Title

Theoretical aspects of underwater photography. I. Graphical-numerical calculation of the luminance field about a submerged point source

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Publication Date

1959-06-01

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Theoretical Aspects of Underwater Photography

I. GRAPHICAL-NUMERICAL CALCULATION OF THE LUMINANCE FIELD ABOUT A SUBMERGED POINT SOURCE

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June 1959
Index Number NS 714-100

Bureau of Ships
Contract NObs-72092

SIO REFERENCE 59-30

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Approved for Distribution:

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PREFACE

Fundamental studies of underwater photography by means of artificial light currently in progress in the Visibility Laboratory have revived interest in an internal memorandum written in 1953 in the course of solving a specific Navy photographic problem. A reproduction of that memorandum is hereby issued as a Laboratory report. An added appendix indicates applications of the memorandum to related but different underwater photographic problems, and extends the discussion to include effects due to multiply scattered light.

ABSTRACT

The primary luminance distribution was calculated about a given source submerged in an homogeneous hydrosol with a non-isotropic volume scattering function. The computations were carried out for a given set of distances from the source, and at each distance the primary luminance was computed for a given range of angles of sight from the observer-source direction. These computations allowed certain specific recommendations to be made as to what film speeds were necessary in order that the field luminance about the given point source be recorded on a photographic plate.

GRAPHICAL-NUMERICAL CALCULATION OF THE PRIMARY LUMINANCE

FIELD ABOUT A SUBMERGED POINT SOURCE

R. W. Preisendorfer

VISIBILITY LABORATORY 11 NOVEMBER 1953

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1. STATEMENT OF THE PROBLEM

Let M^* be an homogeneous hydrosol with attenuation coefficient α' and volume scattering coefficient $\sigma(\theta)$. Let a sphere of small* but finite diameter D be immersed in M^* with center at s. Let the sphere have a self-luminous Lambert surface with luminance B_s. If an observer is at a point o in M^* which is at a distance R_o from s, and the line of sight of the observer is directed at an angle ϕ, $0 \leq \phi \leq \pi$, with the line os, required: the primary luminance $B^{(1)}(\phi)$ of the field,** as seen from o in the direction ϕ.

2. DERIVATION OF FINITE SUMMATION FORMULA FOR PRIMARY LUMINANCE

2.1 Relation between B_s and the Total Luminous Flux Output F_s of Sphere.

Some sources of luminous energy are defined by giving their total luminous flux output instead of their inherent luminance. A relation between F_s and B_s of the given spherical source is needed for use in the complete solution of the stated problem. There are two basic ways to obtain the relation between F_s and B_s. The first proceeds as follows: Let R = D/2 be the radius of the sphere. Then

$$B_s = \Delta I_\theta / \Delta A \cos\theta \qquad (2.1.1)$$

* Small compared to the distance R_o, i.e., $5D \leq R_o$. See Summary Sheet No. 23 of ((3)).

** Primary Luminance is defined in Summary Sheet No. 74 of ((3)).

where ΔI_θ is the luminous intensity in the direction θ to the normal of an element of area ΔA of the surface of the given sphere. Since ΔI_θ obeys Lambert's law we may write

$$\Delta I_\theta = \Delta I_o \cos\theta \tag{2.1.2}$$

so that the element of luminous flux ΔF_s emitted by ΔA is

$$F_s = 2\pi \int_0^{\pi/2} \Delta I_o \cos\theta \sin\theta \, d\theta$$

$$= \pi \Delta I_o. \tag{2.1.3}$$

Since

$$B_s = \Delta I_o / \Delta A \tag{2.1.4}$$

we may write

$$F_s = \pi B_s \Delta A \tag{2.1.5}$$

so that the total luminous flux output of the sphere is

$$F_s = \pi^2 D^2 B_s. \tag{2.1.6}$$

The second basic method of deriving the relation between F_s and B_s is as follows: Let $\Delta\omega_s$ be the solid angle subtended by the sphere at o. Then

$$\Delta\omega_s = \pi D^2 / 4R_o^2. \tag{2.1.7}$$

The illuminance ΔE_s produced at o by the luminous flux from the given sphere on a surface normal to the line os (imagining $\beta' = 0$ in M^*) is:

$$\Delta E_s = F_s / 4\pi R_o^2. \tag{2.1.8}$$

Since

$$B_s = \Delta E_s / \Delta\omega_s \tag{2.1.9}$$

we have

$$B_s = \frac{(F_s / 4\pi R_o^2)}{(\pi D^2 / 4R_o^2)} = \frac{F_s}{\pi^2 D^2} . \tag{2.1.10}$$

