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Online Electricity Pricing from Frequency Measurements

Xinwei Liu†, Vladimir Dvorkin†

***Abstract*—Frequency dynamics in power systems reflect active power imbalance in real time, thereby providing an instantaneous signal to inform electricity pricing. However, existing real-time markets operate on much slower timescales and fail to exploit this signal. In this letter, we develop integrated market–frequency dynamics that enable *online* pricing directly from frequency measurements. Representing the real-time market as a dynamic price-discovery process, and integrating this process with the grid frequency dynamics, we derive an explicit price formation mechanism from frequency measurements. This mechanism manifests as a distributed PID-like controller for each generator, where frequency response is driven and remunerated by electricity prices derived solely from local frequency measurements.**

***Index Terms*—Economic dispatch, electricity pricing, cost recovery, frequency control, online optimization**

I. Introduction

Electricity as a commodity is inextricably linked to the physical system, where power imbalances directly translate into measurable changes in system frequency. However, electricity pricing operates on substantially slower timescales than frequency dynamics. Motivated by reliability, electricity is traded *offline*, with transactions settled well before operation, from days ahead (bilateral trading, day-ahead markets) to near real-time markets (5–15 minutes before operation) [1], and any frequency deviation is addressed in real-time by fast automatic control rather than by markets. This timescale separation decouples price signals from the actual grid state and prevents the delivery of theoretical market efficiency to real-time operations. Indeed, real-time prices fixed for 5–15 minutes fail to internalize the full variability of renewable generation and stochastic loads evolving on subminute timescales.

In this letter, we bridge the gap between real-time markets and operations by developing *online* real-time electricity pricing that acts on frequency timescales and provably delivers the theoretical properties of offline markets in real-time. Our work is inspired by the literature on optimal frequency control [2]–[4], which integrates economic dispatch objectives into automatic generator control, and by recent research on incentive alignment between private (e.g., profit) and system (e.g., stability) objectives in such control [5]. We build on these foundations by making the economic meaning of frequency feedback explicit: we derive a frequency-based price signal with an explicit PID structure that reproduces the dual price dynamics of economic dispatch. This price is obtained from local frequency measurements, and frequency/balance restoration is achieved through a distributed generator response to these prices. Furthermore, we optimize the generator response to guarantee the profitability under online electricity pricing.

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Notation: Bold lowercase $\mathbf{x}$ (uppercase $\mathbf{X}$) letters denote vectors (matrices), calligraphic letters $\mathcal{X}$ are reserved for sets. Notation $\mathbf{x}^\star$ denotes an optimal value, $\mathbf{x}_i$ is the i^{th} element of a vector $\mathbf{x}$, $\Pi_{\mathcal{P}}[\mathbf{x}]$ denotes the projection of $\mathbf{x}$ onto the set $\mathcal{P}$, and $\dot{\mathbf{x}}$ denotes the time derivative of $\mathbf{x}$.

II. Integrated Market-Frequency Dynamics

Consider a vector of fixed generator dispatch $\mathbf{p}^\star$ from the day-ahead market to meet the expected electricity demand d. In real time, when the demand deviates by δ, the goal of the real-time economic dispatch (ED) problem is to find a cost-optimal generator regulation $\mathbf{r}$ that offsets this deviation. This can be formulated as the following optimization problem:

$$\underset{\mathbf{r}\in\mathcal{P}}{\text{minimize}} \quad c(\mathbf{r}) := \tfrac{1}{2}\|\mathbf{p}^\star + \mathbf{r}\|_{\mathbf{C}}^2 + \mathbf{c}^\top(\mathbf{p}^\star + \mathbf{r}) \tag{1a}$$

$$\text{subject to} \quad \mathbf{1}^\top(\mathbf{p}^\star + \mathbf{r}) - d - \delta = 0 \quad : \lambda \tag{1b}$$

where $\mathcal{P} := \{\mathbf{r} \,|\, \underline{\mathbf{p}} \leqslant \mathbf{p}^\star + \mathbf{r} \leqslant \overline{\mathbf{p}}\}$ defines the feasible set for generator regulation. The objective is to minimize the real-time generation cost (1a), subject to generator constraints and active power balance condition (1b). The dual variable λ is the marginal electricity price; for zero demand deviation, the optimal price is that from the day-ahead market $\lambda^\star = \lambda^{\text{da}}$.

