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ABSTRACT

A classical calculation has been carried out to determine the effect of nucleon structure upon scattering of photons. This structure is provided by a pseudoscalar field with gradient coupling to an extended source of size $a\mu = \pi/2$. The nucleon is coupled directly to the electromagnetic field by means of its Dirac magnetic moment.

The results for both charge-symmetric and neutral pion-nucleon coupling are obtained. It is found in both cases that the scattering exhibits a resonance near $\omega/\mu = 1.4$ for coupling constant $g^2/4\pi = .3$.

The angular distribution is symmetric about $\theta = \pi/2$ for the neutral case, but shows a forward peaking below resonance in the charge-symmetric theory. This peaking shifts to backward angles above resonance.

The general features of this calculation indicate that observation of the scattering of photons in the appropriate energy region might provide a sensitive means of studying the structure of nucleons.

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I. Introduction

A fundamental problem of current physical research is the investigation of the nature of elementary particles. Nucleons (protons and neutrons) are important as the constituents of atomic nuclei and an understanding of their properties is a necessary basis for the theory of nuclear structure. Information concerning the nature of nucleons may be obtained through study of their electromagnetic properties, e.g. anomalous magnetic moments[1] and the neutron-electron interaction.[2] Both of these effects have been studied theoretically on the assumption that they arise out of the nucleon-meson interaction.[3,4]

The structure given to the nucleon by this interaction will also modify its ability to scatter light. Corrections to the Klein-Nishina cross section for the scattering of light from Dirac particles have been calculated by Sachs and Foldy,[5] and Minami,[6] using weak coupling perturbation theory. The present paper deals with a classical calculation in which some of the meson effects can be included to all orders in the meson-nucleon coupling coupling constant, in particular those having to do with the gyration of the nucleon spin and isotopic spin. Such a treatment leads to results qualitatively different from the weak coupling calculations.

Experiments on the scattering of light from nucleons at energies up to several times the pion rest energy can be expected to provide information about the virtual meson cloud surrounding the nucleon.

II. Description of the Model

The nucleon has an intrinsic angular momentum or spin of $\frac{1}{2}\hbar$, ($\hbar = h/2\pi$ where h is Planck's constant), and obeys Fermi-Dirac statistics. Consequently, if it were coupled to no other fields, its field amplitude would satisfy the free Dirac equation. To describe the production, absorption, and scattering of pions by nucleons, the nucleon and pion fields are coupled together by a relativistically invariant interaction. It is generally assumed that this interaction provides a major part of the nuclear forces acting between nucleons, at least in the energy region well below the rest energy of heavier mesons. We wish now to consider the way in which the pion-nucleon coupling modifies the photon-nucleon scattering process.

The scattering of light by a free Dirac particle is given to order α^2 ($\alpha = e^2/4\pi\hbar c$ is the electromagnetic fine structure constant) by the Klein-Nishina cross section formula.[7] Because of the pion-nucleon coupling the nucleon is surrounded by a virtual pion cloud. This cloud affects the scattering of light directly through its charge and current and indirectly through its dynamical interaction with the spin and isotopic spin of the nucleon.

To distinguish between the two modes by which the scattering is modified, calculations have been performed for both charge symmetric and neutral pions. Since the latter are not directly coupled to the electromagnetic field, and comparison of the two models will permit a separation of the effects.

The problem will be set up in a relativistically covariant manner and equations of motion will be obtained for the nucleon, pion, and photon field amplitudes. Terms involving nucleon recoil will then be neglected and the nucleon source density will be identified. Equations of motion for the expectation values of the nucleon spin and isotopic spin can then be derived. These spins are the dynamical variables in terms of which the static nucleon is described. The pion-nucleon coupling can then be treated to all orders of the coupling constant. In this way both weak and strong coupling limits can be evaluated in the static nucleon limit.

The meson field will be chosen to correspond to pions and therefore will be a pseudoscalar field describing particles of mass $\mu \cong 140$ Mev/c^2 and spin zero. The pion-nucleon coupling is taken to be pseudovector which reduces to $\sigma \cdot \nabla \phi$ in the static limit.

III. Derivation of the Static Nucleon Limit

In order to trace the connection between the covariant field equations and those describing the static nucleon approximation we begin with an appropriate Lagrange density from which the relativistic field equations for the nucleon, photon, and pion may be derived by the use of Hamilton's principle.

The Lagrange density describing these fields will be taken to be

$$\mathcal{L}(x) = -\bar{\psi}\left[\gamma_\nu D_\nu + M\right]\psi - \tfrac{1}{2}\left[\Pi_\nu\phi \cdot \Pi_\nu\phi + \mu^2\phi^2\right] - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} - \Gamma_\nu \cdot \Pi_\nu\phi \tag{1}$$

where $\psi(x)$, the nucleon field amplitude, is a two-component symbol in the isotopic spin space of the nucleon and a four-component spinor in ordinary spin space. The amplitude $\bar{\psi}(x) = \psi^*(x)\gamma_4$, where $\psi^*(x)$ is the Hermitian conjugate of $\psi(x)$ and the γ_ν ($\nu = 1, 2, 3, 4$) are a set of Hermitian operators in spin space which satisfy the relations

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu} \qquad (\mu, \nu = 1, 2, 3, 4). \tag{2}$$

The operator

$$D_\nu = \partial_\nu - ie\,\tau_p A_\nu \qquad (\nu = 1, 2, 3, 4) \tag{3}$$

where $\partial_\nu \equiv \partial/\partial x_\nu$, with $x_4 = i x_0$. The proton electric charge is denoted by e, $A_\nu(x)$ is the four-potential of the electromagnetic field, and τ_p is the operator in the isotopic spin space of the nucleon having eigenvalues of $+1$ and 0 for the proton and neutron respectively. (We have chosen natural units such that $\hbar = 1$, $c = 1$). The nucleon and pion rest masses are M and μ, respectively.

The pion field amplitude, $\phi(x)$, is a three-component vector in the isotopic spin space of the pion. The operator Π_ν acting on a vector $\vec{\chi}$ in the isotopic spin space of the pion yields

$$\Pi_\nu \vec{\chi} = \partial_\nu \vec{\chi} + eA_\nu(x)\, \vec{\chi} \times \vec{\ell} \quad (\nu = 1, 2, 3, 4) \tag{4}$$

where $\vec{\chi} \times \vec{\ell}$ is the vector product of $\vec{\chi}$ and a unit vector $\vec{\ell}$ with components $(0, 0, 1)$ in the representation in which

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The isotopic spin operator $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ is a vector in pion isotopic spin space, having as components the Pauli spin matrices in nucleon isotopic spin space. With $\vec{\ell}$ defined in this way, τ_p may be expressed as

$$\tau_p = \tfrac{1}{2}(1 - \vec{\tau}\cdot\vec{\ell}) . \tag{5}$$

(From this point on the arrows on isotopic vectors will be omitted, and juxtaposition of two such vectors will denote the inner product.)

The electromagnetic field $F_{\mu\nu}$ is given by the antisymmetric tensor

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x). \tag{6}$$

The final term in the Lagrange density is the pion-nucleon charge symmetric interaction, where

$$\Gamma_\nu = \frac{ig}{\mu} \bar{\psi} \gamma_\nu \gamma_5 \tau \psi \tag{7}$$

is a pseudovector in configuration space and a three-component vector in pion isotopic spin space. The Hermitian operator

$$\gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4$$

anticommutes with the γ_ν $(\nu = 1, 2, 3, 4)$ and satisfies the condition $\gamma_5^2 = 1$. Finally, g is the pion-nucleon pseudovector coupling constant, and a factor $1/\mu$ is included to make it have the dimensions of a charge.

In the case where only neutral pions are coupled to the nucleon, its isotopic spin will be a constant of the motion. The classical equations for the neutral case will be inferred from those of the charge symmetric theory in order not to duplicate the derivation.

In order that the variation of $\int d^4x \, \mathcal{L}(x)$ vanish when $\psi(x)$, $\bar{\psi}(x)$, $\phi(x)$, and $A_\nu(x)$ are varied independently, the field amplitudes must satisfy the equations

$$\left[\gamma_\nu D_\nu + M + \frac{ig}{\mu} \gamma_\nu \gamma_5 \tau \cdot (\Pi_\nu \phi) \right] \psi = 0 \quad , \tag{8}$$

$$\left[(\Pi_\nu)^2 - \mu^2 \right] \phi = - \Pi_\nu \Gamma_\nu , \tag{9}$$

and

$$\partial_\mu F_{\mu\nu} = -ie \; \bar{\psi} \gamma_\nu \tau_p \psi + e\phi \times \ell \cdot \left[\Pi_\nu \phi + \Gamma_\nu \right] . \tag{10}$$

To obtain the static approximation for the nucleon field, we make the usual decomposition leading to the nonrelativistic limit

$$\psi(x) = \Omega(x) + \chi(x) \tag{11}$$

where

$$\Omega(x) = \tfrac{1}{2}(1 + \gamma_4)\psi(x) \tag{12a}$$

and

$$\chi(x) = \tfrac{1}{2}(1 - \gamma_4)\psi(x) \quad . \tag{12b}$$

Making use of the anticommuting properties of the γ_ν (ν = 1, 2, 3, 4) and γ_5 , and multiplying equation (8) from the left by $\frac{1}{2}(1 + \gamma_4)$:

$$\left[D_4 + M + \frac{ig}{\mu} \gamma_k \gamma_5 \tau \Pi_k \phi \right] \Omega + \left[\gamma_k D_k + \frac{ig}{\mu} \gamma_5 \tau \Pi_4 \phi \right] \chi = 0 \tag{13}$$

where the Roman subscript k is summed from 1 to 3 only.

Similarly, multiplying equation (8) from the left by $\frac{1}{2}(1 - \gamma_4)$:

$$\left[\gamma_k D_k - \frac{ig}{\mu} \gamma_5 \tau \Pi_4 \phi \right] \Omega + \left[-D_4 + M + \frac{ig}{\mu} \gamma_k \gamma_5 \tau \Pi_k \phi \right] \chi = 0 \quad . \tag{14}$$

The energy of the nucleon may be written as the sum of its rest energy and a term W which includes its kinetic energy and interaction energy, and which remains finite as M approaches infinity.

$$\partial_4 \psi = -(M + W)\,\psi \,. \tag{15}$$

Substituting into equations (13) and (14), we obtain

$$\left[-W - ieA_4 \tau_p + \frac{ig}{\mu}\gamma_k \gamma_5 \, \tau \Pi_k \phi\right]\Omega + \left[\gamma_k D_k + \frac{ig}{\mu}\gamma_5 \tau \, \Pi_4 \phi\right]\chi = 0 \tag{16}$$

and

$$\left[\gamma_k D_k - \frac{ig}{\mu}\gamma_5 \tau \, \Pi_4 \phi\right]\Omega + \left[W + 2M + ieA_4 \tau_p + \frac{ig}{\mu}\gamma_k \gamma_5 \, \tau \, \Pi_k \phi\right]\chi = 0 \,. \tag{17}$$

If we expand $\psi(x)$ in inverse powers of M

$$\psi(x) = \sum_{n=0}^{\infty} \frac{1}{M^n}\, \psi_{(n)}(x) \,, \tag{18}$$

the equations for $\psi_{(0)}$, obtained by equating equal powers of M in equations (16) and (17), are

$$\left[-W - ieA_4 \, \tau_p + \frac{ig}{\mu}\,\gamma_k \gamma_5 \, \tau \, \Pi_k \, \phi\right]\Omega_{(0)} = 0 \tag{19}$$

and

$$\chi_{(0)} = 0 \,. \tag{20}$$

In the limit of a fixed nucleon of infinite mass

$$\psi(x) \xrightarrow[M\to\infty]{} \Omega_{(0)} + \chi_{(0)} = \Omega_{(0)} \quad . \tag{21}$$

Redefining the zero of energy to eliminate the infinite mass term, W is then to be identified with the energy operator $\frac{i\partial}{\partial t} = -\frac{\partial}{\partial x_4}$.

