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Cilium Driven Flow Near a Free Surface

A Thesis submitted in partial satisfaction
of the requirements for the degree

Master of Science
in
Mechanical Engineering

by

Ryan Michael Abell

Committee in charge:

Professor Emilie Dressaire, Chair
Professor Paolo Luzzatto-Fegiz
Professor Yangying Zhu

June 2026

The Thesis of Ryan Michael Abell is approved.

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June 2026

Cilium Driven Flow Near a Free Surface

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by

Ryan Michael Abell

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Abstract

Cilium Driven Flow Near a Free Surface

by

Ryan Michael Abell

Cilia are microscopic hair-like structures which transport fluid in biological systems. In mucociliary clearance, cilia pump a multilayered film of fluid to remove contaminants from the lungs. At points deep in the lungs (where the mucus layer becomes thin), the cilia interact near the free surface, an effect which has been largely unexplored in an experimental set-up. We first develop an analytic model for the flow response to cilium motion confined between a free-slip surface and a no-slip boundary. Next, we design an experimental set-up which can approximate the motion of biologic cilia using a rotating magnet and a polymer seeded with iron particles. Through particle image velocimetry (PIV), we show deviation from theory in the response of the fluid, specifically that under certain conditions the flow can completely reverse from the expected direction. Lastly, we show there is a transient response occurring which would violate the conditions in a purely Stokes flow and we posit that this could be due surface effects. These results have implications for ciliated organisms moving near free surfaces and for mucociliary transport in the regions of the bronchi where the mucus layer is thin.

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Chapter 1

Introduction

Cilia are hairlike structures which are ubiquitous in nature. Broadly, cilia can be divided into two types: non-motile and motile. Non-motile cilia perform sensing functions such as olfactory cilia, which are responsible for the sense of smell [1]. Motile cilia are often used for the propulsion of an organism within a fluid or for propulsion of a fluid in an organism. Paramecium, for example, is a genus of single celled organisms, commonly found in bodies of water, whose surface is covered with cilia which beat to produce propulsion [2]. Internal motile cilia move fluid around for a variety of purposes. For instance, cilia help circulate cerebrospinal fluid (CSF), which delivers nutrients to the brain [3]. Another example of motile cilia comes from the early stages of embryonic development, where cilia pump fluid from right to left as an early step of asymmetry being established in the human body [4], which involves both motile cilia for fluid propulsion and non-motile cilia for flow sensing. Lastly, we note cilia's important role in the biological function which is the primary inspiration for this research: mucociliary clearance.

Mucociliary clearance is the process by which the cilia lining the walls of the lungs remove contaminants. As the lungs expand, particulates enter and stick to the layer of mucus lining the surface of the airways. This mucus is continuously pumped by cilia

moving in metachronal waves to the trachea where it is swallowed. The mucus has a viscosity $\mu \approx 48.2 \text{ mPa s}$ while being transported by cilia [5], although it is shear thinning [6]; for standard flow in the tracheobronchial regions of the lungs, the shear rate does not exceed the critical shear rate [7, 8]. Beneath that layer of mucus is a less viscous fluid which contains cilia, called the “periciliary layer” or the PCL. The PCL is of viscosity similar to that of water $\mu = 1 \text{ mPa-s}$ [5]. Although this layer is substantially less viscous, the fluid remains in the Stokes flow regime due to the small size of the cilia, 6-7 μm in length, and 0.2-0.3 μm in diameter [9]. The height of the PCL is slightly higher than the cilia at around 7 μm , while the mucus thickness varies from the bronchioles where it is 0.1 μm thick, to the trachea where it reaches upwards of 100 μm [10]. Notably, where the mucus layer is thin the cilia are moving close to a free surface. This interaction is the primary inspiration for this work, as there is limited literature covering this biologically relevant interaction.

Various methods have been devised to create “magnetically actuated cilia,” or thin flexible fibers which can be moved with an external magnet to manipulate fluids. One method is to use a mold with a ferrofluid that crosslinks. The mold can be created with lithography using SU8 or other photoresist [11]. Alternatively, one can use polycarbonate track etched (PCTE) porous membranes as a mold for a lithography free approach [12]. Another common method is to fabricate the system from the bottom up where cilia are formed with a magnet that induces spiking in the ferrofluid through the Rosensweig instability [13]. For an alternative, cleanroom-free method, one can perform polymer extrusion, where an external magnet draws out fibers from a ferrofluid, which stick to another surface [14]. For a recent review of cilia fabrication including methods and applications see Park et al. [15]. In this work we create a single cilium with a borosilicate glass micro-capillary.

A common (experimentally feasible) approximation of the motion of cilia in nature is

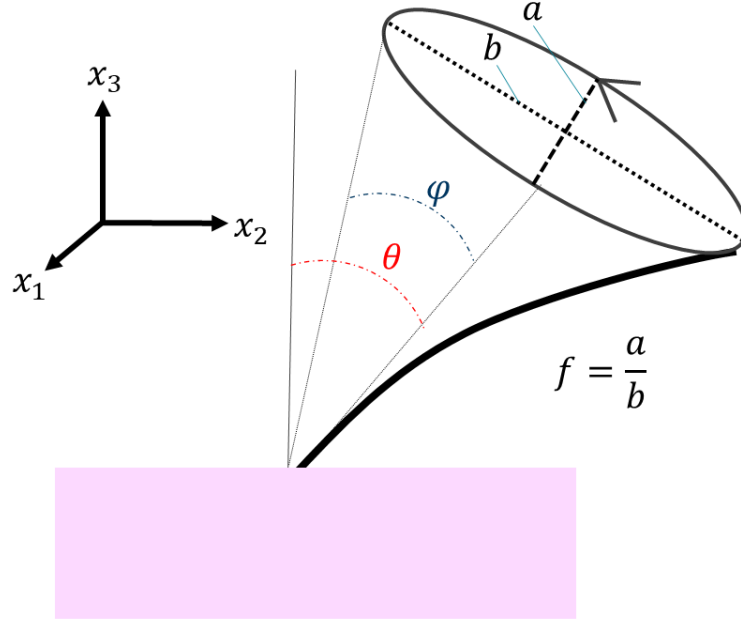


Figure 1.1: Tilted conical motion with elliptical correction.

a “tilted conical motion” as shown in figure 1.1 [16]. In this trajectory, cilia trace out a cone in space, where the “tilt angle,” denoted θ , represents the angle between the cone’s central axis and the normal to the surface. The “opening angle,” denoted ϕ , represents the angle from the cone’s central axis to the furthest extent of the cilia. If the tip of the cilium does not trace out a perfect circle, a correction can be made based on the aspect ratio of the ellipse f [14]. This tilted-cone trajectory generates a net flow even at low Reynolds numbers. This due to the principle that a point force near a surface will generate less flow than a point force further from a surface, mathematically represented by the half domain stokeslet [17] versus the infinite domain Oseen tensor.

In tilted conical motion, the effective stroke is when the cilium is tall and moving in the direction of the generated net flow. Here, the tip of the cilium is far from the solid surface and hence generates substantial flow. During the recovery stroke, the cilium tip is closer to the solid surface and hence generates less flow. When averaged over the

course of a cycle, the cilium generates a net flow in the direction of the effective stroke due to the asymmetry with the boundary conditions. For the geometry presented in 1.1, the generated flow rate is out of the page and is proportional to $\omega \sin(\theta) \sin^2(\phi)$, where ω represents the beat frequency [18]. We note this model has been experimentally verified [16, 14] for unconfined domains.

Additional efforts have been made to more rigorously characterize the flow near cilia. Shields et al. showed that cilia can create forward flow above their tips while simultaneously creating an enhanced diffusion region near the no-slip surface [19]. Other studies have investigated the dynamics of particles in the vicinity of a cilium, showing an absence of net flow when the tilt angle is sufficiently reduced [20]. Lastly, computational studies have delved into the nature of the cilia in the biologically relevant case of mucociliary clearance. Specifically, it has been shown that under certain conditions, backflow is possible in the near field within PCL, while the mucus layer can continue moving forward [21]. It is important to note the phenomenon cited here is local and is not predicted to continue in the far field in the absence of additional cilia.

