UC Berkeley
UC Berkeley Electronic Theses and Dissertations

Title

Essays in Time Series Econometrics: Nonlinear, Nonstationary GMM Estimation, Credit Shock Transmission, and Global VAR Models

Permalink

https://escholarship.org/uc/item/8mh861wd

Author

Han, Fei

Publication Date

2012

Peer reviewed|Thesis/dissertation

eScholarship.org Powered by the California Digital Library
University of California

Essays in Time Series Econometrics: Nonlinear, Nonstationary GMM Estimation, Credit Shock Transmission, and Global VAR Models

by

Fei Han

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy

in

Agricultural and Resource Economics

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Peter Berck, Chair
Professor Michael Jansson
Professor Maximilian Auffhammer

Spring 2012

Essays in Time Series Econometrics: Nonlinear, Nonstationary GMM Estimation, Credit Shock Transmission, and Global VAR Models

Copyright 2012

by

Fei Han

Abstract

Essays in Time Series Econometrics: Nonlinear, Nonstationary GMM Estimation, Credit Shock Transmission, and Global VAR Models

by

Fei Han

Doctor of Philosophy in Agricultural and Resource Economics

University of California, Berkeley

Professor Peter Berck, Chair

This dissertation consists of three chapters dealing with different topics in time series econometrics including generalized method of moments (GMM) estimation and vector autoregressions (VAR). These econometric models have revolutionized empirical research in macroeconomics. Previous work by Hansen and Singleton (1982) showed that the GMM method can be applied to estimate nonlinear rational expectations models in a simple way that the models need not even be solved. The seminal work of Sims (1980) has demonstrated how VAR models can be used for macroeconomic forecasting and policy analysis. The objective of this dissertation is to provide some new econometric tools for applied research in macroeconomics using time series data.

The first chapter develops an asymptotic theory for the GMM estimator in nonlinear econometric models with integrated regressors and instruments. We establish consistency and derive the limiting distribution of the GMM estimator for asymptotically homogeneous regression functions. The estimator is consistent under fairly general conditions, and the convergence rates are determined by the degree of the asymptotic homogeneity of regression functions. Similar to linear regressions, we find that the limiting distribution is generally biased and non-Gaussian, and that instruments themselves cannot eliminate the bias even when they are strictly exogenous. Therefore, GMM yields inefficient estimates and invalid t- and chi-square test statistics in general. By implementing the fully modified method developed by Phillips and Hansen (1990), we obtain an efficient GMM estimator which has an unbiased and mixed normal limiting distribution.

In the second chapter, we develop a novel shock identification strategy in the context of two-country/block structural vector autoregressive (SVAR) models to identify the transmission of credit shocks. Specifically, we investigate how credit shocks originating in the U.S. or euro area affect domestic economic activity in emerging Asia.

Shocks within each block are identified using sign restrictions, whereas shocks across the two blocks are identified using a recursive structure (block Cholesky decomposition). This strategy not only enables us to distinguish the external credit shock from the other structural shocks, but also captures the responses of the domestic country. The main findings include that the transmission of credit shocks across countries through the channel of credit contagion is fast and protracted. The adverse effects of external credit tightening are mitigated by domestic credit policy easing in China, but lead to significant decreases in credit and GDP growth in the other emerging Asian countries. We also find that the external credit shocks play a non-negligible role in driving economic fluctuations in emerging Asia, although the role is smaller in China.

In the last chapter, we use a global vector autoregressive (GVAR) model to forecast the principal macroeconomic indicators of the original five ASEAN member countries (i.e. Indonesia, Malaysia, Philippines, Singapore, and Thailand). The GVAR model is a compact model of the world economy designed to explicitly model the economic and financial interdependencies at national and international levels. Our GVAR model covers twenty countries which are grouped into nine countries/regions. After applying vector error correction model (VECM) to estimate parameters in the GVAR, we generate twelve one-quarter-ahead forecasts of real GDP growth, inflation, short-term interest rates, real exchange rates, real equity prices, and world commodity prices over the period 2009Q1-2011Q4, with four out-of-sample forecasts during 2009Q1-2009Q4. Forecast evaluation based on the panel Diebold-Mariano (DM) tests shows that the forecasts of our GVAR model tend to outperform those of country-specific VAR models, especially for short-term interest rates and real equity prices. These results suggest that the interdependencies among countries in the global financial market play an important role in macroeconomic forecasting.

Contents

Contents ..... **i**

List of Tables ..... **ii**

List of Figures ..... **iii**

1 Efficient GMM Estimation in Nonlinear Models with Integrated Regressors and Instruments **1**
1.1 Introduction 2
1.2 The Model and Assumptions 6
1.3 Consistency 11
1.4 Asymptotic Distributions 14
1.5 Efficient Estimation 18
1.6 Conclusions 21
1.7 Mathematical Proofs 22

2 The Transmission of External Credit Shocks to Emerging Asia: A Two-Country Structural VAR Approach **47**
2.1 Introduction 48
2.2 The Theory of Sign Restrictions 52
2.3 The Model 54
2.4 Data 65
2.5 Results 66
2.6 Robustness 72
2.7 Conclusions 77

3 ASEAN-5 Macroeconomic Forecasting Using a GVAR Model **111**
3.1 Introduction 112
3.2 The GVAR Model 113
3.3 Data 119
3.4 Tests 119
3.5 Forecast and Evaluation 121
3.6 Conclusions 123

List of Tables

2.1 Sign Matrix: Baseline Models 83
2.2 Sign Matrix: Alternative Approach 84
2.3-2.8 Forecast Error Variance Decomposition 85-90

3.1 Trade Weights Based on Direction of Trade Staitistics 128
3.2 Cointegration Rank Statistics for the U.S. Model 129
3.3 Cointegration Rank Statistics for Non-US Countries/East Asia 130
3.4 VARX* Order and Number of Cointegration Relationships in Country-Specific Models 131
3.5 ADF-GLS Unit Root Test Statistics 132
3.6 *F*-statistics for Testing Weak Exogeneity of Country-Specific Foreign Variables and Commodity Prices 133
3.7 *F*-statistics for Tests of Residual Serial Correlation in Country-Specific VARX* Models 133
3.8 Contemporaneous Effects of Foreign Variables on Domestic Counterparts 134
3.9 Panel DM Statistics for GVAR Forecasts Relative to Benchmark Models 135

List of Figures

2.1 Real GDP and Real Credit Growth Rates in Emerging Asia 91
2.2(a)-(d) Identified USEUR Shocks (Full Sample) 92-95
2.3(a)-(d) Impulse Responses of Domestic Variables to USEUR Shocks: Baseline Models (Full Sample) 96-99
2.3(e) Impulse Responses of Domestic Variables to a Domestic Adverse Credit Market Shock: Baseline Models (Full Sample) 100
2.4(a)-(d) Identified USEUR Shocks (Subsample) 101-104
2.5(a)-(d) Impulse Responses of Domestic Variables to USEUR Shocks: Baseline Models (Subsample) 105-108
2.5(e) Impulse Responses of Domestic Variables to a Domestic Adverse Credit Market Shock: Baseline Models (Subsample) 109
2.6 Impulse Responses of Domestic Variables to a USEUR Adverse Credit Market Shock: Alternative Approach (Full Sample) 110

3.1 One-Quarter-Ahead Forecasts of Real GDP Growth 136
3.2 One-Quarter-Ahead Forecasts of Inflation 137
3.3 One-Quarter-Ahead Forecasts of Short-Term Interest Rates 138
3.4 One-Quarter-Ahead Forecasts of Real Exchange Rates 139
3.5 One-Quarter-Ahead Forecasts of Real Equity Prices 140

Acknowledgements

There have been a lot of people that helped achieve this goal. First and foremost, I would like to express my deepest gratitude to my advisor, Professor Peter Berck, for sharing with me his research ideas, and standing behind my work with great patience and care. I want to thank him for supporting me and providing me with an excellent atmosphere for doing research. He is always available, patient, insightful, and most importantly, a good friend to me. I would never have been able to finish my dissertation without his support and advice.

I would like to give special thanks to my two committee members, Professors Michael Jansson and Maximilian Auffhammer. First of all, I want to thank them for being always available and patient, and for sparing no effort to help me with every detailed problem I encountered in my research. Every talk with them inspired me with new research ideas. Without their excellent and insightful guidance, completion of this dissertation seems impossible to me. Second, with their valuable comments and suggestions, I now see the potential to turn some of the dissertation chapters into publishable papers.

I also want to thank Thiam Hee Ng, my co-aurthor of the last chapter of this dissertation. The last chapter was completed when I was an intern in the Asian Development Bank, and as my supervisor and co-author, Thiam Hee Ng provided me with great help and guidance on the last chapter. I also want to thank Selim A. Elekdag, an economist in the International Monetary Fund, who offered valuable comments and suggestions on the second chapter.

A lot of professors were so kind and gave me valuable advice and comments. Jeffrey LaFrance, Jeffrey Perloff, Leo K. Simon, Sofia Berto Villas-Boas, Brian D. Wright, Christian Traeger, and the entire faculty of the Agricultural and Resource Economics Department raised questions, and provided me with great comments. My classmates have also been extremely helpful; Di Zeng, Phu V. Le, and the rest of my classmates who have frequently served as patient ears to my dissertation problems.

Finally, I want to thank my family and especially my parents. I want to thank them for always encouraging me to pursue my dreams and standing behind me whenever I need. I want to thank them for their faith in me no matter what happens that provided me with enough strength and courage to tackle challenges ahead of me.

Chapter 1

Efficient GMM Estimation in Nonlinear Models with Integrated Regressors and Instruments

1.1 INTRODUCTION

Since the seminal work of Hansen (1982), generalized method of moments (GMM) estimation has been widely used in many types of econometric models. One of the main advantages of GMM is that it can produce efficient estimates of the parameters in nonlinear dynamic models with endogenous regressors by making use of exogenous instrumental variables (IV). However, as might be anticipated, both nonlinearity and dynamics create a number of technical issues. The first of these is nonstationarity, a property most economic time series are widely believed to possess, such as trend stationarity (with deterministic trends) and integration (with stochastic trends). Originally, GMM estimation theory was developed only for strictly stationary and ergodic time series (Hansen, 1982) for which all measurable functions are also strictly stationary and ergodic, so that applications of strong laws and central limit theorems (CLT) are straightforward. Some attempts have been made to extend GMM estimation to models with trend-stationary variables. For instance, Wooldridge (1994) developed asymptotics under high level conditions that encompass some interesting cases of deterministically trending variables. Andrews and McDermott extended the GMM asymptotic theory to situations where deterministic trends (but not stochastic trends) appear in nonlinear models by using triangular array asymptotics. However, traditional CLT approaches have still been used in these models with little progress made to incorporate integrated time series into the general GMM framework. Another technical issue is introduced by nonlinearity. The complete theory of linear regression with integrated time series was developed using Brownian motions over twenty years ago (e.g., Park and Phillips, 1988, 1989). It has also been well known since then that nonlinear models with integrated time series might behave quite differently from their linear counterparts because the asymptotics of functions of Brownian motions are dependent upon the functional forms. All these issues pose considerable challenges for developing a GMM asymptotic theory in nonlinear models with integrated time series.

The purpose of this paper is to develop an asymptotic theory for GMM estimation in nonlinear models with integrated regressors and instruments, and to provide a practical approach to produce efficient GMM estimates (in the sense of Phillips, 1991) for applied research. The mechanism of our asymptotic analysis relies on the asymptotics of sample moments of nonlinear functions of integrated time series developed by Park and Phillips (1999, 2001)[1].

The first technical issue of integrated time series is treated in practice largely by pre-filtering such as differencing before implementing GMM estimation. The Monte Carlo simulation results in Doorn (2003) nonetheless suggest that filters may induce bias in the parameter estimates and increase the root mean squared error of

[1]Hereafter, the latter paper will be referred to as PP.

the estimates. Moreover, differencing might even create the 'weak instruments' or 'weak identification' problem which would invalidate the standard GMM asymptotic theory.[2] A simple example helps illustrate this problem. World oil prices are frequently used as instruments in GMM and IV regressions due to their exogeneity in small open-economy models, and are generally believed to be an integrated process of order 1, i.e. $I(1)$ process. However, if the innovations that generate oil prices are weakly correlated with endogenous variables, first-order differencing might create weak instruments. Kline (2008) used first-order differenced oil prices as instruments to estimate a nonlinear labor-market model, and found that the GMM value function was actually flat around the estimates, suggesting a possible identification problem. In this case, oil price levels could be a better choice for instruments because they are asymptotically correlated with any endogenous integrated time series as long as they all carry full rank stochastic trends, as a beneficial artifact of spurious regression theory. Yet, little is known about the asymptotics of nonlinear IV or GMM estimators when there are integrated regressors and instruments.

Asymptotics of linear IV and GMM estimators had been developed in early 1990s. Phillips and Hansen (1990) found that the linear IV estimator is T-consistent, but involves nuisance parameters and is asymptotically biased, referred to as the 'second-order bias'. These nuisance parameters invalidate traditional t- and chi-square tests. To illustrate the second-order bias, consider the following IV regression with $I(1)$ processes:

$$\begin{aligned} y_t &= \beta' x_t + u_{0t}, \\ x_t &= x_{t-1} + u_{1t}, \\ z_t &= z_{t-1} + u_{2t}, \end{aligned}$$

where u_{0t} is a scalar stationary time series, and u_{1t} and u_{2t} are both vector stationary time series. We assume that these error processes satisfy the functional CLT, and they can be autocorrelated or correlated with each other. Therefore, there could be both endogeneity and serial correlation. At the moment, let's assume that $(x_t', z_t')'$ is a full rank $I(1)$ process, i.e. the number of unit roots in the stochastic process $(x_t', z_t')'$ is equal to the dimension of $(x_t', z_t')'$ (and thus the elements of $(x_t', z_t')'$ are not cointegrated). As shown in Phillips and Hansen (1990), the limiting distribution of the IV estimator of β involves elements in the long-run covariance matrix of $(u_{0t}, u_{1t}', u_{2t}')'$. These elements, called the nuisance parameters, induce bias and asymmetry to the limiting distribution.

Several methods have been proposed to solve this problem: see Phillips and Hansen (1990), Phillips and Loretan (1991), Saikkonen (1991), Park (1992), and

[2]Interested readers may refer to Stock, Wright, and Yogo (2002) for a survey of the weak instrument and weak identification problem in GMM estimation.

Stock and Watson (1992). Among them, the fully modified (FM) estimator proposed by Phillips and Hansen (1990) seems to be particularly useful in practice because it allows running regressions much like least squares that yield asymptotically efficient estimates. Specifically, the FM estimator achieves efficiency by making two corrections to the dependent variable in order to eliminate the two sources of second-order bias, endogeneity and serial correlation of regressors and instruments.[3] Phillips and Hansen (1990) also found that the instruments are insufficient to eliminate the second-order bias effects asymptotically even if they are strictly exogenous. Applying the FM corrections to linear IV and GMM estimators, Kitamura and Phillips (1997) and Quintos (1998) developed efficient estimation for linear models with integrated regressors and instruments. These efficient estimators are called the fully modified IV (FM-IV) and fully modified GMM (FM-GMM) estimators, respectively.

However, no significant progress had been made on nonlinear regressions with integrated time series until late 1990s. The seminal work of Park and Phillips (1999) established the asymptotics of sample moments of nonlinear transformations of integrated time series. In their later work, PP developed an asymptotic theory for the NLS estimator in nonlinear regressions with exogenous scalar $I(1)$ regressors. Their approach builds upon the local time and the occupation time formula of Brownian motions. The local time $L(t, x)$ of one-dimensional Brownian motion B measures the time that is spent by B in the vicinity of x over the time interval $[0,\, t]$, and can be expressed as [4]

$$L(t,\, x) = \lim_{\epsilon \to 0^+} \frac{1}{2\epsilon} \int_0^t 1\big(|B(s) - x| < \epsilon\big) ds,$$

where $1\,(\cdot)$ is the indicator function. The occupation time formula says that, for any locally integrable transformation T on $\mathbb{R}$,

$$\int_0^t T\big(B(r)\big) dr = \int_{-\infty}^{\infty} T(s) L(t, s) ds.$$

PP found that the limiting distribution of NLS estimator, just like the OLS estimator in linear regressions, involves nuisance parameters, and is asymptotically biased. Chang, Park and Phillips (2001) developed an efficient nonlinear least squares (EN-NLS) estimator in a more general nonlinear model with deterministic trends, exogenous stationary and integrated regressors. The EN-NLS estimator is essentially a NLS estimator with the second-order bias corrected by the FM approach. De Jong (2003) and Chang and Park (2005) relaxed the exogenous assumption imposed in PP and Change, Park, and Phillips (2001), and developed an asymptotic theory of the

[3] See Kitamura and Phillips (1997) for an overview of the fully modified approach.

[4] Interested readers may refer to Park and Phillips (1999) and the cited references there for the details of Brownian local time L.

NLS estimator with endogenous scalar regressors. Christopeit (2009) extended the asymptotics of nonlinear transformations of univariate integrated time series developed by PP to the multivariate case.

This paper develops an asymptotic theory for the GMM estimator which includes IV estimator as a special case in nonlinear models with integrated time series. In this perspective, our work may be considered as a continuation of linear regressions in Phillips and Hansen (1990) and Kitamura and Phillips (1997) to nonlinear regressions, or continuation of the NLS estimator developed by PP and Change, Park, and Phillips (2001) to the GMM estimator. Similar to the linear IV and GMM estimator analyzed by Kitamura and Phillips (1997), we find second-order bias in the limiting distribution of the GMM estimator, and instruments themselves cannot eliminate the bias even when they are strictly exogenous. An efficient GMM estimator is obtained by applying the FM corrections. As in the stationary GMM theory, we find an 'optimal' choice for the weighting matrix which minimizes the (conditional) variance of the efficient GMM estimator. The resulting optimal efficient GMM estimator is consistent, and has a mixed normal limiting distribution under mild regularity conditions on the nonlinear functions and some identification conditions.[5] Moreover, the optimal efficient GMM estimator is in fact the same as the fully modified nonlinear instrumental variables (FM-NLIV) estimator.

To focus on asymptotic theory of GMM estimator, we simplify our problem by restricting to a certain class of nonlinear functions and scalar regressors. Of course, the unknown parameters and instruments can be multivariate. PP considered two classes of functions: integrable and asymptotically homogeneous functions. However, this paper only focuses on the latter ones which are more difficult to analyze because the convergence rates of sample moments of these functions depend on the true values of unknown parameters. The integrable functions can be easily incorporated into our analysis. Moreover, although this paper only considers nonlinear functions of scalar $I(1)$ regressors, it is straightforward to extend our work to nonlinear models with deterministically trending and multivariate integrated regressors using the same method as in Chang, Park, and Phillips (2001).

The paper is organized as follows. Section 1.2 outlines the model and assumptions, and provides the definitions needed from PP. Section 1.3 gives a preliminary asymptotic distribution for sample moments that provides the foundation of our analysis, and provides the consistency of the GMM estimator. The asymptotic distribution theory is developed in Section 1.4. Section 1.5 considers issues of efficient estimation and optimal choice of weighting matrix, and develops a practical approach to estimation for empirical implementation. Section 1.6 concludes and proofs are collected

[5]Since standard identification conditions involve integrals of nonlinear functions of multidimensional Brownian motions, they are generally difficult to verify in practice. Employing the representation theory of Gaussian processes, we provide stronger, yet simple and easy-to-check, conditions on nonlinear functions to replace those identification conditions.

together in Section 1.7.

Notation in this paper follows PP and Chang, Park, and Phillips (2001). For a vector $x = (x_i)$ or a matrix $A = (a_{ij})$, the absolute value $|\cdot|$ is taken element by element. Therefore, $|x| = (|x_i|)$ and $|A| = (|a_{ij}|)$. For a vector $x = (x_i)$, the notation $\|\cdot\|$ stands for the standard Euclidean norm $\|x\| = \sqrt{\sum_i x_i^2}$, and for a matrix $A = (a_{ij})$, it stands for the induced norm defined by $\|A\| = \sup_{x \neq 0} \|Ax\|/\|x\|$. The same notation is also used to signify the supremum of a function, and in particular, for a function f, which can be vector- or matrix-valued, $\|\cdot\|_C$ signifies the supremum norm over a subset C of its domain, so that $\|\cdot\|_C = \sup_{x \in C} \|f(x)\|$. The subset C, over which the supremum is taken, will not be specified if it is clear from the context. The indicator function is written as $1(\cdot)$, and the identify matrix is denoted by I. We write $\otimes$ to denote the Kronecker product, and $vec(A)$ to stack the columns of a matrix A into a column vector. $\mathbb{R}_+$ stands for the set of positive real numbers, and the inequalities '> 0' and '≥ 0' denote positive definite (p.d.) and positive semidefinite (p.s.d.) respectively when applied to matrices. We use the symbols $\xrightarrow{a.s.}$, $\xrightarrow{p}$, $\xrightarrow{d}$, and $\stackrel{d}{=}$ to signify convergence almost surely, convergence in probability, convergence in distribution, and equality in distribution, respectively. Finally, the symbolism $\mathbf{MN}(0, V)$ signifies the mixed normal distribution $\mathbf{MN}(0, V) \stackrel{d}{=} \int_{V>0} \mathbf{N}(0, V) d\mathbf{P}(V)$.

1.2 THE MODEL AND ASSUMPTIONS

The model we consider is a nonlinear regression model for y_t given by

$$y_t = f(x_t, \theta_0) + \varepsilon_t, \tag{1.1}$$

where $f : \mathbb{R} \times \mathbb{R}^k \to \mathbb{R}$ is known, x_t is a scalar regressor, and ε_t is the regression error. θ_0 is a k-dimensional true parameter vector that lies in the parameter space Θ. We adopt the assumption in PP throughout the paper that Θ is compact and convex, and the true parameter value θ_0 is an interior point of Θ, which is, as noted in PP, a standard assumption for stationary nonlinear regression. Suppose x_t is generated by a scalar $I(1)$ process:

$$x_t = x_{t-1} + u_t, \tag{1.2}$$

Note that model (1.1)-(1.2) is the same as that considered in PP if the regressor x_t is exogenous. However, as long as x_t can be contemporaneously correlated with the regression error ε_t, or in other words, $\mathbf{E}(x_t \varepsilon_t) \neq 0$, our model is different from theirs. Now suppose there exists an $m \times 1$ vector ($m \geq k$) of instrumental variables z_t, an $I(1)$ random vector given by

$$z_t = z_{t-1} + v_t. \tag{1.3}$$

The initialization of the system (1.1)-(1.3) is at $t = 0$ and x_0 can be any $O_p(1)$ random variable, z_0 can be any $O_p(1)$ random vector.

For the time series ε_t, u_t, and v_t, respectively, we define the stochastic processes B_T, U_T and V_T on $[0, 1]$ by

$$B_T(r) = \frac{1}{\sqrt{T}} \sum_{t=1}^{\lfloor Tr \rfloor} \varepsilon_t, \quad U_T(r) = \frac{1}{\sqrt{T}} \sum_{t=1}^{\lfloor Tr \rfloor} u_t, \quad V_T(r) = \frac{1}{\sqrt{T}} \sum_{t=0}^{\lfloor Tr \rfloor} v_t$$

where T is the sample size, and $\lfloor Tr \rfloor$ denotes the largest integer not exceeding Tr.

Define $w_t = (\varepsilon_t, u_t, v'_{t+1})'$ and the filtration $\mathcal{F}_{T,t} = \sigma\{(w_s)_{-\infty}^{t}\}$, i.e. the σ-field generated by $\{w_s\}_{s \leq t}$. We now impose the following assumptions on the innovation process w_t.

Assumption EC (Error condition):
(a). $(w_t, \mathcal{F}_{T,t})$ *is a strictly stationary and ergodic martingale difference sequence with* $\mathbf{E}(w_t w'_t | \mathcal{F}_{T,t-1}) = \Sigma$ *a.s., and* $\sup_{t \geq 1} \mathbf{E}(\|w_t\|^q | \mathcal{F}_{T,t-1}) < \infty$ *for some* $2 < q < \infty$.
(b). $(B_T, U_T, V'_T)' \xrightarrow{d} (B_\varepsilon, B_x, B'_z)'$ *as* $n \to \infty$, *where* $(B_\varepsilon, B_x, B'_z)'$ *is an* $(m + 2) \times 1$ *vector Brownian motion with covariance matrix* Ω.
(c). $\Omega > 0$.

Assumptions EC (a)-(c) are satisfied by a wide variety of data generating processes. Assumption EC(a) implies that the instruments z_t are predetermined, i.e. $\mathbf{E}\left(z_t \mid \mathcal{F}_{T,t-1}\right) = z_t$, and thus we have $\mathbf{E}(z_t \varepsilon_t) = 0$. We partition Σ conformably with w_t as

$$\Sigma = \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{u\varepsilon} & \sigma'_{v\varepsilon} \\ \sigma_{u\varepsilon} & \sigma_u^2 & \sigma'_{vu} \\ \sigma_{v\varepsilon} & \sigma_{vu} & \Sigma_{vv} \end{bmatrix}.$$

The martingale difference assumption on the error processes can be easily relaxed to stationary $MA(\infty)$ processes, which does not affect our analysis except the covariance matrices. Under certain circumstances, the error processes can even have more general properties. For instance, in the linear cointegrating regression theory, serial correlation in the errors and cross correlation between the errors and regressors are both allowed. Also, in De Jong (2003) which considers the NLS estimator with endogenous scalar $I(1)$ regressors, ε_t is assumed to be a $L_2 - NED$ (near epoch dependent) of size -1 on some mixing process. A full generalization of our theory allowing for correlated errors would involve a substantial additional level of complexity and is not attempted here. However, it does not seem overly restrictive at this point to assume the absence of serial correlation in the errors, especially given our flexible nonlinear specification of the regression function and the presence of endogeneity of the regressor. Condition (b) is fairly standard, and is the usual assumption imposed

to analyze linear models with integrated time series. Condition (c) excludes cointegration among the elements of z_t and between x_t and z_t, or in other words, $(x_t, z_t')'$ carries full rank stochastic trends.

For subsequent use, we denote $\zeta_t = (\varepsilon_t, u_t, v_t')'$, and decompose the long-run covariance matrix given in Assumption EC(c) as follows:

$$\Omega = \Omega_0 + \Lambda + \Lambda',$$

where $\Omega_0 = \mathbf{E}(\zeta_t \zeta_t')$ and $\Lambda = \sum_{i=1}^{\infty} \mathbf{E}(\zeta_t \zeta_{t+i}')$. Define the 'one-sided long-run covariance matrix' of ζ_t as

$$\Delta = \Omega_0 + \Lambda = \sum_{i=0}^{\infty} \mathbf{E}(\zeta_t \zeta_{t+i}').$$

We partition Ω, Ω_0, Λ, and Δ conformably with ζ_t. Thus, in the case of Ω we write

$$\Omega = \begin{bmatrix} \omega_{\varepsilon\varepsilon} & \omega_{x\varepsilon} & \omega_{z\varepsilon}' \\ \omega_{x\varepsilon} & \omega_{xx} & \omega_{zx}' \\ \omega_{z\varepsilon} & \omega_{zx} & \Omega_{zz} \end{bmatrix}.$$

It is straightforward to see that Assumption EC(c) implies that the variance of the limiting Brownian motion B_ε is strictly positive, i.e. $\omega_{\varepsilon\varepsilon} > 0$, and the the covariance matrix of the limiting Brownian motion $(B_x, B_z')'$, i.e. the $(m+1)\times(m+1)$ submatrix

$$\Phi = \begin{bmatrix} \omega_{xx} & \omega_{zx}' \\ \omega_{zx} & \Omega_{zz} \end{bmatrix} > 0.$$

As shown in Lemma A2, Assumption EC(c) is one of the sufficient conditions to make sure that the usual asymptotic relevance condition for valid instruments is satisfied. Moreover, Assumption EC(a) implies that $\Omega = \Sigma$, and the one-sided long-run covariance matrices of $(\varepsilon_t, v_t')'$ and $(\varepsilon_t, u_t')'$ are $\Delta_{z\varepsilon} = 0$ and $\Delta_{x\varepsilon} = \sigma_{u\varepsilon}$ respectively.

The stochastic process $(B_T, U_T, V_T')'$ takes values in $D[0,1]^{(m+2)}$, where $D[0,1]$ denotes the space of *cadlag* functions defined on the unit interval $[0,1]$. We equip $D[0,1]^{(m+2)}$ with the uniform metric, and by the Skorohod representation theorem, there is a common probability space (Ω, F, P) supporting $(B_T^0, U_T^0, V_T^{0\prime})'$ and $(B_T, U_T, V_T')'$ such that

$$(B_T, U_T, V_T')' \overset{d}{=} (B_T^0, U_T^0, V_T^{0\prime})', \qquad \text{and} \qquad (B_T^0, U_T^0, V_T^{0\prime})' \xrightarrow{a.s.} (B_\varepsilon, B_x, B_z')'$$

in $D[0,1]^{(m+2)}$ with the uniform topology. The notation in PP, $(B_T, U_T, V_T')' = (B_T^0, U_T^0, V_T^{0\prime})'$, is adopted throughout the paper to avoid repetitious embedding of $(B_T, U_T, V_T')'$ in the probability space (Ω, F, P) where $(B_T^0, U_T^0, V_T^{0\prime})'$ is defined.

In this paper, we consider the estimation of model (1.1) by GMM, which is defined

as the minimizer of a (random) quadratic function over $\theta \in \Theta$, i.e.,

$$\hat{\theta}_T \equiv \underset{\theta\in\Theta}{\operatorname{argmin}} \; m_T(\theta)' \, W_T \, m_T(\theta), \quad w.p. \longrightarrow 1, \tag{1.4}$$

where W_T is an $m \times m$ symmetric (random) matrix for each T, and

$$m_T(\theta) = \frac{1}{T} \sum_{t=1}^{T} \Big[z_t \left(y_t - f(x_t, \theta) \right) \Big].$$

Assumption WM (Weighting Matrix): *W_T is an a.s. positive semidefinite (random) matrix for each T, and $W_T \xrightarrow{p} W$ as $T \to \infty$ where W is an a.s. positive definite (random) $O_{a.s.}(1)$ matrix.*

It should be noted that Assumption WM is more general than its counterpart in the traditional GMM estimator since the limiting matrix W here can be random rather than merely deterministic. If we choose $W_T = \left[\frac{1}{T^2} \sum_{t=1}^{T} z_t z_t' \right]^{-1}$, then the GMM estimator becomes the nonlinear instrumental variables (NLIV) estimator. In this case, Assumption WM is automatically satisfied because $\frac{1}{T^2} \sum_{t=1}^{T} z_t z_t' \geq 0$ *a.s.* as long as there is no cointegration among elements of z_t, and $\frac{1}{T^2} \sum_{t=1}^{T} z_t z_t' \xrightarrow{a.s.} \int_0^1 B_z(r) B_z(r)' dr > 0$ *a.s.* as shown by Phillips and Hansen (1990). Since the NLIV is just a special case, one may expect to obtain an efficiency gain in estimation if we choose the optimal weighting matrix. However, Kitamura and Phillips (1997) showed that the FM-GMM estimator for the coefficients of integrated regressors using the optimal weighting matrix is asymptotically equivalent to the FM-IV estimator in linear regressions, and as found in this paper, the result still holds in nonlinear case.

In this paper, we consider the class of asymptotically homogeneous functions for f. It is also straightforward to incorporate the class of integrable functions into our analysis, as has been done by PP for NLS estimator. For subsequent use, let's briefly overview the definitions of regular function and asymptotically homogeneous function defined in PP.

Definition 2.1 (from PP): A transformation S on $\mathbb{R}$ is said to be *regular* if and only if
(a). it is continuous in a neighborhood of infinity, and
(b). for any compact subset C of $\mathbb{R}$ given, there exist for each $\epsilon > 0$ continuous functions $\underline{S}_\epsilon$, $\overline{S}_\epsilon$, and $\delta_\epsilon > 0$ such that $\underline{S}_\epsilon(x) \leq S(y) \leq \overline{S}_\epsilon(x)$ for all $|x - y| < \delta_\epsilon$ on C, and such that $\int_C \left(\overline{S}_\epsilon - \underline{S}_\epsilon \right)(x) \, dx \to 0$ as $\epsilon \to 0$.

By definition, the class of regular transformations includes locally bounded mono-

tone functions and continuous functions. As noted in PP, any regular transformation is locally bounded and hence locally integrable, and therefore the occupation time formula applies. It is worth noticing that for any locally integrable transformation T on $\mathbb{R}$, the function $T\big(B(\cdot)\big)$ is square integrable on the interval [0, 1] and thus belongs to the $L_2[0,\,1]$ space.

DEFINITION 2.2 (from PP): A function F is *regular* on Π if
(a). $F(\cdot,\pi)$ is *regular* for all $\pi \in \Pi$, and
(b). for all $x \in \mathbb{R}$, $F(x,\cdot)$ is equicontinuous in a neighborhood of x.

It should be noted that F can be vector-valued functions, in which case F is regular on Π means that each element of F is regular on Π. This paper considers a special class of functions, the asymptotically homogeneous functions. As pointed out in Section 1.1, the other class of functions considered in PP, the integrable functions, are easier to analyze because the convergence rates of sample means of integrable functions do not depend on unknown parameters θ_0.[6]

DEFINITION 2.3 (from PP): Suppose $F, H : \mathbb{R}_+ \times \Theta \to \mathbb{R}_d$, $\kappa : \mathbb{R}_+ \times \Theta \to \mathbb{R}_+^{d^2}$ is a nonsingular square matrix, $R : \mathbb{R} \times \mathbb{R}_+ \times \Theta \to \mathbb{R}_d$, and

$$F(\lambda x, \theta) = \kappa(\lambda, \theta) H(x, \theta) + R(x, \lambda, \theta),$$

then we say that F is *H-regular* on Θ if
(a). H is *regular* on Θ, and
(b). $R(x, \lambda, \theta) = a(\lambda, \theta) A(x, \theta)$, where $\kappa(\lambda, \theta)^{-1} a(\lambda, \theta) \to 0$ *uniformly* in $\theta \in \Theta$ as $\lambda \to \infty$, and $\sup_{\theta \in \Theta} A(x, \theta) = O(e^{c|x|})$ as $|x| \to \infty$ for some $c \in \mathbb{R}_+$, i.e. $\sup_{\theta \in \Theta} A(x, \theta)$ belongs to a class of locally bounded transformations that are exponentially bounded.

We call κ, H, and R the *asymptotic order*, *limit homogeneous function*, and *remainder term* of F respectively. If κ does not depend upon θ, then F is said to be *H_0-regular*.

As in Definition 2.2, F can be vector-valued functions, in which case each element of F is H-regular, the limit homogeneous function H is a vector-valued function with the same dimensions as F, and the asymptotic order κ is a nonsingular square matrix. Moreover, by definition, for any $\theta \in \Theta$, $H(\cdot, \theta)$ is a locally integrable function, and therefore $H(B(\cdot), \theta)$ belongs to the $L_2[0,\,1]$ space for any one-dimensional Brownian motion $B(\cdot)$. We make a distinction between the H_0-regular and the general H-regular functions when showing consistency in the next section, because the sufficient conditions to ensure consistency for the general H-regular functions are

[6] Obviously, the convergence rates of sample means of asymptotically homogeneous functions depend on unknown parameters according to the definition below.

different from and much more restrictive than those for the H_0-regular functions. However, the distinction is unnecessary for the asymptotic distributions, since the sufficient conditions on H-regular functions are mild enough to accommodate most of the H-regular functions that are used in nonlinear analysis.

1.3 CONSISTENCY

This section shows the consistency of the GMM estimator $\hat{\theta}_T$ defined in (1.4). The conditions for consistency are easy to check and, in particular, do not require differentiability of the regression function. They are satisfied for most of the asymptotically homogeneous functions used in practice. However, there are some regression functions that are not covered by the conditions we impose for the consistency results in this section. They will be considered in the next section, where we derive the asymptotic distributions of the GMM estimator without assuming consistency but assuming differentiability of the regression function. In order to avoid notational confusion, we use f to signify the scalar-valued function considered in (1.1) in the rest of the paper.

Let's consider the GMM estimator defined in (1.4) with x_t being endogenous and f being H-regular on the parameter space Θ as defined in Definition 2.3 with asymptotic order κ, limit homogeneous function h, and remainder term R. If we take $\lambda = \sqrt{T}, x = x_t/\sqrt{T}$, then

$$f(x_t, \theta) = \kappa(\sqrt{T}, \theta) h\left(\frac{x_t}{\sqrt{T}}, \theta\right) + R\left(\frac{x_t}{\sqrt{T}}, \sqrt{T}, \theta\right).$$

By definition, the residual $R\left(\frac{x_t}{\sqrt{T}}, \sqrt{T}, \theta\right)$ is negligible in the limit, and hence the behavior of $f(x_t, \theta)$ is dominated by the first term on the right-hand side.

The following two lemmas are useful to show the consistency of the GMM estimator.

LEMMA 3.1: *Let Assumption EC holds, and let F be H-regular on Θ with asymptotic order κ and limit homogeneous function H. Then as $T \to \infty$*

$$T^{-\frac{3}{2}} \kappa(\sqrt{T}, \theta)^{-1} \sum_{t=1}^{T} F(x_t, \theta) z_t' \xrightarrow{a.s.} \int_0^1 H\big(B_x(r), \theta\big) B_z(r)' dr \tag{1.5}$$

uniformly in $\theta \in \Theta$.

Lemma 3.1 describes the asymptotic behavior of the sample mean of $z_t F(x_t, \theta)$ normalized by its asymptotic order $T^{\frac{3}{2}} \kappa(\sqrt{T}, \theta)$, which is an IV analogue of the sam-

ple mean of $F(x_t, \theta)$ normalized by its own asymptotic order $\kappa(\sqrt{T}, \theta)$ as studied in PP.

LEMMA 3.2: *Let* $Q_T(\theta) = m_T(\theta)' W_T m_T(\theta)$, *and* $D_T(\theta, \theta_0) = Q_T(\theta) - Q_T(\theta_0)$, *then* $\hat{\theta}_T \xrightarrow{p} \theta_0$ *if either* (a) *or* (b) *below is satisfied:*
(a). *For some sequence* ρ_T *of numbers,* $\rho_T^{-1} D_T(\theta, \theta_0) \xrightarrow{p} D(\theta, \theta_0)$ *uniformly in* θ *as* $T \to \infty$, *where* $D(\cdot, \theta_0)$ *is continuous and has unique minimum* $\theta_0 \quad a.s.$
(b). *For any* $\delta > 0$,

$$\liminf_{T \to \infty} \left[\inf_{\{\theta \in \Theta : \|\theta - \theta_0\| \geq \delta\}} D_T(\theta, \theta_0) \right] > 0 \quad \textit{in probability.} \tag{1.6}$$

Remarks: Conditions (a) and (b) are both sufficient to ensure consistency, as shown in earlier work by Wu (1981) and Potscher and Prucha (1997). Condition (b) is a consequence of the so-called *uniquely identifiable* condition which can be used to show the consistency of the M-estimator.[7] This condition is a more primitive condition than uniform convergence which is used in the classical proof of the consistency of the traditional GMM estimator when x_t and z_t are both stationary. However, we cannot invoke uniform convergence to show consistency here because the convergence rates of sample means are now dependent upon the unknown parameter θ.

The following theorems 3.3 and 3.4 establish the consistency of the GMM estimator with asymptotically homogeneous functions based on the two lemmas above. Conditions (a) and (b) in Lemma 3.2 are invoked to show the consistency with a distinction between H_0-regular and H-regular functions.

THEOREM 3.3: *Let Assumptions EC and WM hold, and let* f *be* H_0*-regular on* Θ *with asymptotic order* κ *and limit homogeneous function* h. *Suppose* (a) *and either* (b1) *or* (b2) *below are satisfied:*
(a). $\kappa(\lambda)$ *is bounded away from zero as* $\lambda \to \infty$.
(b1). *For any* $\theta \neq \theta_0$ *and any* $\delta > 0$,

$$\int_{|s| \leq \delta} \big(h(s, \theta) - h(s, \theta_0)\big)^2 ds > 0$$

(b2). *For any* $\theta \neq \theta_0$, $g(\theta) \neq g(\theta_0)$ *a.s., where*

$$g(\theta) = \int_0^1 B_z(r) h\big(B_x(r), \theta\big) dr.$$

[7]Note that the GMM estimator belongs to the M-estimator class.

In particular, we have $D(\theta,\theta_0) = [g(\theta) - g(\theta_0)]' W [g(\theta) - g(\theta_0)]$ *with* $\rho_T = T\kappa(\sqrt{T})^2$.

Remarks: Condition (a) is the same as condition (a) in Theorem 4.2 of PP, and ensures that there is sufficient variability in the regression function asymptotically to generate a signal stronger than the noise. Condition (b2) is in fact the usual asymptotic relevance condition for valid instruments (or the identification condition in the standard GMM estimation). Comparing to the linear IV case, condition (b2) is essentially the asymptotic relevance condition that $\int_0^1 B_z(r)B_x(r)'dr$ has full column rank *a.s.* which was shown by Phillips and Hansen (1990) using the fact that two independent $I(1)$ processes appear correlated even in the limit, a result from the spurious regression theory. However, this condition is awkward and cumbersome because it involves the integral of multidimensional Brownian motions and the occupation time formula does not apply. We then replace it with a stronger, yet easier to verify, identification condition in (b1). It is straightforward to see that condition (b1) is sufficient for identification of the NLS estimator as shown by PP, however, the argument does not follow automatically for the identification of GMM estimator. This is shown by making use of the representation theory of Brownian motions in terms of orthonormal system, as described in details in the proof of Lemma A2 in Section 1.7.

EXAMPLE 3.1: Consider the linear function $f(x,\theta) = \theta x$ with $\theta \in \Theta \subset \mathbb{R}\backslash\{0\}$. It is obvious that f is H_0-regular with asymptotic order $\kappa(\lambda,\theta) = \lambda$ and limit homogeneous function $h(x,\theta) = \theta x$. It is straightforward to see that conditions (a) and (b1) are both satisfied.

THEOREM 3.4: *Let Assumptions EC and WM hold, and let* f *be* H*-regular on* Θ *with asymptotic order* κ *and limit homogeneous function* h*. Then the GMM estimator defined in (4) is a consistent estimator of* θ_0*, i.e.* $\hat{\theta}_T \xrightarrow{p} \theta_0$*, if* (a) *and either* (b1) *or* (b2) *below are satisfied:*
(a). *For any* $\bar{\theta} \neq \theta_0$*, and* $\bar{p}, \bar{q} > 0$*, there exist* $\epsilon > 0$ *and a neighborhood* N *of* $\bar{\theta}$ *such that as* $\lambda \to \infty$*,*

$$\inf_{\substack{|p'Wp - \bar{p}| < \epsilon \\ |q'Wq - \bar{q}| < \epsilon}} \inf_{\theta \in N} \Big[p\kappa(\lambda,\theta) - q\kappa(\lambda,\theta_0)\Big]' W \Big[p\kappa(\lambda,\theta) - q\kappa(\lambda,\theta_0)\Big] \to \infty$$

where p *and* q *are both* $m \times 1$ *vectors.*
(b1). *For any* $\theta \in \Theta$ *and any* $\delta > 0$*,*

$$\int_{|s| \leq \delta} h(s,\theta)^2 ds > 0.$$

(b2). *For any* $\theta \in \Theta$, $g(\theta) \neq 0$ *a.s.*

Remarks: Condition (a) is a regularity condition similar to that in PP. Just like in Theorem 3.3, condition (b2) is the usual asymptotic relevance condition for valid instruments (or the identification condition in the standard GMM estimation), and is satisfied if (b1) holds.

EXAMPLE 3.2: Consider the Box-Cox transformation $f(x,\theta) = \left(|x|^{\theta} - 1\right)/\theta$ with $\theta \in \Theta \subset \mathbb{R}_+$. It is straightforward to see that f is H-regular with asymptotic order $\kappa(\lambda,\theta) = \lambda^{\theta}$ and limit homogeneous function $h(x,\theta) = |x|^{\theta}/\theta$, respectively. We can easily show that Box-Cox transformation satisfies both conditions (a) and (b1). Obviously, condition (b1) is satisfied due to the assumption that all $\theta > 0$. To see that condition (a) is also satisfied, set $0 < \epsilon < \min(\bar{p}, \bar{q})$, and for any given $\bar{\theta} \neq \theta_0$, let the neighborhood be $N = \left\{\theta \in \Theta : \left|\theta - \bar{\theta}\right| < \frac{1}{2}\left|\bar{\theta} - \theta_0\right|\right\}$.

1.4 ASYMPTOTIC DISTRIBUTIONS

This section of the paper derives the asymptotic distribution of the GMM estimator $\hat{\theta}_T$ defined in (1.4). As in standard GMM estimation theory, we require conditions on the regression function that ensure it is sufficiently smooth as a function of the unknown parameter θ. Assuming differentiability of the regression function also allows us to establish the consistency of the GMM estimator in models where the results in the previous section are not applicable. For such models, the results in this section will give consistency as well as the asymptotic distribution of the GMM estimator.

We follow the notations in PP, and define

$$\dot{f} = \frac{\partial f}{\partial \theta}, \quad \ddot{F} = \frac{\partial^2 f}{\partial \theta \partial \theta'}, \quad \ddot{f} = vec\left(\ddot{F}\right),$$

where $\dot{f}$ is a $k \times 1$ vector, $\ddot{F}$ is the Hessian, a $k \times k$ matrix, and $\ddot{f}$ is a $k^2 \times 1$ vector. In what follows, we denote by $\dot{h}$ the limit homogeneous function of H-regular $\dot{f}$. Moreover, the asymptotic orders of H-regular functions $\dot{f}$ and $\ddot{f}$ will be written as $\dot{\kappa}$ and $\ddot{\kappa}$ respectively. Whenever $\dot{f}$ and $\ddot{f}$ are introduced, we assume that they exist.

Let $\dot{Q}_T(\theta)$ and $\ddot{Q}_T(\theta)$ be the first and second derivatives of $Q_T(\theta)$ with respect to θ, i.e. $\dot{Q}_T(\theta) = \partial Q_T/\partial\theta$ and $\ddot{Q}_T(\theta) = \partial^2 Q_T/\partial\theta\partial\theta'$. Ignoring a constant which is unimportant, we have

$$\begin{aligned}
\dot{Q}_T(\theta) &= M_T(\theta)' W_T m_T(\theta), \\
\ddot{Q}_T(\theta) &= M_T(\theta)' W_T M_T(\theta) + \left[\left(m_T(\theta)' W_T\right) \otimes I_k\right] \frac{\partial^2 m(\theta)}{\partial\theta\partial\theta'},
\end{aligned}$$

where

$$M_T(\theta) = \frac{\partial m_T(\theta)}{\partial \theta} = -\frac{1}{T}\sum_{t=1}^{T} z_t \dot{f}(x_t, \theta)',$$

is a $m \times k$ matrix, and with some algebra,

$$\frac{\partial^2 m(\theta)}{\partial \theta \partial \theta'} = -\frac{1}{T}\sum_{t=1}^{T} \left[z_t \otimes \ddot{F}(x_t, \theta) \right].$$

As in standard nonlinear regression, the asymptotic distribution of $\hat{\theta_T}$ is our model can be obtained from the first order Taylor expansion of $\dot{Q}_T$ around θ_0, which is written as

$$\dot{Q}_T(\hat{\theta}_T) = \dot{Q}_T(\theta_0) + \ddot{Q}_T(\bar{\theta}_T)(\hat{\theta}_T - \theta_0) \tag{1.7}$$

where $\bar{\theta}_T$ lies on the line connecting $\hat{\theta}_T$ and θ_0. We have $\dot{Q}_T(\hat{\theta}_T) = 0$ if $\hat{\theta}_T$ is an interior solution to the minimization problem (1.4).