If we choose the spin representation so that $\gamma_k = -i\beta\,\gamma_k$, $\gamma_4 = \beta$ where α_k, β are the usual anticommuting Dirac matrices, then it follows that

$$\gamma_k\,\gamma_5 = \gamma_k\;\gamma_1\,\gamma_2\,\gamma_3\,\gamma_4 = \beta\,\alpha_k\alpha_1\alpha_2\alpha_3 = i\beta\sigma'_k \tag{22}$$

where

$$\alpha_k = \left(\begin{array}{c|c} 0 & \sigma_k \\ \hline \sigma_k & 0 \end{array}\right), \quad \beta = \left(\begin{array}{c|c} I & 0 \\ \hline 0 & -I \end{array}\right), \quad \sigma'_k = \left(\begin{array}{c|c} \sigma_k & 0 \\ \hline 0 & \sigma_k \end{array}\right) \tag{23}$$

are 4x4 matrices expressed in terms of the 2x2 unit matrix I and the Pauli matrices σ_k, which satisfy the relations

$$\sigma_i\,\sigma_j = i\,\varepsilon_{ijk}\sigma_k \quad . \tag{24}$$

(ε_{ijk} is the completely antisymmetric pseudotensor which vanishes unless all subscripts are distinct, in which case it equals +1 (-1) for even (odd) permutations of (i, j, k).)

The equation (19) for the static nucleon amplitude has been obtained by neglecting all inverse powers of M, the nucleon rest mass. One such term, the Dirac magnetic moment interaction, may be written in the form

$$-\frac{e}{4M}\sigma_{\mu\nu}F_{\mu\nu} = \frac{e}{2M}\vec{\sigma}\cdot\vec{H} \tag{25}$$

where

$$\sigma_{\mu\nu} = \varepsilon_{\mu\nu\lambda}\sigma_{\lambda} \qquad (\mu, \nu, \lambda = 1, 2, 3)$$

and $\vec{H} = \operatorname{curl}\vec{A}$ is the magnetic field. Since we can consider the nuclear magneton $\mu_0 = \frac{e}{2M}$ to remain finite for a static nucleon, we shall include this spin-dependent term in the nucleon field equation, even though it is of order M^{-1}, to obtain

$$i\frac{\partial}{\partial t}\psi = \mathcal{H}_s\psi \tag{26}$$

where

$$\mathcal{H}_s = -ieA_4\gamma_p - \mu_0\gamma_p\vec{\sigma}\cdot\vec{H} - \frac{g}{\mu}\gamma\vec{\sigma}\cdot\vec{\Pi}\phi \ ; \tag{27}$$

and we have replaced Ω_0 by ψ which therefore has only two nonvanishing spin components,

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\\ 0\\ 0\end{pmatrix} \tag{28}$$

so that $\beta\sigma_k'\psi$ simplifies to $\sigma_k\psi$.

The nucleon spins are the only dynamical variables describing the nucleon. It is therefore desirable to identify a spin density

$\sigma_\nu(x)$ and an isotopic spin density $\tau_k(x)$, and to express all other nucleon-dependent quantities in terms of these.

The nucleon-dependent terms occurring in the pion and photon field equations (9) and (10) are of two types, which may be simplified to the following form by the use of equations (22), (23), and (28),

$$\Gamma_\nu = \frac{ig}{\mu} \bar{\psi} \gamma_\nu \gamma_5 \tau \psi \xrightarrow[M \to \infty]{} -\frac{g}{\mu} \psi^* \sigma_\nu \tau \psi \tag{29}$$

where

$$\sigma_4 \equiv 0 .$$

The second type of term is

$$\bar{\psi} \gamma_\nu \tau \psi \xrightarrow[M \to \infty]{} \delta_{4\nu} \psi^* \tau \psi + \frac{i}{2M} \partial_\mu \psi^* \sigma_{\mu\nu} \tau \psi . \tag{30}$$

To elaborate these expressions, we begin with the anticommutator between $\psi_{\rho i}(x)$ and $\psi^*_{\rho' i'}(x')$ at equal times, where the ρ's and i's are spin and isotopic spin indices respectively,

$$\left[\psi_{\rho i}(x), \psi^*_{\rho' i'}(x) \right]_+ = \delta_{\rho\rho'} \delta_{ii'} \delta(x - x') ; \tag{31}$$

this allows us to write

$$\delta(x - x') \psi^*(x) \sigma_\nu \tau \psi(x) = \psi^*_{\rho i}(x) \sigma^\nu_{\rho\lambda} \delta_{\lambda\mu} \delta_{i\rho} \delta(x - x') \tau_{jk} \psi_{\mu k}(x') .$$

$$= \psi^*(x) \sigma_\nu \psi(x) \psi^*(x') \tau \psi(x') + \psi^*(x) \sigma_\nu \psi^*(x') \psi(x) \tau \psi(x') . \tag{32}$$

The last term of this equation has two $\psi(x)$ factors on the right which annihilate nucleons or create antinucleons. The creation of antinucleons will correspond to vacuum fluctuations in the nucleon field. In deriving the classical model, we shall neglect all such effects. The remaining term that annihilates two nucleons gives zero when acting on a one-nucleon state and so vanishes in the present case. If we now identify the spin and isotopic spin densities to be

$$T(x) = \psi^*(x)\, \tau\, \psi(x) \tag{33}$$

and

$$\vec{S}(x) = \psi^*(x)\, \vec{\sigma}\, \psi(x) \quad , \tag{34}$$

equation (32) may be written in the form

$$\delta(x - x')\, \psi^*(x)\, \sigma_\nu\, \tau\, \psi(x) = S_\nu(x)\, T(x') + \text{(2 nucleon terms)} \ . \tag{35}$$

Neglecting vacuum fluctuations as was proposed above, we notice that the product term on the left has been separated into an ordinary spin density multiplying an isotopic spin density. This product is seen to vanish unless the two densities are taken at the same point, $x = x'$, and furthermore this point must be the origin in order that a function of $(x - x')$ on the left will be equal to a product of separate functions of x and x' on the right. This will occur if $\vec{S}(x)$ and $T(x)$ are each proportional to $\delta(x)$, and so we write

$$\vec{S}(x) = \vec{\sigma}(x)\ \delta(x) \tag{36}$$

and

$$T(x) = \tau(x)\ \delta(x) \tag{37}$$

whereupon equation (35) takes the form

$$\psi^*(x)\sigma_\nu \; \tau\psi(x) = \sigma_\nu(x) \; \tau(x) \; \delta(x) \quad . \tag{38}$$

The infinites presented by $\delta(x)$ may be removed by spreading out the point nucleon to a source of finite size, which we accomplish by replacing the delta function by an extended source $U(x)$ with the same volume integral. Equations (36) through (38) are then

$$\vec{S}(x, t) = \vec{\sigma}(x, t) \, U(x) \quad , \tag{39}$$

$$T(x, t) = \tau(x, t) \, U(x) \quad , \tag{40}$$

and

$$\psi^*(x)\sigma_\nu \tau \psi(x) = \sigma_\nu(x) \; \tau(x) \; U(x), \tag{41}$$

where the fixed source is normalized to one nucleon,

$$\int d^3x \;\; U(x) = 1 \; . \tag{42}$$

The equations of motion for $\vec{\sigma}(x)$ and $\tau(x)$ may now be derived from the equation for $\psi(x)$. For $\vec{\sigma}(x)$, we find, using equations (39) and (41) and dividing both sides by $U(x)$,

$$\begin{aligned} \dot{\vec{\sigma}}(x) &= i\,\psi^*(x) \left[\mathcal{H}_s, \vec{\sigma}\right]_- \psi(x) \\ &= 2\,\psi^*(x)\left[\frac{g}{\mu}\,\tau\,\vec{\sigma}\times\vec{\Pi}\phi(x) + \mu_o\,\tau_p\,\vec{\sigma}\times\vec{H}(x)\right]\psi(x) \\ &= \vec{\sigma}(x) \times 2\left[\frac{g}{\mu}\,\tau(x)\,\vec{\Pi}\,\phi(x) + \mu_o\,\tau_p\,\vec{H}(x)\right] \end{aligned} \tag{43}$$

where the dot over $\vec{\sigma}$ signifies differentiation with respect to the time.

Similarly for $\Upsilon(x)$,

$$\dot{\Upsilon}(x) = i\,\psi^*(x)\left[\mathcal{H}_s,\ \Upsilon\right]_-\psi(x)$$

$$= 2\,\psi^*(x)\left[\frac{g}{\mu}\ \Upsilon\times\vec{\sigma}\cdot\vec{\Pi}\phi - \frac{\mu_0}{2}\ \Upsilon\times\ell\,\vec{\sigma}\cdot\vec{H} - i\,\frac{e}{2}\,A_4\,\ell\right]\psi(x) \tag{44}$$

$$= \Upsilon(x)\times 2\left[\frac{g}{\mu}\,\vec{\sigma}(x)\cdot\Pi\phi(x) - \frac{\mu_0}{2}\,\ell\,\vec{\sigma}(x)\cdot\vec{H}(x) - i\,\frac{e}{2}\,\ell\,A_4(x)\right].$$

The introduction of the spin densities will also modify slightly the form of the pion and photon equations. In place of equations (9) and (10) we have, by direct substitution from equations (39) through (41),

$$\left[(\Pi_\nu)^2 - \mu^2\right]\phi(x) = \frac{g}{\mu}\ \Pi_\nu\left[\sigma_\nu(x)\ \Upsilon(x)\ U(x)\right] \tag{45}$$

where Π_ν acts upon the entire bracket expression, and for the photon field

$$\partial_\mu F_{\mu\nu}(x) = -\,ie\,\delta_{4\nu}\ \Upsilon_p(x)\ U(x) + \mu_0\,\partial_\mu\left[\Upsilon_p(x)\ \sigma_{\mu\nu}(x)\ U(x)\right]$$

$$+\ e\phi(x)\times\ell\cdot\left[\Pi_\nu\phi(x) - \frac{g}{\mu}\ \Upsilon(x)\,\sigma_\nu(x)\ U(x)\right] \tag{46}$$

where the Dirac magnetic moment term has been added for reasons discussed previously.