Most relevant to this work, an investigation revealed the dynamics of cilia confined between two nearby no-slip planes [22]. As experimentally and theoretically shown, this can lead to a net backflow of fluid being pumped, even far away from the cilia. The model here provides a starting point to analyze the cilia with a free surface confinement.

Our study investigates the effect of a free surface near the tip of a magnetically actuated cilium during pumping. We show that in realistic laboratory conditions, one can achieve a net backwards flow despite a free-slip model predicting only forward fluid motion. Free surfaces can often act drastically different in experiments compared to theory driven by the free-slip assumption. An early example is the rising dynamics of bubbles covered in surfactants. Boussinesq proposed in 1913, an effective “surface viscosity” which could account for the difference in the velocity of rising fluid droplets compared

to the Hadamard–Rybczynski equation [23]. More recently, trace amounts of surfactants have been uncovered to be the cause of super-hydrophobic surfaces underperforming theoretical predictions in drag reduction [24]. In this case, the shear flow over the plastron causes surfactants to accumulate at the stagnation point leading to a Marangoni stress opposing the bulk fluid flow. Lastly, flagellated organisms such as *E. coli* accumulate near no-slip surfaces and swim in a clockwise direction due to an asymmetry in their body forcing [25]. From this same theoretical framework, flagellated swimmers near a free surface are expected to rotate in a counterclockwise direction; however, experiments do not consistently show this behavior. This has been explained by presence of a complex interface, which can modify the expected circulation behavior in conjunction with the ratio of the fluid viscosity at the interface and the geometry of the specific organism (accounting for different behavior within the same population) [26]. The same work notes that the presence of surfactant densely packed at the free surface is not well captured by a partial-slip (as modeled in polymer depletion layers), due to the constraint of 0 divergence in the surface plane.

To investigate the effect of a free surface near a cilium, we begin by developing an experimental set-up which can mimic to motion of cilia and the resulting flow profile at low Reynolds numbers.

Chapter 2

Magnetically Actuated Cilium Experiments

2.1 Experimental Methods

To fabricate a magnetically actuated cilium, we use the Zhermack Elite Double 8, vinylpolysiloxane (VPS), 2 part silicone elastomer in conjunction with iron particles. An overview of the procedure is shown in figure 2.1. The base and the catalyst component crosslink at room temperature without ultraviolet exposure. To fabricate the cilium, first the catalyst is mixed with Iron(II,III) oxide ($Fe_2O_3 < 5\mu m$) powder depending on the target particle loading. This is mixed by hand for 2 minutes before degassing in a vacuum chamber for 10 minutes. Next, the base is mixed into the ferrofluid, forming a 1:1 base:catalyst ratio. A 0.3mm borosilicate capillary is inserted into the resulting fluid and a vacuum is used to bring the fluid into the capillary. Separately, the catalyst and base are mixed and poured into a mold which will form the non-magnetic base of the cilium. A magnet (K&J Magnetics, 1.25" diameter x 1" height, N42, 5597 Gauss surface field) is placed below the base which aligns the particles during the curing process.

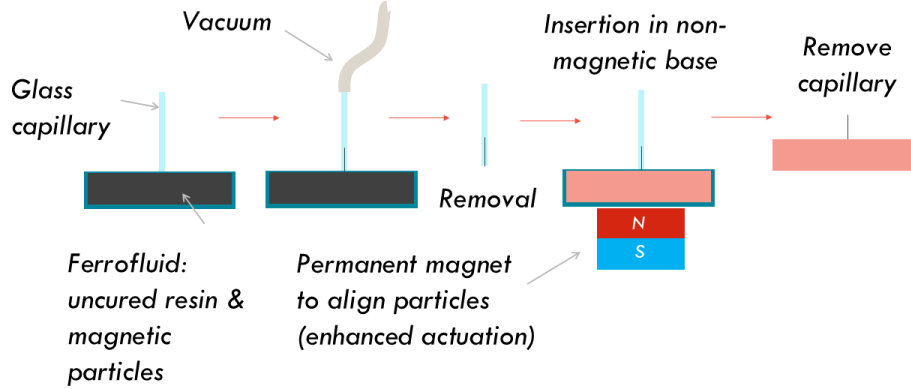
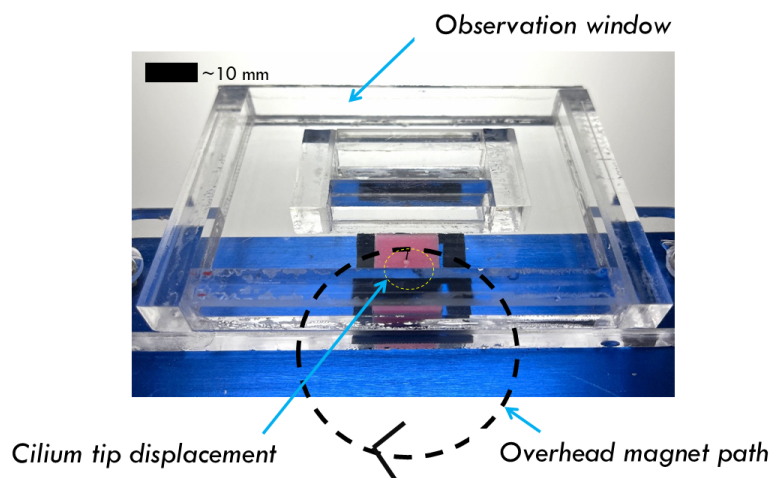


Figure 2.1: Fabrication procedure for the presented magnetically actuated cilium.

After the ferrofluid has cured for 10 minutes, the capillary is lowered into the base with an adjustable z-stage. After a 10+ hour cure, the cilium is removed by breaking the capillary with a pair of tweezers, while being submerged in ethanol. The ethanol helps prevent glass adhesion to the cilium. While higher levels of iron loading can increase actuation, higher iron loading can also weaken the cilium and make it prone to damage during unmolding.

To test our magnetically actuated cilium, we utilize a closed track system fabricated from solvent-glued laser-cut acrylic shown in figure 2.2. The magnet passes above the base of the cilium and revolves around an axis offset from the cilium. This offset causes the cilium to track the magnet in space thus moving in a tilted conical motion.

In this work, we measure flow velocities with particle image velocimetry (PIV) to track fluid motion. The PIV relied on $10\mu\text{m}$ silver coated hollow glass spheres submerged in glycerol to ensure Stokes flow. Using a mirror, we shine a laser to illuminate the vertical central plane of the observation window as shown in figure 2.3. Before each experiment, the fluid was allowed to rest overnight to prevent residual motion. Analysis was performed with Dantec Dynamics software.



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Figure 2.2: Overview of the fluid track.

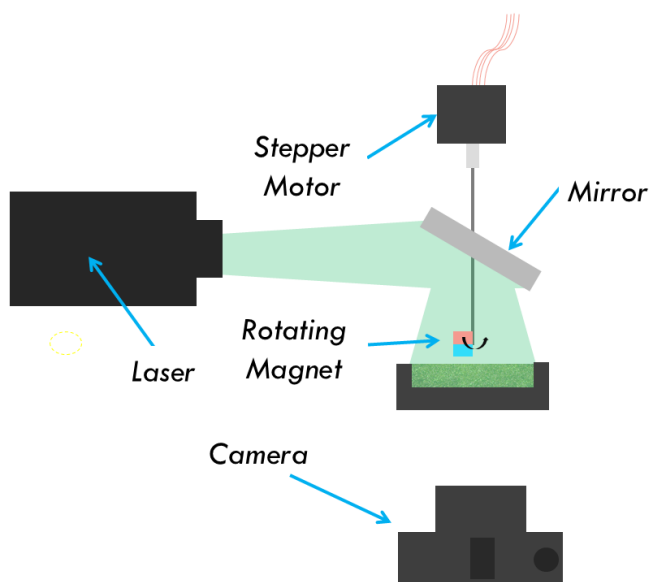


Figure 2.3: Diagram of the PIV set-up.

2.2 Cilium Motion

To analyze the flow response to a magnetically actuated cilium, the cilium motion must be parameterized to the tilted conical motion. In this system, the magnetic force which deforms the cilium is not much larger than the loss due to viscous dissipation, as shown in figure 2.5. This means that the motion of the cilium is dependent on the frequency of the forcing. We start by tracking the motion of the cilium using a Phantom high speed camera as shown in figure 2.4.

