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"SMALL", "LARGE", AND "VERY LARGE" TRANSVERSE MOMENTA IN A UNIFIED HYDRODYNAMICAL DESCRIPTION

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IN A UNIFIED HYDRODYNAMICAL DESCRIPTION

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ABSTRACT

Two apparently unrelated effects, viz. the behavior of transverse momentum spectra at "large" p_T in p-p collisions and the "enhancement" of such spectra in p-nucleus collisions, are shown to follow in a natural way from a hydrodynamical model in which the space-time evolution of the system is taken into account.

Up to $p_T \sim 5$ GeV/c a single value (close to $u^2 \sim 1/7$) for the velocity of sound in hadronic matter gives a consistent description of all experimental facts. Recent observations at very large p_T (5-15 GeV/c) require a jump to $u^2 \sim 1/4$, suggesting the possibility of a phase transition of the second kind.

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Strong interaction physics can hardly be understood without an adequate explanation for the specific features of the transverse momentum distribution $f(p_T)$[1] of secondaries produced in high-energy hadronic collisions, viz.:

i) its exponential shape below $p_T \simeq 1$ GeV/c with a slope of ~ 6 $(\text{GeV/c})^{-1}$ which is independent of the cms energy $\sqrt{s}$ of the reaction

ii) a striking deviation from exponentiality beyond 1 GeV/c; local logarithmic slopes show a significant increase with $\sqrt{s}$

iii) resumption of exponential behavior beyond $p_T \simeq 5$ GeV/c with an (almost energy independent) slope of ~ 1.3 $(\text{GeV/c})^{-1}$.[2]

iv) a significant target dependence of $f(p_T)$ beyond ~ 1 GeV/c in p-nucleus collisions.

Feature i) could be explained so far only by thermodynamical[3-5] and hydrodynamical[6] models. For the more recently observed ii) and iii), various explanations have been suggested, which fall into two main classes, viz.:

a) statistical[7-8] and hydrodynamical[9] models

b) constituent models[10]

As to iv) it has been interpreted in terms of either b) (above)[11] or

c) models based on coherence effects,[12-13] or

d) multiple nucleon scattering processes.[14]

There has been, so far, no successful attempt to find a unique mechanism responsible for i) through iv), and the opinion dominates that these effects reflect different phenomena. Moreover, most of the fits obtained in the abovementioned theoretical papers are far from satisfactory although they apply only to either limited ranges of p_T

or to particular aspects of $f(p_T)$.

It is the purpose of this paper to show that a hydrodynamical model[15] (which also explains a variety of effects like rapidity distributions and energy dependence of multiplicities in p-p collisions[16-18], rapidity distributions and A dependence of multiplicities in p-nucleus collisions[19-21]) can account for all experimental facts, (i)-(iv), over the whole range of p_T.

The hydrodynamical model (h.m.) of Landau contains as an essential ingredient Pomeranchuk's observation[3] that in hadron-hadron collisions the system is initially at such a high pressure that the mean free path of the created particles is much smaller than the dimensions of the system; thus no emission of particles can take place before the system has expanded and hence cooled down to a "decay" temperature $T_c \sim m_\pi$. This explains why the bulk of the particles have limited transverse momenta ($\langle p_T \rangle \sim 0.3$ GeV/c).

It is clear, however, that emission at $T > T_c$ cannot be absolutely forbidden and this must lead to leakage of particles from the excited system before expansion has ended. This idea, which has been familiar to those working in this field for a long time, was stated explicitely by Gorenstein et al.[9] and used in an attempt to explain the behavior of f at large p_T. However, because of the approximations used, the formula for f derived in ref. 9 applies only to large p_T and therefore it was not clear at all whether the h.m. can indeed predict $f(p_T)$ over the whole accessible range of p_T. Furthermore, proton-nucleus collisions were ignored in ref. 9, too.

The approach considered in ref. 9 is a special case of what

Safari and Squires[22] define as "multi-temperature" distributions; they showed that this kind of single-particle distribution leads to constraints in the two-particle distribution which are in agreement with experimental facts.