Let f be H-regular, then for an appropriately chosen normalizing sequence of matrices ν_T (ν_T can depend on both T and θ_0), it follows immediately from the sample mean asymptotics in Section 1.3 that $\nu_T^{-1}\dot{Q}_T(\theta_0) \xrightarrow{d} \dot{Q}(\theta_0)$ for some random vector $\dot{Q}(\theta_0)$. Also, if we define

$$\ddot{Q}_T^0(\theta_0) = M_T(\theta_0)' W_T M_T(\theta_0),$$

then $\nu_T^{-1}\ddot{Q}_T^0(\theta_0)\nu_T^{-1\prime} \xrightarrow{p} \ddot{Q}(\theta_0)$ for some random matrix $\ddot{Q}(\theta_0)$ due to Lemma 3.1. Therefore, under suitable conditions that ensure $\nu_T^{-1}\ddot{Q}_T^0(\bar{\theta}_T)\nu_T^{-1\prime} = \nu_T^{-1}\ddot{Q}_T^0(\theta_0)\nu_T^{-1\prime} + o_p(1)$ and $\ddot{Q}(\theta_0) > 0$ *a.s.*, we may expect from (1.7) that

$$\begin{aligned}
\nu_T'(\hat{\theta}_T - \theta_0) &= -\left(\nu_T^{-1}\ddot{Q}_T(\bar{\theta}_T)\nu_T^{-1\prime}\right)^{-1}\nu_T^{-1}\dot{Q}_T(\theta_0) \\
&= -\left(\nu_T^{-1}\ddot{Q}_T^0(\theta_0)\nu_T^{-1\prime}\right)^{-1}\nu_T^{-1}\dot{Q}_T(\theta_0) + o_p(1) \\
&\xrightarrow{d} -\ddot{Q}(\theta_0)^{-1}\dot{Q}(\theta_0),
\end{aligned}$$

as $T \to \infty$.

This idea is summarized and presented as a lemma in the following. The last condition develops from a trivial modification to the approach by Wooldridge (1994) for nonstationary regressions.

LEMMA 4.1: *Suppose that*
(a). $\nu_T^{-1}\dot{Q}_T(\theta_0) \xrightarrow{d} \dot{Q}(\theta_0)$ *as* $T \to \infty$,
(b). $\nu_T^{-1}\ddot{Q}_T(\theta_0)\nu_T^{-1\prime} = \nu_T^{-1}\ddot{Q}_T^0(\theta_0)\nu_T^{-1\prime} + o_p(1)$ *for large* T,
(c). $\nu_T^{-1}\ddot{Q}_T^0(\theta_0)\nu_T^{-1\prime} \xrightarrow{p} \ddot{Q}(\theta_0)$,

(d). $\ddot{Q}(\theta_0) > 0$ *a.s., and*
(e). *there exists a sequence* μ_T *such that* $\mu_T \nu_T^{-1} \xrightarrow{a.s.} 0$, *and such that* $\mu_T^{-1}\big(\ddot{Q}_T(\theta) - \ddot{Q}_T(\theta_0)\big)\mu_T^{-1\prime} \xrightarrow{p} 0$ *uniformly in* $\theta \in \Theta_T$ *where* $\Theta_T = \big\{\theta : \big\|\mu_T'(\theta - \theta_0)\big\| \leq 1\big\}$,
then $\nu_T'(\hat{\theta}_T - \theta_0) \xrightarrow{d} -\ddot{Q}(\theta_0)^{-1}\dot{Q}(\theta_0)$.

Once we show that the conditions of Lemma 4.1 are satisfied for our model, the limiting distribution of $\hat{\theta}_T$ can be obtained readily. Before presenting the asymptotic theory, we need more assumptions on the functional form of f.

ASSUMPTION H: *The function* f *is* H*-regular on* Θ *with asymptotic order* κ *and limit homogeneous function* h, *and satisfies* (a), (b), *and either* (c1) *or* (c2) *below:*
(a). $\dot{f}$ *is* H*-regular on* Θ *with asymptotic order* $\dot{\kappa}$ *and limit homogeneous function* $\dot{h}$.[8]
(b). *Let* $N(\tau) = \{\theta : \|\theta - \theta_0\| < \tau\}$ *for* $\tau > 0$, *then there exists* $\varepsilon > 0$ *such that as* $\lambda \to \infty$

$$\lambda^{-1+\varepsilon}\left\|\dot{\kappa}(\lambda, \theta_0)^{-1}\right\| \to 0, \tag{1.8}$$

and

$$\lambda^{\varepsilon}\left\|\left|(\dot{\kappa} \otimes \dot{\kappa})(\lambda, \theta_0)^{-1}\right|\left(\sup_{|s| \leq \bar{s}} \sup_{\theta \in N(\tau)} \left|\ddot{f}(\lambda s, \theta)\right|\right)\right\| \to 0, \tag{1.9}$$

for any $\bar{s} > 0$.
(c1). *For any* $\delta > 0$

$$\int_{|s| \leq \delta} \dot{h}(s, \theta_0)\dot{h}(s, \theta_0)' ds > 0.$$

(c2). $G(\theta_0)$ *has full column rank* k *a.s., where*

$$G(\theta) = \int_0^1 B_z(r)\dot{h}\big(B_x(r), \theta_0\big)' dr.$$

Remarks: Conditions (a) and (b) are essentially the same as those on f in PP and Chang, Park, and Phillips (2001), and as pointed out in PP, it is indeed quite easy and straightforward to show that they both hold for many H-regular functions that are used in nonlinear analysis. Condition (c2) is essentially the traditional rank condition which is usually required for identification by classic GMM or IV estimations. A simple example of the linear IV regression well explains this identification condition. Consider the simplest linear IV regression with one unknown parame-

[8]Note that $\dot{\kappa}$ and $\dot{h}$ are merely notations for the asymptotic order and limit homogeneous function of $\dot{f}$, and are different from the derivatives of κ and h, the asymptotic order and limit homogeneous function of f, with respect to θ.

ter and one regressor, i.e. $f(x,\theta) = \theta x$. Thus, $\dot{h}(x,\theta) = x$, and condition (c2) becomes $rank(\int_0^1 B_z(r)B_x(r)dr) = 1$ *a.s.*, which is satisfied due to the assumption that $(B_x, B_z')'$ carries full rank stochastic trends. It is also worth noticing that the traditional order condition $m \geq k$, a necessary condition for identification, is automatically satisfied if condition (c2) holds. However, this condition can be extremely difficult to check due to the integrals of multi-dimensional Brownian motions. Similar to conditions (b1) and (b2) in Theorems 3.3 and 3.4, we provide a stronger, yet easier to verify, identification condition in (c1) by applying the Loéve-Karhunen representation to the random matrix $\int_0^1 B_z(r)\dot{h}\big(B_x(r),\theta_0\big)'$. This result is presented as Lemma A3 in Section 1.7.

The following theorem presents the asymptotic distribution of the GMM estimator $\hat{\theta}_T$ with asymptotically homogeneous functions.

THEOREM 4.2: *Let Assumptions EC, WM, and H hold. Then as* $T \to \infty$

$$\sqrt{T}\dot{\kappa}(\sqrt{T},\theta_0)'(\hat{\theta}_T - \theta_0) \xrightarrow{d} \Big[G(\theta_0)'WG(\theta_0)\Big]^{-1}G(\theta_0)'W\Big[\int_0^1 B_z(r)dB_\varepsilon(r)\Big]. \quad (1.10)$$

The convergence rates of the GMM estimator for asymptotically homogeneous functions are determined by their asymptotic orders, and are given by $\sqrt{T}\dot{\kappa}(\sqrt{T},\theta_0)'$. For functions with increasing asymptotic orders, they are therefore faster than the usual rate $\sqrt{T}$. It is obvious that the linear function provided in Example 3.1 satisfies all the required conditions of Theorem 4.2. For the Box-Cox transformation $f(x,\theta) = \big(|x|^\theta - 1\big)/\theta$ in Example 3.2, it is straightforward to see that its derivative with respect to θ, $\dot{f}(x,\theta)$, is H-regular with asymptotic order and limit homogeneous function given by $\dot{\kappa}(x,\theta) = \lambda^\theta \log(\lambda)$ and $\dot{h}(x,\theta) = |x|^\theta/\theta$, respectively. We may easily show that such an f satisfies the conditions of Theorem 4.2. It is obvious that condition (c1) in Assumption H holds. To see that condition (b) in Assumption H is satisfied, simply let ε and τ in (1.8) and (1.9) be any numbers such that $0 < \varepsilon + \tau < \theta_0$ for any $\theta_0 > 0$.

It is worth noticing that the limiting distributions are generally non-Gaussian. They are biased (second-order bias), and dependent upon nuisance parameters. The source of the nuisance parameters is the dependence between the limiting Brownian motions B_ε and B_x. This dependence may, in turn, be interpreted as a form of conventional simultaneous equations bias arising from the endogeneity of the regressor x_t in (1.1). However, as we have seen from the above theorem, traditional method of dealing with this bias, like IV or GMM estimations, do not eliminate it. The only case in which the dependency disappears and the limiting distributions are mixed normal occurs when the integrated regressor and instruments are all strictly exogenous - a case where instrumentals are unnecessary. This finding is in coincidence with that in

the linear IV or GMM case. In fact, when $f(x_t, \theta) = \theta x_t$, the limiting distribution in (1.10) reduces to that of the linear IV estimator analyzed by Phillips and Hansen (1990).

1.5 EFFICIENT ESTIMATION

In this section, we develop an efficient GMM estimator for our model (1.1) by the fully modified (FM) method developed by Phillips and Hansen (1991). The usual GMM estimator $\hat{\theta}_T$ considered in Section 1.4 is generally not efficient, because it involves nuisance parameters[9] and does not utilize the presence of the unit root in the regressor. Inefficiency of the estimator also results in invalidity of the usual t-, chi-square, or IV validity tests on the parameter in the asymptotically homogeneous regression functions. We can obtain a more efficient estimator (in the sense of Phillips, 1991) along the lines of Phillips and Hansen (1990) and Chang, Park, and Phillips (2001). The estimator has a mixed normal limiting distribution and thus yields asymptotically valid t- or chi-square tests in the usual manner.

Let $l_t = (x_t, z'_{t+1})'$, $B_l = (B_x, B'_z)'$, $\sigma_{l\varepsilon} = (\sigma_{x\varepsilon}, \sigma'_{z\varepsilon})'$, and

$$\Sigma_{ll} = \begin{bmatrix} \sigma_u^2 & \sigma'_{vu} \\ \sigma_{vu} & \Sigma_{vv} \end{bmatrix}$$

The FM method requires making the following corrections to y_t and ε_t:

$$y_t^+ = y_t - \hat{\sigma}'_{l\varepsilon}\hat{\Sigma}_{ll}^{-1}\Delta l_t, \quad \text{and} \quad \varepsilon_t^+ = \varepsilon_t - \hat{\sigma}'_{l\varepsilon}\hat{\Sigma}_{ll}^{-1}\Delta l_t, \tag{1.11}$$

where

$$\hat{\sigma}_{l\varepsilon} = \frac{1}{T-1}\sum_{t=1}^{T-1}\hat{\varepsilon}_t\Delta l_t, \quad \hat{\Sigma}_{ll}^{-1} = \frac{1}{T-1}\sum_{t=1}^{T-1}\Delta l_t\Delta l'_t, \quad \text{and} \quad \Delta l_t = l_t - l_{t-1}, \tag{1.12}$$

with the first step GMM residual $\hat{\varepsilon}_t$ which can be obtained by simply choosing $W_T = I_m$ for any sample size T. Then consider the regression

$$y_t^+ = f(x_t, \theta_0) + \varepsilon_t^+, \tag{1.13}$$

in place of (1.1) and the GMM estimator defined in (1.4) with y_t replaced by y_t^+. The efficient estimator that we propose is the GMM estimator $\hat{\theta}_T^+$ of θ_0 computed from the transformed regression (1.13), which we call *efficient GMM* estimator. This estimator is closely related to the FM-OLS method by Phillips and Hansen (1990),

[9] More specifically, these nuisance parameters are the covariances between the limiting Brownian motions $(B_x, B'_z)'$ and B_ε.

which yields efficient estimators for the coefficients in the linear IV regression. Just as the FM-OLS method, the efficient GMM estimator corrects the long-run dependency between the regression errors and the innovations of the integrated regressor and the instrumental variables. However, the martingale difference assumption on the regression errors is maintained while we achieve the goal. As pointed out by Chang, Park, and Phillips (2001), this assumption is more important for nonlinear nonstationary regressions, in contrast to the linear IV regression where more general error processes are allowed.

Theorem 5.1 below presents the asymptotic theory for the efficient GMM estimator $\hat{\theta}_T^+$. Define the following one-dimensional Brownian motion

$$B_+ = B_\varepsilon - \sigma'_{l\varepsilon}\Sigma_{ll}^{-1}B_l,$$

which is independent of B_l, and has the variance

$$\sigma_+^2 = \sigma_\varepsilon^2 - \sigma'_{l\varepsilon}\Sigma_{ll}^{-1}\sigma_{l\varepsilon},$$

i.e. the long-run conditional variance of B_ε given B_l.

THEOREM 5.1: *Let Assumptions EC, WM, and H hold. Then as* $T \to \infty$

$$\sqrt{T}\dot{\kappa}(\sqrt{T}, \theta_0)'(\hat{\theta}_T^+ - \theta_0) \xrightarrow{d} \mathbf{MN}\big(0\,, V(\theta_0)\big), \tag{1.14}$$

where

$$V(\theta_0) = \sigma_+^2\Big[G(\theta_0)'WG(\theta_0)\Big]^{-1}G(\theta_0)'W\Big[\int_0^1 B_z(r)B_z(r)'dr\Big]WG(\theta_0)\Big[G(\theta_0)'WG(\theta_0)\Big]^{-1}$$

It is worth noticing that the efficient GMM estimator has an unbiased and mixed normal limiting distribution. The second-order bias and non-normality has been thus removed. This, in particular, implies that the t-, chi-square, and IV validity tests are now possible for the parameters using this efficient GMM estimator.

When the innovations of the integrated regressor and the instrumental variables are not martingale difference sequences but $MA(\infty)$ processes, for instance, $\Delta l_t = \Psi(L)\eta_t = \sum_{s=0}^{\infty}\psi_s\eta_{t-s}$ with mild conditions on $\Psi(L)$, then the above analysis is also applicable except replacing the innovations Δl_t with the estimates of the 'original innovations' η_t. There are many ways of obtaining these estimates. One way is, as taken by Chang, Park, and Phillips (2001), to use the residuals from a simple $VAR(p)$ regression of Δl_t on its own lagged values with $p = T^\delta$ for some $\delta \in (0,\, 1/8)$. Then the asymptotic distribution of the efficient GMM estimator remains the same as (1.14).

As in the traditional GMM estimation with stationary regressors, we can also find

an optimal weighting matrix $W^* > 0$ *a.s.* which minimizes the asymptotic variance of the efficient GMM estimator in a matrix sense, and a series of matrices $W_T^* \geq 0$ *a.s.* such that $W_T^* \xrightarrow{p} W^*$ as $T \to \infty$. The following corollary presents the asymptotic distribution of the efficient GMM estimator with the optimal choice of weighting matrix.

COROLLARY 5.2: *Let Assumptions EC, WM, and H hold. Then the optimal efficient GMM estimator $\hat{\theta}_T^{+*}$ defined as the efficient GMM estimator with minimum asymptotic conditional variance relative to the σ-field generated by $\{B_l(r) : 0 \leq r \leq 1\}$ has the following asymptotic distribution:*

$$\sqrt{T}\dot{\kappa}(\sqrt{T}, \theta_0)'(\hat{\theta}_T^{+*} - \theta_0) \xrightarrow{d} \mathbf{MN}\big(0,\, V^*(\theta_0)\big), \tag{1.15}$$

where

$$V^*(\theta_0) = \sigma_+^2 \Big(G(\theta_0)' \big[\int_0^1 B_z(r) B_z(r)' dr \big]^{-1} G(\theta_0) \Big)^{-1}.$$

In particular, the optimal efficient GMM estimator can be obtained by setting

$$W = W^* = \Big[\int_0^1 B_z(r) B_z(r)' dr \Big]^{-1}.$$

The corollary above implies that the optimal choice of W_T is

$$W_T^* = \big(\frac{1}{T^2} \sum_{t=1}^{T} z_t z_t' \big)^{-1},$$

which is a consistent estimator of W^*. This implies that the optimal GMM efficient estimator is exactly the FM-NLIV estimator. As is well known, if the regression function is scalar-valued and all the variables are stationary, then the NLIV estimator coincides with the optimal GMM estimator, and as we see here, this conclusion also holds in the nonstationary world. Moreover, this result is also consistent with the finding in the linear GMM estimator with integrated regressors and instruments analyzed by Kitamura and Phillips (1997). In particular, when f is a linear function $f(x, \theta) = \theta x$, then the limiting distribution of the optimal efficient GMM estimator in (1.15) has the same distribution as the linear FM-IV or FM-GMM estimator in Kitamura and Phillips (1997).

Before concluding this section, we provide a procedure below to empirically implement the optimal efficient GMM estimator for applied researchers who want to use integrated time series such as oil prices as instruments in GMM estimation.

1. Find the parameter values that minimize the 'naive' GMM objective function

(1.4) where $W_T = I_m$ for any sample size T, and calculate the GMM residual $\hat{\varepsilon}_t$;

2. Use the residuals from Step 1 to compute the long-run covariance matrices, i.e. $\hat{\sigma}_{l\varepsilon}$ and $\hat{\Sigma}_{ll}^{-1}$, according to (1.12), and then calculate the FM corrected dependent variable y_t^+ according to (1.11);

3. Replace y_t by the FM corrected dependent variable y_t^+ in the GMM objective function (1.4) with $W_T = \left(\frac{1}{T^2}\sum_{t=1}^{T} z_t z_t'\right)^{-1}$, and find the parameter values that minimize the corrected objective function. This completes the optimal efficient GMM estimation.

1.6 CONCLUSIONS

This paper develops an asymptotic theory for GMM estimator in nonlinear models with asymptotically homogeneous regression functions and integrated time series. It is straightforward to extend our theory to nonlinear models with integrable functions. The GMM estimator is consistent under fairly general conditions, and the convergence rates are determined by the degree of the asymptotic homogeneity of regression functions. Similar to IV and GMM estimation in linear regressions, we find second-order bias in the limiting distribution of the GMM estimator in nonlinear regressions, and that instruments themselves cannot eliminate the bias in nonlinear models even when they are strictly exogenous. Therefore, the GMM estimator is generally inefficient and produces invalid test statistics. By making use of the FM method developed by Phillips and Hansen (1991), we propose an efficient GMM estimator which is asymptotically unbiased and has a mixed normal distribution. Our approach is simple to implement, and produces efficient estimates and hence valid t-, chi-square, and IV validity test statistics. In this perspective, our work can be considered as an extension of the FM-IV and FM-GMM estimation in linear regressions analyzed by Phillips and Hansen (1990) and Kitamura and Phillips (1997) to nonlinear regressions.

As in stationary GMM asymptotic theory, we find that there is an 'optimal' choice of weighting matrix which minimizes the (conditional) variance of the efficient GMM estimator. Moreover, the optimal efficient GMM estimator is, in fact, the same as the FM-NLIV estimator. This result is the same as in the linear case where, as shown by Kitamura and Phillips (1997), the FM-IV and FM-GMM estimators are asymptotically equivalent in linear regressions.

From a practical point of view, our work provides a simple approach to produce efficient estimates for applied researchers who want to use integrated time series such as oil prices as instruments in GMM estimation. In particular, this suggests that one could use oil price levels as instruments when the first-order differences of oil

prices are weakly correlated with endogenous variables and the 'weak instruments' or 'weak identification' problem occurs. Extensions of our theory to accommodate more general error processes and a system of nonlinear equations, are an ongoing area of research for the author.

1.7 MATHEMATICAL PROOFS

LEMMA A1. *Let Assumption EC(b) holds. If f is H-regular on Θ with asymptotic order κ and limit homogeneous function h, then $g(\theta)$ defined in (6) is continuous a.s. on Θ.*

Proof: It suffices to show the case where z_t or $B_z(\cdot)$ is a scalar.

From Lemma A3(a) in PP, for any $\theta \in \Theta$, there exists a neighborhood N_0 of θ such that $\sup_{\theta \in N_0} h(\cdot,\theta)^2$ is regular.

Since

$$\left|B_z(r)h\big(B_x(r),\theta\big)\right| \leq \frac{1}{2}B_z(r)^2 + \frac{1}{2}h\big(B_x(r),\theta\big)^2 \leq \frac{1}{2}B_z(r)^2 + \frac{1}{2}\sup_{\theta\in N_0} h\big(B_x(r),\theta\big)^2,\ a.s.$$

and since $\sup_{\theta\in N_0} h\left(\cdot,\theta\right)^2$ is regular by Lemmas A2 and A3(a) in PP and therefore locally bounded, then by the occupation time formula,

$$\begin{aligned}&\int_0^1 \left[\frac{1}{2}B_z(r)^2 + \frac{1}{2}\sup_{\theta\in N_0} h\big(B_x(r),\theta\big)^2\right] dr \\ &= \frac{1}{2}\int_{-\infty}^{\infty} s^2 L_z(1,s)ds + \frac{1}{2}\int_{-\infty}^{\infty}\sup_{\theta\in N_0} h(s,\theta)^2 L_x(1,s)ds < \infty,\ a.s.\end{aligned}$$

where $L_x(1,\cdot)$ and $L_z(1,\cdot)$ are the local times of the limiting Brownian motions B_x and B_z respectively.

Therefore the continuity of $g(\theta) = \int_0^1 B_z(r)h\big(B_x(r),\theta\big)dr$ on Θ is an immediate consequence of the Dominated Convergence Theorem, and follows immediately from the $a.s.$ integrability of $s^2L_z(1,s)$ and $\sup_{\theta\in N_0} h(s,\theta)^2 L_x(1,s)$. Note that s^2 and $\sup_{\theta\in N_0} h(s,\theta)^2$ are both locally bounded functions of s, and hence locally integrable, and both $L_z(1,s)$ and $L_x(1,s)$ have compact support $a.s.$

LEMMA A2: *Let Assumptions EC holds. If f is H-regular on Θ with asymptotic order κ and limit homogeneous function h, then (b1) is a sufficient condition for (b2) in both Theorems 3.3 and 3.4.*

Proof: We will show that (b1) is a sufficient condition for (b2) in Theorem 3.4,

and it is straightforward to show that the argument also holds in Theorem 3.3 in the same manner.

Since the covariance matrix of the limiting Brownian motion $(B_x, B_z')'$

$$\Phi = \begin{bmatrix} \omega_{xx} & \omega_{zx}' \\ \omega_{zx} & \Omega_{zz} \end{bmatrix} > 0,$$

due to Assumption EC(c), then by Lemma 3.1 in Phillips (1989), we can decompose the (conditional) distribution of Brownian motion B_z conditioning on the filtration generated by B_x into two parts:

$$B_z \mid \mathcal{F}_x \stackrel{d}{=} \omega_{zx}\omega_{xx}^{-1} B_x + \Omega_{zz\cdot x}^{\frac{1}{2}} M, \tag{1.16}$$

where $\Omega_{zz\cdot x} = \Omega_{zz} - \omega_{zx}\omega_{xx}^{-1}\omega_{zx}' > 0$, $M \stackrel{d}{=} BM(I_m)$ and is independent of B_x, and $\mathcal{F}_x$ is the sub-σ-field of $\mathcal{F}$ that is generated by $\{B_x(r) : 0 \le r \le 1\}$. To be consistent with notation in Phillips (1989), we use the symbol '$\cdot \mid \mathcal{F}_x$' to signify the conditional distribution relative to $\mathcal{F}_x$.

Then the (conditional) distribution of $g(\theta)$ conditioning on $\mathcal{F}_x$ is:

$$\begin{aligned} g(\theta) \mid \mathcal{F}_x &= \int_0^1 B_z(r) h\big(B_x(r), \theta\big) dr \mid \mathcal{F}_x \\ &\stackrel{d}{=} \omega_{xx}^{-1}\omega_{zx} \int_0^1 B_x(r) h\big(B_x(r), \theta\big) dr + \Omega_{zz\cdot x}^{\frac{1}{2}} \int_0^1 M(r) h\big(B_x(r), \theta\big) dr \end{aligned}$$

It then easily follows that

$$\begin{aligned} g(\theta) \mid \mathcal{F}_x \stackrel{d}{=} \mathbf{N}\Bigg(& \omega_{xx}^{-1}\omega_{zx} \int_0^1 B_x(r) h\big(B_x(r), \theta\big) dr, \\ & \Omega_{zz\cdot x}^{\frac{1}{2}} \mathbf{Var}\Big[\int_0^1 M(r) h\big(B_x(r), \theta\big) dr \mid \mathcal{F} \Big] \Omega_{zz\cdot x}^{\frac{1}{2}} \Bigg). \end{aligned}$$

Since

$$\mathbf{Pr}\Big[\big|g(\theta)\big| = 0 \Big] = \mathbf{E}\Big[\mathbf{Pr}\big(|g(\theta)| = 0 \mid \mathcal{F}_x \big) \Big],$$

and $\Omega_{zz\cdot x} > 0$, then it suffices to show that for any $\theta \in \Theta$

$$\mathbf{Var}\Big[\int_0^1 B_x(r) h\big(B_x(r), \theta\big) dr \mid \mathcal{F}_x \Big] > 0 \quad a.s. \tag{1.17}$$

The rest of the proof is done by making use of the general representation theory of a stochastic process in terms of an orthonormal system; see, for example, Phillips (1998). It directly follows from the definition of the vector Wiener process M that

each element of M has the same autocovariance function $\gamma(r,s) = r \wedge s$. Thus, by the Loéve-Karhunen representation, $M(r)$ has on the interval [0, 1] the orthogonal expansion

$$M(r) = \sum_{j=1}^{\infty} \sqrt{\lambda_j}\varphi_j(r)\xi_j, \tag{1.18}$$

with

$$\int_0^1 \varphi_i(s)\varphi_j(s)ds = \begin{cases} 1, & \text{if } i = j; \\ 0, & \text{otherwise.} \end{cases}$$

where the λ_j are the eigenvalues and the φ_j are the orthonormalized eigenfunctions of the autocovariance function of each element in M, $r \wedge s$, and the ξ_j are $m \times 1$ vectors of independently and identically distributed (i.i.d.) $\mathbf{N}(0, I_m)$ and are independent of B_x. That ξ_j is identically distributed as $\mathbf{N}(0, I_m)$ directly follows from $M \stackrel{d}{=} BM(I_m)$. More specifically, the eigenvalues and eigenfunctions in the Loéve-Karhunen expansion of $M(r)$ are

$$\lambda_j = \frac{4}{(2j-1)^2\pi^2}, \quad \phi_j(r) = \sqrt{2}\sin[(k-1/2)\pi r],$$

giving the following L_2-representation

$$M(r) = \sqrt{2}\sum_{j=1}^{\infty} \frac{\sin[(k-1/2)\pi r]}{(k-1/2)\pi}\xi_j.$$

As pointed out in Phillips (1998), the above series representation of $M(r)$ is convergent $a.s.$ and uniformly in $r \in [0, 1]$, and thus does converge pointwise. Obviously, the eigenvalues λ_j are strictly positive for all $j \geq 1$.

We equip $L_2[0, 1]$ space by the inner product

$$\langle f,\ g\rangle = \int_0^1 f(x)g(x)dx$$

for any $f, g \in L_2[0, 1]$, and thus it is a Hilbert space. Since $\{\varphi_j\}_{j=1}^{\infty}$ is a complete orthonormal system in $L_2[0, 1]$, and $h\big(B_x(r), \theta\big) \in L_2[0, 1]$, then we have the following expansions for $h\big(B_x(r), \theta\big)$ and $\int_0^1 h\big(B_x(r), \theta\big)^2 dr$ by the definition of complete orthonormal system in Hilbert space; see, for example, Debnath and Mikusiński

(2005). For any $\theta \in \Theta$

$$h\big(B_x(r),\theta\big) = \sum_{j=1}^{\infty}\Big[\int_0^1 h\big(B_x(s),\theta\big)\varphi_j(s)ds\Big]\varphi_j(r), \tag{1.19}$$

$$\int_0^1 h\big(B_x(r),\theta\big)^2 dr = \sum_{j=1}^{\infty}\Big[\int_0^1 h\big(B_x(s),\theta\big)\varphi_j(s)ds\Big]^2. \tag{1.20}$$

Now we can write

$$\begin{aligned}
&\int_0^1 M(r)h\big(B_x(r),\theta\big)dr \;\Big|\; \mathcal{F}_x \\
&\stackrel{d}{=} \int_0^1 \big[\sum_{j=1}^{\infty}\sqrt{\lambda_j}\varphi_j(r)\xi_j\big]\Big[\sum_{i=1}^{\infty}\Big(\int_0^1 h(B_x(s),\theta)\varphi_i(s)ds\Big)\varphi_i(r)\Big]dr \;\Big|\; \mathcal{F}_x \\
&\stackrel{d}{=} \sum_{j=1}^{\infty}\sqrt{\lambda_j}\Big(\int_0^1 h\big(B_x(s),\theta\big)\varphi_j(s)ds\Big)\xi_j \;\Big|\; \mathcal{F}_x.
\end{aligned}$$

Thus the conditional variance in (1.17) can be written as

$$\mathbf{Var}\Big[\int_0^1 M(r)h\big(B_x(r),\theta\big)dr \;\Big|\; \mathcal{F}_x\Big] = \Big[\sum_{j=1}^{\infty}\lambda_j\Big(\int_0^1 h\big(B_x(s),\theta\big)\varphi_j(s)ds\Big)^2\Big]\cdot I_m \tag{1.21}$$

due to the fact that the ξ_j are i.i.d. $\mathbf{N}(0,\, I_m)$.

On the other hand, as shown in PP, it follows from condition (b1) and the occupation time formula that

$$\int_0^1 h\big(B_x(s),\theta\big)^2 ds > 0 \quad a.s.,$$

and by (1.20), we have

$$\sum_{j=1}^{\infty}\Big[\int_0^1 h\big(B_x(r),\theta\big)\varphi_j(r)dr\Big]^2 > 0 \quad a.s.$$

Therefore

$$\sum_{j=1}^{\infty}\lambda_j\Big(\int_0^1 h\big(B_x(s),\theta\big)\varphi_j(s)ds\Big)^2 > 0 \quad a.s.$$

since $\lambda_j > 0$ for all $j \geq 1$. Now it is straightforward to see that

$$\mathbf{Var}\Big[\int_0^1 M(r)h\big(B_x(r),\theta\big)dr \;\Big|\; \mathcal{F}_x\Big] > 0 \quad a.s.,$$

which, as argued above, implies that (b1) is a sufficient condition for (b2) in Theorem 3.4. In the same manner, one can easily show that the argument is also true in Theorem 3.3 by replacing $h(B_x(\cdot),\theta)$ with $\big(h(B_x(\cdot),\theta) - h(B_x(\cdot),\theta_0)\big)$. This completes the proof.

LEMMA A3: *Let Assumption EC holds. If f is H-regular on Θ with asymptotic order κ and limit homogeneous function h, then (c1) is a sufficient condition for (c2) in Assumption H.*

Proof: Suppose condition (c1) in Assumption H holds. It is equivalent to show that $G(\theta_0)' = \int_0^1 \dot{h}\big(B_x(r),\theta_0\big)B_z(r)'dr$ has rank k *a.s.* Decomposing B_z as in (1.16), we have

$$\begin{aligned}&\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)B_z(r)'dr \\ &= \int_0^1 \dot{h}\big(B_x(r),\theta_0\big)B_x(r)dr\cdot\omega_{zx}'\omega_{xx}^{-1} + \int_0^1 \dot{h}\big(B_x(r),\theta_0\big)M(r)'dr\cdot\Omega_{zz\cdot x}^{\frac{1}{2}}.\end{aligned}$$

Thus for any $k\times k$ matrix P,

$$\begin{aligned}&P\cdot\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)B_z(r)'dr \\ &= P\cdot\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)B_x(r)dr\cdot\omega_{zx}'\omega_{xx}^{-1} + P\cdot\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)M(r)'dr\cdot\Omega_{zz\cdot x}^{\frac{1}{2}}. \quad (1.22)\end{aligned}$$

If we can show that

$$\mathbf{Var}\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)M(r)'dr \;\Big|\; \mathcal{F}_x\Big] > 0 \quad a.s., \quad (1.23)$$

then we can set P to equal

$$P = \left\{\mathbf{Var}\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)M(r)'dr \;\Big|\; \mathcal{F}_x\Big]\right\}^{-\frac{1}{2}}.$$

Thus each row of $P\cdot\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)M(r)'dr$ is independently distributed as a multivariate normal random vector with covariance matrix I_m, ensuring that the

matrix $P \cdot \int_0^1 \dot{h}\big(B_x(r), \theta_0\big) M(r)' dr$ has rank k *a.s.* Moreover, since this matrix is stochastically independent of the first term on the right hand side of (1.22), we can conclude that $\int_0^1 \dot{h}\big(B_x(r), \theta_0\big) B_z(r)' dr$ has rank k *a.s.* Therefore, it suffices to show (1.23) to complete the proof.

Since each element of $\dot{h}$ belongs to $L_2[0,\, 1]$, and $\{\varphi_j\}_{j=1}^{\infty}$ is a complete orthonormal system in the $L_2[0,\, 1]$ Hilbert space defined in Lemma A2, we can have the following expansions for $\dot{h}\big(B_x(r), \theta_0\big)$ and $\int_0^1 \dot{h}\big(B_x(r), \theta_0\big) \dot{h}\big(B_x(r), \theta_0\big)' dr$:

$$\dot{h}\big(B_x(r), \theta_0\big) = \sum_{j=1}^{\infty} \Big[\int_0^1 \dot{h}\big(B_x(s), \theta_0\big) \varphi_j(s) ds \Big] \varphi_j(r), \tag{1.24}$$

$$\begin{aligned} \int_0^1 \dot{h}\big(B_x(r), \theta_0\big) \dot{h}\big(B_x(r), \theta_0\big)' dr = \sum_{j=1}^{\infty} \Big[\Big(\int_0^1 \dot{h}\big(B_x(s), \theta_0\big) \varphi_j(s) ds \Big) \cdot \\ \Big(\int_0^1 \dot{h}\big(B_x(s), \theta_0\big)' \varphi_j(s) ds \Big) \Big]. \end{aligned} \tag{1.25}$$

In an analogous way as in the proof of Lemma A2, we can have

$$\int_0^1 \dot{h}\big(B_x(r), \theta_0\big) M(r)' dr \;\Big|\; \mathcal{F}_x \overset{d}{=} \sum_{j=1}^{\infty} \sqrt{\lambda_j} \Big(\int_0^1 \dot{h}\big(B_x(s), \theta_0\big) \varphi_j(s) ds \Big) \xi_j' \;\Big|\; \mathcal{F}_x,$$

and hence the conditional variance in (1.23) can be written as

$$\begin{aligned} & \mathbf{Var}\Big[\int_0^1 \dot{h}\big(B_x(r), \theta_0\big) M(r)' dr \;\Big|\; \mathcal{F}_x \Big] \\ = & \sum_{j=1}^{\infty} m \lambda_j \Big[\int_0^1 \dot{h}\big(B_x(s), \theta_0\big) \varphi_j(s) ds \Big] \Big[\int_0^1 \dot{h}\big(B_x(s), \theta_0\big)' \varphi_j(s) ds \Big] \end{aligned} \tag{1.26}$$

due to the fact that the ξ_j are i.i.d. $\mathbf{N}(0,\, I_m)$.

On the other hand, as shown in PP, it follows from condition (c1) and the occupation time formula that conditioning on $\mathcal{F}_x$ [10]

$$\int_0^1 \dot{h}\big(B_x(s), \theta_0\big) \dot{h}\big(B_x(s), \theta_0\big)' ds > 0,$$

[10] More precisely, it should be conditioning on $\mathcal{F}_x^0$, a filtration equal to $\mathcal{F}_x$ minus a set of probability zero where the local time $L_x(1,\, s)$ of Brownian motion B_x at $s = 0$ is zero. Since the set of probability zero does not affect the conditional probabilities in the following, we denote both filtrations by $\mathcal{F}_x$ for notational simplicity.

and by (1.25), we have

$$\sum_{j=1}^{\infty}\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)\varphi_j(r)dr\Big]\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)'\varphi_j(r)dr\Big] > 0.$$

Since this series representation of matrices converges to a positive definite matrix uniformly in $r \in [0,\,1]$ conditioning on $\mathcal{F}_x$, then there exists $J \geq 1$ such that

$$\sum_{j=1}^{J}\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)\varphi_j(r)dr\Big]\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)'\varphi_j(r)dr\Big] > 0.$$

It then follows that

$$\sum_{j=1}^{J} m\Big(\min_{1\leq j\leq J}(\lambda^2)\Big)\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)\varphi_j(r)dr\Big]\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)'\varphi_j(r)dr\Big] > 0,$$

due to the fact that $\lambda_j > 0$ for all $j \geq 1$. Since

$$\sum_{j=1}^{J} m\Big(\lambda_j^2 - \min_{1\leq j\leq J}(\lambda^2)\Big)\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)\varphi_j(r)dr\Big]\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)'\varphi_j(r)dr\Big] \geq 0,$$

we have

$$\sum_{j=1}^{J} m\lambda_j^2\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)\varphi_j(r)dr\Big]\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)'\varphi_j(r)dr\Big] > 0,$$

and therefore

$$\sum_{j=1}^{\infty} m\lambda_j^2\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)\varphi_j(r)dr\Big]\Big[\int_0^1 \dot{h}\big(B_x(r),\theta_0\big)'\varphi_j(r)dr\Big] > 0.$$

Observe that

$$\begin{aligned}&\mathbf{Pr}\big(\text{The conditional variance in (22)} > 0\big)\\ &= \mathbf{E}\Big[\mathbf{Pr}\big(\text{The conditional variance in (22)} > 0 \;\Big|\; \mathcal{F}_x\big)\Big] = 1,\end{aligned}$$

which establishes (1.23) and completes the proof.

LEMMA A4: *Let Assumption EC(b) holds. If S is a regular transformation on $\mathbb{R}$,*

then

$$\int_0^1 B_z(r)^+ S\big(U_T(r)\big)dr \xrightarrow{a.s.} \int_0^1 B_z(r)^+ S\big(B_x(r)\big)dr,$$
$$\int_0^1 B_z(r)^- S\big(U_T(r)\big)dr \xrightarrow{a.s.} \int_0^1 B_z(r)^- S\big(B_x(r)\big)dr,$$
$$\int_0^1 B_z(r) S\big(U_T(r)\big)dr \xrightarrow{a.s.} \int_0^1 B_z(r) S\big(B_x(r)\big)dr$$

where $B_z(r)^+ = max\big(B_z(r), 0\big)$, *and* $B_z(r)^- = max\big(-B_z(r), 0\big)$.

Proof: It suffices to show the first two results, and the last result is obvious because $B_z(r) = B_z(r)^+ - B_z(r)^-$. In what follows, we assume w.l.o.g. that z_t or $B_z(\cdot)$ is scalar-valued by taking each component separately.

Let $B_z(r)^* = B_z(r)^+$ or $B_z(r)^-$, and let $C = [s_{min} - 1, s_{max} + 1]$ where $s_{min} = \inf_{0\le r\le 1} B_x(r)$ and $s_{max} = \sup_{0\le r\le 1} B_x(r)$. From the proof of Theorem 3.2 in PP, we may take T sufficiently large so that $\sup_{0\le r\le 1} |U_T(r) - B_x(r)| < \delta_\epsilon$ for any $\delta_\epsilon > 0$ and so that both U_T and B_x are in C *a.s.* Since $B_z(r)^* \geqslant 0$ for any $r \in [0,1]$, then

$$B_z(r)^* \underline{S}_\epsilon\big(B_x(r)\big) \le B_z(r)^* S\big(U_T(r)\big) \le B_z(r)^* \underline{S}_\epsilon\big(B_x(r)\big) \tag{1.27}$$

for large T. However,

$$\begin{aligned}
\int_0^1 B_z(r)^* \left(\overline{S}_\epsilon - \underline{S}_\epsilon\right)(B_x(r))\, dr &\le \left[\sup_{0\le r\le 1} B_z(r)^*\right] \int_0^1 \left(\overline{S}_\epsilon - \underline{S}_\epsilon\right)(B_x(r))\, dr \\
&= \left[\sup_{0\le r\le 1} B_z(r)^*\right] \int_{-\infty}^{\infty} \left(\overline{S}_\epsilon - \underline{S}_\epsilon\right)(s) L_x(1,s) ds \\
&= \left[\sup_{0\le r\le 1} B_z(r)^*\right] \left[\sup_s L_x(1,s)\right] \int_C \left(\overline{S}_\epsilon - \underline{S}_\epsilon\right)(s) ds \\
&\xrightarrow{a.s.} 0,
\end{aligned} \tag{1.28}$$

as $\epsilon \to 0$ since $\sup_{0\le r\le 1} B_z(r)^* = O_{a.s.}(1)$ and $\sup_s L_x(1,s) = O_{a.s.}(1)$ by Lemma A4 in PP. The stated result now easily follows from (1.27) and (1.28).

Proof of Lemma 3.1: In what follows, we assume w.l.o.g. that F and z_t are both scalar-valued. It is straightforward to extend the proof to the case where F and z_t are vector-valued.

The asymptotically homogeneous function F can be written as

$$F(x_t,\theta) = \kappa(\sqrt{T},\theta)H\left(\frac{x_t}{\sqrt{T}},\theta\right) + R\left(\frac{x_t}{\sqrt{T}},\sqrt{T},\theta\right),$$

where $R\left(\frac{x_t}{\sqrt{T}},\sqrt{T},\theta\right)$ is of order smaller than $\kappa(\sqrt{T},\theta)$ for all $\theta \in \Theta$, and can be decomposed as

$$R\left(\frac{x_t}{\sqrt{T}},\sqrt{T},\theta\right) = a\left(\sqrt{T},\theta\right)A\left(\sqrt{T},\theta\right),$$

where $a\left(\sqrt{T},\theta\right) = o\left(\kappa(\sqrt{T},\theta)\right)$ on Θ, and $\sup_{\theta\in\Theta} A(x,\theta) = O\left(e^{c|x|}\right)$ as $|x| \to \infty$ for some $c \in \mathbb{R}_+$ by Definition 2.3. Then we can rewrite $g_T(\theta)$ as

$$\begin{aligned} g_T(\theta) &= \frac{1}{T}\kappa(\sqrt{T},\theta)^{-1}\sum_{t=1}^{T}\frac{z_t}{\sqrt{T}}F(x_t,\theta) \\ &= \frac{1}{T}\sum_{t=1}^{T}\frac{z_t}{\sqrt{T}}H\left(\frac{x_t}{\sqrt{T}},\theta\right) + \frac{1}{T}\kappa(\sqrt{T},\theta)^{-1}\sum_{t=1}^{T}\frac{z_t}{\sqrt{T}}R\left(\frac{x_t}{\sqrt{T}},\sqrt{T},\theta\right) \end{aligned}$$

Let's first look at the second term on the right-hand side. We can easily show that the second term $\xrightarrow{a.s.} 0$ uniformly in Θ following the same proof as in Lemma A5 in PP and using the fact

$$\frac{1}{T}\sum_{t=1}^{T}\left|\frac{z_t}{\sqrt{T}}\right| S\left(\frac{x_t}{\sqrt{T}}\right) = O_{a.s.}(1), \tag{1.29}$$

where $S(x) = \sup_{\theta\in\Theta}\|A(x,\theta)\|$. Note that $S(x)$ exists because $\sup_{\theta\in\Theta} A(x,\theta)$ is locally exponentially bounded. To see why (1.29) is true, notice that by Chebyshev's inequality

$$\begin{aligned} \frac{1}{T}\sum_{t=1}^{T}\left|\frac{z_t}{\sqrt{T}}\right| S\left(\frac{x_t}{\sqrt{T}}\right) &\le \sqrt{\left[\frac{1}{T}\sum_{t=1}^{T}\left(\frac{z_t}{\sqrt{T}}\right)^2\right]\left[\frac{1}{T}\sum_{t=1}^{T}S\left(\frac{x_t}{\sqrt{T}}\right)^2\right]} \\ &\le \sqrt{\left\|S_0\right\|_C\left\|S^2\right\|_C} < \infty,\ a.s. \end{aligned}$$

for large T, where $S_0(x) = x^2$ is a regular function by Lemma A1 in PP and thus locally bounded, and $S(x)^2$ is a locally bounded function because the class of locally bounded functions is closed under multiplication.

