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Injection and Plasma Waveguides for Multi-Stage Laser Plasma Acceleration

by

Kelly Swanson

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University of California, Berkeley

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Spring 2019

Injection and Plasma Waveguides for Multi-Stage Laser Plasma Acceleration

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by

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Abstract

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Doctor of Philosophy in Physics

University of California, Berkeley

Doctor Wim Leemans, Chair

Laser plasma accelerators (LPAs) have shown promise as the next generation of compact particle accelerators. When a high intensity laser pulse propagates through a plasma, the laser can drive a large-amplitude wave which supports accelerating and focusing electric fields. Acceleration gradients three orders of magnitude greater than those found in conventional accelerators have been demonstrated. However, before an LPA-based high energy collider can be built, several advancements are still necessary, including further increases in beam energy and improvements in beam quality.

While electron beams have been accelerated to 8 GeV in a 20 cm LPA, the energy gain in a single LPA stage is limited by the laser energy available. To reach the energies needed for high energy physics, coupling of many LPAs is required. In addition, the beams must be injected and accelerated with high beam quality to reach the luminosities needed at the collision point to produce enough events to search for new physics. While enormous progress has been seen in the acceleration of electrons, positron acceleration is more challenging because most of the existing LPA structures are less suitable for positively charged particles.

In this thesis, we demonstrate staging of two laser plasma accelerators, injecting electrons in one stage and post-accelerating them in a second. We discuss the development of a high-quality injector stage and demonstrate techniques for controlling the injected electron beam parameters by modifying the plasma density profile. Finally, we detail a novel LPA structure which has demonstrated low-power laser guiding and has the potential to accelerate both electrons and positrons with high quality.

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Chapter 1

Introduction

1.1 Particle Physics and Conventional Accelerators

Particle beams and radiation produced from accelerators have applications in a variety of fields including medical, nuclear and biological physics. The largest accelerators are used to study the constituents of matter, colliding particle beams at high energies to produce new particles and investigate their structure. By digging through the chaos of these collisions, we can test Standard Model predictions and look for physics beyond the Standard Model. Are there extra dimensions of space? Why are there so many kinds of particles? Where is all the antimatter? For the past century, particle accelerators have been a powerful tool, but they are starting to reach their limits.

Conventional accelerators are based on radio-frequency (RF) cavities that generate electromagnetic fields and accelerate charged particles. In general, there are two types of particle accelerators: linear and circular. In a linear accelerator, or linac, particles pass through the RF cavities once, and the length of the accelerator is determined by the desired final particle energy and the acceleration gradient. However, RF cavities are limited to acceleration gradients on the order of 100 MV/m because higher fields will rip electrons from the metallic or superconducting cavity walls and cause damage. [1] The Compact Linear Collider (CLIC), one of the potential next generation colliders, is designed to accelerate electrons and positrons up to 3 TeV with a 100 MV/m acceleration gradient. [2]

To increase the beam energy without linearly increasing the acceleration length, particles can also be accelerated in circular accelerators, or synchrotrons. In a syn-

chrotron, magnets steer the particles in a circular orbit, allowing the beams to pass through the RF cavities multiple times. This orbit can accelerate particles to higher energies with smaller acceleration gradients. The Large Hadron Collider accelerates protons up to 7 TeV with a 5 MV/m acceleration gradient.

The energy gain in a synchrotron is limited by radiation loss and achievable magnetic field strength. As a particle accelerates around a ring, it will emit synchrotron radiation with an energy loss W_s :

$$W_s \propto \frac{\gamma^4}{\rho}, \quad (1.1)$$

where $\gamma = W/mc^2$ is the particle's relativistic factor, W is the particle energy, m is the particle mass, and ρ is the synchrotron radius. This energy loss must be restored in the RF cavities, and the collider components must be able to handle the radiation. Since the energy loss is strongly dependent on γ , the final beam energy for a given ring size is limited. In the Large Electron-Positron Collider, the world's largest lepton accelerator, particles with 208 GeV center-of-mass energy radiated 3 GeV per turn. To increase beam energy, larger rings are required, but eventually the size of the ring becomes prohibitive.

As can be seen from Eq. 1.1, the energy loss due to synchrotron radiation is larger for leptons relative to hadrons. Heavier particles such as protons radiate much less synchrotron radiation for the same energy. Instead, acceleration of heavier particles is not limited by synchrotron radiation but by the size of the accelerator and the field strength of the magnets. The final particle energy W scales as:

$$W \propto B\rho, \quad (1.2)$$

where B is the magnetic field strength. The final beam energy can be increased by either building more powerful magnets or a larger accelerator. An LHC main dipole generates an 8 tesla magnetic field. Quadrupole magnets designed for the High Luminosity LHC, an upgrade to the LHC, produce 12 tesla fields, and the Future Circular Collider, a post-LHC accelerator designed with a collision energy of 100 TeV, is planned with 16 tesla fields and fits in a 100 km tunnel. [3], [4] However, these options are expensive, approaching impractical.

1.2 Laser Plasma Accelerators and Requirements for a Particle Collider

Plasma-based accelerators are an alternative for the next generation of compact accelerators. Instead of using electromagnetic fields confined in a radio-frequency cavity, laser plasma accelerators (LPAs) use the radiation pressure of an intense laser pulse to drive a plasma wave. [5] When a short and intense laser pulse propagates through a plasma, the radiation pressure pushes electrons to the front and side of the laser. Because ions are much heavier, they are considered immobile at the laser intensities used in this thesis ($10^{18} - 10^{19} \text{ W/cm}^2$). The laser thus induces a charge separation and strong electric fields which pull the electrons back towards the laser propagation axis to produce an oscillating density perturbation co-propagating behind the laser (see Fig. 1.1).

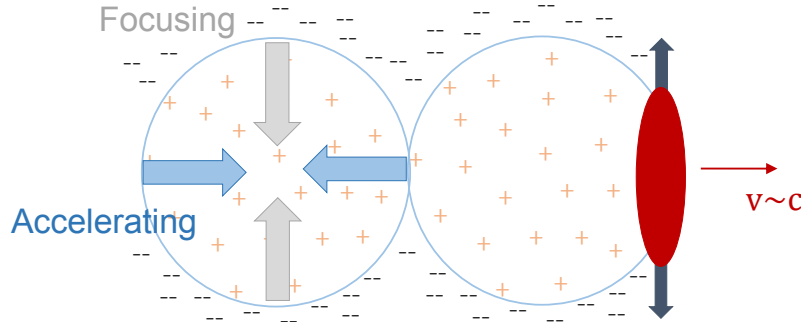


Figure 1.1: Laser plasma acceleration scheme.

The accelerating field E_0 scales with the plasma density n_0 as:

$$E_0[\text{GV/m}] \approx 96\sqrt{n_0(10^{18}\text{cm}^{-3})}. \quad (1.3)$$

For a plasma density of 10^{18} cm^{-3} and optimal laser parameters, LPAs can produce acceleration gradients on the order of 100 GV/m, three orders of magnitude greater than those capable of being sustained in RF cavities.

These high fields make plasma a very attractive medium for building a high energy collider, and in a series of experiments over the past decade the energy was increased

at Lawrence Berkeley National Laboratory from approximately 100 MeV in 2004 [6], to 1 GeV in 2006 [7], to 4.3 GeV in 2014 [8], and most recently to 8 GeV in 2019 [9]. Although these large gradients are very promising, several advancements are required before LPAs are suitable for high energy particle colliders, including further increases in beam energy, improvements in beam quality, and advances in laser technology.

While the acceleration gradients can reach hundreds of gigavolts per meter, there are limitations to how much energy gain can be achieved in a single LPA stage. Since the laser loses energy as it drives the plasma wave, the final particle beam energy gain is eventually limited by the laser energy available. Once the laser energy has been depleted, it can no longer drive a strong enough wake to continue accelerating particles. At this point, a new laser pulse must be coupled into a new LPA stage for further acceleration.

In addition, high luminosities at the collision point are needed to search for rare phenomena. The number of events per second N_{events} for a certain particle production pathway is given by $N_{events} = L \cdot \sigma_{events}$ where L is the beam luminosity and σ_{events} is the event cross-section, or probability that an event occurs. The unit for cross-section is barns, or 10^{-24} cm², and cross-sections for particles such as the Standard Model Higgs are on the order of femto-barns. To produce enough events for a graduate student to finish a thesis, high beam luminosity is required.¹

To achieve high beam luminosities, the LPA must maintain beam quality. One intrinsic measure of beam quality is emittance, defined as the area of the beam in position and momentum phase space. A low-emittance beam increases the likelihood of particles interacting at the collision point. Emittance is dependent on the technique used to inject particles into the plasma wave, which determines the initial beam quality, as well as any mechanisms that cause emittance growth in each LPA stage and during the drift space between stages. Factors such as energy spread, nonlinear focusing forces, and Coulomb scattering off plasma particles can all degrade emittance.

1.3 Outline

This dissertation focuses on the first demonstration of staging two laser plasma accelerators to reach higher energies and novel techniques for injecting and acceler-

¹The LHC had an average beam luminosity of 10^{34} cm⁻² s⁻¹ in 2018 [4], and LEP had a maximum luminosity of 10^{32} cm⁻² s⁻¹ [10].

ating particles with high beam quality. The outline of this thesis is as follows. In Chapter 2, a theoretical background of laser plasma accelerators is presented, including a description of plasma waves and mechanisms for injecting particles into the wave. In addition, limitations to the final beam energy are introduced that will need to be addressed if LPAs are to be applicable to high energy physics. Specifically, the laser will diffract as it propagates, decreasing intensity and limiting the length over which a wake can be driven. In addition, laser energy is lost while driving the wake, putting an upper limit on the length of a single LPA stage.

In Chapter 3, we demonstrate staging of two laser plasma accelerators in order to overcome laser energy loss. In this experiment, one laser pulse injects and accelerates electrons in a gas jet LPA stage. The electrons are then focused into a plasma channel LPA stage which is driven by a second, independent laser pulse for further acceleration. Here, we confirm energy gain in the second stage and use the electron beam to probe and better understand the LPA wakefield structure. This chapter includes experimental results which were published in:

- Steinke, S. and van Tilborg, J. and Benedetti, C. and Geddes, C. G. R. and Schroeder, C. B. and Daniels, J. and **Swanson, K.** and Gonsalves, A. and Nakamura, K. and Matlis, N. and Shaw, B. and Esarey, E. and Leemans, W. P., *Multistage Coupling of Independent Laser-Plasma Accelerators*, Nature **530** (2016).
- Steinke, S and van Tilborg, J. and Benedetti, C. and Geddes, C. G. R. and Daniels, J. and **Swanson, K.** and Gonsalves, A. and Nakamura, K. and Shaw, B. and Schroeder, C. B. and Esarey, E. and Leemans, W. P., *Staging of Laser-Plasma Accelerators*, Physics of Plasmas **5**, 056705 (2016).

While post-acceleration was achieved in the second LPA stage, the capture efficiency of the electron beam was low due to poor beam quality. In Chapter 4, we develop a high-quality electron injector by tailoring the plasma density profile. We demonstrate reproducible injection of beams with low energy spreads, divergence, and pointing fluctuation over a tunable range of beam energies. Such an injector would be suitable for a first stage in a high energy accelerator or for other applications that require high beam quality such as the production of x-ray pulses from coherent undulator radiation or Thomson backscattering. This chapter includes experimental results published in:

- **Swanson, K. K.** and Tsai, H.-E. and Barber, S. and Lehe, R. and Mao, H.-S. and van Tilborg, J. and Nakamura, K. and Geddes, C. G. R. and Schroeder, C. and Esarey, E. and Leemans, W. P., *Control of Tunable, Monoenergetic Laser-Plasma-Accelerated Electron Beams Using a Shock-Induced Density Downramp Injector*, Phys. Rev. Accel. Beams **20**, 051301 (2017).
- Barber, S. K. and van Tilborg, J. and Schroeder, C. B. and Lehe, R. and Tsai, H.-E. and **Swanson, K. K.** and Steinke, S. and Nakamura, K. and Geddes, C. G. R. and Benedetti, C. and Esarey, E. and Leemans, W. P., *Measured Emittance Dependence on the Injection Method in Laser Plasma Accelerators*, Phys. Rev. Lett. **119**, 104801 (2017).
- Tsai, H.-E. and **Swanson, K. K.** and Barber, S. and Lehe, R. and Mao, H.-S. and Mittelberger, D. E. and Steinke, S. and Nakamura, K. and van Tilborg, J. and Schroeder, C. and Esarey, E. and Geddes, C. G. R. and Leemans, W. P., *Control of Quasi-Monoenergetic Electron Beams From Laser-Plasma Accelerators with Adjustable Shock Density Profile*, Physics of Plasmas **25**, 043107 (2018).
- Mao, H.-S. and Fan-Chiang, L. and Tsai, H.-E. and **Swanson, K. K.** and Barber, S. K. and Steinke, S. and van Tilborg, J. and Geddes, C. G. R. and Leemans, W. P. *Gas Density Structure of Supersonic Flows Impinged On By Thin Blades for Laser-Plasma Accelerators*, (in progress).

Chapter 5 will detail the design and development of a cryogenically-formed, variable-radius plasma channel. Many different structures for laser guiding to overcome diffraction have been successfully demonstrated, including the one used in the staging experiment, but they do not efficiently accelerate positively charge particles or preserve beam emittance. Here, we describe the development of a new structure which has the potential to overcome these limitations. With this cryogenically-formed channel, we establish the tunability of the channel properties such as channel diameter and demonstrate low-power laser guiding.

Finally, we summarize our results in Chapter 6 with a note on potential directions for future work.

Chapter 2

Laser Plasma Acceleration

In this chapter, we present a theoretical description of laser plasma acceleration. We study the motion of plasma particles in the fields of a laser and describe the wakefields excited behind the pulse. We then detail different methods for injecting electrons into the wake and the impact they have on beam quality. Finally, effects which can limit the particle energy gain will be discussed along with techniques to overcome them.

2.1 Plasmas and Plasma Frequency

A plasma is a collection of unbound positively and negatively charged particles in a state of quasi-neutrality. Because the charged particles are free, they can move around, generating and interacting through long-range electric and magnetic fields.¹ In contrast to a gas or liquid in which particles interact mostly through collisions, these electromagnetic fields allow for dynamic, collective behaviors that are not seen in other states of matter.

Of particular importance to a laser plasma accelerator is the capability of a plasma to sustain waves. In a neutral plasma when there is no disturbance, the number of ions equals the number of electrons in any given volume such that the total force

¹Not all ionized gases qualify as a plasma. To be a plasma, the ionized gas must satisfy three conditions: 1) the gas is dense enough that the Debye length is much shorter than the dimensions of the system and any large charge concentrations or external potentials become shielded by the plasma electrons, 2) there are enough particles within that shield ($N_D \gg 1$ where N_D is the number of particles in a Debye sphere), and 3) the mean time between particle collisions τ is small relative to the plasma oscillation frequency ω ($\omega\tau > 1$). [11]

on all the particles is zero. However, if an electron is displaced from its equilibrium position, the resulting positive charge will exert a restoring force, causing the electron to oscillate about its equilibrium position. Because the electrons are strongly coupled, they will all oscillate together at a frequency which depends on the plasma properties.

For example, consider an idealized plasma with an equal number of electrons and ions, all initially motionless and uniformly distributed, with density n_0 . This is called a cold plasma. Because the ion mass is much larger than the electron mass, the electron dynamics occur on a much shorter timescale and the ions can be considered fixed over this duration. Assume all the electrons in a sheet are suddenly displaced by a small distance x (see Fig 2.1). Because of this displacement, a charge density $\rho = -en_0x$ develops in the region with excess electrons, and a charge density of $\rho = en_0x$ develops in the region with excess ions, where e is the electric charge. Together, the charge separation produces an electric field $E = -\rho/\epsilon_0 = -en_0x/\epsilon_0$. The acceleration felt by a single electron with mass m_e in the sheet as a result of this field is:

$$\frac{d^2x}{dt^2} = \frac{e}{m_e}E = -\frac{e^2n_0}{m_e\epsilon_0}x, \quad (2.1)$$

which describes the motion of a simple harmonic oscillator with frequency ω_p :

$$\omega_p = \sqrt{\frac{e^2n_0}{m_e\epsilon_0}}, \quad (2.2)$$

where ω_p is called the plasma frequency.² The plasma frequency determines one of the fundamental time-scales in plasma physics, where $1/\omega_p$ is the minimum time it takes the plasma to respond to a force. For a plasma density of 10^{18} cm^{-3} , the plasma frequency is 56 THz, and the plasma will respond on the order of tens of femtoseconds.

2.2 Electromagnetic Waves Propagating Through a Vacuum and a Plasma

The laser plasma accelerators developed and used in this thesis are driven by intense, femtosecond-scale laser pulses. The spatial profile of these pulses is usually

²Alternative derivation shown in Appendix 1.

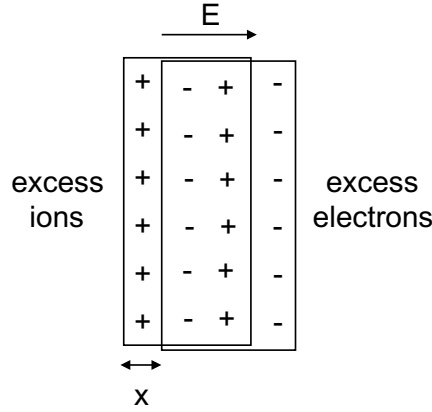


Figure 2.1: Model for simplified derivation of the plasma frequency.

approximated as a Gaussian³ whose intensity $I(r, z)$ can be written as:

$$I(r, z) = I_0 \frac{w_0^2}{w(z)^2} \exp\left(\frac{-2r^2}{w(z)^2}\right), \quad (2.3)$$

where r is the radial distance from the center of the beam, $w(z)$ is the laser spot size at longitudinal position z , w_0 is the laser spot size at focus ($z = 0$), and I_0 is the laser intensity at the center of the beam at focus ($r = 0, z = 0$). In vacuum, the beam spot size $w(z)$ evolves with z as:

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}, \quad (2.4)$$

where

$$z_R = \frac{\pi w_0^2}{\lambda}, \quad (2.5)$$

and λ is the laser wavelength. The parameter z_R , called the Rayleigh length, is the distance along z at which the intensity has decreased by half and is a measure of how long the laser pulse will remain intense. For a spot size of $50 \mu\text{m}$, which is typical for an LPA experiment, the Rayleigh length is approximately 10 mm for a wavelength of 800 nm.

³In actuality, many laser systems have a super-Gaussian profile after amplification, leading to side lobes at focus.

The propagation of a low amplitude electromagnetic wave, such as a laser pulse, in a cold plasma⁴ is described using the dispersion relation: [11]

$$\omega^2 = \omega_p^2 + c^2 k^2, \quad (2.6)$$

where ω is the laser angular frequency, $k = 2\pi/\lambda$ is the laser wavenumber, and c is the speed of light. From Eq. 2.6, it can be seen that k becomes imaginary when $\omega_p > \omega$. At these frequencies, the electrons can move fast enough to cancel out the electromagnetic field within one laser period, and the propagating electromagnetic wave exponentially decays with a penetration depth, or the depth at which the intensity falls to $1/e$, equal to c/ω_p . The condition that $\omega > \omega_p$ determines the critical density n_c for a wave to propagate through a plasma:

$$n_c = \frac{m_e \omega^2 \epsilon_0}{e^2}. \quad (2.7)$$

When $n_0 < n_c$, the plasma is called underdense, and when $n_0 > n_c$ the plasma is called overdense.

From the dispersion relation, it can be seen that the phase velocity of a wave $v_p = \omega/k$ propagating through an underdense plasma is given by:

$$v_p = \sqrt{c^2 + \frac{\omega_p^2}{k^2}} > c, \quad (2.8)$$

and the group velocity $v_g = d\omega/dk$ is given by:

$$v_g = \frac{c^2 k}{\sqrt{\omega_p^2 + c^2 k^2}} = c \sqrt{1 - \frac{\omega_p^2}{\omega^2}} < c. \quad (2.9)$$

Both the phase and group velocities are functions of the plasma density and therefore changes in the plasma density will affect the pulse propagation.

2.3 Ponderomotive Force

While the plasma affects the propagation of an electromagnetic field, the EM field will in turn affect the plasma. As a laser propagates through an underdense plasma,

⁴It is often assumed that before the arrival of the laser pulse the plasma is cold. As will be discussed later, the waves excited in the plasma by a laser pulse propagate at approximately the speed of light. Relative to this speed, the initial thermal velocities of the particles can be neglected, and the plasma is said to have zero temperature.

charged particles move under the influence of the laser's electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields as described by the Lorentz force:

$$\frac{d\mathbf{p}}{dt} = q \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right), \quad (2.10)$$

where $\mathbf{p}$ is the particle momentum, q the particle charge, and $\mathbf{v}$ the particle velocity. When the particle velocity is much less than the speed of light $v \ll c$, the Lorentz force simplifies to $d\mathbf{p}/dt = q\mathbf{E}$. For an EM field of the form $\mathbf{E} = E_L \sin(kz - \omega t)\hat{x}$, an electron will oscillate linearly along the polarization direction at the same frequency (shown in Fig. 2.2(a)). This electron has a normalized momentum of $p_x/m_e c = v_x/c = a_0 \cos(kz - \omega t)$ where a_0 is the normalized vector potential defined as:

$$a_0 = \frac{eE_L}{\omega m_e c}. \quad (2.11)$$

In more commonly used units, $a_0 \approx 8.5 \times 10^{-10} \lambda [\mu m] \sqrt{I \left[\frac{W}{cm^2} \right]}$.

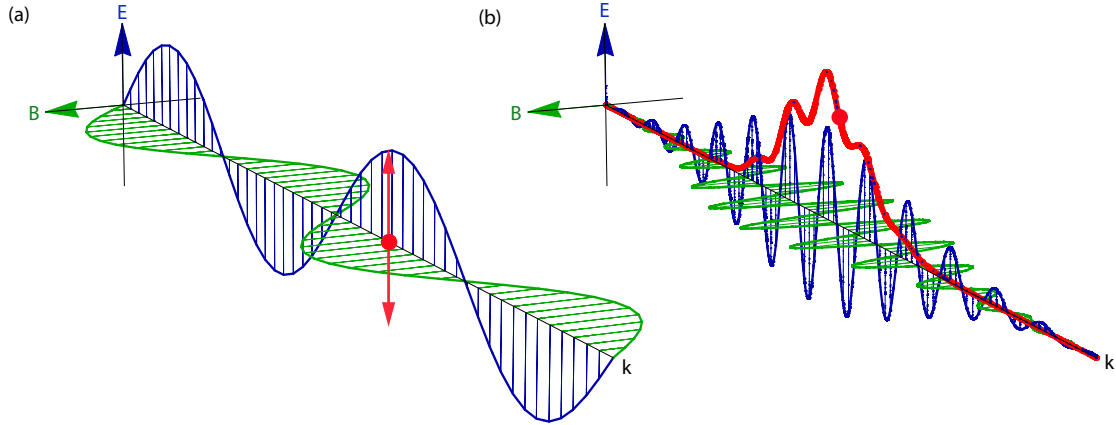


Figure 2.2: Electron motion in a: (a) plane laser field with $a_0 < 1$ and (b) Gaussian laser field. The laser's electric field is shown in blue, magnetic field in green, and electron motion in red.

As the electric field amplitude increases (a_0 increases) and the particle velocity approaches the speed of light, the $\mathbf{v} \times \mathbf{B}$ term in the Lorentz equation becomes important, and the motion of the electrons becomes more complicated. The normalized vector potential roughly differentiates the non-relativistic $a_0 < 1$ and the relativistic $a_0 > 1$ regimes.⁵ In these two regimes, the collective motion of the plasma electrons is

⁵The $a_0 < 1$ regime is also called the linear regime and $a_0 > 1$ the nonlinear regime. Both terms will be used in this thesis.

very different. A normalized vector potential $a_0 = 1$ corresponds to a laser intensity of $2 \times 10^{18} \text{ W/cm}^2$ for an 800 nm laser wavelength.

For a non-uniform laser profile, the electrons will be pushed away from regions of high intensity. For example, a Gaussian laser pulse has an inhomogeneous field with a peak intensity on axis. In this field, an electron will feel a larger outward force during the half-cycle it is in the higher field region and a smaller restoring force during the half-cycle it is in the lower field region. Thus, the electron's center of oscillation feels a net outward force (see Fig 2.2(b)). This force, called the ponderomotive force $\mathbf{F}_p$, in the linear regime is given by:⁶ [12]

$$\mathbf{F}_p = -\frac{mc^2 \nabla a^2(r)}{4}. \quad (2.12)$$

The ponderomotive force is independent of the sign of the particle charge; therefore, both ions and electrons will be pushed away from the high intensity regions. However, combining Eq. 2.11 and Eq. 2.12, it can be seen that $\mathbf{F}_p$ is inversely proportional to the particle mass. As a result, the electrons will be accelerated m_p^2/m_e^2 more than the ions. Therefore, for the intensities used in these experiments and on the time scale of the laser pulse duration, the ions remain stationary, and the ponderomotive force of the laser pulse generates a charge separation behind it.

2.4 Plasma Wakes in a Uniform Plasma

The charge separation created by the ponderomotive force induces electric fields which pull the electrons back towards the laser axis. The electrons overshoot the axis, causing them to oscillate about the laser axis with a frequency equal to the electron plasma frequency ω_p . As the laser propagates through the plasma with group velocity $v_{g,L}$, each electron starts oscillating behind it. The first electron starts oscillating at $z = 0, t = 0$, and the next electron starts oscillating at $z = \Delta z, t = \Delta t = \Delta z/v_{g,L}$. Therefore, the density oscillation becomes a wave propagating behind the laser with phase velocity $v_p = \Delta z/\Delta t = v_{g,L}$ and wavelength $\lambda_p = 2\pi v_p/\omega_p$. The laser group velocity thus determines the phase velocity of the plasma wave. Approximating $v_{g,L}$ as the speed of light,

$$\lambda_p[\mu m] \approx \frac{3.3 \times 10^{10}}{\sqrt{n_0(cm^{-3})}}. \quad (2.13)$$

⁶Derived in Appendix 1.

For a plasma density of 10^{18} cm^{-3} , the plasma wavelength is $33 \text{ } \mu\text{m}$.

The plasma density profile $n(r, z)$ and the wake fields $E(r, z)$ and $B(r, z)$ depend on the laser intensity and temporal profile. To simplify the problem, it is often assumed that the fields depend only on the radial coordinate and the variable $\zeta = z - v_{g,L}t$. Because the pulse evolves on a scale much longer than the plasma response, the laser pulse changes very little during the time a plasma particle sees it.⁷ Therefore, it can be assumed that the pulse is non-evolving, and the fields can be written in coordinates co-moving with the laser, ζ .

A cold, non-relativistic plasma can only support a traveling wave up to a certain maximum electric field amplitude. Past this limit, the fluid velocity exceeds the wake phase velocity and the plasma wave breaks, losing coherence. In 1 dimension, when $a_0 < 1$, the maximum peak amplitude of the electric field in the plasma wave E_0 can be estimated using the Poisson equation $\nabla \cdot \mathbf{E} = -en_0/\epsilon_0$. Assuming the field has the form $E_z = E_0 \sin(k_p \zeta)$ where $k_p = \omega_p/v_p$ and $v_p \approx c$ then:

$$E_0[V/m] = \frac{cm_e \omega_p}{e} \approx 96 \sqrt{n_e[cm^{-3}]}. \quad (2.14)$$

This maximum sustainable field is called the cold nonrelativistic wave breaking field. [14] For a density of 10^{18} cm^{-3} , $E_0 \approx 100 \text{ GV/m}$. In the non-linear regime, the electric field can exceed this limit. In 1 dimension, it can be shown that the maximum electric field amplitude is given by $E_{WB} = \sqrt{2(\gamma_p - 1)}E_0$ where $\gamma_p = (1 - v_p^2/c^2)^{-1/2} \simeq \omega/\omega_p$ is the relativistic Lorentz factor for the plasma wave. [15] E_{WB} is called the cold, relativistic wave breaking field.

In the linear regime, for a given laser intensity profile, the density perturbation behind the laser δn and the longitudinal field E_z can be derived using the Lorentz force, Poisson's equation, and the continuity equation: [16]

$$\frac{\partial}{\partial t} \frac{\delta n}{n_0} = -\nabla \cdot \mathbf{v}. \quad (2.15)$$

From these equations, the motion of the electrons in response to a laser field is given by: [17]

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2 \right) \frac{\delta n}{n_0} = \frac{c^2}{2} \nabla^2 a^2, \quad (2.16)$$

⁷The pulse evolves on the scale of the Rayleigh length $\propto w_0^2/\lambda$ and on the time scale $t \propto w_0^2/(v_{g,L}\lambda)$. If the laser evolution time scale is much larger than $1/\omega_p$ then this approximation holds. [13]

which describes a harmonic oscillator with oscillation frequency ω_p driven by the ponderomotive force of the laser. The general solution for the density perturbation $\delta n/n_0$ behind the laser pulse is given by:

$$\frac{\delta n}{n_0} = \frac{c^2}{2\omega_p} \int_0^t \sin[\omega_p(t-t')] \nabla^2 a^2(r, t') dt'. \quad (2.17)$$

Using Poisson's equation, the accelerating field induced by the density perturbation becomes:

$$E_z = -\frac{m_e c^2 \omega_p}{2e} \int_0^t \sin[\omega_p(t-t')] \nabla a^2(r, t') dt'. \quad (2.18)$$

The specific solutions for Eq. 2.17 - 2.18 can be found by assuming a particular laser temporal profile. If $a_0^2 \ll 1$, and $\delta n \ll n_0$, then analytical solutions can be found. For a half sinusoidal pulse with vector potential $a = a_0 \sin(\zeta/c\tau_L)$ when $\zeta/c\tau_L \in [0, \pi]$ and $a = 0$ elsewhere, the density perturbation and longitudinal electric field behind the pulse become:

$$\frac{\delta n}{n_0} = -\frac{\pi a_0^2}{8} \sin(k_p \zeta), \quad (2.19)$$

$$\frac{E_z}{E_0} = -\frac{\pi a_0^2}{8} \cos(k_p \zeta). \quad (2.20)$$

Here, the condition for maximum wake excitation efficiency, $c\tau_L \approx \lambda_p$, was assumed. When this condition is not true, the plasma oscillations are no longer resonant with the laser pulse, leading to decreased efficiency and reduced wake amplitude.

As seen in Eq. 2.20, the accelerating field is proportional to the laser intensity. To excite larger amplitude wakes, lasers are often focused to small spot sizes to increase the intensity. However, when the laser spot size starts to approach the plasma wavelength, the radial extent of the laser will influence the wakefield, and transverse plasma motion becomes important. In two dimensions, the transverse fields can be calculated using the Panofsky-Wenzel theorem $\partial E_r / \partial z = \partial E_z / \partial r$. [18], [19] For a pulse with a Gaussian radial profile $a \propto a_0 e^{-r^2/w_0^2}$, the longitudinal and radial fields become: [17]

$$\frac{E_z}{E_0} = -\frac{\pi a_0^2}{8} e^{-2r^2/w_0^2} \cos(k_p \zeta), \quad (2.21)$$

$$\frac{E_r}{E_0} = \frac{\pi a_0^2}{8} \frac{4r}{w_0^2 k_p} e^{-2r^2/w_0^2} \sin(k_p \zeta). \quad (2.22)$$

The density perturbation and fields behind the laser pulse are shown in Fig. 2.3 where the laser, centered at $k_p \zeta = 0$, propagates from left to right and has the form

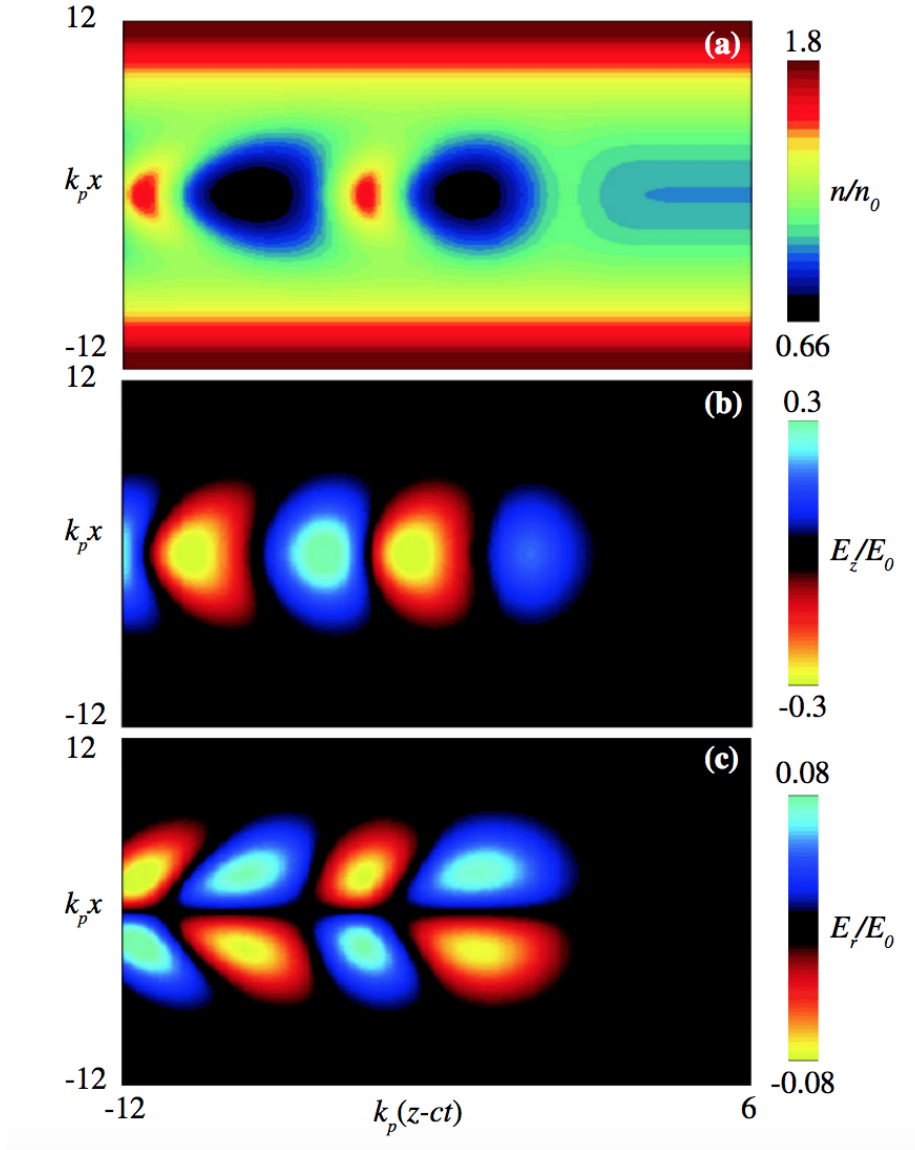


Figure 2.3: Plasma wave driven in the linear regime: (a) Density perturbation n/n_0 , (b) longitudinal electric field E_z/E_0 , and (c) transverse electric field E_r/E_0 . Figure taken from reference [20].

$a = a_0 \exp(-r^2/w_0^2 - z^2/4L^2)$ with $a_0 = 1$, $k_p r_L = 5$, and $k_p L = 1$. The laser propagates in a matched parabolic plasma channel. In this regime, the accelerating and decelerating fields are symmetric as well as the focusing and defocusing fields. There exists a phase region which is both accelerating and focusing for electrons as

well as a phase region suitable for accelerating and focusing positrons.

In the linear regime, the laser pulse perturbs only a small fraction of the electron density, $\delta n/n_0 < 1$, but as the laser intensity increases, the ponderomotive force completely blows out the electrons, forming a cavity of ions surrounded by a sheath of electrons. [21]–[23] This regime is often called the bubble or blow-out regime. When $a_0 \geq 1$ and $\delta n/n_0 \gg 1$, analytical solutions are available for only a few cases. With other parameters, simulations are required to obtain the density and wake fields. Fig. 2.4 shows the induced density perturbation in the nonlinear regime for a laser pulse of the form $a = a_0 \exp(-r^2/w_0^2 - z^2/4L^2)$ where $a_0 = 4$, $k_p w_0 = 5$ and $k_p L = 1$. [20] The laser propagates from left to right with the laser centered at $k_p \zeta = 0$.

Inside the ion cavity, the longitudinal field is uniform as a function of radial coordinate. As a result, particles at different radii experience the same accelerating field which minimizes the beam energy spread.⁸ In addition, the radial fields are linear, preserving beam emittance during acceleration. However, in this regime, the fields are no longer symmetric for electrons and positrons. When all the electrons are expelled from the ion cavity, the wake is always defocusing for positrons. At lower laser intensities, not all of the electrons are expelled, enabling a regime where there is a small phase region which is both accelerating and focusing for positively charged particles. This phase region is located at the back of the cavity where the electron density is high and the accelerating field is non-linear. Therefore, even in this regime, it is harder to accelerate positrons while maintaining high beam quality.

2.5 Particle Injection

As the wake propagates past a plasma electron, the longitudinal field can accelerate the electron. A particle injected into the correct phase of the plasma wake with enough initial velocity can interact with the accelerating field long enough to achieve substantial energy gain. If the electron can be accelerated to the wake phase velocity by the time it reaches the back of the wake, it has enough momentum to keep up with the wake and be injected. Many techniques have been theoretically or experimentally studied to inject particles into the wake including, among others,

⁸Other effects including dephasing (described in Sec. 2.6) and beam loading can induce energy spread.