Now, an approach to $f(p_T)$ based on the h.m., if successful, would have the heuristic advantage of not being an ad hoc model invented just in order to explain the particular class of effects connected with $f(p_T)$, since, as already mentioned, it has already been shown able to explain a variety of characteristics of strong interactions.

We are interested in the probability of particle emission at different temperatures T, hence at different times t. We shall use the one-dimensional solution[23] of the Khalatnikov equation for the relativistic hydrodynamical potential χ in order to derive T(t) and thus to describe the evolution of the system.[24]

The expression which follows is valid both for p-p and p-nucleus collisions. In the latter case, the incident proton is assumed to collide with a nuclear tunnel of length l which, in turn, depends on the impact parameter b.

T(t) is given implicitly by

$$t(T)_{y=0} = \frac{\ell+d}{4uw}\left\{\int_0^{\tau} e^{-wt}\, I_o\,[(w-1)t]\; dt + e^{-w\tau}\, I_o\,[(w-1)\tau]\right\} + \frac{\ell-d}{2u^2w}\left(1 - e^{-\tau}\right) \tag{1}$$

where d is the proton diameter, I_o is the modified Bessel function

$$w \equiv \frac{1+u^2}{2u^2},\ \tau \equiv \ln(T/T_o)\,, \tag{2}$$

u is the velocity of sound and T_o is the initial temperature given (in units of m_π) by

$$T_o = \left(\frac{\varepsilon_o}{\lambda}\right)^{\frac{1}{2w}} \quad ; \tag{3}$$

ε_o is the energy density,

$$\varepsilon_o = (E/V_o) = \frac{E^2 w m_\pi{}^3}{\pi M_p} \quad ; \tag{4}$$

E is the total available energy in the system in which target and projectile have equal and opposite velocities, V_o the normalization volume, m_π and M_p are the pion and proton rest masses, respectively; λ is a function of u evaluated by Cooper et al.[26] for an interacting Bose gas. Values for $T_c(u)$ have also been taken from this reference and approximated by a smooth function.

Numerical evaluation of eq. (1) shows that in a very good approximation the temperature is a decreasing power function of time

$$T \sim t^{-\beta} \tag{5}$$

where β is close to 1/7 and is a weak function of ℓ/d.

Strictly speaking, eqs. (1) and hence (5) are valid only for

$$\ell \leqslant \ell_c \equiv d\,\frac{1 + u}{1 - u} \tag{6}$$

For $\ell > \ell_c$ the solution is much more involved.[20,21] In the present paper we limit ourselves to this simpler case and will use for proton-nucleus collisions the solution valid for $\ell \leqslant \ell_c$ for $\ell > \ell_c$, as well. As will seen below, the fits obtained justify this approximation.[27]

The invariant cross section $f(p_T)$ reads

$$f(p_T)_{y=0} \sim \frac{1}{p_T} \int_0^{t_c} dt \int_{m_\pi}^{\infty} dm \int_0^{R} b^2 db\, F\,(t)\ \phi_{BE}\,(p_T, T(t), m) \tag{7}$$

where

$$\phi_{BE}\,(p_T,\, T,\, m) \equiv \frac{p_T{}^2}{e^{\frac{\sqrt{p_T{}^2+m^2}}{T}} - 1} \tag{8}$$

is the Bose-Einstein distribution[28] and t_c is the "moment of decay" defined by

$$T(t_c) = T_c; \tag{9}$$

R is the target radius, and m the mass of the secondary.[29] F(t) is the decay probability per time interval, i.e. a function which describes the time evolution of the leakage process. The simplest assumption about F, used hereafter, is that F is a constant; this implies equal emission probabilities in equal time intervals.

