Arguing and Reasoning in a Technology-Based Class

Baruch Schwarz
The Hebrew University
Jerusalem, Israel
msschwar@pluto.huji.ac.il

Rina Hershkowitz
The Weizmann Institute
Rehovot, Israel
nthershk@wiccmail.weizmann.ac.il

Abstract

This study has the descriptive aim of showing if and how epistemic procedures typical to mathematical reasoning can be practiced by children when they are in a social situation that supports their individual linguistic and cognitive activity. The present paper consists of a fine-grained analysis confronting argumentative skills and epistemic actions of a group of four students functioning in a Grade 9 mathematics class. The four students were presented with a mathematical problem-situation typical of a one year long experiment whose domain was an introductory course about functions. This activity was typical in the sense that: (i) it demanded inquiry; (ii) students worked in groups; (iii) they had computerized tools at their disposition; (iv) they were invited to discuss their work in a whole class forum. The role of the technological tools as a trigger for the application of argumentative skills is investigated.

Introduction

Research in school discourse and in knowledge acquisition belong to two distinct traditions. School discourse has been the object of many socio-linguistic studies (e.g., Cazden, 1986; Sinclair & Coulthard, 1975). However, such studies have been interested in the conversational features of this form of discourse, and did not focus on how school discourse leads to knowledge acquisition. Similarly, most of teaching-learning studies focusing on knowledge acquisition in particular domains neglected the role of discourse in this process. However, pioneering studies have already focused on the relation between reasoning in a particular domain and argumentative skills (e.g., Pontecorvo & Girardet, 1993; Resnick, Salmon, Zeitz, Wathen, & Holowchak, 1993), in varied domains such as historical reasoning or nuclear power policy. Moreover, investigating the role that technological tools can play in the construction of this relation is also quite a new endeavor (see the studies undertaken by Meira, 1991, Roschelle, 1992, and Schwarz, in press).

The Activity Theory: A Methodology for Analyzing Teaching-Learning Settings with a Vygotskian Perspective

Our approach is to analyze school discourse through Leont'ev's method (1981) that frames the teaching activity, and its specific actions and operations.

The activity construct in Leont'ev's sense refers to the most global level of analysis. It explains the sociocultural interpretation imposed on the context by the participants. It is characterized by a discursive interaction and a cognitive function pursued by the teacher, who proposes (explicitly or implicitly) her or his general goals to the children's group and often recycles them in the course of the discussion.

The second level of the Activity Theory, the level of actions is embedded in the activity. Actions are driven by a goal about which the participants can share awareness. It explains where there is a cultural inter-personal mediation between teacher and child or among children working together. It consists of reasoning sequences in which particular epistemic actions are pursued. In the mathematical domain, examples of such reasoning sequences are planning, or constructing an hypothesis.

Within these reasoning sequences, Leonte'ev's third level of analysis looks at the molecular operations carried out through the idea units. Each idea unit is submitted to a double categorization, looking at the specific argumentative operations and at the epistemic operations used by children.

Defining the Levels of Analysis of a Mathematical School Setting

We used Leont'ev's methodology to analyze a school session in a technology-based mathematics class. At the activity level, the teacher conducted a one year long introductory course about functions, in a parochial school, with forty-two 9th grade girls. The teacher based her instruction on problems characterized by: (i) open ended problem-situations demanding the use of inquiry skills; (ii) work in groups, students being encouraged to discuss the solution paths; (iii) the availability of computerized tools; (iv) aftermath reflection in whole class forum, and/or in written group reports. Details about the experiment are given elsewhere (Hershkowitz and Schwarz, 1995). One typical problem-situation, "Overseas Inc.", is described here in terms of Leonte'ev's levels. A distinction is being made between two types of operations: (a) the argumentative operations that give an account of the collective discursive activity, and (b) the epistemic operations through which the knowledge domain is analyzed.

The Activity around the "Overseas Inc." Problem-Situation

The Overseas Inc. problem-situation is initiated by a homework assignment given to all the individuals of the class. Its formulation is:

The freight company "Overseas Inc." uses containers to ship goods by sea from country to country. The containers *are big boxes made of wood. Their base needs to be a square, and their volume must be 2.25 m^3. The containers must also be open at their top. Can you find two or three examples of such containers?*

You may use paper to construct such a container, or draw it and label its dimensions. In that case, you may use a 1/20 reduction (1m for the container= 5cm of the paper).

