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Los Angeles

**Compact radiation sources for increased access
to high brightness x-rays**

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Physics

by

Finn Henry O'Shea

2013

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ABSTRACT OF THE DISSERTATION

**Compact radiation sources for increased access
to high brightness x-rays**

by

Finn Henry O'Shea

Doctor of Philosophy in Physics

University of California, Los Angeles, 2013

Professor James B. Rosenzweig, Chair

The successful operation of the x-ray free electron lasers at LCLS and SACLA are a boon for science. The increase in brightness of 10 orders of magnitude over synchrotron sources as well as the sub-picosecond time profile of the x-rays are opening new avenues of research in fields ranging from biology to solid state physics. However, synchrotrons and free electron lasers that produce x-rays are expensive, with price tags that measured hundreds of millions. Further, the standard unit of measure for the scale of these sources is kilometers. The sheer size and prohibitive cost of these devices means that such sources are out of the reach of universities and smaller laboratories.

The focus of this dissertation is in increasing access to x-ray sources by making them both smaller and, perhaps more importantly, cheaper. Current limitations to source size reduction are discussed which leads to the conclusion that smaller x-rays sources require short period undulators. In this context, two approaches to increasing access to x-rays are covered. The first is direct decrease in the period length of undulators through more advanced design and materials. This path begins with a discussion of the design and construction of a 9 mm period

prototype. An analysis of the benefits of such a device, in reduced undulator and accelerator lengths at existing free electron lasers, is explored. And finally, the operation of the undulator in a realistic scenario is experimentally explored in a scaled experiment at optical frequencies.

The second method for decreasing the period length of the light source is to replace the undulator with a laser, making an inverse Compton scattering source. The relationship between undulator radiation and the inverse Compton scattering process is examined, as well as the characteristics of the source itself. Lastly, as a demonstration of the function of the inverse Compton scattering source at Brookhaven National Laboratory as a diagnostic tool rather than an experiment itself, the 9 keV x-rays from the source are Bragg reflected from a Silicon crystal as a precursor to a pump-probe experiment which uses the inverse Compton scattered x-rays as a diagnostic. The experiment shows that the characteristics of the produced x-ray beam can be predicted by the input parameters.

With sources like the LCLS accepting one quarter of proposals for beam time, it is clear that there is demand for high brightness x-ray sources. Both of these technologies have the potential to increase access not just to x-rays but also to the sources themselves, potentially allowing proliferation of the number of locations for users to access diagnostic tools as well as creating a community of university scale operators.

The dissertation of Finn Henry O'Shea is approved.

Troy Carter

Pietro Musumeci

Benjamin J. Schwartz

James B. Rosenzweig, Committee Chair

University of California, Los Angeles

2013

For my family and friends.

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CHAPTER 1

Introduction

Electromagnetic radiation and particle accelerators have been inexorably linked since the first observation of synchrotron radiation from the General Electric synchrotron in 1947 [1]. Since that time, radiation production has gone from a side benefit produced by circular high energy physics machines to a specialized tool used at the cutting edge of modern science, with the three largest synchrotrons (3rd generation sources: SPring-8, APS and ESRF) producing hundreds of publications yearly in widely disseminated journals such as *Nature* and *Science* (see, for example, [2]). Further, the successes of the LCLS at the SLAC National Accelerator Laboratory [3] and the SACLA at SPring-8 [4] have brought to the arsenal of science the world's first 4th generation hard x-ray sources, the x-ray free electron laser (FEL). These offer, along with transverse coherence of the photon beam, an increase in peak brightness of 10 orders of magnitude over synchrotron sources. However, because the x-ray gain process starts from noise, the so called self-amplified-spontaneous-emission (SASE) mode, and the electron bunch is much longer than the photon pulse created by the FEL process, the photons are not longitudinally coherent [5, 6]. In addition to longitudinal coherence, users desire femtosecond or shorter pulses (for the purposes of this dissertation I am going to use 'very short' to mean a few femtosecond or shorter) to study processes dominated by electron motion at the atomic or molecular level [7, 8].

The push for lightsources in the x-ray frontier has come at a price, the ma-

chines are on the kilometer and billion dollar scales. The reason for the scale of the machines is the beam energy required to produce photons in the x-ray spectrum. Positive interference between the photons created at each successive period of the undulator occurs at the radiation wavelength

$$\lambda_r \propto \frac{\lambda_u}{2\gamma^2}, \quad (1.1)$$

where λ_u is the undulator period and γ is the electron beam energy normalized to the electron rest mass (see Eq. 3.21). It is clear from the above equation that the only way to reduce the required beam energy, while still producing Ångstrom or shorter wavelength radiation, is to reduce the undulator period.

A majority of insertion devices¹ are made with permanent magnets based on the work by Halbach [9] because permanent magnets have higher energy density available for field production at the centimeter scale [10, 11]; as electromagnets are scaled down the current density becomes too high for practical operation. However, short period, high magnetic field devices require small undulator gaps. Modern undulators have period lengths of a few centimeters (e.g the LCLS undulator has a 3.0 cm period [12]) because of the scaling of the magnetic fields requires that in order to have an undulator gap large enough to fit a $\sim$ cm diameter vacuum chamber pipe, there must be an approximately cm long period (see Chapter 2).

This limitation isn't fundamental, it is possible to make very small diameter vacuum chamber pipes that will allow some amount of charge to pass. The limitation comes from demands on the light that the sources produce. The two

¹The collection of all the different types of undulators and wigglers from regular planar devices to superconducting wavelength shifters is called insertion devices because they are placed in the straight sections of synchrotrons and are designed to effect the electron beam as little as possible.

most important figures of merit for photon sources are brightness and coherence (see section 1.4). We leave the discussion of coherence for later and focus on brightness. Brightness is the density of the electrons (or photons) in phase space.

The desire for bright sources is because higher densities in photon phase space generally lead to larger signal to noise ratios in the experiments using the photons. High brightness also means that only the desired photons –for example, photons of the correct energy in a narrow bandwidth or the desired polarization– can be selected, reducing the heat load on the experimental target. What this means for electron beam-based lightsources is that, because the brightness of the light depends on the brightness of the electron beam, they require as much charge as possible squeezed into the smallest possible phase space area. It is the reaction of these very intense electron beams to the surrounding environment that places the limits on the undulator gap and thus its period.

The two most stringent constraints on electron beam brightness come from wakefields and space charge effects. While space charge effects are important near the cathode when the beam energy is low [13, 14], wakefields are a fact of life for an electron beam of any energy [15]. Wakefields are the reaction of the electron beam to the presence of a boundary (usually a metal wall) which affects the distribution of electric and magnetic fields around the bunch, they usually depend strongly on the inverse of the radius of the vacuum pipe (for instance the resistive wall wakefield scales as $1/\text{radius}$ [16]). This means that smaller pipes leads to larger wakefields which can degrade the brightness. Because wakefields are a collective effect, they also depend on the total amount of charge, limiting the brightness directly.

Because of the sensitivity of the wakefields to vacuum chamber radius it is advantageous to put the undulator inside the vacuum chamber which effectively

increases the physical aperture seen by the electron beam. Moving from 'standard' to in-vacuum devices does allow an increase in peak magnetic field but that increased magnetic field comes with the drawback of poorly understood demagnetization mechanisms that are increasingly relevant as a result of the higher electron and photon beam exposure that occurs without the protection of the vacuum chamber. Hence, one of the critical issues for the adoption of in-vacuum undulators is control of the demagnetization of the magnets.

Another constraint on lightsource brightness is the intensity of radiation emitted per electron. As we will see in Chapter 3, the spectral intensity depends on the strength of the magnets and period length such that $\lambda_u B$ is constant as the period decreases. This means that the magnets that go into shorter period undulators have to be very strong, both in the field that they produce and their ability to resist demagnetization.

To increase both the peak field and the coercivity of the magnets, it has been proposed that the undulator be cryogenically cooled [17]. In most relevant materials, in fact, the rate of increase of coercivity is higher than that of remanent field, allowing the selection of materials that maximize room temperature remanence, limited only by the ability to assemble and correct the undulator [18, 19]. As neodymium-iron-boron (NdFeB) magnets have higher remanent field, they are the typical choice for cryogenic undulators over samarium-cobalt (SmCo), which has shown the highest resistance to demagnetization [20]. However, NdFeB magnets suffer from a spin axis reorientation at $\sim 135\text{K}$, which reduces the useful remanent field at lower temperature by changing the direction of the easy magnetic axis [21]. The coercivity, in contrast, continues to increase with decreasing temperature. To take advantage of the increasing coercivity, a number of groups have been pursuing the use of materials that do not have a spin axis reorienta-

tion [18, 22, 23]. These materials substitute praseodymium for neodymium either partially [24] or entirely [17].

To meet the demands of electron beam exposure, we have developed, in collaboration with Helmholtz-Zentrum Berlin and Ludwig-Maximilians University, a cryogenic undulator using a new magnet material: $(\text{Pr}_{0.8}\text{Nd}_{0.2})_2\text{Fe}_{14}\text{B}$. This material does not show spin axis reorientation down to 10K [19]. The motivation for the development of this undulator was originally as a radiator for a laser wake-field accelerator (LWFA) [25]. In order to make soft x-rays (~ 1 keV) from the high charge, sub-GeV electron beams from the LWFA, the undulator needed to have sub-cm periods. This meant moving into a relatively unexplored regime in insertion device technology: sub-centimeter periods. In addition, the relatively poor pointing stability of the LWFA source required a robust undulator that could be regularly exposed directly to the electron beam [26]. This application is an excellent example of a technological niche that needed to be filled.

In the field of short period undulators, there are a few alternatives to permanent magnet based undulators at various stages of development. These options include inverse Compton scattering sources [27] and RF undulators [28]. However, both of these technologies require significant external infrastructure and cost, either in the form of very high-power lasers or high frequency RF power from klystrons to achieve the necessary $K \geq 1$, nor have they demonstrated the MHz repetition rate capability available at modern synchrotrons using permanent magnet undulators. Additionally, superconducting technology has the potential to surpass cryogenic permanent magnet undulators in radiation quality in sub-cm period undulators, but there are significant technical and operational hurdles to overcome [11].

The focus of this dissertation is in increasing access to x-ray sources by making

them both smaller and, perhaps more importantly, cheaper. Current limitations to source size reduction are discussed (Chapters 1 and 3) which leads to the conclusion that smaller x-rays sources require short period undulators. In this context, two approaches to increasing access to x-rays are covered. The first is direct decrease in the period length of undulators through more advanced design and materials. This path begins with a discussion of the design and construction of a 9 mm period prototype (Chapter 2). An analysis of the benefits of such a device, in reduced undulator and accelerator lengths at existing free electron lasers, is explored (Chapter 4). And finally, the operation of the undulator in a realistic scenario is experimentally explored in a scaled experiment at optical frequencies (section 5.2).

The second method for decreasing the period length of the lightsource is to replace the undulator with a laser, making an inverse Compton scattering source (Chapter 3). The relationship between undulator radiation and the inverse Compton scattering process is examined, as well as the characteristics of the source itself. Lastly, as a demonstration of the function of the inverse Compton Scattering source at Brookhaven National Laboratory as a diagnostic tool rather than an experiment itself, the 9 keV x-rays from the source are Bragg reflected from a Silicon crystal as a precursor to a pump-probe experiment which uses the inverse Compton scattered x-rays as a diagnostic (section 5.1). The experiment shows that the characteristics of the produced x-ray beam can be predicted by the input parameters.

1.1 Jackson, Schwinger, Landau basics

In this dissertation we rely on classical dynamics, drawing upon sources such as texts written by Jackson [29], Schwinger [30] and Landau [31]. As a matter of

preference we use Gaussian units while deriving formulae and SI units for results. The change between the two can be found in the Appendix of [29].

We begin by stating without proof both Maxwell's equations and the Lorentz force law:

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{j}, \quad (1.2a)$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad (1.2b)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (1.2c)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (1.2d)$$

$$\frac{d\mathbf{p}}{dt} = \mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}). \quad (1.3)$$

Throughout this dissertation bold quantities denote vectors. $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields respectively. ρ is the charge density, $\mathbf{j}$ the current density, c the speed of light and ∇ is the gradient operator. Here, q is the charge of the particle of interest, usually the charge of an electron (termed $-e$): -1.602×10^{-19} C, while $\mathbf{v}$ and $\mathbf{p}$ are its velocity and momentum respectively.

Eqs. 1.2a and 1.2b along with the vector identity $\nabla \cdot (\nabla \times \mathbf{a}) = 0$ can be used to show that charge is conserved:

$$\nabla \cdot \mathbf{j} + \frac{\partial \rho}{\partial t} = 0, \quad (1.4)$$

indeed this is ensured by Maxwell's famous correction of Ampère's law.

The more important proof of energy conservation is a little more involved. It begins with the inner product of Eq. 1.3 with a particle's velocity to find the rate

of change the particle's energy $\frac{d\mathcal{E}}{dt} = \mathbf{v} \cdot \mathbf{F} = q\mathbf{v} \cdot \mathbf{E}$. If we are interested in the energy change of a great many particles, the right hand side of the conservation of energy includes a sum over the particles and the sum can be taken to an integral when the particles are described by a distribution, in which case the energy change can be written as an integral $\frac{d\mathcal{E}}{dt} = \int_{\mathcal{V}} \mathbf{j} \cdot \mathbf{E} d^3\mathbf{x}$ where $\mathbf{j}$ can now represent an arbitrary current contained within some volume $\mathcal{V}$. When the current is expanded using Eq. 1.2a and several identities of vector calculus the result is an equation that is interpreted as the conservation of energy

$$\int_{\mathcal{V}} \mathbf{j} \cdot \mathbf{E} d^3\mathbf{x} + \frac{d}{dt} \frac{c}{8\pi} \int_{\mathcal{V}} (B^2 + E^2) d^3\mathbf{x} + \frac{c}{4\pi} \int_{\mathcal{A}} (\mathbf{E} \times \mathbf{B}^*) \cdot d\mathbf{a} = 0, \quad (1.5)$$

where the first term is the rate of change of the energy stored by the particles, the second is the rate of change of the energy of the fields and the third term, most important later when looking at the detection of radiation from accelerating charged particles, is the rate of energy flow out of the volume $\mathcal{V}$ with surface area $\mathcal{A}$. This last term defines the Poynting vector $\mathbf{S} = \frac{c}{4\pi} \mathbf{E} \times \mathbf{B}^*$ which has units of power per unit area ($\mathbf{A}^*$ is the complex conjugate of $\mathbf{A}$). Only those parts of the Poynting vector normal to the surface $\mathcal{A}$ escape or enter the volume.

The interaction of the terms can be illustrated using the examples of spontaneous undulator radiation and the free electron laser. If we take for the volume $\mathcal{V}$ a cylinder of the radius of the beam pipe, the surface area $\mathcal{A}$ is made up of three parts: the cylindrical beam pipe wall and the upstream and downstream end caps. For a spontaneous source the particle energy and the field energy density terms are usually treated as much smaller than the effects caused by the Poynting term. These terms are usually accounted for with the radiation integrals [16]. The Poynting term is accounted for by energy deposition in the walls of the vacuum beam pipe (element impedance) and the energy that leaves the

system through the downstream end cap which is the detected radiation. Only when the free electron laser is studied is the conservation of energy strictly enforced with the first and second terms exchanging energy while the third leeches energy away from the beam and fields via wakefields, element impedances and diffraction effects.

Because this dissertation will concentrate on the Poynting term it is useful to determine what $\mathbf{E}$ and $\mathbf{B}$ look like at some detector somewhere relative to the charged particles creating them. To start off, Maxwell's equations are converted into a set of equations for the scalar (Φ) and vector ($\mathbf{A}$) potentials. By virtue of Eq. 1.2d we can assume a vector potential for the magnetic field

$$\mathbf{B} = \nabla \times \mathbf{A}. \quad (1.6)$$

Substituting this into 1.2c we find that the electric field is given by

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}. \quad (1.7)$$

Because the potentials are not the physical observables, the fields are, we are free to add the gradient of any scalar function to the vector potential so long as we subtract the time derivative of this scalar function from the scalar potential. This choice of scalar potential is called Gauge freedom and the typical choice for radiation is the Lorentz Gauge: $\nabla \cdot \mathbf{A} = -\frac{1}{c} \frac{\partial \Phi}{\partial t}$. In this case, Eqs. 1.2a and 1.2b become respectively,

$$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{4\pi}{c} \mathbf{j} \quad \text{and} \quad (1.8a)$$

$$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -4\pi\rho. \quad (1.8b)$$

For a single electron we take $\rho = -e \delta^3(\mathbf{x} - \mathbf{r}(t))\delta(t - t_e)$ and $\mathbf{j} = -e \mathbf{v} \delta^3(\mathbf{x} - \mathbf{r}(t))\delta(t - t_e)$, where $\mathbf{v} = \frac{d\mathbf{r}(t)}{dt}$, $\mathbf{r}(t)$ is the position of the electron at time t and t_e is the time of photon emission. The solution to these equations is [29, 30, 31]:

$$\mathbf{A}(\mathbf{x}, t) = \frac{-e\boldsymbol{\beta}(\tau)}{(1 - \boldsymbol{\beta}(\tau) \cdot \hat{\mathbf{n}})R(\tau)} \quad \text{and} \quad (1.9a)$$

$$\Phi(\mathbf{x}, t) = \frac{-e}{(1 - \boldsymbol{\beta}(\tau) \cdot \hat{\mathbf{n}})R(\tau)}, \quad (1.9b)$$

where $\hat{\mathbf{n}}$ is the unit vector from the electron to the detector at time τ , $R(\tau) = |\mathbf{x} - \mathbf{r}(\tau)|$ is the distance between the electron and detector and τ is called the retarded time. Because of the finite speed of light, if we are detecting electromagnetic waves we expect that the electron that emitted the photons we detect at time t is not in the same place as it was at the time it emitted the photons. The relationship between the two times is given by the time it takes the photons to travel from the electron to the detector: $t = \tau + \frac{R(\tau)}{c}$. Eqs. 1.9 are called the Liénard-Wiechart potentials.

At this point all of the physics (which is by no means trivial!) contained in Eqs. 1.9 is that potentials are proportional to the charge of the emitting particle, that the finite speed of light causes the potentials to be much larger when emitted in the direction of motion of the electron (this is the $1 - \boldsymbol{\beta}(\tau) \cdot \hat{\mathbf{n}}$ in the denominator) and that the field is proportional to $1/r$. The last part is important because we know that the fields of a static electron decay with distance as $\sim 1/r^2$, which means that we will be able to detect radiation at distances at which it would be impossible to detect static charge. There is more to come on this subject in Chapter 3.

1.2 Particle Motion in Accelerator Fields

The linearity of Maxwell's equations allows us to separate the fields that we can directly control, such as RF structures and quadrupoles, from those we cannot directly control, such as space charge and wakefields. As such, in designing the lattice of a particle accelerator we can use the sourceless Maxwell equations and put off the more difficult problems for later. The result is single particle optics, so called because of the similarities with classical ray optics. At this point we will ignore the time dependence of the applied fields and limit the discussion to static beamline elements. Under these conditions both the electric and magnetic field are divergent-less and irrotational and either can be written as the Laplace equation familiar from electrostatics by defining a potential $(\mathbf{E}, \mathbf{B}) = -\nabla\Phi$ [32, 33].

$$\nabla^2\Phi = 0. \tag{1.10}$$

When the potential is sufficiently two dimensional, as in the case of multipole magnets used in accelerator physics, Laplace's equation is easily solved [29]. The solutions must not be divergent on axis so the relevant remaining terms in the solution are

$$\Phi(r, \phi) = \sum_{n=1}^{\infty} r^n (A_n e^{in\phi} + B_n e^{-in\phi}). \tag{1.11}$$

This is the solution that produces all the standard accelerator magnets, dipoles, quadrupoles, sextupoles and so on, save for the solenoid [34], because the solenoid requires the fields in the longitudinal direction to satisfy Eq. 1.2d. Magnets are used over electrostatic devices because the limits of technology cause the Lorentz force to favor magnets for steering highly relativistic charged particle

beams. Throughout the course of this dissertation we will be encountering dipole magnets, quadrupole magnets and insertion devices (which act like a mixture of the previous two), it is useful to define what these mean and in the context of the forces of these magnetic fields on a charged particle beam define some commonly used beam properties used in beam physics. For simplicity of the definition of these magnets we define a right handed coordinate system in which the beam moves primarily along the s -axis (called the longitudinal direction), the y -axis is directed upward and the x -axis is the horizontal axis (collectively called the transverse directions), which is normal to both the previous vectors. The other assumption that is made is that the particle is moving primarily in the longitudinal direction and that the ratio of transverse momenta to longitudinal momenta is small - called the paraxial approximation.

To introduce the transfer matrix formalism we start with a field free region of length Δz , called a drift. In a drift the particle sees no fields and therefore no forces, e.g. $\frac{dp_x}{dt} = \gamma m(x''\dot{s}^2 + x'\ddot{s}) = 0$ where primes denote derivatives with respect to s and dots derivatives with respect to time. We assume that the longitudinal acceleration of the particle is negligible such that $\ddot{s} \simeq 0$ and therefore the particle's final coordinates can be related to its initial coordinates by

$$\begin{pmatrix} x \\ x' \end{pmatrix} = M(\Delta z) \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} = \begin{pmatrix} 1 & \Delta z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}. \quad (1.12)$$

$M(\Delta z)$ is called the transfer matrix for a drift.

1.2.1 Dipoles and Chicanes

The simplest element is the dipole magnet, which is described by $n=1$ in Eq. [1.11](#). Proper choice of the constants A_1 and B_1 ensures that $\Phi = B_0 y$ and therefore

the magnetic field is constant everywhere within the magnet and pointing in the y -direction. Examination of Eq. 1.3 shows that such a field will deflect a particle in the horizontal direction. Dipole magnets are used both to steer the beam and to compress it longitudinally. The other independent solution with $n=1$ is a constant field in the x -direction, called a vertical dipole.

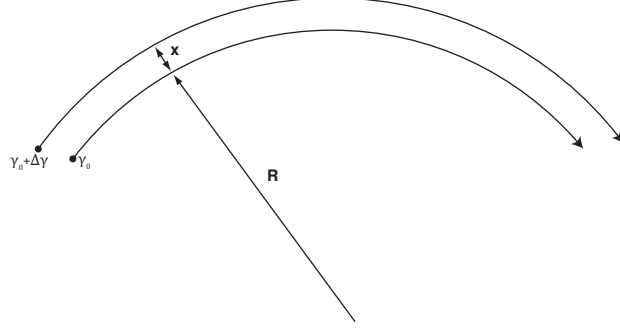


Figure 1.1: Trajectory of a particle with horizontal offset x and energy difference $\Delta\gamma$ relative to the design particle in a dipole.

To solve for the motion of a particle in the dipole we examine the motion of two particles. The first one is the reference particle, the one that the dipole was designed to deflect to some other horizontal coordinate. It has position $\mathbf{r}_0 = s\hat{\mathbf{s}}$ where the direction $\hat{\mathbf{s}}$ is defined by the uniform circular motion such that $\dot{\mathbf{r}}_0 = \dot{s}\hat{\mathbf{s}}$. Given this set of coordinates we have $1/R = -qB_0/(\gamma_0 m \dot{s})$ for the reference particle. The second particle is the particle of interest and we define its position, offset in the horizontal plane, using the reference particle, $\mathbf{r} = \mathbf{r}_0 + x\hat{\mathbf{x}}$ (see Fig. 1.1). We will ignore the vertical direction because in this simple example it acts like a drift. Further, the particle has a slightly different energy than the reference particle, such that $\gamma = \gamma_0(1 + \Delta\gamma/\gamma_0)$ and $\Delta\gamma/\gamma_0 \ll 1$. The equation of motion for the particle of interest in the horizontal direction is then $x'' + x/R^2 = (\Delta\gamma/\gamma_0)/R$. The term to the right of the equal sign causes particles of different energies to take different paths through a dipole magnet and is called dispersion. For the

moment we ignore it to readily solve the homogeneous equation:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos(s/R) & R \sin(s/R) \\ -\frac{1}{R} \sin(s/R) & \cos(s/R) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}. \quad (1.13)$$

Dividing the equation of motion by $\Delta\gamma/\gamma_0$ gives the equation for dispersion, D , which is the particle's horizontal offset due to the difference in momentum from the reference particle. The total offset of the particle is given by $x(s) + D(s)\Delta\gamma/\gamma_0$ with $D(s) = D_0 \cos(s/R) + D'_0 \sin(s/R) + R(1 - \cos(s/R))$.

Along with dispersion, another effect of the different paths taken by particles of different energies is the relative longitudinal motion of the particles. In accelerator physics parlance this relative motion is called R_{56} (or momentum compaction) because of the position of the matrix element when the transfer matrix method is expanded to include all three dimensions. When the particle energy is defined relative to a reference particle, $\gamma = \gamma_0(1 + \Delta\gamma/\gamma_0) = \gamma_0(1 + \delta)$, we have $R_{56} = \frac{ds}{d\delta}|_{\delta=0}$. A common beam line element used to compress an electron beam is called the chicane, it is a series of 4 dipoles separated by drifts. The simplest version is the symmetric chicane, in which the drifts are much longer than the dipole magnets, which do not have large bend angles. In this case $R_{56} \approx -2L\theta_0^2$, where θ_0 is the bend angle of the reference particle in one of the dipoles and L is the drift length between the dipoles (see Fig. 1.2). The negative sign means that this chicane will compress a beam with negative chirp (lower energy particles come at the front of the bunch and higher energy at the back). When the drift between the magnets is much shorter than the magnets the momentum compaction is $R_{56} \approx -\frac{4}{3}a\theta_0^2$, where a is the length of one of the dipoles.

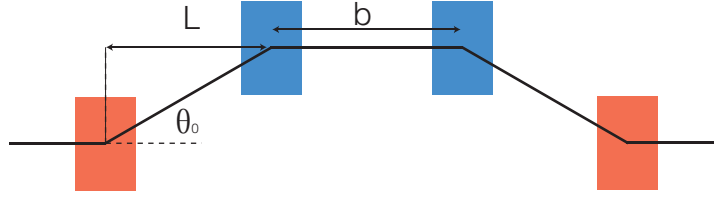


Figure 1.2: Schematic of a typical four dipole bunch compressor chicane for which $R_{56} \approx -2L\theta_0^2$.

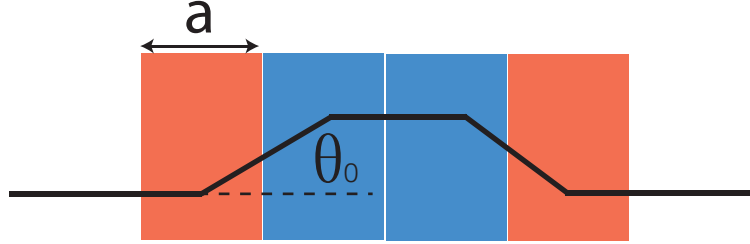


Figure 1.3: Schematic of a Neptune style four dipole short chicane for which $R_{56} \approx -\frac{4}{3}a\theta_0^2$.

1.2.2 Quadrupoles

When we take the $n=2$ term from Eq. 1.11 we find a field that depends on the coordinates linearly. In this case, the periodicity of the fields with $e^{i2\phi}$ means that the field has 4 zeros within the 2π span of the azimuthal coordinate. Within the previously defined coordinate system the zeros can occur along the x - and y -axis, a case called a skew quadrupole, or along the lines $y=x$ and $y=-x$, a scenario called a regular quadrupole. The former is called skew because the orientation of the fields will cause offsets in one coordinate to focus in the other transverse coordinate, seldom desirable. In the latter case, the fields are $\mathbf{B} = 2(B_1 - A_1)(y\hat{\mathbf{x}} + x\hat{\mathbf{y}})$, where typically $2(B_1 - A_1) \equiv B'$, where B' is the gradient of the quadrupole. Assuming a small deflection of the particle the resulting equations of motion are $x'' = -k^2x$ and $y'' = k^2y$, with $k^2 = qvB'/pc$ as the

focusing strength of the quadrupole. It can be seen that a quadrupole is focusing in one direction and defocusing in the other.

The transfer matrix for a focusing quadrupole is given by

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos(ks) & \frac{1}{k} \sin(ks) \\ -k \sin(ks) & \cos(ks) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}. \quad (1.14)$$

Where it has been assumed that the longitudinal acceleration is negligible. In the defocusing direction

$$\begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} \cosh(ks) & \frac{1}{k} \sinh(ks) \\ k \sinh(ks) & \cosh(ks) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}. \quad (1.15)$$

1.2.3 Linear Beam Optics

It is not a coincidence that all of the transfer matrices defined in Eqs. 1.12, 1.13, 1.14 and 1.15 have unit determinant. Because the equations of motion are symplectic the density of particles in phase space is constant through transport within all of these beam line elements [33, 35]. A proof of this will also introduce the beta function, a ubiquitous part of accelerator physics, so it is worth performing. Transverse particle motion which involves drifts, dipoles (ignoring dispersion) and quadrupoles only make up what is known as linear beam optics, which is governed by a second order ordinary differential equation, $x''(s) + K(s)x(s) = 0$. In this equation $K(s)$ is piecewise constant. This differential equation is known as Hill's equation. If we want to find the solution to this equation for a group of particles rather than a single particle it is convenient to define the second moment of the distribution in the usual fashion:

$$\sigma(s)^2 = \langle x^2 \rangle = \int \int x^2 f(x, x'; s) dx dx'. \quad (1.16)$$

Here, $f(x, x'; s)$ is the distribution of particles in x and x' at position s along the beamline. Taking two derivatives of this equation yields the equation that is used to both define emittance and the beta function:

$$\frac{d^2 \sigma(s)}{ds^2} + K(s) \sigma(s) = \frac{\varepsilon^2}{\sigma(s)^3}. \quad (1.17)$$

The numerator of the right side of the above equation defines square of the geometric emittance, $\varepsilon^2 = \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2$. Taking the derivative of this quantity with respect to s shows that the emittance is a constant of the beam evolution along s . The emittance is the configuration space density of the particles and its consistency is a result of Liouville's theorem. In fact, this method of solving Hill's equation is called Milne's method (and Eq. 1.17 is sometimes called Milne's equation) [36]. A special property of Milne's equation is that a single solution of it can generate the general solution of Hill's equation, which means that by knowing the solution to Eq. 1.17 we know what every particle is doing along the beam line. Incidentally, if the beamline is periodic, such as in a synchrotron ring, the solution to Eq. 1.17 is determined uniquely by the periodicity and the beamline itself, while for a linear beamline, initial conditions are required to uniquely define the solutions. The transformation $\beta(s) \equiv \sigma(s)^2 / \varepsilon$ defines the beta function [37]. With this definition, the definition of $x(s)$ becomes

$$x(s) = \sqrt{\varepsilon \beta(s)} \sin(\mu(s) + \phi), \quad (1.18)$$

where the constants of integration are found by requiring that Eq. 1.16 remain true and

$$\mu(s) = \int_0^s \frac{ds'}{\beta(s')}. \quad (1.19)$$

The last equation defines the betatron phase advance of the beamline. It should be clear from the previous derivation that the beta function is the envelope of a group of particles as it travels along the beamline, defined in this case to be the envelope that contains one standard deviation of the particles.

Taking the derivative of Eq. 1.18 and substituting in the same equation to make use of $x(s)$ we find

$$x'(s) = \frac{x(s)}{\beta(s)} \frac{1}{2} \frac{d\beta(s)}{ds} + \sqrt{\frac{\varepsilon}{\beta(s)}} \cos(\mu(s) + \phi). \quad (1.20)$$

When these two equations are combined using the trigonometric identity $\sin^2 x + \cos^2 x = 1$, the result is

$$\gamma(s)x(s)^2 + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2 = \varepsilon. \quad (1.21)$$

The above equation makes use of a few definitions:

$$\varepsilon^2 = \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2, \quad (1.22a)$$

$$\beta(s) \equiv \frac{\sigma^2}{\varepsilon} = \frac{\langle x^2 \rangle}{\varepsilon}, \quad (1.22b)$$

$$\alpha(s) \equiv -\frac{1}{2} \frac{d\beta}{ds} = -\frac{\langle xx' \rangle}{\varepsilon}, \quad (1.22c)$$

$$\gamma(s) \equiv \frac{1 + \alpha(s)^2}{\beta(s)} = \frac{\langle x'^2 \rangle}{\varepsilon}. \quad (1.22d)$$

The second equivalence in Eq. 1.22c comes from the derivative of Eq. 1.16, while the definition of $\gamma(s)$ comes from solving the other three equations.² These

²It is unfortunate that physicists aren't more creative with their symbol choices as the

functions are known as Twiss parameters or Courant-Snyder parameters [37]. To simplify notation we are now going to drop the s -dependence in the Twiss parameters.

Eq. 1.21 describes an ellipse of area $\pi\varepsilon$. When $\alpha = 0$, not only do the axes of the ellipse coincide with the x and x' axes, but the axes of each particle do as well (cf Eqs. 1.18 and 1.20). As a consequence of the conservation of the density in configuration space the particle trajectories in that space cannot cross each other and so all particle trajectories in that space must be scalar multiples of each other. In other words, the trajectories in configuration space are all ellipses of the same ratio of major to minor radius and rotation angle of the ellipse, but with different magnifications. The power of this solution method is that once a single particle's trajectory is known, all the others are also known. It may seem odd that Eq. 1.22d is written in terms of other variables, but there are only three parameters required to define an ellipse which is centered on the origin. Beam physicists choose the area within the ellipse (ε), the projection of the ellipse onto the x -axis (β) and the rotation angle of the ellipse (α) and so the projection of the ellipse onto the x' -axis (γ) comes along for the ride. It is, however, mathematically simpler to use all four parameters to describe the beam because it cleans up the notation, as we are about to find out.

Now that we know how the particles must move given a particular beamline we can figure out how a beam of particles evolves in a way similar to the transfer matrix for x and x' . A particle's coordinates at any point along the beamline can be related to coordinates earlier in the beamline via a transfer matrix: $x(s) =$

symbols presented here can be mighty confusing. β is used as the normalized particle velocity and the normalized transverse envelope size, while α is the rate of change of β , the fine structure constant and also the momentum compaction, and γ is the normalized particle energy and the final Twiss parameter. There isn't much to be done about these conventions except hope that the context will make the different uses clear.

$m_{11}x_0 + m_{12}x'_0$ and $x'(s) = m_{21}x_0 + m_{22}x'_0$. These identities can be plugged into Eq. 1.16 and solved using the definitions in Eq. 1.22 to find the final beta function given the initial Twiss parameters:

$$\beta(s) = m_{11}^2\beta_0 - 2m_{11}m_{12}\alpha_0 + m_{12}^2\gamma_0. \quad (1.23)$$

This relation requires that the density in configuration space is conserved such that $f(x, x'; s)dx dx' = f_0(x_0, x'_0; 0)dx_0 dx'_0$. Similarly, the relations for the other Twiss parameters can be found and the result can be put into matrix form:

$$\begin{aligned} \begin{pmatrix} \beta(s) \\ \alpha(s) \\ \gamma(s) \end{pmatrix} &= \mathcal{M}(s) \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix} \\ &= \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{11}m_{21} & (m_{11}m_{22} + m_{12}m_{21}) & -m_{12}m_{22} \\ m_{21}^2 & -2m_{21}m_{22} & m_{22}^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}. \end{aligned} \quad (1.24)$$

This equation is very useful for simple transport lines, e.g. a single focusing element, as will be seen in the next section.

1.2.4 The Constant Focusing Channel

We will see later that when the transverse forces in an undulator are period averaged the undulator, like a quadrupole, represents a beam line element with constant focusing strength so we look closely at the dynamics of such an element here. The transfer matrix for a focusing element is given by Eq. 1.14. Plugging these elements in the Twiss parameter transfer matrix (Eq. 1.24) returns:

$$\mathcal{M}(s) = \begin{pmatrix} \cos^2(ks) & -\sin(2ks)/k & \sin^2(ks)/k^2 \\ k \sin(2ks)/2 & \cos(2ks) & -\sin(2ks)/(2k) \\ k^2 \sin^2(ks) & k \sin(2ks) & \cos^2(ks) \end{pmatrix}. \quad (1.25)$$

where k is the focusing strength. $\beta(s)$ oscillates with a period of $2k$ starting from some set of initial conditions $\beta(0)$ and $\alpha(0)$. The beam will be focused from the maximum value, $\beta_{\max}$, to the minimum value, $\beta_{\min}$, when the argument of the sinusoids advances such that $2ks = \pi$. Because quadrupoles are fixed length objects with adjustable gradients, this equality (along with initial conditions) tells the operator what the maximum useful gradient is, cranking up the gradient any more will cause the beam to get larger at the exit, not smaller.

In the context of a quadrupole one might want to know the minimum β -function that can be produced with a quadrupole of a given gradient. The general solution to the maximum and minimum β -function can be found by realizing that at an extrema $\alpha(s_e) = 0$, and therefore the 1st and 3rd rows of Eq. 1.25 are compliments of each other and, when they are multiplied by factors appropriate to make them unitless, they can be added to find $k^2\beta_0\beta_e + \frac{\beta_0}{\beta_e} = k^2\beta_0^2 + 1 + \alpha_0^2$. β_e is the extrema value of the β -function. The solution to this quadratic equation is fairly complicated in general but an important subclass which demonstrates many of the properties of the lens is of a collimated beam ($\alpha_0 = 0$) entering the lens. In this case $\beta_{\max} = \beta_0$ and we can quickly find that $\beta_{\min} = 1/(k^2\beta_{\max})$. The second equation is always true because of the assumed linearity of the transport equations which means that if the focusing is strong and/or the maximum β -function is large there can be very large betatron oscillations.

We will see later that large betatron oscillations cause an increase in the gain length of the free electron lasing process by shifting particles around the

pondermotive bucket and it is, therefore, desirable to minimize this motion. To do that in a focusing channel the entering beam is matched to the betatron envelope in the channel so that the phase space is round and does not rotate. The condition for matching is found by setting $\beta_{\max} = \beta_{\min}$ with $\alpha(s) = 0$ for all s (this is the condition that the phase space ellipse not rotate). We already know how the betas are related when $\alpha_0 = 0$ so it is easily found that

$$\beta_{\text{matched}} = 1/k. \quad (1.26)$$

This is the value of input β -function that will cause the least amount of transverse envelope motion - that is, none.

What happens if the channel isn't focusing but defocusing? The problem can be done very similarly to that of the focusing one so we will concentrate on the physical reasoning. The most important point to realize is that we want to have a minima in the betatron motion somewhere downstream of the entrance of the channel, otherwise the beam size can grow a great deal through the channel; this means that the beam entering the defocusing channel should have a positive α_0 . Because the beam envelope cannot oscillate in the defocusing case the optimization goals are different than the focusing case. Instead of matching the beam to prevent phase space rotation we have to settle for minimizing the relative growth of $\beta(s)$. The option that minimizes $\frac{\beta_{\max}}{\beta_{\min}}$ is to place $\beta_{\min}$ at the longitudinal center of the channel. Regardless of where the minima is placed the matching condition requires

$$\alpha_0 = \frac{k\beta_0}{2CS} \left(C^2 + S^2 \pm \sqrt{(C^2 + S^2)^2 - 4 \frac{C^2 S^2}{k\beta_0} \left(k\beta_0 + \frac{1}{k\beta_0} \right)} \right). \quad (1.27)$$

In this equation $C = \cosh(kL)$ and $S = \sinh(kL)$ where L is the distance from

the front of the defocusing channel to the position of the minimum in the betatron motion. In order for α_0 to be real it must be true that $k\beta_0 \geq \sinh(2kL)$; in fact the minimum β_0 , and thus minimum $\beta_{\min}$, occurs for the equality.

1.3 Dynamics in Insertion Devices

Insertion devices, otherwise known as undulators, wigglers, wundulators, and likely a great many other names, are periodic structures of magnets, either permanent or electromagnetic, which sometimes include iron. As we will see later the insertion device modulates the electron's longitudinal velocity such that it can interact coherently with electromagnetic radiation - a common theme in radiation production in lightsources. We will start with a greatly simplified model to illustrate the essential physical features of the particle's motion and then add increasing complexity to end with a three dimensional description of undulator dynamics.

Undulator radiation seems to have been first described by Ginzburg [38] and was later elucidated and demonstrated by Motz [39, 40]. Since this description insertion devices have become ever more complex in search of more efficient or better tailored radiation production [10, 41]. For the most part, however, undulators fall into two categories: planar where the principle magnetic field is linearly polarized and helical where the the principle magnetic field is made up of two linear fields 90° out of phase with each other. Because the undulator used in the work is planar we will derive the dynamics in the undulator assuming a planar configuration, noting differences with the helical type when necessary.