These two equations, together with the spin density equations (43) and (44), characterize the interaction of the pion and photon field with an extended nucleon source. The way in which this extension has been carried out is not unique and is to be considered merely as a heuristic device rather than as a precise derivation.

The equations at which we have arrived involve point interactions between certain spin densities and the other field quantities. It will now be shown that the introduction of these spin densities has preserved the differential conservation of the electromagnetic current density. From equation (46) for the electromagnetic field, the current density is seen to be

$$J_\nu(x) = ie\,\delta_{4\nu}\,\tau_p(x)\,U(x) - \mu_0\,\partial_\mu\left[\tau_p(x)\,\sigma_{\mu\nu}(x)\,U(x)\right]$$
$$- e\,\phi(x) \times \ell \cdot \left[\Pi_\nu\phi(x) - \frac{g}{\mu}\,\tau(x)\,\sigma_\nu(x)\,U(x)\right] . \tag{47}$$

In order that the charge and current be conserved pointwise within the nucleon source, the divergence of $J_\nu(x)$ must vanish. To verify this we differentiate equation (47), obtaining

$$\frac{1}{e}\,\partial_\nu J_\nu(x) = \dot{\tau}_p(x)\,U(x) - \partial_\nu\,\phi(x) \times \ell \cdot \left[\Pi_\nu\phi(x) - \frac{g}{\mu}\,\tau(x)\,\sigma_\nu(x)\,U(x)\right]$$
$$- \phi(x) \times \ell \cdot \left[\partial_\nu\,\Pi_\nu\,\phi(x) - \frac{g}{\mu}\,\partial_\nu(\tau(x)\,\sigma_\nu(x)\,U(x))\right] \tag{48}$$

where the divergence of the second term in $J_\nu(x)$, the Dirac moment term, vanishes because of the antisymmetry of $\sigma_{\mu\nu}(x)$.

To evaluate the right-hand side of this expression we notice that

$$\dot{\tau}_p(x) = -\tfrac{1}{2}\,\dot{\tau}(x)\cdot \ell = \frac{g}{\mu}\,\tau(x) \times \ell\ \sigma(x)\cdot \Pi\, \phi(x) \;; \tag{49}$$

using the definition of $\tau_p(x)$ and equation (44), the equation of motion for the isotopic spin density. In the second term of equation (48),

$$\begin{aligned} -\partial_\nu \phi(x) \times \ell \cdot \Pi_\nu \phi(x) &= -\partial_\nu\ \phi(x) \times \ell \left[\partial_\nu \phi(x) + eA_\nu \phi(x) \times \ell\right] \\ &= -eA_\nu(x)\,\partial_\nu\ \phi(x) \times \ell \cdot \phi(x) \times \ell \;, \end{aligned} \tag{50}$$

since in the first term $\partial_\nu \phi(x) \times \ell$ is orthogonal to $\partial_\nu \phi(x)$. The last term on the right of equation (48) can be rearranged into the form

$$\begin{aligned} -\phi(x) \times \ell \cdot \Big[\partial_\nu\, \Pi_\nu\ \phi(x) - \frac{g}{\mu}\,\partial_\nu\,(\tau(x)\,\sigma_\nu(x)\ \ U(x)\Big] \\ = -\phi(x) \times \ell \cdot \Big\{\Big[(\Pi_\nu)^2 - \mu^2\Big]\phi(x) - \frac{g}{\mu}\,\Pi_\nu\Big[\tau(x)\,\sigma_\nu(x)\ U(x)\Big] \\ - eA_\nu(x)(\Pi_\nu\,\phi(x)) \times \ell + \frac{eg}{\mu}\,A_\nu(x)\,\tau(x) \times \ell\ \sigma_\nu(x)\ U(x)\Big\} \\ = eA_\nu(x)\ \phi(x) \times \ell\Big[\partial_\nu\ \phi(x) \times \ell - \frac{g}{\mu}\,\tau(x) \times \ell\ \sigma_\nu(x)\ U(x)\Big], \end{aligned} \tag{51}$$

where we have used the pion field equation (45) and the isotopic spin dependence of $\Pi_\nu\, \phi(x)$ to arrive at the last line.

Substituting these results into equation (48) for the divergence of the current, we have

$$\begin{aligned}
\frac{1}{e}\partial_\nu J_\nu(x) &= \frac{g}{\mu}\,\tau(x)\times\ell\;\sigma_\nu(x)\,\Pi_\nu\,\phi(x)\,U(x) \\
&\quad + \partial_\nu\phi(x)\times\ell\left[\frac{g}{\mu}\,\tau(x)\,\sigma_\nu(x)\,U(x) - eA_\nu(x)\phi(x)\times\ell\right] \\
&\quad + eA_\nu(x)\phi(x)\times\ell\cdot\left[\partial_\nu\phi(x)\times\ell - \frac{g}{\mu}\,\tau(x)\times\ell\,\sigma_\nu(x)U(x)\right]
\end{aligned} \tag{52}$$

$$\begin{aligned}
&= \frac{g}{\mu}\,\tau(x)\times\ell\;\sigma_\nu(x)\,\Pi_\nu\,\phi(x)\,U(x) \\
&\quad - \frac{g}{\mu}\,\tau(x)\times\ell\;\sigma_\nu(x)\left[\partial_\nu\,\phi(x) - eA_\nu(x)\;\phi(x)\times\ell\right]U(x) \\
&= 0
\end{aligned}$$

from the definition of the operator Π_ν. We have thus shown that this model of the static nucleon, in which the spin and isotopic spin are interpreted as densities that vary over the extension of the nucleon, conserves charge from point to point within the fixed source. In this model the spin densities at different nucleon points are independent dynamical variables and the spins are therefore represented as field quantities.

It should be noted that equations (43) and (44) for the spin densities are nonlinear simultaneous integrodifferential equations, and attempts to solve them in the present application have proven uniformly unsuccessful. Expansion of $\sigma(x)$ and $\tau(x)$ in terms of their radial and angular dependence have shown that approximations based on linearization must necessarily lose many important aspects of the spin behavior. Nevertheless, in order to make the problem mathematically tractable, we shall henceforth retain only the space-independent parts of $\sigma(x)$ and $\tau(x)$ and obtain approximate equations of motion for σ and τ by multiplying equations (43) and (44) by $U(x)$ and integrating over space to obtain

$$\dot{\vec{\sigma}} = \vec{\sigma} \times 2 \int d^3x \left[\frac{g}{\mu} \, \tau \vec{\Pi} \phi(x) + \mu_0 \, \tau_p \vec{H}(x) \right] U(x) \tag{53}$$

and

$$\dot{\tau} = \tau \times 2 \int d^3x \left[\frac{g}{\mu} \, \vec{\sigma} \cdot \vec{\Pi} \, \phi(x) - \frac{\mu_0}{2} \, \ell \, \vec{\sigma} \cdot \vec{H}(x) - i \, \frac{e}{2} \, \ell \, A_4(x) \right] U(x) \, . \tag{54}$$

These, it will be recognized, are the more usual classical spin equations. We look upon them as approximate counterparts to the spin density equations. Because of the nonlocal interaction implied in their integral structure, the continuity equation is no longer satisfied by the electromagnetic current density. Current is conserved over the nucleon source as a whole, but not in a differential sense.

These equations, together with those for the pion and photon fields, will be made the starting point for the next section, in which the Compton scattering will be calculated in this charge-symmetric theory.

IV. Compton Scattering in the Charge Symmetric and Neutral Theory

Let us take for the Lagrangian describing the coupled photon, pion, and nucleon fields

$$\mathcal{L} = \int d^3x \left\{ -\tfrac{1}{4} F_{\mu\nu}(x) F_{\mu\nu}(x) - \tfrac{1}{2}\left[\Pi_\nu \phi(x)\, \Pi_\nu \phi(x) + \mu^2 \phi^2(x) \right] + \left[\frac{g}{\mu}\, \tau\, \sigma_\nu \Pi_\nu \phi(x) + \tfrac{1}{2}\mu_0\, \tau_p\, \sigma_{\mu\nu} F_{\mu\nu}(x) + ie\, \delta_{4\nu} A_\nu(x)\, \tau_p \right] U(x) \right\} \tag{55}$$

where the various symbols have been defined in the previous section. In addition we assume that the nucleon spin and isotopic spin have the Poisson brackets,

$$\left\{ \sigma_\mu , \sigma_\nu \right\}_{P.B.} = 2\, \varepsilon_{\mu\nu\lambda}\, \sigma_\lambda \tag{56}$$

and

$$\left\{ \tau_i , \tau_j \right\}_{P.B.} = 2\varepsilon_{ijk}\, \tau_k \ . \tag{57}$$

The spin equations of motion are of the form

$$\dot{\sigma}_\mu = \left\{ \sigma_\mu , \mathcal{H} \right\}_{P.B.} = \left\{ \mathcal{L} , \sigma_\mu \right\}_{P.B.} \tag{58}$$

since the spin dependent parts of the Hamiltonian, $\mathcal{H}$, and the Lagrangian, $\mathcal{L}$, differ only in sign.

Varying the Lagrangian with respect to $A_\nu(x)$, we find that the photon field satisfies the equation,

$$\partial_\mu F_{\mu\nu}(x) = -ie\delta_{4\nu}\tau_p U(x) + \mu_o \tau_p \sigma_{\mu\nu} \partial_\mu U(x) +$$

$$+ e\phi(x) \times \ell \left[\Pi_\nu \phi(x) - \frac{g}{\mu} \tau \sigma_\nu U(x) \right] . \tag{59}$$

Similarly, varying with respect to $\phi(x)$, we obtain

$$((\Pi_\nu)^2 - \mu^2)\,\phi(x) = \frac{g}{\mu} \sigma_\nu \Pi_\nu (\tau U(x)) . \tag{60}$$

Introducing the D'Alembertian operator $\Box^2 = \partial_\mu \partial_\mu$ and rearranging, this becomes

$$(\Box^2 - \mu^2)\,\phi(x) = \frac{g}{\mu} \sigma_\nu \Pi_\nu (\tau U(x)) - 2eA_\nu(x)\,\partial_\nu \phi(x) \times \ell$$

$$- e^2 A_\nu(x) A_\nu(x) (\phi(x) \times \ell) \times \ell , \tag{61}$$

where the first term on the right couples the pion to the nucleon spin and isotopic spin,and the remaining terms couple it through its charge to the electromagnetic field.

From the Poisson brackets for the spins we find the equations governing their time dependence. Thus for the ordinary spin we have

$$\dot{\vec{\sigma}} = \left\{ \mathcal{L} , \vec{\sigma} \right\}_{P.B.} = \vec{\sigma} \times 2 \int d^3x \left[\frac{g}{\mu} \tau \vec{\Pi} \phi(x) + \mu_o \tau_p \vec{H}(x) \right] U(x) . \tag{62}$$

and for the isotopic spin

$$\dot{\vec{\tau}} = \left\{ \mathcal{L} , \vec{\tau} \right\}_{P.B.} = \vec{\tau} \times 2 \int d^3x \left[\frac{g}{\mu} \vec{\sigma} \cdot \vec{\Pi} \phi(x) - \frac{\mu_o}{2} \ell \vec{\sigma} \cdot \vec{H}(x) + \frac{e}{2} \ell A_o(x) \right] U(x) \tag{63}$$

where we employ three-dimensional vector notation in configuration space and isotopic spin space and write the magnetic field $\vec{H}(x) = \mathrm{curl}\ \vec{A}(x)$ and the potential $A_o(x) = - i\ A_4(x)$.