This problem is typically solved every 5 to 15 minutes, but we aim at solving it on substantially faster timescales. Towards this goal, we analyze its partial Lagrangian function:

$$\mathcal{L}^{\text{ED}}(\mathbf{r}, \lambda) = c(\mathbf{r}) + \lambda(\delta - \mathbf{1}^\top\mathbf{r}), \tag{2}$$

where we used the fact that $\mathbf{1}^\top\mathbf{p}^\star - d = 0$. The optimal solution to (1) can be characterized as the equilibrium point of the following primal-dual dynamical system:

$$\dot{\mathbf{r}} = \Pi_{\mathcal{P}}\left[-\nabla_{\mathbf{r}}\mathcal{L}^{\text{ED}}(\mathbf{r}, \lambda)\right] = \Pi_{\mathcal{P}}\left[\lambda\mathbf{1} - \nabla_{\mathbf{r}}c(\mathbf{r})\right], \tag{3a}$$

$$\dot{\lambda} = \nabla_\lambda\mathcal{L}^{\text{ED}}(\mathbf{r}, \lambda) = \delta - \mathbf{1}^\top\mathbf{r}, \tag{3b}$$

where generators continuously update their regulation $\mathbf{r}$ by projecting their best response to the current price (given their marginal cost $\nabla_{\mathbf{r}}c(\mathbf{r})$) onto the feasible set $\mathcal{P}$. The regulation price λ is continuously updated according to the active power imbalance, and system (3) resembles the classic Walrasian tâtonnement (price adjustment) process [6].

The power imbalance can also be described by the frequency deviation ω. For a copper-plate model, the frequency dynamic is abstracted by the linearized swing equation [7]:

$$\dot{\omega} = \tfrac{1}{M}\left(\mathbf{1}^\top\mathbf{r} - D\omega - \delta\right), \tag{4}$$

where M and D are system inertia and damping, respectively.

To integrate the two dynamical systems, consider the Lyapunov function associated with the frequency dynamic:

$$\mathcal{L}^{\text{FD}}(\omega) = \tfrac{1}{2DM}(D\omega - \mathbf{1}^\top\mathbf{r} + \delta)^2 \tag{5}$$

such that, treating $\mathbf{r}$ and δ as exogenous signals, the frequency dynamic can be written as the gradient flow $\dot{\omega} = -\nabla_\omega \mathcal{L}^{\text{FD}}(\omega)$, which recovers the swing equation.

Then, we introduce the composite Lagrangian function of the integrated market-frequency dynamics:

$$\mathcal{L}(\mathbf{r}, \omega, \lambda, \gamma) \coloneqq \mathcal{L}^{\text{ED}}(\mathbf{r}, \lambda) + \gamma \mathcal{L}^{\text{FD}}(\omega) \tag{6}$$

which augments the Lagrangian of the ED problem with a frequency constraint penalized by an auxiliary dual variable γ. The primal-dual dynamics seeking the saddle point of the composite Lagrangian function (6) takes the form:

$$\begin{aligned}
\dot{\mathbf{r}} &= \Pi_{\mathcal{P}} \left[-\nabla_{\mathbf{r}} \mathcal{L}(\mathbf{r}, \omega, \lambda, \gamma) \right] \\
&= \Pi_{\mathcal{P}} \left[\lambda \mathbf{1} - \tfrac{\gamma}{DM} (\mathbf{1}^\top \mathbf{r} - D\omega - \delta) \mathbf{1} - \nabla_{\mathbf{r}} c(\mathbf{r}) \right], && \text{(7a)} \\
\dot{\omega} &= -\nabla_\omega \mathcal{L}(\mathbf{r}, \omega, \lambda, \gamma) = \tfrac{\gamma}{M} (\mathbf{1}^\top \mathbf{r} - D\omega - \delta), && \text{(7b)} \\
\dot{\lambda} &= \nabla_\lambda \mathcal{L}(\mathbf{r}, \omega, \lambda, \gamma) = \delta - \mathbf{1}^\top \mathbf{r}, && \text{(7c)} \\
\dot{\gamma} &= \nabla_\gamma \mathcal{L}(\mathbf{r}, \omega, \lambda, \gamma) = \tfrac{1}{2DM} (D\omega - \mathbf{1}^\top \mathbf{r} + \delta)^2. && \text{(7d)}
\end{aligned}$$

Compared to system (3), generators respond to the composite price informed by both active power imbalance via equation (7c) and frequency deviation via equation (7d) on the same timescale. An important result from [2]–[4] is that this system converges to the social optimum. We briefly revisit this result.