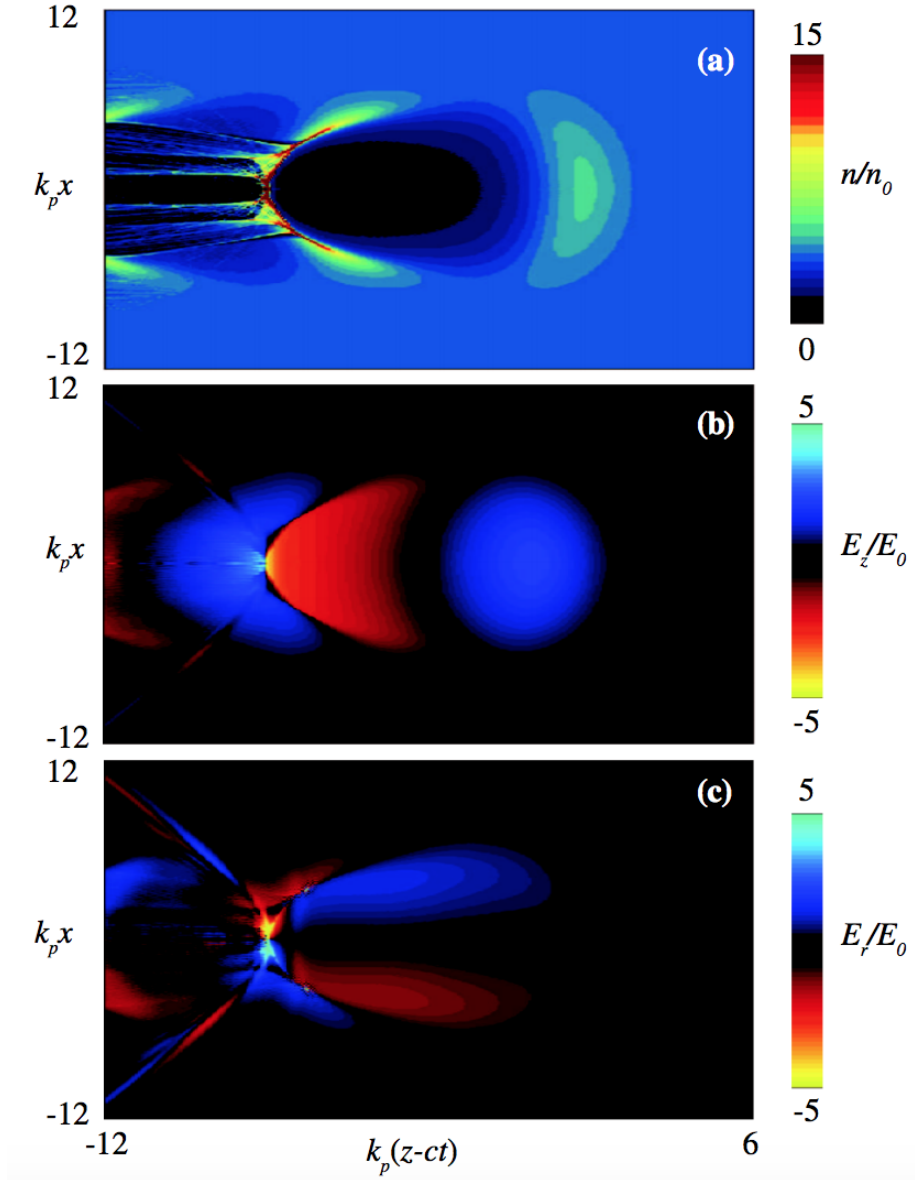


Figure 2.4: (a) Density perturbation, (b) accelerating field E_z , and (c) focusing field E_r in the nonlinear regime with a Gaussian laser pulse and $a_0 = 4$. Figure taken from reference [20].

self-injection, ionization injection and density downramp injection.⁹

⁹Other techniques, which were not implemented in this thesis, include colliding pulse and two-color laser-ionization injection which use two laser beams to decouple the injection from the acceleration process. In the former, the first laser beam drives the wakefield while the second beam, colliding with the first, heats the electrons which gain enough momentum to catch up to the wake and be

2.5.1 Self-Injection

Self injection occurs when background plasma electrons are trapped in the wake due to high amplitude fields. The motion of electrons under the influence of a sinusoidal wake in 1D is shown in Fig. 2.5 as a function of the phase in the wake $k_p\zeta$ and the electron relativistic factor $\gamma = (1 - v_z^2/c^2)^{-1/2}$. [15] The wake has a phase velocity v_p such that $\gamma_p = (1 - v_p^2/c^2)^{-1/2} = 25$.¹⁰

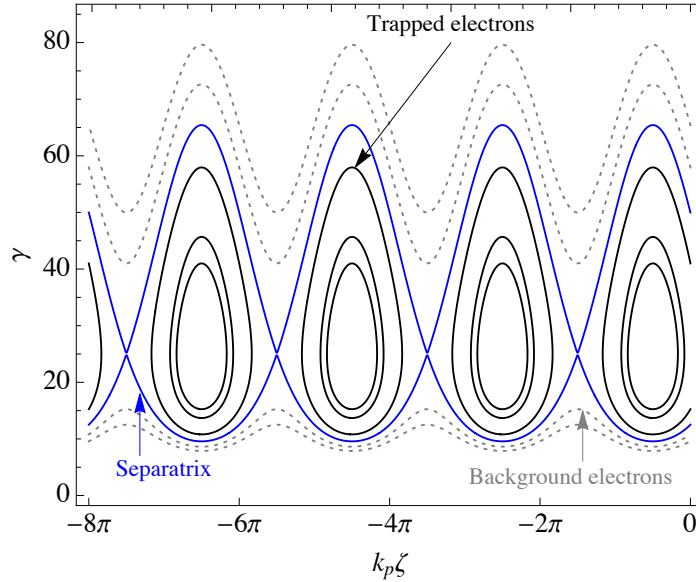


Figure 2.5: Electron trajectories in a sinusoidal plasma wave.

Background electrons with low initial velocity slip backwards relative to the wake. While they are perturbed by the accelerating fields, they do not gain enough energy to catch up. Electrons which start with high enough initial velocity can be accelerated such that $\gamma > \gamma_p$ before the electron has slipped out of the wake. In the first wavelength behind the laser, the accelerating phase of the wake is in the phase region $k_p\zeta \in [-3\pi/2, \pi/2]$. If the electron has not been accelerated to the wake phase veloc-

injected. [24]–[26] Two-color laser-ionization injection uses a long wavelength pulse to generate a large ponderomotive force but small peak electric field and a short wavelength pulse with a small ponderomotive force but large peak electric field. The long wavelength pulse generates the wake without ionizing while the short wavelength pulse, co-propagating with the long wavelength, ionizes bound electrons. [27] Two-color laser-ionization has been theoretically demonstrated to produce very low emittance beams (~ 0.03 mm mrad).

¹⁰Derived in Appendix 1.

ity by the time it reaches $k_p\zeta = -3\pi/2$, it will not be injected.¹¹ The separatrix is defined as the critical initial velocity and phase which separates the untrapped and trapped electrons. The minimum accelerating field required to accelerate electrons from rest to a trapped orbit is the wavebreaking field E_0 given by Eq. 2.14.

In 3D, radial effects can also contribute to particle injection. [28] As electrons are pushed outward as a result of the laser's ponderomotive force, they traverse the sheath of electrons surrounding the ion cavity, experience a focusing force and are pulled back into the accelerating phase of the wake. If these electrons can gain enough momentum by the time they reach the back of the wake, they can be trapped.

Self-injection is experimentally simpler than many of the other techniques, but it requires an intense pulse to generate large amplitude fields. As self-injection relies on the evolution of the laser pulse as well as the wake structure, this technique is difficult to control and predict. In addition, if the laser remains intense enough to self-trap over an extended distance, electrons will be continuously injected. [29], [30] Electrons can be injected into the same phase of the wake over an extended distance, increasing the total beam energy spread as they are accelerated for varying amounts of time, or injected into different phases of the wake, leading to a spread in energies as the electrons experience different accelerating fields. Quasi-monoenergetic electron bunches with a few percent energy spread have been demonstrated by operating near the injection threshold or controlling the acceleration so as to bunch the beam in phase. [6], [31], [32] However, these techniques limit the tunability and operating range of the injector.

2.5.2 Ionization Injection

The laser intensity required to inject electrons can be reduced by using higher atomic number gases in a technique called ionization injection. [33]–[35] In this method, the gas is a mixture of elements that have multiple ionization states with a large difference in the ionization potential between two closed shells, such as helium and nitrogen.¹² In nitrogen, the L-shell electrons require 98 eV to ionize to N^{5+} , and the K-shell electrons require 552 eV and 667 eV of energy to ionize to N^{6+} and N^{7+} , respectively. Due to the large gap in ionization potentials between the L and

¹¹The wake structure repeats every 2π , leading to multiple buckets behind the laser in which particles can be accelerated.

¹²A mixture of gases rather than pure nitrogen is used because a high concentration of nitrogen will cause the laser to defocus. This effect will be examined in Sec. 2.7.1.

K shells, the leading edge of the laser pulse fully ionizes the helium and the outer L-shell nitrogen electrons, forming the plasma wake, while the inner-shell electrons are ionized only near the peak of the laser pulse.

Two processes contribute to the ionization process: barrier suppression and tunnel ionization.¹³ These regimes assume that the laser's electric field is quasi-static, the variation in the laser field is much slower than the ionization rate, and the field is spatially constant on the atomic length scale. Ionization through barrier suppression occurs when the laser's electric field E_L modifies the Coulomb potential of the atom such that the electron has enough energy to overcome the potential barrier. The laser intensity required for barrier suppression ionization is given by:

$$I[W/cm^2] = 4 \times 10^9 \left(\frac{U_i^4[eV]}{Z^2} \right). \quad (2.23)$$

The ionization potentials and required laser intensities to ionize various charge states are shown in Table 2.1 for hydrogen, helium, and nitrogen.

Charge state	Ionization Potential (eV)	Ionization Intensity (W/cm ²)
H ¹⁺	13.6	1.4×10^{14}
He ¹⁺	24.6	1.5×10^{15}
He ²⁺	54.4	8.8×10^{15}
N ¹⁺	14.5	1.8×10^{14}
N ²⁺	29.6	7.7×10^{14}
N ³⁺	47.4	2.3×10^{15}
N ⁴⁺	77.5	9.0×10^{15}
N ⁵⁺	97.9	1.5×10^{16}
N ⁶⁺	552.1	1.0×10^{19}
N ⁷⁺	667.0	1.6×10^{19}

Table 2.1: Ionization potentials and laser intensities for hydrogen, helium, and nitrogen.

Atoms can still be ionized at lower laser energies as the potential barrier becomes distorted by the laser's field, and the probability of an initially bound electron tunneling through the potential barrier increases. The probability of an electron to be

¹³Other processes for ionizing an atom include direct ionization where the photon energy exceeds the binding energy of the electron (the photon energy at the wavelengths used in this thesis is too low for this process to contribute) and multi-photon ionization where the electron gains enough energy through absorption of multiple photons.

ionized p is equal to:

$$p = 1 - \exp \left(- \int_{-\infty}^{\infty} W[E(t)] dt \right), \quad (2.24)$$

where the ionization rate W can be calculated using the Ammosov, Delone, and Krainov (ADK) model: [36]

$$W = \sqrt{\frac{3n^{*3}F}{\pi Z^3}} \frac{3FD^2}{8\pi Z} \exp \left(-\frac{2Z^3}{3n^{*3}F} \right), \quad (2.25)$$

and $n^* = Z/\sqrt{2E_i}$ is the effective principle quantum number, $D = (4eZ^3/Fn^{*4})^{n^*}$, and F and E_i are the laser field strength time averaged over the fast oscillations and ionization potential in atomic units.¹⁴ The dependence of the ionization rate on the laser field strength results in a sharp transition from un-ionized to ionized for a particular charge state. Taking an electric field with an envelope of the form $E(t) = E_0 \exp(-2 \ln 2t^2/\tau^2)$, the ionization probability as a function of laser intensity for different charge states of nitrogen is shown in Fig. 2.6 for a laser pulse duration of 30 fs and normalized vector potential $a_0 = 2$.

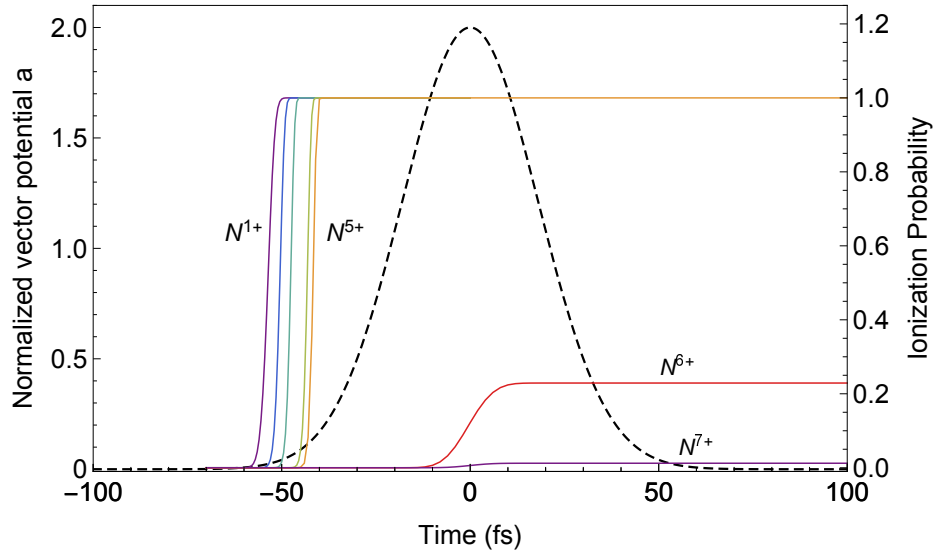


Figure 2.6: Ionization probabilities (solid) for the various charge states of nitrogen as a function of laser intensity. The laser temporal profile is shown as the dashed, black line.

¹⁴Electric field strength in atomic units = 5.142×10^{11} V/m, ionization potential in atomic units = 27.211 eV.

The helium and outer shell electrons are ionized near the foot of the laser pulse where, as can be seen in Fig. 2.4, they will experience an initial decelerating field. The inner bound electrons are ionized only near the peak of the pulse where they experience less of the decelerating phase of the wake and are more easily injected. By choosing a high Z species such as nitrogen, the laser intensity required to trap electrons can be reduced relative to self-injection. However, just as in self-injection, the inner-shell electrons will be injected as long as the laser intensity remains above the ionization threshold. Thus, for the high intensities required for large accelerating fields, electrons will be injected over an extended distance producing beams with a large energy spread. [33], [35] While the energy spread can be reduced by limiting the distance over which the laser remains intense, the operating range of this technique then becomes limited. [37]

2.5.3 Density Downramp Injection

A tailored density profile can decrease energy spread by reducing the injection length. [38], [39] The plasma wake phase velocity $\beta_p = v_p/c$ depends on the plasma density:

$$\beta_p = \frac{v_g}{c} \left[1 - \xi \frac{1}{n(z)} \frac{dn(z)}{dz} \right]^{-1}. \quad (2.26)$$

Along a density downramp, the laser propagates from a peak plasma density n_1 to a plateau plasma density n_2 where $n_2 < n_1$. As shown in Fig. 2.7, along the downramp where $dn/dz < 0$, the wake phase velocity decreases, reducing the velocity threshold β_{thresh} for an electron to be trapped relative to a uniform plasma. Injection is localized to the downramp, limiting the distance over which electrons can be injected. Density downramp injection has been demonstrated in a variety of plasma structures, and energy spreads of a few percent have been achieved. [40]–[42] In Chapter 4, shock-induced density downramp injection will be explored, and new techniques for controlling the electron beam quality will be demonstrated.

2.6 Energy Gain Limitations

Once particles are injected into the wake, the high acceleration gradients enable substantial energy gain. However, a number of effects can limit the final beam energy: laser diffraction, particle dephasing, and laser depletion. [17], [43]

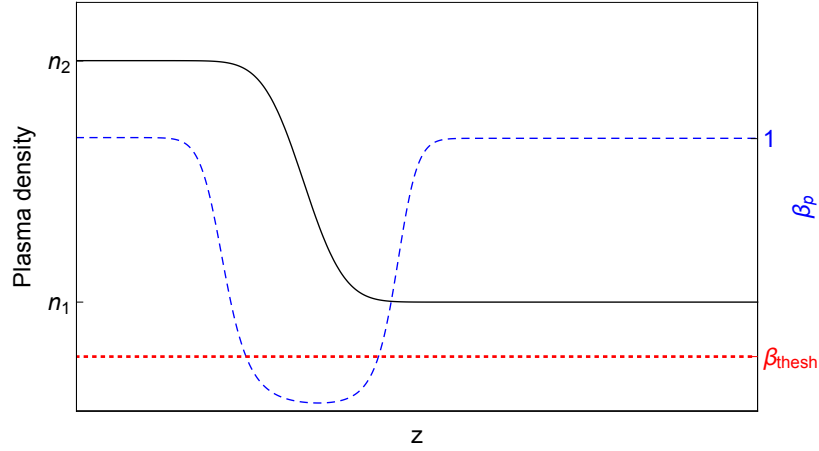


Figure 2.7: Density downramp $n(z)$ (solid, black line), plasma wake phase velocity $\beta_p(z)$ (dashed, blue line), and minimum electron velocity required to be injected β_{thresh} (dotted, red line).

2.6.1 Laser Diffraction

Of the energy limitations, laser diffraction is the most stringent. As a focused laser propagates through vacuum, it undergoes diffraction, decreasing the peak intensity. As the laser intensity decreases so does the wake amplitude and thus the accelerating field. The distance over which diffraction becomes important is the Rayleigh length given in Eq. 2.5. For a $50 \mu\text{m}$ spot size and 800 nm laser wavelength, the Rayleigh length is approximately 10 mm . Without any form of laser guiding, the final electron energy in the linear regime is limited to hundreds of MeV. The Rayleigh length can be extended by increasing the beam spot size, but for a given laser energy, the spot size is limited by the intensity required. Instead, diffraction can be overcome by guiding the laser pulse, as will be explained in Sec. 2.7.

2.6.2 Particle Dephasing

As the electrons gain energy, they become relativistic. Since the phase velocity of the plasma wave is less than the speed of light, the electrons will eventually slip out of the accelerating phase of the wake into the decelerating phase. At this point the electrons begin to lose energy. The distance over which this slippage occurs is called the dephasing length L_d . [17] In a three-dimensional plasma wave, since the wave is both focusing and accelerating over $\lambda_p/4$, the dephasing length is estimated as the distance the electrons travel before they slip forward one-quarter period of a plasma

wave: [5], [44]

$$L_d \approx \frac{\lambda_p^3}{2\lambda^2} \begin{cases} 1 & \text{for } a_0^2 \ll 1 \\ \sqrt{2}a_0/\pi & \text{for } a_0^2 \gg 1 \end{cases} \quad (2.27)$$

This approximation assumes a constant amplitude wake which may not be valid for a plasma wave that evolves as it propagates or when taking 3D effects such as diffraction into account.

In the linear regime, the dephasing length is approximately 2.8 cm for an 800 nm laser pulse propagating through a uniform plasma with density 10^{18} cm^{-3} . The dephasing length can be increased by decreasing the plasma density n_0 or tailoring the plasma density profile such that the electrons always remain in the accelerating phase. The latter can be achieved by longitudinally tapering the plasma density. [45], [46] If the plasma density increases with distance, as the electrons slip forward, the plasma wavelength decreases and the accelerating phase region shrinks, allowing the electrons to remain in the accelerating phase as they gain energy.

2.6.3 Pump Depletion

As the laser propagates through the plasma, it excites a wake and loses energy until the plasma wave can no longer be sustained. [47], [48] The characteristic length for energy loss, called the pump depletion length L_{pd} , depends on the plasma density and the laser intensity: [49]

$$L_{pd} \approx \frac{\lambda_p^3}{\lambda^2} \begin{cases} 2/a_0^2 & \text{for } a_0^2 \ll 1 \\ \sqrt{2}a_0/\pi & \text{for } a_0^2 \gg 1 \end{cases} \quad (2.28)$$

For an 800 nm wavelength laser pulse with intensity $1.4 \times 10^{18} \text{ W/cm}^2$ propagating through uniform plasma with density 10^{18} cm^{-3} , the depletion length is approximately 17.6 cm. The pump depletion length can be extended by increasing the laser intensity or decreasing the plasma density. Eventually, however, the laser will lose too much energy to continuously excite a wake, and a new laser needs to be coupled in. Thus, depletion represents the ultimate fundamental limit for the energy gain achievable in a single stage energy gain. The first experiment demonstrating coupling of two laser plasma accelerators to overcome depletion will be discussed in Chapter 3.

2.6.4 Energy Gain Scaling

With techniques for overcoming diffraction such that the laser can remain intense over many Rayleigh lengths, the energy gain in a single LPA stage is limited by electron beam dephasing and laser energy depletion. In the linear regime ($a_0^2 \ll 1$), while $L_d \ll L_{dp}$ for a longitudinally uniform plasma, dephasing can be mitigated by introducing tailoring the plasma density such that it increases with propagation distance. With this profile, laser depletion puts an upper limit on the acceleration length. In the nonlinear regime ($a_0^2 \gg 1$), $L_d \sim L_{dp}$, and no density tapering is needed. With the beam propagation limited by the laser depletion length, the laser will have transferred all its energy to the plasma wave before acceleration is terminated, leading to more efficient acceleration.

The energy gain in a single stage ΔW is the product of the accelerating field E_z and the length of acceleration L_{acc} :

$$\Delta W = eE_z L_{acc}. \quad (2.29)$$

Taking the accelerating field to be proportional to the cold, non-relativistic wave breaking field E_0 given by Eq. 2.14, the accelerating field scales with density as $n_0^{1/2}$. With an acceleration length limited by the laser depletion, the energy gain scales as n_0^{-1} , and higher energies can be achieved in a single stage by decreasing the plasma density. Figure 2.8 shows the energy gain achieved in various experiments demonstrating the n_0^{-1} fit.

2.7 Laser Guiding Techniques

Laser guiding to overcome diffraction can be achieved by tailoring the plasma index of refraction. [56], [57] The refractive index $\eta(r)$ in a plasma for small a_0 can be derived from the plasma dispersion relation:

$$\eta(r) = \frac{ck}{\omega} = \frac{c}{v_p} = \sqrt{1 - \frac{\omega_p(r)^2}{\omega^2}} = \sqrt{1 - \frac{\omega_p^2}{\omega^2} \frac{n(\mathbf{r})}{\gamma(r)n_0}}, \quad (2.30)$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the electron relativistic factor and can be approximated as $\gamma \approx 1 + a_0^2/2$. If the plasma density profile has the form $n(r) = n_0 + \Delta n(r) + \delta n(r)$ where $\Delta n(r)$ is the density perturbation caused by a pre-formed plasma channel and

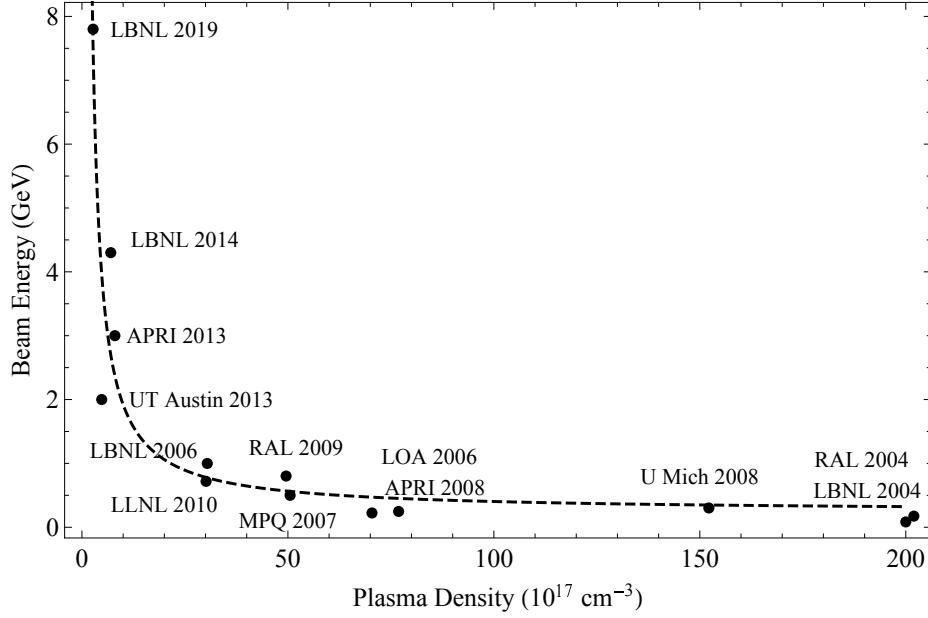


Figure 2.8: Experimental energy gain as a function of plasma density (points) and $1/n_0$ scaling (dashed line). [7]–[9], [31], [50]–[55]

$\delta n(r)$ is the density perturbation caused by the plasma wave, then the refractive index can be expanded as:

$$\eta(r) = 1 - \frac{\omega_p^2}{2\omega^2} \left[1 + \frac{\Delta n(r)}{n_0} + \frac{\delta n(r)}{n_0} - \frac{a_0(r)^2}{2} \right]. \quad (2.31)$$

When the refractive index is maximum on-axis, the laser wavefronts will curve inward, counteracting diffraction and guiding the laser. From Eq. 2.31, a maximum index of refraction on-axis can be achieved with either a minimum plasma density or a maximum laser intensity on-axis.

2.7.1 Relativistic Self-Guiding

When the laser is intense, electrons oscillate in the laser field with enough velocity to modify the plasma index of refraction: [58], [59]

$$\eta(r) = 1 - \frac{\omega_p^2}{\omega^2} \left[1 - \frac{a_0(r)^2}{2} \right]. \quad (2.32)$$

With a Gaussian laser pulse, this effect is strongest on-axis, leading to a higher refractive index and causing the laser to focus. This process is called relativistic

self-guiding. The threshold power required to overcome diffraction is given by the critical power $P_c \approx 17(\omega/\omega_p)^2$ [GW]. [60] When $P < P_c$, diffraction dominates, when $P = P_c$ the change in refractive index is perfectly matched to the laser diffraction and the laser self-guides, and when $P > P_c$ the laser focuses. For a plasma density of $n_0 = 10^{18} \text{ cm}^{-3}$, the critical power for self-focusing is 3 TW with an 800 nm laser pulse.

Because the plasma responds on the plasma frequency timescale, self-guiding does not occur for pulses with durations shorter than ω_p^{-1} , even when $P > P_c$. [13] Equation 2.32 does not take into account the change in density δn caused by the ponderomotive force of the laser. The driven plasma wake increases the plasma density at the front of the pulse which cancels out the effect due to relativistic self-guiding, and the front edge of the pulse diffracts. Relativistic self-guiding typically only applies to the main body of long pulses. [13] Similarly, if an intense laser pulse propagates through a gas or partially-ionized plasma, the laser can ionize atoms. With a higher intensity on axis, the pulse will generate a plasma density maximum on-axis, leading to ionization-induced defocusing. [61] For this reason, the amount of dopant gas used in ionization injection is usually only a few percent of the total volume of gas.

2.7.2 Grazing Incidence Guiding

An external channel allows the guiding to be decoupled from the laser. For a cylindrical waveguide, with a core index of refraction η_{core} surrounded by a cladding with index of refraction $\eta_{cladding}$, a laser pulse will be guided by total internal reflection off the channel wall if $n_{cladding} < n_{core}$. However, for high intensity pulses, the core can only be filled with either a gas or plasma, resulting in $\eta_{cladding} > \eta_{core}$. While total internal reflection is not possible in this case, the pulse can be guided by partial reflections at the capillary wall. [62], [63] If the angle of incidence is close to grazing, the losses at the wall are small. In this regime, a Gaussian beam with radius w_0 will be guided in a cylindrical capillary with 98% transmission in the fundamental mode if the capillary radius r_c satisfies $w_0 = 0.645r_c$. [64] The rest of the energy is coupled into higher order modes or contained in the wings of the Gaussian pulse and are thus lost at the capillary entrance. Laser pulses with intensities on the order of 10^{18} W/cm^2 have been guided through 8 cm-long, hydrogen-filled, glass capillaries with 90% transmission. [65]

Grazing incidence guiding does not depend on the laser power or plasma density,

allowing for particle acceleration at low densities. However, because the capillary radius is not much larger than the laser spot size, the wings of the laser pulse can damage the capillary walls and entrance. [63] The damage threshold for fused silica is on the order of 10^{14} W/cm². [66] If the laser pulse spot size is matched to the channel radius, the intensity at the channel wall of the fundamental mode is a factor of 10^{-4} lower than the intensity on axis. Therefore, a glass capillary can guide only up to 10^{18} W/cm². In addition, gas on-axis can cause ionization-induced defocusing, putting an upper limit on the density inside the channel.

2.7.3 Parabolic Pre-Formed Waveguides

Another guiding technique uses a pre-formed plasma channel. In the low-power limit $P \ll P_c$ and low intensity limit $a_0^2 \ll 1$, the index of refraction given by Eq. 2.31 simplifies to:

$$\eta(r) = 1 - \frac{\omega_p^2}{2\omega^2} \left[1 + \frac{\Delta n(r)}{n_0} \right]. \quad (2.33)$$

A laser pulse with a Gaussian radial profile will be guided inside a channel which has a parabolic plasma density profile of the form:

$$n(r) = n(0) + \Delta n \frac{r^2}{w_0^2}, \quad (2.34)$$

where $\Delta n = n(w_0) - n(0) = 1/(\pi r_e w_0^2)$ is the density drop from the channel radius to the channel center, w_0 is the laser spot size at focus, and $r_e = e^2/(4\pi\epsilon_0 m_e c^2)$ is the classical electron radius. [56], [67] Inside the channel, the pulse propagates with constant intensity if its spot size is equal to the matched spot size:

$$R_M = \left(\frac{\pi \Delta n r_e}{r_{ch}^2} \right)^{-1/4}, \quad (2.35)$$

where r_{ch} is the channel radius. If the spot size is not matched, the laser can still be guided, but the spot will oscillate in size between w_0 and R_M^2/w_0 .

A pre-formed channel allows for guiding of pulses with power less than the critical power for self-focusing and, with larger channel diameters, extends the lifetime of the waveguides. At lower intensities, particles can also be accelerated without self-injection, operating in a dark-current-free mode.

2.7.4 Hollow and Near-Hollow Plasma Channels

While guiding pulses over many Rayleigh lengths has been demonstrated in parabolic plasma channels, the on-axis plasma density causes emittance degradation through Coulomb scattering off background ions. Instead, hollow plasma channels or near-hollow plasma channels can guide a laser pulse while having unique properties for accelerating particles. [68]–[70]

A near-hollow plasma channel with radius r_w has a plasma density n_w at the walls and density n_c inside the channel:

$$n(r) = \begin{cases} n_{ch} & \text{for } r < r_w \\ n_w & \text{for } r \geq r_w \end{cases} \quad (2.36)$$

where $n_{ch} \ll n_w$. In a hollow plasma channel, $n_{ch} = 0$. The index of refraction is given by:

$$\eta = \frac{c}{v_p} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = \sqrt{1 - \frac{e^2 n(r)}{\epsilon_0 m_e \omega^2}}. \quad (2.37)$$

When the channel density is much less than the wall density, $n_{ch} > n_w$, and the channel will act like an optical fiber and confine the laser pulse.

A short laser pulse propagating through a near-hollow plasma channel will excite wakefields inside the wall which extend into the channel to provide focusing and accelerating forces for a particle beam, as well as wakefields inside the channel due to the channel density. When $n_{ch} \ll n_w$, the accelerating field E_z inside the channel dominates the wall fields and can be written as:

$$\frac{E_z}{E_w} = -\frac{\Omega^2}{4} \int_{-\infty}^{\zeta} k_w \cos[\Omega k_w (\zeta - \zeta')] a^2(r_c, \zeta') d\zeta', \quad (2.38)$$

where $E_w = m_e c^2 k_w / e$, $k_w^2 = n_w e^2 / \epsilon_0 m_e c^2$, and

$$\Omega = \left[1 + \frac{k_w r_c K_0(k_w r_c)}{2K_1(k_w r_c)} \right]^{-1/2}. \quad (2.39)$$

The accelerating field E_z is independent of transverse position, minimizing the bunch energy spread and emittance degradation.

The focusing field inside the channel is the sum of the driven wakefield inside the channel and the fields induced in the wall:

$$\frac{(E_r - B_\theta)}{E_w} = \frac{E_c k_c r}{2E_w} - \frac{k_w^2 r}{16\gamma_p^2} \Omega^3 \int_{-\infty}^{\zeta} \sin[\Omega k_w (\zeta - \zeta')] a^2(r_c, \zeta') d\zeta', \quad (2.40)$$

where $E_c = m_e c^2 k_c / e$. Inside the channel, the focusing force is linear with radial coordinate, preserving beam emittance during acceleration. In addition, if $n_{ch}/n_w \gg a_0^2(r_c)/(8\gamma_p^2)$, then the focusing field from the channel density dominates the field from the wall. In this case, the focusing field can be tuned to match the beam using the on-axis plasma density. Figure 2.9 shows the normalized accelerating field E_z/E_w and normalized focusing field $(E_r - B_\theta)/E_w$ for varying channel densities excited by a resonant Gaussian laser pulse with $a_0 = 1$. [71]

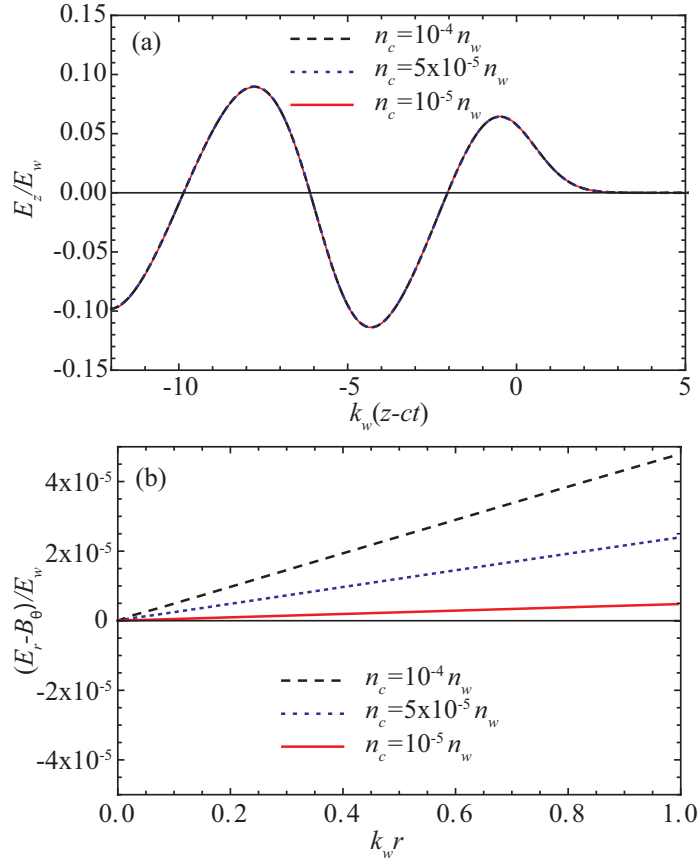


Figure 2.9: (a) Normalized accelerating field E_z/E_w and (b) normalized focusing fields $(E_r - B_\theta)/E_w$ at the peak E_z for a near-hollow plasma channel with varying channel densities. Figure taken from reference [71].

In addition, the decreased density inside the channel mitigates emittance growth due to Coulomb scattering off background ions, providing beam quality suitable for high energy physics applications, and the wakefields are symmetric for both electrons and positrons. Despite the successes of accelerating electrons in LPAs, positron ac-

celeration has been less explored. While LPA targets such as discharge capillaries and gas jets, as will be described in Chapter 2, have produced high quality electron beams, they are less suitable for positrons because of the asymmetric response of the wakefields to positively and negatively charged particles. The wakefields in a near-hollow plasma channel have a symmetric response to both electrons and positrons, and Chapter 5 will explore a new waveguide which can produce this desired density profile.

Chapter 3

Staging of Two Laser Plasma Accelerators

The energy gain of a particle beam is the product of the accelerating field and the length of acceleration. Laser diffraction is the most stringent limitation to the acceleration length, but many techniques have demonstrated guiding of a laser pulse over an extended distance. [6], [8], [9] In the linear regime $a_0^2 \ll 1$, the energy gain is limited by dephasing. With a plasma density of 10^{18} cm^{-3} , the dephasing length is 3 cm while the depletion length is 11.6 cm. Because the depletion length is much longer than the dephasing length, the acceleration ends before all of the laser energy has been transferred to the plasma. More efficient acceleration occurs in the nonlinear regime $a_0^2 \gg 1$ where the dephasing and depletion lengths are approximately equal. However, at these intensities, electrons can be self-injected over extended distances, increasing beam energy spread. In the quasi-linear regime ($a_0^2 \sim 1$), the depletion and dephasing lengths are still fairly equal and higher accelerating gradients can be achieved relative to the linear regime while not being high enough to self-inject. This regime is most suitable for efficiently accelerating dark-current-free particle beams.

The depletion L_d and dephasing L_{dp} lengths both scale with density n_0 as $L_d \approx L_{dp} \propto n_0^{-3/2}$. While decreasing the plasma density increases the acceleration length, keeping the pulse guided becomes more difficult at lower densities. For example, the critical power to self-guide P_c scales as n_0^{-1} . Decreasing the density increases the laser power, and thus energy, required to self-guide. When using a pre-formed plasma channel to counteract diffraction, a constant laser intensity during propagation requires that the laser spot size be equal to the matched spot size which scales with

density as $n_0^{-1/4}$. To continue decreasing density requires a larger spot size which in turn requires more laser energy to keep the pulse intense. Thus, while the accelerating length can be made longer, the particle energy gain in a single stage is limited by the laser energy available.

After the depletion length, a new laser is needed to continue driving a wake. This chapter demonstrates the coupling of two laser plasma accelerators, each with its own independent pulse. Section 3.1 describes chirped pulse amplification which allows for the high intensity laser pulses used in many LPA experiments, including this one. The laser system is detailed in Section 3.2, and Section 3.3 describes the LPA targets. The experimental set-up is given in Section 3.4. In Section 3.5, we successfully demonstrate coupling of electron beams from one LPA stage into another for post-acceleration.

3.1 Chirped Pulse Amplification

To excite plasma waves with large field amplitudes for particle injection and acceleration, laser intensities greater than 10^{18} W/cm² are needed. At these intensities, nonlinear effects such as self-phase modulation¹ or self-focusing inside materials can negatively distort the beam as it propagates through the laser system. Self-focusing in solid materials such as glass limit the laser to power levels on the order of megawatts per pulse. In addition, to reach these high intensities, pulse durations on the order of tens of femtoseconds are needed to resonantly drive the plasma wave, leading to fluences on the order of hundreds of J/mm² and far exceeding the damage threshold for many optics.² While increasing the beam diameter reduces the fluence, larger optics to accommodate the beam are required, increasing the expense and size of the laser system. Complimentary to increasing the beam size, a technique called chirped pulse amplification (CPA) can be used to amplify pulses to high intensities without inducing nonlinear pulse distortion or damage in the system's optical components. [72]

In CPA, as shown in Fig. 3.1, a pulse is stretched out in time before propagating

¹The laser's electric field can induce a change in a material's refractive index which is proportional to the laser intensity. This change in refractive index leads to a phase shift which affects the pulse's frequency.

