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Anomalous electron-ion energy coupling in electron drift wave turbulence

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Anomalous electron-ion energy coupling in electron drift wave turbulence

A dissertation submitted in partial satisfaction of the
requirements for the degree
Doctor of Philosophy

in

Physics

by

Lei Zhao

Committee in charge:

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2013

The dissertation of Lei Zhao is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

University of California, San Diego

2013

DEDICATION

To my mom and dad - with love

EPIGRAPH

I am a slow walker, but I never walk backwards.

—Abraham Lincoln

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ACKNOWLEDGEMENTS

After six years studying in UCSD, I changed from a layperson of plasma physics to know something about nuclear fusion. Everything happened just like yesterday, I even haven't thought about it and it is time to graduate. Before starting a new stage in my life, I would like to thank all the people who helped and are helping me here. First of all, I would like to thank my advisor Prof. Pat Diamond for his guidance and encouragement at every stage of my PHD study. Without his patience on my research, I couldn't finish this thesis. In particular, his attitude of hard work and rigorous approach to science will benefit me in my future research.

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ABSTRACT OF THE DISSERTATION

Anomalous electron-ion energy coupling in electron drift wave turbulence

by

Lei Zhao

Doctor of Philosophy in Physics

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Professor Patrick H. Diamond, Chair

Turbulence is a ubiquitous phenomenon in nature, and it is well known that turbulence couples energy input to dissipation by cascade processes. Plasma turbulence plays a critical role in tokamak confinement. Magnetized plasma turbulence is quasi-2D, anisotropic, wave -like and 2 fluid (i.e. electrons and ions) in structure. Thus , weakly collisional plasma turbulence can mediate electron-ion energy transfer.

The issue of anomalous electron-ion energy coupling is particularly important for low collisionality, electron heated plasmas, such as ITER. In this work, we reconsider the classic problem of "turbulent heating " and energy transfer pathways in drift wave turbulence. The total turbulent heating, composed of quasilinear electron cooling, quasilinear ion heating, nonlinear ion heating and zonal flow frictional heating, is

analyzed. In Chapter 2, the electron-ion energy exchange via linear wave-particle resonance will be computed. To address net heating, we show the turbulent heating in an annulus arises due to a wave energy flux differential across this region. We show this net heating is proportional to the Reynolds work on the zonal flow. Zonal flow friction heats ions, thus the turbulence-zonal flow interaction enters as an important energy transfer channel.

Since zonal flows are nonlinearly generated, it follows that we should apply weak turbulence theory to calculate the nonlinear ion turbulent heating via the virtual mode resonance in the electron drift wave turbulence, which will be discussed in Chapter 3. We defines a new collisionless turbulent energy transfer channel through nonlinear Landau damping in the electron-ion energy coupling process. The result shows that nonlinear ion heating can exceed quasilinear ion heating, so that nonlinear heating becomes the principal collisionless wave energy dissipation channel in electron drift wave turbulence. This follows since the beat mode resonates with the bulk of the ion distribution, in contrast to the linear resonance which is located on the tail. This result also suggests that zonal flow shearing is not necessarily the only saturation mechanism of importance, especially for very low collisionality. This observation brings a new perspective on electron heat transport where ions, play a role as an energy "sink" in a collisionless plasma, such as ITER. In addition, it is shown that the electron turbulent energy transfer to ions in a collisionless plasma can be the same order as electron heat transport losses. Thus, it is necessary to consider the influence of collisionless energy transfer to determine the total energy budget in ITER.

Chapter 1

Introduction

Solving the energy crisis is a popular and controversial topic which has been discussed almost daily in the media. In particular, the realization of an abundant, clean energy source has been considered a long term driving force for the development of the economy. Nuclear fusion has been singled out as one of the most promising candidates for a future energy source since it has the advantage of being safe, sustainable and environmentally friendly[1, 2]. However the challenges of how to peacefully exploit the potential of controlled fusion energy have occupied scientists and engineers for more than half a century. So far, the two most prominent paths to realize fusion are magnetic and inertial confinement. The first one uses a magnetic field to confine charged particles (a hot plasma) and increase plasma confinement time to achieve ignition[3, 4]. The second path focuses on compressing a deuterium and tritium target with the use of high power lasers[5]. For each confinement method, many trials have already taken place. The National Ignition Facility (NIF) [6] is an updated inertial confinement device whose construction was completed at Lawrence Livermore National Laboratory (LLNL) in 2009 , but ignition experiments haven't achieved the expected aims as of yet. On the other hand, the International Thermonuclear Experimental Reactor (ITER)[7], which will be the largest magnetic confinement device ever built, is under construction in the south of France, and is expected to have the first plasma discharge by 2020. How to improve the performance of ITER and lower the cost of future fusion devices? It is an urgent task to consider for both theorists and experimentalists in magnetic fusion.

There are many critical issues unresolved in ITER, such as how to access the so called high confinement mode (H-mode) [8] as well as understand plasma transport (energy, momentum and particle loss of the plasma). Before going into the details of these problems, we will first briefly review the history and basic function of a tokamak [9, 10], which is the most promising magnetic confinement device being pursued. The term " tokamak" is a seven letter Russian acronym which translates as" toroidal chamber with magnetic coils" [10]. The initial physical concept of the tokamak appeared in the early 1950's. In the Soviet Union I. Tamm and A. Sakharov sketched a design for a Tokamak, a device with toroidal plasma current. How does a tokamak confine and contain hot plasmas? The general idea is shown in Fig. 1.1. The toroidal field coils produce a field that travels along the axis of the torus. Most of the plasma will move along this toroidal magnetic field line. However, the toroidal field decreases from the center to the edge, such that the magnetic field curvature and gradient give rise to vertical drifts that are in opposite directions for electrons and ions. The resulting charge separation causes particles to be transported outward due to the $E \times B$ drift. Thus, an additional magnetic field components are required to confine plasma, which is a small magnetic filed in the poloidal direction compared to the strong toroidal magnetic field ($B_\phi \gg B_\theta$). Thus a net helical magnetic field is generated such as that sketched in Fig. 1.1. The helicity of field line averages out the drift so that the plasma remains confined. Here the poloidal magnetic field is generated by the transformer coils in the center of torus and by the toroidal current in the plasma itself.

The hydrogen isotopes deuterium D and tritium T were chosen as the fusion fuel since they have the largest reaction cross section $\langle\sigma\rangle$ (cross section represents the possibility that a reaction occurs). The reaction of interest can be expressed as

$$D + T \longrightarrow He^4 + n + 17.6Mev \tag{1.1}$$

where He is helium, also referred to as an α particle, and n is a neutron. Two lighter nuclei collide and fuse into a heavier nucleus, releasing energy at the same time. The high energy α particles will be slowed down by collisions with electrons, and hence transfer their energy back to the plasma, supporting other nuclear reactions. The neutrons, in contrast, will escape from the plasma, allowing their energy to be used to produce electricity, which is the goal of fusion.

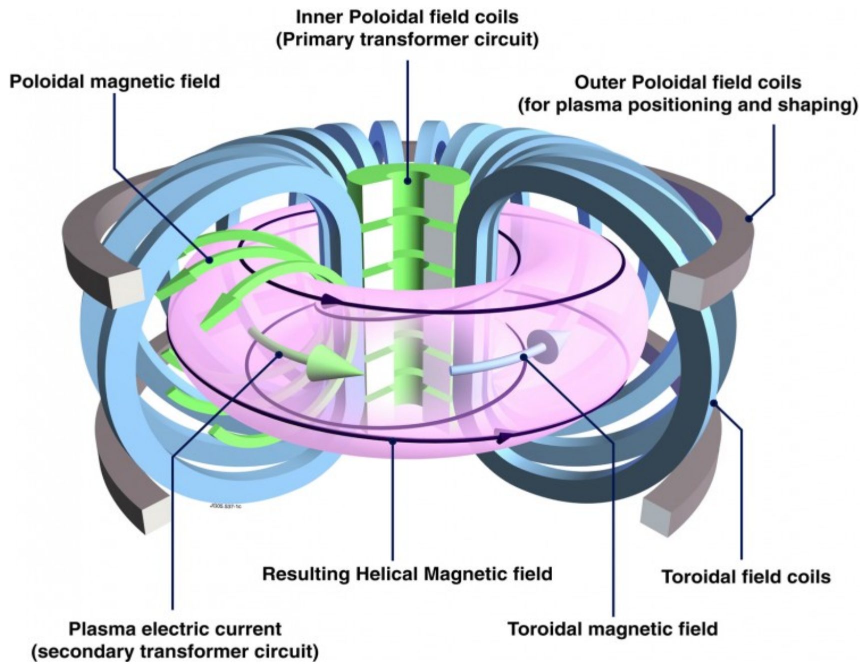

Figure 1.1: Diagram of tokamak geometry. This figure appears courtesy of the European Fusion Development Agreement [11].

In order to induce a self-sustaining reaction, such that the energy deposited by the fusion products is larger than all losses, a criterion needs to be satisfied. The Lawson criterion for the ignition of a Deuterium-tritium (D-T) plasma is given by

$$n_i \tau_E T_i > 3 \times 10^{21} m^{-3} keV s \tag{1.2}$$

where n_i is the ion density, τ_E is the energy confinement time and T_i is the ion temperature. In an ITER burning plasma [12, 13], ignition will be achieved for the experimental target parameters: $n = 1.1 \times 10^{20} m^{-3}$, with a plasma inductive burn time $t = 400s$ and ion temperature $T_i = 10$keV, leading to the fusion power gain of $Q \geq 10$ (Q is the ratio of fusion power produced in a nuclear fusion reactor to the power required to maintain the plasma in steady state).

However, the Lawson criterion hasn't been achieved in present tokamaks. The main issue is how to improve the plasma confinement time τ_E which is given by

$$\tau_E = \frac{U}{P_{loss}} \tag{1.3}$$

where U is the amount of energy in the plasma, P_{loss} is the power loss. In a tokamak, plasma is in a non-equilibrium state, being hotter and denser in the core than the edge. Thus the free energy from the gradients of the temperature and density will drive heat and particle fluxes across the magnetic field to bring the plasma into equilibrium. This is a natural relaxation process which results in particles/energy being transported from the plasma core to the edge. So the greater the transport, the lower the confinement time and the harder it is to reach ignition. How to increase the plasma confinement time? One route is to build larger tokamaks which can increase the number of plasma interactions before the plasma is lost. This will result in an increase in the cost of tokamaks. Or we need a better understanding of plasma transport process. In section 1.2., we will briefly introduce some basic concepts of plasma transport phenomena, with particular emphasis on turbulent energy transport.

How to understand plasma energy transport in a burning plasma, such as ITER? In section 1.3, we will look at this problem from a new perspective, turbulence mediated energy transfer between electrons and ions. We just mentioned that α particles, which provide the energy source for the self-sustaining nuclear reaction, will heat electrons first [12, 14]. Thus energy must be transferred from electrons to the fuel ions, in order to sustain the reaction. Considering the electron heat balance equation [15, 16]

$$\frac{3}{2} n \frac{\partial T_e}{\partial t} + \nabla \cdot Q_e = \left\langle \tilde{E} \cdot \tilde{J} \right\rangle + P_{e,i} + \cdots \tag{1.4}$$

electron energy can be lost via the divergence of the heat flux $\nabla \cdot Q_e$ or energy can be *transferred* to other species through either collisional transfer $P_{e,i}$ or collisionless energy transfer $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle$ [15][16][17]. Here energy exchanged via electron-ion binary collisions can be described by Braginskii's equation [18], given by

$$W_i = \frac{3 n_e m_e}{m_i} \nu_e T_e \left(1 - \frac{T_i}{T_e} \right) \tag{1.5}$$

which is the energy per unit time transferred from electrons to ions. ν_e is the collision rate for electrons and is defined as

$$\nu_{ee} = 2.9 \times 10^{-6} \times n_e \times \lambda \times T_e{}^{-3/2} \approx 2.9 \times 10^3 sec^{-1} \tag{1.6}$$

where ITER parameters have been used [12, 13]: plasma density $n_e \approx n_i = 1.1 \times 10^{14} cm^{-3}$, the Coulomb logarithm is defined as $\lambda = ln\Lambda \equiv ln(r_{max}/r_{min}) \sim 10$, plasma temperature $T_e \approx T_i = 10keV$, the major radius $R = 6.2m$, the minor radius $r = 2m$, and the safety factor is $q = 2$ in the core. So, the normalized collisionality is obtained as $\nu_{\star e} = \epsilon^{-3/2}\nu_{ee}Rq/v_{the} = 4.3 \times 10^{-3}$ in ITER. This collisionality is much lower than the one in present thermal transport experiments where $\nu_\star > 0.01$ in the core of DIII-D [19]. Thus, the collisional energy transfer $P_{e,i}$ is relatively small and *collisionless* energy transfer $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle$ can potentially dominate the energy transfer process in a low collisionality plasma, such as ITER. So far this collisionless energy transfer has not received sufficient attention in the analysis of the present day electron thermal transport experiments.

What does "collisionless" energy transfer mean? Put in a simple way, it implies turbulence mediated energy transfer between electrons and ions. This energy coupling process can involve wave-particle or turbulence-zonal flow interactions [20, 21, 22]. In an electron heated plasma [12, 14], free energy in ∇n and ∇T_e can be released to the wave to drive wave instabilities and turbulence. On the other hand, wave energy can be dissipated by ion Landau damping through either linear or nonlinear wave-particle resonances [23, 24, 25, 26, 27, 28]. Thus ions absorb energy from the wave and stabilize the wave. Basically, waves act as a medium for the inter-species exchange of energy between electrons and ions. In addition, the nonlinear excitation of zonal flows (see Sec. 1.3.1) provides an alternate means through which turbulence may mediate the exchange of energy between electrons and ions. Here, zonal flows driven by the background turbulence act as a repository of fluctuation energy [20]. The zonal flow population will be subsequently damped by friction with ions [21, 22], thus providing an additional means for wave energy to be transferred to the ion population. This latter mechanism introduces a *new* collisionless energy tranfer channel which will be described in detail below. Since zonal flows can dominate the wave saturation balance condition[29], zonal flow frictional damping can also provide a significant energy transfer channel in low collisionality plasmas, such as ITER (see 1.3.2).

The anomalous electron-ion energy coupling can be conveniently summarized by expressing it in a general form. In the energy balance equation, the total turbulent heating for species α can be written schematically as

$$\left\langle \tilde{E} \cdot \tilde{J}_\alpha \right\rangle = A_L^\alpha I + B_{NL}^\alpha I^2 + C_{ZF}^\alpha I^2, \tag{1.7}$$

where the turbulence intensity is defined as $I = \sum_k |e\tilde{\phi}_k/T_e|^2$. The coefficient $A_L^\alpha = \sum_k A_k|e\tilde{\phi}_k/T_e|^2/\sum_k |e\tilde{\phi}_k/T_e|^2$ represents electron and ion quasilinear turbulent heating, $B_{NL}^\alpha = \sum_{k,k'} B_{k,k'}|e\tilde{\phi}_k/T_i|^2|e\tilde{\phi}_{k'}/T_i|^2/I^2$ describes nonlinear ion heating through nonlinear Landau damping [23, 24, 25, 26, 27, 28]. The hot electrons (energy "source", $\left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle < 0$) lose energy to drift waves and the ions gain energy (energy "sink", $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle > 0$) through ion Landau damping (including linear and nonlinear wave-particle resonances). The coefficient $C_{ZF} = \sum_{k,k'} C_{k,k'}|e\tilde{\phi}_k/T_i|^2|e\tilde{\phi}_{k'}/T_i|^2/I^2$ is determined by heating through zonal flow formation.

This thesis will mainly discuss anomalous electron-ion energy coupling with an emphasis on ITER plasmas. Electron/ion thermal energy will not only be lost via turbulent transport processes[30, 31, 32, 33, 34]. Rather, energy transfer between electrons and ions will also occur. In particular, it was shown that the rate of collisionless energy transfer can be the same order as the turbulent transport for collisionless trapped electron modes (CTEM)[34, 35]. Hence it is necessary to consider how collisionless energy transfer influences turbulent energy transport processes in a collisionless plasma, such as ITER. More pragmatically, transport models for low collisionality plasmas must properly account for energy transfer.

1.1 Transport basics

Note that the Lawson ignition condition is closely related to the plasma global energy confinement time, which is largely decided by plasma transport. The transport theory mainly calculates the particle and heat flux Γ and Q which are often assumed to have the form[9]

$$\Gamma = -D\nabla_{\perp} n, \tag{1.8}$$

$$Q = -\chi\nabla_{\perp} T, \tag{1.9}$$

where D is the particle diffusion coefficient and χ is the thermal diffusivity. It is not possible to measure the coefficients D and χ from experiment directly. The particle and heat flux can be calculated from the particle or heat balance equation[9]

$$\frac{\partial n}{\partial t} + \nabla \cdot T = S_{particle} \tag{1.10}$$

$$\frac{\partial}{\partial t}\frac{3nT}{2} + \nabla \cdot Q = S_{heat} + P_{ie} \tag{1.11}$$

where $S_{particle}$ is the particle source, S_{heat} is the heating source and P_{ie} represents the energy exchanged between electrons and ions. In this thesis, we focus on the plasma heat transport in a tokamak. There are a few typical theories used to describe plasma transport phenomena which are classical transport, neoclassical transport and turbulent transport. In essence, classical transport is due to binary collisions in a straight magnetic field[9]. For instance, for ion heat transport driven by ion-ion collisions

$$\chi_{collision} \approx \rho_i^2 \nu_{ii} \tag{1.12}$$

where ρ_i is the ion Larmor radius, used as the step length for diffusion, and ν_{ii} is the ion- ion collision frequency. When considering collisional transport in the more complex magnetic topology present in a tokamak, the above argument requires significant modification. This has given rise to the study of so-called neoclassical transport [9, 36, 37]. The curved magnetic field $B \propto 1/R$ decreases with major radius R. Since energy and the adiabatic invariant $\mu = mv_{\perp}^2/2B$ are conserved, particles with $|v_{\parallel}|/v_{\perp} \leq \epsilon^{1/2}$ will be trapped, reflected back and forth on the outside of the torus by the magnetic mirror effect. Here $\epsilon = r/R < 1$ is the inverse aspect of ratio of the tokamak, and r is the minor radius. Thus the magnetic drift combined with the mirror motion of a trapped particle induce a " banana orbit " [9, 36] with a width

$$\delta r_b \sim \rho_p \epsilon^{1/2} \tag{1.13}$$

where $\rho_p = v_{thi}/\Omega_p$ is the gyroradius evaluated using the poloidal magnetic field B_θ, $\Omega_p = eB_\theta/m$ (see Fig. 1.2). Thus the neoclassical transport is given by

$$\chi_{neo} \approx f_t(\delta r_b)^2 \nu_{eff} \tag{1.14}$$

where $f_t \approx (2\epsilon)^{1/2}$ represents the fraction of trapped ions, $\nu_{eff} = \nu/\omega_b$ is the effective frequency of collisions for trapped ions, ω_b is the trapped particles bounce frequency. Note that the poloidal field is smaller than the toroidal field $B_\theta \ll B_\phi$, so the diffusive size is bigger than the ion gyroradius $\delta r_b \gg \rho_i$, the neocalssical transport of trapped particles is therefore much faster than classical transport. However, the transport level in experiments is generally observed to be one order of magnitude higher than the prediction from neoclassical theory [37]. Thus anomalous transport [30, 31, 32, 33, 34] has received more and more attention since the thermal diffusivity $\chi_{tur} \gg \chi_{neo}$, where the thermal diffusivity χ_{tur} is predicted to exhibit so-called "gyro-Bohm" scaling, i.e.

$$\chi_{tur} \sim \frac{T_i}{eB}\frac{\rho_i}{a} \tag{1.15}$$

Anomalous transport is driven by plasma microinstabilities which are active on scales small compared to the system size. Elements of this rather complex subject will be reviewed in the next subsection.

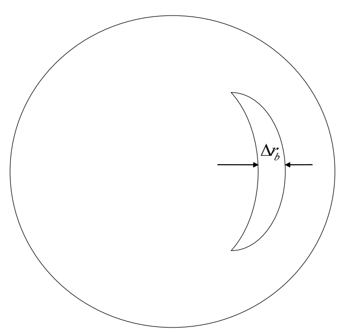

Figure 1.2: Poloidal cross section of a banana orbit in a tokamak.

1.2 Anomalous transport in tokamak plasmas

The anomalous transport alluded to above arises due to plasma microinstabilities which are ubiquitous in tokamak plasmas. As a representative example, it is useful to describe the specific case of electrostatic drift waves [38, 39]. Here we consider electrostatic turbulence ($\tilde{\mathbf{E}} = -\nabla\tilde{\phi}$) which is typical of the turbulent transport in the tokamak core [40]. A critical element in determining the stability of drift waves corresponds to the description of the electron response to electrostatic fluctuations. The electron response is typically treated as either " adiabatic" or " non-adiabatic"[38]. The adiabatic response is appropriate when $\omega \ll k_{\parallel}v_{the}$, implying that the electron response along the field line is much faster than the frequency of the mode. For a non-adiabatic response, the electron's motion is impeded such that it can not respond instantaneously to the electrostatic perturbation, i.e. due to collisions, electron trapping, etc. To get instability, we need a non-adiabatic electron response since an adiabatic response is a reversible process which doesn't contribute extra information to the plasma system. Examples of mechanisms for producing a non-adiabatic response are given by :

a). For large collisionality, the electron is slowed down by collisions along the field line. This gives rise to a resistive drift wave instability" such as typical Hasegawa-Wakatani instability[41, 42].

b). For low collisionality, in a torus, a substantial fraction of electrons are trapped, giving rise to an instability referred to as a collisionless trapped electron mode (CTEM)[41]. In our research, we will mainly discuss the collisionless energy transfer channels induced by CTEM which is a particularly relevant instability for the low collisionality plasmas we will be interested in treating.

All these plasma instabilities can drive turbulence and turbulent transport. The presence of nested magnetic flux surfaces in a tokamak plasma, results in plasma losses only arising due to cross field radial transport. In a turbulent plasma, this cross field transport is primarily due to $E \times B$ drift motion associated with the turbulent fluctuations. The particle flux is given by

$$\Gamma = \langle \tilde{v}_{E\times B,r}\tilde{n}\rangle \tag{1.16}$$

where $\tilde{v}_{E\times B,r} = (c/B)E_\theta$, $\tilde{n}$ is a density fluctuation. The plasma heat flux generated by the drift wave turbulence is written as

$$Q = \left\langle \tilde{v}_{E\times B,r}\tilde{T} \right\rangle \tag{1.17}$$

The divergence of the heat flux $\nabla \cdot Q$ represents the turbulent transport. However, in electron heat transport experiments, some non-diffusive transport phenomena have attracted more attention [43, 44] (i.e. electron temperature profile manifests a property which is known as "profile consistency" or "profile stiffness" [45]. Profile stiffness means that the response of ∇T to increases in heating power or changes in deposition position is small). The general energy balance equation is given by

$$\frac{3}{2}\partial_t\left(nT_\alpha\right) + \nabla \cdot \left(-n\chi\nabla T_\alpha\right) = P_{eff}$$

$$P_{eff} = -\nabla \cdot \left(nV_\alpha T_\alpha\right) + \langle \tilde{E} \cdot \tilde{J}_\alpha \rangle \mp n\nu \frac{m_e}{m_i}\left(T_e - T_i\right) + \cdots, \alpha = e, i \tag{1.18}$$

where $-n\chi\nabla T_\alpha$ is the diffusive part of the heat flux and $nV_\alpha T_\alpha$ is a non-diffusive contribution to the heat flux. The collisioness energy coupling term $\langle \tilde{E} \cdot \tilde{J}_\alpha \rangle$ has the form of a local energy source or sink. The last term in Eq. (1.18), $n\nu(m_e/m_i)\left(T_e - T_i\right)$ is the electron and ion energy transfer by collisions. Note that, both the non-diffusive term [46, 47, 48] and collisionless energy transfer term can be related to the electron temperature evolution ("stiffness") experimental phenomenon. They are physically distinct and independent processes, which can, in principle, co-exist, especially in a low collisionality plasma. Which term has more efficiency on the temperature profile stiffness? That is one of our motivations to understand the collisionless energy coupling. In the next section, we will give a brief introduction of the anomalous electron-ion coupling which is a critical issue in a burning plasma, like ITER.

1.3 Basics of turbulent energy transfer

In the energy balance Eqn. (1.18), the plasma energy can be lost through turbulent transport or energy can be transferred to other species. How to understand turbulence mediated energy transfer channels? We can first get some hints from the energy dissipation mechanisms in 3D eddy turbulence and quasi-2D turbulence[49, 50, 51].

Turbulence is a ubiquitous phenomenon in nature. It has been observed in planetary atmospheres, laboratory plasmas[49, 50, 51], oceans... It is hard to precisely define what turbulence is, however we can identify several basic characteristics of turbulence. It exhibits strong disorder over a broad range of space and time scales. There are nonlinear interactions over multiple scales. How to recognize turbulence? The important tool is to look at the vorticity ($\vec{\omega} = \vec{\nabla} \times \vec{v}$) and vorticity evolution (vortex stretching, tilting...). In 1941, Kolmogorov proposed the energy cascade theory in 3D fluid turbulence[49]. Turbulent energy can be transferred from large scale eddies to small scale eddies through triad interactions throughout the inertial range. Vortex stretching plays a critical role here. In general, a vortex is " frozen" into a fluid element such that the vortex can be stretched in one direction which leads to the radial length scale of the vortex decreasing in the direction perpendicular to the stretching due to volume conservation. When stirring large scale eddies, they become unstable and break down into small scale eddies and this process continues until the eddies are smaller enough to feel the viscosity effect. Then the eddy's kinetic energy can be dissipated into heat. So turbulence is closely related to dissipation. If we seek to understand coupling between dissipative channels, then we must understand turbulence energy coupling. In 3D eddy turbulence, energy forward cascades and is dissipated via viscosity on small scales. For quasi-2D turbulence, energy can be both dissipated at large and small scales which is one of the focuses of our research topic: collisionless turbulent energy transfer between electrons and ions in drift wave turbulence.

In real life, quasi-2D turbulence includes the plasma turbulence in tokamaks, geostrophic turbulence on giant planets, turbulence in the solar tachocline[52]... The quasi-2D effects are enforced by strong magnetization $\vec{B}$ in tokamak turbulence, strong rotation $\vec{\Omega}$ in geostrophic turbulence or stratification in the solar tachocline. Also there are multi-players involved in these systems: eddy turbulence, waves and zonal flows which result in a more complicated description than 3D fluid turbulence (only nonlinear interaction between eddies).