Theorem 1 (Social optimum)**.** *The dynamical system* (7) *converges to the solution of the optimization problem*

$$\begin{aligned}
&\underset{\mathbf{r} \in \mathcal{P}, \omega}{\textit{minimize}} && c(\mathbf{r}) && \text{(8a)} \\
&\textit{subject to} && \mathbf{1}^\top \mathbf{r} - \delta = 0, && \text{(8b)} \\
& && D\omega - \mathbf{1}^\top \mathbf{r} + \delta = 0, && \text{(8c)}
\end{aligned}$$

which simultaneously ensures power balance and zero frequency deviation at the minimum generation cost.

The proof follows similar ideas in [2]–[4], so we only provide the intuition. The dynamical system in (7) can be seen as a mixed-saddle flow dynamics of the following problem:

$$\begin{aligned}
&\underset{\mathbf{r} \in \mathcal{P}, \omega}{\text{minimize}} && c(\mathbf{r}) && && \text{(9a)} \\
&\text{subject to} && \mathbf{1}^\top \mathbf{r} - \delta = 0, && : \lambda && \text{(9b)} \\
& && \tfrac{1}{2DM} (D\omega - \mathbf{1}^\top \mathbf{r} + \delta)^2 = 0, && : \gamma && \text{(9c)}
\end{aligned}$$

where the last constraint (9c) is always satisfied when enforcing (8c) instead. That is, the stationary point of (9) is the optimal solution of (8); moreover, if the dynamical system (7) converges to the stationary point of (9), it also converges to the socially optimal solution of (8), as desired.

Unlike power balance, which can be temporarily violated, the swing equation is a physical law and thus requires the hard constraint $\gamma = 1$. This recovers the original swing equation (4) in (7b) and simplifies the dynamics in (7):

$$\begin{aligned}
\dot{\mathbf{r}} &= \Pi_{\mathcal{P}} \left[(\lambda - \tfrac{1}{D} \dot{\omega}) \mathbf{1} - \nabla_{\mathbf{r}} c(\mathbf{r}) \right], && \text{(10a)} \\
\dot{\omega} &= \tfrac{1}{M} (\mathbf{1}^\top \mathbf{r} - D\omega - \delta), && \text{(10b)} \\
\dot{\lambda} &= \delta - \mathbf{1}^\top \mathbf{r}, && \text{(10c)}
\end{aligned}$$

where generators in (10a) optimally respond to power imbalance and frequency deviations according to marginal generation cost. Theorem 1 holds for the reduced dynamics as well.

III. Online Pricing from Frequency Measurements

In this section, we establish that the classic Walrasian tâtonnement price adjustment process [6] realizes within generator frequency control as a cost-optimal, local PID-like controller enabling real-time pricing on frequency timescales.

Towards this goal, first observe that the electricity price update in (10c) is given by the active power imbalance, which can also be expressed from the swing equation (10b) via frequency variables. The two yield

$$\dot{\lambda} = -D\omega - M\dot{\omega}, \tag{11}$$

where the price is driven by both the frequency deviation and the rate of frequency change. Suppose the initial system state corresponds to the day-ahead schedule, so that the price and frequency deviation are respectively $\lambda(0) = \lambda^{\text{da}}$ and $\omega(0) = 0$. Then, integrating both side of (11) from 0 to t yields

$$\lambda(t) = \lambda^{\text{da}} - D \textstyle\int_0^t \omega(\tau) d\tau - M\omega(t). \tag{12}$$

Substituting this to (10a) leads to the following best generator response to frequency measurements alone:

$$\begin{aligned}
\dot{\mathbf{r}}(t) &= \Pi_{\mathcal{P}} \Big[\big(\lambda^{\text{da}} + \pi(\omega(t)) \big) \mathbf{1} - \nabla_{\mathbf{r}} c(\mathbf{r}(t)) \Big], && \text{(13a)} \\
\text{with } \pi(\omega(t)) &\coloneqq -\underbrace{M\omega(t)}_{\text{P}} - \underbrace{D \textstyle\int_0^t \omega(\tau) d\tau}_{\text{I}} - \underbrace{\tfrac{1}{D} \dot{\omega}(t)}_{\text{D}}. && \text{(13b)}
\end{aligned}$$

This regulation includes the frequency-dependent price adjustment $\pi(\omega(t))$ of the day-ahead price with the following intuition. At nominal frequency (zero frequency deviation), $\pi(\omega(t)) = 0$ and generators maintain their best response to the day-ahead price λ^{da}, and thus the original generation schedule. For negative frequency deviations, $\pi(\omega(t)) > 0$ adjusts the day-ahead price upwards incentivizing generators to meet the growing demand. For positive frequency deviations, $\pi(\omega(t)) < 0$ reduces the day-ahead price prompting generators to regulate downwards to meet the reduced demand. Regulation (13) thus integrates the classic Walrasian tâtonnement price adjustment process within generator frequency response.