Now let's look at the first term on the right-hand side. Using the Skorohod

representation theorem, we can write

$$\frac{1}{T}\sum_{t=1}^{T}\frac{z_t}{\sqrt{T}}H\left(\frac{x_t}{\sqrt{T}},\theta\right)\overset{d}{=}\int_0^1 V_T(r)H\left(U_T(r),\theta\right)dr.$$

Now it suffices to show

$$\sup_{\theta\in\Theta}\left|\int_0^1 V_T(r)H\left(U_T(r),\theta\right)dr-\int_0^1 B_z(r)H\left(B_x(r),\theta\right)dr\right|\xrightarrow{a.s.}0. \tag{1.30}$$

It easily follows that

$$\begin{aligned}
&\sup_{\theta\in\Theta}\left|\int_0^1 V_T(r)H\left(U_T(r),\theta\right)dr-\int_0^1 B_z(r)H\left(B_x(r),\theta\right)dr\right| \\
\leq\ &\sup_{\theta\in\Theta}\left|\int_0^1 \left(V_T(r)-B_z(r)\right)H\left(U_T(r),\theta\right)dr\right| \\
&+\sup_{\theta\in\Theta}\left|\int_0^1 B_z(r)H\left(U_T(r),\theta\right)dr-\int_0^1 B_z(r)H\left(B_x(r),\theta\right)dr\right|
\end{aligned} \tag{1.31}$$

First let's look at the first term on the right-hand side of (1.31). Since

$$\begin{aligned}
&\sup_{\theta\in\Theta}\left|\int_0^1 \left(V_T(r)-B_z(r)\right)H\left(U_T(r),\theta\right)dr\right| \\
\leq\ &\sup_{0\leq r\leq 1}\left|V_T(r)-B_z(r)\right|\int_0^1\sup_{\theta\in\Theta}\left|H\left(U_T(r),\theta\right)\right|dr \\
\leq\ &\sup_{0\leq r\leq 1}\left|V_T(r)-B_z(r)\right|\sup_{x\in C}\left(\sup_{\theta\in\Theta}\left|H(x,\theta)\right|\right),
\end{aligned}$$

due to the fact that $\sup_{\theta\in\Theta}|H(\cdot,\theta)|$ is locally bounded as shown in Lemma A3(b) in PP (and hence $\sup_{x\in C}\sup_{\theta\in\Theta}|H(x,\theta)|=O_{a.s.}(1)$). Moreover, we have

$$\sup_{0\leq r\leq 1}\left|V_T(r)-B_z(r)\right|\xrightarrow{a.s.}0,$$

since $V_T(\cdot)\xrightarrow{a.s.}B_z(\cdot)$. Therefore, it directly follows that the first term

$$\sup_{\theta\in\Theta}\left|\int_0^1\big(V_T(r)-B_z(r)\big)H\big(U_T(r),\theta\big)dr\right|\xrightarrow{a.s.}0. \tag{1.32}$$

Now let's look at the second term on the right-hand side of (1.31). To show that the second term $\xrightarrow{a.s.} 0$, we need to show

$$\int_0^1 B_z(r)H\big(U_T(r),\theta\big)dr \xrightarrow{a.s.} \int_0^1 B_z(r)H\big(B_x(r),\theta\big)dr \tag{1.33}$$

uniformly in $\theta \in \Theta$. Fix an arbitrary $\theta_0 \in \Theta$. Due to Lemma A3(a) in PP, there exists a neighborhood N_0 of θ_0 such that $\sup_{\theta\in\Theta} H(\cdot,\theta)$ and $\inf_{\theta\in\Theta} H(\cdot,\theta)$ are both regular for any neighborhood $N \subset N_0$ of θ_0. Therefore, it follows from Lemma A4 that

$$\int_0^1 B_z(r)^* \sup_{\theta\in N} H\big(U_T(r),\theta\big)dr \xrightarrow{a.s.} \int_0^1 B_z(r)^* \sup_{\theta\in N} H\big(B_x(r),\theta\big)dr,$$

where $B_z(r)^* = B_z(r)^+$ or $B_z(r)^-$. Since $B_z(r)^* \geqslant 0$, we have

$$\int_0^1 \sup_{\theta\in N} B_z(r)^* H\big(U_T(r),\theta\big)dr \xrightarrow{a.s.} \int_0^1 \sup_{\theta\in N} B_z(r)^* H\big(B_x(r),\theta\big)dr, \tag{1.34}$$

$$\int_0^1 \inf_{\theta\in N} B_z(r)^* H\big(U_T(r),\theta\big)dr \xrightarrow{a.s.} \int_0^1 \inf_{\theta\in N} B_z(r)^* H\big(B_x(r),\theta\big)dr. \tag{1.35}$$

Let $N_\delta \subset N_0$ be the δ-neighborhood of θ_0. Then we have

$$\left|\sup_{\theta\in N_\delta} H(x,\theta) - \inf_{\theta\in N_\delta} H(x,\theta)\right| \to 0,$$

as $\delta \to 0$, due to the continuity of $H(x,\cdot)$ for any x. Moreover, since $\big[\sup_{\theta\in N_\delta} H(\cdot,\theta) - \inf_{\theta\in N_\delta} H(\cdot,\theta)\big]$ is regular and hence locally integrable (and therefore the occupation time formula applies), and $\sup_{\theta\in\Theta}|H(\cdot,\theta)|$ is locally bounded as shown in Lemma A3(a) in PP, it therefore follows from the occupation time formula and dominated convergence that

$$\begin{aligned}
&\int_0^1 \sup_{\theta\in N_\delta} B_z(r)^* H\big(B_x(r),\theta\big)dr - \int_0^1 \inf_{\theta\in N_\delta} B_z(r)^* H\big(B_x(r),\theta\big)dr \\
&= \int_0^1 B_z(r)^* \left[\sup_{\theta\in N_\delta} H\big(B_x(r),\theta\big) - \inf_{\theta\in N_\delta} H\big(B_x(r),\theta\big)\right] dr \\
&\leq \left[\sup_{0\leq r\leq 1} B_z(r)^*\right] \int_0^1 \left[\sup_{\theta\in N_\delta} H\big(B_x(r),\theta\big) - \inf_{\theta\in N_\delta} H\big(B_x(r),\theta\big)\right] dr \\
&= \left[\sup_{0\leq r\leq 1} B_z(r)^*\right] \int_{-\infty}^{\infty} \left[\sup_{\theta\in N_\delta} H(s,\theta) - \inf_{\theta\in N_\delta} H(s,\theta)\right] L_x(1,s)ds \\
&\xrightarrow{a.s.} 0,
\end{aligned}$$

as $\delta \to 0$. Now we may now easily deduce that $\int_0^1 B_z(r)^* H\big(U_T(r),\theta\big)dr$ is *strongly asymptotically uniform stochastically equicontinuous*. Since (1.33) holds pointwisely for each $\theta \in \Theta$ due to Lemma A4, then we can conclude from Theorem 21.8 in Davidson (1994) that there exists a neighborhood of θ_0 where

$$\int_0^1 B_z(r)^* H\big(U_T(r),\theta\big)dr \xrightarrow{a.s.} \int_0^1 B_z(r)^* H\big(B_x(r),\theta\big)dr \tag{1.36}$$

holds uniformly in θ. Since θ_0 was chosen arbitrarily and Θ is compact, (1.33) holds uniformly on Θ. Finally we can easily deduce (1.30) since $B_z(r) = B_z(r)^+ - B_z(r)^-$. This completes the proof.

Proof of Lemma 3.2: See Wu (1981) and Section 4.2 of Potscher and Prucha (1997).

Proof of Theorem 3.3: It follows from the definition of $D_T(\theta,\theta_0)$ that, $\forall\theta \in N$, $D_T(\theta,\theta_0) = A_T(\theta,\theta_0) - 2B_T(\theta,\theta_0)$, where

$$A_T(\theta,\theta_0) = \left[\frac{1}{T}\sum_{t=1}^{T} z_t\left(f(x_t,\theta) - f(x_t,\theta_0)\right)\right]' W_T \left[\frac{1}{T}\sum_{t=1}^{T} z_t\left(f(x_t,\theta) - f(x_t,\theta_0)\right)\right]$$

$$B_T(\theta,\theta_0) = \left(\frac{1}{T}\sum_{t=1}^{T} z_t'\varepsilon_t\right) W_T \left[\frac{1}{T}\sum_{t=1}^{T} z_t\left(f(x_t,\theta) - f(x_t,\theta_0)\right)\right]$$

From Assumption EC, we have

$$\frac{1}{T}\sum_{t=1}^{T} z_t\varepsilon_t = O_p(1),$$

and therefore,

$$\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T} z_t\varepsilon_t = o_p(1). \tag{1.37}$$

It then follows from Assumption WM and Lemma 3.1 that

$$\frac{1}{T\kappa(\sqrt{T})} B_T(\theta,\theta_0) = o_p(1),$$

uniformly in $\theta \in \Theta$. Furthermore, if we define $\rho_T = T\kappa(\sqrt{T})^2$, then since κ is bounded away from zero by condition (a), we have

$$\rho_T^{-1} D_T(\theta,\theta_0) = \rho_T^{-1} A_T(\theta,\theta_0) + o_p(1),$$

uniformly in $\theta \in \Theta$. It then follows directly from Lemmas 3.1 that

$$\rho_T^{-1} D_T(\theta, \theta_0) \xrightarrow{p} D(\theta, \theta_0),$$

uniformly in $\theta \in \Theta$ where $D(\theta, \theta_0)$ is defined in the theorem. It then follows from Assumption WM, Lemma A2 that $D(\cdot, \theta_0)$ has a unique minimum θ_0 *a.s.* when and only when the identification condition (b1) or (b2) holds. The continuity of $D(\cdot, \theta_0)$ is due to Lemma A1. This completes the proof.

Proof of THEOREM 3.4: It follows from Assumption WM, condition (b1) or (b2), and Lemma A2 that $g(\theta)'Wg(\theta) > 0$ *a.s.* for any $\theta \in \Theta$.

Let $\delta > 0$ be given, and define $\Theta_0 = \{\|\theta - \theta_0\| \geqslant \delta\} \cap \Theta$. Fix an arbitrary $\bar{\theta} \in \Theta_0$, and let N be the neighborhood of $\bar{\theta}$ given by condition (a). Thus $g(\bar{\theta})'Wg(\bar{\theta}) > 0$ *a.s.* and $g(\theta_0)'Wg(\theta_0) > 0$ *a.s.*. Then we can set $\bar{p} = g(\bar{\theta})'Wg(\bar{\theta})$ and $\bar{q} = g(\theta_0)'Wg(\theta_0)$. For large T, there exists ϵ such that

$$\sup_{\theta \in N} |g_T(\theta)'Wg_T(\theta) - \bar{p}| < \epsilon,$$

since $g_T(\theta) \xrightarrow{a.s.} g(\theta)$ uniformly in $\theta \in \Theta$, and $g(\theta)$ is continuous from Lemma A1. Moreover, we also have $\left|g_T(\theta_0)'Wg_T(\theta_0) - \bar{q}\right| < \epsilon$ for sufficiently large T. Therefore we have

$$\begin{aligned}
&\inf_{\substack{|p'Wp - \bar{p}| < \epsilon \\ |q'Wq - \bar{q}| < \epsilon}} \inf_{\theta \in N} \left[p\kappa(\sqrt{T}, \theta) - q\kappa(\sqrt{T}, \theta_0)\right]' W \left[p\kappa(\sqrt{T}, \theta) - q\kappa(\sqrt{T}, \theta_0)\right] \\
&\leq \left[g_T(\theta)\kappa(\sqrt{T}, \theta) - g_T(\theta_0)\kappa(\sqrt{T}, \theta_0)\right]' W \left[g_T(\theta)\kappa(\sqrt{T}, \theta) - g_T(\theta_0)\kappa(\sqrt{T}, \theta_0)\right] \quad (1.38)
\end{aligned}$$

for large T and any $\theta \in N$.

$\forall \theta \in N$, $D_T(\theta, \theta_0) = A_T(\theta, \theta_0) - 2B_T(\theta, \theta_0)$ where $A_T(\theta, \theta_0)$ and $B_T(\theta, \theta_0)$ are defined in the proof of Theorem 3.3. Let's first look at $A_T(\theta, \theta_0)$, which can be written as

$$\begin{aligned}
&A_T(\theta, \theta_0) \\
&= T \left[g_T(\theta)\kappa(\sqrt{T}, \theta) - g_T(\theta_0)\kappa(\sqrt{T}, \theta_0)\right]' W_T \left[g_T(\theta)\kappa(\sqrt{T}, \theta) - g_T(\theta_0)\kappa(\sqrt{T}, \theta_0)\right]
\end{aligned}$$

Define

$$\begin{aligned}
&A_T^0(\theta,\theta_0)\\
\equiv&\left[\frac{1}{T}\sum_{t=1}^{T} z_t\left(f(x_t,\theta)-f(x_t,\theta_0)\right)\right]' W \left[\frac{1}{T}\sum_{t=1}^{T} z_t\left(f(x_t,\theta)-f(x_t,\theta_0)\right)\right]\\
=&T\left[g_T(\theta)\kappa(\sqrt{T},\theta)-g_T(\theta_0)\kappa(\sqrt{T},\theta_0)\right]' W \left[g_T(\theta)\kappa(\sqrt{T},\theta)-g_T(\theta_0)\kappa(\sqrt{T},\theta_0)\right],
\end{aligned}$$

then from (1.38) and condition (a), we have

$$\begin{aligned}
&\frac{A_T^0(\theta,\theta_0)}{T}\\
=&\left[g_T(\theta)\kappa(\sqrt{T},\theta)-g_T(\theta_0)\kappa(\sqrt{T},\theta_0)\right]' W \left[g_T(\theta)\kappa(\sqrt{T},\theta)-g_T(\theta_0)\kappa(\sqrt{T},\theta_0)\right]\\
\xrightarrow{a.s.}&\infty,
\end{aligned}$$

uniformly in $\theta \in N$.

Since $W > 0$ $a.s.$, it has a unique Cholesky decomposition

$$W = LL',$$

where L is a lower triangular matrix with $a.s.$ strictly positive diagonal entries. Thus the above equation can be written as

$$\frac{A_T^0(\theta,\theta_0)}{T} = \left\| L'\left[g_T(\theta)\kappa(\sqrt{T},\theta)-g_T(\theta_0)\kappa(\sqrt{T},\theta_0)\right]\right\|^2 \xrightarrow{a.s.} \infty,$$

and thus we have

$$\left\| L'\left[g_T(\theta)\kappa(\sqrt{T},\theta)-g_T(\theta_0)\kappa(\sqrt{T},\theta_0)\right]\right\| \xrightarrow{a.s.} \infty.$$

On the other hand, since $W_T \geq 0$, it has a Cholesky decomposition

$$W_T = L_T L_T',$$

where L_T is a lower triangular matrix and is not unique in general. Since $W_T \xrightarrow{p} W$ by Assumption WM, thus, $L_T \xrightarrow{p} L$ as $T \to \infty$ by the continuous mapping theorem. Therefore, for sufficiently large T, the diagonal elements of L_T are all strictly positive with probability approaching to one ($w.p.a.1$), and thus L_T is invertible $w.p.a.1$ as

$T \to \infty$. Then by

$$\begin{aligned}
&\left\| L' \left[g_T(\theta)\kappa(\sqrt{T},\theta) - g_T(\theta_0)\kappa(\sqrt{T},\theta_0) \right] \right\| \\
&= \left\| L' \left(L_T'\right)^{-1} L_T' \left[g_T(\theta)\kappa(\sqrt{T},\theta) - g_T(\theta_0)\kappa(\sqrt{T},\theta_0) \right] \right\| \\
&\leq \left\| L' \left(L_T'\right)^{-1} \right\| \cdot \left\| L_T' \left[g_T(\theta)\kappa(\sqrt{T},\theta) - g_T(\theta_0)\kappa(\sqrt{T},\theta_0) \right] \right\|,
\end{aligned}$$

we have

$$\begin{aligned}
&\left\| L_T' \left[g_T(\theta)\kappa(\sqrt{T},\theta) - g_T(\theta_0)\kappa(\sqrt{T},\theta_0) \right] \right\| \\
&\geq \left\{ \left\| L' \left(L_T'\right)^{-1} \right\| \right\}^{-1} \left\| L' \left[g_T(\theta)\kappa(\sqrt{T},\theta) - g_T(\theta_0)\kappa(\sqrt{T},\theta_0) \right] \right\| \xrightarrow{p} \infty,
\end{aligned}$$

uniformly in $\theta \in N$. Therefore it follows that

$$\frac{A_T(\theta,\theta_0)}{T} \xrightarrow{p} \infty, \tag{1.39}$$

uniformly in $\theta \in N$.

Now let's look at the second term $B_T(\theta,\theta_0)$. Since

$$\frac{1}{T^{\frac{3}{2}}} \sum_{t=1}^{T} z_t \varepsilon_t = o_p(1),$$

and $W_T \xrightarrow{p} W$, then we have

$$\Gamma_T = W_T - W_T \left(\frac{1}{T^{\frac{3}{2}}} \sum_{t=1}^{T} z_t \varepsilon_t \right) \left(\frac{1}{T^{\frac{3}{2}}} \sum_{t=1}^{T} z_t' \varepsilon_t \right) W_T \xrightarrow{p} W.$$

Therefore, in the same manner as in deriving (1.39), we can obtain

$$\begin{aligned}
&A_T(\theta,\theta_0) - \frac{B_T(\theta,\theta_0)^2}{T} \\
&= \left[\frac{1}{T} \sum_{t=1}^{T} z_t \left(f(x_t,\theta) - f(x_t,\theta_0) \right) \right]' \Gamma_T \left[\frac{1}{T} \sum_{t=1}^{T} z_t \left(f(x_t,\theta) - f(x_t,\theta_0) \right) \right] \\
&\xrightarrow{p} \infty.
\end{aligned}$$

Thus we have

$$T \cdot A_T(\theta, \theta_0) - B_T(\theta, \theta_0)^2 \xrightarrow{p} \infty.$$

Therefore, $B_T(\theta, \theta_0)^2 \leq T \cdot A_T(\theta, \theta_0), a.s.$ for large T. Then we can obtain, for large T,

$$A_T(\theta, \theta_0)^{-2} B_T(\theta, \theta_0)^2 \leq A_T(\theta, \theta_0)^{-2} \cdot T \cdot A_T(\theta, \theta_0) = \left[\frac{A_T(\theta, \theta_0)}{T}\right]^{-1} \xrightarrow{p} 0,$$

which implies

$$A_T(\theta, \theta_0)^{-1} \big| B_T(\theta, \theta_0) \big| = o_p(1). \tag{1.40}$$

Now it follows from (1.39) and (1.40) that

$$\begin{aligned} D_T(\theta, \theta_0) &= A_T(\theta, \theta_0) - 2B_T(\theta, \theta_0) \\ &\geq A_T(\theta, \theta_0) \Big[1 - 2A_T(\theta, \theta_0)^{-1} \big| B_T(\theta, \theta_0) \big| \Big] \\ &= A_T(\theta, \theta_0) \left[1 + o_p(1)\right] \xrightarrow{p} \infty, \end{aligned}$$

uniformly in $\theta \in N$. Since Θ_0 is compact and $\bar{\theta}$ was chosen arbitrarily, we may deduce that

$$\inf_{\theta \in \Theta_0} D_T(\theta, \theta_0) \xrightarrow{p} \infty,$$

from which the stated result follows immediately.

Proof of Lemma 4.1: See Theorem 8.1 of Wooldridge (1994).

Proof of Theorem 4.2: Write $\dot{\kappa}_T = \dot{\kappa}(\sqrt{T}, \theta_0)$ to simplify notation. Let $\nu_T = \sqrt{T}\dot{\kappa}_T$. Since $\dot{\kappa}_T$ is nonsingular by definition, we can define $G_T(\theta) = M_T(\theta)\nu_T^{-1\prime}$. It then follows from Lemma 3.1 that $G_T(\theta) \xrightarrow{a.s.} G(\theta)$ uniformly in $\theta \in \Theta$. On the other hand, Assumption EC implies that

$$m_T(\theta_0) = \frac{1}{T} \sum_{t=1}^{T} z_t \varepsilon_t \xrightarrow{d} \int_0^1 B_z(r) dB_\varepsilon(r).$$

Therefore, it directly follows that the conditions in Lemma 4.1(a) and (c) are satisfied with $\ddot{Q}(\theta_0) = G(\theta_0)' W G(\theta_0)$ and

$$\dot{Q}(\theta_0) = G(\theta_0)' W \Big[\int_0^1 B_z(r) dB_\varepsilon(r) \Big].$$

Now It is straightforward to see that under Assumption WM, condition (d) is satisfied if the identification conditions in Assumption H(c1) or H(c2) holds. Note that by Lemma A3, Assumption H(c1) implies H(c2). Therefore we only need to prove that the conditions in Lemma 4.1(b) and (e) hold.

To show Lemma 4.1(b), observe that

$$\begin{aligned}
&\nu_T^{-1}\big(\ddot{Q}_T(\theta_0) - \ddot{Q}_T^0(\theta_0)\big)\nu_T^{-1\prime} \\
&= \nu_T^{-1}\bigg\{-\frac{1}{T}\sum_{t=1}^{T}\Big[(m_T(\theta_0)'W_T)\otimes I_k\Big]\cdot\Big[z_t\otimes\ddot{F}(x_t,\theta_0)\Big]\bigg\}\nu_T^{-1\prime} \\
&= -\frac{1}{T}\sum_{t=1}^{T}\bigg\{\nu_T^{-1}\Big[(m_T(\theta_0)'W_T z_t)\ddot{F}(x_t,\theta_0)\Big]\nu_T^{-1\prime}\bigg\}.
\end{aligned}$$

Let $\bar{s} = \max(s_{max}, -s_{min}) + 1$, where s_{max} and s_{min} are defined in the proof of Lemma A4. Then we have for large T

$$|\ddot{f}(x_t,\theta_0)| \le \sup_{|s|\le\bar{s}}\big|\ddot{f}(\sqrt{T}s,\theta_0)\big|,$$

and hence

$$\begin{aligned}
&\bigg\|vec\Big\{\nu_T^{-1}\big(\ddot{Q}_T(\theta_0) - \ddot{Q}_T^0(\theta_0)\big)\nu_T^{-1\prime}\Big\}\bigg\| \\
&= \bigg\|-\frac{1}{T}\sum_{t=1}^{T}vec\Big\{\nu_T^{-1}\Big[(m_T(\theta_0)'W_T z_t)\ddot{F}(x_t,\theta_0)\Big]\nu_T^{-1\prime}\Big\}\bigg\| \\
&= \bigg\|\frac{1}{T}\sum_{t=1}^{T}(\nu_T\otimes\nu_T)^{-1}\ddot{f}(x_t,\theta_0)\big(m_T(\theta_0)'W_T z_t\big)\bigg\| \\
&\le \frac{1}{\sqrt{T}}\bigg\|\big|(\dot{\kappa}_T\otimes\dot{\kappa}_T)^{-1}\big|\Big(\sup_{|s|\le\bar{s}}\big|\ddot{f}(\sqrt{T}s,\theta_0)\big|\Big)\bigg\|\cdot\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\Big|m_T(\theta_0)'W_T z_t\Big| \\
&\le \frac{1}{\sqrt{T}}\bigg\|\big|(\dot{\kappa}_T\otimes\dot{\kappa}_T)^{-1}\big|\Big(\sup_{|s|\le\bar{s}}\big|\ddot{f}(\sqrt{T}s,\theta_0)\big|\Big)\bigg\|\cdot\big\|m_T(\theta_0)\big\|\cdot\big\|W_T\big\|\cdot\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}\|z_t\|.
\end{aligned}$$

It follows from Assumption H(b) that as $T\to\infty$

$$\frac{1}{\sqrt{T}}\bigg\|\big|(\dot{\kappa}_T\otimes\dot{\kappa}_T)^{-1}\big|\Big(\sup_{|s|\le\bar{s}}\big|\ddot{f}(\sqrt{T}s,\theta_0)\big|\Big)\bigg\|\to 0,$$

due to the fact that $\theta_0 \in N(\tau)$ for any $\tau > 0$. Moreover, we have as $T \to \infty$

$$\frac{1}{T^{\frac{3}{2}}} \sum_{t=1}^{T} \|z_t\| \leq \sum_{i=1}^{m} \Big(\frac{1}{T^{\frac{3}{2}}} \sum_{t=1}^{T} \|z_t^{(i)}\| \Big) \xrightarrow{d} \sum_{i=1}^{m} \int_0^1 \big\|B_z^{(i)}(r)\big\| dr = O_p(1),$$

where $z_t^{(i)}$ and $B_z^{(i)}(r)$ are the i-th elements of vectors z_t and $B_z(r)$ respectively. Meanwhile, we have $m_T(\theta_0) = O_p(1)$ as $T \to \infty$. Then it follows directly from Assumption WM that

$$\Big\| vec\Big\{ \nu_T^{-1} \big(\ddot{Q}_T(\theta_0) - \ddot{Q}_T^0(\theta_0) \big) \nu_T^{-1\prime} \Big\} \Big\| \to 0,$$

as $T \to \infty$, and hence the condition in Lemma 4.1(b) holds.

To show the condition in Lemma 4.1(e), let ε be given by Assumption H(b). Fix σ such that $0 < \sigma < \varepsilon/8$, and define $\mu_T = T^{-\sigma}\nu_T$ so that $\mu_T \nu_T^{-1} \xrightarrow{a.s.} 0$ as required by Lemma 4.1(e). Let Θ_T and $N(\tau)$ be defined as in Lemma 4.1(e) and Assumption H(b) respectively. As shown in PP, if (1.8) in Assumption H(b) holds, then $\Theta_T \subset N(\tau)$ for any $\tau > 0$. We can decompose $\ddot{Q}_T(\theta) - \ddot{Q}_T(\theta_0)$ into five parts:

$$\ddot{Q}_T(\theta) - \ddot{Q}_T(\theta_0) = \ddot{D}_{1T}(\theta) + \ddot{D}_{1T}(\theta)' + \ddot{D}_{2T}(\theta) + \ddot{D}_{3T}(\theta) + \ddot{D}_{4T}(\theta),$$

where

$$\begin{aligned}
\ddot{D}_{1T}(\theta) &= M_T(\theta_0)' W_T \big(M_T(\theta) - M_T(\theta_0) \big), \\
\ddot{D}_{2T}(\theta) &= \big(M_T(\theta) - M_T(\theta_0) \big)' W_T \big(M_T(\theta) - M_T(\theta_0) \big), \\
\ddot{D}_{3T}(\theta) &= -\frac{1}{T} \sum_{t=1}^{T} \ddot{F}(x_t, \theta) \Big(\big(m_T(\theta) - m_T(\theta_0) \big)' W_T z_t \Big), \\
\ddot{D}_{4T}(\theta) &= -\frac{1}{T} \sum_{t=1}^{T} \big(\ddot{F}(x_t, \theta) - \ddot{F}(x_t, \theta_0) \big) (m_T(\theta_0)' W_T z_t).
\end{aligned}$$

After applying the first-order and second-order Taylor expansions to $\dot{f}$ and $\ddot{f}$ respectively, we can obtain that for any $\theta \in \Theta_T$

$$\Big\| \big(M_T(\theta) - M_T(\theta_0) \big) \mu_T^{-1\prime} \Big\| \leq \frac{1}{T} \sum_{t=1}^{T} \Big\| z_t \ddot{f}(x_t, \bar{\theta})' (\mu_T \otimes \mu_T)^{-1\prime} \Big[I_k \otimes \big(\mu_T'(\theta - \theta_0) \big) \Big] \Big\|, \quad (1.41)$$

and

$$\big\|m_T(\theta)-m_T(\theta_0)\big\| \le T^{\sigma}\big\|G_T(\theta_0)\big\| + \frac{1}{2}\sum_{t=1}^{T}\Big\| z_t\Big[\big(\mu_T'(\theta-\theta_0)\big)\otimes\big(\mu_T'(\theta-\theta_0)\big)\Big]'\cdot (\mu_T\otimes\mu_T)^{-1}\ddot{f}(x_t,\bar{\bar{\theta}})\Big\|, \tag{1.42}$$

where $\bar{\theta}$ and $\bar{\bar{\theta}}$ both lie in the line segment connecting θ and θ_0. Since we have for large T

$$\sup_{\theta\in\Theta_T}\big|\ddot{f}(x_t,\theta)\big| \le \sup_{|s|\le\bar{s}}\sup_{\theta\in\Theta_T}\big|\ddot{f}(\sqrt{T}s,\theta)\big|,$$

for all $t=1,\dots,T$. After some algebra using (1.41) and (1.42), we can obtain that for any $\theta\in\Theta_T$

$$\Big\|\mu_T^{-1}\ddot{D}_{1T}(\theta)\mu_T^{-1\prime}\Big\| \le \frac{T^{3\sigma}}{\sqrt{T}}\big\|G_T(\theta)\big\|\cdot\big\|W_T\big\|\cdot\Big\|\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}|z_t|\Big\|\cdot\Big\|\Big|\big(\dot{\kappa}_T\otimes\dot{\kappa}_T\big)^{-1}\Big|\sup_{|s|\le\bar{s}}\sup_{\theta\in\Theta_T}\Big|\ddot{f}(\sqrt{T}s,\theta)\Big|\Big\|, \tag{1.43}$$

$$\Big\|\mu_T^{-1}\ddot{D}_{2T}(\theta)\mu_T^{-1\prime}\Big\| \le \frac{T^{4\sigma}}{T}\big\|W_T\big\|\cdot\Big\|\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}|z_t|\Big\|^2\cdot\Big\|\Big|\big(\dot{\kappa}_T\otimes\dot{\kappa}_T\big)^{-1}\Big|\sup_{|s|\le\bar{s}}\sup_{\theta\in\Theta_T}\Big|\ddot{f}(\sqrt{T}s,\theta)\Big|\Big\|^2, \tag{1.44}$$

$$\Big\|\mu_T^{-1}\ddot{D}_{3T}(\theta)\mu_T^{-1\prime}\Big\| \le \frac{T^{3\sigma}}{\sqrt{T}}\big\|G_T(\theta)\big\|\cdot\big\|W_T\big\|\cdot\Big\|\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}|z_t|\Big\|\cdot\Big\|\big(\dot{\kappa}_T\otimes\dot{\kappa}_T\big)^{-1}\sup_{|s|\le\bar{s}}\sup_{\theta\in\Theta_T}\Big|\ddot{f}(\sqrt{T}s,\theta)\Big|\Big\| \tag{1.45}$$

$$+\frac{T^{4\sigma}}{2T}\big\|W_T\big\|\cdot\Big\|\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}|z_t|\Big\|^2\cdot\Big\|\Big|\big(\dot{\kappa}_T\otimes\dot{\kappa}_T\big)^{-1}\Big|\sup_{|s|\le\bar{s}}\sup_{\theta\in\Theta_T}\Big|\ddot{f}(\sqrt{T}s,\theta)\Big|\Big\|^2, \tag{1.46}$$

$$\Big\|\mu_T^{-1}\ddot{D}_{4T}(\theta)\mu_T^{-1\prime}\Big\| \le \frac{2T^{2\sigma}}{\sqrt{T}}\big\|m_T(\theta_0)\big\|\cdot\big\|W_T\big\|\cdot\Big\|\frac{1}{T^{\frac{3}{2}}}\sum_{t=1}^{T}|z_t|\Big\|\cdot\Big\|\Big|\big(\dot{\kappa}_T\otimes\dot{\kappa}_T\big)^{-1}\Big|\sup_{|s|\le\bar{s}}\sup_{\theta\in\Theta_T}\Big|\ddot{f}(\sqrt{T}s,\theta)\Big|\Big\|, \tag{1.47}$$

from which we may easily deduce that $\|\mu_T^{-1}\ddot{D}_{iT}(\theta)\mu_T^{-1\prime}\| = o_{a.s.}(1)$ for all $i=1,\dots,4$, uniformly in $\theta\in\Theta_T$ due to Assumption H(b). Now the condition in Lemma 4.1(e)

follows immediately from (1.43)-(1.47). This completes the proof.

Proof of THEOREM 5.1: The efficient GMM estimator $\hat{\theta}_T^+$ can be written as

$$\hat{\theta}_T^+ \equiv \underset{\theta\in\Theta}{\operatorname{argmin}}\ m_T^+(\theta)'\, W_T\, m_T^+(\theta), \quad w.p. \longrightarrow 1,$$

where

$$m_T^+(\theta) = \frac{1}{T}\sum_{t=1}^{T}\Big[z_t\left(y_t^+ - f(x_t,\theta)\right)\Big].$$

Since $\hat{\varepsilon}_t^*$ is the first step GMM residual, it is a consistent estimator for the regression error ε_t. It thus follows from Assumption EC(a) that as $T\to\infty$

$$\hat{\sigma}_{l\varepsilon} \xrightarrow{p} \sigma_{l\varepsilon}, \quad \text{and} \quad \hat{\Sigma}_{ll} \xrightarrow{p} \Sigma_{ll}.$$

Then by the definitions of ε_t^+ and B_+, we have as $T\to\infty$

$$\begin{aligned}
m_T^+(\theta_0) &= \frac{1}{T}\sum_{t=1}^{T} z_t\varepsilon_t^+ \\
&= \frac{1}{T}\sum_{t=1}^{T} z_t(\varepsilon_t - \sigma_{l\varepsilon}'\Sigma_{ll}^{-1}\Delta l_t) - \Big(\frac{1}{T}\sum_{t=1}^{T} z_t\Delta l_t'\Big)(\hat{\Sigma}_{ll}^{-1}\hat{\sigma}_{l\varepsilon} - \Sigma_{ll}^{-1}\sigma_{l\varepsilon}) \\
&= \frac{1}{T}\sum_{t=1}^{T} z_t(\varepsilon_t - \sigma_{l\varepsilon}'\Sigma_{ll}^{-1}\Delta l_t) + o_p(1) \\
&\xrightarrow{d} \int_0^1 B_z(r)dB_+(r),
\end{aligned}$$

due to the fact that

$$\frac{1}{T}\sum_{t=1}^{T} z_t\Delta l_t' \xrightarrow{d} \int_0^1 B_z(r)dB_l(r)' = O_p(1).$$

Now since the derivative of $m_T^+(\theta)$ with respect to θ is not affected by the correction, i.e. $\partial m_T^+(\theta)/\partial\theta$ is still equal to $M_T(\theta)$ defined in Section 1.4, then it is straightforward to apply our proof of Theorem 4.2 to the modified problem (1.13) except simply replacing $m_T(\theta)$ and $B_\varepsilon(r)$ with $m_T^+(\theta)$ and $B_+(r)$ respectively. Thus,

due to the fact that $B_+ \stackrel{d}{=} BM(\sigma_+^2)$ and is independent of B_l, we have as $T \to \infty$

$$\sqrt{T}\dot{\kappa}(\sqrt{T}, \theta_0)'(\hat{\theta}_T - \theta_0) \xrightarrow{d} \Big[G(\theta_0)'WG(\theta_0)\Big]^{-1} G(\theta_0)'W\Big[\int_0^1 B_z(r)dB_+(r)\Big]$$
$$\stackrel{d}{=} \mathbf{MN}\big(0,\ V(\theta_0)\big), \tag{1.48}$$

where

$$V(\theta_0) = \sigma_+^2\Big[G(\theta_0)'WG(\theta_0)\Big]^{-1} G(\theta_0)'W\Big[\int_0^1 B_z(r)B_z(r)'dr\Big]WG(\theta_0)\Big[G(\theta_0)'WG(\theta_0)\Big]^{-1}$$

This completes the proof.

Proof of Corollary 5.2: It suffices to show that

$$\mathbf{Pr}\Big(V(W) - V(W^*) \geq 0\Big) = 1, \tag{1.49}$$

where

$$V(W) = \sigma_+^2\Big[G(\theta_0)'WG(\theta_0)\Big]^{-1} G(\theta_0)'W\Big[\int_0^1 B_z(r)B_z(r)'dr\Big]WG(\theta_0)\Big[G(\theta_0)'WG(\theta_0)\Big]^{-1}$$

By the traditional proof of optimal weighting matrix in the stationary GMM estimation (see, for example, Hall, 2005, Section 3.6), one can easily show that $V(W) - V(W^*) \geq 0$ *a.s.*, conditional on $\mathcal{F}_l$ which is the σ-field generated by $\{B_l(r) : 0 \leq r \leq 1\}$. It thus directly follows that

$$\mathbf{Pr}\Big(V(W) - V(W^*) \geq 0\Big) = \mathbf{E}\Big[\mathbf{Pr}\Big(V(W) - V(W^*) \geq 0 \ \Big|\ \mathcal{F}_l\Big)\Big] = 1,$$

which establishes (1.49) and completes the proof.

REFERENCES

Anderson, R.G., and R.H. Rasche, 2001, The Remarkable Stability of Monetary Base Velocity in the United States, 1919-1999, Working paper, Federal Reserve Bank of St. Louis.

Andrews, D.W., and C.J. McDermott, 1995, Nonlinear Econometric Models with Deterministically Trending Variables, *The Review of Economic Studies*, No.3.

Bae, Y., and R.M. De Jong, 2007, Money Demand Function Estimation by Nonlinear Cointegration, *Journal of Applied Econometrics*, Vol.22, pp.767-793.

Bernstein, D.S., 2005, *Matrix Mathematics: Theory, Facts, and Formulas with Application to Linear Systems Theory*, Princeton, N.J.: Princeton University Press.

Billingsley, P., 1968, *Convergence of Probability Measures*, John Wiley & Sons, Inc..

Billingsley, P., 1995, *Probability and Measure*, 3rd Edition, New York: Wiley.

Cajels, E., and M.W. Watson, 1997, Estimating Deterministic Trends in the Presence of Serially Correlated Errors, *The Review of Economics and Statistics*, Vol.79, No.2.

Chang, Y., and J.Y. Park, 2005, Endogeneity in Nonlinear Regressions with Integrated Time Series, Working paper.

Chang, Y., Park, J.Y., and P.C.B. Phillips, 2001, Nonlinear Econometric Models with cointegrated and deterministically trending regressors, *Econometric Journal*, Vol.4.

Christopeit, N., 2009, Weak Convergence of Nonlinear Transformations of Integrated Processes: The Multivariate Case, *Econometric Theory*, Vol.25.

Cooley, T.F., and M. Ogaki, 1996, A Time Series Analysis of Real Wage, Consumption, and Asset Returns, *Journal of Applied Econometrics*, Vol.11, No.2.

Corbae, D., Durlauf, S.N., and B.E. Hansen, 2006, *Econometric Theory and Practice: Frontiers of Analysis and Applied Research*, Cambridge University Press.

Davison, J., 1994, *Advanced Texts in Econometrics: Stochastic Limit Theory*, Oxford University Press.

De Jong, R.M., 2002, Nonlinear Minimization Estimators in the Presence of Cointegrating Relations, *Journal of Econometrics*, Vol.110.

De Jong, R.M., 2004, Addendum to 'Asymptotics for Nonlinear Transformations of Integrated Time Series', *Econometric Theory*, Vol.20.

Debnath, L., and P. Mikusiński, 2005, *Introduction to Hilbert Spaces with Applications*, 3rd Edition.

Doorn, D., 2003, Three Essays on Trend Analysis and Misspecification in Structural Econometric Models, Ph.D. thesis, North Carolina State University.

Eichenbaum, M., 1989, Some Empirical Evidence on the Production Level and Production Cost Smoothing Models for Inventory Investment, *The American Economic Review*, Vol.79, No.4.

Gallant, A.R., and H. White, 1988, *A Unified Theory of Estimation and Inference for Nonlinear Dynamic Models*, Basil Blackwell.

Graham, A., 1981, *Kronecker Products and Matrix Calculus: With Applications*,

Chichester: Horwood; New York: Halsted Press.
Hall, A.R., 2005, *Advanced Texts in Econometrics: Generalized Method of Moments*, Oxford University Press.
Hamilton, J.D., 1994, *Time Series Analysis*, Princeton University Press.
Hansen, L.P., 1982, Large Sample Properties of Generalized Method of Moments Estimators, *Econometrica*, Vol.50, No.4, pp.1029-1054.
Hansen, L.P., and K.J. Singleton, 1982, Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models, *Econometrica*, Vol.50, No.5, pp.1269-1286.
Horn, R.A., and C.R. Johnson, 1991, *Topics in Matrix Analysis*, Cambridge; New York: Cambridge University Press.
Johansen, S., 1988, Statistical Analysis of Cointegration Vectors, *Journal of Economic Dynamics and Control*, Vol.12, pp.231-254.
Karatzas, I., and S.E. Shreve, 1991, *Brownian Motion and Stochastic Calculus*, Springer.
Kitamura, Y., and P.C.B. Phillips, 1995, Efficient IV Estimation in Nonstationary Regression: An Overview and Simulation Study, *Econometric Theory*, Vol.11, No.5.
Kitamura, Y., and P.C.B. Phillips, 1997, Fully Modified IV, GIVE and GMM Estimation with Possibly Non-stationary Regressors and Instruments, *Journal of Econometrics*, Vol.80.
Kline, P., 2008, Understanding Sectorial Labor Market Dynamics: An Equilibrium Analysis of the Oil and Gas Field Services Industry, Working Paper, University of California, Berkeley.
Kuo, H., 2006, *Introduction to Stochastic Integration*, Springer.
Loève, M., 1963, *Probability Theory*, 3rd Edition, New York: Van Nostrand.
Maddala, G.S., Rao, C.R., and H.D. Vinod, 1993, *Handbook of Statistics: Volume 11, Econometrics*, North-Holland.
Matyas, L., 1999, *Generalized Method of Moments Estimation*, Cambridge University Press.
Moon, H.R., Schorfheide, F., 2002, Minimum Distance Estimation of Nonstationary Time Series Models, *Econometric Theory*, Vol.18, No.6.
Newey, W.K., 1991, Uniform Convergence in Probability and Stochastic Equicontinuity, *Econometrica*, Vol.59, No.4.
Newey, W.K., and D.L. McFadden, 1994, Large Sample Estimation and Hypothesis Testing, Chapter 36 in *Handbook of Econometrics*, Vol.IV, Edited by R.F. Engle and D.L. McFadden.
Park, J.Y., 1992, Canonical Cointegrating Regressions, *Econometrica*, Vol.60, pp.119-143.
Park, J.Y., and P.C.B. Phillips, 1988, Statistical Inference in Regressions with Integrated Processes: Part 1, *Econometric Theory*, Vol.4, No.3.

Park, J.Y., and P.C.B. Phillips, 1989, Statistical Inference in Regressions with Integrated Processes: Part 2, *Econometric Theory*, Vol.5, No.1.
Park, J.Y., and P.C.B. Phillips, 1999, Asymptotics for Nonlinear Transformations of Integrated Time Series, *Econometric Theory*, Vol.15, No.3.
Park, J.Y., and P.C.B. Phillips, 2001, Nonlinear Regressions with Integrated Time Series, *Econometrica*, Vol.69, No.1.
Perron, P., and T. Yabu, 2009, Estimating Deterministic Trends with an Integrated or Stationary Noise Component, *Journal of Econometrics*, Vol.151.
Phillips, P.C.B., 1987, Time Series Regression with a Unit Root, *Econometrica*, Vol.55, No.2.
Phillips, P.C.B., 1988, Weak Convergence to the Matrix Stochastic Integral $\int_0^1 BdB'$, *Journal of Multivariate Analysis*, Vol.24.
Phillips, P.C.B., 1989, Partially Identified Econometric Models, *Econometric Theory*, Vol.5, No.2, pp.181-240.
Phillips, P.C.B., 1991, Optimal Inference in Cointegrated Systems, *Econometrica*, Vol.59, No.2.
Phillips, P.C.B., 1995, Fully Modified Least Squares and Vector Autoregression, *Econometrica*, Vol.63, No.5.
Phillips, P.C.B., 1998, New Tools for Understanding Spurious Regressions, *Econometrica*, Vol.66, No.6, pp.1299-1325.
Phillips, P.C.B., and S.N. Durlauf, 1986, Multiple Time Series Regression with Integrated Processes, *The Review of Economic Studies*, Vol.53, No.4.
Phillips, P.C.B., and B.E. Hansen, 1990, Statistical Inference in Instrumental Variables Regression with $I(1)$ Processes, *The Review of Economic Studies*, Vol.57, No.1.
Phillips, P.C.B., and M. Loretan, 1991, Estimating the Long-Run Economic Equilibria, *The Review of Economic Studies*, Vol.58, No.3.
Phillips, P.C.B., and J.Y. Park, 1988, Asymptotic Equivalence of Ordinary Least Squares and Generalized Least Squares in Regressions with Integrated Regressors, *Journal of the American Statistical Association*, Vol.83, No.401.
Phillips, P.C.B., and V. Solo, 1992, Asymptotics for Linear Processes, *The Annals of Statistics*, Vol.20, No.2.
Pollard, D., 1984, *Convergence of Stochastic Processes*, Springer-Verlag.
Potscher, B.M., and I.R. Prucha, 1997, *Dynamic Nonlinear Econometric Models: Asymptotic Theory*, Springer.
Pullman, N.J., 1976, *Matrix Theory and Its Applications: Selected Topics*, New York: M. Dekker.
Quintos, C.E., 1998, Analysis of Cointegration Vectors Using the GMM Approach, *Journal of Econometrics*, Vol.85.
Reed, A.J., Levedahl, J.W., and C. Hallahan, 2005, The Generalized Composite Commodity Theorem and Food Demand Estimation, *American Journal of Agricultural*

Economics, Vol.87, No.1, pp.28-37.
Ross, S.M., and E.A. Pekoz, 2007, *A Second Course in Probability*, ProbabilityBookstore.com.
Saikkonen, P., 1991, Asymptotically Efficient Estimation of Cointegration Regressions, *Econometric Theory*, Vol.7, No.1.
Seber, G.A.F., 2008, *A Matrix Handbook for Statisticians*, Hoboken, N.J.: Wiley-Interscience.
Spitzer, J.J., 1976, The Demand for Money, the Liquidity Trap, and Functional Form, *International Economic Review*, Vol.17, No.1.
Stock, J.H., and M.W. Watson, 1993, A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems, *Econometrica*, Vol.61, pp.783-820.
Stock, J.H., Wright, J.H., and M. Yogo, 2002, A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments, *Journal of Business and Economic Statistics*, Vol.20, No.4.
Takács, L., 1993, On the Distribution of the Integral of the Absolute Value of the Brownian Motion, *The Annals of Applied Probability*, Vol.3, No.1, pp.186-197.
Vogelsang, T.J., 1998, Trend Function Hypothesis Testing in the Presence of Serial Correlation, *Econometrica*, Vol.66, No.1.
White, H., 1984, *Asymptotic Theory for Econometricians*, Academic Press.
Wu, C., 1981, Asymptotic Theory of Nonlinear Least Squares Estimation, *The Annals of Statistics*, Vol.9, No.3. Wooldridge, J.M., 1994, Estimation and Inference for Dependent Processes, Chapter 45 in *Handbook of Econometrics*, Vol.IV, Edited by R.F. Engle and D.L. McFadden.
Zinde-Walsh, V., and P.C.B. Phillips, 2003, Fractional Brownian Motion as a Differentiable Generalized Gaussian Process, Cowles Foundation Discussion Paper No.1391, Cowles Foundation for Research in Economics, Yale University.

Chapter 2

The Transmission of External Credit Shocks to Emerging Asia: A Two-Country Structural VAR Approach

2.1 INTRODUCTION

The global financial crisis of 2007-2009 that originated in U.S. credit markets rapidly spread across borders and led to recessions in most countries. The global reach and depth of the crisis are without precedent in the post-World War II period. This triggered an intensive discussion about the importance of credit shocks for economic fluctuations, and their transmission across countries. This paper uses a novel shock identification strategy in the context of two-country/block structural vector autoregressive (SVAR) models to study the transmission of the shocks originating in the U.S. or euro-area credit markets to economic activity in emerging Asia. Important regional differences are found between China and the other emerging Asian countries. In particular, domestic credit policy plays a pivotal role in offsetting adverse external credit shocks in China, suggesting policy options for other countries.

Many empirical studies have provided evidence regarding the linkages between economic fluctuations and disturbances in credit markets. For example, Bernanke and Gertler (1989) and Borio, Furfine, and Lowe (2001) examine the pro-cyclical nature of credit cycles and economic fluctuations, albeit mostly for single country cases. Ortiz (2008) estimates a dynamic stochastic general equilibrium (DSGE) model with credit market imperfections and finds that credit market shocks and monetary policy shocks are both important factors behind economic fluctuations in the United States. Bordo and Haubrich (2010) analyze cycles in money, credit and output between 1875 and 2007 in the United States, and show that episodes of financial stress exacerbate cyclical downturns.

Recently, a large body of literature proposes VAR models to analyze credit shocks, and finds that they play a significant role in explaining the economic fluctuations in developed countries, especially during financial crises. For example, Meeks (2009) concentrates on the U.S. and documents that credit shocks play an important role during financial crises, but that they have a lesser role during more tranquil periods. Gilchrist, Yankov and Zakrajšek (2009) and Gilchrist and Zakrajšek (2010) report that credit market spreads had a significant impact on business cycles in the U.S. during the period 1990-2008. Helbling *et al.* (2010) provide a global perspective of credit shocks originating in the U.S. and find that these shocks have a significant impact on the evolution of global activity, proxied by the real GDP growth in G-7 countries.

This paper is closely related to Helbling *et al.* (2010). However, there are several significant differences. First, our novel shock identification strategy enables us to identify not only credit shocks and aggregate supply shocks,[1] but a full set of foreign/external and domestic shocks (aggregate supply, aggregate demand, monetary

[1]Helbling *et al.* (2010) only identify these two shocks.

policy, and credit shocks)[2]. The identification of additional shocks should also help to identify the former two shocks. Second, their paper answers the question whether global credit shocks and U.S. credit shocks affect global economic fluctuations, while this paper tries to not only qualify but also quantify the effects of external credit shocks on domestic real economies. Third, their SVAR is a closed economy framework, while the general two-country structural framework developed in this paper enables us to distinguish external credit shocks from their domestic counterparts, and thus captures how domestic credit responds to external credit shocks. As we will see in our main results, the response of domestic credit actually plays an important role in the transmission of external credit shocks because domestic credit policy easing can offset the adverse effects of external credit tightening. We use the term credit policy easing to denote directly increasing the quantity of credit through buying private sector assets including residential mortgage-backed securities, or loosening in non-interest-rate monetary instruments such as required reserve ratio and window guidance. Finally, instead of looking at the effects of U.S. credit shocks on economic fluctuations in G-7 countries, this paper examines the transmission of shocks originating in the U.S. or euro-area credit markets[3] to emerging market economies in Asia (emerging Asia).[4] It is worth mentioning that our framework allows for different transmission channels through which external credit shocks might affect domestic economic fluctuations. For instance, one channel is credit contagion through the financial linkages between external and domestic countries, or, more specifically, through capital flows in global financial markets.[5] Another possible channel is through trade linkages, or the decrease in foreign demand due to external credit tightening. The results of this paper, however, emphasize the importance of the credit contagion channel, and suggest that the adverse effects can be offset by domestic credit policy easing.