²High-reflectance mirrors have a damage threshold on the order of hundreds of mJ/cm² for an 800nm, 30 fs pulse.

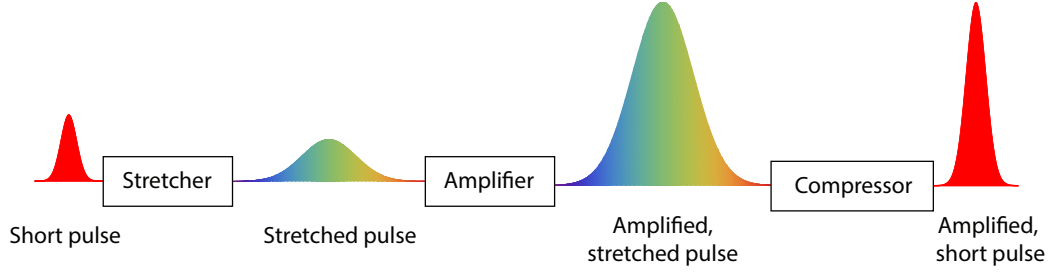


Figure 3.1: Chirped pulse amplification scheme.

through the rest of the laser system. Ultrashort pulses with durations on the order of tens of femtoseconds can have bandwidths on the order of tens of nanometers. A pair of gratings disperses the pulse such that longer wavelengths travel further than shorter wavelengths. At the output of the grating pair, the longer wavelengths arrive before the shorter wavelengths, leading to a stretched pulse with a linearly-varying instantaneous frequency (chirp). To stretch the pulse by hundreds of picoseconds, different wavelengths must experience about 5 cm of path length difference.

The stretched pulse has a lower peak intensity and propagates safely through the rest of the system. Once stretched, the pulse energy is increased in a gain medium pumped to an excited state by an outside light source. As the main pulse passes through the gain medium, it triggers stimulated emission, causing energy amplification. A second grating pair after amplification reverses the chirp induced by the first pair, compressing the pulse back to its original duration. Chirped pulse amplification enables the production of high intensity laser pulses which, after being focused to small spot sizes on the order of tens of microns, can have intensities up to 10^{24} W/cm². [73] These pulses are then intense enough to drive a strong plasma wave for particle injection and acceleration.

3.2 TREX Laser System

The laser system used in these experiments was the TREX laser at the Berkeley Lab Laser Accelerator (BELLA) Center. Pulses were generated in a Ti:sapphire oscillator with 805 nm central wavelength, 15 fs FWHM pulse duration and 4 nJ/pulse energy. Two grating pairs stretched the pulse to a duration of 220 ps. The pulse

was amplified in a regenerative amplifier³ which increased the pulse energy to 1 mJ. At this point, 80% of the energy was picked off and sent to a test stand for low energy experiments and a laser machining set-up where LPA targets are machined in-house. The remaining energy was sent to a cross-polarized wave (XPW) generator for temporal and spatial contrast improvement.⁴ In this process, the laser energy decreased by a factor of 10 to 0.02 mJ.

After the XPW, the pulse energy was increased to 3.5 J/pulse through a series of amplification stages, and a second grating pair then compressed the pulse to 45 fs with 1.8 J of energy.⁵ A deformable mirror after the compressor, allowed for optimization of the beam focus to correct the laser phase front of aberrations caused by imperfections in the beamline.⁶ The deformable mirror compensated for these aberrations and delivered a higher spatial quality beam on target. The beam is then split using a 70:30 beamsplitter to drive two LPA stages. The pulse energy and duration in each stage of the TREX laser system are shown in Table 3.1.

Stage	Energy/pulse	Pulse duration
Ti:Al ₂ O ₃ Oscillator	4 nJ	15 fs
Stretcher	4 nJ	220 ps
Regenerative amplifier	1 mJ	220 ps
XPW	0.02 mJ	220 ps
Pre-amplifier 0	1 mJ	220 ps
Pre-amplifier 1	30 mJ	220 ps
Pre-amplifier 2	50 mJ	220 ps
Main amplifier	3.5 J	220 ps
Compressor	3.5 J	45 fs

Table 3.1: Energy and pulse duration of each stage in the TREX laser system.

³In a regenerative amplifier, the pulse passes through the gain medium multiple times, enabling large energy gains in a smaller footprint.

⁴Cross-wave polarization is nonlinear process which occurs in anisotropic crystals. The output wave has the same frequency as the input but with a different linear polarization direction and an intensity with a cubic dependence on the input intensity. As a result, the low intensity pre-pulses and laser pedestal are reduced relative to the peak of the pulse, improving temporal contrast.

⁵Each grating has approximately 90% efficiency, and compressor has an overall efficiency of 70%.

⁶For example, thermal lensing at higher laser energies can be induced in the amplifier crystals. Although the crystals are cooled to minimize temperature variations, the laser heats the material such that the material is hotter along the beam axis and cooler at the edges. This temperature change induces a modification of the crystal's refractive index, inducing unwanted focusing of the laser pulse.

3.3 Gas Jet and Capillary LPA Targets

There are many designs for LPA targets including solid foils, gas jets, gas cells, and discharge capillaries. [31], [42], [74]–[77] In this experiment, two different targets were used: a de Laval nozzle gas jet as the first stage and a gas-filled discharge capillary as the second. A de Laval nozzle is a tube which is pinched in the middle (the nozzle throat) to form a region with a decreasing cross-sectional area followed by a region with an increasing cross-sectional area (shown in Fig. 3.2). This profile controls the gas flow such that it propagates from a subsonic velocity in the converging section to supersonic in the diverging section. The flow exiting the nozzle has little divergence, enabling a localized region of gas with high densities. To ensure smooth flow and to prevent gas disturbances from developing inside the nozzle, an optimal contour which is approximately parabolic can be determined using the Method of Characteristics. However, to simplify the machining of the nozzle, the shapes of the two sections are approximated as cones joined at their smallest cross-sectional area.

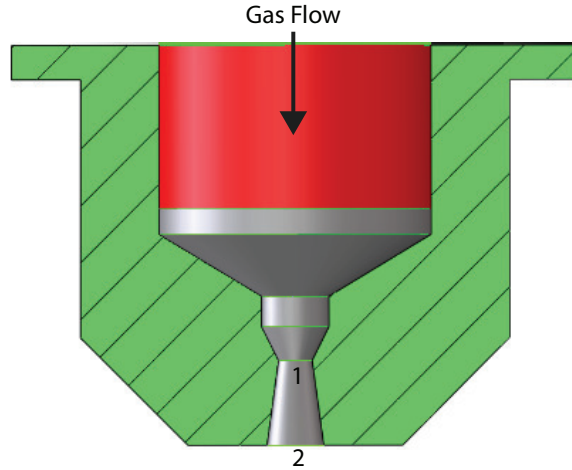


Figure 3.2: Gas jet model: (1) nozzle throat (500 μm diameter) and (2) nozzle exit (800 μm diameter). Gas is supplied from the top of the jet.

The flow properties depend on the type of gas and the ratio between the nozzle throat and the nozzle exit cross-sectional areas:

$$\frac{A}{A_t} = \left(\frac{\kappa + 1}{2} \right)^{-\frac{\kappa+1}{2(\kappa-1)}} \frac{\left(1 + \frac{\kappa-1}{2} M^2 \right)^{\frac{\kappa+1}{2(\kappa-1)}}}{M}, \quad (3.1)$$

where A is the cross-sectional area of the exit, A_t the cross-sectional area of the throat, $\kappa = c_p/c_v$ is the specific heat ratio, and M is the Mach number. [78] The ratio of the exit to throat area defines the Mach number, and once the Mach number is defined, all the other flow properties including pressure and temperature at the exit of the gas jet can be determined. The first stage gas jet had an 800 μm exit diameter and 500 μm throat area for a Mach number of 2.4.

The second LPA stage was a gas-filled, discharge capillary which produced an approximately parabolic plasma channel capable of guiding a laser pulse for many Rayleigh lengths. [77], [79] As discussed in Chapter 2, laser diffraction limits the beam energy gain unless the plasma index of refraction can be tailored to guide the laser over an extended distance. A number of techniques have demonstrated guiding of high intensity laser pulses, including pre-formed plasma channels with a minimum plasma density on-axis. [8], [9] Gas-filled, discharge capillaries are an attractive design for guiding laser pulses because they do not require a second laser pulse, can be scaled to long lengths, and do not rely on the drive laser to ionize the gas. The capillaries used in thesis were semi-cylindrical grooves machined in-house into two sapphire plates. The length of the sapphire plates varied from 1.5 cm to 6 cm. The plates were glued together to form channels with diameters ranging from 250 μm to 1 mm. The channel was continuously filled with with gas, either hydrogen or helium, through two gas slots located near the ends of the plate to establish a constant pressure between the two slots. Sensing slots enabled measurements of the pressure inside the channel.

Two stainless steel electrodes positioned at both ends of the channel provided a large electric potential difference which caused the gas to breakdown and become conductive, propagating current through the capillary. The voltage across the electrodes was controlled using a 20-30 kV pulser which provide an approximately sinusoidal current pulse of duration on the order of 1 μs . At these voltages, the gas is completely ionized to prevent ionization-induced defocusing. During the initial stages of the discharge, the radial plasma density and temperature are constant. As the current increases and full ionization is reached, thermal conduction between the plasma and the cold capillary wall becomes important. The radial temperature distribution becomes non-uniform as heat is transferred to the wall. Through the ideal gas law, this non-uniform radial temperature profile, where the plasma is hot in the center and cold at the wall, leads to a density distribution. Near the peak of the discharge current, the plasma reaches a quasi-steady-state where Ohmic heating is balanced with

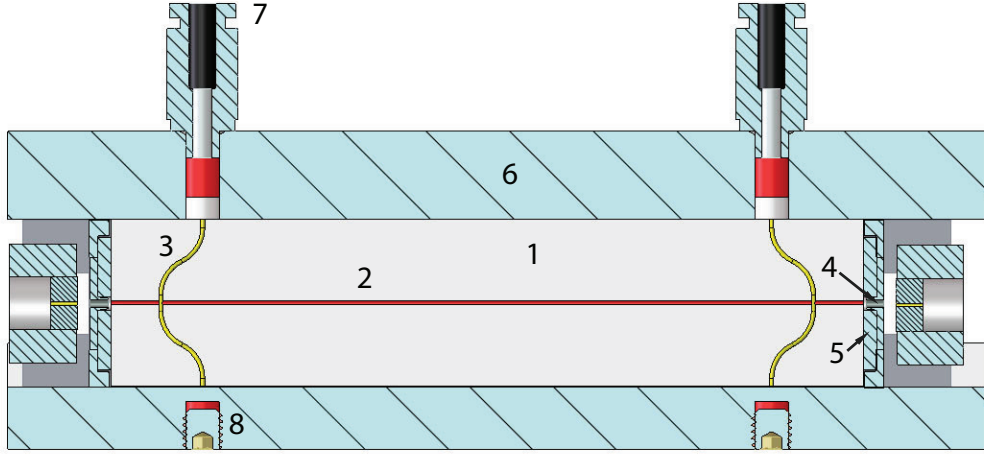


Figure 3.3: Cross-section of a gas-filled discharge capillary model: (1) sapphire plates, (2) capillary channel, (3) gas slots, (4) electrode, (5) ceramic disk, and (6) acrylic base. Gas enters the channel through (7), and the pressure inside the channel is measured through the sensing slots (8).

the conductive heat transfer to the wall. At this time, the electron density profile is parabolic with a minimum density on axis. [79] As the laser pulse travels through the capillary in picoseconds, the density profile was chosen by changing the timing between the discharge and the laser.

To ensure the discharge traveled through the channel and did not short elsewhere, a ceramic disk surrounded the electrodes. The sapphire plates and electrodes were all mounted in an acrylic holder.

3.4 Experimental Configuration

Figure 3.4 shows the beamline after the compressor. Pulse 1, comprising 70% of the laser energy, injected and accelerated electrons in the first LPA stage. An off-axis parabola focused Pulse 1 to an 18 μm FWHM beam waist, corresponding to a $4 \times 10^{18} \text{ W/cm}^2$ peak intensity on target (normalized vector potential $a_0 = 1.4$). Pulse 2, consisting of 30% of the laser energy, excited a plasma wake in the second LPA stage. Using a second off-axis parabola, Pulse 2 was focused to a spot size of 18 μm for a peak intensity of $1.4 \times 10^{18} \text{ W/cm}^2$ ($a_0 = 0.8$). At this intensity, the pulse excited a wake in the linear regime with no electron injection in the second stage.

The timing between the two laser pulses was controlled by a delay stage on the Pulse 1 beamline. Changing the path length of the Pulse 1 beamline changed the arrival time of Pulse 1 relative to Pulse 2, and tens of femtosecond accuracy in timing was achieved.

The LPA targets and electron beam diagnostics are shown in Fig. 3.5. The gas was a mixture of 99% Helium, 1% Nitrogen for ionization injection with a backing pressure of 150 psi for a plasma density of $5 \times 10^{18} \text{ cm}^{-3}$ as measured using interferometry. [80] The gas jet was positioned using three linear stages such that the laser propagated through the center of the jet.

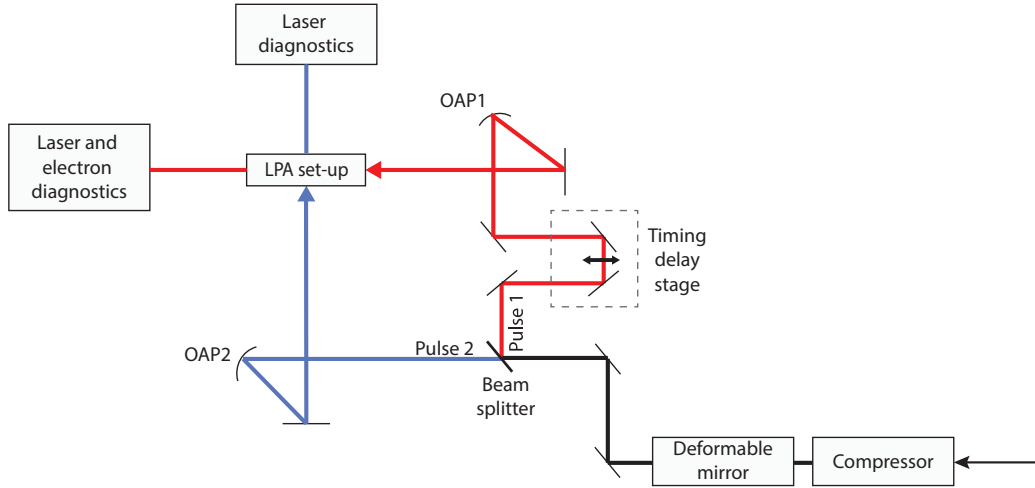


Figure 3.4: Staging beamline.

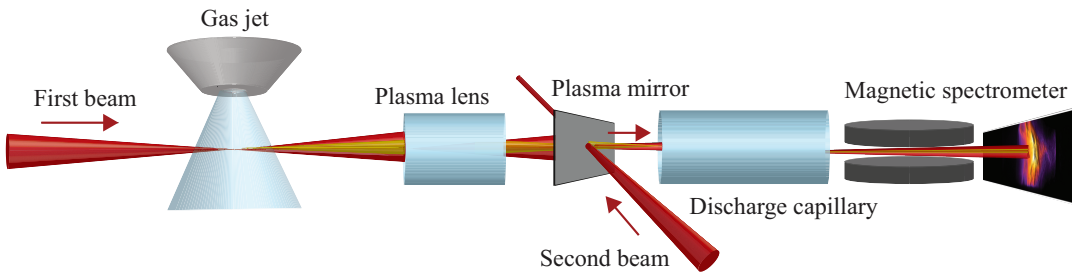


Figure 3.5: Staging experimental set-up schematic.

Once the electrons exit the gas jet and leave the focusing fields in the plasma wave, they begin to diverge. Without these fields to confine the beam, the spot size increases

as the beam drifts through free space. For the beam to be efficiently captured in the second stage, the beam spot size must be on the order of the transverse plasma wake size. In addition, for reduced emittance degradation during acceleration, the beam must be matched to the focusing forces in the plasma such that the beam size stays constant during acceleration. The spot size σ of a Gaussian beam in a linear focusing force $F_r/(\gamma m_e c^2) = -Kr$ evolves as:⁷

$$\frac{d^2\sigma}{dx^2} + K\sigma = \frac{\epsilon_n^2}{\gamma^2\sigma^3}, \quad (3.2)$$

where ϵ_n is the beam normalized emittance and γ is the beam relativistic factor. The spot size remains constant during acceleration when:

$$\sigma^2 = \frac{\epsilon_n}{\gamma\sqrt{K}}. \quad (3.3)$$

If the focusing force is approximated as the result of an ion column, then $F_r = -m_w\omega_p^2 r/2$ and $K = k_p^2/(2\gamma)$. The matched spot size becomes:

$$\sigma = \sqrt{\frac{2}{\gamma}} \left(\frac{\epsilon_n}{k_p} \right). \quad (3.4)$$

If the beam is mismatched to the channel, it will oscillate in size during acceleration and, coupled with an energy spread, lead to emittance growth. For the parameters $\gamma = 500$, $n_0 = 10^{18} \text{ cm}^{-3}$, and $\epsilon_n = 1 \text{ mm mrad}$, then the radius of the beam at the entrance of the second stage needs to be $18 \text{ }\mu\text{m}$.

To counteract divergence during the drift, an electron beam focusing component was required. Solenoid magnets have weak focusing at large beam energies due to their $1/\gamma^2$ scaling, making them useful in the few MeV or less energy range. [81] Small-bore quadrupole magnets have much larger focusing fields but only focus along one axis and defocus along the other. A combination of magnets is required to generate a net focusing force in both axes. While focusing gradients on the order of 500 T/m have been achieved, the multiple magnet design increases the overall device footprint to greater than tens of centimeters. [82], [83]

Instead, a single active plasma lens using the magnetic field induced by a discharge current in a gas-filled capillary can provide much larger, radially-symmetric focusing strengths. [84] The discharge current induces an azimuthal magnetic field B_ϕ based on Ampere's law:

$$B_\phi(r) = \frac{\mu_0}{r} \int_0^r J(r')r'dr', \quad (3.5)$$

⁷Neglecting space charge effects which are on the order of $1/\gamma^2$.

where $J(r)$ is the current density. If the current is parallel to the capillary axis and radially uniform, the magnetic field B_ϕ and field gradient $\partial B_\phi/\partial r$ become $B_\phi = \mu_0 J r/2$ and $\partial B_\phi/\partial r = \mu_0 I_0/(2\pi R^2)$ where R is the channel radius and I_0 the peak current. The focusing strength K of the plasma lens is proportional to the magnetic field gradient:

$$K = \frac{\partial B_\phi(r)}{\partial r} \frac{e}{m_e c \gamma}. \quad (3.6)$$

For a uniform peak current of 300 A in a capillary with a 125 μm radius, focusing forces on the order of kT/m have been demonstrated. [84]

The active plasma lens used in this experiment was a 15 mm length discharge capillary with a 500 μm diameter. Hydrogen gas filled the channel through two gas slots spaced 2 mm from the ends, and the discharge pulse was approximately sinusoidal with a 650 A peak current.

The electrons were focused into the second stage by the active plasma lens, and Pulse 2 was coupled in. At intensities on the order of $10^{15} - 10^{16}$ W/cm², the laser will ionize a solid target. A conventional mirror would have to be placed far away from focus such that the beam size is large enough that the intensity is below the mirror damage threshold. To keep the overall footprint of the experiment small, a plasma mirror was used instead. When a high intensity laser pulse hits a solid surface, the foot of the pulse ionizes the target, and a plasma is produced. Once the plasma reaches critical density, the remainder of the pulse reflects off the surface. [85] The plasma expands by only a small fraction of a laser wavelength during the pulse duration and thus behaves like a high-flatness mirror for most of the pulse.⁸ A plasma mirror can be placed close to focus, decreasing the distance between each stage and allowing for a compact set-up.

In this experiment, a VHS tape served as the plasma mirror, reflecting off the first laser and coupling in a second entering at 90 degrees. The tape-drive system spooled the tape to provide a fresh surface for each laser shot. This system had been demonstrated to reflect laser pulses with a peak reflectance of 82%, 1 mrad pointing stability, and high beam quality for thousands of shots at a 1 Hz repetition rate. [86]

The second stage was a 33 mm length gas-filled discharge capillary machined with a 250 μm diameter channel. Helium gas flowed into the channel through two gas

⁸Plasma mirrors are also used for contrast enhancement as the low intensity pre-pulses and pedestal are transmitted through the plasma mirror before critical density is reached and only the main body of the pulse is reflected.

slots spaced 2 mm from the ends, and a discharge pulse with a peak current of 325 A ionized the gas. The discharge was timed such that Pulse 2 arrived at the peak of the discharge current when the density profile was approximately parabolic.

The tape-drive, plasma lens and second LPA stage were placed on a hexapod which allowed for six-axis adjustment of position and angle. In addition, the plasma lens and second LPA stage were placed on separate alignment rafts which enabled independent fine-tuning of the capillaries' positions relative to each other. The system was initially aligned in air to a continuous diode laser and then checked under vacuum using the TREX laser at low power. The two capillaries were aligned by maximizing the energy transmission. A photo of the LPA set-up is shown in Fig. 3.6.

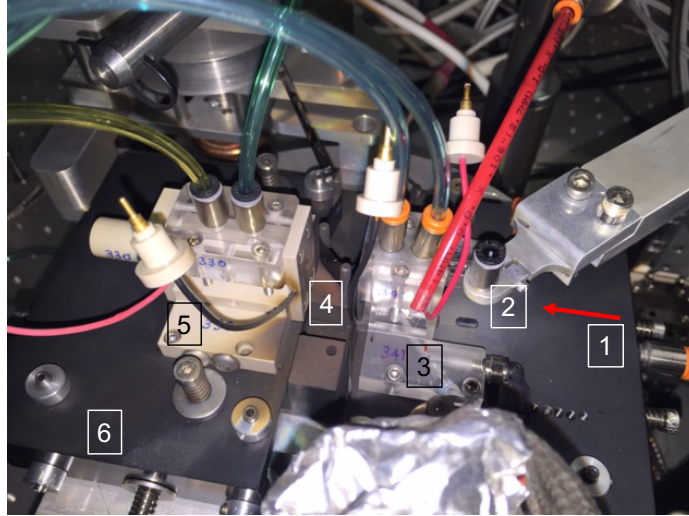


Figure 3.6: Photo of staging setup: (1) Pulse 1 beam, (2) gas jet, (3) plasma lens, (4) tape drive system, (5) discharge capillary, and (6) alignment raft.

After the interaction point, the electron beam propagated to a magnetic spectrometer consisting of a 1 T permanent dipole magnet which measured the beam energy and charge. The dipole magnetic field caused the electron trajectories to bend, with the higher electron energies bending at a larger radius than the lower energies. The beam deflects onto a series of scintillating screens which were imaged by triggered CCD cameras and measured energy dispersion in one direction and divergence in the other. The beam charge was proportional to the scintillating light intensity.

3.5 Experimental Results

Electrons were injected in the first stage through ionization injection with 120 MeV mean energy, 60% FWHM energy spread, 33 pC charge, 4 mrad divergence, and 0.3 mrad pointing stability. Figure 3.7 shows an example energy spectrum. These beams were consistently produced over hours, and the stability of the first stage enabled the electrons to be captured every shot by the plasma lens which focused the beam 25 mm downstream into the second stage. However, because the focusing strength of the lens scales as $1/\gamma$, different electron energies were focused at different longitudinal locations. The plasma lens current was chosen such that the electrons around 100 MeV were focused at the entrance of the second stage. All other energies were defocused and thus not efficiently captured.

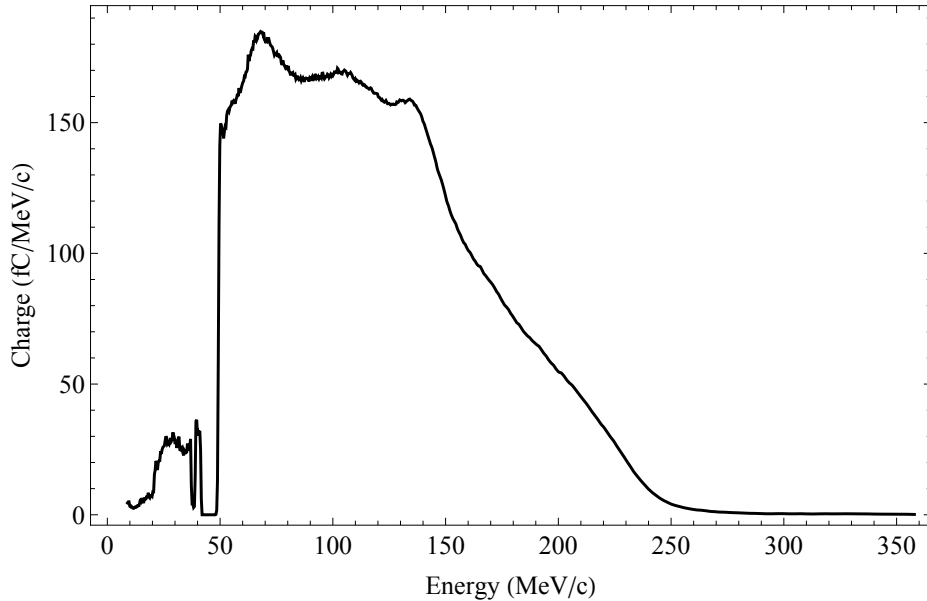


Figure 3.7: Example energy spectrum from first stage gas jet. The drop in energy at 40-50 MeV corresponds to a gap in the scintillating screens.

The electron energy gain in the second stage depended on the phase of the wake it was injected into. The arrival time of the electron beam relative to the second laser pulse was scanned using the delay stage to probe the wakefield structure. Figure 3.8 shows the beam energy and charge as a function of delay between the arrival of the electron beam and second laser pulse.

When the second laser pulse arrived after the electron beam (positive delays), the electrons propagated through the capillary undisturbed. At these delays, the

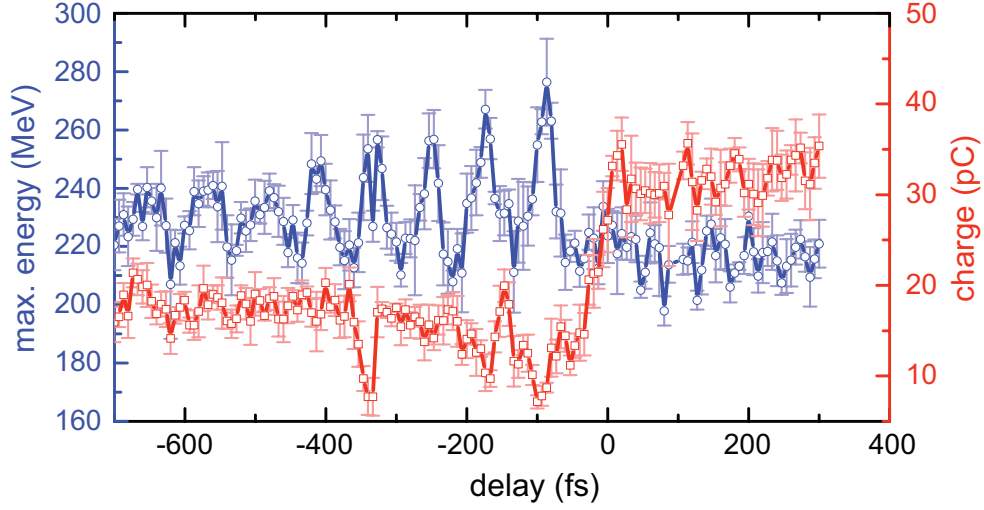


Figure 3.8: Electron beam energy and charge as a function of delay between the first and second laser pulse. Figure taken from reference [87].

energy and charge remain unaffected by the second stage. When the second laser pulse arrived before the electron beam (negative delays), the electron energy and charge were modulated by the wake's accelerating fields. As the delay was increased, the electron beam moved from the decelerating phase to the accelerating phase and back, causing the beam energy to increase and decrease again with a frequency of 80 ± 6 fs, consistent with a plasma wavelength $\lambda_p = 24 \mu\text{m}$ for a channel density of $1.9 \pm 0.3 \times 10^{18} \text{ cm}^{-3}$.

Figure 3.9 shows the energy spectrum as a function of transverse angle where 0 mrad corresponds to the laser propagation direction. The spectrum of the electron beam propagating without the influence of a wake in the second stage is shown in Fig. 3.9(a). Figure 3.9(b) shows the energy spectrum at delay -107 fs, where the beam is injected in an accelerating phase of the wake, having subtracted Fig. 3.9(a). It can be seen that a small fraction of the beam is accelerated to higher energies. Only 1.2 pC of charge was successfully trapped in the second stage wake, representing a trapping efficiency of 3.5% of the input electron beam. Figure 3.9(c) is the energy spectrum at delay -193 fs, having subtracted Fig. 3.9(a). Only a fraction of the beam is trapped in the second stage wake to be decelerated.⁹

⁹The reduced energy can also represent electrons that were deflected by the focusing fields in the wake. This off-axis deflection entering the magnetic spectrometer can manifest as a decrease in energy.

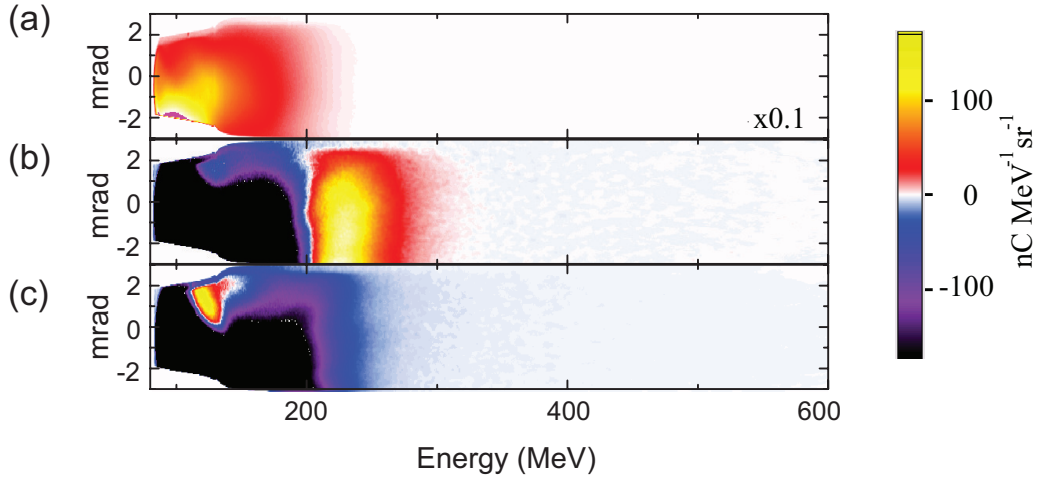


Figure 3.9: Electron beam spectra: (a) spectrum of beam unaffected by second stage (reference), (b) spectrum of beam at delay -107 fs with reference subtracted, (c) spectrum of beam at delay -193 fs with reference subtracted. Figure taken from reference [87].

3.6 Summary and Conclusions

In this chapter, we demonstrated staging of two laser plasma accelerators each driven by its own laser pulse. The gas jet first stage produced stable electron beams with 120 MeV mean energy and 33 pC of charge. The beams were focused into a second, gas-filled discharge capillary using an active plasma lens, and a plasma mirror reflected out the first pulse and coupled in the second.

The wakefield structure in the second stage was investigated by scanning the arrival of the electron beam relative to the second laser pulse. When the electron beam arrived before the laser pulse, the beam remained unaffected by the second stage. When the laser pulse arrived before the electron beam, the beam energy and charge become modulated depending on the phase of the wake the beam is injected into. When injected into the first plasma wavelength accelerating phase, the beam experienced a 100 MeV energy gain.

However, only 3.5% of the initially injected beam was post-accelerated in the second stage. To reduce the size and cost of an LPA-based accelerator, in addition to the high acceleration gradients generated in the plasma, efficiency energy transfer is required. When propagating from one LPA stage to another, a significant percentage

of the beam must be captured to avoid unacceptable losses over many stages.

The electrons were injected with a large initial energy spread which has negative consequences for beam degradation and beam transport. An intrinsic measure of beam quality is its emittance, or the beam's spread in position and momentum space. The emittance ϵ_x along the transverse x-axis is defined as:

$$\epsilon_x = \frac{1}{m_e c} \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle x p_x \rangle^2}, \quad (3.7)$$

where x is a particle's position, p_x a particle's momentum, and $\langle \rangle$ represents the average over the beam. Eq. 3.7 implies that $\epsilon_x^2 m_e^2 c^2 \geq \langle x^2 \rangle \langle p_x^2 \rangle$, and therefore the emittance places a limit on the beam divergence and size even with beam manipulation. For example, while the beam divergence can be reduced using an active plasma lens, it comes at the cost of an increased beam size.

However, Eq. 3.7 assumes the beam is monoenergetic. If the beam has an energy spread, the emittance evolves as:

$$\frac{d\epsilon_x}{dt} = \frac{2}{m_e^3 c^2} \left(\langle p_x^2 \rangle \left\langle \frac{x p_x}{\gamma} \right\rangle - \left\langle \frac{p_x^2}{\gamma} \right\rangle \langle x p_x \rangle \right). \quad (3.8)$$

The energy spread coupled with the beam divergence increases the beam emittance as the beam propagates.

In addition, the focal length of the active plasma lens, which counteracted beam diffraction during the drift between the first and second stages, scales with beam energy as $1/\gamma$. To be efficiently captured in the second stage, the beam size should be on the order of the plasma wake transverse size. If the beam is larger, the charge outside this area will not experience the wake focusing fields. A beam with a spread in energies will cause some electrons to be focused at the entrance of the second stage while other energies are defocused, leading to a low capture efficiency.

Reducing the energy spread, divergence and emittance of the beam will help mitigate these losses, and the dominate contributor to the energy spread is the technique used to inject electrons in the first stage. Ionization injection often produces beams with a large energy spread due to the extended distance over which the electrons are injected. Chapter 4 will present a new technique utilizing density downramp injection to deliver a small energy spread beam useful for many applications, including a high energy, multi-stage accelerator.

Chapter 4

Plasma Density Downramp Injection

In Chapter 3, we demonstrated coupling of two independent laser plasma accelerators. Electrons were injected and accelerated in a gas jet and then further accelerated in a gas-filled discharge capillary. As a result of the injection mechanism used, the beam had a 60% initial energy spread which contributed to a significant decrease in the second stage trapping efficiency due to the focal length chromaticity of the active plasma lens.

Density downramp injection is a technique which has been experimentally proven to produce small energy spread beams by limiting the distance over which the electrons can be injected. In this chapter, we demonstrate control of the properties of electron beams injected along a plasma density downramp through precision tailoring of the density profile. Of particular importance to a high energy accelerator with many stages, this technique enables an injector with low energy spread. While density downramp injection has been studied experimentally and quasi-monoenergetic beams were produced, little investigation has been done to improve the spatial quality of the electron beams or reduce their absolute energy spread. [38], [40], [42], [88]

Here, we advance the understanding of injection along a density downramp and demonstrate novel techniques for controlling the quality of the electron beam. Section 4.1 details the gas jet target used to produce the plasma density downramp, and Section 4.2 demonstrates controlled injection of quasi-monoenergetic electron beams and the techniques used to tailor the electron beam properties.

4.1 Gas Jet Characterization

The gas jet used in these experiments is the same one used in the staging experiment described in Chapter 3. To create a density downramp, a razor blade was placed in the gas flow (see Fig. 4.2). Small perturbations in the gas can only travel with a maximum velocity equal to the speed of sound. An object placed in the flow acts as an obstruction, leading to changes in the flow properties. These changes travel through the fluid as pressure waves moving at the speed of sound, both upstream and downstream of the object. In subsonic flow, the pressure waves, and thus information about the object, can travel upstream, and the flow has time to smoothly respond. In supersonic flow, because information about the object would have to travel faster than the speed of sound to propagate upstream, the fluid does not have time to respond and therefore piles up and compresses at the object. These compression waves merge and cause the flow velocity, pressure, temperature and density to change over a very short distance. This change is defined as a shock. [89] Before the shock, the fluid is unaware of the object, and after the shock the flow is subsonic.

A density change develops across the shock which can be used for downramp injection. The width of the shock determines the length over which electrons are injected and thus the beam energy spread, and varies depending on the type of shock that forms. When the fluid hits the blade, it can form a bow shock. This type of shock is similar to the curved ridge of water in front of a fast-moving boat created when the bow of the boat pushes through the water. Changes in the fluid properties can occur over distances on the order of the gas mean free path, leading to a sharp density drop over a distance less than λ_p across the shock front. The ratio of the density before the shock n_1 and after n_2 can be related to the initial Mach number M : [40]

$$\frac{n_1}{n_2} = 1 - \frac{2}{\kappa - 1} \left(1 - \frac{1}{M^2} \right), \quad (4.1)$$

where $\kappa = c_P/c_V$ is the specific heat ratio.

Certain types of shocks are commonly seen in nozzles. In these experiments where the gas jet is in vacuum, the ambient pressure is on the order of a few mTorr while the gas jet pressure is on the order of hundreds of psi. Because the gas pressure at the exit of the nozzle is greater than the ambient pressure, the gas expands once it leaves the nozzle. The flow diverges at the jet edge as a fan of expansion waves, and the gas pressure decreases. When the jet pressure equals the ambient pressure, the expansion

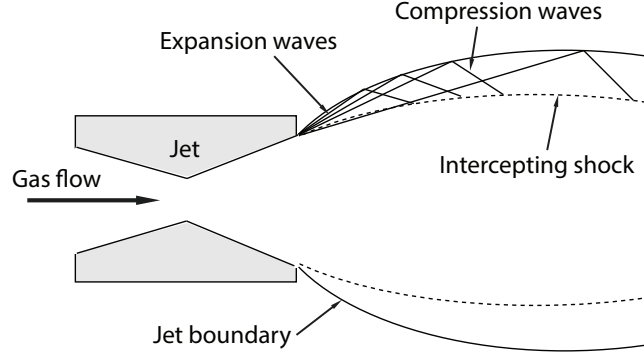


Figure 4.1: A simplified schematic showing the expansion waves developing at the jet edge and the compression waves coalescing into an intercepting shock.

waves reflect into compression waves which coalesce into an intercepting shock as shown in Fig. 4.1. The strength of the shock depends on the number of compression waves, causing a more complicated relationship between density and Mach number. As a result, the length over which the intercepting shock occurs can be much longer than the gas mean free path.

