustic pressure field in the j th segment, $p^j(r, z)$, can be written as a superposition of the normal modes local to that segment:

$$p^j(r, z) = \sum_{m=1}^M \left[a_m^j \widehat{H}_m^{(1)}(r) + b_m^j \widehat{H}_m^{(2)}(r) \right] \Psi_m^j(z), \quad (\text{A.1})$$

where the $\Psi_m^j(z)$ are the normalized modal eigenfunctions and $\widehat{H}_m^{(1),j}(r)$ and $\widehat{H}_m^{(2),j}(r)$ govern the modal phase evolution along the segment:

$$\widehat{H}_m^{(1),j}(r) = \frac{H_0^{(1)}(k_{r,m}^j r)}{H_0^{(1)}(k_{r,m}^j r_{j-1})}, \quad \widehat{H}_m^{(2),j}(r) = \frac{H_0^{(2)}(k_{r,m}^j r)}{H_0^{(2)}(k_{r,m}^j r_j)}, \quad (\text{A.2})$$

with $k_{r,m}^j$ the modal radial wavenumbers, a_m^j the outgoing modal amplitude, and b_m^j the incoming modal amplitude for mode m in the j th segment. In the first segment:

$$\widehat{H}_m^{(1),1}(r) = H_0^{(1)}(k_{r,m}^1 r), \quad \widehat{H}_m^{(2),1}(r) = H_0^{(2)}(k_{r,m}^1 r), \quad (\text{A.3})$$

To determine the relationship between the modal amplitudes in each segment, the global solution must satisfy the continuity of pressure and radial particle velocity along each interface.

Imposing the interface conditions at r_j eventually gives:

$$a_\ell^{j+1} + b_\ell^{j+1} \widehat{H}_\ell^{(2),j+1}(r_j) = \sum_{m=1}^M \left[a_m^j \widehat{H}_m^{(1),j}(r_j) + b_m^j \right] \widetilde{C}_{\ell m} \quad (\text{A.4})$$

$$\widetilde{C}_{\ell m} = \int \frac{\Psi_\ell^{j+1}(z) \Psi_m^j(z)}{\rho_{j+1}(z)} dz \quad (\text{A.5})$$

$$a_\ell^{j+1} - b_\ell^{j+1} \widehat{H}_\ell^{(2),j+1}(r_j) = \sum_{m=1}^M \left[a_m^j \widehat{H}_m^{(1),j}(r_j) - b_m^j \right] \widehat{C}_{\ell m} \quad (\text{A.6})$$

$$\widehat{C}_{\ell m} = \frac{k_{r,m}^j}{k_{r,\ell}^{j+1}} \int \frac{\Psi_\ell^{j+1}(z) \Psi_m^j(z)}{\rho_j(z)} dz. \quad (\text{A.7})$$

Collecting the equations for each mode combination into matrices:

$$\mathbf{a}^{j+1} + \mathbf{H}_2^{j+1} \mathbf{b}^{j+1} = \widetilde{\mathbf{C}}^j \left(\mathbf{H}_1^j \mathbf{a}^j + \mathbf{b}^j \right) \quad (\text{A.8})$$

$$\mathbf{a}^{j+1} - \mathbf{H}_2^{j+1} \mathbf{b}^{j+1} = \widehat{\mathbf{C}}^j \left(\mathbf{H}_1^j \mathbf{a}^j - \mathbf{b}^j \right) \quad (\text{A.9})$$

where $\mathbf{H}_1^j$ and $\mathbf{H}_2^{j+1}$ denote diagonal matrices with entries $\widehat{H}_m^{(1),j}(r_j)$ and $\widehat{H}_m^{(2),j+1}(r_j)$, respectively. The two equations in (A.9) define the pressure- and radial-velocity-matching maps $\widetilde{\mathbf{C}}^j$ and $\widehat{\mathbf{C}}^j$, respectively. Their sum and difference produce the direction-preserving coupling matrix $\mathbf{C}_f^j$ and the direction-reversing coupling matrix $\mathbf{C}_b^j$, as developed in the main text.

REFERENCES

- [1] D. S. Brothers, C. Ruppel, J. W. Kluesner, U. S. ten Brink, J. D. Chaytor, J. C. Hill, B. D. Andrews, and C. Flores, “Seabed fluid expulsion along the upper slope and outer shelf of the u.s. atlantic continental margin,” *Geophysical Research Letters*, vol. 41, pp. 96–101, 2014.
- [2] L. H. King and B. MacLean, “Pockmarks on the scotian shelf,” *Geological Society of America Bulletin*, vol. 81, no. 10, pp. 3141–3148, 1970.
- [3] M. Hovland and A. G. Judd, *Seabed Pockmarks and Seepages: Impact on Geology, Biology, and the Marine Environment*. London: Graham & Trotman, 1988.
- [4] M. Hovland, J. V. Gardner, and A. G. Judd, “The significance of pockmarks to understanding fluid flow processes and geohazards,” *Geofluids*, vol. 2, no. 2, pp. 127–136, 2002.
- [5] J. Gafeira, D. Long, and D. Diaz-Doce, “Semi-automated characterisation of seabed pockmarks in the central north sea,” *Near Surface Geophysics*, vol. 10, no. 4, pp. 301–312, 2012.
- [6] R. P. Stokey, A. Roup, C. von Alt, B. Allen, N. Forrester, T. Austin, R. Goldsborough, M. Purcell, F. Jaffre, G. Packard, and A. Kukulya, “Development of the REMUS 600 autonomous underwater vehicle,” in *OCEANS 2005 MTS/IEEE*, vol. 2, pp. 1301–1304, 2005.
- [7] M. S. Ballard, D. D. Garcia, K. M. Lee, G. R. Venegas, A. R. McNeese, P. S. Wilson, and J. D. Chaytor, “Direct measurements of sediment geoacoustic properties in the new england mud patch and shelf break,” *The Journal of the Acoustical Society of America*, vol. 156, no. 4, pp. 2889–2902, 2024.
- [8] D. P. Knobles, P. S. Wilson, J. A. Goff, L. Wan, M. J. Buckingham, J. D. Chaytor, and M. Badiy, “Maximum entropy derived statistics of sound-speed structure in a fine-grained sediment inferred from sparse broadband acoustic measurements on the new england continental shelf,” *IEEE Journal of Oceanic Engineering*, vol. 45, no. 1, pp. 161–173, 2020.

