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Effect of Nozzle Geometry on the Performance of Pulsed-Jet Propulsion at Low Reynolds Number

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Abstract

A pulsed-jet propulsive system which includes a deformable body and a nozzle affixed to it is proposed and numerically examined. The impact of nozzle geometry on the performance of the system during a single discharge at low Reynolds number is studied. A force decomposition algorithm is employed to decouple the thrust into three parts: normal stress at exit, jet flux, and time derivative of internal fluid momentum. This approach enables the investigation of underlying physics in thrust generation. Since its jet flow features negative radial velocity, the system with converging nozzles is found to produce over-pressure (dominant component of the exit normal stress) on the nozzle exit, making positive contributions to thrust. The diverging nozzles, with reversed radial velocity, tend to induce negative pressure on the exit, which diminishes the thrust generation. On the other hand, the contribution from exit pressure only accounts for a relatively small portion of the thrust, which is found to be dominated by the jet flux momentum. As a result, with fixed stroke ratio and time history of deformation the overall thrust generated by the system is determined mostly by the exit size of the nozzle D' . Specifically, the time-averaged thrust $\bar{F}_t$ is found to be proportional to $1/D'^4$. The geometry of the nozzle also affects the efficiency. For example, the system without a nozzle turns out to be most efficient, followed by one with a short and divergent nozzle.

Keywords: Squid, thrust generation, nozzle geometry, jet propulsion

1. Introduction

In contrast to the underwater locomotion mechanisms based on body undulation and/or flapping of appendages utilized by fish and other aquatic animals, species of invertebrates often employ an alternative, and equally effective, swimming technique using pulsed jetting enabled through cyclic body deformations. Examples of jet-propelled swimmers include jellyfish, cephalopods (octopus, squids, cuttlefish), and salps [1, 2, 3, 4].

Among these creatures, squids stand out owing to their impressive bursting speed [1, 2]. In the high-speed regime squid locomotion is powered by intermittent jets as water is discharged from its mantle cavity, which serves as a pressure chamber. Although a squid is also equipped with a pair of fins capable of generating propulsion force via undulating and flapping motions, their performance in high speed is severely compromised due to the lack of structural stiffness. Instead, high-speed swimming is achieved almost entirely through pulsed jetting enabled by a propulsion system including a mantle cavity, a funnel tube, and a musculature including two groups of muscles, radial and circular [5]. It has been pointed out that this locomotion mode significantly reduces the hydrodynamic load on any parts of the body, especially the bending load, so that it is particularly suitable for soft-bodied creatures such as marine invertebrates or soft-bodied swimming robots that are being developed [6, 7, 8].

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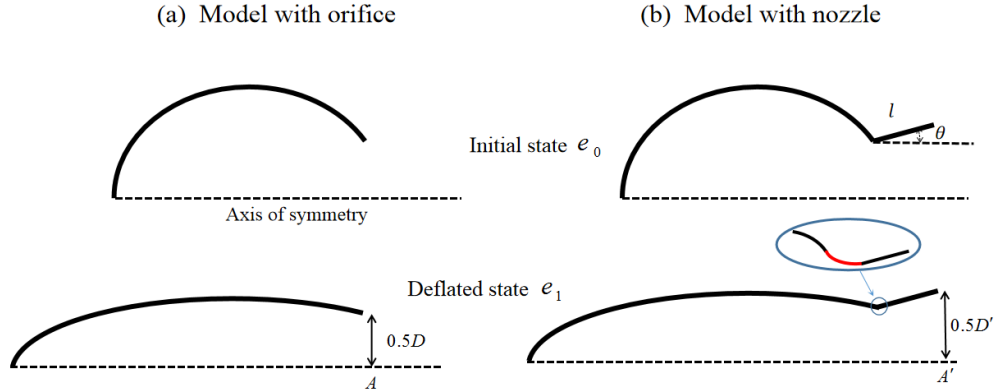


Figure 1. Rendition of the axisymmetric squid-inspired system with orifice(a) and nozzle tube(b).

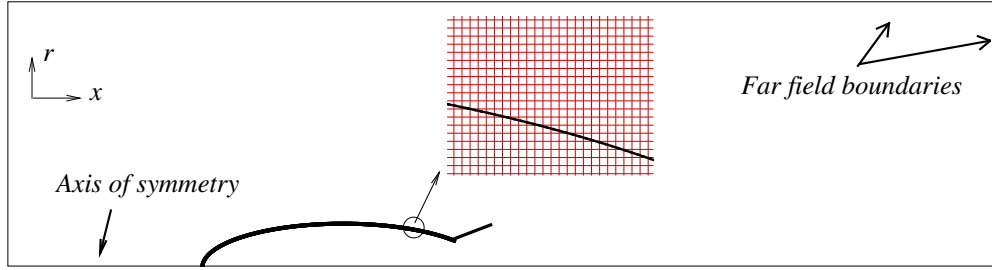


Figure 2. Computational domain ($24D \times 6D$) and fluidic mesh.

A key component of a squid's jet propulsion system is the funnel tube, which functions as a nozzle from which the jet flow is formed. Other species, such as some species of jellyfish, have similar morphological features formed by velum (which essentially forms a converging nozzle) so that they are also capable of generating high-speed jets for propulsion. A squid's funnel tube can be pointed at a range of directions so that maneuvers can be achieved through thrust vectoring. The exact shape and size of the nozzle are also important factors in determining the performance. It is thus of interest to examine effects of the design of a nozzle on the propulsion performance through pulsed jetting.

The dynamics of pulsed-jet propulsion utilized in squid swimming have been investigated through laboratory observations and measurements of live animals [9, 10, 11], experiments on bio-inspired jetting tubes or robotic devices [12, 13, 6, 7, 8], and numerical studies [14, 15, 16, 17, 18, 19, 20]. However, few of these studies examine the effect of detailed geometry of the nozzles on the dynamics of the system. One exception is the experiment conducted by Krieg & Mohseni [21]. By measuring the thrust generation of a piston jetting device with either tubular or conical nozzles, they concluded that conical tubes outperform tubular ones in terms of force generation.

The purpose of this study is to explore the possibility of performance enhancement of bio-inspired pulsed-jetting systems by using nozzles with various geometries. Towards this end, we conduct a numerical investigation concentrating on the force generation capacity of a pulsed-jet system including a pressure chamber enclosed within a deformable shell and a nozzle. The configuration of our system is inspired by a recently reported squid-like robot [8]. Our numerical model is based on the immersed-boundary framework, in which the axisymmetric Navier-Stokes equations are solved using a cartesian computational grid and the fluid-structure coupling is enabled through a penalty algorithm. This method is applied to systematically simulate the dynamics of the system with different nozzle characteristics (e.g. length and angle).

The rest of the paper is organized as following. We first describe the physical problem, including the geometry and kinematics of the swimmer with deformable body. This is followed by formulations of mathematical equations and numerical algorithm. The numerical results are then presented. Finally, conclusions are drawn.

2. Problem statement

Figure 1(a) displays the geometry of the axisymmetric cephalopod-inspired propulsive system [22]. The model consists of a pressure chamber enclosed within an outer shell, which is represented as part of an ellipsoid with an open end (the orifice). The orifice diameter is D , accounting for 10% of the contour length of the entire shell L . By deforming the ellipsoidal shell (achieved by increasing the eccentricity from $e_0 = 0.8$ to $e_1 = 0.95$), the fluid inside the chamber will be ejected through the orifice and generate thrust force. The system is tethered at the exit plane during the deflation process. There is no incoming flow.

Let V_j denote the spatially averaged horizontal flow speed at the exit, due to conservation of mass we have

$$V_j(t) = -\frac{4}{\pi D^2} \frac{d\forall(e)}{dt}, \quad (1)$$

where $\forall(e)$ is the internal body volume depending on eccentricity e . Herein we are able to determine the time history of eccentricity $e(t)$ once the V_j profile is prescribed. In the current study, we focus on a single deflation with sinusoidal jet velocity profile expressed by

$$V_j(t) = \frac{1}{2} \tilde{V}_j \left[1 - \cos \left(\frac{2\pi t}{T} \right) \right], \quad (2)$$

where $\tilde{V}_j$ is the upper-bound jet speed and T is the time duration of the body deformation so that

$$T = \frac{\forall(e_0) - \forall(e_1)}{\frac{1}{2} \tilde{V}_j \frac{\pi D^2}{4}}. \quad (3)$$

As noted in the introduction, this work aims at studying the effects of nozzle configuration upon the propulsion capability of the aforementioned pulsed-jet propeller. Towards this end we attach an axisymmetric nozzle to the ellipsoidal body at the orifice (Fig. 1(b)). To minimize energy loss the nozzle and the body are joined smoothly as demonstrated in the inset. The nozzle configuration is characterized by two independent parameters, i.e. the contour length l and angle of inclination θ . Hereby a variety of nozzles with different l and θ will be tested while the body geometry (including the size of the orifice) and deformation remain unchanged. An important dimensionless parameter, the Reynolds number Re_j , is defined as $Re_j \equiv \rho \tilde{V}_j D / \mu$, where ρ is the fluid density and μ represents the dynamic viscosity. Herein, we choose $Re_j = 50$ for all simulations in the current study. The stroke ratio, defined as $4 \frac{\forall(e_0) - \forall(e_1)}{\pi D^3}$, is 10.4.

3. Mathematical formulations and numerical approach

3.1. Governing equations

The fluid dynamics is governed by the incompressible axisymmetric Navier-Stokes equations. Within the immersed-boundary framework, an additional forcing term is incorporated into the governing equations to enforces the boundary conditions associated with the solid body and its deformation.

Let $\mathbf{x} \equiv (x, r)$ be a cylindrical frame with x in the direction of the jet and r the radial direction, $\mathbf{u}(\mathbf{x}, t) \equiv (u_x, u_r)$ corresponds to velocity components in each direction, $p(\mathbf{x}, t)$ is the pressure distribution, and $\mathbf{f}(\mathbf{x}, t) \equiv (f_x, f_r)$ stands for the force density exerted by the immersed boundary Γ_{ib} on the surrounding flow. In the immersed-boundary approach the Navier-Stokes equations in the fluid domain Ω_f and the continuity equation are

$$f_x = \frac{\partial u_x}{\partial t} + \mathbf{u} \cdot \nabla u_x + \frac{\partial p}{\partial x} - \frac{1}{Re_j} \left[\frac{\partial^2 u_x}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_x}{\partial r} \right) \right] \quad (4)$$

$$f_r = \frac{\partial u_r}{\partial t} + \mathbf{u} \cdot \nabla u_r + \frac{\partial p}{\partial r} - \frac{1}{Re_j} \left[\frac{\partial^2 u_r}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) - \frac{u_r}{r^2} \right] \quad (5)$$

$$0 = \frac{\partial u_x}{\partial x} + \frac{1}{r} \frac{\partial (r u_r)}{\partial r}. \quad (6)$$

The corresponding boundary conditions are

- Axis of symmetry: $\frac{\partial u_x}{\partial \mathbf{n}} = 0$, $u_r = 0$, $\frac{\partial p}{\partial \mathbf{n}} = 0$;
- Far field: $\frac{\partial \mathbf{u}}{\partial \mathbf{n}} = 0$, $\frac{\partial p}{\partial \mathbf{n}} = 0$,

where $\mathbf{n}$ is the outward normal unit vector of the domain boundaries.