These equations (59) through (63) are nonlinear. But if we treat the electric charge e as a perturbation and expand the field quantities in powers of e, we obtain in any given order a set of linear equations which we may proceed to solve.

In this expansion, the n'th order term of any quantity will have the subscript n, e.g.:

$$A^\nu(x) = A_o^\nu(x) + eA_1^\nu(x) + e^2A_2^\nu(x) + \ldots , \tag{64}$$

$$\phi(x) = \phi_o(x) + e\phi_1(x) + \ldots , \tag{65}$$

and similarly for the spins.

Equating coefficients of $e^o = 1$, we obtain the following zero-order field and spin equations:

$$\Box^2 A_o(x) = 0 \qquad\qquad \partial_\nu A_o^\nu(x) = 0 , \tag{66a,b}$$

$$(\Box^2 - \mu^2)\, \phi_o(x) = \frac{g}{\mu} \tau_o \, \vec{\sigma}_o \cdot \vec{\nabla} U(x) \quad , \tag{67}$$

$$\dot{\vec{\sigma}}_o = \vec{\sigma}_o \times 2 \frac{g}{\mu} \int d^3x \; \tau_o \vec{\nabla} \phi_o(x)\; U(x) \; , \tag{68}$$

$$\dot{\tau}_o = \tau_o \times 2 \frac{g}{\mu} \int d^3x \; \vec{\sigma}_o \cdot \vec{\nabla} \phi_o(x)\; U(x) \; , \tag{69}$$

where we have restricted the electromagnetic potential to the Lorentz gauge by requiring its divergence to vanish.

The solution of the photon equation (66), representing a transverse plane wave of momentum $\vec{\omega}$, is

$$A_o^{\nu}(x) = \varepsilon_o^{\nu} \, e^{i(\vec{\omega}\cdot\vec{x} - \omega_o t)} \tag{70}$$

where

$$\vec{\omega} \cdot \vec{\omega} = \omega_o^{\,2} \tag{71}$$

and we choose $\varepsilon_o^4 = 0$. The gauge condition, (66b), implies the orthogonality of $\vec{\varepsilon}$ and $\vec{\omega}$,

$$\vec{\varepsilon}_o \cdot \vec{\omega} = 0 \; . \tag{72}$$

To fix the normalization, we take $\vec{\varepsilon}_o$ to be a unit vector, $\varepsilon_o^2 = 1$.

Transforming to momentum space,

$$A_o^{\nu}(\vec{k}) = (2\pi)^{3/2} \, \varepsilon_o^{\nu} \, \delta(\vec{k} - \vec{\omega})\, e^{-i\,\omega_o t} \quad , \tag{73}$$

where we have used the three-dimensional Fourier transformation

$$F(\vec{k}) = \frac{1}{(2\pi)^{3/2}} \int d^3x \; e^{-i\,\vec{k}\cdot\vec{x}} \, F(\vec{x}) \,, \tag{74}$$

with the inverse transformation

$$F(x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \; e^{i\,\vec{k}\cdot\vec{x}} \, F(\vec{k}) \,. \tag{75}$$

The pion equation (67), becomes in momentum space

$$\left(k^2 + \frac{\partial^2}{\partial t^2} + \mu^2\right) \phi_0(\vec{k}, t) = -i \frac{g}{\mu} \tau_0 \vec{\sigma}_0 \cdot \vec{k} \, U(\vec{k}) \,. \tag{76}$$

The boundary conditions which we impose on $\phi_0(\vec{k})$ follow from the fact that it represents the pion field surrounding a nucleon in its ground state. In order that this state correspond to the lowest energy of the pion-nucleon system, we choose the static solution of equation (76) and add none of the homogeneous solution which would represent free mesons. The zero-order pion field then has the form

$$\phi_0(\vec{k}) = -i \frac{g}{\mu} \frac{\tau_0 \vec{\sigma}_0 \cdot \vec{k} \, U(\vec{k})}{k^2 + \mu^2} \,. \tag{77}$$

The pion field is seen to have the angular dependence of a p state, arising from the $\vec{\sigma} \cdot \vec{\nabla}$ coupling to a spherically symmetric source density $U(x)$.

Continuing with the zero-order equations, we transform the spin equations to momentum space to obtain

$$\dot{\vec{\sigma}}_o = \vec{\sigma}_o \times 2i \frac{g}{\mu} \int d^3k \, \vec{k} \, \tau_o \, \phi_o(\vec{k}) \, U(-\vec{k}) \tag{78}$$

and

$$\dot{\tau}_o = \tau_o \times 2i \frac{g}{\mu} \int d^3k \; \vec{\sigma}_o \cdot \vec{k} \, \phi_o(\vec{k}) \, U(-\vec{k}) \; . \tag{79}$$

Looking at equation (78), and remembering that the angular dependence of $\phi_o(\vec{k})$ is $\vec{\sigma}_o \cdot \vec{k}$, we see that the integral must be a vector directed along $\vec{\sigma}_o$ and therefore

$$\dot{\vec{\sigma}}_o = 0 \;, \qquad \text{and} \qquad \vec{\sigma}_o = \text{constant.} \tag{80}$$

Furthermore, in isotopic spin space $\phi_o(\vec{k})$ is parallel to τ_o, which implies

$$\dot{\tau}_o = 0 \;, \qquad \text{and} \qquad \tau_o = \text{constant.} \tag{81}$$

In the zero-order solutions then, the nucleon spins are fixed vectors independent of the time.

The first-order field equations obtained by equating coefficients of e^1 in equations (59) through (63) are found to be

$$\Box^2 A_1^{\nu}(x) = -i \, \delta_{4\nu} \, \tau_{po} \, U(x) + \frac{1}{2M} \tau_{po} \, \vec{\sigma}_o \times \vec{\nabla} \, U(x) \;, \tag{82}$$

$$(\Box^2 - \mu^2) \, \phi_1(x) = \frac{g}{\mu} (\vec{\sigma}_o \tau_1 + \vec{\sigma}_1 \tau_o) \cdot \vec{\nabla} U(x)$$
$$+ \frac{g}{\mu} \vec{\sigma}_o \cdot \vec{A}_o(x) \, \tau_o \times \ell \; U(x) - 2\vec{A}_o(x) \cdot \vec{\nabla} \phi_o(x) \times \ell \;, \tag{83}$$

$$\dot{\vec{\sigma}}_1 = \vec{\sigma}_1 \times 2\,\frac{g}{\mu} \int d^3x\; \tau_o\, \vec{\nabla}\, \phi_o(x)\; U(x)$$

$$+ \vec{\sigma}_o \times 2 \int d^3x \left[\frac{g}{\mu} \left(\tau_1 \vec{\nabla} \phi_o(x) + \tau_o \vec{\nabla} \phi_1(x) \right) + \frac{\tau_{po}}{2M}\, \vec{H}_o(x) \right] U(x) \;, \tag{84}$$

and

$$\dot{\tau}_1 = \tau_1 \times 2\,\frac{g}{\mu} \int d^3x\;\; \vec{\sigma}_o \cdot \vec{\nabla}\, \phi_o(x)\; U(x)$$

$$+ \tau_o \times 2 \int d^3x \left[\frac{g}{\mu} \left(\vec{\sigma}_o \cdot \vec{\nabla} \phi_1(x) + \vec{\sigma}_o \cdot \vec{A}_o(x) \phi_o(x) \times \ell \right) - \frac{\vec{\sigma}_o \cdot \vec{H}_o}{4M}\,(x)\, \ell \right] \cdot U(x) \;. \tag{85}$$

where certain terms from equations (59) through (63) are zero because of the isotopic spin dependence of $\phi_o(x)$ and because $A_o^4(x)$ has been chosen to be zero.

The boundary conditions to be imposed on the solution of these equations follow from the fact that in the absence of the incident plane wave $\vec{A}_o(x)$, the pion field and nucleon spins must reduce to their zero-order solutions. This requires that only the inhomogeneous solutions proportional to $\vec{A}_o(x)$ be chosen, which further implies that the time dependence of the first order pion field and nucleon spins will be $e^{-i\omega t}$ where ω is the photon energy.

Transforming to momentum space and making use of the fact that only a single frequency is excited by the incident photon, we get from equations (82) through (85)

$$k^2 A_1^{\nu}(\vec{k}) = i\tau_{po}\, U(\vec{k})\left(\delta_{4\nu} - \frac{1}{2M}(\vec{\sigma}_o \times \vec{k})_\nu\right), \tag{86}$$

$$\begin{aligned}(k^2 - \omega^2 + \mu^2)\,\phi_1(\vec{k}) = & -i\,\frac{g}{\mu}\,(\vec{\sigma}_o\cdot\vec{k}\,\tau_1 + \vec{\sigma}_1\cdot\vec{k}\,\tau_o)\,U(\vec{k}) \\ & - \frac{g}{\mu}\,\vec{\sigma}_o\cdot\vec{\varepsilon}_o\,\tau_o \times \ell\; U(\vec{k}-\vec{\omega}) \\ & 2\,i\,\vec{\varepsilon}_o\cdot\vec{k}\,\phi_o(\vec{k}-\vec{\omega}) \times \ell\ ,\end{aligned} \tag{87}$$

$$\begin{aligned}-i\,\omega\,\vec{\sigma}_1 = & \; 2\,i\,\frac{g}{\mu}\int d^3k\;\tau_o\left[\left(\phi_o(\vec{k})\,\vec{\sigma}_1 + \phi_1(\vec{k})\,\vec{\sigma}_o\right)\times\vec{k}\right]U(-\vec{k}) \\ & + (2\pi)^{3/2}\,\frac{i}{M}\,\tau_{po}\,\vec{\sigma}_o \times (\vec{\omega}\times\vec{\varepsilon}_o)\,U(-\vec{\omega}),\end{aligned} \tag{88}$$

and

$$\begin{aligned}-i\omega\tau_1 = & \; 2\,i\,\frac{g}{\mu}\int d^3k\;\vec{\sigma}_o\cdot\vec{k}\,(\tau_1 \times \phi_o(\vec{k}) + \tau_o \times \phi_1(\vec{k}))\,U(-\vec{k}) \\ & + \tau_o \times 2\,\frac{g}{\mu}\int d^3k\;\vec{\sigma}_o\cdot\vec{\varepsilon}_o\,\phi_o(\vec{k}-\vec{\omega}) \times \ell\; U(-\vec{k}) \\ & - (2\pi)^{3/2}\,\frac{i}{2M}\,\tau_o \times \ell\;\vec{\sigma}_o\cdot\vec{\omega}\times\vec{\varepsilon}_o\,U(-\vec{\omega})\ .\end{aligned} \tag{89}$$

From equation (80), we see that the first-order electromagnetic potential is time-independent and yields the static electric and magnetic fields arising from the charge and magnetic moment of the proton. There is therefore no scattering of electromagnetic radiation in this order and the lowest nonvanishing order in which it occurs will be e^2.