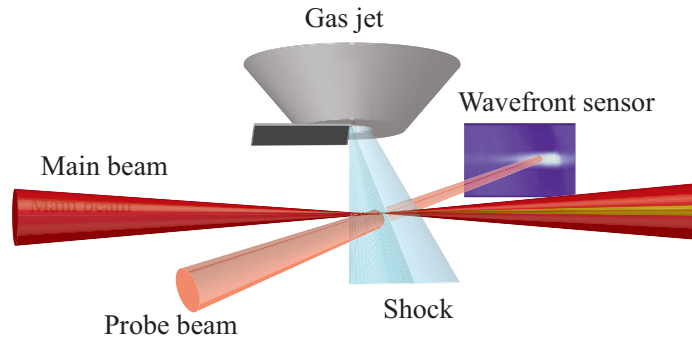


Figure 4.2: Schematic of the experimental setup. A razor blade was inserted into the flow of a supersonic gas jet, forming a shock. The main laser beam was focused into the shock region, producing electron beams. A probe beam perpendicular to the shock front was imaged onto a wavefront sensor for density measurements.

To characterize the effects of the razor blade on the gas flow, a probe beam perpendicular to the main beam, which drives the plasma wave and accelerates electrons, propagates through the gas jet and was imaged using a Phasics SID4 wavefront sensor (see Fig. 4.2) [80]. The sensor had a 3 mm field of view and a $30\text{ }\mu\text{m}$ imaging resolution. Unlike an interferometer which combines two beams to form an interference

pattern, a wavefront sensor requires only one beam propagating through the sample. The sensor measures the phase front curvature of the probe using a modified Shack-Hartmann mask consisting of an array of microlens. As a wavefront hits the microlens array, each lenslet focuses a portion of the wavefront onto a CCD camera. The local tilt of the wavefront will appear as a change in each lenslet's focal position relative to a flat wavefront. From all the lenslets, the overall wavefront can be approximated.

The phase of the probe beam propagating through vacuum was subtracted from the phase of the probe beam propagating through the gas jet to obtain an overall phase shift due to the gas and plasma. Figure 4.3 shows an example Phasics phase image of the plasma ionized by the main beam. The razor blade is above the image, and the laser propagates from left to right 1 mm below the blade. Gas fills the whole image, and the shock front is seen as a discontinuity in phase. The driver and probe beam were timed such that the probe arrived after the main beam when the shock width was the smallest.

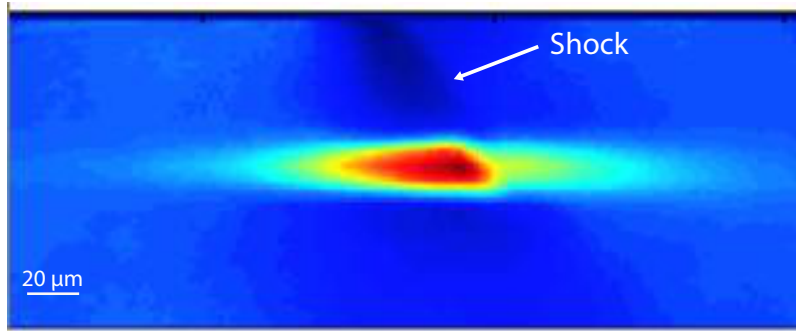


Figure 4.3: Example phase image showing the plasma and shock front. The neutral gas is shown in dark blue while the plasma is shown in the green to red region. The blade is 1 mm above the laser, out of view.

As can be seen in Fig. 4.3, the shock front was not necessarily perpendicular to the laser propagation direction. It was observed that the shock front angle was dependent on the location of the blade in the flow. Figure 4.4 shows this angle as a function of the percentage of the jet the blade is covering, or blade coverage. Because the shock front angle becomes negative at smaller blade coverages, the density feature observed cannot be a bow shock as was originally assumed since a negative shock front angle is unphysical.

In order to better understand the gas flow over the razor blade and this blade-dependent shock angle, two-dimensional simulations were performed using a com-

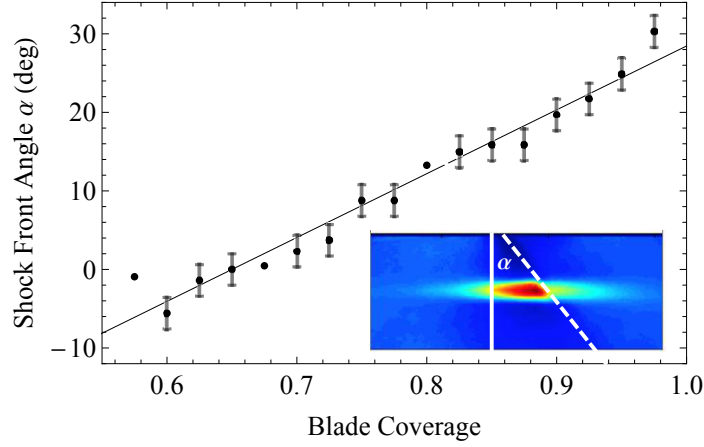


Figure 4.4: Shock front angle versus blade coverage. The inset shows an example phase image from the wavefront sensor where the shock front angle α has been defined.

compressible fluid dynamics solver in OpenFOAM. [90]. The gas jet was modeled using the same parameters as the experiment. In these simulations, it was seen that when the blade coverage is small, a bow shock develops at the blade tip. However, this regime only occurs when the razor blade is almost completely removed from the gas, a position not ideal for laser plasma acceleration. As the blade coverage increases, the entire gas cone exiting the nozzle changes. The blade forces the main plume of gas to flow in a direction different than the gas jet axis. The bow shock is now positioned close to the jet and out of reach of the main beam. Instead, the laser propagates through an intercepting shock as shown in Fig. 4.5. The direction of the expanding flow depends on the fraction of the gas jet covered by the blade, leading to a blade position-dependent intercepting shock. [91] When the blade coverage decreases, the intercepting shock starts to bend underneath the blade, leading to the negative shock front angles observed in the experiments.

From the wavefront sensor phase images, the density can be derived using an Abel transform. The phase image is a 2D projection of the 3D gas jet. For an arbitrary jet density distribution, the jet can be rotated to obtain projections of the jet profile for a series of views. Given enough angles, the 3D profile can be reconstructed from these 2D images. The reconstruction can be simplified, however, if the gas jet is axisymmetric. In this case, one projection is enough to determine the density.

From one projection, an Abel transform calculates the density from the phase shift. [92], [93] The phase shift caused by the neutral gas at the densities used in these experiments does not have an index of refraction much different than vacuum,

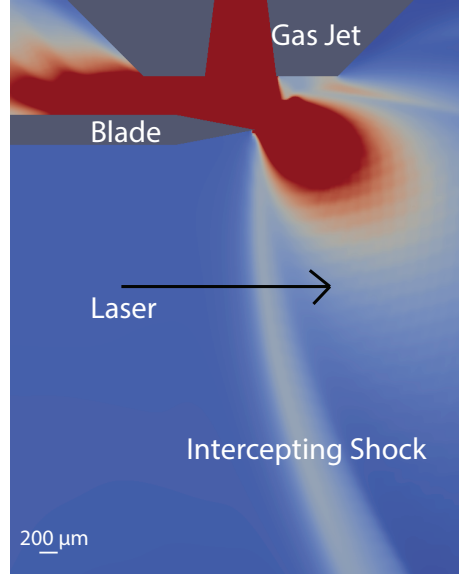


Figure 4.5: OpenFOAM simulation of the razor blade in the gas flow showing an intercepting shock front. The color scale has been saturated to better see the intercepting shock. Taken from reference [91].

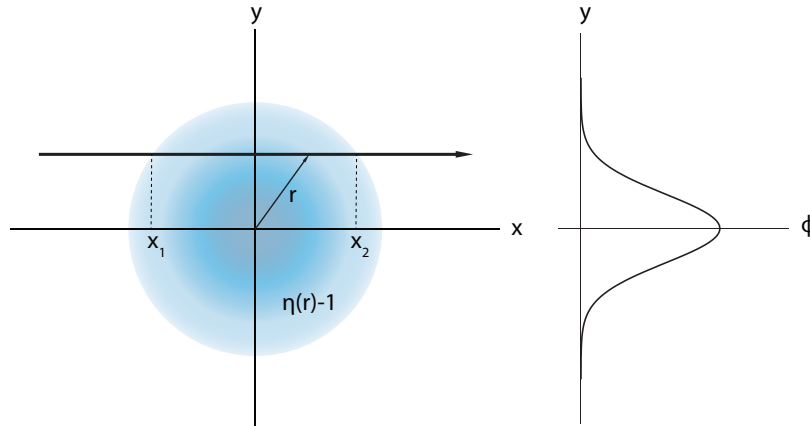


Figure 4.6: Simplified schematic of an Abel transform.

and very little phase shift was measured in the gas. The phase shift is much greater when the laser ionizes the gas due to the plasma index of refraction. As can be seen in Fig. 4.3, the shock front appears as a faint phase shift in the gas but as a much larger shift in the plasma. For this reason, the plasma region is used when deriving the plasma density.

To calculate the density from the measured phase shift, consider a gas jet pointed

in the z -direction and a probe ray propagating in the $+x$ direction, as shown in Fig. 4.6. A ray, traveling from x_1 to x_2 through the gas accumulates a total phase shift ϕ :

$$\phi = \frac{\omega}{c} \int_{x_1}^{x_2} (1 - \eta_p(r, z)) dx, \quad (4.2)$$

where $\eta_p(r, z)$ is the index of refraction profile along the path of the ray. The plasma index of refraction is given by $\eta_p^2 = 1 - \omega_p^2/\omega^2 = 1 - n_0/n_c$ where n_c is the critical density. Close to the laser focus, it can be assumed that the plasma is fully ionized. Therefore, the phase shift as a function of plasma density becomes:

$$\phi = \frac{\omega}{c} \int_{x_1}^{x_2} \left(1 - \sqrt{1 - \frac{n_0(r, z)}{n_c}} \right) dx. \quad (4.3)$$

The Abel transform $F(y)$ of a function $f(r)$ is defined as:

$$F(y) = 2 \int_y^\infty \frac{f(r)r}{\sqrt{r^2 - y^2}} dr, \quad (4.4)$$

and the inverse Abel transform as:

$$f(r) = -\frac{1}{\pi} \int_r^\infty \frac{dF}{dy} \frac{dy}{\sqrt{y^2 - r^2}}. \quad (4.5)$$

Using the relation $r^2 = x^2 + y^2$, the Abel transform of Eq. 4.3 becomes:

$$\Phi(r, z) = -\frac{1}{\pi} \int_r^\infty \frac{d\phi}{dy} \frac{1}{\sqrt{y^2 - r^2}} dy, \quad (4.6)$$

where $\Phi(r, z) = \omega(1 - \sqrt{1 - n_0/n_c})/c$. Solving for the density:

$$n_0(r, z) = n_c \left[1 - \left(1 - \frac{c\Phi}{\omega} \right)^2 \right]. \quad (4.7)$$

The Abel transform assumes radial symmetry, so a single transform cannot be performed on a phase image containing a tilted shock front. To maintain symmetry, these phase images were partitioned into two regions as shown in the inset of Fig. 4.7. In each of these regions, the tilted shock front is not included and symmetry can be assumed. The plasma density cannot be extended outside these regions as the tilted shock front then breaks the symmetry. Density profiles were obtained by combining the two Abel-inverted images and taking a lineout across the center. The downramp

length was measured directly from the wavefront images. Figure 4.7 shows an example reconstructed density lineout which indicates the peak density n_1 , plateau density n_2 , high density length L_{high} which controls the injection, and plateau density length L_{acc} over which the electrons are accelerated.

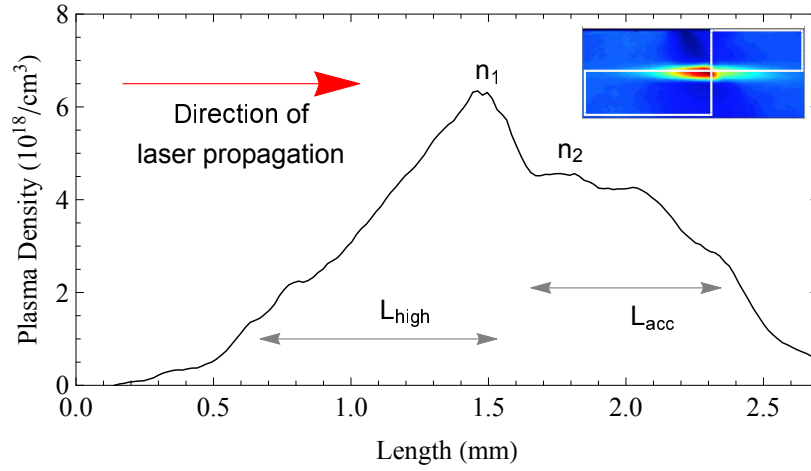


Figure 4.7: Example density lineout. Inset shows regions on which Abel transforms were performed.

Stacked density lineouts for various blade coverages are shown in Fig. 4.8. It can be seen that the location of the shock moved with the blade: as the blade moved further into the gas, the shock front moved with it, reducing L_{acc} . In this manner, L_{acc} could be tuned up to 2 mm. The density downramp width, defined as the distance between the peak density n_1 and the plateau density n_2 , was constant at 100 μm over the blade coverage range. The density transition length of the intercepting shock was larger than the mean free path of the gas and thus larger than λ_p . During the experiments, it was thought that the shock width would increase with distance away from the blade tip as the gas expanded, and so the laser was focused close to the blade. The simulations later showed that an intercepting shock does not form immediately at the blade tip. Because the parameters of the shock depend on how the gas expands and recompresses into an intercepting shock, the shock width depends on the laser distance from the blade. [91] As the compression waves combine, the shock becomes stronger, and the shock width decreases. However, in these experiments, the effect of distance from the blade was not investigated.

It can also be seen in Fig. 4.8 that the densities n_1 and n_2 were both functions of the blade position. The plateau density n_2 varied from 0.8 to $3.5 \times 10^{18} \text{ cm}^{-3}$ while

the peak density n_1 varied from 2.7 to $8.7 \times 10^{18} \text{ cm}^{-3}$ as the blade coverage changed from 0.25 to 1 . This density shift is a result of the blade altering the gas cone coming out of the nozzle. As the blade redirects the gas flow, the amount of gas in the new nozzle changes, affecting the input gas density which feeds the shock. Thus, the shock densities become dependent on the blade location.

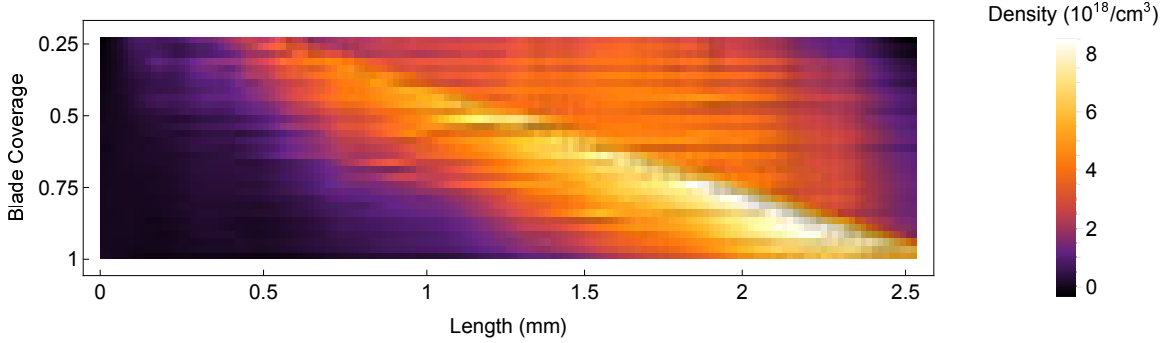


Figure 4.8: Stacked lineouts of density as a function of blade position.

4.2 Experimental Configuration and Results

The laser system used for these experiments is the same used in the staging experiment described in Chapter 3. The gas jet setup is shown in Fig. 4.2. Electrons were injected along the downramp by focusing the main beam¹ onto the shock front. The electron beam energy spectrum was measured using a magnetic spectrometer, and a removable scintillating screen measured the electron beam profile. The scintillation light was imaged using a triggered CCD camera, and aluminum foil was placed in front of the screen to prevent it from being damaged by the laser. Using the distance between the screen and the target, the spot size and location measured by the screen yields the beam divergence and pointing, respectively.

Because the shock front moved with the blade, the laser focal location was adjusted to keep the laser focused onto the shock region. This optimization resulted in the most stable electron beams with the most charge. Electron beams were produced every shot with down to 0.3 mrad r.m.s. pointing fluctuation, an average of 2 pC charge fluctuation and 6% energy fluctuation. Figure 4.9(a) shows the energy spectra

¹Pulse 1 in the staging experiment.

for 50 consecutive shots at a blade coverage of 0.8 demonstrating the stability of this mechanism.

By changing the location of the blade, the electron energy was tuned from 32 to 180 MeV, as shown in Fig. 4.9(b). The experimental energy gain W can be compared with the gain calculated using the simple model $W \approx eE_z L_{acc} \propto \int_0^{L_{acc}} \sqrt{n_2(z)} dz$ where $E_z \propto 1/\lambda_p$ [17], L_{acc} is defined as the distance it takes n_2 to decrease to $n_2/4$, and both L_{acc} and n_2 are functions of the blade coverage. At the densities used in this experiment, acceleration was not limited by laser depletion, and the electrons were accelerated over the whole L_{acc} . [17] Using this scaling law, the calculated electron beam energies based off measured n_2 and L_{acc} were fit to the experimental values, and these fitted energies are shown in the shaded region of Fig. 4.9(b). While the plateau density n_2 changed with blade coverage, the energy gain was dominated by the change in acceleration length: as the blade coverage increased, the acceleration length and thus the beam energy decreased. The experimental electron energies are in good agreement with this trend, demonstrating the tunability of this injection technique.

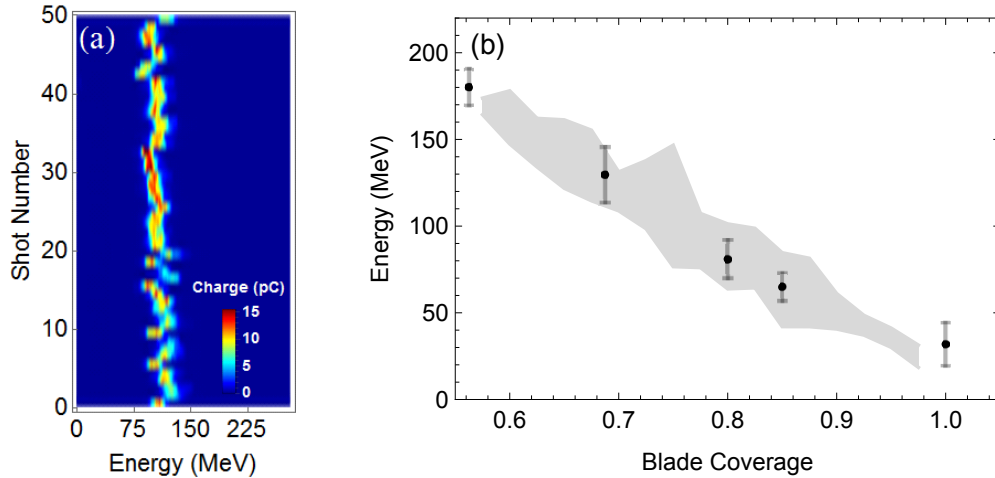


Figure 4.9: (a) Energy spectra for 50 consecutive shots with a blade coverage of 0.8, (b) mean energy of the electron beams versus blade coverage. Experimental results displayed as points and energies calculated from measured density profiles displayed as the shaded region.

Electrons will be injected within the laser volume with enough intensity, leading to injection off-axis. The strong focusing forces present in the wake cause these electrons

to oscillate about the laser axis. These oscillations are called betatron oscillations. Using a simplified model, the motion of an electron in an accelerating field E_z and a radially focusing field due to an ion column can be written as:

$$m \frac{d\gamma \vec{v}}{dt} = eE_z \hat{z} - \frac{m_e \omega_p^2 r}{2} \hat{r}, \quad (4.8)$$

where γ and $\vec{v}$ are the electron Lorentz factor and velocity, respectively. This equation describes a damped harmonic oscillator where the position of the electron $x_\perp(t)$ with initial off-axis displacement $x_{\perp,0}$ is given by: [94]

$$x_\perp(t) = x_{\perp,0} \left(\frac{\gamma_0}{\gamma(t)} \right)^{1/4} \cos \left(\int_{t_0}^t \frac{\omega_p^2}{2\gamma(t')} dt' \right). \quad (4.9)$$

Within the injection volume, a uniform distribution of electrons in the transverse plane is expected. Therefore, the beam size is the sum of all the electron positions, and from Eq. 4.9, the transverse size of the accelerating electron bunch will decrease with energy as $\gamma^{-1/4}$. The divergence, $\theta_\perp \approx dx_\perp/d(ct)$ is given by:

$$\theta_\perp = -\frac{k_p x_{\perp,0}}{\sqrt{2\gamma_0}} \left(\frac{\gamma_0}{\gamma} \right)^{-3/4} \sin \left(\int_{t_0}^t \frac{\omega_p^2}{2\gamma(t')} dt' \right). \quad (4.10)$$

The collective angle of all the injected electrons determines the average divergence of the beam. Thus, the beam divergence is proportional to $\gamma^{-3/4}$. Fig. 4.10 shows the beam FWHM divergence averaged in the vertical and horizontal directions as a function of the beam energy. A $\gamma^{-3/4}$ fit to the divergence data is shown, and this scaling agrees well with the data. This model assumes the electron beam divergence and transverse focusing force balance each other, resulting in a beam size matched to the plasma wave. Therefore, within the errors, the injected electrons appear to undergo matched propagation.

The total injected charge Q can be estimated by considering the injection volume V and plasma density n_0 : $Q \propto n_0 V$. Before the shock, the laser propagates through the plasma, driving a wake with plasma wavelength $\lambda_{p,1}$, and after the shock, the wavelength has increased to $\lambda_{p,2}$. The electrons are injected as the bubble is expanding along the downramp, leading to an injection volume which is the difference between the wake volume before and after the shock. Assuming the laser makes an approximately cylindrical plasma with length proportional to λ_p and cross-sectional area proportional to w_0^2 where w_0 is the beam waist at the shock, then the injection volume is given by $V \propto w_0^2 (\Delta \lambda_p)$ where $\Delta \lambda_p = \lambda_{p,2} - \lambda_{p,1}$. Simulations, discussed

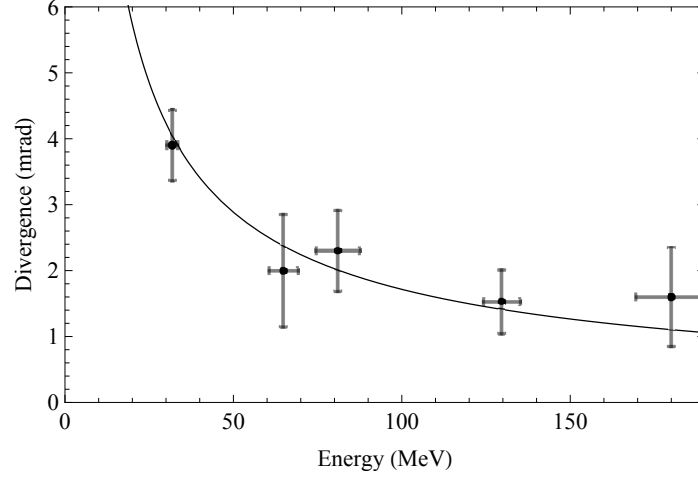


Figure 4.10: Divergence versus energy with experimental values displayed as points and $\gamma^{-3/4}$ fit as solid line.

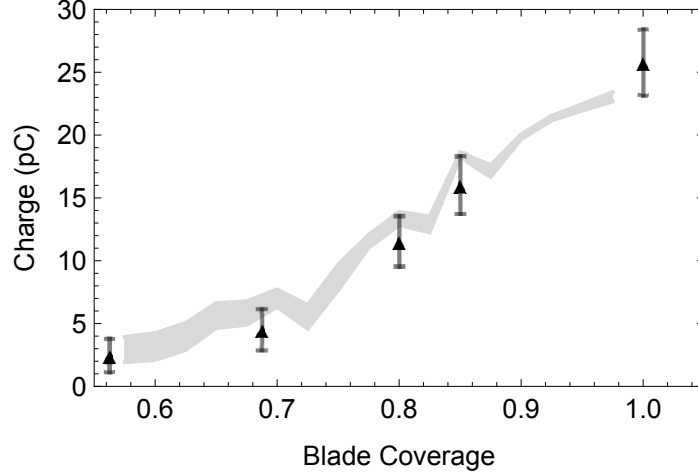


Figure 4.11: Mean charge of the electron beams versus blade coverage. Experimental results displayed as points and values calculated from measured density profiles displayed as shaded region.

below, showed that w_0 weakly depends on blade position and was therefore assumed constant. The fit of $n_1 \Delta \lambda_p$, using measured n_1 and n_2 values, to the experimental data is shown in the gray area in Fig. 4.11.

As electrons are injected, the field created by the electron bunch will modify the initial driven wakefield. Because the electrons are out of phase with the plasma wave, the plasma wake amplitude decreases. Trailing electrons in the bunch witness a weaker accelerating field, causing a decrease in beam energy and increase in energy

spread. This effect is called beam loading. The maximum number of electrons that can be injected into the wake before the beam field cancels out the plasma wakefield is defined as: [95]

$$N_{max} = \frac{n_0 A_b}{k_p} \frac{E_z}{E_0}, \quad (4.11)$$

where A_b is the transverse area of the electron beam. For the plasma densities measured in the experiment, an 18 μm spot size, and an accelerating field $E_z/E_0 \approx 1$, the maximum number of electrons that can be injected is approximately 2 nC.² The charge observed in the experiment is much less than this limit, and beam loading effects are not suppressing charge injection or affecting the energy spread.

As shown in Fig. 4.4, for any given blade position, the shock front angle α may not be perpendicular to the laser which has several repercussions for the laser and electron beam propagation and quality. As the laser propagates through the gas, it becomes refracted due to both the overall shape of the gas profile as well as the shock front. The Snell-Descartes law relates the angle of refraction θ_2 light undergoes when passing through an interface from one medium with density n_1 to another medium with density n_2 given an initial propagation angle θ_1 :

$$\theta_2 = \sin^{-1} \left(\frac{n_1}{n_2} \sin \theta_1 \right). \quad (4.12)$$

The electron beam centroid follows the direction of the laser, leading to an off-axis deflection when the laser refraction angle is nonzero. An attempt was made to directly measure the laser steering using a camera imaging near the target plane. However, at a distance downstream of the gas jet which would enable detection of a shift in the laser position, the laser had diffracted enough that it could only be faintly seen on the CCD camera. Therefore, the expected total refraction of the laser as it propagated through the gas was calculated using measured density profiles and shock front angles.

The electron beam deviation was measured on the scintillating screen, and this steering is compared to the calculated laser steering in Fig. 4.12. The calculated laser refraction agrees well with the experimentally measured electron beam steering. To keep the electron beam propagating along the laser axis, which is necessary for experiments with components downstream of the LPA, a shock front perpendicular to the laser axis is required.

²The electric field amplitude is justified by the measured densities and the simulations discussed later.

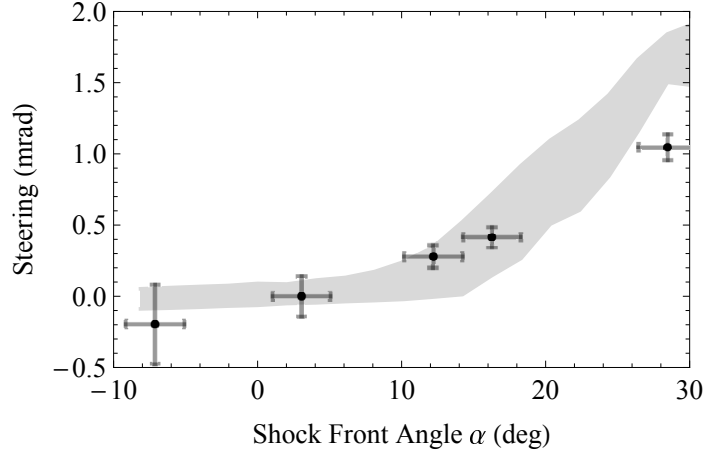


Figure 4.12: Electron beam steering versus shock front angle. Experimental data shown as points while the gray area displays laser refraction calculated from measured density profiles. Electrons propagate on axis when the shock front is nearly perpendicular to laser.

The beam ellipticity, defined as the difference between the divergence in the vertical and horizontal axes, is also influenced by the shock front angle. As the laser is steered, the electron beam position becomes displaced from the laser axis, inducing betatron oscillations. The larger the laser refraction, the more the electrons are displaced from the axis, and from Eq. 4.9, the larger the betatron amplitude. As this effect occurs only along the shock, the betatron oscillations are induced only in the vertical direction, leading to a beam size increase along one axis. This effect manifests as an elliptical beam. As shown in Fig. 4.13, the beam displacement increases as the shock becomes more tilted.

In this experiment, the laser polarization is also in the vertical direction, inducing ellipticity when the electrons interact with the laser's electric field. As the plasma wavelength λ_p approaches the size of the laser $c\tau$, where τ is the pulse duration, the electron beam can spatially overlap with the laser. [96] The electrons will oscillate along the laser polarization direction, leading to an increase in beam size along that axis. When the plateau density approaches $5 \times 10^{18} \text{ cm}^{-3}$, λ_p is comparable to the laser duration, suggesting that at higher densities, electron beam ellipticity can be attributed to the laser. At lower densities, λ_p is much longer than the laser duration, and the laser's electric field has a reduced influence. In this regime, ellipticity is induced by a tilted shock front. These results demonstrate that ellipticity is minimized when the shock front is nearly perpendicular to the laser propagation direction.

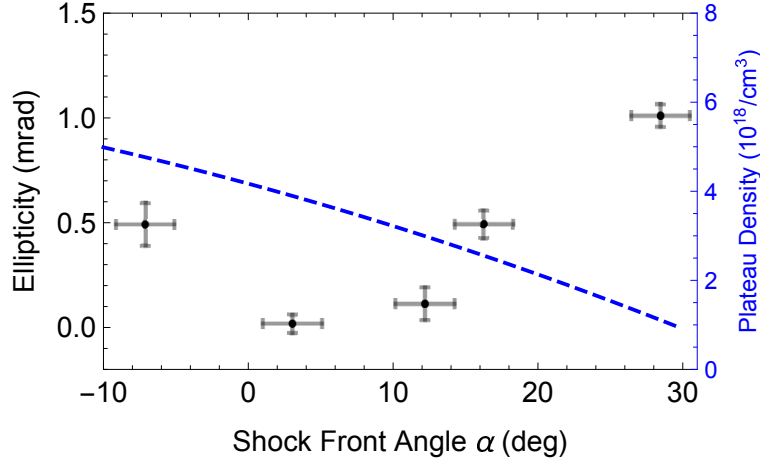


Figure 4.13: Ellipticity (black points) and n_2 (dashed line) versus shock front angle.

Both the beam quality and energy are controlled by the location of the blade in the gas flow. For a desired energy gain, the blade position is determined by the acceleration length needed to achieve that energy. At this blade location, the shock front may be at an angle α , inducing unwanted beam steering and quality degradation. A tilted shock front can be compensated for by tilting the gas and blade configuration by angle α to achieve a straight shock at the desired location. In this manner, a high quality beam can be injected over a tunable range of energies.

The absolute energy spread ΔE of the electrons was minimized by changing the length of the high density region L_{high} which is defined as the distance from where the density first exceeds 10^{18} cm^{-3} , when the laser begins to self-focus, to the density peak (see Fig. 4.7). Decreasing the backing pressure or the vertical distance between the jet assembly and the laser caused L_{high} to decrease. It was observed that a decrease in L_{high} reduced ΔE as shown in Fig. 4.14. The backing pressure and jet-laser height also affect n_1 , but ΔE was dominated by L_{high} rather than n_1 . [97] By reducing L_{high} , the absolute energy spread was minimized to $\Delta E = 4 \text{ MeV}$ for energies on the order of 100 MeV, allowing percent-level energy spreads to be attained.

As shown in Fig. 4.15, the energy spread was not dependent on the blade coverage. From Fig. 4.14, every 1 mm increase in up-ramp length increases the energy spread by 12 MeV. Over the blade coverages investigated, the upramp length was weakly dependent on blade coverage: L_{high} changed by 0.15 mm from a blade coverage of 0.25 to 1, corresponding to a 1.8 MeV increase in energy spread. This increase is less than the observed fluctuation. As ΔE was not dependent on the final energy

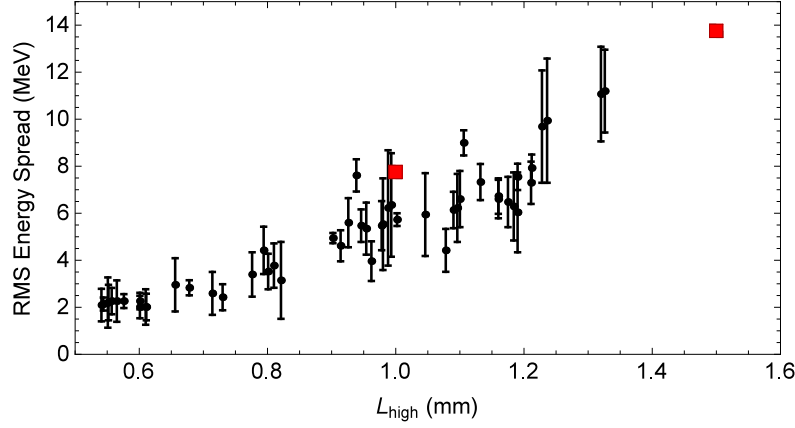


Figure 4.14: Energy spread of the electron beams versus the width of the high density region. Black circles represent experimental data, and red squares represent simulation results.

W , the energy spread was conserved during the acceleration process, allowing for independent adjustment of the beam energy and energy spread.

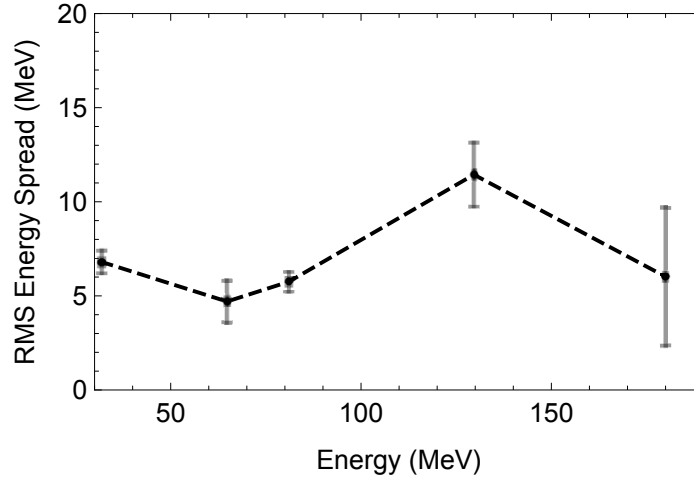


Figure 4.15: Energy spread as a function of energy.

Particle-in-cell simulations using the spectral, quasi-3D code FBPIC provided more insight into the injection process. [98] The effect of the up-ramp was examined by simulating two density profiles. The upramp regions matched two experimental profiles while the downramp regions were set to be identical to decouple upramp and downramp effects on the electron beams. The two profiles are shown in Fig. 4.16(a) where the wide profile has a 1.5 mm up-ramp length and the narrow has a 1 mm

up-ramp length.

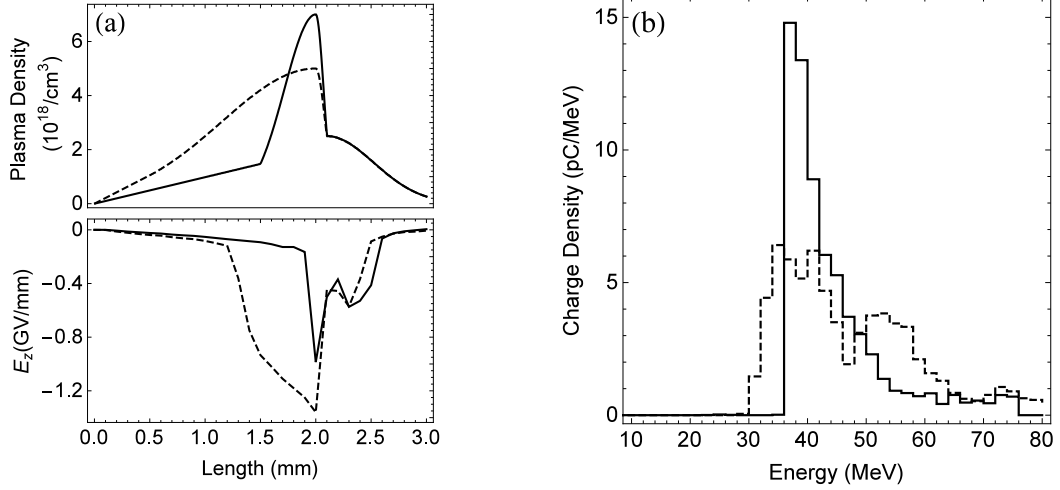


Figure 4.16: (a) Particle-in-cell simulations demonstrating the peak longitudinal E_z field at the back of the plasma bubble for a wide up-ramp (dashed) and narrow up-ramp (solid) density profile, (b) Energy spectrum of the injected electrons at the end of the downramp for the wide up-ramp (dashed) and narrow up-ramp (solid) profile.