The body kinematics are defined in an one-dimensional Lagrangian coordinate s , physically representing the arc-length along the body measured from one end. Lagrangian variables $\mathbf{X}(s, t)$ and $\mathbf{V}(s, t)$ respectively depict the instantaneous position and velocity of the body (the immersed boundary), which are both prescribed in §2. Other Lagrangian variables include $\mathbf{F}(s, t)$ and $\mathbf{U}(s, t)$, which are the force per unit length applied by the body to the surrounding flow and the interpolated flow velocity in the Lagrangian frame, respectively. The interaction algorithm, which enables communication between the Eulerian fluidic grid and the Lagrangian structural grid, is formulated as the following integral equations

$$\mathbf{f}(\mathbf{x}, t) = \int_{\Gamma_{ib}} \mathbf{F}(s, t) \delta(\mathbf{X}(s, t) - \mathbf{x}) ds, \quad (7)$$

$$\mathbf{U}(s, t) = \int_{\Omega_f} \mathbf{u}(\mathbf{x}, t) \delta(\mathbf{x} - \mathbf{X}(s, t)) d\mathbf{x}, \quad (8)$$

where δ is the Dirac Delta function. For numerical purpose, a discretized and regularized delta function δ_h is employed so that

$$\delta_h(\mathbf{x}) = h^{-2} \phi\left(\frac{x}{h}\right) \phi\left(\frac{r}{h}\right), \quad (9)$$

where h is the Eulerian grid size and $\phi(a)$ is defined as

$$\phi(a) = \begin{cases} \frac{1}{8}(3 - 2|a| + \sqrt{1 + 4|a| - 4a^2}) & 0 \leq |a| < 1 \\ \frac{1}{8}(5 - 2|a| + \sqrt{-7 + 12|a| - 4a^2}) & 1 \leq |a| < 2 \\ 0 & 2 \leq |a| \end{cases} \quad (10)$$

We apply a penalty algorithm to calculate the interacting force $\mathbf{F}(s, t)$ through mitigating the discrepancy between the fluid velocity $\mathbf{U}$ at the fluid-body interface and the body velocity $\mathbf{V}$

$$\mathbf{F}(s, t) = \alpha \int_0^t [\mathbf{U}(s, \tau) - \mathbf{V}(s, \tau)] d\tau + \beta [\mathbf{U}(s, t) - \mathbf{V}(s, t)], \quad (11)$$

where α and β are negative parameters with sufficiently large absolute values so that the difference between $\mathbf{U}$ and $\mathbf{V}$ is minimized when the penalty force $\mathbf{F}(s, t)$ converges after iterations [22]. In current study, we choose $\alpha = -10^3 \rho \tilde{V}_j^2 / D$ and $\beta = -150 \rho \tilde{V}_j$.

3.2. Numerical algorithm

The governing equations (Eqns.(4)-(6)) are solved to update the fluid velocity, pressure and interacting force at $(n+1)$ th time step using data from its previous time step n . Following Kim *et al.* [23], a conventional fractional-step method is applied in this study. The fluid-dynamics equations are temporally discretized with the Crank-Nicholson scheme and spatially discretized using the finite difference method within a rectangular domain filled with staggered grid (see Fig. 2). The discrete governing equations are presented as

$$\begin{aligned} \frac{u_x^{n+1} - u_x^n}{\Delta t} + \frac{\mathcal{H}(u_x^{n+1}) + \mathcal{H}(u_x^n)}{2} &= -\mathcal{G}_x(p^{n+1/2}) + \frac{\mathcal{L}_x(u_x^{n+1}) + \mathcal{L}_x(u_x^n)}{2Re_j} + f_x^n, \\ \frac{u_r^{n+1} - u_r^n}{\Delta t} + \frac{\mathcal{H}(u_r^{n+1}) + \mathcal{H}(u_r^n)}{2} &= -\mathcal{G}_r(p^{n+1/2}) + \frac{\mathcal{L}_r(u_r^{n+1}) + \mathcal{L}_r(u_r^n)}{2Re_j} + f_r^n, \\ \mathcal{D}_x(u_x^{n+1}) + \mathcal{D}_r(u_r^{n+1}) &= 0, \end{aligned} \quad (12)$$

where the operators are defined as

$$\begin{aligned} \mathcal{G}_x &= \mathcal{D}_x = \frac{\partial}{\partial x}, & \mathcal{G}_r &= \frac{\partial}{\partial r}, & \mathcal{D}_r &= \frac{1}{r} \frac{\partial}{\partial r} r, & \mathcal{H} &= u_x \frac{\partial}{\partial x} + u_r \frac{\partial}{\partial r}, \\ \mathcal{L}_x &= \frac{\partial^2}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r}, & \mathcal{L}_r &= \frac{\partial^2}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{1}{r^2}. \end{aligned}$$

The presence of convective terms makes the problem nonlinear since the velocities are unknown, e.g. $\mathcal{H}(u_x^{n+1}) = u_x^{n+1} \frac{\partial u_x^{n+1}}{\partial x} + u_r^{n+1} \frac{\partial u_x^{n+1}}{\partial r}$. Hereby we linearize the equations by employing the scheme of Beam & Warming [24] with second-order temporal accuracy.

$$\frac{\mathcal{H}(u_x^{n+1}) + \mathcal{H}(u_x^n)}{2} = \mathcal{N}_{xx}(u_x^{n+1}) + \mathcal{N}_{xr}(u_r^{n+1}), \quad (13)$$

$$\frac{\mathcal{H}(u_r^{n+1}) + \mathcal{H}(u_r^n)}{2} = \mathcal{N}_{rx}(u_x^{n+1}) + \mathcal{N}_{rr}(u_r^{n+1}), \quad (14)$$

in which the linear convective operators only include the velocity at n time step:

$$\begin{aligned} \mathcal{N}_{xx} &= \frac{\partial}{\partial x} u_x^n + \frac{1}{2r} \frac{\partial}{\partial r} r u_r^n, & \mathcal{N}_{xr} &= \frac{1}{2r} \frac{\partial}{\partial r} r u_x^n, \\ \mathcal{N}_{rx} &= \frac{1}{2} \frac{\partial}{\partial x} u_r^n, & \mathcal{N}_{rr} &= \frac{1}{2} \frac{\partial}{\partial x} u_x^n + \frac{1}{r} \frac{\partial}{\partial r} r u_r^n \end{aligned}$$

Assuming $\delta u_x^{n+1} = u_x^{n+1} - u_x^n$, $\delta u_r^{n+1} = u_r^{n+1} - u_r^n$, $\delta p = p^{n+1/2} - p^{n-1/2}$, we then rewrite the discretized and linearized equations in matrix form as

$$\begin{bmatrix} A_x & \mathcal{N}_{xr} & \mathcal{G}_x \\ \mathcal{N}_{rx} & A_r & \mathcal{G}_r \\ \mathcal{D}_x & \mathcal{D}_r & 0 \end{bmatrix} \begin{bmatrix} \delta u_x^{n+1} \\ \delta u_r^{n+1} \\ \delta p \end{bmatrix} = \begin{bmatrix} R_x \\ R_r \\ 0 \end{bmatrix}, \quad (15)$$

where

$$\begin{aligned} A_x &= \frac{1}{\Delta t} \left[I + \Delta t \left(\mathcal{N}_{xx} - \frac{1}{2Re_j} \mathcal{L}_x \right) \right], \\ A_r &= \frac{1}{\Delta t} \left[I + \Delta t \left(\mathcal{N}_{rr} - \frac{1}{2Re_j} \mathcal{L}_r \right) \right], \\ R_x &= \frac{1}{Re_j} \mathcal{L}_x(u_x^n) + f_x^n - \mathcal{N}_{xx}(u_x^n) - \mathcal{N}_{xr}(u_r^n) - \mathcal{G}_x(p^{n-1/2}), \\ R_r &= \frac{1}{Re_j} \mathcal{L}_r(u_r^n) + f_r^n - \mathcal{N}_{rx}(u_x^n) - \mathcal{N}_{rr}(u_r^n) - \mathcal{G}_r(p^{n-1/2}). \end{aligned}$$

To solve Eq. (15), inversion of the large sparse coefficient matrix is needed, which requires significant *CPU* time and memory. Instead, we use *LU* decomposition to factorize the coefficient matrix so that velocity and pressure are decoupled, yielding

$$\begin{bmatrix} A_x & 0 & 0 \\ \mathcal{N}_{rx} & A_r & 0 \\ \mathcal{D}_x & \mathcal{D}_r & -\Delta t(\mathcal{D}_x \mathcal{G}_x + \mathcal{D}_r \mathcal{G}_r) \end{bmatrix} \begin{bmatrix} I & \Delta t \mathcal{N}_{xr} & \Delta t \mathcal{G}_x \\ 0 & I & \Delta t \mathcal{G}_r \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \delta u_x^{n+1} \\ \delta u_r^{n+1} \\ \delta p \end{bmatrix} = \begin{bmatrix} R_x \\ R_r \\ 0 \end{bmatrix} + err(\Delta t^2), \quad (16)$$

where the second-order residual error term is

$$err = \Delta t \begin{bmatrix} \left(\mathcal{N}_{xx} - \frac{1}{2Re_j} \mathcal{L}_x \right) \mathcal{N}_{xr}(\delta u_r^{n+1}) + \left(\mathcal{N}_{xx} - \frac{1}{2Re_j} \mathcal{L}_x \right) \mathcal{G}_x(\delta p) \\ \mathcal{N}_{xr} \mathcal{N}_{rx}(\delta u_r^{n+1}) + \left(\mathcal{N}_{rr} - \frac{1}{2Re_j} \mathcal{L}_r \right) \mathcal{G}_r(\delta p) + \mathcal{N}_{rx} \mathcal{G}_x(\delta p) \\ \mathcal{D}_x \mathcal{N}_{xr}(\delta u_r^{n+1}) \end{bmatrix}.$$

Note that since δu_x^{n+1} , δu_r^{n+1} and δp are $O(\Delta t)$, the residual error term is $O(\Delta t^2)$, making the above approximation second order in time.

Let δu_x^* and δu_r^* be the intermediate values of the velocity increments at $(n+1)$ th time step, Eq. (16) can be rewritten as

$$\begin{bmatrix} A_x & 0 & 0 \\ \mathcal{N}_{rx} & A_r & 0 \\ \mathcal{D}_x & \mathcal{D}_r & -\Delta t(\mathcal{D}_x \mathcal{G}_x + \mathcal{D}_r \mathcal{G}_r) \end{bmatrix} \begin{bmatrix} \delta u_x^* \\ \delta u_r^* \\ \delta p \end{bmatrix} = \begin{bmatrix} R_x \\ R_r \\ 0 \end{bmatrix}, \quad (17)$$

and

$$\begin{bmatrix} I & \Delta t \mathcal{N}_{xr} & \Delta t \mathcal{G}_x \\ 0 & I & \Delta t \mathcal{G}_r \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \delta u_x^{n+1} \\ \delta u_r^{n+1} \\ \delta p \end{bmatrix} = \begin{bmatrix} \delta u_x^* \\ \delta u_r^* \\ \delta p \end{bmatrix}. \quad (18)$$

The following steps are taken to update the flow field:

Step 1: solve the intermediate velocity increments via $A_x \delta u_x^* = R_x$ and $A_r \delta u_r^* = R_r - \mathcal{N}_{rx} \delta u_x^*$;

Step 2: solve δp from the elliptic equation $\Delta t (\mathcal{D}_x \mathcal{G}_x + \mathcal{D}_r \mathcal{G}_r) \delta p = \mathcal{D}_x \delta u_x^* + \mathcal{D}_r \delta u_r^*$;

Step 3: update the velocity increments at $n + 1$ time level via $\delta u_x^{n+1} = \delta u_x^* - \Delta t \mathcal{G}_r \delta p$ and $\delta u_r^{n+1} = \delta u_r^* - \Delta t \mathcal{N}_{xr} \delta u_x^{n+1} - \Delta t \mathcal{G}_x \delta p$;

Step 4: update the velocities and pressure at time level $n + 1$ via $u_x^{n+1} = u_x^n + \delta u_x^{n+1}$, $u_r^{n+1} = u_r^n + \delta u_r^{n+1}$, and $p^{n+1/2} = p^{n-1/2} + \delta p$.