It should also be noted that the credit shock considered in this paper is distinguished from the monetary policy shock especially the monetary policy-induced credit supply shock. We follow the terminology in Meeks (2009) and Helbling *et al.* (2010), and define the credit shock to be a credit supply contraction as opposed to an en-

[2]In this context, foreign/external shocks are those originating in the U.S. or euro area, and domestic shocks are those originating in emerging Asia. We split emerging Asia into two groups: China and other emerging Asian countries.

[3]An example is credit tightening caused by deleveraging in euro-area banks due to the ongoing euro-zone debt crisis.

[4]We focus on emerging Asia because, as argued in Kose and Prasad (2010), emerging Asian countries, especially China, have weathered the global recession initiated by U.S. credit shocks better than the advanced economies and most other emerging market economies, in terms of economic growth. Therefore, it is important in terms of policy implications to examine both the transmission of external credit shocks and how emerging Asia responds to these shocks.

[5]This credit contagion channel can be very important in propagating monetary policy decisions in one country to the real economy in another country if the contagion is fast and long-lasting.

dogenous decline in credit due to lenders reducing credit in response to expectations of an increase in future default rates and/or a decline in future productivity. Thus, a contractionary monetary policy shock implies an increase in short-term interest rates, however, an adverse credit shock would lead to a decrease in interest rates owing to quantitative monetary easing in response to declines in economic activity following the unexpected tightening in credit markets, as identified and discussed in most DSGE and empirical literature, such as Ortiz (2008), Meeks (2009), and Helbling *et al.* (2010).

We classify emerging Asia into two groups. One is China, and the other group consists of the remaining countries (denoted as 'EMAS-9'[6]). There are three reasons why we separate China from the other emerging Asian countries instead of taking the entire region as one single economy. First, as argued in Zhang (2011) and Du (2010), unlike the other emerging Asian countries, the quantity of credit in China has been directly controlled by the government (or controlled through a heterodox combination of the non-interest-rate monetary instruments[7]) rather than indirectly managed through interest rate policies. As a result, China has more capacity to undertake direct domestic credit loosening. Second, Kose and Prasad (2010) find that China displays more resilience to global recessions than the other emerging Asian economies. Third, as Figure 2.1 shows, quarter-over-quarter real GDP and credit growth is different in China than in the other emerging Asian countries. In particular, during the 1997-1998 Asian financial crisis and the latest 2007-2009 global financial crisis, real credit growth is much higher in China than in EMAS-9, and real GDP was not affected as much in China as in EMAS-9.

Against this background, this paper uses a structural two-country/block framework to study the transmission of external credit shocks to domestic real economies in emerging Asia, with particular attention to the responses of domestic real GDP and credit growth.[8] Specifically, we focus on the following questions:

- How large and persistent are the effects of adverse external credit shocks on the growth of GDP and credit in emerging Asia?
- Do these shocks affect the real economy in China differently than they affect

[6]Nine other emerging Asian countries (EMAS-9) are considered in this paper; they include the main emerging Asian countries. See Section 2.4 for a detailed description of these nine countries.

[7]These monetary instruments include, for instance, required reserve ratio and window guidance, which can affect the volume of credit more directly than interest rate policies. Western central banks rarely use the required reserve ratio because it would cause immediate liquidity problems for banks with low excess reserves, but Chinese central bank frequently uses these monetary instruments. For example, the Chinese required reserve ratio was raised 10 times in 2007.

[8]There are two reasons why we also pay particular attention to domestic credit in emerging Asia. First, we need to ensure that the identified external credit shock is not commingled with a domestic credit shock. Second, in terms of policy implications, it is important to examine the response of domestic credit to external credit shocks.

other emerging Asian countries? If so, how important are China's domestic credit policies in mitigating the effects of adverse external credit shocks on GDP growth?

- Do external credit shocks play a significant role in driving GDP and credit growth in emerging Asia?

The main conclusions of this paper are as follows:

- First, external credit shocks have significantly negative and protracted effects on credit growth in emerging Asia, suggesting the importance of the credit contagion channel in the transmission of external credit shocks to emerging Asia.

- Second, significant differences are found between China and other emerging Asian countries in the short and medium run. In particular, the external credit shocks affect credit growth in China and other emerging Asian countries in opposite directions. The shocks lead to a two-quarter increase in credit growth in China, in contrast with a protracted decrease in credit growth in the other emerging Asian countries, implying that China has more capacity to stimulate its economy by credit policy easing when facing external credit tightening. As a result, GDP growth in China is not significantly affected, but GDP growth in the other Asian countries falls significantly.[9] This finding implies that other economies could effectively mitigate the adverse effects of external credit shocks by quantitatively easing credit.

- Third, although domestic shocks are dominant, external credit shocks play a non-negligible role in driving economic fluctuations in emerging Asia. The role of external credit shocks relative to domestic shocks is smaller in China than in the other emerging Asian countries. This is consistent with the previous finding that the effects of adverse external credit shocks are mitigated by credit policy easing in China but not in the other emerging Asian countries.

Structural econometric models are used to generate these conclusions. Specifically, the empirical methodology is based on two novel two-country/block SVAR models with (i) China (or EMAS-9) taken as the domestic block, and (ii) the United States and the euro area (henceforth, USEUR) taken as the external or foreign block.[10] These are the baseline USEUR-CHN model and the baseline USEUR-EMAS-9 model. We consider these two models in order to make comparisons between China and other

[9]Policymakers should be cautious about implementation of credit policy easing. This issue will be discussed in Section 2.5.

[10]We use credit shocks originating in the U.S. and/or the euro area to capture and proxy external credit shocks.

emerging Asian countries. Either China or EMAS-9 should not be modeled as a small open economy for several reasons: China and some EMAS-9 countries (such as India and Indonesia) are populous, have made large contributions to global growth (particularly in 2009-2010), have a large and growing impact on global commodity price fluctuations, and include economies that are global financial centers. In this context, a two-country/block setup which allows feedback across blocks seems warranted. The variables in the VARs include quarterly real GDP, CPI, short-term interest rates, and bank credit to private sectors for each block.[11]

Within the two-country structural VAR framework, this paper also contributes to the literature by proposing a novel identification strategy, where shocks within each block are identified using sign restrictions, and shocks across the two blocks are identified using a recursive structure (block Cholesky decomposition). This identification, which we call block Cholesky-sign restriction identification, is warranted by the hypothesis that shocks from the foreign USEUR block can affect the China (or EMAS-9) block contemporaneously, whereas propagation of shocks in the other direction occurs with a one quarter lag.[12]

The rest of the paper is organized as follows. Section 2.2 provides a literature review on sign restrictions which serves as the econometric background for our model. Section 2.3 discusses the data, and Section 2.4 introduces our baseline structural model, discusses the methodological contribution of this paper, and sketches the scheme of estimation and inference. Section 2.5 discusses the empirical results, and Section 2.6 checks their robustness. Section 2.7 concludes the paper and suggests policy implications.

2.2 THE THEORY OF SIGN RESTRICTIONS

Helbling *et al.* (2010) and others have used VAR models to better comprehend credit shocks. However, all of these papers employ one-country VAR models and cannot distinguish an external credit shock either from its domestic counterpart or from the other external structural shocks, such as a monetary policy shock. In this section, we start with an overview of two-country VAR models and their shock identification problems, and then briefly discuss the intuition of our identification strategy and its contribution to the literature.

[11]We also include exchange rates to assess the robustness of our results in Section 2.6.

[12]This hypothesis is motivated by the fact that the U.S. and European central banks conduct monetary policy relatively free of international considerations, and is verified by a test procedure proposed by Canova (2005); see Section 2.3 for a detailed discussion.

2.2.1 Two-Country VAR Models

Before continuing, it should be emphasized that neither China nor EMAS-9 can be modeled as a small open economy. As argued above, a two-country (block) setup that allows feedback across blocks seems warranted in this context. Since we are interested in the responses of the Chinese (or EMAS-9) real economy to external credit shocks, the conceptual framework is a global economy with two blocks, namely, China (or EMAS-9) and the rest of the world (initially assumed to be adequately summarized by the United States and the euro area, or USEUR), whereby shocks originating in each of the two blocks can be transmitted to each other.

Many previous studies have used structural VARs to model two-country setups. Clarida and Gali (1994) popularized a two-country framework whereby each variable include in the VAR is measured in relative terms. Farrant and Peersman (2006) present a modern incarnation of this idea using sign restrictions. A shortcoming of such an approach is that estimation using relative variables implies that the propagation mechanism is the same for shocks originating in both blocks. Moreover, using relative variables does not provide any information about the relevance of shocks for the level variables, such as real GDP and credit. Although Peersman (2011) elaborates on Mountford (2005) and Farrant and Peersman (2006) and models a two-country VAR in level variables using symmetric and asymmetric shocks, his approach is inappropriate for our context, for reasons discussed below.

2.2.2 Shock Identification Strategies

The identification strategy used in structural VARs can have an important influence on the results. Clarida and Gali (1994) use a set of long-run zero restrictions. The latter, however, are often criticized in the literature. For instance, Faust and Leeper (1997) show that substantial distortions might arise due to small sample biases and measurement errors when zero restrictions in the long run are used. In addition, some equilibrium growth models or models with hysteresis effects allow for permanent real effects of nominal shocks such as monetary policy shocks. To avoid these issues, we take the sign restriction approach as formalized and pioneered by Faust (1998), Uhlig (2005), and Canova and De Nicolo (2002). Peersman (2005) shows that the impulse responses obtained with zero restrictions can be situated in the tails of the distributions of all possible impulse responses that are produced by sign restrictions. However, there are two identification problems we need to solve before our analysis.

The first problem involves the sign restriction approach. In a review of the sign restrictions literature, Fry and Pagan (2010) emphasize a critical issue which they call the *multiple models problem*. Specifically, most of the papers using sign restrictions use the median to summarize the impulse responses satisfying the imposed sign

restrictions. The risk, however, is clear from the example they give, namely that reporting medians would be similar to presenting the responses to technology shocks from a real business cycle model, and the responses to monetary shocks from a monetary model. This implies that the identified structural shocks may be correlated, thereby making any inferences based on impulse responses and variance decompositions economically meaningless. To solve this multiple models problem, Fry and Pagan (2010) propose a median target (MT) method by presenting the impulse responses generated from a single set of orthogonal shocks that are as close to the medians as possible, instead of the medians themselves.

The second problem is about shock identification in two-country SVAR models. Identification is an issue for any two-country SVAR model in level variables because it is difficult to distinguish domestic shocks from their external counterparts. Peersman (2011) proposes a two-country SVAR in level variables, and uses symmetric and asymmetric shocks to achieve identification. One issue with this setup is that a purely common shock, such as a hike in global commodity prices, may have opposite effects depending on whether a country is a net commodity importer or exporter. Second, because there are a few important net exporters of key global commodities (for instance, Indonesia and Malaysia) within emerging Asia, this would make the interpretation of symmetric versus asymmetric shocks even more difficult. In addition, because there are too many sign restrictions (45 in total), the shocks have to be identified one by one,[13] and therefore the results are subject to the multiple models problem as mentioned above.

In the next section, we propose a different identification strategy from previous studies, which enables us to build two-country VAR models in level instead of relative variables, and identify all shocks simultaneously. As mentioned above, two models will be considered. The first is the USEUR-CHN model, and consists of the (PPP-GDP-weighted average of the) United States and the euro area as the external or foreign country/block, and China as the domestic country/block. In the second model, the USEUR-EMAS-9 model, the external block is the same as in the USEUR-CHN model, and the domestic block is constructed as the PPP-GDP-weighted averages of all the countries in EMAS-9.

2.3 THE MODEL

This paper develops an intuitive shock identification strategy that we call the block Cholesky-sign restriction identification within the context of two-country VAR models. Specifically, within each block, sign restrictions are employed; across blocks,

[13]In practice, it is usually infeasible to identify all the shocks at the same time in two-country SVAR models with level variables because of a large number of sign restrictions generated by the doubled number of structural shocks.

however, a recursive causal structure (block Cholesky) is assumed. Shock identification is jointly imposed, but nonetheless is computationally less expensive than some other recent approaches using sign restrictions to identify shocks in two-country SVARs. As discussed below, a lighter computational burden can be critical in that it facilitates quantitative results that are economically meaningful. Before describing the identification strategy, we start with our baseline VAR specification.

2.3.1 Baseline Empirical Specification

Because we fit the same specification to both the USEUR-CHN model and the USEUR-EMAS-9 model, we present the baseline specification only of the former. The baseline two-country structural VAR model for USEUR-CHN incorporates a parsimonious set of macroeconomic variables needed to investigate Chinese credit dynamics, namely, the quarterly logarithmic growth rates of real GDP and real credit, CPI inflation, and the level of short-term interest rates for both blocks, as shown in the SVAR(p) model below:

$$A_0 Z_t = c + \sum_{i=1}^{p} A_i Z_{t-i} + \varepsilon_t, \tag{2.1}$$

where c is a (8×1) vector of constants, A_i is a (8×8) matrix of autoregressive coefficients, A_0 is a (8×8) structural coefficient matrix, and ε_t is a (8×1) vector of structural disturbances or shocks which are uncorrelated and normalized to have unit variances, i.e. $\mathbf{E}(\varepsilon_t \varepsilon_t') = I_8$. The endogenous variables, Z_t, that we include in the baseline specification are

$$Z_t = \begin{pmatrix} \Delta \log(Real\ GDP_t^{USEUR}) \\ \Delta \log(CPI_t^{USEUR}) \\ i_t^{USEUR} \\ \Delta \log(Real\ Credit_t^{USEUR}) \\ \Delta \log(Real\ GDP_t^{CHN}) \\ \Delta \log(CPI_t^{CHN}) \\ i_t^{CHN} \\ \Delta \log(Real\ Credit_t^{CHN}) \end{pmatrix}.$$

It should be noted that domestic real credit growth is also included as an endogenous variable in order to ensure that the identified external credit shock is not commingled with its domestic counterpart.

The reduced form of model (2.1) can be written as

$$Z_t = Bc + \sum_{i=1}^{p} BA_i Z_{t-i} + B\varepsilon_t, \tag{2.2}$$

where $B = A_0^{-1}$ is the impulse matrix, which is a matrix version of the impulse vector defined in Uhlig (2005), and $B\varepsilon_t$ is the reduced-form residuals. Lag length p is determined by the standard likelihood ratio tests and the Akaike information criterion as is usual in the sign restriction literature, and turns out to be one in both USEUR-CHN and USEUR-EMAS-9 models.

2.3.2 Conventional Sign-Restriction Identification

Identification of the baseline model (2.1) boils down to the identification of matrix A_0, or equivalently the impulse matrix B, which, without further restrictions, is unidentified. For example, the Cholesky decomposition identification assumes that B is a lower triangular matrix. We identify all eight structural shocks, namely, aggregate supply, aggregate demand, monetary policy, and credit shocks for each of the two blocks USEUR and CHN.

Let Ω be the estimated variance-covariance matrix of the residuals $B\varepsilon_t$ in the reduced-form model (2.2), and F be the lower triangular in the Cholesky decomposition for Ω, i.e. $FF' = \Omega$, then because ε_t is assumed to be uncorrelated and have unit variances, we have $BB' = \Omega$. Using the observation that any two decompositions $\Omega = AA'$ and $\Omega = \tilde{A}\tilde{A}'$ have to satisfy the condition that

$$\tilde{A} = AQ,$$

for some orthonormal matrix Q, i.e. $QQ' = Q^2 = I$,[14] we can conclude that the structural matrix B in (2.2) satisfies $B = FQ_1$ for some orthonormal matrix Q_1.

On the other hand, suppose Q_2 is an arbitrary orthonormal matrix with the same dimension as the matrix B, then we can rewrite model (2.2) as:

$$Z_t = Bc + \sum_{i=1}^{p} BA_i Z_{t-i} + FQ_0'\tilde{\varepsilon}_t, \tag{2.3}$$

where $Q_0 = Q_2 Q_1'$ is also orthonormal, and the variance of the new shocks $\tilde{\varepsilon}_t = Q_2\varepsilon_t$ is $\mathbf{Var}(\varepsilon_t) = \mathbf{Var}(Q_2\varepsilon_t) = Q_2 \cdot I \cdot Q_2' = I$. In other words, we construct a new set of uncorrelated shocks with unit variances. In general, the orthonormal matrix Q_0 affects both the contemporaneous effects of shocks on variables and the standard

[14] Interested readers may refer to Uhlig (2005) for more details.

deviations of shocks. Thus the impulse responses generated by the new set of shocks $\tilde{\varepsilon}_t$ change.

2.3.3 The Block Cholesky-Sign Restriction Identification

The proposed identification strategy is summarized in Table 2.1, with the details as follows:

- *Sign restrictions are used to identify shocks within each block.* As discussed earlier, these methods were popularized by Faust (1998), Uhlig (2005), Canova and De Nicolo (2002), and Peersman (2005).[15] Consider the foreign USEUR block comprising four equations determining the dynamics of real GDP growth, CPI inflation, the level of the short-term interest rate, and real credit growth. Underpinned by a four-equation New Keynesian framework along the lines of Woodford (2003) and Helbling *et al.* (2010), aggregate supply, aggregate demand, monetary policy, and credit shocks[16] are identified by the first three equations. Specifically, while an aggregate demand shock implies a positive co-movement of output and prices, the opposite is true for the aggregate supply shock, in line with the standard textbook aggregate supply-aggregate demand model. The reaction of short-term interest rates can be used to differentiate an aggregate demand shock from a monetary policy shock. While the short-term interest rate decreases in the face of an adverse aggregate demand shock, it increases in response to contractionary monetary policies implemented to prevent macroeconomic overheating (see, for example, Peersman, 2005). It should also be noted that, under the baseline specification, the last variable, real credit growth, is assumed to be pro-cyclical. For credit shocks, we follow the findings in Meeks (2009), Helbling *et al.* (2010), and Zhang (2011), and assume that an adverse credit shock[17] leads to a decrease in real credit growth, leaving output, prices, and short-term interest rates non-increasing.[18] The non-increasing response of short-term interest rates is consistent with a systematic easing of

[15] Other notable studies using sign restrictions include Peersman (2011), Canova (2005), Farrant and Peersman (2006), and those listed in Fry and Pagan (2007, 2010). Bjørnland and Halvorsen (2008) also combine sign and short-term (zero) restrictions, which we discuss in further detail, along with important differences in methodologies.

[16] Note that the credit shock is identified in a different way from the other three shocks. This is motivated by the innovation proposed by Peersman (2005) to distinguish oil price shocks from other aggregate supply shocks, and is discussed in detail in Section 2.3.

[17] As argued in Helbling *et al.* (2010), our adverse credit shock reflects a credit supply contraction, as opposed to an endogenous decline in credit due to lenders reducing credit in response to expectations of an increase in future default rates and/or a decline in future productivity.

[18] For example, Zhang (2011) finds that positive (domestic) credit shocks make external funds more available to firms, which helps relax their borrowing constraints, stimulating investment and output.

monetary policy in reaction to the unexpected tightening in credit markets.[19] These restrictions are also motivated by the DSGE models with financial frictions such as Gilchrist *et al.* (2009). Therefore, the credit shock behaves very much like an aggregate demand shock, and cannot be disentangled from the aggregate demand shock using only the first three variables. To achieve identification, we utilize the innovation introduced by Peersman (2005)[20], and assume that the credit shock would affect real credit growth more than would an aggregate demand shock.

- *A recursive structure (block Cholesky decomposition) is imposed to identify shocks across the two blocks.* Recursive identification was popularized by Sims (1980). In the baseline specification, it is assumed that shocks from the foreign USEUR block can affect the China (or EMAS-9) block contemporaneously, whereas propagation of shocks in the other direction occurs with a one quarter lag. This hypothesis is motivated by the fact that the U.S. or European central banks conduct monetary policy relatively free of international considerations. To verify this hypothesis, I employ a test procedure proposed in Canova (2005): Run a VAR for each of the two country pairs, namely, USEUR-CHN and USEUR-EMAS-9, and then examine the exogeneity of the current China (or EMAS-9) variables with respect to the USEUR block. Confirming a priori expectations, the null hypothesis that current values of China (or EMAS-9) variables have zero coefficients in the USEUR block is not rejected for either pair.

Overall, using sign restrictions within blocks, and the recursive structure (block Cholesky decomposition) across blocks, implies that emerging Asia is not only affected by external shocks, but can in turn influence the USEUR block. Our results are not influenced by the ordering of variables within each block.

Addressing the Multiple Models Problem

In a review of the sign restrictions literature, Fry and Pagan (2010) emphasize a critical issue which they call the multiple models problem. Specifically, most literature uses the median (and/or various percentiles) to summarize the impulse responses satisfying the imposed sign restrictions. The risk has been discussed above: identified structural shocks may be correlated, thereby distorting any inferences gleaned via impulse responses and variance decompositions.

To solve this multiple models problem, Fry and Pagan (2010) propose a median

[19]Most DSGE models with the financial accelerator mechanism, as well as empirical literature using VAR models, produce this result; see, for example, Ortiz (2008), Meeks (2009), and Helbling *et al.* (2010).

[20]Peersman (2005) proposed this innovation to distinguish oil price shocks from the other aggregate supply shocks.

target (MT) method by presenting the impulse responses generated from a single set of orthogonal shocks which are as close to the medians as possible, instead of the medians themselves. We follow their advice and present the impulse responses and variance decompositions using the MT method (hereafter called MT impulse responses and MT variance decompositions) along with the 16th and 84th percentiles error bands, as is usual in the literature.

Does a computationally less expensive method matter?

One contribution of our proposed identification scheme is that it is computationally less expensive than other identification schemes using sign restrictions. As illustrated in an alternative approach in Section 2.6 to assess the robustness of our results, the computational efficiency is very important for generating economically meaningful results. The new set of sign restrictions we impose in the alternative approach are shown in Table 2.2.[21] However, it is computationally very expensive: numerous attempts (measured in days) yield about 1 accepted draw out of over 10 million.[22] Our alternative is a parsimonious nine-variable two-country/block system with sign restrictions imposed only in the first period. As can be seen, even small-to-medium-scale VAR identified using sign restrictions can quickly become computationally overbearing. Furthermore, the small number of accepted draws raises the question of whether using the Fry-Pagan MT method makes sense in this situation. This highlights why the computational gains from our proposed identification scheme (which yields runs in about an hour or two) is potentially so important. Combining sign restrictions with a recursive structure (block Cholesky decomposition) also facilitates the utilization of the Fry-Pagan MT method needed for meaningful impulse response functions and variance decompositions.

To further underscore the potential importance of computations savings under our proposed identification strategy, consider Peersman (2011). This recent paper develops a two-country SVAR with 7 variables and 7 structural shocks identified using 45 sign restrictions. We conjecture from the paper that there were very few valid draws when all the sign restrictions were imposed jointly, and this is why the shocks are identified one by one in the paper. In other words, if the impulse responses to an individual shock are consistent with the imposed sign restrictions for a shock, then the result for the specific shock is accepted (in contrast to Peersman, 2005). This is critical, because to apply the Fry and Pagan (2010) MT method, we need to find a single set of orthogonal shocks. To do so, the shocks need to be identified simultaneously, which in the case of Peersman (2011) seems infeasible. This would imply that the structural shocks used for impulse response functions and variance decom-

[21] This alternative identification strategy will be specified in details in Section 2.6.

[22] Therefore, we have to follow Peersman (2011) and identify all the nine shocks one by one; see Section 2.6 for a detailed discussion.

positions would likely be correlated, and therefore such quantitative findings might not be economically meaningful. The shortcoming in the example above highlights the value of the computational savings as it facilitates the use of the MT method, thereby allowing for economically meaningful quantitative inferences.

Two Other Noteworthy Points

- First, during the final drafting stages of this paper, we read in detail Bjørnland and Halvorsen (2008), who also combine sign restrictions and short-term (zero) restrictions. In the broadest terms, we both innovate by proposing that sign restrictions be combined with a recursive ordering. We can differentiate our paper in several ways: (i) We use a two-country setup that could be seen as a more natural case when sign restrictions are combined with a recursive structure; also, our recursive ordering is across countries, which is why we use the term block Cholesky. (ii) Their paper restricts some of the contemporaneous impulse responses to zero instead of positive or negative by restricting the rotation matrices,[23] and is only applicable to their specific question, while our methodology is more general and can be applied to any pair of countries. (iii) The Fry-Pagan MT method underpins our quantitative results, and, in this regard, we emphasize the computational savings gained by combining the block Cholesky decomposition with the popular sign restrictions that facilitate the MT method needed for meaningful quantitative results.

- Second, like Peersman (2005) and Farrant and Peersman (2006), this paper initially identifies all the structural shocks in the systems, including the credit shock that is emphasized in Helbling *et al.* (2010). It should also be noted that exchange rate dynamics are partially captured by the interest rate differential between domestic and international short-term interest rates, as shown in Clarida and Gali (1994). Also, when discussing robustness, we include real exchange rates and show that a pure sign restriction approach developed from Bjørnland and Halvorsen (2008) reinforces our main results.

2.3.4 Technical Details of the Block Cholesky-Sign Restriction Identification

Here, in line with one of the main contributions of this paper, we develop the following block Cholesky-sign restriction method. Consider a general two-country

[23] A similar method is proposed as an alternative identification strategy after we include real exchange rates in the model, and is discussed in Section 2.6.

Structural VAR(p) model

$$\begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \varepsilon_t^X \\ \varepsilon_t^Y \end{pmatrix}, \quad t = 1, \ , 2, \ , \ldots, \ , T, \tag{2.4}$$

where $A_{ij}(L) = A_{ij}^{(0)} + A_{ij}^{(1)}L + A_{ij}^{(2)}L^2 + \ldots + A_{ij}^{(p)}L^p$ with $i, j \in 1, 2$. $A_ij^{(k)}$' s are the structural coefficient matrices, and L is the lag operator. X_t is an $m \times 1$ vector and denotes the block for the variables of the more 'exogenous' country (which will be called country X afterwards). Y_t is an $n \times 1$ vector and denotes the block for the variables of the other country (which will be called country Y). ε_t^X and ε_t^Y are the structural shocks with dimensions $m \times 1$ and $n \times 1$ respectively.

Thus the baseline model (2.1) can be written as model (2.4) with

$$X_t = \begin{pmatrix} \Delta \log(Real\ GDP_t^{USEUR}) \\ \Delta \log(CPI_t^{USEUR}) \\ i_t^{USEUR} \\ \Delta \log(Real\ Credit_t^{USEUR}) \end{pmatrix}, \quad Y_t = \begin{pmatrix} \Delta \log(Real\ GDP_t^{CHN}) \\ \Delta \log(CPI_t^{CHN}) \\ i_t^{CHN} \\ \Delta \log(Real\ Credit_t^{CHN}) \end{pmatrix},$$

$$\varepsilon_t^X = \begin{pmatrix} \varepsilon_t^{AS,USEUR} \\ \varepsilon_t^{AD,USEUR} \\ \varepsilon_t^{MP,USEUR} \\ \varepsilon_t^{C,USEUR} \end{pmatrix}, \quad \varepsilon_t^Y = \begin{pmatrix} \varepsilon_t^{AS,CHN} \\ \varepsilon_t^{AD,CHN} \\ \varepsilon_t^{MP,CHN} \\ \varepsilon_t^{C,CHN} \end{pmatrix},$$

where $\varepsilon_t^{C,USEUR}$ and $\varepsilon_t^{C,CHN}$ are the credit shocks originating in USEUR and CHN respectively.

Define

$$B_{ij}(L) = -\left[A_{ij}^{(1)} + A_{ij}^{(2)}L + \ldots + A_{ij}^{(p)}L^{p-1}\right], \quad i, j \in \{1, 2\},$$

then $A_{ij}(L) = A_{ij}^{(0)} - B_{ij}(L)L$, and model (2.1) can be written as

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{bmatrix} B_{11}(L) & B_{12}(L) \\ B_{21}(L) & B_{22}(L) \end{bmatrix} \begin{pmatrix} X_{t-1} \\ Y_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^X \\ \varepsilon_t^Y \end{pmatrix}, \quad t = 1, 2, \ldots, T, \tag{2.5}$$

where, for notational simplicity, we denote $H_{ij} = A_{ij}^{(0)}$ with $i, j \in \{1, 2\}$.

Before proceeding, it would be useful to highlight three important assumptions:

1. The structural shocks $(\varepsilon_t^{X\prime}, \varepsilon_t^{Y\prime})'$ follows i.i.d. $(0, I_{m+n})$;

2. The structural coefficient matrices H_{11} and H_{22} are invertible;

3. The structural coefficient matrix $H_{12} = 0$.

The first assumption is standard in the SVAR literature, and normalizes the uncorrelated structural shocks so that they all have unit variances. The third assumption is exactly the assumption of the recursive (block) Cholesky decomposition. The first two assumptions imply that the entire structural coefficient matrix

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

is invertible, and that there is no contemporaneous effect of ε_t^Y on X_t. The latter, along with sign restrictions in country X, gives us the identification of the corresponding structural shocks across the two countries.[24]

The reduced-form VAR (2.2) can be written as

$$\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{bmatrix} H_{11} & 0 \\ H_{21} & H_{22} \end{bmatrix}^{-1} \begin{bmatrix} B_{11}(L) & B_{12}(L) \\ B_{21}(L) & B_{22}(L) \end{bmatrix} \begin{pmatrix} X_{t-1} \\ Y_{t-1} \end{pmatrix} + \begin{bmatrix} H_{11} & 0 \\ H_{21} & H_{22} \end{bmatrix}^{-1} \begin{pmatrix} \varepsilon_t^X \\ \varepsilon_t^Y \end{pmatrix}, \quad (2.6)$$

for $t = 1, 2, \ldots, T$. Then by the formula of the inverse of partitioned matrix, the reduced-form VAR can be written as

$$\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{bmatrix} H_{11}^{-1} & 0 \\ -H_{22}^{-1} H_{21} H_{11}^{-1} & H_{22}^{-1} \end{bmatrix} \begin{bmatrix} B_{11}(L) & B_{12}(L) \\ B_{21}(L) & B_{22}(L) \end{bmatrix} \begin{pmatrix} X_{t-1} \\ Y_{t-1} \end{pmatrix} + \begin{pmatrix} e_t^X \\ e_t^Y \end{pmatrix}, \quad (2.7)$$

for $t = 1, 2, \ldots, T$, where $(e_t^{X\prime}, e_t^{Y\prime})'$ is the reduced-form residuals:

$$\begin{pmatrix} e_t^X \\ e_t^Y \end{pmatrix} = \begin{bmatrix} H_{11}^{-1} & 0 \\ -H_{22}^{-1} H_{21} H_{11}^{-1} & H_{22}^{-1} \end{bmatrix} \begin{pmatrix} \varepsilon_t^X \\ \varepsilon_t^Y \end{pmatrix}, \quad t = 1, 2, \ldots, T.$$

Because we can estimate the reduced-form VAR and obtain the estimated variance-covariance matrix for residuals, denoted by Σ which can be partitioned conformably with $(\varepsilon_t^{X\prime}, \varepsilon_t^{Y\prime})'$ as

$$\Sigma = \begin{bmatrix} \Sigma_X & Cov \\ Cov' & \Sigma_Y \end{bmatrix},$$

then, by assumption 1, we can obtain the following three formulae:

$$\begin{cases} H_{11}^{-1} H_{11}^{-1\prime} = \Sigma_X, \\ -H_{22}^{-1} H_{21} = Cov' \Sigma_X^{-1}, \\ H_{22}^{-1} H_2 2^{-1\prime} = \Sigma_Y - Cov' \Sigma_X^{-1} Cov. \end{cases} \quad (2.8)$$

[24] It may be useful to indicate that Canova (2005), for example, assumed that the entire polynomial $A_{12}(L) = 0$ - implying a small-open economy setup - which would not be a reasonable assumption in our two-country SVAR setup containing the USEUR and CHN (or EMAS-9) blocks.

Denote the impulse response function of vector z_t to a one standard deviation shock in the structural disturbance or residual vector v_t^j by a series of matrices, $IRF_z^{v,j}(i)$, where $i = 0, 1, 2, \ldots$ is the time horizon (in quarters), $z, j \in \{X, Y\}$, and $v \in \{\varepsilon, e\}$. For example, the (k, l)-th element in the matrix $IRF_X^{\varepsilon,X}(i)$ is the impulse response of the k-th variable in X_t to a one standard deviation shock in the l-th structural disturbance in ε_t^X after i quarters. Notice that the impulse response functions of z_t to residual shock e_t^j, namely, $IRF_X^{e,X}(i)$,$IRF_Y^{e,X}(i)$,$IRF_X^{e,Y}(i)$ and $IRF_Y^{e,Y}(i)$, can be calculated by the Wold decomposition theorem using the estimated reduced-form VAR coefficient matrices.

Now our objective is to find expressions for $IRF_X^{\varepsilon,X}(i)$, $IRF_X^{\varepsilon,Y}(i)$, $IRF_Y^{\varepsilon,X}(i)$, and $IRF_Y^{\varepsilon,Y}(i)$ for each i based on the reduced-form VAR estimates. According to the Wold decomposition theorem, we have

$$\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \sum_{i=0}^{\infty} \begin{bmatrix} IRF_X^{e,X}(i) & IRF_X^{e,Y}(i) \\ IRF_Y^{e,X}(i) & IRF_Y^{e,Y}(i) \end{bmatrix} \begin{pmatrix} e_{t-i}^X \\ e_{t-i}^Y \end{pmatrix}, \quad t = 1, 2, \ldots T. \tag{2.9}$$

It follows from assumption 3 that $IRF_X^{e,Y}(0) = 0$. Using the relationship between structural shocks and residuals (2.8), we can rewrite (2.9) as

$$\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \sum_{i=0}^{\infty} \begin{bmatrix} \left(IRF_X^{e,X}(i) + IRF_X^{e,Y}(i) \cdot Cov' \cdot \Sigma_X^{-1}\right) H_{11}^{-1} & IRF_X^{e,Y}(i) \cdot H_{22}^{-1} \\ \left(IRF_Y^{e,X}(i) + IRF_Y^{e,Y}(i) \cdot Cov' \cdot \Sigma_X^{-1}\right) H_{11}^{-1} & IRF_Y^{e,Y}(i) \cdot H_{22}^{-1} \end{bmatrix} \begin{pmatrix} \varepsilon_{t-i}^X \\ \varepsilon_{t-i}^Y \end{pmatrix}.$$

The Cholesky decomposition for Σ_X and $(\Sigma_Y - Cov' \cdot \Sigma_X^{-1} \cdot Cov)$ can be written as

$$\begin{cases} \Sigma_X = F_X \cdot F_X', \\ \Sigma_Y - Cov' \cdot \Sigma_X^{-1} \cdot Cov = F_Y \cdot F_Y', \end{cases}$$

where F_X and F_Y are both lower triangular matrices. Then by (2.9), and using the fact that any two decompositions $\Sigma_X = AA'$ and $\Sigma_X = \tilde{A}\tilde{A}'$ must satisfy that $\tilde{A} = AQ$ for some orthonormal matrix Q, we have

$$\begin{cases} H_{11}^{-1} = F_X \cdot Q_X, \\ H_{22}^{-1} = F_Y \cdot Q_Y, \end{cases} \tag{2.10}$$

where Q_X and Q_Y are some orthonormal matrices which can be generated by either the Givens matrices or the Householder transformations.[25] Now we can obtain the

[25]Interested reader may refer to Fry and Pagan (2007) for the details about how to generate orthonormal matrices.

expression for impulse response functions of all variables to structural shocks by

$$\begin{cases} IRF_X^{\varepsilon,X}(i) = \left[IRF_X^{e,X}(i) + IRF_X^{e,Y}(i) \cdot Cov' \cdot \Sigma_X^{-1}\right] F_X Q_X, \\ IRF_Y^{\varepsilon,X}(i) = \left[IRF_Y^{e,X}(i) + IRF_Y^{e,Y}(i) \cdot Cov' \cdot \Sigma_X^{-1}\right] F_X Q_X, \\ IRF_X^{\varepsilon,Y}(i) = IRF_X^{e,Y}(i) \cdot F_Y Q_Y, \\ IRF_Y^{\varepsilon,Y}(i) = IRF_Y^{e,Y}(i) \cdot F_Y Q_Y, \end{cases} \tag{2.11}$$

for all $i = 0, 1, 2, \ldots$.

Then we keep the Q_X's and Q_Y's such that the associated impulse response functions satisfy all the sign restrictions that we impose on the variables of both countries. Note that no restrictions are imposed on the responses of variables in one block/country to the shocks in the other,[26] which are determined by the data. For example, in the baseline USEUR-CHN model (2.1), the sign restrictions we impose are shown in Table 2.1. Because we employ two different rotations, the number of sign restrictions that each rotation needs to satisfy is cut by half. This is why our proposed identification strategy is computationally less expensive than alternative identification schemes using sign restrictions in many other two-country SVARs.

2.3.5 Estimation and Inference

We use a Bayesian approach following Uhlig (2005), Farrant and Peerman (2006), and Peersman (2011) to estimate model (2.2), the reduced-form VAR. The prior and posteriors belong to the Normal-Wishart family. Because there are infinite admissible decompositions for each draw from the posterior when using sign restrictions, we use the following procedure.

1. Take a draw from the posterior for the usual unrestricted Normal-Wishart posterior for the VAR parameters.

2. For each draw in step 1, we take a draw from the standard multivariate normal distribution and use the Householder transformations[27] to generate the rotation matrices for H_{11}^{-1} and H_{22}^{-1} according to (2.10).

3. Compute the associated impulse response functions for all variables according to (2.11).

[26] The only cross-block/country sign restrictions are the block Cholesky restrictions that contemporaneous responses of USEUR variables to CHN (or EMAS-9) shocks are zero.

[27] We use the Householder transformations instead of Givens matrices because Rubio-Ramirez *et al.* (2005) show that, as the size of the VAR grows, the Householder approach is computationally more efficient than the Givens approach.

4. If the impulse response functions satisfy all imposed sign restrictions in Table 2.1, then the results for that draw are accepted. Otherwise, the draw is rejected, and go back to step 2.

5. Repeat steps 2 - 4 one thousand times.

6. Repeat steps 1 - 5 one thousand times, and this gives us one million candidate draws in total.[28]

7. Compute the Fry-Pagan MT impulse response functions based on all the accepted draws.

We present only the results with contemporaneous sign restrictions for all the shocks, but they are invariant to different horizons over which the sign restrictions are imposed. We conduct sensitivity exercises to check the robustness of our results to horizon assumptions, different sample periods, and alternative identification restrictions. All of our main results are robust to these variations.

2.4 DATA

The database includes quarterly real GDP, CPI, short-term interest rates, credit, and exchange rates for all of the countries. The full sample is a quarterly dataset spanning the period 1989:Q1-2010:Q4[29] and includes the following countries: The foreign block in both models (USEUR-CHN model and USEUR-EMAS-9 model) consists of the (PPP-weighted average of the) U.S. and euro area (USEUR); the domestic block in the USEUR-CHN model is China; the domestic block in the USEUR-EMAS-9 model contains Hong Kong SAR, Korea, Singapore, Taiwan Province of China, Indonesia, Malaysia, the Philippines, Thailand, and India. As with the foreign block (USEUR), individual EMAS-9 time series are combined using PPP-GDP weights to form the EMAS-9 aggregates in the USEUR-EMAS-9 model. The average PPP-GDP weights for these nine countries during the sample period are 5%, 16%, 3%, 10%, 13%, 5%, 6%, 7%, and 36% respectively.

Quarterly logarithmic growth rates of real GDP, CPI, and real credit, along with the level of short-term interest rates, are used in the baseline structural VARs.[30] First-order differencing is standard, but following Clarida and Gali (1994), Peersman (2005), and others, the level of interest rates are used. Real credit is defined as the stock of credit scaled by CPI. The credit series are from the IMF's International Financial Statistics (IFS) database (IFS line 22d and line 42d when a sufficiently long

[28] The overall acceptance rate is around 1 out of 500 candidate draws.

[29] A subsample (1998Q1 - 2010Q4) is also used in Section 2.6.

[30] Exchange rates are included in the extended models in Section 2.6, but not in the baseline models since their effects should be partially captured by interest rate differentials.

time series was available). This corresponds to the aggregate claims on the private sector by deposit money banks and is standard in other studies (see, for example, Elekdag and Wu, 2011, or Mendoza and Terrones, 2008). The rest of the series are standard and were compiled from the IMF's World Economic Outlook (WEO), IFS databases and Haver Analytics.

2.5 RESULTS

This section presents the main findings of the paper, starting with a discussion of impulse response functions of domestic variables to USEUR shocks, with particular emphasis on adverse credit shocks. As shown below, and reinforced through robustness checks, an adverse USEUR credit shock has a significantly positive effect on Chinese real credit on impact (but no significant effect in the medium run). By contrast, there is a significantly negative effect on EMAS-9 real credit in the medium run (but no significant effect on impact). In addition, Chinese real GDP is not significantly affected by the adverse shock, while EMAS-9 real GDP has a significant decrease. These findings confirm that external credit shocks have been influential in driving domestic credit dynamics and economic activity in emerging Asia. They also are consistent with the fact that the quantity of credit in China, unlike EMAS-9, has been directly controlled by the government; hence, it is a policy instrument, as pointed out in Dickinson and Liu (2008) and Zhang (2011). The subsequent forecast error variance decompositions show that domestic factors seem to be more influential than external factors in driving real credit growth in both China and EMAS-9. Therefore, domestic credit policy could play a pivotal role in offsetting adverse effects of external credit shocks.

2.5.1 Impulse Response Functions

We use the impulse response analysis to provide qualitative and quantitative answers to the main questions in this paper. We also use this analysis to evaluate the effects of adverse domestic credit shocks on the economies of the U.S. and euro area; this secondary analysis identifies consistent responses to shocks in non-Asian, non-emerging economies. The impulse response functions of all endogenous variables to one standard deviation external shocks are shown in Figures 2.2(a)-2.3(d), including the Fry-Pagan MT (solid lines), the ordinary median (dashed lines), and the 16th and 84th percentiles (dotted lines) as is usual in literature. These are the results from the baseline SVAR models, with contemporaneously imposed sign restrictions. For instance, the graphs at the lower left and lower right corners in Figure 2.3(d) show that a one standard deviation adverse USEUR credit shock results in an increase of about 0.28 (0.17) percentage points in Chinese real credit growth rate on impact by

the MT method (by the median), in contrast to a decrease of 0.15 percentage points in EMAS-9 real credit growth rate by the MT method (by the median).

Are we identifying the same USEUR shocks in the two baseline models?

Because we have two baseline models, the USEUR-CHN model and the USEUR-EMAS-9 model, the identified USEUR shocks must be the same across the two baseline models so that we can make comparisons. The dynamic reactions of USEUR variables to their own aggregate supply (AS), aggregate demand (AD), monetary policy (MP), and credit shocks are shown in Figures 2.2(a)-(d) for both models.

There are several points worth noticing in these graphs. First, the shocks affect USEUR variables on impact are by construction, owing to the imposed sign restrictions; however, the identification scheme does not restrict the strength or duration of the impact. Second, these impulse responses are consistent with the effects of structural shocks in most DSGE models. The graphs can be summarized as follows. A negative AS shock such as a cost-pushing shock leads to a (significant) fall in real GDP growth for 6 quarters in the USEUR-CHN model and 5 quarters in the USEUR-EMAS-9 model; such a shock leads to 6-quarter rises in both inflation and interest rates on impact and a fall in real credit growth in both models. A negative AD shock leads to falls in all macro variables including real credit growth in both models; in particular, there is a protracted effect on domestic short-term interest rates, which implies a hump-shaped inflation reaction. A contractionary MP shock leads to falls in real GDP growth and inflation, and to a rise in interest rates and a fall in real credit growth on impact in both models. An adverse credit shock behaves qualitatively as an AD shock; quantitatively, however, the negative response of real credit growth to an adverse credit shock is larger and lasts longer than the response to a negative AD shock in both models. Third, both the MT and median impulse responses are qualitatively the same across the two baseline models; in particular, the median impulse responses are quantitatively very similar. These results imply that the identified external (USEUR) shocks are very similar across the two baseline models. The identified adverse credit shocks originating in USEUR also have significantly negative and protracted effects (5 quarters) on real credit growth within the USEUR economies.

We now focus on the impact of USEUR shocks on domestic[31] variables with particular attention to the adverse credit shock, given its relevance in terms of global liquidity shocks and Chinese credit policy, and its importance in global recessions as discussed in Helbling *et al.* (2010).

[31]Domestic country refers to China in the USEUR-CHN model and to EMAS-9 in the USEUR-EMAS-9 model.

Domestic Impulse Responses to USEUR Shocks

The impulse response functions of domestic variables to all USEUR shocks are shown in Figures 2.3(a)-(d). Recall that, as emphasized by Fry and Pagan (2010), the percentiles - including the median - convey the distribution across models and therefore have nothing to do with sampling uncertainty.

We can make the following observations from these graphs. A USEUR adverse AS shock has: (i) a (significantly) positive effect on inflation on impact in both China and EMAS-9, (ii) a negative effect on Chinese interest rates for 4 quarters, and, in particular, (iii) a positive lag effect on Chinese real credit in the medium run but no significant effect on EMAS-9 real credit growth. A USEUR adverse AD shock leads to: (i) a fall in the real GDP growth of China on impact and a fall in that of EMAS-9 for two quarters, (ii) a protracted fall in domestic short-term interest rates, which implies a hump-shaped inflation reaction in both CHN and EMAS-9, and (iii) a two-quarter increase in the real credit growth of China, in contrast with a 4-quarter decrease in that of EMAS-9. A USEUR contractionary MP shock leads to: (i) falls in both Chinese and EMAS-9 real GDP growth and inflation on impact, (ii) an increase in EMAS-9 interest rates as opposed to a fall in Chinese interest rates on impact, and (iii) a one-quarter-lagged increase in Chinese real credit, but no significant response of EMAS-9 real credit growth. The lagged increase in Chinese real credit might be due to the contemporaneous decrease in interest rates. Finally, a USEUR adverse credit shock leads to: (i) a fall in EMAS-9 real GDP growth for three quarters but no significant response of Chinese real GDP growth, (ii) a fall in EMAS-9 interest rates in the long run but no significant change in Chinese interest rates, and (iii) a significant two-quarter rise in the real credit growth of China on impact in contrast with a significant three-quarter fall in that of EMAS-9 in the medium run.

Unlike the case of contractionary MP shocks, Chinese interest rates are not significantly affected on impact by adverse credit shocks. Therefore, the real credit increase must be due to something else, possibly credit policy easing: making use of non-interest-rate monetary instruments to directly increase the quantity of bank credit. This finding emphasizes the importance of the credit contagion channel in the transmission of external credit shocks to both China and EMAS-9, despite their different responses. As noted below, it is also consistent with the fact that the Chinese central bank frequently uses monetary instruments. The finding is invariant to the use of a subsample and to alternative specifications and identification strategies.

What role for domestic credit policy?

As discussed in the introduction, credit policy easing refers to directly increasing the quantity of bank credit through buying private sector assets, including residential mortgage-backed securities, or loosening in non-interest-rate monetary instruments, such as required reserve ratio and window guidance. Credit policies are distinguished

from conventional interest rate policies. While credit policies could increase the quantity of credit and stimulate investment more directly, they might also cause immediate liquidity problem for banks with low excess reserves, and might cause a moral hazard problem, namely, risk-seeking behavior and rising non-performing loans of banks.

China uses credit policy easing instead of interest rate policies in response to adverse external credit shocks, as opposed to EMAS-9, whose real credit growth declines in the medium run. Although western central banks rarely alter the required reserve ratio because of the liquidity problem, this method is frequently used by the Chinese central bank to fight inflation, due to the large amount of savings held by the four big state-owned banks in China.