What is tokamak turbulence? It is driven by temperature or density gradient relaxation[34, 53]. It is quasi-2D since the turbulence doesn't vary much along the magnetic field (see Fig.1.3 [54]). This leads to strongly anisotropic structures, where the modes vary on a length scale comparable to the ion gyroradius in the poloidal and radial direction, but are nearly constant along the magnetic field line. Plasma turbulence involves two species: electrons and ions. As in the case of quasi-2D turbulence described above, energy may be dissipated at both large and small scales. Here, energy dissipation on the large scales is mainly due to the drift wave and zonal flow interaction, where energy is transferred from microscale turbulence to large scale flows and then dissipated by zonal flow friction. Turbulence and zonal flow consist of a self-organized system. We will discuss these in the following section.

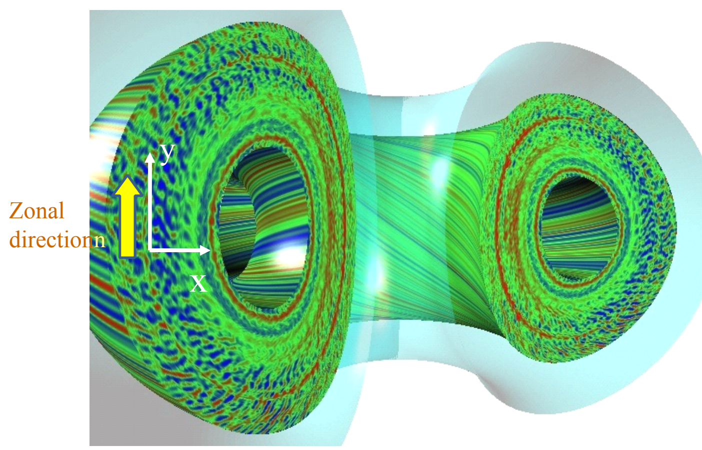

Figure 1.3: GYRO ITG turbulence, toroidal view. This figure appears courtesy of the General Atomic [54].

Drift waves are strongly dispersive whose dispersion relation can be described as [38] :

$$\omega = \frac{\omega_\star}{1 + k_\perp^2 \rho_s^2} \tag{1.19}$$

where $\omega_\star = k_y \rho_s c_s / L_n$ is the electron diamagnetic frequency, the scale length for the density gradient is given by $L_n^{-1} = --(1/n)(dn/dx), \rho_s = c_s/\Omega_i$. Here, the density gradient ∇n provides the relaxation drive for the wave instabilities and turbulence.

What is a zonal flow [20, 55, 56]? A zonal flow is an $n = m = 0$ toroidally, poloidally symmetric $E \times B$ shear flow. It is constant on a magnetic surface but rapidly varies in the radial direction, as illustrated in Fig.1.4. Zonal flows shear turbulence eddies and suppress turbulence and turbulent transport which is a main characteristic of zonal flow (see Fig.1.5)[20, 57, 58, 59, 60].

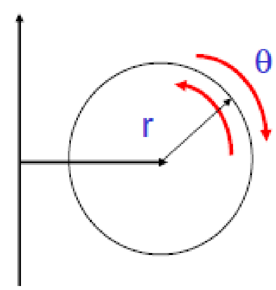

Figure 1.4: Zonal flow is constant in the poloidal direction and varies in the radial direction.

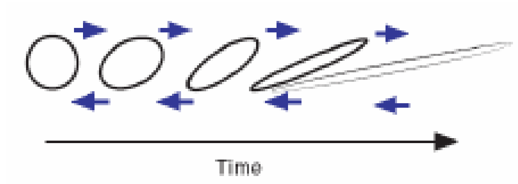

Figure 1.5: Zonal flow shearing turbulence eddies [20] .

How are zonal flows generated? Zonal flows are nonlinearly driven by Reynolds stress. The zonal flow evolution can be written as

$$\frac{\partial \langle v_\theta \rangle}{\partial t} = -\frac{\partial}{\partial r} \langle \tilde{v}_r \tilde{v}_\theta \rangle - \nu_{coll} \langle v_\theta \rangle \tag{1.20}$$

The first term on the RHS of Eqn. (1.20) is the Reynolds force which drives zonal flow generation. The second term is a weak damping through zonal flow friction ν_{coll}[21, 61, 62]. In Fig. 1.6, we show simulation results for the zonal flow generation in a tokamak [63]. The turbulence without zonal flow is shown on the LHS of Fig. 1.6. Here, the turbulence exhibits radially extended structure which allow for the efficient radial transport of heat. On the RHS of Fig. 1.6, the zonal flow is allowed to develop in turbulence. Since zonal flows have strong shearing effect, the radial extent of the turbulent eddies is significantly reduced. So we can see the turbulent radial structure was torn apart , leading to a significant reduction of the turbulence and turbulent transport.

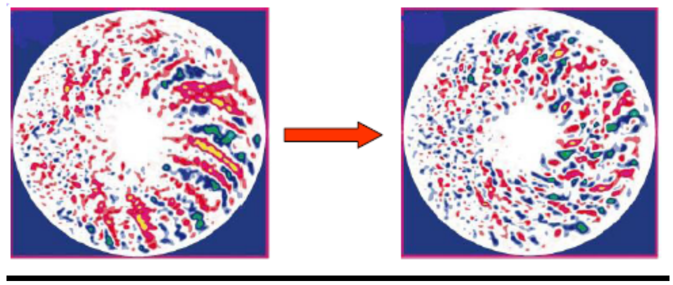

Figure 1.6: The poloidal contour shots: turbulence without the zonal flow on the left side of the figure; turbulence with the zonal flow on the right side of the figure [63].

Why are zonal flows important? Zonal flows provide an "energy sink" and naturally retain energy released by gradient driven turbulence[20]. Also zonal flow dissipate wave energy through zonal flow friction. In contrast to 3D eddy turbulence, where turbulent energy is only dissipated on small scales, energy can also be dissipated at large scales in tokamak turbulence due to the zonal flow frictional damping. Here the scale of the microturbulence is at the ion gyroradius ρ_i and zonal flow is at the mesoscale which is around a few ion gyroradius ρ_i. The energy transfer is from short wave length drift waves to long wave length zonal flows through non-local interaction. This is also the inspiration of our research. We consider the electron-ion energy coupling in the drift wave turbulence. Hot electrons may lose energy to the wave and drive wave instabilities and turbulence. Cold ions obtain energy from the wave through wave-particle resonance or zonal flow friction effect. Then turbulence plays an important role of mediating the collisionless energy transfer between electrons and ions. The idea of the turbulence energy transfer through zonal flow frictional damping can be seen in Fig. 1.7. In the next section, we will describe in more detail how turbulence and zonal flow interactions induced collisionless energy transfer in drift wave turbulence.

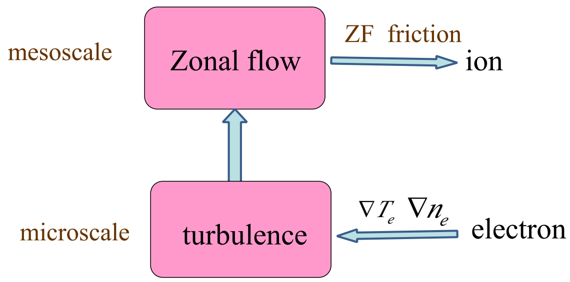

Figure 1.7: Turbulence and zonal flow interaction mediated the energy transfer between electrons and ions.

1.3.1 Turbulence and zonal flow interaction

The classical theorem for wave energy evolution in a strongly magnetized plasma is the wave energy balance theorem[64, 65] $\partial_t W + \nabla \cdot \mathbf{S} + \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = 0$, where W is the wave energy density, $\mathbf{S}$ represents the wave energy density flux, and $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle$ is the turbulent heating induced by current fluctuations[16]. It has been argued that there is no net turbulent heating and only electron-ion energy exchange through the wave-particle quasilinear resonance[15, 17]. In that calculation, *periodic boundary conditions* in the radial direction were utilized, such that boundary contributions to the turbulent heating vanished. However, we consider the turbulent heating taking place within a region of finite radial extent given by,

$$\int dr \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = -\tilde{\phi}\tilde{J}_r|_{r_1}^{r_2} + \int dr (\nabla \cdot \tilde{J})\tilde{\phi} \neq 0 \tag{1.21}$$

where the turbulent heating can be written as $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle = \sum_{\alpha=e,i} \left\langle \tilde{E} \cdot \tilde{J}_\alpha \right\rangle$, $\tilde{J}_r$ is the radial current fluctuation in an annular region , and the width of the annular region is $r_1 < r < r_2$. The $\langle \cdots \rangle$ defines an average in the θ, ϕ direction. The first term on the RHS of equation (1.21) corresponds to a surface term at the annular boundary which can give rise to a *net* turbulent heating. The second term vanishes since $\nabla \cdot \tilde{\mathbf{J}}=0$ (plasma quasi-neutrality). Thus, the boundary effect in a finite annular region will give rise to *net heating*. We also can look at this point from the perspective of a wave energy theorem. At steady state, the volume integral of the turbulent heating in an annulus gives rise to a wave energy flux differential, $\int dr \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = S|_{r_1}^{r_2}$. The radial component of the wave energy density flux S_r can be proportional to the Reynolds stress [65, 20]. Reynolds stress drives zonal flow [20]. Thus the wave energy flux differential is seen to be directly link to the zonal flow generation. So we need reconsider the energy coupling problems, which should include the nonlinear energy coupling through the turbulence and zonal flow interaction.

We consider ion polarization current contribution to turbulent heating in an annulus. The annulus has a width 2Δ at the local center r_0. The ion polarization current is given by

$$\tilde{\mathbf{J}}^i_{\perp pol} = en\tilde{V}^i_{\perp pol} = nm_i \frac{c^2}{B^2}\left[\frac{\partial}{\partial t}\tilde{\mathbf{E}}_\perp + \langle V_\theta \rangle \frac{\partial}{\partial \theta}\tilde{\mathbf{E}}_\perp + \hat{r}\tilde{V}_r \frac{\partial}{\partial r}\langle E_r \rangle\right] \tag{1.22}$$

where $\tilde{V}^i_{\perp pol}$ is the ion polarization drift velocity. Thus the ion polarization drift induced turbulent heating can be written as :

$$\begin{aligned}\int_0^{2\pi R} dz \int_0^{2\pi} r_0 d\theta \int_{r_0-\Delta}^{r_0+\Delta} \left\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}^i_{\perp pol} \right\rangle dr &= n_i m_i A \int_{r_0-\Delta}^{r_0+\Delta} dr \langle V_\theta \rangle' \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle \\ &= n_i m_i A (\langle V_\theta \rangle \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle)|_{r_0-\Delta}^{r_0+\Delta} - \int_{r_0-\Delta}^{r_0+\Delta} dr \, \langle V_\theta \rangle \frac{\partial}{\partial r} \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle)\end{aligned} \tag{1.23}$$

where the surface area is a constant and is given by a number A. (Note: we assumed the fluctuation spectrum to be stationary here). The first term in equation (1.23) is the energy flux differential at the boundary, which gives rise to a *net* turbulent heating. The second term in the RHS of equation (1.23) is the Reynolds work of the turbulence on the mean flow, which is directly linked to the zonal flow drive [65].

In the zonal flow evolution equation (1.20), $\partial \langle v_\theta \rangle / \partial t = -\partial \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle / \partial r - \nu_{coll} \langle v_\theta \rangle$, the zonal flow is driven by the divergence of the Reynolds stress and damped through frictional effect [20]. At steady state, Reynolds work on the zonal flow must balance zonal flow friction [21], $\partial \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle / \partial r = -\nu_{col} \langle V_\theta \rangle$, then the ion polarization drift induced turbulent heating in the equation (1.23) can be approximated as

$$\left\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}^i_{\perp pol} \right\rangle \approx n m_i A \int_{r_0-\Delta}^{r_0+\Delta} dr \nu_{col} \langle V_\theta \rangle^2 \tag{1.24}$$

which is zonal flow frictional damping ($\sim \nu_{col} \langle V_\theta \rangle^2 \sim |e\tilde{\phi}/T|^4$).

Hence, the ion polarization drift induces turbulent heating over an annular region which gives rise to a net heating that is ultimately due to the zonal flow friction. This process of energy transfer via zonal flows has not previously been accounted for in analyses of energy coupling. In addition, zonal flow frictional heating is estimated below by using the mixing length approximation [10, 66], where it is shown to provide a significant energy transfer channel in low collisionality plasmas, like ITER. In ITER, collisionless energy transfer can be of equal importance as the collisional transfer ($P_{i,e} = n\nu \frac{m_e}{m_i} (T_e - T_i)$). As T_i and T_e get closer, the collisional energy transfer can drop and collsionless energy transfer will dominate the energy transfer channels. Thus, it is worth comparing the collisionless energy transfer with turbulent transport. Also The collisionless energy transfer must be accounted for in the energy transport models. The details of the calculation will be discussed in Chapter 2.

1.3.2 Quasilinear wave-particle interaction

Collisionless energy transfer involves linear and nonlinear wave-particle interaction[23, 24, 25, 26, 27, 28]. Within quasilinear theory, the key point is the mean field relaxation. In other words, how the mean distribution function (i.e. whose moments describe $\langle n \rangle$, $\langle T \rangle$, $\langle P \rangle$) evolves in the presence of turbulence. We look at the application of quasilinear theory to drift waves. The ion drift kinetic equation can be written as

$$\frac{\partial f_i}{\partial t} + v_\parallel \nabla_\parallel f_i + \frac{c\hat{b} \times \nabla \tilde{\phi}}{B} \cdot \nabla f_i - \frac{e}{m} \nabla_\parallel \phi \frac{\partial f_i}{\partial v_\parallel} = 0 \tag{1.25}$$

Taking the ensemble average of this equation, we obtain

$$\frac{\partial \langle f_i \rangle}{\partial t} = -\left\langle \frac{c\hat{b} \times \nabla \tilde{\phi}}{B} \cdot \nabla \tilde{f}_i \right\rangle + \left\langle \frac{e}{m} \nabla_\parallel \tilde{\phi} \frac{\partial \tilde{f}_i}{\partial v_\parallel} \right\rangle \tag{1.26}$$

where the ion distribution function consists of a mean and fluctuation part: $f_i = \langle f_i \rangle + \tilde{f}_i$. For simplicity, we take $\langle f_i \rangle = \left\langle f_i(x, v_\parallel) \right\rangle, \hat{b} = \hat{z}$. Linearizing equation (1.25) in Fourier space, we have

$$\tilde{f}_i = \frac{i}{\omega - k_\parallel v_\parallel}\left(-\frac{c\hat{b} \times \nabla \tilde{\phi}}{B} \cdot \nabla \langle f_i \rangle + \frac{e}{m} \nabla_\parallel \tilde{\phi} \frac{\partial \langle f_i \rangle}{\partial v_\parallel}\right) \tag{1.27}$$

Plugging equation (1.27) into (1.26), we see that the mean distribution function will evolve according to

$$\frac{\partial \langle f_i \rangle}{\partial t} = \frac{\partial}{\partial v} D_{vv}^{(2)} \frac{\partial \langle f_i \rangle}{\partial v} + \frac{\partial}{\partial v} D_{vx}^{(2)} \frac{\partial \langle f_i \rangle}{\partial x} + \frac{\partial}{\partial x} D_{xv}^{(2)} \frac{\partial \langle f_i \rangle}{\partial v} + \frac{\partial}{\partial x} D_{xx}^{(2)} \frac{\partial \langle f_i \rangle}{\partial x} \tag{1.28}$$

where the diffusion coefficients $D_{vv}^{(2)} = Re \sum_k (e/m)^2 k_\parallel^2 |\tilde{\phi}|^2 i/(\omega - k_\parallel v_\parallel)$ describes diffusion in velocity space caused by random velocity scattering. $D_{xx}^{(2)} = Re \sum_k (c/B)^2$ $k_\theta^2 |\tilde{\phi}|^2 \ (i/\omega - k_\parallel v_\parallel)$ is diffusion in real space, which arises due to random $E \times B$ scattering. $D_{xv}^{(2)} = D_{vx}^{(2)} = Re \sum_k (ec/mB) k_\theta k_\parallel |\tilde{\phi}|^2 (i/\omega - k_\parallel v_\parallel)$ is the cross field diffusion, $D_{xv}^{(2)}$ which results from sheared acceleration.

Taking the energy integral of the diffusion equation (1.28), the resonant ion kinetic energy evolution equation can be written as

$$\begin{aligned} \frac{\partial E_{res}}{\partial t} &= \int dv \frac{1}{2} m v^2 \frac{\partial}{\partial t} \langle f_i \rangle \\ &= -\int dv m v D_{vv}^{(2)} \frac{\partial}{\partial v} \langle f_i \rangle - \int dv m v D_{vx}^{(2)} \frac{\partial}{\partial x} \langle f_i \rangle \\ &+ \frac{\partial}{\partial x} \int dv \frac{1}{2} m v^2 \left(D_{xv}^{(2)} \frac{\partial}{\partial v} \langle f_i \rangle + D_{xx}^{(2)} \frac{\partial}{\partial x} \langle f_i \rangle \right) \end{aligned} \tag{1.29}$$

From the total ion energy balance, we have:

$$\frac{\partial E}{\partial t} + \nabla \cdot Q + \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle = 0 \tag{1.30}$$

where E represents the kinetic energy of ions, Q is the spatial heat flux and $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle$ is the turbulent heating. Since total wave and resonant particles energy is conserved, combining Eqn.(1.28) and Eqn. (1.30), we have the turbulent heating

$$\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(2)} = -\int dvmvD_{vv}^{(2)} \frac{\partial}{\partial v} \langle f_i \rangle - \int dvmvD_{vx}^{(2)} \frac{\partial}{\partial x} \langle f_i \rangle \tag{1.31}$$

Given the diffusion coefficient in Eqn. (1.29), we can obtain the quasilinear ion turbulent heating. This is one way to calculate the turbulent heating from the mean particle diffusion equation. Another way to calculate the turbulent heating is through the current fluctuation $\tilde{J}$ which is given by

$$\tilde{J} = \sum_{e,i} e \int d^3v v \tilde{f}_i = \sigma^t \tilde{E} \tag{1.32}$$

where the σ^t is the plasma conductivity due to the turbulence. In Eqn. (1.32), the ion distribution function $\tilde{f}_i$ can be obtained from linearizing ion drift kinetic equation, thus the ion turbulent heating can be written as :

$$\left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)} = \sum_k -e \int dv_z v_z \tilde{E}_z \tilde{f}_i \tag{1.33}$$

Note that the Eqn. (1.31) and (1.33) will have the same results and be discussed in Chapter 2 and Chapter 3. In addition, the quasilinear turbulent heating calculation shows that electrons will lose energy to the drift wave through inverse Landau damping and give rise to the electron cooling $\left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle < 0$. In contrast, ions will obtain energy from the wave via ion Landau damping such that $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle > 0$. This is one of collisionless energy transfer channels.

1.3.3 Nonlinear wave-particle interaction

In the last two sections, the anomalous electron-ion energy coupling was shown to be mediated through quasilinear wave-particle resonance and turbulence and zonal flow interaction. Zonal flows are nonlinearly generated, i.e. they scale as $\mathcal{O}|e\tilde{\phi}/T|^4$, thus it is necessarily to extend the calculation to include nonlinear wave-particle interactions in the parallel magnetic field direction[23, 24, 25, 26, 27, 28, 29]. The nonlinear ion heating can be calculated through weak turbulence theory. We consider two primary modes (ω, k), (ω', k') which drive the beat mode generation (ω'', k'') if the resonance conditions $\omega'' = \omega - \omega'$ and $k'' = k' - k$ are satisfied. Naively, this beat mode contribution appears smaller than the quasilinear ion turbulent heating since $\left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel i} \right\rangle_{NL} / \left\langle \tilde{E}_\parallel \cdot \tilde{J}_\parallel i \right\rangle_{QL} \sim |e\tilde{\phi}_k/T_i|^2$. However, the relevant comparison can be written as $\left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel i} \right\rangle^{(4)} / \left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)} \sim |e\tilde{\phi}/T_i|^2 \exp(\omega^2/k^2{}_\parallel V^2{}_{thi})$. The exponential factor is large while the turbulence intensity is small. Thus *nonlinear ion turbulent heating is surely important and it is not necessarily small*.

Examining the average ion distribution function profile (see Fig. 1.8), the linear Landau damping is located on the tail of the ion distribution which means the resonance is weak[53]. However, the beat mode can resonate with the bulk of the ion distribution, which makes nonlinear Landau damping stronger that would be naively anticipated. Physically, the wave is excited by electrons where the wave's phase velocity satisfies $v_{thi} < \omega/k < v_{the}$ (Note: $v_{the} \gg v_{thi}$, due to $m_i \gg m_e$). So, it is harder for ions to couple to the electron drift waves and the quasilinear ion Landau damping is weak. But the beat mode's phase velocity can be much smaller which is comparable to the ion thermal velocity, so it is much easier for beat modes to interact with ions. Thus *the beat mode resonance is a strong nonlinear effect*. Hence, nonlinear ion heating is an important energy transfer channel and should not be neglected.

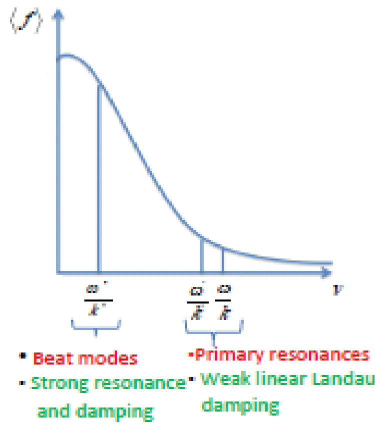

Figure 1.8: Beat mode resonance is a strong nonlinear effect compared with the weak quasilinear resonance effect.

Following the energy integral of the ion diffusion equation (1.29), we can extend the quasilinear diffusion coefficients to nonlinear order. The fourth order diffusion coefficient can be written as: $D_{vv}^{(4)} = \int_0^\infty \left\langle \tilde{F}^{(2)}(\tau)\tilde{F}^{(2)}(t+\tau)\right\rangle d\tau$ in velocity space, $D_{xx}^{(4} = \int_0^\infty \left\langle \tilde{v}^{(2)}(\tau)\tilde{v}^{(2)}(t+\tau)\right\rangle d\tau$ in real space and the cross term diffusion $D_{xv}^{(4)} = D_{vx}^{(4)} = \int_0^\infty \left\langle \tilde{F}^{(2)}(\tau)\tilde{v}^{(2)}(t+\tau)\right\rangle d\tau$ combines the velocity and real space effects, where $\tilde{F}^{(2)}$ and $\tilde{v}^{(2)}$ are second order in the electrostatic potential fluctuation $|\tilde{\phi}|^2$. Note that the second order perturbation contributed by the $E \times B$ scattering is much bigger than the parallel force acceleration since $k_\perp \gg k_\parallel$. Thus the nonlinear ion heating can be written as :

$$\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(4)} = -\int dvmvD_{vv}^{(4)} \frac{\partial}{\partial v} \langle f_i \rangle - \int dvmvD_{vx}^{(4)} \frac{\partial}{\partial x} \langle f_i \rangle \tag{1.34}$$

The details of the nonlinear diffusion coefficients will be discussed in Chapter 3.

1.3.4 Organization of this thesis

In the remainder of this thesis, the turbulent heating and anomalous electron-ion energy coupling will be analyzed in drift wave turbulence. In Chapter 2, the wave energy flux differential is shown to give rise to a *net* heating which is due to zonal flows. Zonal flow frictional heating contributes an important collisionless energy transfer channel. Also this channel is significant compared to the energy dissipated by ion Landau damping through the linear wave-particle resonance in a low collisionality plasma, like ITER. In addition, the collisionless energy transfer can dominate the energy transfer channel in ITER. The rate of the energy transfer can be the same order as the turbulent energy transport in CTEM. Thus, the collisionless energy transfer effect has to be considered in a transport model. In Chapter 3, we discuss another significant collisionless energy transfer channel which is nonlinear ion Landau damping. Nonlinear ion heating is not necessarily small compared with the quasilinear ion heating due to the strong $E \times B$ scattering and the exponential effect. Thus, a general anomalous electron -ion coupling forms will be given in this thesis. Considering the turbulence mediated electron-ion energy exchange term in the energy balance equation, some puzzling thermal transport experiments and simulation results can be explained in present tokamaks.

Chapter 2

Collisionless inter-species energy transfer and turbulent heating in drift wave turbulence

2.1 Introduction

2.1.1 A general turbulent energy transfer and transport problem

We reconsider the classic problems of calculating "turbulent heating " and collisionless inter-species transfer of energy in drift wave turbulence. This issue is of interest for near future low collisionality electron heated plasmas, such as ITER, where collisionless energy transfer from electrons to ions is likely to be significant. In the heat balance equation [15, 16],

$$\frac{3}{2}n\frac{\partial T_\alpha}{\partial t} + \nabla \cdot Q_\alpha = \left\langle \tilde{E} \cdot \tilde{J}_\alpha \right\rangle \mp n\nu\frac{m_e}{m_i}(T_e - T_i) + \cdots , \alpha = e, i \tag{2.1}$$

where Q_α is the heat flux, $\left\langle \tilde{E} \cdot \tilde{J}_\alpha \right\rangle$ is turbulent dissipation and corresponds to turbulent heating [16], and the last term on the right hand side represents the collisional transfer of energy between particle species. Generally, turbulent energy can be exchanged between electrons and ions by collisional or collisionless energy transfer [15, 16, 17]. The familiar term $n\nu\frac{m_e}{m_i}(T_e - T_i)$ describes collisional energy transfer through electron and ion binary collisions whereas $\left\langle \tilde{E} \cdot \tilde{J}_\alpha \right\rangle$ describes turbulent heating for a single species, such as electron turbulent cooling ($\left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle < 0 \rightarrow$ electrons lose energy to the drift wave so the wave is destabilized), or ion turbulent heating ($\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle > 0 \rightarrow$ ion gain energy from the drift wave and wave is tabilized). If we consider inter-species turbulent heating for electron heated plasmas, electron and ion collisionless energy transfer could occur where the hot electrons act as the local energy "source " and cold ions as the "energy sink". This collisionless turbulent energy transfer is especially important in a burning plasma, since the fuel ions (deuterium, tritium) can be heated by this inter-species turbulent energy transfer process. Also we note that any energetic particles (α particles) produced in the nuclear reaction will heat electrons first [14, 12]. Thus, the energy flow can be transferred from hot electrons to cooler ions again, to allow the reaction to sustain itself. Hence, the electron and ion collisionless energy coupling will be a critical issue for burning plasmas, such as ITER.

To see this, we track the energy exchanged between electrons and ions. There exist two stages of energy transfer processes before and after the nuclear reaction (see Fig. 2.1). We ignore the energy loss due to electron and ion radiation in these processes. During the first stage, electrons can be heated by the auxiliary energy system ECRH (Electron Cyclotron Resonance Heating) and then lost by electron heat flux Q_e or transferred to ions through collisionless or collisional energy transfer channels. Next, the fuel ions become hot enough and can reach nuclear reaction ignition. Once the energetic particles are produced in the nuclear reaction, the second energy flow will be generated and the process of energy transport or transfer can continue with more and more reactions occurring. But what is the ultimate fate of the energy? We need to understand the collisionless energy transfer mechanism and what roles turbulent energy transfer and turbulent transport play in the energy budget.