The price adjustment (13b) contains the three canonical components of PID control: a proportional term (P) reacting to instantaneous frequency deviation, an integral term (I) accumulating frequency error, and a derivative term (D) responding to the rate of frequency change. Provided that the day-ahead price and the current system inertia and damping are known, it can be realized locally by generators from frequency measurements alone. The day-ahead prices are publicly announced well ahead of real-time operation. Recent work on converter probing and measurement-based identification demonstrates that system-level inertia and damping can be periodically estimated and broadcast by system operators [8].

In practice, regulation is implemented in discrete time:

$$\begin{aligned}
\hat{\mathbf{r}}_i(t+1) &= \mathbf{r}_i(t) + \eta_i \big(\lambda^{\text{da}} + \pi(\omega(t)) - \nabla_{\mathbf{r}} c_i(\mathbf{r}_i(t)) \big), && \text{(14a)} \\
\text{followed by} \quad \mathbf{r}_i(t+1) &= \Pi_{\mathcal{P}_i} \Big[\hat{\mathbf{r}}_i(t+1) \Big], && \text{(14b)}
\end{aligned}$$

for each generator i, as illustrated in Fig. 1. The step size $\eta_i > 0$ is the design parameter controlling the speed of generator response to frequency-dependent prices. The selection of the step size is economically motivated: small η_i may prevent

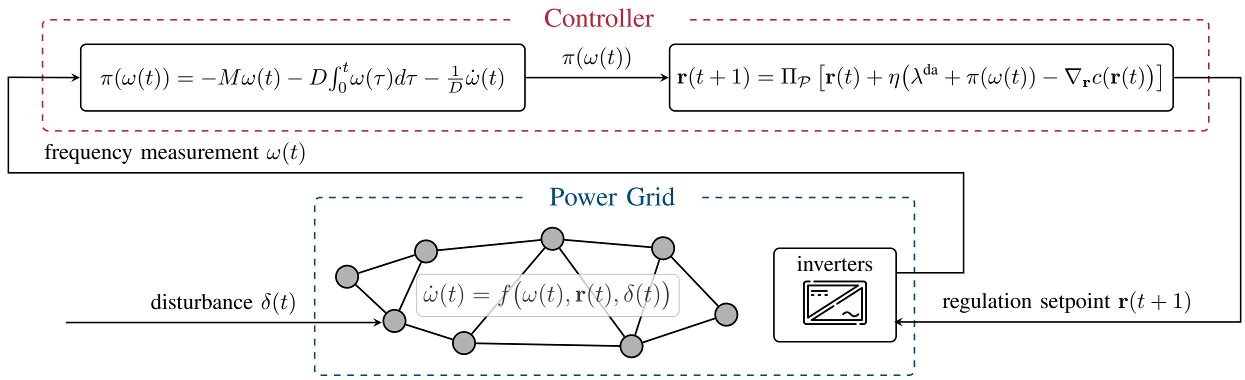

Fig. 1. Block diagram of the controller. The controller gets the local frequency measurement, computes the frequency-dependent price adjustment to the day-ahead price, then determines and implements the optimal regulation of active power output.

generators from responding fast enough to rapidly changing prices, while large η_i may increase costs faster than electricity price adjusts. Accordingly, the following result establishes that for fully dispatchable generators, the step size can be optimally chosen to guarantee cost recovery at each time step.

Theorem 2 (Cost recovery)**.** *The step size $\eta_i = \mathbf{C}_{ii}^{-1}$, inversely proportional to the quadratic cost coefficient, guarantees non-negative profit for each generator i at every time step.*

Proof. We need to establish that

$$\big(\lambda^{\text{da}} + \pi(\omega(t))\big)\big(\mathbf{p}_i^{\star} + \mathbf{r}_i(t)\big) - c_i\big(\mathbf{r}_i(t)\big) \geqslant 0, \tag{15}$$

i.e., the profit function is nonnegative at any time step t. For notational convenience, define:

$$\mathbf{g}_i(t) = \mathbf{p}_i^{\star} + \mathbf{r}_i(t), \quad \lambda^{\text{rt}}(t) = \lambda^{\text{da}} + \pi(\omega(t)), \tag{16}$$

as real-time generation and price, respectively.