The simulations showed that the laser self-focuses until it reaches the density downramp, increasing the peak longitudinal accelerating field E_z . This effect can be seen in Fig. 4.16(a). Due to the difference in densities, the wide and narrow up-ramp profiles induce differing amounts of self-focusing. Most noticeably, at the start of the downramp when the electrons are first being injected, the wide up-ramp profile produces a larger amplitude E_z field while the E_z fields are similar along the rest of the profile. Fig. 4.16(b) shows the final energy spectra of the electrons injected in the two different profiles. The wide up-ramp profile produced electron beams with an energy spread of 16.0 MeV while the narrow up-ramp profile beams had a spread of 8.5 MeV which is consistent with the experimental data as shown in Fig. 4.14. It can be seen that the wide up-ramp profile injected more electrons with energies greater than 50 MeV compared to the narrow up-ramp profile. These electrons were located in the first $0.5 \mu\text{m}$ of the bunch and injected in the larger amplitude E_z field region. When these electrons are removed, the absolute energy spread becomes comparable for the two profiles. Therefore, the larger range in E_z values for the wide up-ramp induced a larger energy spread by accelerating the head of the electron bunch to

higher energies. These results explain the experimental finding that energy spread is correlated with up-ramp width.

4.3 Summary and Conclusions

In this chapter, experiments developing a stable, high-quality injector using the TREX laser were presented. To accelerate small energy spread beams, density down-ramp injection was investigated by inserting a razor blade into the gas flow of a supersonic de Laval nozzle to form a shock. The parameters of the shock were measured as a function of the blade's position in the flow and correlated with the injected electron beam properties. Using these relationships, we demonstrated control over the beams by tuning the density profile. The energy of the electrons is determined by the acceleration length after the shock, and the injected charge is governed by the magnitude of the density drop. In addition, the length of the up-ramp allowed control over the electron beam's energy spread. Decreasing this length reduced the absolute energy spread to 4 MeV while maintaining independently-tunable energies in the range of 30-180 MeV.

The shock front angle influenced the beam pointing by inducing laser steering, and impacted the beam ellipticity by increasing the magnitude of the betatron oscillations. The highest quality electron beams were produced when the shock front was perpendicular to the laser propagation direction. By adjusting the shock front angle, this technique can produce stable, high-quality electron beams over a tunable range of energies suitable for a wide range of applications. Such an injector would enable higher capture efficiency when used as a first stage in a multi-stage accelerator. In addition, this compact source can provide electrons for various applications that require a high-quality beams including the production of x-ray pulses from coherent undulator radiation or Thomson backscattering [99]–[102] which can be used for imaging the structural dynamics of chemical and biological systems, high-resolution lithography or characterization of nuclear materials [103]–[105].

Chapter 5

A Cryogenically-Formed Discharge Capillary

With a high-quality injector, particle beams with low energy spread, emittance and divergence can be produced. If these beams are injected into a second LPA stage for further acceleration, that stage must also maintain beam quality. For a particle accelerator with many stages, high luminosity at the collision point places stringent requirements on each individual stage. Using techniques such as density downramp injection and two-color ionization injection, laser plasma accelerators can produce beams with a small initial emittance on the order of those produced in a conventional RF accelerator. [27] A challenge is to maintain this emittance during acceleration.

When a beam with a nonzero energy spread accelerates, if the beam size is mismatched to the plasma focusing forces, the emittance can increase by orders of magnitude as a result of betatron decoherence. [106] As derived in Section 3.4, the matched spot size σ is defined as:

$$\sigma = \frac{\epsilon_n}{\gamma\sqrt{K}}, \quad (5.1)$$

where $K = k_p^2/(2\gamma)$ is the strength of the plasma focusing force. For higher beam energies, weaker focusing is required to satisfy the matching condition and prevent emittance degradation. As a laser pulse propagates through a homogenous or parabolic plasma channel, the transverse electric fields in the plasma wake depend on the gradient of the laser intensity profile, and tailoring the laser's profile can reduce the focusing force. For example, with a flat-top pulse $a^2 = a_0^2$, the transverse ponderomotive force becomes $F_{p,\perp} \propto \nabla_{\perp} a^2 = 0$. Simulations have shown that higher order laser modes can also shape the transverse profile of the laser, allowing for control over

and reduction of the focusing fields. [107] Without a tailored pulse, depending on the laser and plasma parameters, the magnitude of the focusing force, while zero on-axis, can be on the same order as the accelerating force off-axis. These large fields place stringent requirements on the beam spot size, especially at higher energies.

In addition, as a particle beam accelerates in a homogeneous or parabolic plasma channel, the beam Coulomb scatters off background ions, inducing emittance degradation as the beam propagates. A large focusing force can compensate for the emittance growth due to scattering, but it then contributes to emittance growth through beam mismatch. Hollow and near-hollow plasma channels were proposed in 1995 as a new technique to guide a laser pulse while providing favorable accelerating and focusing fields for both electrons and positrons. [69]–[71], [108] A near-hollow plasma channel with radius r_w has a density profile of the form:

$$n(r) = \begin{cases} n_{ch} & \text{for } r < r_w \\ n_w & \text{for } r \geq r_w \end{cases} \quad (5.2)$$

where $n_{ch} \ll n_w$. When $n_{ch} = 0$, the channel is hollow. As a laser propagates through this structure, it both drives a plasma wake inside the channel when n_{ch} is non-zero and surface currents in the wall which induce fields that extend into the channel. The accelerating field is transversely uniform and dominated by the plasma density in the wall n_w while the focusing field is linear and dominated by the plasma density in the channel n_{ch} . Independent control over the transverse and longitudinal electric fields allow the wakefield to be matched to the particle beam during acceleration while maintaining a high acceleration gradient. Large acceleration gradients with weak focusing forces for emittance preservation can be achieved. The reduced density on-axis in the channel also mitigates the emittance growth due to Coulomb scattering. [109]

Hollow plasma channels have been theoretically investigated for decades, but this technique has not been extensively investigated experimentally because of the difficulties in producing the required density profile. [108], [110], [111] In the few experiments performed, the channel was generated by hollow Bessel beams using phase masks and special conical lenses called axicons. [112]–[114] The off-axis region of the laser had enough intensity to ionize neutral gas, but the center of the pulse did not, generating a hollow plasma channel as it propagated. Experiments demonstrating the formation of these structures have shown that wakefields can be excited in the channel by a positron beam. [112], [113] This method, however, does not allow for independent

adjustment of the wall and channel density. In addition, the channel has neutral atoms on-axis which will contribute to emittance degradation and limit the current of the beam propagating through the channel so as not to ionize the gas.

In this chapter, we describe the development of a cryogenically-formed, variable-radius waveguide whose flexibility allows it to be applicable to various laser guiding and acceleration regimes, including as a hollow plasma channel. These waveguides were formed by flowing a gas with freezing temperature T_f through a capillary which was cooled down to a temperature less than T_f . As the gas propagated through the capillary, it froze to the walls and formed an ice shell. Shell thicknesses on the order of 100 μm were grown within a few seconds. A second species of gas with a freezing temperature lower than T_f was introduced into the capillary to add density on-axis. A discharge pulse ionized the ice shell and gas, and guiding of a low-power laser through the plasma channel was demonstrated.

In Section 5.1, the basic concepts behind the cryogenically-formed waveguide are discussed. The method used to measure the ice layer is reviewed in Section 5.2. Section 5.3 summarizes several of the attempts made to deposit the ice and the techniques developed to consistently freeze radially and longitudinally uniform ice shells. In Section 5.4, we demonstrate low-power laser guiding for varying ice thicknesses, and in Section 5.5, we elaborate on the potential improvements and applications for this cryogenically-formed capillary. Finally, Section 5.6 reviews the potential next steps for this technique.

5.1 Theory of Freezing in Pipe Flows

The problem of solidification of a fluid, or deposition of a gas, flowing over a cooled surface has been extensively studied for a variety of applications including freezing of foods, frost formation on airplane wings or ice formation and blockage in pipes. [115]–[117] These problems are difficult to analyze because the position of the gas-solid interface as a function of time is not known a priori and must be solved along with the temperature profile. In the case of solidification inside a channel, the gas flows through a continuously changing domain, adding a layer of complexity. Therefore, exact solutions to the solidification problem are known for only a few cases, and simulations must be performed for more complex conditions and geometries.

When there is no fluid flow, the problem can be reduced to a Stefan-Neumann

problem. [118], [119] Freezing of a fluid with flow in a tube has been examined for both steady-state and transient conditions. [120]–[125] Ozisik and Mulligan developed an approximate analytical model for the transient freezing of flow through a tube with a circular cross-section, and a solution for the interface position given certain simplifying assumptions has been studied and compared with experiments. [126], [127] In the rest of this section, the work of Ozisik is extended to the proposed cryogenically-cooled capillary to study the ice growth.

5.1.1 Physical Model

The theoretical model and coordinate system describing gas flow in a channel with surface solidification is shown in Fig. 5.1. The channel has a circular cross-section with radius r_w . At time $t = 0$ both the channel and gas are at the freezing temperature of the gas T_f . For times $t > 0$, the capillary wall temperature T_w is decreased to a temperature below T_f , and the gas enters the channel into the test section at $z = 0$ with constant temperature T_0 and a full-developed profile.¹

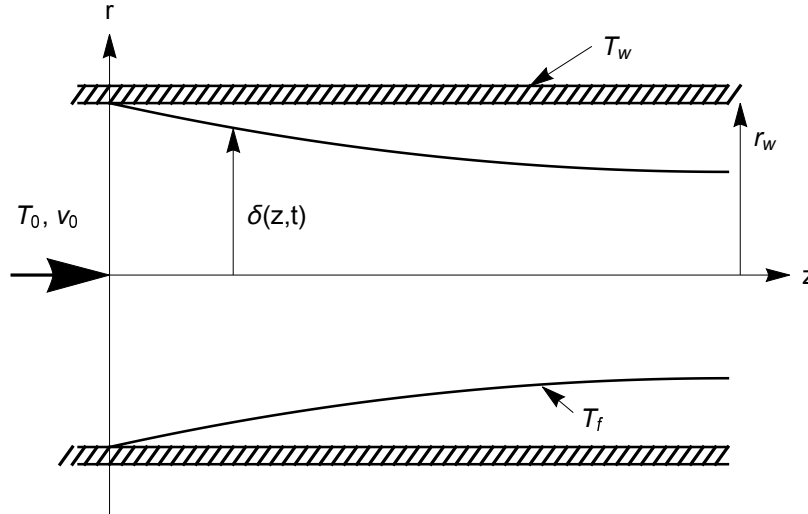


Figure 5.1: Theoretical model of gas depositing onto tube wall.

As the gas flows through the channel, the bulk temperature decreases until the gas begins to freeze to the walls. The phase transition region, where both solid

¹At the entrance of the tube, boundary effects at the inner wall propagate into the flow. After a certain distance, the flow reaches a stable profile, and the flow is called fully-developed.

and gas co-exist is called the interface. The interface thickness can vary from a few Angstroms to centimeters, depending on the solid microstructure which can be very complex. However, most materials under regular freezing conditions will solidify with an interface of negligible thickness. Therefore, this transition can be considered sharp, separating the gas phase from the solid.

At times $t > 0$, deposition of the gas onto the capillary wall begins, and the diameter of the solid-gas interface $\delta(z, t)$ decreases. The rate that heat can be transferred through the solid and the gas determines the thickness of the ice layer and is a function of both time and position along the length of the tube.

In the following model for the interface position, several simplifying assumptions were made:

1. The properties of the gas and solid are constant.
2. The solid state heat conduction reaches a quasi-static state.
3. The radius of curvature of the solid is large enough such that axial heat conduction in the solid can be neglected.
4. Transient effects in the gas due to deposition are negligible.
5. Velocity at the inlet of the capillary is fully-developed and constant.
6. The gas flow inside the capillary is proportional to the ratio of capillary cross-sectional area and ice channel cross-sectional area.

Assumption 1 establishes that the material properties, except for density, remain constant in each of the phases but allows for a discontinuity in values across the interface. While the properties of both the gas and solid are functions of temperature, the effect is largest when considering the gas density during the freezing process. The change in properties such as thermal conductivity, specific heat, and viscosity among others is small in comparison. For example, the thermal conductivity and specific heat capacity of the solid over the experimental temperature range varies on the order of 10%. Density between the two phases is also assumed constant which is necessary to prevent movement of material during freezing. Assumption 1 is presumed in order to simplify the heat transfer and gas flow, but Section 5.1.4 will examine how density changes affect the predicted ice growth.

Assumptions 2-6 are more reasonable and will be discussed below as they become applicable.

5.1.2 Governing Equations

In the solid phase, heat $\mathbf{q}$ is transferred through conduction and is proportional to the temperature gradient as described by Fourier's law $\mathbf{q} = -k_s \nabla T_s$, where T_s is the solid temperature and k_s is the solid thermal conductivity. For a time-dependent temperature, the solid heat equation is given by:

$$\rho c_{p,s} \frac{dT_s}{dt} = k_s \nabla^2 T_s, \quad (5.3)$$

where ρ is the density and $c_{p,s}$ is the solid specific heat. The heat equation has been simplified by assuming that the properties of the solid are constant.² Under Assumptions 2 and 3³, the temperature in the solid phase reaches steady state $dT_s/dt = 0$, and there is no azimuthal heat transfer $dT_s/d\theta = 0$. The former condition is true if the ice growth rate is sufficiently small. Therefore, the solid heat equation becomes:

$$\frac{d}{dr} \left[r \frac{dT_s}{dr} \right] = 0, \quad (5.4)$$

for $r \in [\delta, r_w]$. The interface temperature is constant⁴ and equal to the freezing temperature of the gas T_f , and the wall is maintained at a constant temperature $T_w \leq T_f$. Thus, the solid phase has the following boundary conditions:

$$T_s(r_w) = T_w \quad (5.5a)$$

$$T_s(\delta) = T_f \quad (5.5b)$$

In the gas phase, the distribution of heat as a function of time depends on convection as the fluid transports energy, viscous effects, and work caused by a temperature-dependent pressure. Neglecting the viscous⁵ and pressure work terms⁶, the transient temperature distribution in the gas is given by: [129]

$$\rho c_{p,g} \frac{\partial T_g}{\partial t} + \rho c_{p,g} \mathbf{v} \cdot \nabla T_g = -\nabla \cdot \mathbf{q}, \quad (5.6)$$

where $c_{p,g}$ is the specific heat of the gas, $\mathbf{v}$ is the flow velocity, T_g is the gas temperature, and $\mathbf{q}$ is the net heat flux. [89] The rate of heat transfer between the interface

²Assumption 1.

³This condition is true if ratio of wall thickness to capillary diameter is small. [128]

⁴This condition is not generally true as in the case of a supercooled liquid or a large radius of curvature.

⁵Valid for large Reynolds numbers.

⁶Assumption 1.

and the gas (first term) occurs through transport of the gas (second term) and conduction through the boundary (third term). The heat equation has been simplified by assuming that the properties of the gas are constant.

Convection in the gas phase can take place through natural convection, where movement is caused by temperature-induced differences in density, or forced convection, where movement is caused by pressure differences induced by an external source such as a vacuum pump. The magnitude of the movement caused by natural versus forced convection can be determined by the Rayleigh number Ra :

$$Ra = \frac{\Delta \rho g l^3}{\alpha \mu}, \quad (5.7)$$

where $\Delta \rho$ is the density difference between two sections of the gas caused by a temperature difference, g is the gravitational acceleration, l is the characteristic length scale, α is the thermal diffusivity, and μ is the dynamic viscosity. The thermal diffusivity of the gas is defined as $\alpha_g = k_g / \rho c_{p,g}$. Table 5.1 gives various properties of nitrous oxide, the gas used in the experiment, and typical parameters for the channel. When the Rayleigh number is less than a critical value, natural convection can be

Property	Symbol	Value
Dynamic viscosity (Pa·s)	μ	$1.4 \cdot 10^{-5}$ (at 300 K)
Kinematic viscosity (m^2/s)	ν	$7.92 \cdot 10^{-6}$ (at 300 K)
Solid thermal conductivity ($\text{mW}/\text{m}\cdot\text{K}$)	k_s	539 (at 100 K) [130]
Gas thermal conductivity ($\text{mW}/\text{m}\cdot\text{K}$)	k_g	17.4 (at 300 K) [131]
Solid specific heat capacity ($\text{kJ}/\text{kg}\cdot\text{K}$)	$c_{p,s}$	0.98 (at 100 K) [132]
Gas specific heat capacity ($\text{kJ}/\text{kg}\cdot\text{K}$)	$c_{p,g}$	0.88 [131]
Latent heat (kJ/mole)	L	25.1
Molecular weight (g/mole)	M	44
Solid density (g/cm^3)	ρ	1.16
Solid refractive index	n_i	1.318 [133]
Channel wall diameter (mm)	r_w	1
Channel length (mm)	L_c	60

Table 5.1: Nitrous oxide and capillary properties.

neglected. For the experimental parameters which will be described in Sec. 5.3, forced convection is the dominant heat transfer mechanism.

The thermal transfer due to convection is dependent on whether the flow is turbulent or laminar. Assuming that the flow is fully-developed and laminar, each gas

particle moves along the longitudinal direction with constant velocity and no movement in the radial direction. Turbulent flow is characterized by gas motion which is chaotic, with a pressure and flow velocity sensitive to the initial conditions. The Reynolds number is a dimensionless quantity which helps predict whether flow is laminar or turbulent and is defined as:

$$Re = \frac{\rho v D}{\mu}, \quad (5.8)$$

where D is the characteristic dimension of the flow.⁷ If the Reynolds number is less than 2300, the flow is often considered laminar. For nitrous oxide gas with a density less than 0.1 kg/m^3 and the parameters given in Table 5.1, the flow is laminar, and the heat transfer can be considered axisymmetric.

The conductive rate of heat transfer is given by:

$$\mathbf{q} = -k_g \nabla T_g, \quad (5.9)$$

where k_g is the gas thermal conductivity. The ratio of heat transfer through convection versus conduction is given by the Peclet number Pe :

$$Pe = \frac{Dv}{\alpha}. \quad (5.10)$$

The Peclet number is large for the parameters used in these experiments, and thus the longitudinal conduction in the gas is negligible: $-k_g(\partial T_g/\partial z) \approx 0$. Therefore, the heat flux simplifies to:

$$\mathbf{q} = -k_g \frac{\partial T_g}{\partial r} \hat{r}. \quad (5.11)$$

Equation 5.11 combined with Eq. 5.6 gives the following temperature differential equation in the gas phase:

$$\frac{\partial T_g}{\partial t} + v_z \frac{\partial T_g}{\partial z} = \kappa_g \left(\frac{\partial^2 T_g}{\partial r^2} + \frac{1}{r} \frac{\partial T_g}{\partial r} \right), \quad (5.12)$$

for $r \in [0, \delta]$ and $\kappa_g = k_g/\rho c_{p,g}$. The associated boundary conditions are:

$$T_g(\delta, z, t) = T_f, \quad (5.13a)$$

$$T_g(r, 0, t) = T_0, \quad (5.13b)$$

$$T_g(r, z, 0) = T_f. \quad (5.13c)$$

⁷In a tube with a circular cross-section, D is the tube diameter.

As the ice layer grows thicker, the cross-sectional area through which the gas flows decreases. Fully-developed, laminar flow in a tube will have a velocity distribution which is zero at the boundary and maximum on-axis. However, it is often convenient to work with an average velocity which is constant as a function of radius if the cross-sectional area is constant. For flow through a pipe, the average velocity is half of the maximum velocity. Using the continuity equation for steady, one-dimensional flow propagating through a section of tube with a cross-sectional area A_1 to a section with area A_2 , the fluid velocity v_z satisfies the condition $A_1 v_{z,1} = A_2 v_{z,2}$. Thus, as the ice shell increases in thickness, the velocity through the capillary can be defined as:

$$v_z = v_0 \frac{r_w^2}{\delta^2}, \quad (5.14)$$

where v_0 is the average flow velocity.

Eq. 5.3 can be used to find the temperature distribution in the solid phase and Eq. 5.12 in the gas phase. The interface is the boundary for both phases. The heat flowing in and out of the interface is balanced with the amount of heat needed to convert the gas into a solid. The interface boundary condition is given by:

$$\rho L \frac{d\delta}{dt} = [\vec{q} \cdot \vec{n}]_{\text{solid}}^{\text{liquid}}, \quad (5.15)$$

where L is the latent heat per unit mass, and $d\delta/dt$ is the velocity of the phase front due to the freezing. Since conduction through the gas and solid are the only heat fluxes normal to the surface, the interface condition, known as the Stefan condition, becomes:

$$\rho L \frac{d\delta}{dt} = k_s \frac{\partial T_s}{\partial r} - k_l \frac{\partial T_g}{\partial r}, \quad (5.16)$$

evaluated at $r = \delta$ and $\delta(z, 0) = r_w$.

The heat equations and boundary and initial conditions can be written in dimen-

sionless form by converting to the following variables:

$$R = \frac{r}{r_w} \quad (5.17a)$$

$$\tau = \frac{\alpha_l}{r_w^2} t \quad (5.17b)$$

$$\theta = \frac{T - T_f}{T_0 - T_f} \quad (5.17c)$$

$$Z = \frac{\alpha_l}{r_w^2 v_z} z \quad (5.17d)$$

$$K_l = \frac{T_0 - T_f}{\alpha_l \rho \lambda} k_l \quad (5.17e)$$

$$K_s = \frac{T_0 - T_f}{\alpha_s \rho \lambda} k_s \quad (5.17f)$$

$$\Delta = \frac{\delta}{r_w} \quad (5.17g)$$

With these changes in variables, the temperature equation for the gas phase becomes:

$$\frac{\partial \theta_l}{\partial \tau} + \frac{1}{\Delta^2} \frac{\partial \theta_l}{\partial Z} = \frac{\partial^2 \theta_l}{\partial R^2} + \frac{1}{R} \frac{\partial \theta_l}{\partial R}, \quad (5.18)$$

with the boundary and initial conditions:

$$\theta_l(\Delta, Z, \tau) = 0, \quad (5.19a)$$

$$\theta_l(R, 0, \tau) = 1, \quad (5.19b)$$

$$\theta_l(R, Z, 0) = 0. \quad (5.19c)$$

The temperature equation for the solid phase simplifies to:

$$\frac{d}{dR} \left(R \frac{d\theta_s}{dR} \right) = 0, \quad (5.20)$$

subject to the boundary conditions:

$$\theta_s(1) = \theta_w, \quad (5.21a)$$

$$\theta_s(\Delta) = 0. \quad (5.21b)$$

Finally, the Stefan condition becomes:

$$\frac{\partial \Delta}{\partial \tau} = K_s \frac{\partial \theta_s}{\partial R} - K_l \frac{\partial \theta_l}{\partial R}, \quad (5.22)$$

evaluated at $R = \Delta$ and $\Delta(Z, 0) = 1$. Equations 5.18-5.22 can be solved using the techniques explored in Ozisik. [126]

5.1.3 Steady-State and Transient Deposition Results

The solid phase has the solution:

$$\theta_s(R, \Delta) = \theta_w \left[1 - \frac{\ln R}{\ln \Delta} \right], \quad (5.23)$$

and the thickness will reach a steady state given by:

$$\Delta(Z, \infty) = e^{-B/A}, \quad (5.24)$$

where $B = k_s(T_f - T_w)/k_l(T_0 - T_f)$ is the ratio of the sensible cooling⁸ to the sensible heating of the gas. For a freezing temperature of 153 K and an initial gas temperature of 300 K, $B = 5$ corresponds to a wall temperature of 140 K. Figure 5.2 shows the steady-state thickness as a function of the dimensionless parameters B and Z . As the gas propagates through the channel, its temperature decreases through heat transfer with the interface until it reaches its freezing temperature and deposits onto the wall. As the longitudinal distance Z increases, the ice thickness increases starting from zero at the inlet.

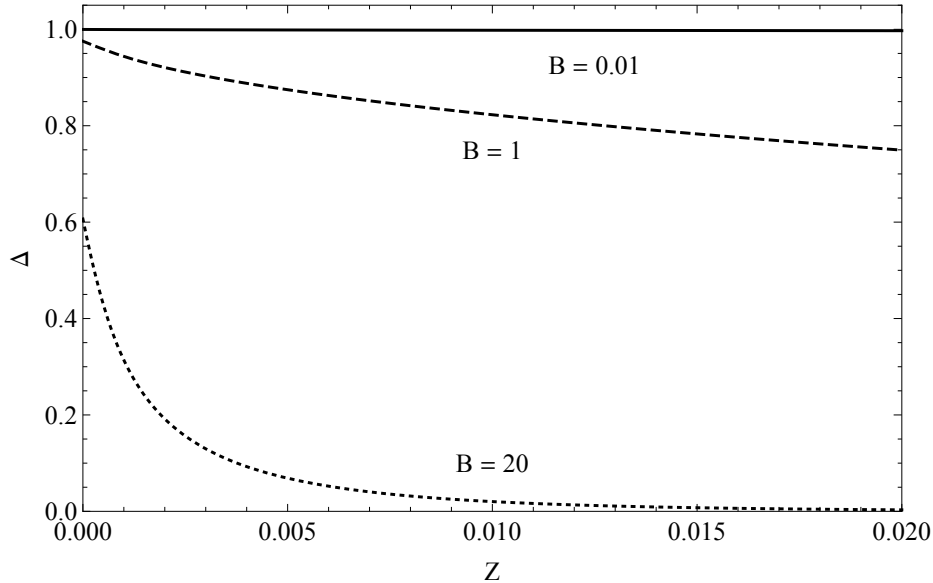


Figure 5.2: Normalized ice thickness as a function of time for different positions along the tube for varying values of B .

⁸Sensible heat, in contrast to latent heat, is the heat exchanged by a body which causes the body to change temperature.

The parameter B determines the amount of solidification. A larger value of B for a fixed freezing temperature corresponds to either a lower wall temperature T_w or a lower initial gas temperature T_0 . The former increases the heat transfer between the wall and the interface while the latter means the gas requires less cooling before it reaches the freezing temperature. Both factors increase the rate of freezing and result in thicker ice layers. In these experiments, the inlet gas temperature remained constant, and the parameter B was varied by adjusting the wall temperature. It can be seen that as B increases, the ice shell becomes thicker and more longitudinally non-uniform.⁹

The interface position can be found by numerically solving the integral:

$$K_l(\tau - Z) = - \int_{\Delta}^1 \frac{\xi d\xi}{A(Z) + \frac{B}{\ln \xi}}, \quad (5.25)$$

for $\tau \geq \Delta^2 Z$, where A is defined as:

$$A(Z) = 2 \sum_{m=1}^{\infty} e^{-\gamma_m^2 Z} \quad (5.26)$$

, and γ_m are the zeros of the zeroth order Bessel function $J_0(\gamma_m) = 0$.

Figure 5.3 shows the normalized ice thickness Δ along the length of the capillary as a function of time for $B = 5$. The ice shell grows thicker with time, and the growth rate increases with thickness as a result of the inside surface area of the ice shell decreasing. For earlier times, the longitudinal ice thickness is relatively constant. When $\tau = 0.005$, the ice thickness from $Z = 0.015$ to $Z = 0.2$ varies by only 2%.

The impact of the dimensionless parameter B on the freezing rate for a fixed position $Z = 0.1$ is shown in Fig. 5.4. As B increases, the heat transfer between the gas and the interface increases, and the ice shell grows faster. For a more longitudinally uniform ice growth, B should be kept small, requiring that the wall temperature remain close to the freezing temperature of the gas.

The dependence of the ice thickness on gas density is also shown in Fig. 5.4. As the density increases, the ice shell becomes thicker as there are now more molecules to freeze. This effect is largest near the entrance of the capillary as well as for larger values of B . It should be noted that the heat transfer between the gas and the solid increases at larger B and can compensate for the increased heat carried in through the gas.

⁹It should be noted that the absolute thickness values obtained with this model cannot be directly compared with experiments since the flow rate does not remain constant when the ice is very thick.

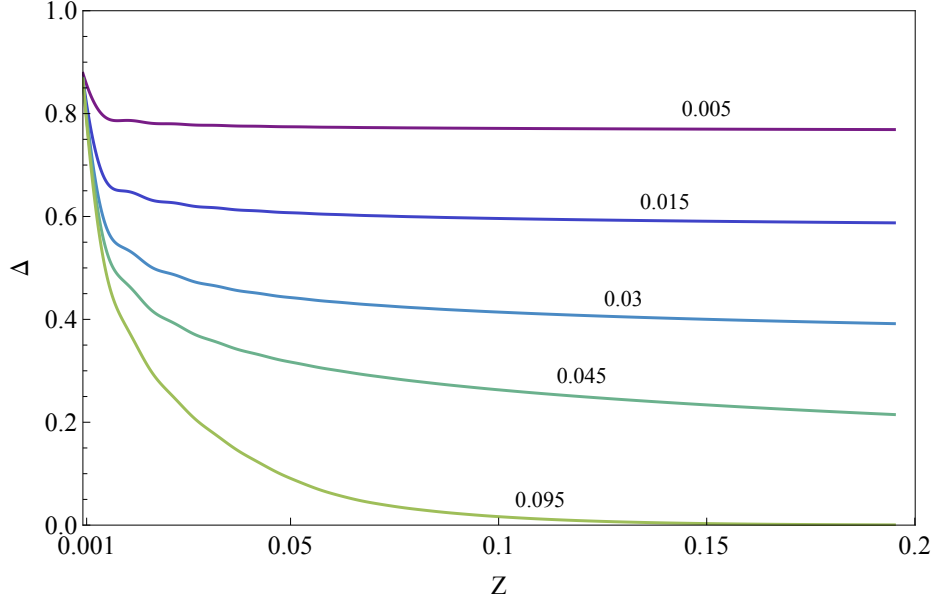


Figure 5.3: Normalized ice thickness Δ as a function of normalized position Z along the tube for $B = 5$ for different normalized times τ .

5.1.4 Discussion of Assumptions

Several assumptions were made to simplify the model of gas deposition onto the inner surface of a cooled channel. While these assumptions preclude us from directly comparing the deposition results with experimentally measured thicknesses, the model provides an overall understanding of the growth and how it depends on various factors such as time, position along the channel, gas pressure, initial gas temperature, and wall temperature. This model assumes a constant density between the two phases, an assumption which can be acceptable when considering the case of a liquid freezing to a surface but means certain dynamics are ignored when considering a gas phase. Because the solid density is much larger than the gas density, the gas freezes and contracts during solidification, resulting in thicknesses less than predicted under an assumption of constant density.

The model also assumes that the gas flow is only dependent on the radius of the channel. This assumption is true if the ice thickness is small, and the pressure drop caused by the reduction of the channel radius is small, or if the flow rate is high. As the fluid velocity increases, the heat transfer rate decreases and less ice deposits. There exists a flow velocity above which the channel will never freeze shut. The flow velocity can be related to the pressure drop between the entrance of the channel P_1

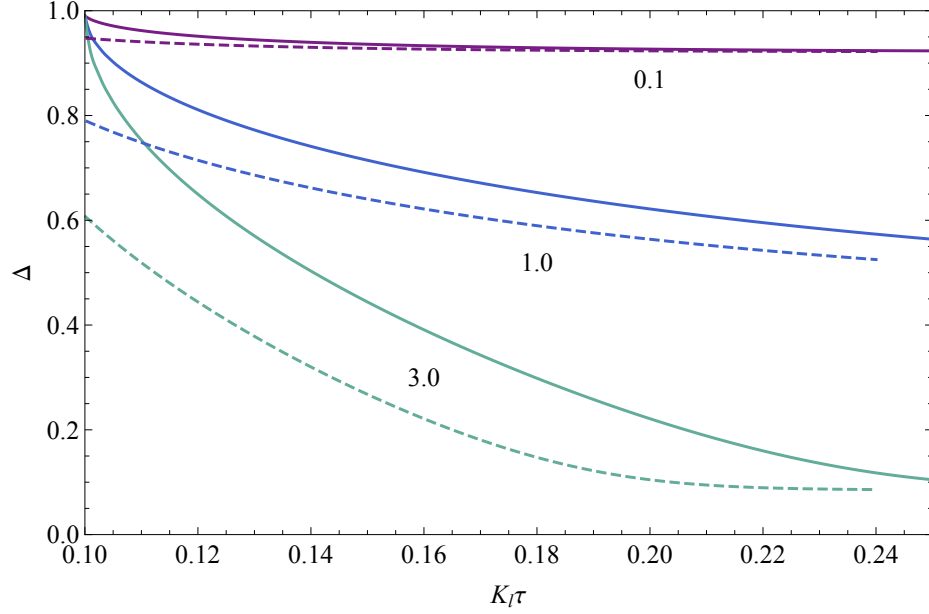


Figure 5.4: Normalized ice thickness as a function of time for different B values. The longitudinal position is fixed at $Z = 0.1$. Dashed lines correspond to a 25% increase in density relative to the solid lines.

and the exit P_2 . Laminar flow of a compressible gas through a pipe has a flow rate proportional to the pressure drop: $P_1 - P_2$. As the pressure drop decreases, the flow rate decreases and ice deposits at a faster rate. If the wall temperature is below the freezing temperature and the flow velocity is less than the critical velocity, complete blockage of the channel will occur.

Finally, the theory assumes laminar flow which is valid for gas densities below a certain threshold. However, as the pressure is increased, the flow can become turbulent, and the gas no longer flows in smooth parallel layers but in a chaotic manner instead. The fluctuations in velocity allow for rapid radial heat transfer between the gas and the interface. Turbulent flow, as a result, will produce thicker ice layers than this model predicts.

5.2 Spectral Domain Optical Coherence Tomography

To diagnose the ice shells, a commercially-available optical coherence tomography (OCT) imaging system was used to measure the ice thickness along the length of the channel. [134], [135] Optical coherence tomography is a non-invasive technique used to measure internal structures in materials by measuring the time delay and intensity of backscattered light. OCT is an emerging technology most utilized in biomedical imaging applications due to its micron-level resolution for high-quality images and fluence levels low enough to be used near sensitive tissues. [136]–[139] In addition, most OCT systems are optical-fiber based, making them flexible and easy to use. With more sophisticated techniques, OCT is capable of measuring both internal structures and flow velocity.

Figure 5.5 shows a simplified layout of the Thorlabs Ganymede 220 OCT system used in these experiments. Measurements were performed using a Michelson-type interferometer with a broadband superluminescent diode with center wavelength 900 nm. The central wavelength was chosen such that the nitrous oxide ice layers would not completely absorb the incident light. The source was split into two beams, one incident onto the sample and the other propagating through a reference arm. Light was backscattered off interfaces with different indices of refraction in the sample, and this backscattered light was recombined with the reference arm at the beam splitter. When the optical path length difference between the two arms is less than the coherence length of the source, the two beams interfere.

The coherence length L_c scales as $\lambda^2/\Delta\lambda$ where λ is the central wavelength and $\Delta\lambda$ is the source bandwidth. With a broad bandwidth light source, fine resolution can be achieved, and different layers in the sample can be resolved. The OCT system used in these experiments had a resolution of 3 μm in air.

After light from the two arms recombine, there are a number of techniques for determining depth information from the interferogram. In Time-Domain OCT (TD-OCT), at each position along the channel, the reference mirror position is scanned, resulting in interference when the reference mirror position corresponds to an interface position in the sample.

In Spectral-Domain OCT (SD-OCT), the reference arm is kept fixed, and the light is spectrally dispersed after the two beams are recombined using a diffraction

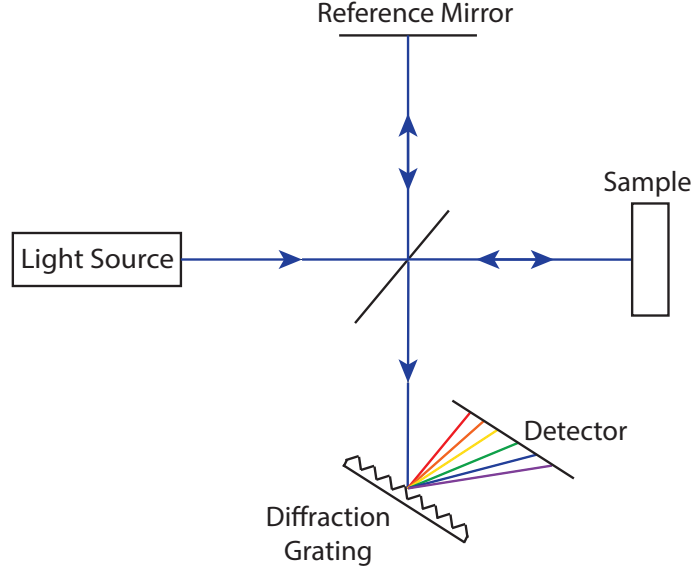


Figure 5.5: OCT schematic with light source split into a reference arm and sample arm. After light from the two arms recombines, the spectrum is dispersed to obtain the intensity as a function of wavelength.

grating to obtain the interferogram intensity as a function of wavelength. [140] The recombined spectrum $I(k)$ is a sum of all the waves reflected from the N layers in the sample:

$$I(k) = S(k) \left[R_r + \sum_{n=1}^N R_{s,n} + 2 \sum_{n=1}^N \sqrt{R_r R_{s,n}} \cos(k \Delta d_{rn}) + 2 \sum_{n \neq m=1}^N \sqrt{R_{s,n} R_{s,m}} \cos(k \Delta d_{nm}) \right], \quad (5.27)$$

where k is the light source wavenumber, $S(k)$ is the spectral intensity distribution of the source, R_r is the reflectivity of the reference arm, $R_{s,n}$ and $R_{s,m}$ are the reflectivity of the sample off the n^{th} and m^{th} layer, Δd_{rn} is the optical path length difference between the n^{th} layer and the reference arm, and Δd_{nm} is the optical path length difference between the n^{th} and m^{th} layers.

From the third term in Eq. 5.27, it can be seen that the oscillation frequency will depend on the optical path length difference between the reference and the sample layers. Fourier transforming the intensity distribution $I(k)$ allows for retrieval of the

sample depth profile:

$$F[I(k)] = \Gamma(z) * \left[R_r + \sum_{n=1}^N R_n + 2 \sum_{n=1}^N \sqrt{R_r R_n} \delta(z \pm \Delta d_{rn}) + 2 \sum_{n \neq m=1}^N \sqrt{R_n R_m} \delta(z \pm \Delta d_{nm}) \right], \quad (5.28)$$

where $\Gamma(z)$ is the Fourier transform of the source spectrum. The first two terms are DC terms, the third term represents the depth information of each of the sample layers while the fourth term corresponds to the autocorrelation noise between different layers in the sample. The last term is usually small.