To illustrate the magnitude of the motion we start with two dimensional motion in the horizontal plane for a single electron assumed to be traveling in

a field $\mathbf{B} = B_0 \cos(k_u z) \hat{\mathbf{y}}$, with $k_u = 2\pi/\lambda_u$ defining the longitudinal magnetic periodicity. The coordinate system used here is such that the beam travels in the z -direction with the x -direction being the horizontal plane of deflection and the y -direction perpendicular to both. The xz -plane defines the midplane of a planar undulator, the plane half way between the top and bottom jaws of the undulator. Inserting the assumed field into the Lorentz force equation (Eq. 1.3) and assuming that the z -velocity is constant while integrating the x -direction equation of motion results in following equations of motion:

$$\beta_x = \frac{K}{\gamma} \sin(k_u z), \quad (1.28a)$$

$$\beta_z = \sqrt{\beta^2 - \beta_x^2} = \beta \sqrt{1 - \frac{K^2}{(\beta\gamma)^2} \sin^2(k_u z)}, \quad (1.28b)$$

$$K = \frac{eB_0}{k_u mc}. \quad (1.28c)$$

It can be seen that if K , the undulator parameter, is much less than $\beta\gamma$ the approximation of the longitudinal velocity as constant is acceptable. Typically K is less than 10 (less than 3 in fact), while γ can be very large for ultra relativistic beams. In the context of this dissertation $K \sim 1$ while $\gamma \sim 100$ so that the second term under the radical is quite small. Only when the difference of the particle velocity from the speed of light is important do we need to worry about the consequences of approximating the speed of the particle as constant. Using the binomial expansion on the radical and keeping only the linear terms shows the small longitudinal velocity modulation

$$\beta_z \simeq \bar{\beta}_z + \frac{K^2}{4\beta\gamma^2} \cos(2k_u z), \quad (1.29)$$

where $\bar{\beta}_z = 1 - 1/(2\gamma^2) - K^2/(4\beta\gamma^2)$ is the average longitudinal velocity. It

can be noticed immediately that the longitudinal velocity modulation occurs at twice the frequency of the horizontal oscillation. When the particle motion is observed in the frame of average velocity this produces the "figure-of-eight" motion characteristic of undulator dynamics [42, 43]. See Fig. 1.4. The scale of the transverse motion is indeed small for a relativistic particle. For a 9 mm period length the maximum horizontal offset is $K/(\gamma k_u) = 14 \mu\text{m}$, while the maximum deflection angle is $K/\gamma = 10 \text{ mrad}$.

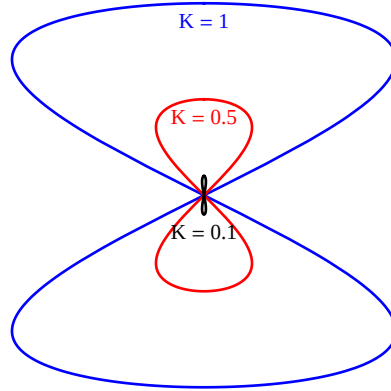


Figure 1.4: Schematic of the figure-8 motion caused by the longitudinal velocity modulation at twice the undulator frequency. All of the curves are to scale.

A complete solution of the particle motion requires we find the fields in an undulator consistent with Maxwell's equations. The region in the center of the undulator is free from both charge and current (we have set the problems of space charge and wakefields aside using the linearity of the equations) and we can therefore use equations 1.2a and 1.2b without sources. Because the known conditions are on the fields, solving for them directly is most straight forward using separation of variables. That $\mathbf{B}$ is irrotational gives the solution

$$\begin{aligned}
\frac{\mathbf{B}}{B_0} = & \frac{k_x}{k_y} \sinh(k_x x) \sinh(k_y y) \cos(k_u z) \hat{\mathbf{x}} \\
& + \cosh(k_x x) \cosh(k_y y) \cos(k_u z) \hat{\mathbf{y}} \\
& - \frac{k_u}{k_y} \cosh(k_x x) \sinh(k_y y) \sin(k_u z) \hat{\mathbf{z}},
\end{aligned} \tag{1.30}$$

while the divergentless condition requires

$$k_x^2 + k_y^2 = k_u^2, \tag{1.31}$$

where it should be remembered that k_x and k_y can be imaginary. B_0 is the peak on-axis field. Indeed, in equation 1.30 the vertical ($\hat{\mathbf{y}}$) field grows as you leave the z -axis in the horizontal plane, so k_x must be imaginary in this case. It is useful to see the fields in vector potential form to find that there is no A_z ,

$$A_x = \frac{B_0}{k_u} \cosh(k_x x) \cosh(k_y y) \sin(k_u z), \tag{1.32a}$$

$$A_y = - \frac{B_0 k_x}{k_u k_y} \sinh(k_x x) \sinh(k_y y) \sin(k_u z). \tag{1.32b}$$

Using the Hamiltonian formalism we will find the three dimensional equations of motion in the limit that the particles are relativistic and the motion in the z -direction dominates. Because the period averaged motion within the undulator will turn out to be a simple focusing or defocusing channel, the previously defined parameters, in section 1.2.4, can be used to determine how the beam reacts to the undulator.

To solve the problem at hand, we begin by writing down the relativistic Legrangian to find the conjugate momenta (always capital P) $P_i \equiv \partial \mathcal{L} / \partial \dot{x}_i$ [44]. Then the Hamiltonian can be found either by performing the Legendre transformation or through recognizing the relationship between H and the total energy,

which is conserved in the case of only magnetic fields [43, 44]. Using cartesian coordinates the conjugate momenta are found to be $\mathbf{P} = \mathbf{p} + e\mathbf{A}/c$ where the lower case $\mathbf{p}$ is the mechanical momentum and the Hamiltonian is

$$H(\mathbf{x}(t), \mathbf{P}(t); t) = \sqrt{c^2(\mathbf{P} - e\mathbf{A}/c)^2 + m_0^2 c^4}, \quad (1.33)$$

the functional dependencies have been omitted on the right side for clarity. In the case of a Hamiltonian that does not explicitly depend on time and for which there is no scalar potential, H can be equated with the energy of the particle $H = \gamma mc^2$ [44].

The usual machinery of the Hamiltonian formulation uses time as the independent variable, the seconds tick by and the coordinates and their conjugate momenta evolve according to Hamilton's equations. In some cases it is useful to use the longitudinal coordinate z as the independent variable. In this case the new Hamiltonian is $\mathcal{H} = -P_z$ and the three sets of conjugate coordinates are (x_i, P_i) and $(t, -H)$ [45]. In this case, because $A_z = 0$ the conjugate and mechanical momenta are equal. Equation 1.33 can be directly solved for P_z :

$$\mathcal{H}(\cdots; z) = -\sqrt{(\gamma^2 - 1)m^2 c^2 - (P_x - eA_x/c)^2 - (P_y - eA_y/c)^2}. \quad (1.34)$$

Using Hamilton's equations [44]:

$$-\frac{dH}{dz} = 0, \quad (1.35a)$$

$$\frac{dt}{dz} = \frac{\gamma m}{p_z}, \quad (1.35b)$$

$$\frac{dx}{dz} = \frac{p_x}{p_z}, \quad (1.35c)$$

$$\frac{dy}{dz} = \frac{p_y}{p_z}, \quad (1.35d)$$

$$\frac{dP_x}{dz} = \frac{e}{c} \left\{ \frac{p_x}{p_z} \frac{\partial A_x}{\partial x} + \frac{p_y}{p_z} \frac{\partial A_y}{\partial x} \right\}, \quad (1.35e)$$

$$\frac{dP_y}{dz} = \frac{e}{c} \left\{ \frac{p_x}{p_z} \frac{\partial A_x}{\partial y} + \frac{p_y}{p_z} \frac{\partial A_y}{\partial y} \right\}. \quad (1.35f)$$

The first equation is the conservation of energy in a pure magnetic field, and it can be used to set $H = \gamma mc^2$. The third and fourth equations can be combined with the second to show that $\frac{dz}{dt} = \left(\frac{dt}{dz}\right)^{-1}$. The last two equations are the Lorentz force law for either transverse direction.

To integrate equations 1.35e and 1.35f analytically, a set of approximations must be made. The first is the paraxial approximation: $p_x, p_y \ll p_z \approx \gamma mc$. This means that the particles' angles with respect the z-axis (which we call forward) are never large; the beam mostly travels forward along the z-axis. The next approximation is that $k_x x, k_y y \ll 1$. This means that the particles stay near (relative to the transverse rate of change of the fields) the z-axis and do not see much of the transverse field curvature evident in equation 1.30.

Careful expansion of the equations in the small quantities leads to a pair of complicated coupled equations. Solution will reveal that the two equations are uncoupled when the beam enters the undulator on axis and with the proper angle (these are the second and first integrals of the undulator, respectively, and will be encountered later). The task now is to remove the fast oscillations that are known to occur in the undulator (cf equation 1.28a) to look at the slower evolution of

the particle motion. To lowest order the equations are identical to equations 1.28 with $y_f = 0$ and $x_f = \frac{K}{\gamma k_u} \cos(k_u z)$, where the f stands for fast motion which is the oscillation of the electrons with a frequency $k_u c$. The next nonzero order is second in all the small quantities (here the s stands for slow, meaning motion that takes place over many periods):

$$\begin{aligned} \frac{\gamma m c^2}{e B_0} \frac{d^2 x_s}{dz^2} = & - \left[\left\{ \frac{dx_f}{dz} + \frac{dx_s}{dz} \right\} \cos(k_u z) \right] \left\{ \frac{dx_f}{dz} + \frac{dx_s}{dz} \right\} \\ & - \left(\frac{k_x^2 (x_f + x_s)^2}{2} + \frac{k_y^2 y_s^2}{2} \right) \cos(k_u z) - k_u y_s \left\{ \frac{dy_s}{dz} \right\} \sin(k_u z), \end{aligned} \quad (1.36)$$

$$\begin{aligned} \frac{\gamma m c^2}{e B_0} \frac{d^2 y_s}{dz^2} = & \left\{ \frac{dx_f}{dz} \frac{dy_s}{dz} + \frac{dx_s}{dz} \frac{dy_s}{dz} \right\} \cos(k_u z) \\ & + k_x^2 (x_f y_s + x_s y_s) \cos(k_u z) - k_u y_s \left\{ \frac{dx_f}{dz} + \frac{dx_s}{dz} \right\} \sin(k_u z), \end{aligned} \quad (1.37)$$

when these equations are period averaged ($\langle f \rangle = \frac{1}{\lambda_u} \int_{-\lambda_u/2}^{\lambda_u/2} f(x) dx$) a great many of the terms drop out and equations 1.36 and 1.37 become

$$\frac{d^2 x_s}{dz^2} = \frac{K^2 k_x^2}{2 \gamma^2} x_s, \quad (1.38a)$$

$$\frac{d^2 y_s}{dz^2} = - \frac{K^2 k_y^2}{2 \gamma^2} y_s. \quad (1.38b)$$

As promised, the motion in the undulator is focusing vertically and defocusing horizontally. The above equations assume that the magnets and poles of the undulator are flat, which will result in a peak field that decreases as a function of x from the central axis of the undulator. When the poles are wide compared to the gap this decrease is small in the region of interest and the limit of $k_x \rightarrow 0$ can be taken.

There are now some higher order terms to add to equation 1.28b, $x' = x'_f + x'_s$ and $y' = y'_s$. We will see later that the free electron laser works by causing longitudinal microbunching in the electron density distribution. Because of this, if the electrons move too much within the pondermotive bucket (that is, they shift forward and backward for reasons not associated with the FEL process) then the gain length will be increased as the transverse motion competes with the microbunching process. To minimize the gain length the motion in both transverse directions should be coupled such that the total transverse velocity is a constant over the length of the undulator. If the poles are suitably shaped the undulator can be focusing in both directions ($k_x = k_y = k_u/\sqrt{2}$) and in this case the period averaged transverse velocity will be a constant [46, 47]. When the beam is properly matched, the transverse orbits become circles instead of ellipses with changing major and minor axes. Incidentally, helical undulators do this naturally, the price being a more complex insertion device.

For a particle that enters the undulator off axis due to betatron motion (what we earlier called slow motion, subscript s) the amplitude of the wiggle motion of the electron is changed (cf. Eq. 1.30). The total trajectory of the electron is

$$x(z) = -\frac{K}{\gamma k_u} \cos(k_u z) \left(1 + \frac{k_x^2 x_s^2}{2} + \frac{k_y^2 y_s^2}{2} \right) + x_s \quad \text{and} \quad (1.39)$$

$$y(z) = y_s.$$

Where the betatron motion can come from either the natural focusing in the undulator or some external strong focusing. This motion beyond the wiggling is going to cause the particles that are not on an ideal orbit (wiggling only) to slow down compared to the ideal orbit. This means a correction to Eq. 1.28b is in order. By assuming that the longitudinal velocity will remain close enough to c we find that $\beta_{x,y} \approx d(x,y)/dz$ and the transverse velocities are, when period

averaged, given by

$$\begin{aligned}\langle \beta_x^2 \rangle &\approx \langle (x'_f + x'_s)^2 \rangle = \frac{K^2}{2\gamma^2} + k_{n_x}^2 x_s^2 + k_{n_y}^2 y_s^2 + p_x^2 \quad \text{and} \\ \langle \beta_y^2 \rangle &\approx \langle (y'_s)^2 \rangle = p_y^2.\end{aligned}\tag{1.40}$$

Here we have defined $p_x \equiv dx_s/dz$ and $k_{n_x} \equiv \frac{K^2 k_x^2}{2\gamma^2}$ with similar expressions for y_s . In which case the longitudinal velocity is

$$\begin{aligned}\beta_z &= \sqrt{\beta^2 - \beta_x^2 - \beta_y^2} \approx \beta - \frac{\beta_x^2 + \beta_y^2}{2\beta} \\ \langle \beta_z \rangle &= \bar{\beta}_z - \frac{k_{n_x}^2 x_s^2 + k_{n_y}^2 y_s^2 + p_x^2 + p_y^2}{2}.\end{aligned}\tag{1.41}$$

This correction to the average longitudinal velocity is critical. When the free-electron laser mechanism is used to microbunch an electron beam the wavelength of the laser is typically at the $\sim 1 \mu\text{m}$ level, which means that the 'extraneous' transverse motion must not cause the electrons to slip more than a fraction of a μm or the bunching rate will be adversely affected. In an x-ray free-electron laser the requirements are even more stringent.

1.4 Brightness and Coherence

The two quantities that are most important to lightsource users are brightness and coherence. Brightness, which is applied to both electron and photon beams, gets its name from the compactness of phase space of the particles. Coherence is the term used for the phase relationship between particles in a beam, or wavefronts in the case of classical waves. If the phase relationship is known, the beam is called coherent, otherwise it is somewhere between incoherent and partially coherent. Transverse coherence (the degree of coherence along directions perpendicular to

the direction of the beam motion) and longitudinal coherence (the degree of coherence along the direction of motion) are usually separated so that one can speak of a transversely coherent but longitudinally incoherent beam.

Users care about brightness because it tells them how much useful light they will have. Synchrotron sources are incredibly intense, the monochromator crystals used for selecting a narrow photon bandwidth are frequently cryogenically cooled to both ensure stable crystal lattice spacing and to prevent the crystal from melting. This also prevents the users sample from being unnecessarily damaged by photon flux at unwanted photon energies, as the monochromator selects a narrow radiation bandwidth for the downstream experiment. However, without a sufficiently bright initial source there may be too few photons for the experiment after the unwanted photons are filtered out.

1.4.1 Brightness

The six dimensional peak brightness for a beam that is Gaussian in all dimensions is defined as [48]³

$$\mathcal{B}_0 = \frac{N_e}{(2\pi)^3 \epsilon_{n,x} \epsilon_{n,y} \epsilon_{n,z}}, \quad (1.42)$$

where N is the number of particles and $\epsilon_{n,j}$ is the normalized emittance in the j^{th} direction. The transverse geometric emittances are defined in Eq. 1.22a. To get the normalized emittance they are multiplied by $\beta\gamma$ because the angle with respect to the z-axis is given by, e.g. $x' = p_x/p_z$, the geometric emittance gets smaller as the beam is accelerated. The normalized emittance is therefore a Lorentz invariant. The definition of the normalized emittance is

³Each one of the dimensions is normalized as $\sqrt{2\pi}\sigma$, some authors do not include these in the definition of brightness.

$$\epsilon_{n,x} = \beta\gamma\epsilon_x = \frac{1}{mc} \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle xp_x \rangle^2}, \quad (1.43)$$

which is analogous to Eq. 1.22a but uses phase space parameters instead of trace space parameters. For the other dimensions, switch out y or z for x .

Eq. 1.42 is a Lorentz invariant because all of the constituent pieces of the equation are Lorentz invariants. The total number of particles, N_e , is trivially so and the normalized emittances only contain conjugate coordinates, so they must be constants of the motion as they commute with the Hamiltonian, which is the accelerator physics version of Liouville's theorem [44]. On that note, it is important to equate the momentum used in Eq. 1.43 with the conjugate momentum and not just the mechanical momentum. Usually the emittance is measured or analyzed in a drift so the conjugate and mechanical momentum are one and the same.

Defined as Eq. 1.42, brightness is the phase space density of the particles of interest. It tells the reader how densely the electrons or photons are packed into phase space. The principle goal of lightsource optimization is higher brightness in both the electron and photon beams. Because the photon beam brightness depends on the electron beam brightness, optimization of the electron beam brightness is a key concern of accelerator physicists. In fact, the improving brightness of electron beams is the basis for the success of the SASE free-electron laser.

An example of the importance of brightness is the scaling of the free-electron laser, discussed in more detail below. A useful requirement for first order design is [48]

$$\left(\frac{r_e}{16ec}\right)^{1/3} (KN_u \lambda_u^{1/3}) \left[\frac{1}{\gamma} \left(\frac{I}{\epsilon_n}\right)^{1/3} \right] \geq 1, \quad (1.44)$$

which is a prerequisite for the free-electron laser to lase. This assumes a round, matched electron beam. In this equation, I is the peak current of the electron beam -in the previous notation this is $eN_e/(\sqrt{2\pi}\sigma_t)$ -, r_e is the classical electron radius, and N_u is the number of undulator periods. The last term on the left side of Eq. 1.44 is the transverse brightness, $B \equiv I/\epsilon_n$.

As an example of the power of increasing brightness, assume we want to create a 1 Å free-electron laser. Knowing this and substituting the resonance condition for γ (see Eq. 3.21), Eq. 1.44 becomes $1.78 \times 10^{-7} N_u \lambda_u^{-1/6} B^{1/3} \geq 1$, assuming $K = 1$. Fig. 1.5 shows a parametric plot of the region where Eq. 1.44 is satisfied.

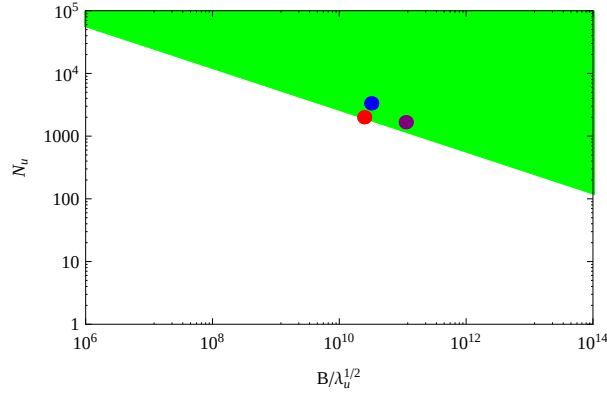


Figure 1.5: Parametric region plot showing the green shaded region where Eq. 1.44 is satisfied for a free-electron laser that lases at 1 Å with an undulator of $K = 1$. The red dot indicates the operating point for LCLS at 1.5 Å [3] and the blue dot indicates the operating point for SACLA at 0.73 Å [4]. The purple point is for the low charge LCLS operation case outlined in section 4.3, the resonant wavelength is 1.4 Å.

One of the remarkable features of Fig. 1.5 is that the number of periods is nearly constant at approximately 10^3 for all of the cases investigated. This isn't an accident and we will see why in Chapter 4, but for now we note that the

approximately constant number of periods to saturation means that length of undulator section scales directly down with the period length. More importantly, the shorter periods require lower energies to reach a given wavelength and this means shorter sections of expensive RF linacs. This is why the short period is so critical. One might think that just increasing the brightness would allow much fewer periods and shorter saturation lengths such that there is no need to move to shorter periods, but it is the decrease in length of the linac that leads to the real cost savings.⁴

In Chapter 4 we will see that there is far more going on than is shown in Eq. 1.44, but it does show that increasing the brightness of the electron beam does allow for decreasing the length of the free-electron laser undulator. In addition, reducing the period length reduces the required energy. This suggests that the push for smaller free-electron lasers benefits from using the brightest electron beams possible and the shortest undulator periods possible (while maintaining $K \sim 1$).

Because the brightness of the electron beam is a Lorentz invariant the highest possible brightness is limited by the process which produces the initial beam. From RF photoinjector sources the scaling of the transverse emittance with charge is $\epsilon_n \propto Q^{2/3}$ (for beam dimensions it is $\sigma_{x,y,z} \propto Q^{1/3}$) and therefore, the 5D brightness (ignoring energy spread) scales as

$$B \propto \frac{Q}{\sigma_z \epsilon_n^2} \propto Q^{-2/3} \quad (1.45)$$

for a fixed wavelength photoinjector [13, 50]. The scaling is faithful for beam charges less than approximately 1 nC, where the beam size is very small such

⁴In the LCLS CDR, the authors claim that building the 15 GeV linac for LCLS would cost about \$300 million, doubling the cost of LCLS [49].

that space charge forces dominate the evolution of the beam in the photoinjector. Indeed it might seem obvious that to obtain the highest brightness beams space charge forces would have to dominate near the cathode, otherwise the injector isn't being pushed hard enough!

The scaling given by Eq. 1.45 suggests that the way to increased brightness is through reducing the charge instead of increasing it, a somewhat counter intuitive result. The reason is that by decreasing the charge the space charge forces which must dominate the evolution of the beam in the photoinjector for a high brightness beam are decreased faster than the total amount of charge. In Chapter 4 we will present examples of relatively short free-electron lasers that take advantage of the scaling in Eq. 1.45 and short period undulators to reduce the total foot print of the device.

Brightness for photon beams is defined slightly differently from Eq. 1.42 because the photons are usually used to do something which requires a certain photon energy. The definition uses relative energy spread ($\sigma_E/\langle E \rangle$) rather than absolute energy spread. Because relative energy spread is not a Lorentz invariant, the brightness defined in this way is not an invariant either, this doesn't matter for photons but becomes problematic when this definition is used for electron beams. The units for 'photon brightness' are usually [photons/(s mm² mrad² 0.1% bandwidth)].

1.4.2 Coherence

Coherence is defined by the ability of the photons in one part of the beam to interfere with photons from another part of the beam. Suppose that a monochromatic light is shined into a Michelson-Morley interferometer (that is a telling name!) with both legs of the split beam being equally long, the wave will inter-

ferre strongly with itself and there will be strong interference rings with totally destructive nulls and very bright maxima. This beam is totally longitudinally coherent. It is also transversely coherent in this example, however transverse coherence is discussed later. If the light is a plane wave, it will be coherent no matter what the difference between the distances travelled along the different legs of the interferometer. The so-called coherence length, the length of the part of the beam over which the interference is complete, of a monochromatic plane wave is infinite.

Suppose, instead, the light is a pulse of Gaussian shape such that

$$E(t) \propto \exp\left(-\frac{t^2}{4\sigma_t^2} - i\omega_0 t\right), \quad (1.46)$$

where σ_t is the rms length of the longitudinal intensity profile, $|E(t)|^2$ and ω_0 is the center frequency. Clearly the coherence time for this electric field must be finite, for if the relative delay is set to say $10\sigma_t$ there cannot possibly be any interference. The coherence time is defined as [51]

$$\tau_c \equiv \int_{-\infty}^{\infty} \left| \frac{\int_{-\infty}^{\infty} E(t+\tau)E^*(t)dt}{\int_{-\infty}^{\infty} E(t)E^*(t)dt} \right|^2 d\tau \quad (1.47)$$

where the integrand is the autocorrelation over the time delay, τ , of the interferometer normalized to the case of zero delay. If we rename the normalized autocorrelation function $R(\tau)$ we can easily verify that its Fourier transform partner is the normalized spectral density of the pulse

$$g(\omega) \propto \exp\left(-\frac{(\omega - \omega_0)^2}{2\sigma_\omega^2}\right), \quad (1.48)$$

where $\sigma_\omega = (2\sigma_t)^{-1}$ is the spectral width of the pulse. The coherence time is $\tau_c = 2\sqrt{\pi}\sigma_t = \sqrt{\pi}/\sigma_\omega$ for the Gaussian example given above. The inverse relationship

between the coherence time and the width of the spectral density means that longitudinal coherence is inversely proportional to the bandwidth, which is given by σ_ω in the Gaussian example. Therefore, the longitudinal coherence can be increased by using a monochromator to decrease the bandwidth. This does not, however, change the number of photons within the passed bandwidth and can lead to too few useful photons.

The inverse relationship between coherence time and bandwidth can be physically elucidated by Parseval's theorem, which requires that the coherence time be also proportional to $\int_{-\infty}^{\infty} |g(\omega)|^2 d\omega$. This leads to two complementary interpretations of what the coherence time means. The time based interpretation is that the coherence time is the amount of relative delay in which the autocorrelation of the signal is significant compared to the intensity of the same signal. The frequency domain interpretation is that the coherence time is distance, in time, within a signal over which the signal is approximately monochromatic. This is why coherence is an important part of brightness. When the coherence length is long, not only is there a known phase relationship between different times of the pulse but the pulse also contains fewer frequencies which is precisely why the coherence time is long in the first place. Saying the beam contains fewer frequencies for long coherence time is just another way of saying that the bandwidth is small relative to a shorter coherence length.

Longitudinal coherence is important for any experiment where the length of the photon pulse is comparable to the length of time that the process under study requires (for example, chemical reactions taking ~ 10 fs). If the experiment requires the creation of interference patterns and the coherence length of the detection laser is very short there will not be very many photons at each time to interfere with each other and the contrast between maxima and minima will be

very low.

Similarly for transverse coherence, the determining experiment is what happens to the radiation when a flat screen with two small holes is placed in the radiation path and the radiation is allowed to interfere. The distance between the two holes at which the interference pattern becomes less distinct, D , defines the transverse coherence area πD^2 . For a large assortment of independent radiators that are far apart relative to the distance to the screen the wavefronts from all the different radiators will reach the two holes with very different phases. This means small transverse coherence area and therefore low transverse coherence. Conversely, if the radiators are very close together compared to the distance to the screen the wavefronts of all the different radiators reach the two holes with similar phases and the coherence area, and thus the coherence, gets larger.

Transverse coherence is important to experiments such as coherent diffraction imaging [52] and phase contrast imaging [53], both of which use the transverse coherence to produce interference at the detector. Clearly the more distinct the peaks of the interference pattern the more robust the signal is, which –by the definition of coherence– demands a very coherent signal.

CHAPTER 2

Magnetic Materials and Insertion Device Technology

As was presented in Chapter 1, the usefulness of a magnet in an insertion device is generally defined by the properties of remanent field and coercivity. To better understand how the interplay between these properties effects the design of insertion devices we investigate here what magnetization is, how microscopic magnetic properties affect macroscopic properties and what those macroscopic properties mean for undulators. That said, magnet creation is still an experimentalist's realm and there are plenty of topics which are thought to be understood qualitatively rather than quantitatively. The fact that 4th generation light sources have the potential to increase the understanding of magnetism and magnets means that the improvement of insertion devices may ultimately depend on how well current knowledge of magnetism is applied to producing coherent radiation.

The motivation for undulator construction is quite straight forward. The simplest goal is to produce magnetic fields that produce motion in a relativistic electron beam that causes interference effects amongst the wavefronts of the radiation to interfere constructively. Specifically, the goal for an undulator is to reproduce the sinusoidal trajectory given by Eq. 1.28 such that the interference condition given by Eq. 3.21 is true for the desired wavelength. There are all kinds of secondary goals, like control of the radiation and trajectory integrals,

but they are taken care of after the spectral goals have been reached.

As we will see in Chapter 3, undulator radiation is a special form of bending magnet radiation, so it is useful to explore why one would bother making a special form of radiation that comes for free from making electrons travel in a circle, as in a synchrotron. The principle reason is greater spectral intensity as the radiation is concentrated in a narrower bandwidth. Because the spectral intensity from an undulator scales as the square of the number of periods, compare Eq. 3.37 for undulator radiation to Eq. 3.17 for bending magnet radiation, a straight section of a synchrotron with lots of short periods can produce many orders of magnitude brighter radiation at the frequencies that satisfy the interference condition.

Additionally, the spectral reach of bending magnet radiation is limited by the critical frequency, Eq. 3.12, which scales like $\gamma^2 B$. This means that in order to produce higher energy photons, the electron beam energy or the magnetic field of the bending magnet must be increased. This leads to the creation of devices called wavelength shifters that are a single super conducting magnet of very high field with a half field strength pole of opposite polarity before and after. This is one of the reasons the Sirius synchrotron is planning to use bending magnets that contain a high field slice [54]. For undulators the frequency (ignoring harmonics) scales as γ^2/λ_u so higher photon energies can be accessed by shorter period undulators. Indeed, this is what has motivated the development of the short period undulator we discuss here.

There are two types of undulator when they are categorized by the source of the magnetic field. The first is current based undulators including electromagnet and superconducting undulators. The second is permanent magnet based undulators including pure permanent magnet designs and hybrid designs which use steel poles. We will concentrate exclusively on permanent magnet based undulators

because the scaling of the required current in short period undulators requires very intense current to compete with the energy stored in permanent magnets, such that the field ratio of current based undulators to permanent magnet based undulators is approximately [10]

$$\frac{B_I}{B_M} \approx 0.11\mu_0\lambda_u\frac{J}{B_r}, \quad (2.1)$$

where J is the current density and B_r is the remanent field of the permanent magnet. For example, with $B_r = 1.4$ T in a 1 cm period undulator, the required current density is over 1000 A/mm² for equal peak fields. This current density can only be reached by superconducting magnets because of Joule heating in regular conductors. A recent parameter study comparing superconducting technology to permanent magnet technology at short periods (5-15 mm) indicates that, for the time being, permanent magnet based undulators are the preferred choice [11].

2.1 General Considerations

The materials chosen for magnets are hard, which means that the magnetization curve looks like the blue line in Fig. 2.1. That is, they maintain magnetization after the applied magnetizing field has been turned off. The figure is focused on the second quadrant of the magnetization curve because that is where the operating points are. The features of the curve are a near unit permeability at zero H which continues back along the H -axis to a sharp "knee" in the curve that takes the curve to the region of very large permeability. The region of near unit permeability is called the linear regime and the area near the knee is called the nonlinear regime. The distinction is important because while the material is handled, assembled, heated and cooled, if the material remains in the linear

regime it can always be returned to any other operating point in this region without affecting the magnetization level. When the magnet enters the nonlinear regime, simply changing the operating point back to the former linear regime does not recover the full magnetization because of hysteresis effects.

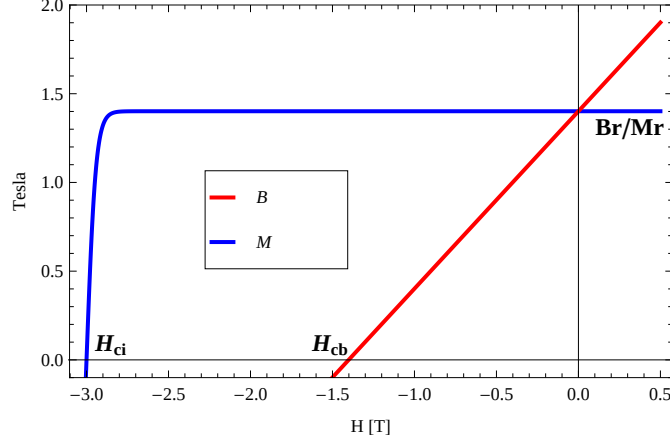


Figure 2.1: MvH and BvH curves for materials used in permanent magnet based undulators.

The point where the magnetization crosses zero is called the intrinsic coercivity, H_{ci} , while the point on the magnetic field curve that crosses zero field is called the coercivity, H_{cb} . The point at $H = 0$ is the remanent magnetization (M_r) or induction (B_r). All of these parameters depend on temperature [55]. The gradients are negative, such that heating the magnets up reduces them all to different degrees, as we will see in section 2.2 on materials. In the linear regime, $\mu = 1 + \delta$, where $\delta \ll 1$ for a good permanent magnet material. This causes most of the flux from the magnet to come from the ends of the magnet, rather than escaping along the whole length [56].

The simplest approximation of the shape of magnets in an undulator is a rectangular parallelepiped. In any type of permanent magnet based undulator there will be at least two magnets per period that are magnetized in the thin

direction, that is approximately 1/4 period thick. To see how this geometry affects the choice of materials we make the approximation that the transverse dimensions are much larger than the dimension in the direction of magnetization, such that Eq. 1.2d becomes (with $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$)

$$\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M} \rightarrow \frac{dH_z}{dz} = -\frac{dM}{dz} \quad (2.2)$$

and there is a constant $\mathbf{H}$ inside the magnet, that opposes the magnetization. The H inside the magnet always opposes the magnetization because the magnetic field is divergent-less.

When the magnet is not exposed to any external fields it will find its natural working point (H_d, M_d) . This point is always in the second quadrant because of the aforementioned direction of H with respect to the magnetization. In general, this point is defined as

$$H_d = -NM_d, \quad (2.3)$$

which defines the demagnetization factor N , which is a purely geometric factor depending only on the shape of the magnet and the orientation of the magnetization inside the magnet. For instance the demagnetization factor in the previous example is 1, while for a uniformly magnetized sphere it is 1/3. The demagnetization factor is only uniform for 3D shapes (as opposed to the semi-infinite sheet used in Eq. 2.2) for a uniformly magnetized ellipsoid. For other 3D shapes the demagnetization factor is larger closer to the surface of the magnet. Note that N must be between 0 and 1, otherwise the magnetic field and H would be oriented in the same direction inside the magnet and the requirements of a divergent-less magnetic field could not be met.

Because of the natural tendency of magnets to demagnetize themselves, as shown by the demagnetization factor, the magnetic material for permanent magnet based undulators must be very hard. Indeed, unless $|H_{ci}| > |H_d|$ the magnet can enter the nonlinear regime and begin to demagnetize itself.

2.1.1 Pure Permanent Magnet Undulators

The analytical solution for the case of a pure permanent magnet undulator was first derived by Halbach [9, 57, 58]. He assumed an infinitely wide undulator of period length λ_u , magnet height h and gap g (shown in Fig. 2.2) and found that

$$B_z + iB_y = 2iB_r \sum_{n=1}^{\infty} \cos[nk(z + iy)] \times e^{-n\pi g/\lambda_u} (1 - e^{-n2\pi h/\lambda_u}) \frac{\sin(n\pi\delta/M)}{n\pi/M}. \quad (2.4)$$

Here M is the number of equal length magnets per period with magnetization direction rotating by $2\pi/M$ per magnet, summation is over $n = 1, 1 + 2M, 1 + 3M, \dots$ and $\delta\lambda_u/M$ is the length of each magnet so that longitudinal gaps between the magnets can be taken into account. The coordinate system is the same as given by Eq. 1.30. Eq. 1.30 is equal to Eq. 2.4 when the limit of $k_x \rightarrow 0$ is taken, which is the assumption of infinitely wide magnets. n is actually the harmonic number of the field and from the summation we know that only odd harmonics exist. Solution of the motion of an electron shows that the undulator parameter is extended to include the harmonics via

$$K_{eff} = \frac{e\lambda_u}{2\pi m_0 c} \sqrt{\sum_{n=1,3,5\dots}^{\infty} \left(\frac{B_n}{n}\right)^2}, \quad (2.5)$$

where B_n is the field strength of the n^{th} harmonic. Clearly the fundamental

($n = 1$) is the most important component of the field.

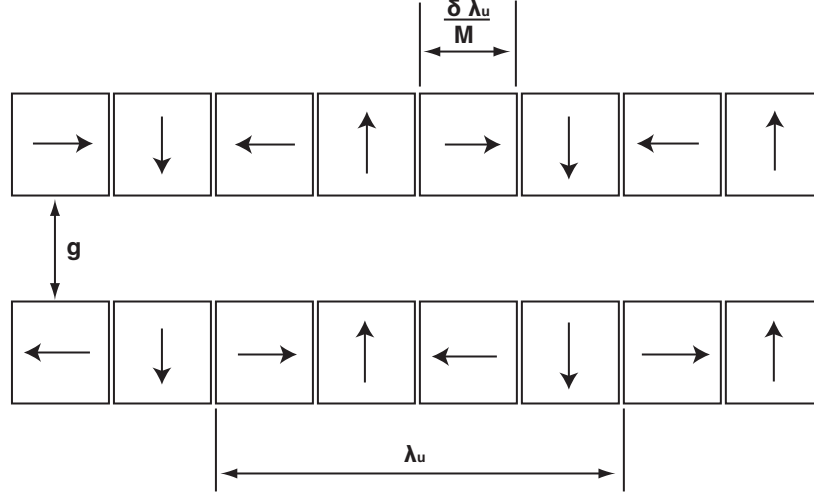


Figure 2.2: Schematic of a pure permanent magnet undulator. The arrows within the blocks indicate the direction of magnetization.

There are a few things to learn about undulator design from Eq. 2.4. The first, and most important is that the gap to period ratio should be small for large fields. Increasing the gap by 10% can cost 6% of the fundamental when $g/\lambda_u = 0.2$. Next is the height of each of the magnets, shown in Fig. 2.3. For magnets that are as tall as a period the field is only 0.2% lower than maximum, while for $h = \lambda_u/4$ the field is an impressive 79% of maximum. Designing an undulator with $h = \lambda_u/4$ might seem like a poor decision, except that by doing so all the magnets are identical when $M = 4$ which can simplify aspects of construction.

The final parameter is the number of magnets per period, M . When the air gaps are eliminated $M = 4$ results in a 10% loss in the fundamental field strength, while $M = 8$ leads to a 2.5% reduction. Nevertheless, 4 is often the choice made because it simplifies the design both conceptually and during assembly. Further, the blocks will be λ_u/M long (ignoring the gaps), which will lead to a larger

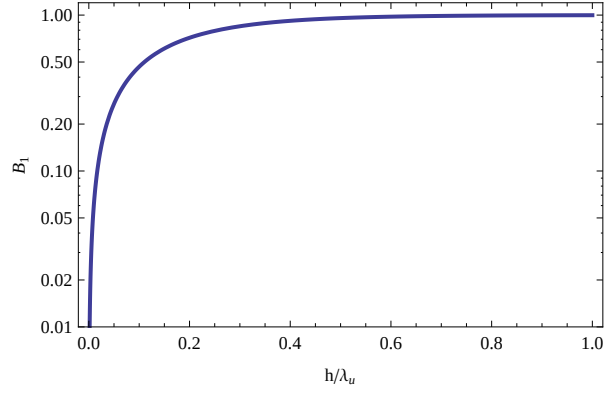


Figure 2.3: Effect of magnet height, h , on the fundamental harmonic of undulator field strength for a pure permanent magnet undulator. All other effects in Eq. 2.4 have been ignored.

demagnetizing factor in the longitudinally magnetized blocks for shorter magnets, which may complicate the design or construction.

The one design consideration not contained in Eq. 2.4 is the magnet width. The constraint of that parameter requires numerical simulation. Fig. 2.4 shows the effect of pole width for different gap to period ratios. For the simulation $M = 4$, $\lambda_u = 10$ mm, $h = 10$ mm and $\delta = 1$. The simulation was done using the package Radia [59, 60]. It is clear from the figure that once the magnet width is greater than the period length there is little to be gained in the peak field by making the magnets wider.

2.1.2 Hybrid Undulators

Hybrid undulators replace the vertically polarized magnets in the lattice with poles of nonlinear material to increase the field strength on axis [10]. The gain in field over a pure permanent magnet device is greatest for small gap to period ratios, reaching nearly 50% in peak field for $g/\lambda_u = 0.2$. There is, however, higher

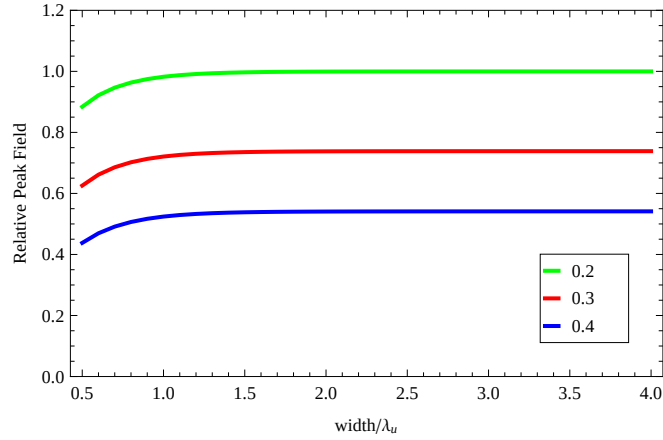


Figure 2.4: Effect of magnet width on the peak of undulator field strength for varying gap to period ratio for a pure permanent magnet undulator. The curves are all normalized to $\text{width}/\lambda_u = 10$ at $g/\lambda_u = 0.2$. The simulations were done using the code Radia.

harmonic content in hybrid devices. There is also a new set of free parameters in the dimensions of the pole pieces, in particular, the poles are usually not the same longitudinal length as the magnets. When the poles are made thinner than $\lambda_u/4$, the demagnetization factor of the magnets is reduced but there is also an increase in the amount of field that gets channeled into the higher harmonics.

The key design goal of a high performance hybrid design is scantily saturating the tips of the poles. If the pole tips aren't saturated the material being used is probably too expensive. If the pole tips are driven well past saturation the poles are not providing any field enhancement past the point of saturation and a material with higher saturation induction should be selected, if available. This design goal means that the optimization requires parameter scans which elucidate the optimal combination of materials and shapes for a desired peak field. Typically, this means that the poles are thinner than the magnets. This improves the field over a pure permanent magnet device when the working point of the pole

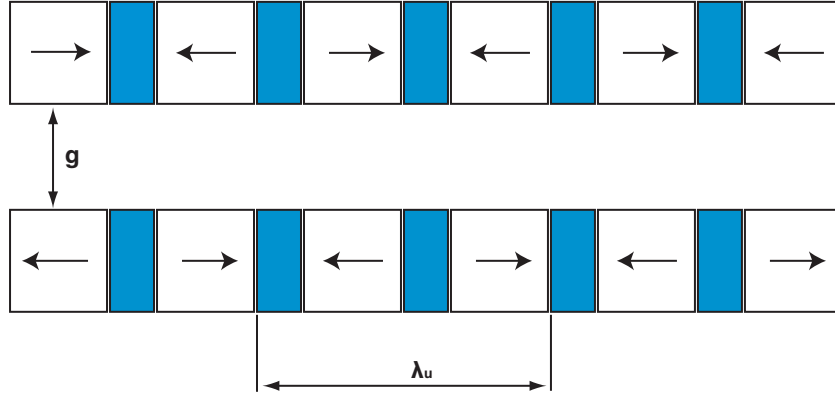


Figure 2.5: Schematic of a hybrid permanent magnet undulator. The arrows within the white blocks indicate the direction of magnetization of the magnets. The blue blocks are soft magnetic material.

results in higher magnetization. For example, permanent magnets are limited to a maximum remanent field of approximately 1.5 T, while Vanadium Permendur will saturate at 2.35 T.