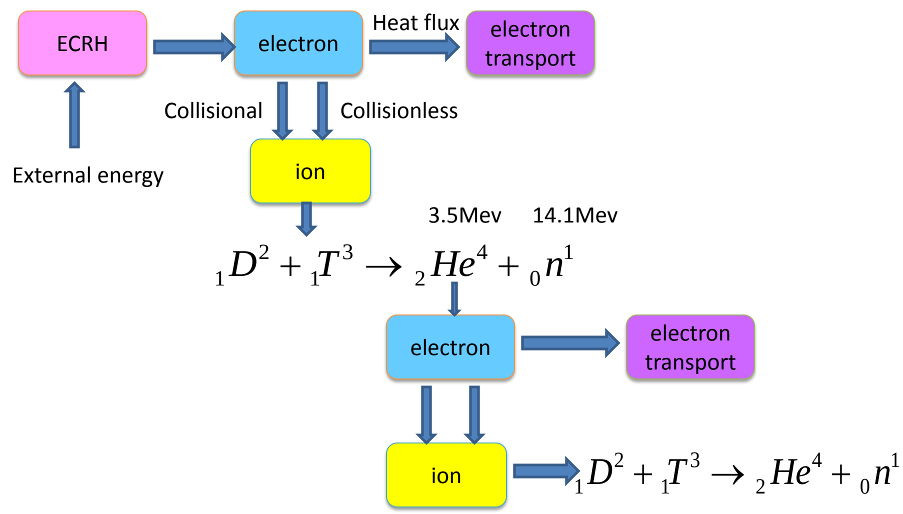

Figure 2.1: There are two stages in the energy flow before and after the nuclear reaction.

2.1.2 Net turbulent heating

Does turbulence heat a given volume of plasma? Manheimer et al. argued that there was no net turbulent heating of plasma and only an exchange of energy between electrons and ions [15]. In that calculation, *periodic boundary conditions* in the radial direction were utilized, such that boundary contributions to the turbulent heating vanished. However, we consider the turbulent heating taking place within a region of finite radial extent given by,

$$\int dr \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = -\tilde{\phi}\tilde{J}_r|_{r_1}^{r_2} + \int dr (\nabla \cdot \tilde{J})\tilde{\phi} \neq 0 \tag{2.2}$$

where the turbulent heating can be written as $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle = \sum_{\alpha=e,i} \left\langle \tilde{E} \cdot \tilde{J}_\alpha \right\rangle$, $\tilde{J}_r$ is the radial current fluctuation in an annular region , and the width of the annular region is $r_1 < r < r_2$. The $\langle \cdots \rangle$ defines an average in the θ, ϕ direction. The first term on the RHS of equation (2.2) corresponds to a surface term at the annular boundary which can give rise to a *net* turbulent heating. The second term vanishes since $\nabla \cdot \tilde{\mathbf{J}}$=0 (plasma quasi-neutrality). Thus, the boundary effect in a finite annular region will give rise to *net heating*. This can also be understood from the wave energy theorem which is written as [64, 65],

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{S} + \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = 0 \tag{2.3}$$

where W is wave energy density and $\mathbf{S}$ represents wave energy density flux. At steady state, taking a volume integral of this theorem in an annular region, we have at stationarity,

$$\int_{r_1}^{r_2} dr \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = -S_r|_{r_1}^{r_2} \tag{2.4}$$

Equation (2.4) suggests that a wave energy flux differential across an annular region $r_1 < r < r_2$ gives rise to a *net* heating within that region. Aspects of this point have been addressed in Refs. [16, 17]. In addition, this wave energy flux can be related to the Reynolds stress by noting the wave energy density flux can be written as [65, 20],

$$S_r = V_{gr,r} W_k = -2 \frac{\rho_s^2 k_r k_\theta v_\star W_k}{(1 + k_\perp^2 \rho_s^2)^2} \tag{2.5}$$

where $V_{gr,r} = d\omega/d\mathbf{k}$ is the wave group velocity. W_k is the wave energy density in Fourier space. In the electrostatic drift wave, $V_{gr,r} = -(d\chi/d\mathbf{k})/(d\chi/d\omega) = -(2\rho_s^2 k_r \omega)$ $/(1 + k_\perp^2 \rho_s^2)$ and the susceptibility is $\chi(k,\omega) = 1 + k_\perp^2 \rho_s^2 - \omega_\star/\omega - i\delta_\omega$. The electron drift wave frequency is $\omega = \omega_\star/(1 + k_\perp^2 \rho_s^2)$ and the electron diamagnetic frequency is $\omega_\star = k_\theta v_\star$. Since the Reynolds stress can be written as $\left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle = \sum_k -(c^2/B^2) k_r k_\theta |\tilde{\phi}|^2$, the energy flux differential is seen to be directly proportional to the Reynolds stress, which drives zonal flow generation.

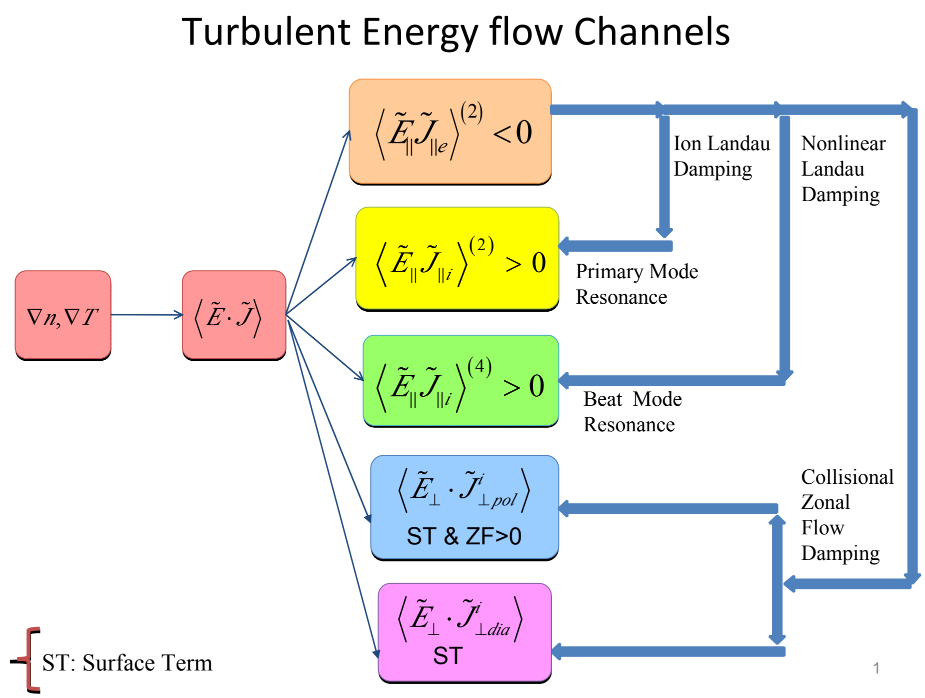

Figure 2.2: Turbulent energy flow channels: "energy source" of quasilinear electron cooling$\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)}$; "energy sink" of quasilinear ion turbulent heating $\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)}$, nonlinear ion heating$\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \right\rangle^{(4)}$, ion polarization drift and ion diamagnetic drift induced heating $\left\langle \tilde{E}_{\perp} \cdot \tilde{J}^{i}_{\perp pol} \right\rangle$ and $\left\langle \tilde{E}_{\perp} \cdot \tilde{J}^{i}_{\perp dia} \right\rangle$.

2.1.3 Collisionless turbulent energy transfer channels

Now we look at the collisionless interspecies energy transfer channels. The turbulent energy flow channels considered within this manuscript are shown in Fig. 2.2. Free energy in ∇n and ∇T will drive microturbulence which gives rise to turbulent heating $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle$. The dominant parallel components of the turbulent heating are composed of quasilinear electron cooling$\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)}$, quasilinear ion heating $\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)}$, and nonlinear ion turbulent heating $\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \right\rangle^{(4)}$. The perpendicular components considered are given by the ion polarization drift and ion diamagnetic drift induced turbulent heating $\left\langle \tilde{E}_{\perp} \cdot \tilde{J}^{i}_{\perp pol} \right\rangle$ and $\left\langle \tilde{E}_{\perp} \cdot \tilde{J}^{i}_{\perp dia} \right\rangle$. The ion polarization drift induced turbulent heating in an annulus contributes a wave energy flux differential term at the boundary, also a Reynolds work of turbulence on the mean flow. This flux differential, intimately linked to the turbulent Reynolds stress and hence zonal flow formation, will be shown to give rise to net turbulent heating. This process of energy transfer via zonal flows has not previously been accounted for in analyses of energy transfer. On the right side of the Fig. 2.2, we describe three kinds of energy dissipation channels for electron drift wave turbulence. The first two are wave energy dissipated through ion Landau damping at quasilinear order and nonlinear ion Landau damping (the beat mode resonance) [24, 25, 26, 27, 28]. The third one is by zonal flow frictional damping at steady state, which is due to the turbulence and zonal flow interaction [20, 21, 22]. Basically, the electrons will lose energy to the wave and the ions will gain energy from the wave, thus providing collisionless energy transfer channels mediated by electron drift waves and zonal flow.

For each turbulent heating term, there will be a corresponding γ_{growth} for wave growth or $\gamma_{damping}$ for wave dissipation in the saturation balance condition. Nonlinear saturation in a turbulent state implies energy transfer from source $(\nabla T_e, \nabla n)$ to sink [29]. Schematically, saturation implies some fluctuation energy balance condition must be satisfied, so we have

$$0 = \gamma = \gamma_L^{electron} + \gamma_L^{ion} + \gamma_{NL}^{ion} + \gamma_{zonalflow} + \cdots \tag{2.6}$$

For quasilinear electron cooling terms, the electrons will lose energy to the drift wave and drive wave instability, which gives rise to $\gamma_L^{electron} > 0$. In contrast, for quasilinear ion heating, the ions gain energy from the wave through ion Landau damping , so $\gamma_L^{ion} < 0$. Also the wave energy can be dissipated through nonlinear ion Landau damping and $\gamma_{NL}^{ion} < 0$ [26, 27, 28, 29]. In particular, wave energy can be dissipated by zonal flow friction, so that $\gamma_{zonalflow} < 0$ in the saturation balance[8]. Here $\gamma_{zonalflow}$ represent a nonlinear saturation mechanism. If $\gamma_{zonalflow}$ dominates the saturation balance [20, 62], corresponding to turbulent heating, the energy coupled to the zonal flow must contribute to energy transfer channels as well. (Note: the nonlinear ion turbulent heating and its contribution for turbulent energy transfer channel will be discussed in another paper, for simplicity).

This paper is organized as follows. In section II, we will calculate the parallel quasilinear turbulent heating terms $\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)}$ and $\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)}$ which effect a collisionless energy transfer from electrons to ions, mediated by the background waves. In section III, the ion polarization drift and ion diamagnetic drift induced turbulent heating are calculated. $\left\langle \tilde{E}_{\perp} \cdot \tilde{J}^{i}_{\perp pol} \right\rangle$ accounts for the energy flux differential term at the boundaries and the net Reynolds work of turbulence on the zonal flow. And $\left\langle \tilde{E}_{\perp} \cdot \tilde{J}^{i}_{\perp dia} \right\rangle$ contributes a heat flux differential at boundary ($\sim \chi_{tur} n \nabla \tilde{T}$), which transports the turbulent energy through ion diffusion. In section IV and V, we will estimate all of the turbulent heating terms by using a mixing length estimation for $|e\tilde{\phi}/T|^2$. At different collisionalities, we will compare the ratios of the energy dissipation channels through linear Landau damping and zonal flow frictional damping. By applying ITER like parameters [12, 13], we can see that the zonal flow frictional damping can be a significant energy dissipation channel in low collisionality drift wave turbulence in ITER plasmas. In sections VI and VII, we will continue to explore the implications of our results for ITER plasmas. For realistic cases, we will extend our discussion to CTEM(collisionless trapped electron mode) [26, 27, 28, 29]. Firstly we will compare two energy transfer channels: collisional and collisionless. Subsequently, we compare the collisionless energy transfer with the turbulent energy transport losses through the CTEM heat flux. At the end, we will summarize the turbulent heating and collisionless energy transfer channels in drift wave turbulence.

2.2 Quasilinear turbulent heating for drift wave turbulence

Within this section, we calculate the turbulent heating for both electrons and ions by using quasilinear theory. In the limit of vanishing collisionality, the stability of a wave can be determined by computing the transfer of energy between resonant particles and waves. For the case of collisionless drift waves considered below, resonant electrons will lose energy to the wave (resulting in electron cooling), whereas ions generally gain energy (via ion Landau damping) , and are thus heated. Any imbalance in the rate of electron cooling versus ion heating will generally lead to wave growth or damping.

Generally, the turbulent heating is driven by current fluctuations which can be written as

$$\tilde{J} = \sum_{e,i} e \int d^3 v v \tilde{f} = \sigma^t \tilde{E} \tag{2.7}$$

where the σ^t is the plasma conductivity due to the turbulence. The calculation of the turbulent heating requires computing a plasma conductivity which related to the particle distribution function $\tilde{f}$. We will be interested in describing the evolution of collisionless drift waves in an electron heated plasma in a simplified geometry. The evolution of electrons will be described by a drift kinetic equation (DKE), namely

$$\frac{\partial \tilde{f}_e}{\partial t} - i\frac{c}{B}k_y \frac{\partial \langle f_e \rangle}{\partial x}\tilde{\phi}_k + v_z \frac{\partial \tilde{f}_e}{\partial z} - \frac{e}{m_e}\tilde{E}_z \frac{\partial \langle f_e \rangle}{\partial v_z} = 0. \tag{2.8}$$

The fluctuation of the electron distribution can be separated into adiabatic and non-adiabatic parts

$$\tilde{f}_e = \frac{e\tilde{\phi}_k}{T_e}\langle f_e \rangle + \tilde{g}_k \tag{2.9}$$

Substituting (2.9) into (2.8) , yields

$$\frac{\partial \tilde{g}_k}{\partial t} + v_z \frac{\partial \tilde{g}_k}{\partial z} = -\frac{\partial}{\partial t}\frac{e\tilde{\phi}_k}{T_e}\langle f_e \rangle - v_z \frac{\partial}{\partial z}\frac{e\tilde{\phi}_k}{T_e}\langle f_e \rangle - \frac{e}{m_e}\frac{\partial}{\partial z}\tilde{\phi}_k \frac{\partial \langle f_e \rangle}{\partial v_z} + i\frac{c}{B}k_y \frac{\partial \langle f_e \rangle}{\partial x}\tilde{\phi}_k \tag{2.10}$$

We take the electron equilibrium distribution function as a local Maxwellian in one dimension, i.e$\langle f_e \rangle = n_0(x)(m_e/2\pi T_e)^{1/2}\exp(-v_z^2/V_{the}^2)$ and $\partial \langle f_e \rangle / \partial v_z = (-2v_z/V_{the}^2)$ $\langle f_e \rangle$. Substituting $\langle f_e \rangle$ into equation (2.10) yields:

$$\frac{\partial \tilde{g}_k}{\partial t} + v_z \frac{\partial \tilde{g}_k}{\partial z} = -\frac{\partial}{\partial t}\frac{e\tilde{\phi}_k}{T_e}\langle f_e \rangle + i\frac{c}{B}k_y \frac{\partial \langle f_e \rangle}{\partial x}\tilde{\phi}_k \tag{2.11}$$

Fourier transforming equation (2.11) gives

$$\begin{aligned}\tilde{g}_k &= \frac{-\omega\frac{e\tilde{\phi}_k}{T_e}\langle f_e \rangle - \frac{c}{B}k_y \frac{\partial \langle f_e \rangle}{\partial x}\tilde{\phi}_k}{\omega - k_z v_z} \\ &= \frac{(\omega_{\star e} - \omega)\frac{e\tilde{\phi}_k}{T_e}\langle f_e \rangle}{\omega - k_z v_z}\end{aligned} \tag{2.12}$$

Here we define the diamagnetic frequency $\omega_{\star e} = -k_y(T_e c/eB)(1/n)(dn/dx) = k_y \rho_s C_s/L_n$, and the scale length for the density gradient is given by $L_n^{-1} = -(1/n)(dn/dr)$, $\rho_s = C_s/\Omega_i$. Then the quasilinear turbulent heating for electrons can be written as

$$\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)} = \sum_k -e \int dv_z v_z \tilde{E}_z \tilde{g}_k \tag{2.13}$$

Here the contribution of the electron quasilinear turbulent heating is in the parallel electric field direction. Substituting (2.12) into (2.13), we obtain

$$\begin{aligned}\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)} &= \sum_k -e \int dv_z v_z (ik_z \tilde{\phi}_{-k}) \frac{(\omega_{\star e} - \omega)}{\omega - k_z v_z} \frac{e\tilde{\phi}_k}{T_e} \langle f_e \rangle \\ &= \sum_k -e \int dv_z v_z (ik_z \tilde{\phi}_{-k})(\omega_{\star e} - \omega) \frac{(-i\pi)}{|k_z|} \delta(\omega/k_z - v_z) \frac{e\tilde{\phi}_k}{T_e} \langle f_e \rangle \\ &= \sum_k \sqrt{\pi} n T_e \left| \frac{e\tilde{\phi}_k}{T_e} \right|^2 \frac{\omega}{|k_z| V_{the}} (\omega - \omega_{\star e}) \exp\left[-\frac{(\omega/k_z)^2}{V_{the}^2} \right]\end{aligned} \tag{2.14}$$

The calculation of the δ function integral $\int dv_z \delta(\omega/k_z - v_z) \langle f_e \rangle = \langle f_e \rangle |_{v_z=\omega/k_z}$was applied to equation (2.14), where the electron drift waves resonate with the background electrons if the phase velocity satisfies $v_z = \omega/k_z$. We also note that the quasilinear turbulent heating for electrons $\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)}$ is determined by the resonant electron density n, temperature T_e and turbulent intensity $|e\tilde{\phi}_k/T_e|^2$.

Considering the drift wave dispersion relationship $\omega = \omega_{\star e}/(1 + k_\perp^2 \rho_s^2)$, then we have $\omega(\omega - \omega_{\star e}) < 0$ which implies a competition between stabilization and destabilization effects for the electron drift wave. On one hand, the ω is the Landau resonance frequency and the wave was stabilized by the Landau damping. On the other hand, free energy was released by the density gradient relaxation related to the diamagnetic frequency $\omega_{\star e}$, which drives wave instability. Here, the electron drift wave was destabilized since $\omega(\omega - \omega_{\star e}) < 0$ which flips the sign of the electron dissipation and so gives rise to inverse Landau damping. So we obtain the quasilinear electron cooling $\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)} < 0$ and the energy exchange between electrons and drift waves through Landau resonance.

Similarly, we calculate the quasilinear turbulent heating for ions. Utilizing a

DKE for ions

$$\frac{\partial \tilde{f}_i}{\partial t} - i\frac{c}{B}k_y\frac{\partial \langle f_i \rangle}{\partial x}\tilde{\phi}_k + v_z\frac{\partial \tilde{f}_i}{\partial z} + \frac{e}{m_e}\tilde{E}_z\frac{\partial \langle f_i \rangle}{\partial v_z} = 0. \tag{2.15}$$

Linearizing equation (2.15), we have

$$\tilde{f}_i = \frac{\frac{e\tilde{\phi}_k}{T_i}k_z v_z\langle f_i\rangle + \frac{e\tilde{\phi}_k}{T_e}\omega_{\star e}\langle f_i\rangle}{\omega - k_z v_z} \tag{2.16}$$

Substituting equation (2.16) into the turbulent heating for ions, yields

$$\begin{aligned}\left\langle \tilde{E}_{\|}\cdot\tilde{J}_{\|i}\right\rangle^{(2)} &= \sum_k e\int dv v_z\tilde{E}_z\tilde{f}_i \\ &= \sum_k \sqrt{\pi}nT_i|\frac{e\tilde{\phi}_k}{T_e}|^2\frac{\omega}{|k_z|V_{thi}}\left(\omega + \frac{T_i}{T_e}\omega_{\star e}\right)\exp\left[-\frac{(\omega/k_z)^2}{V_{thi}^2}\right] \end{aligned} \tag{2.17}$$

Here the quasilinear ion heating $\left\langle \tilde{E}_{\|}\cdot\tilde{J}_{\|i}\right\rangle^{(2)} > 0$. The ions gain energy from the drift wave through ion Landau damping. In the parallel magnetic field direction , the total quasilinear turbulent heating is $\left\langle \tilde{E}_{\|}\cdot\tilde{J}_{\|}\right\rangle^{(2)} = \left\langle \tilde{E}_{\|}\cdot\tilde{J}_{\|e}\right\rangle^{(2)} + \left\langle \tilde{E}_{\|}\cdot\tilde{J}_{\|i}\right\rangle^{(2)}$. The energy flow is described by $electron \longrightarrow drift\ wave \longrightarrow ion$. The energy in the hot electrons is lost to the drift wave through inverse Landau damping and the cold ions gain energy via Landau damping. Basically, energy was transferred from electrons to ions via wave-particle resonance (Landau damping). This is one of the wave energy dissipation channels.

So far we have reviewed the calculation of turbulent heating at quasilinear order. We note that the quasilinear electron cooling generally doesn't cancel the quasilinear ion heating and the energy leftover in the drift wave will drive the wave instability, which is written as $\gamma_L = \gamma_e - \gamma_i > 0$. So, there must be other turbulent heating and energy transfer channels available to the system to dissipate turbulence energy and stabilize the drift wave. In the next section, we will show that the ion polarization drift and ion diamagnetic drift induce net ion turbulent heating which contribute another important energy transfer channel in the direction perpendicular to the magnetic field.

2.3 Ion polarization and ion diamagnetic current contribution to turbulent heating

Since the $E \times B$ drifts of the ions and electrons cancel automatically, and $m_e \ll m_i$ (which allows the electron polarization drift to be ignored) , we only need to consider the perpendicular turbulent current carried by the ion polarization drift and the ion diamagnetic drift in the direction perpendicular to the magnetic field, which is written as

$$\tilde{\mathbf{J}}^i_{\perp} = \tilde{\mathbf{J}}^i_{\perp pol} + \tilde{\mathbf{J}}^i_{\perp dia} \tag{2.18}$$

First we compute the ion polarization contribution to the turbulent heating defined by

$$\tilde{Q}^i_{\perp pol} = \left\langle \tilde{\mathbf{E}}_{\perp} \cdot \tilde{\mathbf{J}}^i_{\perp pol} \right\rangle \tag{2.19}$$

The ion polarization drift velocity is given by $\tilde{V}^i_{\perp pol} = (c/B\Omega_i)d\tilde{E}_{\perp}/dt$, and we define $d/dt = \partial/\partial t + \mathbf{V}\cdot\nabla$, $V = \langle V_y\rangle + \tilde{V}_{\perp}$, $E_{\perp} = \langle E_{\perp}\rangle + \tilde{E}_{\perp}$, so we have the ion polarization current

$$\tilde{\mathbf{J}}^i_{\perp pol} = en\tilde{V}^i_{\perp pol} = nm_i\frac{c^2}{B^2}\left[\frac{\partial}{\partial t}\tilde{\mathbf{E}}_{\perp} + \langle V_y\rangle \frac{\partial}{\partial y}\tilde{\mathbf{E}}_{\perp} + \hat{x}\tilde{V}_x\frac{\partial}{\partial x}\langle E_x\rangle\right] \tag{2.20}$$

where we have neglected quadratic terms in fluctuation amplitude for simplicity. Considering the stationary limit, and substituting the ion polarization current into the expression for turbulent heating, yields

$$\left\langle \tilde{\mathbf{E}}_{\perp} \cdot \tilde{\mathbf{J}}^i_{\perp pol} \right\rangle = \left\langle n_i m_i \frac{c^2}{B^2}\tilde{\mathbf{E}}_{\perp}\cdot\frac{\partial \tilde{\mathbf{E}}_{\perp}}{\partial y}\right\rangle \langle V_y\rangle + \left\langle n_i m_i\frac{c^2}{B^2}\tilde{E}_x\tilde{V}_x\right\rangle \frac{\partial\langle E_x\rangle}{\partial x} \tag{2.21}$$

Noting the definition of the $E \times B$ flow $V_y = -(c/B)E_x$ and simplifying, allows

$$\left\langle \tilde{\mathbf{E}}_{\perp} \cdot \tilde{\mathbf{J}}^i_{\perp pol} \right\rangle = \frac{1}{2}n_i m_i\frac{c^2}{B^2}\left\langle \frac{\partial}{\partial y}|\tilde{\mathbf{E}}_{\perp}|^2\right\rangle \langle V_y\rangle - n_i m_i\frac{c}{B}\left\langle \tilde{V}_y\tilde{V}_x\right\rangle\frac{\partial\langle E_x\rangle}{\partial x} \tag{2.22}$$

for the simplified geometry considered here, $n_i = n_i(x)$ and $B = B(x)$. The first term can be seen to vanish identically upon averaging, hence we can write

$$\left\langle \tilde{\mathbf{E}}_\perp \cdot \tilde{\mathbf{J}}^i_{\perp pol} \right\rangle = -n_i m_i \frac{c}{B} \left\langle \tilde{V}_y \tilde{V}_x \right\rangle \frac{\partial \langle E_x \rangle}{\partial x} \tag{2.23}$$

which is Reynolds work on the $E \times B$ flow. This is simply the work associated with the flow generation.

We note that this relation is exact only for the simplified slab geometry considered within this work. A more realistic geometry [i.e. with $B = B(r,\theta)$] will generally give rise to additional geometrical contributions and small corrections. Here we consider the geometry in a local annular region with the center at r_0 (see Fig.2.3), so the coordinates can be described by (r, θ, z). For our simple case, the magnetic field is considered as a constant. Since the Reynolds stress in slab geometry is given by $\left\langle \tilde{V}_x \tilde{V}_y \right\rangle = -(c^2/B^2) \left\langle \partial_x \tilde{\phi} \partial_y \tilde{\phi} \right\rangle$, then in a local annulus the Reynolds stress can be written as $\left\langle \tilde{V}_x \tilde{V}_y \right\rangle = \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle$. Now we take the mean poloidal velocity to be the mean $\mathbf{E} \times B$ velocity , so $\langle V_\theta \rangle \sim \langle V_E \rangle = -(c/B) \langle E_r \rangle$. Then the turbulent heating can be obtained as

$$\left\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}^i_{\perp pol} \right\rangle = n_i m_i \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle \langle V_\theta \rangle' \tag{2.24}$$

If we define an average over an annular region with the width 2Δ at the local centre r_0 , this average can be written as

$$\langle \cdots \rangle = \int_0^{2\pi R} dz \int_0^{2\pi} r_0 d\theta \int_{r_0-\Delta}^{r_0+\Delta} (\cdots) dr \tag{2.25}$$

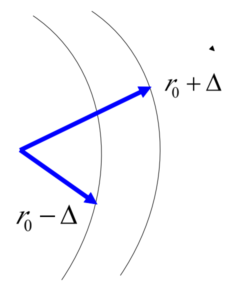

Figure 2.3: Energy flux differential in an annular region. The annular width is 2Δ.