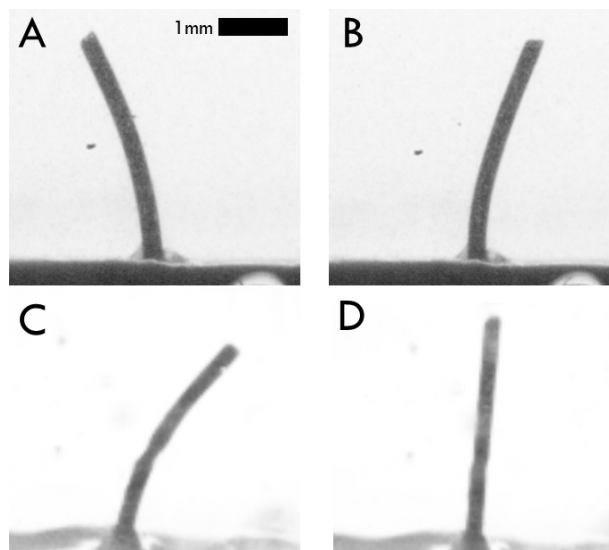


Figure 2.4: Video of cilium motion at 0.5Hz. **a**: Side view, leftmost deflection. **b**: Side view, rightmost deflection. **c**: Front view, recovery stroke. **d**: Front view, effective stroke.

This information can be used to extract the tilt angle θ , opening angle ϕ , and the aspect ratio f as shown in figure 1.1. The tilt angle and opening angle can be directly measured from the front view of the cilium. The aspect ratio requires the side view of the cilium. To eliminate the need to scale distances in each video, we calculate the aspect ratio with the following formula:

$$f = \frac{\tan \theta_1 + \tan \theta_2}{\sqrt{2 - 2 \cos(2\phi)}}, \quad (2.1)$$

where θ_1 and θ_2 are the angles formed between the maximum point of deflection and the vertical axis as viewed from the side during the rightmost extension and leftmost extension of the cilium respectively [14].

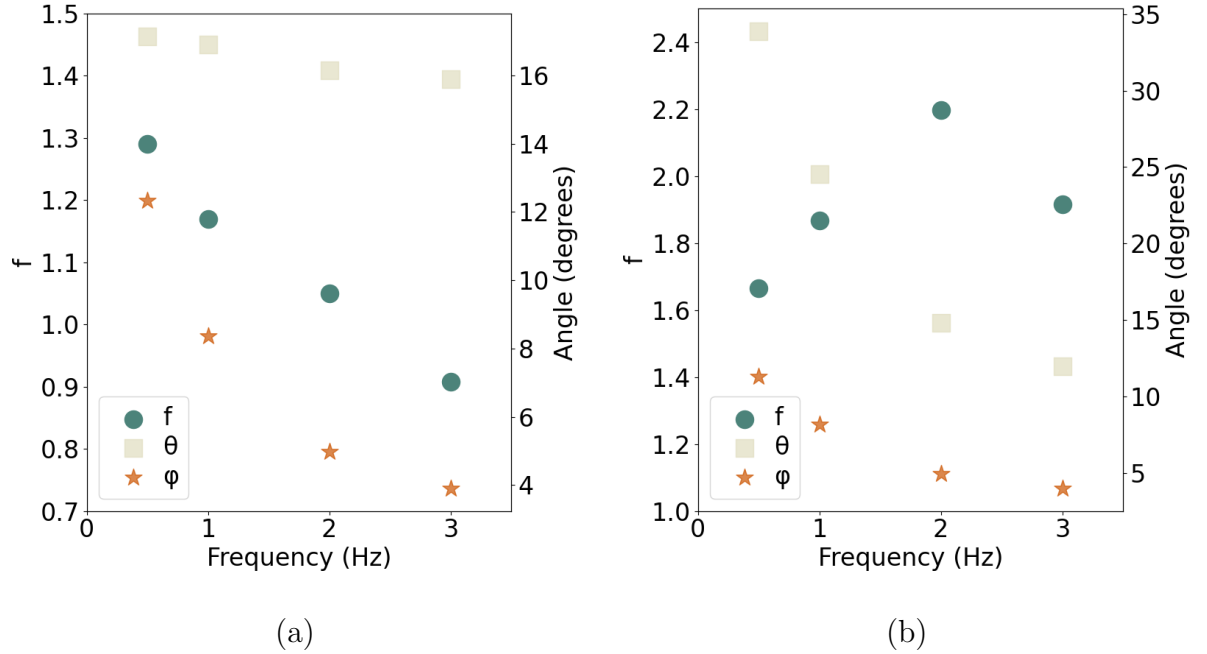


Figure 2.5: Cilium motion including opening angle ϕ , tilt angle θ , and aspect ratio f . **a:** Cilium fabricated from ferrofluid resin consisting of mass ratio 2:4:4 (iron particles, VPS base, VPS catalyst). **b:** Cilium fabricated from ferrofluid resin consisting of mass ratio 3:4:4 (iron particles, VPS base, VPS catalyst).

We utilize two different iron loadings in the elastomer to change the cilium motion as shown in figure 2.5. For both mass loadings, we see similar opening angles, however, the tilt angles and aspect ratios are substantially higher for the higher mass loading. This indicates that the higher loading — 3:4:4 (iron particles, VPS base, VPS catalyst) - creates a cilium which tracks closer to the magnet’s motion for the same frequency. The

tradeoff here is the 3:4:4 loading is a less reliable manufacturing method than the 2:4:4 resin which breaks less frequently during unmolding.

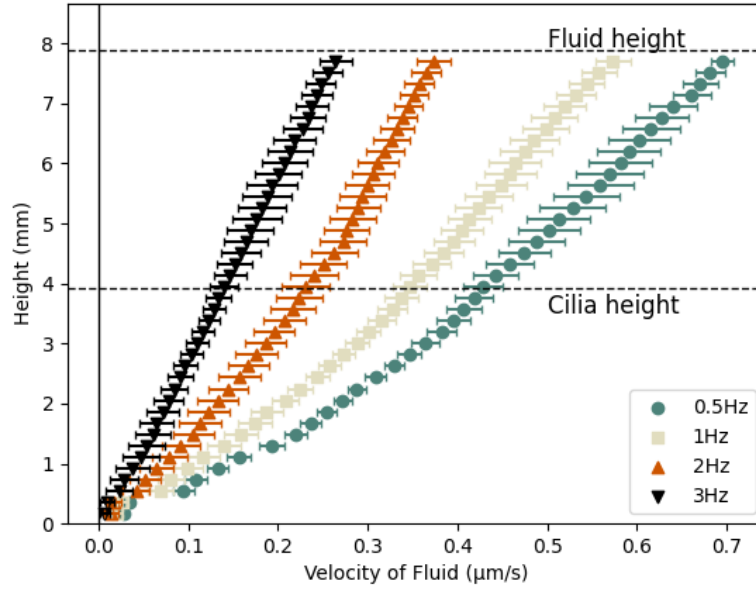
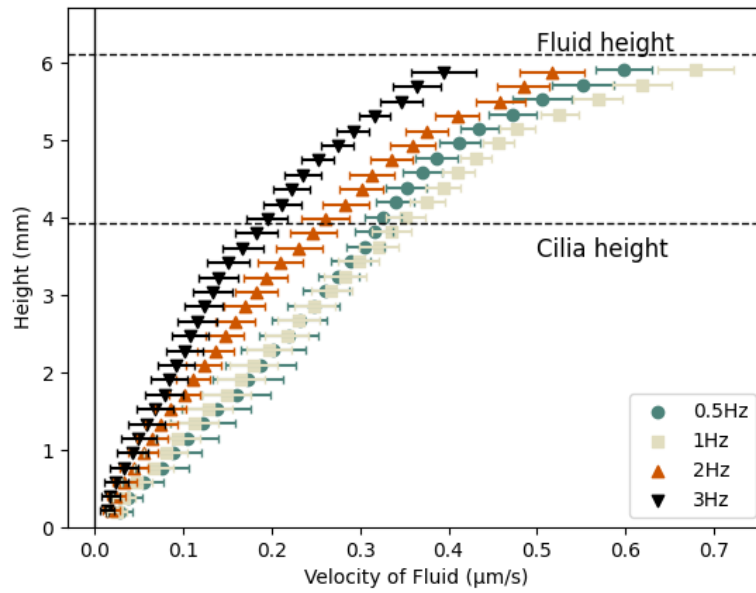
Regardless of mass loading, the tilt angle and opening angle decrease with frequency. This is due to the viscosity of the fluid preventing the cilium from tracking the magnet in space. In the literature, this relationship has been shown to cause the flow rate to decrease with frequency or to behave in a non-monotonic manner[14]. While the opening angle and tilt angle decrease consistently with frequency, the aspect ratio shows some non-monotonic behavior at high frequencies.