The use of these 'quantity policies' instead of interest rate policies to control bank credit in China has been reported in other literature. For example, Dickinson and Liu (2005) argue that an alternative interest rate measure, the central bank lending rate, was marginally effective in controlling bank credit in the late 1990s. More recently, Zhang (2011) finds that interest rate spread does not significantly co-move with real credit in China, and argues that the quantity of credit has been directly controlled by the government (or controlled by adjusting the required reserve ratio) rather than indirectly managed through interest rate policies.

The impulse responses to adverse external credit shocks highlight the effects of credit policy easing in China. Chinese interest rates do not decline on impact; its real credit increases in response to the shock rather than decreasing, as in EMAS-9. This is consistent with the fact that the quantity of credit is directly controlled by the government in China but not in EMAS-9, where governments usually use interest rate policies to manage credit. Further, the adverse external credit shock does not lead to a fall in the real GDP growth of China, in contrast with a significant slump in that of the other emerging Asian countries. Because interest rates do not decline significantly in China, this result implies that credit policy rather than interest rate policy is playing a role in stimulating investment and output. As a result, Chinese real GDP growth is not significantly affected, while EMAS-9 real GDP growth falls for three periods. Because interest rates do not respond significantly on impact in either China or EMAS-9, it seems that credit policy easing is the only difference between China and EMAS-9; this might be the reason that Chinese real GDP is not significantly affected. It is also worth noticing that adverse external credit shocks have a 3-quarter significantly negative effect on EMAS-9, implying that the transmission of credit shocks to other countries is fast and not short-lived.

This analysis sheds light on why Chinese real GDP growth did not decline as much as EMAS-9, as shown in Figure 2.1. One possible reason is that, while the Chinese government relaxed domestic credit constraints, EMAS-9 did not or was not able to do so. The evidence is that real credit growth increased rapidly in China but decreased dramatically in EMAS-9 during that period in Figure 2.1. This

credit-stimulus argument is further confirmed by either using a subsample or fitting alternative specifications and identification strategies; this will be discussed in detail later.

Credit policy easing could play a significant role which interest rate policy cannot play during a global credit crisis, especially for emerging market countries. The emerging market countries have attracted abundant foreign investment in the form of either footloose portfolio investment or Foreign Direct Investment (FDI). When an emerging market country is facing an adverse credit shock originating in the U.S. or the euro area, there might be a 'sudden stop' of foreign investment in the country, as discussed in Calvo *et al.* (2004), or even large capital outflows due to the global tightening, leaving the country deep in recession. In this case, interest rate policies do not seem to work well. The reason is the following. If the central bank tries to stimulate investment by lowering short-term interest rates, then there would be larger capital outflows (except in the U.S., which is considered a harbor for international capital) despite capital controls, which might exacerbate the global tightening crisis. However, if the central bank could ease its credit policy to directly relax its credit constraint and expand credit supply, then these policies would make up for the decreased foreign capital and stimulate investment, leaving the country less affected.[32] A recent example of easing domestic credit would be the $1.25 trillion purchase of mortgage-backed securities (MBS) made by the U.S. Federal Reserve in order to support the sagging mortgage market and keep the economy from plunging into depression. Similarly, when the economy is overheating, the government or central bank could restrict the quantity of credit supplied and thus discourage investment.

As is clear from our analysis, credit policy easing is effective in terms of offsetting the adverse effects of external credit tightening. However, policymakers should be cautious about implementing credit policy easing. Directly increasing the volume of credit is a double-edged sword, and might be associated with the moral hazard problem. For instance, Chinese banks have long been criticized because of their large amount of non-performing loans. Therefore, it would also be essential for central banks to help banks improve their risk evaluation and management. Nevertheless, during global credit crises, credit policy easing, if properly implemented, might be effective to avoid a deep recession for some countries.

Domestic Impulse Responses to Domestic Shocks

The impulse responses of EMAS domestic variables to EMAS domestic shocks resemble those of USEUR variables to their own domestic shocks. For instance, the impulse responses to EMAS domestic credit shock shown in Figure 2.3(e) are very

[32]This might not be feasible for some countries when their domestic banks are also experiencing the 'painful' process of deleveraging. However, it is feasible for China, where the four big state-owned banks have a huge amount of savings, and are strictly regulated by the government.

similar to those in Figure 2.2(d). As in the USEUR, adverse shocks originating in emerging Asian credit markets have significantly negative and lasting effects on domestic real GDP and credit growth (3 quarters for GDP, and 4 quarters for credit) in both China and EMAS-9. This suggests that countercyclical credit policies are implemented in China only as a response to external credit shocks, not internal ones. The adverse domestic credit shock has a significantly negative effect on real GDP growth in both China and EMAS-9, and a protracted effect on domestic short-term interest rates, which implies a hump-shaped inflation reaction.

2.5.2 Forecast Error Variance Decompositions

Are external credit shocks important drivers of GDP and credit growth in China and EMAS-9? To provide a quantitative answer, we use the (forecast error) variance decomposition of real GDP and credit growth generated from the two baseline models. The MT and median variance decompositions for real GDP growth and real credit growth are presented in Tables 2.3 and 2.4 respectively. The medians are reported in parentheses. As argued before, medians do not provide much information due to the multiple models problem; therefore, we only focus on the MT variance decompositions.[33]

We find that domestic factors are dominant but that external credit shocks play a small role in China, and a non-negligible role in EMAS-9, in explaining the variation of real GDP and credit growth. In China, domestic shocks in total explain 86 (74) percent of the variation of real GDP (credit) growth in the long run, much more important than external shocks. In EMAS-9, domestic shocks explain 84 (76) percent of the variation in real GDP (credit) growth. In particular, domestic AS and AD shocks explain the most of real GDP variation in China, while domestic AS and MP shocks explain most of the real GDP variation in EMAS-9. For real credit, domestic AS and MP shocks explain the most (except domestic credit shock) in both China and EMAS-9. It is worth noticing that external credit shocks are relatively more important for the real GDP growth of EMAS-9 (6 percent) than for that of China (only 2 percent). However, they explain much more of the variations in the real GDP growth in both China and EMAS-9 when a subsample (1998Q1-2010Q4) is used in Section 2.6. These findings confirm a non-negligible role played by external credit shocks in explaining GDP growth in EMAS-9, and are consistent with the impulse response results above.

As a summary of the main results, we find that:

- Adverse credit shocks have significantly negative and protracted effects on domestic real credit growth, no matter whether they originate in USEUR or

[33] In fact, as shown in the two tables, the median variance decompositions are more or less the same as those using the Fry-Pagan MT method.

emerging Asia.

- External credit shocks affect the credit growth in China and the other emerging Asian countries in opposite directions. Specifically, these shocks lead to a two-quarter increase in the real credit growth of China in contrast with a protracted decrease in that of the other emerging Asian countries, implying that China stimulates its economy through credit policy easing immediately after adverse external credit shocks while EMAS-9 countries do not (or cannot due to domestic liquidity tightening). Thus, the real GDP growth of China is not significantly affected by external credit tightening, but that of the other emerging Asian countries slumps significantly. Therefore, credit policy easing might be relatively effective during a global credit crisis.

- Domestic factors are more dominant than external factors in driving real GDP and credit growth in both China and EMAS-9. However, external credit shocks do play a non-negligible role in explaining GDP growth in EMAS-9.

2.6 ROBUSTNESS

One natural question is whether the above results change over time. There were several important policy shifts in the Chinese credit market in the mid-1990s. Jiang and Zeng (2008) find that credit has started to play a significant role in monetary policy transmission in China since 1998. To assess whether our results are sensitive to the time frame, we apply the same baseline specification and identification strategy to a subsample from 1998Q1 to 2010Q4 in Subsection 2.6.1. Another question is whether these findings are robust to alternative specifications and identification strategies. In Subsection 2.6.2, we consider an alternative specification with exchange rates where a new shock identification strategy is needed.

2.6.1 Subsample (1998Q1 - 2010Q4)

Structural changes are often considered in the empirical literature on SVAR. We check whether and how our results vary over time by studying a subsample period, 1998Q1-2010Q4, during which credit remained relatively stable and important in the monetary policy transmission in China (Jiang and Zeng, 2008). We follow the same specification and identification scheme as before. The results are largely consistent with those from the full sample.

The impulse responses of foreign variables to their own shocks are shown in Figures 2.4(a)-(d). We find that, as in the full sample, the identified USEUR shocks are also very similar across USEUR-CHN and USEUR-EMAS-9 models. However, one difference is worth noticing. The AD shock now leads to real credit tightening

two quarters later and lasts for three quarters, as opposed to in the full sample case where the tightening effect is only on impact, which might imply that Chinese credit policy became more counteractive after 1998.

The impulse responses of domestic variables to external shocks and their own credit shocks are shown in Figures 2.5(a)-(e). We can see that the first two results from the full sample are not affected at all. Chinese real credit increases for two quarters to offset the adverse effect of an external credit shock, and interest rates are not significantly affected. EMAS-9 real credit falls one quarter later and lasts for three quarters. Moreover, real GDP growth is not significantly affected in China, but slumps significantly on impact in EMAS-9. Therefore, our first three results from the full sample are supported by the subsample as well.

The variance decomposition results are also largely consistent with those in the full sample. Tables 2.5 and 2.6 present the variance decompositions of real GDP and credit growth. It is worth noticing that the last result from the full sample is not only unaffected, but also reinforced because the external credit shocks now play a non-negligible role in China. Specifically, the external credit shocks in the long run explain 8 percent of the variation in Chinese real GDP and 11 percent of that in EMAS-9. These are very similar to the findings by Helbling *et al.* (2010),[34] confirming the important role that the disturbances from the U.S. or euro area credit markets play in explaining the economic activity of emerging Asia. This finding might also suggest that China is becoming more and more involved in global financial markets. Moreover, the external credit shocks explain more of the variation of Chinese real credit growth in the subsample (15 percent) than in the full sample (2 percent). This is another signal that China is more involved in global financial markets, and suggests that credit policy easing might be a more frequently used policy tool of the Chinese central bank to offset the adverse effects of external credit tightening.

In summary, all our main results presented in last section still hold in the subsample period of 1998Q1-2010Q4. In particular, the external credit shocks play an important role in explaining the real GDP growth in all emerging Asian countries, although the role is smaller in China.

2.6.2 An Alternative Approach: Extended Two-country Structural VAR

One concern of our baseline specification might be the absence of exchange rates. If the fluctuations in exchange rates are mainly driven by shocks other than the shock originating in the foreign exchange market itself, then exchange rate dynamics can be well captured by the interest rate differential between domestic and international

[34]Helbling *et al.* (2010) find that the U.S. credit shocks account for 11 percent of the variation in global GDP.

short-term interest rates; this is shown in Clarida and Gali (1994). Many empirical studies have tried to analyze the relative importance of different shocks in explaining exchange rate fluctuations. However, they disagree in their results.[35] Therefore, it is important to check whether our results differ when exchange rates are included. As suggested by Clarida and Gali (1994), Farrant and Peersman (2006), and Peersman (2011), we use the real exchange rates.

Thus, the endogenous vector, Z_t, in the USEUR-CHN model becomes

$$Z_t = \begin{pmatrix} \Delta \log(Real\ GDP_t^{USEUR}) \\ \Delta \log(CPI_t^{USEUR}) \\ i_t^{USEUR} \\ \Delta \log(Real\ Credit_t^{USEUR}) \\ \Delta \log(Real\ GDP_t^{CHN}) \\ \Delta \log(CPI_t^{CHN}) \\ i_t^{CHN} \\ \Delta \log(q_t) \\ \Delta \log(Real\ Credit_t^{CHN}) \end{pmatrix},$$

where q_t is the real exchange rate of China (or EMAS-9) *vis-à-vis* the (PPP-GDP-weighted average real exchange rate of) the U.S. and euro area. As before, the same variables and sign restrictions are considered in both the extended USEUR-CHN and USEUR-EMAS-9 models.

One difficulty of including exchange rates in our baseline two-country VAR model is that it is implausible to assume that the exchange rate shock has immediate impact on one country while leaving the other unaffected. We follow Farrant and Peersman (2006) and Peersman (2011) to use the real exchange rates and assume that an exogenous exchange rate shock (depreciation in country X) has a positive effect on output, prices, and nominal interest rates in country X, while there is a fall of all three variables in country Y. If we keep the sign restrictions in our baseline model, we end up with a new set of restrictions, shown in Table 2.2. Therefore, the block Cholesky-sign restriction identification strategy does not apply here. We now proceed to an alternative identification strategy developed from Bjoørnland and Halvorsen (2008) which can be easily applied here. We will refer to it as the 'pure sign restrictions'.

The 'zero' restrictions in Table 2.2 imply that the impulse matrix B should have

[35] See Farrant and Peersman (2006) for a survey of this issue.

the following form

$$B = \begin{bmatrix} \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & 0 & 0 & 0 & 0 & \mathrm{X} \\ \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & 0 & 0 & 0 & 0 & \mathrm{X} \\ \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & 0 & 0 & 0 & 0 & \mathrm{X} \\ \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & 0 & 0 & 0 & 0 & \mathrm{X} \\ \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} \\ \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} \\ \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} \\ \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} \\ \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} \end{bmatrix}_{(9\times 9)} \tag{2.12}$$

where 'X' denotes non-zero elements. Because in sign restrictions the impulse matrix B is generated by $B = FQ$ where F is the lower triangular matrix in the Cholesky decomposition of Ω and Q is some orthonormal matrix. Because all the diagonal elements of F are positive, then the form of B above implies that the orthonormal matrix Q must have the same form as the impulse matrix B.

As mentioned above, there are two methods to generate the orthonormal matrix Q. The Householder transformations are not applicable because we need to restrict some of the elements in the orthonormal matrix to be zero, but we can use the Givens rotation after some modifications similar to Bjørnland and Halvorsen (2008). More specifically, an unrestricted orthonormal matrix P can be generated by $P = \prod_{m,n} Q_{(m,n)}(\theta)$ with $Q_{(m,n)}(\theta)$ being rotation matrices of the form:

$$Q_{(m,n)}(\theta) = \begin{bmatrix} 1 & \dots & 0 & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \dots & \cos\theta & \dots & -\sin\theta & \dots & 0 \\ \vdots & \vdots & \vdots & 1 & \vdots & \vdots & \vdots \\ 0 & \dots & \sin\theta & \dots & \cos\theta & \dots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 & \dots & 1 \end{bmatrix}_{(9\times 9)}$$

i.e. the matrix is the identity matrix where the (m, m) and (n, n) elements are replaced by $\cos\theta$, and the (m, n) and (n, m) elements are replaced by $-\sin\theta$ and $\sin\theta$ respectively, where $m < n$ and θ lies between 0 and π. $Q_{(m,n)}(\theta)$ is called a Givens rotation, and is clearly an orthonormal matrix. It should be noted that θ can vary for different $Q_{(m,n)}$. In practice, one can generate the unrestricted orthonormal matrix P by the product of all Givens matrices. For example, because we have nine

variables in our model, there are 36 rotations in total and

$$P = \prod_{i=1}^{8} \prod_{j=i+1}^{9} Q_{(i,j)}(\theta_{i,j}),$$

where $\theta_{i,j}$ is uniformly distributed over $(0, \pi)$ for all i, j.

However, we cannot use the entire set of Givens matrices in our model, because we need to generate restricted orthonormal matrices. By the property of the orthonormal matrix, we can see that, if Q is generated according to the following formula, then it is orthonormal, satisfies all the zero restrictions in (2.12), and imposes no extra restrictions on the rest of the elements:

$$Q = \left(\prod_{i=1}^{8} \prod_{j=i+1 j \neq 5,6,7,8}^{9} Q_{(i,j)}(\theta_{i,j}) \right)', \tag{2.13}$$

where $\theta_{i,j}$ is uniformly distributed over $(0, \pi)$ for all i, j. Now we can compute the impulse response functions using these restricted orthonormal matrices.

However, since there are too many sign restrictions now (39 in total, see Table 2.2), it is very difficult to obtain one draw that satisfies all the restrictions. Numerous attempts indicated that the overall acceptance rate is around 1 out of over 10 million candidate draws (which took days, even using state-of-the art server-based hardware). Recall that this is quite a parsimonious nine-variable two-country SVAR. Therefore, even slight extensions could make the curse of dimensionality more acute, rendering computation of meaningful quantitative results infeasible. This practical reality again highlights the contribution of our block Cholesky-sign restriction identification strategy introduced in Subsection 2.3.3.

Therefore, we follow Peersman (2011) and identify the nine shocks with 39 sign restrictions shock by shock. Specifically, we use the following procedure. We first take a draw from the posterior for the usual unrestricted Normal-Wishart posterior for the VAR parameters. For each draw, we take another draw from the uniform distribution over $(0, \pi)$ and generate (restricted) orthonormal matrices according to (2.13). Then we can proceed to construct impulse response functions. If the impulse responses to an individual shock are consistent with the imposed conditions for this shock, the results for the specific shock are accepted. Otherwise, the draw is rejected.[36] We set the time horizon upon which the sign restrictions are imposed to be the same as in our baseline model. It is obvious that the Fry-Pagan MT method cannot be applied because we are only identifying one shock at a time. Therefore, the shocks identified might not be orthogonal to each other in the median impulse responses or median variance decompositions.

[36]The overall acceptance rate is around 1 out of 150 candidate draws.

Figure 2.6 presents the impulse responses of EMAS domestic variables to an external (USEUR) adverse credit shock using this alternative specification and identification strategy. The other impulse responses are consistent with those in the baseline models as well. Since the Fry-Pagan MT method cannot be applied as argued previously, only medians and the 16th and 84th percentiles error bands are provided. As before, there is a two-quarter increase in Chinese real credit growth on impact, as opposed to a lagged decrease in that of EMAS-9. As a result, EMAS-9 real GDP growth plummets significantly on impact, while Chinese real GDP growth is not significantly affected. In other words, including exchange rates does not alter the first three main results, at least not qualitatively.

The variance decompositions of EMAS domestic real GDP and credit growth are reported in Tables 2.7 and 2.8. Although the (median) variance decompositions are not very informative owing to the multiple models problem, they might serve as a benchmark for assessing the importance of exchange rates in the dynamics of real GDP and credit growth. Exchange rate shocks are not important either for real GDP growth or for real credit growth in emerging Asian countries. They only explain 4 percent at most of real GDP variation in China; the number is a little larger for EMAS-9 (11 percent) due to their more flexible exchange rate regimes relative to China. For real credit growth, exchange rate shocks only account for 5 percent at most in EMAS-9 and an even smaller percentage for China (3 percent). These findings not only show the robustness of the last main result from our baseline models, but also provide evidence for the implicit assumption in the baseline models that exchange rate dynamics are partially captured by interest rate differentials between domestic and external short-term interest rates.

2.7 CONCLUSIONS

This paper contributes to the literature by proposing a novel shock identification strategy in the context of two-country structural vector autoregressive (SVAR) models to study whether and how external credit shocks originating in the U.S. or the euro area affect domestic real economies in emerging Asia, with emphasis on the responses of domestic real GDP and credit growth. Unlike most other studies in the current literature, the proposed identification strategy enables us to distinguish the external credit shock not only from other external shocks, but also from its domestic counterpart.

In particular, we present a two-country SVAR whereby shocks within each block are identified using sign restrictions, whereas shocks across the two blocks are identified using a recursive structure (block Cholesky decomposition). Specifically, underpinned by a four-equation New Keynesian framework along the lines of Woodford (2003) and Helbling *et al.*(2010), a monetary policy, aggregate demand, aggregate

supply, and credit shocks are identified within each block using sign restrictions. However, the classical recursive (Cholesky) structure is imposed to identify shocks across the two blocks.[37] Therefore, while one block can affect the other contemporaneously, feedback in the opposite direction occurs with a one period lag. Such feedback is important because it does not seem realistic to posit either China or the other emerging Asian countries (which include economies such as India) as a small open economy.

Another contribution of our proposed identification scheme is that despite jointly imposing a combination of sign restrictions with a recursive structure (block Cholesky decomposition), it is computationally less expensive than alternative identification schemes using sign restrictions in many other two-country SVARs. This is important because it facilitates the utilization of the Fry-Pagan MT method needed to resolve the *multiple models problem*, thereby generating meaningful impulse response functions, forecast error variance decompositions, and other quantitative results.

The main conclusions of this paper are as follows: First, adverse credit shocks have significantly negative and protracted effects on domestic credit growth. Second, important regional differences are found between China and the other emerging Asian countries. In particular, the external credit shocks affect credit growth in China and the other emerging Asian countries in opposite directions. Specifically, these shocks lead to a two-quarter increase in the credit growth of China in contrast with a protracted decrease in other emerging Asian countries, indicating a countercyclical credit policy response in China. As a result, the growth of China is not affected by external credit tightening, but that of the other emerging Asian countries falls significantly. Third, the external credit shocks play a growing and non-negligible role in driving economic fluctuations in emerging Asia, although the role is smaller for China. As for policy implications, the resilience of China's growth to external credit shocks suggests that credit policy easing might be appropriate for some economies with well developed financial markets in view of the current exceptionally uncertain global growth prospects and the ongoing deleveraging process in the euro-area banking sector.

[37] Other notable studies using sign restrictions include Peersman (2005, 2011), Canova (2005), Farrant and Peersman (2006), and those listed in Fry and Pagan (2007, 2010). Bjørnland and Halvorsen (2008) also combined sign and short-term (zero) restrictions, but, as discussed in Section 2.3, there are significant differences between our methodology and theirs.

REFERENCES

Barajas, A., G. Dell'Ariccia, and A. Levchenko, 2011, Credit Booms: The Good, the Bad, and the Ugly, forthcoming, IMF Working Paper (Washington: International Monetary Fund).

Bernanke, B.S., and M. Gertler, 1989, Agency Costs, Net Worth, and Business Fluctuations, *American Economic Review*, Vol.79, pp.14-31.

Bernanke, B.S., M. Gertler, and S. Gilchrist, 1999, The Financial Accelerator in a Quantitative Business Cycle Framework, in *Handbook of Macroeconomics*, Vol.1C of the *Handbooks in Economics*, Vol.15. Amsterdam: Elsevier, pp.1341-93.

Bernanke, B.S., and C.S. Lown, 1991, The Credit Crunch, *Brookings Papers on Economic Activity*, No.2, pp.205-47.

Bjørnland, H.C., and J.I., Halvorsen, 2008, How Does Monetary Policy Respond to Exchange Rate Movements? New International Evidence, Working Paper 2008/15, Norges Bank.

Bordo, M.D., and J.G. Haubrich, 2010, Credit crises, money and contractions: An historical view, *Journal of Monetary Economics*, Vol.57, pp.1-18.

Borio, C., C. Furfine, and P. Lowe, 2001, Procyclicality of Financial Systems and Financial Stability, BIS Papers No.1 (Basel, Switzerland: Bank for International Settlements).

Caballero, R., and A. Krishnamurthy, 1998, Emerging Market Crises: An Asset Markets Perspective, Working Papers 98-18, MIT.

Calvo, G.A., A. Izquierdo, and L.F. Mejia, 2004, On the empirics of Sudden Stops: The Relevance of Balance Sheet Effects, NBER Working Paper No. 10520 (Cambridge, MA).

Calza, A., and J. Sousa, 2005, Output and Inflation Responses to Credit Shocks: Are There Threshold Effects in the Euro Area?, Working Paper Series No. 481, European Central Bank.

Campbell, J.Y., 2003, Consumption-based asset pricing, Chapter 13 in Constantinides, G. M., Harris, M. and Stulz R. (eds.), *Handbook of the Economics of Finance*, Amsterdam: Elsevier North-Holland.

Canova, F., 2005, The Transmission of US Shocks to Latin America, *Journal of Applied Econometrics*, Vol.20(2), pp.229-251.

Canova, F., and G. De Nicolo, 2002, Monetary Disturbances Matter for Business Fluctuations in the G-7, *Journal of Monetary Economics*, Vol.49, pp.1131-59.

Canova, F and G. De Nicoló, 2003, On the sources of business cycles in the G-7, *Journal of International Economics*, Vol.59, pp.77-100.

Céspedes, L.F., R. Chang, and A. Velasco, 2004, Balance Sheets and Exchange Rate Policy, *American Economic Review*, Vol.94, No.4, pp.1183-1193.

Clarida, R., and J. Gali, 1994, Sources of Real Exchange Rate Fluctuations: How Important are Nominal Shocks? *Carnegie-Rochester Conference on Public Policy*,

Vol.41, pp.1-56.
Cochrane, J.H., 2006, Financial Markets and the Real Economy, in John H. Cochrane, ed., *Financial Markets and the Real Economy*, Vol.18 of the *International Library of Critical Writings in Financial Economics*, London: Edward Elgar, pp.237-327.
Cushman D.O., and T. Zha, 1997, Identifying Monetary Policy in a Small Open Economy Under Flexible Exchange Rates, *Journal of Monetary Economics*, Vol.39, pp.433-488.
Du, Y., 2010, Monetary Policy and Bank Loan Supply in China, Bachelor Thesis, Williams College.
Elekdag, S., and F. Han, 2011, What Drives Credit Growth in Emerging Asia?, IMF Working Paper No.12/43 (Washington: International Monetary Fund).
Elekdag, S., and Y. Wu, 2011, Rapid Credit Growth: Boon or Boom-Bust? IMF Working Paper No.11/241 (Washington: International Monetary Fund).
Farrant, K. and G. Peersman, 2006, Is the Exchange Rate a Shock Absorber or Source of Shocks? New Empirical Evidence, *Journal of Money, Credit, and Banking*, Vol.38, pp.939-62.
Faust, J., 1998, The Robustness of Identified VAR Conclusions about Money, *Carnegie-Rochester Conference on Public Policy*, Vol.49, pp.207-244.
Fisher, I., 1933, The Debt-Deflation Theory of Great Depressions, *Econometrica*, Vol.1 (October), pp.337-357.
Fry, R., and A. Pagan, A., 2007, Some Issues in Using Sign Restrictions for Identifying Structural VARs, NCER Working Paper Series 14, National Centre for Econometric Research.
Fry, R., and A. Pagan, 2010, Sign Restrictions in Structural Vector Autoregressions: A Critical Review, CAMA Working Papers 2010-22, Australian National University, Centre for Applied Macroeconomic Analysis.
Gilchrist, S., V. Yankov, and E. Zakrajšek, 2009, Credit Market Shocks and Economic Fluctuations: Evidence from Corporate Bond and Stock Markets, NBER Working Paper 14863.
Gilchrist, S., and E. Zakrajšek, 2010, Credit Spreads and Business Cycle Fluctuations, unpublished, Boston University, Department of Economics.
Gourinchas, P.-O., R. Valdes, and O. Landerretche, 2001, Lending Booms: Latin America and the World, *Economia*, Vol.1 (Spring), pp.47-99.
Helbling, T., R. Huidrom, M.A. Kose, and C. Otrok, 2010, Do Credit Shocks Matter? A Global Perspective, IMF Working Paper No.10/261 (Washington: International Monetary Fund).
Jiang, Q., and L. Zeng, 2008, The Credit Channel of Monetary Policy in China: Theoretical Model and Empirical Analysis, *Collected Essays on Finance and Economics*, No.3, pp.40-46.
Kose, A., and E. Prasad, 2010, Resilience of Emerging Market Economies to Eco-

nomic and Financial Developments in Advanced Economies, Economic Papers 411, Directorate-General for Economic and Financial Affairs, October 2010.
Kiyotaki, N., and J. Moore, 1997, Credit Cycles, *Journal of Political Economy*, Vol.105, No.2, pp.211-248.
Levy-Yeyati, E., and F. Sturzenegger, 2005, Classifying Exchange Rate Regimes: Deeds vs. Words, *European Economic Review*, Vol.49, No.6, pp.1603-1635.
Meeks, R., 2009, Credit shocks: Evidence from Corporate Spreads and Defaults, Working Paper 0906, Federal Reserve Bank of Dallas.
Mendoza, E.G., 2010, Sudden Stops, Financial Crises and Leverage, *American Economic Review*, Vol.100, No.5, pp.1941-1966.
Mendoza, E.G., and M. Terrones, 2008, An Anatomy of Credit Booms: Evidence from Macro Aggregates and Micro Data, NBER Working Paper No. 14049 (Cambridge, Massachusetts: National Bureau of Economic Research).
Modigliani, F., and M. Miller, 1958, The Cost of Capital, Corporation Finance and the Theory of Investment, *American Economic Review*, Vol.48, No.3, pp.261-297.
Mountford, A., 2005, Leaning into the wind: A structural VAR investigation of UK monetary policy, *Oxford Bulletin of Economic and Statistics*, Vol.67, pp.597-621.
Ortiz, A., 2008, Credit Market Shocks, Monetary Policy, and Economic Fluctuations, Oberlin College.
Peersman, G., 2005, What caused the early millennium slowdown? Evidence based on vector autoregressions, *Journal of Applied Econometrics*, Vol.20, pp.185-207.
Peersman, G., 2011, The Relative Importance of Symmetric and Asymmetric Shocks: The Case of the United Kingdom and the Euro Area, *Oxford Bulletin of Economic and Statistics*, Vol.73, No.1, pp.104-18.
Reinhart, C. and K.S. Rogoff, 2009, *This Time is Different: Eight Centuries of Financial Folly*, Princeton Press.
Rubio-Ramirez, J.F., D.F.Waggoner and T. Zha, 2005, Markov-Switching Structural Vector Autoregressions: Theory and Application, Working Paper 2005-27, Federal Reserve Bank of Atlanta.
Schneider, M., and A. Tornell, 2004, Balance Sheet Effects, Bailout Guarantees and Financial Crises, *Review of Economic Studies*, Blackwell Publishing, Vol.71, pp.883-913.
Sims, C.A., 1980, Macroeconomics and Reality, *Econometrica*, Vol.48(1), pp.1-48.
Tornell, A., and F. Westermann, 2002, Boom-Bust Cycles in Middle Income Countries: Facts and Explanation, *IMF Staff Papers*, Vol.49 Special Issue, pp.111-153.
Uhlig, H., 2005, What are the effects of monetary policy on output? Results from an agnostic identification procedure, *Journal of Monetary Economics*, Vol.52, pp.381-419.
Woodford, M., 2003, Interest and Prices: Foundations of a Theory of Monetary Policy (Princeton: Princeton Press).

Zhang, Y., 2011, Credit Market Imperfection and Sectoral Asymmetry of Chinese Business Cycle, IMF Working Paper No. 11/118 (Washington: International Monetary Fund).
Zhang, Y., 2009, Chinese Credit Market Asymmetries, Ph.D. Dissertation.
Zhao, Z., Z. Yu, and L. Miao, 2007, The Nonlinearity between Credit Market and Macroeconomic Fluctuations in China: An Empirical Research from the Perspective of Financial Accelerator, *Economic Research Journal*, Vol.6, pp.27-38.

Table 2.1. Sign Matrix: Baseline Models

	AS^{USEUR}	AD^{USEUR}	MP^{USEUR}	C^{USEUR}	AS^{CHN}	AD^{CHN}	MP^{CHN}	C^{CHN}
$\Delta log(Real\ GDP^{USEUR})$	< 0	< 0	< 0	< 0				
$\Delta log(CPI^{USEUR})$	> 0	< 0	< 0	< 0		0		
i^{USEUR}	> 0	< 0	> 0	< 0				
$\Delta log(Real\ Credit^{USEUR})$	?	?	?	$\ll 0$				
$\Delta log(Real\ GDP^{CHN})$	?	?	?	?	< 0	< 0	< 0	< 0
$\Delta log(CPI^{CHN})$	?	?	?	?	> 0	< 0	< 0	< 0
i^{CHN}	?	?	?	?	> 0	< 0	> 0	< 0
$\Delta log(Real\ Credit^{CHN})$	?	?	?	?	?	?	?	$\ll 0$

NOTE: The first '< 0' represents the sign for the impulse response of USEUR real GDP growth to USEUR aggregate supply shock, i.e.

$$IRF^{AS,USEUR}_{GDP,USEUR}(i) < 0,$$

and the big '0' at the upper-right corner denote block Cholesky decomposition. The sign '$\ll 0$' means that the impulse response of USEUR (CHN) real credit growth to domestic credit shock in USEUR (CHN) is less than 0, and greater than that to domestic aggregate demand shock in magnitude, i.e.

$$IRF^{C,USEUR}_{Credit,USEUR}(i) < 0,\ \left|IRF^{C,USEUR}_{Credit,USEUR}(i)\right| > \left|IRF^{AD,USEUR}_{Credit,USEUR}(i)\right|;$$

$$IRF^{C,CHN}_{Credit,CHN}(i) < 0,\ \left|IRF^{C,CHN}_{Credit,CHN}(i)\right| > \left|IRF^{AD,CHN}_{Credit,CHN}(i)\right|.$$

The last inequality is exactly how we differentiate domestic real credit shock from domestic aggregate demand shock. The questions mark '?' means that no restrictions is imposed. Note that since we have 21 sign restrictions per period to impose in the system, we only consider contemporaneous sign restrictions ($i = 0$) in this paper to ease the computational burden. The same sign restrictions apply to the baseline USEUR-EMAS-9 model.

Table 2.2. Sign Matrix: Alternative Approach

	AS^{USEUR}	AD^{USEUR}	MP^{USEUR}	C^{USEUR}	AS^{CHN}	AD^{CHN}	MP^{CHN}	C^{CHN}	q
$\Delta log(Real\ GDP^{USEUR})$	< 0	< 0	< 0	< 0	0	0	0	0	> 0
$\Delta log(CPI^{USEUR})$	> 0	< 0	< 0	< 0	0	0	0	0	> 0
i^{USEUR}	> 0	< 0	> 0	< 0	0	0	0	0	> 0
$\Delta log(Real\ Credit^{USEUR})$	?	?	?	$\ll 0$	0	0	0	0	?
$\Delta log(Real\ GDP^{CHN})$	?	?	?	?	< 0	< 0	< 0	< 0	< 0
$\Delta log(CPI^{CHN})$	?	?	?	?	> 0	< 0	< 0	< 0	< 0
i^{CHN}	?	?	?	?	> 0	< 0	> 0	< 0	< 0
$\Delta log(q)$	?	≤ 0	≥ 0	?	?	≥ 0	≤ 0	?	≤ 0
$\Delta log(Real\ Credit^{CHN})$	?	?	?	?	?	?	?	$\ll 0$	?

NOTE: q is the (bilateral) real exchange rate of CHN *vis-á-vis* the (PPP-GDP-weighted real exchange rates of) USEUR. The first '< 0' represents the sign for the impulse response of USEUR real GDP growth to USEUR aggregate supply shock, i.e.

$$IRF^{AS,USEUR}_{GDP,USEUR}(i) < 0,$$

and the 0's denote '$= 0$' in sign restrictions instead of the block-Choleksy scheme. The sign '$\ll 0$' means that the impulse response of USEUR (CHN) real credit growth to domestic credit shock in USEUR (CHN) is less than 0, and greater than that to domestic aggregate demand shock in magnitude, i.e.

$$IRF^{C,USEUR}_{Credit,USEUR}(i) < 0,\ \left|IRF^{C,USEUR}_{Credit,USEUR}(i)\right| > \left|IRF^{AD,USEUR}_{Credit,USEUR}(i)\right|;$$

$$IRF^{C,CHN}_{Credit,CHN}(i) < 0,\ \left|IRF^{C,CHN}_{Credit,CHN}(i)\right| > \left|IRF^{AD,CHN}_{Credit,CHN}(i)\right|.$$

The last inequality is exactly how we differentiate domestic real credit shock from domestic aggregate demand shock. The questions mark '?' means that no restrictions is imposed. Note that since we have 21 sign restrictions per period to impose in the system, we only consider contemporaneous sign restrictions ($i = 0$) in this paper to ease the computational burden. The same sign restrictions apply to the baseline USEUR-EMAS-9 model.

Table 2.3. Forecast Error Variance Decomposition of Domestic Real GDP Growth: Baseline Models (Full sample: 1989Q1 – 2010Q4)

	CHN Real GDP Growth		EMAS (w/o CHN) Real GDP Growth	
	0 quarters	16 quarters	0 quarters	16 quarters
USEUR shocks	12 (8)	14 (15)	14 (16)	16 (22)
AS	0 (1)	2 (3)	0 (1)	1 (3)
AD	8 (3)	6 (5)	6 (9)	7 (10)
MP	5 (2)	4 (4)	2 (4)	2 (5)
Credit	0 (2)	2 (4)	6 (2)	6 (4)
Domestic shocks	**88 (92)**	**86 (85)**	**86 (84)**	**84 (78)**
AS	33 (39)	29 (32)	46 (54)	48 (48)
AD	37 (38)	33 (32)	6 (13)	5 (11)
MP	5 (5)	11 (11)	20 (12)	18 (11)
Credit	13 (10)	14 (10)	14 (5)	13 (9)

NOTE: These are the forecast error variance decompositions using the Fry-Pagan MT method, and the median values (normalized to sum to 100) are given in parentheses. *AS*, *AD*, *MP* and *Credit* denote the aggregate supply shock, aggregate demand shock, monetary policy shock, and credit market shock respectively.

Table 2.4. Forecast Error Variance Decomposition of Domestic Real Credit Growth: Baseline Models (Full sample: 1989Q1 – 2010Q4)

	CHN Real Credit Growth		EMAS-9 Real Credit Growth	
	0 quarters	16 quarters	0 quarters	16 quarters
USEUR shocks	**8 (4)**	**26 (29)**	**2 (5)**	**24 (17)**
AS	3 (1)	5 (6)	0 (1)	4 (3)
AD	3 (1)	10 (8)	1 (3)	9 (8)
MP	0 (1)	9 (5)	0 (1)	2 (2)
Credit	2 (1)	2 (9)	1 (1)	10 (4)
Domestic shocks	**92 (96)**	**74 (71)**	**98 (95)**	**76 (83)**
AS	26 (27)	18 (15)	4 (7)	16 (13)
AD	8 (12)	6 (8)	12 (17)	9 (13)
MP	14 (11)	18 (17)	18 (13)	11 (13)
Credit	44 (45)	32 (31)	64 (58)	40 (43)

NOTE: These are the forecast error variance decompositions using the Fry-Pagan MT method, and the median values (normalized to sum to 100) are given in parentheses. *AS*, *AD*, *MP* and *Credit* denote the aggregate supply shock, aggregate demand shock, monetary policy shock, and credit market shock respectively.

Table 2.5. Forecast Error Variance Decomposition of Domestic Real GDP Growth: Baseline Models (Subsample: 1998Q1 – 2010Q4)

	CHN Real GDP Growth		EMAS (w/o CHN) Real GDP Growth	
	0 quarters	16 quarters	0 quarters	16 quarters
USEUR shocks	27 (28)	30 (37)	14 (21)	25 (38)
AS	0 (2)	3 (7)	2 (2)	9 (10)
AD	18 (19)	15 (14)	2 (5)	4 (9)
MP	6 (3)	5 (7)	1 (4)	1 (6)
Credit	3 (4)	8 (9)	10 (10)	11 (13)
Domestic shocks	**73 (72)**	**70 (63)**	**86 (79)**	**75 (62)**
AS	16 (18)	11 (16)	40 (35)	35 (24)
AD	25 (35)	22 (25)	41 (31)	36 (20)
MP	19 (15)	23 (12)	3 (8)	2 (8)
Credit	12 (4)	14 (10)	2 (5)	2 (10)

NOTE: These are the forecast error variance decompositions using the Fry-Pagan MT method, and the median values (normalized to sum to 100) are given in parentheses. *AS*, *AD*, *MP* and *Credit* denote the aggregate supply shock, aggregate demand shock, monetary policy shock, and credit market shock respectively.

Table 2.6. Forecast Error Variance Decomposition of Domestic Real Credit Growth: Baseline Models (Subsample: 1998Q1 – 2010Q4)

	CHN Real Credit Growth		EMAS (w/o CHN) Real Credit Growth	
	0 quarters	16 quarters	0 quarters	16 quarters
USEUR shocks	13 (12)	40 (43)	5 (5)	26 (29)
AS	0 (2)	7 (9)	1 (1)	8 (6)
AD	1 (2)	5 (12)	0 (1)	7 (8)
MP	0 (2)	12 (9)	1 (1)	4 (4)
Credit	12 (6)	15 (13)	3 (1)	7 (11)
Domestic shocks	**87 (88)**	**60 (57)**	**95 (95)**	**76 (71)**
AS	0 (9)	8 (12)	9 (7)	15 (9)
AD	20 (19)	19 (16)	20 (17)	16 (13)
MP	31 (19)	16 (10)	28 (25)	19 (21)
Credit	36 (41)	17 (19)	38 (45)	26 (29)

NOTE: These are the forecast error variance decompositions using the Fry-Pagan MT method, and the median values (normalized to sum to 100) are given in parentheses. *AS*, *AD*, *MP* and *Credit* denote the aggregate supply shock, aggregate demand shock, monetary policy shock, and credit market shock respectively.

Table 2.7. Forecast Error Variance Decomposition of Domestic Real GDP Growth: Alternative Approach (Full sample: 1989Q1 – 2010Q4)

	CHN Real GDP Growth		EMAS-9 Real GDP Growth	
	0 quarters	16 quarters	0 quarters	16 quarters
USEUR shocks	**25**	**27**	**13**	**16**
AS	0	2	3	4
AD	13	11	2	1
MP	6	9	1	3
Credit	6	5	8	8
EX shocks	**3**	**4**	**11**	**9**
Domestic shocks	**72**	**69**	**76**	**75**
AS	13	18	28	32
AD	24	20	23	19
MP	12	16	9	8
Credit	13	15	16	16

NOTE: Median values of the posterior, normalized to sum to 100. *AS*, *AD*, *MP* and *Credit* denote the aggregate supply shock, aggregate demand shock, monetary policy shock, and credit market shock respectively. EX shocks are the exchange rate shocks.

Table 2.8. Forecast Error Variance Decomposition of Domestic Real Credit Growth: Alternative Approach (Full sample: 1989Q1 – 2010Q4)

	CHN Real GDP Growth		EMAS-9 Real GDP Growth	
	0 quarters	16 quarters	0 quarters	16 quarters
USEUR shocks	**3**	**26**	**10**	**15**
AS	1	1	1	1
AD	2	3	5	7
MP	0	6	0	1
Credit	0	16	3	6
EX shocks	**0**	**3**	**4**	**5**
Domestic shocks	**97**	**71**	**86**	**80**
AS	13	6	1	6
AD	8	4	12	10
MP	14	18	13	11
Credit	61	43	60	53

NOTE: Median values of the posterior, normalized to sum to 100. *AS*, *AD*, *MP* and *Credit* denote the aggregate supply shock, aggregate demand shock, monetary policy shock, and credit market shock respectively. EX shocks are the exchange rate shocks.