To solve the spin equations, let us eliminate the first-order pion field $\phi_1(\vec{k})$ with the aid of equation (87). The spin equations then take the form,

$$-i\omega\vec{\sigma}_1 = \vec{\sigma}_o \times \vec{\sigma}_1 N(\omega) + i\,\vec{\sigma}_o \times (\vec{\omega} \times \vec{\mathcal{E}}_o)\, G(\vec{\omega}) , \tag{90}$$

$$-i\omega\tau_1 = \tau_o \times \tau_1 N(\omega) \quad i\,\tau_o \times (\tau_o \times \ell)\, K(\vec{\omega}) + i\,\tau_o \times \ell\, L(\vec{\omega}), \tag{91}$$

where the angular integrations have been performed to obtain

$$N(\omega) = \frac{8\pi}{3}\frac{g^2}{\mu^2}\int_0^\infty dk\, k^4\, U^2(k)\left[\frac{1}{k^2 - \omega^2 + \mu^2} - \frac{1}{k^2 + \mu^2}\right] \tag{92}$$

and the remaining symbols represent the expressions

$$G(\vec{\omega}) = (2\pi)^{3/2}\,\frac{\tau_{po}}{M}\, U(-\vec{\omega}) , \tag{93}$$

$$K(\vec{\omega}) = 2\frac{g^2}{\mu^2}\int d^3k\, U(-\vec{k})\, U(\vec{k}-\vec{\omega})\left\{\frac{2\,\vec{\mathcal{E}}_o\cdot\vec{k}\;\vec{\sigma}_o\cdot\vec{k}\;\vec{\sigma}_o\cdot(\vec{k}-\vec{\omega})}{\left[k^2-\omega^2+\mu^2\right]\left[(\vec{k}-\vec{\omega})^2+\mu^2\right]} - \frac{\vec{\sigma}_o\cdot\vec{\mathcal{E}}_o\;\vec{\sigma}_o\cdot\vec{k}}{\left[k^2-\omega^2+\mu^2\right]} - \frac{\vec{\sigma}_o\cdot\vec{\mathcal{E}}_o\;\vec{\sigma}_o\cdot(\vec{k}-\vec{\omega})}{\left[(\vec{k}-\vec{\omega})^2+\mu^2\right]}\right\}, \tag{94}$$

$$L(\vec{\omega}) = -\frac{(2\pi)^{3/2}}{2M} \vec{\sigma}_o \cdot \vec{\omega} \times \vec{\mathcal{E}}_o U(-\vec{\omega}) . \tag{95}$$

In arriving at these results, we have made use of the fact that $\vec{\sigma}_o$ and τ_o are unit vectors and that the first-order spins $\vec{\sigma}_1$ and τ_1 are orthogonal to the zero-order spins $\vec{\sigma}_o$ and τ_o, respectively. Proceeding to solve the vector equations (90) and (91) in terms of the quantities N, G, K, and L, we resolve $\vec{\sigma}_1$ and τ_1 into components as follows, with $\vec{\xi}_o = \vec{\omega} \times \vec{\mathcal{E}}_o$,

$$\vec{\sigma}_1 = S_A \vec{\sigma}_o \times \vec{\xi}_o + S_B \vec{\sigma}_o \times (\vec{\sigma}_o \times \vec{\xi}_o) , \tag{96}$$

and

$$\tau_1 = T_A \tau_o \times \ell + T_B \tau_o \times (\tau_o \times \ell). \tag{97}$$

Substituting into equations (90) and (91) we find for the components of the ordinary spin,

$$-i \omega S_A = - N(\omega) S_B + i G(\vec{\omega}) \tag{98}$$

$$-i \omega S_B = N(\omega) S_A \tag{99}$$

and for the components of isotopic spin,

$$-i \omega T_A = - N(\omega) T_B + i L(\vec{\omega}) \tag{100}$$

$$-i \omega T_B = N T_A + i K(\vec{\omega}). \tag{101}$$

The solution of the first pair of equations (98) and (99) yields

$$S_A = \frac{\omega G}{N^2 - \omega^2} ; \qquad S_B = \frac{i N G}{N^2 - \omega^2} , \tag{102}$$

while from equations (100) and (101), we find

$$T_A = \frac{-i N K + \omega L}{N^2 - \omega^2} ; \qquad T_B = \frac{i N L + \omega K}{N^2 - \omega^2} . \tag{103}$$

At this point, let us evaluate the expression $N(\omega)$ given in equation (92). The first integral which occurs in this expression may be written in the form

$$\int_0^\infty \frac{dk\ k^4 U^2(k)}{k^2 - \omega^2 + \mu^2} = \int_0^\infty dk\ U^2(k) \left[k^2 + (\omega^2 - \mu^2) - \frac{(\omega^2 - \mu^2)^2}{k^2 - \omega^2 + \mu^2} \right] . \tag{104}$$

In the last term, which converges without the $U^2(k)$, we replace the latter by $1/(2\pi)^3$, while in the first two terms we make the same substitution but integrate only up to a cutoff momentum $k_c = \frac{\pi}{2a}$. In evaluating the last integral, poles appear in the denominator for $\omega > \mu$. To define this integral we deform the contour away from the real axis to yield a result which corresponds to an outgoing spherical wave.

$$\int_0^\infty \frac{dk}{k^2 - \omega^2 + \mu^2 - i\epsilon} = i \frac{\pi}{2} \frac{1}{(\omega^2 - \mu^2)^{\frac{1}{2}}} \tag{105}$$

which gives for $N(\omega)$,

$$N(\omega) = \frac{2}{3}\frac{g^2}{4\pi}\frac{1}{\mu^2 a}\left[\omega^2 - a\mu^3 + ia(\omega^2 - \mu^2)^{3/2}\right]. \tag{106}$$

The remaining first-order equation (87) for the pion field has the solution

$$\phi_1(\vec{k}) = -i\frac{g}{\mu}\frac{(\vec{\sigma}_o\cdot\vec{k}\,\tau_1 + \vec{\sigma}_1\cdot\vec{k}\,\tau_o)\,U(k)}{k^2 - \omega^2 + \mu^2} - \frac{g}{\mu}\frac{\vec{\sigma}_o\cdot\vec{\mathcal{E}}_o\,\tau_o \times \ell\;U(\vec{k}-\vec{\omega})}{k^2 - \omega^2 + \mu^2}$$
$$+ \frac{2\,\vec{\mathcal{E}}_o\cdot\vec{k}\;\vec{\sigma}_o\cdot(\vec{k}-\vec{\omega})\,\tau_o \times \ell\;U(\vec{k}-\vec{\omega})}{\left[k^2 - \omega^2 + \mu^2\right]\left[(\vec{k}-\vec{\omega})^2 + \mu^2\right]} . \tag{107}$$

To obtain the scattering cross section, we must evaluate the asymptotic form of $\check{A}_2(x)$. The boundary conditions on $\check{A}_2(x)$ should correspond to an outgoing spherical wave. We know that the time dependence of $\check{A}_2$ is $e^{-i\omega t}$ because this is the only frequence excited in $\check{j}_2(x)$, and so we may separate it off and write

$$(\nabla^2 - \omega^2)\,\check{A}_2(x) = -\check{j}_2(x) . \tag{108}$$

The solution corresponding to an outgoing spherical wave is then

$$\check{A}_2(x) = \frac{1}{4\pi}\int d^3x'\,\frac{e^{i\omega|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}\,\check{j}_2(x') \tag{109}$$

where

$$\left| \vec{x} - \vec{x}' \right| = \sqrt{x^2 \quad x'^2 - 2\, \vec{x} \cdot \vec{x}'} \;.$$

Defining the unit vector $\vec{n} = \frac{\vec{x}}{x}$ we may write

$$\left| \vec{x} - \vec{x}' \right| = x \sqrt{1 - 2 \frac{\vec{n} \cdot \vec{x}'}{x} \quad \frac{x'^2}{x^2}} = x - \vec{n} \cdot \vec{x} + 0\left(\frac{1}{x}\right) , \tag{110}$$

and so

$$\overset{\vee}{A}_2(x) \sim \frac{1}{4\pi} \frac{e^{i\omega x}}{x} \int d^3x' \, e^{-i\omega \vec{n} \cdot \vec{x}'} \, \overset{\vee}{j}_2(x') + 0\left(\frac{1}{x^2}\right) , \tag{111}$$

which may be written

$$\overset{\vee}{A}_2(x) \sim \frac{e^{i\omega x}}{x} \sqrt{\frac{\pi}{2}} \, \overset{\vee}{j}_2(\vec{\omega}_f) \tag{112}$$

where $\vec{\omega}_f = \vec{n}\,\omega$ and $\overset{\vee}{j}_2(\vec{\omega}_f)$ is the three-dimensional Fourier transform of $\overset{\vee}{j}_2(x)$.

Returning to the electromagnetic field equation (59), we see that

$$\begin{aligned} \overset{\vee}{j}_2(x) = {} & i\, \delta_{4\nu} \tau_{p1} U(x) - \frac{1}{2M} \left(\tau_{po} \sigma_1^{\mu\nu} + \tau_{p1} \sigma_o^{\mu\nu} \right) \partial_\mu U(x) \\ & - \phi_o(x) \times \ell \left[\partial_\nu \phi_1(x) + \overset{\nu}{A_o}(x) \, \phi_o(x) \times \ell - \frac{g}{\mu} \tau_1 \sigma_o^\nu \, U(x) \right] \\ & - \phi_1(x) \times \ell \left[\partial_\nu \phi_o(x) - \frac{g}{\mu} \tau_o \sigma_o^\nu \, U(x) \right] . \end{aligned} \tag{113}$$

Transforming to momentum space, inserting the expression for $\phi_1(\vec{k})$ from equation (104) and for τ_1 from equation (97), and evaluating at the point $\vec{\omega}_f$, we have

$$j_2^\nu(\vec{\omega}_f) = i\, U(\vec{\omega}_f)\Big[\delta_{4\nu}\tau_{p1} - \frac{1}{2M}(\tau_{po}\sigma_1^{\mu\nu} + \tau_{p1}\sigma_o^{\mu\nu})\omega_f^\mu\Big]$$

$$+ \frac{g^2}{\mu^2}(\tau_o \times \ell)^2 \int \frac{d^3k}{(2\pi)^{3/2}} \Bigg\{ \frac{\mathcal{E}_o^\nu \vec{\sigma}_o\cdot(\vec{\omega}_f - \vec{k})\, \vec{\sigma}_o\cdot(\vec{k}-\vec{\omega})U(\vec{\omega}_f - \vec{k})\cdot U(\vec{k}-\vec{\omega})}{[(\vec{\omega}_f - \vec{k})^2 + \mu^2][(\vec{k}-\vec{\omega})^2 + \mu^2]}$$

$$+ \frac{i\,T_A\,\sigma_o^\nu \vec{\sigma}_o\cdot(\vec{k}-\vec{\omega}_f)\, U(\vec{k}-\vec{\omega}_f)\, U(\vec{k})}{[(\vec{k}-\vec{\omega}_f)^2 + \mu^2]}$$

$$+ \frac{U(\vec{\omega}_f - \vec{k})}{(k^2 - \omega^2 + \mu^2)}\left[\frac{(2k - \omega_f)_\nu\, \vec{\sigma}_o\cdot(\vec{\omega}_f - \vec{k})}{(\vec{k}-\vec{\omega}_f)^2 + \mu^2} + \sigma_o^\nu\right]\times$$

$$\left[i\,T_A\, \vec{\sigma}_o\cdot\vec{k}\, U(\vec{k}) + \vec{\sigma}_o\cdot\vec{\mathcal{E}}_o\, U(\vec{k}-\vec{\omega}) - \frac{2\,\vec{\mathcal{E}}_o\cdot\vec{k}\,\vec{\sigma}_o\cdot(\vec{k}-\vec{\omega})\, U(\vec{k}-\vec{\omega})}{(\vec{k}-\vec{\omega})^2 + \mu^2}\right]\Bigg\}. \tag{114}$$

In the evaluation of these integrals, as well as those in $K(\vec{\omega})$, we shall replace the nucleon source by a cutoff momentum $k_c = \pi/2a$. Here a is an appropriate cutoff radius, which we shall take $\frac{1}{2}$ times the nucleon Compton wave length. Since the integrals are divergent without such a cutoff, we shall approximate them by retaining only the leading term proportional to k_c. For $\omega << k_c$, this will indicate the order of magnitude of the integrals.