2.3 PIV Results

We start by using the high mass loading cilium and performing a sweep over different fluid heights. The cilium length is 3.92 mm and we further define the confinement $C=L/H$ where L is the length of the cilium and H is the height of the fluid. We start with a low confinement, $C=0.50$, with velocity profiles plotted in figure 2.6. Here, the length of cilium is half of the height of the fluid.

From this figure, one can see the no-slip condition at the bottom as well as the fastest velocity at the top of the profile as expected. Additionally, the speed of the flow is decreasing with frequency, due to the effects of viscosity on the tilt and opening angle discussed in section 2.2.

Next we lower the height of the free surface to investigate $C=0.64$ in figure 2.7. The velocity still generally decreases with frequency, although, we start to see deviation from the expected flow profile for a free surface. While the no-slip condition is observed on the bottom, the top surface has an apparent shear stress as evident by $\frac{dV}{dx_3} \neq 0$ at $x_3 = H$. Furthermore, there is a change in the sign of the curvature in the profile from $C=0.50$ to $C=0.64$.

Figure 2.6: Velocity profile $C=0.50$ with a free surface.Figure 2.7: Velocity profile $C=0.64$ with a free surface.

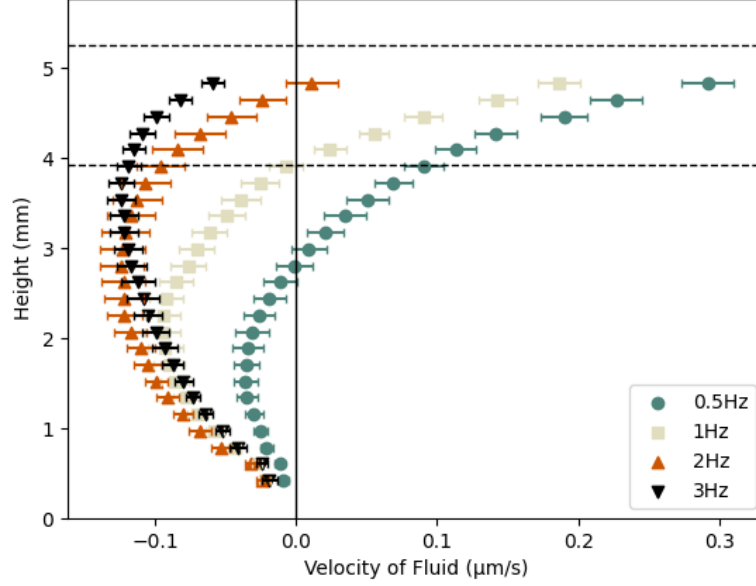


Figure 2.8: Velocity profile $C=0.75$ with a free surface.

Lastly, we study a high confinement value $C=0.75$. The resulting flow profile is plotted in figure 2.8. Here, the deviation from the expected flow profile becomes undeniable. Not only are we seeing a complete change from the expected half parabola, we now distinctly see a multilayered flow, with fluid moving in the direction opposite to the effective stroke. This is a complete reversal from what is possible in theoretical model discussed in section 1. This model indicates a flow rate proportional to $\omega \sin(\theta) \sin^2(\phi)$. Physically, θ and ϕ cannot add to more than 90° , therefore the resulting flow can never be negative. Intuitively, this is captured by the theory as the effective stroke moves fluid further from the no slip boundary than the recovery stroke, generating more flow in the direction of the effective stroke than in the direction of the recovery stroke.

2.4 Preliminary Transient Analysis

To investigate the root cause of this unexpected behavior, we analyze the response of the flow rate over time. These trials were done with a low iron loading cilium (2:4:4), with cilium length 3.85 mm. We actuate the cilium for 40 minutes with 15 minutes of rest before and after each trial. Trials are conducted at 0.5 Hz. The results are shown in figure 2.9.

From the transient results, we can note a few things of interest. The experiments performed here are done at low Reynolds number ($Re \sim 10^{-3}$), as calculated by the diameter and velocity of the cilium indicating the Stokes flow regime. Therefore, it is expected that the fluid should respond instantly to changes in forcing: when the motor is turned off, we expect the velocity to immediately drop to 0. Instead, the net flow becomes negative when the motor is turned off, regardless of the confinement. After some time, the velocity returns to 0. Additionally, at the highest confinement, we clearly see a “ramp up” period before the velocity reaches a steady state value.

To understand this behavior we start by developing a model which accounts for the presence of a surface near the moving cilium.

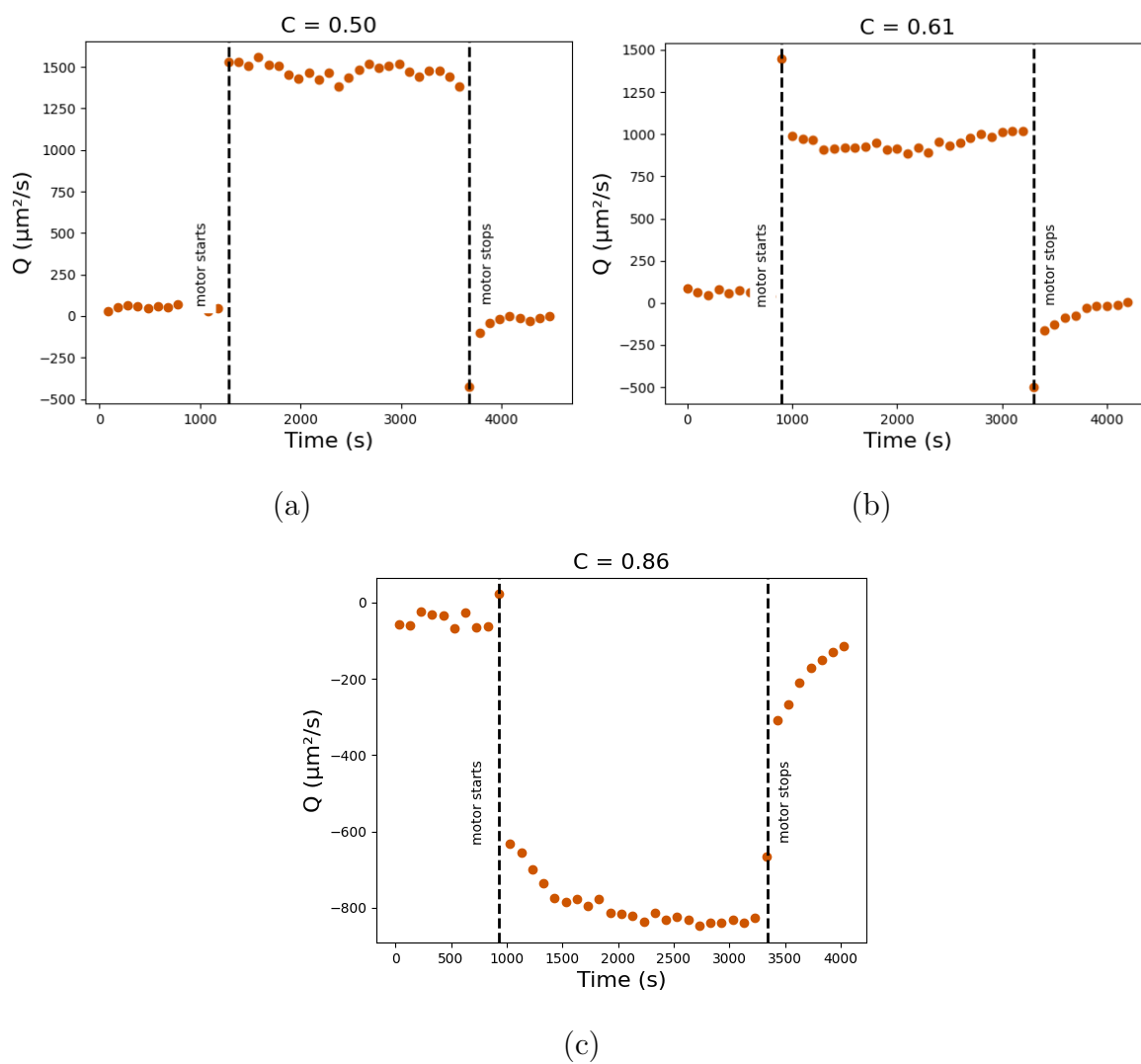


Figure 2.9: Transient results across multiple confinements at 0.5Hz.