We first observe that regulation update (14) can be obtained as the solution to the following optimization problem:

$$\underset{\mathbf{g}_i}{\text{maximize}} \ \Big[\mathbf{g}_i(t) + \eta_i\big[\lambda^{\text{rt}}(t) - \mathbf{C}_{ii}\mathbf{g}_i(t) - \mathbf{c}_i\big]\Big]\mathbf{g}_i - \tfrac{1}{2}\mathbf{g}_i^2 \tag{17a}$$

$$\text{subject to } \underline{\mathbf{p}}_i \leqslant \mathbf{g}_i \leqslant \overline{\mathbf{p}}_i \quad : \underline{\boldsymbol{\alpha}}_i, \overline{\boldsymbol{\alpha}}_i, \tag{17b}$$

as verified by its Karush-Kuhn-Tucker conditions. We show that objective (17a) is nonnegative and use this to bound the generator's profit.

The dual problem associated with problem (17) is

$$\underset{\underline{\boldsymbol{\alpha}}_i, \overline{\boldsymbol{\alpha}}_i \geqslant 0}{\text{minimize}} \quad \tfrac{1}{2}\mathbf{g}_i^{\star}(\underline{\boldsymbol{\alpha}}_i, \overline{\boldsymbol{\alpha}}_i)^2 - \underline{\mathbf{p}}_i\underline{\boldsymbol{\alpha}}_i + \overline{\mathbf{p}}_i\overline{\boldsymbol{\alpha}}_i \tag{18}$$

where the quadratic term $\mathbf{g}_i^{\star}(\underline{\boldsymbol{\alpha}}_i, \overline{\boldsymbol{\alpha}}_i)^2$ comes from the stationarity condition of problem (17), i.e.,

$$\mathbf{g}_i^{\star}(\underline{\boldsymbol{\alpha}}_i, \overline{\boldsymbol{\alpha}}_i) = \mathbf{g}_i(t) + \eta_i\big[\lambda^{\text{rt}}(t) - \mathbf{C}_{ii}\mathbf{g}_i(t) - \mathbf{c}_i\big] - \underline{\boldsymbol{\alpha}}_i + \overline{\boldsymbol{\alpha}}_i.$$

The quadratic term in (18) is nonnegative. For fully dispatchable generators with $\underline{\mathbf{p}}_i = 0$, the second term vanishes. The third term is nonnegative by dual feasibility $\overline{\boldsymbol{\alpha}}_i \geqslant 0$ and $\overline{\mathbf{p}}_i$. Thus the dual objective is nonnegative. By strong duality, the primal objective (17a) is also nonnegative:

$$\Big[\mathbf{g}_i(t) + \eta_i\big[\lambda^{\text{rt}}(t) - \mathbf{C}_{ii}\mathbf{g}_i(t) - \mathbf{c}_i\big]\Big]\mathbf{g}_i - \tfrac{1}{2}\mathbf{g}_i^2 \geqslant 0. \tag{19}$$

Adding and subtracting the term $\frac{1}{2}\mathbf{C}_{ii}\mathbf{g}_i^2$ corresponding to the quadratic part of the cost function, and rearranging the terms helps isolating the profit on the left-hand side:

$$\begin{aligned}&\lambda^{\text{rt}}(t)\mathbf{g}_i - \tfrac{1}{2}\mathbf{C}_{ii}\mathbf{g}_i^2 - \mathbf{c}_i\mathbf{g}_i \geqslant \\ &-\tfrac{1}{\eta_i}\mathbf{g}_i(t)\mathbf{g}_i + \mathbf{C}_{ii}\mathbf{g}_i(t)\mathbf{g}_i + \tfrac{1}{2\eta_i}\mathbf{g}_i^2 - \tfrac{1}{2}\mathbf{C}_{ii}\mathbf{g}_i^2.\end{aligned} \tag{20}$$

Setting the step size $\eta_i = \mathbf{C}_{ii}^{-1}$ makes the right-hand side vanish, yielding

$$\lambda^{\text{rt}}(t)\mathbf{g}_i - \tfrac{1}{2}\mathbf{C}_{ii}\mathbf{g}_i^2 - \mathbf{c}_i\mathbf{g}_i \geqslant 0. \tag{21}$$