2.2 Formula for $B^{(1)}(\phi)$.

For the geometrical relations used in the following derivation refer to Figures 2.2.1 and 2.2.2.

Let p be a point in M^* at which the unattenuated luminous flux from s is scattered through an angle θ to o. The unattenuated illuminance $E^{(o)}$ from s at p on a surface normal to the line sp is:

$$E^{(o)} = F_s \, e^{-\beta' R_1} / 4\pi R_1^2. \tag{2.2.1}$$

We now determine a region of M^* about p of finite volume dv by means of an elementary rectangular solid angle centered on the line sp. Let the vertical angular height of the solid angle be a_v and the horizontal angular width be a_h measured in radians. The volume dv is determined by the intersection of this solid angle and a solid angle of suitable dimensions a_v', a_h' with vertex at o. It is easy to see that

$$dv = R_1^2 \, a_h \, a_v \, dR_1, \tag{2.2.2}$$

where dR_1 is the increment of length of R_1 cut off by the solid angle with vertex at o. See Figure 2.2.2. Some of the unattenuated luminous flux at p is redirected by scattering through an angle θ to o. The primary luminous intensity $dI_\theta^{(1)}$ of dv in the direction po as induced by the $E^{(o)}$ given in (2.2.1) is:

$$dI_\theta^{(1)} = E^{(o)} \sigma(\theta) \, dv \tag{2.2.3}$$

by definition of $\sigma(\theta)$. Hence

$$dI_\theta^{(1)} = (F_s / 4\pi)\, a_h\, a_v\, \sigma(\theta)\, e^{-\beta R_1}\, dR_1 . \qquad (2.2.4)$$

As seen from o, the element of volume dv is a rectangle of height $dR_1 \sin\theta$ and width $a_h R_1$, so that the solid angle $d\Omega_\theta$ subtended at o by dv is:

$$d\Omega_\theta = a_h R_1\, dR_1 \sin\theta / R_2^2 . \qquad (2.2.5)$$

The unattenuated illuminance $dE_2^{(1)}$ produced at o (on a surface whose normal lies along po) by this unattenuated luminous flux from the illuminated element of volume dv is:

$$dE_2^{(1)} = dI_\theta^{(1)}\, e^{-\beta' R_2} / R_2^2 . \qquad (2.2.6)$$

The contribution $B_p^{(1)}$ to the primary luminance $B^{(1)}(\psi)$ by the illuminated element of volume dv is then:

$$B_p^{(1)} = \frac{dE_2^{(1)}}{d\Omega_\theta} = \frac{dI_\theta^{(1)}\, e^{-\beta' R_2} / R_2^2}{a_h R_1\, dR_1 \sin\theta / R_2^2} \qquad (2.2.7)$$

that is,

$$B_p^{(1)} = \frac{(F_s / 4\pi)\, a_h\, a_v\, \sigma(\theta)\, e^{-\beta'(R_1 + R_2)}\, dR_1}{a_h R_1\, dR_1 \sin\theta}$$

or

$$B_p^{(1)} = F_s\, a_v\, \sigma(\theta)\, e^{-\beta'(R_1 + R_2)} / (4\pi R_1 \sin\theta). \tag{2.2.8}$$

Now

$$R_o \sin\psi = R_1 \sin\theta \tag{2.2.9}$$

which allows (2.2.8) to be written

$$B_p^{(1)} = F_s\, a_v\, \sigma(\theta)\, e^{-\beta'(R_1 + R_2)} / (4\pi R_o \sin\psi). \tag{2.2.10}$$