Figure 2.1. Real GDP and Real Credit Growth Rates in Emerging Asia
(Quarter-over-quarter percent change)

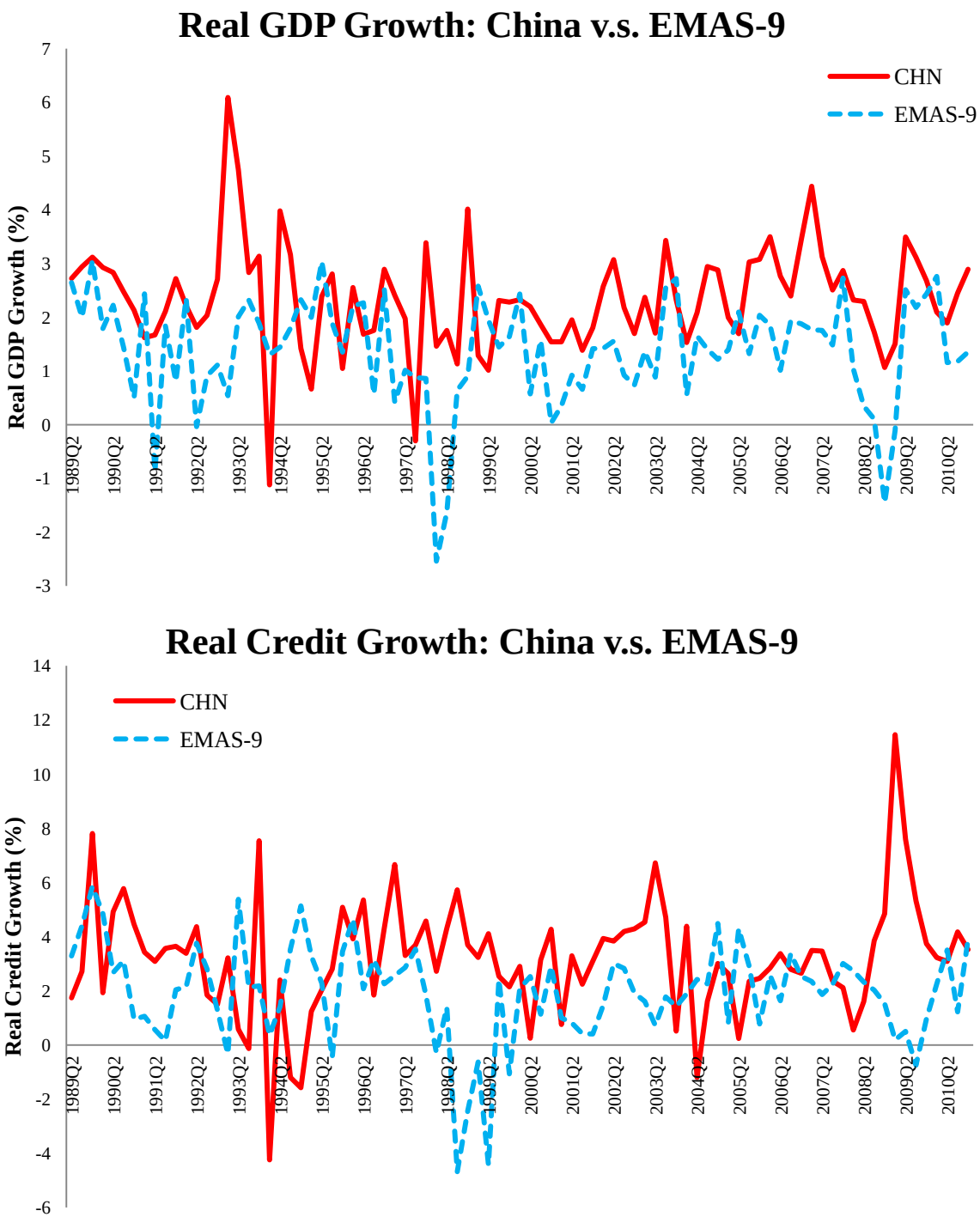
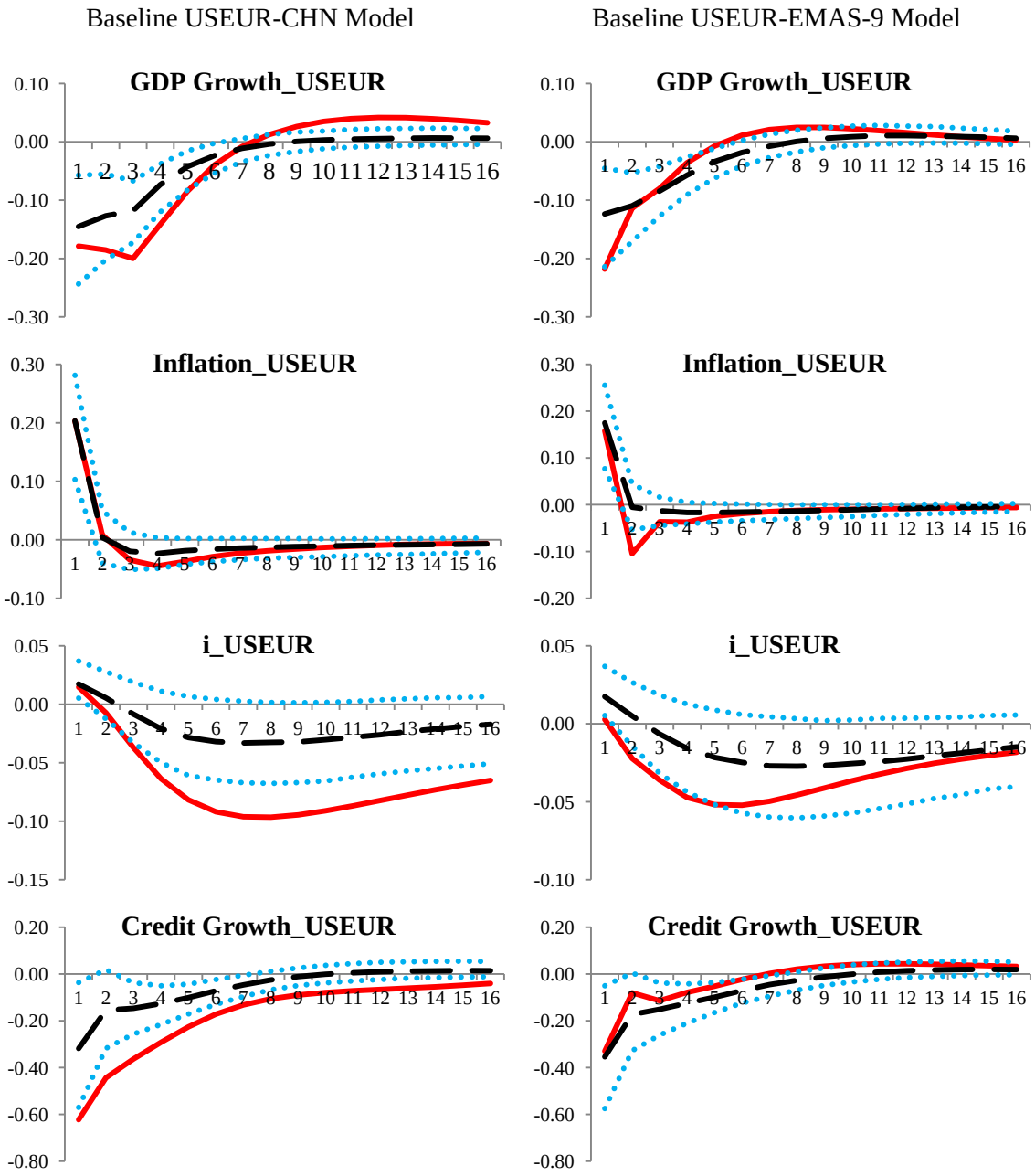
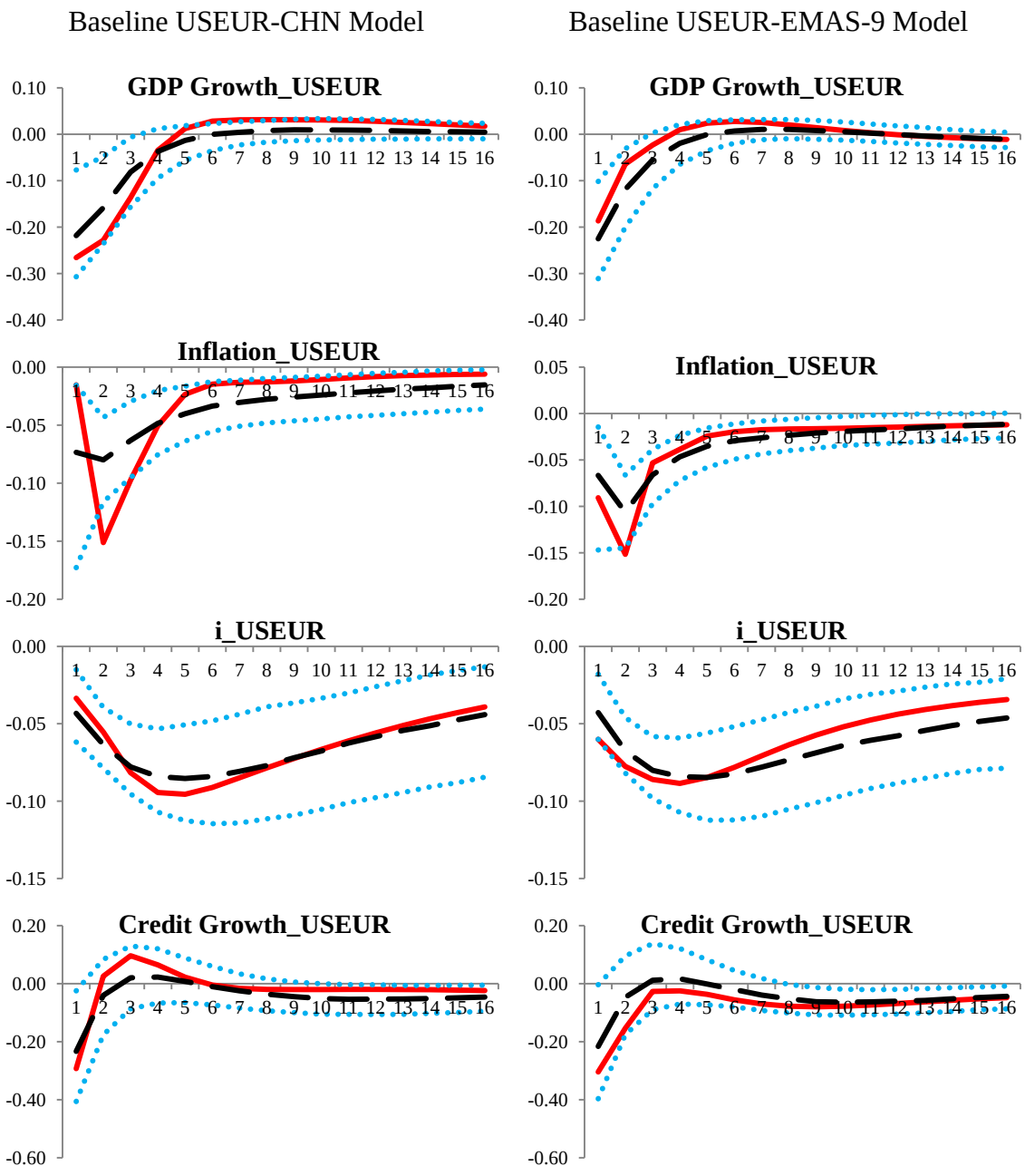


Figure 2.2(a). Identified USEUR Adverse **Aggregate Supply** Shocks:
Baseline Models (Full sample: 1989Q1 – 2010Q4)



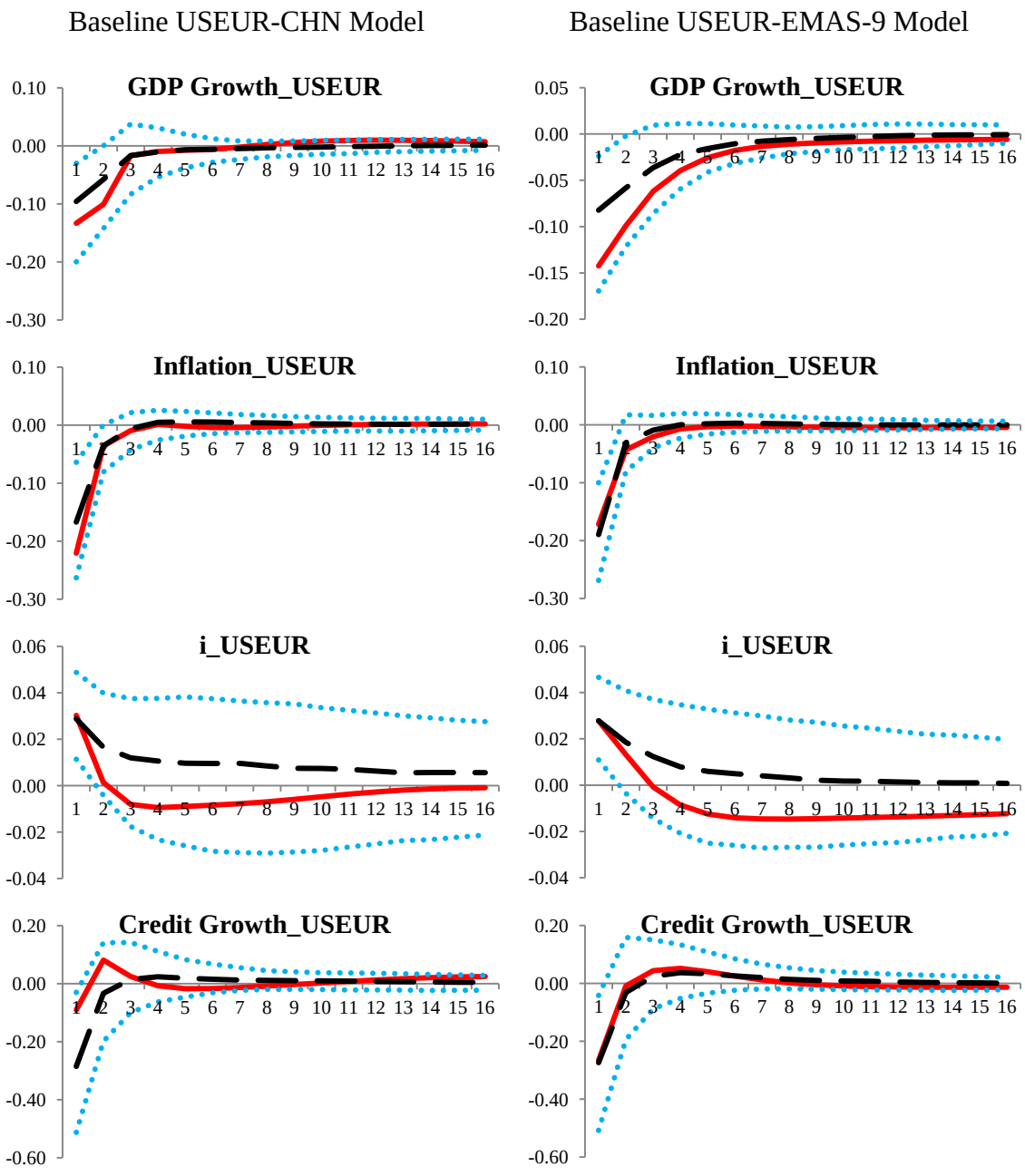
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.2(b). Identified USEUR Adverse **Aggregate Demand** Shocks:
Baseline Models (Full sample: 1989Q1 – 2010Q4)



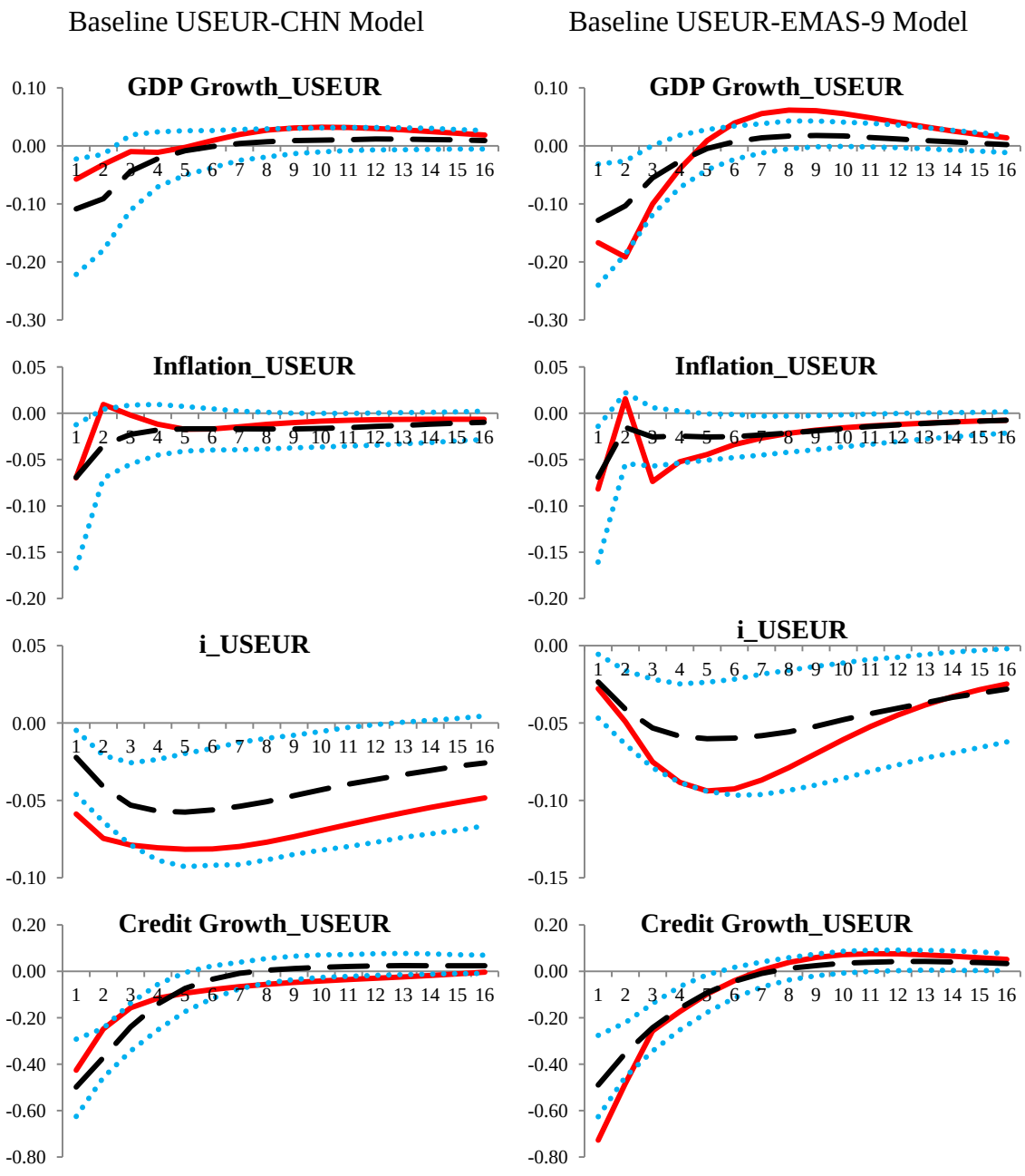
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.2(c). Identified USEUR Contractionary **Monetary Policy** Shocks:
Baseline Models (Full sample: 1989Q1 – 2010Q4)



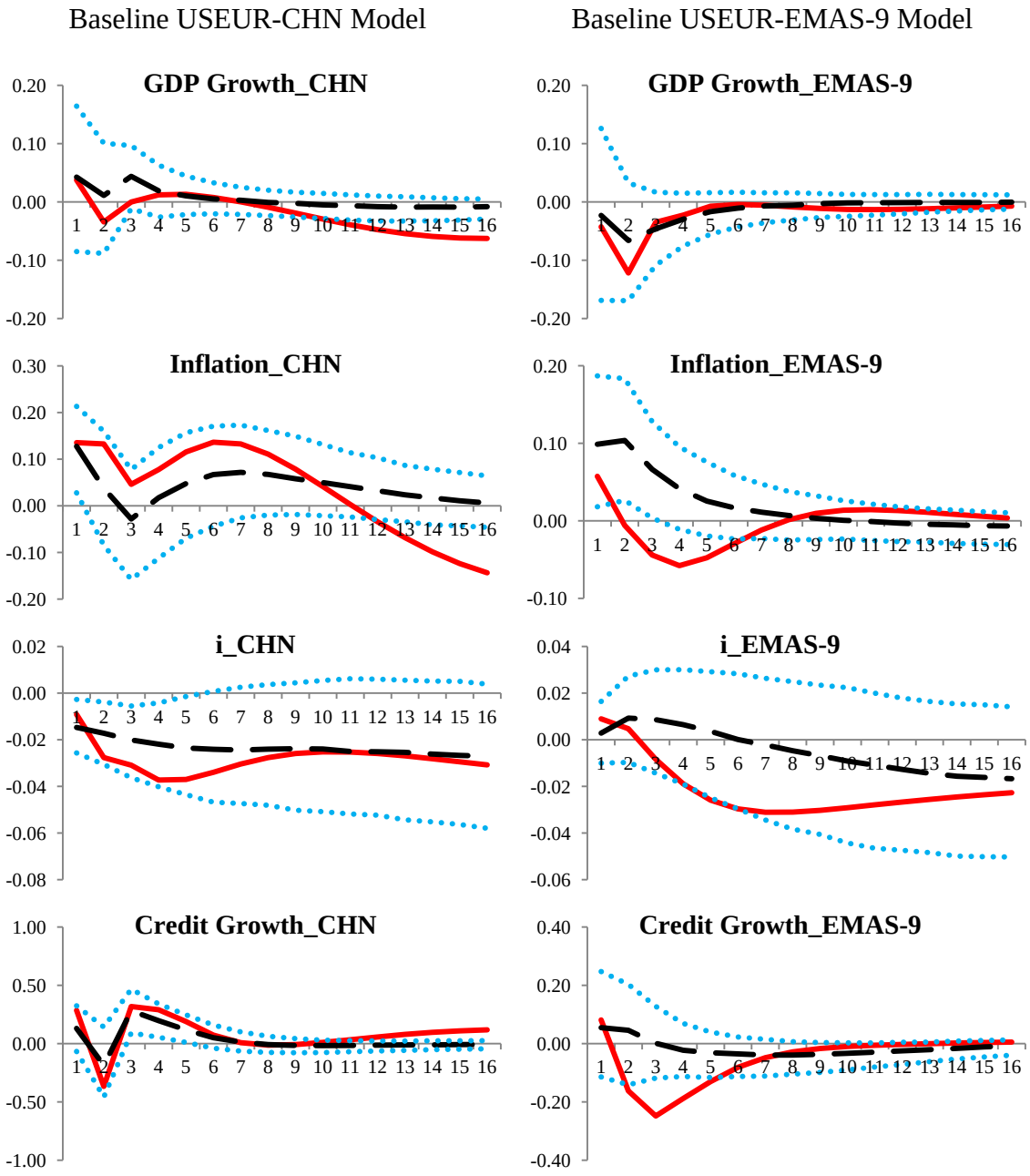
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.2(d). Identified USEUR Adverse **Credit Market** Shocks:
Baseline Models (Full sample: 1989Q1 – 2010Q4)



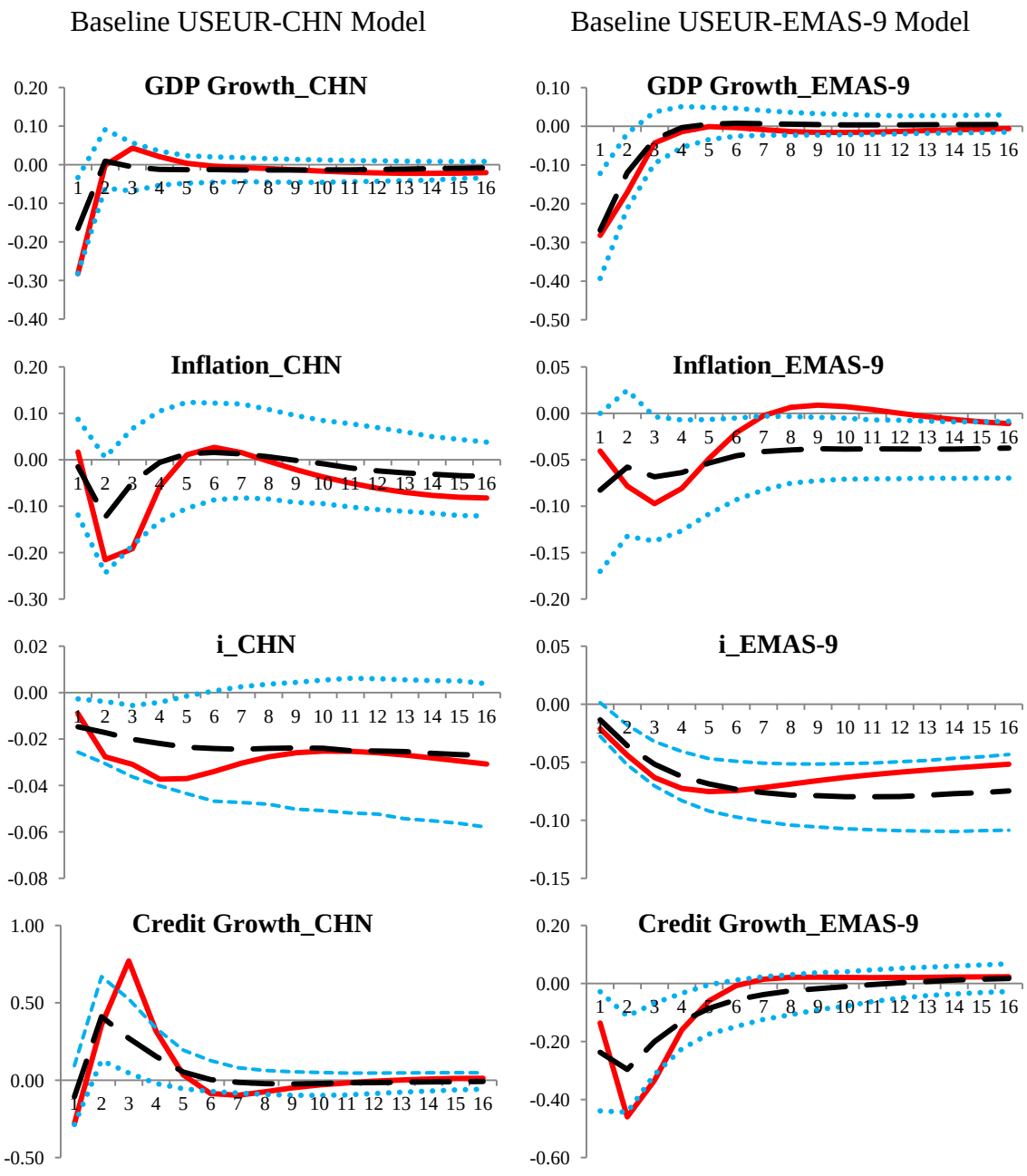
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.3(a). Impulse Responses of
Domestic Variables to USEUR Adverse **Aggregate Supply** Shocks: Baseline Models
(Full sample: 1989Q1 – 2010Q4)



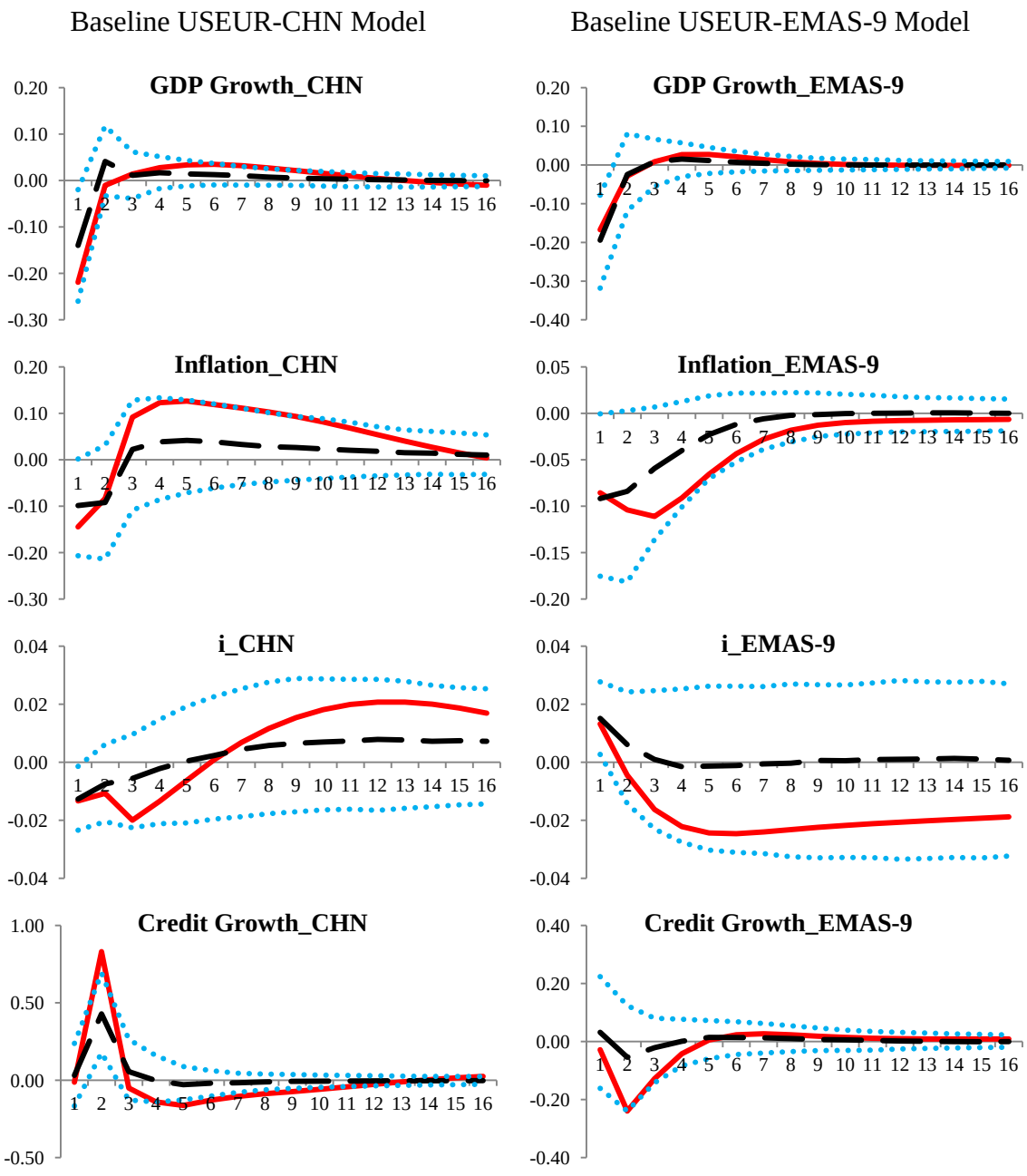
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.3(b). Impulse Responses of
Domestic Variables to USEUR Adverse **Aggregate Demand** Shocks: Baseline Models
(Full sample: 1989Q1 – 2010Q4)



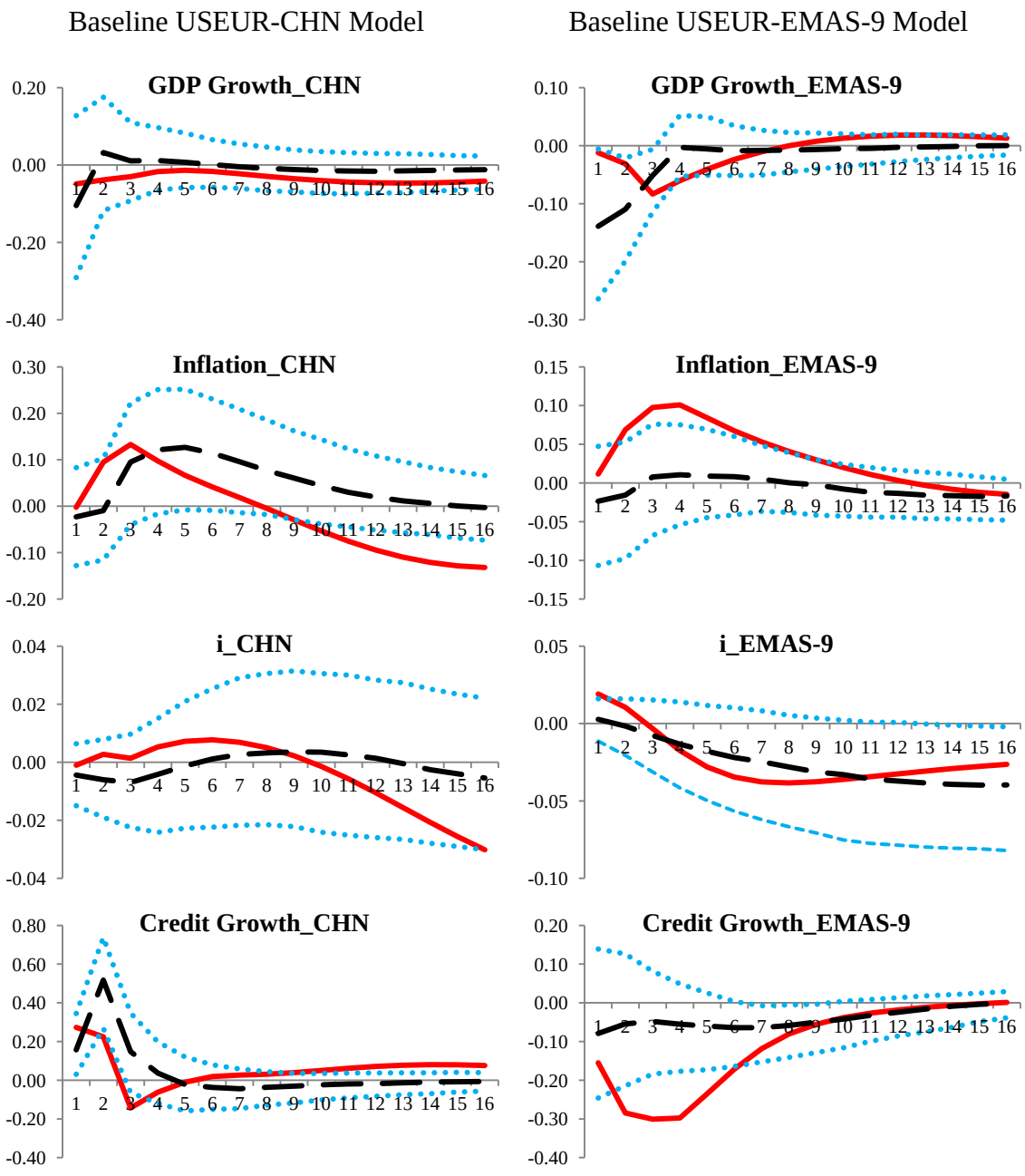
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.3(c). Impulse Responses of
Domestic Variables to USEUR Contractionary **Monetary Policy** Shocks: Baseline Models (Full sample: 1989Q1 – 2010Q4)



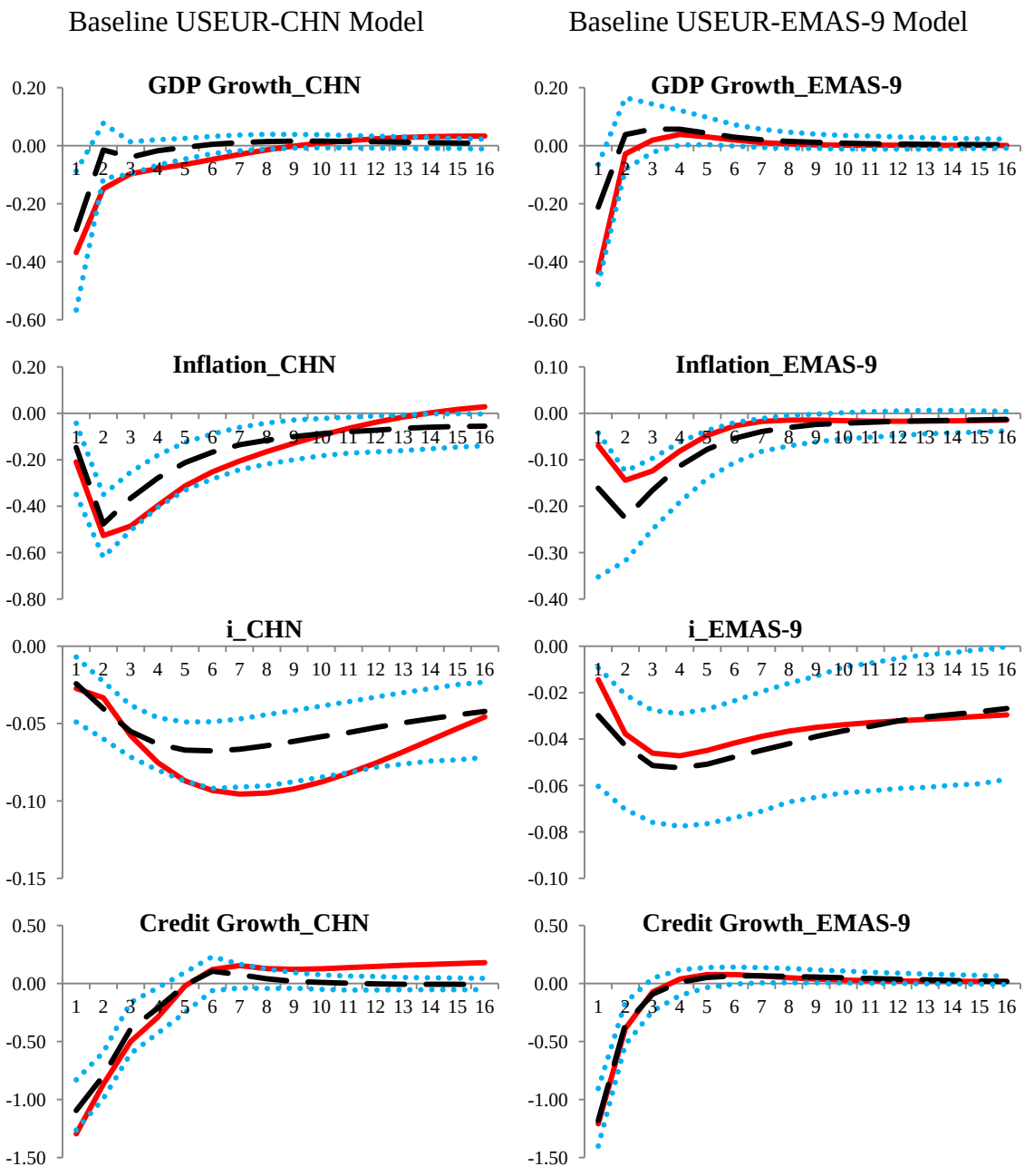
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.3(d). Impulse Responses of
Domestic Variables to USEUR Adverse **Credit Market** Shocks: Baseline Models
(Full sample: 1989Q1 – 2010Q4)



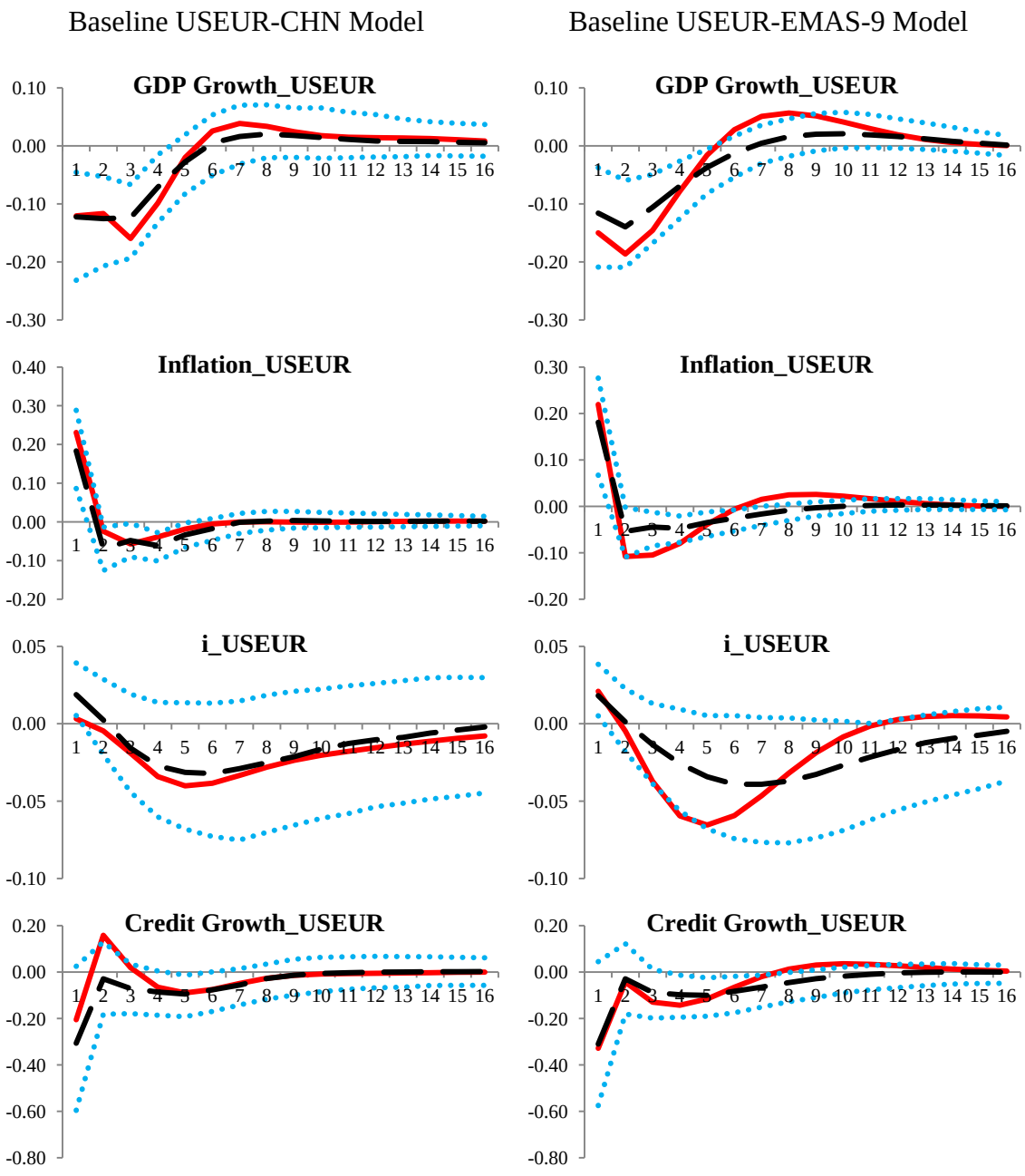
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.3(e). Impulse Responses of
Domestic Variables to a **Domestic** Adverse **Credit Market** Shock: Baseline Models
(Full sample: 1989Q1 – 2010Q4)



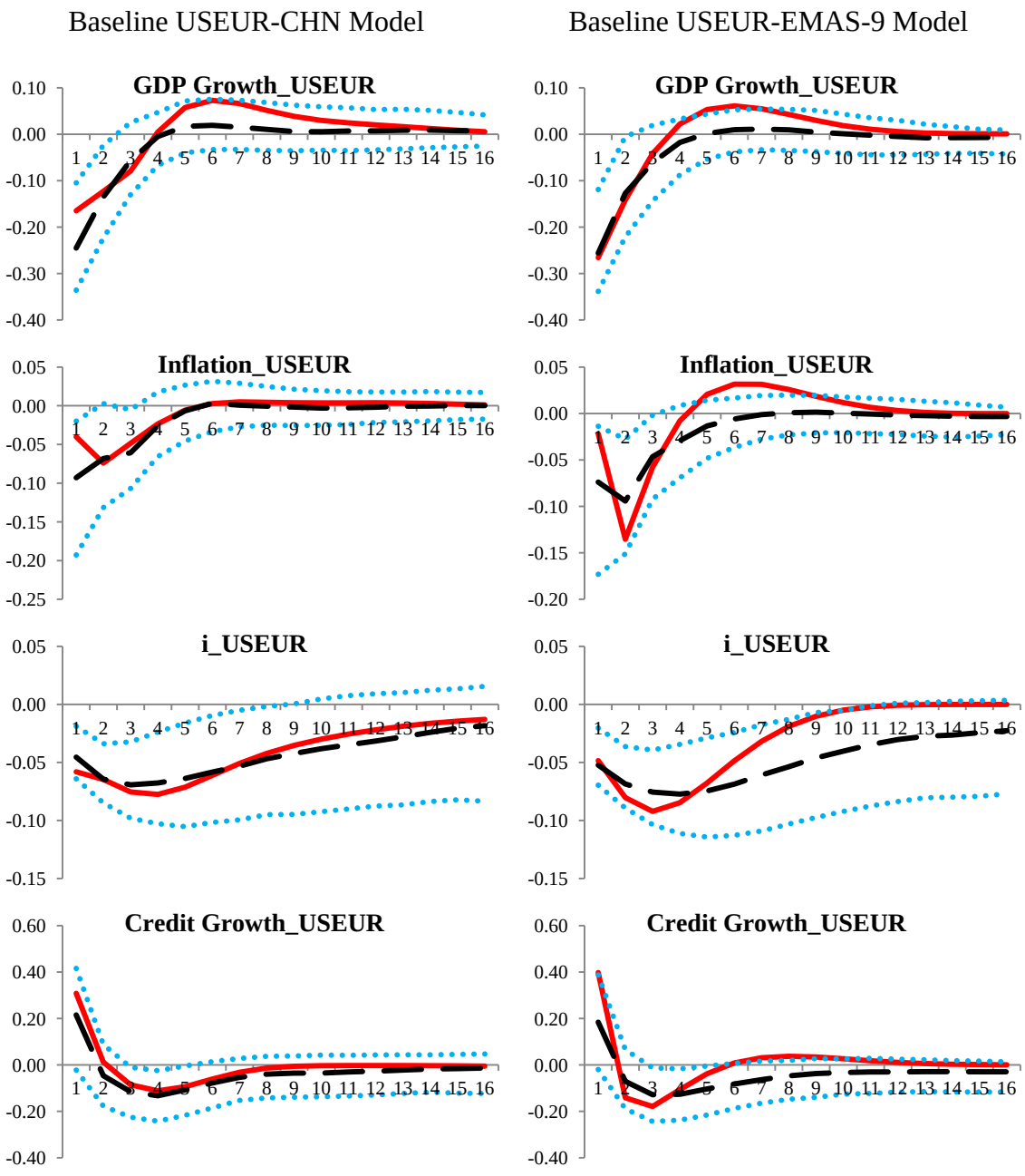
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.4(a). Identified USEUR Adverse **Aggregate Supply** Shocks:
Baseline Models (Subsample: 1998Q1 – 2010Q4)



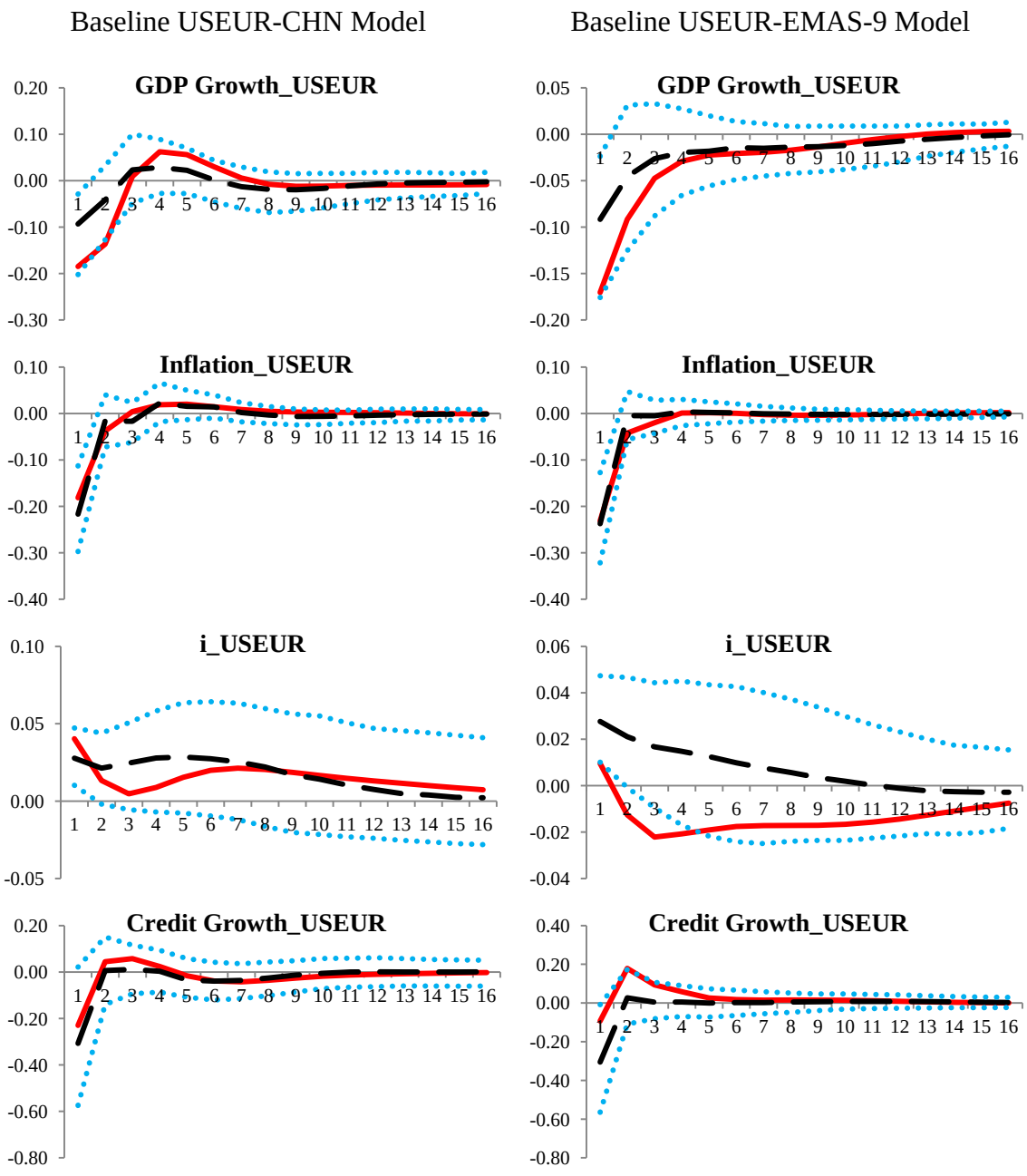
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.4(b). Identified USEUR Adverse **Aggregate Demand** Shocks:
Baseline Models (Subsample: 1998Q1 – 2010Q4)



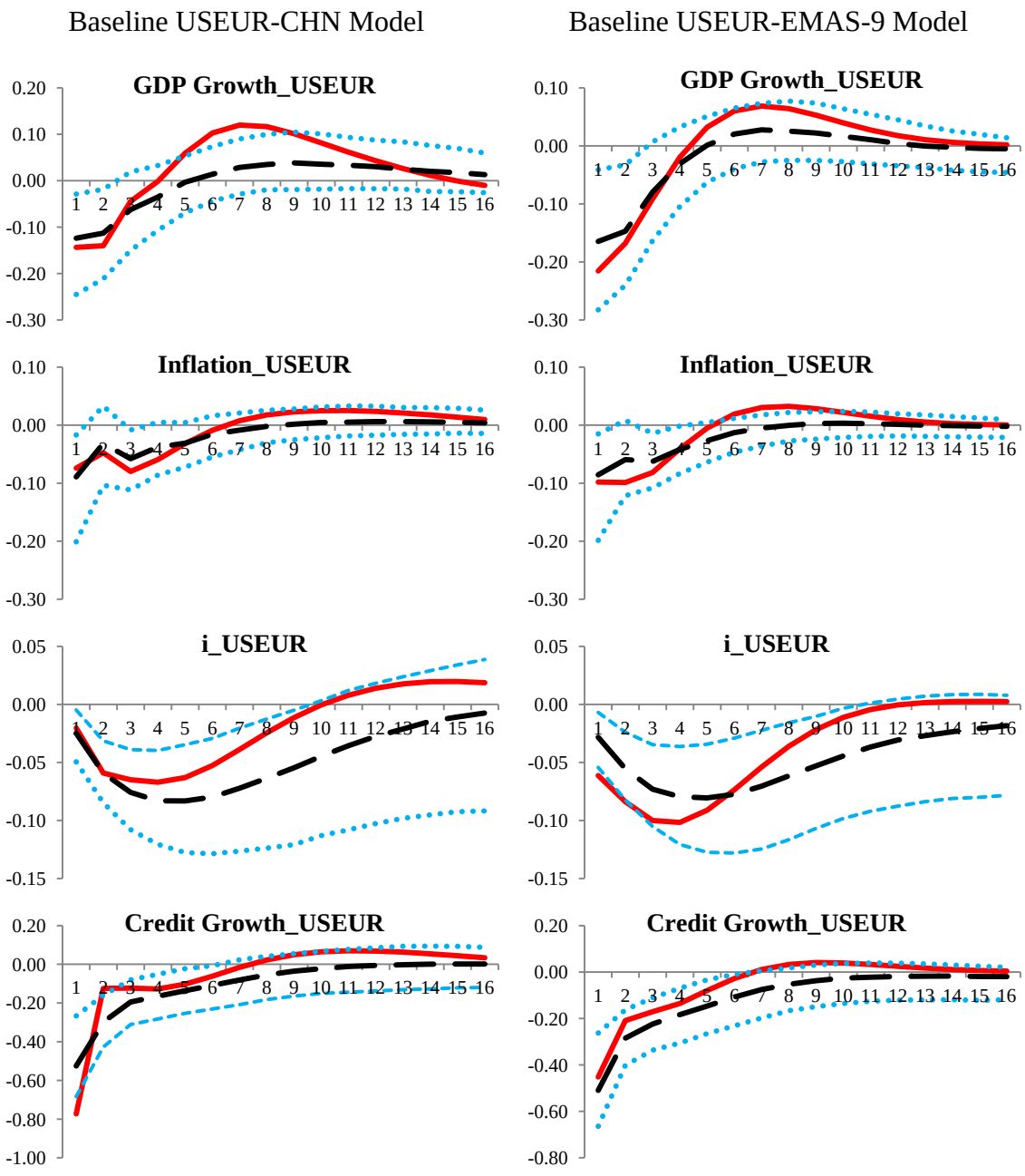
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.4(c). Identified USEUR Contractionary **Monetary Policy** Shocks:
Baseline Models (Subsample: 1998Q1 – 2010Q4)