The integration over the impact parameter b is evaluated as follows: For the p-p case the only dependence on b is contained in T_o via the available energy E (eq. (3); indeed

$$E = K(b)\sqrt{s} \tag{10}$$

where K is the inelasticity of the collision. Since K is known from experiment to be approximately uniformly distributed between 0 and 1, integration over b is equivalent to integration over K which we approximate by fixing the integrand at the mean value of $K^{1/w}$

$$\langle K^{\frac{1}{w}} \rangle = \frac{1 + u^2}{1 + 3u^2}\ . \tag{11}$$

For proton-nucleus collisions the h.m. assumes K = 1 in the tunnel (and 0 outside). Now b comes in via ℓ, and we approximate integration over b by fixing the integrand at $\ell = \ell(\langle b \rangle)$.[31]

We have applied the results of the model discussed above to the analysis of p-p collisions at the CERN ISR[32-33,2] and p-nucleus collisions at FNAL.[34] Besides normalization the only quantity to be fitted is u. The results of the fits for the p_T-range 0-5 GeV/c are shown in figs. 1-3. Fig. 4 shows a complete picture of the pion p_T-spectrum at $\sqrt{s}$ = 53 GeV (ISR) from 0 to 15 GeV/c with the newest data[2] included.

Our results can be summarized as follows:

1) From $p_T \sim 0.1$ up to $p_T \sim 5$ GeV/c the data for both p-p and p-A collisions (in the energy range covered by FNAL and ISR experiments) can be well fitted by our model with a value of u in the narrow range $(1/\sqrt{6.4} - 1/\sqrt{6.8}$; this range is compatible with values obtained for u from the h.m. when analyzing rapidity distributions in p-p[18] and p-nucleus[21] collisions.[35]

2) The fits are rather sensitive to small (~ 5%) variations in u^2.[36]

3) While in most other models, "new physics" are invoked to explain the departure from a simple exponential in p_T beyond ~ 1 GeV/c in the hydrodynamical approach the "large p_T" region (1-5 GeV/c) appears as a smooth and natural continuation of the "low p_T" region.

4) Beyond, say 4-5 GeV/c our eq. (7) turns for all practical purposes into an exponential

$$f(p_T) \sim e^{-\frac{p_T}{T_o}} \tag{12}$$

As can be seen from fig. 4, in the "very large p_T" region (5-15 GeV/c) the (most recent) data deviate strongly from this asymptotic form. However, they are remarkably well fitted by an exponential ($\chi^2 \sim 4$ with 11 degrees of freedom) with a higher initial temperature T_o ($\sim 5m_\pi$ instead of $\sim 2\ m_\pi$). Such a high T_o can be understood in our model if u jumps from a value of $\sim 1/\sqrt{7}$ to $\sim 1/\sqrt{4}$.

It is gratifying to observe that the h.m. with only one free parameter, viz. u (which, however, is already pinned down to within a few percent of our fitted value by independent experimental facts) gives such a consistent description of the p_T spectra over 9 orders of magnitude in cross section.[37]

This situation should be compared, e.g to fits to parton model predictions used in ref. 32; in spite of the large number of parameters fitted and the limited range in p_T covered these fits yielded in no way a better consistency.

Incidentally, one notices that the large value of u^2 ($\sim 1/4$) required to explain the spectra in the "very large p_T" region (5-15 GeV/c) is consistent with the energy dependence of total multiplicities in the same (p-p) reactions. This might not be surprising since both phenomena are determined essentially by the initial value T_o of the temperature. It is conceivable and even predicted by theoretical arguments[38] that u depends on temperature and might even undergo a jump as a consequence of a phase transition of second kind. Indeed[38] a sudden change is predicted from a lower value

$$u^2 = 1/3 - \delta \tag{13}$$

to the ideal Bose gas value of $u^2 = 1/3$. The gap parameter δ is determined by the coupling and the characteristics of the symmetry group.

Thus, in the approach suggested here, "new physics" appear beyond 5 GeV/c and not earlier as in other models. Obviously it cannot be excluded that details of nucleon structure (partons?) are responsible for the change in behavior of p_T-spectra in the "very large p_T" region, and for the jump to the ideal gas value of 1/3. Parton effects have also been invoked by Eilam and Zarmi[13] in order to explain discrepancies between $f(p_T)$ in p-nucleus collisions and the predictions of the coherent tube model beyond ~ 4 GeV/c.