Most pole materials are made of some kind of steel, which means that the material is isotropic. Dysprosium poles are a recent exception [61, 62]. If the pole is isotropic the flux lines must be controlled to pass through the magnetic gap, otherwise there can be significant flux leakage through the sides and gap of the magnetic lattice as the flux from neighboring poles takes the path of lowest reluctance.¹ This means that the poles should have transverse dimensions smaller than the magnets.

¹Reluctance, $\mathcal{R}$, is the magnetic circuit analog to resistance, which obeys $\mathcal{F} = \phi\mathcal{R}$. $\mathcal{F}$ is the magnetomotive force taking the place of EMF and ϕ is the flux.

2.2 Magnetic Materials and Magnets

2.2.1 Pole Materials

Hybrid undulator poles increase the peak on axis field when they have saturation induction larger than the magnet that would be filling the space in a pure permanent magnet device. There is a menagerie of pole materials being developed today, including Dysprosium [61], Holmium [62] and high temperature superconductors [63]. However, none of these technologies is yet mature enough for use in undulators and still require a great deal of testing. Accordingly, all existing hybrid undulators use some form of steel for the pole material.

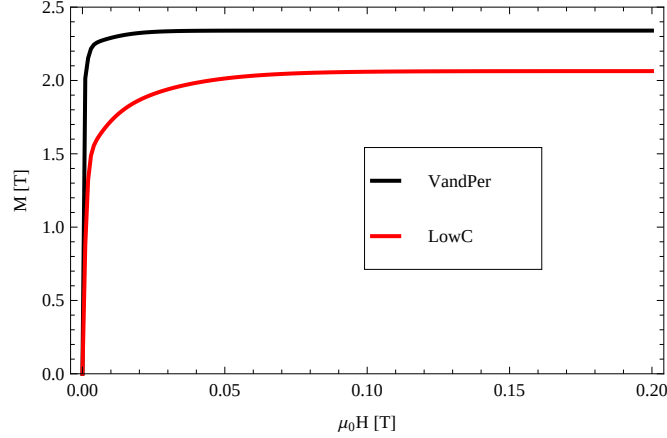


Figure 2.6: Comparison of the magnetization curves between Vanadium Permendur and low cobalt steel.

The steels used all share the common properties of at least 2 T saturation magnetization, which is much larger than the 1.0-1.5T available in permanent magnets, and large initial permeability, $\mu_i \sim 10^4$. Fig. 2.6 shows the magnetization curves for Vanadium Permendur (49% Fe, 49% Co, 2% V) and a low carbon steel (like ARMCO). It is clear from this figure that Vanadium Permendur is the

best choice to maximize on axis field because of the larger saturation magnetism and sharper knee.

2.2.2 Magnet Materials

All magnets used in undulators are either SmCo_5 , $\text{Sm}_2\text{Co}_{17}$, $\text{Nd}_2\text{Fe}_{14}\text{B}$, $\text{Pr}_2\text{Fe}_{14}\text{B}$ or some combination of the two like $(\text{Pr}_{0.8}\text{Nd}_{0.2})_2\text{Fe}_{14}\text{B}$ [24]. This is because these ferromagnets have large remanent magnetizations from 1-1.5 T and large coercivities, such that the knee usually shows up in the 3rd quadrant of the BH curve. Of these materials samarium is particularly expensive and the magnets using it usually have remanent magnetizations near the bottom of the available range [56].

All of these materials rely on crystal anisotropy to produce the coercivity that makes them hard [56]. This anisotropy is a dependence of magnetization on direction of applied field that is attributed to the spin-orbit coupling of the electrons in the atoms making up the material. Because it isn't possible to calculate the strength of the anisotropy or even the direction of the anisotropy based on first principles [64], we will sketch out the models that cause magnetization and anisotropy before discussing what the anisotropy means for the properties in neodymium and praseodymium based magnets as a function of temperature.

To begin, magnetization comes from the spins of unpaired electrons in the atoms of the material [56]. For simplicity, we will talk about a single unpaired electron associated with each atom, however, by Hund's rules it is certainly more likely that strongly magnetic materials have multiple unpaired electrons per atom. It is the nearest neighbor force between electrons in different atoms that is believed to cause ferromagnetism. There are three forces that electrons in neighboring atoms apply to each other: the magnetic dipole-dipole force be-

tween spins, the electrostatic force between charges and the exchange interaction between fermions. The magnetic dipole fields between two electrons will tend to cause them to align opposite to each other to minimize the energy of the system. But this effect is small as the magnetic moments of the electrons are approximately equal to the Bohr magneton and the dipole fields decay with distance as $1/r^3$ [29]. The electrostatic force is much stronger at interatomic distances.

The exchange interaction is a different beast all together. It is the manifestation of the purely quantum mechanical exclusion principle [65]. The result of the exclusion principle is that the energy of the system of two electrons depends both on the distance between the two but also on the relative spin. Parallel spins are favored energetically when the particles are farther apart and opposite spins are favored when the particles are closer together. Clearly this isn't the whole story, because there has to be something keeping these two identical particles close to each other, the bonding that keeps the material solid. What can be said is that ferromagnetic materials contain unpaired electrons that have spins oriented in the same direction. Further, this means that ferromagnetic materials should have larger distances between these 'active' electrons than non-ferromagnetic materials because of the exclusion principle. This is indeed the case [56].

The anisotropy is caused by the electrostatic affect of all of the neighboring atoms on each other. The fields of these neighboring atoms distort the orbitals of each other which acts to give the electrons in these orbitals preferred axes of motion. This then acts, through spin-orbit coupling, to cause the electron spins to have a preferred direction as well [64]. This preference in direction is the anisotropy. Therefore, there are two different pieces of the ferromagnetic puzzle. The first is that the combination of the exchange interaction and the electrostatic force cause the spins of the unpaired electrons of different atoms

to be parallel to each other. The second is that the coupling of these spins to the lattice of the material through spin-orbit coupling causes these spins to have preferred directions.

For a uniaxial crystal, like neodymium and praseodymium based magnets, the anisotropy energy of a single crystal is [64, 66]

$$E = K_1 \sin^2(\theta) + K_2 \sin^4(\theta), \quad (2.6)$$

where K_1 and K_2 are the anisotropy constants, and θ is the angle between the crystal c-axis and the direction of magnetization. This equation is used for determining the angle of the easy axis of magnetization relative to the c-axis. Taking the derivative of Eq. 2.6 shows that the turning points of the potential energy are given by

$$\sin(\theta) = \sqrt{\frac{-K_1}{2K_2}}. \quad (2.7)$$

When $K_1 > 0, |2K_2|$ the easy axis is always along the c-axis (0 or π). If K_1 is negative and K_2 is positive than the easy axis shifts away from the c-axis by the angle given in Eq. 2.7. This is the cause of the loss of flux when neodymium magnets are cooled below 135K [21, 66, 67]. The change in easy axis direction, that is the change in the values of K_1 and K_2 , seems to be an effect of the neodymium because the reorientation does not happen in magnets containing praseodymium [68]. This is complicated by the fact that magnets with rare earth fractions up to 20% neodymium also do not show the reorientation [24, 69].

This spin reorientation transition is important because because the coercivity and remanent field of rare earth based magnets have negative temperature coefficients [70]. In the context of undulators the usefulness of increases in these

two properties with cooling was first recognized in [17]. More recent work has given quantitative values to the increase in coercivity and remanent field, indicating that the coercivity grows approximately 6 times faster than remanent field [22, 24]. What this means for undulators is that materials can be optimized for higher remanent field at room temperature and the cooling can be used to increase the coercivity to the levels required for operation. The trade-off between remanent field and coercivity is because additions to the magnet that increase anisotropy are not magnetic, like Dysprosium. This means that a larger fraction of the material does not contribute to the remanent field. This can be partially ameliorated by advanced magnet making techniques, such as Grain Boundary Diffusion [71].

It is the appearance of a spin reorientation transition in neodymium that lead to the choice of using praseodymium based magnets, despite the greater availability of neodymium [56]. Both materials show increasing coercivity and magnetization with decreasing temperature but the advantage of using praseodymium based magnets is that they can be cooled down below 130K. This is advantageous because the undulator can be designed for direct cooling with liquid nitrogen or helium without the need for heaters and further, the conductivity of copper is much larger in the 30-50K range allowing for minimization of thermal gradients that can affect undulator performance [72]. Direct comparison of $\text{Nd}_2\text{Fe}_{14}\text{B}$ with $\text{Pr}_2\text{Fe}_{14}\text{B}$ at their respective optimal operating temperatures shows that there is little difference between the remanent field of the two, while the coercivity is much higher for the latter [22].

2.2.3 Radiation Exposure

As was seen earlier in this chapter, short period undulators require small gaps to create large fields which are crucial to achieving large undulator parameters – in Chapters 3 and 4 it will become clear that $K \geq 1$ is a good estimate for defining large K . Accordingly, as the magnet gap gets smaller it becomes advantageous to put the whole undulator in the vacuum chamber instead of building a small beam pipe that will fit inside a few mm gap. First, because the chamber walls can use up an appreciable fraction of physical aperture and second because in synchrotrons where injection procedures require larger vertical space, the beam pipe may not allow optimal filling of the ring. In these cases it becomes necessary, if short period undulators are desired, to put the undulator inside the vacuum chamber to allow gap adjustment for beam clearance.

Small undulator gaps lead to greater radiation exposure because the halves of the device are closer to the electron beam. Small gaps also present a smaller aperture for upstream radiation sources such that hard x-rays from upstream bending magnets can irradiate the undulator. A large number of studies have been done on the effect of radiation on demagnetizing rare earth permanent magnets [20]. The effect seems to be one of local heating which causes reversed domain nucleation. The incident radiation heats a small volume of the magnet to above the Curie temperature and the local demagnetizing field causes the magnetization vector to flip. Because magnetization can be restored via remagnetization, it does not appear that there is any permanent damage to the lattice structure.

The local heating model is consistent with the increase of resistance to demagnetization by radiation in colder magnets [73, 20]. The colder magnets have a higher integrated heat capacity to reach the Curie temperature and more energy is required to heat the magnet up to the point of demagnetization [62]. If this is

indeed what causes radiation induced demagnetization, then the value of the coercivity is irrelevant because it is zero at the Curie temperature. Nevertheless, the coercivity and the integrated heat capacity increase with decreasing temperature and there is a positive correlation between the increasing coercivity and resistance to radiation caused demagnetization. The increased radiation resistance is another reason to choose cryogenically cooled magnets over room temperature magnets for small gap undulators. The increased radiation resistance will increase operational lifetime in undulators that have strict field tolerance requirements as the total allowed Amp hours of exposure is increased over non-cryogenic undulators [74].

2.3 Cryogenic Permanent Magnet Undulator - 9 mm

Cryogenic Permanent Magnet Undulator - 9 mm period (CPMU9) was started as a project to build a soft x-ray undulator radiation source for a Laser Wakefield Accelerator (LWFA) at the Max Planck Institute for Quantum Optics (MPQ) [25]. The goal was to produce 1 nm photons. At the time, the maximum beam energy reported from such an accelerator was approximately 1 GeV [75]. This meant that the period length of the undulator had to be sub-cm. Further, to produce a large number of photons $K \sim 1$ was desired (see Chapter 3), which would require a still shorter period length to reach 1 nm photons.

The downstream side of these LWFA tubes can be high radiation environments as the structures can produce large energy spread electron beams [76, 77], can have poor pointing stability [78] and can even produce radiation from the betatron motion of the electron beam in the plasma [79]. This means that the short period undulator should be radiation resistant. The combination of large field, short period, and radiation resistance naturally lead to the decision to make a

CPMU device. Because of the spin reorientation transition of neodymium based magnets, an alternative was sought to make operation straightforward at liquid nitrogen temperatures or below, which would presumably lead to an increase in radiation resistance over warmer magnets [24]. Figure 2.7 shows the change in the magnetization curve for the material at 300K and 30K.

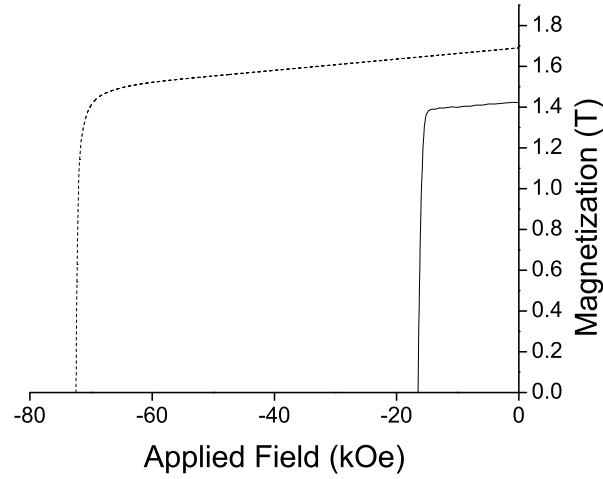


Figure 2.7: Magnetization curves for $(\text{Pr}_{0.8}\text{Nd}_{0.2})_2\text{Fe}_{14}\text{B}$ at 300K and 30K.

Scaling of the operational parameters of the MPQ LWFA had led to a scenario in which a 1.74 GeV electron beam was feasible [80], which corresponds to a 15 mm period undulator to produce 1 nm radiation. However, because the current state-of-the-art energy was nearly a factor of two too low, an aggressive design was sought so that the undulator would produce harder x-rays as the LWFA developed. To limit the demagnetizing factor and increase K , a 9 mm period was chosen. This period length was relatively conservative compared to a previous $\text{Nd}_2\text{Fe}_{14}\text{B}$ undulator built for MPQ which has a 5 mm period and $K = 0.55$ [25].

The undulator design process was an iteration between increasing knowledge of the new material and magnetic design [19]. The final design is magnets that are

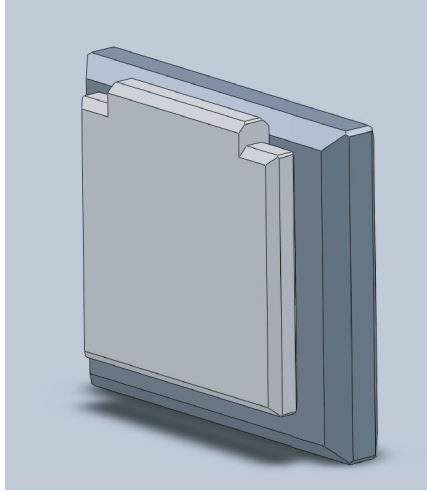


Figure 2.8: The magnetically important geometry of CPMU9, showing one half period from the bottom jaw of the undulator. The lighter grey material is the pole in front of the magnet.

16 mm tall $\times$ 14 mm wide $\times$ 2.8 mm thick. The poles are 12 mm $\times$ 12 mm $\times$ 1.6 mm (same directions as magnet). Fig. 2.8 shows the magnetic geometry of the undulator. The poles and magnets are strategically chamfered to reduce reverse fields as the magnetic fields of the magnets will attempt to close through the same jaw of the undulator. Further, sharp points in high permeability materials concentrate flux [29].

The design of a precision cut trough in a single copper jaw was chosen to increase the thermal contact between the magnets and the jaw which required that the poles be clamped from the top. To accomplish the clamping without reducing the physical aperture of the already small gap 2 mm wide $\times$ 1 mm deep "shoulders" are cut from the gap side of the poles, see Fig 2.8. The structure is made of copper because of its excellent thermal conduction which increases with decreasing temperature down to approximately 30K [72].

Because of the shape of the field in the undulator (given by Eq. 1.30) each

Table 2.1: CPMU9 parameters from simulation using Radia.

Parameter	Value
λ_u (mm)	9
Num. Per.	20
gap (mm)	2.5
B_{eff} (T) @ 300 K	1.06
K_{eff} @ 300 K	0.89
B_{eff} (T) @ 30 K	1.18
K_{eff} @ 30 K	0.99

pair of poles, one in the upper jaw and one in the lower jaw, are polarized in the same direction so that they are strongly attracted to each other, the force is greater than 20 N per pole in the CPMU9. The magnetic properties of CPMU9 are given in table 2.1. The gap is maintained by a strong back spacer machined to hold the jaws 2.5 mm apart. Because of the approximately 1 kN force on each jaw from the attraction between poles across the gap the strong back had to be massive. It also serves as a heat conduction path to allow temperature equalization between the two jaws. Fig. 2.9 shows an image of the undulator.

2.3.1 Pulse Wire Measurements

In section 1.3 the dynamics of electrons in a sinusoidal magnetic field was explored, however there was no explanation of how the electrons went from a field free region to the sinusoidal region. The horizontal velocity of an electron that passes through vertical field $B_y(z)$ is given by

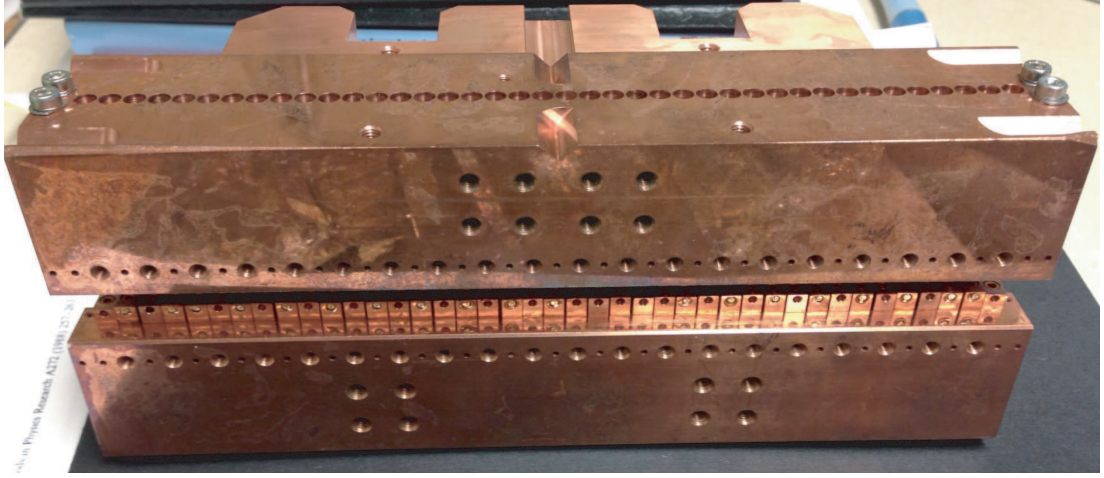


Figure 2.9: Picture of CPMU9. The strongback can be partially seen at the top of the picture. The undulator is upside down to show the grooves that constrain the movement of the plastic balls that the undulator sits on (top side of the top jaw). The aluminum screws on the left and right control the vertical position of the first and last metal pole for compensation of the first integral (see subsection [2.3.1](#)).

$$\beta_x = \frac{e}{\gamma m_0 c} \int_{-\infty}^z B_y(z') dz', \quad (2.8)$$

where it has been assumed that the field is zero at $z = -\infty$, that $\beta_x(0) = 0$ and that the particle is relativistic such that $v_z = c$ is approximately constant over the integral. The angle of the particle's motion relative to the z -axis, which is the straight path the electron would take if $B_y = 0$, is given by $x' = \beta_x/\beta_z$ which means that the angle of the electron is given by

$$x'(z) = \frac{dx}{dz}(z) = \frac{e}{\gamma m_0 c} I_y, \quad (2.9)$$

where I_y is called the first integral of the motion. This is horizontal angle of the electron as a function of its previous path. If Eq. [2.9](#) is integrated again, the

horizontal position of the electron with respect to the axis is found as

$$x(z) = \frac{e}{\gamma m_0 c} \int_{-\infty}^z \int_{-\infty}^{z'} B_y(s) ds dz' \equiv \frac{e}{\gamma m_0 c} II_y, \quad (2.10)$$

where II_y is the second integral of the motion. These two parameters give the initial conditions for the electrons as they reach the first period of the undulator.

Just building an undulator without worrying about the electron trajectory will not result in electrons that travel on axis, the ends of the undulator must be carefully designed [10, 41]. Whereas there are known solutions for pure permanent magnet devices [41], for hybrid devices the compensation process can be involved for a device that must operate at several gaps [10]. Because the prototype CPMU9 was designed as a radiator for a LWFA, it was decided to compensate (that is, zero) the first integral of the device while leaving the second integral uncompensated. This resulted in a simpler design in which the first and last pole were retracted from the gap a fixed amount and no new pieces had to be designed.

A typical approach to measuring the field integrals without use of a Hall probe is the pulse wire method [81]. In this method, a thin wire of high conductivity and yield strength is stretched through the undulator. When a current pulse is applied to the wire it excites acoustic waves on the wire which are detected by using the transverse motion of the wire to interrupt a laser beam that is incident on a photodiode. The acoustic wave on the wire contains information on the local field magnitude, as can be seen in Fig. 2.10. This is important because it is not enough to correct the integrals of the whole undulator if the beam is to oscillate on axis, both ends must be compensated individually. This is an advantage of the pulse wire method over the moving wire method [10]. The moving wire method only shows the total integrals over the whole undulator.

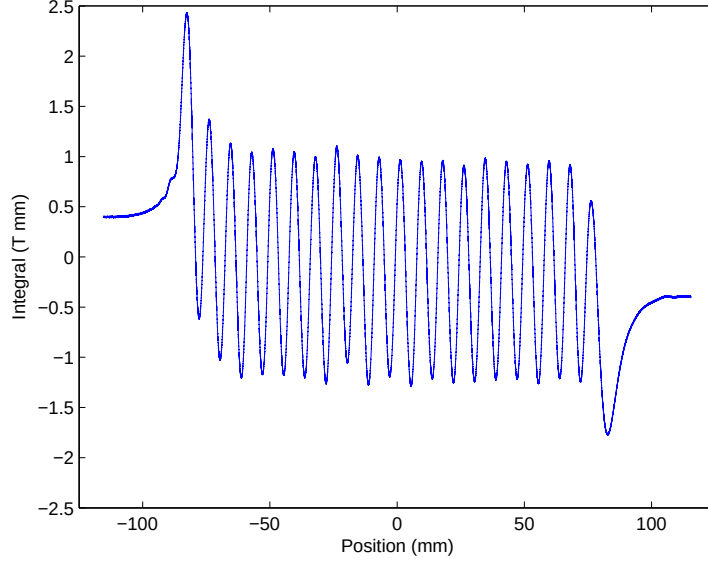


Figure 2.10: Pulsewire result showing the first integral of CPMU9 at room temperature. The total first integral at room temperature is 0.8 T mm, which is consistent with simulation and corresponds to a total kick by the undulator of 4.8 mrad. The large spikes at the beginning and end of the signal are the entrance and exit fields of the undulator while the smaller oscillations are the periodic undulator field.

The field of the CPMU9 was measured at Helmholtz-Zentrum Berlin where a custom designed cryogenic measurement bench was used to measure the field using a Hall probe [62]. Because the first integral is compensated at cryogenic temperatures, at room temperature the integral is non-zero because of the reduction in flux through the first and last poles. A pulse wire measurement performed at UCLA and shown in Fig. 2.10 confirms this.

CHAPTER 3

Charged Particle Radiation

The common feature shared by all electromagnetic radiation types generated by dedicated accelerator sources, that is sources designed to produce electromagnetic radiation, is that they all involve the acceleration of electrons ¹. The totality of these radiation types can be broken down into two main categories: radiation due directly to the beam electrons, such as synchrotron radiation, and radiation due to the interaction of the beam electrons and other (usually "stationary" in the lab frame) target electrons, such as transition radiation.

A taxonomy of radiation types reveals that all of the cases of beam electron acceleration can be described as a special case of the more general synchrotron radiation, the principle difference is the motion of the electrons. In synchrotron radiation the particle is expected to be moving in a circle of radius given by the static magnetic field. In undulator radiation the particles follow a sinusoidal trajectory found by linking the arc lengths of half periods of reversing polarity. In edge radiation the position of the particle is tracked as it moves along a straight line from a field free region into a bending magnet. Similarly for transition undulator radiation the particles follow a straight line through an undulator and the difference in longitudinal velocity between inside and outside of the undulator fields causes the particles to radiate. The connection between inverse Compton

¹Electrons, being the lightest, stable, charged particles known to us are the natural choice for producing acceleration based radiation.

Table 3.1: Taxonomy of radiation types.

Beam Electrons	Target Electrons
Synchrotron	Transition
Undulator	Diffraction
Edge	Čerenkov
Transition Undulator	Wakefields
Inverse Compton Scattering	Bremsstrahlung

scattering and synchrotron radiation is made by analyzing the beam motion in its rest frame where the fields from a counter propagating laser and an undulator are very similar [82].

The target based radiations are divided into two categories, surface currents and material polarization. Transition radiation and diffraction radiation occur when an electron pass through a screen or through a hole in a screen, respectively. The screen is usually metal and the electrons in the metal rush around to cancel the surface fields on the metal and these moving electrons radiate. Čerenkov radiation occurs as a particle moves through a material at greater than the phase velocity of light in that material. When this condition is met, the material cannot polarize sufficiently quickly to shield the moving charge and the changing polarization state of the material (accelerating electrons) radiates. Wakefields are a mixture of the previous two, some beam line element like a beam pipe or bellows interrupts the fields of the moving charge relative to its free space configuration and the charges moving in the element to satisfy Maxwell’s equations radiate. Bremsstrahlung is a different beast in that the acceleration of the beam particles comes from collision with the electromagnetic fields of atoms and molecules, it is included for completeness but will take us too far afield so we won’t be mentioning

it further.

All of the radiation types in the left column of table 3.1 can be neatly related by the fact that they are all calculated using Liénard-Wiechart potentials. Since we are only interested in radiation fields we drop the velocity field term and write the electric field of an accelerating charge detected at a point $\mathbf{x}$ at time t as

$$\mathbf{E}(\mathbf{x}, t) = \frac{e}{c} \left[\frac{\hat{\mathbf{n}} \times \{(\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}})^3 R(\tau)} \right]_{ret}, \quad (3.1a)$$

$$\mathbf{B}(\mathbf{x}, t) = [\hat{\mathbf{n}} \times \mathbf{E}(\mathbf{x}, t)]_{ret}. \quad (3.1b)$$

In this equation, $\hat{\mathbf{n}} \equiv \frac{\mathbf{R}(\tau)}{R(\tau)}$ is the unit vector pointing from the accelerating charge to the point where the radiation is being collected and τ is the retarded time which is related to the current time at the detector by $t = \tau + R(\tau)/c$. The subscript *ret* means that all of the quantities are evaluated at the retarded time. We note here that due to the $(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}})^3$ in the denominator of Eq. 3.1 the radiation from a relativistic electron is directed in the direction of motion. The radiation will be most intense when $1 - \beta \cos(\theta) \approx 0$, which occurs at $\theta \sim 1/\gamma$ for ultra relativistic particles.

The quantity of interest, however, isn't the electric field itself, but the amount of energy crossing a differential area, dA , surrounding the point $\mathbf{x}$ per unit time, which is just the Poynting vector. It is easy to show that $\hat{\mathbf{n}}$, $\mathbf{E}$ and $\mathbf{B}$ are all normal to each other (as it should be for radiation!) and that the Poynting vector is equal to

$$\mathbf{S} = \frac{dW}{dt dA} \hat{\mathbf{n}} = \frac{c}{4\pi} (\mathbf{E}(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}, t)^*) \hat{\mathbf{n}}. \quad (3.2)$$

Because the electric field in the time domain, $\mathbf{E}$, is related to the electric field in

the frequency domain, $\mathbf{E}_\omega$, via a Fourier transform and negative frequency has no physical meaning so that we can take $-\omega = \omega$, the magnitude of the Poynting vector can be transformed into ²

$$\frac{dW}{d\omega dA} = \frac{c}{2\pi} |E_\omega|^2. \quad (3.3)$$

The Fourier transform in Eq. 3.3 involves the time at the detector, t , which is related to the time τ at the particle using the retarded time relation given above. Because we know what the particle is doing in terms of time τ the Fourier transform must be converted using

$$\begin{aligned} dt &= d\tau \left(1 + \frac{1}{c} \frac{dR(\tau)}{d\tau} \right) \\ &= d\tau \left(1 + \frac{1}{c} \left[\mathbf{R} \cdot \frac{d\hat{\mathbf{n}}}{d\tau} + \hat{\mathbf{n}} \cdot \frac{d\mathbf{R}}{d\tau} \right] \right) = d\tau (1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}}). \end{aligned} \quad (3.4)$$

The last equality comes about because the unit vector $\hat{\mathbf{n}}$ only changes normal to $\mathbf{R}$ and $\mathbf{R} = \mathbf{x} - \mathbf{r}(\tau)$. Using this identity, Eq. 3.3 becomes

$$\frac{dW}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c} R(\tau)^2 \left| \int_{-\infty}^{\infty} \left[\frac{\hat{\mathbf{n}} \times \{(\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}})^2 R(\tau)} \right] e^{-i\omega(\tau + R(\tau)/c)} d\tau \right|^2, \quad (3.5)$$

where we have used the definition of the solid angle subtended at the point $\mathbf{x}$, seen from the point $\mathbf{r}(\tau)$, due to area dA , $dA = R(\tau)^2 d\Omega$.

That is as far as we can go using this formulation without knowing what the particle motion looks like. The most important part of the above integral (in Eq. 3.5) is the phase term because in all of the following cases an infinitesimal

²The Fourier transforms are defined as $E(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E_\omega e^{i\omega t} d\omega$ and $E_\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t) e^{-i\omega t} dt$. In this case, the Dirac delta function can be defined as $\delta(\omega - \omega^*) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i(\omega - \omega^*)t} dt$.

piece of the path taken can be treated similarly for the part of the integrand in the square brackets, it is how the photons from these different parts of the path add up in phase that is most important in determining the characteristics of the radiation. There is, however, an important aspect of the radiation production that has been glossed over presently, we look at that now.

3.1 The importance of acceleration direction

One of the features that all of the radiation types in the left column of Table 3.1 have in common is that the relevant acceleration is normal to the direction of motion. This is because a particle emits far more radiation when accelerated normal to its direction of motion rather than along the direction of motion. The simplest way to see this is to start in a frame of reference in which the particle is at rest except for a small, non-relativistic acceleration. Starting with Eq. 3.1a and assuming that the range of motion is small such that we can ignore the changing distance between the charge and the detector we find ($R^2 \mathbf{S} = \frac{dP}{d\Omega}$) [29]

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c} \left| \dot{\boldsymbol{\beta}}^2 - (\hat{\mathbf{n}} \cdot \dot{\boldsymbol{\beta}})^2 \right| \quad (3.6a)$$

$$= \frac{e^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \sin^2(\theta), \quad (3.6b)$$

where θ is the angle between the direction of observation and acceleration. When integrated over all angles this is the Larmor result [29]:

$$P = \frac{2e^2}{3c^3} |\dot{\mathbf{v}}|^2. \quad (3.7)$$

The Larmor equation is explicitly non-relativistic, but because the relativistic result must reduce the non-relativistic form in the limit that the particle velocity

is small (indeed, this is how we got the Larmor formula), it can be used to find the relativistically correct result. To generalize, the acceleration is replaced by its Lorentz invariant form $|\dot{\mathbf{v}}|^2 \rightarrow -\frac{1}{m_0^2} \left| \frac{dP_\mu}{dt'} \frac{dP^\mu}{dt'} \right|$ where t' is the proper time of the particle (that is the time in the frame where the particle is at "rest" – where the Larmor formula is true) and P is the 4-momentum of the particle. Noting that the proper time is defined such that when changing to any other frame $dt' = dt/\gamma$, we find that the total energy radiated by a particle whose normalized velocity and acceleration in the lab frame are given by $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$, respectively, is

$$P = \frac{2e^2}{3c} \gamma^6 \left[\dot{\boldsymbol{\beta}}^2 - (\boldsymbol{\beta} \times \dot{\boldsymbol{\beta}})^2 \right]. \quad (3.8)$$

It is obvious that when $\gamma = 1$ we recover Eq. 3.7. More important is the enormous increase in radiated power as the relativistic factor, γ , increases. When $\gamma = 10$ the particle emits one million times more radiation when compared to the stationary particle. Using this formula, we can discover how the amount of radiation varies with the direction of acceleration relative to the velocity.

When the particle is accelerated along the direction of motion the calculation of the radiated power is straightforward because the equations of motion are one dimensional, leading to the following relation for the radiated power:

$$P = \frac{2e^2}{3m_0^2 c^3} \left| \left(\frac{d\mathbf{p}}{dt'} \right)^2 - \frac{1}{c^2} \left(\frac{d\mathcal{E}}{dt'} \right)^2 \right| = \frac{2e^2}{3m_0^2 c^3} \left(\frac{d\mathbf{p}}{dt} \right)^2 = \frac{2e^2 c}{3m_0^2 c^4} \left(\frac{d\mathcal{E}}{dx} \right)^2. \quad (3.9)$$

This shows that the rate of radiation is proportional to the square of the rate of energy gain for the particle. A typical linear accelerator will have a gradient less than 50 MV/m resulting in 10 $\mu\text{eV}/\text{m}$ of energy loss for a relativistic particle, which is a relative energy change of 10^{-12} , totally undetectable.

For an electron accelerated in a constant magnetic field, the acceleration is perpendicular to the velocity and the velocity is constant such that Eq. 3.8 can be written as

$$P = \frac{2e^2}{3c} \gamma^4 \dot{\beta}^2 = \frac{2e^2 c}{3} \frac{\gamma^4 \beta^4}{\rho^2} = \frac{2e^2}{3m_0^2 c^3} \gamma^2 \left(\frac{d\mathbf{p}}{dt} \right)^2, \quad (3.10)$$

where ρ is the bend radius of the electron (see Eq. 3.11). Eq 3.10 is a factor of γ^2 larger than Eq. 3.9, this shows the power of acceleration normal to the velocity. For a highly relativistic electron beam on a tight bending radius, an enormous amount of radiation power can be created.

Eqs. 3.7, 3.9 and 3.10 all contain vectors to allow the inner product of that vector ($\dot{\mathbf{v}}$ or $\mathbf{p}$) with a unit vector for evaluation of the radiated power with respect to a certain direction. However, this feature also elucidates an important aspect of radiation depending on the dynamics of the particle. Because the power is proportional to the square magnitude of the electric field (see Eq. 3.2), the aforementioned relations show that the magnitude of detected electric field is proportional to the apparent acceleration of the particle. This apparent acceleration is why t shows up in these formulae and not τ .

3.2 Radiation types of interest in beam physics

3.2.1 Synchrotron Radiation

Perhaps the simplest form of radiation from accelerators is synchrotron radiation. The particle engages in uniform circular motion and emits radiation because it is accelerating normal to the direction of motion. For this problem we take the path of the electron to be a circle given by the Lorentz force equation,

$$B\rho = \frac{p}{e}. \quad (3.11)$$

where B is the magnitude of the magnetic field, ρ is the bending radius and p is the particle momentum. The left side of Eq. 3.11 is often called the particle's rigidity because it is a measure of how much magnetic field is required to bend the trajectory of the particle. The rotational frequency of the particle is $\omega_0 = \frac{eB}{\gamma m_0}$.

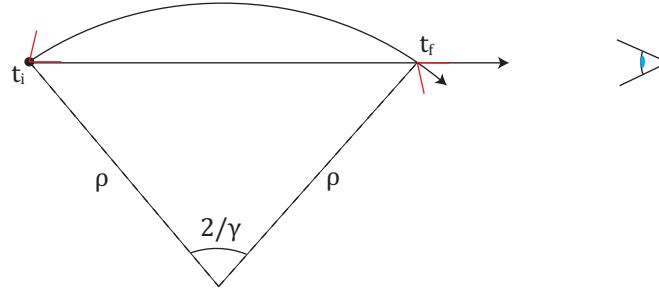


Figure 3.1: Schematic of synchrotron radiation emission in a bend. The electron travels along the arc of radius ρ while the radiation takes the 'short cut' of a straight line to the viewer. γ is usually several thousand, so the scale of the image is exaggerated. The red lines show the opening angle of the radiation at the initial and final points, labeled t_i and t_f .

Before solving the equations of motion it is useful to estimate the length of time over which the detector will see radiation. To do that we look at Fig. 3.1 and see that the particle moves through $\sim 2/\gamma$ radians of arc that the detector actually receives photons from. The first photon detected travels along a straight line from the point of emission to the detector, while the particle travels along an arc of radius ρ . The photon takes $t_i = L/c \simeq \frac{2\rho}{\gamma c}(1 - \frac{1}{6\gamma^2})$ to get to the point of last detected emission while the electron takes $t_f = S/(\beta c) \simeq \frac{2\rho}{\gamma c}(1 + \frac{1}{2\gamma^2})$, for a total time difference of $\Delta t = t_f - t_i = \frac{4\rho}{3\gamma^3 c}$. The length of the pulse at the observer is compressed by a factor of γ^3 which is an enormous number in

synchrotrons where γ can be many thousands. We note that the length of the pulse will determine its spectral content. When working in frequency space we find the so-called critical frequency, $\omega_c = 2/\Delta t$, or

$$\omega_c = \frac{3\gamma^3 c}{2\rho}. \quad (3.12)$$

This quantity is very useful for scaling the properties of the radiation produced in a bending magnet, it is a measure of the frequencies produced. The cause of this was described by K.-J. Kim as a time compression due to relativistic effects [42], in fact, he calls Eq. 3.4 the "time compression factor". Figure 3.2 shows the difference in the trajectory of the particle (on the left), using Eq. 3.13a, and the apparent trajectory of the particle (on the right) due to viewing the radiative fields. The compression of time causes the trajectory to pinch into a point. It is this apparent time compression that upshifts the observed frequency of the radiation by γ^2 , from $2\rho/\gamma c$ to ω_c .

The solution to the equations of motion is the familiar uniform circular motion (ignoring energy loss from radiation), if we choose the coordinate system centered on the particle at $\tau = 0$ and such that $\hat{\mathbf{x}}$ points in the direction of acceleration, $\hat{\mathbf{z}}$ in the direction of motion and $\hat{\mathbf{y}}$ perpendicular to both and the particle orbits in the xz -plane, the trajectory of the particle is given by

$$\mathbf{r}(\tau) = \rho(1 - \cos(\omega_0\tau))\hat{\mathbf{x}} + \rho\sin(\omega_0\tau)\hat{\mathbf{z}} \quad \text{and} \quad (3.13a)$$

$$\mathbf{v}(\tau) = \rho\omega_0\sin(\omega_0\tau)\hat{\mathbf{x}} + \rho\omega_0\cos(\omega_0\tau)\hat{\mathbf{z}}. \quad (3.13b)$$

Previously, it was shown that the opening angle of the radiation is of the order of $1/\gamma$, this means that the path length over which the particle emits radiation that is detected at a distant screen is small for a relativistic particle,

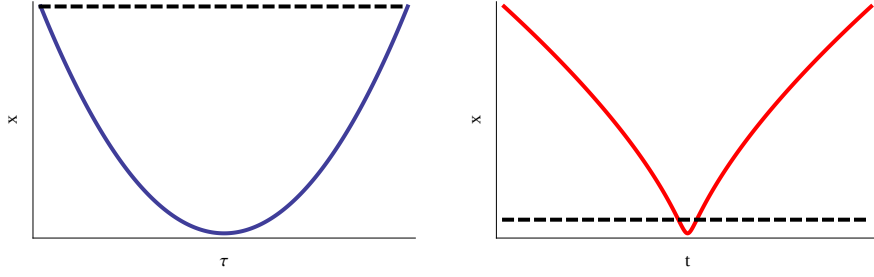


Figure 3.2: Time compression in synchrotron radiation due to relativistic effects as seen by an observer in the plane of motion. The figure on the left is the x -trajectory of the particle (following Eqs. 3.13), while the figure on the right is the apparent trajectory of the particle seen through observing the radiation field. At time $\tau = t = 0$ the particle is pointing directly at the observer. The dashed black line shows the same x position in both plots. The pinching of the trajectory from a circular section to a point is obvious. It is this observed acceleration that is responsible for the polarization, frequency and intensity properties of synchrotron radiation.

$S \sim \rho/\gamma$. Modern synchrotron sources that make use of bending magnet radiation have circumferences of around 100 m and 2 GeV electron beams, and therefore $S \sim 4$ mm. In this case we can assume the particle is stationary and cancel the $R(\tau) \rightarrow R$ in the numerator and denominator of Eq. 3.5. The path length in the phase of the radiation is not assumed stationary, because the particle moves very large distances compared to the wavelength of the radiation we are interested in.

Because the path length is going to sweep the radiation past a fixed point in the plane of motion, effectively integrating over the horizontal coordinate, we can choose the unit vector to the detection point as being in the xy -plane without loss of any information – this actually happens at bending magnets in synchrotrons, a carelessly placed detector will be ‘swept’ by the radiation producing a horizontal

stripe. So, we take $\hat{\mathbf{n}} = \sin(\psi)\hat{\mathbf{y}} + \cos(\psi)\hat{\mathbf{z}}$. Where ψ is a latitude angle with respect to the plane of motion.