The surface area is a constant and is given by a number $A(r_0)$, $A(r_0) = \int_0^{2\pi R} dz \int_0^{2\pi} r_0 d\theta$. Then, we only need to consider the radial integral to compute the turbulent heating within an annulus, which is,

$$\begin{aligned}
&\int_0^{2\pi R} dz \int_0^{2\pi} r_0 d\theta \int_{r_0-\Delta}^{r_0+\Delta} \left\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}^i_{\perp pol} \right\rangle dr = n_i m_i A \int_{r_0-\Delta}^{r_0+\Delta} dr \langle V_\theta \rangle' \left\langle \tilde{V_r} \tilde{V_\theta} \right\rangle \\
&= n_i m_i A (\langle V_\theta \rangle \left\langle \tilde{V_r} \tilde{V_\theta} \right\rangle)|_{r_0-\Delta}^{r_0+\Delta} - \int_{r_0-\Delta}^{r_0+\Delta} dr \, \langle V_\theta \rangle \frac{\partial}{\partial r} \left\langle \tilde{V_r} \tilde{V_\theta} \right\rangle)
\end{aligned} \tag{2.26}$$

The first term in equation (2.26) is the energy flux differential at the boundary, which gives rise to a *net* turbulent heating. This arises since wave propagation can carry energy through the annular boundary. The finite wave energy flux S_r is proportional to the Reynolds stress (see equation (2.5)) which drives zonal flow formation [65]. So zonal flow generation is the destination of net turbulent heating.

The second term in the RHS of equation (2.26) is the Reynolds work of the turbulence on the mean flow, which is directly linked to the zonal flow drive. For the simplified advective nonlinearity of the incompressible fluid momentum balance equation (including friction) in this local annular region [21, 22], we have

$$\frac{\partial}{\partial t} \langle V_\theta \rangle = -\frac{\partial}{\partial r} \left\langle \tilde{V_r} \tilde{V_\theta} \right\rangle - \nu_{col} \langle V_\theta \rangle \tag{2.27}$$

Since at steady state, Reynolds work on the zonal flow must balance zonal flow friction, $\partial\left\langle \tilde{V}_r\tilde{V}_\theta\right\rangle/\partial r = -\nu_{col}\left\langle V_\theta\right\rangle$, then the ion polarization drift induced turbulent heating in the equation (2.26) can be approximated as

$$\left\langle \tilde{\mathbf{E}}\cdot\tilde{\mathbf{J}}^i_{\perp pol}\right\rangle \approx nm_iA\int_{r-\Delta}^{r+\Delta}dr\nu_{col}\left\langle V_\theta\right\rangle^2 \tag{2.28}$$

which is zonal flow frictional damping ($\sim \nu_{col}\left\langle V_\theta\right\rangle^2 \sim |e\tilde{\phi}/T|^4$). Hence, the ion polarization drift induced turbulent heating over an annular region gives rise to a net heating which is ultimately due to the zonal flow friction. Then this net heating can be dissipated by the zonal flow frictional damping at steady state, which gives rise to another electron-ion energy transfer channel. This process of energy transfer via zonal flow has not previously been accounted for in analyses of energy coupling.

Now we calculate the ion diamagnetic current induced turbulent heating in the direction perpendicular to the magnetic field. We have the diamagnetic drift velocity induced by the gradient of ion pressure fluctuation $\nabla\tilde{P}$ in the direction perpendicular to $\mathbf{B}$,

$$\tilde{\mathbf{V}}^i_{\perp dia} = \frac{c}{en}\frac{\mathbf{B}\times\nabla\tilde{P}_i}{B^2} \tag{2.29}$$

The corresponding diamagnetic current is

$$\tilde{\mathbf{J}}^i_{\perp dia} = en\tilde{V}^i_{\perp dia} = c\frac{\mathbf{B}\times\nabla\tilde{P}_i}{B^2} \tag{2.30}$$

Then we obtain the divergence of the diamagnetic current as:

$$\nabla_\perp\cdot\tilde{\mathbf{J}}^i_{\perp dia} = \nabla_\perp\cdot\frac{c}{B^2}[\tilde{P}_i\nabla\times\mathbf{B} - \nabla\times(\tilde{P}_i\mathbf{B})] = 0 \tag{2.31}$$

where $\mathbf{B}$ is assumed to be a straight magnetic field. Further, the turbulent heating induced by the diamagnetic current can be calculated and is given by

$$\begin{aligned}\left\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}^i_{\perp dia} \right\rangle &= \left\langle -\nabla_\perp \cdot (\tilde{\phi}\tilde{\mathbf{J}}^i_{\perp dia}) + \phi \nabla \cdot \tilde{\mathbf{J}}^i_{\perp dia} \right\rangle \\ &= -\frac{1}{r}\frac{\partial}{\partial r} r \left\langle \tilde{\phi} \frac{c}{B}\frac{1}{r}\frac{\partial \tilde{P}_i}{\partial \theta} \right\rangle \\ &= -\frac{1}{r}\frac{\partial}{\partial r} r \left\langle \tilde{V}_{E\times B}\tilde{P}_i \right\rangle \end{aligned} \tag{2.32}$$

Averaging over the equation (2.32) in an annulus , we obtain

$$\left\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}^i_{\perp dia} \right\rangle = -A \int_{r-\Delta}^{r+\Delta} dr \frac{1}{r}\frac{\partial}{\partial r} r \left\langle \tilde{V}_{E\times B}\tilde{P}_i \right\rangle = -A \left\langle \tilde{V}_{E\times B}\tilde{P}_i \right\rangle |_{r-\Delta}^{r+\Delta} \tag{2.33}$$

Obviously equation (2.33) is the ion heat flux differential across the annulus, which means the ion turbulent energy can be eliminated through a heat flux. Similarly, we can calculate the electron diamagnetic flow induced turbulent heating, which is a electron heat flux differential across the annulus and is given by

$$\left\langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}^e_{\perp dia} \right\rangle = A \left\langle \tilde{V}_{E\times B}\tilde{P}_e \right\rangle |_{r-\Delta}^{r+\Delta} \tag{2.34}$$

Here the total heat flux ($Q = -\chi_T \nabla T - \chi_{neo} \nabla T$), including the turbulence and neoclassical parts, is a constant. In a turbulent transport dominated annular region, the turbulent heat flux $Q_{tur} = Q^e_{tur} + Q^i_{tur}$ must also be a constant. Then, the diamagnetic flow contribution to the turbulent heat flux differential can be ignored in an annular region. However, if we consider a barrier region (i.e, ITB), the heat flux differential will be finite, and so will result in heating in the barrier region.

So far the whole process of turbulent energy flow due to the electron and ion transfer has been completed. In electron heated plasma, electrons can lose energy to the wave and drive wave instability and turbulence, so electrons are cooled. And the ions gain energy from the wave through wave Landau damping and the ions are heated. Meanwhile, we note that the energy flux differential can contribute a net heating, which drives zonal flow generation in an annular region. This net energy can be dissipated by zonal flow frictional damping, giving rise to another collisionless energy transfer channel ($electrons \longrightarrow ions$). As for the diamagnetic flow contributed turbulent heating, we can ignore that since the turbulent heat flux is not changed in the annular region of interest.

Now we see that the simplified electron and ion turbulent energy coupling can be written into a form:

$$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle = A_L I + B_{NL} I^2 + C_{ZF} \frac{\langle V_\theta \rangle^2}{C_s^2}, \tag{2.35}$$

Here the turbulence intensity is defined as $I = \sum_k |e\tilde{\phi}_k/T_e|^2$, and the coefficients A_L, B_{NL}, C_{ZF} have dimensions of a power density. $A_L = \sum_k A_k |e\tilde{\phi}_k/T_e|^2 / \sum_k |e\tilde{\phi}_k /T_e|^2$ is set by the electron and ion quasilinear turbulent heating, $B_{NL} = \sum_{k,k'} B_{k,k'} |e\tilde{\phi}_k/T_i|^2 |e\tilde{\phi}_{k'}/T_i|^2 / I^2$ describes the nonlinear ion turbulent heating through the nonlinear Landau damping (here we ignore this nonlinear effect, but will analyze it in a future paper) [24, 26, 27, 28], and the coefficient C_{ZF} is determined by heating through zonal flow formation[65, 20, 22].

In relation to turbulent heating, we also analyzed two basic wave energy dissipation channels, which are quasilinear Landau damping and zonal flow frictional damping. In the next section, we will estimate the size of each turbulent heating channel by using the mixing length approximation [66, 10] and then comparing these two dissipation channels by considering ITER parameters [12, 13].

2.4 Application of the results to ITER

In table 1, we listed all of the turbulent heating terms before and after using the mixing length approximation for fluctuation levels [66, 10]. Now we will discuss how to estimate each turbulent heating term in detail. For the quasilinear electron cooling term is given by

$$\left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)} = \sum_k \sqrt{\pi} n T_e \left| \frac{e\tilde{\phi}_k}{T_e} \right|^2 \frac{\omega}{|k_z| V_{the}} (\omega - \omega_{\star e}) \exp\left[-\frac{(\omega/k_z)^2}{V_{the}^2} \right] \tag{2.36}$$

Here, the turbulence intensity can be estimated as $e\tilde{\phi}_k/T_e \sim \rho_\star$ [66, 10], the wave vector $k_z \sim 1/Rq$, and the dispersion relationship is taken as $\omega = \omega_{\star e}/(1 + k_\perp^2\rho_s^2)$ in drift wave turbulence (note: $\omega = \omega_k$). Also the exponential factor $\exp[-(\omega/k_z)^2/V_{the}^2] \approx 1$ since $\omega/k_z \ll V_{the}$. The quasilinear electron cooling then can be estimated as

$$\left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)} = -\sum_k \sqrt{\pi} n T_e \frac{Rq}{V_{the}} \rho_\star^2 \omega_{\star e}^2 \left(\frac{k_\perp \rho_s}{1 + k_\perp^2 \rho_s^2} \right)^2 \tag{2.37}$$

We define the function $F_1(k_\perp\rho_s) = [k_\perp\rho_s/(1 + k_\perp^2\rho_s^2)]^2$ which is a dimensionless number dependent on the finite Larmor effect $k_\perp\rho_s$, so the simplified quasilinear electron cooling can be written as

$$|\left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)}| \sim n T_e \rho_\star^2 \frac{Rq\omega_{\star e}^2}{V_{the}} F_1(k_\perp\rho_s) \tag{2.38}$$

Similarly, the quasilinear ion heating term can be estimated as

$$\begin{aligned}
\left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)} &= \sum_k \sqrt{\pi} n T_i \left| \frac{e\tilde{\phi}_k}{T_i} \right|^2 \frac{\omega}{|k_z| V_{thi}} \left(\omega + \frac{T_i}{T_e} \omega_{\star e} \right) \exp\left[-\frac{(\omega/k_z)^2}{V_{thi}^2} \right] \\
&= \sum_k \sqrt{\pi} n T_i \left| \frac{T_e}{T_i} \right|^2 \rho_\star^2 \frac{Rq}{V_{thi}} \omega_{\star e}^2 \left(\frac{1}{1 + k_\perp^2 \rho_s^2} \right) \left(\frac{1}{1 + k_\perp^2 \rho_s^2} + \frac{T_i}{T_e} \right) \\
&\exp\left[-(\frac{k_\perp \rho_s}{1 + k_\perp^2 \rho_s^2})^2 (\frac{Rq}{a})^2 \frac{T_e}{T_i} \right] \\
&\approx n T_i \rho_\star^2 \frac{Rq\omega_{\star e}^2}{V_{thi}} F_2(k_\perp\rho_s)
\end{aligned} \tag{2.39}$$

where $F_2(k_\perp\rho_s)$ is another dimensionless number which is determined by finite Larmor radius $k_\perp\rho_s$,

$$F_2(k_\perp\rho_s) = \left(\frac{1}{1 + k_\perp^2 \rho_s^2} \right) \left(\frac{1}{1 + k_\perp^2 \rho_s^2} + \frac{T_i}{T_e} \right) \exp\left[-\left(\frac{k_\perp \rho_s}{1 + k_\perp^2 \rho_s^2} \right)^2 \left(\frac{Rq}{a} \right)^2 \frac{T_e}{T_i} \right] \tag{2.40}$$

Now we estimate the ion polarization drift and ion diamagnetic drift induced turbulent heating. The energy flux differential in an annular region gives rise to the energy in the zonal flow which can be written as,

$$\left\langle \tilde{E} \cdot \tilde{J}^i_{\perp pol} \right\rangle = nm_i A \int_{r_0-\Delta}^{r_0+\Delta} dr \nu_{col} \left\langle V_\theta \right\rangle^2 \tag{2.41}$$

Here the zonal flow can be treated as an $E \times B$ flow, so $\langle V_\theta \rangle \sim \langle V_{E\times B} \rangle \sim -(C/B) \langle E_r \rangle$. At steady state, the radial electric field $E_r \approx (1/n)(\nabla P/e) \approx -(T_e/e)(1/L_n)$. The long wavelength zonal $\mathbf{E} \times \mathbf{B}$ flow can then be approximated as the diamagnetic flow,

$$\langle V_E \rangle \sim \frac{c}{B}\frac{T_e}{e}\frac{1}{L_n} = \frac{\rho_s}{L_n} C_s \tag{2.42}$$

Then an estimate of the energy in the zonal flow can be obtained, which is given by

$$\left\langle \tilde{E} \cdot \tilde{J}^i_{\perp pol} \right\rangle \approx nm_i \nu_{col} (\frac{\rho_s}{L_n} C_s)^2 (A\Delta) \tag{2.43}$$

Defining a dimensionless number for collisionality $\nu_{\star i} = \epsilon^{-3/2} \nu_{ii} Rq/V_{thi}$, the effective collisionality $\nu_{col} \approx \nu_{ii}/\epsilon$, the net turbulent heating is given by (note: we assume the volume of annulus to be $A\Delta \approx 1$)

$$\left\langle \tilde{E} \cdot \tilde{J}^i_{\perp pol} \right\rangle \approx n \nu_{\star i} \rho_\star^2 \epsilon^{1/2} m_i C_s^2 \frac{V_{thi}}{Rq} \tag{2.44}$$

So far we have estimated all of the turbulent heating terms by using mixing length theory (see Table 2.1). The total electron and ion energy coupling in equation (2.35) can also be obtained (note: we didn't consider the nonlinear ion turbulent heating term for the reason of simplicity) as:

$$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle = A_L I + C_{ZF} \frac{\langle V_\theta \rangle^2}{C_s^2} \tag{2.45}$$

where $A_L = \sum_k A_k |e\tilde{\phi}_k/T_e|^2 / \sum_k |e\tilde{\phi}_k/T_e|^2$, $A_k(n, T_e, T_i, V_{the}, V_{thi}, \omega_\star, k_\perp \rho_s) = -n T_e (Rq/V_{the}) {\omega_{\star e}}^2 F_1(k_\perp \rho_s) + n T_i (Rq/V_{thi}) {\omega_{\star e}}^2 F_2(k_\perp \rho_s)$ and $C_{ZF}(n, m_i, q, V_{thi}, \epsilon, C_s, \nu_{\star i}) = n m_i C_s^2 \nu_{\star i} \epsilon^{1/2} V_{thi}/(Rq)$. By using ITER parameters [12, 13], we can see the physical implications of results of the calculation in the next section.

Table 2.1: Overview of results: estimation of the turbulent heating contributions

Turbulent heating Contribution	Analytical Theory Prediction	Scaling, Using mixing length approximations for fluctuation levels: $\frac{e\tilde{\phi}}{T_e} \sim \rho_\star$
$\left\|\left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)}\right\|$	$nT_e \left\| e\tilde{\phi}/T_e \right\|^2 (\omega - \omega_{\star e})\omega/(\|k_z\|V_{the})$	$nT_e\rho_\star^2(Rq/V_{the})\omega_{\star e}^2 F_1(k_\perp\rho_s)$
$\left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)}$	$nT_i \left\| e\tilde{\phi}/T_i \right\|^2 (\omega + \frac{T_i}{T_e}\omega_{\star e})\omega/(\|k_z\|V_{thi}) \exp -(\frac{\omega}{k_z V_{thi}})^2$	$nT_i\rho_\star^2(Rq/V_{thi})\omega_{\star e}^2 F_2(k_\perp\rho_s)$
$\left\langle \tilde{E}_\perp \cdot \tilde{J}^i_{\perp pol} \right\rangle$	$nm_i\nu_{col}\langle V_\theta\rangle^2$	$n\rho_\star^2\nu_{\star i}\epsilon^{1/2}m_i C_s^2 V_{thi}/(Rq)$

2.5 Basic comparison of dissipation channels

In the last section, turbulent heating terms which include the quasilinear electron cooling, quasilinear ion heating and zonal flow frictional heating have been estimated using the mixing length approximation [66, 10]. These calculations are specially important for ITER, as a low collisionality plasma. Then we will use ITER parameters [12, 13] to determine each contribution to turbulent heating, and compare the ratios of the various energy dissipation channels at different collisionalities. There are two kinds of basic turbulent energy dissipation channels in an electron drift wave. One is through linear ion Landau damping and another one is the net turbulent heating, which is due to the zonal flow frictional damping at steady state. Basically, the quasilinear electron cooling plays the role of the"energy source", while the ion turbulent heating channels act as an " energy sink". The ratios of two kinds of energy dissipation channels are listed in Table 2.2.

Table 2.2: Basic comparison of dissipation channels

ITER Parameters	R=6.2m, a=2m, q=2
$Ratio = \frac{\left\langle \tilde{E}\cdot\tilde{J}_i \right\rangle}{\left\langle \tilde{E}\cdot\tilde{J}_e \right\rangle}$	Short wavelength: $k_\perp \rho_s \sim 1$
$\left\langle \tilde{E}_\parallel \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)}$	1.6×10^{-2}
$\left\langle \tilde{E}_\perp \cdot \tilde{J}^i_{\perp pol} \right\rangle$	$0.8\nu_\star$

The following gives the details of the comparisons. The ratio of the energy dissipated by the ion Landau damping to the energy released by electron cooling is given by

$$\frac{\left|\left\langle \tilde{E} \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)}\right|}{\left|\left\langle \tilde{E} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)}\right|} = \frac{V_{the}}{V_{thi}} \frac{F_2(k_\perp \rho_s)}{F_1(k_\perp \rho_s)} \tag{2.46}$$

For ITER parameters, the electron temperature and ion temperature $T_e \approx T_i = 10kev$, the major radius $R = 6.2m$, the minor radius $r = 2m$, and the safety factor is $q = 2$ at the core [12, 13]. Also we consider the influence of finite Larmor radius effects, where the short wave length $k_\perp \rho_s \sim 1$ for the most unstable mode. Then, the ratio in equation

(2.46) is,

$$\frac{\left|\left\langle \tilde{E} \cdot \tilde{J}_{\parallel i} \right\rangle^{(2)}\right|}{\left|\left\langle \tilde{E} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)}\right|} = 1.6 \times 10^{-2} \tag{2.47}$$

Now we consider the ratio of the turbulent energy dissipated by the zonal flow frictional damping to the electron cooling. This is given by

$$\frac{\left|\left\langle \tilde{E} \cdot \tilde{J}^{i}_{\perp pol} \right\rangle\right|}{\left|\left\langle \tilde{E}_{\parallel} \cdot \tilde{J}_{\parallel e} \right\rangle^{(2)}\right|} = \frac{4}{\pi}\nu_{\star}\epsilon^{\frac{1}{2}}\sqrt{\frac{m_i}{m_e}}\sqrt{\frac{T_i}{T_e}}\left(\frac{a}{Rq}\right)^{2} \approx 0.8\nu_{\star} \tag{2.48}$$

The result shows that the energy dissipation through the zonal flow frictional damping channel is also a dimensionless number which is determined by the collisionality $\nu_{\star i}$. So far we estimated the ratios of energy dissipation channels, and now we will compare them at different collisionality in ITER. Using the collisionality $\nu_{\star i} \approx 10^{-3}$ (ITER parameters were used to calculate $\nu_{\star i}$)[5][18] in the ratio of the zonal flow frictional damping channel in equation (2.48), we can see that this energy dissipation channel is not big. It is about 5% compared to the ratio of the energy dissipated by the ion linear Landau damping. However, if we consider the collisionaltiy $\nu_{\star i} = 10^{-2}$, zonal flow frictional damping can be a significant energy dissipation channel for the collisionless drift wave and the ratio rises to 50% , as compared to the ion linear Landau damping. The first case corresponds to the collisionality in the core in the ITER and second one is appropriate close to the edge. In addition, we need to clarify why we discuss frictional zonal flow damping for a "collisionless" drift wave. Since $\omega_{\star} \gg \nu_{\star} > 0$, "collisionless" drift wave means the collisionaltiy is low compare to the drift wave frequency. But even weak collisionality is still significant for zonal flow frictional damping. Since the zonal flow frequency is approximately zero, even low collisionality will have a strong effect on the zonal flow via frictional damping. So zonal flows indeed play a very important role in collisionless energy transfer for low collisionlity ITER plasmas.

2.6 Implications: collisionless turbulent energy transfer in ITER plasma

We calculated all of turbulent heating terms in electron drift wave and then estimated them by using a mixing length approximation [66, 10]. By using ITER parameters, we compared the ratios of the energy dissipation channels at different collisionalities. We realized that the turbulent heating not only involves energy exchange between electrons and ions at the quasilinear level but also produce net heating which is the energy flux differential in an annulus. The latter is related to the energy in the zonal flow. The net energy dissipated by the zonal flow frictional damping for a collisionless drift wave is a significant energy dissipation channel in ITER plasma. In the following, we will continue to explore the implication of our results for ITER. For a realistic case, we will extend our discussion to the CTEM (collisionless trapped electron mode) [26, 27, 28]. Consider the electron turbulent energy transport process in CTEM, the electron heat balance equation can be written as:

$$\frac{3}{2}n\frac{\partial T_e}{\partial t} + \nabla \cdot Q_e = \left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle - n\nu\frac{m_e}{m_i}(T_e - T_i) + \cdots \tag{2.49}$$

The free energy in the electrons $(\nabla T, \nabla n)$ can be lost through the electron heat flux $\nabla \cdot Q_e$ in the turbulent energy transport process, or energy can be transferred to ions through collisional transfer $n\nu\frac{m_e}{m_i}(T_e - T_i)$ and collisionless energy transfer process $\left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle$. The energy flow in the electron turbulent energy transport is shown in Fig.2.4. First, we compare two energy transfer processes which are collisional and collisionless energy transfer. Since the collisionality is small $(\nu_\star \sim 10^{-3})$ in ITER, the collisional energy transfer is small and collisionless energy transfer may dominate in the energy transfer process. Then we will compare the energy transfer in the collisionless transfer process with the energy transported by the heat flux Q.

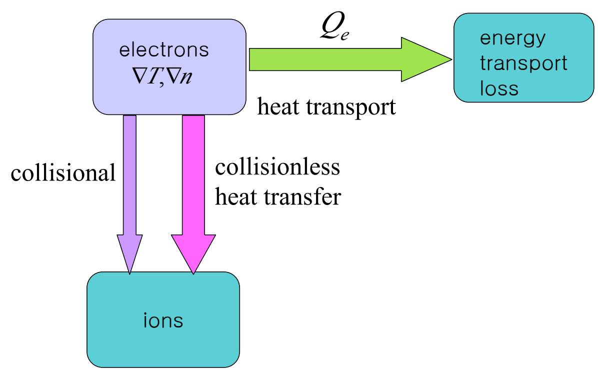

Figure 2.4: The electron turbulent energy flow. The energy can be transported by the electron heat flux Q_e or energy can be transferred to the ions by the collisional or collisionless energy transfer channels.

The collisionality is defined as $\nu_{\star e} = \epsilon^{-3/2}\nu_{ee}Rq/V_{the}$. Here ν_{ee} is the collision rate for electrons and is given by

$$\nu_{ee} = 2.9 \times 10^{-6} \times n_e \times \lambda \times T_e^{-3/2} \approx 2.9 \times 10^3 sec^{-1} \tag{2.50}$$

The plasma density $n_e \approx n_i = 1.1 \times 10^{14} cm^{-3}$, the Coulomb logarithm is defined as $\lambda = ln\Lambda \equiv ln(r_{max}/r_{min}) \sim 10$, plasma temperature $T_e \approx T_i = 10kev$ [12, 13]. So, the normalized collisionality is obtained as $\nu_{\star e} = 4.3 \times 10^{-3}$ in ITER. Now we find the collisionality $\nu_\star$ at the crossover of *collisional* turbulent energy *transfer* and *collisionless* turbulent energy *transfer*. The energy exchange in the electron-ion collision process is given by Braginskii's equation [18],

$$W_i = \frac{3n_e m_e}{m_i}\nu_e T_e \left(1 - \frac{T_i}{T_e}\right) \tag{2.51}$$

which is the energy per unit time transferred from electrons to ions.