On the day the assignment is due, the students (equipped with the models of containers they brought to school) are given a worksheet. Its formulation is:

As wood is expensive, the company is interested in designing ideal containers with as little wood as possible. Can you figure out how the ideal container looks? Guess and explain. Can you help the company to find out the exact dimensions of the ideal container?

The teacher invited the students to organize themselves in groups of four, and to work in any way they wanted. They had graphical calculators at their disposal. The groups worked basically alone, although the teacher was available to help. At the end of the activity, the girls were invited to participate in an open discussion about the process that they undertook during the construction of their hypotheses and of their solution paths within the different groups. Group reports were collected before the discussion began.

The Actions of the Activity

At the second level of analysis, argumentative phases in which a dominant collective goal-mediated action is pursued were identified. Thirteen such actions were observed for "Overseas Inc.":

The teacher gives first a *preparing task* (1), a homework assignment consisting of a worksheet in which students are asked to construct several models. During the *presentation* (2), the teacher asks the students first to hypothesize the solution. Then, students organize themselves in groups of four. The four girls whose work was analyzed, first undertake *computations* (3) while manipulating the models they have brought in. They collaborate to jointly add up all the elements of the surface. They distribute their efforts and group the computations of each of the models in a common table. Then, two of the girls formulate their own *hypotheses* (4), and the group tries to understand these conflicting hypotheses. These interactions cause the participants to justify and to *defend* (5) these hypotheses. One of the girls decides to find out *which of them is right* (6). Gradually, the students jointly *construct an hypothesis* (7). They come to understand that each of the previous hypotheses is "locally" right, but that none of them takes into account the overall variation of the surface. After a short re*treat to the old hypotheses* (8), the students recognize that they cannot go further in the elaboration of a better hypothesis and they jointly decide to turn to the *solution* (9). They return to the *table* (10), but at that time in a more controlled way: They define their role: one makes variations for small decreasing values of the side, another for big increasing values of it. They finally decide to turn to an alg*ebraic formula of the surface* (11), and they use the *graphical calculator* (12) to display the graph of the function, and read its minimum. In the last part of the lesson, the teacher asks the groups of students to discuss about the strategies used by each of them. This segment is a *synthesis* (13) in which students reflect upon their and others work, the teacher being a moderator among the contributions of the participants.

The analysis of the work of four girls solving "Overseas Inc." is done in the next subsection, at the level of operations, argumentative and epistemic. This analysis is preceded here by a categorization of these operations.

General Categorization of Argumentative Operations

The argumentative operations are those listed by Toulmin (1958), and adopted by Pontecorvo and Girardet (1993) in their analysis of arguing in historical topics. These are:

Claim:	Any clause that states a position (that can be claimed).
Justification:	Any clause that furnishes adequate grounds or warrants for a claim.
Concession:	Any clause that concedes something to an addressee, admitting a point claimed in the dispute.
Opposition:	Any claim that denies what has been claimed by another, with or without giving reasons.
Counter-opposition:	Any claim that opposes another's opposition, which can be more or less justified.

General Categorization of Epistemic Operations

These operations are grounded on the explanation procedures in terms of the mathematical content to which they refer. Schoenfeld (1992) recognized several kinds of epistemic operations that correspond to the explanation procedures that are used for interpreting and solving mathematical problems. The first kind consists of higher level metacognitive procedures, which are the basis of mathematical interpretative activity; they deal with regulatory processes and with evaluation of the adequacy of moves (or control). For example, these operations can consist of choosing or evaluating which strategy to choose for tackling a particular problem, or of deciding to leave a heuristic method after a long enough search. The second kind includes heuristics, such as the use of analogies, the search for patterns, hypothesizing, or simplifying a problem. The third kind of epistemic operations is the appeal to resources. Resources are a very rich list of facts, theorems, definitions, procedures, etc. that are at the disposition of the learner. Some examples are: reading graphs, inferring the order of magnitude of a variable quantity from a table or a formula, or knowing the solution of a particular task previously done. A full list of resources is impossible to write down, because it depends on the task, and on cognitive development: a procedure can be a heuristic for some, while for others, it is a fact. The list given in the following is then specific to the four girls solving "Overseas Inc.". It is organized into metacognitive /regulatory operations (I), heuristics (II), and appeal to resources (III).

Reflection: (I) Reflecting upon one's previous moves, or planning further ones.

Control (I) Asserting/checking with an evaluative dimension.