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24. Most of the data are available at $y = 0$ and this is why we restrict our discussion to that case. Predictions for $y \neq 0$ can be worked out along the same lines since the solution of the hydrodynamical equations is known for an arbitrary y.

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28. We restrict the discussion to boson secondaries.

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36. It seems premature to ascribe any significance to the small (if systematic) increase of the fitted u values with A in view both of experimental uncertainties and of the approximations made in the present calculations.

37. The decrease with p_T of the logarithmic slope (fig. 2) with increasing A, or conversely the increase of α if the parameterization $f(p_T) \sim A^{\alpha(p_T)}$ is used, emerges in a natural way in the hydrodynamical context.

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FIGURE CAPTIONS

Fig. 1 Three-dimensional plot of $\log_{10}[f(p_T)]$ for pp(ISR) collisions at four energies. Here and in the following figures the invariant cross section is in $mb/(GeV/c)^3$; characters related to the target (*B* = Be, *T* = Ti) identify experimental points; their size is not related to the value of experimental errors. Thin curves delimit the 0.5% confidence interval on u^2.

Fig. 2 Same as fig. 1 for a constant energy ($\sqrt{s}$ = 23 GeV) and four different target nuclei (target scale is $\log_{10} A$).

Fig. 3 Same as fig. 1 for a W target at three energies.

Fig. 4 $f(p_T)$ in pp (ISR) collisions at 53 GeV covering the whole range of p_T (0-15 GeV/c). The points beyond 5 GeV (π^0 from the most recent experiment[2]) are independently fitted by an exponential.

TABLE 1. Fitted values of $1/u^2$ and figures of merit FOM = $(\chi^2-N)/\sqrt{2N}$ where N is the number of degrees of freedom. FOM should be asymptotically normal (0,1).

Target	$\sqrt{s}$	$(1/u^2)$fitted	FOM
H (ISR	23	6.80	5.8
H (ISR)	31	6.68	7.5
H (ISR)	45	6.68	4.7
H (ISR)	53	6.61	2.2
Be	23	6.90	22.4
Ti	23	6.68	9.4
W	18	6.80	6.1
W	23	6.48	3.5
W	27	6.44	4.4