Now we estimate the collisionless energy transfer in CTEM. The toroidal geometry of the magnetic surfaces can be described by coordinates(r, θ, ξ), which are the minor radius, the poloidal angle, and the toroidal angle,respectively. Also the equilibrium magnetic field can be written as $\mathbf{B} = B_0 \hat{e}_\xi + (\epsilon/q)\hat{e}_\theta$, where $\epsilon = r/R$ is the inverse aspect ratio. For trapped electrons, poloidal motion is prohibited but slow precessional motion in the toroidal direction occurs. For electrostatic perturbations, the dynamics of the non-adiabatic response for trapped electrons is given by the bounce averaged kinetic equation [26, 27, 28], which is:

$$\bar{\bar{g}}_{k,\omega} = -\frac{e}{T_e}\langle f_e\rangle \frac{\omega - \omega_{\star e}\left[1 + \eta_e(\frac{E}{T_e} - \frac{3}{2})\right]}{\omega - \bar{\omega}_d + i\nu_{eff}} \left\langle e^{-inq\theta}\tilde{\phi}_{k,\omega}\right\rangle_b \tag{2.52}$$

where $\bar{\omega}_d \approx G(k_\theta \rho_s C_s/R)(E/T_e)$ is the orbit-averaged trapped electron precession frequency, G is the magnetic field geometry effect and $G \approx 1$ for deeply trapped electrons [67]. The effective collision frequency ν_{eff} is ignored in the collisionless regime, and $\eta_e = L_n/L_{T_e}$. The $\langle\cdots\rangle_b$ means bounce average and was taken as $\langle\cdots\rangle_b = (\oint\frac{d\theta}{v_\parallel}\cdots)/(\oint\frac{d\theta}{v_\parallel})$. The bounce averaged potential fluctuation is $\left\langle e^{-inq\theta}\tilde{\phi}_{k,\omega}\right\rangle_b \approx \tilde{\phi}_{k,\omega}$, since the finite orbit width effects of trapped electrons are neglected. Similar to the quasilinear electron cooling in electron drift waves, a realistic application is the quasilinear trapped electron cooling in CTEM. The turbulent heating for quasilinear trapped electrons is given by

$$\left\langle \bar{\bar{E}} \cdot \bar{\bar{J}}\right\rangle_b^{(2)} = \sum_k -e\int d^3v \bar{v}_d \bar{\bar{E}}\bar{\bar{g}}_{k,\omega} \tag{2.53}$$

where the bounce averaged curvature and ∇B drift velocity is $\bar{v}_d = \bar{\omega}_d/k_\theta$ and $\bar{\bar{E}}$ is the bounce averaged electric field. The velocity integration over the trapped-electron population is written as

$$d^3v \backsimeq 4\pi\left(\frac{\epsilon}{2}\right)^{\frac{1}{2}} v^2 dv d\kappa^2 \left(\kappa^2 - sin^2\frac{\theta}{2}\right)^{-1/2} \tag{2.54}$$

where κ is the pitch angle variable related to the azimuthal angle θ_0 of the turning point of a trapped electron and $\kappa = \sin\theta_0$. We can convert the integral in velocity space into one in energy space (for the kinetic energy $E = m_e v^2/2$), where we find

$$\int d^3v = \left(\frac{\epsilon}{2}\right)^{1/2} \int 4\pi v^2 dv \int_{\kappa_0}^{1} d\kappa^2 \left(\kappa^2 - \sin^2\frac{\theta}{2}\right)^{-\frac{1}{2}}$$
$$\approx 2\pi \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{2}{m_e}\right)^{3/2} \int E^{1/2} dE \tag{2.55}$$

The fraction of the trapped electrons $f_t = (\epsilon/2)^{1/2} \int_{\kappa_0}^{1} d\kappa^2 \left(\kappa^2 - \sin^2\frac{\theta}{2}\right)^{-1/2} \approx (\epsilon/2)^{1/2}$ was approximated in equation (2.55), for simplicity. Then the bounce averaged turbulent heating from the curvature and ∇B drift current is:

$$\left\langle \bar{\tilde{E}} \cdot \bar{\tilde{J}} \right\rangle_b^{(2)} = \sum_k -2\pi \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{2}{m_e}\right)^{3/2} e \int E^{1/2} dE \bar{v}_d \bar{\tilde{E}} \bar{\tilde{g}}_{k,\omega}$$
$$= \sum_k -2\pi \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{2}{m_e}\right)^{3/2} e \int E^{1/2} dE \left(\frac{\bar{\omega}_d}{k_\theta}\right) \left(ik_\theta \tilde{\phi}_{-k}\right) \left(-\frac{e\tilde{\phi}_k}{T_e} \langle f_e \rangle \frac{\omega - \omega_{\star T}}{\omega - \bar{\omega}_d}\right)$$
$$= \sum_k 2\pi i \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{2}{m_e}\right)^{3/2} T_e \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \int E^{3/2} dE \left(\frac{k_\theta \rho_s C_s}{RT_e}\right) \left(\frac{\omega - \omega_{\star T}}{\omega - \frac{k_\theta \rho_s C_s}{R}\frac{E}{T_e}} \langle f_e \rangle\right) \tag{2.56}$$

where we define $\omega_{\star T} = \omega_{\star e}[1 + \eta_e(E/T_e - 3/2)]$ in equation (2.56). The integral is

$$\int dE \frac{\langle f_e \rangle}{\omega - (k_\theta \rho_s C_s E/RT_e)} = \frac{-i\pi}{|k_\theta \rho_s C_s/RT_e|} \langle f_e \rangle_{E = \frac{\omega RT_e}{k_\theta \rho_s C_s}}$$

The wave-trapped electron resonance gives $\omega = \bar{\omega}_d$, so energy can be obtain at the resonance $E = (\omega RT_e)/(k_\theta \rho_s C_s) \approx RT_e/2L_n$. The drift wave frequency $\omega = \omega_{\star e}$ $/(1 + k_\perp^2 \rho_s^2)$, the unstable wave number $k_\perp \rho_s \approx 1$, and $\omega_{\star e} = k_\theta \rho_s C_s/L_n \approx C_s/L_n$ were considered for CTEM. Then, the quasilinear turbulent cooling for trapped electron can be estimated as

$$
\begin{aligned}
\left\langle \bar{\tilde{E}} \cdot \bar{\tilde{J}} \right\rangle_b^{(2)} &= \sum_k 2\pi i \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{2}{m_e}\right)^{3/2} T_e \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \left(\frac{RT_e}{2Ln}\right)^{3/2} (-i\pi)(\omega - \omega_{\star T}) \\
&\langle f_e \rangle |_{E=\frac{RT_e}{2Ln}} \\
&= \sum_k 2\pi^{1/2} \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{R}{2L_n}\right)^{3/2} n_0 T_e \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \left(\frac{C_s}{L_n}\right) \\
&\left(-\frac{1}{2} - \eta_e \frac{R}{2L_n} + \eta_e \frac{3}{2}\right) \exp\left(-\frac{R}{2L_n}\right)
\end{aligned}
\tag{2.57}
$$

Here the electron equilibrium distribution function is still taken to be a local Maxwellian, i.e $\langle f_e \rangle = n_0(x)(m_e/2\pi T_e(x))^{3/2} \exp(-E/T_e)$. The quasilinear turbulent cooling for trapped electrons is $\left\langle \bar{\tilde{E}} \cdot \bar{\tilde{J}} \right\rangle_b^{(2)} (\epsilon, n, R/L_n, R/L_T, \rho_\star, T_e, k_\perp \rho_s)$, which is a function dependent on the all the parameters.

Now we compare the collisional turbulent energy transfer (2.51) to the magnitude of quasilinear trapped electron cooling (2.57), and we can obtain the collisionality at the crossover, which is a dimensionless number given by

$$
\begin{aligned}
\nu_\star \approx& \left(\frac{Rq\epsilon^{-3/2}}{V_{the}}\right) \frac{2}{3}\pi^{1/2} \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{R}{2L_n}\right)^{3/2} \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \left(\frac{C_s}{L_n}\right) \left(\frac{1}{2} + \frac{R\eta_e}{2L_n} - \frac{3\eta_e}{2}\right) \frac{m_i}{m_e} \\
&\left(1 - \frac{T_i}{T_e}\right)^{-1} \exp(-\frac{R}{2L_n}) \\
=& \frac{1}{12}\pi^{1/2} \sqrt{\frac{m_i}{m_e}} \left(\frac{q}{\epsilon}\right) \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \left(\frac{R}{L_n}\right)^{3/2} \left(\frac{R}{L_n} + \frac{R^2}{L_n L_T} - 3\frac{R}{L_T}\right) \left(1 - \frac{T_i}{T_e}\right)^{-1} \\
&\exp\left(-\frac{R}{2L_n}\right)
\end{aligned}
\tag{2.58}
$$

Here the collisionality $\nu_\star(\epsilon, q, R/L_n, R/L_T, \rho_\star, T_i/T_e)$. can be determined if all parameters are given. For simplicity, we assume $T_i/T_e < 1$ for an electron heated plasma and analyze how the ratio of T_i/T_e affects collisionality $\nu_\star$. We also note that the collisionality $\nu_\star$ at crossover is sensitive to the local parameters R/L_n and R/L_T

[68, 69]. Qualitatively, the relation between them is described in Fig.2.5 where we take the ITER parameters (the inverse aspect ratio $\epsilon \approx 1/3$, safety factor $q = 2$)[12, 13], the mixing length approximation $|e\tilde{\phi}_k/T_e| \sim 10^{-3}$, the temperature ratio $T_i/T_e = 1/2$, the ranges $3 < R/L_T < 13$ and $3 < R/L_n < 13$. For typical parameters $R/L_T = 10$, $R/L_n = 4$, we have

$$\nu_\star \approx 1.2 \times 10^{-3} \tag{2.59}$$

Obviously, the collisionality in ITER $\nu_\star \approx 10^{-3}$ is same order as the collisionality at the crossover of collisional energy transfer and collisionless energy transfer. In other words, the collisionality is low enough such that the collisionless turbulent energy transfer and the collisional inter-species coupling process are *both* important for the energy transfer process!

However, as the ions gain more and more energy, the difference between T_i and T_e will decrease, and the collisional energy transfer will drop. For example, $T_i \approx 0.95T_e$ corresponding to the collisionality at the crossover $\nu_\star = 1.2 \times 10^{-2}$, in equation (2.58). Thus, the collisionless energy transfer process will ultimately control electron-ion energy transfer in ITER.

We examined the turbulent energy transfer mechanism for CTEM and found that the collisionless energy transfer is anticipated to be the dominant electron-ion coupling process in the heat balance . Furthermore, in the next section, we will compare the rate of the electron turbulent energy lost by turbulent *transport* through the electron heat flux with the rate of the electron energy *transferred* to the ions in the CTEM.

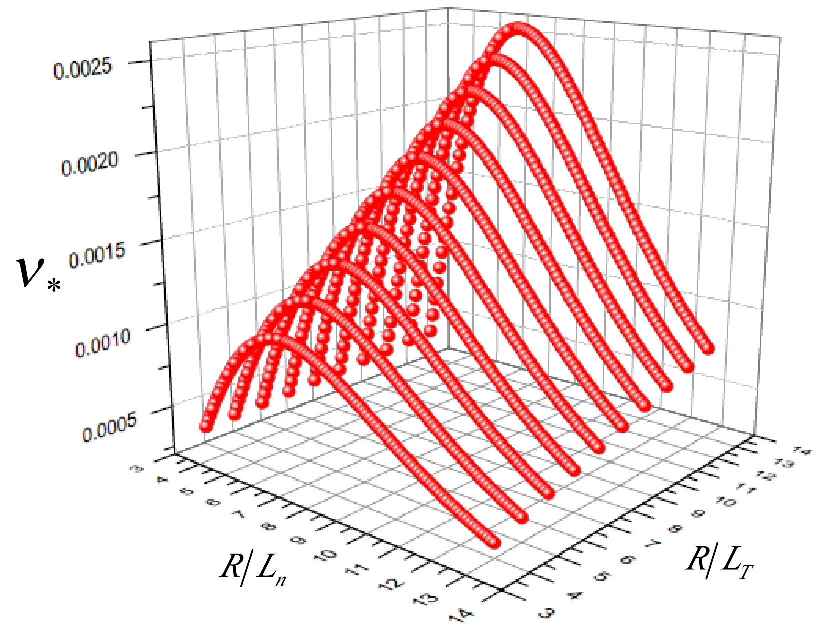

Figure 2.5: The collisionality $\nu_\star$ at crossover depends on the local parameters R/L_T and R/L_n. For $\epsilon \approx 1/3$, $q = 2$, $\rho_\star = 10^{-3}$, $T_i/T_e = 1/2$, $R/L_T = 10$, $R/L_n = 4$, the crossover collisionality is $\nu_\star = 1.2 \times 10^{-3}$.
.

2.7 Turbulent energy transfer vs turbulent energy transport

The effective way to compare the rate of the electron energy lost by the turbulent transport to the rate of the collisionless energy transfer by comparsion of the volume integral of the electron cooling, to the surface integrated electron heat flux in CTEM. In the electron heat balance equation (2.49), we ignore the collisional energy transfer term for low collisionality CTEM and have,

$$\frac{3}{2} n \frac{\partial T_e}{\partial t} + \nabla \cdot Q_e = \left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle + Source + \cdots \tag{2.60}$$

At steady state, the volume integral of the annulus for equation (2.60), we obtain

$$AQ_e|_{boundary} = \int d^3r \left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle \tag{2.61}$$

where A is a constant surface area and the volume integral over the annular region can be estimated as $\int d^3r = A\Delta r$, where Δr is the annular thickness. Then the equation can be simplified to

$$Q_e|_{boundary} = \Delta r \left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle \tag{2.62}$$

Here Q_e is the electron heat flux, and we consider the quasilinear trapped electron heat flux in CTEM which is given by[28, 46]

$$Q_e = \left\langle \tilde{V}_r \tilde{P}_e \right\rangle = -\frac{c}{B} \sum_k k_\theta \, \mathrm{Im}(\tilde{P}_e^{(1)} \tilde{\phi}) \tag{2.63}$$

The pressure fluctuation $\tilde{P}$ is written as,

$$\begin{aligned} \tilde{P}_e^{(1)} &= \int d^3v \frac{1}{2} m_e v^2 \bar{\bar{g}}_{k,\omega} \\ &= 2\pi \left(\frac{2}{m_e}\right)^{3/2} \left(\frac{\epsilon}{2}\right)^{1/2} \int dE E^{3/2} \left(-\frac{e\tilde{\phi}_k}{T_e} \langle f_e \rangle \frac{\omega - \omega_{\star e}\left[1 + \eta_e(\frac{E}{T_e} - \frac{3}{2})\right]}{\omega - \bar{\omega}_d} \right) \end{aligned} \tag{2.64}$$

The bounce averaged trapped electron distribution function $\bar{\bar{g}}_{k,\omega}$ in equation (2.52) has been used here [26, 27, 28]. Considering the resonance of the wave with trapped electron precession motion (i.e $\omega = \omega_d = (k_\theta \rho_s C_s/R)(E/T_e)$) and noting the electron diamagnetic frequency $\omega_{\star e} = k_\theta \rho_s C_s / L_n$, the pressure fluctuation is seen to be

$$\mathrm{Im}\, \tilde{P}_e^{(1)} = 2\pi^2 \left(\frac{2}{m_e}\right)^{3/2} \left(\frac{\epsilon}{2}\right)^{1/2} \left(\frac{\omega R T_e}{L_n \omega_{\star e}}\right)^{3/2} \left(\frac{e\tilde{\phi}_k}{T_e}\right) \frac{(\omega - \omega_{\star T})}{\left|\frac{L_n \omega_{\star e}}{RT_e}\right|} \langle f_e \rangle_{E = \frac{RT_e}{2L_n}} \tag{2.65}$$

The quasilinear electron heat flux in equation (2.63) can thus be calculated as:

$$
\begin{aligned}
Q_e = & \sum_k 2\pi^2 \left(\frac{2}{m_e}\right)^{3/2} \left(\frac{\epsilon}{2}\right)^{1/2} (\frac{RT_e}{2L_n})^{3/2} \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \frac{V_{the}^2 k_\theta}{\Omega_e} \left|\frac{RT_e}{L_n}\right| \left[\frac{1}{2} + \eta_e(\frac{E}{T_e} - \frac{3}{2})\right] \\
& \langle f_e \rangle_{E=\frac{RT_e}{2L_n}} \\
& = \sum_k 2\pi^{1/2} \left(\frac{\epsilon}{2}\right)^{1/2} (\frac{R}{2L_n})^{3/2} n_0 T_e \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \frac{V_{the}^2 k_\theta}{\Omega_e} \left|\frac{R}{L_n}\right| \left[\frac{1}{2} + \eta_e(\frac{E}{T_e} - \frac{3}{2})\right] \\
& \exp\left(-\frac{R}{2L_n}\right)
\end{aligned}
\tag{2.66}
$$

The trapped electron distribution function at resonance $\langle f_e \rangle_{E=\frac{RT_e}{2L_n}} = n_0(x)(m_e/2\pi T_e(x))^{3/2} \exp(-R/2L_n)$ was used in equation (2.66). Comparing equation (2.57) with equation (2.66), the ratio of the electron energy lost by turbulent transport to the collisionless energy transfer can be written as (note: we estimated $\omega_{\star e} = k_\theta \rho_s C_s/L_n \approx C_s/L_n$ in equation (2.57), as before)

$$
\begin{aligned}
\frac{\Delta r \left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle}{Q_e|_{boundary}} &= \frac{L_n}{R} \frac{\Omega_e}{V_{the}^2 k_\theta} \omega_{\star e} \Delta r \\
&= \frac{\Delta r}{R} \\
&\approx \frac{a}{R}
\end{aligned}
\tag{2.67}
$$

where the annular width $\Delta r \approx a$, so the radial annular integral can extend to the whole minor radius. Thus the ratio of transfer to transport loss is given by $a/R \sim o(1)$ which suggests that electron turbulent energy *transfer* to ions in a collisionless plasma can be the same order as electron heat *transport* loss! Hence the collisionless electron heat transfer by turbulence is surely a critical element of any transport analysis model for a low collisionality, electron heated plasma. It is necessary to consider the influence of the collisionless energy transfer to determine the total electron heat budget. This issue is especially relevant to ITER plasmas.

2.8 Conclusion

In this paper , we considered two problems: the net turbulent heating and inter-species collisionless energy transfer channels in electron drift wave turbulence. The principal results of this analysis are:

1) We extended the classical calculation of the turbulent heating within the quasi-linear framework to the nonlinear level. The turbulent heating includes quasilinear electron cooling, quasilinear ion heating, and ion polarization drift and ion diamagnetic drift induced turbulent heating. The volume integral of the ion polarization drift in an annulus give rises to the net turbulent heating, which occurs through the zonal flow. This net heating is dissipated by the zonal flow frictional damping. Thus it constitutes an important collisionless energy transfer channel working through zonal flow generation. The process of energy transfer via the zonal flow has not previously been accounted for in analyses of energy transfer from electrons to ions. We ignore the small effect of the ion diamagnetic drift induced heat flux differential in an annulus.

2) We identified three kinds of collisionless turbulent energy transfer channels in electron drift wave turbulence. The hot electrons can transfer turbulent energy to cold ions through wave-particle interaction (ion Landau damping, ion Nonlinear Landau damping) and turbulence-zonal flow interaction (zonal flow frictional damping). Here we focus more on the nonlinear turbulent energy transfer through the zonal flow channel, since it can dominate the nonlinear saturation balance. The nonlinear collisionless heat transfer through the nonlinear Landau damping will be discussed in a future paper.

3) By using a mixing length approximation, we estimated all of the turbulent heating ratios. Using ITER parameters, we discussed the implication of our results. The comparison of the ratios of the energy dissipation channels showed that the zonal flow frictional damping can be a significant energy dissipation channel for the low collisionality drift wave in ITER plasma.

4) We explored the meaning of the collisionless turbulent energy transfer channels in a more realistic case, namely for CTEM. For ITER plasma, the collisionality is low enough such that the collisionless energy transfer may ultimately dominate the energy transfer process. Also we compared the rate of the energy lost through collisionless energy *transfer* with the electron turbulent energy *transport* in CTEM. The ratio is

order unity, which means the collisionless turbulent energy transfer can be comparable to the turbulent energy transport in the heat balance. Then in future large, collisionless tokamaks, we have to consider the influence of the collisionless energy transfer as well as the turbulent energy transport.

The collisionless electron-ion coupling model may be related to some experimental phenomena, such as the electron temperature profile " stiffness" where the temperature profile reacts weakly to changes in auxiliary heating deposition [44]. One of the possible causes of such behavior is the nondiffusive term in the heat flux, which is an inward flow and carries energy from edge to the core [46]. Another reason may due to electron-ion energy transfer in the core, where the electron energy is dissipated through collisionless energy transfer. These two effects are different and independent. Both must be examined to see which one is more efficient in future experiments. The proper analysis of these two effects will be presented in a future paper

Acknowledgments

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Chapter 3

Nonlinear ion heating in electron drift wave turbulence

3.1 Introduction

In a low collisionality, electron heated plasma (like ITER)[14, 12], the issue of anomalous electron-ion energy coupling is particularly important to the analysis of electron/ion thermal transport processes. In the electron/ion heat transport equation

$$\frac{3}{2}n\frac{\partial T_\alpha}{\partial t} + \nabla\cdot Q_\alpha = \left\langle \tilde{E}\cdot\tilde{J}\right\rangle_\alpha \mp \frac{3}{2}n\nu\frac{m_e}{m_i}(T_e - T_i) + \cdots ; \alpha = e, i \qquad (3.1)$$

electron/ion energy can be lost via spatial transport (i.e.$\nabla\cdot Q_\alpha$) or be *transferred* to other species by either collisional $3/2n\nu m_e/m_i(T_e - T_i)$ or collisionless energy transfer $\left\langle \tilde{E}\cdot\tilde{J}\right\rangle_\alpha$. In previous papers [70, 71], we argued that the inter-species transfer of energy to be dominated by collisionless processes in low collisionality, ITER like plasmas. In addition, it was shown that the rate of collisionless energy transfer can be the same order as turbulent transport for collisionless trapped electron mode (CTEM) turbulence. This suggests that for the case of CTEM, a substantial fraction of the electron energy is transferred to ions, resulting in a significant reduction in the amount of energy carried by the electron heat flux. Hence it is necessary to consider how collisionless energy transfer influences turbulent energy transport processes in a collisionless plasma, such as ITER. More pragmatically, transport models for low collisionality plasmas must be improved so as to properly account for energy transfer.

How can electrons transfer energy to ions in a collisionless plasma? The answer will involve wave-particle interaction and turbulence -zonal flow interaction. The classical theorem for wave energy evolution in a strongly magnetized plasma is the wave energy balance theorem $\partial_t W + \nabla \cdot \mathbf{S} + \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = 0$, where W is the wave energy density, $\mathbf{S}$ represents the wave energy density flux, and $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle$ is the turbulent heating induced by current fluctuations[16]. It has been argued that there is no net turbulent heating and only electron-ion energy exchange through the wave-particle quasilinear resonance[15, 17]. In previous works [70, 71], we considered turbulent heating in an annulus. We showed that for the general case where an energy flux differential across this annular region was non-zero, a net turbulent heating *must* arise. It was subsequently shown that this heating was linked to the Reynolds work on the zonal flow. Zonal flow friction heats ions [21], thus the turbulence-zonal flow interaction enters as an important energy transfer channel. Since zonal flows are nonlinearly generated, it follows that we should also apply weak turbulence theory to calculate the turbulent ion heating via beat mode resonance in electron drift wave turbulence [24]. This will allow *all* $o|e\tilde{\phi}/T|^4$ contributions to be considered. This nonlinear turbulent ion heating contributes a new collisionless turbulent energy transfer channel for electron-ion energy coupling, which is shown to be significant in a number of relevant regimes.

Intuitively, the transfer of energy through the linear wave-particle resonance has been considered the most efficient collisionless mechanism for mediating inter-species energy exchange[15]. However, nonlinear wave-particle coupling is a strong nonlinear effect and will play a significant role in the inter-species energy transfer. Historically, nonlinear wave-particle processes have often been studied as mechanisms through which electrostatic microturbulence may nonlinearly saturate. Chen et al. [29], using a local approximation to the drift wave eigenmode structure, found that forward scattering by the trapped electron nonlinearity is dominant for weakly dissipative trapped electron modes. In contrast, utilizing weak turbulence theory, Gang and Diamond et al [26, 27], emphasized the importance of inverse scattering by ion nonlinearity for collisionless trapped electron modes (CTEM) in sheared slab geometry. In related work, Hahm [28] found that both the ion and electron nonlinearities play an important role in determining the saturated turbulence spectrum for CTEM in toroidal geometry. These works, and countless others like them, all focused on nonlinear saturation and its implications for turbulent *transport*. Here, we employ weak turbulence theory in order to describe the role played by the ion nonlinearity as an energy *transfer* and dissipation channel for simple electron drift wave turbulence (driven only by the density gradient, for simplicity). The results obtained indicate that energy transfer is an important component of the overall electron energy balance. The results also remind us not to ignore nonlinear wave-ion processes in favor of zonal flows.

In previous papers [70, 71], we discussed the collisionless electron and ion energy transfer channels and turbulent heating in electron drift wave turbulence. In an electron heated plasma, electrons can transfer energy to ions through collisional energy transfer (electron-ion binary collisions) or collisionless energy transfer(wave-particle interaction, turbulence and zonal flow interaction [20], etc). In a low collisionality, electron heated plasma (like ITER) [14, 12], collisional energy transfer is weak, so that collisionless energy transfer will likely dominate the electron-ion energy coupling process. In Ref. [70], we analyzed the wave energy dissipation channels related to linear Landau damping and zonal flow frictional damping. We also elucidated the nature of net turbulent heating and its unavoidable connection to drift wave radiation. Here, we extend the heating calculation to incorporate ion heating by nonlinear wave-particle interaction in electron drift wave turbulence. Perhaps surprisingly, this nonlinear turbulent ion heating due to nonlinear ion Landau damping is a stronger effect than quasilinear Landau damping. Since the beat mode resonates with the bulk ion distribution whereas the linear drift wave resonance sits at the tail of the ion distribution (see Fig. 3.1), the efficiency of the linear resonance is smaller by a factor of $exp(-(\omega/k)^2/v_{thi}^2) \ll 1$ as compared to the nonlinear case. Thus, even though the nonlinear ion Landau damping term is formally $\sim \mathcal{O}\left|e\tilde{\phi}/T_i\right|^4$, whereas the quasilinear contribution is formally $\sim \mathcal{O}\left|e\tilde{\phi}/T_i\right|^2$, the net effect is $\sim \left|e\tilde{\phi}/T_i\right|^4 exp(-(\omega''/k'')^2/v_{thi}^2)$, which is not necessarily smaller than the quasilinear result. This is because $\omega'' \leq k_{\parallel}^2 v_{th}^2$, so the gain in the exponential factor can outweigh the additional spectral intensity factor of $|e\tilde{\phi}/T|^2$.

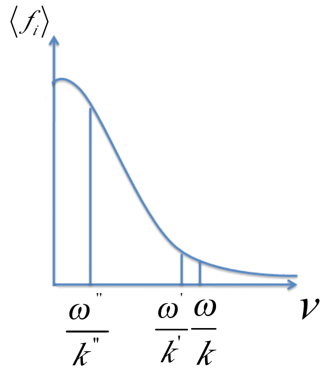

Figure 3.1: The beat mode resonance ω''/k''correspond to the bulk of ion distribution function and quasilinear resonance ω/k,ω'/k' sit at the tail in velocity space .

Nonlinear turbulent ion heating along with quasilinear electron cooling, quasilinear ion heating and heating due to zonal flow friction[70, 71], define the energy balance at saturation. The total turbulent heating for species α can be written schematically as [70, 71]

$$\left\langle \tilde{E} \cdot \tilde{J}_\alpha \right\rangle = A_L^\alpha I + B_{NL}^\alpha I^2 + C_{ZF}^\alpha I^2, \tag{3.2}$$

where the turbulence intensity is defined as $I = \sum_k |e\tilde{\phi}_k/T_e|^2$. The coefficient $A_L^\alpha = \sum_k A_k |e\tilde{\phi}_k/T_e|^2 / \sum_k |e\tilde{\phi}_k/T_e|^2$ represents electron and ion quasilinear turbulent heating, $B_{NL}^\alpha = \sum_{k,k'} B_{k,k'} |e\tilde{\phi}_k/T_i|^2 |e\tilde{\phi}_{k'}/T_i|^2 / I^2$ describes nonlinear ion heating through nonlinear Landau damping, and C_{ZF}^α accounts for zonal flow frictional damping. This simple electron and ion energy coupling process is analogous to radiative coupling, where the source and sink act as radio antennas. The radiative source radiates an electromagnetic wave which is picked up by the sink, through resonance. Similarly here, the hot electrons(energy "source") lose energy to drift waves and the ions gain energy (energy "sink") through ion Landau damping (including linear and nonlinear wave-particle resonances) in Fig.3.2.