Predication (I) Asserting without any evaluative dimension.

Hypothesis (II) Figuring out a fact, a rule or a law. Hypothesizing may be grounded on numerical data, previous knowledge, or experience.

Extreme cases (II) Use of extreme cases in order to find out a law.

Analogy (II) Search for an analogical case

Change of rep. (II) The change of external representation as a strategic move to see a problem from a new perspective.

Appeal to resources (III) (retrieving of information considered as relevant to the topic by the speaker). It can be: *Definition* (A statement about the nature of an object, or a quantity), *Exemplar cases*, (numeric data, models,..), *Rules, General Principles, Authority* (expert, previously solved problems,...), *Procedures* (reading graphs; computations, etc.).

Again, these epistemic operations were carried out in a social interaction setting by particular linguistic and cognitive operations, which can be identified as argumentative operations because of their linkage of social arguing and individual reasoning.

Our hypothesis is that children as novices in the mathematical domain can learn to master these latter operations by practicing them in appropriate learning environments, especially in environments such as that created in this experiment.

Examples of Interaction

In the following examples, we present some excerpts of interaction between the four participants, Hanna, Miriam, Liat and Osnat. The four girls were videotaped, then their discourse was transcribed. The protocols are accompanied by VCR-time (first column), argumentative operations (third column), and epistemic operations (fourth column). It is important to notice that the participants used three models they constructed at home: a "long" box with narrow base, a "short" box with large base, and a box with comparable dimensions. The first excerpt (Table 1) shows the actions of formulating and defending own hypotheses (actions 4-5). This excerpt begins by a phase during which students investigate the nature of the problem. They clarify that the surface includes the lateral faces and the base, and that the problem (to find as less wood as possible) means to minimalize the surface. At 48:10, Liat concludes that surface and quantity of wood mean the same. At 48:15, Osnat relates to a hypothesis that was not uttered, but it might be that she derived it from her interpretation of Liat claim. She justifies the hypothesis by appealing to a principle: as the sides contribute four times to the overall surface (as opposed to the base), the height has to be minimal. Liat changes representation (she uses the models) to complete the justification. However, this move is problematic in the short run: She shows that the short model has a very big base. Hanna opposes to Osnat hypothesis, the shorter is the height, the bigger is the base, and consequently the larger is the surface.

This excerpt shows two main points. First, the students were very often engaged in metacognitive/regulatory ("control", "reflection") and heuristic operations ("hypothesis", "change of representation", "extreme cases"). Such operations characterize high-level mathematical reasoning, and are generally difficult for students. Second, the children conducted an autonomous collective discourse in which mathematical reasoning was supported by the application of rich argumentative skills. Quantitative considerations about the distribution of epistemic and argumentative operations are beyond the scope of this descriptive paper.

A second excerpt (Table 2) shows two roles of computerized tools in relation to argumentative and epistemic operations. This excerpt (actions 9-12) occurs after Osnat claims and warrants that the surface is smaller when the sides are smaller, and Miriam opposes that the surface is smaller when the base is smaller. After the participants realize that two hypotheses formulated previously are "locally" right only, Liat counteropposes the two hypotheses and formulates her own hypothesis. The counter-opposition drawn by Liat makes a compromise between the two previous hypotheses. It is interesting that Osnat who held one of the "old" hypotheses hurries to check and justify Liat's hypothesis by changing representation with her graphical calculator. As for the first excerpt, the four students very often engage in meta-cognitive and strategic epistemic operations. For example, all Hanna's interventions deal with planning the solution or with choosing the right variable (the side of the base or the height). The last "change representation" operation is different from the first one; it does not come to justify an argument, but to solve a problem whose planning is clear.