To make the equation simpler we can take advantage of the fact that the part in square brackets in Eq. 3.5 is a perfect differential with respect to the retarded time, τ , and write the whole expression as [29]³

$$\begin{aligned} \frac{dW}{d\omega d\Omega} = & \frac{e^2}{4\pi^2 c} \left| \frac{\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta})}{1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}}} e^{-i\omega(\tau + R(\tau)/c)} \right|_{-\infty}^{-\infty} + i\omega \int_{-\infty}^{\infty} [\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta})] e^{-i\omega(\tau + R(\tau)/c)} d\tau \Big|^2, \end{aligned} \quad (3.14)$$

after integrating by parts. It is important to recognize that this method works because the motion at the end points don't contribute to the radiation collected by the detector and the boundary value terms in Eq. 3.14 can be ignored. Because all of the arguments of the sinusoids are small, $\psi, \omega_0\tau, 1/\gamma \ll 1$, we expand everything in the integral to linear order, except that in the phase, which gets expanded to third order and find

$$\frac{dW}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} \left[-\frac{c\tau}{\rho} \hat{\mathbf{x}} + \psi \hat{\mathbf{y}} \right] e^{-i\Phi(\tau)} d\tau \right|^2 \quad (3.15)$$

with the phase given by

$$\Phi(\tau) = \omega \left(\frac{\tau}{2\gamma^2} (1 + \gamma^2 \psi^2) + \frac{c^2 \tau^3}{6\rho^2} \right), \quad (3.16)$$

where the term x/c was discarded because the distance from the origin to the detector is arbitrary, it is just a phase offset. The most obvious consequence of

³ $\frac{\hat{\mathbf{n}} \times \{(\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}})^2} = \frac{d}{d\tau} \left[\frac{\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta})}{1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}}} \right]$

Eq. 3.15 is that there is no electric field to lowest order in the $\hat{\mathbf{z}}$ direction, as expected for radiation in the same direction. The integrals are done in lots of places [29, 30, 41, 42], the result is a spectral intensity of

$$\frac{dW}{d\omega d\Omega} = \frac{3e^2\gamma^2}{4\pi^2c} \left(\frac{\omega}{\omega_c}\right)^2 (1 + \gamma^2\psi^2)^2 \left[K_{2/3}^2(u) + \frac{\gamma^2\psi^2}{1 + \gamma^2\psi^2} K_{1/3}^2(u) \right], \quad (3.17)$$

where $u = \left(\frac{\omega}{2\omega_c}\right) (1 + \gamma^2\psi^2)^{3/2}$ and K_x is the modified Bessel function of order x . The first term in the square brackets is for the $\hat{\mathbf{x}}$ polarized radiation (also called σ -polarization) and the second term is the $\hat{\mathbf{y}}$ polarized radiation (also called π -polarization). There is an amazing array of properties from bending magnet radiation [29, 30, 41, 42], but we will concentrate on the properties that will effect the results of undulator radiation, which is really just a special kind of bending magnet radiation (or if you dig even deeper, it is all Compton scattering [82]).

Most apparent in Eq. 3.17 is that there is no vertically polarized radiation in the plane of motion. The simple physical explanation for this is that when the detector is in the plane of motion it cannot see any vertical curvature, it only sees the particle moving horizontally so the apparent acceleration is only in the horizontal plane and thus the radiation is only polarized in that direction. When the particle is viewed from out of the plane of motion the apparent acceleration is partially in the vertical direction and that causes the observer to see both polarizations, the radiation is actually elliptically polarized.⁴ It is clear from Figs. 3.3 and 3.4 that at larger angles, ψ , the spectrum is dominated by frequencies below the critical frequency and that higher frequencies are concentrated near the plane of motion.

Integrating Eq. 3.17 over all frequencies and angles shows that 7/8 of the

⁴There is a factor i difference between E_σ and E_π [41].

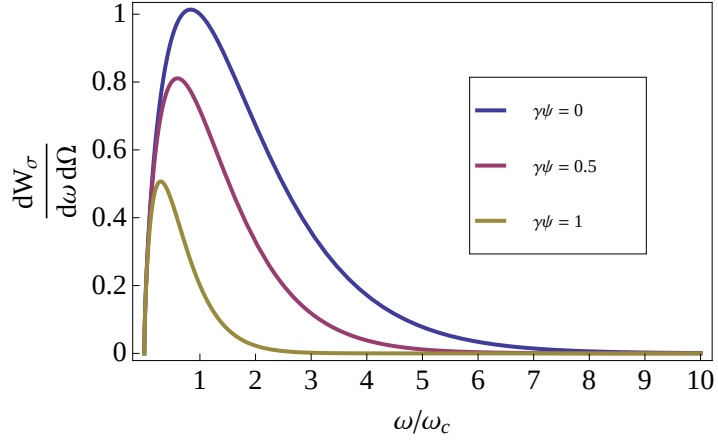


Figure 3.3: σ -polarized synchrotron radiation as a function of frequency for a number of different angles. The output is normalized to $\frac{dW_\sigma}{d\omega d\Omega} \Big|_{\omega=\omega_c}$.

power radiated is radiated in the σ -polarization and that the critical frequency is at the 50% energy point, half the energy emitted (in both polarizations combined) is at frequencies lower than ω_c and half is at higher frequencies, a feature that is sometimes used to *define* the critical frequency [41].

3.2.2 Edge Radiation

Edge radiation is a natural consequence of the design of synchrotrons. These machines are designed to have a set of dipole magnets to bend the trajectory of an electron into a circle, but there are spaces between the dipoles for other things like quadrupoles, sextapoles, and undulators. Edge radiation happens where the field of the dipole goes from the constant field of the dipole to free space, or vice-versa [83].⁵ In this case we start at Eq. 3.5 and assume the edge is both short compared to the distance to the detector and that the position of the detector can be described by the angles ψ and θ , where ψ is the same angle of latitude as

⁵Actually, it happens at the edges of other magnets too, but the canonical example is bending magnets.

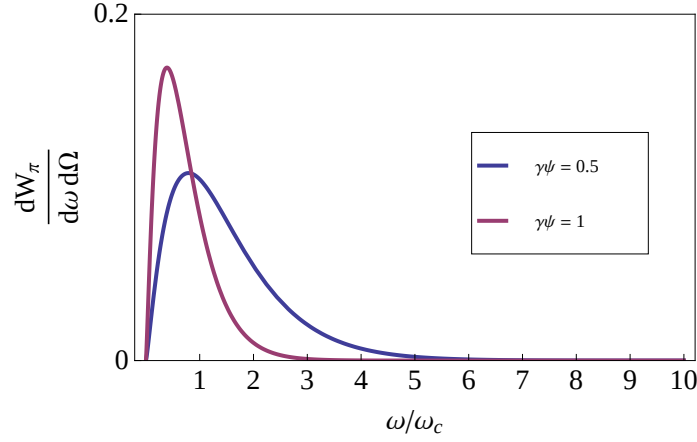


Figure 3.4: π -polarized synchrotron radiation as a function of frequency for a number of different angles. The output is normalized to $\frac{dW_\sigma}{d\omega d\Omega}|_{\omega=\omega_c}$. Note that the scale is $1/5$ of Fig. 3.3.

in the synchrotron case and θ is the angle from the edge to the detector in the plane of motion (see Fig. 3.5), such that $\hat{\mathbf{n}} = \theta\hat{\mathbf{x}} + \psi\hat{\mathbf{y}} + (1 - \theta^2/2 - \psi^2/2)\hat{\mathbf{z}}$. Expanding everything to second order in terms of order $1/\gamma$ returns

$$\frac{dW}{d\omega d\Omega} = \frac{e^2 \gamma^4}{\pi^2 c} (|F_\sigma|^2 + |F_\pi|^2) \quad (3.18a)$$

$$F_\sigma = -\frac{1}{\rho} \int_{-\infty}^{\infty} f(z') \frac{1 + \gamma^2 \psi^2 - \gamma^2 (\theta - \beta_x)^2}{[1 + \gamma^2 \psi^2 + \gamma^2 (\theta - \beta_x)^2]^2} e^{i\Phi} dz' \quad (3.18b)$$

$$F_\pi = \frac{2}{\rho} \int_{-\infty}^{\infty} f(z') \frac{\gamma^2 \psi (\theta - \beta_x)}{[1 + \gamma^2 \psi^2 + \gamma^2 (\theta - \beta_x)^2]^2} e^{i\Phi} dz' \quad (3.18c)$$

$$\Phi(z) = \frac{\omega}{2c\gamma^2} \int_0^z [1 + \gamma^2 \psi^2 + \gamma^2 (\theta - \beta_x)^2] dz' \quad (3.18d)$$

$$\beta_x(z) = \frac{1}{\rho} \int_{-\infty}^z f(z') dz', \quad (3.18e)$$

where $\rho = \frac{\gamma m_0 c^2}{e B_0}$ is the bending radius in the dipole and $f(z')$ is a unit less function that makes the change between the dipole field and zero field region: $B(z') = B_0 f(z')$, and we take $\beta_x(-\infty) = 0$. F_σ and F_π are the normalized elec-

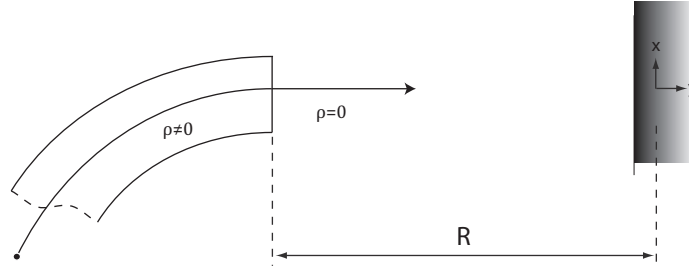


Figure 3.5: Diagram of edge radiation showing an electron leaving a dipole magnet. At the transition from constant magnetic field to field free region edge radiation is radiated by the electron to the the detector screen on the right.

tric field vectors. The phase relationship indicates that, unlike bending magnet radiation, the fields are radially polarized, this is because the acceleration is no longer along a circular arc. The motion this time is a transition from circular to linear, in the simplest sense it is a single linear kick in the plane of motion, which is why the polarization is also linear. Because edge radiation is just a special case of synchrotron radiation, meaning that each infinitesimal piece of path length is a piece of a circular trajectory with local radius of curvature, there is still no vertically polarized radiation in the plane of motion. A typical edge radiation distribution is shown in Figs. 3.6a and 3.6b, the radiation is emitted in a cone of opening angle $\sim 1/\gamma$ and the radiation is depressed on-axis ($\psi = \theta = 0$), but not zero. The radiation on the side of positive θ is the onset of synchrotron radiation, after a bend through an angle of $\sim 5/\gamma$ the radiation is pure synchrotron radiation and the edge can be ignored, this justifies the dropping of the boundary conditions when integrating Eq. 3.5 by parts to get Eq. 3.14.

An interesting result occurs when the edge radiation formalism is applied to an undulator that is short compared to the distance from the undulator to the detector. The electron motion looks like an edge where the electron beam acceleration isn't in one direction but oscillatory. If all you care about are wavelengths

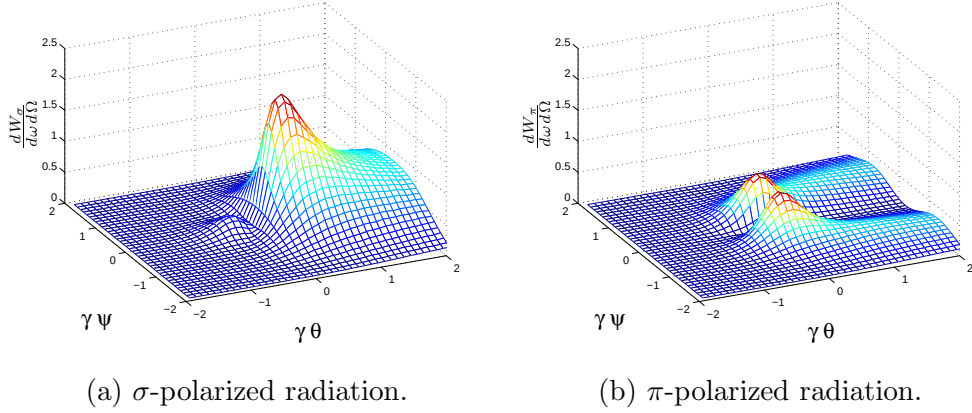


Figure 3.6: 100 nm radiation emitted by a single electron traversing a Gaussian shaped edge of length $\sigma = 2$ cm. The beam energy is 1 GeV.

long compared to the phase advance of the radiation relative to the electron beam (this is given by Eqs. 3.16 and 3.18d for synchrotron and edge radiation, respectively), then you can treat the undulator as a single linear kick with properties similar to edge radiation [84]. The principle difference is that the symmetry of the motion causes horizontal symmetry in the radiation which equalizes the two peaks seen in Fig. 3.6a, which also makes the radiation on-axis a true null. The radiation is still radially polarized in the case of a planar undulator.

3.2.3 Transition Radiation

Transition radiation occurs when a charged particle passes from a material of one permittivity to another [29, 30, 31, 85, 86]. Because transition radiation is used as a diagnostic we will sketch out the properties in the simplest case to demonstrate the physical properties of the radiation without getting mired in the details of a more faithful calculation. The simplest case of transition radiation is a particle going from vacuum into a metal with infinite conductivity. The charges

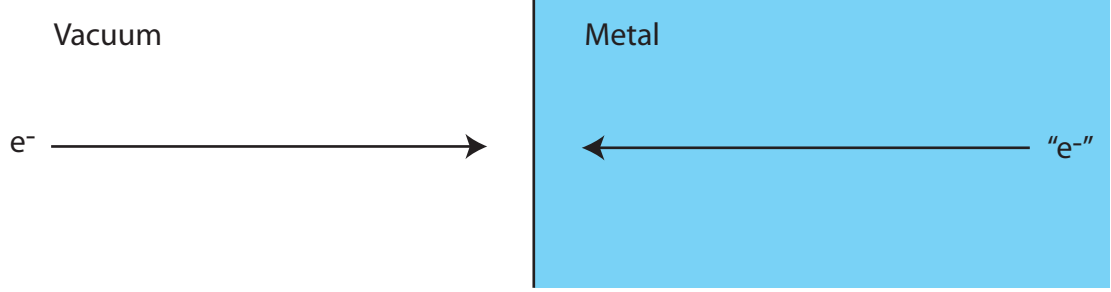


Figure 3.7: Diagram of transition radiation. The real electron moves from left to right until the surface of the metal where it meets the virtual electron moving from right to left. In this model, the radiation detected in vacuum comes from the virtual electron.

in the metal are modeled as an image charge of equal magnitude moving in the opposite direction from the other side of the boundary. As we saw earlier, the radiation from a relativistic particle is concentrated near the direction of motion, so an observer looking from the outside the metal actually sees the radiation fields from this virtual charge. The origin of the radiated fields is really the electrons in the metal moving around, conspiring to cancel the electric field of the charged particle parallel to the metal surface. This illustrates the power of the virtual particle method, the radiation fields actually come from the charges in the metal! The particles "annihilate" at the interface, and it is this apparent acceleration that causes the radiation.

Using Eq. 3.14, noting that $R(\tau)$ is a constant we find that

$$\frac{dW}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c} \left| \frac{\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta})}{1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}}} e^{-i\omega(\tau + R(\tau)/c)} \right|_{\tau=0}^2. \quad (3.19)$$

Contra the synchrotron radiation case, we only care about the radiation that comes from the instant the particle stops moving (the boundary conditions), which is defined to happen at $\tau = 0$, because the electron is not accelerating at

any other time.⁶ In this case,

$$\begin{aligned}
\frac{dW}{d\omega d\Omega} &= \frac{e^2}{4\pi^2 c} \left| \frac{\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta})}{1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}}} e^{-i\omega R(0)/c} \right|^2 \\
&= \frac{e^2}{4\pi^2 c} \frac{\beta^2 \sin^2 \Theta}{(1 - \beta \cos \Theta)^2} \\
&\simeq \frac{e^2 \gamma^2}{\pi^2 c} \frac{\gamma^2 \Theta^2}{(1 + \gamma^2 \Theta^2)^2},
\end{aligned} \tag{3.20}$$

where Θ is the angle between the direction of initial velocity and $\hat{\mathbf{n}}$ and the last line comes from assuming a very relativistic particle. The previous analysis was done for the particle moving the forward direction but, as previously stated, the backward moving image charge is the one that produces the radiation of interest (or we can reverse the domain composition such that the electron moves from the metal to vacuum). To find the radiation from that charge, we take $\beta \rightarrow -\beta$. Because the transition zone was assumed to be a delta function, Eq. 3.20 predicts an infinite amount of energy when integrated over frequency, this is clearly physically impossible. To resolve this we assume that the minimum allowed frequency is inversely proportional to the width of the transition (which can be figured using the surface roughness of the metal). While the maximum frequency is given by the plasma frequency of the electrons in the metal, frequencies higher than this cannot be created because the electrons cannot move fast enough to create them. All of the frequencies within this interval are created with equal intensity.

Once again, because the motion is not curved, the radiation is linearly polarized, but in this case the radiation intensity is symmetric about the axis of

⁶If the electron were moving between two materials and traveling faster than the phase velocity of light in one or both of the materials you would get Čerenkov radiation. See, for example [85, 86, 87].

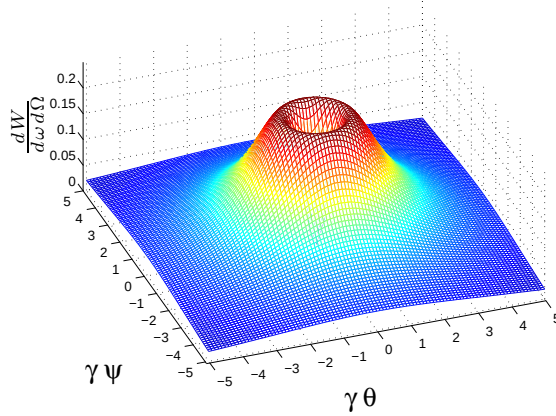


Figure 3.8: Distribution of transition radiation from an electron passing through a perfectly conducting metal sheet. For this image $\gamma^2\Theta^2 = \gamma^2\psi^2 + \gamma^2\theta^2$ (see Eqs. 3.20 and 3.18).

motion because there is nothing to break the symmetry, unlike the edge radiation. There is a null on axis for transition radiation because the electron motion does not sweep through the detector solid angle. Further of note is the scale of the radiation, transition radiation grows with the square of the normalized energy while edge radiation grows as γ^4 , the difference between the two is the same as the more generalized result in Eqs. 3.9 and 3.10, respectively, because the transition radiation is a specific case of longitudinal acceleration and edge radiation is a specific case of acceleration perpendicular to the velocity. The similarities between edge radiation and transition radiation can make it difficult to tell the two apart if there is reason to suspect that either could be present - for example, if a transition radiation screen is located downstream of a chicane.

3.3 Undulator Radiation

Undulator radiation is a special case of synchrotron radiation in which the electron oscillates in an alternating field that causes the radiation to interfere with itself and the combination leads to unique and very beneficial properties that have made it the backbone of synchrotron radiation sources. We saw in Eq. 1.28 that the sinusoidal magnetic field of the undulator leads to similarly sinusoidal motion in the electron. Using Fig. 3.9 we see that the difference in arrival time of two photons emitted one undulator period apart is $\Delta t = R_2/c + \lambda_u/(\bar{\beta}_z c) - R_1/c$ where $\bar{\beta}_z$ is defined below Eq. 1.29 and $R_2 = \sqrt{R_1^2 + \lambda_u^2 - 2R_1\lambda_u \cos \Theta} \simeq R_1 - \lambda_u \cos \Theta$ with the last equality coming from assuming $\lambda_u \ll R_1$. In order for the two photons (or wave fronts) to interfere constructively the phase difference between the two must be an integer multiple of 2π , $\omega \Delta t = 2\pi n$. Assuming all the terms of order $1/\gamma$ are much less than one, we find

$$\frac{k_u}{k_{r,n}} = \frac{\lambda_{r,n}}{\lambda_u} = \frac{1 + K^2/2 + \gamma^2 \Theta^2}{2n\gamma^2}. \quad (3.21)$$

Where $\omega_x/c = k_x = 2\pi/\lambda_x$, K is defined in Eq. 1.28c and the extra subscript n denotes the harmonic number. We can already see that the size of the electron oscillations, given by K , causes the electrons to traverse each period more slowly, resulting in constructive interference between longer wavelength photons. Further, when the photons are viewed away from the axis of electron motion the photon wavelength is also increased because the photons travel faster than the electrons (even along a straight line) and the increase in relative path length is larger when viewed off axis, on axis $R_1 - R_2 = \lambda_u$. The integer n is the harmonic number of the radiation, physically it is a result of the non-dispersive medium in which the photons are propagating, because all the photons travel at the speed

of light, c , the higher frequency photons in the harmonics can also positively interfere.

It is also possible to find Eq. 3.21 by shifting frames into the average rest frame of the electron, allowing for the electron motion due to the electric field of the apparent light pulse due to the undulator magnetic field and then shift back into the lab frame. Ignoring the details of the motion we find that the wavelength of the undulator shrinks to $\lambda' = \lambda_u/\gamma_z$ in the rest frame of the electron, and a photon emitted on axis has wavelength $\lambda'' = \frac{\lambda'_u}{\gamma_z^2(1+\beta_z)}$ in the lab frame.⁷ γ_z is the relativistic factor for $\beta = \bar{\beta}_z$ and is given by $\gamma_z^2 = \frac{1+K^2/2}{\gamma^2}$. The wavelength of the emitted photon is therefore given by Eq. 3.21. In this interpretation, the properties of the resonance condition come from a Lorentz contraction of the undulator period in the average rest frame of the electron and a Doppler shift of the emitted photon's wavelength from the average rest frame back to the lab frame.

Another property of the radiation that can be deduced from the simple interference model is the width of the harmonics of the radiation, which comes from the length of the undulator. If the undulator is N_u periods long than the length of the pulse received by the detector is $\Delta T \simeq \frac{N_u \lambda_u}{c} (1 + 1/(2\gamma^2) + K^2/(4\gamma^2) - \cos \Theta) = \frac{2\pi N_u n}{\omega_r}$. For a radiation pulse that is a square shape in either the frequency or time domain the full-width-at-half-maximum (FWHM) time-bandwidth product [88] is $\Delta\omega \Delta T \geq 0.9 \, 2\pi$, with equality for a transform limited pulse. In this case it is the motion in the time domain that is a square shape (we are ignoring the entrance and exit of the undulator), so we expect the shape in frequency space to be a sinc function of FWHM width at each harmonic of

⁷The general formula for the relativistic Doppler shift is $\omega' \equiv \gamma\omega(1 - \beta \cos \theta)$ [29].

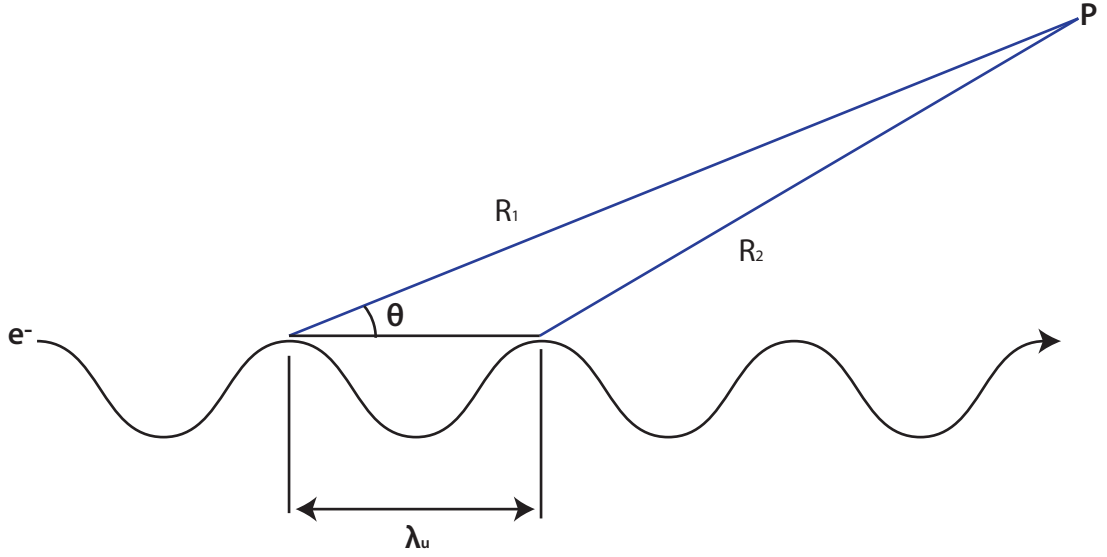


Figure 3.9: Interference between photons emitted at the same phase in the sinusoidal motion of the electron. The detector is located at point P.

$$\frac{\Delta\omega}{\omega_r} = \frac{0.9}{nN_u}. \quad (3.22)$$

The higher harmonics have narrower bandwidths because the fixed length undulator contains n times more radiation periods. Now that we have some idea of what to expect from the calculation for synchrotron radiation, we can proceed with the full calculation.

We start with the spectral intensity (Eq. 3.14) because we are ignoring the entrance and exit of the undulator (that was taken care of in subsection 3.2.2 on edge radiation). Once again, we define the unit vector to the detector as $\hat{\mathbf{n}} = \theta\hat{\mathbf{x}} + \psi\hat{\mathbf{y}} + (1 - \theta^2/2 - \psi^2/2)\hat{\mathbf{z}}$ and this time we take $\boldsymbol{\beta} = -\frac{K}{\gamma}\sin(k_uz)\hat{\mathbf{x}} + \left(\bar{\beta}_z + \frac{K^2}{4\beta\gamma^2}\cos(2k_uz)\right)\hat{\mathbf{z}}$. Multiplying out the double cross product returns

$$\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta}) = \left(\theta + \frac{K}{\gamma}\sin(k_uz)\right)\hat{\mathbf{x}} + \psi\hat{\mathbf{y}}. \quad (3.23)$$

In the above equation we have thrown out all terms smaller than first order in θ , ψ or $1/\gamma$, which further means $\bar{\beta}_z = 1$. Therefore, the spectral intensity becomes

$$\frac{dW}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{-T/2}^{T/2} \left[\left(\theta + \frac{K}{\gamma} \sin(k_u z) \right) \hat{\mathbf{x}} + \psi \hat{\mathbf{y}} \right] e^{-i\omega(\tau + R(\tau)/c)} d\tau \right|^2. \quad (3.24)$$

Next, we handle the limits of integration by noting that instead of integrating from $-T/2$ to $+T/2$, the length of the whole undulator, we can integrate over a single period, such that

$$\begin{aligned} \int_{-T/2}^{T/2} \dots d\tau &= \left(1 + e^{-i\omega(\frac{\lambda_u}{\beta_z c} + \frac{R_1 - R_0}{c})} + e^{-i\omega(\frac{2\lambda_u}{\beta_z c} + \frac{R_2 - R_0}{c})} + \dots \right) \int_{-\frac{\lambda_u}{2\beta_z c}}^{\frac{\lambda_u}{2\beta_z c}} \dots d\tau \\ &= \sum_{m=0}^{N_u-1} e^{-imx} \int_{-\frac{\lambda_u}{2\beta_z c}}^{\frac{\lambda_u}{2\beta_z c}} \dots d\tau \\ &= \frac{1 - e^{-iN_u x}}{1 - e^{-ix}} \int_{-\frac{\lambda_u}{2\beta_z c}}^{\frac{\lambda_u}{2\beta_z c}} \dots d\tau, \end{aligned} \quad (3.25)$$

where the arbitrary phase from the first term has been set to zero and the last line follows from the definition of the geometric series. x is the time delay (written as a distance) of the arrival of photons from the beginning of each period at the detector and is given by $x = \omega(\lambda_u/c)(1/\bar{\beta}_z - \cos \Theta)$, expanded as was done previously for each period, such that $R_m - R_0 = m\lambda_u \cos \Theta$. When expanded for small terms ($1/\gamma$ and θ) to second order we get $x = 2\pi\omega/\omega_{r,1}$, with $\omega_{r,1}$ given by Eq. 3.21. We find the spectral intensity is

$$\frac{dW}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} N_u^2 S(\omega) \left| \int_{-\frac{\lambda_u}{2\beta_z c}}^{\frac{\lambda_u}{2\beta_z c}} \left[\left(\theta + \frac{K}{\gamma} \sin(\bar{\beta}_z \omega_u \tau) \right) \hat{\mathbf{x}} + \psi \hat{\mathbf{y}} \right] e^{-i\omega(\tau + R(\tau)/c)} d\tau \right|^2, \quad (3.26)$$

with⁸

$$S(\omega) = \text{sinc}^2 \left[\frac{N_u \pi \omega}{\omega_{r,1}} \right]. \quad (3.27)$$

Eq. 3.27 is called the *lineshape function*, it is the result of interference between many periods of the undulator. To find the width of the line we find the half width and multiply by 2: $\text{sinc}^2(\pi y) = 1/2$ when $y \simeq 0.45$, such that $\frac{\omega_{1/2}}{\omega_{r,n}} = \frac{0.45}{N_u n}$ and $\frac{\Delta\omega}{\omega_{r,n}} = \frac{0.9}{N_u n}$, just as was found earlier with a time-bandwidth argument. It appears that the radiation from a undulator is time-bandwidth (also called Fourier transform) limited, the frequency content from undulator radiation is as narrow as the physical world will allow it to be.

Because the lineshape function multiplies the whole spectral intensity it does its spectral narrowing of the radiation (as compared to synchrotron radiation) over the whole radiation distribution. Later, when we calculate the integral to find the radiation distribution it should be kept in mind that, at each point in that distribution, the central wavelength is given by Eq. 3.21 with a FWHM given by Eq 3.27. This means if we want to limit the bandwidth of the radiation collected we need to use an aperture to prevent the longer wavelengths from being collected. How small can this aperture be made? The most efficient answer is to make the aperture large enough to accept all the radiation with the same bandwidth that is emitted on axis ($\theta = 0$).

$$8 \frac{\sin(N_u y)}{\sin y} = N_u \text{sinc}(N_u y)$$

$$\begin{aligned}
\frac{\delta\omega}{\omega_{r,1}} &\simeq \frac{0.45}{N_u} \\
\frac{\omega_{r,1}(0) - \omega_{r,1}(\Theta)}{\omega_{r,1}(\Theta)} &\simeq \frac{0.45}{N_u} \\
\frac{1 + K^2/2 + \gamma^2\Theta^2}{1 + K^2/2} - 1 &\simeq \frac{0.45}{N_u}
\end{aligned} \tag{3.28}$$

and therefore,

$$\Theta_D \simeq \sqrt{0.45 \frac{1 + K^2/2}{\gamma^2 N_u}}. \tag{3.29}$$

What this means is that an aperture of opening angle Θ_D is the minimum aperture size that can be used to limit the bandwidth of the radiation, any smaller and the aperture is cutting off photons without actually decreasing the bandwidth. This does not mean, however, that the radiation distribution gets smaller in angle as you add periods to the undulator. The resonance condition in Eq. 3.21 is still true and the transverse distribution of radiation on the detector is given by the parts of Eq. 3.26 that we haven't solved yet. Up to this point we did not need to assume anything about the motion of the electron in the magnetic field except for the periodicity, we could have come just as far without multiplying out the triple cross product in Eq. 3.23. What we have assumed is that the electron travels a period of distance λ_u with average speed $\bar{\beta}_z c$. What the exact nature of the periodic motion is, is irrelevant, as long as the motion is periodic, the lineshape function ensures that there will be narrow (for large N_u) lines at all harmonics of the period length. We move now to evaluating the spectral intensity for an electron in an undulator.

The first thing to notice is that, once again, there is no vertically ($\hat{\mathbf{y}}$) polarized radiation in the plane of motion. But it is also obvious that the only reason

there is any horizontally ($\hat{\mathbf{x}}$) polarized radiation on axis is because the electron accelerates in the horizontal plane. The goal now is to understand how the phases of the photons emitted by the electron add up at the detector. The trajectory of the electron is

$$\mathbf{r}(\tau) = \frac{cK}{\gamma\bar{\beta}_z\omega_u} \cos(\bar{\beta}_z\omega_u\tau)\hat{\mathbf{x}} + \left(\bar{\beta}_zc\tau + \frac{cK^2}{8\gamma^2\bar{\beta}_z\omega_u} \sin(2\bar{\beta}_z\omega_u\tau)\right)\hat{\mathbf{z}}, \quad (3.30)$$

and then the phase is⁹

$$\begin{aligned} \frac{\Phi(\tau)}{\omega} &= \tau + R(\tau)/c = \tau + \hat{\mathbf{n}} \cdot (\mathbf{x}/c - \mathbf{r}(\tau)/c) \\ &= \frac{\omega_u}{\omega_r}\tau - \frac{K^2}{8\gamma^2\bar{\beta}_z\omega_u} \sin(2\bar{\beta}_z\omega_u\tau) - \frac{\theta K}{\gamma\bar{\beta}_z\omega_u} \cos(\bar{\beta}_z\omega_u\tau). \end{aligned} \quad (3.31)$$

Which means there will be cosines in the exponential in Eq. 3.26. To take care of them we use the Jacobi-Anger expansion [89]:

$$e^{iz \cos \varphi} = \sum_{n=-\infty}^{\infty} i^n J_n(z) e^{in\varphi} \quad \text{and} \quad (3.32a)$$

$$e^{iz \sin \varphi} = \sum_{n=-\infty}^{\infty} J_n(z) e^{in\varphi}. \quad (3.32b)$$

Using this we write the exponential part of Eq. 3.26 as

$$e^{-i\frac{\omega\bar{\omega}}{\bar{\beta}_z\omega_r}\tau} \left[\sum_{p=-\infty}^{\infty} J_p\left(\frac{\omega K^2}{8\gamma^2\bar{\omega}}\right) e^{i2p\bar{\omega}\tau} \right] \left[\sum_{q=-\infty}^{\infty} (i)^q J_q\left(\frac{\omega\theta K}{\gamma\bar{\omega}}\right) e^{iq\bar{\omega}\tau} \right], \quad (3.33)$$

where $\bar{\omega} = \bar{\beta}_z\omega_u$. Starting with the vertical ($\hat{\mathbf{y}}$ or π) polarized term we find that, because we are integrating over a period, the integral is zero unless all the stuff

⁹Some constants have been dropped from Eqs. 3.30 and 3.31 because constants in the phase don't matter.

in front of the τ in the exponential is equal to zero, in which case the integral evaluates as $2\pi/\bar{\omega}$. In this case, the term $\omega/(\bar{\beta}_z\omega_r)$ is equal to the harmonic number n . Indeed, it had to be, otherwise the interference condition between the periods given by the lineshape function and the intra-period interference condition given by Eq. 3.33 wouldn't overlap in frequency space and undulator radiation would be difficult to detect. The simple physical interpretation is that the motion must be periodic, which we assumed early on, to have the large growth in intensity. So we have $n - 2p - q = 0$ and the vertically polarized radiation is written as ¹⁰

$$\frac{dW_\pi}{d\omega d\Omega} = \frac{e^2 n^2 \gamma^2}{c(1 + K^2/2 + \gamma^2 \Theta^2)^2} N_u^2 S(\omega) \left| 2\gamma\psi \sum_{p=-\infty}^{\infty} J_{n+2p}(v) J_p(\xi) \right|^2, \quad (3.34)$$

with $\xi = \frac{nK^2}{4(1+K^2/2+\gamma^2\Theta^2)}$ and $v = \frac{2n\gamma\theta K}{1+K^2/2+\gamma^2\Theta^2}$. There is a similar term for the horizontal ($\hat{x}$ or σ) polarized term, but there is also a term with a sine in the integrand, resulting in terms that require $n - 2p - q \pm 1 = 0$. In total the horizontally polarized radiation is given by

$$\begin{aligned} \frac{dW_\sigma}{d\omega d\Omega} &= \frac{e^2 n^2 \gamma^2}{c(1 + K^2/2 + \gamma^2 \Theta^2)^2} N_u^2 S(\omega) \\ &\times \left| \sum_{p=-\infty}^{\infty} \left[2\gamma\theta J_{n+2p}(v) - K(J_{n+2p-1}(v) + J_{n+2p+1}(v)) \right] J_p(\xi) \right|^2. \end{aligned} \quad (3.35)$$

Eqs. 3.34 and 3.35 are not defined at any particular frequency, the resonance condition is still true, so when we investigate the properties of undulator radiation it is important to remember that we are often not looking at a single frequency. The three conditions on the periodicity of the motion ($n - 2p - q = 0$ and

¹⁰Using the identity $J_{-m}(z) = e^{-i\pi m} J_m(z)$ and then taking $p \rightarrow -p$ to flip the order of the summation.

$n - 2p - q \pm 1 = 0$) have shown that the frequency of the radiation is strongly peaked around integer values of the resonance frequency, which were called $\omega_{r,n}$ earlier, so we take this opportunity to rewrite the lineshape function as

$$S(\omega) = \sum_{n=1}^{\infty} \text{sinc}^2 \left[N_u \pi \frac{\omega - \omega_{r,n}(\Theta)}{\omega_{r,1}(\Theta)} \right]. \quad (3.36)$$

Which is a series of peaks of width $\sim 1/N_u$ at each harmonic number as shown in Fig. 3.10.

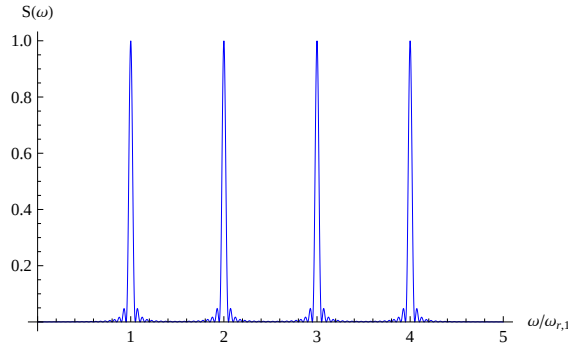


Figure 3.10: Line shape function for a 20 period undulator, the fundamental and the first three harmonics are shown. The lines are very well separated, even with a relatively low value for N_u .

Plots of the lowest three harmonic flux distributions are shown in Fig. 3.11. Undulator radiation is linearly polarized because the radiation from the first half of each period (when considering the fundamental harmonic) is polarized in one circular direction (say right handed) and the radiation from the second half of the period is polarized in the opposite direction. At large distances and for frequencies obeying the resonance condition the wavefronts from each half are π out of phase with each other, keeping in mind that the resonance condition requires that the phase difference across a whole period be 2π , and this causes the two oppositely circularly polarized wavefronts to add constructively to form

a linearly polarized wavefront. Asymmetric wigglers avoid this cancellation by designing the lattice such that field in one direction is larger than the other [90].

At each harmonic the σ -polarized radiation is an order of magnitude larger than the π polarized radiation. It is not surprising at this point that there is no vertically polarized radiation in the plane of motion ($\psi = 0$), because undulator radiation is simply adding up synchrotron radiation in a special way and synchrotron radiation doesn't have any vertically polarized radiation in the plane of motion. But why are there no even harmonics on axis? An observer looking on axis sees an electric field that is purely an odd function of time and because of this the frequency content cannot have any even components. When the observer is looking from off axis, the electric field is neither even nor odd as a function of time and so off axis there can be both even or odd harmonics, although the specific phase relationships may suppress either or both at any given location of observation.

Another way to imagine the lack of even harmonics on axis is to think of each period of undulator producing two photons per period which are born π out of phase with each other, if they are observed along the axis there is no extra distance to cause the phase shift required for constructive interference. This is also why there are n maxima for the σ -polarized radiation. n was the integer number of 2π between the phases of photons emitted at the beginning and end of each period. In order to see the larger phase advances $n > 1$, the observer must move off axis to allow the photons enough relative distance difference to advance another $(n - 1) \times 2\pi$. This is why there are $n - 1$ off axis maxima for the odd harmonics and n off axis maxima for the even harmonics.