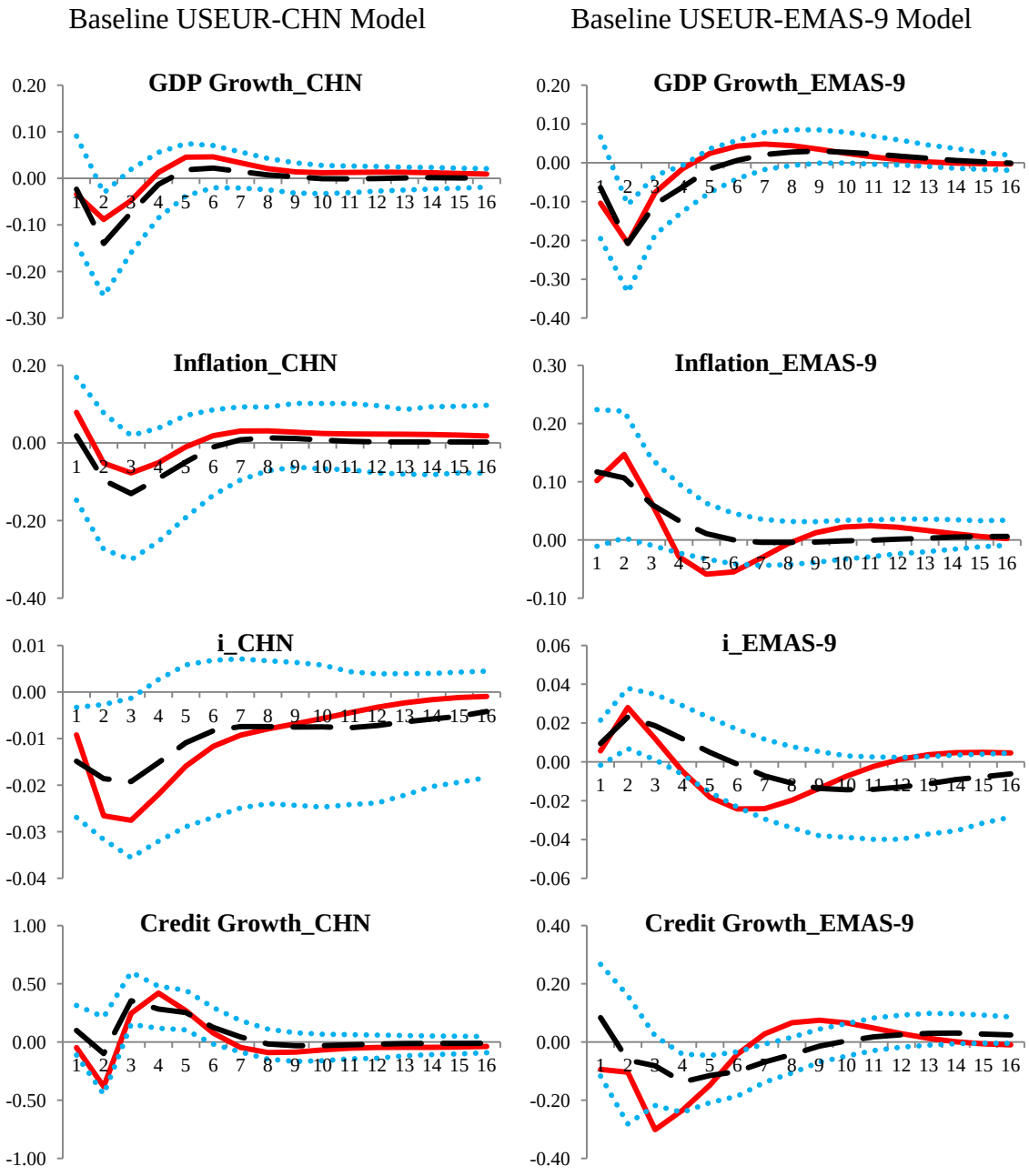
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.4(d). Identified USEUR Adverse **Credit Market** Shocks:
Baseline Models (Subsample: 1998Q1 – 2010Q4)



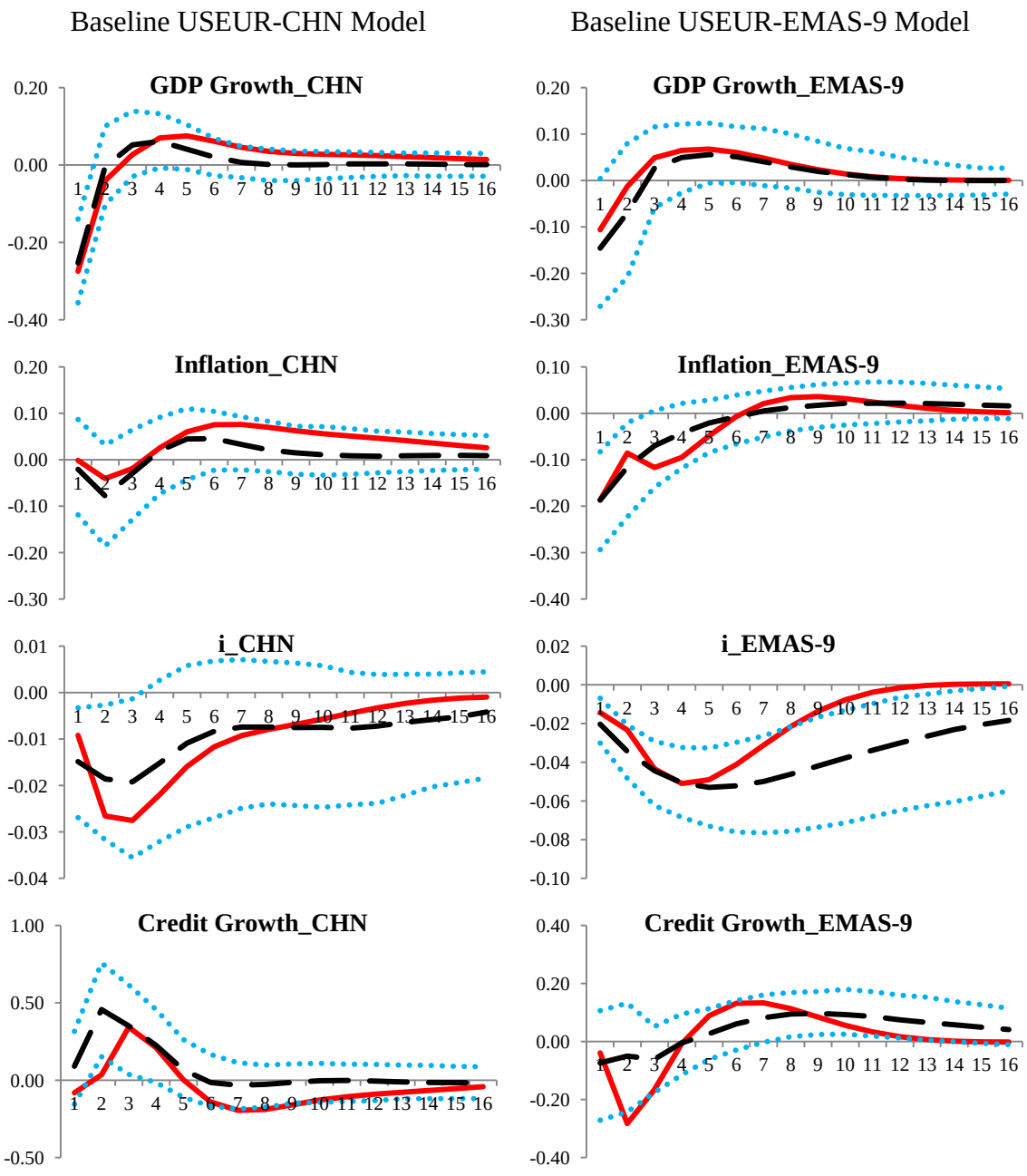
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.5(a). Impulse Responses of
Domestic Variables to a USEUR Adverse **Aggregate Supply** Shock: Baseline Models
(Subsample: 1998Q1 – 2010Q4)



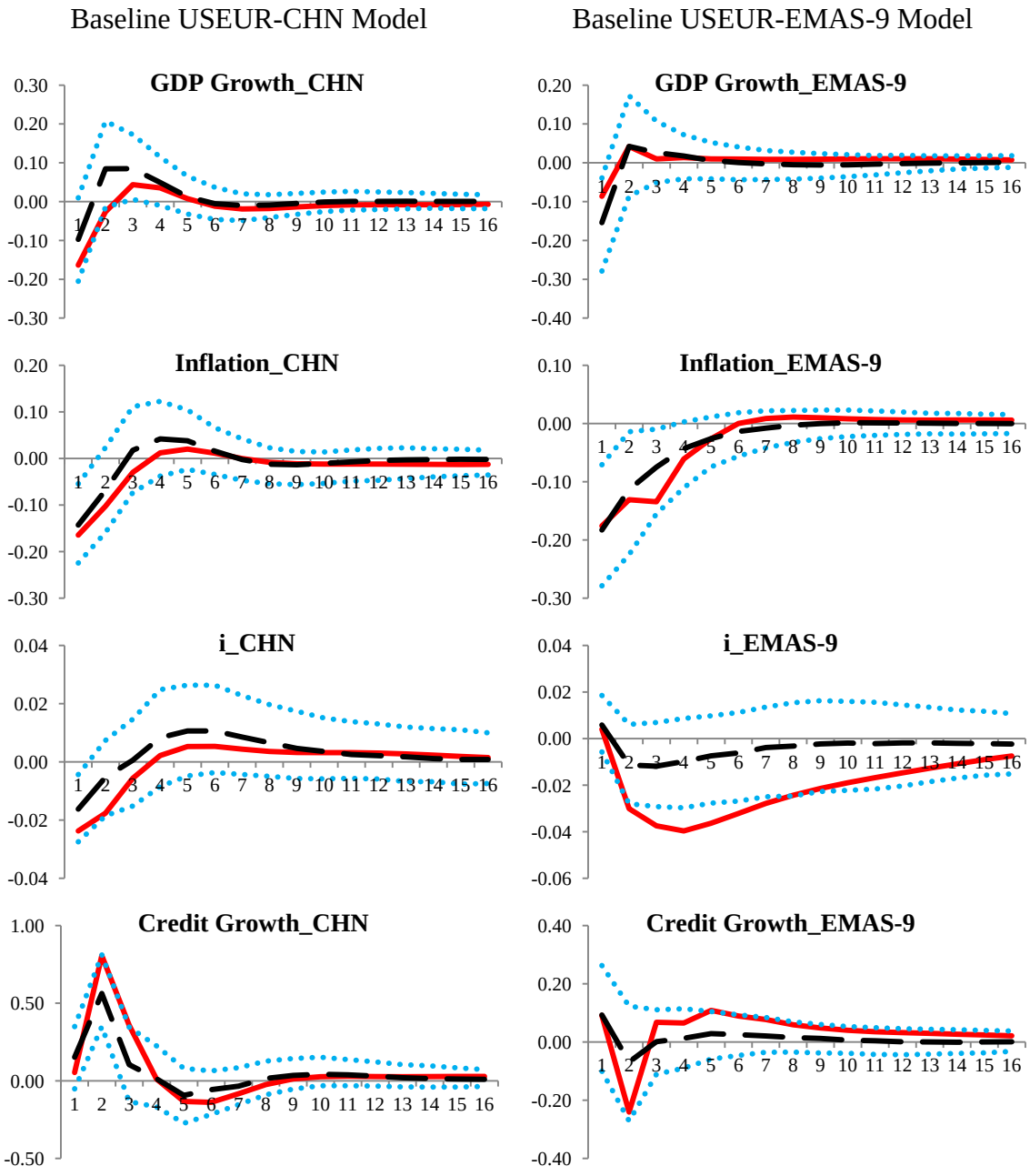
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.5(b). Impulse Responses of
Domestic Variables to a USEUR Adverse **Aggregate Demand** Shock: Baseline Models
(Subsample: 1998Q1 – 2010Q4)



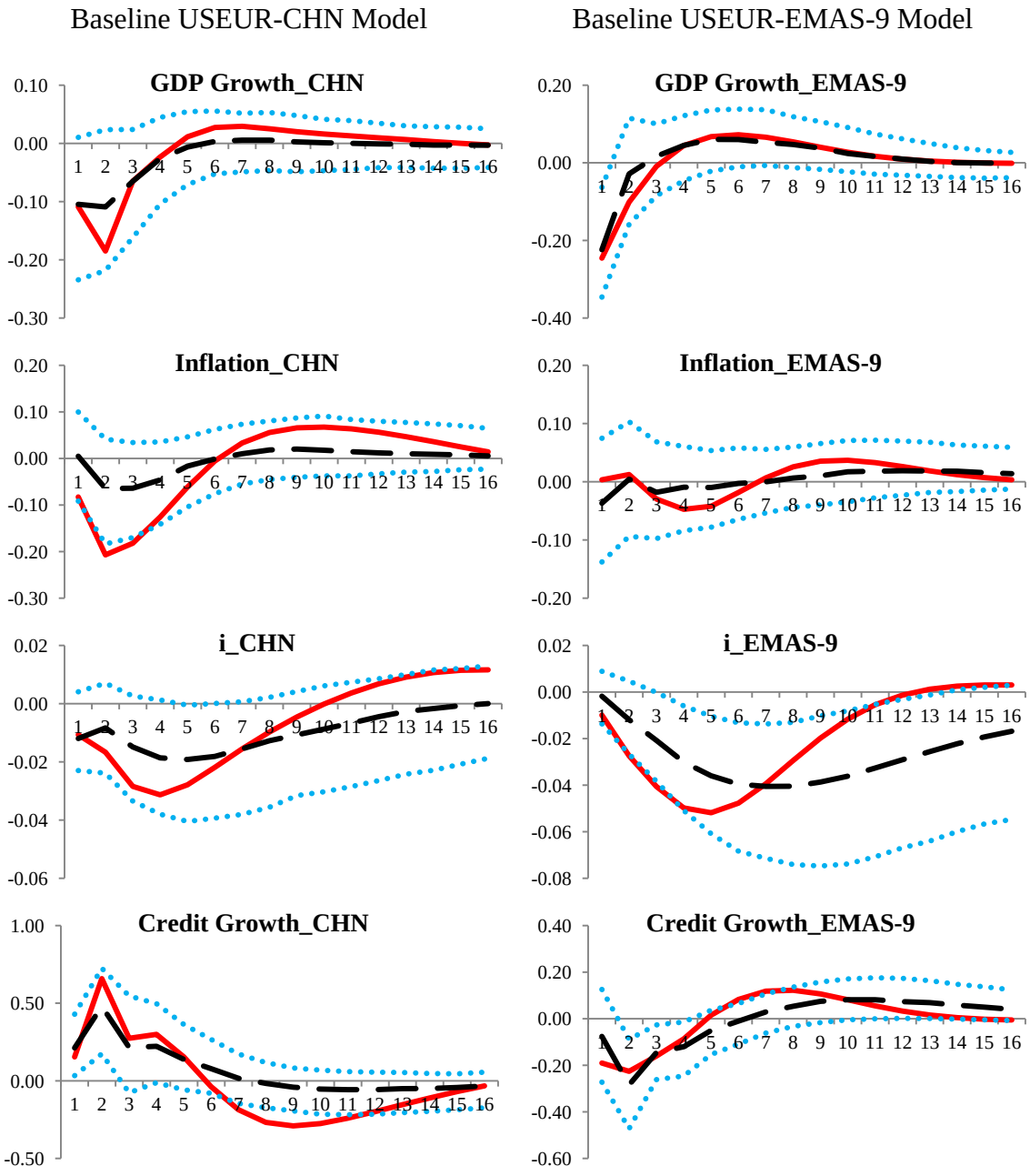
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.5(c). Impulse Responses of
Domestic Variables to a USEUR Contractionary **Monetary Policy** Shock: Baseline Models (Subsample: 1998Q1 – 2010Q4)



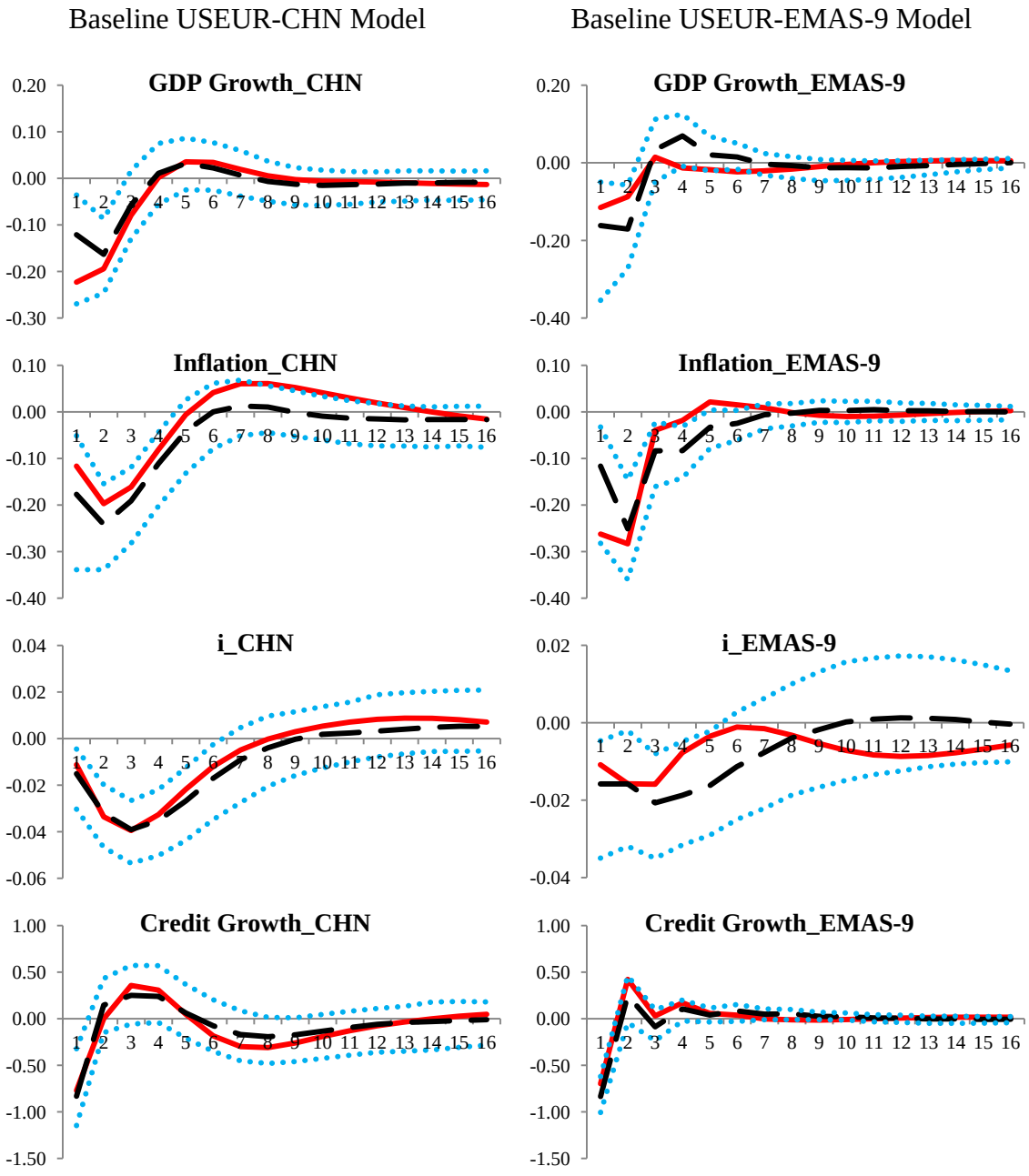
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.5(d). Impulse Responses of
Domestic Variables to a USEUR Adverse **Credit Market** Shock: Baseline Models
(Subsample: 1998Q1 – 2010Q4)



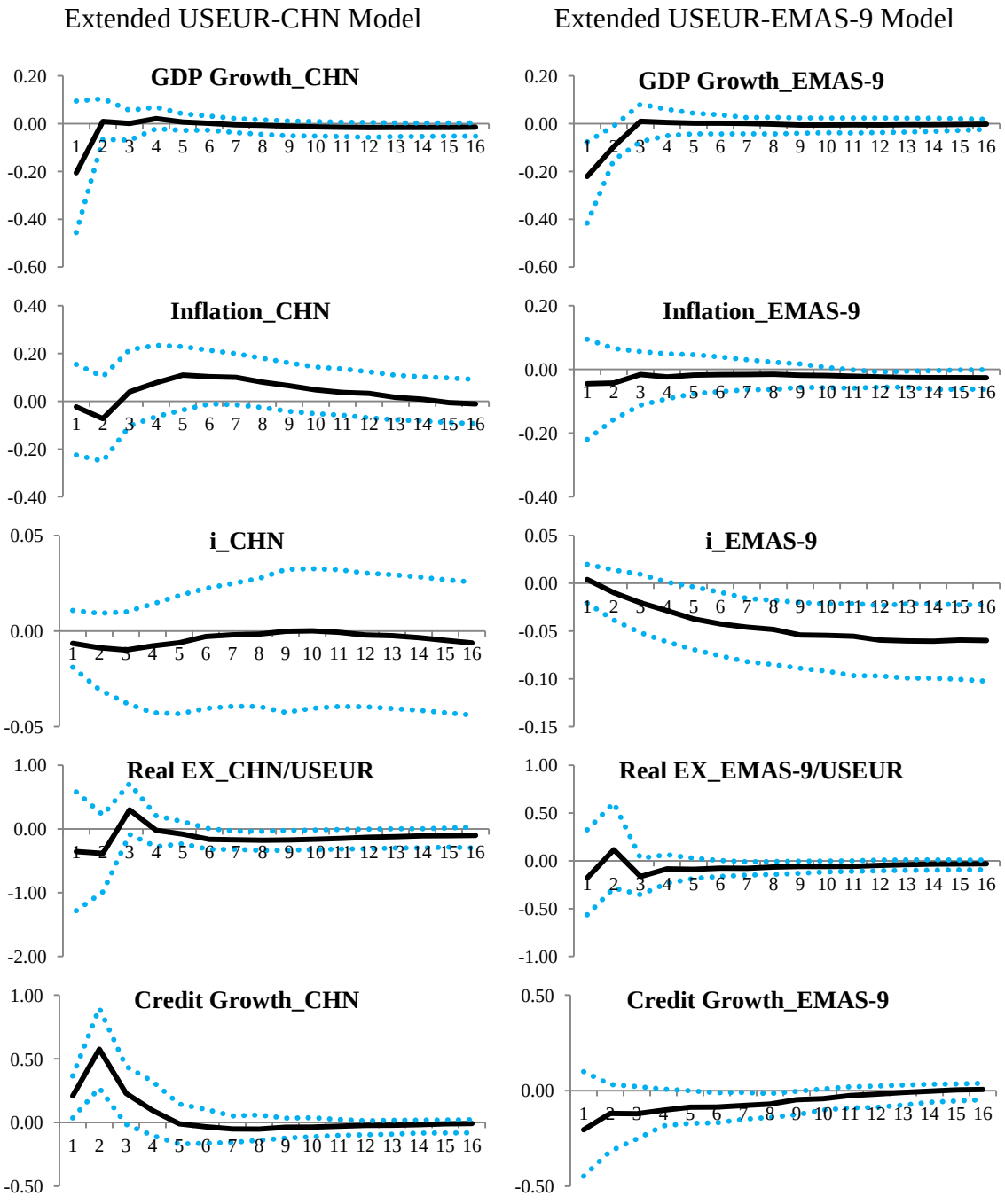
NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.5(e). Impulse Responses of
Domestic Variables to a **Domestic** Adverse **Credit Market** Shock: Baseline Models
(Subsample: 1998Q1 – 2010Q4)



NOTE: These are the impulse responses to one standard error shocks. The solid, dashed, and dotted lines are the median impulse responses using the Fry-Pagan MT method, the ordinary median impulse responses, and the 16th and 84th percentiles error bands, respectively.

Figure 2.6. Impulse Responses of
Domestic Variables to a USEUR Adverse **Credit Market** Shock: Alternative Approach
(Full sample: 1989Q1 – 2010Q4)



NOTE: These are the impulse responses to one standard error shocks. The solid lines are the ordinary median impulse responses, and dotted lines are the 16th and 84th percentiles error bands.

Chapter 3

ASEAN-5 Macroeconomic Forecasting Using a GVAR Model*

*This chapter is co-authored with Thiam Hee Ng.

3.1 INTRODUCTION

There are various time series models for macroeconomic forecasting, which can be generally classified into two different approaches: structural approach and reduced-form approach. Although the structural approach is model-oriented and embeds more economic structures, it usually requires building a complex economic model with multiple parameters. As a result, it is more sensitive parameter estimates and underlying assumptions about the economy relative to the reduced-form approach. It is also computationally more intense and difficult to implement in practice-especially when doing macroeconomic forecasting for more than one country. On the other hand, the reduced-form approach is usually more data-oriented and does not incorporate many economic structures, but it is easier to implement with its smaller computational requirements.

Most time series models belong to the reduced-form approach. For univariate forecasting, autoregressive moving variable (ARMA) and autoregressive integrated moving variable (ARIMA) models are frequently used in literature for stationary and nonstationary time series, respectively; autoregressive conditional heteroskedasticity/generalized autoregressive conditional heteroskedasticity (ARCH/GARCH) models are useful to model time series with time-varying conditional variance, and have great power for estimation and forecasting in finance. For multivariate stationary time series, the vector autoregressive (VAR) model, a generalization of the univariate autoregressive model, is able to capture the evolution and interdependencies among multiple time series. Sims (1980) proposed a Cholesky decomposition method to solve the well-known identification problem of the original VAR system. However, this VAR approach has been criticized as being devoid of any economic content.

Therefore, many economists and econometricians have been trying to come up with new techniques to incorporate the pros of the structural approach into VAR. Sims (1986) and Bernanke (1986) proposed to impose economic restrictions on the regression innovations - known as the Structural Vector Autoregression (SVAR) model. However, one needs to impose too many economic restrictions - $(n^2-n)/2$ restrictions - in an n-variable VAR in order to achieve identification, which is quite difficult and sometimes impossible when there is more than one country in the sample.

Besides these technical issues, another important feature of macroeconomic forecasting we need to consider is the increased globalization and interdependency of the world economy. This has important consequences for conducting monetary and financial policies by central bankers and risk management by commercial bankers. The main motivation for this paper is to forecast the main macroeconomic variables for ASEAN-5. Since these five countries all mid-sized, open economies and are highly affected by other world economic powers such as the United States, it is necessary for us to take into account these external impacts.

In this paper, we employ the global vector autoregressive (GVAR) model orig-

inally introduced by Pesaran *et al.* (2004) and further developed by Dees *et al.* (2007) and Pesaran *et al.* (2009) to construct a macroeconomic forecasting model for the five original ASEAN member countries, i.e. Indonesia, Malaysia, Philippines, Singapore, and Thailand. The advantage of the GVAR model is that it not only incorporates economic structures and global interdependencies of the world economy into the VAR model, but also avoids the identification problem in VAR. Furthermore, there are major differences in the cross-country correlations of various real variables. For instance, equity returns are much more closely correlated across countries than real GDP growth and inflation. This suggests that different channels of transmission should be considered. The GVAR approach allows us to model these different types of links directly, using trade-weighted observable macroeconomic aggregates and financial variables.

The plan of the paper is as follows. Section 3.2 presents the GVAR model, its assumptions, and the estimation strategy. Section 3.3 describes the data used. Section 3.4 presents tests for two assumptions of GVAR, and contemporaneous effects of foreign variables on their domestic counterparts. Section 3.5 presents the forecasting results and evaluation. Section 3.6 offers some concluding remarks.

3.2 THE GVAR MODEL

There are two steps in constructing a GVAR model: building the country-specific models and transforming them into a global VAR model. In this section, we provide an overview of the GVAR framework describe the country-specific models and explain how the global VAR is constructed. We can thus ensure that the forecasts obtained for different countries are internally coherent within the GVAR modeling framework.

3.2.1 Country-Specific VARX* Models

The country-specific model is a VARX* model[1] for each individual country/region. The endogenous variables in most country-specific models include the following core variables:

$$y_{it} = \log(\frac{GDP_{it}}{CPI_{it}}), \pi_{it} = \log(\frac{GDP_{it}}{CPI_{i,t-1}}), e_{it} = \log(\frac{E_{it}}{CPI_{it}}),$$
$$q_{it} = \log(\frac{EQ_{it}}{CPI_{it}}), r_{it} = 0.25 \cdot \log(1 + \frac{R^S_{it}}{100}), p^W_t = \log(P^W_t),$$

[1]VARX* is a vector autoregressive (VAR) model with weakly exogenous variables.

where

GDP_{it} = nominal gross domestic product of country i during period t, (in local currency)

CPI_{it} = consumer price index for country i at time t (with the base year at 100),

E_{it} = exchange rate of country $i$currency at time t in US dollars,

EQ_{it} = nominal equity price index of country i at time t,

R^S_{it} = nominal short-term interest rate per annum of country i at time t, (in percent)

P^W_{it} = world commodity price index at time t.

The typical maturity of short-term interest rates is 3 months. Full details of data sources are provided in Appendix. The U.S. is indexed as country 0, and the exchange rate of the U.S., E_{0t}, is taken to be 1. In the country-specific model for each country/region other than the U.S., the endogenous variables are $(y_{it}, \pi_{it}, r_{it}, e_{it}, q_{it})$; while for the US model, the endogenous variables are $(y_{0t}, \pi_{0t}, r_{0t}, q_{0t}, p^W_t)$. Note that the endogeneity of the world commodity price in the US model reflects the large size of the US economy (it alone accounts for about one-quarter of world output in nominal terms). The real equity price is also included as an endogenous variable to capture the financial market shocks.[2]

The (weakly) exogenous variables in the country-specific VARX* models are trade-weighted foreign core macro-variables (denoted by an'*'). In most country-specific models, foreign variables for country i are constructed as

$$y^*_{it} = \sum_{j=0}^{N} w_{ij} y_{jt}, \quad \pi^*_{it} = \sum_{j=0}^{N} w_{ij} \pi_{jt}, \quad e^*_{it} = \sum_{j=0}^{N} w_{ij} e_{jt},$$
$$q^*_{it} = \sum_{j=0}^{N} w_{ij} q_{jt}, \quad r^*_{it} = \sum_{j=0}^{N} w_{ij} r_{jt},$$

where the weights w_{ij} for $i, j = 0, \ldots, N$[3] are trade weights between country i and country j computed using the simple average of monthly total trade of a country/region during the 2007-2009 period. $w_{ii} = 0$ for any country i. Table 3.1 shows the trade weights within all countries examined here. We use the exogenous variables in Dees *et al.* (2007). In the country-specific model for each country or regional

[2]Interested readers may refer to Dees *et al.* (2007) for the choice of variables. The long-term interest rate is not included in this paper as an endogenous variable as data for ASEAN-5 are unavailable.

[3]In this paper, N is equal to 8, the total number of countries/regions except the U.S.

economy other than the U.S., the exogenous variables are $(y^*_{it}, \pi^*_{it}, r^*_{it}, q^*_{it}, p^W_t)$, while for the U.S. model, the exogenous variables are $(y^*_{0t}, \pi^*_{0t}, e^*_{0t})$.[4] The inclusion of only three foreign variables in the U.S. model reflects the importance of U.S. financial markets within the global financial system.

After specifying the variables included in each country model, we impose the following assumptions imposed by Dees *et al.* (2007): (i) All the country-specific variables are $I(1)$, (ii) the country-specific exogenous variables are weakly exogenous, and (iii) the parameters of the country-specific models remain stable over time. The former two assumptions are tested in Section 3.4. We then proceed to select the order of the individual country VARX*(p_i, q_i) models, where p_i denotes the lag order of endogenous variables (or domestic variables) and q_i denotes the lag order of exogenous variables (or foreign variables). In the empirical analysis that follows, we examine the case where p_i is selected according to the Akaike information criterion (AIC). Due to data limitations, we set q_i, the lag order of the foreign variables, to be 1 for all countries/regions. For the same reason, the maximum p_i is not allowed to be greater than two.

Based on the AIC results, a VARX*(2,1) model is fitted to all nine countries/ regions except Japan, Singapore, and Thailand, where a VARX*(1,1) model is fitted. Therefore, for all the countries except the latter three, the country-specific VARX*(2,1) models can be written as

$$X_{it} = h_{i0} + h_{i1}t + \Phi_{i1}X_{i,t-1} + \Phi_{i2}X_{i,t-2} + \Psi_{i0}X^*_{it} + \Psi_{i1}X^*_{i,t-1} + \varepsilon_{it}, \tag{3.1}$$

where t is a linear time trend. The variables X_{it} and X^*_{it} of the U.S. are different from those of the other five countries. More specifically, $X_{0t} = (y_{0t}, \pi_{0t}, r_{0t}, q_{0t}, p^W_t)'$ and $X^*_{0t} = (y^*_{0t}, \pi^*_{0t}, e^*_{0t})'$ for the U.S., and $X_{it} = (y_{it}, \pi_{it}, r_{it}, e_{it}, q_{it})'$ and $X^*_{it} = (y^*_{it}, \pi^*_{it}, r^*_{it}, q^*_{it}, p^W_t)'$ for the eurozone, the People's Republic of China (PRC), Indonesia, Malaysia, and the Philippines.

The VARX*(1,1) specification fitted to Japan, Singapore, and Thailand is

$$X_{it} = h_{i0} + h_{i1}t + \Phi_{i1}X_{i,t-1} + \Psi_{i0}X^*_{it} + \Psi_{i1}X^*_{i,t-1} + \varepsilon_{it}, \tag{3.2}$$

where the variables X_{it} and X^*_{it} are the same for all the three countries, $X_{it} = (y_{it}, \pi_{it}, r_{it}, e_{it}, q_{it})'$ and $X^*_{it} = (y^*_{it}, \pi^*_{it}, r^*_{it}, q^*_{it}, p^W_t)'$. It is obvious that the VARX*(1,1) specification in (3.2) can be written as (3.1) with $\Phi_{i2} = 0$.

The error term ε_{it} is assumed to be a serially uncorrelated and weakly dependent process across all i's or countries, such that for each t and i, and a set of granular

[4] Although foreign inflation fails the exogeneity test within our sample period, it is still included because it passes the test for a longer data period.

weights w_{ij}, we have

$$\varepsilon_{it}^{*} = \sum_{j=0}^{N} w_{ij}\varepsilon jt \xrightarrow{p} 0,$$

as $N \to \infty$.[5]

3.2.2 Estimation Strategy

There are two main system approaches to estimate the country-specific VARX* models (3.1) and (3.2). The first one is a fully parametric approach based on the vector autoregressive error correction model (VECM) developed by Johansen (1988, 1992) and Pesaran *et al.* (2000). The second one is a semi-parametric procedure based on a triangular formulation of a vector correction model developed by Phillips (1991, 1995). There are pros and cons of the two approaches. Although the latter is more robust to error distributions, it is also more data demanding comparing to the former. Due to data limitations, in this paper we use the former approach developed by Pesaran *et al.* (2000).

Let $Z_{it} = (X_{it}', X_{it}^{*\prime})'$, a vector of both exogenous and endogenous variables for country i at time t, and let k_i and k_i^{*} denote the numbers of domestic and foreign variables in country i respectively. Then the VECM with exogenous variables (denoted by VECMX*) for both the VARX*(2,1) specification (3.1) and the VARX*(1,1) specification (3.2) can be written as

$$\Delta X_{it} = c_{i0} + \alpha_i \beta_i' Z_{i,t-1}^{*} + \Psi_{i0}\Delta X_{it}^{*} + \Gamma_i \Delta X_{i,t-1} + \varepsilon_{it}, \tag{3.3}$$

where $\Gamma_i = 0$ for the countries with VARX*(1, 1) specification, $Z_{it}^{*} = (t, Z_{it}')'$, α_i is a $k_i \times r_i$ matrix with rank r_i, and β_i is a $(k_i + k_i^{*}) \times r_i$ matrix with rank r_i. By partitioning β_i as $\beta_i = (\beta_{it}', \beta_{ix}', \beta_{ix^{*}}')'$ conformable to Z_{it}^{*}, the r_i error-correction terms defined by the above equation can be written as

$$\beta_i' Z_{i,t-1}^{*} = \beta_{it}'(t-1) + \beta_{ix}' X_{i,t-1} + \beta_{ix^{*}}' X_{i,t-1}^{*}, \tag{3.4}$$

which clearly allows for the possibility of cointegration both within X_{it} and between X_{it} and X_{it}^{*}, and consequently across X_{it} and X_{jt} when $i \neq j$.

Under all the assumptions given in Subsection 3.2.1, the estimation of VECMX* in (3.3) is carried out in three steps. First, β_i is estimated by the maximum likelihood (ML) estimator $\hat{\beta}_i$ proposed in Case IV (unrestricted intercepts and restricted

[5]This assumption and the definition of granular weights are not important for our analysis, so we only present the assumption here. However, interested readers may refer to Pesaran *et al.* (2009) and Pesaran *et al.* (2004) for a detailed discussion.

trends)[6] of Pesaran *et al.* (2000). Second, r_i, the rank of β_i, is determined by the maximum eigenvalue and the trace statistics developed by Pesaran *et al.* (2000). Third, as shown by Dees *et al.* (2007), $(c_{i0}, \alpha_i, \Psi_{i0}, \Gamma_i)$ ($\Gamma_i = 0$ in the VARX*(1,1) model) can be consistently estimated by the ordinary least squares (OLS) estimator in regressions of ΔX_{it} on intercepts, the estimated error-correction terms $(\beta_i' \hat{Z}^*_{i,t-1})$, ΔX^*_{it}, and $\Delta X_{i,t-1}$. Note that there is no $\Delta X_{i,t-1}$ in VARX*(1,1) models.

The maximum eigenvalues and the trace statistics for all the country-specific models are summarized in Tables 3.2 and 3.3. It is known that both of these statistics tend to over-reject in small samples, with the extent of over-rejection being much more serious for the maximum eigenvalue as compared with trace statistics. However, Monte Carlo simulations conducted by Cheung and Lai (1993) showed that the maximum eigenvalue test is generally less robust to departures from normal errors than trace statistics. Therefore, we focus on the inferences from trace statistics. Table 3.4 shows the lag orders and the numbers of cointegration relationships for the country-specific VARX* models.

After estimating all the coefficients in (3.3), we can transform them to obtain all the coefficient estimates in the original VARX* models. First, partition $\alpha_i \beta_i'$ as $\alpha_i \beta_i' = (g_{i1}, g_{i2}, g_{i3})$ conformably with $(t-1, X_{t-1}', X_{t-1}^{*\prime})'$.[7] Second, it is straightforward to show that the relationship between the coefficients in (3.1) or (3.2) and those in (3.3) is

$$\begin{cases} h_{i0} = c_{i0} - g_{i1}, \\ h_{i1} = g_{i1}, \\ \Phi_{i1} = I_{k_i} + g_{i2} + \Gamma_i, \\ \Phi_{i2} = -\Gamma_i, \\ \Psi_{i1} = g_{i3} - \Psi_{i0}, \end{cases}$$

for each country i where $\Gamma_i = 0$ for countries with the VARX*(1,1) specification.

3.2.3 Solution of the GVAR Model

Although estimation is done country by country, the GVAR model needs to be solved simultaneously for all endogenous variables in the global economy. The VARX*(2,1) model (3.1) and the VARX*(1,1) model (3.2) can both be written as

$$A_i Z_{it} = h_{i0} + h_{i1} t + B_i Z_{i,t-1} + C_i Z_{i,t-2} + \varepsilon_{it}, \tag{3.5}$$

[6] Here, restricting the time trend is to avoid a quadratic trend in the original model of levels.

[7] The symbol for estimates, $\hat{}$, is omitted for notational simplicity.

where

$$A_i = (I_{k_i}, -\Phi_{i0}), \quad B_i = (\Phi_{i1}, \Psi_{i1}),$$
$$C_i = \begin{cases} (\Phi_{i2},\, 0_{k_i \times k_i}), & \text{for VARX*(2,1) models,} \\ 0, & \text{for VARX*(1, 1) models,} \end{cases}$$
$$k_i = \text{ number of endogenous variables in country } i.$$

It is obvious that A_i, B_i, and C_i are all $k_i \times (k_i + k_i^*)$ matrices. Let $X_t = (X_{0t}', X_{1t}', \ldots, X_{Nt}')'$ be the $k \times 1$ global vector of endogenous variables with $k = \sum_{i=0}^{N} k_i$. It is straightforward to see that the link between X_{it} and the variables in the i-th country-specific model Z_{it} can be expressed by the identity

$$Z_{it} = W_i X_t, \quad i = 0, 1, \ldots, N, \tag{3.6}$$

where W_i is a $(k_i + k_i^*) \times k_i$ 'link' matrix defined by the trade weights. Thus, using the identity, model (3.5) can be written as

$$A_i W_t X_t = h_{i0} + h_{i1} t + B_{i1} W_i X_{t-1} + B_{i2} W_i X_{t-2} + \varepsilon_{it},$$

where $A_i W_i$ and $B_{i1} W_i$ are both $k_i \times k$ matrices. Stacking these equations now yields

$$G X_t = h_0 + h_1 t + H_1 X_{t-1} + h_2 X_{t-2} + \varepsilon_t, \tag{3.7}$$

where

$$h_0 = \begin{pmatrix} h_{00} \\ h_{10} \\ \vdots \\ h_{N0} \end{pmatrix}, \quad h_1 = \begin{pmatrix} h_{01} \\ h_{11} \\ \vdots \\ h_{N1} \end{pmatrix}, \quad G = \begin{pmatrix} A_0 W_0 \\ A_1 W_1 \\ \vdots \\ A_N W_N \end{pmatrix},$$
$$H_1 = \begin{pmatrix} B_{01} W_0 \\ B_{11} W_1 \\ \vdots \\ B_{N1} W_N \end{pmatrix}, \quad H_2 = \begin{pmatrix} B_{02} W_0 \\ B_{12} W_1 \\ \vdots \\ B_{N2} W_N \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_{0t} \\ \varepsilon_{1t} \\ \vdots \\ \varepsilon_{Nt} \end{pmatrix}.$$

It is obvious that G is a $k \times k$ matrix and is generally of full rank and hence invertible. Therefore, the global VAR(2) (GVAR) model can be written as

$$X_t = f_0 + f_1 t + F_1 X_{t-1} + F_2 X_{t-2} + u_t, \tag{3.8}$$

where $f_0 = G^{-1} h_0$, $f_1 = G^{-1} h_1$, $F_1 = G^{-1} H_1$, $F_2 = G^{-1} H_2$, and $u_t = G^{-1} \varepsilon_t$. The GVAR model (3.8) are now ready to be solved recursively forward for macroeconomic

forecasting in the usual manner.

3.3 DATA

The quarterly data set used for estimation and forecasting in this paper covers the period 1991Q1-2009Q4, extending the data set in Pesaran *et al.* (2004) four more years. More specifically, the data used for estimation cover 1991Q1-2008Q4, and the out-of-sample one quarter ahead forecasts are from 2009Q1 to 2009Q4.

The main data source is the CEIC database, which contains the International Financial Statistics (IFS) database from the International Monetary Fund and national statistics from countries' central banks. The nine countries or regional economies considered in this paper are the U.S., eurozone (Austria, Belgium, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain), the People's Republic of China (PRC), Japan, and the ASEAN-5. A detailed description of the data which include more countries/regions is provided in Appendix.

3.4 TESTS

3.4.1 Unit Root Tests

Although the GVAR methodology can be applied to stationary and/or integrated variables, the assumption made by Pesaran *et al.* (2004), Dees *et al.* (2007), and Pesaran *et al.* (2009) that all variables in the country-specific models are integrated of order one, i.e. $I(1)$, still plays an important role. This assumption allows us to distinguish short- and long-run relationships and interpret the latter as cointegration. Therefore, we begin our tests by examining the integration properties of the individual series under consideration. Due to the widely accepted poor power performance of the traditional augmented Dickey-Fuller (ADF) tests, we follow the existing literature and use the ADF-GLS statistics developed by Elliot, Rothenberg, and Stock (1996) for all the time series. The results are presented in Table 3.5. With only a few exceptions, the $I(1)$ assumption cannot be rejected for most of the endogenous and exogenous variables under consideration.[8]

3.4.2 Testing Weak Exogeneity

The main assumption underlying the estimation strategy is the weak exogeneity of X_{it}^{*} with respect to the long-run parameters of the VECMX* model defined by

[8]Nevertheless, some of the variables used in the country-specific models seem to be $I(2)$, for instance, U.S. inflation, and some even seem to be $I(3)$ such as Japanese inflation.

(3.3). Following Dees *et al.* (2007), we can test the weak exogeneity by testing the joint significance of the estimated error-correction terms defined in (3.4) for the country-specific foreign variables and world commodity prices. In particular, for each l-th element of X^*_{it}, the following regression is carried out:

$$\Delta X^*_{it,l} = \mu_{il} + \sum_{j=1}^{r_i} \gamma_{i,j,l} ECM^j_{i,t-1} + \sum_{k=1}^{S_i} \phi_{ik,l} \Delta X_{i,t-k} + \nu_{i1,l} \Delta \tilde{X}^*_{i,t-1} + \zeta_{it,l}, \qquad (3.9)$$

where $ECM^j_{i,t-1}$, $j = 1, 2, \ldots, r_i$ are the estimated error-correction terms corresponding to the r_i cointegrating relationships found for the i-th country model, $s_i = p_i$ (the lag order of endogenous variables in the i-th country model), and $\Delta \tilde{X}^*_{it} = (\Delta X^{*\prime}_{it}, \Delta e^*_{it}, \Delta p^W_t)'$. Note that in the case of the U.S. model, the term Δe^*_{it} is implicitly included in ΔX^*_{it}.

The test for weak exogeneity is an F-test of the joint hypothesis that $\gamma_{i,j,l} = 0$ in the above regression for all $j = 1, 2, \ldots, r_i$. The test results are presented in Table 3.6. We can see that the weak exogeneity hypothesis is only rejected for inflation in the U.S. model and the short-term interest rates in the Thai model among all time series of the countries. As expected, foreign real equity prices and foreign short-term interest rates cannot be considered as weakly exogenous and have not been included in the U.S. model, which justifies the importance of the U.S. financial markets in the global financial system.