Equation (2.2.10) thus gives the characteristic form for the component $B_p^{(1)}$ of $B^{(1)}(\psi)$. If a set $dv_1, dv_2, \ldots, dv_k$ of volumes at points $p_1, p_2, \ldots, p_k$ along the line of sight each subtended a vertical angular height a_v at s and if the scattering angle θ_i is associated with p_i, and if R_{1i} and R_{2i} are the distances sp_i and p_io respectively, and if the resulting luminance component of $B^{(1)}(\psi)$ associated with p_i is denoted by $B_{p_i}^{(1)}$, then we may approximate $B(\psi)$ by writing

$$B^{(1)}(\psi) = \sum_{i=1}^{k} B_{p_i}^{(1)} = \frac{F_s\, a_v}{4\pi R_o \sin\psi} \sum_{i=1}^{k} \sigma(\theta_i)\, e^{-\beta'(R_{1i} + R_{2i})}. \tag{2.2.11}$$

The computation of $B^{(1)}(\psi)$ for given F_s, ψ and R_o is reduced to a finite summation of the products $\sigma(\theta_i)\, e^{-\beta'(R_{1i} + R_{2i})}$. The choice of the angle a_v depends on how fine the subdivisions of the line of sight are to be chosen. The dimensions of the physical situation should suggest an appropriate choice of a_v. See Figure 2.2.3.

3. PHYSICAL AND GEOMETRICAL DATA FOR NUMERICAL EXAMPLE.

This report is based on a request to the Visibility Laboratory for an answer to a specific problem which had the form of the statement given in section 1. For the specific problem submitted optical constants for the hydrosol M^* were chosen as $\beta' = .051$ / ft which is equivalent to an horizontal hydrological range of 77 feet. The volume scattering function $\sigma(\theta)$ was taken from graphical data for ocean water given in the figures on page 71 in ((1)). The source was assumed to have a diameter of 0.20 feet and a luminous energy output of 1.2×10^5 lumen-seconds which is the luminous energy output of a #50 photoflash. A luminous energy output was assumed instead of a luminous flux (which is energy per unit of time) since the detector of the field luminance $B(\psi)$ was to be a photographic plate of suitable speed. Consequently, if F_s has units of lumen-seconds, $B^{(1)}(\psi)$ as given in (2.2.11) will have units of luminance-seconds and is so recorded in TABLE 4.1 and plotted in Figure 5.1. The calculations were performed for the following values of R_o: 40, 100, 200, and 300 feet; and for the following values of ψ: 0°, 2.5°, 5°, 10°, 20°, 25°, and 30°.

4. RESULTS TABULATED

TABLE 4.1

ψ	LUMINANCE-SECONDS			
	R_o 40 ft	R_o 100 ft	R_o 200 ft	R_o 300 ft
0°	3.9×10^{4}	2.04×10^{3}	1.37×10^{1}	9.3×10^{-2}
2.5°	1.07×10^{1}	2.12×10^{-1}	6.41×10^{-4}	2.50×10^{-6}
5°	4.60×10^{0}	8.68×10^{-2}	2.64×10^{-4}	9.10×10^{-7}
10°	1.69×10^{0}	3.24×10^{-2}	9.27×10^{-5}	3.40×10^{-7}
15°	7.67×10^{-1}	1.45×10^{-2}	3.73×10^{-5}	1.39×10^{-7}
20°	3.70×10^{-1}	7.15×10^{-3}	1.97×10^{-5}	7.30×10^{-8}
25°	1.93×10^{-1}	3.66×10^{-3}	9.94×10^{-6}	3.50×10^{-8}
30°	1.03×10^{-1}	1.89×10^{-3}	5.00×10^{-6}	1.78×10^{-8}


LUMINANCE SECONDS
ANGULAR DISTANCE FROM POINT SOURCE
0° 5° 10° 15° 20° 25° 30°
Ro = 40'
Ro = 100'
Ro = 200'
Ro = 300'

FIG. 5.1

6. SUMMARY AND CONCLUSIONS

The primary field luminance about a submerged point source has been calculated in a given hydrosol for various distances from the given point source and for various angular distances about the line connecting the source with the observer. For details, see section 3. The results are tabulated in TABLE 4.1 and Fig. 5.1. For the physical and geometrical conditions defined in section 3, some statements about the film speeds needed for film plates exposed to the luminance field may be made:

For R_o = 40 ft, the source and aureole are expected to be recorded on Eastman Panatomic X or Eastman Portrait Panchromatic.