Chapter 3

Towards a Theoretical Model

3.1 Predicted Flow With a Free-Slip Surface

To a first approximation, a free surface acts like a free-slip condition. This assumes that the deformation of the surface is negligible. Our mathematical framework closely follows the derivation by Sun et al. [22]. First, we parametrize the motion of the cilium as shown in equations 3.1. We note the use of capital letters (X_1, X_2, X_3) to denote a point on the cilium and lowercase letters (x_1, x_2, x_3) to denote a point of interest in space.

$$\begin{aligned} X_1(s, t) &= sf \sin \phi \sin \omega t \\ X_2(s, t) &= s \cos \phi \sin \theta - s \sin \phi \cos \omega t \cos \theta \\ X_3(s, t) &= s \cos \phi \cos \theta + s \sin \phi \cos \omega t \sin \theta \end{aligned} \tag{3.1}$$

Here, we introduce s , which represents the distance along the length of the cilium, ω is the actuated beat frequency and t is time.

Next we require a stokeslet to give us the velocity response in direction i due to a unit force in j . In this case, we are interested in the net flow in the x_1 direction which corresponding to the direction of the effective stroke. We use the leading order terms for

the far field stokeslet of a thin film free-slip geometry as derived in Mathijssen et al. [27], equation B4, given here in equation 3.2.

$$\mathcal{F}_{ij} = \frac{3x_3}{\pi\mu} \left(1 - c\frac{x_3}{H}\right) \frac{X_3(s, t)}{H} \left(1 - c\frac{X_3(s, t)}{H}\right) \left[\frac{\delta_{\alpha\beta}}{2\rho^2} - \frac{r_\alpha r_\beta}{\rho^4}\right] \delta_{i\alpha} \delta_{j\beta} + O(e^{-\rho/H}) \quad (3.2)$$

Note that the forcing in the x_3 direction is symmetric with respect to the boundary. For example, any force acting in direction x_3 at some time t_0 is associated with a corresponding, equal, and opposite force with respect to the boundaries, on the other side of the cycle at $t_1 = \tau - t_0$, where τ is the period of the cilium. While the presence of sidewalls could theoretically cause asymmetry in the power vs recovery stroke, as the recovery stroke comes slightly closer to the sidewalls compared to the effective stroke, we assume this is negligible compared to the interaction with the lower boundary.

In equation 3.2, i and j represent the directions of the velocity and applied forcing respectively, $c = 1/2$, ρ represents the planar distance

$\rho = \sqrt{(X_1 - x_1)^2 + (X_2 - x_2)^2}$, H is the height of the fluid, μ is the dynamic viscosity, and δ is the kronecker symbol. Using Einstein notation, α and β are repeated indices, each taking values of 1 and 2. We note that this only differs from the parallel plate stokeslet as derived by Liron and Mochon [28] by the factor c .

In the far field, we neglect higher order terms, including the $\frac{r_\alpha r_\beta}{\rho^4}$ and $O(e^{-\rho/H})$. From here, we are left with an equation proportional to $1/\rho^2$, because this stokeslet represents the case where the geometry is infinite in the x_1 and x_2 directions and the momentum therefore dissipates radially. In most experimental cases, the sidewalls confine the flow and therefore the velocity cannot decay radially by conservation of mass. We remove the dependency on ρ and replace it with a prefactor R to account for the impact of these side walls. This same assumption is made and validated with simulations by Sun et al. [22] for a channel with a no-slip top boundary. Additionally, this parameter accounts for

the fact that the flow rate scales linearly with velocity. The stokeslet can be simplified as follows:

$$\mathcal{F}_{1,1} = \frac{3Rx_3}{2\pi\mu} \left(1 - \frac{x_3}{2H}\right) \frac{X_3(s,t)}{H} \left(1 - \frac{X_3(s,t)}{2H}\right) \quad (3.3)$$

Lastly we utilize slender body theory to get the forcing in the x_1 direction:

$$f_1(s,t) = C_N \frac{dX_1(s,t)}{dt} = C_N s f \omega \sin \phi \cos \omega t \quad (3.4)$$

where C_N is the resistance coefficient normal to the cilium length. We additionally note that the normal force coefficient is accepted to be proportional to the viscosity, indicating the net flow should not depend on viscosity for a given cilium trajectory.

Multiplying the stokeslet by the forcing gives the velocity response at any point in space. We then integrate along the length of the cilium to account for all point forces and we integrate to average over the time of one beat cycle as shown in equation 3.5. This should approximate the velocity profile in the far field flow profile for the free-slip boundary condition:

$$\begin{aligned} V_{FS}(x_3) &= \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \int_0^L \mathcal{F}_{11}(x_3, s, t) f_1(s, t) ds dt \\ V_{FS}(x_3) &= \frac{C_N f \omega R}{32\pi\mu} \frac{L^3}{H^3} x_3 (2H - x_3) (4H - 3L \cos \theta \cos \phi) \sin \theta \sin^2 \phi \end{aligned} \quad (3.5)$$

Lastly, we can integrate along the height of the fluid to achieve our net flow rate.

$$\begin{aligned} Q &= \int_0^H V_{FS}(x_3) dx_3 \\ Q &= \frac{C_N f \omega R L^3 H}{12\mu} \sin \theta \sin^2 \phi \left(1 - \frac{3L}{4H} \cos \theta \cos \phi\right). \end{aligned} \quad (3.6)$$

We compare our result to other works with different boundary conditions in table 3.1.

The key difference between the parallel plate and the free-slip condition is that the coefficient of $\cos \theta \cos \phi$ has changed from $3L/2H$ to $3L/4H$. As L/H becomes small,

Table 3.1: Predicted generated flow from a cilium in tilted conical motion.

Geometry	Stokeslet	Predicted Flow
Semi-infinite	Volumetric response $\frac{X_3}{\pi\mu}$ [17]	$\frac{C_N\omega L^3}{6\pi\mu} \sin^2 \phi \sin \theta$ [18]
Parallel plate	Velocity response $\frac{3Rx_3}{2\pi\mu} (1 - \frac{x_3}{H}) \frac{X_3}{H} (1 - \frac{X_3}{H})$ [28]	$\frac{C_N f \omega R L^3 H}{12\mu} \sin \theta \sin^2 \phi (1 - \frac{3L}{2H} \cos \theta \cos \phi)$ [22]
Free-slip	Velocity response $\frac{3Rx_3}{2\pi\mu} (1 - \frac{x_3}{2H}) \frac{X_3}{H} (1 - \frac{X_3}{2H})$ [27]	Current Work $\frac{C_N f \omega R L^3 H}{12\mu} \sin \theta \sin^2 \phi (1 - \frac{3L}{4H} \cos \theta \cos \phi)$