Time		Protocol	Arg. Operator	Epistemic Operator
47:06	*[Miriam reads aloud the question from the worksheet]*			
	Hanna:	How much did you get? We got 19.		**Control**
47:08	Liat:	The less the surface is, the less wood there will be.	**Claim**	**Appeal def.**
47:14	Osnat:	I'll tell you something		
47:20	Miriam:	*[grasps the "long model"]*		**Change of rep.**
	The surface is not only this, it is everything		**Oppose**	**Appeal definition**
	[She points at the base than at the lateral faces]			
	Osnat:	No! I did not say that...		
	Miriam:	Oh! I did not understand really...	**Concession**	**Reflection**
	Osnat:	One moment how could we have less wood?		
		here it has to be the smallest		**Control**
		[Osnat points at the sides of the large box]		
48:10	Liat:	When the surface is smaller, there is less wood. Right?	**Claim**	
		See, we see that here *[the height]*, there is 1/2	**Hypothesis**	**Num. data, extr. cases**
		and there *[the base side]*, 3.		
48:30	Hanna:	Why is it so?		**Control**
48:35	Osnat:	It's because here..if it will be the smallest possible,	**Justification**	**Start Hypothesis**
		we multiply it by four, we need to multiply by four.		**Appeal to principles**
	Liat:	Because here *[Liat points to the base]*, it will be larger,	**Justification**	**Change of rep.**
		and here *[Liat points to the sides]*, it will be smaller.		
	Osnat:	Multiplied by four, it must be as small as possible		**Appeal principles**
	Liat:	Yeah! The smaller it will be, Yeah		
	Hanna:	Smaller is the height, bigger is the surface	**Opposition**	**Hypothesis**
	Miriam:	Didn't we say the contrary,	**Counter-Opposition**	
		that the smaller is the height,	**Claim**	**Hypothesis**
		the smaller is the surface		
	Osnat:	But this is exactly what we said.		
	Hanna:	One moment, how did we get it that way?		**Control**
	[Liat writes down the hypothesis on the worksheet].			
	Miriam:	..[inaudible]...bigger		
	Osnat:	OK., it's exactly what we said		**Control**
	Liat:	Tell it in the other way, shorter is the container...	**Claim**	**Inference**

Table 1

1:03:58	Liat:	It's as if it is not constant, it has to be like that	**Counter-opp.**	**Hypothesis**
		[Liat draws in the air a sketch of a "parabola"]		
1:04:06	Osnat:	It's worthy to do a table, first we will make big the height	**Justification**	**Change of rep.**
		[The four students construct a table with four values in		
		the graphical calculator,		**Appeal proc.**
		and plot the corresponding points on a graph]		**Appeal proc.**
1:05:25	Hanna:	Let's think in a logic way.		**Control**
1:05:57	Liat:	Yeah, but it does not need to be a linear function.	**Counter-opp**	**Appeal prop.**
1:06:04	Miriam:	Yeah, it does not have to go up with the same rate.	**Justification**	**Appeal prop.**
1:06:16	Hanna:	We have to think which container is the ideal one.		**Control**
1:06:25	Osnat:	If it's what you think, then it's a parabola, and the	**Claim**	**App.Condition**
		apex is where it seems horizontal and stable.		
1:06:58	Miriam:	The y-axis is the surface, and the x axis is the side of the base		**Control/Plan**
1:07:07	Osnat:	We have to be more goal oriented !		**Reflection**
		First, we make the base grow, that is to say the side of the base,		**Control**
		and second, the side of the height.		
1:07:37	Miriam:	The variable is the base		**Control/Plan**
1:07:47	Hanna:	In my opinion, the side of the base and the height, and	**Opp./Justifi**	**Control/Plan**
		then we'll solve this.		
		[The four students take their graphical calculator, enter the		**Change rep.**
		formula expressing the height as a function of the side of the base,		**Appeal proc.**
		then enter the surface as a function of the side, draw the graph of		
		the function and read the minimum]		

Table 2

In the two excerpts presented here, the technological tools seem secondary, being involved with the "changing representation" epistemic operation only. However, the high level of mathematical reasoning the students attained (see Hershkowitz & Schwarz, 1995), was linked to the fact that the students knew they did not have to carry out technical tasks, and that any claim could be warranted with the computerized tools at their disposal.

Concluding remarks

The Activity Theory has descriptive power: it was a suitable frame to describe the relations between argumentative and mathematical epistemic operations. The examples of data presented here show the concomitance of high-level mathematical reasoning and rich argumentation. Studying more specific relations between mathematical reasoning and argumentation at the level of their operations is a domain of research that needs to be investigated in further research. Place limitations dictated a somehow discrete picture of the interactions between participants. Such an approach misses some features of the dynamics of the group interactions, in which operations contributed by individuals cannot be isolated. The computerized tools were important in the sense that students could use "change of representation" as a strategic move for controlling or checking hypotheses, and for justifying (warranting or backing) arguments. In other words, the computerized tools enabled the students to use "change of representation" both from an epistemic and an argumentative perspective. It is then not surprising that, as shown in Hershkowitz and Schwarz (1995), change of representation was often contiguous to "Aha" occurrences.

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