The validity and accuracy of the numerical algorithm has been demonstrated in our previous work [22]. Canonical problems such as uniform flow past a stationary sphere have been tested using this fluid solver. Our results show good agreement with the experimental data. The details are included in that publication [22].

4. Results

The simulations were performed in a rectangular domain, as displayed in Fig. 2. The fluidic grid size, structural grid size and time step are chosen to be $\Delta x = \Delta r = 0.02D$, $\Delta s = 0.0167D$ and $\Delta t = 5.0 \times 10^{-4} D / \tilde{V}_j$, respectively. Convergence tests have been conducted to ensure the results are not sensitive to spatial and temporal steps [22]. We will simulate the dynamics of the bio-inspired propeller with various nozzle configurations. Unless otherwise specified, all results will be presented in normalized forms by using reference variables $\tilde{V}_j$, ρ and D . With this normalization, in the following results the speeds are normalized by $\tilde{V}_j$, the forces by $\rho D^2 \tilde{V}_j^2$, and the pressure by $\rho \tilde{V}_j^2$.

The propulsion performance is characterized by the overall thrust F_t and power expenditure P . Since the propeller is tethered during the jetting process (which means the drag force is negligible), the thrust can be approximated by the net hydrodynamic force acting on the body, which is obtained by integrating F_x (the horizontal component of the structure-fluid interaction force obtained in Eq. (11)) along the body. Let F_1 and F_2 be the hydrodynamic force acting on the deformable body Γ_b and the nozzles Γ_n , respectively. We have

$$F_1 = - \int_{\Gamma_b} 2\pi r F_x(s, t) ds, \quad F_2 = - \int_{\Gamma_n} 2\pi r F_x(s, t) ds. \quad (19)$$

The overall thrust F_t is then the summation of F_1 and F_2 , i.e. $F_t = F_1 + F_2$. Note that by definition positive F_t , F_1 and F_2 all point towards the $-x$ direction. The instantaneous power expenditure P is calculated by

$$P = \int_{\Gamma_b} 2\pi r \mathbf{F} \cdot \mathbf{U} ds. \quad (20)$$

Conventionally, the energetics of biological or bio-inspired swimming systems are characterized by using the cost of transport (CoT) or the propulsion efficiency. Hereby CoT is the ratio between energy expenditure and travelled distance. Propulsion efficiency is the ratio between useful power (thrust times forward speed) and power expenditure. These quantities cannot be applied in our problem since there is neither forward motion nor incoming flow. Instead, to evaluate the energetics of the system we define a dimensional quantity called the *cost of impulse* η , which is the ratio between the overall energy expenditure and the generated impulse calculated as

$$\eta = \frac{\bar{P}}{\bar{F}_t}, \quad (21)$$

where $\bar{F}_t$ is the time-averaged thrust and $\bar{P}$ is the mean power expenditure. Higher η corresponds to lower effectiveness in energy usage.

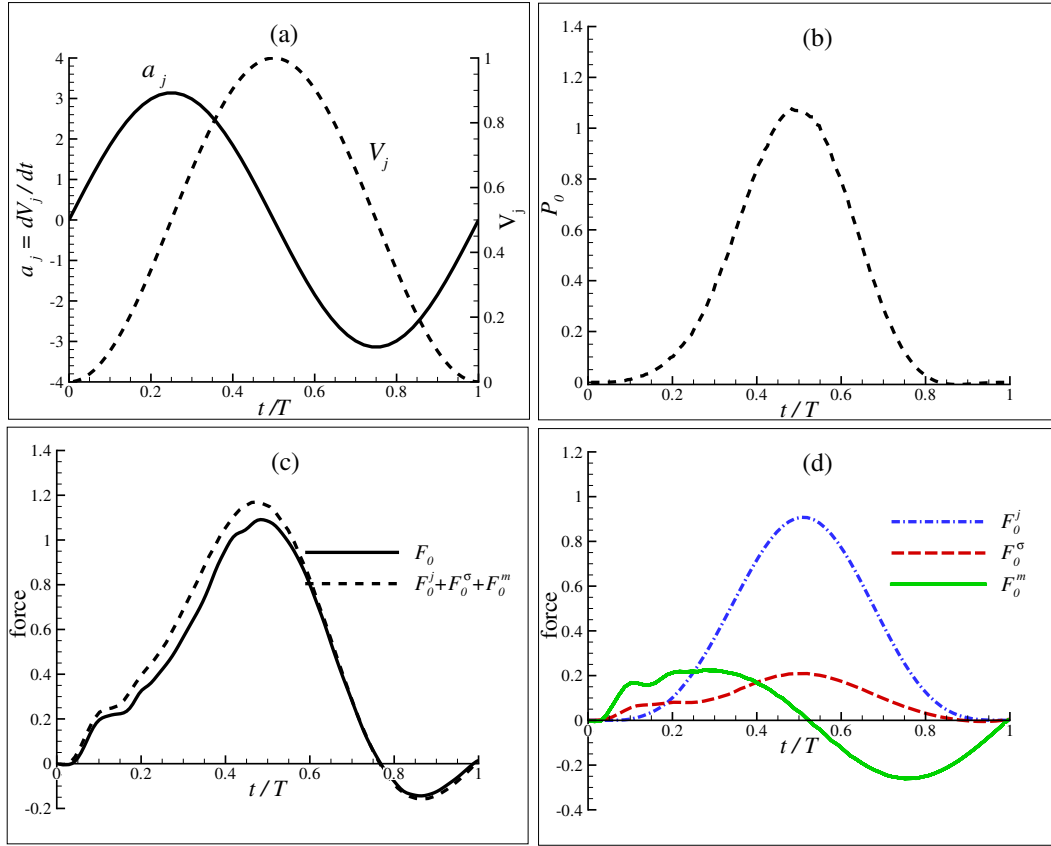


Figure 3. Thrust generation by jetting through the orifice: (a) The prescribed jet speed V_j and jet acceleration a_j ; (b) Time history of the power expenditure P_0 ; (c) The comparison between thrust F_0 and $F_0^j + F_0^\sigma + F_0^m$; (d) Time histories of the thrust components F_0^j , F_0^σ , F_0^m .

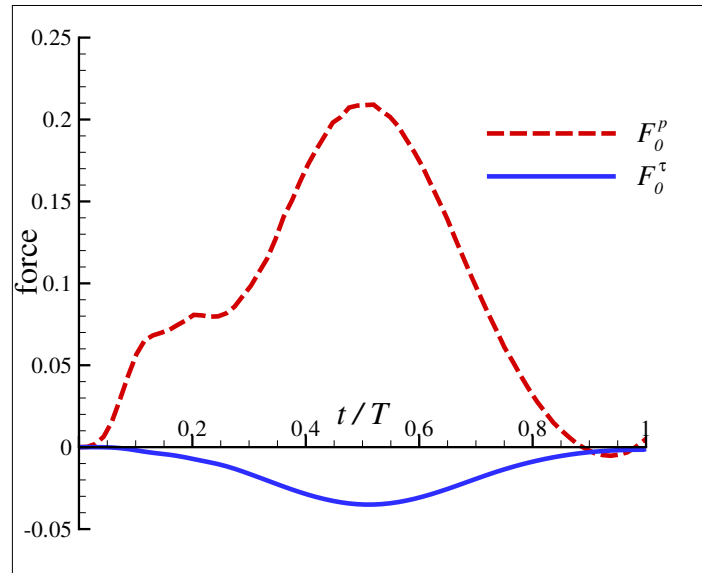


Figure 4. Comparison between F_0^P and F_0^τ for the case shown in Fig. 3.

4.1. Jetting through the orifice

As mentioned earlier, the present research focuses on the effect of different nozzle configurations upon the propulsion capability of the squid-inspired propeller. Towards this end, we first analyze the dynamics of the model with no nozzle as shown in Fig. 1a. The results will be used as a reference for the subsequent simulations with various nozzles included. Hereafter quantities with subscript ‘0’ are related to the system without nozzles (i.e. orifice jetting). Those with subscript ‘1’ are related to quantities on the deformable body, while subscript ‘2’ is used to depict quantities on the nozzle.

The jet speed profile $V_j(t)$ and jet acceleration $a_j = dV_j/dt$ are delineated in Fig. 3a. Figures 3b,c present the time histories of the power expenditure P_0 and thrust force F_0 (these two quantities are actually P and F_1 for the case of orifice jetting). The instantaneous power cost P_0 , as expected, is proportional to V_j^3 according to the plug flow approximation [25]. It is also found that positive thrust is created during most of the process except in a relatively short period near the end, where negative thrust is present. Based on a control volume analysis [22], there are three distinct contributions to the thrust generation, the jet flux momentum F_0^j , the excess normal stress at the orifice exit F_0^σ , and the fluid momentum inside the body F_0^m , i.e.

$$F_0 \approx \iint_A \rho u_x^2 dS + \iint_A (-\sigma - p_\infty) dS + \frac{d}{dt} \iiint_\Omega \rho u_x dV = F_0^j + F_0^\sigma + F_0^m, \quad (22)$$

where A denotes the orifice exit plane, p_∞ is the background pressure, Ω represents the fluidic volume inside the pressure chamber. $\sigma = -p + \tau$ is the normal fluid stress, where τ denotes the normal viscous stress. These three contributions are plotted in Fig. 3d. As demonstrated, F_0^j is proportional to V_j^2 and F_0^σ , more or less, related to V_j whereas F_0^m mainly depends on jet acceleration. The above thrust decomposition strategy can be verified by consistency of the thrust F_0 (calculated by Eq. (19)) and the summation of these three components, as shown in Fig. 3c. The small difference between the two quantities results from the small but finite drag the propeller sustains since the centroid of the body translates with non-zero velocity during the deformation. Given the fact that the body becomes less blunt over time (as the body eccentricity increases from $e_0 = 0.8$ to $e_1 = 0.95$, the semi-minor axis decreases from $1.3D$ to $0.82D$), the discrepancy is not pronounced after $T/2$ due to the diminishing drag. The presence of the negative thrust in $[0.8T, T]$ is attributed to the contribution from F_0^m , which becomes negative as the jet is decelerating.

It is also important to note that only F_0^j and F_0^σ contribute to the average thrust $\bar{F}_0$ because the time-averaged F_0^m is approximately zero for the specified jet profile we use in this study. After normalization, the time-averaged thrust, power expenditure and the cost of impulse in this particular case are $\bar{F}_0 = 0.42$, $\bar{P}_0 = 0.35$, $\eta_0 = 0.831$, respectively. Additionally, Fig. 4 illustrates that the over-pressure contribution (referred to as $F_0^p = \iint_A (p - p_\infty) dS$) to F_0^σ is predominant over the viscous part (referred to as $F_0^\tau = \iint_A (-\tau) dS$), i.e. $F_0^\sigma \approx F_0^p$. This turns out to be the scenario in all the cases considered.