3.4.3 Other Features of the Country-Specific Models

Due to data limitations and the relatively large number of endogenous and exogenous variables involved, we are forced to set the lag order of exogenous variables for all country-specific models at one. It is therefore important to check the adequacy of the country-specific models in dealing with the complex dynamic interrelationships that exist in the world economy. Table 3.7 presents the F-statistics for Breusch-Godfrey LM tests of serial correlation of order 4 among the errors of the error-correction regressions for all 45 endogenous variables in the GVAR model.

Considering the relative simplicity of the underlying models, it is comforting that 35 of the 45 regressions pass the residual serial correlation test at the 95% level. In particular, if we focus on ASEAN-5, only 4 out of the 20 regressions fail to pass the test at the 95% level. These test results, together with the weak exogeneity of the foreign variables, also allow for consistent estimation of the contemporaneous effects of foreign-specific variables on their domestic counterparts (at least for the ones where the error serial correlation test is not statistically significant).

3.4.4 Contemporaneous Effects of Foreign Variables on Domestic Counterparts

Table 3.8 presents the contemporaneous effects of foreign variables on their domestic counterparts together with Newey-West heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimator. These estimates can be interpreted as impact elasticities between domestic and foreign variables. In Singapore, for example, a 1% change in foreign real GDP in a given quarter leads to an increase of 1.14% in domestic real GDP in the same quarter. Similar foreign elasticities are obtained across different countries/regions.

Most of these elasticities are significant and have a positive sign, which are consistent with the results in Dees *et al.* (2007), except the foreign short-term interest rates of the PRC and the foreign inflation of Japan. Focusing on ASEAN-5, foreign real GDP in Malaysia, Singapore, and Thailand, and foreign inflation in Malaysia, the Philippines, and Thailand have significant and positive contemporaneous effects on their domestic counterparts. In addition, the foreign equity prices in all ASEAN-5 countries have significant and positive effects on their domestic counterparts, suggesting contemporaneous financial links are likely to be very strong among ASEAN-5 economies through the equity market channel. Another interesting finding is that neither foreign real GDP nor foreign inflation has a significant effect on their domestic counterparts in the PRC and Indonesia-which may imply the two economies are not as open as the other countries/regions under consideration.

3.5 FORECAST AND EVALUATION

3.5.1 Forecast

We compute the one-quarter-ahead forecasts for 2009Q1-2011Q4. Figure 3.1 presents real GDP growth forecasts for all the countries under consideration. We can see that the real GDP growth forecasts for all countries expect Thailand have a clear downward trend in 2011. Figures 3.2-3.5 present forecasts for inflation, short-term interest rates, real exchange rates, and real equity prices respectively.

It is necessary to evaluate how well these forecasts perform compared with other models. We consider two benchmark models used in the forecast evaluation. We apply the panel Diebold and Mariano (DM) test developed by Pesaran *et al.* (2009) which allows us to statistically test the GVAR forecasts against our benchmark models for a given variable. Note that we have only four one-quarter-ahead forecasts (obtained over 2009Q1-2009Q4) for each of the variables and for each country, which is insufficient for statistical testing of forecasts for individual countries. However, by pooling forecast errors for the same variable across different countries, the panel DM

test is able to take into account the panel nature of the pooled series.

3.5.2 Benchmark Models

We compare the forecast performance of the GVAR model with that of forecasts from country-specific VAR(2) models, with and without trend. The specifications of the two benchmark models are:

$$\begin{cases} \text{Country-specific VAR(2): } X_{it} = a + \gamma_1 X_{i,t-1} + \gamma_2 X_{i,t-2} + \xi_{it}; \\ \text{Country-specific VAR(2) with trend: } X_{it} = a + bt + \gamma_1 X_{i,t-1} + \gamma_2 X_{i,t-2} + \eta_{it}; \end{cases}$$

where $i = 1, 2, \ldots, N$.

We choose VAR(2) models for two reasons: (i) they usually perform very well in forecasting; and more importantly, (ii) the only feature they don't possess compared with the GVAR model is global interdependency. Thus, if the GVAR model outperforms these VAR(2) models, then the global interrelationships should be considered important in forecasting. The forecast is based on the conditional expectation in the usual manner for VAR models.

3.5.3 Forecast Evaluation

To illustrate how the panel DM test works, let's consider the following loss differential of forecasting the variable j in country i, using method A relative to method B

$$z_{ijt} = (e^A_{ijt})^2 - (e^B_{ijt})^2,$$

where

$$e_{ijt} = y_{ijt} - \hat{y}_{ij,t|t-1,t-2}$$

is the one-quarter-ahead forecast error of y_{ijt} formed at time $t-1$. y_{ijt} is the actual value at time t, and $\hat{y}_{ij,t|t-1,t-2}$ is the forecast formed at time $t-1$. $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, k$, and $t = 1, 2, \ldots, n$ where $m = 9$ is the total number of countries, k is the total number of variables, and $n = 4$ is the forecast sample. Let method A stand for the GVAR model and method B stand for the benchmark models.

The panel DM statistic developed by Pesaran *et al.* (2009) is defined as follows. For a given variable (say, real GDP growth), consider

$$z_{it} = \alpha_i + \varepsilon_{it},$$

with the following null and alternative hypotheses

$$H_0 : \ \alpha_i = 0, \quad \text{v.s.} \quad H_1 : \ \alpha_i < 0 \text{ for some } i,$$

where the variable index j is suppressed for notational simplicity. As shown by Pesaran *et al.* (2009), if we assume that ε_{it} follows independently and identically distributed (i.i.d.) $\mathbf{N}(0, 1)$, then under the null hypothesis H_0, the DM statistic

$$\bar{DM} = \frac{\bar{z}}{\sqrt{V(\bar{z})}} \sim \mathbf{N}(0, 1),$$

where

$$\bar{z} = \frac{1}{m}\sum_{i=1}^{m} \bar{z}_i, \quad V(\bar{z}) = \frac{1}{mn}\left(\frac{1}{m}\sum_{i=1}^{m} \hat{\sigma}_i^2\right),$$

with the over-time sample mean and variance defined as

$$\hat{\sigma}_i^2 = \frac{1}{n-1}\sum_{t=1}^{n}(z_{it} - \bar{z}_i)^2, \quad \bar{z}_i = \frac{1}{n}\sum_{t=1}^{n} z_{it}.$$

It should be noted that the panel DM test defined above is a one-sided test, and therefore the relevant 1% and 5% critical values are -2.326 and -1.645, respectively. Since we assume that method A denotes the GVAR model, a positive value of the panel DM statistic shows evidence against the null hypothesis that the GVAR model outperforms the benchmark models in terms of forecasting.

Table 3.9 presents the panel DM statistics for the one-quarter-ahead GVAR forecasts relative to the two benchmark models. We can see that there is no evidence against the GVAR forecasts as the statistics for all variables are negative. In particular, the statistics for the short-term interest rates compared with both benchmarks are significantly negative at the 5% level, suggesting that the forecasts from our GVAR model are very likely to beat those from the two benchmark VAR(2) models with or without trend.

3.6 CONCLUSIONS

This paper examines and evaluates the macroeconomic forecasts for ASEAN-5 countries in the context of a global vector autoregressive (GVAR) model covering twenty countries grouped into nine country/regions. We generate twelve one-quarter-ahead forecasts of real GDP growth, inflation, short-term interest rates, real exchange rates, real equity prices, and world commodity prices over the period 2009Q1-2011Q4, with four out-of-sample forecasts during 2009Q1-2009Q4. The out-of-sample forecasts are compared with country-specific vector autoregressive models with and without trend. The forecast evaluation results indicate that the GVAR forecasts tend to outperform forecasts based on individual country-specific models (VAR(2) benchmark models), especially for short-term interest rates and real equity prices, implying the

importance of interdependencies among countries in global financial markets.

There are many extensions we can make in the future. One is to apply Phillips' semi-parametric approach, the fully modified vector autoregression (FMVAR)-robust to error distributions-to estimate the country-specific VARX* models, and compare the results with the results in this paper. One extension is to include more countries in the sample as well as to extend the sample period. Another extension is to estimate a variety of models with different lag orders p_i and q_i, and estimation windows, and to use the Bayesian model averaging to integrate the uncertainties that prevail across models and across estimation windows.[9] Obviously, these three extensions can be made simultaneously.

[9]Interested readers may refer to Pesaran *et al.* (2009) for more details on Bayesian model averaging.

APPENDIX: DATA DESCRIPTION

A1: Real GDP

Real gross domestic product (GDP) data of all the countries come from the CEIC database except those of the People's Republic of China (PRC), which is obtained from Oxford Economics. More specifically, the International Monetary Fund's International Financial Statistics (IFS) seasonally adjusted real GDP series is used for Canada, and the real GDP series from IFS is used for Indonesia. The real GDP series from Oxford Economics is used for the PRC, and the seasonally adjusted nominal GDP series (deflated by the seasonally adjusted Harmonized Consumer Price Index) from the European Central Bank is used for the eurozone. Data from the Economic and Social Research Institute are used for Japan's seasonally adjusted real GDP, Malaysia's real GDP data are from the Department of Statistics, the Philippines seasonally adjusted real GDP data are from the National Statistical Co-ordination Board, and Singapore's Ministry of Trade provided its seasonally adjusted real GDP data. Thailand's National Economic and Social Development Board is the source for its real GDP series (1993Q1-2009Q4)-and for the rest of the sample period, the annual real GDP series from the same data source is used to interpolate the quarterly series. The U.S. Bureau of Economic Analysis provided its seasonally adjusted real GDP data.

The procedure proposed in Dees *et al.* (2007) was used to assess the joint significance of seasonal components, and seasonal adjustments were then applied to the real GDP series using the US Census Bureau's X12 program in Eviews 5.0 software for the following countries: the PRC, Indonesia, Malaysia, and Thailand, whose seasonal components all have great joint significance above the critical level.

Interpolation from annual to quarterly series was conducted for Thailand (1991Q1-1992Q4) using the exponential interpolation procedure described in Supplement A of Dees *et al.* (2007) as quarterly data were unavailable.

A2: Consumer Price Index (CPI)

IFS CPI data from the CEIC database were used for Indonesia, Japan, Malaysia, the Philippines, Singapore, Thailand, and the U.S. For the PRC, the HAVER Analytics data from Pesaran *et al.* (2009) (Consumer Price Index (SA, 2000 = 100), source: China National Bureau of Statistics and HAVER Analytics) was used for 1991Q1-2005Q4. The rate of percent changes from IFS data was applied to extend the series to the sample period. For the eurozone, the Harmonized Consumer Price Index (HICP) series in the CEIC database was collected from European Central Bank.

Seasonal adjustments were applied to CPI data for the eurozone, Japan, Thailand, and the U.S., as described above. Seasonal adjustments were not applied to the PRC, Indonesia, Malaysia, the Philippines, and Singapore, because their seasonal

components did not have great significance.

A3: Short-term Interest Rates

IFS data in the CEIC database are used as the main source for short-term interest rates, with a typical maturity of 3 months. The Money Market Rate series from IFS is used for Indonesia, Japan, the Philippines, Singapore, and Thailand. The Treasury Bill Rate series from IFS is used for Malaysia, and the U.S. The Time Deposit Rate (3 months) series in the CEIC database collected from the People's Bank of China is used for the PRC. The Euro Interbank Rate (3 months) series for 12 eurozone countries in the CEIC database collected from European Central Bank. Unlike in Pesaran *et al.* (2009), we did not use any combined or artificially generated series for short-term interest rates.

A4: Exchange Rates

The exchange rate of the U.S. is normalized to be 1. The Official Rate (period average, USD per national currency) series from the IFS is used for all the others except the eurozone. The FX Reference Rate (ECB) series in CEIC collected from European Central Bank is used for the eurozone. No combined or artificially generated data are used.

A5: Equity Price Indices

The IFS Share Price Index series in the CEIC database is used for the PRC, Japan, Malaysia, Singapore, and US. The Equity Market Index series in the CEIC database is the main source for the other countries. The Equity Market Index series from Dow Jones Euro Stoxx is used for the eurozone. The Equity Market Index series from the Jakarta Stock Exchange is used for Indonesia. The Equity Market Index series from Philippine Stock Exchange is used for the Philippines. The Equity Market Index series from The Stock Exchange of Thailand is used for Thailand. No combined or artificially generated data are used.

A6: Fuel and Non-fuel Commodity Price Index

The quarterly Thomson Reuters/Jefferies CRB Index series from Bloomberg is used, which includes 19 commodities representing all commodity sectors-energy (39%), grains and agricultural products (34%), base metals (13%), precious metals (7%), and livestock (7%).

REFERENCES

Bernanke, B., 1986, Alternative Explanations of the Money-Income Correlation, *Carnegie-Rochester Conference Series on Public Policy*, Vol.25, pp.49-99.

Cheung, Y.W., and K.S., Lai, 1993, Finite-Sample Sizes of Johansen's Likelihood Ration Tests for Conintegration, *Oxford Bulletin of Economics and Statistics*, Vol.55, pp.313-328.

Dees, S., di Mauro, F., Pesaran, M.H., and L.V. Smith, 2007, Exploring the International Linkages in the Euro Area: A Global VAR Analysis, *Journal of Applied Econometrics*, Vol.22(1), pp.1-38.

Elliot, G., Rothenberg, T.J., and J.H. Stock, 1996, Efficient Tests for an Autoregressive Unit Root, *Econometrica*, Vol.64(4), pp.813-836.

Johansen, S., 1988, Statistical Analysis of Cointegration Vectors, *Journal of Economic Dynamics and Control*, Vol.12, pp.231-254.

Johansen, S., 1992, Cointegration in Partial Systems and the Efficiency of Single-equation Analysis, *Journal of Econometrics*, Vol.52(3), pp.389-402.

Koop, G., Pesaran, M.H., and S. M. Potter, 1996, Impulse Response Analysis in Nonlinear Multivariate Models, *Journal of Econometrics*, Vol.74(1), pp.119-147.

Pesaran, M.H., Schuermann, T., and L.V. Smith, 2009, Forecasting Economic and Financial Variables with Global VARs, *International Journal of Forecasting*, Vol.25(4), pp.642-675.

Pesaran, M.H., Schuermann, T., and S.M. Weiner, 2004, Modeling Regional Interdependencies Using a Global Error-correcting Macroeconometric Model, *Journal of Business and Economic Statistics*, Vol.22(2), pp.129-162.

Pesaran, M.H., and Y. Shin, 2002, Long-run Structural Modeling, *Econometric Reviews*, Vol.21(1), pp.49-87.

Pesaran, M.H., Shin, Y., and R.J. Smith, 2000, Structural Analysis of Vector Error Correction Models with Exogenous $I(1)$ Variables, *Journal of Econometrics*, Vol.97(2), pp.293-343.

Phillips, P.C.B., 1991, Optimal Inference in Cointegrated Systems, *Econometrica*, Vol.59(2), pp.283-306.

Phillips, P.C.B., 1995, Fully Modified Least Squares and Vector Autoregression, *Econometrica*, Vol.63(5), pp.1023-1078.

Sims, C.A., 1980, Macroeconomics and Reality, *Econometrica*, Vol.48, pp.1-48.

Sims, C.A., 1986, Are Forecasting Models Usable for Policy Analysis?, *Quarterly Review of the Federal Reserve Bank of Minneapolis*, Vol.10, pp.2-16.

Table 3.1. Trade Weights Based on Direction of Trade Statistics

	US	eurozone	PRC	Japan	Indonesia	Malaysia	Philippines	Singapore	Thailand
US	0.000	0.346	0.317	0.149	0.024	0.051	0.020	0.0537	0.0390
eurozone	0.429	0.000	0.330	0.119	0.021	0.031	0.009	0.0373	0.0247
PRC	0.307	0.281	0.000	0.233	0.026	0.046	0.025	0.0474	0.0352
Japan	0.271	0.168	0.338	0.000	0.051	0.047	0.024	0.0413	0.0607
Indonesia	0.123	0.125	0.152	0.228	0.000	0.085	0.016	0.2140	0.0574
Malaysia	0.178	0.137	0.161	0.160	0.053	0.000	0.021	0.2123	0.0773
Philippines	0.215	0.152	0.144	0.207	0.028	0.061	0.000	0.1308	0.0619
Singapore	0.161	0.139	0.170	0.105	0.131	0.202	0.033	0.0000	0.0607
Thailand	0.157	0.134	0.185	0.260	0.053	0.097	0.028	0.0853	0.0000

NOTE: US = United States; PRC = People's Republic of China. Trade weights are computed as shares of exports and imports, displayed in rows by region (such that a row, but not a column, sum to 1). Source: *Direction of Trade Statistics*, 2007–2009, International Monetary Fund.

Table 3.2. Cointegration Rank Statistics for the U.S. Model
(3 exogenous variables)

			Critical Values*	
H_0	H_1	US	95%	90%
		Maximum Eigenvalue Statistics		
r = 0	r = 1	72.11	46.84	43.92
r = 1	r = 2	48.93	40.98	38.04
r = 2	r = 3	**24.81**	34.65	31.89
r = 3	r = 4	19.09	27.80	25.28
r = 4	r = 5	8.92	20.47	18.19
		Trace Statistics		
r = 0	r ≥ 1	173.87	120.00	114.70
r = 1	r ≥ 2	101.76	90.02	85.59
r = 2	r ≥ 3	**52.82**	63.54	59.39
r = 3	r ≥ 4	28.01	40.37	37.07
r = 4	r ≥ 5	8.92	20.47	18.19

NOTE: * The critical values are obtained from Table 6 (d) in Pesaran *et al*. (2000) as the country-specific models in Section 2 belong to case IV in their paper. US = United States.

Table 3.3. Cointegration Rank Statistics for Non-US Countries/East Asia
(5 exogenous variables)

H_0	H_1	eurozone	PRC	Japan	Indonesia	Malaysia	Philippines	Singapore	Thailand	Critical Values* 95%	90%
					Maximum Eigenvalue Statistics						
r = 0	r = 1	89.80	85.20	104.20	77.74	72.91	74.65	53.80	77.92	52.63	49.59
r = 1	r = 2	52.58	54.63	85.79	53.02	53.07	50.00	**44.53**	58.73	46.66	43.66
r = 2	r = 3	43.29	**33.15**	**30.35**	43.84	**38.92**	**28.40**	35.16	40.49	40.12	37.28
r = 3	r = 4	**27.45**	24.17	28.03	34.79	27.72	23.79	28.32	38.26	33.26	30.54
r = 4	r = 5	21.11	20.30	17.81	**26.48**	19.40	18.55	11.07	**20.39**	25.70	23.11
					Trace Statistics						
r = 0	r ≥1	234.22	217.45	266.18	235.87	212.01	195.39	172.87	235.79	141.20	135.80
r = 1	r ≥2	144.42	132.25	161.99	158.14	139.11	120.74	119.07	157.87	107.60	102.50
r = 2	r ≥3	91.84	77.62	**76.20**	105.11	86.04	**70.74**	**74.55**	99.14	76.82	72.33
r = 3	r ≥4	**48.55**	**44.46**	45.85	61.28	**47.12**	42.34	39.39	58.66	49.52	46.10
r = 4	r ≥5	21.11	20.30	17.81	**26.48**	19.40	18.55	11.07	**20.39**	25.70	23.11

NOTE: * The critical values are obtained from Table 6(d) in Pesaran *et al.* (2000) as the country-specific models in Section 2 belong to case IV in their paper. PRC= People's Republic of China.

Table 3.4. VARX* Order and Number of Cointegrating Relationships in Country-Specific Models

	VARX*(p_i, q_i)		
Country/Region	p_i	q_i	# Cointegrating relationships
US	2	1	2
eurozone	2	1	3
PRC	2	1	3
Japan	1	1	2
Indonesia	2	1	4
Malaysia	2	1	3
Philippines	2	1	2
Singapore	1	1	2
Thailand	1	1	4

NOTE: US = United States; PRC = People's Republic of China.

Table 3.5. ADF-GLS Unit Root Test Statistics (based on AIC order selection)

Domestic Variables	US	eurozone	PRC	Japan	Indonesia	Malaysia	Philippines	Singapore	Thailand
y	-0.58	-2.06	-1.14	-2.41	-2.48	-1.91	-1.31	-1.90	-1.68
Δy	-2.29	-4.95	-3.66	-4.97	-6.51	-3.91	-6.15	-5.95	-4.81
$\Delta^2 y$	-13.49	-6.70	-12.50	-6.84	-14.04	-3.51	-12.00	-12.40	-10.20
π	-1.80	-2.52	-1.68	-0.88	-3.82	-1.80	-1.34	-3.26**	-2.19
$\Delta\pi$	-1.49*	-8.98	-7.28	-1.14*	-9.35	-4.61	-0.74*	-10.52	-4.12
$\Delta^2\pi$	-8.95	-9.99	-11.40	-1.78*	-16.55	-3.25	-0.51*	-13.62	-7.75
r	-2.52	-1.45	-2.16	-1.45	-2.36	-2.11	-2.35	-3.13**	-2.81
Δr	-3.09	-3.19	-2.37	-3.31	-4.62	-8.24	-9.59	-4.69	-6.47
$\Delta^2 r$	-9.64	-6.38	-4.14	-2.70	-10.24	-15.74	-6.41	-6.57	-10.56
e		-1.49	-0.98	-1.87	-1.79	-2.10	-1.90	-1.63	-1.14
Δe		-3.11	-4.17	-5.50	-3.76	-4.83	-5.06	-4.90	-4.46
$\Delta^2 e$		-5.29	-14.66	-8.07	-14.78	-8.98	-10.43	-7.40	-10.39
q	-0.42	-1.14	-2.44	-2.70	-2.35	-2.49	-1.60	-2.64	-1.59
Δq	-4.35	-5.26	-5.67	-4.79	-4.51	-5.39	-6.25	-5.39	-6.86
$\Delta^2 q$	-9.41	-9.00	-7.05	-5.90	-5.49	-8.11	-7.14	-8.37	-9.01
y^*	-1.71	-1.23	-2.18	-1.45	-1.52	-1.77	-1.48	-2.28	-2.10
Δy^*	-2.52	-2.89	-2.70	-2.85	-2.53	-2.26	-2.00*	-3.53	-2.99
$\Delta^2 y^*$	-8.80	-10.46	-6.60	-10.42	-10.46	-8.67	-8.88	-6.96	-10.03
π^*	-2.27	-2.23	-1.60	-2.42	-2.68	-3.10**	-2.93	-3.85**	-3.02
$\Delta\pi^*$	-9.63	-9.72	-1.63*	-8.92	-8.53	-8.45	-8.53	-8.97	-9.21
$\Delta^2\pi^*$	-14.21	-13.60	-5.17	-12.99	-12.90	-14.80	-12.88	-13.25	-16.16
r^*		-2.32	-1.30	-1.58	-1.39	-1.60	-1.25	-1.90	-1.41
Δr^*		-2.74	-2.23	-1.60*	-3.19	-2.57	-2.36	-2.04*	-2.01*
$\Delta^2 r^*$		-6.95	-6.30	-3.15	-7.33	-6.99	-7.91	-4.01	-4.31
e^*	-0.58	-0.57	-1.45	-0.88	-0.97	-0.92	-0.93	-1.29	-0.97
Δe^*	-4.37	-6.84	-4.13	-3.25	-4.89	-5.16	-4.89	-3.47	-5.30
$\Delta^2 e^*$	-7.52	-12.66	-6.23	-11.28	-7.11	-8.53	-7.52	-11.27	-8.35
q^*		-2.04	-1.87	-2.13	-2.49	-2.43	-2.38	-2.69	-2.52
Δq^*		-2.08*	-4.18	-2.08*	-4.23	-4.42	-4.15	-4.68	-4.12
$\Delta^2 q^*$		-7.96	-6.15	-9.27	-9.13	-9.47	-9.02	-8.78	-9.32
p	-1.70	-1.70	-1.70	-1.70	-1.70	-1.70	-1.70	-1.70	-1.70
Δp	-2.24	-2.24	-2.24	-2.24	-2.24	-2.24	-2.24	-2.24	-2.24
$\Delta^2 p$	-9.17	-9.17	-9.17	-9.17	-9.17	-9.17	-9.17	-9.17	-9.17

NOTE: US = United States; PRC = People's Republic of China. * denotes fail to reject the null hypothesis of unit root at the 95% confidence level, and ** denotes reject the null hypothesis of unit root at the 95% confidence level.

Table 3.6. *F*-statistics for Testing Weak Exogeneity of Country-Specific Foreign Variables and Commodity Prices

Country	Distribution	Foreign variables					
		y^*	π^*	r^*	e^*	q^*	P
US	$F(2, 53)$	0.72	3.86*		0.93		
eurozone	$F(4, 54)$	1.24	0.39	1.08		0.67	0.52
PRC	$F(3, 49)$	0.56	1.90	0.87		1.01	0.95
Japan	$F(3, 55)$	1.93	1.19	0.22		0.68	0.10
Indonesia	$F(3, 55)$	0.82	0.23	1.15		0.49	0.14
Malaysia	$F(3, 55)$	1.53	0.62	1.16		2.65	1.76
Philippines	$F(2, 50)$	0.53	0.80	2.41		0.16	0.27
Singapore	$F(2, 56)$	0.05	0.33	0.15		0.96	1.39
Thailand	$F(4, 54)$	1.12	2.22	3.85*		0.87	2.03

NOTE: US = United States; PRC = People's Republic of China. * denotes statistical significance at the 95% confidence level.

Table 3.7. *F*-statistics for Tests of Residual Serial Correlation in Country-Specific VARX* Models

Country		y	π	r	e	Q	p
US	$F(4, 55)$	3.84*	1.29	0.57		1.40	3.20*
eurozone	$F(4, 57)$	0.54	3.67*	2.68*	1.65	0.12	
PRC	$F(4, 52)$	1.05	0.27	1.62	0.14	3.51*	
Japan	$F(4, 59)$	1.44	2.15	2.15	2.80*	2.34	
Indonesia	$F(4, 58)$	1.15	0.68	1.17	1.35	0.54	
Malaysia	$F(4, 58)$	0.24	2.10	1.52	2.96*	0.79	
Philippines	$F(4, 53)$	0.36	0.82	1.59	0.93	3.69*	
Singapore	$F(4, 59)$	1.98	0.64	0.82	0.12	0.99	
Thailand	$F(4, 57)$	6.07*	2.12	2.49	1.78	2.63*	

NOTE: US = United States; PRC = People's Republic of China. * denotes statistical significance at the 95% confidence level.

Table 3.8. Contemporaneous Effects of Foreign Variables on Domestic Counterparts

Country	Domestic variables			
	y	π	r	q
US	0.49***	0.22***		
	(0.15)	(0.08)		
eurozone	020	0.29***	0.45***	0.45***
	(0.15)	(0.04)	(0.16)	(0.17)
PRC	-0.09	0.14	-0.16**	-0.14
	(0.18)	(0.34)	(0.08)	(0.29)
Japan	0.40*	-0.14**	-0.04	0.07
	(0.22)	(0.06)	(0.05)	(0.18)
Indonesia	-0.16	1.14	5.26**	1.33**
	(0.54)	(0.77)	(2.57)	(0.16)
Malaysia	1.31***	0.94***	0.21	0.72***
	(0.21)	(0.26)	(0.15)	(0.19)
Philippines	0.11	1.38***	2.76***	1.24***
	(0.25)	(0.30)	(0.92)	(0.14)
Singapore	1.14***	0.04	0.13	0.93***
	(0.31)	(0.09)	(0.11)	(0.12)
Thailand	1.02**	0.31**	3.77***	1.19***
	(0.46)	(0.15)	(0.50)	(0.26)

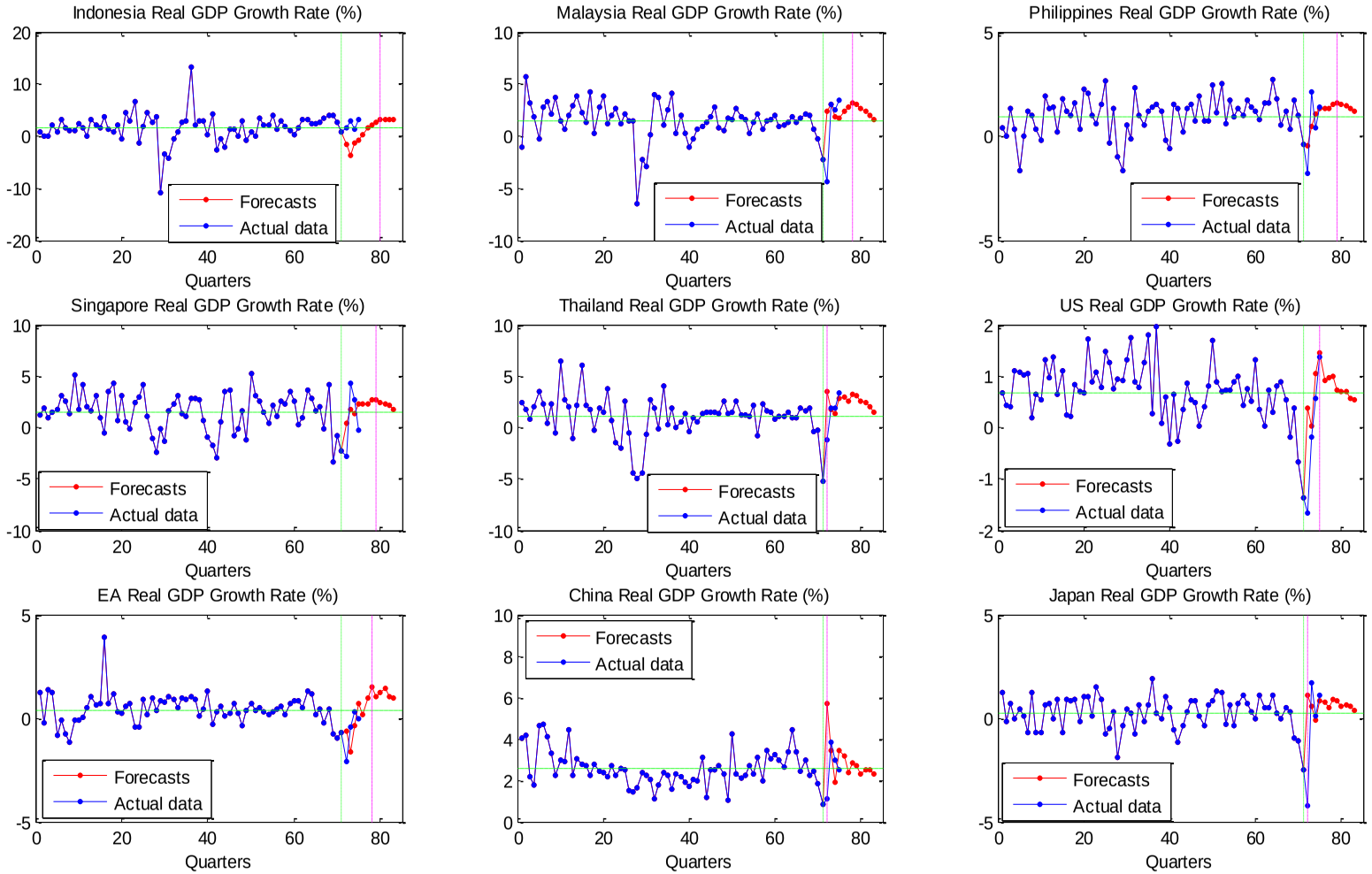
NOTE: US = United States; PRC = People's Republic of China. Newey-West HAC standard errors are given in parentheses. * denotes statistical significance at the 90% confidence level, ** denotes statistical significance at the 95% confidence level, and *** denotes statistical significance at the 99% confidence level.

Table 3.9. Panel DM Statistics for GVAR Forecasts Relative to Benchmark Models (2009Q1–2009Q4)

Benchmark Model	Real GDP Growth Rate	Inflation	Short-Term Interest Rate	Real Exchange Rate	Real Equity Price
Country-specific VAR(2)	-0.29	-0.21	-2.09*	-0.48	-0.96
Country-specific VAR(2) with trend	-0.25	-0.82	-2.20*	-0.60	-1.11

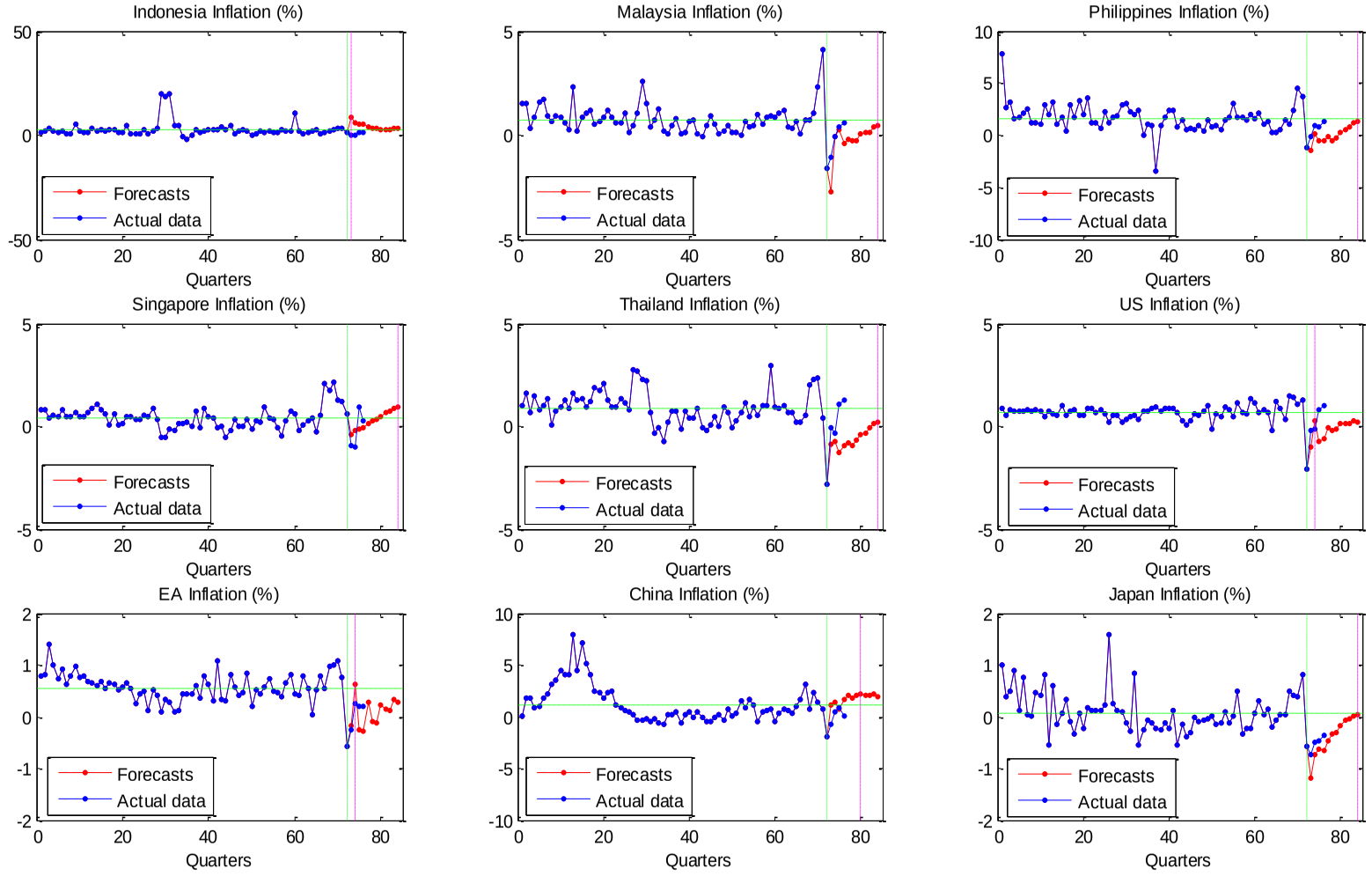
NOTE: * denotes statistical significance at the 95% confidence level. Negative values of the panel DM statistics imply that our GVAR model outperforms the benchmark models in terms of forecasting.

Figure 3.1. One-Quarter-Ahead Forecasts of Real GDP Growth



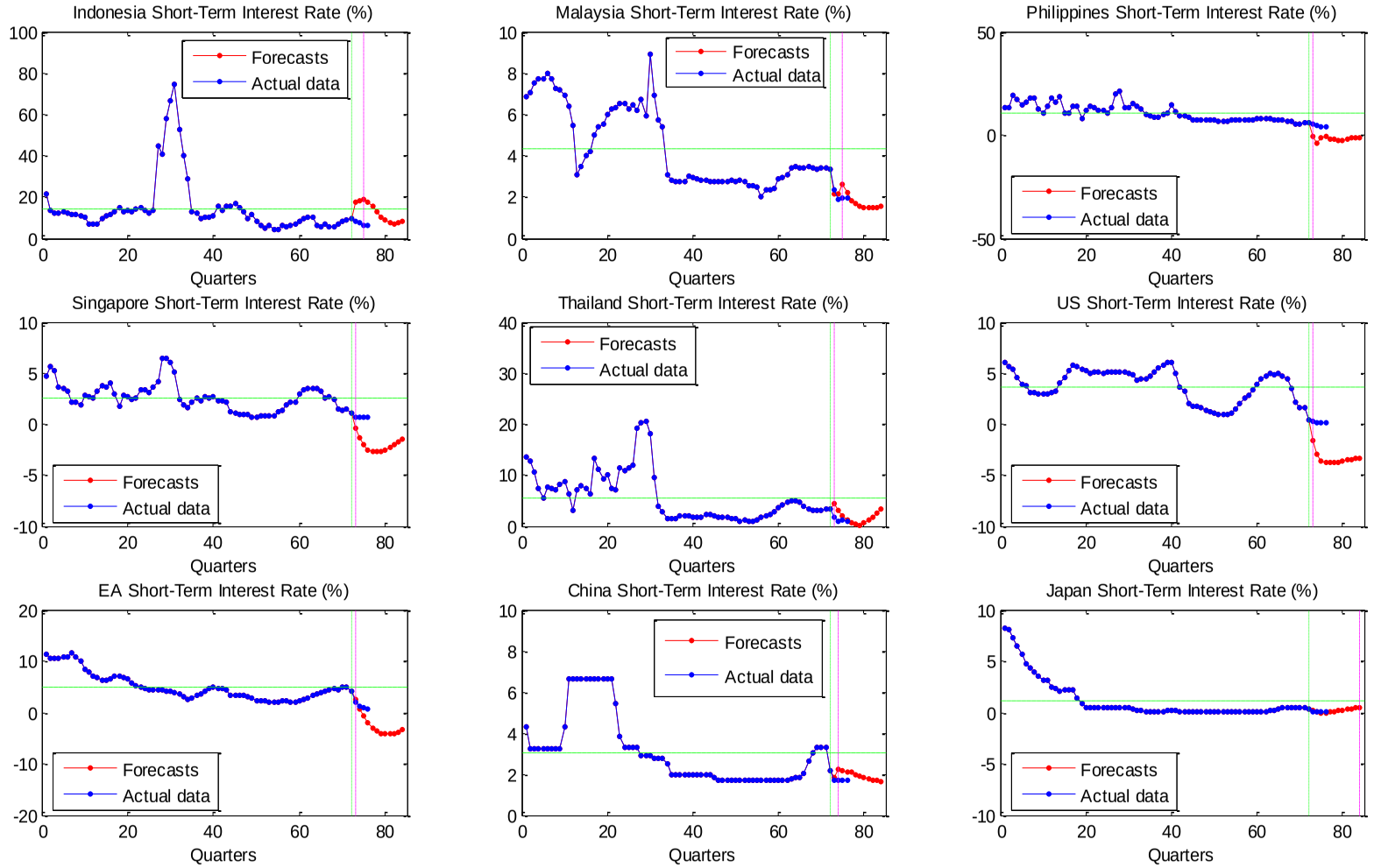
NOTE: EA = eurozone, PRC = People's Republic of China.

Figure 3.2. One-Quarter-Ahead Forecasts of Inflation



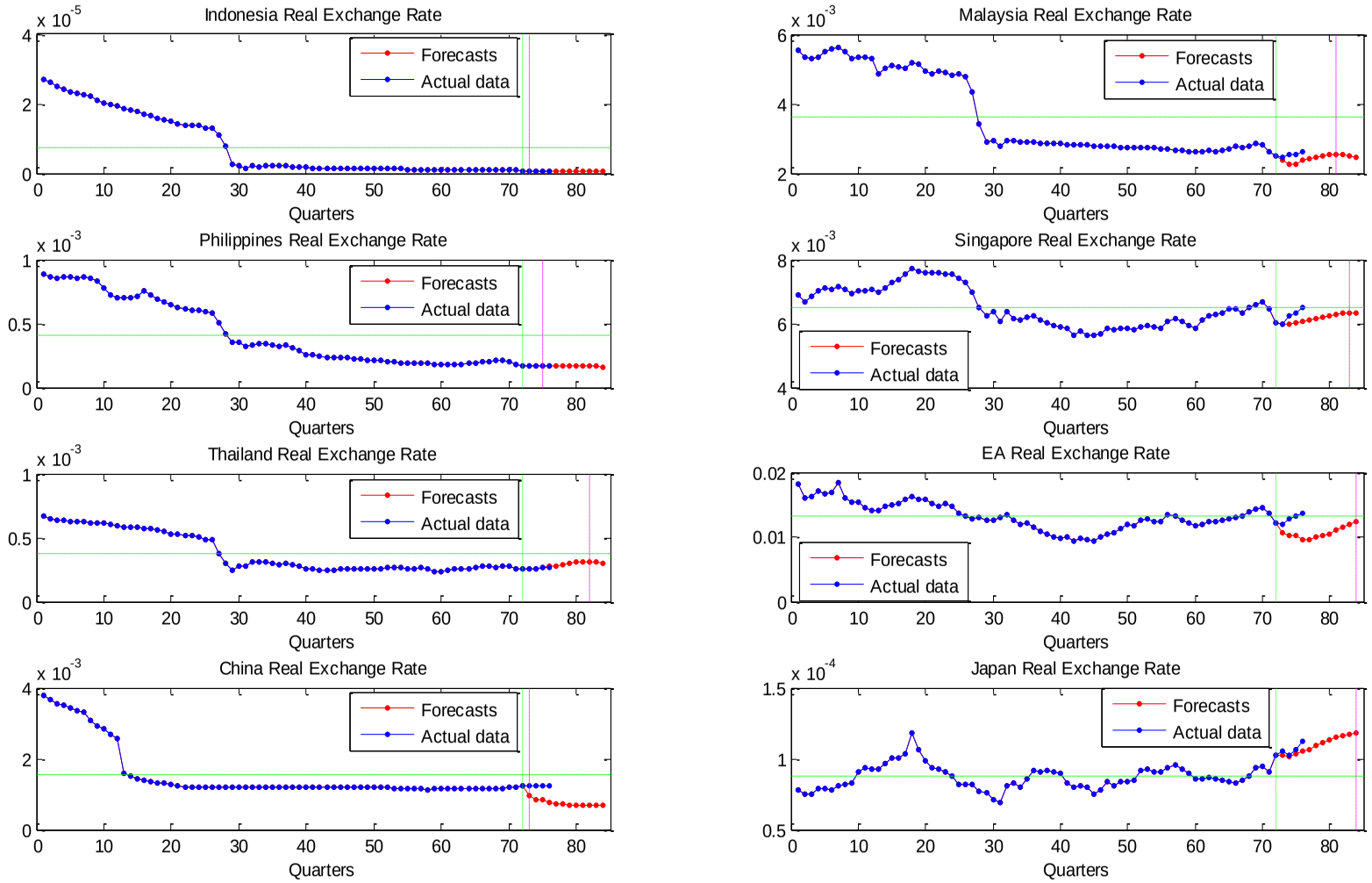
NOTE: EA = eurozone, PRC = People's Republic of China.

Figure 3.3. One-Quarter-Ahead Forecasts of Short-Term Interest Rates



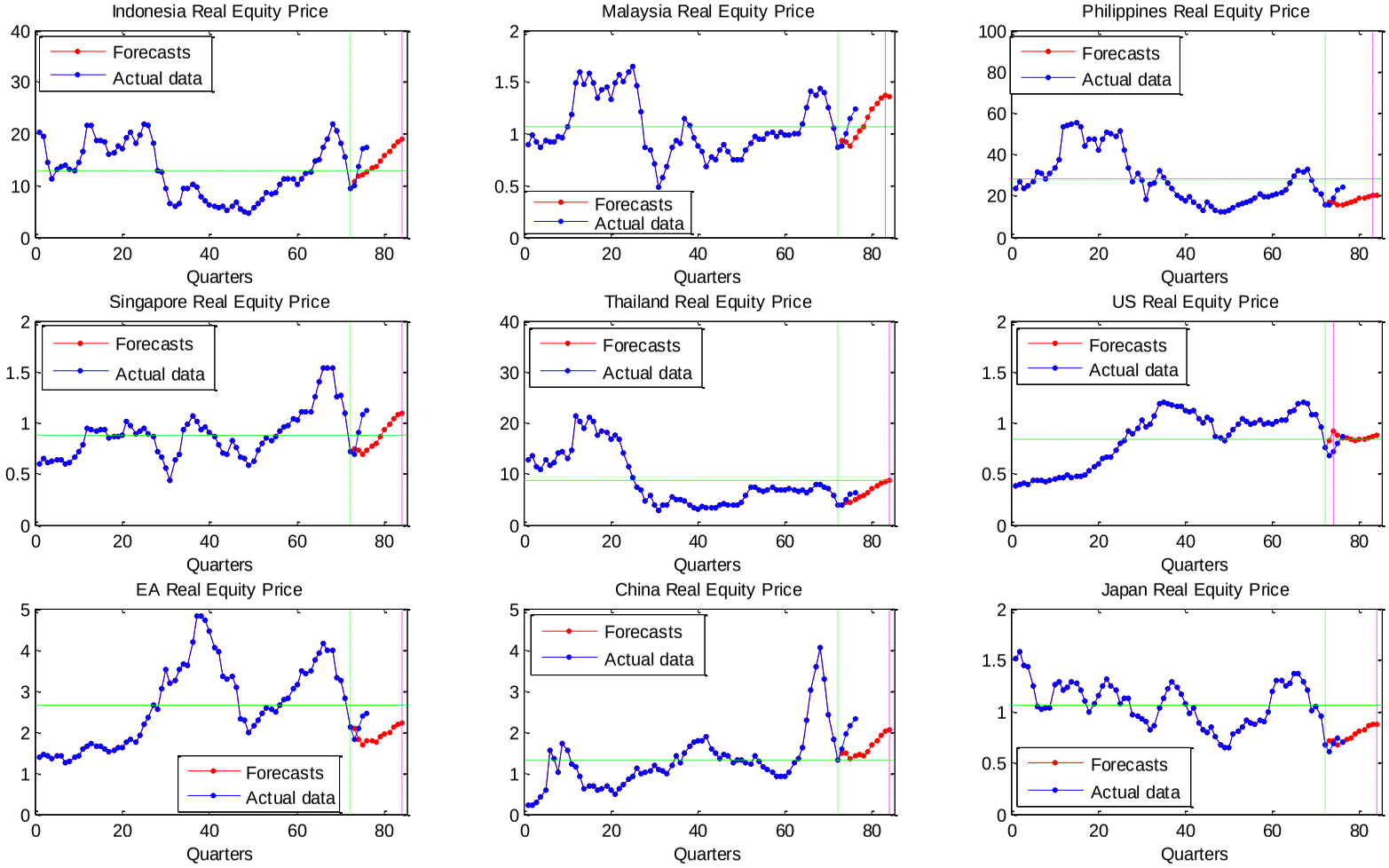
NOTE: EA = eurozone, PRC = People's Republic of China.

Figure 3.4. One-Quarter-Ahead Forecasts of Real Exchange Rates



NOTE: EA = eurozone, PRC = People's Republic of China.

Figure 3.5. One-Quarter-Ahead Forecasts of Real Equity Prices



NOTE: EA = eurozone, PRC = People's Republic of China.