Substituting (16) recovers (15) as desired. □

IV. Numerical Results

We present numerical results in a stylized single-area power system with five generators. The nominal demand $d = 200$ MW, and the maximum capacity of fully dispatchable generators is set to 50 MW. The linear cost coefficients are set to \$27.4/MWh, and the quadratic costs range from \$0.01 to \$0.015 per MWh^2 across the generators. The inertia and damping constants are chosen to be $M = 12$ s and $D = 35$ MW/Hz. The step size η_i is chosen for every generator according to Theorem 2. The frequency dynamics are simulated on a 0.05 s timescale, and the sampling (measurement) time is 0.25 s. We test the controller in two settings: constant load disturbance, where step disturbances settle to steady state over time, and time-varying with an evolving stochastic demand process.

A. Constant Disturbance

We introduce two events: a step decrease in demand of 30 MW at $t = 30$ s, followed by generator outage at $t = 5$ minutes, both having impacts on frequency and market dynamics as shown in Fig. 2. The system frequency is successfully restored after the two events (top-left) as generators respond to dynamic prices (top-right). As expected, the price dynamically decreases after the first event and increases after the second event. The generator response (bottom-left) is consistent with the expected frequency response, and profits (bottom-right) intuitively follow the demand pattern. This correspondence between physical and market variables proves the ability of the integrated dynamics to translate physical measurements into efficient prices, providing sufficient incentives for distributed generators to restore frequency to the nominal value online.

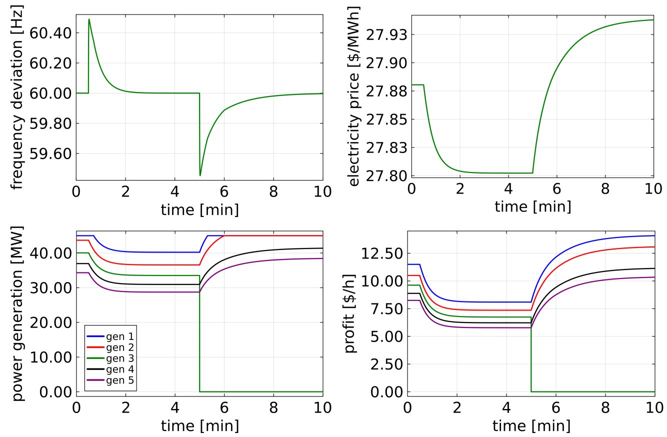

Fig. 2. Frequency, electricity price, generation, and profit dynamics.

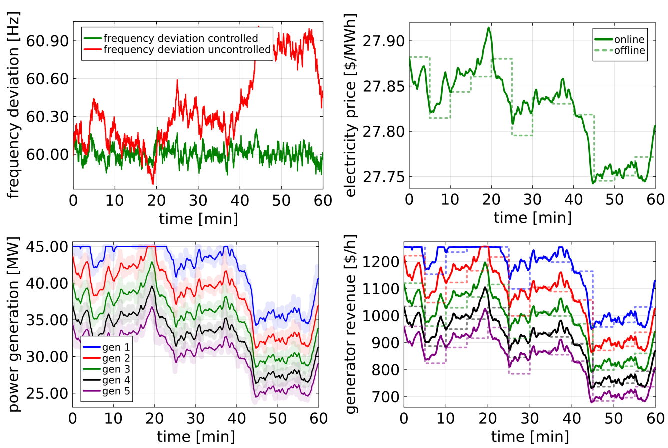

Fig. 3. Frequency and market dynamics in time-varying setting.

B. Time-Varying Disturbance

We model the stochastic demand as a Wiener process with zero drift and a standard deviation of 1 MW. Fig. 3 illustrates frequency and market dynamics. The controller maintains the frequency around 60 Hz in a time-varying simulation. The top-right plot illustrates the contrast between offline pricing on a 5-minute basis and online pricing of the integrated market-frequency dynamics. Observe that online pricing respects intra-interval frequency changes ignored in the offline market. The bottom-left plot shows resultant generation dynamics: the pale background trajectories are the offline solution, as if we were able to solve economic dispatch at every simulation time step. This illustrates the efficient tracking performance of the controller. The revenues in online and offline settings are given on the bottom-right panel. The online revenues on the frequency (measurement) timescale internalize the variability of frequency dynamics, in contrast to the slow offline revenue dynamics, which are decoupled from actual operation.