The Fourier transform of the source spectrum $\Gamma(z)$ determines the resolution of the system. If the source has a Gaussian spectral intensity distribution, then $\Gamma(z)$ will also be Gaussian. In this case, the depth resolution is given by:

$$\Delta z = \frac{2 \ln 2}{\pi} \frac{\lambda^2}{\Delta \lambda}, \quad (5.29)$$

where λ is the source wavelength and $\Delta \lambda$ is the source spectral bandwidth. Because SD-OCT does not require the reference mirror to be scanned, an increased imaging speed of over 100x the speed of TD-OCT can be achieved. This advantage enables the ice thickness to be measured during deposition with reduced motion artifacts. The sample beam path contains two actuated mirrors which enable the beam to be scanned in two axes transverse to the beam. The depth information at each position can be stitched together to provide a three dimensional profile. Figure 5.6 shows an example two-dimensional OCT image of a finger tip where the structure beneath the finger surface including epidermis and dermis layers of the skin as well as sweat glands can be seen.

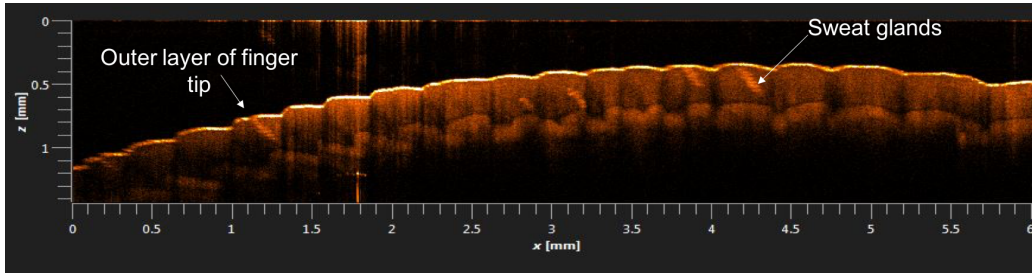


Figure 5.6: OCT image of a finger tip showing the two upper layers of the finger and the sweat glands.

5.3 Capillary Design and Ice Growth

In the designs tested in this thesis, the capillary was cooled by liquid nitrogen which has a temperature of 77 K. The gas to be frozen was nitrous oxide N_2O which is relatively inexpensive, safe, and freezes at a temperature above 77 K for a wide range of pressures.¹⁰ Figure 5.7 shows a schematic of the capillary and OCT set-up. Because the effective focal length of the OCT lens was 36 mm and the OCT scanning system could not be installed in vacuum, the capillary was mounted inside a vacuum chamber a millimeter away from a window with the OCT directly on the other side.

An Alicat pressure controller regulated the nitrous oxide pressure inside the capillary. As various designs were tested, a valve for better control over the N_2O gas flow was added as well as a valve connected to a vacuum pump for evacuating the capillary of residual gas. Eventually a helium line was added to facilitate discharge and a baratron to measure the pressure inside the capillary.

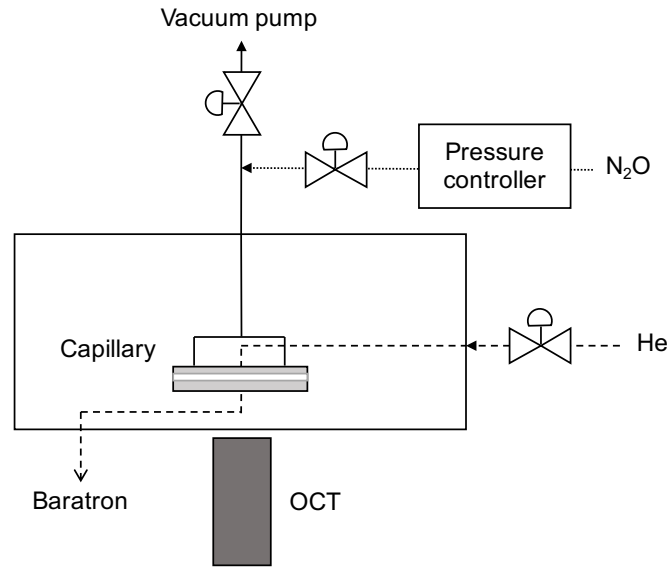


Figure 5.7: The OCT and capillary set-up including the valves and pressure controllers for regulating the gas flow and a baratron for measuring the pressure inside the channel.

¹⁰Other gases such as carbon dioxide could have been used.

5.3.1 Aluminum and Acrylic Capillaries

To determine whether the nitrous oxide would freeze to the walls of a capillary, a simplified model was first tested. This version was a 9 cm long aluminum block with a 3 mm diameter channel machined down the center surrounded by two 3 mm diameter channels for liquid nitrogen flow (see Fig. 5.8(a)). 40 Torr of nitrous oxide was continuously flowed through two gas slots into the channel as the aluminum block was cooled. Figure 5.8(b) shows a representative image of the ice shell 25 s after deposition began. Over the entire freezing process until the channel froze shut, the ice shell was radially uniform.

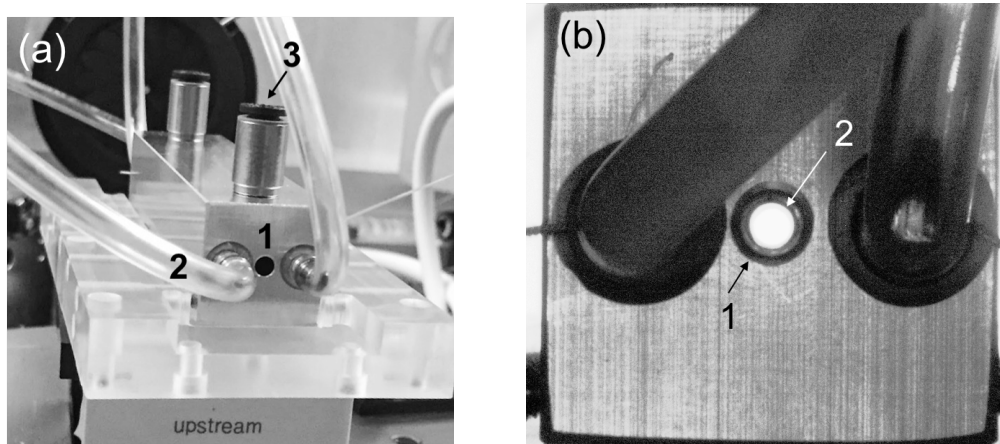


Figure 5.8: (a) Aluminum capillary: 1) aluminum body with central channel, 2) liquid nitrogen plastic lines, 3) nitrous oxide gas slots, (b) Ice growth: 1) aluminum capillary wall, 2) inner ice shell wall.

The channel radius during deposition is shown in Fig. 5.9. The gas started freezing to the channel wall at time $t = 0$, and the ice shell thickness grew at a rate of $122 \mu\text{m/s}$. After 1 minute, the gas flow was stopped. It can be seen that the channel radius remained constant and did not melt. After 3.6 minutes, the flow was initiated again, and the ice continued to deposit until the channel froze shut.

With the capillary made of aluminum, the longitudinal growth of the ice could not be observed, and so an acrylic version was made with the same dimensions. Figure 5.10(a) shows the ice growth in the acrylic capillary. It was seen that the shell thickness was no longer radially uniform. Simulations of the temperature distribution over time in both the acrylic and aluminum capillaries were performed using the

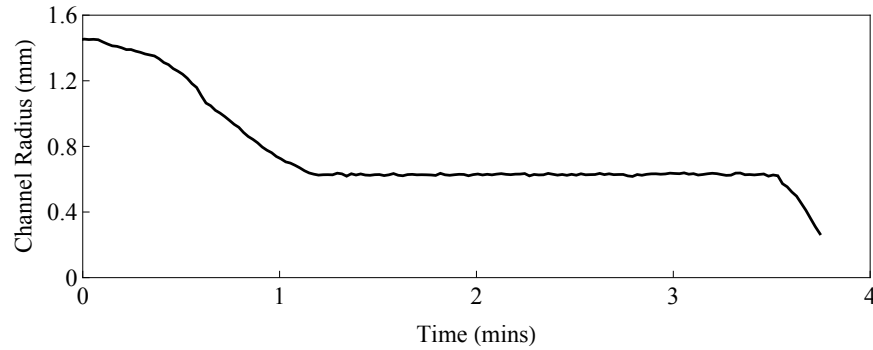


Figure 5.9: Ice shell inner radius during deposition. From 1 minute to 3.4 minutes, the nitrous oxide gas flow was stopped to observe the ice.

commercially-available, finite element software ANSYS. The capillary was modeled with a constant 77 K boundary condition on the liquid nitrogen lines. Transient thermal analysis was performed over a period of 20 s for both capillaries. At the end of the simulations, the temperature in the aluminum capillary varied by less than 2 K around the main channel. Figure 5.10(b) shows the temperature distribution in the acrylic capillary after the same period of time. It can be seen that the temperature around the main channel varies from 100 K to 230 K and that the region of the channel closer to the liquid nitrogen lines reached a lower temperature.

As detailed in Sec. 5.1, a lower wall temperature will cause ice to deposit faster. Therefore, where the capillary is closer to the liquid nitrogen, a thicker ice layer develops. Aluminum has a thermal conductivity of 205 W/(m·K) while acrylic has a thermal conductivity of 0.2 W/(m·K). Because of the acrylic's lower thermal conductivity, the cooling distribution becomes more important. Successive capillaries were made with either radially symmetric cooling or from materials with high thermal conductivity.

A second acrylic capillary with four liquid nitrogen lines machined at the far corners of the block was tested. This configuration provided a more uniform temperature distribution around the central channel and resulted in radially uniform ice growth. To isolate the gas slots and prevent gas from freezing before it reached the channel, the slots were thermally separated from the rest of the capillary as shown in Fig. 5.11(a). The capillary was cooled to 108 K and 120 psi of gas was continuously flowed through the channel.

Figure 5.11(b) shows the longitudinal ice deposition after 86 s of growth. It can be seen that the ice thickness increases in the flow direction until a plug forms. The

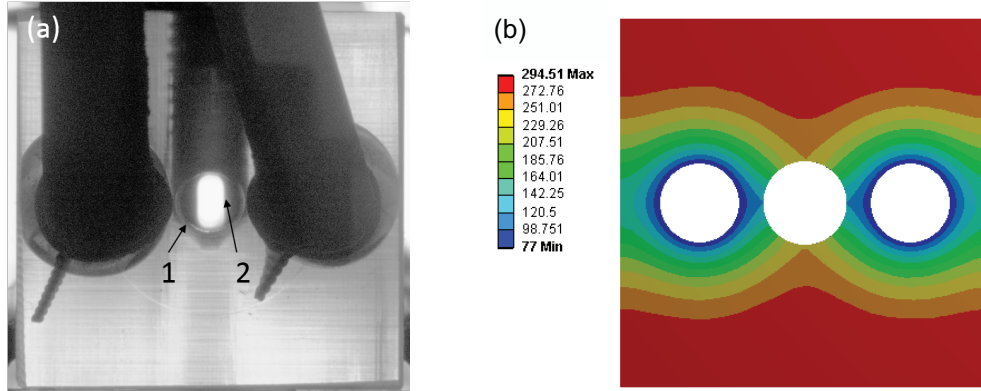


Figure 5.10: (a) Acrylic capillary with asymmetric ice growth: 1) acrylic capillary wall, 2) inner ice shell wall. (b) Asymmetric temperature distribution of acrylic capillary after 20 s of cooling.

ice growth shown in Fig. 5.8(b) and Fig. 5.10(a) are these plugs. Since thicker ice layers grow faster, the plugs dominate the longitudinal ice growth. As nitrous oxide molecules freeze at the entrance of the cap and at the plug, the gas density decreases with propagation and regions further from the entrance do not experience as much growth.

Once the plugs started to develop, no appreciable ice deposition occurred further into the channel. The plugs were the result of a wall temperature (108 K) much lower than the freezing temperature of the gas (approximately 193 K) as well as reduced gas flow. Because the wall temperature was low, the gas bulk temperature decreased quickly and ice deposited close to the entrance of the channel. In addition, the gas entered through both gas slots and filled the capillary, forming a stagnant channel of gas after a few hundred μs . [141] By this time, no appreciable ice layer had formed, but the flow rate had decreased as a result of the reduced pressure drop between the channel entrance and center. As the flow velocity decreases, the amount of heat that can be transferred to a volume of gas increases, and the gas freezes faster. Both of these effects contributed to the plugs growing at the entrance, preventing ice formation further into the capillary.

Attempts to prevent the plugs from forming were made by locally heating the ends of the channel using resistive heaters placed on top of the channel near the plug location. However, because the wall was thick, the heat spread over a wider distance than desired and resulted in no ice growth over an extended region. For

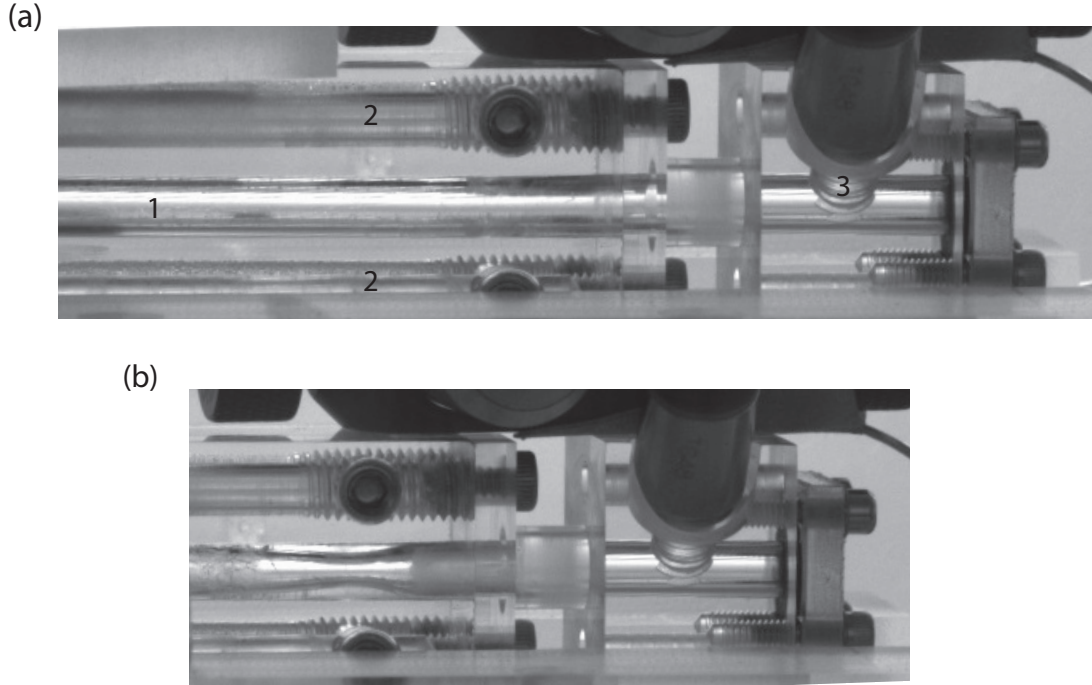


Figure 5.11: (a) End of second acrylic capillary with no ice deposition showing: 1) central channel, 2) liquid nitrogen lines, and 3) separated gas slot, (b) after 86 s of ice growth.

finer temperature control, the channel was then made out of a glass tube.

5.3.2 Glass Capillaries

To ensure symmetric cooling, the capillary was made using a 9 cm-long quartz tube with a 1.5 mm inner diameter and 7.5 mm outer diameter. Quartz was chosen over other types of transparent materials such as borosilicate glass because of its lower coefficient of thermal expansion, making the capillary more resistant to changes in temperature. N_2O gas of varying pressure flowed into the capillary through two glass tubes of 2 mm inner diameter fused to the central channel (see Fig. 5.12) For most of the tests, the glass capillary ends were closed while the pressure was varied to protect the chamber vacuum pumps.

The temperature of the glass tube was controlled using the cooling ring shown in Fig. 5.12. The rings are machined from oxygen free, high thermal conductivity (OFHT) copper. One end of the cooling ring is clamped to a copper liquid nitrogen tube, and a resistive heater is attached to the other end. The entire copper ring was

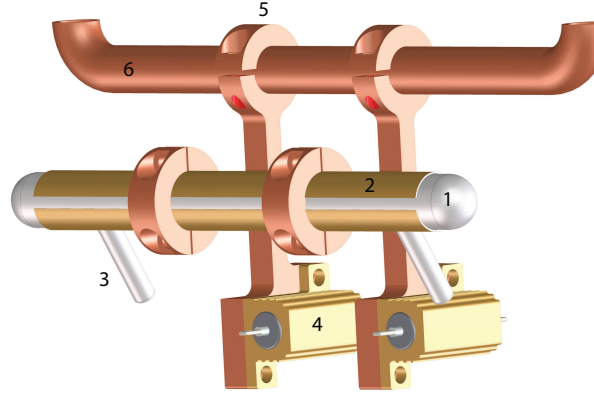


Figure 5.12: Glass capillary: 1) glass tube, 2) copper jacket, 3) gas slots, 4) resistive heaters, 5) cooling ring, and 6) liquid nitrogen lines.

then clamped around the glass tube. To provide longitudinally uniform cooling, a thin jacket of OFHT copper was placed around the glass tube such that the cooling ring clamped the jacket to the glass. The jacket had a slit through which the ice could be observed. A thermocouple attached to the copper ring measured the temperature, and the liquid nitrogen cooled the capillary at a rate of 1 K/s. By adjusting the resistive heater voltage, the temperature at the glass tube could be controlled.

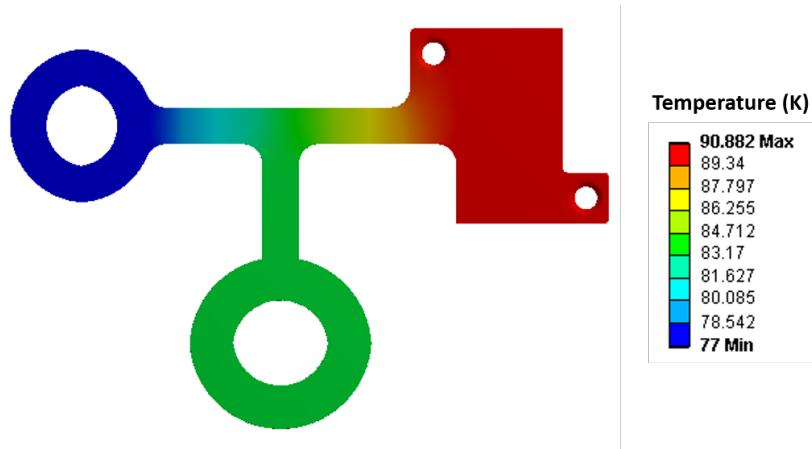


Figure 5.13: Thermal analysis of the copper ring with 24 W of heating showing the even temperature distribution around the ring clamped to the glass tube.

Figure 5.13 shows the temperature distribution of the cooling ring with 24 W of heating power. At this power, the temperature is a constant 83 K along the ring

clamped to the glass, providing radially symmetric cooling. The intent was to add a series of cooling clamps along the length of the channel and control the temperature of each independently, providing a more complicated longitudinal temperature distribution if desired.¹¹ For improved thermal contact, a layer of indium was added between the glass and the copper jacket.

Different techniques for depositing ice onto the capillary walls were tried. One option is to first fill the channel with nitrous oxide at pressure p and then decrease the wall temperature. When the temperature reaches the freezing temperature, N_2O deposits onto the wall. The relationship between the nitrous oxide pressure and freezing temperature is shown in Fig. 5.14. [142] The temperature T_t and pressure p_t at which all three phases co-exist in thermal equilibrium, called the triple point, is 12.7 psi and 182.3 K, respectively. [131]

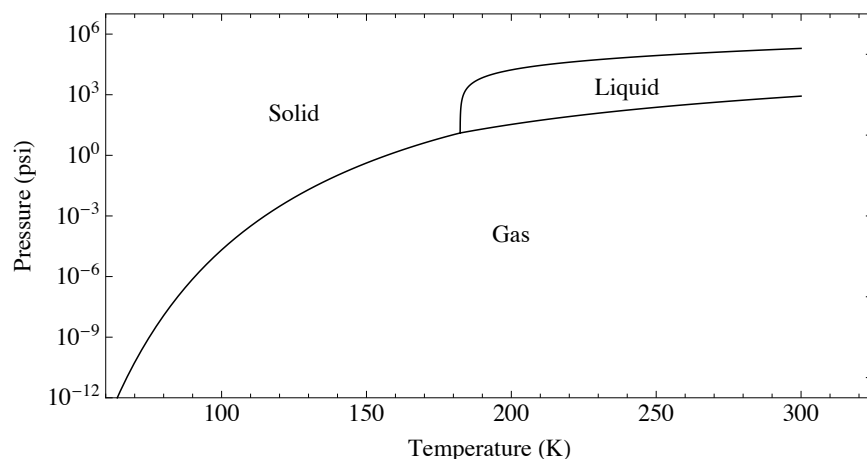


Figure 5.14: Phase diagram of nitrous oxide.

If $p > p_t$, the gas will first liquify as it cools. Condensation begins at the location of the cooling rings as these points reach the condensation temperature first. In these tests, a clearly-defined liquid front was observed. Once the freezing temperature was reached, the liquid rapidly froze. Without more sophisticated temperature regulation enabling the liquid to slowly freeze, this process was non-reproducible and difficult to control. Uniform, reproducible ice layers were never measured. Therefore, the technique of filling the channel with gas and then decreasing the wall temperature was considered unsuitable for this application given the temperature control available.

¹¹It was later shown that this control was not necessary.

If the pressure is lower than the critical pressure p_t , then the gas can transition to a solid without passing through the liquid phase. However, using the solid density of nitrous oxide (given in Table 5.1), this pressure corresponds to an ice thickness layer of a few microns, too thin for many applications.

Instead, the channel wall temperature was first decreased to a temperature below the N_2O freezing temperature, and then nitrous oxide was added into the channel at the desired pressure. A fast-response¹² valve after the pressure controller further regulated how much gas flowed through the channel by changing the time the valve was held open.

In the first set of experiments, the capillary was cooled until the temperature started to level off around 108 K, and then gas was added into the channel through both gas slots. With the cooling rings, radially symmetric ice growth was observed, and the longitudinal ice deposition was then addressed. The OCT system was installed during the glass capillary experiments, and more precise measurements of the ice growth could be taken. Figure 5.15 shows an example two-dimensional OCT image of ice deposition along a section of the capillary, measured at the channel centerline. The glass and ice interface locations are clearly defined. For each longitudinal position, a lineout was taken and peaks in the intensity were identified as the glass and ice interfaces. The ice thickness layer is defined as the difference in these two positions. Only the two interfaces can be seen in the OCT image, showing no substructure when the ice freezes.

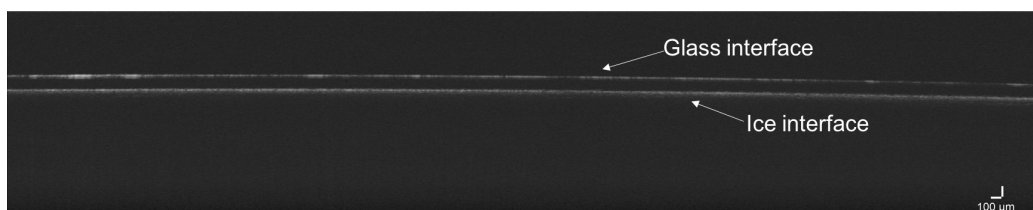


Figure 5.15: OCT image showing a layer of ice deposited inside a capillary.

The ice growth as a function of time for a 7.5 mm length section of capillary starting at the gas slot is shown in Fig. 5.16. Each horizontal lineout corresponds to a thickness distribution along the capillary. The N_2O valve was open for only 500 ms, but it can be seen that the gas thickness continued to increase at the end of the channel for longer than 1 minute. Because the wall temperature was much lower

¹²Tens of milliseconds.

than the freezing temperature of the N_2O , the gas froze rapidly as soon as it hit glass, which occurred inside the gas slots. As ice continued to deposit, plugs developed inside the slots, resulting in thicker ice layers near the ends. However, as the OCT images showed, a uniform ice layer approximately $100\ \mu\text{m}$ thick was deposited at early times.

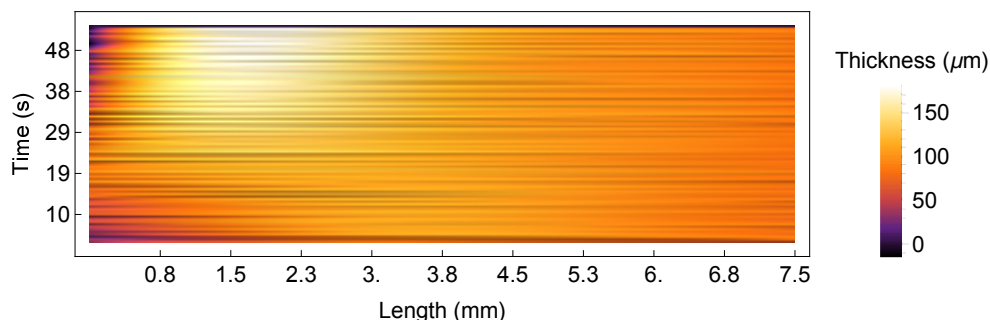


Figure 5.16: Ice deposition over time along a 7.5 mm region of the capillary starting at the gas slot. The gas valve was held open for 500 ms.

To prevent the gas at later arrival times from depositing at the ends, the channel was evacuated after the first initial deposition of ice. The gas valve, while controlling the amount of gas entering the capillary, did not effectively control the duration of the gas pulse. As the valve opened, gas expanded into the tubing leading to the capillary, causing the gas to spread out and resulting in a longer pulse duration than the gas valve opening time. A second valve connected to a vacuum pump enabled better control over the pulse duration.

The time between the closing of the gas valve and opening of the vacuum valve determined the effective deposition time: the shorter the time between the valves, the shorter the deposition time. Because gas arriving at later times preferentially deposited at the ends of the capillary, shorter gas pulses prevented plugs from developing inside the gas slots. Without the vacuum valve, the difference in thickness from the entrance of the channel and a point 12 mm inside the channel was $200\ \mu\text{m}$. Waiting 2 s before opening up the vacuum valve, decreased the difference in thickness to $60\ \mu\text{m}$. Waiting 1 s or less resulted in longitudinally uniform ice growth. In subsequent experiments, the vacuum valve was opened immediately after the gas valve closed.

Like the second acrylic capillary, however, no appreciable ice layer was detected between the two gas slots. The ice would freeze up to 15 mm into the channel and

then the thickness would rapidly decrease to nothing over a couple centimeters. This effect was attributed to the lack of flow rate during most of the freezing process and the build up of ice in the gas slots and channel entrance. To prevent non-uniform ice deposition, gas flow needs to be maintained. Instead of having gas enter through both gas slots, gas was flowed into the channel through one slot and the second gas slot was left open into the vacuum. In this configuration, the gas flow rate was high. Figure 5.17 shows the ice thickness for 50 psi of pressure and a 0.5 s gas valve opening time along the length of the glass tube. The solid line corresponds to an averaged thickness over successive segments of 60 μm , showing large-scale variations in the ice shell. The ice layer deposited with an average of 133 μm uniform thickness along the length of the tube. The gray shaded region corresponds to fluctuations in the measured ice thickness due to scattering off the rough ice surface on the order of 10 μm .

The region from 12 mm to 16 mm corresponds to the location of the copper ring where the ice thickness could not be measured.¹³ The measured ice thickness around the copper ring was used to predict the ice underneath the ring. The left side of the capillary was partially blocked by the window mount, resulting in a slightly thicker layer due to reduced gas flow. Once the ice fully deposited, the thickness remained constant.

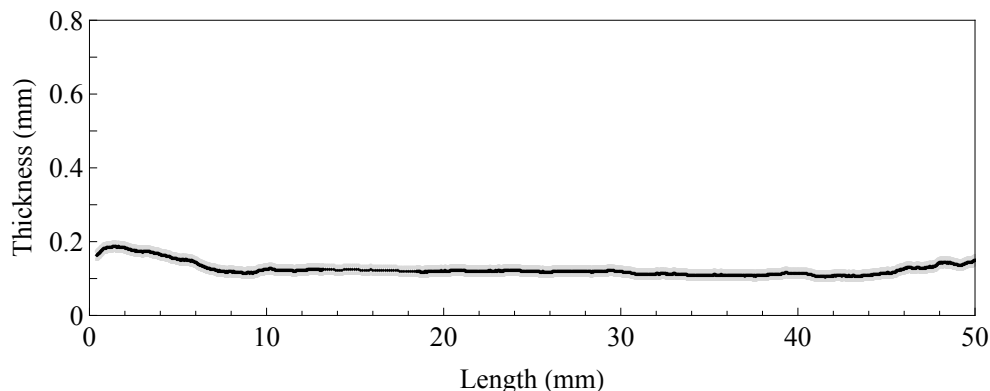


Figure 5.17: Longitudinally uniform ice growth in the glass capillary with gas flow.

To deposit thicker ice layers, more gas was flowed through the channel. One technique investigated using the glass capillary was to send in another burst of N_2O . Once the first layer had deposited, opening the gas valve a second time added more

¹³It was determined that only one copper ring was needed to uniformly cool the length of the glass tube.

gas into the capillary which froze to the existing ice shell and increased the layer thickness. Figure 5.18 shows the results of three successive gas bursts. The three layers of growth can clearly be seen in the OCT image as shown in Fig. 5.18(a). The longitudinal ice thickness remains uniform for each deposition cycle as shown in Fig. 5.18(b).

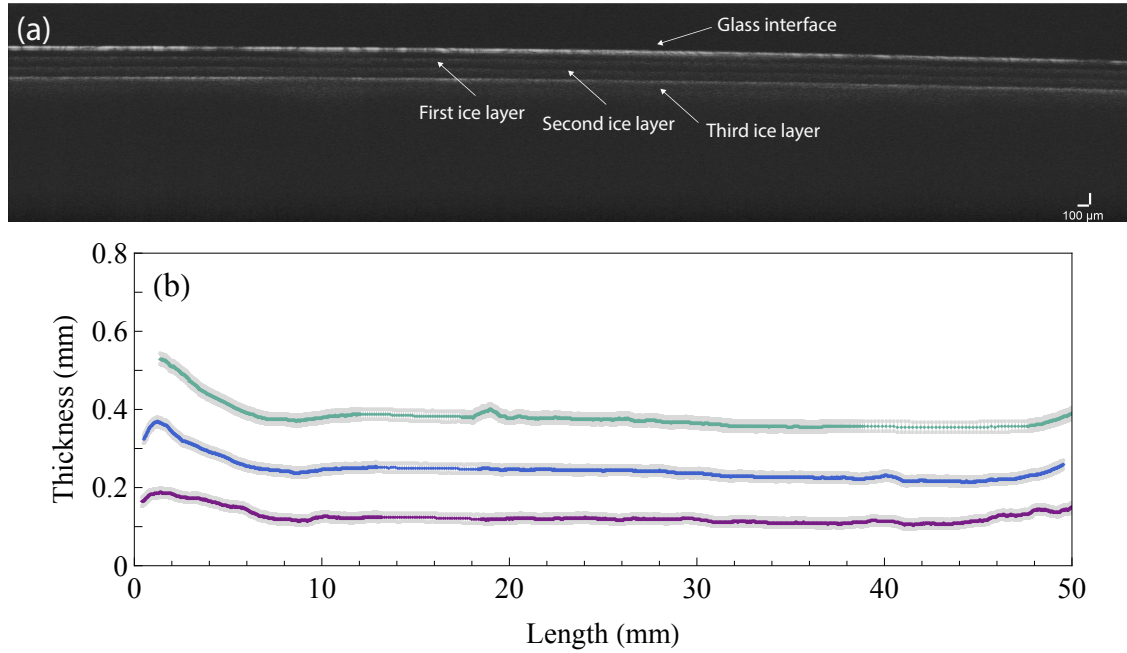


Figure 5.18: Multiple bursts of N_2O gas: (a) OCT image showing the three layers of growth and (b) thickness along the length of the capillary.

In general, the ice thickness can also be controlled by the pressure, but when the flow rate is high, the thickness is very weakly dependent on the gas density as shown in Fig. 5.4. The average thickness of the ice shell as a function of N_2O pressure for a 0.5 s valve opening time is shown in Fig. 5.19. Over the length of the capillary, the thickness was uniform. The error bars correspond to fluctuations in the measured ice thickness due to the rough surface. It can be seen that over a large pressure range (1 psi to 100 psi), the thickness is weakly dependent on pressure.

The glass capillary design demonstrated longitudinally uniform ice growth for a range of ice thicknesses. However, in other experiments investigating guiding in a gas-filled discharge capillary where the capillary was made out of glass, the glass shattered after a few shots of high-intensity laser pulses. For this reason, studies of

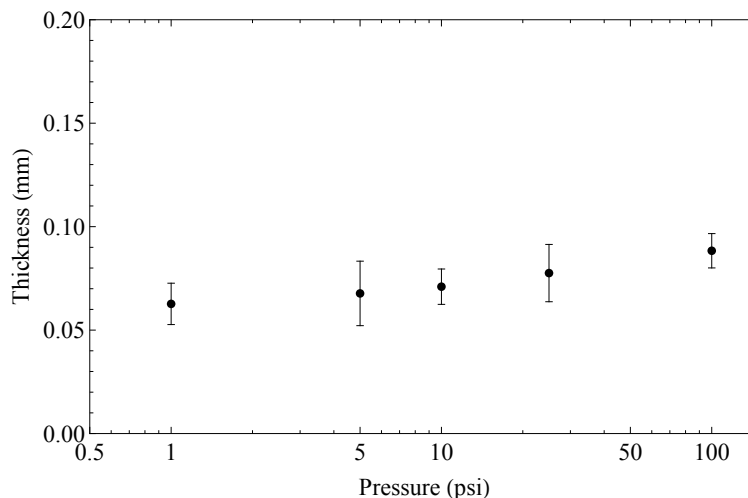


Figure 5.19: Average thickness as a function of N_2O pressure.

ice shells in glass capillaries were suspended, and capillaries made out of sapphire were investigated.

5.3.3 Sapphire Capillaries

Because sapphire has a high thermal conductivity ($k = 23.1 \text{ W/mK}$), it can be cooled efficiently. Sapphire capillaries have also already been shown to survive discharge and high-intensity laser pulses.¹⁴ The capillary design was adapted from the gas-filled discharge capillary used in Chapter 3. The channel is laser machined into two 6 cm sapphire blocks which are then glued together to form a 1 mm diameter channel with an approximately circular cross-section. Two gas slots of 1 mm diameter were machined perpendicular to the main channel, 6 mm from the end. Nitrous oxide and helium gas flowed into the channel through these slots. A third gas slot was machined in the center of the channel but was kept plugged during these experiments.¹⁵ At the output of one of the gas slots, a baratron was added to measure the pressure inside the capillary.

A thermocouple attached to the outside of the sapphire wall measured the capillary temperature. Liquid nitrogen cooled the capillary with a cooling rate of 1.5 K/s. To provide a level of temperature control with enough heating to raise the capillary temperature from 77 to 153 K, the freezing temperature of N_2O at 10 psi, two 24

¹⁴See Chapter 3.

¹⁵While the middle gas slot did not influence the ice growth, it did impact the discharge ablation.

W ceramic resistive heaters were clamped to two liquid nitrogen copper tubes. Each resistive heater plus liquid nitrogen tube assembly was clamped to two thin plates of copper. The plates bracketed the top and bottom of the sapphire blocks to provide uniform cooling along almost the entire length of the capillary (see Fig. 5.20). For temperature control, the resistive heater voltage could be adjusted.

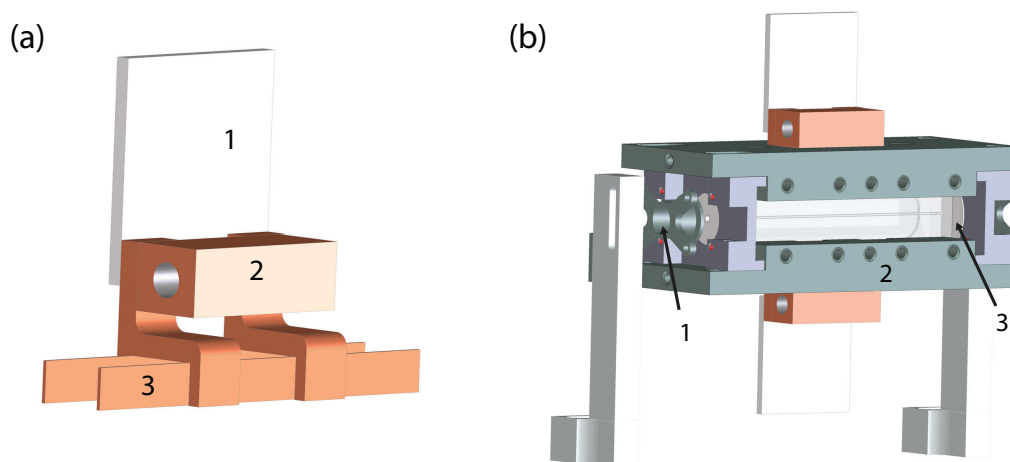


Figure 5.20: (a) Model of heating-cooling assembly: 1) resistive heater, 2) liquid nitrogen line, 3) copper plates, (b) Model of entire capillary: 1) electrode holder, 2) mount, 3) ceramic disk.

The sapphire plates and copper cooling assembly were both mounted in an acrylic holder. As will be further detailed later, discharge and plasma channel formation were tested in the sapphire capillary. Therefore, a ceramic disk was glued at each end of the capillary to prevent the discharge from arcing to places other than down the channel. Two stainless steel electrodes were positioned inside the ceramic disk to provide the electric potential. The electrodes were mounted separately from the rest of the capillary.