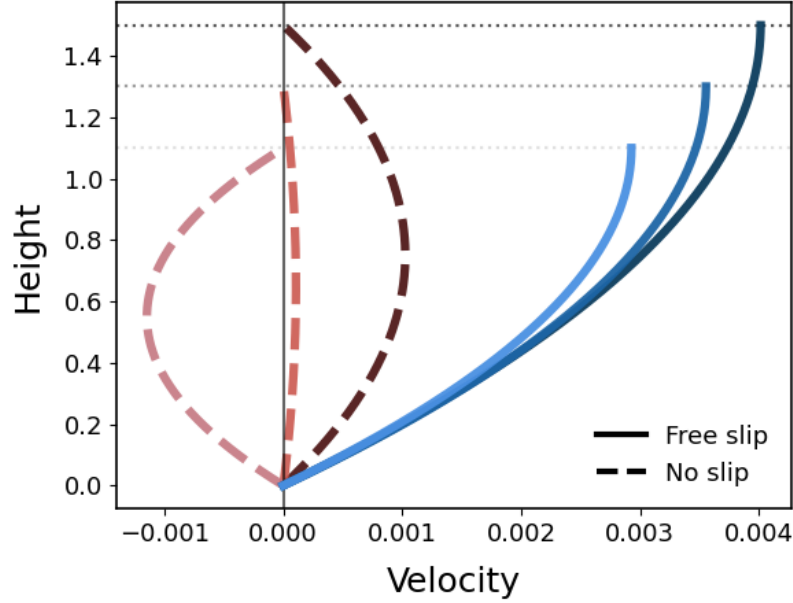


Figure 3.1: Expected flow profile for free-slip (solid line) vs no-slip (dashed line) boundary conditions for varying heights of fluid with $L = 1$, $\frac{C_N f \omega R}{\pi \mu} = 1$, and $\theta = \phi = \pi/8$. Horizontal dotted lines represent varying fluid heights.

the term in parentheses goes to 1, returning to the unconfined form of the semi-infinite domain. The $3/4$ prefactor prevents the second term from being larger than 1, indicating that this model does not predict backflow in the open channel case.

To illustrate this, we arbitrarily set $L = 1$, $\frac{C_N f \omega R}{\pi \mu} = 1$, and $\theta = \phi = \pi/8$ and plot the resultant velocity profile at various confinements in figure 3.1. We refrain from assigning units to this as it is purely intended to show the expected velocity profile shape. This uses the free-slip velocity profile given in equation 3.5. To plot the no-slip velocity profile, we use the no-slip profile as given below:

$$V_{NS}(x_3) = \frac{C_N f \omega R}{4\pi\mu} \frac{L^3}{H^3} x_3 (H - x_3) (2H - 3L \cos \theta \cos \phi) \sin(\theta) \sin^2 \phi \quad (3.7)$$

For the free-slip condition, the expected velocity profile is positive regardless of the height of the fluid. In the no-slip condition, this leads to a flow reversal when the upper boundary gets too close to the cilium during the effective stroke. There, the recovery stroke is the portion of the beat cycle which moves fluid at the farthest point from the boundaries and therefore the recovery stroke generates more flow than the effective.

3.2 Comparison to model

While the theoretical model cannot capture the negative observed at high confinements, we can check for agreement at low and moderate confinements which observe positive flow.

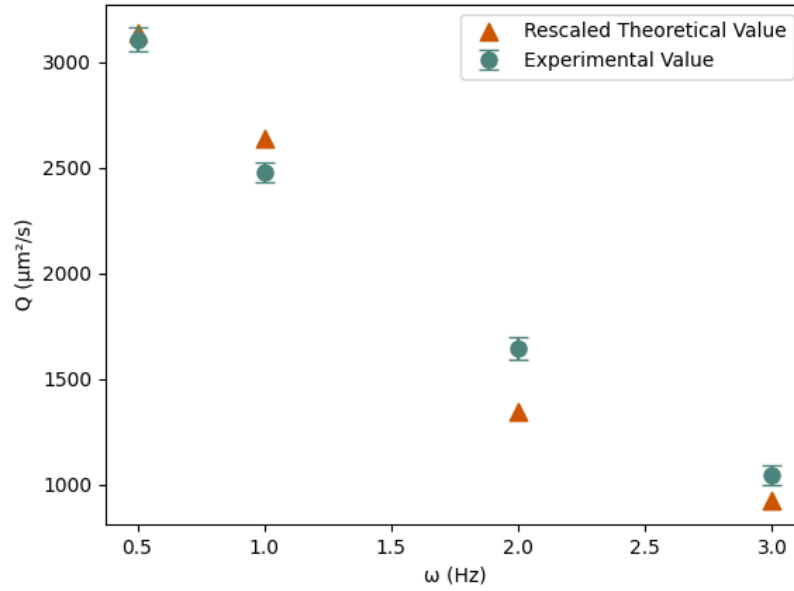


Figure 3.2: C=0.50 experimental flow rate vs theory - best fit.

First, we integrate the experimental velocity profiles to get the in plane flow rate and compare to the theoretical values from equation 3.6, using a least squares fit to determine $C_N * R$. We observe good agreement between the theoretical and experimental flow rates in figure 3.2. Note that the theory predicts forward flow with a decreasing magnitude as found.

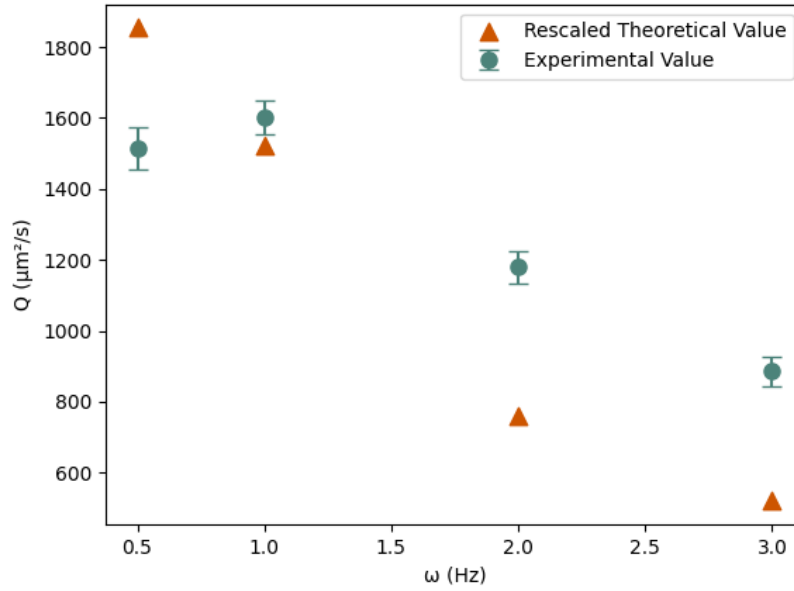


Figure 3.3: $C=0.64$ experimental flow rate vs theory - best fit.

We additionally plot the resulting flow rate vs the theoretical model for the moderate confinement experiments in figure 3.3. Here, there is a substantial deviation from the expected flow rate at intermediate confinement.

Performing the same analysis on the high confinement experiments would result in a negative value of $C_N * R$, which is a non-physical result. To gain insight into the source of this discrepancy, we look at the shape of the flow profiles themselves.

3.3 Partial-Slip

As noted in section 3.1, the velocity profiles do not show a free-slip condition at the top surface as evident by $\frac{dV}{dx_3} \neq 0$ at $x_3 = H$. A possible explanation for this is that the surface is observing some kind of partial-slip behavior, where there is a shear stress being applied to the top which does not necessarily cause the fluid to come to a stop. Some systems that observe this behavior include nanofluidics, where electroosmotically driven flow has a boundary velocity [29] or in polymer depletion layers where the flow will move with some apparent shear stress quantified by the “slip-length” [30].

While the exact nature of the partial-slip is unknown, we start by assuming our flow profile can be represented by a superposition of the free-slip and no-slip profiles as evident by the linearity of Stokes flow. Mathematically, this is represented by the following equation:

$$V_{PS}(x_3) = \alpha * V_{FS}(x_3) + (1 - \alpha)V_{NS}(x_3) \quad (3.8)$$

Where α represents some value between 0 and 1. By adjusting the values of α and H we can recover flow profiles similar in shape to those observed experimentally. This is plotted in figure 3.4.

Conditions 1, 2, and 3, are similarly shaped flow profiles to the experimental low confinement, medium confinement, and high confinement trials respectively. At low confinements, we only see a small shear stress at the surface. At medium confinements, the surface shear stress increases and we observe a change in sign of the curvature of the profile. At high confinements, we can recover the multilayered flow, with backflow in the bulk and forward flow near the surface. To achieve this similarity, we decrease α as the confinement increases. This indicates that the free surface is acting more as a no-slip boundary as the free surface approaches the tip of the cilium.