For R_o = 100 ft, the source and aureole are expected to be recorded out to the angular distance of 2.5° on Panatomic X.

For R_o = 200 ft, the source and aureole are expected to be recorded out to 1.25° on Panatomic X.

For R_o = 300 ft, only source should make an impression on Eastman Super XX.

The above recommendations are based on the following relation:

$$\text{WESTON FILM SPEED} = 0.8\, f^2 / Bt \qquad (6.1)$$

as given on pages 8-10 in ((2)), where

f is the diaphragm opening

B is the luminance of the scene

t is the time of exposure to luminance field or flash duration.

The characteristics of the lens for the recording camera were assumed to be:

focal length -- 12 inches

lens diameter -- 4.8 inches

diaphragm opening -- 2.5

The product Bt was taken from the data shown in Figure 5.1; the film speed and hence the type of film were determined by using (6.1).

APPENDIX

We append three general observations to the present work. First of all, we note that even though the results and conclusions of the present report are restricted to a specific problem of underwater photography (See Sec. 6), the method of approach considered here is amenable to wider applications. In particular , equation (2.2.11) summarizes a useful approximate method for the determination of the primary component of the steady state luminance distribution generated by an isotropic point source in an homogeneous optical medium with a general volume scattering function σ . Furthermore, equation (2.2.11) can be extended to the case of <u>nonhomogeneous</u> media with <u>anisotropic point sources</u> thus:

$$B^{(1)}(\psi) = \frac{A_s}{R_o \sin\psi} \sum_{i=1}^{k} B_s(\theta_i - \psi)\, \sigma_i(\theta_i)\, T_i \; ,$$

where T_i is the beam transmittance of the path Op_iS, A_s is the area of a great circle on the non-isotropic spherical source, $B_s(\theta_i - \psi)$ is the luminance of the source in the direction $(\theta_i - \psi)$ (See Figs. 2.2.1 and 2.2.3), and $\sigma_i(\theta_i)$ is the value of σ at p_i for the scattering angle θ_i .

Secondly, knowledge of the primary luminance distribution allows one to compute such auxiliary photometric quantities as the primary component of the illuminance on arbitrarily oriented plane receivers, or primary scalar illuminance (associated with spherical receivers). Thus, the tabulations of luminance distributions are potentially more useful than tabulations of illuminance quantities and, for this reason, should always be given first priority in the initial phases of planning of any numerical tabulation project.

Finally, we observe that the primary component of the luminance distribution, which is the main object of study of the present report, is of course only one of a denumerably infinite number of scattering-order components of the <u>observable</u> luminance distribution at point O. The extent to which $B^{(1)}(\psi)$ contributes to the observable luminance at O depends principally on the ratio of the total scattering coefficient $s = \int_{4\pi} \sigma \, d\Omega$ to the volume attenuation coefficient β', namely on $\tilde{\omega}_0 = s/\beta'$. An order of magnitude estimate of the observable luminance $B(\psi)$ may be made by noting that[4]

$$B^{(j+1)}(\psi) \cong \tilde{\omega}_0 B^{(j)}(\psi) , \qquad j = 1, 2, \cdots ,$$

where the symbol $\cong$ denotes "is of the order of magnitude of." It follows that for $\psi > 0$,

$$B(\psi) = \sum_{j=1}^{\infty} B^{(j)}(\psi) \cong \frac{B^{(1)}(\psi)}{1 - \tilde{\omega}_0} .$$

Thus, for the present medium, in which $\tilde{\omega}_0$ is approximately 0.4, we estimate that $B(\psi) \cong 1.7 \, B^{(1)}(\psi)$.