With gas flowing through both gas slots, the capillary quickly reached a stagnant pressure. As was demonstrated with the acrylic and glass capillaries, reduced gas flow with a wall temperature much lower than the freezing temperature of N_2O resulted in non-uniform longitudinal ice growth. In most cases, the ice never reached the center of the channel. More uniform layers were demonstrated by increasing the gas flow. However, as shown in Fig. 5.4, the deposition rate as a function of position along the

channel is also dependent on the wall temperature. This relationship was investigated using the sapphire capillary.

The capillary was cooled first, and gas was added into the channel at a given pressure and duration once the desired temperature had been reached. The ice thickness along the length of the capillary is shown in Fig. 5.21 for wall temperatures of 143 K and 153 K at 25 psi pressure and 500 ms gas valve opening time. It can be seen that the longitudinal distribution is dependent on the capillary wall temperature. For lower wall temperatures, the ice thickness at any point grows at a much faster rate, leading to non-uniform longitudinal growth. Because the gas begins to freeze inside the gas slots, the ice thickness is largest where the slots meet the main channel (at 4 mm and 50 mm). To maintain a constant ice thickness along the length of the capillary, the wall temperature should be close to the freezing temperature. This temperature was determined by experimentally measuring the temperature at which ice would just start to deposit: at a warmer temperature there would be no ice growth, and at a colder temperature non-uniformity would start to develop.

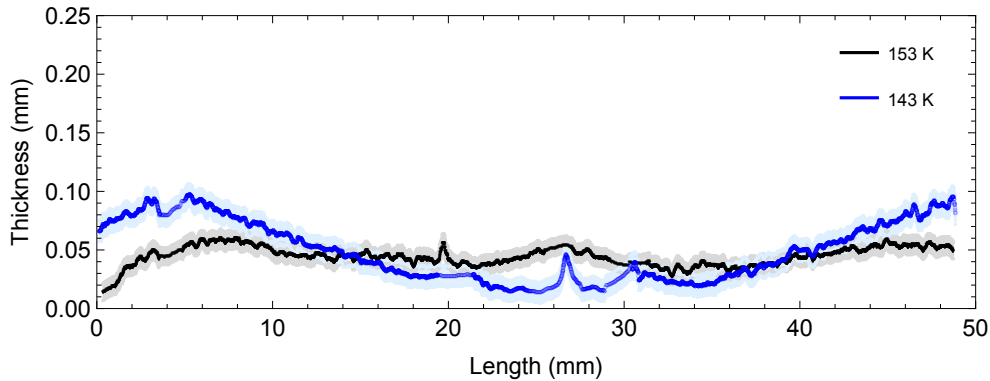


Figure 5.21: Thickness for varying wall temperatures: 153 K (black) and 143 K (blue).

Once the optimal wall temperature was determined, the ice thickness was tuned using the gas valve opening time. With more gas flowing through the capillary, more ice was deposited, leading to thicker layers. Figure 5.22 shows the ice layer along the length of the capillary for a 25 psi backing pressure, 153 K wall temperature, and 0.5 s and 1.5 s opening time. A 112 μm ice layer deposited when the valve was held open for 1.5 s while a 44 μm ice layer deposited when the valve was held open for 0.5 s. The ice thickness fluctuations caused by surface roughness is 8.6 μm and 7.6 μm for 1.5 s and 0.5 s opening time, respectively. The gas slots were located at 3.5 mm and

26.0 mm. Neglecting the end 5 mm where the end of the capillary distorts the ice growth, the 1.5 s (0.5 s) layer has a maximum thickness variation of $17\text{ }\mu\text{m}$ ($29\text{ }\mu\text{m}$).

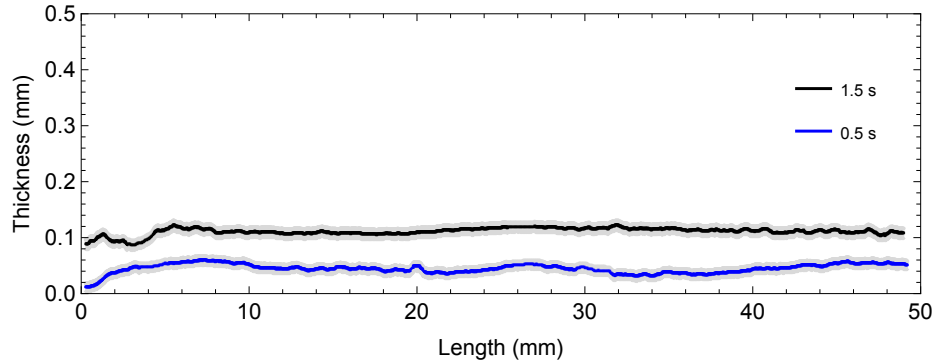


Figure 5.22: Thickness as a function of gas valve opening time.

It can be seen that the ice thickness increases uniformly across the channel with opening time. Figure 5.23 shows the thickness as a function of opening time ranging from 0.2 s to 2.2 s. The error bars correspond to fluctuations in the measured thickness as a result of the rough ice surface. For each second the gas valve remained open, $67\text{ }\mu\text{m}$ of ice deposited.

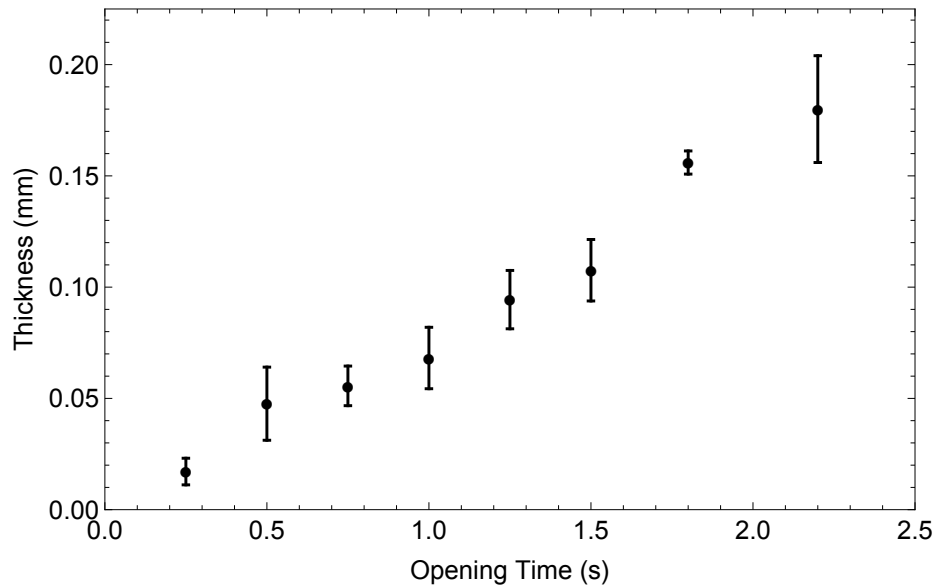


Figure 5.23: Average thickness as a function of gas valve opening time.

After an ice layer had developed and was measured, either flowing room temperature air through the liquid nitrogen lines or increasing the voltage on the resistive

heater resulted in an increase of the capillary temperature and melted the ice. Once the ice had fully melted, another ice shell could be regrown. Figure 5.24 shows two ice growth cycles, both with 25 psi of pressure and a 1.5 s gas valve opening time, resulting in 107 μm and 112 μm layer thicknesses. With the same initial parameters, the ice shells are reproducible.

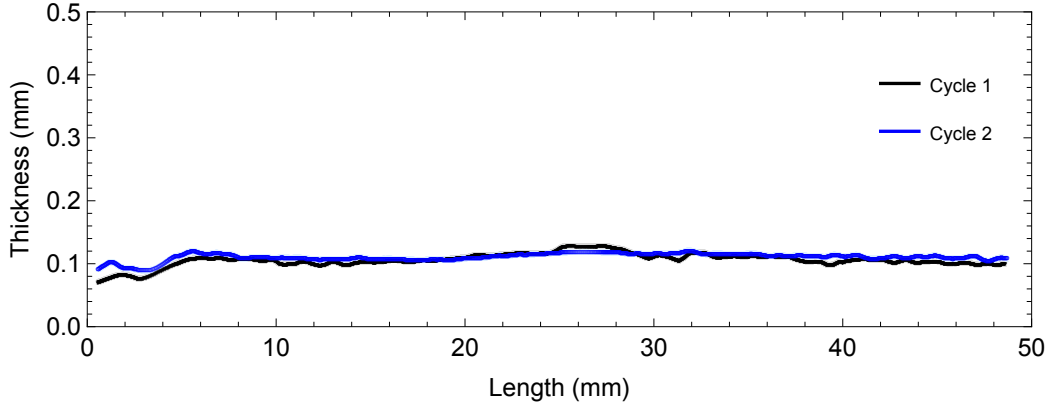


Figure 5.24: Two ice shells grown under the same conditions.

The ice thickness measurements shown are all located between the gas slots unless otherwise stated. It was observed that the ice in the region from the gas slots to the end of the channel would begin to melt away after the initial deposition. The melting would initiate at the end of the channel and slowly propagate inward over a few seconds, leaving a zone free of ice. Several effects could contribute to the ends melting. The ceramic disks at the ends of the capillary transferred heat from the capillary mount, which remained within a few degrees of room temperature during these tests, to the sapphire. In addition, the copper assembly did not extend all the way to the ends of the sapphire. Thus, the ends were the warmest part of the channel. In this design, a melt zone of 6 mm developed within a few seconds after the initial deposition. When guiding a laser pulse through the channel, entrance effects can impact the guiding properties when the Rayleigh length of the laser is on the order of this melt zone. However, for the guiding experiments described in Sec. 5.4, the Rayleigh length of the laser was 2.5 cm, and this edge melting was considered acceptable. Improved cooling and decreased heat transfer between the ends and the rest of the capillary can potentially improve this effect if it is needed.

5.4 Guiding

In the cryogenically-formed capillary, various guiding mechanisms are possible. Here, we investigate guiding in a parabolic plasma channel produced by a discharge pulse as was initially described in Chapter 3.

5.4.1 Guiding Theory

A discharge pulse propagating through a gas-filled channel induces a dynamic waveguide, evolving to form a plasma channel with a parabolic density profile. [79] At the beginning of the discharge pulse, the plasma temperature, density, and degree of ionization increase homogeneously. Around the time of full ionization, the plasma temperature becomes radially non-uniform as heat is transferred from the plasma to the cold capillary walls. At the peak of the discharge current, the plasma temperature and density reach a quasi-steady state with a maximum temperature (minimum density) on-axis. The radial density profile is approximately parabolic and remains so until recombination occurs.¹⁶

The energy and quality of the particle beams accelerated in these channels are dependent on the channel shape and density. [9], [50] Various techniques including transverse and longitudinal interferometry have measured the channel profile and evolution. [143]–[145] Here, we used the measured spot size and peak intensity of the laser pulse at the exit of the capillary. [146] The matched spot size w_M is one indicator of the channel's guiding properties and is defined as the spot size at which the channel will guide a laser pulse with a constant spot size $w(z)$ during propagation. If the laser is not matched to the channel, the laser intensity will oscillate during propagation, and the plasma wake amplitude, and thus acceleration and focusing properties, will also oscillate. High intensity laser pulses with small spot sizes on the order of tens of microns require waveguides with small matched spot sizes to maintain a constant intensity.

For a Gaussian laser profile of the form $I(r, z) = I_0(w_0/w(z))^2 \exp(-2r^2/w(z)^2)$ where I_0 is the peak intensity on axis and w_0 is the spot size at focus, if the laser is focused at the channel entrance ($w(0) = w_0$), the spot size in the channel evolves as:

$$w(z)^2 = \frac{w_0^2}{2} \left[1 + \frac{R_M^2}{w_0^4} + \left(1 - \frac{R_M^4}{w_0^4} \right) \cos(2k_{\beta c} z) \right], \quad (5.30)$$

¹⁶Longitudinal plasma flow out the ends of the channel is not considered.

where $k_{\beta c} = 2/(kR_M^2)$. [147], [148] Eq. 5.30 assumes that the spot is not influenced by other effects such as self-focusing and a plasma wave. If $w_0 = R_M$, the laser will propagate with constant spot size and intensity. If $w_0 \neq R_M$, the laser can still be guided, but the spot size will oscillate between w_0 and R_M^2/w_0 with oscillation wavelength $\pi^2 R_M^2/\lambda$. Figure 5.25 shows the output spot size and normalized peak intensity $I(0, z)/I_0$ for a beam with an 87 μm FWHM focal spot size propagating through a 6 cm capillary. For a measured exit spot size, Eq. 5.30 can yield multiple solutions. The correct solution among the set can be determined by other beam characteristics such as divergence, but for the following experiments, the output spot size was large enough to not be in this regime.

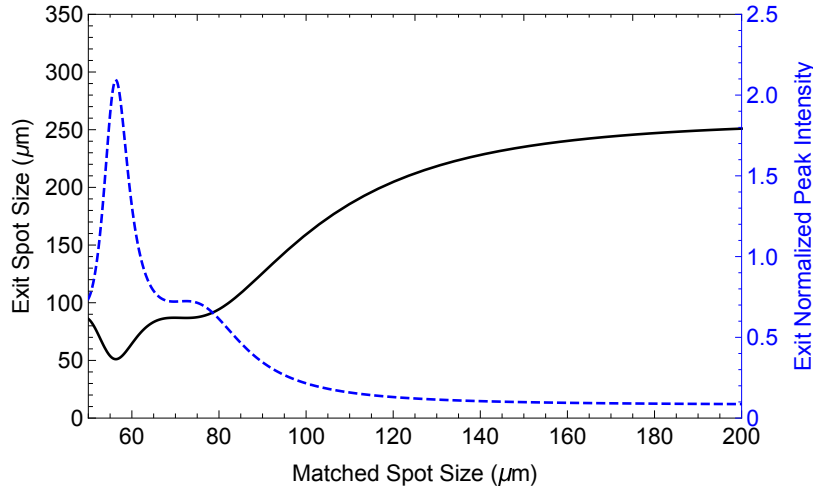


Figure 5.25: The exit spot size (black) and exit normalized peak intensity $I(0, z)/I_0$ as a function of the matched spot size.

A simple model of the matched spot size in the steady-state regime can be formulated by relating the heat flow due to Ohmic heating and conduction to the cold capillary walls: [79]

$$R_M[\mu\text{m}] \approx 1.48 \times 10^5 \frac{\sqrt{r_c[\mu\text{m}]}}{(Zn_i[\text{cm}^{-3}])^{1/4}}, \quad (5.31)$$

where n_i is the gas density before the discharge. From Eq. 5.31, the matched spot size depends on the channel radius and weakly on the initial gas density.

5.4.2 Discharge

Having developed reproducible and uniform ice layers of varying thicknesses, the effect of a discharge pulse on the ice layer was investigated. After the ice had fully deposited and no more growth was observed, helium gas was flowed through the capillary.¹⁷ The added gas caused the capillary temperature to increase: for 20 Torr pressure inside the capillary, a 6 K rise in temperature was measured before steady state was reached. If the capillary temperature was close to the freezing temperature when helium was added, the ice layer would melt. Therefore, the capillary was allowed to cool at least 6 K before adding helium.

A 20 kV pulser generated a large electric field across the electrodes situated on both ends of the capillary. Without an applied electric field, an electron must acquire a minimum amount of energy, called the work function, to escape the surface of the electrode. For most metals, the work function is on the order of a few electronvolts. The induced electric field lowers the potential barrier of the atoms on the electrode surface, and the larger the applied electric field, the more the potential barrier is suppressed, making it easier for electrons to tunnel through. [149] If the helium gas is sufficiently dense, the electrons can collide with and ionize neutral atoms, each of which has a probability γ_{se} of emitting a secondary electron. If each electron can ionize enough atoms and release enough secondary electrons, breakdown occurs, and a discharge pulse propagates through the channel.

The breakdown voltage V_B , or minimum voltage required to make the gas electrically conductive, is dependent on the type of gas, the pressure at the electrodes p , the distance between the electrodes d , and the gas temperature T as given by Paschen's law:

$$V_B = \frac{Bpd/k_B T}{\ln(Apd/k_B T) - \ln[\ln(1 + \frac{1}{\gamma_{se}})]}, \quad (5.32)$$

where γ_{se} is the secondary-electron-emission coefficient, and A and B are constants related to the neutral gas ionization potential and cross-section. The constants A and B can be determined experimentally. [150] Figure 5.26 shows example relationships between the breakdown voltage, pressure and electrode distance for different gases.

There is a limiting value of the combined pressure and electrode distance pd below which breakdown cannot occur, and at both small and large pd values the breakdown

¹⁷ H_2 cannot be used with N_2O because nitrous oxide is an oxidizer.

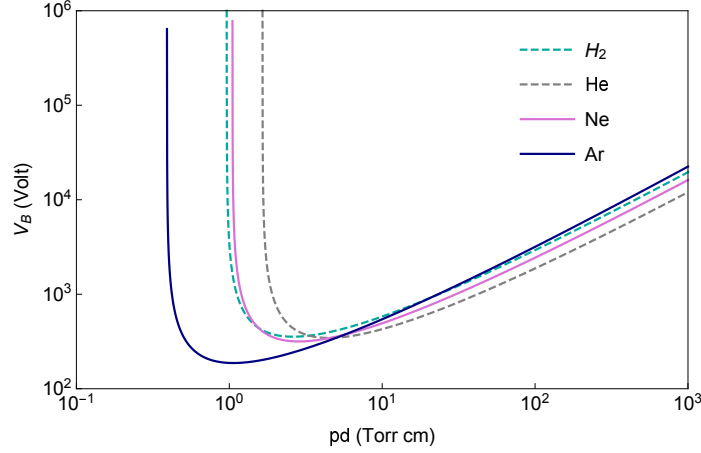


Figure 5.26: Paschen curves for a variety of gases.

voltage is large.¹⁸ When pd is low, either the gas density is low or the electrodes are very close. Even when many secondary electrons are emitted, the probability that they will collide with a neutral atom before reaching the anode is low. As pd increases, the collision probability increases, and the voltage required to initiate breakdown decreases. At large values of pd , electrons undergo many collisions and can't build up enough velocity to ionize neutral atoms. In this regime, a larger voltage is required to accelerate them, and the breakdown voltage increases with increasing pd .

Reducing the temperature of the electrodes decreases the likelihood of breakdown. To prevent the electrodes from getting cold, they were displaced from the ends of the capillary by approximately 0.5 mm. The gas expands as it leaves the channel, decreasing in density by an amount proportional to $1/r^3$ where r is the distance from the channel exit. Therefore, even a small displacement leads to a significantly decreased gas density at the electrodes, and it becomes harder to discharge, manifesting as fluctuations in discharge timing. To reduce these fluctuations, the electrodes should be placed as close to the ends of the capillary as possible while still not touching. The fluctuations can also be improved by increasing the capillary pressure which increases the density at the electrodes. The vacuum pumps, however, placed a 25 Torr upper limit on the capillary pressure. In the following guiding experiments, the pressure was maintained at 20 Torr, and a 500 A discharge pulse, measured by current monitors on

¹⁸Fig. 5.26 cannot be directly compared with experimental values as it does not take into consideration more complicated discharge dynamics or the specific electrode arrangement.

either side of the capillary, ionized the gas. Figure 5.27 shows an example discharge pulse. The best discharge timing fluctuation achieved in a cryogenically-formed capillary was 350 ns. In a 1 mm warm capillary with no gap between the ends of the capillary and the electrodes, the fluctuation in timing was 37 ns.¹⁹

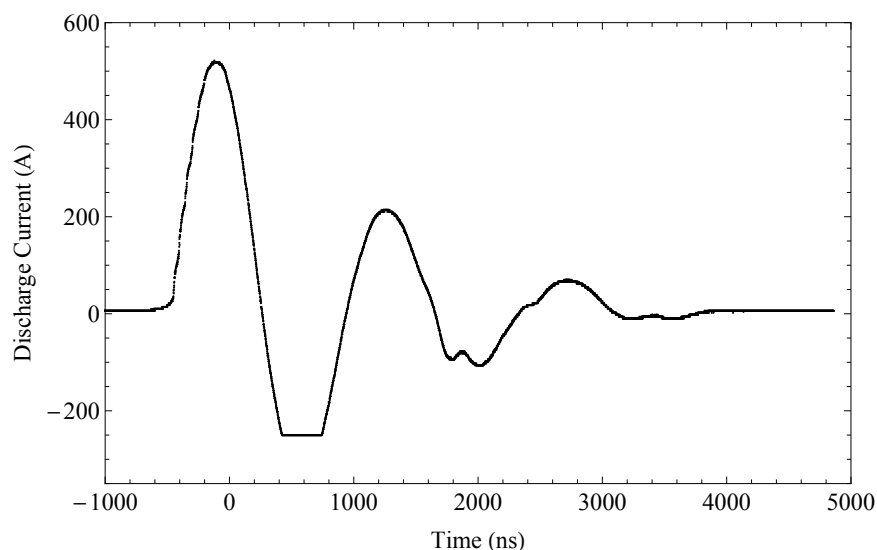


Figure 5.27: Example 500 A discharge pulse.

With gas continuously flowing through the capillary at 20 Torr, the chamber pressure would rise to approximately 10^{-2} Torr. If any metal was placed close to the electrodes, the discharge would short to the metal rather than through the capillary. As such, special care was taken to use non-metallic components such as plastic instead of metal screws. No path was made available from the electrodes to the copper cooling clamps. However, the discharge pulse would arc to the thermocouple and damage the thermocouple reader. To protect the reader, the thermocouple was disconnected before firing the discharge. Thus, the capillary temperature and discharge could not be concurrently measured.

Each discharge shot ablated a fraction of the ice shell. To measure the amount of ice ablated per discharge shot, the thickness after every 100 shots was measured. Figure 5.28 shows the ice thickness as a function of shots for an initial 74 μm and 50 μm thick ice shell. The discharge ablated the ice shell at a rate of 50.6 nm/shot and 46.6 nm/shot, respectively. After 700 shots, approximately 35 μm of ice had ablated.

¹⁹Capillaries with less than 2 ns timing jitter have been achieved with other methods of mounting the electrodes to that ensure the distance between the electrode and the end of the capillary is minimized.

For a helium pressure of 20 Torr and assuming that the helium gas is at 300 K, using Eq. 5.31, this level of ablation corresponds to a $1.8 \mu\text{m}$ increase in matched spot size, a 2% change. Therefore, with a discharge ablation rate of tens of nanometers per shot, the ice shells can last hundreds of shots before a new layer has to be regrown.

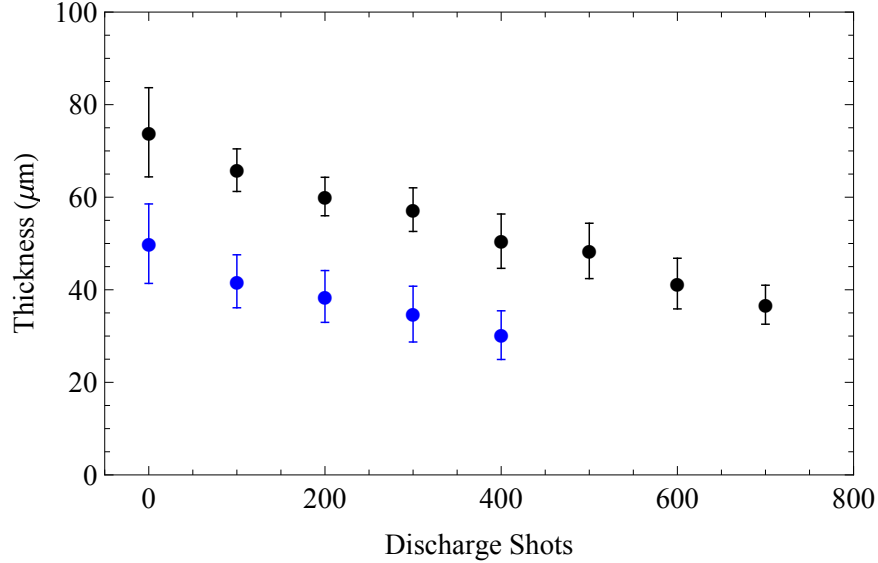


Figure 5.28: Thickness as a function of discharge shots for an initial ice shell $74 \mu\text{m}$ thick (black) and $50 \mu\text{m}$ thick (blue).

Figure 5.29 shows the ice thickness for an initial $80 \mu\text{m}$ ice layer before and after 700 discharge shots. It can be seen that the discharge ablates the ice relatively uniformly along the length of the capillary except near the center gas slot located at position 25-29 mm. As the ice ablated, it flowed up the gas slots and refroze, causing this increase in thickness.

5.4.3 Experimental Set-up

To study the guiding properties of the cryogenically-formed capillary, the capillary and OCT system were moved to a different test chamber with a low-power probe beam picked off from the BELLA laser frontend. [151] The experimental layout is shown in Fig. 5.30. Pulses with 7 nJ/pulse and 800 nm central wavelength were stretched to a pulse duration on the order of 100 ps and then amplified in a regenerative amplifier up to 1 mJ/pulse. At this point, a fraction (~ 37 nJ) of the energy was focused into

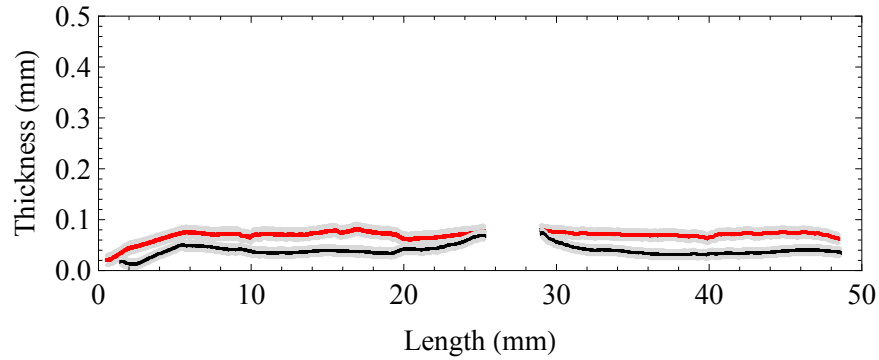


Figure 5.29: Thickness over the length of capillary before (red) and after 700 discharge shots (black). The ice free zone from 25 mm to 29 mm corresponds to the region near the center gas slot.

a 25 m single-mode fiber by a 10x microscope objective. The fiber transported the pulses to an adjacent room with a low-power test bench.

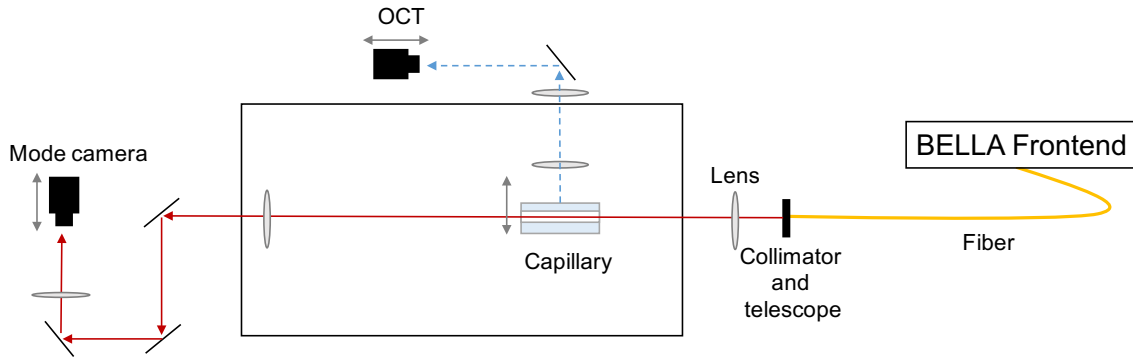


Figure 5.30: Schematic layout of low power laser beamline with capillary and laser diagnostics.

At the end of the fiber, the pulse was split into two beam paths, each with 50% of the total energy. Only one of the beam paths was used in the following guiding experiments. The beam was then collimated and expanded by a Galilean telescope. A 1.68 m focal length lens focused the beam to an $87 \mu\text{m}$ FWHM spot size. Figure 5.31 shows a horizontal lineout of the focused mode across the beam center and the Gaussian fit from which the beam spot size was derived.

The capillary sat on top of a hexapod for six-axis alignment in angle and position to the laser beam line. The capillary was installed such that the laser was focused at the capillary entrance. The OCT system imaged along the length of the capillary

when it was translated 20 mm out of the beam path. Because the laser propagated through the center of the vacuum chamber, 13 in from the nearest window, a one-to-one imaging relay was installed using two 200 mm focal length, 2 inch diameter achromats. The f-number was kept low to collect as much light as possible. At larger f-numbers, the signal reflected off the deposited ice was too weak to measure.

After the capillary, a lens roughly collimated the beam before it left the vacuum chamber and propagated to a CCD camera measuring the beam mode. The camera was installed on a stage which allowed its imaging plane to be translated from the capillary entrance to the capillary exit. The liquid nitrogen was supplied by a 160 L dewar, and the temperature of the sapphire was measured by a thermocouple. A 100 psi Alicat pressure controller regulated the pressure of the nitrous oxide gas flowing into the capillary while a 500 Torr MKS pressure controller regulated the pressure of the helium gas added to initiate discharge. A valve system was installed for further regulating the gas flow as shown in Fig. 5.32. Both N_2O and He flowed into the capillary through the outside gas slots. To evacuate the gas lines, a vacuum pump was connected to the gas line, and a baratron measured the pressure inside the capillary.

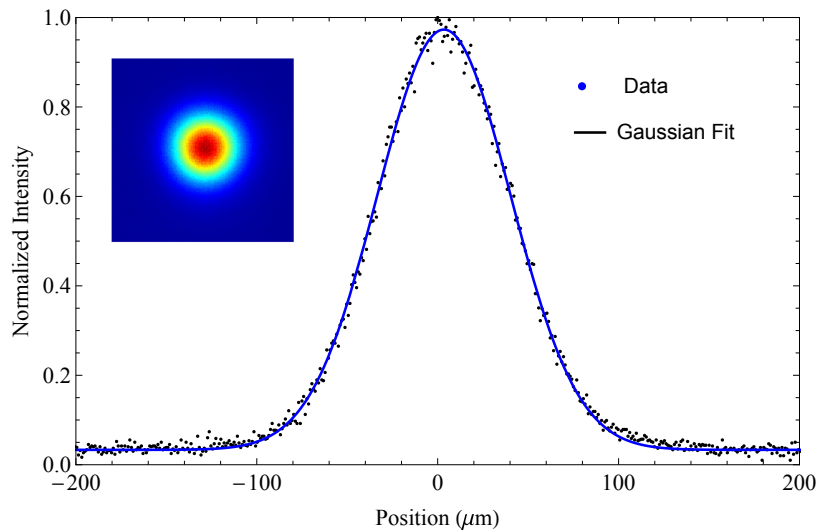


Figure 5.31: Horizontal lineout across the center of the beam at focus (black points) and Gaussian fit (blue line). Inset shows the focused spot.

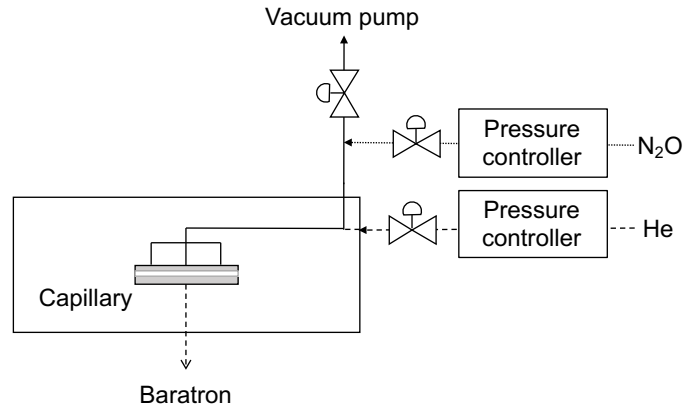


Figure 5.32: Gas distribution system.

5.4.4 Guiding Results

The capillary was roughly aligned by imaging the laser output mode. An ice layer of desired thickness was grown and measured to confirm a uniform ice shell. The probe beam was focused into the entrance of the capillary, and the mode camera positioned to image the capillary exit. 20 Torr of helium gas was continuously flowed through the channel, and the pulser voltage was maintained at 17 kV. Both the laser and discharge fired at a 1 Hz repetition rate.

Figure 5.33 shows the measured output FWHM spot size and peak intensity normalized to the input peak intensity as a function of timing between the arrival of the laser pulse and the peak of the discharge (discharge delay). It can be seen that with a larger capillary radius, there was little change in spot size and intensity over the duration of the discharge delays investigated. As the channel radius decreased, a larger change in output parameters was observed. Figure 5.34 shows representative images for different discharge timings for the varying thicknesses.

Using Eq. 5.30, the matched spot size during the discharge pulse can be calculated from the output spot size and peak intensity. The matched spot size once the density profile has reached steady-state can be calculated from Eq. 5.31 assuming a capillary helium pressure of 20 Torr and gas temperature of 300 K. Figure 5.35 shows the matched spot size as a function of timing in the discharge (points) and once the density profile has reached steady state (lines). Each point is the average of many shots occurring within a 50 ns timing window. When the laser arrived before the start of the discharge pulse, the plasma channel had not yet formed. During the rise

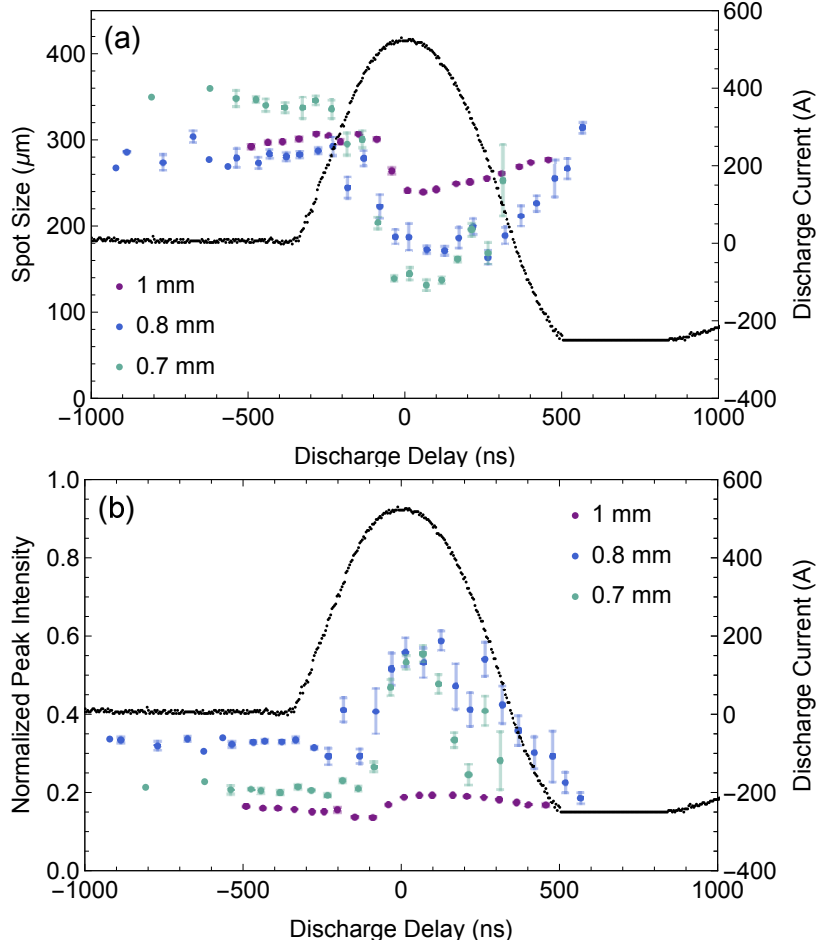


Figure 5.33: Measured beam parameters at the exit of capillary as a function of discharge timing for varying channel diameter (colors): (a) spot size and (b) normalized peak intensity. Discharge pulse shown in black.

time of the discharge, the matched spot size decreases as the channel develops until the peak of the discharge where the largest density drop between the channel wall and center is reached. After this point, the matched spot size increases again. These dynamics can be observed in each of the different channels. The reduction in matched spot size is largest in a small diameter capillary.

The different methods for calculating the matched spot sizes result in different values, especially when the spot size is large. In the channel with a 0.7 mm diameter, the minimum matched spot size as calculated using the output spot size and the normalized peak intensity differ by 12%. However, as the output spot size increases, as in the case of the 1 mm channel, the calculated matched spot sizes differ significantly.

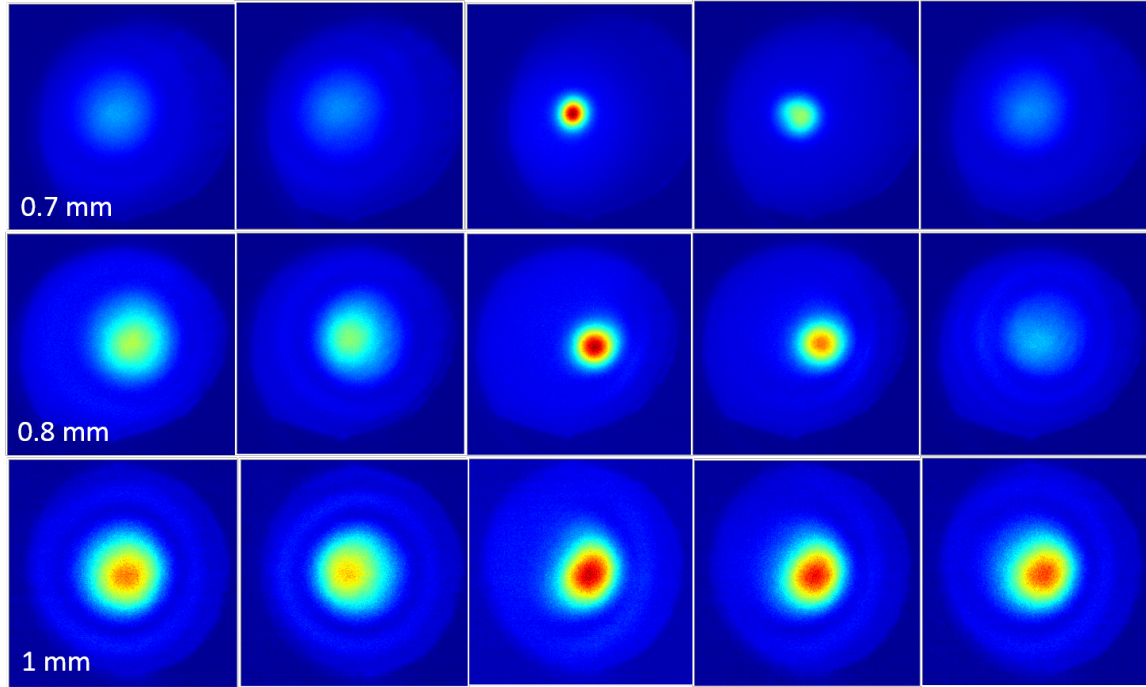


Figure 5.34: Representative images of laser spot at the exit of the capillary in 0.7 mm, 0.8 mm and 1 mm diameter capillaries for approximately (from left to right) -450 ns, -260 ns, 110 ns, 240 ns, 450 ns discharge timings. The lighter blue circle constant in all the images is the capillary opening.