Also of note is the symmetry of the radiation. We need only look at one quadrant of the image (say $\theta, \psi \geq 0$) to know what the radiation looks like over

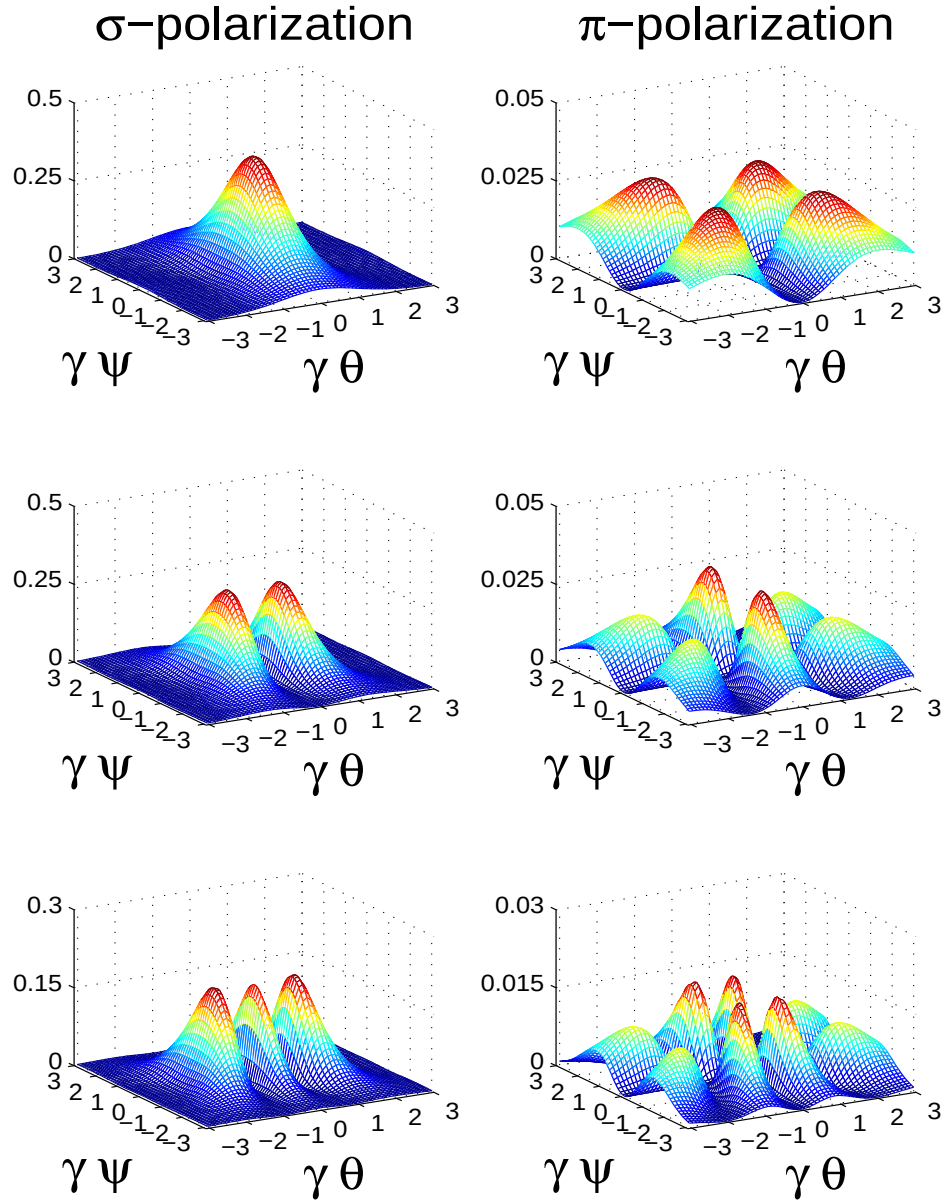


Figure 3.11: Frequency integrated undulator radiation distributions for (top to bottom) $n = 1, 2, 3$. The formulas are given by Eq. 3.35 on the left and Eq. 3.34 on the right. For this image $K = 1$.

the whole screen. This is because the plane of motion is a symmetry plane in the vertical direction and the motion within the plane is symmetric as well. In the case of an asymmetric wiggler the horizontal symmetry is broken but the vertical symmetry remains.

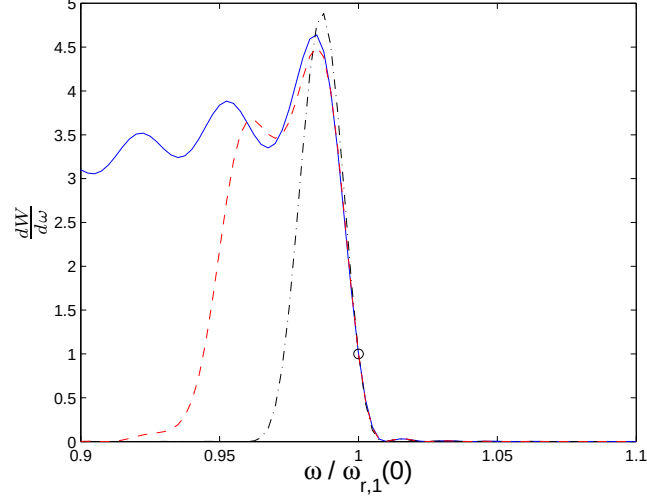


Figure 3.12: Angle integrated flux from a 20 period undulator at constant frequency. The different lines are for different limits to the detection aperture. Blue has no aperture, red dashed has an aperture given by $\gamma\Theta = \sqrt{\frac{1+K^2/2}{N_u}}$ and black dash-dot has an aperture given by Eq. 3.29. The point for $\omega/\omega_{r,1}(0) = 1$ is marked with a black circle and is set to 1 for each graph. The peak in the angle integrated spectrum does not occur at the maximum frequency allowed by the resonance condition.

From the images in Fig. 3.11 it might seem like the most intense wavelength is the fundamental on axis ($\omega_{r,1}(0) = \frac{2\omega_u\gamma^2}{1+K^2/2}$), which also happens to be the shortest wavelength. But this isn't the case. To find the frequency of maximum flux we integrate Eqs. 3.34 and 3.35 numerically over all angles while holding ω to a particular frequency. The result, shown in Fig. 3.12, displays that there is

approximately four times as much flux when looking at a frequency detuned from the on-axis frequency by about $1/N_u$ [41, 42]. This is because the bandwidth of the radiation given by Eq. 3.36 allows for more total radiation to be collected when the radiation is slightly detuned from the on axis resonance wavelength.

Another way of looking at the angular-spectral content of undulator radiation is shown in Fig. 3.13. Subsidiary maxima of the lineshape function (cf Fig. 3.10) appear as the smaller curves under the main red curve.¹¹ It is these subsidiary maxima that cause the oscillations in the low photon frequency side of Fig. 3.12.

The oscillations in Figs. 3.12 and 3.13 occur at distances given by the maxima of the lineshape function which occur approximately when argument of the sinc is equal to $(m + 1/2)\pi$. The frequency of the m^{th} on axis peak of the fundamental is then given by $\omega_m = \omega_{r,1}(0)(1 - (m + \frac{1}{2})/N_u)$, and the distance between peaks is $1/N_u$.¹²

3.3.1 Some useful limits

It can be seen from Eq. 3.35 that the undulator parameter, K , is essential for breaking the symmetry between the vertical and horizontal polarizations and that the same term is the one that produces the spectral intensity on axis. It will be useful in Chapter 4 to know what the spectral intensity on the axis is. To find out we take $\theta = \psi = 0$ to find that $v = 0$ and $\xi = \frac{nK^2}{4+2K^2}$. The only Bessel function that is non-zero when the argument is zero is J_0 , this limits the values that p can take on in Eq. 3.35 and the resulting spectral intensity on axis is

¹¹For obvious reasons we call these boomerangs.

¹²It seems like the subsidiary peaks are an artifact of the calculation and that they would be very difficult to detect. They usually are difficult to detect because they are covered up by other bandwidth effects such as electron beam emittance and energy spread. But when you put a prebunched beam into the undulator such that the bunch radiates coherently, they can actually be seen. See section 3.5.

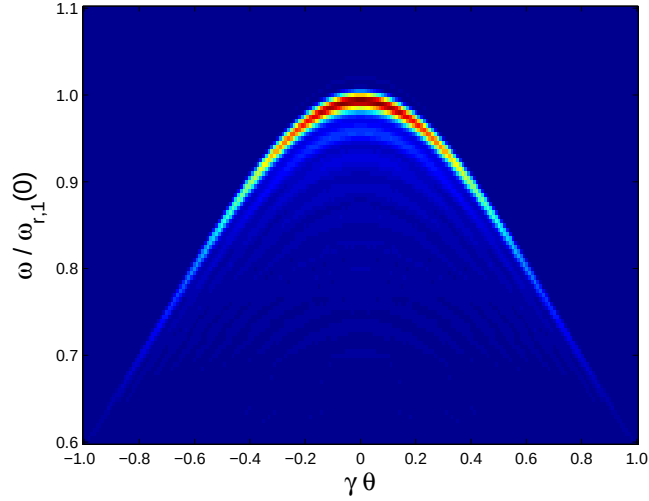


Figure 3.13: The double differential spectrum of undulator radiation from a 20 period undulator ($K=1$) that is integrated over the vertical direction. This false color image shows radiation intensity from lowest (blue) to highest (red). The most intense radiation occurs just below on axis resonance ($\frac{\omega}{\omega_{r,1}(0)} = 1$) because of the bandwidth of the radiation given by Eq. 3.36.

$$\left. \frac{dW}{d\omega d\Omega} \right|_0 = \frac{e^2 n^2 \gamma^2 K^2}{c(1 + K^2/2)^2} N_u^2 S(\omega) \left| J_{\frac{n-1}{2}} \left(\frac{nK^2}{4 + 2K^2} \right) - J_{\frac{n+1}{2}} \left(\frac{nK^2}{4 + 2K^2} \right) \right|^2. \quad (3.37)$$

The radiation is entirely horizontally polarized. The Bessel functions inside the absolute value are often abbreviated as $[JJ]$. Further, $F_n(K) \equiv \frac{n^2 K^2}{(1 + K^2/2)^2} [JJ]^2$ is the dependence of the on axis spectral intensity on K [41]. Fig. 3.14 shows how $\sqrt{n^2 K^2 [JJ]^2}$ and $F_n(K)$ depend on K for the first three odd harmonics. For the fundamental, the highest spectral intensity occurs for $K \approx 1$, while the free-electron laser efficiency (see Chapter 4) grows continuously with increasing K . These functions are why it is important to prevent K from getting too small while reducing the period length.

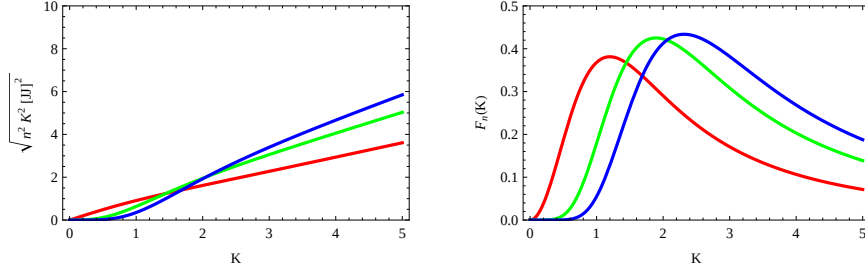


Figure 3.14: The functional forms of $\sqrt{n^2 K^2 [JJ]^2}$ (left) and $F_n(K)$ (right) as functions of K for the first three odd harmonics: $n = 1$ (red), 3 (green) and 5 (blue).

In the limit of $K \rightarrow 0$ the angular dependence of the radiation takes on the form [42, 91]

$$\frac{dP}{d\Omega} \rightarrow \frac{1 + 2\gamma^2(\psi^2 - \theta^2) + (\gamma^2(\theta^2 + \psi^2))^2}{(1 + \gamma^2(\theta^2 + \psi^2))^5}. \quad (3.38)$$

Although it seems somewhat paradoxical that there can be radiation with $K = 0$, and thus no magnetic field, what we are really doing is taking the equation to the limit of a single incident photon. In that case it doesn't make much sense to talk about a macroscopic magnetic field of the type produced by an undulator. The distribution is shown in Fig. 3.15, and it turns out that this distribution is the very same as for polarized Thomson scattering, as we will see in section 3.4.

This limit is interesting because it shows that the shape of the photon distribution from an undulator source, assuming that each electron emits only one photon, does not depend on the transverse motion of the electron. Of course, classically, the motion was critical because an acceleration is required to produce radiation. But in the single photon emission regime, the process is quantum in nature and it turns out that the image in Fig. 3.15 is the relativistic projection of the Compton cross section into the lab frame. In the rest frame of the electron

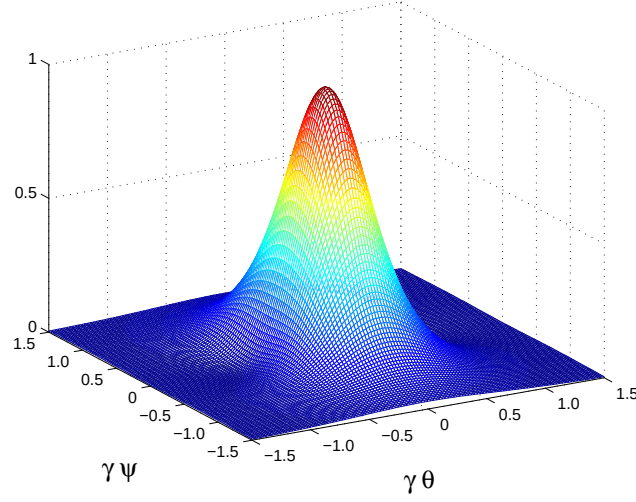


Figure 3.15: Angular radiation pattern in the limit that $K \rightarrow 0$. The distribution is the same as that for Thomson scattering from a linearly polarized photon.

the radiation distribution is that of simple harmonic motion [29].

3.4 Inverse Compton Scattering

Compton scattering occurs when a photon collides with an electron and a new photon, of different energy, flies off in some new direction while the electron recoils to conserve momentum [92]. Thomson scattering is the classical analog in which the electron does not recoil and the final photon is emitted with the same energy as the initial photon [93]. The basic result of Compton scattering comes from analyzing the system shown in Fig. 3.16, in which an electron moving with reduced velocity β (and energy $\gamma m_0 c^2$) collides with a photon of energy $\hbar\omega$. The energy of the scattered photon is given by

$$\hbar\omega' = \frac{(1 - \beta \cos \theta_i) \hbar\omega}{(1 - \beta \cos \theta_f) + (1 - \cos(\theta_i - \theta_f)) \frac{\hbar\omega}{\gamma m_0 c^2}}. \quad (3.39)$$

In the limit $\beta \rightarrow 0$ this reduces to Compton's famous formula. The scattered photons are concentrated in the initial direction of motion of the electron [94]. It also reduces to something remarkably similar to the resonance condition for undulator radiation given by Eq. 3.21, when a head on collision is assumed ($\theta_i = \pi$):¹³

$$\frac{\lambda'}{\lambda} = \frac{1}{4\gamma^2} (1 + \gamma^2 \theta_f^2) + \frac{\hbar\omega}{\gamma m_0 c^2}, \quad (3.40)$$

when θ_f is assumed to be small and terms small compared to 1 are dropped. The last term is usually small because the laser photon energy is of the order of 1 eV, so when γ is a few thousand the term is very small. It is the effect of the electron recoil, which always acts to increase the wavelength of the emitted photon. If we ignore this term, there are still a few differences between Eqs. 3.40 and 3.21. The most obvious omission is the lack of an undulator parameter, that it doesn't show up in Eq. 3.40 makes sense because K comes from the relativistic transverse motion of the electron [95] which we have omitted here. From another point of view it comes from an electron scattering from multiple photons at once [96], which has also been ignored. The case of electrons colliding with single photons and not recoiling, which we assume through out this dissertation, is called linear Compton scattering or Thomson scattering.

The factor of 4 that shows up in the numerator of Eq. 3.40 is larger than the 2 in Eq. 3.21 because the photon is traveling at the speed of light in the lab frame so it is Doppler shifted twice (rather than the simple Lorentz contraction seen in

¹³It has been shown that all synchrotron radiation based radiation, which includes edge and undulator radiation, is equivalent to Compton scattering [82]. For undulator radiation, in particular, this seems manifestly true for weak undulators ($K \ll 1$) because in the frame moving with average velocity $\bar{\beta}_z$, the electron undergoes simple harmonic motion in both cases. If the physics is the same in one frame, the results in another—in this case the lab frame—have to be consistent as well.

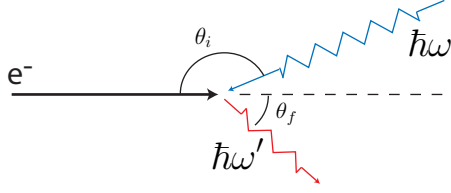


Figure 3.16: Schematic of single-photon-single-electron ICS scattering. The recoil of the electron always causes the scattered photon wavelength to grow compared to the non-recoil case.

the undulator case) and the wavelength of the emitted photon in the lab frame is $\frac{\lambda'}{\lambda} = \frac{1}{\gamma^2(1+\beta)^2} \simeq \frac{1}{4\gamma^2}$.

Before moving on to work out the details of beam-beam ICS it is useful to see how the parameters in Eq. 3.39 affect the detected photon energy when they vary slightly from their assumed values, this will give some estimates of the error encountered when attempting to measure the energy of a photon produced via ICS. For the rest of this dissertation we will assume that the collision is nearly head on ($\theta_i = \pi + \Delta\theta_i$) and that the electron is highly relativistic such that the radiation is predominantly scattered in the initial direction of motion of the electron ($\gamma \gg 1$ and $\theta_f \ll 1$). In this case we find that the uncertainty in the energy of the emitted photon has four contributions from collision angle, emission angle, initial electron energy and initial photon energy. The list of all of these errors, the relative errors they cause in the emitted photon energy and the common causes of the error are shown in Table 3.2.

The first item in Table 3.2 is almost always ignorable in ICS experiments because the angle errors are usually on the order of mrad, which means that the error is on the order of 10^{-8} – much smaller than the other terms in the table. To assure good temporal overlap and large enough luminosity (see Eq. 3.53) in short pulse ICS, the Rayleigh length ($L_R = \frac{\pi w_0^2}{\lambda}$, w_0 is the waist size) of the laser and

Table 3.2: Causes of emitted photon energy deviations, the size of the relative error and common causes in inverse Compton scattering sources. Follows [94] and [97].

Parameter	Relative Error	Common Causes
Collision Angle ($\Delta\theta_i$)	$-\frac{(\Delta\theta_i)^2}{4}$	Misalignment of laser and/or electron beam at the interaction point. Laser focusing.
Electron Energy ($\Delta\gamma$)	$2\frac{\Delta\gamma}{\gamma}$	Electron beam energy spread.
Photon Energy ($\hbar\Delta\omega$)	$\frac{\hbar\Delta\omega}{\hbar\omega}$	Laser bandwidth.
Observ. Angle ($\Delta\theta_f$)	$-\gamma^2(\Delta\theta_f)^2$	Electron beam emittance and opening angle of detection.

the beta function of the electron beam are made approximately equal, usually on the order of a mm. For a CO₂ laser, this corresponds to a waist size of 60 microns and angles, within one Rayleigh length of the interaction point, of less than 130 μ rad. Unless the laser is focused extremely hard the angles created by laser focus are very small. Errors due to misalignment can be much larger, on the order of mrad, and must be handled with a careful alignment procedure.

The energy spread of a RF photoinjector electron beam is typically on the order of 0.1% after the beam is transported through a dispersive section with a collimator [34]. So the error in the detected photon energy is of the same order.

The bandwidth of the laser is limited by the time-bandwidth product. If we assume a Gaussian shaped pulse that limit is $\Delta\omega\Delta T \geq 0.44$. When the length of the laser pulse is taken to be one Rayleigh length the bandwidth is given by $\Delta\omega \geq 0.44c/L_R$. Which leads to a relative bandwidth of $\frac{\Delta\omega}{\omega} \geq 7.4 \times 10^{-4}$. Because there is no upper limit on the bandwidth, it could be very large, but a time-bandwidth limited pulse will have small error due to laser bandwidth.

The last item in the table, the detection angle, is the most important. This term dominates the error of the observed photon energy with everything from electron beam emittance to opening angle of the detector to the transverse resolution of the detector muddying the energy measurement. For example, a detector which detects all the photons within an angle $1/\gamma$ of the axis will see 50% bandwidth, by far the biggest contribution to photon energy error. The photons observed off axis are always of lower energy. Therefore, while the relative error is negative, the effect is to count more (assumed) high energy photons.

More pernicious is that emittance causes lower energy photons to be emitted on axis. Because the angle θ_f in the Compton scattering formula is measured from the direction of electron motion, when an electron travels through the interaction point at an angle ϑ , the photon emitted on the axis of electron beam propagation is of lower energy. To estimate the effect, we look at the angle with which the electron at the root mean square radius (RMS) of the electron bunch travels through the interaction point (see section 1.2.3), this electron has angle $\vartheta = \epsilon/\sigma = \sqrt{\epsilon/\beta}$, where σ is the RMS waist size. The electron beams used in ICS sources typically have normalized emittances of the order 1 mm mrad, leading to an angle of 3 mrad from the beam axis and a relative energy shift of nearly 20% for a 70 MeV electron beam.

These same arguments can be made about undulator radiation, because the resonance condition in Eq. 3.21 has the same functional form. The principle difference is that there are no focusing effects in the undulator magnetic field and the bandwidth of the undulator periodicity (λ_u) is assumed to be zero. The bandwidth of the radiation is given by the interference effect, the angle-energy correlation of emitted photons and the energy spread of the electron beam. This is not to say that focusing the electron beam through an undulator doesn't have

effects on the output radiation, but rather that they were not included in the single electron formalism of the previous section.

3.4.1 The Compton Scattering Cross Section

Unlike undulator radiation, which was solved via the classical motion of the electron in a magnetic field, Compton scattering is a purely quantum event that reduces to a classical analog when the recoil of the electron can be ignored, the classical analog being Thomson scattering [93]. The fully relativistic and quantum Compton scattering cross section is called the Klein-Nishina cross section [98], which is rarely needed in its entirety for accelerator based Compton scattering because the incident photon energies are too low. For example, the recoil term in Eq. 3.39 is of the order 10^{-9} for a $10.6\text{ }\mu\text{m}$ photon and a 70 MeV electron.

The principle advantage of inverse Compton scattering (ICS) over undulator radiation is that the initial wavelength of the laser (as opposed to the period of the undulator) is much shorter, typically by 4 orders of magnitude. The difference in the wavelength regimes is caused by the dearth of undulator technologies at sub-mm periods, while typical lasers operate in the 10 micron (CO_2), 1 micron (Nd:YAG) and 800 nm ($\text{Ti:Al}_2\text{O}_3$) regimes. The drawbacks, are that the electron beam and laser beam must be very tightly focused to get sufficient luminosity at the interaction point such that there are a relatively large number of photons created. Thomson scattering, and Compton scattering for that matter, require high intensity lasers because the Thomson cross section is really small. Thomson scattering occurs when a plane wave is incident on an initially stationary electron. The electric field of the plane wave causes the electron to oscillate non-relativistically and that causes dipole-like radiation.

The total Thomson cross section is found by integrating the latter half of Eq.

3.6 over the whole solid angle and assuming an acceleration from an incident plane wave of $\dot{v} = \frac{eE_0}{m} \sin \omega t$ the result is ¹⁴

$$\sigma_T = \frac{8\pi}{3} r_e^2, \quad (3.41)$$

where $r_e = \frac{e^2}{m_0 c^2} = 2.82 \times 10^{-15}$ m is the classical electron radius. The conversion efficiency for Thomson scattering is really small! Thomson scattering is really just a special case of the radiation predicted by Larmor's formula when the acceleration of the electron is proportional to e/m .

For inverse Compton scattering the electron is not stationary in the lab frame. In order to calculate what the distribution of radiation looks like it is easiest to use a Lorentz invariant form of the scattering cross section. For a head on collision between an unpolarized electron and a linearly polarized photon the differential cross section is [97, 99]

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & \frac{8r_e^2}{X^2} \left(\frac{\hbar\omega'}{m_0 c^2} \right)^2 \\ & \times \left\{ \cos^2(\phi_f) \left[\left(\frac{1}{X} - \frac{1}{Y} \right)^2 + \frac{1}{X} - \frac{1}{Y} \right] + \frac{1}{4} \left(\frac{X}{Y} + \frac{Y}{X} \right) \right\}, \end{aligned} \quad (3.42)$$

with (in the lab frame only)

$$X = \frac{2\gamma\hbar\omega(1+\beta)}{m_0 c^2}, \quad \text{and} \quad (3.43a)$$

$$Y = \frac{2\gamma\hbar\omega'(1-\beta \cos(\theta_f))}{m_0 c^2}, \quad (3.43b)$$

¹⁴The definition of the cross section is the total amount of radiated power (W) divided by the total input intensity (W/m²). The units of cross section are therefore m². The intensity of the incoming plane wave is $u = \frac{cE_0^2}{4\pi} \sin^2(\omega t)$.

where ϕ_f is the azimuthal angle of the detected photon relative to the direction of polarization of the initial photon.

When we assume that the electron is ultra relativistic ($\gamma \gg 1$) and that the radiation is scattered mostly in the forward direction in the lab frame ($\theta_f \ll 1$), which we have seen is the case for ultra relativistic scattering, the differential radiated power takes on the form

$$\frac{dP}{d\Omega} \sim \hbar\omega' \frac{d\sigma}{d\Omega} \rightarrow \frac{1 + 2\gamma^2(\psi^2 - \theta^2) + (\gamma^2(\theta^2 + \psi^2))^2}{(1 + \gamma^2(\theta^2 + \psi^2))^5}. \quad (3.44)$$

This is just Eq. 3.38! It appears that buried inside all of that Bessel function business in the figure-8 motion of the electrons in an undulator is simply Thomson scattering. When K is not zero the motion of the electron is the classical analog (literally!) of the effect of multiple photon scattering [96]. This is an alternative explanation of the existence of harmonics in undulator radiation. Rather than an interference effect between photons emitted at different times during the motion through the undulator, it is a result of two photons from the undulator scattering from the electron at the same time.

We can see now that the peculiar shape of the Thomson scattered radiation (shown in Fig. 3.15) is due to the relativistic contortions of the plain old electric dipole field which occurs in the average rest frame of the electron when the electron is going through simple harmonic motion. The reason the radiation is "pinched" in the direction of the polarization of the undulator or laser is because of the nulls that occur in the electric dipole distribution. The width of the distribution parallel to the laser polarization is $\sim 0.6/\gamma$ while the width perpendicular to the polarization is $\sim 1/\gamma$ [100]. Circularly polarized undulators and lasers don't show this pinching because the radiation is a superposition of two similar distributions rotated 90° relative to each other.

The energy distribution of the radiated photons in this point of view is also entirely due to the relativistic effects. In the rest frame the photons are all the same frequency as the oscillations caused by the incoming wave. The frequencies that we see in the lab are due to the different Doppler shifts of the photons emitted in different directions in the rest frame. The energy distribution of Compton scattered photons is given by [97, 101]

$$\frac{d\sigma}{d(\hbar\omega')} = \frac{8\pi r_e^2}{X(\gamma\beta m_0 c^2 - \hbar\omega)} \left[\left(\frac{1}{X} - \frac{1}{Y} \right)^2 + \frac{1}{X} - \frac{1}{Y} + \frac{1}{4} \left(\frac{X}{Y} + \frac{Y}{X} \right) \right], \quad (3.45)$$

with X given above in Eq. 3.43 and

$$Y = X \left(\frac{\beta\gamma m_0 c^2 - \hbar\omega'}{\beta\gamma m_0 c^2 - \hbar\omega} \right). \quad (3.46)$$

A plot of this function for a 10.6 μm photon scattering from a 70 MeV electron is shown in Fig. 3.17. The minimum intensity occurs for the photons that are emitted in the plane of motion of the electron in the rest frame,¹⁵ the reduced intensity is once again caused by the null in the electric dipole radiation pattern in the direction of dipole motion. The photons in the low energy half of the distribution are those emitted in the opposite direction to the relativistic motion of the electron in the lab frame while those on the high energy half are emitted in the direction of relativistic motion.

¹⁵The dipole oscillates with frequency $\gamma(1 + \beta)\omega$ due to the Doppler upshift of the incident radiation into the electron rest frame and the scattered photons emitted at angle $\pi/2$ relative to the direction of relativistic motion are upshifted by just γ when going back to the lab frame, for a total up shift of $2\gamma^2\omega$. The photons emitted in the direction of relativistic motion in the lab frame are the highest energy photons, they have frequency $4\gamma^2\omega$. These same arguments involving Doppler shifts are why the highest energy photons in a "side on" inverse Compton scattering source have maximum frequency $2\gamma^2\omega$.

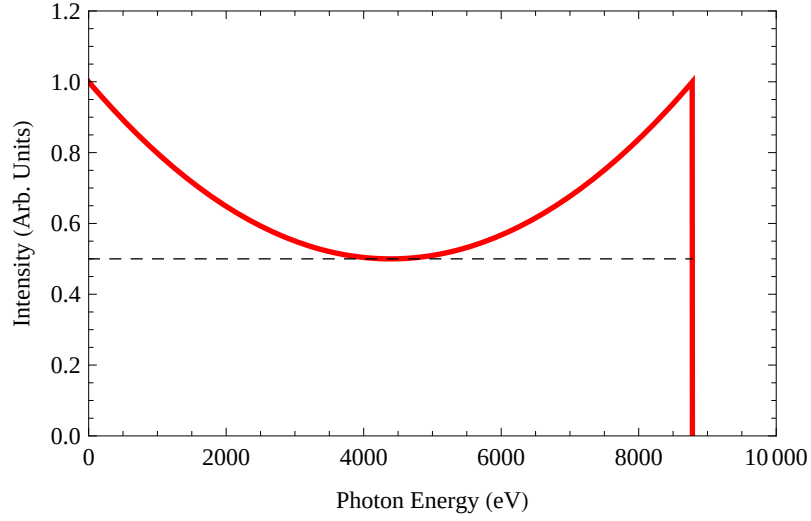


Figure 3.17: Energy distribution in the lab frame of the Compton scattered photons when a $10.6 \mu\text{m}$ photon scatters from a 70 MeV electron. The black dashed line shows intensity of 0.5. The intensity of the photons emitted at zero energy is very slightly higher than the intensity of the photons emitted at maximum energy.

3.4.2 Inverse Compton Scattering of Beams

Classically, when many projectiles are incident on a target, at rest, of cross sectional area σ with relative velocity v_{rel} the number of collisions in time interval dt is $dN_1 = \sigma n_\gamma v_{rel} dt$, where n_γ is the density of projectiles (photons in the Compton scattering case). If there is more than one target, with density n_e in volume dV the total number of scattering events is $dN = n_e dV dN_1 = \sigma v_{rel} n_e n_\gamma dV dt$. When the target is moving relativistically the number of scattering events is given by [31]

$$dN = \sigma n_e n_\gamma c \left(1 - \boldsymbol{\beta} \cdot \frac{\mathbf{k}}{|\mathbf{k}|} \right) dV dt, \quad (3.47)$$

where it has been assumed that the relative velocity of the projectiles and targets

is c , so the projectiles must be photons, and β is the reduced velocity of the targets. The term in parenthesis is a relativistic correction to the rate at which the projectiles flow past the targets. This change is possible because the total number of particles emitted is a Lorentz invariant, a space ship flying by your front lawn may disagree with you about how far apart your fence posts are but not about how many posts there are.

When the projectiles and the targets are moving in the same direction, the rate of collision is decreased by a factor of $1/(2\gamma^2)$ as compared to the case when the targets are rest, when the targets are assumed to move ultra relativistically. Alternatively, when the projectiles and targets are counter propagating, the rate of collision increases by a factor of 2 as compared to the case of targets at rest. In the previous analysis, the effect of the targets screening each other has been ignored.

When we started off with this line of reasoning σ was the physical cross sectional area of the target, but it need not be so any more. The cross section is short hand for all the physics that occurs when a projectile and target meet, whatever that may mean for this particular pair of particles and the process that we are interested in. That is why, in the limit of zero recoil, Compton scattering reduces to Thomson scattering, despite the ability of deriving the Thomson cross section via assuming an impinging plane wave on an electron at rest. All the physics of Thomson scattering is contained within the more general Compton scattering and all of the physics of both is handled inside σ , leaving us free to worry about how the particles are going to get close enough to each other.

Because all of the physics of the process is contained in σ , if we want to know how the results of the scattering events are distributed in some quantity we take the derivative of the cross section only to find it. The other parts of Eq. 3.47 are

all about how a projectile and a target meet, the interesting stuff is all happening in the cross section. For instance if we wanted to know how the results of the scattering events are distributed in both angle and scattered photon energy we would take $\sigma \rightarrow \frac{d\sigma}{d\Omega d(\hbar\omega')}$ and find that

$$\frac{dN}{d\Omega d(\hbar\omega')} = \frac{d\sigma}{d\Omega d(\hbar\omega')} n_e n_\gamma c \left(1 - \boldsymbol{\beta} \cdot \frac{\mathbf{k}}{|\mathbf{k}|} \right) dV dt. \quad (3.48)$$

Now all we have to do is start integrating to eliminate the input parameters that we don't want to see any more, like collision angle and initial photon energy and so on. For beams, n_e and n_γ depend on all sorts of parameters, so eliminating those parameters to find out what happens when two beams collide is where we turn to now.

From Eqs. 3.42 and 3.45 it is clear that the Compton scattering cross section does not depend on position. This makes sense because there is no special position in the universe (so far as we know) so there is no reason for Compton scattering to work differently at any particular location. If we assume that the scattering event occurs at the waist of the two beams, such that the position and angle of each electron or photon is uncorrelated (see section 1.2.3) we can separate the densities such that, for example, $n_e = N_e f_e(\mathbf{r}, \mathbf{p}, t) = N_e f_e(\mathbf{r}, t) g_e(\mathbf{p})$, where f_e is a normalized density function for the electrons. Similarly, for the laser photons, $n_p = N_p f_p(\mathbf{r}, \mathbf{k}, t) = N_p f_p(\mathbf{r}, t) g_p(\mathbf{k})$. In this Eq. 3.48 becomes

$$\frac{dN}{d\Omega d(\hbar\omega')} = N_e N_p \mathcal{L}_{sc} \int \frac{d\sigma}{d\Omega d(\hbar\omega')} g_e(\mathbf{p}) g_p(\mathbf{k}) d^3\mathbf{p} d^3\mathbf{k}, \quad (3.49)$$

where $\mathcal{L}_{sc}$ is the single collision luminosity of the collision, given by¹⁶

¹⁶This is actually the normalized luminosity, the "real" luminosity is $\mathcal{L} = N/\sigma = N_e N_p \mathcal{L}_{sc}$. Eq. 3.47 can be used to define the luminosity, it is a Lorentz invariant defined as the number of reactions per cross section, and it has units 1/area.

$$\mathcal{L}_{sc} = c \int (1 - \beta \cos(\theta_i)) f_e(\mathbf{r}, t) f_p(\mathbf{r}, t) dV dt, \quad (3.50)$$

where θ_i is the incidence angle of the photon as defined at the beginning of this section. This is called the single collision luminosity because we are ignoring multiple collisions between photons and electrons. Is this an appropriate approximation? To check this we find that the mean path length of a photon in the electron distribution before scattering (called the mean free path) is given by $\sigma \lambda_{mfp} n_e = 1$. Typical inverse Compton scattering experiments focus the electron beam to a 10 μm radius spot size with a length of 5 ps. With 200 pC of charge in one bunch, $n_e \simeq 8.5 \times 10^{20} \text{ 1/m}^3$. In this case $\lambda_{mfp} \simeq 1.8 \times 10^7 \text{ m}$, using the Thomson cross section for σ . The cross section is so small that the mean free path is the distance between Los Angeles and Madagascar. This doesn't mean that it is impossible to create higher harmonic photons, just that one has to work especially hard [102].

The integral in Eq. 3.49 is an average of the differential cross section over the electron and photon momentum distributions in the collision of the two beams so we can write it as

$$\left\langle \frac{d\sigma}{d\Omega d(\hbar\omega')} \right\rangle = \int \frac{d\sigma}{d\Omega d(\hbar\omega')} g_e(\mathbf{p}) g_p(\mathbf{k}) d^3\mathbf{p} d^3\mathbf{k}. \quad (3.51)$$

Because all of the scattered photons are collimated in the forward direction for relativistic electrons, we can imagine it is possible to collect all of the photons and therefore use the total cross section inside the integral instead of the differential cross section to calculate the total number of photons created. For an electron *beam* and a laser *beam* colliding at the waist of each the total number of x-rays produced is [97]

$$N_x = N_e N_p \mathcal{L}_{sc} \sigma_T, \quad (3.52)$$

assuming there is no recoil and that the electron and photon beams aren't too divergent so that we can use the Thomson cross section and with $\mathcal{L}_{sc}$, the single collision luminosity of the collision, given by [16]

$$\mathcal{L}_{sc} = \frac{1}{2\pi \sqrt{\frac{\lambda L_R}{4\pi} + \beta_x \epsilon_x} \sqrt{\frac{\lambda L_R}{4\pi} + \beta_y \epsilon_y}}. \quad (3.53)$$

Here, λ and L_R are the wavelength and Rayleigh range of the laser, respectively and β_i and ϵ_i are the β -function and emittance of the electron beam in the i^{th} transverse direction, respectively. The importance of matching the electron beam and laser sizes at the collision point can be seen in Eq. 3.53. Because it is relatively easy to measure the total number of photons in a pulse and the total number of electrons in a bunch, a majority of the error in the calculation of the number of photons produced comes from the luminosity term. Perhaps the beams aren't overlapping exactly or the size of one of the two beams is poorly known in general.

The electron beam and laser distributions take on a simple form after a couple of assumptions are made, the first is that the electron beam is traveling in the $+z$ direction with velocity βc and is relativistic such that the paraxial approximation applies ($x' = p_x/p_z$ and $y' = p_y/p_z$ with $p_z \simeq p$ and $p_x, p_y \ll p_z$). Further, all of the distributions are assumed to be Gaussians. In this case the distribution at a waist, where the position and transverse momentum of any given electron is uncoupled -the beam is not chirped transversely-, is given by

$$f(\mathbf{r}, \mathbf{p}, t) = f(x, y, z, x', y', p, t) = \frac{\exp \left[-\frac{x^2}{2\sigma_x^2} - \frac{x'^2}{2\sigma_x'^2} - \frac{y^2}{2\sigma_y^2} - \frac{y'^2}{2\sigma_y'^2} - \frac{(p-p_0)^2}{2\sigma_p^2} - \frac{(z-\beta ct)^2}{2\sigma_z^2} \right]}{(2\pi)^3 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'} \sigma_p \sigma_z}, \quad (3.54)$$

where p_0 is the central momentum of the electron beam. At a waist we have $\sigma_x^2 = \beta_x \epsilon_{x,rms}$, $\sigma_{x'}^2 = \epsilon_{x,rms} / \beta_x$ and $\sigma_x \sigma_{x'} = \epsilon_{x,rms}$. There are similar equations for the y -direction. We therefore find that

$$\frac{x^2}{2\sigma_x^2} + \frac{x'^2}{2\sigma_x'^2} = \frac{1}{2\epsilon_{x,rms}} \left(\frac{x^2}{\beta_x} + \beta_x x'^2 \right) \equiv \frac{\epsilon_x(x, x')}{2\epsilon_{x,rms}}, \quad (3.55)$$

where the last equality is used to define the emittance of the particle at position x with momentum x' using Eq. 1.21. Away from the waist position the emittance of any particle is also given by Eq. 1.21, such that $\epsilon_x(x, x') \rightarrow \gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2$ and we have for the distribution of electrons (dropping the *rms* subscript now)

$$f_e = \frac{\exp \left[-\frac{\gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2}{2\epsilon_x} - \frac{\gamma_y y^2 + 2\alpha_y y y' + \beta_y y'^2}{2\epsilon_y} - \frac{(p-p_0)^2}{2\sigma_p^2} - \frac{(z-\beta ct)^2}{2\sigma_z^2} \right]}{(2\pi)^3 \epsilon_x \epsilon_y \sigma_p \sigma_z}. \quad (3.56)$$

For the laser beam the standard definition involves allowing the envelope ($\sigma_w(z)$) to vary rather than using an invariant emittance, along with assuming the beam is round. The distribution is

$$f_p(x, y, z, k, t) = \frac{\exp \left[-\frac{x^2 + y^2}{2\sigma_w^2(z)} - \frac{(z+ct)^2}{2\sigma_l^2} - \frac{(k-k_0)^2}{2\sigma_k^2} \right]}{(2\pi)^2 \sigma_l \sigma_k \sigma_w^2(z)}, \quad (3.57)$$

where σ_l is the length of the laser beam and σ_k is the momentum spread and $\sigma_w(z) = \sqrt{\frac{\lambda L_R}{4\pi} \left(1 + \frac{z^2}{L_R^2} \right)}$ is the laser beam radius as a function of distance from the waist point, which is assumed to occur at the origin of coordinates. $L_R =$

$\pi w_0^2/\lambda$ is the Rayleigh range, where $w_0 = 2\sigma_w(0)$ is the waist size of the laser beam. The angle of the incident laser photons has been left out because, as was seen in section 3.4, it is a very small correction unless the laser is very tightly focused.

Under the above assumptions, Eq. 3.48 can be partially solved analytically [97]. There are a few constraints that must be applied to the integration to ensure a physically possible solution. The first is that the angle of emission of the scattered photon must send the photon to the detection point. If we define the solid angle for a distant detector as $d\Omega \rightarrow dx_d dy_d / L^2$ where x_d and y_d are the coordinates of the detection point on the screen and L is the distance between the interaction point and the detection screen, then the angle of emission of the scattered photon must be given by Eq. 3.58 in order to be detected at point $\mathbf{r}_d$. Where $\mathbf{r}$ is the point of emission relative to the origin of coordinates so that $|\mathbf{r}_d - \mathbf{r}| \approx L$, the underlying assumption is that emission occurs very close to the origin while detection happens very far from the origin, with a majority of that distance laying along the direction of electron beam propagation (z -axis in the current coordinates).

$$\frac{\mathbf{k}'}{|\mathbf{k}'|} = \frac{\mathbf{r}_d - \mathbf{r}}{|\mathbf{r}_d - \mathbf{r}|}. \quad (3.58)$$

Using the usual expansion of the emission angle θ_f such that $\theta_f^2 = \theta_x^2 + \theta_y^2$, the above constraint becomes

$$\theta_x + x' = \frac{x_d - x}{L} \quad \text{and} \quad \theta_y + y' = \frac{y_d - y}{L}, \quad (3.59)$$

for each of the directions.

The second constraint is on the scattered photon energy. When a photon of

energy $\hbar\omega$ meets and electron of reduced energy γ and scatters a photon into direction θ_f , the scattered photon energy ($\hbar\omega'$) must obey Eq. 3.39. This is ensured by multiplying Eq. 3.48 by

$$\delta \left(\hbar\omega' - \frac{4\gamma^2\hbar\omega}{1 + \gamma^2\theta_f^2 + \frac{4\gamma^2\hbar\omega}{m_0c^2}} \right). \quad (3.60)$$

Because we want to maintain the dependence on scattered photon energy and detection point on the left side of Eq. 3.48, this last constraint turns into a constraint on the electron energy required to scatter a photon of energy $\hbar\omega'$ into the direction, $\mathbf{r}_d$, given by Eq. 3.58.