Applying this procedure, we obtain

$$\overset{\nu}{j}_2(\vec{\omega}_f) = i\, U(\vec{\omega}_f)\Big[\, \delta_{4\nu}\, \tau_{pl} - \frac{1}{2M}\,(\tau_{po}\, \sigma_1^{\mu\nu} + \tau_{pl}\, \sigma_o^{\mu\nu})\,\omega_f^{\mu}\,\Big]$$

$$+ \frac{1}{(2\pi)^{7/2}}\, \frac{g^2}{\mu^2}\,(\tau_o \times \ell\,)^2\, \frac{\pi}{15\,a}\Big[- \mathcal{E}_o^{\nu} - 3iT_A(\vec{\sigma}_o\cdot\vec{\omega}_f\, \sigma_o^{\nu} - 3\,\omega_f^{\nu})$$

$$+ 3\,\sigma_o^{\nu}\, \vec{\sigma}_o\cdot\vec{\mathcal{E}}_o\Big] . \tag{115}$$

and for $K(\vec{\omega})$,

$$K(\vec{\omega}) = \frac{2}{5}\left(\frac{g^2}{4\pi}\right)\frac{1}{\mu^2 a}\,\vec{\sigma}\cdot\vec{\mathcal{E}}_o\, \vec{\sigma}_o\cdot\vec{\omega} \tag{116}$$

Making use of the fact that in the asymptotic region the electric and magnetic field intensities are equal in magnitude and perpendicular to each other, we can evaluate the magnitude of the Poynting vector for the scattered wave, $\vec{P}_s$, to be

$$\left|\vec{P}_s\right| = e^4\left|\vec{E}_2 \times \vec{H}_2\right| = e^4\left|\vec{H}_2\cdot\vec{H}_2\right| = e^4\, \vec{H}_2\cdot\vec{H}_2^{*} , \tag{117}$$

where

$$\vec{H}_2 = \vec{\nabla}\times\vec{A}_2 \sim \frac{e^{i\omega x}}{x}\sqrt{\frac{\pi}{2}}\; i\,\vec{\omega}_f \times \vec{j}_2(\vec{\omega}_f) + O\!\left(\frac{1}{x^2}\right) . \tag{118}$$

Taking the curl of the asymptotic form of $\check{A}_2(x)$ given in equation (112), we find for the magnetic field,

$$\vec{H}_2(x) \sim \frac{e^{i\omega x}}{x} \left(\frac{e^2}{4\pi}\right) \left\{ \frac{1}{2M} \left[\tau_{po} (\omega^2 \vec{\sigma}_1 - \vec{\sigma}_1 \cdot \vec{\omega}_f \, \vec{\omega}_f) \right.\right.$$

$$+ \tau_{pl} (\omega^2 \vec{\sigma}_o - \vec{\sigma}_o \cdot \vec{\omega}_f \, \vec{\omega}_f) + i \frac{\gamma}{15} (\tau_o \times \ell)^2$$

$$\left.\left. \cdot \left[\vec{\varepsilon}_o \times \vec{\omega}_f + 3 i T_A \, \vec{\sigma}_o \cdot \vec{\omega}_f \, \vec{\sigma}_o \times \vec{\omega}_f - 3 \vec{\sigma}_o \cdot \vec{\varepsilon}_o \, \vec{\sigma}_o \times \vec{\omega}_f \right] \right\} \tag{119}$$

where we have introduced the dimensionless quantity

$$\gamma = \left(\frac{g^2}{4\pi}\right) \frac{1}{(\mu a)} \frac{M}{\mu} \tag{120}$$

and have set the nucleon source density equal to $1/(2\pi)^{3/2}$, the Fourier transform of a point nucleon, wherever it remains in equation (115). This allows us to write in place of equations (93), (95), and (116),

$$G(\vec{\omega}) = \frac{\tau_{po}}{M} , \tag{121}$$

$$K(\vec{\omega}) = \frac{2}{5} \frac{\gamma}{M} \vec{\sigma}_o \cdot \vec{\varepsilon}_o \, \vec{\sigma}_o \cdot \vec{\omega} , \tag{122}$$

and

$$L(\vec{\omega}) = -\frac{1}{2M} \vec{\sigma}_o \cdot \vec{\omega} \times \vec{\varepsilon}_o \tag{123}$$

The differential scattering cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{\langle x^2 \, |\vec{P}_s(\vec{\omega}_f)| \rangle}{|\vec{P}_i(\vec{\omega})|} = \frac{e^4 \langle x^2 \, \vec{H}_2(\vec{\omega}_f)\cdot\vec{H}_2^*(\vec{\omega}_f) \rangle}{\vec{H}_o\cdot\vec{H}_o^*}$$

$$= \frac{e^4}{\omega^2} \langle x^2 \, \vec{H}_2(\vec{\omega}_f)\cdot\vec{H}_2^*(\vec{\omega}_f) \rangle \tag{124}$$

where the brackets $\langle \quad \rangle$ indicate averages to be taken over the nucleon spin, the isotopic spin, and the polarization of the incident photon. In performing these averages, $\vec{\sigma}_o$ and τ_o are treated as classical unit vectors and use is made of the relations

$$\langle \vec{\sigma}_o\cdot\vec{A}_1 \, \vec{\sigma}_o\cdot\vec{A}_2 \rangle = \frac{1}{3} \vec{A}_1\cdot\vec{A}_2 \tag{125}$$

$$\langle \vec{\sigma}_o\cdot\vec{A}_1 \, \vec{\sigma}_o\cdot\vec{A}_2 \, \vec{\sigma}_o\cdot\vec{A}_3 \, \vec{\sigma}_o\cdot\vec{A}_4 \rangle = \frac{1}{15}\Big[\vec{A}_1\cdot\vec{A}_2 \, \vec{A}_3\cdot\vec{A}_4 + \vec{A}_1\cdot\vec{A}_3 \, \vec{A}_2\cdot\vec{A}_4 + \vec{A}_1\cdot\vec{A}_4 \, \vec{A}_2\cdot\vec{A}_3 \Big] \tag{126}$$

where for n $\vec{\sigma}_o$-terms, the coefficient on the right is $\frac{1}{3 \times 5 \ldots \times (n+1)}$. The averages vanish for an odd number of terms. The relations for the τ_o - averages are completely analogous.

To carry out the $\vec{\varepsilon}_o$-averages we note the relation

$$\langle \vec{\varepsilon}_o\cdot\vec{A} \, \vec{\varepsilon}_o\cdot\vec{B} \rangle = \frac{1}{2}\left(\vec{A}\cdot\vec{B} - \frac{\vec{A}\cdot\vec{\omega} \, \vec{B}\cdot\vec{\omega}}{\omega^2}\right) \tag{127}$$

since there are only two independent orientations of $\vec{\mathcal{E}}_o$, which are both perpendicular to $\vec{\omega}$.

A straightforward calculation, carrying out the averages over the nucleon spin $\vec{\sigma}_o$ and photon polarization $\vec{\mathcal{E}}_o$, yields the following expression for the cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{M^2}\left(\frac{e^2}{4\pi}\right)^2 \left\{ \frac{\alpha_1 \gamma^2}{2{,}250} (13 + \cos^2\theta)\, \Gamma_o(\omega) \right.$$

$$+ \left[-\frac{\alpha_1 \gamma^2}{5{,}250} (1 + \cos^2\theta) + \frac{\alpha_2 \gamma}{150} (1 - 3\cos^2\theta) + \frac{\alpha_3}{24}(3 - \cos^2\theta) \right] \Gamma_1(\omega)$$

$$+ \left[\frac{\alpha_1}{960}(3 + \cos^2\theta) + \frac{4\alpha_1\gamma^4}{196{,}875}(3 - 3\cos^2\theta + 4\cos^4\theta) - \right.$$

$$\left. - \frac{\alpha_2}{15}\left(\frac{1}{16} + \frac{\gamma^2}{25}\right)(1 - 3\cos^2\theta) + \frac{\alpha_3}{120}(11 + 7\cos^2\theta) \right] \Gamma_2(\omega)$$

$$\left. + \left[\frac{\alpha_1\gamma}{180}\cos\theta - \frac{4\alpha_1\gamma^3}{39{,}375}(3\cos\theta + 2\cos^3\theta) + \frac{2}{45}\alpha_2\gamma\cos\theta \right] \Gamma_3(\omega) \right\} \tag{128}$$

where the isotopic spin dependence is contained in the α_1, α_2, and α_3 coefficients, which are as follows

$$\alpha_1 = (\tau_o \times \ell)^4 \tag{129}$$

$$\alpha_2 = (\tau_o \times \ell)^2 \tau_{po}^2 = \tfrac{1}{4}\left[1 - (\tau_o \cdot \ell)^2\right](1 - \tau_o \cdot \ell)^2 \tag{130}$$

$$\alpha_3 = \tau_{po}^4 = \frac{1}{16}(1 - \tau_o \cdot \ell)^4 \tag{131}$$

The $\Gamma_n(\omega)$, $(n = 0, 1, 2, 3)$, are dimensionless functions of the photon energy. If we separate N into real and imaginary parts

$$N = R + iJ \ , \tag{132}$$

the $\Gamma_n(\omega)$ take the form

$$\Gamma_o(\omega) = 1 \ , \tag{133}$$

$$\Gamma_1(\omega) = \frac{\omega^6}{M^2} \frac{1}{|N^2 - \omega^2|^2} \ , \tag{134}$$

$$\Gamma_2(\omega) = \frac{\omega^4}{M^2} \frac{R^2 + J^2}{|N^2 - \omega^2|^2} \ , \tag{135}$$

$$\Gamma_3(\omega) = \frac{\omega^2}{M} R \frac{(R^2 + J^2 - \omega^2)}{|N^2 - \omega^2|^2} \tag{136}$$

To find the cross section in the charge-symmetric case, we average the α_1, α_2, and α_3 over all orientations in isotopic spin space and obtain

$$\langle \alpha_1 \rangle = \frac{8}{15}\,, \qquad \langle \alpha_2 \rangle = \langle \alpha_3 \rangle = \frac{1}{5}\,. \tag{137}$$

For the case of a neutral pion field, the nucleon isotopic spin is parallel to ℓ, and τ_{po} is one for a proton and zero for a neutron. We then set

$$\alpha_1 = 0\,, \qquad \alpha_2 = 0\,, \qquad \alpha_3 = 1,(0) \tag{138}$$

for a proton, (neutron). The cross section in this case has the much simpler form,

$$\frac{d\sigma_p}{d\Omega}(\text{neutral}) = \frac{1}{24\,M^2}\left(\frac{e^2}{4\pi}\right)^2 \left\{ (3 - \cos^2\theta)\,\Gamma_1(\omega) + \frac{1}{5}(11 + 7\cos^2\theta)\,\Gamma_2(\omega) \right\}$$

and (139)

$$\frac{d\sigma_n}{d\Omega}(\text{neutral}) = 0\,.$$

It is clear that the cross section for a neutron should vanish in this case, since the neutral pion cloud provides no currents to scatter the photon, and the neutron has no Dirac moment.