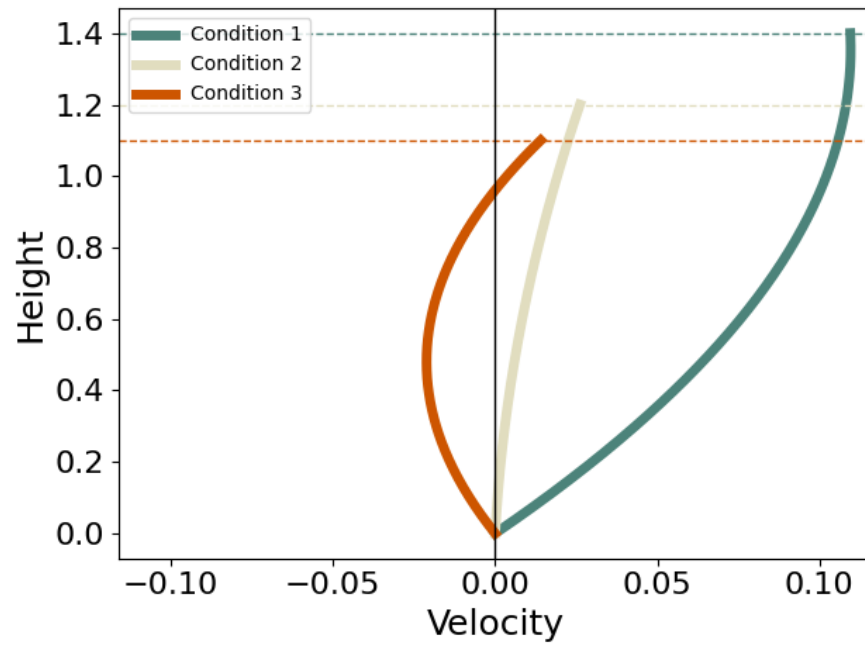


Figure 3.4: Flow profile along 3 different partial-slip conditions with varying α and H . $L = 1$, $\frac{C_N f \omega R}{\pi \mu} = 1$, and $\theta = \phi = \pi/8$ for all conditions. Condition 1: $\alpha = 0.9$ and $H = 1.4$. Condition 2: $\alpha = 0.25$ and $H = 1.2$. Condition 3: $\alpha = 0.15$, and $H = 1.1$.

This is not derived from interfacial mechanics and should be interpreted as a phenomenological model intended only to assess whether an effective transition from free-slip to no-slip behavior can reproduce the measured velocity profiles. This gives clues into the considerations for a full analytic model. Crucially, to develop a full analytic model, one must understand what causes the apparent surface shear stress.

3.4 Possible Explanations for the Observed Surface Shear Stress

There are multiple possible sources of this surface stress. As a preface, it is important to recognize that the velocities in this study are extremely small, under $1\mu\text{m/s}$. Because of this, even small forces may affect the motion of the fluid.

Initially, we considered that the air above the cilium could lead to a shear stress at the surface; however, the viscosity of air is 5 orders of magnitude less than the viscosity of the glycerol and therefore it cannot effect the flow profile. Additionally, the transient behavior discussed in section 2.4 would not be possible as this multi-layered fluid approach would be governed only by the time-independent Stokes equations.

We additionally consider the deformation of the free surface as a possible source of error which would deviate from the assumptions made for the free-slip model. There is some literature indicating that microorganisms can utilize the deformation of a free surface in order to propel themselves parallel to it [31]. While this may impact the dynamics of the generated flow, we note that experimentally there is minimal deformation of the surface and surface deformation effects are local to the cilium and therefore we should not observe an apparent surface shear far from the cilium as observed in experiments. The cause must globally effect the system.

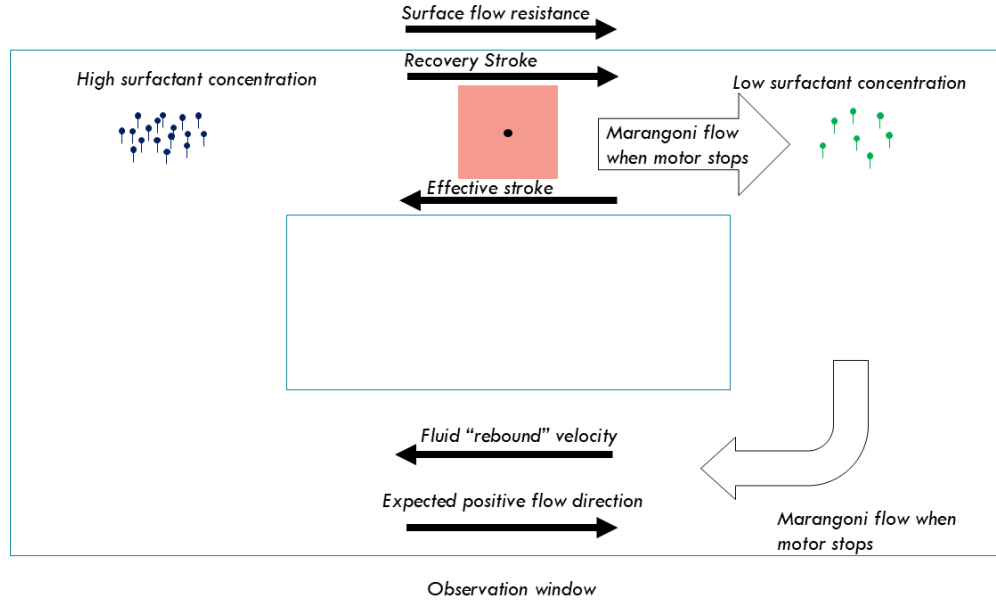


Figure 3.5: Surfactant theory for backflow induced by a magnetically actuated cilium near a free surface.

One possible explanation could be the effect of surfactants as illustrated in figure 3.5. When the motor starts, the surfactants become unevenly distributed across the experimental set-up, the unevenness of the distribution may develop over time which could lead to the ramp-up period seen in the flow rate. This leads to a net force at the surface opposing the effective stroke. At high confinements, this effect could act as a partial-slip boundary, leading to a net backflow produced by the cilium, similar to the parallel plate case. Additionally, the relaxing of these surfactants after the cilium stops moving could cause a Marangoni flow from the low surface tension region to the high surface tension region which could account for the rebound velocity. Critically to this theory, one must determine why surfactants accumulate in specific areas of the loop.

Lastly, we note that fluid interfaces can exhibit two-dimensional viscoelasticity properties, separate from Marangoni stresses [32]. This behavior can be caused by packed

surfactants, particulate contaminants, or complex chemical species. In our system this could be due to trace surfactants in the system or the tracer particles themselves. The introduction of a surface viscosity could explain the resistance to the flow during the effective stroke leading to backflow. Surface elasticity could explain the transient behavior of the flow as the surface relaxes.

We note these are all preliminary theories and more experiments are needed to determine the source of the backflow.

Chapter 4

Conclusion and Future Outlook

To further understand the flow behavior we propose multiple paths of investigation. First, particle image velocimetry should be performed on the fluid near the free surface to determine where fluid moves in the entire system. This could show points of surface divergence after the motor is turned off, which would indicate where surfactants may be accumulating. Furthermore, we can test the addition of known surfactants to the system and observe changes in the surface behavior. Lastly, hydrophobic fluids such as oil should be tested to see if the flow behavior changes with chemical species.

This thesis investigated the effects of a free surface near a cilium pumping fluid in tilted conical motion. First, we developed an analytic model for the theoretical fluid response to a cilium moving in tilted conical motion. We then created manufacturing methods for the magnetically actuated cilium and analyzed the corresponding motion in a closed-loop experimental set-up. Finally, we performed particle image velocimetry experiments in glycerol. Here we showed that the flow response due to cilium motion is non trivial and that surface effects can alter change the flow profile or even reverse it entirely. Even at the Stokes flow, Reynolds numbers studied, we see a transient response in the fluid under steady forcing, which could be indicative of surface effects. These

results could impact the motion of ciliated organisms near free surfaces and has direct implications for the movement of fluid in the bronchi, where a free surface is near the ciliated region.