At long last we are ready to (partially) integrate Eq. 3.48, the procedure is outlined in [97], with the result being

$$\begin{aligned} \frac{dN}{d(\hbar\omega')dx_d dy_d} &= \frac{r_e^2 N_e N_p}{4\pi^3 \hbar c L^2 L_R \sigma_\gamma \sigma_k} \\ &\times \int_0^\infty dk \int_{-\sqrt{4\omega/\omega'}}^{\sqrt{4\omega/\omega'}} d\theta_y \int_{-\theta_{x,max}}^{\theta_{x,max}} d\theta_x \frac{1}{\sqrt{\zeta_x \zeta_y} \sigma_{\theta_x} \sigma_{\theta_y}} \frac{\gamma}{1 + 2\gamma\hbar\omega/(m_0c^2)} \\ &\times \left\{ \frac{1}{4} \left[\frac{4\gamma^2\hbar\omega}{\hbar\omega'(1 + \gamma^2\theta_f^2)} + \frac{\hbar\omega'(1 + \gamma^2\theta_f^2)}{4\gamma^2\hbar\omega} \right] - 2 \cos^2(\phi_f) \frac{\gamma^2\theta_f^2}{(1 + \gamma^2\theta_f^2)^2} \right\} \\ &\times \exp \left[-\frac{(\theta_x - x_d/L)^2}{2\sigma_{\theta_x}^2} - \frac{(\theta_y - y_d/L)^2}{2\sigma_{\theta_y}^2} - \frac{(\gamma - \gamma_0)^2}{2\sigma_\gamma^2} - \frac{(k - k_0)^2}{2\sigma_k^2} \right]. \end{aligned} \quad (3.61)$$

The remaining integral contains a lot of short hand functions, they are given below.

$$\begin{aligned}
\zeta_x &= 1 + \frac{2k\beta_x\epsilon_x}{L_R}, \quad \zeta_y = 1 + \frac{2k\beta_y\epsilon_y}{L_R}, \quad \theta_f^2 = \theta_x^2 + \theta_y^2, \\
\theta_{x,max} &= \sqrt{4\hbar\omega/(\hbar\omega') - \theta_y^2}, \\
\sigma_{\theta_x} &= \sqrt{\frac{\epsilon_x\xi_x}{\beta_x\zeta_x}}, \quad \sigma_{\theta_y} = \sqrt{\frac{\epsilon_y\xi_y}{\beta_y\zeta_y}}, \quad \sigma_\gamma = \frac{\sigma_E}{m_0c^2}, \\
\gamma &= \frac{2\hbar\omega\hbar\omega'/(m_0c^2)}{4\hbar\omega - \hbar\omega'\theta_f^2} \left(1 + \sqrt{1 + \frac{4\hbar\omega - \hbar\omega'\theta_f^2}{4(\hbar\omega)^2\hbar\omega'/(m_0c^2)}} \right), \\
\xi_x &= 1 + \left(\alpha_x - \frac{\beta_x}{L} \right)^2 + \frac{2k\beta_x\epsilon_x}{L_R}, \quad \xi_y = 1 + \left(\alpha_y - \frac{\beta_y}{L} \right)^2 + \frac{2k\beta_y\epsilon_y}{L_R},
\end{aligned} \tag{3.62}$$

and σ_E is the rms energy spread of the electron beam.

From here on the integrations must be done numerically. Looking at all of the contortions in the functions needed, it may seem unnecessary to keep the differential cross section as a function of the scattered photon energy ($\hbar\omega'$), but a great many uses of Compton scattering sources require only a narrow bandwidth of the created photons and it is therefore crucial to be able to separate the different energy photons theoretically to find out how many useful photons are available to the experiment.

The numerical integration of Eq. 3.61 is used in section 5.1 to make a qualitative comparison with the data for both the spectrum and the transverse distribution reflected from a flat silicon crystal of the inverse Compton scattering source at the Accelerator Test Facility at Brookhaven National Laboratory.

3.5 Coherence of Many Particle Emitter

To determine the presence of a correlated energy spread in the electron beam during the undulator testing at NLCTA, coherent transition radiation was used.

It is useful, therefore, to look at what coherent radiation means so that the results can be interpreted. In subsection 3.2.3 we found what transition radiation means for a single electron passing through a change in material from vacuum to a perfect conductor, or vice versa. The description of the radiation produced is given by Eq. 3.20.

We found in Eq. 1.5 that the radiated energy detected from an electron beam is given by integrating the Poynting vector over the area of the detector, using Eq. 3.2 we find

$$\begin{aligned}\frac{dW}{dtd\Omega} &= \frac{c}{4\pi} R^2 (\mathbf{E}(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}, t)^*) \quad \text{and} \\ \frac{dW}{d\omega d\Omega} &= \frac{c}{2\pi} R^2 (\mathbf{E}_\omega \cdot \mathbf{E}_\omega^*).\end{aligned}\tag{3.63}$$

Previously this was used for a single electron to calculate radiation distributions in a variety of situations. But the result is readily generalizable to multiple emitting electrons. The basis of Eq. 3.63, the Poynting vector, simply tells us what to do with radiation moving normal to some boundary, it doesn't matter how that radiation got there. To find out what happens when there are many electrons we insert the electric fields from those many electrons into Eq. 3.63. Because we are considering radiation, the electric fields detected are all part of plane waves of the form $\mathbf{E}(\mathbf{R}, t) = \hat{\mathbf{e}} E_\omega e^{-i\mathbf{k} \cdot \mathbf{R} + i\omega t}$ with Fourier transform $\mathbf{E}_\omega(\mathbf{R}) = \hat{\mathbf{e}} E_\omega e^{-i\mathbf{k} \cdot \mathbf{R}}$. The field of each of the N_e electrons is given by [103]

$$\mathbf{E}_\omega = \sum_{j=1}^{N_e} \hat{\mathbf{e}}_j E_{\omega,j} \exp(-i\mathbf{k}_j \cdot \mathbf{R}_j)\tag{3.64}$$

with the polarization unit vector, $\hat{\mathbf{e}}_j$, given by

$$\hat{\mathbf{e}}_j = \frac{\mathbf{k}_j \times (\mathbf{k}_j \times \mathbf{v}_j)}{|\mathbf{k}_j \times (\mathbf{k}_j \times \mathbf{v}_j)|}.\tag{3.65}$$

Here, $\mathbf{k}_j$, is the direction of propagation of the plane wave of the j^{th} electron, $\mathbf{v}_j$ is the velocity of the same electron and $\mathbf{R}_j = \mathbf{r} - \mathbf{x}_j$ is the vector from the j^{th} electron to the detection point. All of the vectors in Eq. 3.64 are real as they describe physical quantities that can be measured: the polarization of the electric field, the direction of propagation of a plane wave, and the position of an electron. The function of the electric field, given by E_ω , can be complex. Using this information we can write the spectral intensity of the detected radiation as

$$\begin{aligned} \frac{dW}{d\omega d\Omega} &= \frac{c}{2\pi} R^2 \sum_{j=1}^{N_e} \hat{\mathbf{e}}_j E_{\omega,j} e^{-i\mathbf{k}_j \cdot \mathbf{R}_j} \cdot \sum_{m=1}^{N_e} \hat{\mathbf{e}}_m E_{\omega,m}^* e^{+i\mathbf{k}_m \cdot \mathbf{R}_m} \\ &= \frac{c}{2\pi} R^2 \sum_{j=1}^{N_e} \sum_{m=1}^{N_e} (\hat{\mathbf{e}}_j \cdot \hat{\mathbf{e}}_m) E_{\omega,j} E_{\omega,m}^* e^{-i\mathbf{k}_j \cdot \mathbf{R}_j + i\mathbf{k}_m \cdot \mathbf{R}_m}. \end{aligned} \quad (3.66)$$

Eq. 3.66 can be split into two terms, the first is when $m = n$ which is the single electron spectral intensity and $m \neq n$ which is the interference between different electrons. The result is that the spectral intensity becomes

$$\frac{dW}{d\omega d\Omega} = N_e \left. \frac{dW}{d\omega d\Omega} \right|_e + \frac{c}{2\pi} R^2 \sum_{j=1}^{N_e} \sum_{\substack{m=1 \\ m \neq n}}^{N_e} (\hat{\mathbf{e}}_j \cdot \hat{\mathbf{e}}_m) E_{\omega,j} E_{\omega,m}^* e^{-i\mathbf{k}_j \cdot \mathbf{R}_j + i\mathbf{k}_m \cdot \mathbf{R}_m}. \quad (3.67)$$

The question now is what to make of $E_{\omega,j} E_{\omega,m}^*$. We assume that whatever the radiation process is, the radiation by one electron doesn't effect the radiation of another. This assumption amounts to deciding that the only thing the electrons have in common is their proximity, otherwise they act independently. Because the radiation process is the same for all electrons, the spectral content will be the same and we take $\frac{c}{2\pi} R^2 E_{\omega,j} E_{\omega,m}^* \rightarrow \left. \frac{dW}{d\omega d\Omega} \right|_e$ just like we did in the first term. The spectral intensity then becomes

$$\frac{dW}{d\omega d\Omega} = \left[N_e + \sum_{j=1}^{N_e} \sum_{\substack{m=1 \\ m \neq j}}^{N_e} (\hat{\mathbf{e}}_j \cdot \hat{\mathbf{e}}_m) e^{-i\mathbf{k}_j \cdot \mathbf{R}_j + i\mathbf{k}_m \cdot \mathbf{R}_m} \right] \frac{dW}{d\omega d\Omega} \Big|_e. \quad (3.68)$$

The inner product between the two unit vectors is there to make sure that we take into account the polarization of the plane waves as they interfere so that we don't, say, find interference between a horizontally polarized plane wave and vertically polarized one. For the cases of edge radiation, transition radiation and Čerenkov radiation the radiation is radially polarized so the plane waves from each of the electrons are polarized in the same direction when the electrons are traveling in the same direction. However, focusing effects will cause the electrons to move in different directions. As we will later see, we usually focus the electron beam before creating any of these type of radiation so let us assume that the radiation is created at a beam waist. If the beam has emittance $\epsilon_x = 1 \mu\text{m}$ and waist radius $\sigma = 100 \mu\text{m}$ such that the rms divergence of the beam is $\sigma_{x'} = 0.01$ and the reduction in interference due to polarization effects is $\hat{\mathbf{e}}_j \cdot \hat{\mathbf{e}}_m \sim \cos(2\sigma_{x'}) \approx 0.9998$. For our purposes we can safely ignore the effect of polarization. When it is included it is called the synthesis of vectors [103].

Usually we know (or assume) the charge distribution rather than the location of each particle. That means replacing the sums with integrals such that we have

$$\sum_{j=1}^{N_e} e^{-i\mathbf{k}_j \cdot \mathbf{R}_j} \rightarrow \int \rho(\mathbf{R}) e^{-i\mathbf{k} \cdot \mathbf{R}} d\mathbf{R}, \quad (3.69)$$

where $\rho(\mathbf{R}) = N_e f(\mathbf{R})$ is the charge distribution. Being careful not to double count the n^{th} electron left out of the 2^{nd} sum in Eq. 3.68 the spectral intensity of a continuous distribution of many emitting electrons is

$$\frac{dW}{d\omega d\Omega} = \left[N_e + N_e(N_e - 1) \left| \int f(\mathbf{R}) e^{-i\mathbf{k} \cdot \mathbf{R}} d\mathbf{R} \right|^2 \right] \frac{dW}{d\omega d\Omega} \Big|_e. \quad (3.70)$$

The second term shows the power of coherent emission. The spectral intensity grows as the square of the number of electrons, usually numbering in the billions. The second term also contains the Fourier transform of the electron beam in space. This means that the name of the game in creating coherent radiation is to get structures in the beam of the order of the wavelength of the radiation you want to detect.

For coherent transition radiation from an electron beam which has been microbunched in an undulator, the total number of photons created when passing through a vacuum-to-perfect-conductor interface is [104]

$$N \approx \frac{\alpha (N_e b_1)^2}{4\sqrt{\pi} k_r \sigma_z} \left(\frac{\gamma}{k_r \sigma_r} \right)^4. \quad (3.71)$$

Here, $\alpha = e^2/(\hbar c)$ is the fine structure constant, k_r is the wave number of the microbunching wavelength ($n = 1$ in Eq. 3.21), b_1 is the microbunching factor at the fundamental wavelength (covered in section 4), σ_r is the rms radius of the electron beam and σ_z is the rms bunch length. Eq. 3.71 covers an rms bandwidth of $1/(\sqrt{2}\sigma_z)$. It is clear that the number of photons depends crucially on the size of the beam as it passes through the transition and on the wavelength of the radiation desired. The number of photons scales like λ^5 , so the relative output at half of any given wavelength is 32 times smaller! It should be noted that the number of photons doesn't grow without bound. The maximum number of coherent photons is $N_e(N_e - 1)$ and it occurs when the electron beam is small compared to the wavelength of interest, such that Fourier transform factor reaches the limiting value of 1.

Whereas the incoherent radiation has an opening angle of approximately $1/\gamma$ (see subsection 3.2.3), the coherent radiation is limited by the antenna effect which much prefers to emit in the forward direction [105]. The opening angle of coherent radiation is approximately $1/(k_r\sigma_r)$ due to this effect [104].¹⁷ In order to produce a large number of photons $\gamma/(k_r\sigma_r)$ (which is the ratio of the coherent opening angle to the incoherent opening angle) should be of order 1 or greater. Which means that the increase in photons due to the coherence effect is much greater than the decrease in photons due to moving closer to the on axis ($\theta = 0$) null characteristic of transition radiation (see section 3.2.3).

¹⁷The antenna effect also prefers directly backward but there is no radiation there that we care about so it isn't very useful.

CHAPTER 4

The Free-Electron Laser

All the previous radiation types covered have been single electron phenomena. They can be completely described by the effect of outside fields on a single electron which emits radiation because of the acceleration of the electron (classical) or scattering of the electron from the incoming fields (quantum mechanical). The results of the fields from many electrons can be found with formulae like Eqs. 3.61 or 3.70 which add up the electric fields of the radiation from each electron at some distant point where the detector is sitting.

The free-electron laser (FEL) is very different [106, 107]. The gain mechanism for the radiation is deeply collective between the electrons as they interact with each other via the radiation fields they produce. As we saw in section 3.5, when an electron bunch is suitably shaped the radiation power can be quadratic in the beam power. In the free-electron laser the radiation power can grow exponentially from the incoherent power of N_e to the coherent power of N_e^2 , creating the brightest light sources in the world. What is more, these light sources can be completely coherent.

Better still, the free-electron laser can operate at wavelengths for which there is no known gain material besides the electrons. Unlike traditional lasers, the free-electron laser can produce any wavelength that undulator and accelerator technology can combine to achieve, meaning that they can be used to produce hard x-ray lasers. Existing examples of x-ray FELs are LCLS [3] and SACLA

[4]. These lasers are designed based on the principle of self-amplified spontaneous emission (SASE) in which the undulator radiation from the beginning of a really long undulator is amplified by the free-electron laser gain mechanism [5]. This produces transversely coherent but longitudinally incoherent radiation. There have been proposals for how to create longitudinal coherence in such a design. A recently successful example uses a crystal to filter the radiation part way along the undulator, and then uses the filtered radiation as a seed for the growth regime [108].

The other class of free-electron laser is called seeded. In this case either a laser pulse is injected into the undulator with the electron beam or the electron beam is conditioned beforehand to preferentially radiate at a given frequency. The laser seeded free-electron laser depends on a source of light for the seed, this type of FEL is being built at Synchrotrone Trieste using high harmonic generation (HHG) in gases to produce a soft x-ray seed pulse [109]. The electron beam conditioning type of seed is done by microbunching the electron beam in an upstream undulator and using a dispersive section to change the wavelength of the microbunching. The "classic" example is high gain harmonic generation (HG) [110] with a more modern example being echo-enabled harmonic generation (EEHG) [111]. The advantage of beam conditioning methods is that, in principle, they allow up conversion to higher frequencies of the microbunching created using any existing laser seed.

4.1 Small Signal Gain

The simplest example of the FEL mechanism is the small signal gain regime. In this regime we assume that the intensity of the laser doesn't change over the course of the interaction so the most pronounced changes happen with the elec-

trons. This is the interaction that occurs in the modulator sections of advanced seeding schemes [110, 111] and the experimental tool used to evaluate the fitness of CPMU9 as an FEL component in section 5.2. The trajectory of the electrons through the undulator is given by Eqs. 1.40 and 1.41. For simplicity we assume that the electron beam is going straight through the undulator, although the individual electrons certainly are not. Also traveling straight through the undulator is an electric field, in the small signal gain regime the field is from a laser, but in the SASE FEL the field is that radiated by the electrons which is given by Eqs. 3.34 and 3.35. The field has the form $\mathbf{E} = E_0 \hat{\mathbf{x}} \cos[k_1 z - \omega_1 t + \phi_0]$, where $\omega_1 = k_1 c$ is the frequency of the field and ϕ_0 is the initial phase.

The rate of change of the energy of an electron in the combined planar undulator magnetic field and laser electric field is given by

$$\frac{d\gamma}{dz} = \frac{eKE_0}{2\gamma m_0 c^2 \beta_z} \left\{ \cos[(k_1 + k_u)z - \omega_1 t + \phi_0] + \cos[(k_1 - k_u)z - \omega_1 t + \phi_0] \right\}, \quad (4.1)$$

where we have ignored the effect of the higher order terms on the transverse velocity. The first term in the curly braces contains the pondermotive phase of the electric field relative to the electrons,

$$\vartheta = (k_1 + k_u)z - \omega_1 t. \quad (4.2)$$

If we use the distance along the undulator, z , instead of the time as the independent variable, the rate of change of the phase along the undulator is

$$\begin{aligned}
\frac{d\vartheta}{dz} &= k_u + k_1 - \frac{\omega_1}{\langle \beta_z \rangle c} \\
&= k_u - k_1 \left\{ \frac{1}{2\gamma^2} + \frac{K^2}{4\gamma^2} + \frac{k_{n_x}^2 x_s^2 + k_{n_y}^2 y_s^2 + p_x^2 + p_y^2}{2} \right\}.
\end{aligned} \tag{4.3}$$

Here, we have used the period averaged longitudinal velocity. The coupling is strongest when the phase of the radiation slips $2\pi n$ ahead of the phase of the electrons per period, where $n \geq 1$ is any integer. In this case $\frac{k_1}{k_u} \left(\frac{\langle \beta_z \rangle - 1}{\langle \beta_z \rangle} \right) = n$. Keeping in mind that the pondermotive phase has the period averaged longitudinal velocity in it, this means that the electron is at the same phase of the laser field at the same point in each undulator period. When $\langle \beta_z \rangle$ is written out, the condition becomes

$$n \frac{k_u}{k_1} = \frac{1}{2\gamma^2} \left(1 + \frac{K^2}{2} \right) + \frac{k_{n_x}^2 x_s^2 + k_{n_y}^2 y_s^2 + p_x^2 + p_y^2}{2}, \tag{4.4}$$

which is the same as Eq. 3.21 when we allow for the lack of off-axis viewing ($\Theta = 0$) and for the extra slow motion that was not taken into account in Chapter 3. In the resonance condition, the parameter $\bar{\gamma}_z$ is sometimes used, it is defined as $\bar{\gamma}_z = \sqrt{\gamma^2/(1 + K^2/2)}$. When the slow motion is included, the parameter becomes

$$\langle \gamma_z \rangle = \sqrt{\frac{\gamma^2}{1 + \frac{K^2}{2} + \gamma^2 \left(k_{n_x}^2 x_s^2 + k_{n_y}^2 y_s^2 + p_x^2 + p_y^2 \right)}}. \tag{4.5}$$

This is a rather clear indication of what the slow motion does to the average longitudinal motion, failure to take into account the effects of that motion will cause the electrons to not be resonant with the seed laser and the coupling will be poor because the electrons are moving too slowly and are therefore shifting around in the laser phase between undulator periods.

The bandwidth of the interaction is given approximately by the bandwidth of the undulator, $1/N_p$. Physically, this is because radiation wavelengths within that bandwidth will stay resonant with the electrons for the entire length of the undulator. Straying too far from this bandwidth means that more laser power will be needed to complete the interaction before the end of the undulator, which means the undulator is too long –what a waste! Further, the extra phase slippage in the later periods can cause unwanted longitudinal motion of the electrons.

To get more from the equations of motion (Eqs. 4.1 and 4.3) we expand them for particles that will vary in energy a little bit from the resonant energy. If we define the resonant energy (γ_0) as the energy that satisfies Eq. 3.21, then we can define small relative deviations from this energy as $\eta = (\gamma - \gamma_0)/\gamma_0$. For these off resonant energy electrons, the phase relative to the resonant electron is given by

$$\frac{d\theta}{dz} = 2k_u\eta - k_1 \frac{k_{n_x}^2 x_s^2 + k_{n_y}^2 y_s^2 + p_x^2 + p_y^2}{2}. \quad (4.6)$$

While the difference in energy causes a relatively small phase advance difference, the slow motion phase advance can be quite large because $k_1 \gg k_u$.

When the longitudinal motion at twice the undulator frequency is period averaged Eq. 4.1 becomes¹

$$\frac{d\eta}{dz} = -\frac{eK [JJ] E_0}{2\gamma_0^2 m_0 c^2} \sin(\theta) \quad (4.7)$$

where $[JJ]$ is given by Eq. 3.37 and only the slowly varying phase term has been kept. Eqs. 4.6 and 4.7 are called the pendulum equations because of their similarity to the equations of motion for the classical mechanics problem [44].

Under the assumption that the field of the laser doesn't change very much when

¹A constant phase has also been added such that the equation ends up with a sine, which is consistent with the usual way of writing the pendulum equations.

passing through the undulator with the electron beam, the pendulum equations are all we need. The motion of the particles, ignoring the slow motion due to the undulator derived in section 1.3, is given by the Hamiltonian

$$H = k_u \eta^2 - \epsilon \cos(\theta). \quad (4.8)$$

Here,

$$\epsilon = \frac{eK[JJ]E_0}{2\gamma_0^2 m_0 c^2} \quad (4.9)$$

is the rate of change of the relative energy of an electron at pondermotive phase θ as a function of z along the undulator. Fig. 4.1 shows a plot of the phase space for this Hamiltonian. There are three types of particles. The first are trapped particles that are inside the red lines, they remain in a single bucket throughout the interaction. The passing particles are those located outside the red lines, they move past the different buckets. The last particles are those located exactly on the red line, called the separatrix, they don't move at all in phase space and are neither trapped nor passing. The location of the separatrix is given by $H = \epsilon$ or $\eta = \pm \sqrt{\epsilon [1 + \cos(\theta)] / k_u}$. These particles are in the same position that the pendulum is in when it starts out straight up in the air with no kinetic energy: unstable equilibrium. As the particles get closer to the separatrix they move slower in phase space.

For the trapped particles with small oscillations about zero (called the stable fixed point), the equations of motion can be approximated as $\theta'' + k_f^2 \theta = 0$ where,

$$k_f^2 = k_u \frac{eK[JJ]E_0}{\gamma_0^2 m_0 c^2} \quad (4.10)$$

is the 'frequency' of oscillation in the bucket. Maximal microbunching occurs

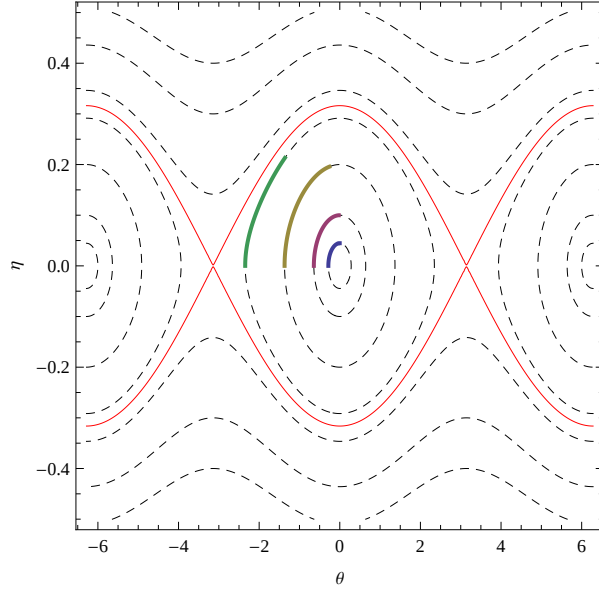


Figure 4.1: Phase space plot of the Hamiltonian for the small signal gain regime of the FEL. The dashed lines are the lines of constant energy and the red line is the separatrix which separates the stable buckets from the passing particles. The other colored lines are the trajectories of the trapped particles starting at $\eta = 0$ with energy equal to the contour each travels on.

when, at the end of the undulator, the particles nearest the stable fixed point at the center of the bucket have gone through approximately one quarter of an oscillation (it is actually closer to $3/8$ of an oscillation). This constrains ϵ such that $\epsilon = \pi^2/(8k_u L_u^2)$. When we next expand the equations of motion, we will assume that $\epsilon N_u L_u \ll \pi^2/8$, this assumption is equivalent to the assumption that the electron bunch is not allowed come anywhere near one quarter longitudinal oscillation. It has also been interpreted that the laser power at injection is much smaller than the laser power at saturation of the FEL [112]. These two interpretations are equivalent because the microbunching is maximized when the FEL saturates [6], which is the assumption we made to limit the oscillation in phase

space.

Microbunching is the name for the variation of beam density (or current) along the length of the beam. It is mathematically defined as

$$b = \frac{1}{N} \sqrt{\left| \sum_{j=1}^N \exp(-i\theta_j) \right|^2}, \quad (4.11)$$

where N is the total number of electrons in a longitudinal slice of the electron beam that is 2π long. From Eq. 4.11 it can be seen that if a large fraction of the electrons gather in part of the bucket the microbunching factor will become quite large, approaching 1. Whereas, if the electrons are evenly spread out, the microbunching factor is small, close to 0.

Fig. 4.1 shows the mechanism for microbunching to occur. Within each bucket the electrons rotate such that, if the mechanism is stopped at the correct time there is a density spike in each (see Fig. 4.2). By the definition of the phase (Eq. 4.2) we know that the buckets, and thus the spikes, occur with a periodicity of λ_1 in the lab frame.

To see how the phase space of the particles evolves analytically, we expand the pendulum equations in terms of ϵ : $\theta = \theta_0 + \epsilon\theta_1 + \epsilon^2\theta_2 + \dots$ and $\eta = \eta_0 + \epsilon\eta_1 + \epsilon^2\eta_2 + \dots$. Separating by order in ϵ we find three sets of equations, given by²

$$(0) \quad \frac{d\theta_0}{dz} = 2k_u\eta_0 \quad \& \quad \frac{d\eta_0}{dz} = 0, \quad (4.12a)$$

$$(1) \quad \frac{d\theta_1}{dz} = 2k_u\eta_1 \quad \& \quad \frac{d\eta_1}{dz} = -\sin(\theta_0), \quad \text{and} \quad (4.12b)$$

$$(2) \quad \frac{d\theta_2}{dz} = 2k_u\eta_2 \quad \& \quad \frac{d\eta_2}{dz} = -\theta_1 \cos(\theta_0). \quad (4.12c)$$

² η_0 should not be confused for the relative energy of the resonant particle, which is 0.

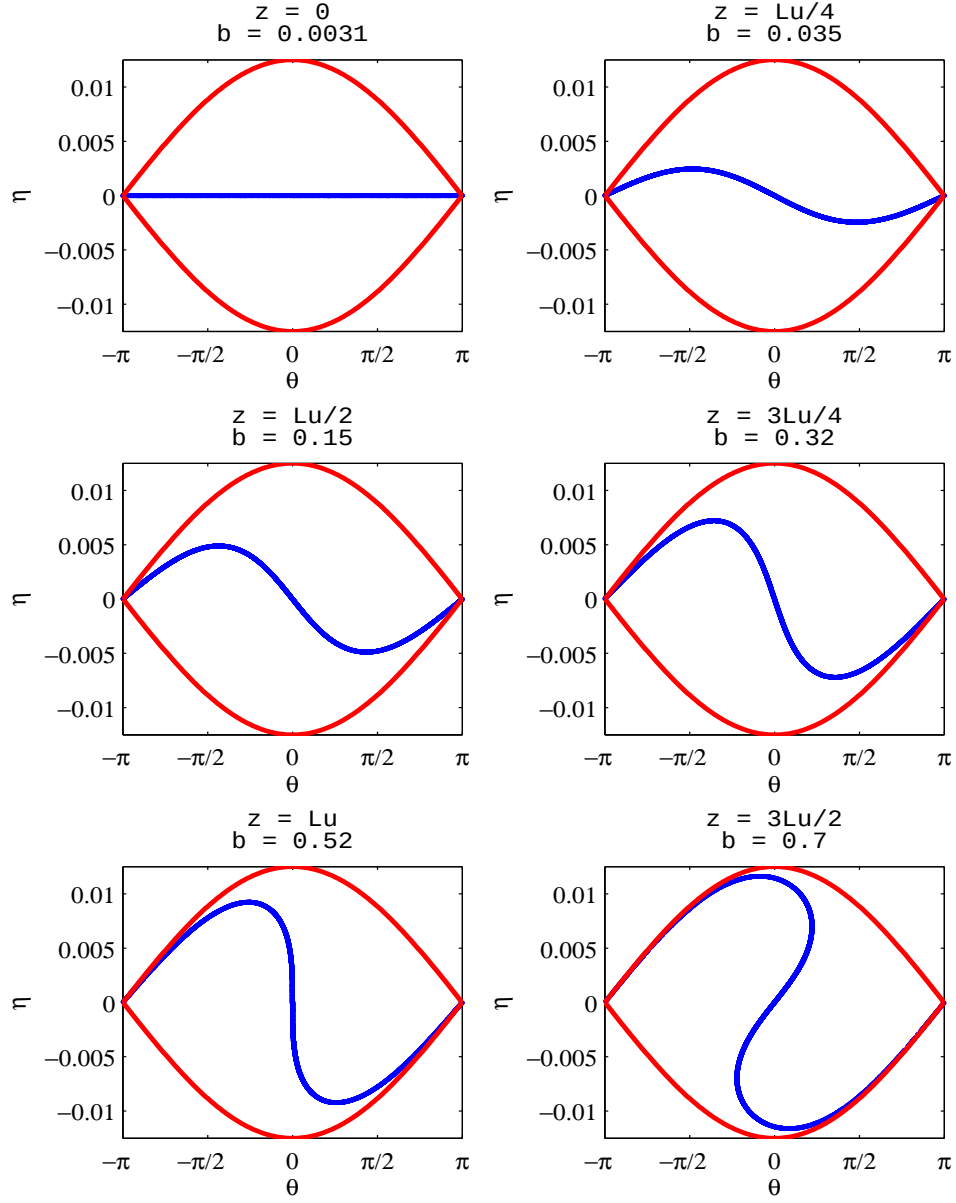


Figure 4.2: Evolution of microbunching in a small signal gain FEL. The mean input energy is $\langle \eta_0 \rangle = 0$. Maximum microbunching (b) occurs after approximately $3/8$ of a longitudinal oscillation.

The zeroth order equations are straight forward, $\theta_0 = 2k_u\eta_0 z + \phi_0$, where η_0 is a constant and ϕ_0 is the initial phase which also appears in Eq. 4.1.

The first order equations show the energy modulation, $\eta_1 = \frac{\cos[2k_u\eta_0 z + \phi_0] - \cos \phi_0}{2k_u\eta_0}$ and $\theta_1 = \frac{\sin[2k_u\eta_0 z + \phi_0] - \sin \phi_0}{2k_u\eta_0^2} - \frac{z}{\eta_0} \cos \phi_0$. However, there is no net energy exchange at this order because $\langle \eta_1 \rangle = 0$, that will occur at the next order. The second order equation for the relative energy is

$$\frac{d\eta_2}{dz} = -\cos[2k_u\eta_0 z + \phi_0] \left\{ \frac{\sin[2k_u\eta_0 z + \phi_0] - \sin \phi_0}{2k_u\eta_0^2} - \frac{z}{\eta_0} \cos \phi_0 \right\}. \quad (4.13)$$

Because we only care about the energy exchange between the laser and the electron beam averaged over a whole period, that is net gain or loss, we average Eq. 4.13 over the initial phase of the electrons, ϕ_0 and then integrate from the beginning of the undulator to the end. Thus, the net energy gain over the length of the undulator is

$$\epsilon^2 \langle \Delta\eta_2 \rangle = \frac{e^2 K^2 [JJ]^2 E_0^2 k_u L_u^3}{4\gamma_0^2 (m_0 c^2)^2} \frac{1}{4} g(x), \quad (4.14)$$

where $x = k_u\eta_0 L_u$ and

$$g(x) = \frac{d}{dx} \left(\frac{\sin^2 x}{x^2} \right) \quad (4.15)$$

is the normalized gain function. A plot of the gain function is shown in Fig. 4.3. To prove the gain curve for the small signal gain FEL is indeed the derivative of the undulator bandwidth, we know that the resonance condition is true for all the different energy electrons such that $\omega = 2\omega_u\gamma^2/(1 + K^2/2)$ and in particular $\omega_0 = 2\omega_u\gamma_0^2/(1 + K^2/2)$ for the resonant energy. In which case $\omega \approx (1 + 2\eta_0)2\omega_u\gamma_0^2/(1 + K^2/2)$ and $x = \pi N_u \left(\frac{\omega - \omega_0}{\omega_0} \right)$. The argument in the derivative of the normalized

gain function agrees with Eq. 3.27. That the gain in the small signal regime depends on the derivative of the undulator radiation bandwidth is called Madey's Theorem [106].

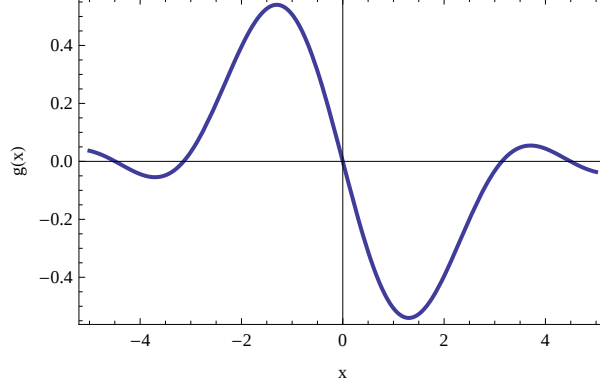


Figure 4.3: Plot of the normalized gain function in the small signal regime. The global maximum is at $x \approx -1.3$ and the global minimum is at $x \approx 1.3$. Because the small signal equations were solved for the electrons, the point of maximum laser gain is the global minimum.

As the equations of motion were solved for the electrons, the maximum gain in the laser energy comes when $x \approx 1.3$ and therefore $g(1.3) \approx -0.54$. The maximum energy gain of the laser in the small signal regime is approximately

$$\Delta E \approx \frac{\pi r_e K^2 [JJ]^2 I_0 k_u L_u^3}{3c\gamma_0}, \quad (4.16)$$

where I_0 is the initial laser intensity. This gain occurs for an electron beam that is initially detuned by $\eta_0 = 1.3/(2\pi N_u)$. Because we assumed that the initial detuning was small, the gain must happen over more than about 10 periods. The evolution of the phase space for an optimally detuned beam is shown in Fig. 4.4. For the experiment at NLCTA outlined in Chapter 5 the change in energy at maximum gain for a single pass is 5 pJ (starting with 30 μ J) or a relative gain

of 2×10^{-7} , which would be very difficult to detect. This is why most small gain devices are configured as oscillators where a cavity for the radiation is set up and the laser gains energy over the course of many electron bunches passing through the undulator [112].

Back in Chapter 1 we introduced the concept of R_{56} which causes particles of different energy to change relative longitudinal position through certain beam line elements, such as chicanes. For a drift section and length L , it is easy to show that $R_{56} = -L/\gamma_0^2$. We can find the R_{56} of a undulator by taking $L \rightarrow L_u$ and $\gamma_0 \rightarrow \bar{\gamma}_z$ which means that $R_{56} = -2\lambda_1 N_u$ for an undulator. This difference in path length due to energy is what causes the microbunching to develop from the energy modulation.

The important point for this section is that by seeding an FEL with both an electron beam and a laser beam we can induce microbunching at the wavelength of the laser seed if the electron beam energy satisfies the resonance condition. The maximal amount of microbunching is close to 1 and it occurs when the particles closest to the stable fixed point go through approximately one quarter oscillation through the entire undulator. The mechanism for microbunching development is that the laser imprints a correlated energy spread, that is an energy spread that depends on longitudinal position, onto the electron beam with periodicity equal to the laser wavelength and this energy modulation interacts with the R_{56} of the undulator to produce a density modulation.

4.2 High Gain FELs

Rather than go through the rather involved derivation of the physics of high gain FELs, we will sketch out the physical principles that work throughout the gain

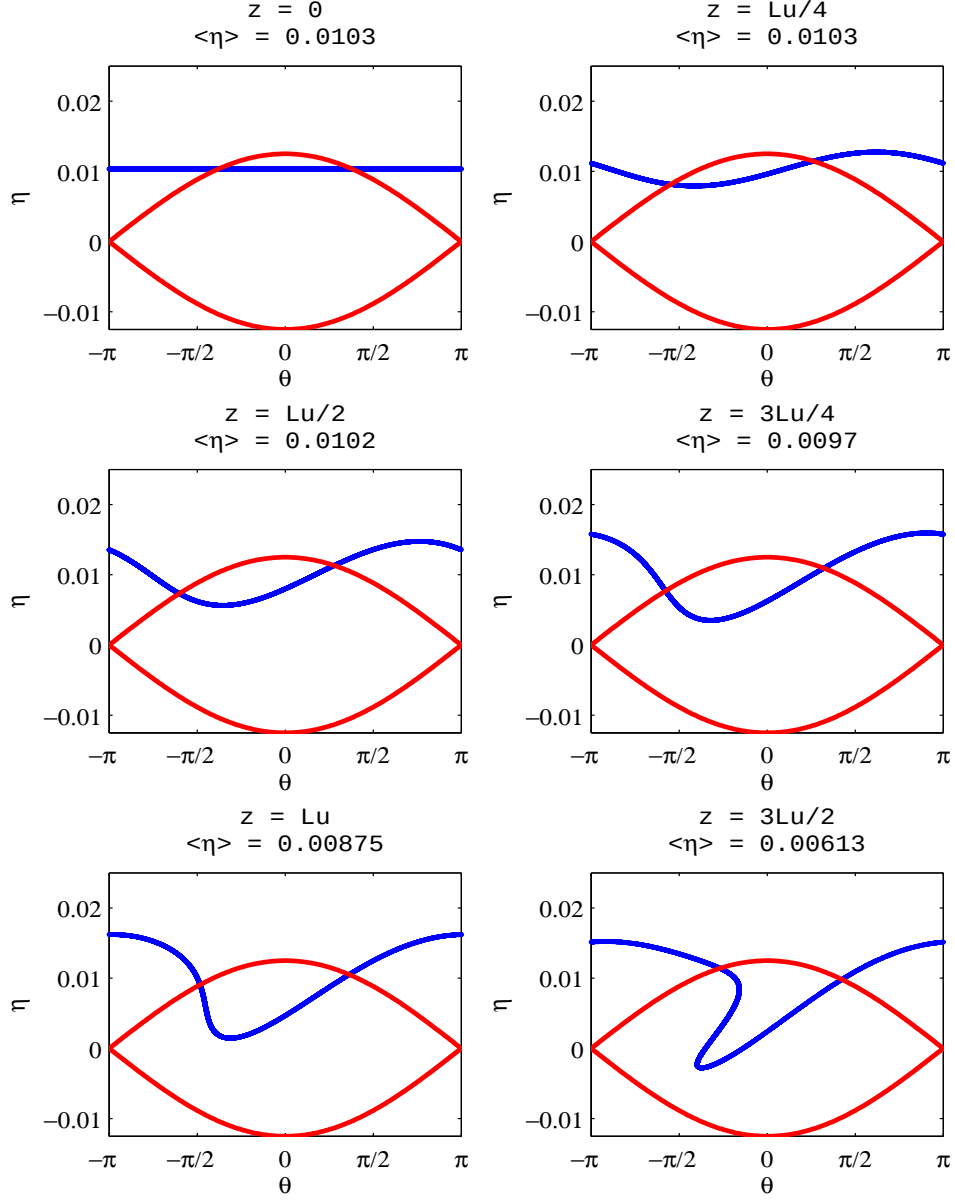


Figure 4.4: Evolution of the longitudinal phase space for an optimally detuned electron beam starting with $\langle \eta_0 \rangle = 1.3/(2\pi N_u)$. The mean relative energy of the electron beam is shown above each image.

process in order to demonstrate the importance of electron beam brightness to the gain mechanism. This section draws on sources from textbooks on FELs [112, 113] and review articles [6] to conference proceedings on high brightness electron beams [114] and many papers [5, 115, 116, 117, 118, 119, 120, 121, 122]. The most fundamental parameter for free-electron lasers is the so-called one-dimensional FEL parameter (sometimes also called Pierce parameter) [5]

$$\rho = \left[\gamma \frac{I}{I_A} \left(\frac{K [JJ]}{1 + K^2/2} \right)^2 \frac{\lambda_1^2}{16\pi^2 \sigma_x^2} \right]^{1/3}, \quad (4.17)$$

where I is the beam current, $I_A = ec/r_e = 17045$ A is the Alfvén current, and σ_x is the horizontal beam size. All of the other symbols have been defined previously. The first term in the brackets is basically the number of electrons, the second term is the unit less scaling of the on axis spectral intensity (see Fig. 3.14) which has maximum of 0.4 for $K = 1.2$, and the last term is the relative size of the photon beam emittance, $\lambda_1/(4\pi)$, squared to the cross sectional area of the electron beam, σ_x^2 . The meaning of the FEL parameter is the ratio of the field energy generated at saturation to the e-beam kinetic energy [112], such that $P_{Laser} \sim \rho(I/e)\gamma m_0 c^2 = \rho P_{beam}$. This is why it is sometimes called the efficiency parameter.