In order to interpret the results contained in equations (128) and (139) for the scattering cross section in the charge-symmetric and neutral theories, respectively, we have plotted the Γ_n $(n = 1, 2, 3)$ as functions of the incident photon energy in Fig. 1. The source radius has been taken to be $\frac{\ }{2}$ times the nucleon Compton wavelength, which corresponds to a cutoff momentum $k_c = M$, and the pion-nucleon coupling constant is chosen to be $g^2/4\pi = .3$. With these vales for the parameters, the cross section formulas may be written in the following form:

$$\frac{d\sigma}{d\Omega}\,(\text{symmetric}) = (.539 \times 10^{-32}\ \text{cm}^2) \sum_{n=0}^{3} C_n(\theta)\,\Gamma_n(\omega) \tag{140}$$

and

$$\frac{d\sigma}{d\Omega}\,(\text{neutral}) = (1.0 \times 10^{-33}\ \text{cm}^2) \sum_{n=1}^{2} D_n(\theta)\,\Gamma_n(\omega) \tag{141}$$

where the $C_n(\theta)$ are given by

$$C_0(\theta) = 1 + .0769 \cos^2\theta \,, \tag{142}$$

$$C_1(\theta) = .127 - .221 \cos^2\theta \,, \tag{143}$$

$$C_2(\theta) = .691 - .205 \cos^2\theta + 1.04 \cos^4\theta \,, \tag{144}$$

$$C_3(\theta) = -.302 \cos^3\theta \,, \tag{145}$$

and the $D_n(\theta)$ are

$$D_1(\theta) = 3 - \cos^2\theta \quad , \tag{146}$$

and

$$D_2(\theta) = \frac{1}{5}\ (11 + 7\cos^2\theta) \quad . \tag{147}$$

The constant term in $C_o(\theta)$ has been normalized to unity. These θ-dependent coefficients are plotted in Figures 2 and 3.

It is seen that $C_3(\theta)$ is the only coefficient which is asymmetric about $\theta = \pi/2$. In addition we note that $\Gamma_3(\omega)$ is negative for small ω and becomes positive for $\omega > \omega_o$ where ω_o is an energy near $1.4\,\mu \cong 200$ Mev. Therefore, in the charge symmetric case, the angular distribution is peaked in the forward direction for energies below ω_o; and this peaking shifts into the backward direction for $\omega > \omega_o$. This may be contrasted with the behavior of the angular distribution in the neutral case. Here the peak appears at $\theta = \pi/2$ for energies below the resonance energy ω_o and this peak diminishes and is replaced by a trough for $\omega > \omega_o$. These angular distributions are illustrated in Figure 4 and 5 for typical energies.

Integrating equations (140) and (141) over solid angle we obtain for the total cross sections,

$$\sigma\,(\text{symmetric}) = (.697 \times 10^{-31}\ \text{cm}^2)\left[1 + .049\,\Gamma_1(\omega) + .805\,\Gamma_2(\omega)\right] \tag{146}$$

and

$$\sigma \text{ (neutral)} = (3.4 \times 10^{-32}\ \text{cm}^2)\left[\Gamma_1(\omega) + \Gamma_2(\omega)\right]. \tag{147}$$

These cross sections are plotted in Figure 6. It is to be noted that the magnitude of the cross section is smaller in the neutral than in the symmetric theory. However both exhibit a resonance peak in the total cross section. This confirms the fact that the pion cloud modifies the scattering of light through the mechanism of inertial effects as well as directly through its charge.

The general features of this calculation indicate that the pion-nucleon coupling may modify the scattering of light in a sensitively energy-dependent manner. Experiments at higher energies could provide interesting new information about this coupling.

It should be pointed out that the neglect of the recoil effects of the nucleon will invalidate the details of these results to some extent. However, it may be expected that the main features of these results, including the peak in the total cross section and a strongly energy-dependent angular distribution, will be valid.

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FIGURE CAPTIONS

Figure 1: Graph of coefficients $\Gamma_n(\omega)$ (n = 1, 2, 3) as functions of ω/μ.

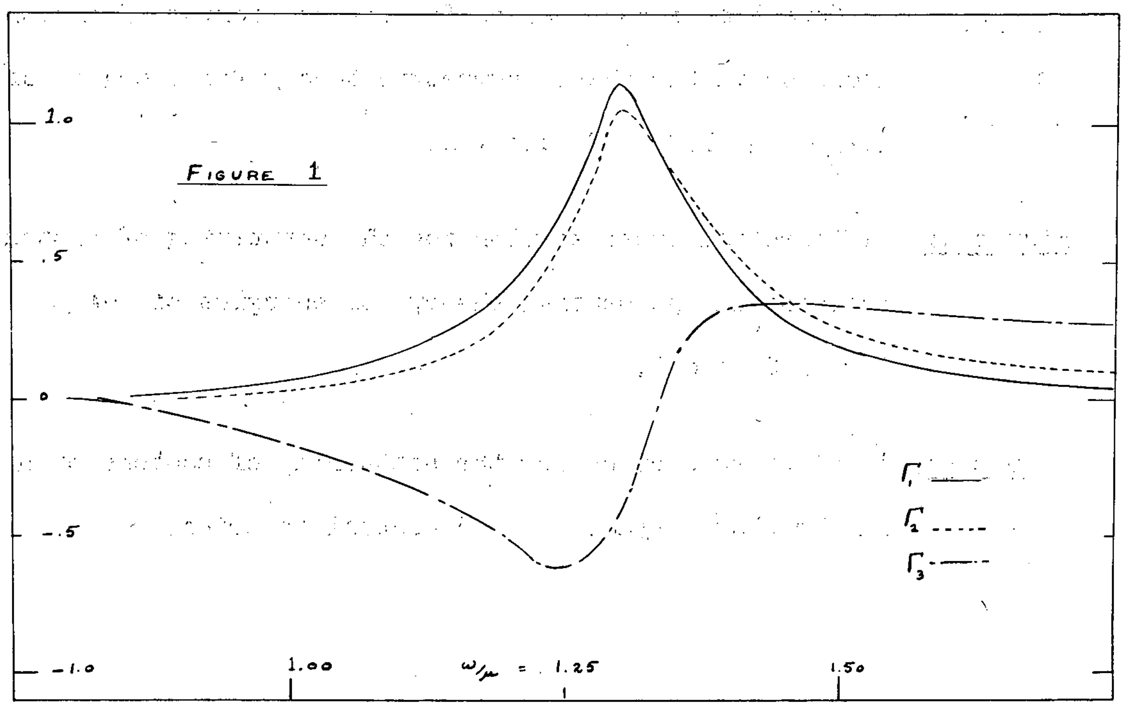
Figure 2: Graph of $C_n(\theta)$ (n = 1, 2, 3) as functions of θ.

Figure 3: Graph of $D_n(\theta)$ (n = 1, 2) as functions of θ.

Figure 4: Differential cross section for the scattering of photons on nucleons in the charge-symmetric theory for energies of ω/μ = 1.2, 1.3, and 1.4.

Figure 5: Differential cross section for the scattering of photons on nucleons in the neutral theory for energies of ω/μ = 1.0, 1.2, and 1.4.

Figure 6: Total cross section for the scattering of photons on nucleons in the charge-symmetric and neutral theories.



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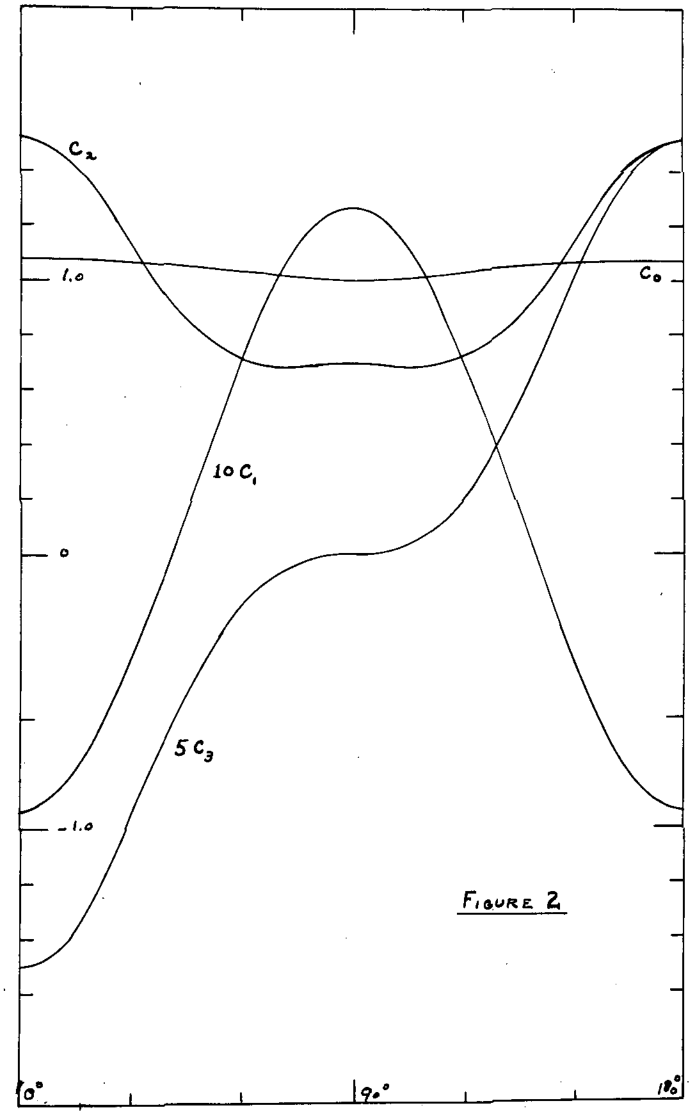