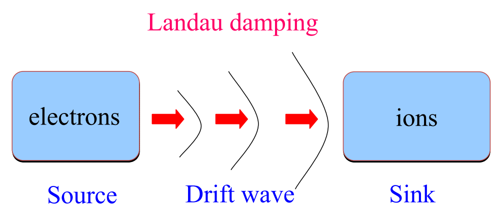

Figure 3.2: Electron and ion energy coupling through electron drift wave turbulence

In addition, this anomalous electron-ion energy coupling equation forces a new perspective on the analysis of electron thermal transport experiments in future large, low collisionality tokamaks. Electron thermal energy will not be lost only via electron transport processes. Rather, collisionless energy transfer between electrons and ions will also occur. Thus the empirical evaluation of the electron thermal diffusivity χ_e will be influenced by the contribution of electron-ion energy *transfer* to the total energy budget. In particular, the anomalous electron-ion energy coupling has not received sufficient attention in the analysis of present day electron thermal transport experiments. These have been analyzed using standard transport codes. Transport codes calculate the input to the energy balance equation, and then solve the energy balance equation for the thermal diffusivity χ. The terms in the energy balance equation are based on the measured parameters : density, temperature etc. Most transport codes do *not* account for collisionless energy transfer. This shortcoming will become more acute as collisionality drops. To this end, some hints from previous, somewhat puzzling experimental phenomena are available. In Alcator C-Mod [72], low density ohmic regimes, the transport code TRANSP [73] shows that the experimental thermal diffusivity is significantly different from the simulation results predicted by GYRO [74, 75], for trapped electron mode (TEM) turbulence [76]. GYRO predicts a much larger ion thermal diffusivity χ_i and smaller electron thermal diffusivity χ_e than the analysis by TRANSP [76]. A possible reason, explained below, may be due to the fact that the electron-ion collisionless energy transfer effect is ignored in the energy balance equation in TRANSP. The original electron energy balance equation in TRANSP can be written as [77, 78]

$$\frac{3}{2}n\frac{\partial T_e}{\partial t}+\nabla\cdot Q_e+\nabla\cdot(\frac{5}{2}n_eT_e\vec{v_e})+\vec{v_i}\cdot\nabla(n_iT_i) = P_{OH}+P_{add}-P_{rad}-P_{ie}+\cdots\ , \quad (3.3)$$

The first term of the equation represents the rate of change of the electron energy and the second term describes electron heat flux ($Q_e = -n_e\chi_e\nabla T_e, \chi_e$ is the electron thermal diffusivity). The third and forth terms correspond to the electron convection losses $\nabla\cdot(\frac{5}{2}n_eT_ev_e)$ and work done by plasma particles against the pressure gradient $\vec{v_i}\cdot\nabla(n_iT_i)$. On the right side of the equation, the energy source terms are primary Ohmic heating P_{OH} and the additional heating source P_{add} (ECRH, ICRH...). The term P_{rad} is the energy loss through radiation. The last term in the energy balance equation is the collisional electron-ion power exchange $P_{ie} = 3/2n\nu m_e/m_i(T_e-T_i)$, which results from electron-ion Coulomb collisions [18]. Given the experimental quantities (electron density n_e, electron temperature T_e, ion temperature T_i, the radiated power P_{rad} , etc...) , we can solve Eq. (3.3) for the electron heat flux Q_e, and the electron thermal diffusivity χ_e . However, the anomalous electron-ion coupling term $\left\langle\tilde{E}\cdot\tilde{J}\right\rangle_\alpha, \alpha = e, i$ was neglected in the energy exchange term P_{ie}. This turbulence mediated energy transfer can be significant in a low density plasma, where the energy transferred by collisions is relatively small.

When we include the anomalous electron-ion energy coupling term in this energy balance equation, we have

$$\frac{3}{2}n\frac{\partial T_\alpha}{\partial t}+\nabla\cdot Q_\alpha+\cdots = \left\langle\tilde{E}\cdot\tilde{J}\right\rangle_\alpha \mp \frac{3}{2}n\nu\frac{m_e}{m_i}(T_e-T_i)+P_{OH}\cdots\,;\alpha = e, i \quad (3.4)$$

The first term on the right side of the equation $\left\langle\tilde{E}\cdot\tilde{J}\right\rangle_\alpha$ $(\alpha = e, i)$ represents the turbulent heating for a single species (electron cooling $\left\langle\tilde{E}\cdot\tilde{J}\right\rangle_e$ or ion heating $\left\langle\tilde{E}\cdot\tilde{J}\right\rangle_i$). If we now consider the electron turbulent *cooling* term in the energy balance, $\left\langle\tilde{E}\cdot\tilde{J_e}\right\rangle < 0$

forces the divergence of heat flux ($\nabla \cdot Q_e$) to be smaller at steady state, and hence the calculated thermal diffusivity ($\chi_e = -Q_e/n\nabla T$) should decrease. In contrast, the prediction for the ion thermal diffusivity χ_i will appear larger, due to the appearance of a related ion turbulent *heating* $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle > 0$ in the ion energy balance equation. This observation appears to be consistent with the thermal diffusivity disagreement between the GYRO simulation results and TRANSP calculations based on the energy balance equation (see Table 3.1), which ignores collisionless coupling [77, 78]. Note that at low density, the collisional coupling is *weak*. Furthermore, we note that the anomalous electron-ion energy coupling is especially important for a low collisionality plasma, like ITER, where collisionless energy transfer can dominate the energy transfer process [70, 71].

In sections II, nonlinear ion heating is calculated in detail. We will first review the quasilinear ion heating based on the ion diffusion equation in electron drift wave turbulence. We then extend the derivation of quasilinear diffusion coefficient $D^{(2)}$ to the nonlinear order $D^{(4)}$ by using weak turbulence theory [24]. Here the electrostatic force $\tilde{F}_z^{(2)}$ in velocity space and velocity $v_x^{(2)}$ in real space corresponds to the second order fluctuation $|\tilde{\phi}|^2$ due to the orbit deviation due to $E \times B$ drift motion in the x, y direction and parallel force $F_{\|}$ scattering in the z direction. The nonlinear velocity diffusion coefficient $D_{vv}^{(4)}$ is calculated in section II, as well.

Table 3.1: A comparison of electron and ion energy balance with or without the turbulent energy transfer. S represents terms on the right side of Eq. (3.3).

	Heat transport without transfer	Heat transport with transfer	Comparison
Heat balance equation	$\partial_t T_\alpha + \nabla \cdot Q_\alpha = S$	$\partial_t T + \nabla \cdot Q_\alpha = \left\langle \tilde{E} \cdot \tilde{J} \right\rangle_\alpha + S$	$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle_\alpha \neq 0$
Electron heat flux	$\nabla \cdot Q_e = S$	$\nabla \cdot Q'_e = \left\langle \tilde{E} \cdot \tilde{J} \right\rangle_e + S$	$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle_e < 0$
Ion heat flux	$\nabla \cdot Q_i = S$	$\nabla \cdot Q'_i = \left\langle \tilde{E} \cdot \tilde{J} \right\rangle_i + S$	$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle_i > 0$
Electron thermal diffusivity χ_e	$\chi_e = -Q_e/n\nabla T_e$	$\chi'_e = -Q'_e/n\nabla T_e$	$\chi_e > \chi'_e$
Ion thermal diffusivity χ_i	$\chi_i = -Q_i/n\nabla T_i$	$\chi'_i = -Q'_i/n\nabla T_i$	$\chi_i < \chi'_i$

General Ion Diffusion Eqn			
$$\frac{\partial\langle f_i\rangle}{\partial t}=\frac{\partial}{\partial v}D_{vv}\frac{\partial\langle f_i\rangle}{\partial v}+\frac{\partial}{\partial v}D_{vx}\frac{\partial\langle f_i\rangle}{\partial x}+\frac{\partial}{\partial x}D_{xv}\frac{\partial\langle f_i\rangle}{\partial v}+\frac{\partial}{\partial x}D_{xx}\frac{\partial\langle f_i\rangle}{\partial x}$$			
Diffusion coefficient	Velocity space $D_{vv}=\int_{-\infty}^{\infty}\frac{\langle\tilde{F}\tilde{F}\rangle}{2}d\tau$	Real space $D_{xx}=\int_{-\infty}^{\infty}\frac{\langle\tilde{v}\tilde{v}\rangle}{2}d\tau$	Cross term $D_{vx}=D_{xv}=\int_{-\infty}^{\infty}\frac{\langle\tilde{F}\tilde{v}\rangle}{2}d\tau$
Quasilinear diffusion	$D_{vv}^{(2)}:\tilde{F}_z^{(1)}\tilde{F}_z^{(1)}$	$D_{xx}^{(2)}:\tilde{v}_x^{(1)}\tilde{v}_x^{(1)}$	$D_{vx}^{(2)}:\ \tilde{F}_z^{(1)}\tilde{v}_x^{(1)}$
Nonlinear diffusion	$D_{vv}^{(4)}:\ \tilde{F}_z^{(2)}\tilde{F}_z^{(2)}$	$D_{xx}^{(4)}:\tilde{v}_x^{(2)}\tilde{v}_x^{(2)}$	$D_{vx}^{(4)}:\ \tilde{F}_z^{(2)}\tilde{v}_x^{(2)}$

Figure 3.3: The decomposition of quasilinear and nonlinear diffusion coeffients in electron drift wave turbulence.

In section III, we will discuss nonlinear diffusion in real space $D_{xx}^{(4)}$, which contributes a heat flux differential at the boundary. In particular, the cross-diffusion term $D_{vx}^{(4)}$ in the nonlinear ion diffusion equation which combines velocity and real space effects will contribute another important nonlinear ion heating effect. Details of the calculation of the quasilinear and nonlinear diffusion coefficients are given in Fig. 3.3. We estimate the nonlinear ion heating in ITER in section IV, where the turbulence intensity is obtained through an approximate saturation balance condition [29] (NB: Here we ignore zonal flows in the saturation balance, for simplicity). The crossover of the turbulence intensity between linear Landau damping and nonlinear Landau damping occurs for $|e\tilde{\phi}/T_e| \ll 10^{-3}$, using ITER parameters [12, 13]. At that fluctuation level, the quasilinear and nonlinear energy dissipation channels are comparable. In addition, the turbulent intensity was estimated using the mixing length approximation [70, 66, 10], to be $|e\tilde{\phi}/T_e| \sim \rho_s/L_n \sim 10^{-3}$ in ITER plasma. Thus, it is likely that the nonlinear heating will exceed quasilinear heating as the dominant mechanism for the electron-ion energy exchange in ITER plasma. This value of $\rho_\star$ implies a level of $|e\tilde{\phi}/T_e|$ which is *above* the value where quasilinear and nonlinear heating cross, so nonlinear ion heating *must* be considered. In the conclusion, we will discuss some possible experimental proposals to identify collisionless energy transfer in present day and in future tokamaks (ITER) and give a summary in section V.

3.2 Nonlinear ion heating

3.2.1 General diffusion equation for resonant particles

In our previous papers [70, 71], we showed that zonal flow frictional damping can be a significant energy dissipation channel in electron drift wave turbulence. Since zonal flows are nonlinearly generated by the fluctuation Reynolds stress [20], the energy in the zonal flow is $\sim \mathcal{O}|e\tilde{\phi}/T|^4$, i.e. fourth order in the electric potential fluctuation. Thus it is necessary to extend the calculation of *quasilinear* heating to *nonlinear* (parallel) heating, in order to address *all* $\mathcal{O}|e\tilde{\phi}/T|^4$ effects consistently. The calculation of quasilinear and nonlinear ion heating is based on the diffusion equation for resonant ions. First, we will derive the general diffusion equation for resonant ions in drift wave turbulence.

The ion drift kinetic equation (DKE) can be written as

$$\frac{\partial f_i}{\partial t} + v_\parallel \nabla_\parallel f_i + \frac{c\hat{b}\times\nabla\tilde{\phi}}{B}\cdot\nabla f_i - \frac{e}{m}\nabla_\parallel \phi \frac{\partial f_i}{\partial v_\parallel} = 0 \tag{3.5}$$

Taking the ensemble average of this equation, we obtain

$$\frac{\partial \langle f_i \rangle}{\partial t} = -\left\langle \frac{c\hat{b}\times\nabla\tilde{\phi}}{B}\cdot\nabla\tilde{f}_i \right\rangle + \left\langle \frac{e}{m}\nabla_\parallel \tilde{\phi}\frac{\partial \tilde{f}_i}{\partial v_\parallel} \right\rangle \tag{3.6}$$

where the ion distribution function consists of a mean and fluctuation part: $f_i = \langle f_i \rangle + \tilde{f}_i$. For simplicity, we take $\langle f_i \rangle = \left\langle f_i(x, v_\parallel) \right\rangle, \hat{b} = \hat{z}$. Linearizing equation (3.5) in Fourier space, we have

$$\tilde{f}_i = \frac{i}{\omega - k_\parallel v_\parallel}(-\frac{c\hat{b} \times \nabla\tilde{\phi}}{B} \cdot \nabla\langle f_i\rangle + \frac{e}{m}\nabla_\parallel\tilde{\phi}\frac{\partial\langle f_i\rangle}{\partial v_\parallel}) \tag{3.7}$$

Plugging equation (3.7) into (3.6), we see that the mean distribution function will evolve according to

$$\frac{\partial\langle f_i\rangle}{\partial t} = \sum_k (\frac{ck_y}{B}\frac{\partial}{\partial x} + \frac{ek_\parallel}{m}\frac{\partial}{\partial v_\parallel})\frac{i}{\omega - k_\parallel v_\parallel}|\tilde{\phi}|^2(\frac{ck_y}{B}\frac{\partial\langle f_i\rangle}{\partial x} + \frac{ek_\parallel}{B}\frac{\partial\langle f_i\rangle}{\partial v_\parallel}) \tag{3.8}$$

Eq. (3.8) is the familiar quasilinear diffusion equation. By calculating $\tilde{f}_i$ to third order in fluctuation level, Eq. (3.8) can be extend to a *nonlinear* diffusion equation [25]. This is:

$$\frac{\partial\langle f_i\rangle}{\partial t} = \frac{\partial}{\partial v}D_{vv}\frac{\partial\langle f_i\rangle}{\partial v} + \frac{\partial}{\partial v}D_{vx}\frac{\partial\langle f_i\rangle}{\partial x} + \frac{\partial}{\partial x}D_{xv}\frac{\partial\langle f_i\rangle}{\partial v} + \frac{\partial}{\partial x}D_{xx}\frac{\partial\langle f_i\rangle}{\partial x} \tag{3.9}$$

where the diffusion coefficients are:

$$D_{vv} = D_{vv}^{(2)} + D_{vv}^{(4)} + \cdots, D_{vx} = D_{xv} = D_{vx}^{(2)} + D_{vx}^{(4)} + \cdots, D_{xx} = D_{xx}^{(2)} + D_{xx}^{(4)} + \cdots \tag{3.10}$$

The coefficients $D_{vv}^{(2)}, D_{vv}^{(4)}$ correspond to quasilinear and nonlinear diffusion in velocity space, respectively, $D_{xx}^{(2)}, D_{xx}^{(4)}$ are the corresponding diffusion coefficients in real space, and $D_{vx}^{(2)}, D_{xv}^{(2)}, D_{vx}^{(4)}, D_{xv}^{(4)}$ are the cross terms. The familiar quasilinear diffusion coefficients $D^{(2)}$ can be calculated from Eq. (3.8) and are given by

$$D_{vv}^{(2)} = Re\sum_k \frac{e^2}{m^2}k_\parallel^2|\tilde{\phi}|^2\frac{i}{\omega - k_\parallel v_\parallel} \tag{3.11}$$

$$D_{xx}^{(2)} = Re\sum_k \frac{c^2}{B^2}k_\theta^2|\tilde{\phi}|^2\frac{i}{\omega - k_\parallel v_\parallel} \tag{3.12}$$

$$D_{xv}^{(2)} = D_{vx}^{(2)} = Re\sum_k \frac{ec}{mB}k_\theta k_\parallel|\tilde{\phi}|^2\frac{i}{\omega - k_\parallel v_\parallel} \tag{3.13}$$

where $D_{vv}^{(2)}$ is diffusion in velocity space (z direction) caused by random velocity scattering. $D_{xx}^{(2)}$ is diffusion in real space, which arises due to random $E \times B$ scattering, and

$D_{xv}^{(2)} = D_{vx}^{(2)}$ is the cross field diffusion, $D_{xv}^{(2)}$ which results from sheared acceleration. We will apply weak turbulence theory to calculate the nonlinear diffusion coefficients later in this paper [24].

Taking the energy integral of the diffusion equation (3.9), the resonant ion kinetic energy evolution equation can be written as

$$\begin{aligned}\frac{\partial E_{res}}{\partial t} &= \int dv \frac{1}{2} m v^2 \frac{\partial}{\partial t} \langle f_i \rangle \\ &= -\int dv m v D_{vv} \frac{\partial}{\partial v} \langle f_i \rangle - \int dv m v D_{vx} \frac{\partial}{\partial x} \langle f_i \rangle + \frac{\partial}{\partial x} \int dv \frac{1}{2} m v^2 \\ &\left(D_{xv} \frac{\partial}{\partial v} \langle f_i \rangle + D_{xx} \frac{\partial}{\partial x} \langle f_i \rangle \right)\end{aligned} \tag{3.14}$$

From the total ion energy balance, we have:

$$\frac{\partial E}{\partial t} + \nabla \cdot Q + \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle = 0 \tag{3.15}$$

where E represents the kinetic energy of ions, Q is the spatial heat flux and $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle$ is the turbulent heating, which can be written as

$$\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle = \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vv}^{(2)} + \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vv}^{(4)} + \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vx}^{(2)} + \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vx}^{(4)} + \cdots \tag{3.16}$$

where the correspondence of the terms in Eq. (3.16) to those in Eq. (3.14) is straightforward. Combining Eq. (3.14), Eq. (3.15) with Eq. (3.16), the quasilinear ion heating corresponds to resonant quasilinear ion kinetic energy diffusion, and is given by $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vv}^{(2)} = -\int dv m v D_{vv}^{(2)} \frac{\partial}{\partial v} \langle f_i \rangle \sim (|\frac{e\tilde{\phi}}{T}|)^2$. The nonlinear ion heating can be written as $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vv}^{(4)} = -\int dv m v D_{vv}^{(4)} \frac{\partial}{\partial v} \langle f_i \rangle$ which is $\mathcal{O}(|\frac{e\tilde{\phi}}{T}|)^4$. The $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vx}^{(2)}$ and $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vx}^{(4)}$ are the quasilinear heating and nonlinear heating from the cross terms in the diffusion equation. The divergence of the energy flux $\nabla \cdot Q^{(2)} = \frac{\partial}{\partial x} \int dv \frac{1}{2} m v^2 (D_{xv}^{(2)} \frac{\partial}{\partial x} \langle f_i \rangle + D_{xx}^{(2)} \frac{\partial}{\partial x} \langle f_i \rangle)$ and $\nabla \cdot Q^{(4)} = \frac{\partial}{\partial x} \int dv \frac{1}{2} m v^2 (D_{xv}^{(4)} \frac{\partial}{\partial x} \langle f_i \rangle + D_{xx}^{(4)} \frac{\partial}{\partial x} \langle f_i \rangle)$ originate from scattering in real space, which leads to heat transport.

3.2.2 Quasilinear ion heating

Now we briefly review the calculation of the quasilinear turbulent heating from the diffusion equation. The quasilinear ion heating results from the random acceleration (due to $\left\langle \tilde{E}_z^2 \right\rangle$) parallel to the magnetic field $\hat{z}$,

$$\begin{aligned}
\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{v_z v_z}^{(2)} &= -\int dv_z m v_z D_{vv}^{(2)} \frac{\partial}{\partial v} \langle f_i \rangle \\
&= \int dv_z \sum_k \frac{e^2}{m^2} k_{zl}^2 |\tilde{\phi}|^2 \frac{i}{\omega - k_z v_z} \langle f_i \rangle \frac{2 m v_z^2}{v_{thi}^2} \\
&= \sum_k \pi n T_i (\frac{e\tilde{\phi}}{T_i})^2 \frac{1}{v_{thi}} \frac{\omega^2}{|k_z|} \langle f_i \rangle_{v_z = \frac{\omega}{k_z}}
\end{aligned} \tag{3.17}$$

(Note: The ion equilibrium distribution function was taken as a local Maxwellian, i.e. $\langle f_i \rangle = n_0(x)(m_i/2\pi T_i)^{1/2} \exp(-v_z^2/v_{thi}^2)$. $\partial \langle f_i \rangle / \partial v_z = (-2 v_z / v_{thi}^2) \langle f_i \rangle$ was also used in equation (3.17)). We calculate the turbulent heating from the cross-term $D_{vx}^{(2)}$,

$$\begin{aligned}
\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vx}^{(2)} &= -\int dv_z m v_z D_{vx}^{(2)} \frac{\partial}{\partial x} \langle f_i \rangle \\
&= -\int dv_z m v_z \sum_k \frac{ec}{mB} k_\theta k_z |\tilde{\phi}|^2 R_e \frac{i}{\omega - k_z v_z} \frac{\partial}{\partial x} \langle f_i \rangle \\
&= -\sum_k \pi n T_i (\frac{e\tilde{\phi}}{T_i})^2 \frac{\omega}{|k_z| v_{thi}} \omega_{\star i} \langle f_i \rangle_{v_z = \frac{\omega}{k_z}}
\end{aligned} \tag{3.18}$$

where the ion diamagnetic frequency was defined as $\omega_{\star i} = k_\theta (T_i c/eB)(1/n)(\partial n/\partial x)$ and we neglect the temperature gradient. Then $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle_{vx}^{(2)} < 0$, so the wave is damped by ion heating, and ions must gain energy from the wave. Thus the total ion quasilinear heating is given by,

$$\left\langle \tilde{E}\cdot\tilde{J}_i\right\rangle_{vv}^{(2)}+\left\langle \tilde{E}\cdot\tilde{J}_i\right\rangle_{vx}^{(2)}=\sum_k \pi n T_i(\frac{e\tilde{\phi}}{T_i})^2\frac{1}{v_{thi}}\frac{\omega^2}{|k_z|}\left\langle f_i\right\rangle|_{v_z=\frac{\omega}{k_z}}-\sum_k \pi n T_i(\frac{e\tilde{\phi}}{T_i})^2 \frac{\omega}{|k_z|v_{thi}}\omega_{\star i}\left\langle f_i\right\rangle_{v_z=\frac{\omega}{k_z}}$$
$$=\sum_k \sqrt{\pi} n T_i(\frac{e\tilde{\phi}_k}{T_i})^2\frac{\omega}{|k_z|v_{thi}}(\omega+\frac{T_i}{T_e}\omega_{\star e})\exp\left[-\frac{(\omega/k_z)^2}{v_{thi}^2}\right] \tag{3.19}$$

This result is consistent with the quasilinear ion heating calculation in previous paper [70, 71], where we calculated $\left\langle \tilde{E}\cdot\tilde{J}_i\right\rangle^{(2)}$ directly by computing the quasilinear ion current.

3.2.3 Nonlinear ion heating corrected by v_z scattering

Now we calculate *nonlinear* ion heating using weak turbulence theory [24]. The foundation of weak turbulence theory is irreversibility via three-wave resonance or nonlinear wave-particle resonance, which are present even in the dissipationless limit. These resonances lead to energy scattering and transfer. Such resonances appear in the theory in the form $\delta(\omega_k+\omega_{k'}-kv_\parallel-k'v_\parallel)$ or $\delta(\omega_k+\omega_{k'}-\omega_{k''})$. Here we assume the fluctuations are sufficiently weak so that the particle orbit deviates only slightly from the unperturbed orbits. In weak turbulence theory, valid when the autocorrelation is short compared to the spectral transfer time, the diffusion coefficient is expanded as $D=D^{(2)}+D^{(4)}+\cdots$ The resonant quasilinear coefficient $D^{(2)}$ describes a Markov process. The $D^{(4)}$ term can also be derived from perturbation theory. $D^{(4)}$ is fourth order in the electric field perturbation and can be interpreted as scattering due to the interaction of two primary modes with frequency and wave number (ω,k) and (ω',k'), which drive the beat mode(ω'',k''), where $\omega''=\omega-\omega'$, $k''=k-k'$. The beat mode resonates with the particles whose phase velocity is given by

$$v_z=\frac{\omega''}{k''} \tag{3.20}$$

This process is known as " nonlinear Landau damping ". Overlap of phase space islands due to beat modes then yields a random walk and diffusion in velocity and position space. Note that $D^{(4)}$ describes a Markov process too.