4.2. Effects of the nozzle geometry

In this section, a variety of nozzles with different combinations of l and θ will be affixed to the body and tested with the prescribed body deformation. We will evaluate the influence of nozzles on the original propulsion system (see Fig. 1a) by using $\lambda_1 = \frac{\bar{F}_1 - \bar{F}_0}{\bar{F}_0}$. The synthetic performance of the entire system (including the deformable body and the nozzle), characterized by $\lambda = \lambda_1 + \lambda_2$ (where $\lambda_2 = \bar{F}_2/\bar{F}_0$), $\bar{P}/\bar{P}_0$ and η/η_0 , will also be examined.

4.2.1. Thrust generation

The thrust decomposition scheme (Eq. (22)) can be applied to the thrust F_1 . After time averaging, we have $\bar{F}_1 \approx \bar{F}_1^j + \bar{F}_1^\sigma + \bar{F}_1^m \approx \bar{F}_1^j + \bar{F}_1^\sigma$ (since $\bar{F}_1^m \approx 0$). Therefore λ_1 can be decomposed into $\lambda_1^j = \frac{\bar{F}_1^j - \bar{F}_0^j}{\bar{F}_0}$ and $\lambda_1^\sigma = \frac{\bar{F}_1^\sigma - \bar{F}_0^\sigma}{\bar{F}_0}$. Since the body deformation remains unchanged, the mean axial flow speed at the orifice is expected to be unchanged as well ($\bar{F}_1^j \approx \bar{F}_0^j$). Subsequently, λ_1^j is small so that λ_1 is dominated by λ_1^σ . Figures 5a,b demonstrate the values of λ_1 and λ_1^σ as functions of l and θ . The consistency between λ_1 and λ_1^σ confirms that after the installation of nozzles, the enhancement of thrust on the deformable body is mainly attributed to the fluid stress (mostly over-pressure) augmentation at the orifice plane A .

Even though with a nozzle added there is significant increase in the thrust on the deformable body ($\bar{F}_1 > \bar{F}_0$ according to Fig. 5a), the nozzle itself may sustain considerable drag force (i.e. $\lambda_2 < 0$) when the flow goes through

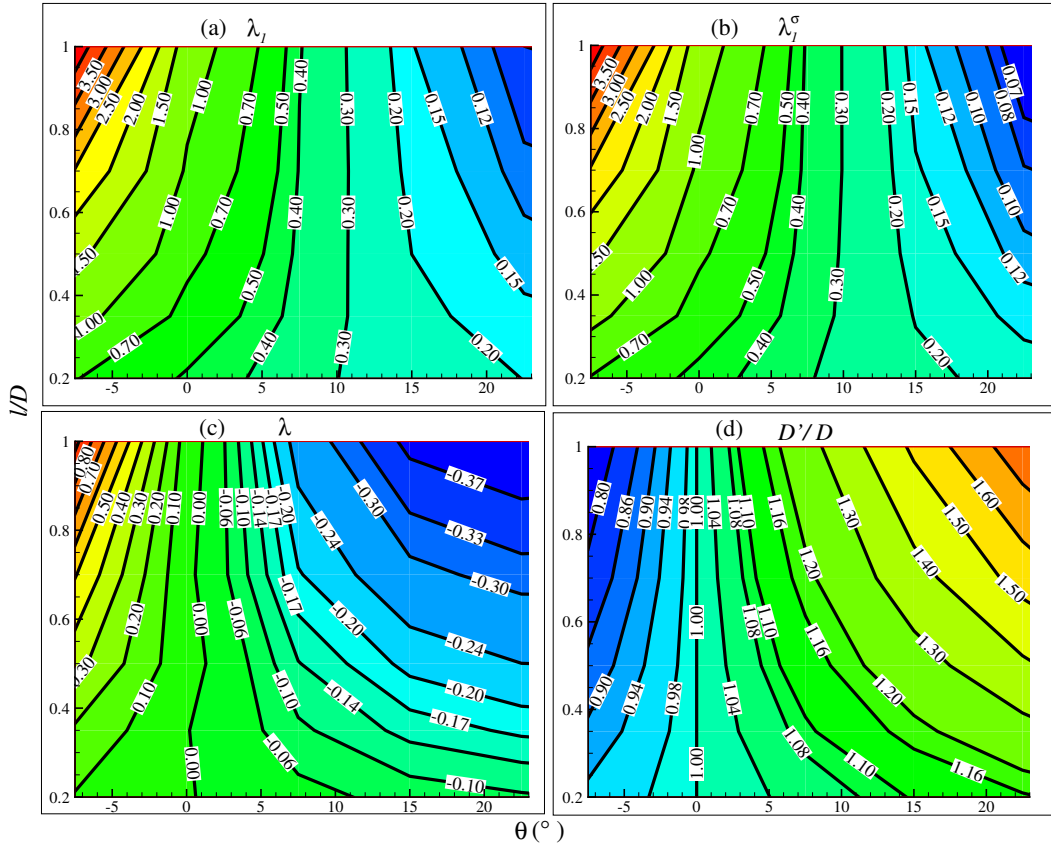


Figure 5. $\lambda_l, \lambda_l^\sigma, \lambda$ and the nozzle exit diameter D' as functions of the nozzle contour length l and the inclination angle θ .

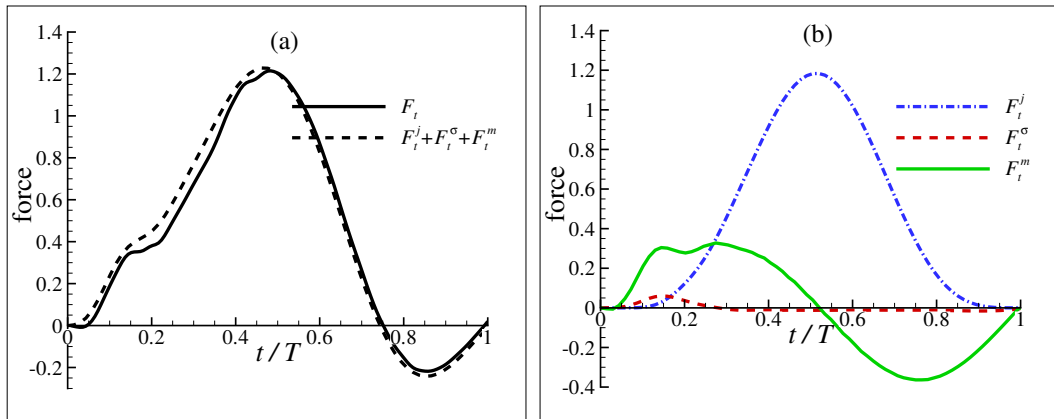


Figure 6. (a) The comparison between F_t and $F_t^j + F_t^\sigma + F_t^m$; (b) Time histories of the thrust components F_t^j, F_t^σ, F_t^m . $l = D, \theta = 0$.

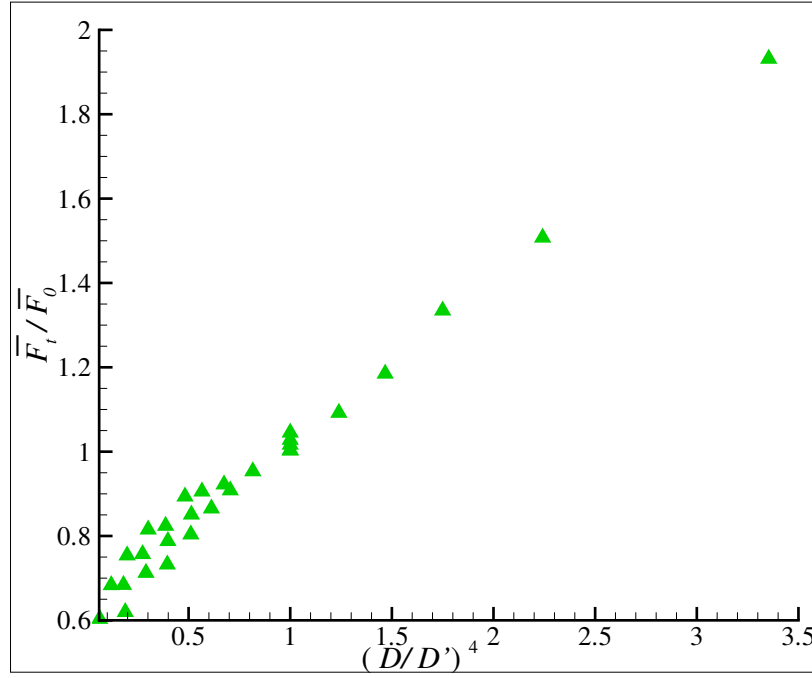


Figure 7. The average thrust of the whole system $\overline{F}_t$ as functions of $1/D'^4$. D' is normalized by D .

them so that it is not clear if there is a net gain in the thrust on the whole body including the deformable part and the nozzle. It is necessary to measure the overall thrust created by the whole system, which is characterized by λ . In Fig. 5c, we present the value of λ as a function of l and θ . It is shown that tubular nozzles ($\theta = 0$) has little impact on the thrust generation of the whole system (i.e. $\lambda \sim 0$), which means that the thrust increase on the deformable part is almost canceled by the drag force acting on the nozzle. Systems with converging nozzles ($\theta < 0$) tend to produce higher thrust ($\lambda > 0$) while those with diverging nozzles ($\theta > 0$) will produce reduced thrust ($\lambda < 0$).

Figure 5d presents the dependence of the nozzle exit diameter D' on l and θ . Note that hereafter the quantities defined at the nozzle exit plane A' are labeled by adding primes to their counterparts at the orifice plane A . Upon closer inspection, it is found that the contour lines of D' (Fig. 5d) are roughly correlated with those of λ (Fig. 5c), which implies that the thrust generation of the entire system is closely related to the nozzle exit diameter D' , or alternatively the jet speed at the nozzle exit V_j' since the body deformation is given. Quantitatively, it is believed that the thrust depends on $V_j'^2$ [26] so that it is proportional to $1/D'^4$ (here $D' = D(1 + 2A_R \tan \theta)$, A_R is the ratio between the longitudinal length and the inlet diameter D of the nozzle). This is confirmed in Fig. 7, in which the relation between average thrust and D' in the cases considered in this study is plotted.