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May 1959

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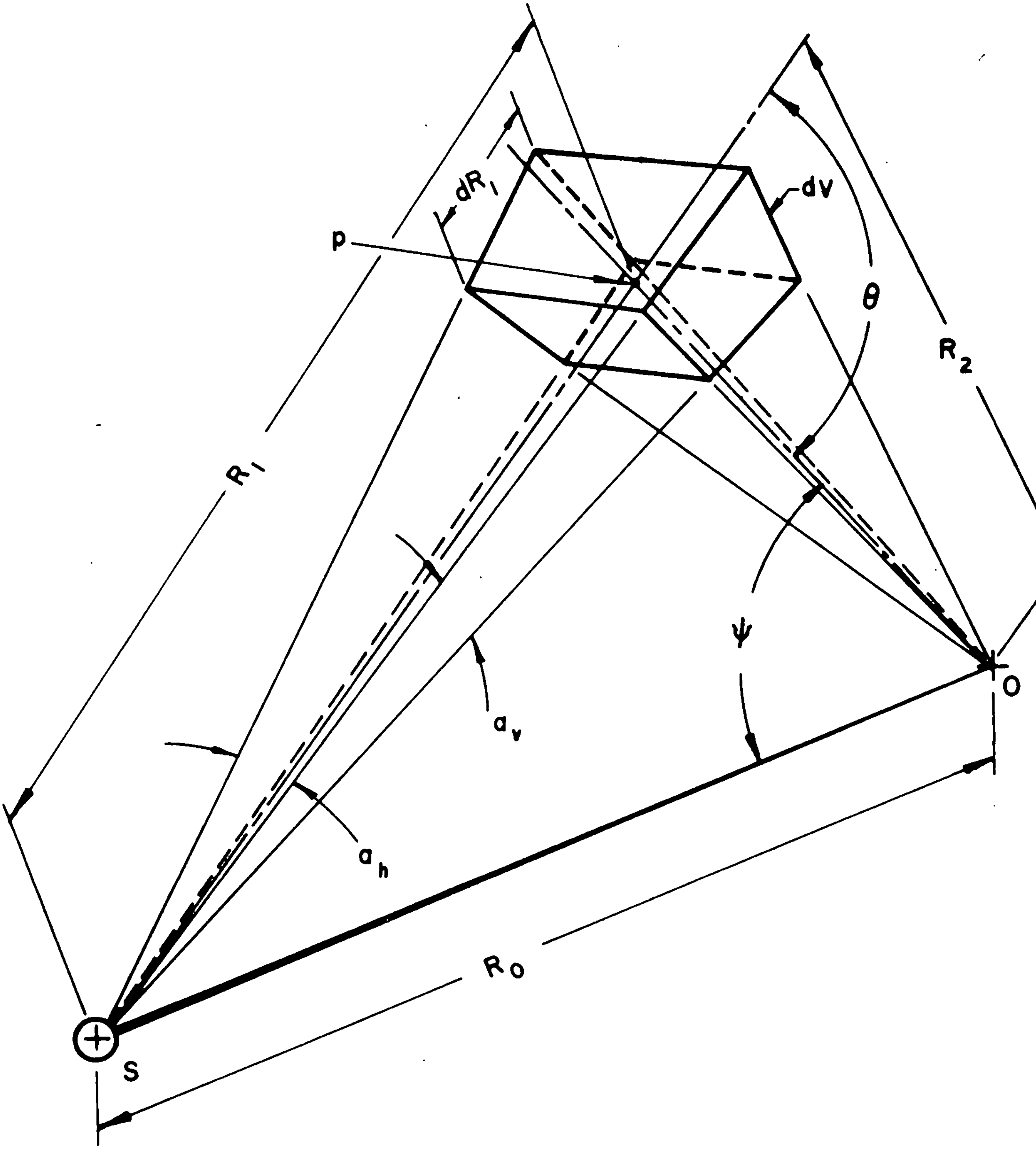
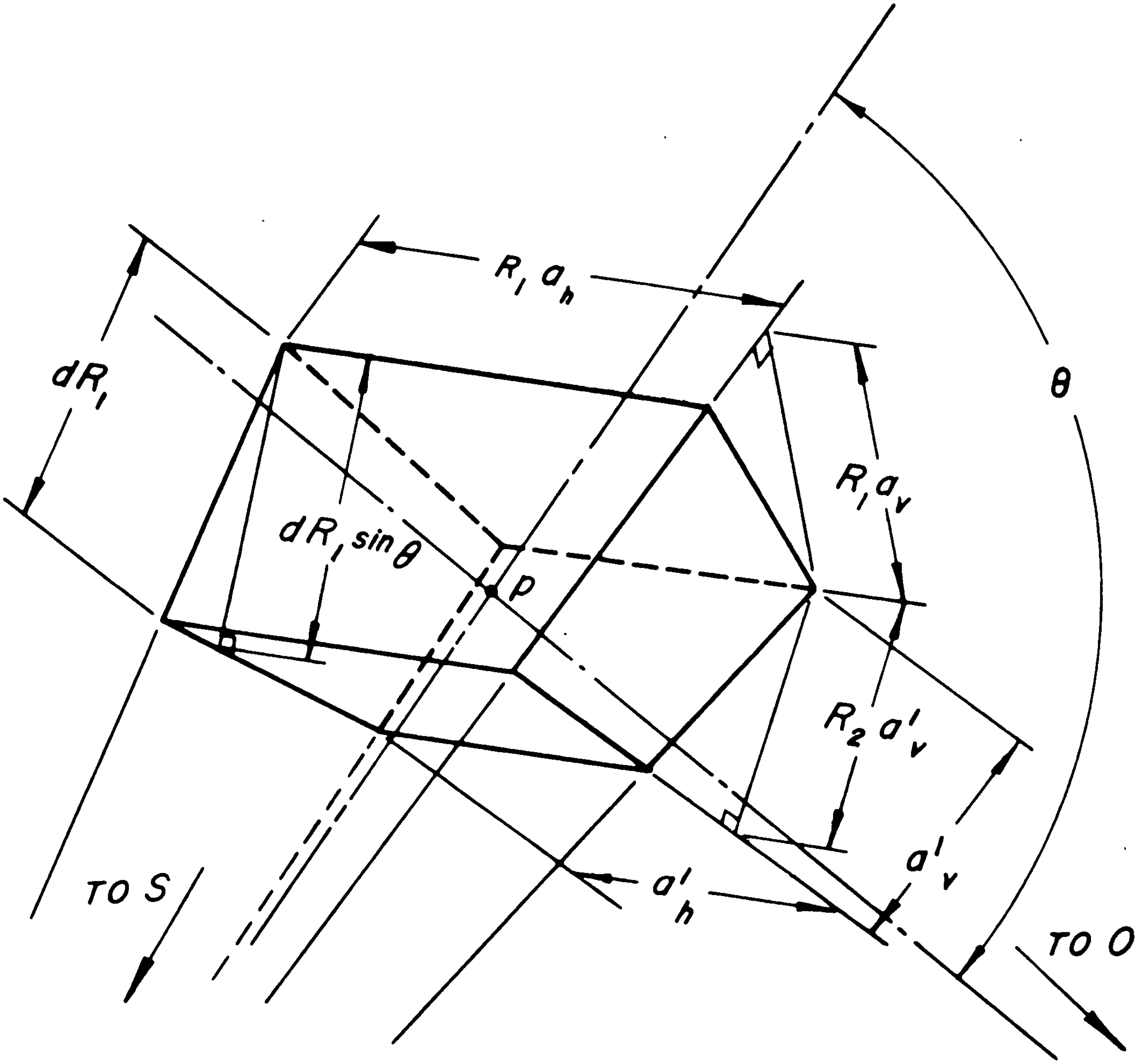