Figure 2

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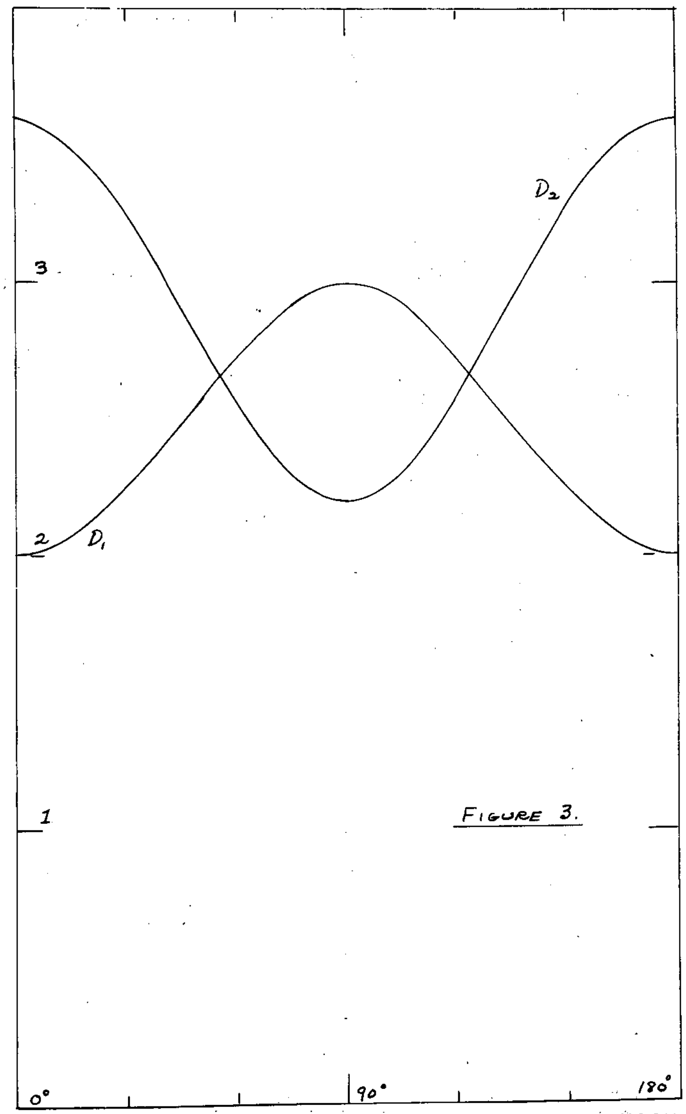

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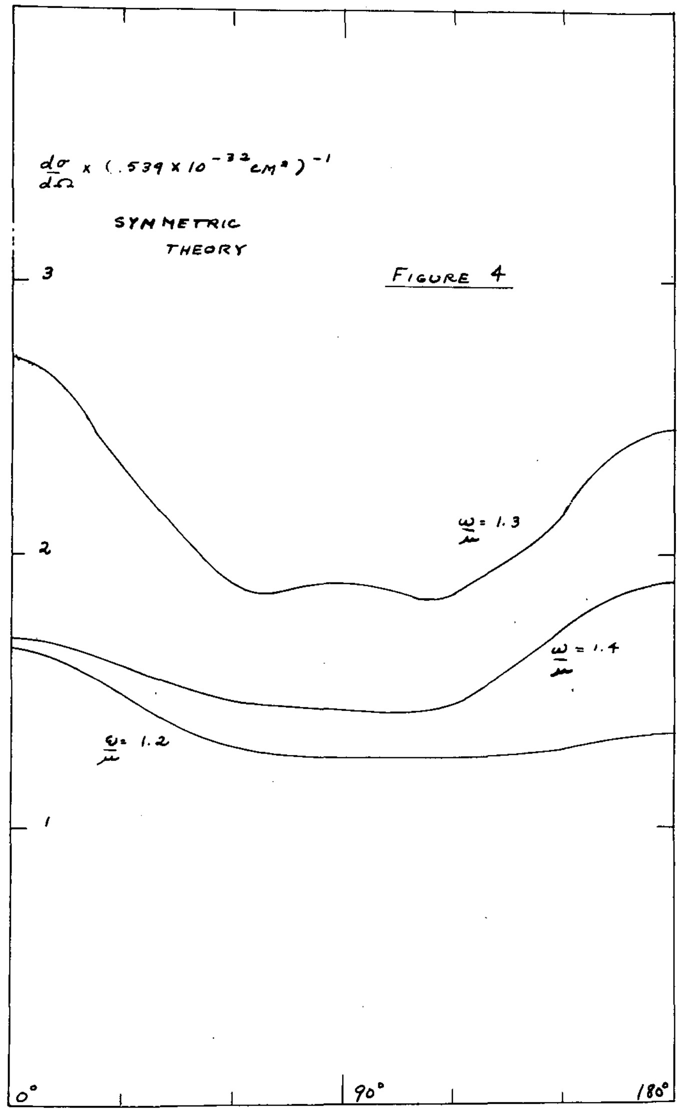
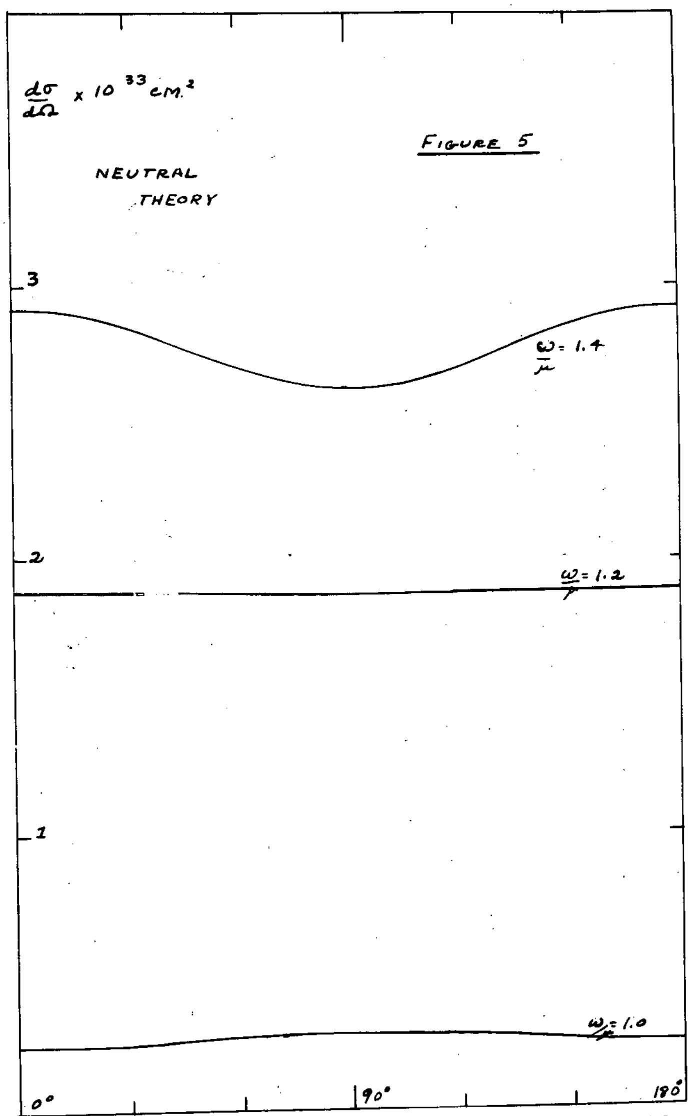
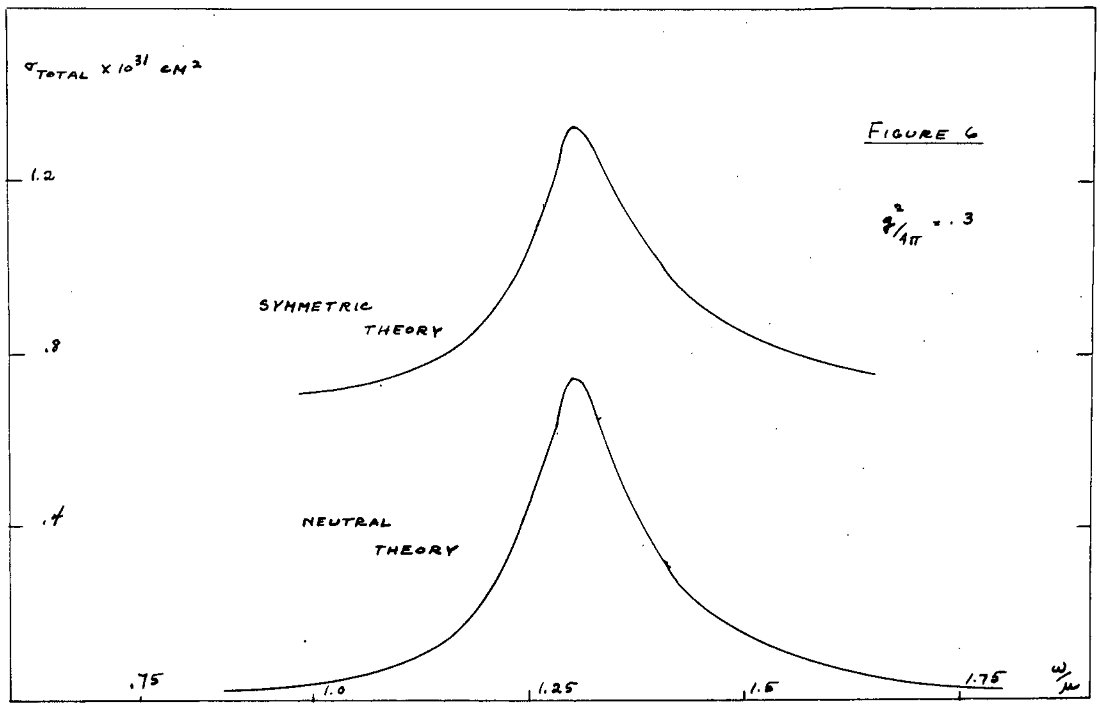

$\frac{d\sigma}{d\Omega} \times (.539 \times 10^{-32} cm^2)^{-1}$
SYMMETRIC THEORY
FIGURE 4
3
2
1
$\frac{\omega}{\mu} = 1.3$
$\frac{\omega}{\mu} = 1.4$
$\frac{\omega}{\mu} = 1.2$
0°
90°
180°
MU-7688


$\frac{d\sigma}{d\Omega} \times 10^{33}$ cm.^{2}
FIGURE 5
NEUTRAL THEORY
3
2
1
$\frac{\omega}{\mu} = 1.4$
$\frac{\omega}{\mu} = 1.2$
$\frac{\omega}{\mu} = 1.0$
0°
90°
180°

MU-7689



$\sigma_{TOTAL} \times 10^{31}$ cm^2
FIGURE 6
1.2
$g^2/4\pi = .3$
SYMMETRIC
THEORY
.8
.4
NEUTRAL
THEORY
.75
1.0
1.25
1.5
1.75
ω/μ

MU-7690