Lastly, we compare profits in both offline and online market settings in Fig. 4 for the same frequency response of generators. Under offline prices that do not internalize the per-instance cost of real-time response to frequency deviations, profits are highly volatile, oscillatory, and frequently negative. All are consequences of timescale separation between frequency and market dynamics. Under online pricing, the profit dynamics are more stable and remain in the non-negative domain. These dynamics confirm the result of Theorem 2, ensuring the profitability of response to frequency-derived prices under optimal step-size selection.

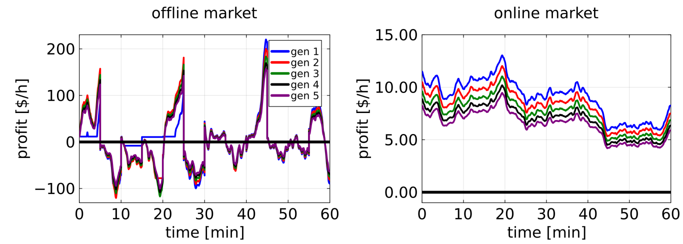

Fig. 4. Offline versus online market profits of each generator.

V. Conclusion

We developed online real-time pricing for electricity on frequency timescales using integrated market-frequency dynamics, realizing it in the form of a local PID controller. Similar to automatic generator control, the sudden changes in load and generation are accommodated in response to frequency deviations. However, in our design, generators respond to local frequency-derived prices that ensure convergence to the social optimum and profitability of frequency response. In future work, we will expand the scope of this letter to transmission network dynamics and focus on computing locational marginal prices from bus frequency measurements. In addition to cost recovery in networked settings, we will focus on establishing the budget balance property of the online market.

VI. AI Usage Disclosure

The authors used AI tools to assist with writing, including spell-checking and streamlining arguments, as well as for coding and debugging. None of the narrative or code was directly produced by AI; it was used solely in an advisory role. All models and ideas are the sole intellectual property of the authors, and no AI contributed to their development.

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I. Review #145A

This is an interesting and well motivated work with a potentially transformational contribution. The paper talks about how frequency provides a useful but underexploited signal that could be utilized by the markets, except that markets operate on much slower timescales, and fail to exploit it. This work deals with the timescale separation problem that currently decouples price signals from the actual grid state, which prevents theoretical market efficiency from being achieved in real time operations. By removing the 5-15 minute intervals for real-time prices, it offers a new mechanism to determine online pricing from frequency measurements for a more direct link between real-time markets and operations.

The result gives a local PID-like control solution where generators respond to the composite price informed by both active power imbalance and frequency deviation on the same timescale. It provides a solution that simultaneously ensures power balance and zero frequency deviation at the minimum generation cost (Theorem 1) with proof described.

A. Paper summary

A controller is developed that gets the local frequency measurement, computes the frequency-dependent price adjustment to the day-ahead price, and determines and implements the optimal regulation of active output power.

Would like to better understand how this relates to AGC/ACE. How is it different/how does it compare? The authors do mention "Our work is inspired by the literature on optimal frequency control [2]–[4], which integrates economic dispatch objectives into automatic generator control." It also says "Similar to automatic generator control, the sudden changes in load and generation are accommodated in response to frequency deviations. However, in our design, generators respond to local frequency-derived prices that ensure convergence to the social optimum and profitability of frequency response." Can you please elaborate on this relationship to AGC?

The authors say they will expand the scope of this letter to transmission network dynamics and focus on computing locational marginal prices from bus frequency measurements. This seems like a good and important direction to investigate next.

B. Comments for authors

This is a potentially mechanism and proof of the idea of online real-time pricing on frequency time scales using integrated market frequency dynamics. It is realized in the form of a local PID controller. The main suggestion is to further explain/compare its relationship with AGC and what its relationship would be if implemented in reality.

C. Summarize your recommendation in one to two sentences

— Katherine Davis

II. Review #145B

A. Paper summary

This paper develops an online method for pricing electricity to balance generation and load during real-time operation, a task typically accomplished by primary and secondary frequency controls without an explicit market. The main contribution is a price signal that reflects PID control structure for frequency deviations. The resulting price provably guarantees convergence to the optimal solution of a swing dynamics-aware economic dispatch and cost recovery for individual generators under a specific condition.