To throw light upon the above phenomenon, we apply the decomposition strategy to the entire propulsive system. Via this approach, the whole-system thrust $F_t = F_1 + F_2$ can be decoupled into three parts: F_t^j, F_t^σ and F_t^m , where F_t^j and F_t^σ are defined at the nozzle exit A' and F_t^m is based on the overall fluidic volume including the fluid inside the pressure chamber and the nozzle. As a demonstration, Fig. 6 displays the instantaneous F_t and its three components for the case of $l = D$, $\theta = 0$. It is clear that F_t is in good agreement with $F_t^j + F_t^\sigma + F_t^m$. In the following we will use this technique to study the force generation at three different scenarios, converging nozzle, tubular nozzle, and diverging nozzle.

a) Converging nozzles

We examine three converging nozzles ($\theta < 0$) with the exit size being fixed at $D' = 0.7D$. The combinations of geometry parameters (l, θ) for these nozzles are, respectively, $(0.4D, -22^\circ)$, $(0.6D, -14.5^\circ)$ and $(D, -8.6^\circ)$. Figure 8a shows that these three cases generated almost the same thrust F_t . With the nozzle exit diameter D' being a constant, F_t^j is nearly unchanged for various converging nozzles as shown in Fig. 8b (the slight difference is attributed to the

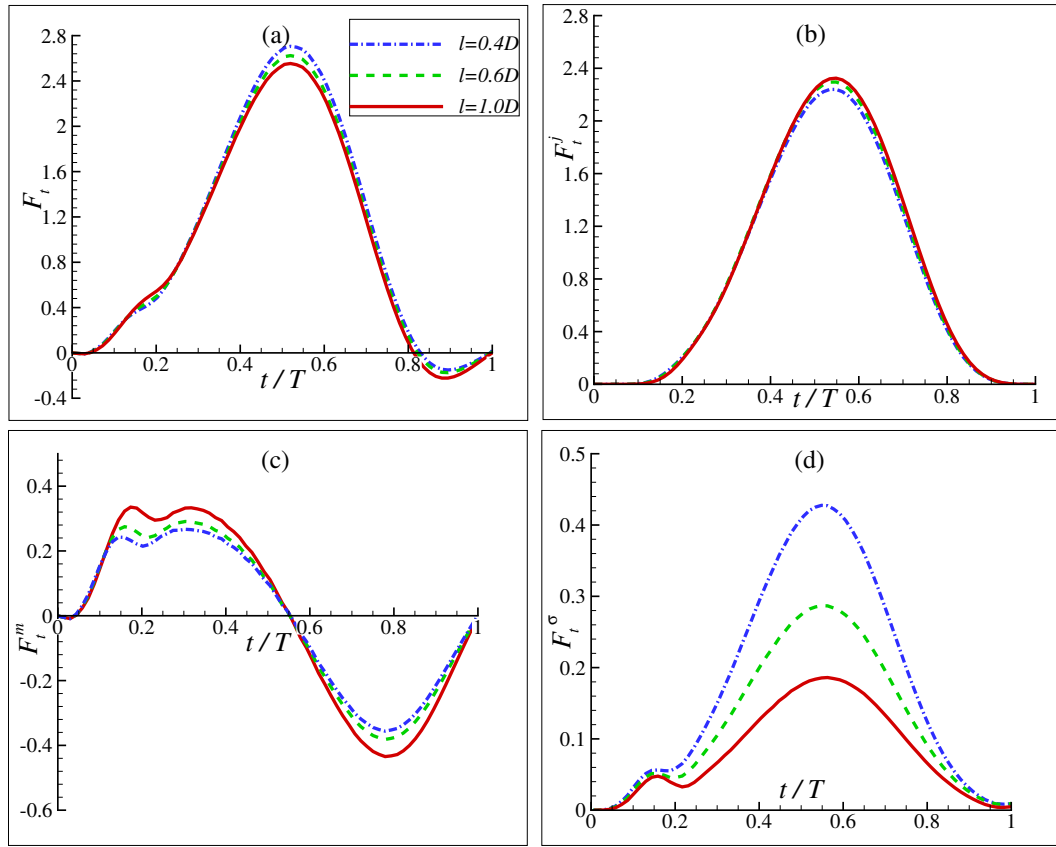


Figure 8. Thrust generation using converging nozzles with $D' = 0.7D$: (a) The whole-system thrust F_t and (b,c,d) their three components.

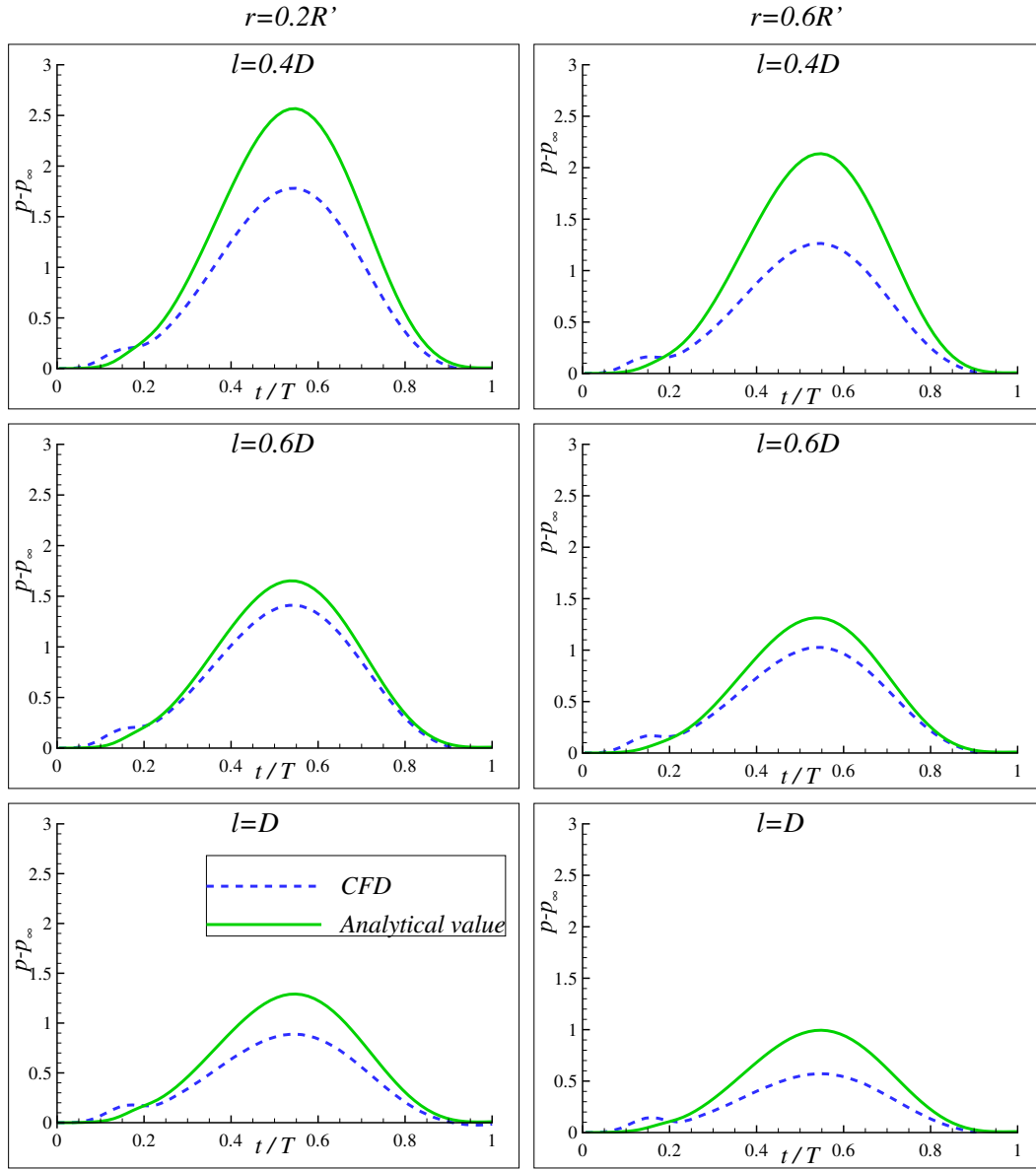


Figure 9. Comparisons between the numerically obtained over-pressures and the analytical values at the monitoring locations for the converging nozzles.

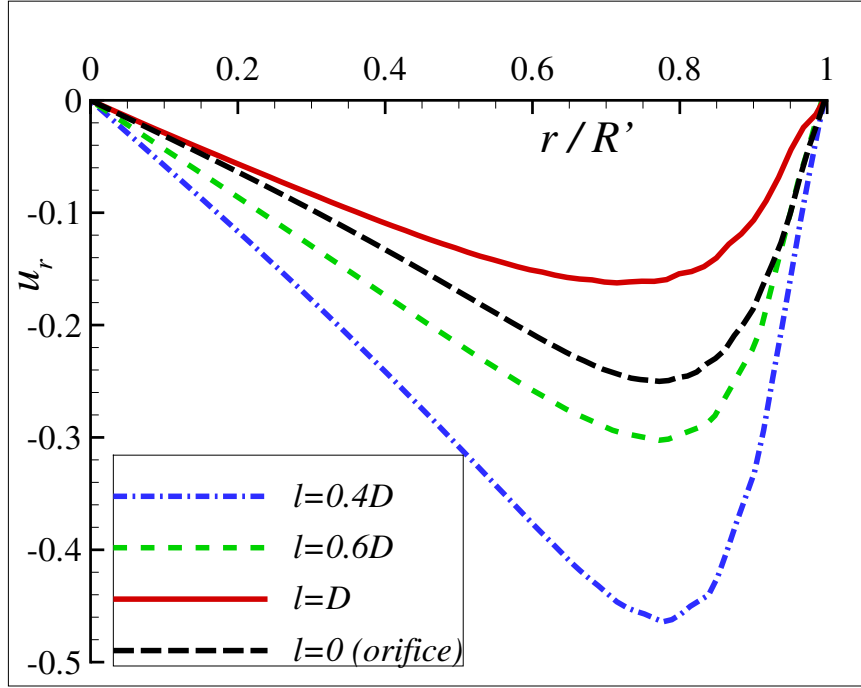


Figure 10. Comparison of radial velocity profiles on the exit of converging nozzles, $t = 0.5T$.

discrepancy in spatial distribution of the axial velocity at A'). Figure 8c demonstrates the time histories of F_t^m , which is found to have similar dependence upon the jet acceleration as F_0^m . Although the magnitude of F_t^m becomes higher as l increases, the time-averaged values are always close to zero. We then present the over-pressure term F_t^σ in Fig. 8d. There is indeed significant differences among the three cases. However, it is observed that F_t^σ only accounts for a small percentage of the overall thrust F_t . Instead, F_t^j is the dominant source in F_t , which depends on the nozzle exit size D' .

Interestingly, the time evolution of F_t^σ displays a twin-peak feature. A minor peak occurs in the initial stage, followed by a major peak near the middle of the jetting process. The magnitude of the first peak varies very little among the three cases, whereas there is a notable difference for the second one. In order to investigate the mechanism of F_t^σ generation during the jet pulsation, in Fig. 9 we plot time histories of the over-pressure $p - p_\infty$ at two representative locations on the nozzle exit, $r = 0.2R'$ and $0.6R'$ (where $R' = 0.5D'$) since F_t^σ is mostly contributed from F_t^p . They are found to experience similar tendencies as F_t^σ . It is thus necessary to study the underlying mechanism of over-pressure on the exit.