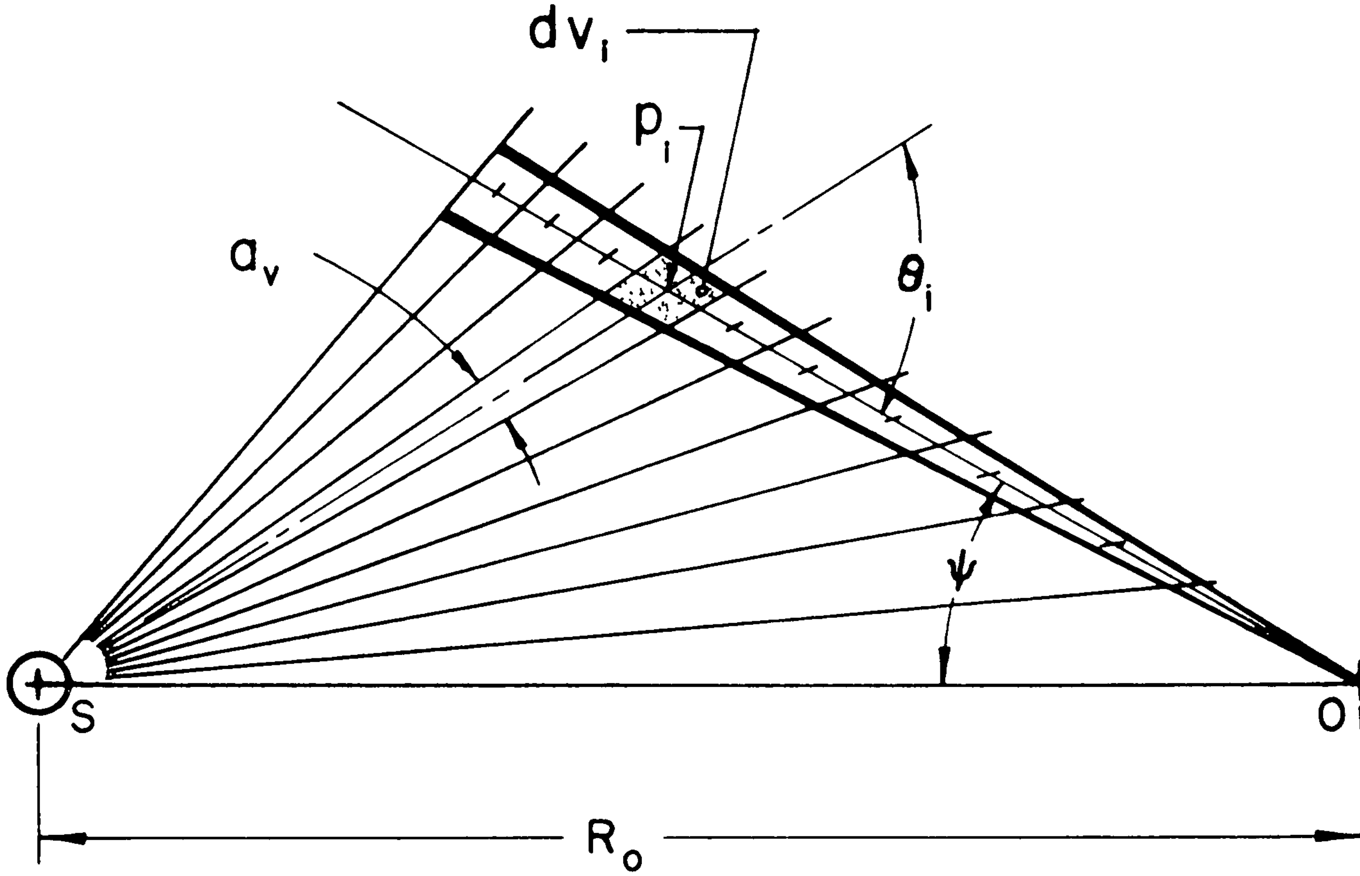
GEOMETRY OF DERIVATION OF $B^{(1)}(\psi)$

FIG. 2.2.1



DETAIL OF dV

FIG. 2.2.2



SHOWING EQUI-ANGULAR SUBDIVISION OF LINE OF SIGHT

FIG. 2.2.3