HYDROGEN

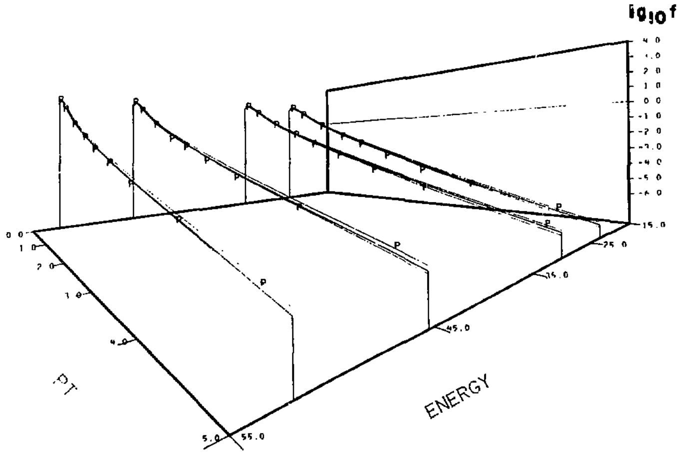

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Fig. 1

P-NUCLEUS

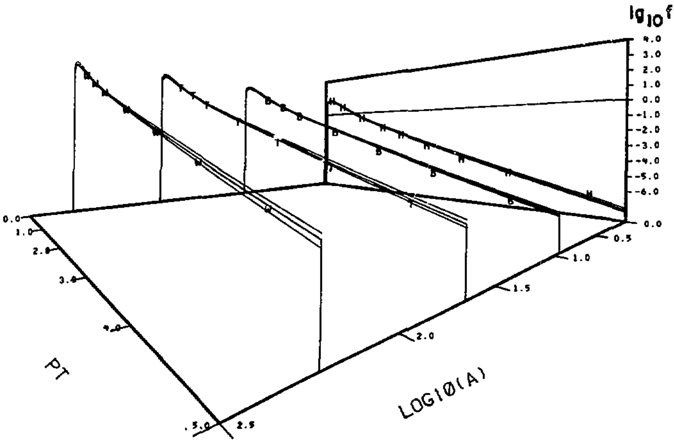

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Fig. 2

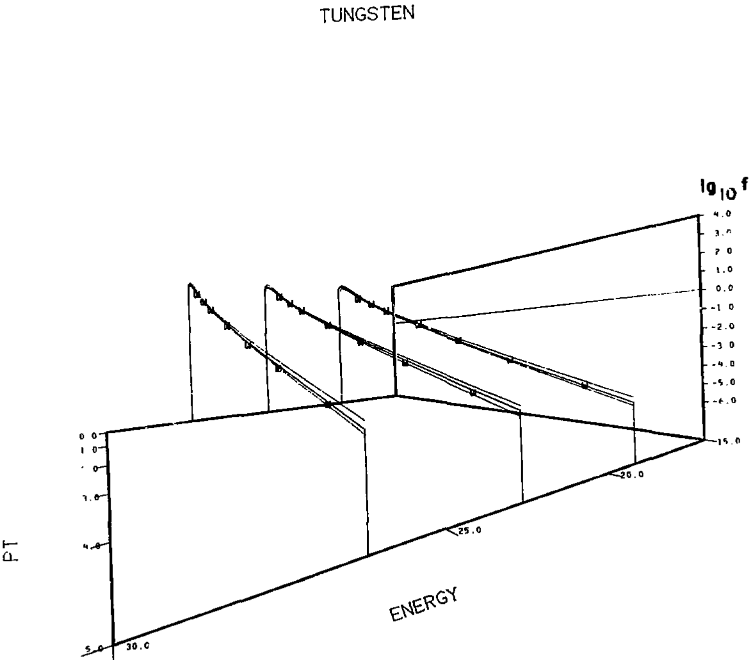

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Fig. 3

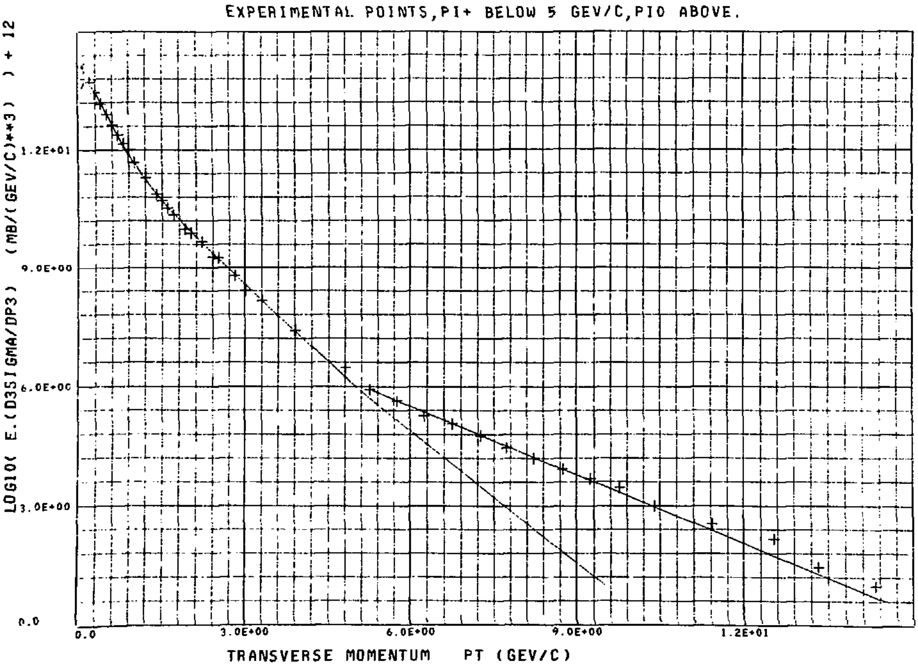

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Fig. 4