In the following we will briefly review the calculation of the fourth order diffusion coefficients $D^{(4)}$ by perturbation theory for electron drift wave turbulence. We will calculate the nonlinear diffusion coefficients in velocity space ($\hat{z}$ direction), real space ($\hat{x}$ direction) and the cross terms individually (see Fig. 3.3). We start with velocity diffusion and consider the perturbed electric field in a three dimensional plasma,

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m}\tilde{\mathbf{E}}(\mathbf{x}, t) = \frac{e}{m}\sum_k \tilde{\mathbf{E}}_{\mathbf{k}} exp[i(\mathbf{k}\cdot\mathbf{x} - \omega t + \phi_k)] \tag{3.21}$$

where the position $\mathbf{x}(x, y, x)$ are three coordinates and the frequency ω satisfies the dispersion relation $\omega = \omega_k$. The acceleration at time t, $\mathbf{F}(t)/m$, depends on the location of the particle $\mathbf{x}(t)$, through the space dependence of the electric field $\mathbf{E}(\mathbf{x}(t), t)$. In the perturbation method, the particle orbit is expanded as

$$\mathbf{x}(t) = \mathbf{x}^{(0)}(t) + \epsilon\mathbf{x}^{(1)}(t) + \epsilon^2\mathbf{x}^{(2)}(t) + \cdots \tag{3.22}$$

where $\mathbf{x}^{(0)}(t) = \mathbf{x} + \mathbf{v}t$ is a straight-line orbit, and $\mathbf{x}^{(1)}(t)$ is the first order correction of the orbit due to the electric field perturbation. Here ϵ is a small constant. Taylor expanding the exponential factor, we have

$$exp[i\mathbf{k}\cdot\mathbf{x}(t)] = exp[i\mathbf{k}\cdot\mathbf{x}^{(0)}(t)][1 + \epsilon i\mathbf{k}\cdot\mathbf{x}^{(1)} + \epsilon^2 i(\mathbf{k}\cdot\mathbf{x}^{(2)} - \frac{1}{2}(\mathbf{k}\cdot\mathbf{x}^{(1)})^2 + \cdots] \tag{3.23}$$

Associated with this, the force (expanded in a series of the electric field amplitude) can be written as

$$\mathbf{F}(t) = \mathbf{F}^{(1)}(t) + \mathbf{F}^{(2)}(t) + \cdots \tag{3.24}$$

The force corresponds to the first order electric field perturbation in the z direction, which is

$$\tilde{F}_z^{(1)} = \sum_k \frac{e}{m}\tilde{E}_z(k)\exp[i(\mathbf{k}\cdot\mathbf{x} + \phi_k)]\exp[i(k_z v_z - \omega)t] \tag{3.25}$$

The first order orbit correction $\mathbf{z}^{(1)}(t)$ can be obtained by integrating $\tilde{F}_z^{(1)}$ twice, and is given by

$$z^{(1)}(t) = -\frac{e}{m}\sum_k \tilde{E}_z(k)\frac{\exp[i(\mathbf{k}\cdot\mathbf{x}+\phi_k)]}{(\omega-k_zv_z)^2}\exp[i(k_zv_z-\omega t)] \tag{3.26}$$

The second order force exerted by the perturbed electric field is given by

$$\begin{aligned}\tilde{F}_z^{(2)} &= \frac{dv_z^{(2)}}{dt}\\ &= \frac{e}{m}\sum_{k'}\tilde{E}_z(-k')\exp[i(-\mathbf{k}'\cdot\mathbf{x}+\phi_{-k'})](-i\mathbf{k}'\cdot\mathbf{x^{(1)}})\exp[i(-k_z'v_z+\omega')t]\\ &= \frac{e}{m}\sum_{k'}\tilde{E}_z(-k')\exp[-i(\mathbf{k}'\cdot\mathbf{x}+\phi_{-k'})][-i(k_x'x^{(1)}+k_y'y^{(1)}+k_z'z^{(1)})]\\ &\exp[i(\omega'-k_z'v_z)t]\end{aligned} \tag{3.27}$$

The first order orbit corrections $z^{(1)}$ contributed by the force perturbation in $\hat{z}$ and $x^{(1)}, y^{(1)}$ will be shown to result from the $E\times B$ drift effect in $\hat{x},\hat{y}$ in the next section. Substituting equation (3.26) into (3.27), we obtain

$$F_{zz}^{(2)} = \sum_{k,k'}(\frac{e}{m})^2\tilde{E}_z(-k')\tilde{E}_z(k)\frac{ik_z'}{(\omega-k_zv_z)^2}e^{i\mathbf{k}''\cdot\mathbf{x}}[e^{i(k_z''v_z-\omega'')t}]e^{i(\phi_k+\phi_{-k'})} \tag{3.28}$$

in which the sum over modes includes the contributions of the beats $exp[i(k_z-k_z')v_z-(\omega-\omega'))t]$. Note that $F_{zz}^{(2)}\sim|\tilde{E}_z|^2$, the second order of the electric field fluctuation. We consider the beat modes $\omega''=\omega-\omega', k''=k-k'$. $F_{zz}^{(2)}$ can also be written as

$$F_{zz}^{(2)} = \sum_{k,k'}A_{2z}(k,-k')\tilde{E}(-k')\tilde{E}(k)e^{[i(k_z''v_z-\omega'')t]}e^{i(\phi_k+\phi_{-k'})} \tag{3.29}$$

where the coefficient A_{2z} is symmetric with respect to interchanging any pair of k. A_{2z} is given by

$$A_{2z} = \frac{e^2}{2m^2}e^{i\mathbf{k}''\cdot\mathbf{x}}\left[\frac{-k_z'}{(k_zv_z-\omega)^2}+\frac{k_z}{(-k_z'v_z+\omega')^2}\right] \tag{3.30}$$

Then the diffusion coefficient $D_{v_{zz}v_{zz}}^{(4)}$ in velocity space produced by random ion parallel acceleration is given by

$$\begin{aligned}
D^{(4)}_{v_{zz}v_{zz}} &= \int_{-\infty}^{\infty} \frac{\left\langle \tilde{F}^{(2)}_{zz}(\tau)\tilde{F}^{(2)}_{zz}(t+\tau)\right\rangle}{2} d\tau \\
&= \frac{1}{2}\int_{-\infty}^{\infty} d\tau \sum_{k,k'} A_{2z}(k,-k') e^{[i(k''_z v_z - \omega'')t]} \tilde{E}_z(-k')\tilde{E}_z(k)\tilde{E}_z(-k)\tilde{E}_z(k') \\
&\cdot \exp[i(\phi_k + \phi_{-k'} + \phi_{-k} + \phi_{k'})] A_{2z}(-k,k') e^{[i(k''_z v - \omega'')(t+\tau)]} \\
&= \sum_{k,k'} (\frac{e}{m})^4 \pi |\tilde{E}_z(k)|^2 |\tilde{E}_z(k')|^2 \left[\frac{k_z - k'_z}{(\omega - k_z v_z)(\omega' - k'_z v_z)}\right]^2 Re \frac{i}{\omega'' - k''_z v_z}
\end{aligned} \tag{3.31}$$

where the second order force $\tilde{F}^{(2)}_{zz}$ in Eq. (3.29) was used to obtain the fourth order diffusion coefficient. Note that the nonlinear diffusion coefficient $D^{(4)}_{v_{zz}v_{zz}}$ in Eq. (3.31) is similar to the nonlinear diffusion in the Vlasov plasma, discussed in Ref [24]. In other words, the nonlinear diffusion in a 1D Vlasov plasma is consistent with diffusion in the parallel direction produced by second order perturbation of electrostatic force in 3D drift wave turbulence.

3.2.4 Nonlinear ion turbulent heating driven by $E \times B$ scattering

Now we consider the nonlinear diffusion in velocity space driven by $E \times B$ drift motion. We treat the $E \times B$ drift velocity ($\hat{x}$, $\hat{y}$ direction) as a first order perturbation in velocity space ($\hat{z}$ direction). The $E \times B$ drift velocity in the $\hat{x}$ direction can be written as

$$\tilde{v}_{E\times B,x} = \frac{dx^{(1)}}{dt} = \sum_k (-ik_y)\frac{c}{B}\tilde{\phi}_k e^{i\mathbf{k}\cdot\mathbf{x^{(0)}}} e^{-i(\omega - k_z v_z)t} \tag{3.32}$$

Integrating Eq. (3.32), we obtain the first order orbit deviation in the $\hat{x}$ direction as

$$x^{(1)} = \sum_k \frac{c}{B}\frac{k_y}{\omega - k_z v_z}\tilde{\phi}_k e^{i\mathbf{k}\cdot\mathbf{x^{(0)}}} e^{-i(\omega - k_z v_z)t} \tag{3.33}$$

We substitute $x^{(1)}$ into Eq. (3.27) and then obtain the second order perturbation force produced by the $E \times B$ drift motion in the $\hat{x}$ direction as:

$$\begin{aligned} F_{zx}^{(2)} &= \sum_{k,-k'} \frac{ec}{mB} \frac{(-ik'_x)k_y}{\omega - k_z v_z} \tilde{E}_z(-k') e^{i\mathbf{k}''\cdot\mathbf{x}} \tilde{\phi}_k e^{-i(\omega'' - k''_z v_z)t} \\ &= \sum_{k,-k'} A_{2x}(k,-k') e^{-i(\omega'' - k''_z v_z)t} \end{aligned} \tag{3.34}$$

Here the coefficient A_{2x} is given by

$$A_{2x}(-k',k) = \sum_{k,k'} \frac{ec}{2mB} \tilde{\phi}_k \tilde{\phi}_{-k'} e^{i\mathbf{k}''\cdot\mathbf{x}} \left(\frac{k'_x k_y k'_z}{\omega - k_z v_z} + \frac{k_x k'_y k_z}{\omega' - k'_z v_z} \right) \tag{3.35}$$

Similarly, the second order perturbation force corrected by the $E \times B$ drift motion in the $\hat{y}$ direction, is given by

$$\begin{aligned} F_{zy}^{(2)} &= \sum_{k,-k'} \frac{ec}{mB} \frac{(ik_x k'_y)}{\omega - k_z v_z} \tilde{E}_z(-k') e^{i\mathbf{k}''\cdot\mathbf{x}} \tilde{\phi}_k e^{-i(\omega'' - k''_z v_z)t} \\ &= \sum_{k,-k'} A_{2y}(k,-k') e^{-i(\omega'' - k''_z v_z)t} \end{aligned} \tag{3.36}$$

The coefficient A_{2y} is given by

$$A_{2y}(-k',k) = -\sum_{k,k'} \frac{ec}{2mB} \tilde{\phi}_k \tilde{\phi}_{-k'} e^{i\mathbf{k}''\cdot\mathbf{x}} \left(\frac{k_x k'_y k'_z}{\omega - k_z v_z} + \frac{k'_x k_y k_z}{\omega' - k'_z v_z} \right) \tag{3.37}$$

(Note: the $E \times B$ drift velocity in the $\hat{y}$ direction, $\tilde{v}_{E\times B,y} = dy^{(1)}/dt = \sum_k (ik_x)(c/B)$ $\tilde{\phi}_k e^{i\mathbf{k}\cdot\mathbf{x^{(0)}}} e^{-i(\omega - k_z v_z)t}$ and first order spatial deviation in y direction $y^{(1)} = \sum_k (-c/B)$ $k_x/(\omega - k_z v_z) \tilde{\phi}_k e^{i\mathbf{k}\cdot\mathbf{x^{(0)}}} e^{-i(\omega - k_z v_z)t}$ were used in Eq. (3.36)). In Eq. (3.34) and Eq. (3.36), $F_{zx}^{(2)}$ and $F_{zy}^{(2)}$ both corresponds to $|\tilde{\phi}|^2$, the second order electric filed perturbations are due to the $E \times B$ drift effects in $\hat{x}$ and $\hat{y}$ direction.

Thus the nonlinear diffusion in velocity space, coupled to $E \times B$ drift motion, is given by

$$\begin{aligned} D^{(4)}_{v_{zx}v_{zx}} &= \int_{-\infty}^{\infty} \frac{\left\langle \tilde{F}^{(2)}_{zx}(\tau)\tilde{F}^{(2)}_{zx}(t+\tau)\right\rangle}{2} d\tau \\ &= \int_{-\infty}^{\infty} d\tau A_{2x}(-k',k)e^{-i(\omega''-k''_z v_z)t}A_{2x}(-k,k')e^{-i(\omega''-k''_z v_z)(t+\tau)} \\ &= \sum_{k,k'}(\frac{ec}{mB})^2|\tilde{\phi}_k|^2|\tilde{\phi}_{k'}|^2(\frac{k_y k'_x k'_z}{\omega - k_z v_z} + \frac{k_x k'_y k_z}{\omega' - k'_z v_z})^2 Re\frac{i}{\omega'' - k''_z v_z} \end{aligned} \tag{3.38}$$

The nonlinear diffusion coefficient $D^{(4)}_{v_{zy}v_{zy}}$ is thus given by

$$D^{(4)}_{v_{zy}v_{zy}} = \sum_{k,k'}(\frac{ec}{mB})^2|\tilde{\phi}_k|^2|\tilde{\phi}_{k'}|^2(\frac{k'_y k_x k'_z}{\omega - k_z v_z} + \frac{k'_x k_y k_z}{\omega' - k'_z v_z})^2 Re\frac{i}{\omega'' - k''_z v_z} \tag{3.39}$$

Finally, the nonlinear heating for ions induced by the parallel acceleration can be calculated as:

$$\begin{aligned} \left\langle \tilde{E}\cdot\tilde{J}_i\right\rangle^{(4)}_{vv} &= -\int dv_z m v_z D^{(4)}_{vv}\frac{\partial}{\partial v}\langle f_i\rangle \\ &= -\int dv_z m v_z (D^{(4)}_{v_{zx}v_{zx}} + D^{(4)}_{v_{zy}v_{zy}} + D^{(4)}_{v_{zz}v_{zz}})\frac{\partial}{\partial v}\langle f_i\rangle \end{aligned} \tag{3.40}$$

Substituting the coefficients (3.31), (3.38) and (3.39) into equation (3.40), we obtain,

$$\begin{aligned} \left\langle \tilde{E}\cdot\tilde{J}_i\right\rangle^{(4)}_{v_{zz}v_{zz}} &= -\int dv_z m v_z D^{(4)}_{v_{zz}v_{zz}}\frac{\partial}{\partial v}\langle f_i\rangle \\ &= \sum_{k,k'}\sqrt{\pi}nT_i v^3_{thi}\frac{(\omega'' k''_z)^2}{|k''_z|}\left|\frac{e\tilde{\phi}_k}{T_i}\right|^2\left|\frac{e\tilde{\phi}'_k}{T_i}\right|^2 k_z^2 k'^2_z \\ &\left[\frac{k_z - k'_z}{(k_z\omega'' - \omega k''_z)(k'_z\omega'' - \omega' k''_z)}\right]^2 exp\left(\frac{-(\omega''/k''_z)^2}{v^2_{thi}}\right) \end{aligned} \tag{3.41}$$

$$\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(4)}_{v_{zx}v_{zx}} = -\int dv_z m v_z D^{(4)}_{v_{zx}v_{zx}} \frac{\partial}{\partial v} \langle f_i \rangle$$
$$= \sum_{k,k'} \sqrt{\pi} n T_i \frac{v_{thi}^3}{\Omega_{ci}^2} \frac{\omega''^2}{|k_z''|} \left| \frac{e\tilde{\phi}_k}{T_i} \right|^2 \left| \frac{e\tilde{\phi}'_k}{T_i} \right|^2 (\frac{k'_x k_y k'_z}{\omega k''_z - k_z \omega''}$$
$$+ \frac{k_x k'_y k_z}{\omega' k''_z - k'_z \omega''})^2 exp \left(\frac{-(\omega''/k''_z)^2}{v_{thi}^2} \right) \tag{3.42}$$

$$\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(4)}_{v_{zy}v_{zy}} = -\int dv_z m v_z D^{(4)}_{v_{zy}v_{zy}} \frac{\partial}{\partial v} \langle f \rangle$$
$$= \sum_{k,k'} \sqrt{\pi} n T_i \frac{v_{thi}^3}{\Omega_{ci}^2} \frac{\omega''^2}{|k_z''|} \left| \frac{e\tilde{\phi}_k}{T_i} \right|^2 \left| \frac{e\tilde{\phi}'_k}{T_i} \right|^2 (\frac{k_x k'_y k'_z}{\omega k''_z - k_z \omega''}$$
$$+ \frac{k'_x k_y k_z}{\omega' k''_z - k'_z \omega''})^2 exp \left(\frac{-(\omega''/k''_z)^2}{v_{thi}^2} \right) \tag{3.43}$$

where the nonlinear ion heating produced by the v_{zz} scattering was shown in Eq. (3.41) and the nonlinear ion heating contributed by $E \times B$ scattering (v_{zx}, v_{zy}) was presented in Eq. (3.42) and Eq. (3.43). Comparing the turbulent heating contributed by the v_{zx}, v_{zy} scattering and the v_{zz} acceleration, we have

$$\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(4)}_{v_{zx}v_{zx}} : \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(4)}_{v_{zy}v_{zy}} : \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(4)}_{v_{zz}v_{zz}} \approx 1 : 1 : (k_{\parallel}/k_{\perp})^2 \tag{3.44}$$

Here $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(4)}_{v_{zx}v_{zx}} \approx \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(4)}_{v_{zy}v_{zy}} >> \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(4)}_{v_{zz}v_{zz}}$ since $k_{\perp} >> k_{\parallel}$ in electron drift wave turbulence. So perpendicular scattering via parallel beat wave resonance is the dominant nonlinear heating process.

3.3 Nonlinear diffusion in real space and cross diffusivities

3.3.1 Nonlinear diffusion in real space

Now we look at nonlinear diffusion in real space. The nonlinear diffusion for fluxes is related to the generalized flux in $\mathbf{x}$ and is given by $D^{(4)}_{xx} = D^{(4)}_{x_{xx}x_{xx}} + D^{(4)}_{x_{xy}x_{xy}} + D^{(4)}_{x_{xz}x_{xz}}$. $D^{(4)}_{xx}$ can be written as

$$D^{(4)}_{xx} = \int_0^\infty d\tau \big\langle \tilde{v}^{(2)}_x(t)\tilde{v}^{(2)}_x(t+\tau)\big\rangle \tag{3.45}$$

Here the velocity correction $\tilde{v}^{(2)}_x$ can be obtained from perturbation theory, which we used to calculate $\tilde{F}^{(2)}_z$ in section II. $\tilde{v}^{(2)}_x$ is given by

$$\tilde{v}^{(2)}_x = \frac{d\tilde{x}^{(2)}}{dt} = \sum_{-k'}(ik'_y)\frac{c}{B}\tilde{\phi}_{-k'}e^{-i\mathbf{k'}\mathbf{x}^{(0)}}e^{i(-k'_zv_z+\omega')t}[-i(k'_xx^{(1)}+k'_yy^{(1)}+k'_zz^{(1)}] \tag{3.46}$$

We note that the first order orbit deviation x_1, y_1 and z_1 will contribute the second order perturbation in $\tilde{v}^{(2)}_x$ in Eq. (3.46), which gives rise to $\tilde{v}^{(2)}_{xx}$, $\tilde{v}^{(2)}_{xy}$ and $\tilde{v}^{(2)}_{xz}$ in real space (for details of the calculation see Appendix A) . Thus the total nonlinear diffusion coefficients which appears in the radial flux can be obtained as:

$$\begin{aligned} D^{(4)}_{x_{xx}x_{xx}} &= \int_0^\infty d\tau \big\langle \tilde{v}^{(2)}_{xx}(t)\tilde{v}^{(2)}_{xx}(t+\tau)\big\rangle \\ &= \sum_{k,k'} k_y^2 k_y'^2 \left(\frac{c}{B}\right)^4 |\tilde{\phi}_k|^2|\tilde{\phi}'_k|^2 \left(\frac{k'_x}{\omega - k_zv_z} + \frac{k_x}{\omega' - k'_zv_z}\right)^2 Re\frac{i}{\omega'' - k''_zv_z} \end{aligned} \tag{3.47}$$

$$\begin{aligned} D^{(4)}_{x_{xy}x_{xy}} &= \int_0^\infty d\tau \big\langle \tilde{v}^{(2)}_{xy}(t)\tilde{v}^{(2)}_{xy}(t+\tau)\big\rangle \\ &= \sum_{k,k'} \left(\frac{c}{B}\right)^4 |\tilde{\phi}_k|^2|\tilde{\phi}'_k|^2 \left(\frac{k_xk_y'^2}{\omega - k_zv_z} + \frac{k'_xk_y^2}{\omega' - k'_zv_z}\right)^2 Re\frac{i}{\omega'' - k''_zv_z} \end{aligned} \tag{3.48}$$

$$
\begin{aligned}
D^{(4)}_{x_{xz}x_{xz}} &= \int_0^\infty d\tau \big\langle \tilde{v}^{(2)}_{xz}(t)\tilde{v}^{(2)}_{xz}(t+\tau)\big\rangle \\
&= \sum_{k,k'} (\frac{ec}{mB})^2 |\tilde{\phi}_k|^2 |\tilde{\phi}_{-k'}|^2 k_z^2 k_z'^2 \left(\frac{k_y'}{(\omega - k_z v_z)^2} - \frac{k_y}{(\omega' - k_z' v_z)^2}\right)^2 \\
& Re \frac{i}{\omega'' - k_z'' v_z}
\end{aligned}
\tag{3.49}
$$

Here $D^{(4)}_{x_{xx}x_{xx}}$ and $D^{(4)}_{x_{xy}x_{xy}}$ are caused by $E \times B$ scattering and $D^{(4)}_{x_{xz}x_{xz}}$ involves parallel acceleration. The coefficients can be scaled as:

$$
D^{(4)}_{x_{xx}x_{xx}} : D^{(4)}_{x_{xy}x_{xy}} : D^{(4)}_{x_{xz}x_{xz}} = 1 : 1 : (k_{\parallel}/k_{\perp})^2 \tag{3.50}
$$

Thus, $D^{(4)}_{x_{xx}x_{xx}} \approx D^{(4)}_{x_{xy}x_{xy}} >> D^{(4)}_{x_{xz}x_{xz}}$, and nonlinear diffusion driven by $E \times B$ scattering is larger than that due to parallel acceleration. The corresponding piece of the ion kinetic energy evolution in Eq. (3.14), especially the divergence of the nonlinear ion heat flux, can thus be written as:

$$
\nabla \cdot Q^{(2)} + \nabla \cdot Q^{(4)} = -\frac{\partial}{\partial x} \int dv \frac{1}{2} m v^2 (D^{(2)}_{xv} + D^{(2)}_{xx} + D^{(4)}_{xv} + D^{(4)}_{xx}) \frac{\partial}{\partial x} \langle f_i \rangle \tag{3.51}
$$

The nonlinear ion heat flux $Q^{(4)}$ might play a role for energy transport as compared to the quasilinear order $Q^{(2)}$. That is another interesting topic, but here we only focus on inter-species nonlinear energy coupling, for simplicity.

3.3.2 Nonlinear ion heating from the cross term diffusion terms

As for the calculation of the nonlinear diffusion $D^{(4)}_{vv}$ in velocity space and $D^{(4)}_{xx}$ in real space, we can obtain the cross term of the nonlinear diffusion which is given by

$$
D^{(4)}_{vx} = \int_0^\infty d\tau \Big\langle \tilde{F}^{(2)}_z(t) \tilde{v}^{(2)}_x(t+\tau) \Big\rangle \tag{3.52}
$$

where $\tilde{F}^{(2)}_z$ includes $(\tilde{F}^{(2)}_{zx}, \tilde{F}^{(2)}_{zy}, \tilde{F}^{(2)}_{zz})$ and $\tilde{v}^{(2)}_x$ is composed of $(\tilde{v}^{(2)}_{xx}, \tilde{v}^{(2)}_{xy}, \tilde{v}^{(2)}_{xz})$ as discussed before. Thus the total $D^{(4)}_{vx}$ will have nine terms. The easy way to obtain the cross diffusion coefficient $D^{(4)}_{vx}$ is to combine the nonlinear diffusion coefficients in velocity and real space, i.e $D^{(4)}_{v_{zx}x_{xx}}$ can be recovered from $D^{(4)}_{v_{zx}v_{zx}}$ in equation (3.38) and $D^{(4)}_{x_{xx}x_{xx}}$ in equation (3.47) since they are both symmetric in structure. Thus the coefficients can be written as:

$$
\begin{aligned}
D^{(4)}_{vx} =& D^{(4)}_{v_{zx}x_{xx}} + D^{(4)}_{v_{zx}x_{xy}} + D^{(4)}_{v_{zy}x_{xx}} + \\
& D^{(4)}_{v_{zy}x_{xx}} + D^{(4)}_{v_{zy}x_{xy}} + D^{(4)}_{v_{zy}x_{xz}} + \\
& D^{(4)}_{v_{zz}x_{xx}} + D^{(4)}_{v_{zz}x_{xy}} + D^{(4)}_{v_{zx}x_{xz}}
\end{aligned}
\tag{3.53}
$$

In section 3.2.4 and section 3.3.1, we showed that $D^{(4)}_{v_{zx}} \approx D^{(4)}_{v_{zy}} \gg D^{(4)}_{v_{zz}}$ and $D^{(4)}_{x_{xx}} \approx D^{(4)}_{x_{xy}} \gg D^{(4)}_{x_{xz}}$ which are due to the $E \times B$ effect being bigger than the parallel scattering effect. Similarly here, in Eq. (3.53) we have $D^{(4)}_{v_{zx}x_{xx}} \approx D^{(4)}_{v_{zx}x_{xy}} \gg D^{(4)}_{v_{zy}x_{xx}}$, $D^{(4)}_{v_{zy}x_{xx}} \approx D^{(4)}_{v_{zy}x_{xy}} \gg D^{(4)}_{v_{zz}x_{xx}}$, and $D^{(4)}_{v_{zz}x_{xx}} \approx D^{(4)}_{v_{zz}x_{xy}} \gg D^{(4)}_{v_{zz}x_{xx}}$. Thus, the cross term diffusion coefficients can be simplified as

$$
D^{(4)}_{v_{zx}x_{xx}} \approx D^{(4)}_{v_{zx}x_{xy}} \approx D^{(4)}_{v_{zy}x_{xx}} \approx D^{(4)}_{v_{zy}x_{xy}} \tag{3.54}
$$

where $D^{(4)}_{v_{zx}} \approx D^{(4)}_{v_{zy}}$ and $D^{(4)}_{x_{xx}} \approx D^{(4)}_{x_{xy}}$ was used in Eq. (3.54). So the cross term of the nonlinear diffusion coefficient can be obtained:

$$
\begin{aligned}
D^{(4)}_{v_{zx}x_{xx}} = \sum_{k,-k,k',-k'} & \frac{e}{m}(\frac{c}{B})^3 |\tilde{\phi}_k|^2 |\tilde{\phi}_{k'}|^2 k_y k'_y (\frac{k_y k'_x k'_z}{\omega - k_z v_z} + \frac{k_x k'_y k_z}{\omega' - k'_z v_z}) \\
& (\frac{k'_x}{\omega - k_z v_z} + \frac{k_x}{\omega' - k'_z v_z}) R_e \frac{i}{\omega'' - k''_z v_z}
\end{aligned}
\tag{3.55}
$$

The associated turbulent ion heating can be written as

$$\begin{aligned}\left\langle \tilde{E}\cdot\tilde{J}_i\right\rangle_{vx}^{(4)} &= -\int dv_z m v_z D_{vx}^{(4)}\frac{\partial\langle f_i\rangle}{\partial x}\\ &= -\int dv_z m v_z (D_{v_{zx}x_{xx}}^{(4)} + D_{v_{zx}x_{xy}}^{(4)} + D_{v_{zy}x_{xx}}^{(4)} + D_{v_{zy}x_{xy}}^{(4)})\frac{\partial\langle f_i\rangle}{\partial x}\\ &\approx \sum_{k,-k,k',-k'}\sqrt{\pi}nT_i\frac{v_{thi}^3}{\Omega_{ci}^2}\frac{T_i}{T_e}\frac{\omega''}{|k_z''|}\omega_{\star e}\left|\frac{e\tilde{\phi}_k}{T_i}\right|^2\left|\frac{e\tilde{\phi}_{k'}}{T_i}\right|^2 k_y' k_z''(\frac{k_x' k_y k_z'}{\omega k_z'' - k_z\omega''}\\ &+ \frac{k_x k_y' k_z}{\omega' k_z'' - k_z'\omega''})\\ &\left(\frac{k_x'}{\omega k_z'' - k_z\omega''} + \frac{k_x}{\omega' k_z'' - k_z'\omega''}\right) exp\left(-\frac{(\omega''/k_z'')^2}{v_{thi}^2}\right)\end{aligned} \tag{3.56}$$

Combining equation (3.42) and equation (3.56), the total nonlinear ion turbulent heating is given by

$$\begin{aligned}\left\langle \tilde{E}\cdot\tilde{J}_i\right\rangle_{vv}^{(4)} + \left\langle \tilde{E}\cdot\tilde{J}_i\right\rangle_{vx}^{(4)} = \sum_{k,k'}\sqrt{\pi}nT_i\frac{v_{thi}^3}{\Omega_{ci}^2}\left|\frac{e\tilde{\phi}_k}{T_i}\right|^2\left|\frac{e\tilde{\phi}_{k'}}{T_i}\right|^2 (\frac{k_x' k_y k_z'}{\omega k_z'' - k_z\omega''}+\\ \frac{k_x k_y' k_z}{\omega' k_z'' - k_z'\omega''})^2\frac{\omega'' k_z''}{|k_z''|}[\frac{\omega''}{k_z''} + \frac{T_i}{T_e}\omega_{\star e}(\frac{k_x' k_y'}{\omega k_z'' - k_z\omega''}+\\ \frac{k_x k_y'}{\omega' k_z'' - k_z'\omega''})/(\frac{k_x' k_y k_z'}{\omega k_z'' - k_z\omega''} + \frac{k_x k_y' k_z}{\omega' k_z'' - k_z'\omega''})]\\ exp\left(-\frac{(\omega''/k_z'')^2}{v_{thi}^2}\right)\end{aligned} \tag{3.57}$$

where the (k_x, k_y, k_z, ω) and $(k_x', k_y', k_z', \omega')$ are arbitrary drift wave modes.