The meaning of each of the three terms is as follows. The first term is the density of the electron beam. As electrons are the gain medium for the laser, more electrons means more gain. Because Eq. 4.17 was found under the assumed absence of space charge forces, there is no limit to what I can be achieved [123]. The emission intensity of the individual electrons is given by the second term. Because the FEL is the manifestation of the coherent emission shown in section 3.5, higher single electron emission means higher coherent emission. That $F_1(K)$ peaks at $K = 1.2$ is the reason that maintaining a relatively large K , and thus

large peak fields, is so crucial for short period undulators.

The last term is the effect of electron beam-laser beam overlap in transverse space on the laser gain. Because it hasn't been explored yet, we investigate it further now.

For a transverse laser mode in one dimension the minimum phase space area is given by the uncertainty principle: $\sigma_r \sigma_{p_r} \geq \hbar/2$. Because photons follow Bose-Einstein statistics, a great many photons can be compacted into this uncertainty limited single photon phase space. Because we normally work in trace space under the paraxial approximation the uncertainty principle can be recast as (defining the divergence of the beam, $\sigma_{r'} \equiv \sigma_{p_r}/p$)

$$\sigma_r \sigma_{r'} \geq \frac{\lambda}{4\pi} \equiv \epsilon_\lambda, \quad (4.18)$$

with λ the wavelength of the photon. The existence of this limit on the product of the trace space dimensions leads to diffraction. As the beam is focused more narrowly in real space the divergence of the beam increases. The relation in Eq. 4.18 reaches the limit of equality when the laser mode is Gaussian, this mode diffracts the least when focused.

Returning to the third term on the right in Eq. 4.17, we find it is equal to $(\epsilon_\lambda/\sigma_x)^2$. The meaning of this term is that the electron beam size should be approximately equal to the laser beam size. If the electron beam is too large it doesn't efficiently feed energy into the desired laser mode. In fact, the electron beam will feed many transverse modes and the coherence will be degraded. However, what is not clear from Eq. 4.17 is that if the electron beam is too small the growth of the desired Gaussian mode will also be slowed. We will see this after introducing another parameter.

The next most important parameter, after the efficiency, is the gain length [5], which is the length along the undulator in which the laser power grows by a factor of e . It is defined as

$$L_{g,1D} = \frac{\lambda_u}{4\pi\sqrt{3}\rho}. \quad (4.19)$$

The gain length is a measure of the FEL's efficient use of the undulator length. As ρ gets larger, the gain length decreases and the length of the undulator required to reach saturation is decreased. From the combination of λ_u and ρ in Eq. 4.19 it is easy to see the impetus for using high brightness electron beams in short period undulators. Increased brightness more efficiently uses the undulator length and shorter periods makes the undulator shorter.

The FEL parameter in Eq. 4.17 and the gain length in Eq. 4.19 were derived under quasi-1D conditions which assumed no diffraction and that the electrons emit a single frequency. A variational fit of the solutions of the FEL equations finds the effect of diffraction and incoherent emission bandwidth on the gain length as [124]

$$\begin{aligned} \frac{L_g}{L_{g,1D}} = & 1 + 0.45 \left(\frac{\epsilon_\lambda L_{g,1D}}{\sigma_x^2} \right)^{0.57} + 0.55 \left(\frac{\epsilon_x L_{g,1D}}{\bar{\beta} \epsilon_\lambda} \right)^{1.6} \\ & + 5.4 \left(\frac{\epsilon_\lambda L_{g,1D}}{\sigma_x^2} \right)^{0.7} \left(\frac{\epsilon_x L_{g,1D}}{\bar{\beta} \epsilon_\lambda} \right)^{1.9}, \end{aligned} \quad (4.20)$$

where the corrections due to energy spread have been ignored and $\bar{\beta}$ is the average beta function in a gain length.³ The first correction term shows the effect of decreasing the beam size too much in an attempt to increase the FEL parameter – which is the folly of attempting to maximize a quasi-1D parameter by taking

³As we saw in Eq. 4.6, it is undesirable to have β change too much or the electrons will slip in phase too much, so taking the average over a gain length isn't a stretch.

extremes in one of the quasi-dimensions. As the beam is reduced, too much of the produced radiation quickly diffracts away from the electron beam and the gain is reduced, as signified by the increased gain length. The second term is the effect of the finite beam emittance on the gain length. As the beam is focused the rms angle of the electrons with respect to the axis of propagation (which is ideally straight down the undulator) increases. This increases the spread of wavelengths emitted on axis because of the Θ in the resonance condition, Eq. 3.21. The effect is to increase the bandwidth of the radiation. For the increase in the gain length to be modest both of these terms should be kept small with respect to unity. These two conditions can be summarized as a triple inequality relating the average beta function, the 1D gain length and the emittances of both the electron and laser beams:

$$\frac{L_{g,1D}}{\bar{\beta}} \leq \frac{\epsilon_n}{\gamma\epsilon_\lambda} < \frac{\bar{\beta}}{L_{g,1D}}. \quad (4.21)$$

The diffraction limit is on the left and the angular spread (emittance) limit is on the right. A careful balance between matching the electron beam size to the diffraction limited beam size of the laser is required to optimized gain.

With the two most important parameters in hand we can now summarize the operation of an FEL from start to finish. The electron beam starts at an electron source where the space charge forces dominate the brightness (subsection 1.4.1) and then it is accelerated to the design energy such that the resonance condition produces the desired wavelength using the resonance condition (Eq. 3.21). At this point it enters the undulator, sometimes with either a laser seed or a microbunching seed of the same wavelength, and the radiation power begins to grow as

$$\frac{dP}{d\omega} = \frac{dP_0}{d\omega} \exp [z/L_g]. \quad (4.22)$$

The dependence on the radiation frequency has been explicitly added because the input power and output power are different for each wavelength.

As the electron and laser beams travel along the undulator, the power grows and so does the fraction of the electron beam that is microbunched. In fact, the rate of growth of the field is proportional to the microbunching factor. As we saw in the linear section, the FEL mechanism causes the energy spread of the electron beam to increase. We also know from Chapter 3 that the energy bandwidth of particles that will remain in resonance over an undulator that is N_s periods long will be $\Delta\omega/\omega = 2\Delta\gamma/\gamma \sim 1/N_s$. So that saturation, that is cessation of exponential gain, occurs when the relative energy bandwidth of the electrons is equal to the incoherent bandwidth of the undulator. If the undulator is longer than N_s some of the electrons have time to slip in phase with the radiation field and then begin to absorb energy from it.

By conservation of energy, if the radiation field is gaining ρP_{beam} then the electron beam is losing the same amount of power. For a laser beam that is initially on resonance such that $\langle\eta_0\rangle = 0$ then in the final period (at saturation) we must have $\langle\eta_s\rangle \simeq \rho$.⁴ To be clear, $\Delta\eta$ is not the energy spread of the electron beam, it is the average relative energy difference between the electrons and the resonant energy. But because we know that the gain has to stop when most of the particles have lost enough energy to have left the bandwidth of the process

⁴Here the brackets denote averaging over all of the particles,

$$\langle\eta\rangle = \frac{1}{N_e} \sum_{j=1}^{N_e} \eta_j \exp(-i\theta_j). \quad (4.23)$$

we see that the relative bandwidth is of the order of ρ at saturation. Analysis of the FEL gain equations finds this very answer [125]. Coupled with the known bandwidth of the undulator due to slippage we find that $\rho \sim 1/N_s$. Therefore, the saturation length is $L_{sat} = N_s \lambda_u \simeq \lambda_u / \rho \simeq 4\pi\sqrt{3}L_g$. Because the FEL parameter is usually of a few parts in a thousand, saturation typically requires about a thousand periods, which is why all of the points in Fig. 1.5 show roughly the same number of periods, because ρ is similar in all three cases.

The input power spectrum is a function only of the the initial conditions and therefore does not depend on any of the preceding parameters. What is normally assumed is that the radiation field has some set of modes given by the initial conditions, such that

$$E_\omega(\mathbf{x}; z) = \sum_{n=0}^{\infty} C_n A_n(\mathbf{x}) \exp \left[\frac{z}{2L_{g,n}} \right] \quad (4.24)$$

where the C_n are the coefficients that solve the initial conditions, $A_n(\mathbf{x})$ is the transverse profile of the mode and the gain length is now explicitly mode dependent. Usually, it is the single lowest mode that is considered because it has the highest gain. But this mode does not cover the whole length of the electron beam longitudinally because the saturation length is finite the radiation will slip along the electron beam for a total length of $L_{sat} = N_s \lambda_1$.

In the case of the seeded FEL, the spectrum of the radiation is dominated by the spectrum of the input radiation or microbunching. This spectrum dominates in the output power spectrum because the seeded frequencies grow much faster than the SASE radiation. Further, if the seed is totally coherent (like a laser) the output radiation is also totally coherent (both longitudinal and transverse).

When the FEL isn't seeded, the radiation is dominated by the shot noise of the electron beam. This shot noise is the result of the non-uniform spacing between

the electrons at frequencies comparable to the wavelength of interest. The noise in the electron beam translates into radiation via incoherent radiation in the undulator. What this means is that the first few gain lengths of the undulator "seed" the FEL modes of Eq. 4.24 for later gain lengths.

For an FEL, the power of what ever signal is put in (whether shot noise or seed) grows by an order of magnitude in power in 2.3 gain lengths. It is at this point that the radiation from the incoherent radiation from the previous sections is much larger than the radiation due to the shot noise itself (which is by definition a factor of 10 smaller). Thus, after 2.3 gain lengths, the electron beam ceases to be a source of shot noise seed and becomes the gain medium alone. The length of electron beam over which the radiation field slips in this distance is called the cooperation length because the electrons within this region are all radiating independently into the incoherent radiation seed for the gain mechanism, it is $L_{coop} \equiv 2.3L_g\lambda_1/\lambda_u \simeq 2.3\lambda_1/(4\pi\sqrt{3}\rho)$.

To see how the cooperation length after 2.3 gain lengths translates into the spectrum at the end of the undulator we investigate the FEL bandwidth. The bandwidth of the radiation in an FEL evolves along the undulator according to [122]

$$\sigma_\omega^2 \simeq \frac{18k_1^2 c^2 \rho^2 L_g}{z} = \left(\rho\omega_1 \sqrt{\frac{L_{sat}}{z}} \right)^2 \quad (4.25)$$

which reduces to $\sigma_\omega \simeq \rho\omega_1$ in the limit of saturation, as was found earlier through an intuitive argument. This means that the coherence time of the FEL, which we saw in section 1.4 is the inverse of the bandwidth of the spectral density, evolves as

$$\omega_1 \tau_c(z) \simeq \frac{1}{\rho} \sqrt{\frac{\pi z}{L_{sat}}} \quad (4.26)$$

for a Gaussian spectral density. From this it can be seen that the cooperation length is just the coherence length of the radiation at 2.3 gain lengths. In this sense, the cooperation length can be thought of the length of the electron beam that can contribute to a single spike of the relatively wide FEL bandwidth at $2.3L_g$. The cooperation length is also explained by the definition of the coherence time ($L_c \equiv c\tau_c$) which uses the autocorrelation. The fields emitted in two different cooperation lengths are not related to each other, so the autocorrelation becomes zero at the points when the relative delay becomes greater than τ_c . What this means is that the incoherent radiation produces, at saturation, a number of independent radiation pulses of bandwidth inversely proportional to the coherence time at saturation.

Which spikes grow varies on a shot-to-shot basis but as the FEL gain mechanism proceeds, the bandwidth narrows and the coherence time grows such that there are approximately $M_L = T_{SASE}/\tau_c(z = L_{sat})$ separate radiation spikes per pulse where T_{SASE} is the total time length of the radiation pulse. The exception is when the SASE pulse length is shorter than one coherence length at saturation, then there can be only one single spike. Therefore, for SASE radiation, the pulse is not longitudinally coherent unless the electron beam is short enough such that $M_L \rightarrow 1$.

In contrast, the transverse coherence is assisted greatly by diffraction. This process known as optical guiding [124]. A common basis for the expansion in Eq. 4.24 is the Hermite-Gaussian set of functions because they are complete [88]. The lowest order mode ($n = 0$) in this set is the Gaussian distribution with emittance given by Eq. 4.18. For a Hermite-Gaussian mode of order n the emittance is

[126]

$$\sigma_r \sigma_{r'} = (2n + 1) \epsilon_\lambda, \quad (4.27)$$

where $n = 0, 1, 2, 3 \dots$. This means that for modes that are approximately the size of the electron beam the Gaussian fundamental mode diffracts the least. By taking $\epsilon_\lambda \rightarrow (2n + 1) \epsilon_\lambda$ in Eq. 4.20 it is clear that the gain length scales approximately as the square root of the order of the Hermite-Gaussian mode, approximately $\sqrt{n}$. Further, the higher order modes are larger, also scaling as $\sqrt{n}$ [88], which reduces the efficiency of coupling to the electron beam because the radiation from the electrons is angle dependent (see Eq. 3.21), while the mode is a single frequency. To underscore the importance of good overlap between the electron beam and the laser mode, it has been found that beams that travel with an offset relative to the optical axis of an FEL oscillator produce intensity distributions similar to higher order modes [127]. Following Huang and Kim [6], we define the transverse mode number as $M_T^2 = (\epsilon_r / \epsilon_\lambda)^2$ where ϵ_r is the emittance of the radiation which will differ from the ideal value because the radiation from an FEL has a finite bandwidth. The difference from 1 is quite small, $M_T^2 - 1 \approx 0.06$ for the LCLS [128]. Thus, the radiation is very nearly totally transversely coherent and contains only the fundamental mode ($n = 1$).

Because the saturation length of FELs is proportional to λ_u / ρ the apposite combination of short period undulators and high brightness electron beams seems clear. Reduced beam energy (for a desired wavelength) and undulator length can result in lower cost FELs. With this in mind we present a number of examples combining these two factors. The undulator that is used in these simulations is the cryogenic permanent magnet undulator described in Chapter 2, with $\lambda_u = 9$ mm and $K = 1.7$. The undulator parameter used in the following analysis is

larger than the device that was built because the prototype CPMU9 device has a 2.5 mm gap, while the device used in the simulations was assumed to have a smaller gap [18].

4.3 High Brightness Electron Beams in a Short Period Undulator

In this section, we illustrate the performance of the combination of high brightness electron beams and short period undulators, in simulation, using five distinct cases: (1) the SPARX FEL, in which a ~ 2 GeV, 1 pC beam is used to create an extremely compact hard x-ray FEL, with harmonics reaching to the current LCLS wavelength; (2) the LCLS beam run at 0.25 pC charge and 4.5 GeV energy, in which one obtains saturation at the current LCLS wavelength in a fraction of the undulator and linac lengths; (3) the same beam run at 13.65 GeV energy, which enables a >80 keV x-ray FEL; (4) the experimentally studied moderate charge (20 pC) beam case at LCLS, which is compressed to 2 fs, in which the emittance effects are notable, and (5) the LWFA case, in which one may have a compact *table-top terawatt* FEL.

The the time dependent FEL code GENESIS 1.3 was used to perform these simulations [129]. A summary of the parameters used in the five cases is shown in table 4.1.

4.3.1 SPARX

We discuss here an example of the exploitation of the 9 mm period, high field undulator introduced in Chapter 2 in the SPARX FEL. For the beam simulations, we follow the acceleration, compression, and lattice program used in the SPARX

Table 4.1: Summary of the results of the GENESIS 1.3 FEL simulations.

		SPARX		LCLS		(T) ³ -FEL
E	GeV	2.1	4.5	13.65	13.65	1.74
I	kA	0.7	0.35	0.35	8.0	160
Q	pC	1.0	0.25	0.25	20	1600
$\epsilon_{n,x(y)}$	10^{-8} m·rad	7.5(3.3)	3.3(3.3)	3.3(3.3)	40(14)	100(100)
λ_r	Å	6.5	1.4	0.15	0.15	9.5
$\bar{\beta}$	m	2.0	4.8	4.8	4.8	4.3
ρ	10^{-3}	1.8	0.75	0.36	0.51	3.6
L_{sat}	m	10	15	40	170	3
Out. Power	GW	2.4	4.5	2.4	1.0 ⁵	1090

technical design report [130]. We concentrate on the case of 1 pC operation, in which sub-fs pulse, single-spike operation at 2 nm fundamental wavelength has been predicted in previous analysis with the standard $\lambda_u = 2.8$ cm undulator [131]. The electron beam is first compressed to $\sigma_z = 4.7$ μ m using velocity bunching [132], and then chicane compressed to an rms bunch length of $\sigma_z = 0.21$ μ m (0.7 fs) at 1.2 GeV, on the way to acceleration up to a final energy of 2.1 GeV. The beam in this case has extremely low emittances, $\epsilon_{n,x(y)} = 7.5(3.3) \times 10^{-8}$ m-rad, and over 700 A peak current after final compression. The larger horizontal emittance is due to coherent synchrotron radiation effects in the chicane [133, 134]. With this limit, the electron beam transverse brightness at the undulator is $B_T = 2 \times 10^{20}$ A-m⁻²rad⁻², which should be compared with the nominal high charge ($Q = 1$ nC) base-line design value of $B_T = 2.8 \times 10^{15}$ A-m⁻²rad⁻². This dramatic difference in transverse brightness produces quantitatively different behavior in the FEL gain process.

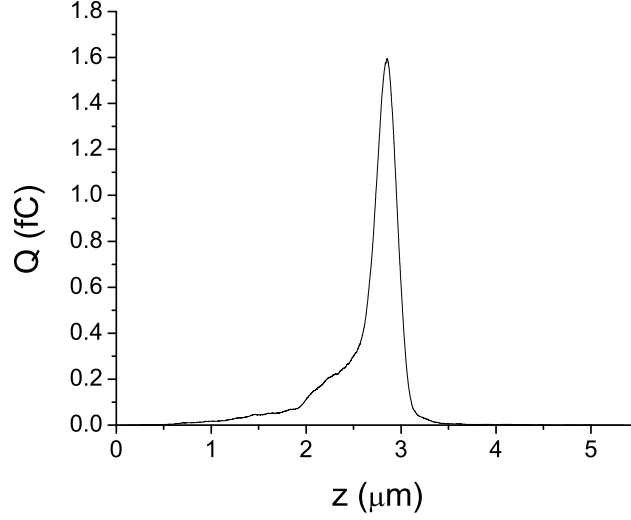


Figure 4.5: Beam profile of the SPARX ultra-short beam as generated with start-to-end simulations.

The beam's longitudinal phase space displays multi-fs tails outside of the lasing core, and thus the rms length of the radiation pulse is yet smaller than that of the electron beam, see Fig. 4.5. In the reference design case discussed in Ref. [131], where an undulator with $\lambda_u = 2.8$ cm and $K = 1.67$, the 2 nm FEL pulse given in GENESIS 1.3 simulations has an rms length of $\sigma_t = 0.48$ fs. This pulse displays only a single spike at saturation, which is achieved in simulation after 30 m. As the beam is significantly higher brightness than that of the high charge base-line design, the saturation length is indeed shorter, decreasing from 50 m in the base-line design case, despite running at higher energy and shorter wavelength, which is 3 nm in the standard case.

With the high field cryogenic undulator, we employ the same simulation beam particles as utilized in Ref. [131] to evaluate the SASE FEL performance. In

⁵At 170 m.

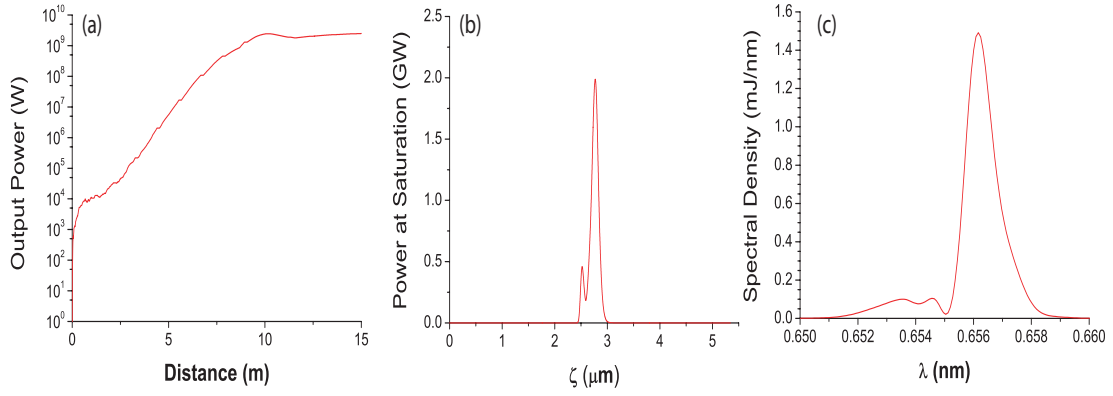


Figure 4.6: GENESIS 1.3 simulation results for the SPARX ultrashort low charge case. Figure (a) shows the exponential high gain regime saturating at 10 m of undulator. Figure (b) shows the gain as a function of distance along the beam. Figure (c) shows the spectrum of the output radiation.

comparison to the base-line design, we have decreased the average beta-function in the undulator, from 5.5 m to 2 m. The tighter focus implied is encouraged by the use of lower emittance beams, as one does not encounter gain degradation from excessive transverse angular motion (this is the right side of Eq. 4.21). The fundamental wavelength is, in this case, 6.56 Å, or a factor of nearly five shorter than obtained with the $\lambda_u = 2.8$ cm undulator. The spectrum at saturation, given by GENESIS 1.3 simulation, is shown in Fig. 4.6c. The evolution of the power along the undulator is illustrated in Fig. 4.6a, showing a gain-length of 43 cm, and saturation of the fundamental at ~ 10 m. Indeed, with such a short gain length, one may in principle decrease the average beta function in the undulator further. In practice, the lower limit on the beta-function is given by the practical consideration of our choice of 1 m undulator sections. Quadratic pole faces will allow for strong focusing and a further decrease in the beta-function (section 1.3). Nonetheless, the presented scenario is a strikingly compact, high gain system. Despite such a short cooperation length, with a well sub-fs electron

beam, quasi-single spike operation (cf. Fig. 4.6b) is obtained, with an rms x-ray pulse length of 0.35 fs. The peak power is over 2 GW, giving 6×10^9 photons at 1.8 keV, a photon beam peak brightness of 3×10^{35} photons/(s mm² mrad² 0.1% BW) .

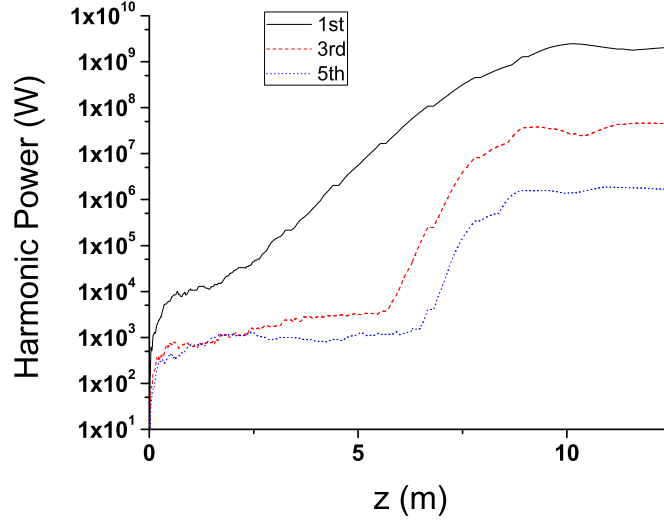


Figure 4.7: Harmonic gain of the SPARX ultrashort low-charge beam lasing in the cryogenic undulator.

Because of the strong gain on the fundamental interaction, the beam is microbunched ever more strongly in the undulator, and the possibility of observing gain on the odd harmonics presents itself [135]. We have studied this phenomenon with GENESIS 1.3 as well, with the results summarized in Fig. 4.7. Note that in this case the 5th harmonic has a wavelength of only 1.3 Å; we may create a SASE FEL with wavelength below that of the LCLS using only 2.1 GeV electrons in this short period, high field, high brightness electron beam scenario. This 10-m undulator system produces over 10⁶ coherent hard X-ray photons.

This compact SASE FEL system illustrates well the marriage of a novel high

field cryogenic undulator design with ultra-high brightness electron beams produced at low charge. In the following example, we emphasize the possibilities opened not just by the beam brightness, but by the achievement of lower emittance per se. We will, from this viewpoint, examine opportunities for pushing the frontier in coherent x-ray production to ever shorter wavelengths.

4.3.2 LCLS ultra-low charge, high brightness beam scenario

In order to study possible single spike behavior in the LCLS, we have examined scenarios based on use of 0.25 pC beams, in which case yet shorter pulses can be achieved. This shortening, in comparison with the SPARX case discussed above, is demanded for single spike performance at shorter wavelength because the cooperation length is much reduced at the LCLS wavelength of 1.5 Å. In the context of the LCLS injector and linac, we have performed start-to-end simulations in which the beam is compressed to $\sigma_t \simeq 0.2$ fs with a peak current of ~ 350 A. With such small charge, the beam longitudinal phase space is quite compact, and no significant tails arising from CSR during compression are observed in the ELEGANT simulations. The transverse phase space is also more well-behaved in the compressor, with all significant emittance growth occurring during velocity bunching; no focusing solenoids are available in the post-gun section of the injector, and the final emittance in both transverse planes is in this case $\epsilon_n = 3.3 \times 10^{-8}$ m-rad. Thus the beam is slightly brighter than in the 1 pC case discussed above, while the 4-dimensional transverse phase space area is diminished by over half. This will prove to be a key advantage in pushing the FEL to lase at unprecedented short wavelengths, well sub-Ångstrom.

To illustrate first the effect of beam brightness and a short period undulator, we study the use of the cryogenic undulator in the same configuration as in

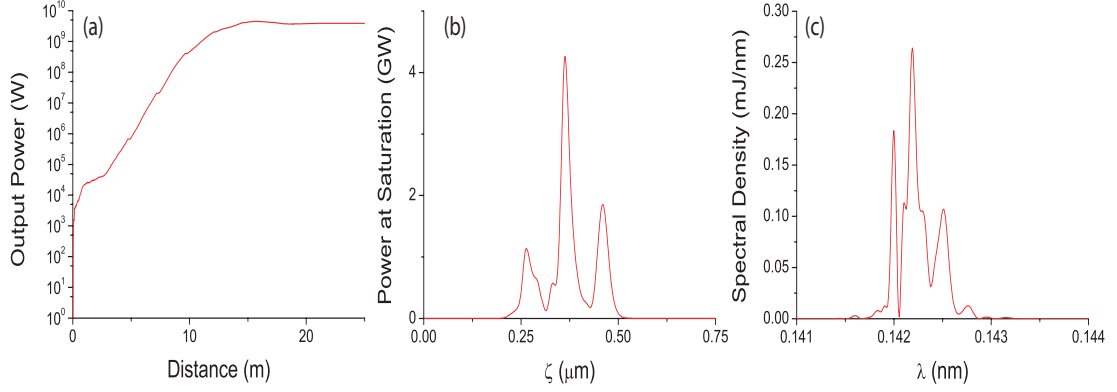


Figure 4.8: GENESIS 1.3 simulation results for the 4.5 GeV LCLS beam. The saturation length of the fundamental mode is 15 m with a gain length of 70 cm (a). At saturation the beam shows 3 spikes in the longitudinal profile, consistent with a 11 nm cooperation length (b). The fundamental interaction wavelength is just over 1.42 Å(c).

the previous section to investigate SASE FEL performance near the nominal LCLS wavelength, obtained at a lower energy of 4.5 GeV. The results of the GENESIS 1.3 simulation of this scenario are shown in Fig. 4.8 (a-c). Figure 4.8(a) shows the evolution of the SASE power along the undulator, in which case we note saturation achieved within 15 m, with a gain length $L_g = 70$ cm. These parameters are a factor of 5 shorter than achieved in the the LCLS standard undulator, high charge case [3]. Despite use of charge 3 orders of magnitude smaller than in the LCLS nominal operating regime, a peak power of 5 GW is achieved in this simulation, corresponding to 2×10^9 1.42 Å photons in less than 0.5 fs FWHM.

With such high gain, and thus shortened cooperation length, the beam does not lase in single-spike mode in this case, as seen in Fig. 4.8(b), but rather has roughly three spikes. These spikes are also displayed, as expected, in the wavelength spectrum shown in Fig. 4.8(c). One may recover single spike perfor-

mance by running at higher charge, with (perhaps purposefully enlarged) higher emittance.

We note that in this case the electron beam emittance remains a factor of ~ 25 below the diffraction limit in Eq. 4.21. Thus one may envision use of higher energy beams with shorter wavelength operation than that expected of the LCLS. In fact, what is particularly striking about the LCLS-derived examples is perhaps not the possibility of dramatic shortening of the undulator and use of lower energy beams, but extending the wavelength reach of the FEL. As an aside we see that, given the scaling of Eq. 4.21 with energy, the middle term is the fastest growing term and that means that increasing the energy of the beam cannot decrease the transverse coherence of the output radiation, if all the other factors are held constant. As such, we have studied use of a nearly full energy, 13.65 GeV, electron beam at the LCLS in tandem with the cryogenic undulator. In this case we expect lasing at a fundamental wavelength of 0.15 Å, or exactly one order of magnitude shorter than the LCLS [3]. At this energy and wavelength, the beam used is much closer to the emittance limit, about a factor of four. The photons at this wavelength have an energy of 83 keV, and lie in a spectral region of high interest for studies of dense, high-Z materials, and may form the basis of large future FEL initiatives. Such incredible x-ray energy can only be achieved by an electron beam with a very small emittance, such as the beams we present here, in order to prevent energy spread caused by quantum fluctuations in the emitted synchrotron radiation to quench the lasing process [136, 137].

To simulate this ultra-short wavelength system, we introduce 2.27 m long undulator sections, with quadrupole focusing to give an average focusing $\bar{\beta} = 4.8$ m. The performance of this FEL is a prime example of the new possibilities afforded by use of high brightness beam, short period undulator system: the gain

length at 0.15 Angstrom is only 2.46 m (Fig. 4.9(a)), and yielding saturation in 40 m. The saturated power in this case is over 2 GW, with about 10^8 photons per pulse. The spectral coherence of this source is as expected for SASE, with the temporal (Fig. 4.9(b)) and wavelength power distributions both showing approximately eight spikes, reflecting the number of cooperation lengths (4 nm, or 14 attoseconds) inside of the lasing pulse.

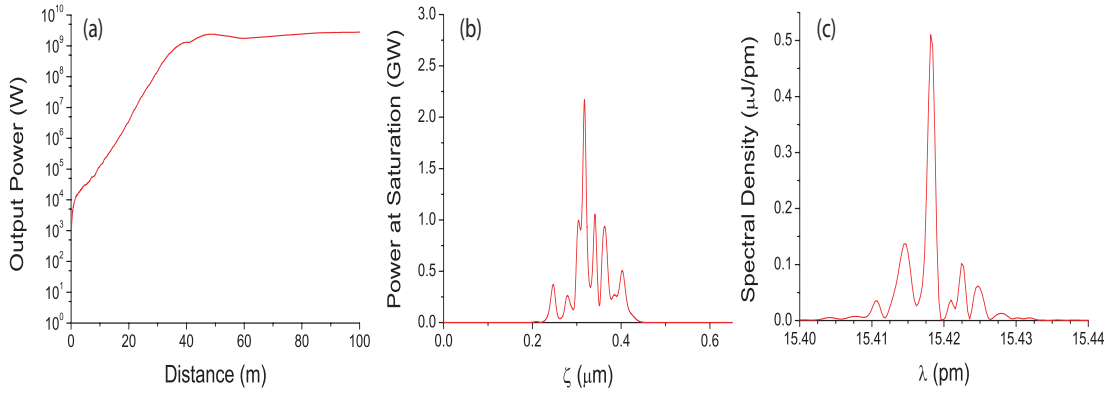


Figure 4.9: GENESIS 1.3 simulation of a 13.65 GeV LCLS beam traveling through the cryogenic high field undulator. Saturation occurs in 40 m (a). The incredibly short fundamental wavelength, 0.15 Å(c), does not allow single spike operation (b).

Lasing at such short wavelengths is indeed predicated on the emittance being as small as predicted from the start-to-end simulations. To illustrate this, we utilize the beam parameters achieved at SLAC in the first attempt to examine low charge, high brightness beam performance [138]. In this case, while the peak current is increased to an estimated 8 kA, the geometric mean of the transverse emittances is also over an order of magnitude higher, as noted above. Thus the beam brightness is degraded, and more importantly, the diffraction criterion is quite strongly violated. As such the SASE FEL gain simulated in GENESIS 1.3 using the cryogenic undulator is also degraded, with the results shown in Figure

4.10. Here one would need a 150 m undulator to approach saturation, which is a much less desirable scenario than given by use of the higher brightness beam. Thus it is advisable to push towards lower charges and therefore emittances, as studied in Ref. [131].

Alternatively, one can regain compliance with Eq. 4.21 by lowering, as studied in the extreme low charge case above, the beam energy. With the electron energy set to 4.5 GeV and the same emittance and peak current, one lases at the nominal LCLS wavelength (1.5 Å), as before, and saturates the FEL at approximately 15 GW peak power in 20 m of undulator. The evolution of the SASE FEL power in GENESIS 1.3 simulation of this case is shown in Fig. 4.11. A similar exercise in examining the performance of the 20 pC beam was reported in Ref. [138] using the existing LCLS undulator. In this case, the 4.3 GeV beam produces a saturating FEL within 25 meters, at a wavelength of 1.5 nm. This comparison nicely demonstrates the advantage of using the short period cryogenic undulator.

4.3.3 Table-top FEL based on laser wakefield accelerator

The original motivation of the undulator design work discussed in Chapter 2 was the desire to build an x-ray SASE FEL with a very compact footprint by utilizing a laser wakefield accelerator (LWFA) as the injector. This injector is currently being used in a soft x-ray undulator light source [25]. In such a scheme, one with >10 GV/m field driven by an ultra-short (10's of fs) laser pulse, one may create >10 GeV electron beams in a dense plasma a few cm in length. This approach is discussed in depth in Refs. [80, 139]. To explore the performance of the current undulator design as applied to this scheme, we use the beam parameters given in [80] for the beam expected from the LWFA driven by a 1 PW laser pulse: 160 kA peak current contained in a 4 fs rms pulse, $\epsilon_n = 1 \times 10^{-6}$ m-rad, energy

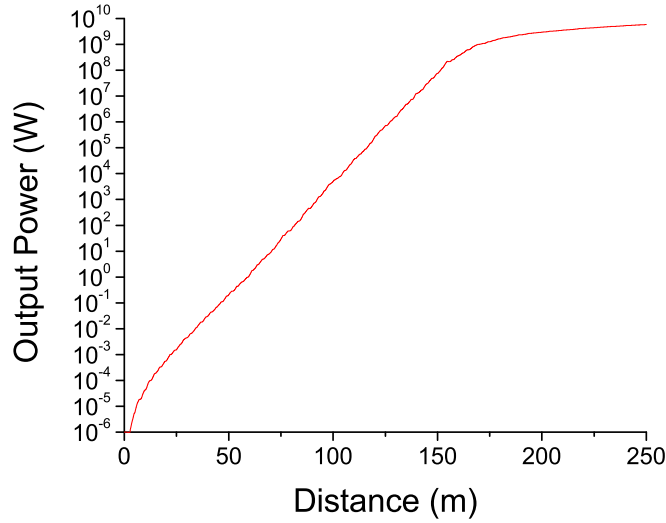


Figure 4.10: GENESIS 1.3 simulation of the 20 pC, 2 fs rms pulse length beam at LCLS, full LCLS energy (14.3 GeV). Violation of the diffraction criterion causes dramatic growth in the gain length.

of 1.74 GeV, with rms energy spread of 0.1%. The beam here is quite bright, competitive with the 0.25 pC LCLS case given above, due to the extremely high predicted current. This is an important distinction. The previous examples show the importance of meeting the diffraction criterion in Eq. 4.21 by controlling the electron beam emittance and thus settling for lower total bunch charge. The result of using only these electrons best tuned to the exponential gain process is a quickly saturating free electron laser. In the current table-top case, however, the opposite approach of brute force has been taken. This free electron laser saturates in a short distance because an enormous number of electrons have been crowded into a rather typical emittance. Therefore, the electrons can constructively add energy to the fundamental mode much faster than diffraction effects can take energy away. This further means that there will be many transverse radiation modes in the far field.

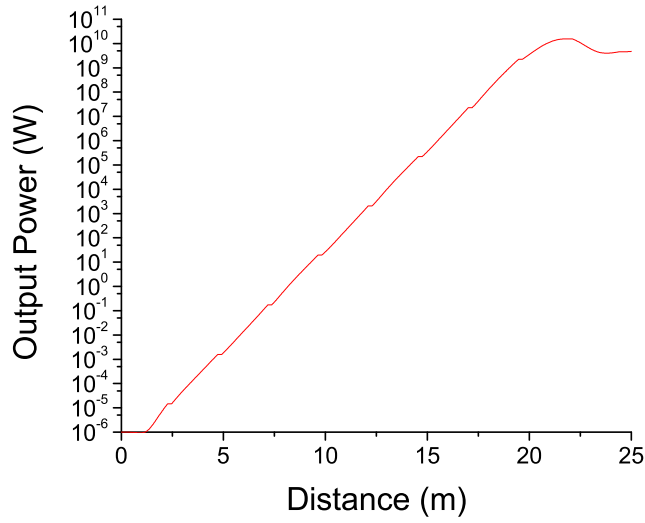


Figure 4.11: GENESIS 1.3 simulation of the LCLS 20 pC beam at derated energy of 4.5 GeV, compliance with diffraction criterion re-established.

The GENESIS 1.3 simulation results for this case are given in Figures 4.12(a-c). They display extremely high gain at a wavelength near 0.95 nm, with a peak saturation power of over 1 TW, well in excess of the 58 GW predicted for the design discussed in Ref. [80]. The total saturated pulse energy is impressive, reaching 6.5 mJ, or 3×10^{13} soft x-ray (1.3 keV) photons. This highly energetic saturation state, further, occurs within 3 m of undulator.

This example serves well in illustrating a vision of an ultra-compact FEL design, one in which the injector is a compact LWFA section, and an undulator that is over an order of magnitude shorter than those in use today. This compelling scenario, should it be realized, will produce the first ever terawatt FEL, and it will fit in a university-scale lab. One might term it, in the tradition of today's ultra-fast laser systems and the LWFA initiative described in Ref. [80], a *table-top terawatt* (T³) SASE FEL.

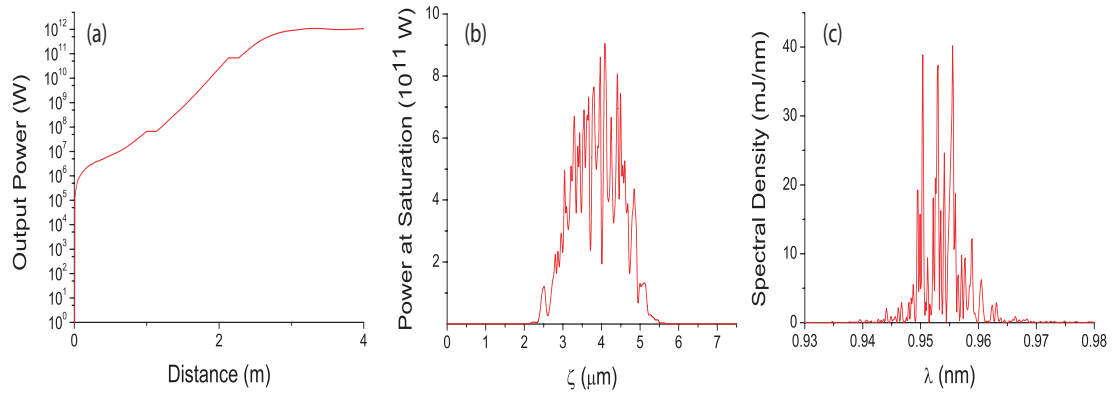


Figure 4.12: GENESIS 1.3 results for the (T^3) SASE FEL. Saturation occurs in 3 m (a), with the soft x-ray energy distributed fairly uniformly over the entire length of the pulse (b). The spectral make-up if the beam is noisy (c), expected from such a large emittance beam.

CHAPTER 5

Experiments

5.1 Inverse Compton Scattering at ATF

High brightness x-ray light sources are an invaluable tool for modern science. However, in addition to all the useful properties of x-rays created by insertion devices in synchrotrons, some applications demand sub-ps x-ray pulses. Short pulse x-rays may be created in synchrotrons using laser slicing of the electron beam [140]. However, along with ultrashort (~ 100 fs level [100]) pulses, Inverse Compton scattering (ICS) sources have other attractive attributes such as straightforward x-ray energy tuning and polarization control as well as large peak flux (see section 3.4). The ICS source also benefits from the scaling of the scattered radiation wavelength with that of the interaction wavelength requiring a lower energy electron to produce a given wavelength as compared to undulators.

Further, unique aspects of ICS probes –production of MeV γ -rays [97, 101] and sub-ps measurement of delay between pump and x-ray probe pulses [141]– can extend the range of processes that can be studied to higher energy and shorter time scales as compared to synchrotron and FEL sources. ICS sources [142], made compact by RF photoinjector and chirped pulse amplification laser technologies [143], provide relatively high peak brightness, narrow spectrum x-rays with average brightness limited by accelerator and laser repetition rates. All of these features benefit research into microscopic processes that evolve at the

ps timescale or shorter, including chemical reactions, structural changes such as deformation and melting, and magnetic damping and demagnetization. Additionally, the possibility of high-brightness MeV γ -ray production by ICS may permit beneficial applications in cargo inspection and remote sensing of nuclear materials [144]. It is in this context that we present the detection of 10^2 diffracted x-rays as evidence of suitability of the ICS source as a time resolved x-ray diagnostic.