Based on the unsteady inviscid flow theory, Krieg & Mohseni [21] derived an analytical model to determine the over-pressure distribution along the exit boundary for axisymmetric jet flow, i.e.

$$\frac{p(r) - p_\infty}{\rho} = \int_r^{R_\infty} \left(\frac{\partial u_r}{\partial t} + \mathbf{u} \cdot \nabla u_r \right) d\varpi, \quad (23)$$

where R_∞ is the infinity in the radial direction, ϖ is the dummy variable for the integration along the radius. The formulation shows that the exit over-pressure is related to the material derivative of the radial velocity on the exit, which indicates that a non-zero radial velocity is required for any non-negligible over-pressure on the nozzle exit. Therefore, we present the radial velocity $u_r(r)$ distributions along the exit radius at time instant $t = 0.5T$ for the three nozzles considered here in Fig. 10, where the corresponding result from the orifice scenario (§4.1) is also presented for comparison. The converging nozzles produce negative radial velocity $u_r < 0$ on the exit due to geometric effect. Likewise, the orifice jetting also experience negative radial velocity because as body shrinks, the orifice forms a conical nozzle, through which the flow is ejected with converging streamlines. This may explain the similarity between the

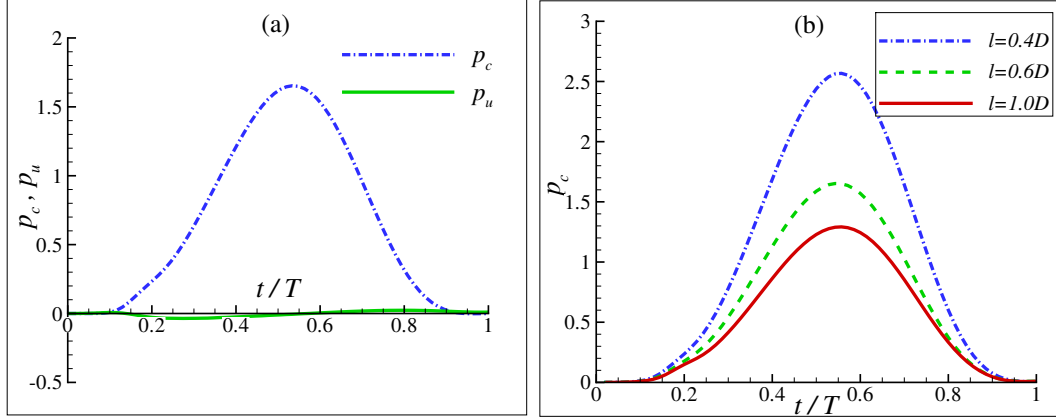


Figure 11. (a) p_c and p_u for a converging nozzle with $l = 0.6D$ and $\theta = -14.5^\circ$; (b) comparison of p_c for various converging nozzles. The monitoring point is $r = 0.2R'$.

time histories of F_t^σ ($\approx F_t^p$) and F_0^σ ($\approx F_0^p$), as displayed in Fig. 8d and Fig. 3d.

Figure 9 also presents the time evolution of the analytically obtained over-pressure obtained using Eq. (23). It is seen that the analytical model can qualitatively predict the tendency since the two results are consistent in phase, especially during the second peak. The quantitative inconsistency might be because viscosity is ignored in the analytical model. Figure 9 shows that more converging nozzles (with shorter l) result in higher exit pressure, which then translates to higher F_t^σ as shown in Fig. 8d. Not surprisingly, the rather large quantitative discrepancy between analytical and numerical results suggests that by neglecting viscosity the analytical model is not capable of accurate predictions in low Reynolds number regime.

Notwithstanding its quantitative accuracy, the analytical model is still useful for physical insights. Let p_u denote the integration of u_r unsteady acceleration, $p_u = \int_r^{R_\infty} \frac{\partial u_r}{\partial t} d\bar{\omega}$, and p_c be the integration of u_r convective acceleration, $p_c = \int_r^{R_\infty} \mathbf{u} \cdot \nabla u_r d\bar{\omega}$, then the analytical over-pressure (Eq. (23)) can be expressed as $p_u + p_c$. Figure 11a displays the time histories of p_c and p_u for a representative case with $r = 0.2R'$, $l = 0.6D$, and $\theta = -14.5^\circ$. It shows that the exit over-pressure is predominantly contributed from the convective acceleration related p_c . Increasing the nozzle convergence (achieved through decreasing l) leads to larger radial velocity u_r , as displayed in Fig. 10. This is accompanied by the augmentation of spatial derivative of u_r along the streamline direction (the convective acceleration) on the exit, leading to p_c enhancement as shown in Fig. 11b, which eventually translates to higher F_t^σ (see Fig. 8d). In addition, $p_c(r)$ can be rewritten as $\int_r^{R_\infty} u_x \frac{\partial u_r}{\partial x} d\bar{\omega} - 0.5u_r^2$, in which the second term is much smaller than the integration term according to our simulations. Since $\frac{\partial u_r}{\partial x} > 0$ is satisfied for converging nozzles, this new expression explains the reason why F_t^p (or alternatively F_t^σ) is more or less proportional to the jet speed, hereby forming the second peak with the prescribed jet speed profile.

The mechanism for the first peak of F_t^σ will be illustrated by using tubular nozzles as an example since in that scenario the twin-peak phenomenon becomes more pronounced.

b) Tubular nozzles

We then examine three tubular nozzles ($\theta = 0$ so that the nozzle exit diameter is a constant, i.e. $D' = D$) with different contour lengths $l = 0.2D, 0.5D, D$. Not surprisingly, Fig. 12a confirms the consistency of the overall thrust F_t in these three cases. The thrust components F_t^j and F_t^m have similar time histories as those with the converging nozzles (see Figs. 12b,c). Indeed, tubular nozzles may share some characteristics with converging nozzles since in both cases converging streamlines are generated at the orifice, which may not be fully flattened in the nozzle when l is not sufficiently long. As a result, with tubular nozzles the other thrust component F_t^σ also behaves similar to that of the converging nozzles (both have twin peaks) as shown in Fig. 12d.

Figure 13 displays the radial velocity $u_r(r)$ at time instant $t = 0.5T$ with the three tubular nozzles. As expected, the radial velocity is significantly reduced after applying tubular nozzles to the orifice. For this reason, small over-pressure is created on the nozzle exit and subsequently F_t^σ is also small, as shown in Fig. 12d. As a result, the

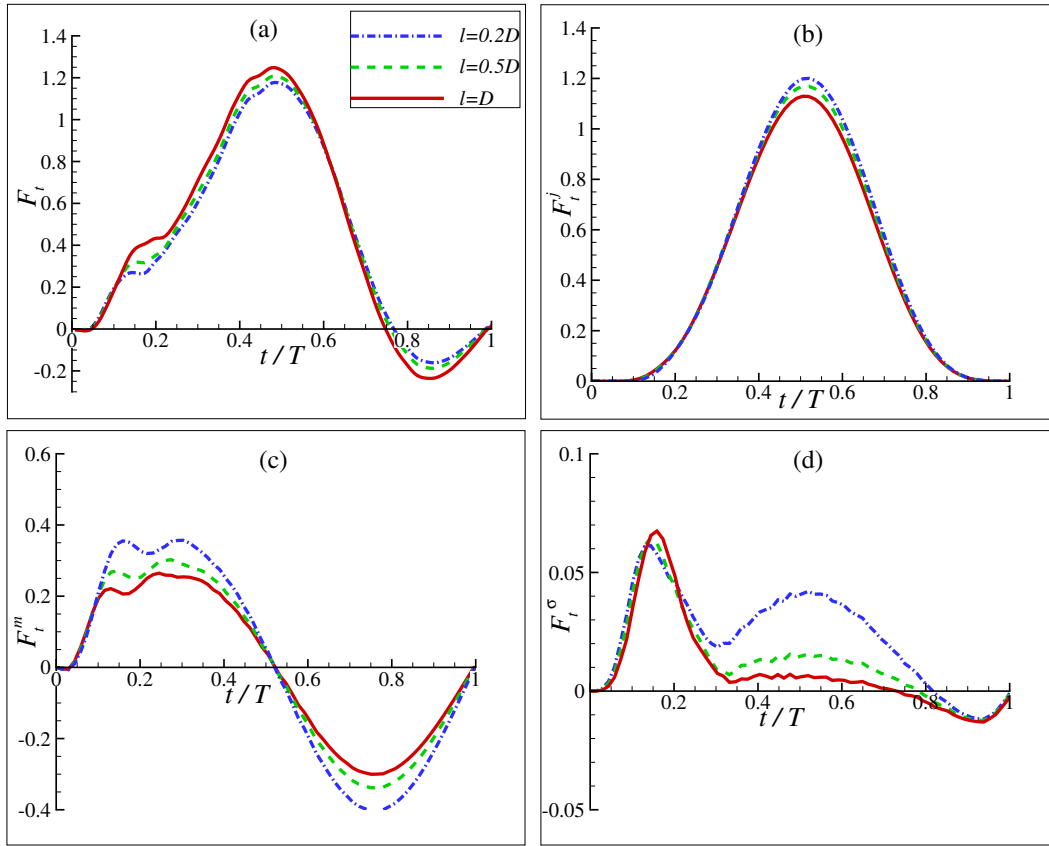


Figure 12. Thrust generation using tubular nozzles: (a) The whole-system thrust F_t and (b,c,d) their three components.

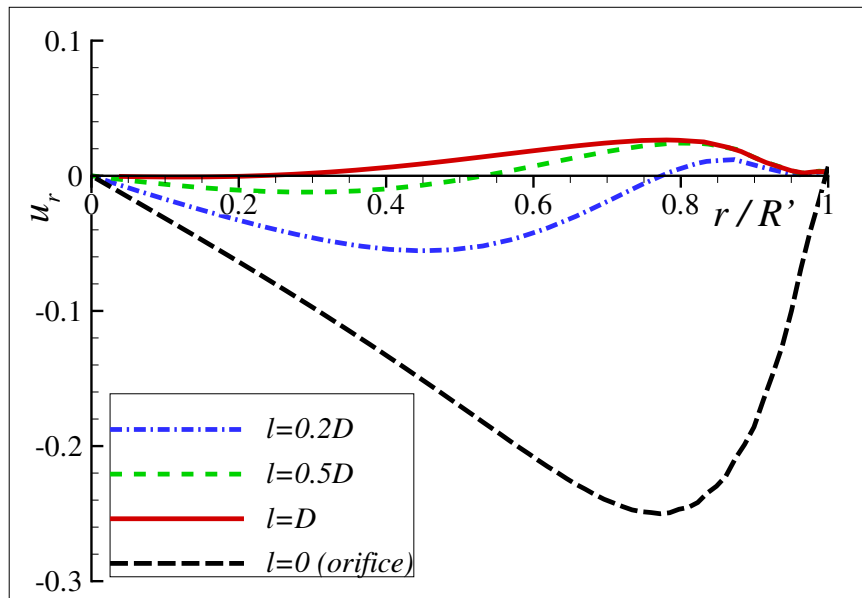


Figure 13. Comparison of radial velocity profiles on the exit of tubular nozzles and orifice, $t = 0.5T$

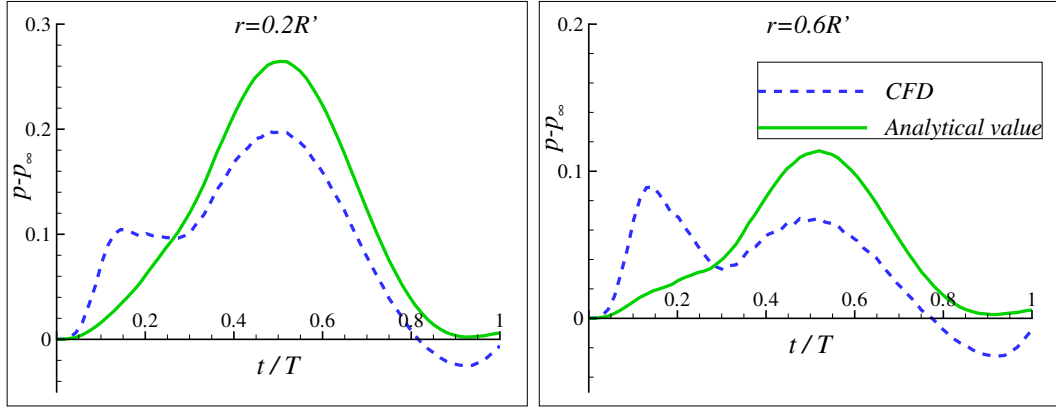


Figure 14. Comparisons between the numerically obtained over-pressures and the analytical values at the monitoring locations. The nozzle is tubular with $l = 0.2D$.