Bibliography

- [1] P. Jenkins, D. McEwen, and J. Martens, *Olfactory cilia: Linking sensory cilia function and human disease*, *Chemical Senses* **34** (04, 2009) 451–464.
- [2] J. S. H. Saikat and S. J., *Paramecium swimming in capillary tube*, *Physics of Fluids* **24** (04, 2012) 041901.
- [3] V. Kumar, Z. Umair, S. Kumar, R. S. Goutam, S. Park, and J. Kim, *The regulatory roles of motile cilia in csf circulation and hydrocephalus*, *Fluids and Barriers of the CNS* **18** (Jul, 2021) 31.
- [4] H. Hamada, C. Meno, D. Watanabe, and Y. Saijoh, *Establishment of vertebrate left–right asymmetry*, *Nature Reviews Genetics* **3** (02, 2002) 103–113.
- [5] D. J. Lubkin, E. A. Gaffney, and J. R. Blake, *A viscoelastic traction layer model of muco-ciliary transport*, *Bulletin of Mathematical Biology* **69** (Jan, 2007) 289–327.
- [6] S. Girod, J. M. Zahm, C. Plotkowski, G. Beck, and E. Puchelle, *Role of the physiochemical properties of mucus in the protection of the respiratory epithelium*, *Eur Respir J* **5** (Apr., 1992) 477–487.
- [7] R. Banerjee, J. R. Bellare, and R. Puniyani, *Effect of phospholipid mixtures and surfactant formulations on rheology of polymeric gels, simulating mucus, at shear rates experienced in the tracheobronchial tree*, *Biochemical Engineering Journal* **7** (2001), no. 3 195–200.
- [8] S. K. Lai, Y.-Y. Wang, D. Wirtz, and J. Hanes, *Micro- and macrorheology of mucus*, *Adv. Drug Deliv. Rev.* **61** (Feb., 2009) 86–100.
- [9] *Airway epithelial cell cilia and obstructive lung disease*, *Cells* **5** (Nov., 2016).
- [10] M. Abrami, A. Biasin, F. Tescione, D. Tierno, B. Dapas, A. Carbone, G. Grassi, M. Conese, S. Di Gioia, D. Larobina, and M. Grassi, *Mucus structure, viscoelastic properties, and composition in chronic respiratory diseases*, *Int J Mol Sci* **25** (Feb., 2024).

- [11] S. Zhang, Z. Cui, Y. Wang, and J. Toonder, *Metachronal actuation of microscopic magnetic artificial cilia generates strong microfluidic pumping, Lab on a Chip* **20** (08, 2020) 3569–3581.
- [12] T. ul Islam, Y. Bellouard, and J. M. J. den Toonder, *Highly motile nanoscale magnetic artificial cilia, Proceedings of the National Academy of Sciences* **118** (2021), no. 35 e2104930118, [<https://www.pnas.org/doi/pdf/10.1073/pnas.2104930118>].
- [13] J. V. I. Timonen, C. Johans, K. Kontturi, A. Walther, O. Ikkala, and R. H. A. Ras, *A facile template-free approach to magnetodriven, multifunctional artificial cilia, ACS Applied Materials & Interfaces* **2** (Aug, 2010) 2226–2230.
- [14] Y. Wang, Y. Gao, H. M. Wyss, P. D. Anderson, and J. M. J. den Toonder, *Artificial cilia fabricated using magnetic fiber drawing generate substantial fluid flow, Microfluidics and Nanofluidics* **18** (Feb, 2015) 167–174.
- [15] S. Park, G. Choi, M. Kang, W. Kim, J. Kim, and H. E. Jeong, *Bioinspired magnetic cilia: from materials to applications, Microsystems & Nanoengineering* **9** (Dec, 2023) 153.
- [16] M. Vilfan, A. Potočnik, B. Kavčič, N. Osterman, I. Poberaj, A. Vilfan, and D. Babič, *Self-assembled artificial cilia, Proceedings of the National Academy of Sciences* **107** (2010), no. 5 1844–1847, [<https://www.pnas.org/doi/pdf/10.1073/pnas.0906819106>].
- [17] J. R. Blake, *A note on the image system for a stokeslet in a no-slip boundary, Mathematical Proceedings of the Cambridge Philosophical Society* **70** (1971), no. 2 303–310.
- [18] D. J. Smith, J. R. Blake, and E. A. Gaffney, *Fluid mechanics of nodal flow due to embryonic primary cilia, J R Soc Interface* **5** (May, 2008) 567–573.
- [19] A. R. Shields, B. L. Fiser, E. E. Evans, M. R. Falvo, S. Washburn, and R. Superfine, *Biomimetic cilia arrays generate simultaneous pumping and mixing regimes, Proceedings of the National Academy of Sciences* **107** (2010), no. 36 15670–15675, [<https://www.pnas.org/doi/pdf/10.1073/pnas.1005127107>].
- [20] G. Kokot, M. Vilfan, N. Osterman, A. Vilfan, B. Kavčič, I. Poberaj, and D. Babič, *Measurement of fluid flow generated by artificial cilia, Biomicrofluidics* **5** (July, 2011) 34103–34109.
- [21] W. Lee, P. G. Jayathilake, Z. Tan, D.-V. Le, H. Lee, and B. Khoo, *Muco-ciliary transport: Effect of mucus viscosity, cilia beat frequency and cilia density, Computers Fluids* **49** (10, 2011) 214–221.

- [22] Y. Sun, T. Wang, T. ul Islam, J. den Toonder, and Y. Wang, *Optimisation of the net flow generated by artificial cilia performing tilted conical motion in confined micro-channels*, *Journal of Fluid Mechanics* **1017** (2025) A20.
- [23] J. V. Boussinesq, *Speed of the slow, uniform fall of a liquid spherical drop in a viscous fluid of lesser specific weight.*, *Computers Fluids* **29** (1913) 364–371.
- [24] F. J. Peaudecerf, J. R. Landel, R. E. Goldstein, and P. Luzzatto-Fegiz, *Traces of surfactants can severely limit the drag reduction of superhydrophobic surfaces*, *Proceedings of the National Academy of Sciences* **114** (2017), no. 28 7254–7259, [<https://www.pnas.org/doi/pdf/10.1073/pnas.1702469114>].
- [25] E. Lauga, W. R. DiLuzio, G. M. Whitesides, and H. A. Stone, *Swimming in circles: Motion of bacteria near solid boundaries*, *Biophysical Journal* **90** (Jan, 2006) 400–412.
- [26] D. Lopez and E. Lauga, *Dynamics of swimming bacteria at complex interfaces*, *Physics of Fluids* **26** (07, 2014) 071902, [<https://pubs.aip.org/aip/pof/article-pdf/doi/10.1063/1.4887255/15788631/0719021online.pdf>].
- [27] A. J. T. M. Mathijssen, A. Doostmohammadi, J. M. Yeomans, and T. N. Shendruk, *Hydrodynamics of micro-swimmers in films*, *Journal of Fluid Mechanics* **806** (2016) 35–70.
- [28] N. Liron and S. Mochon, *Stokes flow for a stokeslet between two parallel flat plates*, *Journal of Engineering Mathematics* **10** (Oct, 1976) 287–303.
- [29] C. L. Rice and R. Whitehead, *Electrokinetic flow in a narrow cylindrical capillary*, *The Journal of Physical Chemistry* **69** (1965), no. 11 4017–4024, [<https://doi.org/10.1021/j100895a062>].
- [30] K. Sorbie, *Depleted layer effects in polymer flow through porous media: I. single capillary calculations*, *Journal of Colloid and Interface Science* **139** (1990), no. 2 299–314.
- [31] D. CROWDY, S. LEE, O. SAMSON, E. LAUGA, and A. E. HOSOI, *A two-dimensional model of low-reynolds number swimming beneath a free surface*, *Journal of Fluid Mechanics* **681** (2011) 24–47.
- [32] L. Scriven, *Dynamics of a fluid interface equation of motion for newtonian surface fluids*, *Chemical Engineering Science* **12** (1960), no. 2 98–108.