3.4 The ratios of energy dissipation channels in ITER plasma

In previous papers[70, 71], the collisionless electron-ion energy coupling was written in a simplified form, i.e.

$$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle = A_L I + B_{NL} I^2 + C_{ZF} I^2, \tag{3.58}$$

where the turbulence intensity is defined as $I = \sum_k |e\tilde{\phi}_k/T_e|^2$, and the coefficients A_L, B_{NL}, C_{ZF} have dimensions of a power density. $A_L = \sum_k A_k |e\tilde{\phi}_k/T_e|^2 / \sum_k |e\tilde{\phi}_k/T_e|^2$ is set by the electron and ion quasilinear heating, and $A_k(n, T_e, T_i, V_{the}, V_{thi}, \omega_\star, k_\perp\rho_s) = -nT_e(Rq/V_{the})\ {\omega_{\star e}}^2 F_1(k_\perp\rho_s) + nT_i(Rq/V_{thi}){\omega_{\star e}}^2 F_2(k_\perp\rho_s)$ (see Ref. [70]). Then $B_{NL} = \sum_{k,k'} B_{k,k'} |e\tilde{\phi}_k/T_i|^2\ |e\tilde{\phi}_{k'}/T_i|^2/I^2$ describes the nonlinear ion heating through nonlinear Landau damping, and $B_{k,k'}$ can be obtained from Eq. (3.57). The coefficient $C_{ZF} = \sum_{k,k'} C_{k,k'} |e\tilde{\phi}_k/T_i|^2 |e\tilde{\phi}_{k'}/T_i|^2/I^2$ is determined by heating through zonal flow formation. Here, we ignore the nonlinear heating due to zonal flows and focus on nonlinear heating. We estimate the nonlinear heating by using the saturation balance equation [29]. For each turbulent heating term, there will be a corresponding γ_{growth} for wave growth, or $\gamma_{damping}$ for wave dissipation in the net saturation balance condition. Nonlinear saturation in a turbulent state implies steady energy transfer from source $(\nabla T_e, \nabla n)$ to sink. Schematically, saturation implies that a fluctuation energy balance condition must be satisfied, so we have

$$\gamma = \gamma_L^{electron} + \gamma_L^{ion} + \gamma_{NL}^{ion} + \gamma_{zonalflow} \approx 0 \tag{3.59}$$

For simplicity, we ignore the zonal flow in the saturation balance. Correspondingly, the divergence of the wave energy flux also need not be considered, since $\partial W/\partial t = 2\gamma W$ [65], then $2\gamma W + \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = 0$ and the *total* turbulent heating $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle = 0$ in the stationary state. In this limit, the total turbulent heating in Eq. (3.58) can be written as

$$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle = A_L I + B_{NL} I^2 = 0 \tag{3.60}$$

Turbulence intensity can be calculated from Eq. (3.60) and is given by $I = |A_L/B_{NL}|$, the ratio of the quasilinear turbulent heating to nonlinear turbulent heating. Here we will discuss the turbulence intensity at the crossover of the nonlinear ion heating with quasilinear ion heating. The quasilinear ion heating in Eq. (3.19) can be approximated as :

$$\begin{aligned}\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(2)} &= \sum_k \sqrt{\pi} n T_i (\frac{e\tilde{\phi}_k}{T_i})^2 \frac{\omega}{|k_z| v_{thi}} (\omega + \frac{T_i}{T_e}\omega_{\star e}) \exp\left[-\frac{(\omega/k_z)^2}{v_{thi}^2}\right] \\ &\sim n T_i \rho_\star^2 \frac{Rq\omega_{\star e}^2}{v_{thi}} F_2(k_\perp \rho_s)\end{aligned} \tag{3.61}$$

where the turbulence intensity $|e\tilde{\phi}/T_e| \sim \rho_s/L_n \sim \rho_\star$ (note: $T_i \approx T_e$ in ITER plasma) was estimated by using mixing length theory, the wave vector $k_z \sim 1/Rq$ and the dispersion relationship is taken as $\omega = \omega_\star/(1 + k_\perp^2 \rho_s^2)$ in drift wave turbulence. The functions $F_2(k_\perp \rho_s)$ are dimensionless, and depend on $k_\perp \rho_s$, as given by

$$F_2(k_\perp \rho_s) = \left(\frac{1}{1 + k_\perp^2 \rho_s^2}\right) \left(\frac{1}{1 + k_\perp^2 \rho_s^2} + \frac{T_i}{T_e}\right) \exp\left[-\left(\frac{k_\perp \rho_s}{1 + k_\perp^2 \rho_s^2}\right)^2 \left(\frac{Rq}{a}\right)^2 \frac{T_e}{T_i}\right] \tag{3.62}$$

We apply ITER parameters into Eq. (3.61), the quasilinear ion heating can be simplified as:

$$\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle^{(2)} \sim P F_2(k_\perp \rho_s) \tag{3.63}$$

where the power density $P = n_i T_i \rho_\star^2 (Rq/v_{thi})(c_s/L_n)^2$. In ITER, the electron temperature and ion temperature $T_e \approx T_i = 10keV$,the major radius $R = 6.2m$, the minor radius $r = 2m$, the safety factor is $q = 2$ at the core, and the turbulence intensity $\rho_\star \sim \rho_s/L_n \sim 10^{-3}$[12, 13].

Thus the quasilinear ion heating in Eq. (3.63) can be shown in Fig. 3.4, which is a function dependent on $k_\perp \rho_s$, for $0 < k_\perp \rho_s < 1$.

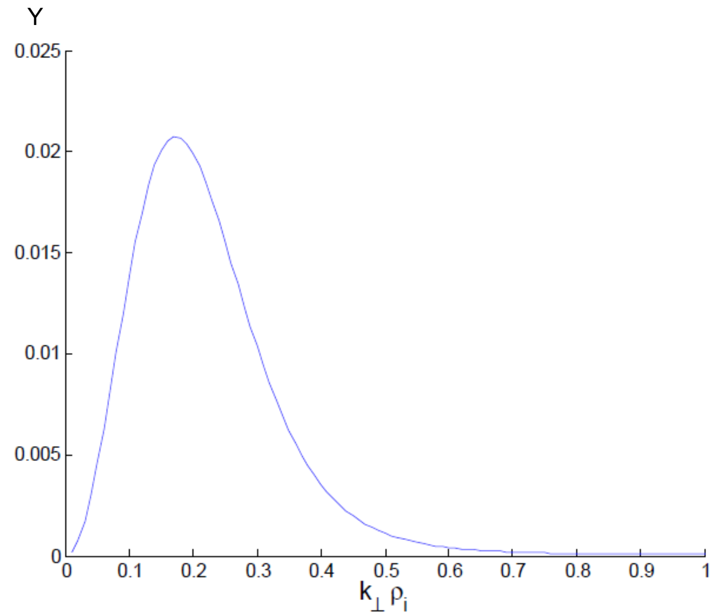

Figure 3.4: The quasilinear ion heating dependence on with $k_\perp \rho_s$, for $0 < k_\perp \rho_s < 1$. The coordinate $Y = F_2(k_\perp \rho_s)$.

In Fig. 3.4. the quasilinear ion heating dropped dramatically as $0.2 < k_\perp \rho_s < 1$, where the exponential effect will be dominant in Eq. (3.63).

Now we compare the equation (3.57) with (3.61), so the value of the turbulence intensity at the crossover of nonlinear ion heating and quasilinear ion heating can be obtained,

$$\frac{\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_i^{(2)}}{\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_i^{(4)}} \sim 1 \tag{3.64}$$

$$\Rightarrow |\frac{e\tilde{\phi}_{k'}}{T_i}|^2 \approx (\frac{c_s^4}{V_{thi}^4})(\frac{\rho_s}{L_n})^2(\frac{Rq}{L_n}) \exp\left[-\left(\frac{k_\perp \rho_s}{1+k_\perp^2 \rho_s^2}\right)^2 \left(\frac{Rq}{a}\right)^2 \frac{T_e}{T_i}\right] \tag{3.65}$$

Note: we only keep basic scalings in Eq. (3.65), where the exponential factor $exp[-(\omega''$

$/k_z'')^2(1/v_{the}^2)] \approx 1$ for nonlinear ion heating Eq. (3.57), since the beat mode can always resonate with the bulk of the ion distribution.

To evaluate the turbulence intensity in Eq. (3.65), ITER parameters were utilized [12, 13] again. The Eq. (3.65) can be approximated as:

$$|\frac{e\tilde{\phi}_{k'}}{T_i}| \approx 6 \times 10^{-3} \exp\left[-36\left(\frac{k_\perp \rho_s}{1+k_\perp^2\rho_s^2}\right)^2\right] \tag{3.66}$$

This result is consistent with the decrease in quasilinear ion heating in the range $0.2 < k_\perp \rho_s < 1$ in Fig. 3.4, where the exponential factor has a dominant effect. In other words, the nonlinear ion heating is not necessarily small compared with quasilinear ion heating even thought it scales as $\sim \mathcal{O}|e\tilde{\phi}/T|^4$. In particular, the nonlinear ion heating contributed by the $E \times B$ scattering introduces the large factor since $(k_\perp/k_z)^2 \sim (L_n/\rho_s)^2$ in Eq. (3.65).

We analyzed nonlinear ion heating which becomes comparable to the quasilinear ion heating for a turbulence intensity $|e\tilde{\phi}_{k'}/T_e| \ll 10^{-3}$. In ITER plasma, the turbulence intensity is estimated to be $|e\tilde{\phi}_k/T_e| \sim \rho_\star \sim 10^{-3}$, using the usual mixing length approximation [70, 66, 10]. This value of $\rho_\star$ implies a level of $|e\tilde{\phi}_k/T|$ which is *above* the value where quasilinear and nonlinear heating cross. Thus, the turbulent ion heating can exceed quasilinear ion turbulent heating, as the dominant energy dissipation channel in collisionless electron drift wave turbulence. The exponential factor has a significant effect.

3.5 Conclusion

In this paper, we focus on the calculation of the turbulent ion heating through nonlinear wave-particle interaction. The principal results of this paper are given below. By constructing the nonlinear diffusion coefficients in velocity and real space, we were able to calculate the nonlinear heating terms. The nonlinear diffusion coefficients $D_{vv}^{(4)}$ and $D_{xx}^{(4)}$ were calculated via $v_\parallel$ scattering and $E \times B$ scattering in velocity and and real space, using perturbation theory to higher order. The coefficients $D_{vv}^{(4)}$ contribute to the nonlinear ion heating $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vv}^{(4)}$. In particular, the cross-term nonlinear diffusion coefficients $D_{vx}^{(4)}$, which combine the velocity and spatial effects, contributed another important nonlinear ion heating term $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vx}^{(4)}$. In addition, the nonlinear electron-ion energy coupling terms $\left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vv}^{(4)} + \left\langle \tilde{E} \cdot \tilde{J}_i \right\rangle_{vx}^{(4)}$, have been shown to enter as an important collisionless energy dissipation channel in ITER-like plasmas. This nonlinear turbulent heating becomes a dominant effect when the turbulence intensity is $|e\tilde{\phi}_{k'}/T_e| < 10^{-3}$. Thus, it is necessary to consider these nonlinear wave-particle coupling effects in the total energy balance, in a low collisionality plasma, like ITER. In addition, we proposed a simple expression for the electron and ion turbulent energy coupling. In Eq. (3.58), all turbulent heating terms were listed. This theoretical model is useful for experimental tests or simulations of the electron heat transport in future large, collisionless tokamaks.

As for the identification of the collisionless turbulent energy transfer channels in actual experiment, here we would like to give some basic suggestions. First, measure the T_i temperature profile and turbulence intensity with increasing electron heating. In an electron heated plasma (i.e. ECRH heating), one might expect to see T_i increase with more heat input. Usually this process is dominated by collisional energy transfer, since the collisionality $\nu_\star$ is high at low temperature. Once the ions are hot enough, the ion temperature T_i will approach the electron temperature T_e, so the collisional energy transfer will drop, and so collisionless energy transfer must control the electron-ion energy coupling process. Obviously, the turbulent heating $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle$ is turbulence intensity $(i.e. \mathcal{O}|e\tilde{\phi}/T|^2$ and $\mathcal{O}|e\tilde{\phi}/T|^4)$ dependent. The thermal diffusivity χ is also. Thus, $T_i \sim \left\langle \tilde{E} \cdot \tilde{J} \right\rangle / \chi$ should not be sensitive to the turbulence intensity if the *collisionless* energy transfer process are dominant. This could be checked by T_i profile and fluctuation measurements.

One can also add a model of the collisionless energy transfer term (i.e. Eq. (3.58)) to the energy balance equation in transport analysis codes to predict the calculated electron and ion thermal diffusivities. The predicted χ_i and χ_e can be compared to nonlinear simulations with collisionless coupling properly calculated. Then, discrepancies like those discussed in the introductions could be eliminated.

Weak turbulence theory was used in the nonlinear electron-ion energy coupling process in our paper [24]. A similar calculation can also be used to calculate nonlinear wave-particle momentum exchange based on the nonlinear diffusion equation derived in this paper. This will enable the calculation of the intrinsic torque due to nonlinear wave-particle interaction. $E \times B$ shear effects will make an important contribution to the beat wave resonance in this case. Given the results for nonlinear heating discussed in this paper, we can expect this nonlinear intrinsic torque to be substantial. This new perspective on intrinsic torque, residual stress and intrinsic rotation will be discussed in a future paper [79, 80, 81, 82].

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Appendix A: Nonlinear diffusion in real space

Now we look at the nonlinear diffusion in real space. The nonlinear diffusion for fluxes is related to the generalized flux in $\mathbf{x}$ and is given by $D^{(4)}_{xx} = D^{(4)}_{x_{xx}x_{xx}} + D^{(4)}_{x_{xy}x_{xy}} + D^{(4)}_{x_{xz}x_{xz}}$. The $D^{(4)}_{xx}$ can be written as

$$D^{(4)}_{xx} = \int_0^\infty d\tau \Big\langle \tilde{v}^{(2)}_x(t)\tilde{v}^{(2)}_x(t+\tau)\Big\rangle \tag{3.67}$$

Here the velocity correction $\tilde{v}^{(2)}_x$ can be obtained from perturbation theory, which we used to calculate $\tilde{F}^{(2)}_z$ before. $\tilde{v}^{(2)}_x$ is given by

$$\tilde{v}^2_x = \frac{d\tilde{x}^{(2)}}{dt} = \sum_{-k'}(ik'_\theta)\frac{c}{B}\tilde{\phi}_{-k'}e^{-i\mathbf{k}'\cdot\mathbf{x}^{(0)}}e^{i(-k'_z v_z+\omega')t}[-i(k'_x x^{(1)} + k'_y y^{(1)} + k'_z z^{(1)}] \tag{3.68}$$

where the first order orbit deviation were given by

$$z^{(1)} = -\frac{e}{m}\sum_k \tilde{E}(k)\frac{\exp[i(\mathbf{k}\cdot\mathbf{x}+\phi_k)]}{(\omega-k_z v_z)^2}\exp[i(k_z v_z-\omega t)] \tag{3.69}$$

$$x^{(1)} = \sum_k \frac{c}{B}\frac{k_y}{\omega-k_z v_z}\tilde{\phi}_k e^{i\mathbf{k}\cdot\mathbf{x^{(0)}}}e^{-i(\omega-k_z v_z)t} \tag{3.70}$$

$$y^{(1)} = \sum_k -\frac{c}{B}\frac{k_x}{\omega-k_z v_z}\tilde{\phi}_k e^{i\mathbf{k}\cdot\mathbf{x^{(0)}}}e^{-i(\omega-k_z v_z)t} \tag{3.71}$$

After substituting $x^{(1)}$,$y^{(1)}$ and $z^{(1)}$ into Eq. (3.68), we obtain,

$$v^{(2)}_{x_{xx}} = \sum_{k,-k'} k_y k'_y(\frac{c}{B})^2|\tilde{\phi}_k||\tilde{\phi}_{k'}|\frac{k'_x}{\omega-k_z v_z}e^{i\mathbf{k}''\cdot\mathbf{x}_0}e^{i(k''_z-\omega'')t} \tag{3.72}$$

And

$$v^{(2)}_{x_{xy}} = -\sum_{k,-k'} k_x k'_y(\frac{c}{B})^2|\tilde{\phi}_k||\tilde{\phi}_{k'}|\frac{k'_y}{\omega-k_z v_z}e^{i\mathbf{k}''\cdot\mathbf{x}_0}e^{i(k''_z-\omega'')t} \tag{3.73}$$

$$v^{(2)}_{x_{xz}} = -\sum_{k,-k'} (\frac{ec}{mB})(ik_z)|\tilde{\phi}_k||\tilde{\phi}_{k'}|\frac{k'_y k'_z}{(\omega-k_z v_z)^2}e^{i\mathbf{k}''\cdot\mathbf{x}_0}e^{i(k''_z-\omega'')t} \tag{3.74}$$

Substituting equation (3.72) into equation (3.67), we obtain

$$
\begin{aligned}
D^{(4)}_{x_{xx}x_{xx}} &= \int_0^\infty d\tau \big\langle \tilde{v}^{(2)}_{x_{xx}}(t)\tilde{v}^{(2)}_{x_{xx}}(t+\tau)\big\rangle \\
&= \int_0^\infty d\tau \left\langle A_{2xx}(k,-k')e^{i\mathbf{k}''\cdot\mathbf{x}_0}e^{i(k_z''-\omega'')t}A_{2xx}(-k,k')e^{-i\mathbf{k}''\mathbf{x}_0}e^{i(k_z''-\omega'')(t+\tau)}\right\rangle \\
&= \sum_{k,k'} k_y^2 k_y'^2 \left(\frac{c}{B}\right)^4 |\tilde{\phi}_k|^2|\tilde{\phi}_k'|^2 \left(\frac{k_x'}{\omega - k_z v_z} + \frac{k_x}{\omega' - k_z' v_z}\right)^2 Re\frac{i}{\omega'' - k_z'' v_z}
\end{aligned}
\tag{3.75}
$$

where the coefficient $A_{2xx}(k,-k')$ is given by

$$
A_{2xx}(k,-k') = \sum_{k,k'} \frac{k_y k_y'}{2}(\frac{c}{B})^2 |\tilde{\phi}_k||\tilde{\phi}_{-k'}| \left(\frac{k_x'}{\omega - k_z v_z} + \frac{k_x'}{\omega' - k_z' v_z}\right) \tag{3.76}
$$

Similarly the nonlinear diffusion coefficient corrected by velocity $v^{(2)}_{x_{xy}}$ scattering in (3.73), is given by

$$
\begin{aligned}
D^{(4)}_{x_{xy}x_{xy}} &= \int_0^\infty d\tau \Big\langle \tilde{v}^{(2)}_{x_{xy}}(t)\tilde{v}^{(2)}_{x_{xy}}(t+\tau)\Big\rangle \\
&= \int_0^\infty d\tau \left\langle A_{2xy}(k,-k')e^{i\mathbf{k}''\cdot\mathbf{x}_0}e^{i(k_z''-\omega'')t}A_{2xy}(-k,k')e^{-i\mathbf{k}''\cdot\mathbf{x}_0}e^{i(k_z''-\omega'')(t+\tau)}\right\rangle \\
&= \sum_{k,k'} \left(\frac{c}{B}\right)^4 |\tilde{\phi}_k|^2|\tilde{\phi}_k'|^2 \left(\frac{k_x k_y'^2}{\omega - k_z v_z} + \frac{k_x' k_y^2}{\omega' - k_z' v_z}\right)^2 Re\frac{i}{\omega'' - k_z'' v_z}
\end{aligned}
\tag{3.77}
$$

The coefficient A_{2xy} is written as

$$
A_{2xy}(k,-k') = -\sum_{k,k'} \frac{1}{2}(\frac{c}{B})^2 |\tilde{\phi}_k||\tilde{\phi}_{-k'}| \left(\frac{k_y'^2 k_x}{\omega - k_z v_z} + \frac{k_y^2 k_x'}{\omega' - k_z' v_z}\right) \tag{3.78}
$$

Then the nonlinear diffusion coefficient in x space corrected by the velocity $v^{(2)}_{x_{xz}}$ scattering in (3.74) can be obtained

$$\begin{aligned}
D^{(4)}_{x_{xz}x_{xz}} &= \int_0^\infty d\tau \big\langle \tilde{v}^{(2)}_{x_{xz}}(t)\tilde{v}^{(2)}_{x_{xz}}(t+\tau)\big\rangle \\
&= \int_0^\infty d\tau \left\langle A_{2xz}(k,-k')e^{i\mathbf{k}''\cdot\mathbf{x}_0}e^{i(k''_z-\omega'')t}A_{2xz}(-k,k')e^{-i\mathbf{k}''\cdot\mathbf{x}_0}e^{i(k''_z-\omega'')(t+\tau)}\right\rangle \\
&= \sum_{k,k'}(\frac{ec}{mB})^2|\tilde{\phi}_k|^2|\tilde{\phi}_{-k'}|^2k_z^2k_z'^2\left(\frac{k'_y}{(\omega-k_zv_z)^2}-\frac{k_y}{(\omega'-k'_zv_z)^2}\right)^2 \\
&Re\frac{i}{\omega''-k''_zv_z}
\end{aligned} \tag{3.79}$$

where the coefficient A_{2xz} is given by

$$A_{2xz}(k,-k') = -\sum_{k,-k'}\frac{1}{2}(\frac{ec}{mB})|\tilde{\phi}_k||\tilde{\phi}_{-k'}|k_zk'_z\left(\frac{ik'_y}{(\omega-k_zv_z)^2}-\frac{ik_y}{(\omega'-k'_zv_z)^2}\right) \tag{3.80}$$

Chapter 4

Summary and discussion

In this thesis, the issue of anomalous electron-ion energy coupling was elaborated completely. A general form for the description of collisionless energy transfer was presented. This theoretical model will bring a new perspective to the study of turbulent transport in present and future tokamak plasmas. The "collisionless" energy transfer effect must be considered in the analysis of the plasma heat transport experiments and simulations in a low collisionality plasma, such as ITER. With the realization of a burning plasma in the near future, this anomalous plasmas energy coupling problem will attract more attention and our theoretical model will provide more useful information. The principal results of this thesis are listed as following.

The classical problems of " turbulent heating" and collisionless energy transfer channels were reviewed. The turbulence mediated the electron-ion energy transfer can dominate the energy transfer process, where the collisional energy transfer is relatively small in a low collisionality plasma, like ITER. Here, the channel of collisionless energy transfer will not only be through quasilinear wave-particle interactions but also through two nonlinear interaction processes. One is wave energy coupling to the zonal flow such that zonal flow friction can heat ions. Thus zonal flow frictional damping is one of the transfer channels and is likely to be significant in an ITER plasma. Another important nonlinear energy coupling process is nonlinear wave-particle interaction. This collisionless energy transfer was shown to be bigger than the quasilinear wave-particle interaction in a low collisionality plasma. Thus the traditional view of the anomalous electron-ion energy coupling only through quasilinear wave-particle interaction has been vitiated. A general form for the description of energy coupling through quasilinear Landau damping, nonlinear Landau damping and zonal flow frictional damping have been discussed in this thesis.

We compared the turbulent energy transfer with turbulent transport in CTEM. It was shown that the rate of the collisionless energy transfer can be the same order as turbulent transport in a low collisionality plasma. Thus, it is necessary to consider the influence of collisionless energy transfer in a transport model. Energy will not totally be lost through diffusion, it may also be transferred to other species. In this thesis, we mainly discussed collisionless energy transfer mechanisms where energy is transferred from hotter electrons to colder ions in an electron drift wave turbulence. Similarly, we can extend the application to ITG turbulence where collisioness energy coupling could also occur between ion-electron. In addition, the collisionless energy coupling between α particles and electrons is also an interesting research topic for a low collisionality ITER plasmas.

Collisionless energy transfer has been shown to provide a robust means of inter-species energy transfer. How to identify it in the present plasma thermal transport experiments? Some possible experimental proposals were presented in this thesis. In our discussion, the plasma heat flux and thermal diffusivity will be determined by the collisionless energy transfer processes in the heat balance equation (i.e. $\nabla \cdot Q \sim \left\langle \tilde{E} \cdot \tilde{J} \right\rangle$). Note that, turbulent heating $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle$ and thermal diffusivity χ are both proportional to the fluctuation levels (i.e.$|\tilde{\phi}|^2$, $|\tilde{\phi}|^4$). Thus, the temperature profile will be proportional to $\left\langle \tilde{E} \cdot \tilde{J} \right\rangle / \chi$. It doesn't depend on the fluctuation levels anymore. This temperature profile evolution will be significantly different with the heat flux decided by plasma collisional processes. By noting that, it is possible to identify the collisionless energy transfer in the plasma thermal transport experiments.

In addition, the electron temperature profile " stiffiness" phenomenon can be related to the heat pinch and the collisionless electron-ion energy coupling processes. They are two independent physical processes and which one is more efficient needs to be further investigated in future experiments.

As for the impact of the collisionless energy transfer model discussed above to the present turbulent transport modeling, we might add these terms in the electron balance equation to predict a new thermal diffusivity. This can be an effective verification and validation between transport codes and experiments.

Finally, turbulence mediated energy coupling has been discussed in this thesis. In particular, two nonlinear energy coupling processes have been identified. Similarly, the nonlinear momentum transport could be explored in future work based on an analogous structure to what we already described in this thesis. This will be another interesting research topic for future work.

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