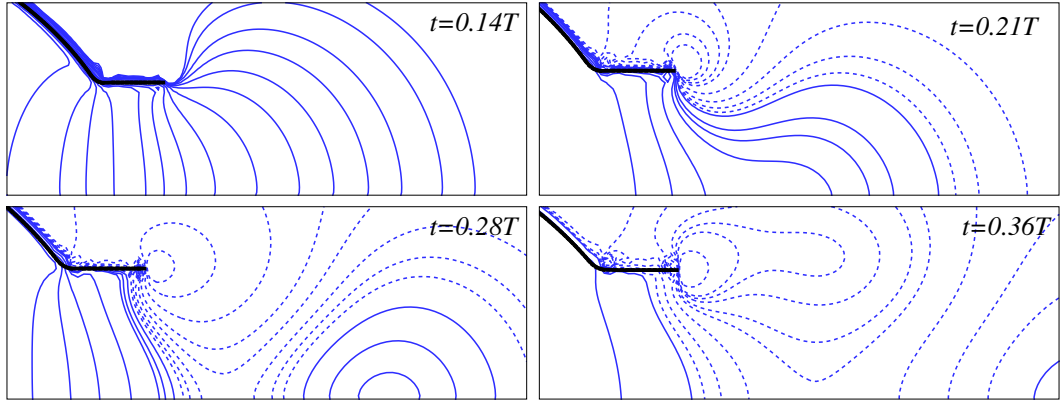


Figure 15. The evolution of the pressure field for the tubular nozzle with $l = 0.2D$. The solid lines represent the positive pressure contours and the dashed lines indicate the negative pressure contours. This figure shows the hinder part of the body (see Fig. 2), the nozzle, and the near-body wake.

second peak is significantly diminished due to the reduced radial velocity, making the twin-peak feature in F_t^σ more pronounced. Figure 14 presents the comparison between the numerically obtained over-pressure and the analytical predictions at the two monitoring locations for a sample case with $l = 0.2D$. Similar to the case with converging nozzles, the analytical model is not able to predict the first peak.

A likely source of the first peak in F_t^σ is the generation and evolution of the leading vortex ring. When the jetting is initiated, a single vortex ring is generated and starts to grow in the wake very close to the exit plane. The pressure on the exit is thus influenced by the coupled effect of the jet flow and the wake flow associated with the vortex ring. In Fig. 15, we exhibit the evolution of the pressure field near the nozzle exit during the initial stage ($t < 0.36T$). It shows that the exit is initially surrounded by positive pressure (e.g. $t = 0.14T$). When it reaches $t = 0.21T$, there appears a negative pressure region close to the nozzle boundary layer, corresponding to the vortex ring generation (negative pressure has been found to coincide with the cores of vortex rings in experiments [27]). Afterwards the negative pressure region grows gradually and eventually almost fills the exit plane. When this vortex-induced negative pressure steps in, the exit pressure will decline, a phenomenon related to the reduced F_t^σ in $0.2T < t < 0.3T$ in Fig. 12d. Similar discoveries have been reported by Gao *et al.* [28]. As the vortex convects downstream, its effect on the exit pressure will be diminished and the radial velocity will become the dominant factor.

Besides the vortex-induced negative pressure, the jet acceleration may also play a role in the formation of the first peak. According to control volume analysis (see Appendix A), to certain extent F_t^σ is dependent on the jet acceleration, particularly during the initial stage when the leading vortex ring is near the exit plane. The time history

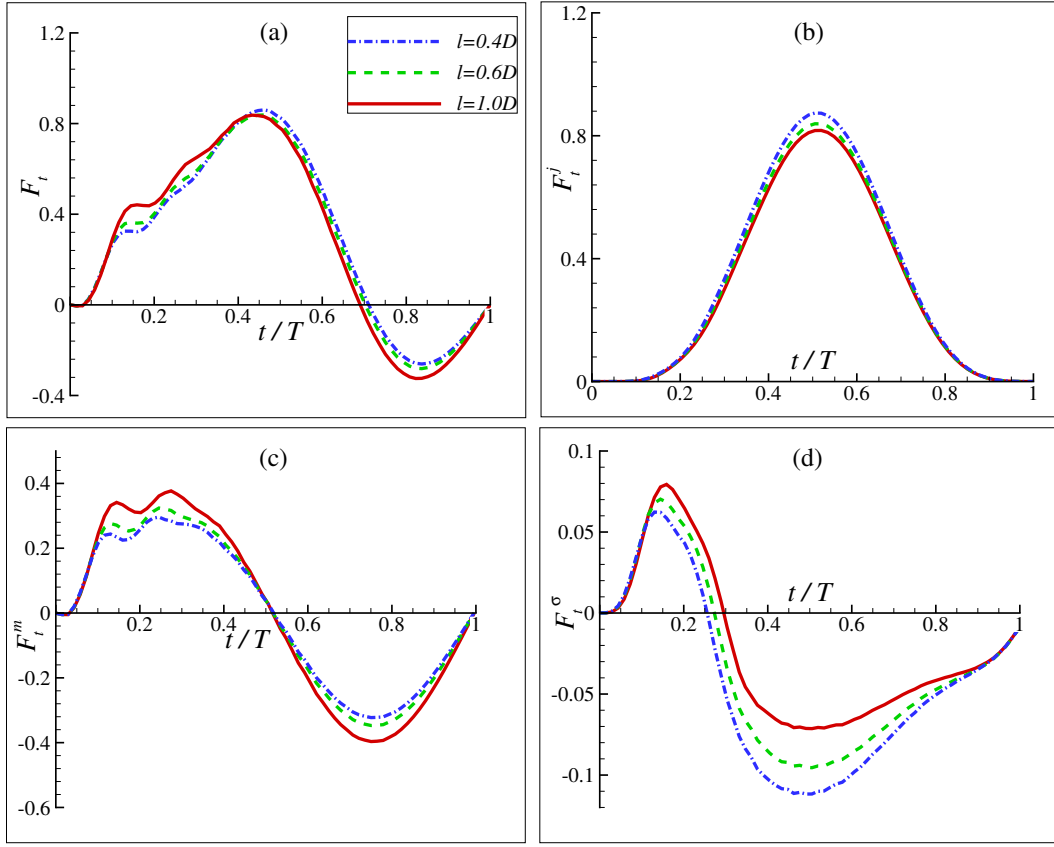


Figure 16. Thrust generation using converging nozzles with $D' = 1.3D$: (a) The whole-system thrust F_t and (b,c,d) their three components.

of the jet acceleration is plotted in Fig. 3a. It suggests that the acceleration peak at $t \approx 0.2T$ may be another source of the first peak of F_t^σ , which occurs near that instant.

c) Diverging nozzles

For diverging nozzles ($\theta > 0$), three configurations, $(l, \theta) = (0.4D, 22^\circ)$, $(0.6D, 14.5^\circ)$, and $(D, 8.6^\circ)$ are chosen with the nozzle exit set to be constant ($D' = 1.3D$). Figure 16 presents the time histories of F_t and its three components F_t^j, F_t^m, F_t^σ . Like the previous cases with converging and tubular nozzles, slight variations of these components among the three cases fail to cause significant difference in F_t . Unlike the converging and tubular nozzles, a diverging nozzle produces positive radial flow speed (see Fig. 17), which creates a negative trough instead of the second peak as displayed in Fig. 16d. For this reason in diverging nozzles $\bar{F}_t^\sigma$ contributes negatively to the thrust.

4.2.2. Vortex ring propagation

Figure 18 displays the wake flow fields at two time instants $t = 0.4$ and $t = 0.6T$ for the pulsed jetting through the orifice and the three types of nozzles. We use the Q-criterion to identify the vorticity region. Since the simulations lie in low Reynolds number regime, the *pinch off* phenomenon (i.e. a secondary vortex ring is induced behind the leading vortex ring) reported in previous studies [12] is not observed in the current investigation. Instead, there is merely a single vortex ring formed by the jet flow and propagates downstream. The spread speed of the vortex ring is closely associated with V_j' , the jet speed at the nozzle exit A' . Accordingly, tubular nozzles hardly affect the vortex motion trajectory since V_j' is the same as V_j at the orifice. On the other hand, the jet speed V_j' is increased by employing converging nozzles so that the vortex propagation is accelerated. With diverging nozzles its motion is decelerated.

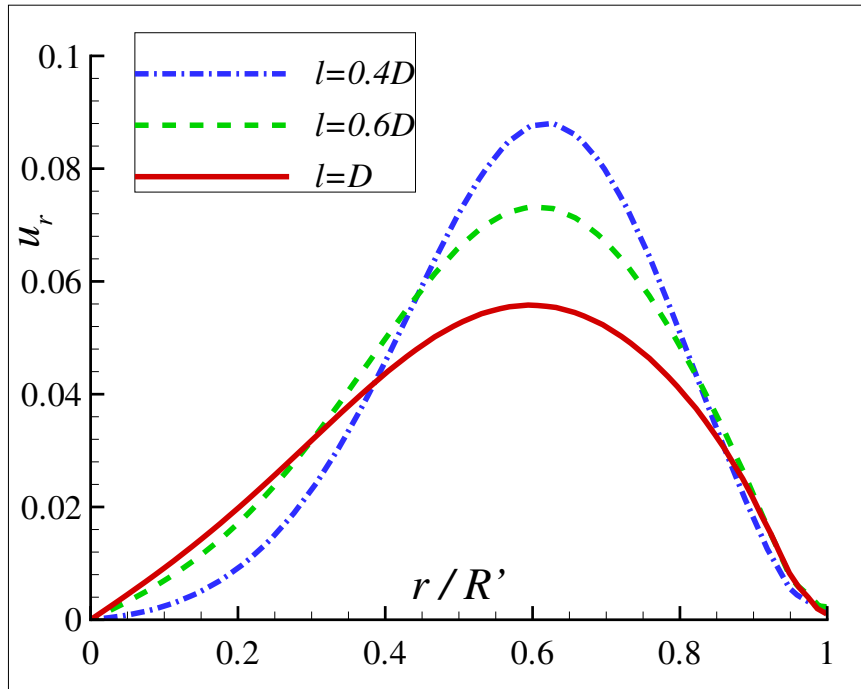


Figure 17. Comparison of radial velocity profiles on the exit of diverging nozzles, $t = 0.5T$.