The pulse length of ICS x-rays is determined by the interaction geometry, which is head-on (180°) in this case (see Fig. 5.1). In this case the x-ray pulse length is determined by the electron pulse duration, which can be made as short as sub-ps using techniques such as magnetic chicane compression [145] or velocity bunching [146]. To demonstrate multi-photon diffraction and detection, we have produced single-shot images of Bragg diffraction of the x-rays created at the ICS source developed at the Brookhaven National Laboratory Accelerator Test Facility (BNL ATF) [147, 100] using a flat silicon crystal as a diffraction element.

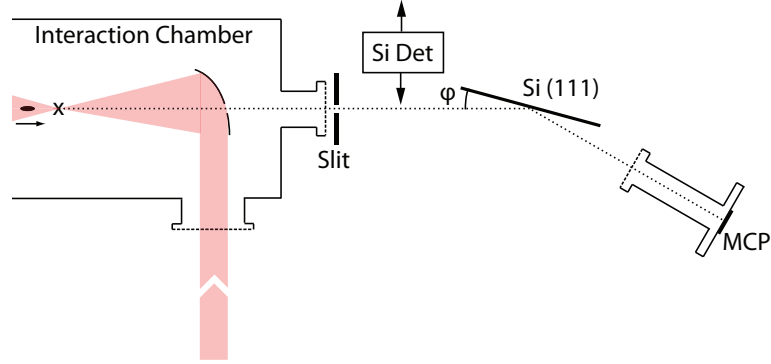


Figure 5.1: Experimental setup of the ICS source at the BNL ATF. The x-rays (dotted line) are generated at the IP (X), collimated by the metal slits, and are then reflected off the silicon crystal, Si (111), for detection at the MCP. A retractable silicon detector (Si Det) is used for total flux measurements.

The experimental setup is illustrated in Fig. 5.1, while the parameters for

the electron beam and the CO₂ laser at the IP given in Table 5.1. The x-rays produced at the IP exit the vacuum system through a 250 μm Beryllium window; their path is determined by the direction of motion of the electrons [94, 97]. After the IP, the electrons are steered to a beam dump with a vertical dipole. An adjustable slit is placed directly downstream of the Be window to select x-rays with small divergence in one transverse dimension. The slit is composed of two 5 mm thick aluminum plates controlled by independent linear translation stages. The x-ray imaging system consists of a chevron-type micro-channel plate (MCP) and a phosphor. The phosphor is imaged with a sensitive megapixel CCD fitted with a 50 mm focal length (f/1.3) lens. The x-rays pass through a second 250 μm Be vacuum window before illuminating the MCP.

A retractable, calibrated silicon diode is used to measure the total intensity of the x-ray flux [148]. The x-rays passed through the vacuum window on the interaction chamber (see Fig. 5.1) and 20 cm of air before detection by the diode. The high average number of x-rays created per shot, 6×10^7 , was made possible by the interaction of a focused, intense electron beam with the large photon density of the 2 J CO₂ laser. We note that the CO₂ laser has the advantage of lower photon energy when compared to solid state lasers, thus leading to larger x-ray counts for a given total laser energy (e.g. a 14-fold increase over 800 nm lasers) [147]. The measured total photon flux of 6×10^7 photons is in excellent agreement with the analytical estimate of 7×10^7 given by Eq. 3.52, that assumes the classical Thomson cross section and a single collision luminosity.

This total flux corresponds to a peak brightness (see subsection 1.4.1) of 3.2×10^{19} *photons/(s mm² mrad² (10%BW))*. ICS sources produce a large relative bandwidth due to the red-shift of off-axis-directed photons. The total bandwidth can be narrowed using an aperture which will simultaneously sacrifice flux (e.g.

Table 5.1: Electron beam and CO₂ laser parameters at the interaction point.

Parameter	Value
Electron Beam	
Energy (MeV)	70.0
Charge (pC)	200
Energy spread rms	0.1%
Bunch length rms (ps)	4.0
Normalized emittance rms (mm mrad)	1
Spot size rms (μm)	30
CO ₂ Laser	
Energy (J)	2
Wavelength (μm)	10.6
Pulse length FWHM (ps)	6.0
Waist size FWHM (μm)	140
Bandwidth FWHM	0.7%

Fig. 3.12). Simulation (following Ref. [94]) indicates that an aperture of 0.5 mrad half angle will pass 2.7% of the photons having a 1.1% relative bandwidth for a peak brightness of $1.6 \times 10^{19} \text{ photons}/(\text{s mm}^2 \text{ mrad}^2 (0.1\% BW))$. Because of the modest efficiency of our detection scheme, we elected not to use an aperture this narrow.

Downstream of the IP, the diffraction of the narrow-band x-rays from a 4 inch diameter silicon (111) crystal (see Fig. 5.1), mounted on a rotation stage, obeys the Bragg criterion: $\varepsilon_x = \hbar\pi c/(d \sin \varphi)$ [149]. Here, $\hbar$ and c are the reduced Planck constant and the speed of light, respectively, $d = 3.135 \text{ \AA}$ is the spacing between the crystal planes and φ is the complement to the angle of incidence of the x-rays on the crystal (as defined in the Bragg formulation). The angular deviation from the Bragg criterion, or Darwin angle [150], is negligible for the Si(111) crystal within the energy range analyzed; we thus assume that all x-rays meeting the Bragg criteria are reflected with equal intensity.

A silicon crystal was chosen because of the wide availability of high quality samples. The sample used is a 0.5 mm thick single side polished crystal with surface roughness less than 5 $\text{\AA}$ on the polished side, as specified by the manufacturer. To keep bending to a minimum the crystal is held in place by set of grooved jaws which are closed only until movement is arrested. Bisection of the laser by the rotated crystal ($\varphi = 0^\circ$) showed no visible bending of the crystal. The crystal was aligned to the x-rays via alignment laser which was also used to define the electron beam path, measured using YAG fluorescent screens.

As the bandwidth of the x-rays is quite large compared to the crystal acceptance, the spectral selection of the diffraction process, along with the upstream slit, limits the number of detected photons. The regularly spaced Laue spots [149] usually present in standard diffraction techniques are not produced; instead, the

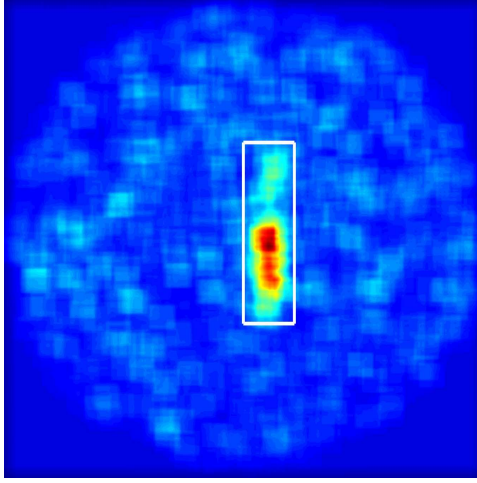


Figure 5.2: A single-shot diffraction image taken at $\varphi = 13.25^\circ$ (8.6 keV), median filtered. The white box indicates the diffraction pattern. The width (horizontal) of the x-ray distribution pattern is due to the slit acceptance while the vertical extent is determined by the Bragg criterion.

photons produce a characteristic diffraction pattern which is a function of the single electron x-ray intensity distribution and the electron beam distribution, filtered by the diffraction process.

An example single shot diffraction image taken at $\varphi = 13.25^\circ$ (8.6 keV) is shown in Fig. 5.2. The pattern is not vertically symmetric because the crystal was tilted slightly downward vertically (relative to the electron propagation through the IP), a systematic effect due to differences in transport between the electron beam and the alignment laser and a deflection of the electron beam by the downstream vertical dipole. This misalignment can be described by a vertical rotation of the crystal that precedes the rotation of the crystal to set the Bragg reflection angle. The diffraction element begins with planes normal to incident x-rays, it is then rotated vertically by an angle η (rotation down corresponds to positive η), after which it is rotated horizontally into a Bragg angle φ . The Bragg

criteria for an incident x-ray is

$$\begin{aligned} \frac{\lambda_x}{2d} = & \cos(\eta) \cos(\varphi) \sin(\psi_x) + \sin(\eta) \sin(\psi_y) \\ & + \cos \eta \sin \varphi \sqrt{1 - \sin^2(\psi_x) - \sin^2(\psi_y)}, \end{aligned} \quad (5.1)$$

with $\lambda_x = hc/\varepsilon_x$ and $\psi_{(x,y)}$ are the horizontal and vertical angle of incidence on the crystal respectively. The reflected x-ray appears on the detecting screen with a vertical offset (relative to ideal alignment) of

$$\vartheta - \vartheta_0 = -\sin(2\eta)(\varphi - \varphi_0), \quad (5.2)$$

for small Bragg angles, measured from a relative zero, ϑ_0 , that occurs at φ_0 . Vertical displacement of the expected null in the x-ray distribution as a function of Bragg angle, φ , indicates that the vertical tilt of the crystal is approximately 40 mrad. This vertical tilt is consistent with the ratio of the intensity of the upper and lower peaks in the diffraction patterns.

The vertical extent of the slit allows the full angular-spectral distribution of the photons to pass through, while the slit limits the horizontal extent to ± 0.5 mrad. The result is that the diffraction pattern displays a vertical distribution described by the parts of the photon beam that match the Bragg criteria for the angle of photon incidence. The largest photon flux after reflection occurs at $\varphi = 13.25^\circ$ (8.6 keV). This angle setting produced the most intense diffraction patterns, indicating that the angle of incidence meets the Bragg criteria near the peak spectral intensity of the x-ray distribution. At this angle 83 ± 22 photons are detected over the full aperture of the slit. A peak counting routine is used to identify counts on the MCP, with a threshold value to prevent counting of local variation in noise. It is assumed that each local maximum is a single photon.

Background counts are measured by averaging the total counts in other areas of equal size on the detector, consistent with background measurements taken with the source turned off.

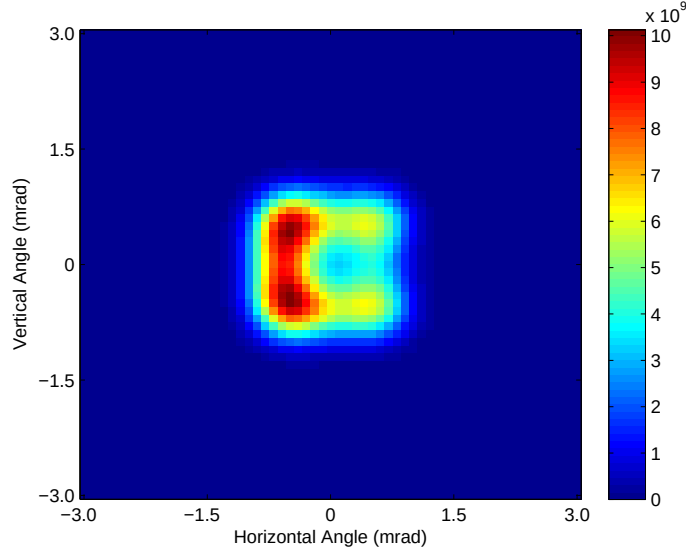
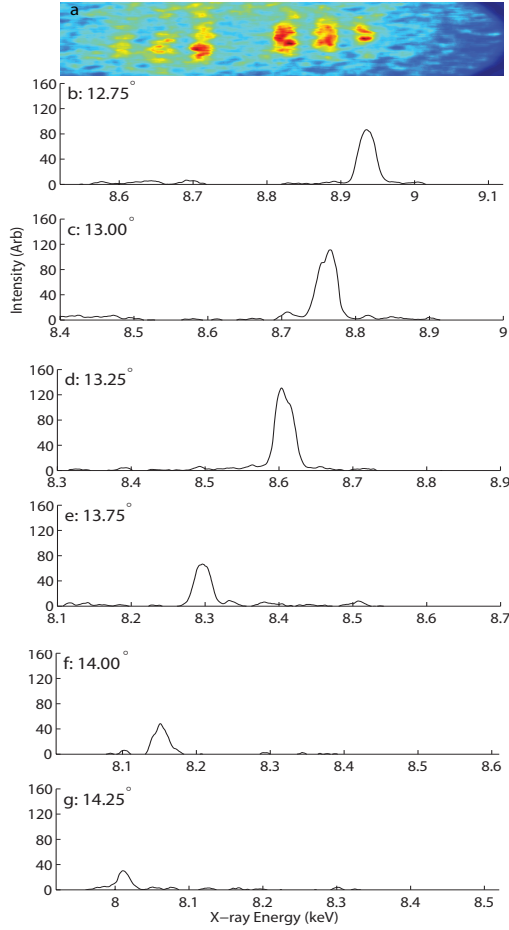


Figure 5.3: Result of simulation of a single shot diffraction pattern for the same conditions as given in Fig. 5.2. The intensity is $photons/rad^2$.

The error in the calculated number of x-rays detected at the MCP is due to differences in laser energy from shot to shot. Including the attenuation of 70 cm of air, the transmission of the beryllium windows (96% [151]) and the efficiency of the MCP ($\approx 1\%$ [100]), we estimate, using the given parameters, that the crystal reflects up to 10^4 photons, while 10^2 are detected. To confirm this estimate, a numerical code based on the semianalytical calculation found in Ref. [97] was used with the x-ray energy related to the emission angle via Eq. 5.1, the result is that 2×10^4 photons are reflected. The photon density distribution can be seen in Fig. 5.3. The difference between the patterns seen in Figures 5.2 and 5.3 are attributed to the deviation of the electron and laser beams from Gaussian distributions, which are assumed in the semianalytical treatment, and



5.4: a) False color addition of 36 images taken at multiple angles and median filtered showing the physical relationship between the diffraction patterns at each angle (φ) setting. The round edge of the MCP can be seen in the corners. The image is 10 mrad tall. The vertical stripe pattern is due to the rotation of the silicon crystal (see Fig. 5.1). b-g) Integrated line outs from each angle (φ) as noted. The integrated area is 2 mrad vertically, centered on the pair of peaks seen in the 12.75° diffraction pattern.

the presence of the slit. The detected flux is a dramatic improvement over the most noted ICS diffraction experiment which yielded only approximately a single detected photon per interaction [140].

Characterization of the ICS x-ray source requires measurements of the electron and laser beam properties at the interaction point (IP), which can be used to supplement direct measurements of the x-ray spectral distribution via simulation [94, 97]. Prior ICS work has studied the electron beam phase space distribution [152], and measured the bandwidth of the x-rays by using the transmission spectra of transition metals to preferentially attenuate the x-rays [100]. We supplement

the previous work by using the diffracted x-rays to quantify the near-axis bandwidth of the ICS photons over a 1 keV range of x-ray energies centered around 8.7 keV and compare the results to simulation [94].

To begin an enumeration of the sources of ICS x-ray bandwidth we start with the angle-energy correlation of a single electron, single photon interaction, given by Eq. 3.39 and [94]:

$$\varepsilon_x = \frac{2\gamma^2\varepsilon_L(1 - \cos\phi)}{1 + a_L^2 + \gamma^2(\theta_x^2 + \theta_y^2)}, \quad (5.3)$$

where ε_x is the x-ray energy, ε_L is the laser photon energy, γ is the relativistic factor of the electron and a_L is the root-mean-square normalized vector potential of the laser at the interaction point. The observation angles of the x-rays, as measured from the direction of propagation of the electron beam, in the horizontal (x) and vertical (y) directions are indicated by $\theta_{(x,y)}$, while ϕ is the angle of incidence of the laser with respect to the direction of forward propagation of the electrons. The source is configured for head-on interaction ($\phi = 180^\circ$). We assume, consistent with measured laser parameters, that a_L is very small and ignorable. Further, we take the limit of Thomson scattering and ignore the recoil of the electrons.

The diffraction criteria given by Bragg's law (Eq. 5.1) and the angle-energy correlation given by Eq. 5.3, along with the acceptance limits given by the upstream slit, create a diffraction pattern which contains two lobes because of a combination of the off-axis energy red shift, non-ideal overlap (ideal meaning two Gaussian beams centered on the same axis) between the interacting electron and laser beams and non-Gaussian shapes in each beam. The vertical misalignment of the silicon crystal suppresses the upper peaks on the low energy side in Fig. 5.4a.

Determination of the spectral intensity of the x-rays requires numerical simulations for realistic cases [94, 97]. Following Ref. [94], we estimate the bandwidth due to the moments in the electron and laser beams: beam divergence at the IP (caused by the tight focus), beam energy spread and laser bandwidth, respectively:

$$\frac{\Delta\varepsilon_x}{\varepsilon_x} \approx \sqrt{4\left(\frac{\epsilon_n}{\sigma}\right)^2 + 4\left(\frac{\Delta\gamma}{\gamma}\right)^2 + \left(\frac{\Delta\varepsilon_L}{\varepsilon_L}\right)^2}, \quad (5.4)$$

where ϵ_n is the normalized emittance and σ is the transverse electron beam size, measured to be approximately symmetric. The laser focus, produced by an off-axis parabolic mirror (f/5), is a negligible source of x-ray bandwidth relative to the above sources. Table 5.2 compares the sources of bandwidth in the detected x-rays, as calculated using Eq. 5.4 and the ICS simulation found in Ref. [94]. The electron spot size is very small, 30 μm rms, compared to the IP-to-detector distance, 2.15 m, so the x-rays are treated as originating from a point source. The bandwidth allowed by the opening angle of the detection scheme is estimated by assuming the radiation distribution is a symmetric Gaussian [42].

Table 5.2: Comparison of rms contributions to x-ray spectrum bandwidth between analytical estimates and simulation, both as found in table 3.2.

Parameter	Estimate	Simulation
Electron Beam Focusing	0.4%	0.3%
Energy Spread	0.2%	0.2%
Laser Bandwidth	0.3%	0.4%
Opening angle of detection	2.5%	2.6%

To measure the bandwidth of the ICS interaction several images were produced at each of multiple crystal angle settings. The sampling of the spectrum

within a window $2 \text{ mrad} \times 1 \text{ mrad}$ centered on the central axis of the electron beam is shown in Fig. 5.4b.

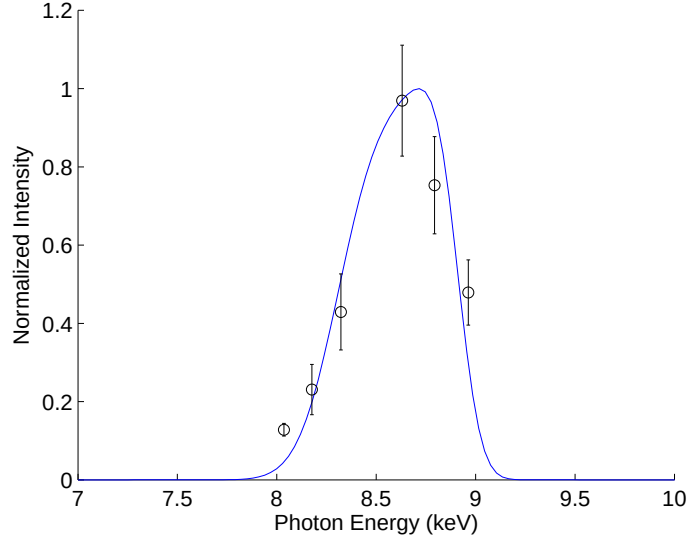


Figure 5.5: Comparison of simulation (blue curve) and the integrated intensity of the recorded diffraction patterns (black circles). The error bars are the standard deviation of the mean normalized intensity at each angle.

Fig. 5.5 shows a comparison between simulation and the measured data of the detected x-rays; the blue line represents the output of simulation of the x-ray flux detected within the $2 \text{ mrad} \times 1 \text{ mrad}$ window using the parameters in Table 5.1, while the circles are derived from median filtering each of the 37 images taken, integrating the total intensity over the $2 \text{ mrad} \times 1 \text{ mrad}$ aperture at each angle setting and taking the mean value, the error bars are the standard deviation of the mean of samples at each angle. The simulation curve is normalized for direct comparison to the measured data, which is normalized such that the 13.25° data point lies on the simulation curve. The simulation curve, produced assuming ballistic motion of the electrons ($a_L = 0$), is not offset in abscissa, consistent with the previous assumption of negligible red-shifting due to nonlinear electron

motion or recoil. The simulation curve yields an rms bandwidth of 2.6% with a peak value of 8.72 keV. It is apparent from table 5.2 that the bandwidth is largely due to the detection aperture which can be decreased at the expense of lowering the total photon count.

In summary, we have produced single-shot multiphoton diffraction images from an x-ray ICS source. The analytical estimates and predictions of simulation agree with the measured properties of the x-rays. This demonstration of the ability to consistently produce and detect 10^2 relatively high peak brightness x-rays with ps-scale pulse length is an important step forward in the research of ultrafast processes (sub-ps) and nuclear material detection that is currently beyond the reach of modern third generation sources. Previously demonstrated electron bunch compression [100] as well as increased detector efficiency will allow the detection of nearly 10^4 photons within a sub-ps pulse, capable of future, time dependent, diffraction studies. The compact size of the source will make tunable, ultrafast, hard x-rays available on a university laboratory scale, so opening the door for a new class of pump-probe experiments in this context.

5.2 Cryogenic Undulator at NLCTA

In this section, we present the results of in-situ testing of the in-vacuum cryogenic permanent magnet undulator CPMU9. Several tests were performed on this device to both characterize the undulator radiation and assess it's applicability as a modulator. To examine the properties as a radiator, incoherent synchrotron radiation is observed at near-infrared wavelengths with the bandwidth of the radiation measured using filters. Further, the resonance condition is used, along with filters, to evaluate the undulator parameter. To demonstrate the utility of the device as a compact current modulator of the electron beam, as may be found

in optical klystron or advanced seeding schemes, we generated a correlated energy modulation in the undulator, imparted on the electron beam via interaction with a $\lambda = 800$ nm linearly polarized laser. The amplitude of the energy modulation is measured in the electron beam spectrometer. Finally, using a conversion of the correlated energy spread to a correlated density modulation through a longitudinally dispersive chicane [110], induced microbunching is observed via coherent optical transition radiation [153].

This in-situ testing provides a framework for evaluating the effectiveness of a short period cryogenic undulator for increasing the capabilities of linear accelerator based radiation sources such as the x-ray FEL or as an inverse FEL (iFEL) component. As the undulator was originally conceived as a radiator for a plasma wakefield accelerator [25], in this study we experimentally address questions concerning the negotiation of the small undulator gap by a high-brightness RF photoinjector electron beam; we further study whether transport of the electron beam can be adapted for effective propagation of the microbunching downstream. This experimental context is of critical relevance to future x-ray FELs, as the combination of high-brightness electron beams and short period undulators has the potential to both increase the photon energy reach of multi-GeV 4th generation light sources to the very hard (well-sub-Angstrom) x-ray regime, and also permit very low energy ($E < 500$ MeV) facilities that can access soft x-rays, or hard X-rays at higher harmonics of the undulator resonance (see Chapters 3 and 4).

The cryogenic in-vacuum undulator testing was performed on the ECHO beam line of the Next Linear Collider Test Accelerator at SLAC National Accelerator Laboratory [111, 154]. The beam line begins with an S-band RF photocathode gun of the BNL/SLAC/UCLA type, which produces a 5 MeV emittance compen-

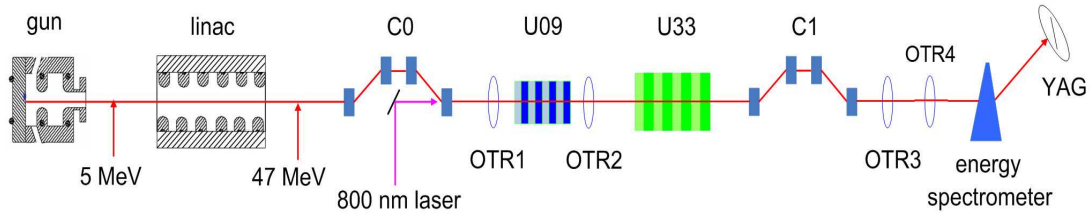


Figure 5.6: Layout of the NLCTA beam line.

sated electron beam 2 picoseconds (ps) full-width-half-maximum (FWHM) long with a total charge of 30 pC. The beam is accelerated in an X-band traveling wave structure to 47 MeV. Because the bunch length is relatively long in terms of X-band RF phase (over 8°), the electron beam is accelerated with considerable nonlinear curvature in energy-time phase space.

CPMU9 is installed just upstream of U33 (which was designed to resonantly couple a 120 MeV electron beam to the 800 nm seed laser). It is cooled by a Gifford-McMahon cycle cryo-refrigerator. The cold head is connected thermally to the undulator via a pair of copper bands made by press riveting thin Cu sheets together, this configuration isolates the undulator from vibrations of the cold head. The copper bands are connected to the undulator via a Cu cooling block that is fastened to the lower jaw. All connections between Cu pieces are made with indium foil. To minimize thermal conduction with the vacuum chamber, the undulator is installed on Teflon spheres which sit in grooves in a polyether ether ketone (PEEK) platform which is fastened to steel rods that are attached to the vacuum chamber. The temperature of the undulator and cooling system is measured in three locations: at the cold head, on the cooling block and on the undulator at the downstream end of the upper jaw of the undulator. The mean temperature of the undulator through out the experiment is 47.3K with a 0.5K gradient between the sensor on the undulator (hotter) and cooling block (colder). See Fig. 5.7 for an image of the undulator installed in the vacuum chamber.

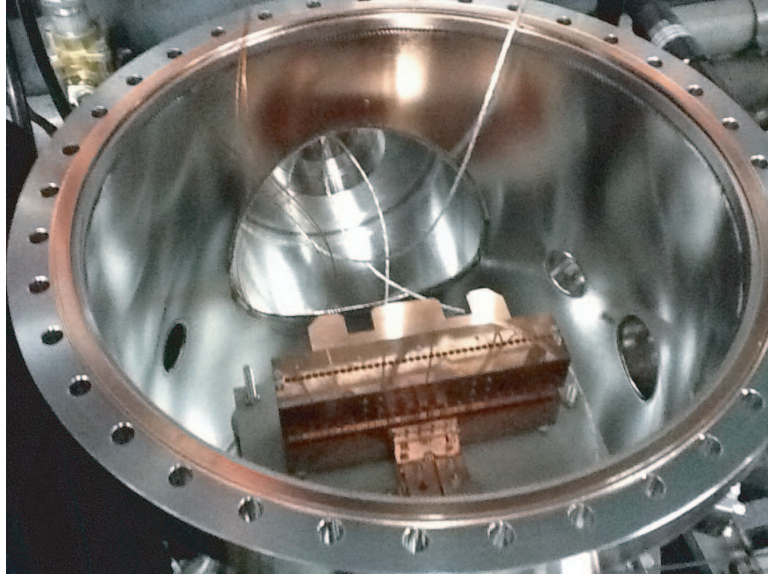


Figure 5.7: The in-vacuum cryogenic undulator CPMU9 is shown during installation at NLCTA. The cooling block and the ends of the copper cooling bands can be seen at the bottom of the picture, while the temperature sensor leads are fed through at the top of the picture. Also shown is the PEEK platform that the undulator is installed on. The copper gasket is the standard size for a 16.5 inch flange.

To measure the bandwidth of the undulator radiation, the beam energy is varied by changing the accelerating gradient in the traveling wave structure and the radiation is observed through a near-infrared filter. The slice energy spread of the electron beam is small (1 keV) [154], and the beam is accelerated near crest to minimize the global energy spread due to RF curvature. To estimate the energy spread of the electron beam we ignore the slice energy spread and assume that all of the energy difference as a function of length along the beam comes from the acceleration in the X-band linacs, $E(z) = E_x \sin(k_{rf}z) + E_s$. Here, E_s is the energy gained in the S-band gun, E_x is the energy gained in the X-band structures and $k_{rf} = 239.4$ 1/m is the wavenumber of the linacs which have

frequency 11.424 GHz. The mean energy of the electron beam that is σ_z long is $\langle E \rangle = E_s + E_x \sin(\phi_0) \exp(-k_{rf}^2 \sigma_z^2 / 2)$ and the rms energy spread is given by

$$\sigma_E^2 = \frac{E_x^2}{2} \{1 - \cos(2\phi_0) \exp(-2k_{rf}^2 \sigma_z^2) - 2 \sin(\phi_0) \exp(-k_{rf}^2 \sigma_z^2)\} \quad (5.5)$$

where ϕ_0 is the location of the bunch center in phase relative to zero acceleration. The minimum energy spread occurs at $\phi_0 = \pi/2$ which is maximum acceleration. For an electron beam that is $k_{rf} \sigma_z = 3.5^\circ$ long, $\sigma_E = 110$ keV at maximum acceleration. The energy spread due to the RF curvature is much larger than the slice energy spread, justifying ignoring the latter.

Further, as $\sigma_E / \langle E \rangle = 0.2\%$, the radiation bandwidth –when measured by adjusting the beam energy and observing the incoherent undulator radiation through a fixed wavelength filter– is dominated by the interference term of the undulator radiation for beam energies below the energy which produces the most intense signal, that is $\gamma < \gamma_r$. When the peak intensity is reached and the observed radiation moves off axis with increasing energy, the dependence of the bandwidth on the interference condition is masked by the photon distribution pattern of undulator radiation (cf Chapter 3). The maximum intensity occurs at a beam energy slightly higher than the undulator resonance condition predicts, by a factor of $1 + 1/N_p$ where N_p is the number of periods, because photons are more readily emitted off axis (at $\theta \neq 0$) (see section 3.3). Fig. 5.8 shows the result of the spectral measurement for two different filters as well as example far field radiation distribution images for each point. The solid curve is given by

$$I(E) = \exp\left(-\frac{(E - E_0)^2}{2\sigma_E^2}\right), \quad (5.6)$$

where

$$\sigma_E = \frac{E_0}{2\sqrt{2\ln(2)}N_p}, \quad (5.7a)$$

$$E_0 = (1 + 1/N_p)m_e c^2 \sqrt{\frac{\lambda_u(1 + K^2/2)}{2\lambda_f}} \quad (5.7b)$$

and λ_f is the central wavelength of the filter. The results are consistent with a spectrum that is dominated by the 5% FWHM bandwidth of the 20 period undulator.

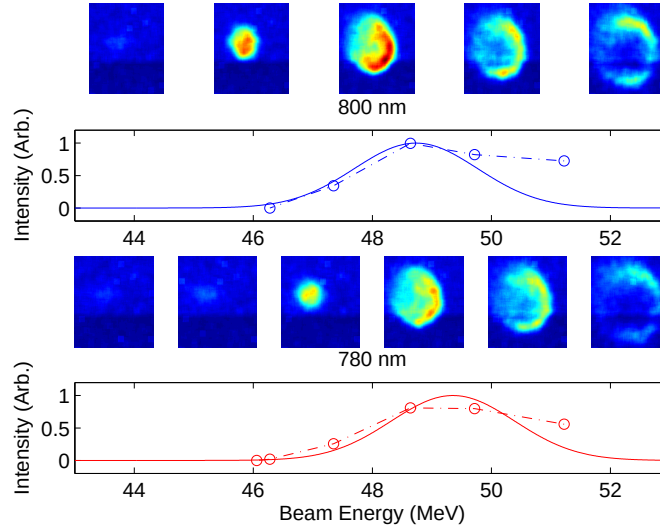


Figure 5.8: Spectral measurement of the radiation produced with the cryogenic undulator using two different filters. The top 5 images correspond to the 5 points measured in the upper (blue) curves for an 800 nm filter of 10 nm FWHM bandwidth, below the images is the plot comparing the measured data (circles with dash-dot line) and a Gaussian of 5% FWHM relative bandwidth. The lower (red) curves and the corresponding 6 images above them are the same for a 780 nm filter of 10 nm FWHM bandwidth.

The undulator parameter K is measured via comparison of the opening angle

of the radiation through a set of filters (780, 800 and 820 nm) by calculating the undulator parameter required to produce the measured angles using the resonance condition (Eq. 3.21),

$$K = \sqrt{\frac{4\gamma^2\lambda_f}{\lambda_u} - 2 - 2(\gamma\theta)^2}. \quad (5.8)$$

In this formula λ_f is given by the filter, γ is measured at the spectrometer and θ is calculated from the images by taking the mean radius of the pixels with intensity larger than half the maximum intensity. The value of the undulator parameter measured in this way is $K=0.943\pm0.064$, in good agreement with the expected value of 0.967 (a 2.5% relative difference), based on the measurement of the field at cryogenic temperatures [155]. The error in the measurement is dominated by the error in the angle measurement due to the wide relative bandwidth of the filters (10 nm). It should be noted that the previous measurements of the undulator field were performed at 20 K while the experiment was run at 47.3 K, as such part of the difference between the two different measurements of the undulator parameter is because the remanent field is slightly lower, by approximately 1%, at 50 K versus 20 K [19]. The thermal contraction of the copper parts that define the 9 mm period results in a several hundred nanometer shrinking of the period from room temperature to cryogenic temperatures [72], ignorable compared to the increase in field from cooling. The beam energy spread and jitter were found to be well below the 1% level at the electron beam spectrometer while the increase in bandwidth due to emittance was even smaller because of the gentle focus of the electron beam in the undulator. The other sources of systematic error include filter center wavelength and the transverse position of the electron beam within the undulator which are estimated to be at the sub-percent level.

For the energy modulation/microbunching measurement, the seed laser is

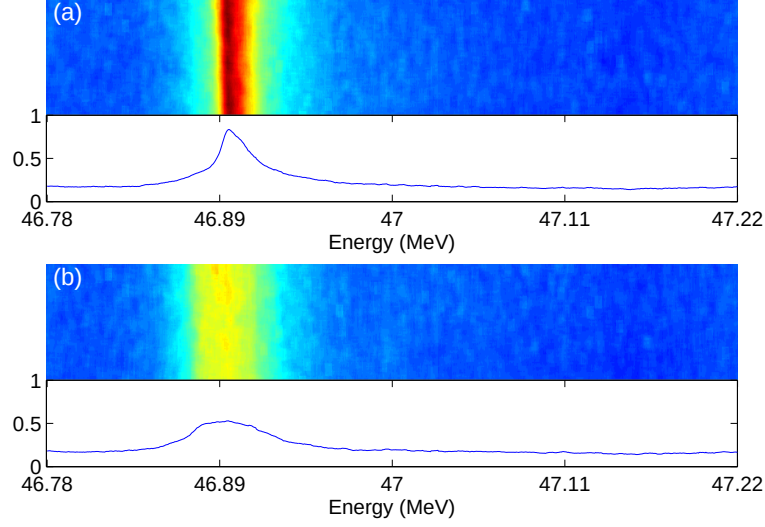


Figure 5.9: Comparison of the beam energy spread with the 800 nm seed laser off (a) and seed laser on (b). In each image pair the upper image is a false color image taken from the magnetic spectrometer while the bottom image is the vertical sum of the shown false color image. The false color images have been median filtered. The false color images have the same scale, as do the integrated line outs. The apparent energy spread of the electron beam increases by $\sigma_{E,mod} = 18 \pm 5$ keV.

30 μ J of 800 nm near-infrared light focused to a 1 mm FWHM spot at the downstream end of CPMU9. The pulse length is 0.5 ps FWHM with a 2 nm FWHM bandwidth. The electron beam and laser beam are aligned transversely by ensuring overlap between the two on the OTR screens just up stream and down stream of CPMU9 (OTR1 and OTR2 in Fig. 5.6). Because the laser pulse is shorter than the electron beam we expect that the imparted correlated energy spread will be a function of position within the electron beam, further the 900 μ m FWHM transverse beam size in the undulator, comparable to the laser spot size, means that there will be significant 3D effects in the energy modulation process. Therefore, to estimate the imparted energy spread GENESIS 1.3 was used,

yielding an increase in energy spread of $\sigma_E = 23$ keV over the whole electron beam, assuming that the laser and electron beam are ideally overlapping and using the measured K parameter of $K = 0.967$. When the GENESIS 1.3 simulations are run using the K measured via radiation $K = 0.943$, the expected increase in energy spread is $\sigma_E = 15$ keV over the whole electron beam. This energy spread is calculated by finding the peak energy modulation using the code and then calculating the energy modulation at other points in the beam. Because the laser and electron beam do not exchange net energy we assume the energy modulation as a function of longitudinal position within the beam is given by integrating Eq. 4.7 holding ϵ and θ constant, such that $\eta_f = -\epsilon L_u \sin(\theta)$. Because ϵ scales with the laser power as $\sqrt{P}$ the mean rms energy spread of the electron beam after energy modulation and assuming zero initial energy spread is

$$\begin{aligned} \langle \sigma_E \rangle &= \int_{-\infty}^{\infty} \left[\sigma_{E,0} \exp\left(-\frac{z^2}{4c^2\sigma_t^2}\right) \right] \frac{\exp\left(-\frac{z^2}{2\sigma_z^2}\right)}{\sqrt{2\pi}\sigma_z} dz, \\ &= \frac{\sigma_{E,0}}{\sqrt{1 + \frac{1}{2} \left(\frac{\sigma_z}{c\sigma_t}\right)^2}}, \end{aligned} \quad (5.9)$$

where σ_z is the rms electron beam length and σ_t is the rms laser beam duration in time.

Fig. 5.9 shows the increase in the energy spread due to the seed laser, as seen in the electron beam energy spectrometer at the end of the NLCTA beam line. The dispersion of the spectrometer is 1.5 m and the energy resolution is about 5 keV, obtained by properly focusing the beam to a small horizontal size on the screen. The increase in the energy spread when the seed laser is turned on is attributable entirely to the correlated energy spread induced in the undulator. The measured increase in the energy spread of the electron beam is

$\sigma_{E,mod} = 18 \pm 5$ keV (100 keV/m) demonstrating the effective coupling and potential use of the cryogenic in-vacuum undulator as a FEL modulator or iFEL component.

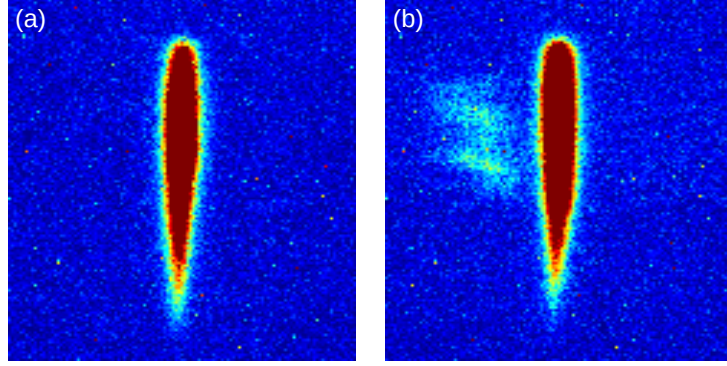


Figure 5.10: Image of the OTR screen after chicane C1. (a) shows the electron beam distribution due to transition radiation while (b) also shows the coherent transition radiation emitted from the screen as the structure to the left of the electron beam.

To confirm that the increase in energy spread is indeed a correlated energy spread, the electron beam is focused on OTR4 (see Fig. 5.6) and the screen is imaged with a CCD camera. When passing through chicane C1 the momentum compaction changes the correlated energy modulation into a density modulation [110]. A non-modulated electron beam will cause the screen to radiate optical transition radiation when passing through the screen while an electron beam that is microbunched will also cause the emission of coherent transition radiation. When the camera is focused on the screen, the image in the former case will be the transverse distribution of the electron beam (see Fig. 5.10a), while the addition of coherent radiation will cause other structures to show up in the image, as can be seen in Fig. 5.10b.

To prevent contamination of the signal from upstream sources, screen OTR3 just upstream of OTR4 is inserted. Because of the lack of overlap between the incoherent transition radiation, which must be coming from the imaged screen because it is persistent when the seed laser is turned off and the upstream OTR3 screen is removed, the observed signal is likely forward coherent transition radiation from OTR3 screen reflected by the downstream OTR4 screen, rather than backward transition radiation from the imaged screen. The weakness of the coherent signal is due to a combination of non-optimum transport of the 47 MeV beam in a beam line designed for 120 MeV operation, and over-modulation in undulator CPMU9 resulting in reduced microbunching at the radiating screens. Despite the fact that –following [110]– the modulation is predicted to be large and the microbunching therefore suppressed, reducing the energy of the seed laser did not improve the signal. Less drastic changes in operating conditions are predicted to be much more effective when using small gap devices [156].

The observed structure disappears when either the laser or the electron beam is stopped before the interaction in undulator CPMU9, confirming that it comes from a correlated energy spread in the electron beam induced in the undulator. Further, the energy spread increase is observed when the laser is blocked with OTR2 between CPMU9 and U33, establishing that the energy modulation interaction is occurring in CPMU9 and not U33, which would require coupling to the much weaker 6th harmonic, which has a null on-axis.

In this section we have presented the in-situ testing of a sub-cm period length cryogenic in-vacuum undulator, motivated by the potential for shorter period undulators to extend the photon energy reach of existing light sources, reduce the required accelerator size for next-generation x-ray FELs, and enable lower energy iFELs and beam modulation schemes. This technology has the potential

to increase access to harder x-rays at large facilities and increase the availability of soft x-rays by scaling down the beam energy required to achieve them from GeV-scale to hundreds of MeV. The favorable scaling of the cryogenic in-vacuum technology has been demonstrated by producing near-infrared radiation at low beam energy and further producing a correlated energy spread via the iFEL effect at a rate of 100 keV/m. The measured bandwidth of incoherent synchrotron radiation produced in CPMU9 is dominated by the 5% FWHM bandwidth of the 20 period undulator due to interference, and the undulator parameter, as measured with the radiation, is consistent with the value based on the measured field at cryogenic temperatures. This proof-of-principle experiment, in sum, gives the technical foundation for employing a new class of high-field, short period, cryogenic undulators based on novel materials.

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