B. Comments for authors

Strengths

[+] Well written with easy-to-follow logical progression of contributions
[+] Mathematically elegant solution with intuitive connection to PID controller
[+] Numerical simulations validate theorems and proofs

Weaknesses

[–] Impractical solution that would require a complete redesign of existing primary and secondary frequency controls. For example, the frequency trajectory plotted in Fig. 2 does not at all reflect typical swing / governor / AGC time-domain response.
[–] Related to the above, there are numerous idealized assumptions that deviate greatly from the reality of power system control and operation.
[–] If I understand correctly, the simulations are performed using the same simplifying assumptions (and hence power system model) upon which the theoretical results are derived, so it is unsurprising that they are validated. The results would be more convincing if the simulations were done under more realistic settings.

C. Summarize your recommendation in one to two sentences

This is a well-written paper that provides a mathematically elegant solution for real-time pricing to balance generation and load during real-time operation. I recommend for the paper to be accepted.

— Anonymous reviewer

III. Review #145C

A. Paper summary

Currently, real-time electricity markets operate at timescales of 5-15 mins, i.e., generation dispatch and electricity prices are adjusted every 5-15mins. Any demand (or renewable generation) fluctuations in between are handled by fast frequency control actions that operate at secs to a couple minutes timescale. This short paper (Letter or Notes) proposes online pricing of electricity that adjust generation levels and prices at the frequency control timescale, based not just on excess demand (as the current real-time electricity market does), but also on frequency deviations at secs timescale.

Starting from the standard economic dispatch problem (1), the Letter adds a term (5) to the Lagrangian that is a quadratic Lyapunov function of the swing equation (4). Then it shows that the traditional electricity prices adjustment can be expressed as a PID controller (12) of the frequency dynamic (swing equation). The generation adjustment follows the traditional approach (13): increase generation if the marginal electricity price is higher than the marginal production price, except that the electricity price now includes not just the day-ahead price, but also online price that responds to frequency dynamics. It shows how to stepsize of the generation adjustment to recover (average) generation cost. Finally, it illustrates the proposal using numerical simulations.

B. Comments for authors

Strengths:

1) The idea of integrating frequency response and electricity pricing is interesting. This is usually considered impractical because dispatching prices and adjusting generation levels typically operate at a slower timescale than what is required for frequency control (for maintaining stability). Nonetheless, it is useful to explore this systematically, not only for intellectual reasons, but also this might one day become practical with new fast acting technologies (e.g., inverter-based resources) proliferate. This is such a paper. Historical examples include: Fred Schweppes proposed using loads (DER) for distributed frequency control in late 1970s (Homeostatic Utility Control) and electricity markets in early 1980s, which took more than 4 decades and 2 decades to realize.
2) The online generation and price adjustment schemes are very clever. It combines naturally both the supply-demand gap and frequency deviation. The design is derived systematically from standard economic principles and frequency (swing) dynamics and therefore rests on a good foundation. Moreover, it leverages the frequency dynamics as part of the price adjustments. It proves the optimality and stability of the feedback process. All fits nicely in the same framework.
3) The truly novel result is Theorem 2 that says that the stepsize (pace) of generation adjustment can be chosen so that the generation cost can be recovered (average cost, not marginal cost). This property does not always hold under marginal pricing schemes, but the Letter shows that it holds in their design, which further justifies their proposal.
4) Simulations illustrates the transient behavior (while stability and optimality properties are asymptotic results). It shows that that the proposed online scheme can indeed stabilize frequency around the nominal value in between traditional dispatch intervals, whereas offline control cannot (Fig 3a). Moreover, the generator profit is much more stable than offline control (Fig 4)
5) The paper is very well written, clean and clear, and easy to understand.

Weakness (It is understandable that a Letter cannot realistically address all the following weaknesses):

1) This Letter is primary a theory paper and therefore it does not address practical issues, such as how will the price and generation adjustments dispatched to the generators, will that be fast enough, etc.
2) The simulations are relatively simple and not comprehensive enough. They illustrate the behavior of the proposal, properties that are guaranteed by the theory, but are insufficient for a credible evaluation of the proposal for practical deployment.

C. Summarize your recommendation in one to two sentences

I recommend acceptance of this Notes. A single idea, clearly articulated, cleanly justified in theory and illustrated in simulations, well written paper, though its practical value is unclear in the short term.

— Anonymous reviewer

IV. Brief response to reviewers (optional)

We thank the three reviewers for their insightful comments, which we will address in the journal extension of this work.

V. Brief summary of changes made to the final paper

No substantive changes made. Reference [2] was updated to a more appropriate citation; no technical content was modified.