This deviation from the theoretical guiding can be attributed to two effects: a non-Gaussian mode and interactions with the capillary wall. When the beam propagates without the channel, the spot size does not evolve according to Eq. 2.4, indicating that the beam contains higher order modes. These modes interfere and result in a more complicated relationship between the measured output parameters and the matched spot size. [148] In addition, the matched spot size derivation assumes a parabolic density profile which is no longer valid for larger spot sizes. As the spot size increases, the beam can sample the non-parabolic density regions of the channel at larger radii and interact with the walls. These effects contribute to the difference between the calculated matched spot sizes using the output spot size and intensity. Other techniques using the transverse oscillations undergone by the laser when the laser is intentionally offset from the channel axis can also determine the matched spot size and are less sensitive to the higher order mode content of the beam. [148] Work on improving the accuracy of the calculated matched spot size in the cryogenically-

formed waveguide will be done in the future.

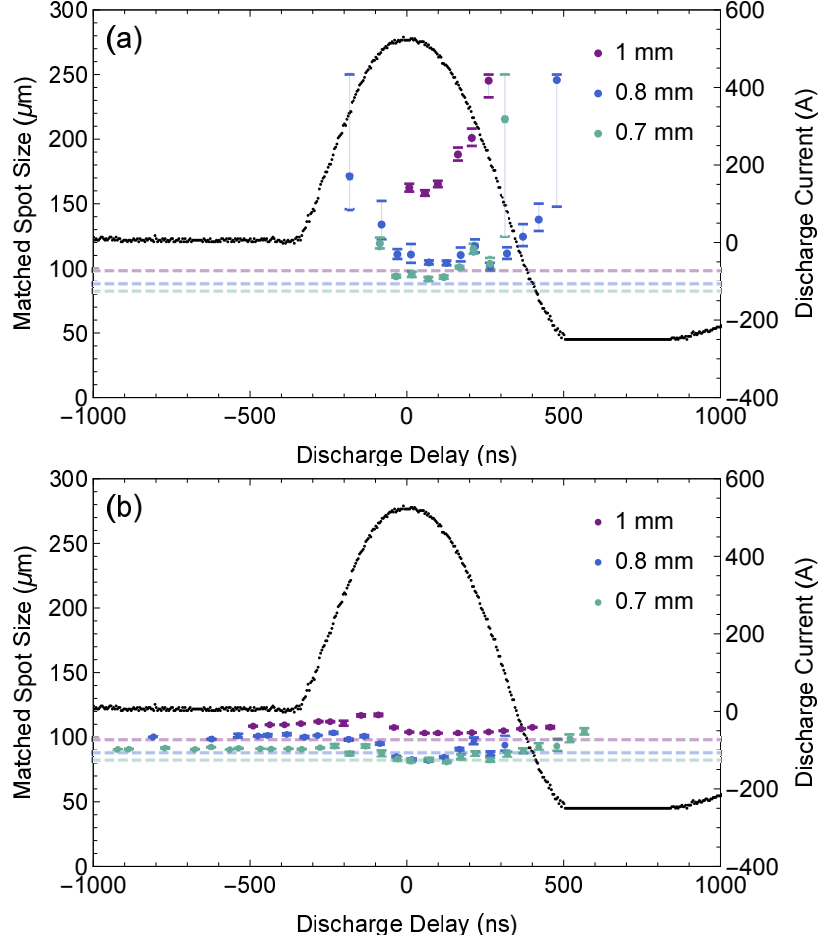


Figure 5.35: Matched spot size calculated from Eq. 5.30 as a function of discharge timing for varying channel diameters (colors) using: (a) the exit spot size and (b) the normalized peak intensity. Discharge pulse shown in black. Matched spot size at steady-state from Eq. 5.31 shown as dashed lines.

5.5 Applications for Laser Plasma Acceleration

5.5.1 Improving Repetition Rate

With a 50 nm/shot ablation rate, the ice shell survived 200 shots before the thickness decreased by 12 μm . Hundreds of shots could be fired before the ice layer had to be regrown. This process consisted of melting the ice layer by flowing room

temperature nitrogen or air through the liquid nitrogen lines to warm up the capillary, and then regrowing a new ice layer. In total, regrowing another ice channel could take several minutes if the capillary temperature was close to the freezing temperature or 20 minutes if the temperature had leveled off around 120 K.

However, the theory described in Sec. 5.1 and preliminary data suggests that the ice layers can be redeposited on top of already frozen ice shells. Instead of thawing the depleted ice shell and regrowing an entirely new one, N_2O can be deposited between shots to recoup the thickness ablated by the discharge and laser. To deposit a layer on top of an existing ice shell requires either better control over the capillary temperature or a high flow rate. Instead of allowing the capillary temperature to decrease over time, as was done in the experiments, finer temperature control could maintain the wall temperature near the freezing temperature (small B). When $T_w \sim T_f$, ice freezes more uniformly, and depositing ice on top of existing ice shells becomes possible. Similarly, a large flow rate leads to slower growth and thus a more uniform ice deposition even when the wall temperature is low (large B). Figure 5.18 demonstrates this technique of freezing successive ice layers. All three layers were frozen when the temperature of the capillary was around 125 K, but the ice froze uniformly as a result of the high flow rate.

If the gas slots are also cold, N_2O will deposit inside the slots and over time can cause a blockage. This problem can be mitigated by either actively warming the slots or, at the very least, separating the gas slots from the main body of the capillary. If these problems are addressed, redeposition may be possible. As shown in Fig. 5.23, for short opening times, the ice grows at a rate of $82 \mu\text{m/s}$. For an ablation rate of approximately 50 nm/shot , it takes 0.6 ms to regrow the amount ablated.²⁰ In addition, during the discharge shot, the temperature of the helium rapidly increases and the plasma expands, flowing into the vacuum chamber and up the gas slots. After the discharge, the helium inside the channel has to be replenished. Simulations of the gas flow after the discharge pulse show that a 33 mm long, 0.5 mm diameter capillary can be refilled with less than 0.2% pressure variation between the capillary center and the gas slots within 0.6 ms. [141] With enough cooling capacity and a wall temperature a few degrees below the freezing temperature, freezing of nitrous oxide and refilling of helium can occur simultaneously without the helium heating the capillary walls and causing the ice to melt. With a gas handling system that operates

²⁰This ablation rate is for a low-energy laser pulse.

fast enough, ice layers can be regrown between each laser shot, and the capillary can operate at a kHz repetition rate. The requirements on the gas system are less stringent with a 1 Hz repetition rate. As the ice shell will last hundreds of shots before the matched spot size changes by a few percent, the ice layer does not have to be replenished every shot. Therefore, by depositing nitrous oxide on top of existing ice shells, the repetition rate of the cryogenically-formed waveguide can be improved over what was demonstrated in these experiments.

Here, waveguide diameters ranging from 950 μm to 600 μm were demonstrated, but the technique for ice deposition can be extended to much smaller diameters. These thicknesses were limited by the ice deposition inside the gas slots, eventually plugging them shut, a problem which can be mitigated by locally heating the gas slots. Smaller diameter ice shells can also be achieved by starting with a smaller diameter capillary. The model presented in Sec. 5.1 depends on the ratio of the solid-gas interface diameter and the capillary diameter. As long as the gas flow conditions still apply, the same ratio achieved in these experiments can be achieved in a smaller diameter capillary, leading to smaller diameter channels.

While only low-power guiding was demonstrated in this thesis, future work will consist of studying the cryogenically-formed waveguide's ability to guide a high-power laser and sustain a wake inside the channel. At higher laser energies, two effects can contribute to a reduced channel life time: direct laser ablation and residual energy in the driven wake. As the parabolic plasma channel develops, the density profile rapidly increases as it approaches the ice shell wall. If the laser is well-guided inside the channel, the density steepness at the wall protects the ice from direct laser ablation. In addition, if the laser drives a wake inside the channel, the wake energy will be transferred to the ice shell, adding heat which can cause the ice to melt. This heat load can be mitigated by extracting the energy with a second, lower-intensity laser pulse propagating behind the first. The second pulse excites a wake inside the channel which destructively interferes with the first and absorbs the excess energy in the wake. [152] However, due to the complexity of this process, the survival of the ice shell with a high-power laser and excited plasma wake will have to be investigated.

5.5.2 Additional Guiding Techniques

Due to the regenerative nature and ability to grow varying thickness ice layers, the applications of the cryogenically-formed waveguide can be extended beyond what has

been demonstrated in this thesis. Unlike in a pre-formed plasma channel generated by a discharge pulse where the laser guiding and particle acceleration properties are both dependent on the plasma density, grazing incidence guiding decouples the two. In this technique, the guiding properties are determined by the waveguide radius. However, to guide by grazing incidence, the radius of the capillary cannot be significantly greater than the spot size of the laser w_0 . To guide the fundamental mode with maximum transmission, the capillary radius r_c must satisfy $w_0 = 0.645r_c$. [62] At these small radii, interaction of the wings with the capillary wall and entrance can create damage. Thus, solid waveguides often have limited lifetimes.

In a cryogenically-formed waveguide, the ice shell thickness can be restored by depositing more ice at the interface. Either the ice layer can be continuously rebuilt as it becomes ablated or the entire depleted ice layer can be melted and a new shell frozen. The regenerative nature of this design enables a longer lifetime waveguide for applications including grazing incidence guiding.

In addition, the radius of the waveguide can be adjusted without machining a whole new capillary by modifying the time over which the ice is deposited. The inner diameter of the ice shell can be made very small, providing flexibility for different experiments. For example, in matched guiding in a pre-formed plasma channel, as given in Eq. 2.35, the matched spot size is dependent on the channel radius and the density drop. The plasma density is often fixed within a certain range to accelerate particles to a desired final energy. Therefore, the ability to tune the channel radius can enable better matching of the guiding channel to the driving laser.

By decoupling the laser guiding properties and particle acceleration, the cryogenically-formed waveguide can be suitable as a near-hollow plasma channel. The density inside the channel is independently controlled from the wall density and can be adjusted to any desired value. Ionization of the wall by either a discharge pulse or a second laser can form a near-hollow plasma channel which potentially can guide a laser pulse and accelerate particles. Unlike previous hollow plasma channel designs, the cryogenically-formed waveguide allows for independent control over the wall and channel densities, and a hollow plasma channel can be achieved. [111]–[113]

5.6 Conclusions

In this chapter, we developed a regenerative, variable-radius, cryogenically-formed discharge capillary. In all the designs detailed, nitrous oxide N_2O gas was flowed through a channel which had been cooled to a temperature below the freezing temperature of N_2O . The nitrous oxide gas froze to the channel walls as it propagated, forming an ice shell. The process of ice formation was studied using an analytical model of gas flow through the channel and heat transfer between the gas and the capillary wall to determine how different parameters of the gas and capillary affect the ice growth. Steady state and transient ice growth along the length of the capillary for varying wall temperatures were obtained.

It was shown that the ice deposition is not, in general, longitudinally uniform. As the gas flows along the channel, its bulk temperature decreases and molecules start to freeze to the wall. The further the gas flows, the lower the bulk temperature and more molecules freeze, leading to thicker ice layers as a function of longitudinal position. The magnitude of non-uniformity is determined by how fast heat is extracted from the gas: the higher the heat transfer, the faster the gas freezes and the larger the difference in thickness between the entrance and exit of the channel.

In the experiments, the wall temperature and flow velocity controlled the rate of heat transfer. As the wall temperature was decreased, the N_2O froze faster, forming thicker and more non-uniform ice layers. The wall should be maintained at a temperature close to the freezing temperature of N_2O to form longitudinally uniform ice shells. An increased flow rate, on the other hand, decreases the heat transfer, and above a critical velocity the channel will never freeze shut. However, as the flow rate is decreased, the ice freezes faster and plugs develop which close shut before the ice has deposited along the whole length of the capillary. A high flow rate should be maintained to ensure longitudinally uniform ice shells.

The model developed to describe the overall trends in the ice growth has several constraints, including the assumption of constant density for both the gas and solid phases and incompressible flow. These assumptions, while explaining several key characteristics of the ice deposition and providing a general understanding of the freezing process, limit the ability to compare the theoretical freezing rates and thicknesses with the experimental data. To better predict the absolute ice thickness, numerical modeling of this problem including density variations should be developed.

Various capillary designs demonstrated both radially and longitudinally uniform

ice growth. The dependence of the ice shell thickness on gas pressure, ice deposition time, wall temperature, and radial temperature distribution were experimentally determined. The radial ice growth was uniform during deposition as long as the temperature around the channel was constant. Several techniques for establishing longitudinal ice deposition were explored, including maintaining the wall at the gas freezing temperature and ensuring a high flow rate. The ice shell thickness was controlled by varying the amount of time the gas flowed through the channel. In this manner, uniform ice shells were deposited with thicknesses varying from 50 μm to 200 μm inside a 1 mm diameter sapphire channel.

Once the ice shell was grown, helium was added into the channel, and a discharge pulse ionized the helium to form a waveguide. Guiding of low-power laser pulses in this channel was demonstrated, and the matched spot sizes as a function of the laser arrival time in the discharge pulse were measured. As the ice thickness increased, the matched spot size decreased, consistent with the guiding expected in a parabolic plasma channel. With the flexibility afforded by this design in both channel density and radius, the cryogenically-formed waveguide can be applicable for not only gas-filled discharge capillary guiding, as was demonstrated, but also grazing incidence guiding and near-hollow or hollow plasma channel acceleration.

Chapter 6

Conclusions

6.1 Summary

Laser plasma accelerators are a promising alternative to conventional radio-frequency cavities because of the high acceleration gradients achievable in a plasma. While acceleration gradients on the order of hundreds of GV/m have been demonstrated, several challenges still exist before an LPA-based high-energy collider can be built. The distance over which a single laser pulse can accelerate particles is limited by the laser energy available: the laser loses energy as it propagates through the plasma until it can no longer sustain the wake. At this point, a fresh laser needs to be coupled in to continue accelerating the beam.

In Chapter 3, we demonstrated staging of two laser plasma accelerators. The first stage was a de Laval nozzle gas jet which injected electrons with 120 MeV mean energy, 60% energy spread, and 33 pC of charge. To counteract diffraction during the drift space, the electron beam was focused into the second LPA, a gas-filled discharge capillary using an active plasma lens. A VHS-tape-based plasma mirror reflected out the first pulse and coupled a second pulse into the capillary to drive a wake in the second stage. By scanning the arrival time of the electron beam relative to the arrival of the second laser, the wakefield structure in the second stage was investigated. When injected into the first accelerating bucket, the beam experienced a 100 MeV energy gain.

However, because of the 60% initial energy spread and the chromaticity of the active plasma lens, only 3.5% of the beam was post-accelerated in the second stage. Such a trapping efficiency makes the beam losses over many stages unacceptable

for a collider. Decreasing the initial energy spread can help mitigate the problem, and Chapter 4 detailed novel techniques for controlling the beam quality of electrons injected along a plasma density downramp. The downramp was produced by placing a razor blade in the gas flow of a de Laval nozzle to form a shock front. The effects of the blade location on the density profile were investigated, and correlations between the density profile and the electron and laser properties were determined. It was observed that the shock front was not always perpendicular to the laser propagation direction. The shock development and evolution as the blade was inserted into the gas flow were simulated, and the formation of an intercepting shock was shown. A tilted shock front induced laser and electron beam steering as well as elliptical electron beam profiles. On-axis laser and electron beam propagation was achieved by setting the shock front perpendicular to the laser propagation axis. The electron beam energy was controlled by the length of the plasma after the downramp and the beam charge by the magnitude of the density drop along the downramp. By tuning the blade location and the gas jet angle, we demonstrated injection and acceleration of beams with a few percent energy spread for a tunable range of energies. Such an injector would enable a larger capture efficiency when used as a first stage in a multi-stage accelerator.

Once beams are injected, they must be accelerated without quality degradation. Effects such as nonlinear forces, energy spread, and Coulomb scattering off background ions can induce emittance degradation as the beam propagates. Hollow plasma channels have been proposed to guide laser pulses and accelerate beams while maintaining low emittance; however, these structures have not been extensively experimentally studied because of the difficulties in creating the required density profile.

In Chapter 5, we described the development of a cryogenically-formed, variable-radius waveguide which can be suitable for hollow plasma channel acceleration. In the designs investigated, nitrous oxide gas was flowed through a capillary whose wall temperature was reduced past the freezing temperature of the gas. The gas froze to the walls of the capillary, forming an ice shell. We explored the different parameters which affected the ice deposition and demonstrated radially and longitudinally uniform growth. The duration of the nitrous oxide flow controlled the ice thickness, and thicknesses from $50\text{ }\mu\text{m}$ to $200\text{ }\mu\text{m}$ were produced in a 1 mm diameter capillary with a growth rate of $82\text{ }\mu\text{m/s}$. Low-power laser guiding was studied by adding helium gas and striking a discharge to produce a parabolic plasma channel. With an ablation rate of approximately 50 nm/shot , the ice shells survived hundreds of discharge shots

before a new shell had to be regrown. Guiding was demonstrated for varying channel diameters and compared with theoretical models.

6.2 Future Work

In this thesis, a proof-of-concept design for a potential hollow plasma channel was developed and low-power guiding was demonstrated. Several shortcomings of this device were identified including capillary lifetime and discharge timing jitter. If better temperature control can be implemented, ice can be deposited on top of already existing layers, improving the repetition rate. Multiple ice layers were deposited in quick succession in the glass capillary, establishing that this technique is feasible. While the current design lasted hundreds of shots before an ice shell had to be regrown, continuous renewal of the ice shell as it is ablated by the discharge allows the capillary to be employed indefinitely. The fluctuations in discharge were attributed to reduced electrode temperature and decreased density as a result of their displacement from the capillary ends. To make the discharge more stable without increasing the capillary pressure, the electrodes can be actively heated. With enough heat, the electrode temperature will not decrease significantly, and the timing jitter can be mitigated.

While low-power guiding was demonstrated, the next steps for this waveguide should also include guiding pulses with higher energies. Investigations into the ablation and repetition rate can provide a better understanding of the limits of this design. As these issues become addressed, the cryogenically-formed, variable-radius waveguide could be suitable for guiding pulses in a variety of regimes and potentially for particle acceleration. While techniques using grazing incidence guiding offer a level of control over the laser guiding and particle acceleration unsupported in parabolic plasma channels, the small channel radii required to match the channel to the laser spot size induce wall and entrance damage. Therefore, these waveguides have a limited lifetime. An ice shell enables longer-lifetime devices as the ice layers can be regrown after ablation. In addition, with in situ control over the channel radius, the matched spot size of the channel can be better matched to the guided laser pulse. In a gas-filled, discharge capillary, as the channel density is decreased, particles can be accelerated to higher energies in a single stage. However, keeping the laser confined in the waveguide becomes more difficult as the matched spot size increases. By decreasing the channel radius, matched guiding for a given laser spot size can be achieved at

lower densities. With no density added on-axis, the cryogenically-formed waveguide can also potentially be suitable for hollow plasma channel acceleration.

The techniques described in this thesis are a step forward in achieving high-quality, high-energy electron and positron beams. As the pace of laser plasma accelerator research continues, an LPA-based high-energy collider becomes more and more likely.

Chapter 7

Appendix I

7.1 Ponderomotive Force

The ponderomotive force is a nonlinear force felt by a charged particle in an oscillating, inhomogeneous electromagnetic field. Consider the electric field $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) \cos(\omega t)$ which oscillates at frequency ω with envelope amplitude $\mathbf{E}(\mathbf{r})$. A particle with charge q and mass m will experience a force caused by this field:

$$m \frac{d\mathbf{v}}{dt} = q [\mathbf{E}(\mathbf{r}) \cos(\omega t) + \mathbf{v} \times \mathbf{B}(\mathbf{r}) \sin(\omega t)],$$

where $\partial \mathbf{B} / \partial t = -\nabla \times \mathbf{E}$. The motion of a particle in the field $\mathbf{r}(t)$ can be split into two time scales: $r = \langle r \rangle + [r]_{os} = r_0 + r_1$ where $\langle \rangle$ represents the equilibrium component, $[]_{os}$ represents the perturbation component. The subscripts 0 and 1 denote the slow and fast varying parts, respectively. Thus, the electric field at the position of the particle can be expanded as $E(\mathbf{r}) = \mathbf{E}(\mathbf{r}_0) + (\mathbf{r}_1 \cdot \nabla) \mathbf{E}(\mathbf{r})|_{\mathbf{r}_0} + \dots = \mathbf{E}_1 + \mathbf{E}_2 + \dots$. Using this expansion, the force equation can be examined order by order. To first order,

$$m \frac{d\mathbf{v}_1}{dt} = q\mathbf{E}_1 = q\mathbf{E}(\mathbf{r}_0) \cos(\omega t). \quad (7.1)$$

Therefore,

$$\mathbf{v}_1 = \frac{q\mathbf{E}(\mathbf{r}_0)}{m\omega} \sin(\omega t), \quad (7.2)$$

$$\mathbf{r}_1 = -\frac{q\mathbf{E}(\mathbf{r}_0)}{m\omega^2} \cos(\omega t). \quad (7.3)$$

The zeroth order solution represents the fundamental oscillation of the particle in the laser's electric field, and it can be seen that the particle will oscillate with the same frequency as the laser.

To second order:

$$\begin{aligned} m \frac{d\mathbf{v}_2}{dt} &= q\mathbf{E}_2 \cos(\omega t) + q\mathbf{v}_1 \times \mathbf{B}_1 \sin(\omega t) \\ &= q\mathbf{E}_2 \cos(\omega t) - \frac{qc}{\omega} \mathbf{v}_1 \times [\nabla \times \mathbf{E}(\mathbf{r}_0)] \sin(\omega t), \end{aligned} \quad (7.4)$$

where $\mathbf{B}_1 = -[\nabla \times \mathbf{E}(\mathbf{r}_0)]/\omega$. Using the relations for $\mathbf{v}_1$ and $\mathbf{r}_1$ found in Eq. 7.2 and Eq. 7.3:

$$\begin{aligned} m \frac{d\mathbf{v}_2}{dt} &= q(\mathbf{r}_1 \cdot \nabla) \mathbf{E}(\mathbf{r}_0) \cos(\omega t) - \frac{q^2}{m\omega^2} \mathbf{E}(\mathbf{r}_0) \times [\nabla \times \mathbf{E}(\mathbf{r}_0)] \sin^2(\omega t) \\ &= -\frac{q^2}{m\omega^2} [\mathbf{E}(\mathbf{r}_0) \cdot \nabla] \mathbf{E}(\mathbf{r}_0) \cos^2(\omega t) - \frac{q^2}{m\omega^2} \mathbf{E}(\mathbf{r}_0) \times [\nabla \times \mathbf{E}(\mathbf{r}_0)] \sin^2(\omega t). \end{aligned} \quad (7.5)$$

Using the cross-product identity $a \times (b \times c) = b(a \cdot c) - c(a \cdot b)$ and averaging over the laser period to get only the slow motion of the particle,

$$F_p = \langle m \frac{d\mathbf{v}_2}{dt} \rangle = -\frac{q^2}{4m\omega^2} \nabla \mathbf{E}^2(\mathbf{r}), \quad (7.6)$$

where F_p is the ponderomotive force of the laser. Defining the normalized vector potential of the laser a_0 :

$$a_0 = \frac{eE}{\omega mc}, \quad (7.7)$$

then

$$F_p = -\frac{mc^2}{4} \nabla a_0^2(\mathbf{r}). \quad (7.8)$$

7.2 Plasma Frequency

The plasma frequency can be derived from the electron equation of motion and continuity equation assuming that there is no magnetic field, no thermal motion, the ions are uniformly distributed and fixed in space, the plasma is infinite, and the electron motion occurs only along one direction. The electron equation of motion is given by: [11]

$$m_e n_e \left[\frac{\partial v_e}{\partial t} + (v_e \cdot \nabla) v_e \right] = -en_e E, \quad (7.9)$$

where n_e is the plasma electron density, v_e is the electron velocity, and E is the electric field caused by a charge separation. Both v_e and E are assumed to be along the x-direction. The continuity equation is given by:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e v_e) = 0. \quad (7.10)$$

The electric field caused by the charge separation can be related to the plasma density through Poisson's equation:

$$\nabla \cdot E = \frac{e(n_i - n_e)}{\epsilon_0}, \quad (7.11)$$

where n_i is the ion density. Assuming that the oscillation amplitude is small, the parameters can be separated into an equilibrium component denoted with a subscript 0 and a perturbation component denoted with a subscript 1: $n_e = n_0 + n_1$, $v_e = v_0 + v_1$, and $E = E_0 + E_1$.

Since the equilibrium component represents the plasma at rest,

$$\nabla n_0 = v_0 = E_0 = 0, \quad (7.12)$$

and $\partial/\partial t = 0$. Therefore, the force equation becomes:

$$m_e \left[\frac{\partial v_1}{\partial t} + (v_1 \cdot \nabla) v_1 \right] = -eE_1. \quad (7.13)$$

To first order, the second term in the brackets can be neglected if the plasma oscillation amplitude is small. Therefore,

$$m_e \frac{\partial v_1}{\partial t} = -eE_1. \quad (7.14)$$

The continuity equation becomes:

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot v_1 = 0, \quad (7.15)$$

and Poisson's equation is given by:

$$\nabla \cdot E_1 = -\frac{en_1}{\epsilon_0}, \quad (7.16)$$

where the plasma at equilibrium is quasi-neutral $n_{i,0} = n_{e,0}$ and the ions are fixed $n_{i,1} = 0$. It will be assumed that the plasma oscillations are sinusoidal, and thus the plasma parameters and fields have a time dependence proportional to $e^{i(kx - \omega t)}$ and spatial dependence only along the x-direction. Therefore, Eq. 7.14-7.16 become:

$$-im_e \omega v_1 = -eE_1, \quad (7.17)$$

$$-i\omega n_1 = -in_0 k v_1, \quad (7.18)$$

$$ikE_1 = -\frac{en_1}{\epsilon_0}. \quad (7.19)$$

Therefore, solving for ω :

$$\omega = \omega_p = \sqrt{\frac{e^2 n_0}{m_e \epsilon_0}}. \quad (7.20)$$

7.3 Hamiltonian Model

Electron injection and acceleration can be analyzed by considering the Hamiltonian H for an electron interacting with a plasma wave:

$$H = \sqrt{1 + u_{\perp}^2 + u_z^2} - \phi(z - v_g t), \quad (7.21)$$

where $u_{\perp} = p_{\perp}/(m_e c)$ and $u_z = p_z/(m_e c)$ are the transverse and longitudinal normalized linear momenta, respectively, and ϕ is the normalized electrostatic potential of the plasma wave such that $E_z = -\partial\phi/\partial\zeta$. [15] The Hamiltonian given in Eq. 7.21 is described by the canonical coordinates $(q, p) = (\zeta, u_z)$. However, the motion of an electron in a plasma wake is often defined in terms of the position behind the laser pulse $\zeta = z - v_g t$. A generating function G can be used to perform a canonical transform from $(z, u_z) \rightarrow (\zeta, u_z)$ such that the form of the canonical equations of motion are preserved. Using the type 2 generating function G which defines the transformation $(q, p) \rightarrow (Q, P)$, the old and new generalized coordinates must satisfy:

$$p = \frac{\partial G}{\partial q}, \quad (7.22)$$

$$Q = \frac{\partial G}{\partial P}, \quad (7.23)$$

$$H' = H + \frac{1}{c} \frac{\partial G}{\partial t}, \quad (7.24)$$

where H' is the new Hamiltonian. Taking the generating function $G = u_z \zeta$, then the new Hamiltonian can be written as:

$$H' = H - \frac{u_z v_g}{c} = \sqrt{1 + u_{\perp}^2 + u_z^2} - \phi(\zeta) - u_z \beta_p, \quad (7.25)$$

where $\beta_p = v_g/c$. The normalized canonical momentum U for an electron is defined as $U = u - a$, where a is the laser field which has a component only in the transverse direction. The time evolution of the canonical coordinates can be determined from the Hamiltonian,

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial \mathbf{q}}, \quad (7.26)$$

$$\frac{d\mathbf{q}}{dt} = \frac{\partial H}{\partial \mathbf{p}}. \quad (7.27)$$

Therefore, the normalized transverse canonical momentum can be written as:

$$\dot{U}_{\perp} = -\frac{\partial H}{\partial r_{\perp}} = 0. \quad (7.28)$$

Therefore, $U_{\perp} = u_{\perp}(\zeta) - a(\zeta) = \text{constant}$. For an electron initially at rest in front of the laser pulse, $u_{\perp}(\zeta) = a(\zeta)$.

Because the H' does not depend on time but only on ζ , energy is conserved and the Hamiltonian is constant along an electron trajectory. For an electron with initial energy H_0 , the longitudinal momentum is given by:

$$u_z(\zeta) = \beta_p \gamma_p^2 (H_0 + \phi) \pm \gamma_p \sqrt{\gamma_p^2 (H_0 + \phi)^2 - \gamma_{\perp}^2}, \quad (7.29)$$

where $\gamma_{\perp} = 1 + u_{\perp}^2$. Therefore, once $\phi(\zeta)$ and $a(\zeta)$ are known, the electron trajectory in (ζ, u_z) space can be determined. For an electron at rest before the arrival of the laser pulse, $u_z = u_{\perp} = 0$ and $H_0 = 1$. These electrons represent the plasma background electrons that are not trapped in the wake.

The separatrix is the trajectory in (ζ, u_z) -space which separates the trapped from the untrapped electrons. These electrons are those which have an initial velocity $u_z = u_p$ at the minimum of the potential $\phi(\zeta_{min}) = \phi_{min} < 0$. Therefore, the Hamiltonian of the separatrix behind the laser pulse is given by:

$$H_{sep} = \frac{1}{\gamma_p} - \phi_{min} \quad (7.30)$$

Electrons will be trapped when the Hamiltonian satisfies $H \leq H_{sep}$ (since $\phi(\zeta) \geq \phi_{min}$).

7.4 Derivation of Energy Conservation

7.4.1 Energy Equation

For a fixed volume V' with surface S , the rate of energy accumulation in V' equals the rate of energy transfer into V' by fluid flowing across the surface (convection) plus the rate of energy transfer into V' by heat and work (including conduction and radiation). The energy per unit volume is given by ρe where ρ is the fluid density and e is the energy per unit mass of the fluid given by:

$$e = u + \frac{1}{2}v^2, \quad (7.31)$$

where u is the internal energy per unit mass and $v^2/2$ is the kinetic energy per unit mass. Equation 7.31 neglects the gravitational potential energy. In an infinitesimal

volume dV' inside volume V' , the amount of energy is equal to $\rho e dV'$. Therefore, the total energy accumulation in V' is given by:

$$E = \int_{V'} \rho e dV', \quad (7.32)$$

and the total rate of energy accumulation becomes:

$$\frac{dE}{dt} = \frac{d}{dt} \int_{V'} \rho e dV' \quad (7.33)$$

To find the amount of energy carried into the volume by convection, consider a differential area dS on the surface of V' . The rate of convective inflow of energy is given by $-\rho e(\mathbf{v} \cdot \mathbf{n})dS$, where $\mathbf{v}$ is the fluid velocity and $\mathbf{n}$ is the unit normal vector to the surface dS . Therefore, the total convective transport of energy into the volume becomes:

$$\text{Rate of convective energy transport into } V' = - \int_S \rho e(\mathbf{v} \cdot \mathbf{n})dS. \quad (7.34)$$

The rate of energy transfer into volume V' by heat and work can be written as the two terms dQ/dt (the total rate of heat transfer) and dW/dt (the total rate of energy expended by the fluid performing work). The transfer of heat into V' through conduction is given by the component of the conductive heat flux $\mathbf{q}$ perpendicular to the surface $-\mathbf{n} \cdot \mathbf{q}$. Therefore, the rate of heat flow by conduction through the surface of the volume can be written as:

$$\text{Rate of conductive energy transport into } V' = - \int_S \mathbf{n} \cdot \mathbf{q} dS. \quad (7.35)$$

Heat can also be added directly to the volume through other mechanisms such as electrical energy, but these terms will be neglected.

The work term is the result of forces acting between material inside the volume and material outside. The force per area (stress) exerted by the material inside V' on material outside V' is given by $-\mathbf{n} \cdot \boldsymbol{\sigma}$ where $\boldsymbol{\sigma}$ is the stress tensor. Therefore, the force exerted by the material in V' over a differential area dS is $-\mathbf{n} \cdot \boldsymbol{\sigma} dS$, and the rate at which work is performed by material is given by $-(\mathbf{n} \cdot \boldsymbol{\sigma}) \cdot \mathbf{v} dS$. Therefore, the total rate of work performed by the material inside volume V' is given by:

$$\text{Rate of work performed by material in } V' = - \int_S \mathbf{n} \cdot \boldsymbol{\sigma} dS \quad (7.36)$$

Combining, Eq. 7.33 with Eq. 7.34- Eq. 7.36, the energy conservation law in integral form becomes:

$$\frac{d}{dt} \int_{V'} \rho e dV' = - \int_S \rho e (\mathbf{v} \cdot \mathbf{n}) dS - \int_S \mathbf{n} \cdot \mathbf{q} dS + \int_S \mathbf{n} \cdot \boldsymbol{\sigma} dS. \quad (7.37)$$

The integral form can be converted to a differential form using Gauss' Divergence Theorem which changes integrals involving a vector $\mathbf{F}$ over surfaces to integrals over volumes:

$$\int_S \mathbf{n} \cdot \mathbf{F} dS = \int_{V'} \nabla \cdot \mathbf{F} dV'. \quad (7.38)$$

Therefore, the differential form of the energy conservation law given in Eq. 7.37 can be written as:

$$\frac{\partial}{\partial t}(\rho e) = -\nabla \cdot (\rho e \mathbf{v}) - \nabla \cdot \mathbf{q} + \nabla \cdot (\boldsymbol{\sigma} \cdot \mathbf{v}). \quad (7.39)$$

The stress tensor can be written as a function of the pressure p and the viscous stress tensor $\boldsymbol{\tau}$. Therefore, the last term in Eq. 7.39 becomes:

$$\nabla \cdot (\boldsymbol{\sigma} \cdot \mathbf{v}) = -\nabla \cdot (p\mathbf{v}) + \nabla \cdot (\boldsymbol{\tau} \cdot \mathbf{v}), \quad (7.40)$$

where the first term on the right-hand side is the pressure work and the second term on the right-hand side of the viscous work. Expanding Eq. 7.40,

$$\nabla \cdot (\boldsymbol{\sigma} \cdot \mathbf{v}) = -p\nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla p + \boldsymbol{\tau} : \nabla \mathbf{v} + \mathbf{v} \cdot (\nabla \cdot \boldsymbol{\tau}). \quad (7.41)$$

The Cauchy momentum equation relates the fluid velocity to the pressure and stress tensor:

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \nabla \cdot \boldsymbol{\tau}, \quad (7.42)$$

neglecting body accelerations such as gravity. Multiplying Eq. 7.42 by $\mathbf{v}$ and rearranging:

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho v^2 \right) + \nabla \cdot \left(\frac{1}{2} \rho \mathbf{v} v^2 \right) = -\mathbf{v} \cdot \nabla p + \mathbf{v} \cdot (\nabla \cdot \boldsymbol{\tau}). \quad (7.43)$$

Subtracting Eq. 7.43 from Eq. 7.39 using the stress tensor given in Eq. 7.40, the internal energy equation can be written as:

$$\frac{\partial}{\partial t}(\rho u) + \nabla \cdot (\rho \mathbf{v} u) = -\nabla \cdot \mathbf{q} - p\nabla \cdot \mathbf{v} + \boldsymbol{\tau} : \nabla \mathbf{v}. \quad (7.44)$$

7.4.2 Enthalpy Equation

The internal energy u can be written in terms of enthalpy h , pressure, and density:

$$u = h - \frac{p}{\rho}. \quad (7.45)$$

Using this definition of enthalpy, Eq. 7.44 can be written as:

$$\frac{\partial}{\partial t}(\rho h) + \nabla \cdot (\rho \mathbf{v} h) = -\nabla \cdot \mathbf{q} + \frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \boldsymbol{\tau} : \nabla \mathbf{v}. \quad (7.46)$$

The right-hand side can be expanded as:

$$\frac{\partial}{\partial t}(\rho h) + \nabla \cdot (\rho \mathbf{v} h) = h \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) \right] + \rho \frac{\partial h}{\partial t} + \rho \mathbf{v} \cdot \nabla h. \quad (7.47)$$

Using the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (7.48)$$

then Eq. 7.47 becomes:

$$\rho \frac{\partial h}{\partial t} + \rho \mathbf{v} \cdot \nabla h = -\nabla \cdot \mathbf{q} + \frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \boldsymbol{\tau} : \nabla \mathbf{v}. \quad (7.49)$$

7.4.3 Temperature Equation

The enthalpy can be related to the temperature T and pressure p using the relation:

$$dh = c_p dT + \frac{1}{\rho} \left[1 + \frac{T}{\rho} \frac{\partial \rho}{\partial T_p} \right] dp. \quad (7.50)$$

Defining

$$\alpha = -\frac{1}{\rho} \frac{\partial \rho}{\partial T_p}, \quad (7.51)$$

then enthalpy can be written as:

$$dh = c_p dT + \frac{1}{\rho} [1 - \alpha T] dp. \quad (7.52)$$

Using this definition and the continuity equation, Eq. 7.49 becomes:

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p \mathbf{v} \cdot \nabla T = -\nabla \cdot \mathbf{q} + \alpha T \left(\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p \right) + \boldsymbol{\tau} : \nabla \mathbf{v}. \quad (7.53)$$

For ideal gases, $\alpha T = 1$ and if the fluid is incompressible then the pressure work term is zero.

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