- [9] P. Pfenninger and Y.-T. Lin, “Geoacoustic inversion of sub-bottom profile data using an autonomous underwater vehicle equipped with a sound source and towed hydrophone array,” *The Journal of the Acoustical Society of America*, vol. 158, no. 3, pp. 2602–2614, 2025.
- [10] G. V. Frisk, *Ocean and Seabed Acoustics: A Theory of Wave Propagation*. Englewood Cliffs, New Jersey: Prentice Hall, 1994.
- [11] F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, *Computational Ocean Acoustics*. New York: Springer, 2 ed., 2011.
- [12] A. D. Pierce, “Extension of the method of normal modes to sound propagation in an almost-stratified medium,” *The Journal of the Acoustical Society of America*, vol. 37, no. 1, pp. 19–27, 1965.
- [13] S. R. Rutherford and K. E. Hawker, “Consistent coupled mode theory of sound propagation for a class of nonseparable problems,” *The Journal of the Acoustical Society of America*, vol. 70, no. 2, pp. 554–564, 1981.
- [14] R. B. Evans, “A coupled mode solution for acoustic propagation in a waveguide with stepwise depth variations of a penetrable bottom,” *The Journal of the Acoustical Society of America*, vol. 74, no. 1, pp. 188–195, 1983.
- [15] M. D. Collins and R. B. Evans, “A two-way parabolic equation for acoustic backscattering in the ocean,” *The Journal of the Acoustical Society of America*, vol. 91, no. 3, pp. 1357–1368, 1992.
- [16] J. F. Lingeitch, M. D. Collins, M. J. Mills, and R. B. Evans, “A two-way parabolic equation that accounts for multiple scattering,” *The Journal of the Acoustical Society of America*, vol. 112, no. 2, pp. 476–480, 2002.
- [17] E. L. Hamilton and R. T. Bachman, “Sound velocity and related properties of marine sediments,” *J. Acoust. Soc. Am.*, vol. 72, no. 6, pp. 1891–1904, 1982.

- [18] J. B. Burland, “On the compressibility and shear strength of natural clays,” *Géotechnique*, vol. 40, no. 3, pp. 329–378, 1990.
- [19] M. O’Regan, M. Forwick, M. Jakobsson, K. Moran, and D. C. Mosher, “Seafloor cratering and sediment remolding at sites of fluid escape,” *Geology*, vol. 43, no. 10, pp. 895–898, 2015.
- [20] M. J. Buckingham, “Wave speed and attenuation profiles in a stratified marine sediment: Geo-acoustic modeling of seabed layering using the viscous grain shearing theory,” *J. Acoust. Soc. Am.*, vol. 148, no. 2, pp. 962–974, 2020.
- [21] D. H. Johnson and D. E. Dudgeon, *Array Signal Processing: Concepts and Techniques*. Prentice Hall, 1993.
- [22] E. L. Hamilton, “Sound velocity–density relations in sea-floor sediments and rocks,” *J. Acoust. Soc. Am.*, vol. 63, no. 2, pp. 366–377, 1978.
- [23] M. B. Porter and E. L. Reiss, “A numerical method for ocean-acoustic normal modes,” *The Journal of the Acoustical Society of America*, vol. 76, no. 1, pp. 244–252, 1984.
- [24] M. B. Porter and E. L. Reiss, “A numerical method for bottom-interacting ocean acoustic normal modes,” *The Journal of the Acoustical Society of America*, vol. 77, no. 5, pp. 1760–1767, 1985.
- [25] B. V. Numerov, “A method of extrapolation of perturbations,” *Monthly Notices of the Royal Astronomical Society*, vol. 84, no. 8, pp. 592–602, 1924.
- [26] L. F. Richardson, “The approximate arithmetical solution by finite differences of physical problems involving differential equations, with an application to the stresses in a masonry dam,” *Philosophical Transactions of the Royal Society of London. Series A*, vol. 210, pp. 307–357, 1911.
- [27] G. H. Golub and C. F. V. Loan, *Matrix Computations*. Baltimore, Maryland: Johns Hopkins University Press, 4 ed., 2013.

- [28] S. Güttel and F. Tisseur, “The nonlinear eigenvalue problem,” *Acta Numerica*, vol. 26, pp. 1–94, 2017.
- [29] L. M. Delves and J. N. Lyness, “A numerical method for locating the zeros of an analytic function,” *Mathematics of Computation*, vol. 21, no. 100, pp. 543–560, 1967.
- [30] V. C. Anderson, “Sound scattering from a fluid sphere,” *The Journal of the Acoustical Society of America*, vol. 22, no. 4, pp. 426–431, 1950.
- [31] J. J. Faran, “Sound scattering by solid cylinders and spheres,” *The Journal of the Acoustical Society of America*, vol. 23, no. 4, pp. 405–418, 1951.
- [32] C. H. Harrison and J. A. Harrison, “A simple relationship between frequency and range averages for broadband sonar,” *The Journal of the Acoustical Society of America*, vol. 97, no. 2, pp. 1314–1317, 1995.