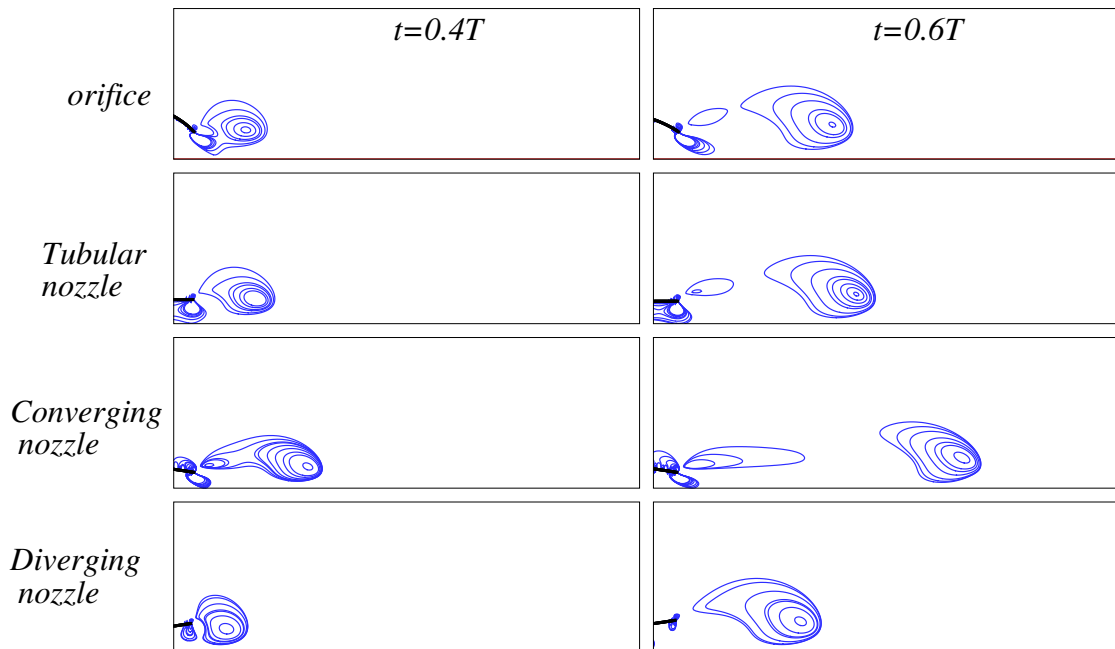


Figure 18. Vortex ring structures at time instants $t = 0.4T$ and $t = 0.6T$, visualized through positive Q . The nozzle configurations are $(l, \theta) = (D, 0^\circ), (D, -8.6^\circ)$ and $(D, 8.6^\circ)$.

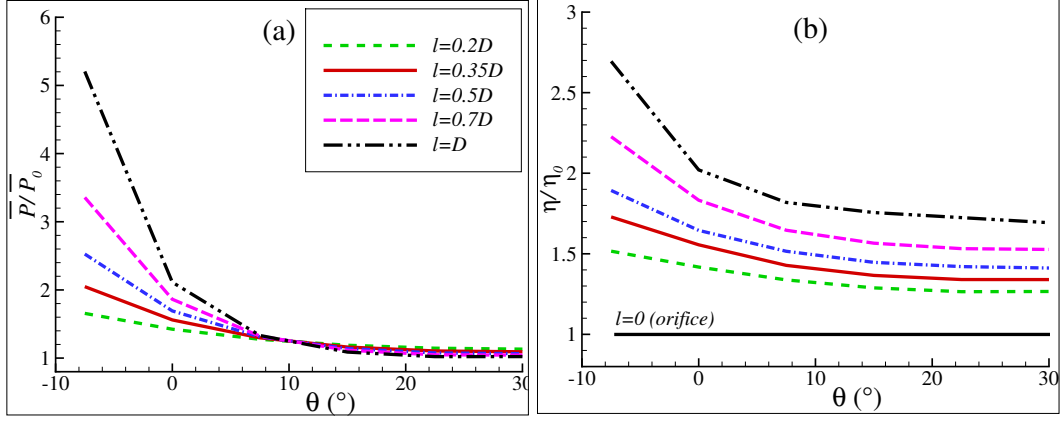


Figure 19. (a) Normalized power expenditure $\bar{P}/\bar{P}_0$ as functions of θ and l , (b) normalized cost of impulse η/η_0 as functions of θ and l .

4.2.3. Cost of impulse

Figure 19a plots the time-averaged power expenditure $\bar{P}$ (normalized by $\bar{P}_0$) as function of θ and l . It shows that when $\theta \leq 0$, $\bar{P} > \bar{P}_0$ so that it takes additional energy to expel fluid through tubular or converging nozzles. For diverging nozzles, however, it depends on the inclination angle θ . Specifically, additional power expenditure is still needed when $\theta < 20^\circ$. Otherwise, when a sufficiently diverging nozzle (e.g. $\theta > 20^\circ$) is installed, $\bar{P}$ is close to $\bar{P}_0$. Figure 19b presents the dependencies of the normalized cost of impulse η/η_0 upon θ and l . Note that higher cost of impulse corresponds to lower efficiency. We find that the shorter the nozzle is, the more efficient the system will be. In fact, the orifice jetting in which $l = 0$ turns out to be the most efficient. Moreover, It is found that the propulsive efficiency tends to be higher as the nozzle gets more diverged till it reaches a threshold (approximately $\theta = 20^\circ$), beyond which the efficiency becomes insensitive to the inclination angle θ .

The energetics of the system can be further improved through various measures. For example, the refilling phase may play a very important role in the efficient locomotion of pulse-jet animals. Indeed, jellyfish exhibit a unique mechanism of passive energy recapture resulting from positive pressure created by a vortex ring underneath the bell during the refilling phase, which may increase the cost of transport by as much as 48% [29]. In addition, other mechanisms such as structural resonance and preferred Strouhal number will further improve the performance [30].

In summary, the geometry of nozzles affects the propulsion performance in terms of thrust generation and efficiency. Specifically, after adding diverging nozzles, both thrust and efficiency are decreased. The usage of tubular nozzles tends to reduce the efficiency while has negligible effect on thrust generation. The converging nozzles, however, will improve the thrust at the cost of efficiency.

5. Conclusions

By using numerical simulations, the propulsive performance of a pulsed-jet system with various types of nozzles has been studied. We aim at exploring how geometrically different nozzles affect the thrust generation and efficiency of the propeller in tethered mode.

First, the original model without any nozzle was tested. A force decomposition strategy was applied to decouple the thrust into three components, namely the jet flux, over-pressure at the orifice, and the axial momentum of fluid inside the chamber. The robustness of the decoupling scheme was validated by the comparison between the actual thrust calculated by *CFD* and the summation of the above three parts obtained from the flow field information. This force decomposition strategy enables a deeper understanding of force generation mechanism during the jetting process.

Systematic computations were conducted to simulate the pulsed jetting of the system with different nozzles added. After installing nozzles, there could be considerable thrust enhancement on the deformable body (without the nozzle), mainly attributed to augmentation of the orifice over-pressure. For the performance of the system including the deformable body and the nozzle, the thrust production is mostly from the jet flux term at the nozzle exit. The other

two components account for relatively small percentage of the overall thrust. As a result, variations in these two components associated with different nozzles do not affect the overall thrust significantly. Therefore the thrust generation of the whole system depends mostly on the nozzle exit size. The time-averaged thrust is found to be proportional to $1/D'^4$, where D' is the diameter of the nozzle exit. Subsequently, tubular nozzles generate thrust close to the one by the system without nozzle. Converging and diverging nozzles result in increased and declined thrust production, respectively. The propulsive efficiency of the whole system was also examined. It is shown that the usage of nozzles makes the jet propulsion less efficient.

The jet-propulsion mechanism of squid and squid larva works over a huge range of Reynolds number ($\mathcal{O}(1 - 10^6)$) [31]. Our axisymmetric model, on the other hand, is only applicable within the low Reynolds number regime since turbulence and three-dimensional flow effects in high Reynolds numbers are not considered. In future studies three-dimensional models with proper consideration of turbulence effects are needed to study the problem in a broad range of Reynolds numbers. Meanwhile, the current study concentrates on a tethered configuration without incoming flow so that the added-mass effect is not important. In reality, the motion of squid may be skewed in time so that the inertia of the fluid (especially the added-mass effect) may contribute significantly to thrust generation [32]. These are well within the capacity of our numerical method. Future studies in this direction will be conducted.

According to previous studies, an important parameter in pulsed jetting problems is the stroke ratio, defined as the length of the jet plug created by a discharging divided by its diameter. This parameter may affect the wake evolution as well as the generation of force. However, the effect of this parameter depends on Reynolds number and jet profile. For example, unlike high Reynolds number cases, in the low Reynolds number regime the pinch-off of the leading vortex ring does not occur [22, 33]. Besides, with a sinusoidal jet profile there does not exist a universal formation number featuring the saturation of the leading vortex ring [22]. Within the Reynolds regime we consider the thrust generation is found to be dominated by the jet flux momentum, which depends on the stroke ratio through its effect on the jet speed. After scaling with the jet speed the thrust shows very little dependence on the stroke ratio [34]. For this reason in the current study we concentrate on just one value of the stroke ratio.

It is necessary to note that in operation a pulsed-jet propeller experiences not only jetting phases but also recovery phases through which the body is inflated to refill the chamber and get ready for the next jetting. These will be directions of future studies .

Appendix A Impulse of an isolated vortex ring

Considering the wake region behind the exit plane as a control volume Ω' as illustrated in Fig. 20, the axial momentum conservation of the control volume yields [22]

$$\frac{\partial}{\partial t} \iiint_{\Omega'} \rho u_x dV \approx \iint_{A'} \rho u_x^2 dS + \iint_{A'} (p - p_\infty - \tau) dS = F_t^j + F_t^\sigma, \quad (24)$$

in which the flow flux and fluid stress term on the control volume boundaries except A' are ignored. For an isolated vortex ring, the momentum can be approximated by $(m + m_e + m_a)W$, where W is the translating speed of the vortex ring centroid in the axial direction, m is the ejected fluid mass (note that $dm/dt = \rho A' V_j'$), m_a is the added mass from the ambient fluid, m_e represents the ambient fluid entrained by the shear layer from the exit boundary layer as it rolls up into the ring [35, 13]. Assuming that the majority of the momentum inside the control volume is stored in the vortex ring bubble (this is reasonable at the initial stage when only a vortex ring is shed and there is no trailing flow following behind the ring), Eq. (24) can be rewritten as

$$\frac{\partial(m + m_e + m_a)W}{\partial t} \approx F_t^j + F_t^\sigma, \quad (25)$$

Therefore

$$\rho A' V_j' W + \frac{d(m_e + m_a)}{dt} W + (m + m_e + m_a) \frac{dW}{dt} \approx F_t^j + F_t^\sigma. \quad (26)$$

Given the fact that the translating speed W is roughly proportional to the jet speed V_j' [36], it is reasonable that the first term of above equation is closely related to the jet flux F_t^j and the second and third terms can be regarded as the source of F_t^σ ,

$$F_t^\sigma \sim \frac{d(m_e + m_a)}{dt} W + (m + m_e + m_a) \frac{dW}{dt}. \quad (27)$$

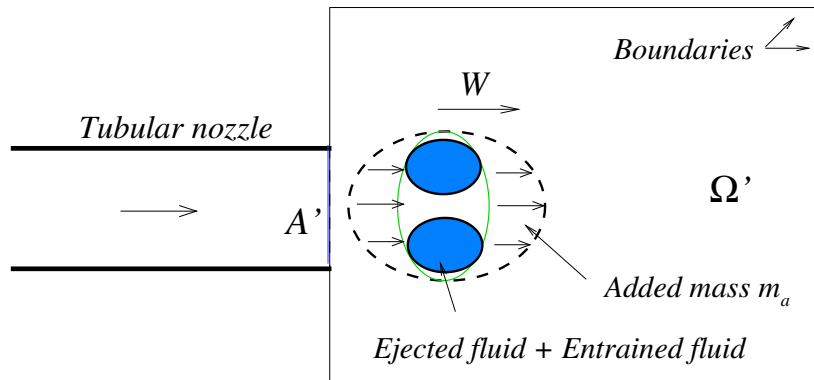


Figure 20. Illustration of the control volume and vortex bubble. This figure is modified from Fig. 10 in reference [13].

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